C. 1

COMPUTER-AIDED ROLLING OF PARTS WITH VARIABLE RECTANGULAR CROSS-SECTION

by

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ABSTRACT

A computer-aided process planning scheme for flat rolling of symmetric parts with variable rectangular cross-section is proposed. As a starting point, El-Kalay and Sparling's formula for spread was found to be suitable for this application and thus was used in developing the method. Two distinct criteria were considered in the analysis, namely: *kinematic* and *dynamic constraints*. In order to control the precision of the rolled parts, provisions were made for specifying the tolerances of the finishing passes in the form of *convexity constraints*.

Numerical formulation was used and the resulting non-linear equations were solved by an iterative method. Based on the process constraints and the numerical solution, a computer algorithm was then developed to determine the number of rolling passes required, as well as the dynamic variation of the roll gap as a function of the rolled length.

Preliminary laboratory experiments were then conducted to verify the validity of the predicted results and the applicability of the spread formula in determining the process behaviour. These experiments led to the modification of the spread formula. Using the modified formula it was found that a good agreement existed between the predicted results and those of the experiments.

Operating aspects were also considered. It was proposed that a control system based on the rolled length would be

both simple and suitable. It was then concluded that for rectangular parts with moderate variation in shape and reasonable complexity, where formed-die rolling and die-forging are also applicable, this method has considerable advantages as it replaces the forging *hardware* with the rolling *software*.

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NOMENCLATURE

h,H	Height(Thickness)
w,W	Width(Breadth)
l,L	Length
A	Cross-sectional area
V	Volume
Δh	Draft
S	Spread(an increase in width)
R	Roll's radius
D	Roll's diameter
l_d	Projected length of arc of contact
a	Angle of bite
ϕ	Rolling angle
δ	Neutral angle
v	Linear velocity of the rolled material
t	Rolling temperature
f	Coefficient of friction
p_ϕ	Local pressure
F	Local horizontal force
P	Separating force
M	Rolling torque

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1. INTRODUCTION

With the rapid development of new materials, mechanization and automation, the processes of manufacturing are becoming more varied. The application of computers in all aspects of engineering, specially in design and manufacturing (*CAD/CAM*), has in a way led to the development of new processes whereby a product can frequently be made in several ways. It is therefore important to understand the many ways in which a product can be processed , the effects that these processes have on the product properties, their advantages and limitations as well as the accuracy that is expected. A fundamental criterion which determines suitable methods of manufacturing, is the correct process of producing the individual part so that it is manufactured no more accurately than necessary and at the lowest cost possible[1].

Amongst all the manufacturing processes, *forming* is the one which is becoming more important . Forming is a method, by which the size or shape of a part is changed by the application of force on the part. Forming is a fast way to change the shape of parts. Generally speaking , if the shape and the required accuracy of a part are such that it can be made by one of the forming operations, then forming is the most economical process to be used[2]. Formed parts have fine-grain structures which increase their toughness and consequently prolong their working lives[3].

Industrial practice uses various forming techniques such as *rolling* , *forging*, *pressing*, *stamping* or *extrusion*. *Rolling* is known as the the most economic method amongst the other forming techniques. It has been practiced since the fourteenth century[4] and still has extensive application in manufacturing. *Flat rolling* is the simplest form of rolling. The mechanism of flat rolling is simple; two circular rotating cylinders draw the work-piece through the opening between them, by means of the friction force between the cylinders and the work-piece , thus reducing the cross-sectional area of the part. If this is done in a temperature above the recrystallization point of the material , it is called *hot rolling*. Hot rolling is commonly used for steel parts.

This thesis presents a computer-aided rolling scheme for manufacturing symmetric parts with variable rectangular cross-section. This form of parts is often used in equipments and machinery. *A taper leaf spring* is a typical example which is becoming increasingly common as the means of road suspension system for trucks and vans. According to Fig. 1-1, a taper leaf spring can replace a stack of five or more conventional constant thickness leaf springs, offering certain economical and mechanical performance advantages. It leads to substantial weight saving for the same load carrying capacity, and better ride characteristics. Moreover, the controlled pattern of the taper leaf spring permits higher working stresses to be used , and gives a longer working life.

A common method to produce such springs is based on the use of dies eccentrically fixed to a pair of rolls, rotation of which progressively forms the thickness of the spring blanks (Fig. 1-2). Closed dies are used in an attempt to limit the lateral spread of the stock. Grinding operations are often needed to remove the forging flash from the rolled material. The cost of maintaining the machine is high, since under the heavy side loads die breakages are frequent. The cost of producing the dies is also high. Furthermore, a separate tool set is required for different spring sizes.

In the method presented here, a taper leaf spring ,for example, can be produced within two operations of flat rolling; a blank of uniform cross-section is first rolled, through a variable roll gap, in its width (Fig. 1-3a), then, retapered from its original thickness side (Fig. 1-3b) in such a way that the initial constant width is regained leaving the part formed only in its thickness. The main features of this method can be summarized as below:

(i) The blank is rolled by two plain rolls, whereby different shapes may be produced from the same tool set.

(ii) Parts with variation both in width and height can be produced by this method.

(iii) Two operations is the minimum required to achieve a certain shape, however, more than two passes are frequently needed depending on the shape, appearance

of the finished part and the process constraints.

(iv) Unlike conventional rolling, the process is of an unsteady state nature, i.e., the roll gap continuously changes to form the desired shape. The ingoing material could be non-uniform as well.

(v) Using a suitable control system, such as a *Micro-processor*, permits fast operating speeds to be used. Simplicity in resetting the system parameters, when design changes occur, can easily be accomplished through the micro-processor. This would result in a higher production rate and lower down time.

(vi) Parts, produced by this method, usually need no further treatment and can be used directly, although further processing could be done, if necessary.

The objectives of this thesis are as follows:

(a) To study the theory of conventional rolling and modify it for application in the unsteady rolling process.

(b) To develop procedures and strategies for determining the particulars of the rolling of parts with variable rectangular cross-section.

(c) To identify and apply the process constraints to the above strategies.

(d) To evaluate the applicability of the developed method experimentally and to apply corrections and

modifications on the basis of the experimental evidence.

A literature survey is presented in chapter two. Some important relations with regard to the geometry, kinematics and dynamics of flat rolling are first described. A study of the geometric deformation of steel under hot flat rolling is presented next. This covers a major part of the chapter which also includes a comparison between the different formlae used to predict the spread in hot flat rolling. The chapter is concluded by discussions leading to the selection of a suitable spread formula for use in this work.

In chapter three, the proposed method is described by presenting a solution for the uniform to uniform deformation. This is then extended to include the deformation of non-uniform parts. All known practical constraints are applied at this stage; however, *the main constraint is of course dictated by the desired shape of the parts*. The numerical approach based on the proposed algorithm is programmed and is described next. Operating aspects are briefly discussed at the end of the chapter.

Chapter four presents some typical results and discussions on basic experiments which were performed on an experimental rolling mill. The first part of this chapter illustrates the results of the experiments, under steady state conditions, conducted to verify the applicability of the method. The second part presents an experimental evaluation of the geometric deformation of the material in unsteady rolling. This chapter is completed by discussions leading to the introduction of an improved formula for spread in the general case.

Two examples with regard to the use of the method are illustrated in chapter five. Finally, in chapter six conclusions are presented along with scope for future work.

There are two appendices. In appendix *A* the numerical method for finding the roots of non-linear equations is described. Appendix *B* describes a curve-fitting method, which was developed to obtain an analytical representation of the variation of roll gap for each rolling pass.

2. LITERATURE SURVEY

2.1 FUNDAMENTALS OF THE ROLLING PROCESS

2.1.1 BASIC CONCEPTS AND GEOMETRIC RELATIONS

A schematic representation of flat rolling is shown in Fig. 2-1. In the successive stages of rolling, the dimensions of a rectangular bar are changed but the volume constancy holds, i.e,

$$V_0=V_1=V_2=...=V_n$$

where

$$V_n=A_n l_n=h_n w_n l_n$$

A_n, l_n, h_n and w_n are the cross-sectional area, length, height and the width of the stock after the n^{th} stage.

The increase in length of the stock after each pass is usually greater than the increase in width. The increase in width or lateral elongation is called *spread*.

Referring to Fig. 2-1, when a uniform bar with initial thickness h_1 enters the rolls , the edge of the bar touches the roll at a point through which passes one arm of the angle having its apex on the roll axis, and the other arm in the plane passing through the roll axes. The included angle, a, is called *the angle of bite*.

The height of the bar leaving the rolls is h_2. l_d denotes the projected length of the arc of contact between the rolls and the metal. h is the height of the bar in the rolls at a distance x from the exit side of the rolls, corresponding to a rolling angle ϕ. The difference between the incoming and outgoing thicknesses, i.e., *the absolute draft*, is

$$\Delta h = h_1 - h_2 \qquad (2\text{-}1)$$

With reference to Fig. 2-1, a geometric relation for rolling with rolls of the same diameter, D, (radius of R) can be derived as

$$R\mathrm{Cos}\alpha = R - (h_1 - h_2)/2$$

The expression for calculating the angle of bite is then found as

$$\mathrm{Cos}\alpha = 1 - (h_1 - h_2)/2R = 1 - \Delta h/D \qquad (2\text{-}2)$$

From this, the absolute draft can be calculated as

$$\Delta h = D(1 - \mathrm{Cos}\alpha) \qquad (2\text{-}3)$$

The projected arc of contact between the metal and the rolls is calculated from the geometrical relationship

$$l_d = \sqrt{R^2 - (R - \Delta h/2)^2} = \sqrt{R \cdot \Delta h - (\Delta h)^2/4} \qquad (2\text{-}4)$$

Equation (2-4) may be assumed without a significant error in a simplified form

$$l_d \simeq \sqrt{R \cdot \Delta h} \qquad (2\text{-}5)$$

This simplification is allowable for small angles of bite. When $\Delta h \leq 0.08R$, the error is less than 1%[5]. A similar expression can be deduced for the rolling angle, ϕ, and thickness, h,

$$\mathrm{Cos}\phi = 1 - (h - h_2)/D \qquad (2\text{-}6)$$

hence

$$h = h_2 + D(1 - \cos\phi)$$

(2-7)

The angle ϕ is calculated from the following relationship

$$\mathrm{Sin}\phi=x/R$$

2.1.2 CALCULATION OF ROLLING SPEED

Rolled stock enters the gap with a speed less than the peripheral roll speed. On the other hand, the exit speed of the stock is greater than the peripheral speed of the rolls (see Fig. 2-2). Thus, there is a plane in the deformation zone, at which the horizontal component of the peripheral speed is equal to the speed of the rolled stock. This plane is called *the neutral plane*, and the value of the rolling angle at this plane is specified by δ. The following equation holds

$$v_\delta=v_r\mathrm{Cos}\delta \qquad (2\text{-}8)$$

- v_δ is the speed of the rolled stock at the neutral plane,
- v_r is the peripheral speed of the rolls.

Applying the constancy of volume

$$V=h_1w_1v_1=h_2w_2v_2=h_\delta w_\delta v_\delta=h_\delta w_\delta v_r\mathrm{Cos}\delta \qquad (2\text{-}9)$$

- v_1 and v_2 are the speeds of the material at the entry and the exit planes, respectively.

Wusatowski[6] used the following relationships for spread of mild steel under hot flat rolling

$$w_2/w_1=(h_2/h_1)^{-W}$$

and

$$w_\delta/w_1=(h_\delta/h_1)^{-W}$$

where

$$W=10^{(-1.269(w_1/h_1)(h_1/D)^{0.556})}$$

The value of the neutral angle, δ, can now be related to the outgoing velocity

$$(w_\delta/w_1)(h_\delta/h_1)v_r\text{Cos}\delta=(w_2/w_1)(h_2/h_1)v_2$$

Equation (2-7) can be written at $\phi=\delta$ as,

$$h_\delta/h_1=[D(1-\text{Cos}\delta)+h_2]/h_1$$

Using the above two equations, the following relationship then holds

$$v_r\text{Cos}\delta/[D(1-\text{Cos}\delta)+h_2]^{(W-1)}=v_2/[h_2^{(W-1)}]$$

from which, the value of the exit velocity can be determined providing that the value of the neutral angle is known. A similar relationship could be written between v_1 and δ.

Koncewicz[7] derived a formula for the determination of the neutral angle; referring to Fig. 2-3, the roll gap can be divided into two zones: the zone of forward slip and the zone of backward slip. For free rolling, without front or back tension,

$$\int_\delta^a dF_1+\int_0^\delta dF_2=0 \qquad (2\text{-}10)$$

- $dF_1=p_\phi R\cdot w_\phi(\text{Sin}\phi-f\text{Cos}\phi)d\phi$, is the horizontal force due to backward slip when $\delta\le\phi\le a$;
- $dF_2=p_\phi R\cdot w_\phi(\text{Sin}\phi+f\text{Cos}\phi)d\phi$ is the horizontal force due to forward slip when $0\le\phi\le\delta$.

In order to solve equation (2-10), Konsewicz assumed p_ϕ and f to be constant along the whole length of the arc of contact. He also suggested a linear variation for the width of the rolled stock along the horizontal length of the roll gap, i.e.,

$$w_\phi = w_2 - (w_2 - w_1)\mathrm{Sin}\phi/\mathrm{Sin}a$$

The value of the neutral angle can then be estimated by integrating equation (2-10) (for detailes see 6, pp 170 to 175). Neither Wusatowski's prediction for spread nor Koncewicz's approach for finding δ are accurate enough. The best way of finding v_1 or v_2 is through direct measurement.

2.1.3 MECHANISM OF BITE AND FRICTION

The maximum angle of a at which free rolling can take place, i.e., without using force to push the metal into the roll gap, is called *the maximum angle of bite*. Referring to Fig. 2-3, at the point of entry, for the element of area dA,

$$(p_\phi \mathrm{Sin}a_{max})dA = (f \cdot p_\phi \mathrm{Cos}a_{max})dA$$

Then

$$\mathrm{Tan}a_{max} = f \qquad (2\text{-}11)$$

So, if $a \leq \mathrm{Tan}^{-1} f$, then the rolls bite the metal, without any backforce or forward tension. This is referred to as free rolling. From the geometry of rolling, one can write

$$\mathrm{Tan}a_{max} = l_d/R\mathrm{Cos}a_{max}$$

and approximately

$$\mathrm{Tan}a_{max} \simeq \sqrt{R \cdot \Delta h_{max}}/(R - \Delta h_{max}/2)$$

or

$$\mathrm{Tan}a_{max} \simeq \sqrt{\Delta h_{max}/R}$$

So the maximum draft for free rolling is

$$\Delta h_{max} = R \cdot f^2 \qquad (2\text{-}12)$$

Referring to Fig. 2-4, the frictional forces change direction at the neutral plane, FF. When the roll gap is completely filled, in order that rolling takes place, the frictional forces assisting rolling at the sector b-c should be greater than the sum of the frictional forces hindering rolling in the sector a-b and the horizontal component of the roll pressure. This results in a new condition which allows free rolling with higher drafts to take place. Recent investigations have confirmed the following inequality as the condition for free rolling (for details see 6, pp 120 to 126 and 16, pp 204 to 206)

$$0<a_{max}\leq 2\mathrm{Tan}^{-1}f \qquad (2\text{-}13)$$

During rolling both the frictional forces and the coefficient of friction vary along the arc of contact. The coefficient of friction, f, increases with the normal force. With ideal lubrication, the coefficient of friction decreases as the velocity of the body increases. Since it is not possible to calculate the coefficient of friction along the arc of contact, the average coefficient of friction is used. There are many ways to find the average coefficient of friction. A common method is to measure the value of the roll load while the plane of no slip is at the exit. This can be achieved by applying a certain amount of back tension to the material (for details see 6, pp 128 to 130 and 16, p 207). Ekelund[8] suggested an emperical formula which expresses the mean coefficient of friction as a function of the rolling temperature. Bachtinov[6,9] then proposed a modification to Ekelund's formula to allow for the influence of the rolling speed as well

$$f=a\kappa(1.05-0.0005t)$$

- $a=1.0$ for cast iron or rough steel rolls,
- $a=0.8$ for chilled and smooth steel rolls,
- $a=0.55$ for ground steel rolls,
- t is the rolling temperature(°C),
- κ is a factor relating f to the peripheral speed of the roll, v_r, according to table 2-1 .

A distinction should be made between the slipping friction and the sticking of metal to the roll which occurs specially in hot rolling (see 16, p 205). Korolev (see 6, p 208) has developed a formula which determines the length of the sticking zone.

2.1.4 PRESSURE DISTRIBUTION, FORCE AND TORQUE

Referring to Fig. 2-5, the variation of roll pressure along the roll gap contains two parts. The lower part ,*ADGEC*, shows work-hardening for ideal (*frictionless*) deformation (it is almost horizontal in hot rolling) , and the upper part ,*DFEGD*, shows the roll pressure necessary to overcome the additional constraint caused by the friction forces. A theoretical approach for the determination of roll-pressure from local stress-evaluation is now briefly described.

The starting point is to develop an equation representing the horizontal equiblibrium of forces in the roll gap. Considering the elemental slice of material in the inset diagram of Fig. 2-1, the horizontal stress σ is assumed to be distributed uniformely over the vertical section. The horizontal forces acting on the element will be in equilibrium if the following relationship holds

$$d(\sigma h)/d\phi+2R(p_{\phi}\sin\phi\pm\tau\cos\phi)=0$$

where τ is the shear stress due to friction; depending on whether the element is in the slipping zone or sticking zone, the resultant frictional force is dependent or independent of the pressure, respectively. The negative sign refers to the condition on the entry side of the neutral point, and the positive sign refers to the condition on the exit side.

Applying the plasticity criterion to the element and by using a numerical method, the distribution of pressure along the roll gap can be determined (refer to 10, 11 and 12). Recently, some efforts have been made to find the three-dimensional distribution of pressure in the roll gap, amongst whom, Lalli[13] and Kobayashi[14] can be named. A typical three-dimensional distribution of pressure is shown in Fig. 2-6.

Having the pressure distribution, the separating force as well as the acting torque can be calculated by integrating over the area of contact (see 16, pp 208 to 111). Due to the difficulty in finding the actual pressure distribution theoretically[10,15] some attempts have been made to derive practical and easy-to-use formulae for calculating the separating force in rolling (see 6, pp 229 to 266). A good estimate of the roll load in flat rolling can be obtained by considering the process as a homogenous compression between two platens. The platens are of length l_d and width of w_{mean}, the mean value of the width of the block before and after rolling . The yield stress of the material, Y, is assumed to be constant along the roll gap; this is almost true for hot rolling[25]. The separating force, P, necessary for the rolling is then

$$P=Y\cdot l_d\cdot w_{mean} \tag{2-14}$$

Substituting the value for l_d and increasing the value of yield stress by the amount of 20% (which was suggested by Orawan[15]) for the contribution of friction, the roll load will then be

$$P=(1.2)Y\cdot w_{mean}\sqrt{R\cdot\Delta h} \tag{2-15}$$

It is true with good approximation to assume that the resultant force acts at the centre of the arc of contact in hot rolling (see 6, p 270). In that case the applied torque will be

$$M=P(l_d/2)=(0.6)Y\cdot w_{mean}R\cdot\Delta h \tag{2-16}$$

2.2 GEOMETRIC DEFORMATION IN FLAT ROLLING

2.2.1 INTRODUCTION

In rolling, *draft* (the change of thickness) is produced by pressure of the rolls. This is normally accompanied by the increase of the length, *elongation*, and the increase of the width, *spread*, of the material being rolled. These are connected and strictly dependent on one another.

Due to the difficulty in predicting theoretically the three-dimensional plastic deformation of material, most of the existing formulae for predicting the spread are empirical. Referring to Fig. 2-7 , the initially plane edge of the block usually becomes convex in shape (other possible modes of deformation will be discussed later). In some of these formulae the maximum value, w_b, and in some others the mean value, w_m, according to the following formula has been used as the rolled width[6]

$$w_m = w_b - (w_b - w_f)/3 \qquad (2\text{-}17)$$

Generally, factors affecting spread can be divided into three main groups[17,18]:

(i) - Geometric factors such as aspect ratio, draft and the bar width to the length of arc of contact ratio .

(ii) - Factors relating to frictional conditions such as roll surface and scale formation .

(iii) -Factors affecting the yield stress of the material such as strain rate, material composition and rolling temperature.

The following section deals with a study and a comparison between the most recent formulae used for predicting the spread of steel in hot flat rolling. The interest is of course in the geometric shape of the material at the entry and exit points.

2.2.2 *FORMULAE FOR SPREAD*

Significant work on spread has been ongoing since 1955. Prior to this, some approximate formulae had been in use in practice. In this regard, Tafel and Sedlaczek's formula (1923), Seibel's formula (1932), Trinks's formula (1933), Bechmann's formula (1950) and Ekelund's formula (1953) can be named as the most well-known (for details see 6, pp 84 to 86 and 4, pp 836 to 836).

In 1955, Z. Wusatowski[19] published a paper in which the following formula for spread was proposed

$$\beta=\gamma^{-W}$$

where

$$W=10^{-1.269(w_1/h_1)(h_1/D)^{0.556}}$$

$$\beta=w_b/w_1$$

$$\gamma=h_2/h_1$$

Wusatowski indicated that his formula was accurate while rolling low carbon steel with light draft ratios ($0.5\le\gamma\le0.9$). He also conducted some experiments to make a comparison between his formula and those proposed previously. His results are listed in table 2-2. According to this table, Wusatowski's formula shows no larger deviation than the other formulae except for the Ekelund's. Wusatowski suggested[20] that the mean error in his formula could be reduced by introducing some correctional factors for rolling temperature, a, rolling velocity, b, roll surface condition, c, and the type of steel to be rolled, d,

$$\beta=a.b.c.d.\gamma^{-W}$$

- $a=1.005$ while rolling at 750° to 900°C.
- $a=1.000$ while rolling above 900°C.
- $c=1.020$ for cast iron and rough steel rolls.
- $c=1.000$ for chilled cast iron and smooth steel rolls.
- $c=0.980$ for ground steel rolls.
- d is chosen from table 2-3.

By using the new formula, Wusatowski could decrease the total mean error from 4.59% to 2.24% which was satisfactory at that time. Wusatowski also suggested that his formula would be valid even for heavy draft ratios ($0.1 \le \gamma \le 0.5$) if values of 3.954 and 0.967 were substituted for the constants 1.269 and 0.556 in his formula, respectively[6].

In the same year, R. Hill[17,18,20] proposed an alternative formula for spread

$$Ln(w_b/w_1)/Ln(h_1/h_2)=0.5\ EXP[-\lambda(w_1/\sqrt{D\cdot\Delta h})]$$

λ is a constant which is selected to fit the experimental data. Hill suggested a value of 0.5 for λ.

A.W. McCrum[18] in 1956, carried out some critical tests to compare the three most recent formulae: Ekelund's, Wusatowski's and Hill's. His experiments were performed on rolling of bars with the same material, the same temperature and constant width, draft, roll radius, frictional conditions and rolling speed, but with different stock heights. The results deviated considerably from those predicted by the formulae given by Ekelund and Wusatowski, but were more agreeable with those of Hill's. McCrum showed that a value of 0.525 for λ would give better results when rolling mild steel[18].

In a growing need for more precise information about the rolling process, a series of experiments was carried out by L.G.M. Sparling[18], under carefully controlled conditions, to see the effects of various factors governing spread. He recommended the following formula

$$S=C\cdot EXP[-K(w_1/h_1)^A(h_1/R)^B(\Delta h/h_1)^G]$$

where

$$S=Ln(w_m/w_1)/Ln(h_1/h_2)$$

w_m is calculated from equation (2-17). Constants C, K, A, B and G are listed in table 2-4.

In Sparling's experiments, all the geometric factors were changed at a constant temperature (*1100±10°C*) and a constant strain rate ($5Sec^{-1}$). The material in Sparling's experiments was mild steel.

There were 35 satisfactory tests. The results extracted from his experiments show that neither the formula of Wusatowski nor that of Hill accurately predicts spread over a wide range of experimental conditions. Table 2-5 shows some relevant results.

In 1968, El-Kalay and Sparling[22] extended their investigations to an area which had been neglected until then; *friction and its effect on spread*. They modified the formula derived earlier by Sparling to give the best fit for their experimental data. The formula is as follows

$$Ln(w_m/w_1)/Ln(h_1/h_2)=A\cdot EXP[-B(w_1/h_1)^C(h_1/R)^D(\Delta h/h_1)^E]$$

the values of constants are listed in table 2-6 according to the frictional conditions.

In the same year a paper was written by A. Helmi and J.M. Alexander[17]. More than 200 tests were performed on mild steel (scale free steel with 0.18% carbon) at constant rolling temperature (1000°C) and constant rolling speed (9m/min). The following conclusions were derived by the authors:

(i)-Wusatowski's formula is basically in error, it can only predict spread under a very limited range of geometric variables (the same deduction as Sparling[18] and McCrum[17]).

(ii)-Hill's formula is basically sound; However, it is slightly in error as it neglects the effect of w_1/h_1 on the spread.

(iii)-El-Kalay and Sparling's formula seems to be the soundest existing.

Helmi and Alexander proposed a range of conditions in which Sparling's formula is more accurate, however, it has not been proved by other works. They also developed an alternative formula for spread

$$S=0.95(h_1/w_1)^{1.1}EXP[-0.707(w_1/(R\Delta h)^{0.5})(h_1/w_1)^{-0.971}]$$

where

$$S=Ln(w_b/w_1)/Ln(h_1/h_2)$$

According to the authors' paper this formula can be applied over a very wide range of geometric variables and its application is unlimited except *when* w_1/h_1 *is less than unity*. It was also verified that the deviation of temperature, roll surface and scale condition from those of the experiments would affect the value of 0.707 in the above expression.

In 1972, J.G. Beese[23] performed some industrial tests to make a comparison between the El-Kalay and Sparling's formula with that of Helmi and Alexander. The results showed that Alexander's estimates of maximum spread were all high (overestimated), while Sparling's estimates of mean spread on small slabs were satisfactory.

A new formula was developed by Beese which was claimed to be accurate for values of w_1/h_1 between 3 to 16.

$$S=0.61(h_1/w_1)^{1.3}EXP[-0.32h_1/(R^{0.5}\Delta h^{0.5})]$$

"S" has the same definition as that of Helmi and Alexander's.

There have also been a few attempts to predict the geometrical deformation of the material under rolling theoretically, for specific cases[18,24]. One typical example is the computer-aided modular upper bound approach, which was developed at Battelle Columbus laboratories, under *NASA* sponsorship to predict the metal flow in the rolling of an airfoil section. In this method the spread profile under rolls, as well as the lateral spread and longitudinal elongation are predicted.

2.3 SUMMARY AND EVALUATION

The first part of this chapter described some basic fundamentals with regard to the process of rolling. Three concepts namely, rolling speed, mechanism of bite and force calculation were discussed. These concepts will be later used to formulate the process constraints in rolling.

The literature survey on spread formulae shows that a considerable amount of study has been done to predict the effects of the geometric factors on spread. However, despite the importance of frictional conditions, little is known about their effects[22]. This study also revealed that although there is no single formula that gives reasonable prediction under all conditions, El-Kalay and Sparling's formula provides satisfactory estimates over a fairly wide range of operating conditions. Helmi and Alexander's formula has been found to overestimate the value of spread, due to the fact that short specimens were used in their experiments; so, the results were greatly influenced by back and front-end spread (see also 23). Besides, the formula is incapable of giving reasonable estimates of spread for blocks with aspect ratios less than unity. No evaluation of the Beese's formula has been reported. However, his formula is only capable of predicting the spread of the slabs with the range of aspect ratios from 3 to 16. El-Kalay and Sparling's formula can predict the spread for both cases when the aspect ratio is less or greater than one, although the range of this capability is still not very well defined. The effects of frictional conditions are also included in this formula. Another feature of El-Kalay and Sparling's folmula is that, it predicts the mean value of the spread. This will be useful when estimating the spread for different modes of deformations (see section 4.2.2). Other formulae like those belonging to Beese or Alexander were developed to estimate the maximum spread for the rhombic cross-section which, as will be seen later, is not the only mode of deformation for the steel stocks.

El-Kalay and Sparling's formula has been generalized and modified such that it could be adapted for different conditions. This modification has been based on the nature of the formula itself, and Sparling's discussions[18]; he suggested that *for most industrial processes*, the spread would be expected to increase with

(1)- decreasing temperature

(2)- decreasing amount of scales

(3)- medium carbon steels used instead of mild steel

(4)- increasing strain rates

(5)- increasing roughness of the rolles.

Conditions *(2)* and *(5)* have already been included in the formula. To satisfy other three conditions, the modified formula has been suggested by Sparling as follow

$$S=A\cdot EXP[-B(w_1/h_1)^C(h_1/R)^D(\Delta h/h_1)^E.f.g.j] \qquad (2\text{-}18)$$

$$S=Ln(w_m/w_1)/Ln(h_1/h_2)$$

where

- f indicates the effects of the composition of the rolled material; $f=1$ for 0.13% carbon, 0.55% manganese mild steel, $f<1$ for medium carbon and stainless steels.
- g indicates the effects of temperature of the rolled material; $g=1$ for 1100°C, $g<1$ for lower temperatures.
- j reflects the effects of mean strain rate during the rolling; $j=1$ for an approximate mean strain rate of $5sec^{-1}$, $j<1$ for higher strain rates.

Therefore, El-Kalay and Sparling's formula, with provisions for modification factors, was deemed suitable to be used as starting point in developing the method of rolling parts with variable rectangular cross-section. It will be seen later that the approach towards improvement, as shown above, is suitable for the more generalized form of rolling, i.e., *the unsteady rolling*.

3. ROLLING OF VARIABLE RECTANGULAR CROSS-SECTIONS

3.1 BASIC FORMULATION FOR DEFORMATION

3.1.1 UNIFORM TO UNIFORM DEFORMATION

3.1.1.1 Method

A uniform block of dimensions HI, WI and LI is to be deformed into a uniform block of desired dimensions HF, WF and LF (Fig. 3-1a). The steps involved in the process are as follows:

(i) The initial blank is rolled on the side WI. The rolled material referred to as *the intermediate material*, will have dimensions HM, WM and LM (Fig. 3-1b).

(ii) The intermediate material is then rolled into the required thickness HF. The rolled material is expected to have the width WF and the length LF (Fig. 3-1c).

The problem is to find the appropriate dimensions of the intermediate material so that the desired shape is formed within the steps stated above. The generalized form of the spread formula, (2-18), is used. Applying this formula to the first pass

$$h_1=WI$$

$$h_2=WM$$

$$w_1=HI$$

$$w_2=HM$$

$$\Delta h=(WI-WM)$$

$$R=RW$$

$$\frac{Ln(HM/HI)}{Ln(WI/WM)}=A\cdot e^{-B(HI/WI)^{C}(WI/RW)^{D}((WI-WM)/WI)^{E}}f.g.j \qquad (3-1)$$

and for the second pass

$$h_1=HM$$

$$h_2=HF$$

$$w_1=WM$$

$$w_2=WF$$

$$\Delta h=(HM-HF)$$

$$R=RH$$

$$\frac{Ln(WF/WM)}{Ln(HM/HF)}=A\cdot e^{-B(WM/HM)^{C}(HM/RH)^{D}((HM-HF)/HM)^{E}}f.g.j \qquad (3-2)$$

where RW and RH refer to the radii of the rolls in the first and the second operation, respectively. The volume constancy also exists during these processes

$$HI.WI.LI=HM.WM.LM=HF.WF.LF \qquad (3-3)$$

Two non-linear equations (3-1) and (3-2) can be solved simultaneously with eqution (3-3) to obtain the intermediate dimensions as well as the length of the final part or the initial material, depending on whichever is unknown.

The incremental search method was used in solving the above equations. The general algorithm is described in appendix A.

3.1.1.2 Number of Possible Solutions

In a two-pass rolling process, depending on which side of the initial material is rolled first, i.e., named WI, and which side is formed last, i.e., termed HF, there would exist, at most, four distinct solutions (see Fig. 3-2):

(i) Case one:

Side of WI is rolled first, HM is then reduced to HF. Convexity (out of flatness) appears on the side of HF.

(ii) Case two:

Side of WI is rolled first, HM is then reduced to WF. Convexity appears on the side of WF.

(iii) Case three:

Side of HI is rolled first, HM is then reduced to HF. Convexity appears on the side of HF.

(iv) Case four:

Side of HI is rolled first, HM is then reduced to WF. Convexity appears on the side of WF.

It is necessary to mention that *the intermediate shape for each case is different*.

3.1.1.3 Existence of the Solutions

It is clear that when the cross-sectional area of the initial blank is less than the cross-sectional area of the desired part, the proposed method is not applicable. This is due to the physical nature of the rolling process which always leads to a reduction in the cross-sectional area. Also, when the cross-sectional area of the initial blank is larger than that of the desired part, a control is still needed to evaluate the existence of the solutions. Referring to Fig. 3-1a, four cases may occur in this regard:

(i) $WI \geq WF$ and $HI \geq HF$

the process is possible.

(ii) $WI = WF$, $HI < HF$ *or* $WI < WF$, $HI \leq HF$

the process is impossible.

(iii) $WI > WF$, $HI < HF$

the process is possible if $HF \leq HS$, where HS is the spread value of HI after a single free rolling in which WI is reduced to WF.

(iv) $WI < WF$, $HI > HF$

the process is possible if $WF \leq WS$, where WS is the spread value of WI after a single free rolling in which HI is reduced to HF.

The physical explanation for the last two cases is that the extent of lateral elongation caused by rolling is less than that of the longitudinal elongation, so the maximum elongation on one side can only be achieved by applying a single pass of reduction to the other side rather than two successive passes involving a reduction on both sides or on one side. An analytical explanation for the above deduction is as follows:

Consider a block with dimensions HI, WI and LI which undergoes a width reduction to intermediate dimensions of WM, HM and LM (Fig. 3-3a.1). Spread occurs along the side HI. The following relations can then be written

$$WI.HI.LI=WM.HM.LM \tag{3-4a}$$

$$WI.HI>WM.HM \tag{3-4b}$$

$$WM<WI \quad , \quad HM>HI \tag{3-4c}$$

$$(HM-HI)/HI=\lambda(LM-LI)/LI \tag{3-4d}$$

where $0\leq\lambda<1$ is defined as a general coefficient relating lateral elongation to the longitudinal elongation. $\lambda=0$, where no spread occurs (*condition of plain strain*).

The intermediate material then undergoes rolling in such a way that its height, HM, is reduced to HF. Spread occurs on the side of WM (Fig. 3-3a.2). The following relations hold

$$WF_1.LF_1.HF=WM.LM.HM \tag{3-5a}$$

$$WM.HM>WF_1.HF \tag{3-5b}$$

$$WF_1>WM \quad , \quad HF<HM \tag{3-5c}$$

$$(WF_1-WM)/WM=\lambda(LF_1-LM)/LM \tag{3-5d}$$

Using equation (3-5a), equation (3-5d) becomes:

$$(WF_1-WM)/WM=\lambda(WM.HM-WF_1.HF)/(WF_1.HF)$$

or

$$WF_1=\{(1-\lambda)WM.HF+\sqrt{[(1-\lambda)^2WM^2.HF^2+4\lambda WM^2.HM.HF]}\}/2HF$$

(3-6)

WF_1 is now compared to WF_2 which is the elongated value of WI when the same initial material undergoes a single free rolling so that its height, HI, is reduced to HF (Fig. 3-3b).

Using a similar approach as for WF_1; WF_2 is calculated as

$$WF_2.HF.LF_2=WI.HI.LI \quad (3\text{-}7a)$$

$$WI.HI>WF_2.HF \quad (3\text{-}7b)$$

$$WF_2>WI \ , \ HF<HI \quad (3\text{-}7c)$$

$$(WF_2-WI)/WI=\lambda(LF_2-LI)/LI \quad (3\text{-}7d)$$

or

$$WF_2=\{(1-\lambda)WI.HF+\sqrt{[(1-\lambda)^2WI^2.HF^2+4\lambda WI^2.HI.HF]}\}/2HF \quad (3\text{-}8)$$

A comparison between equations (3-6) and (3-8) shows that WF_2 is equal to or greater than WF_1.

The effect of a subsequent reduction of one side, on the value of the spread for the other side is investigated experimentally in section 4.3.1.

3.1.2 UNIFORM TO NON-UNIFORM DEFORMATION

A typical example of a part having non-uniform rectangular cross-section is shown in Fig. 3-5. Initial material which is a uniform block is shown in Fig. 3-4. To formulate the problem the desired part is divided into *N* equal length segments (Fig. 3-5).

$$Lf1=Lf2=Lf3=...=Lfn=LF/N$$

Each segment is now assumed to have a uniform cross-sectional area of $Hfj.Wfj$, $(j=1, 2, \ldots, n)$, which can be formed by means of two operations from a corresponding segment in the initial material. Therefore, the initial material should also be divided into N segments in a way that each segment is a volumetric equivalent of its corresponding segment in the desired part, i.e.,

$$Hij.Wij.Lij=Hfj.Wfj.Lfj \qquad (j=1, 2, 3, \ldots, n)$$

where in this case

$$Hij, \ (j=1, 2, \ldots, n)=constant$$

and

$$Wij, \ (j=1, 2, \ldots, n)=constant$$

The lengths of the segments in the initial material would, in general, be unequal.

Applying the same procedure as described in section 3.1.1.1 to each pair of corresponding segments, the following relationships can be written

$$\frac{Ln(Hmj/Hij)}{Ln(Wij/Wmj)}=A.e^{-B}(Hij/Wij)^{C}(Wij/RW)^{D}((Wij-Wmj)/Wij)^{E}f.g.j \qquad (j=1, 2, 3, \ldots, n) \qquad (3-9)$$

$$\frac{Ln(Wfj/Wmj)}{Ln(Hmj/Hfj)}=A.e^{-B}(Wmj/Hmj)^{C}(Hmj/RH)^{D}((Hmj-Hfj)/Hmj)^{E}f.g.j \qquad (j=1, 2, 3, \ldots, n) \qquad (3-10)$$

$$Wij.Hij.Lij=Wmj.Hmj.Lmj=Wfj.Hfj.Lfj \qquad (j=1, 2, 3, \ldots, n) \qquad (3-11)$$

By solving equations (3-9), (3-10) and (3-11) simultaneously for all segments, the dimensions of the segments of the intermediate material are found. Referring to Fig. 3-6, the intermediate material consists of *N*, generally non-equal length, uniform cross-section segments with dimensions Wmj, Hmj and Lmj (j=*1, 2, . . , n*). The incremental values of roll gap variation for the first and the second pass are then Wmj and Hfj (j=*1, 2, . . , n*), respectively. So, in order to form the desired part by means of flat rolling, the roll gap in the first and the second pass should vary accordingly. The variation of roll gap can be controlled via the displacement of the rolled material during each pass (refer to section 3.4).

3. 1. 3 NON-UNIFORM TO NON-UNIFORM DEFORMATION

Normally the initial material is of uniform cross-section. However, sometimes it may be desired to deform a non-uniform material to a non-uniform part, e.g., some pre-manufactured parts are required to be reshaped. The formulation derived in 3.1.2 is applicable in this regard except that Hij and Wij (j=*1, 2, . . , n*) are no longer constant. However, a control is necessary to check the existence of the solutions for every pair of segments (refer to section 3.1.1.3). In this regard, if condition *(ii)* in 3.1.1.3 occurs for even one pair, then a feasible solution does not exist. For the case where conditions *(iii)* or *(iv)* occur and the existence of a solution is in question, an increase in the roll's radius can lead to satisfy the requirements. The variation of spread versus roll's radius for a typical case is shown in Fig. 3-7. It is seen that the increase in roll's radius leads to the increase of the lateral elongtion. This physically means that by increasing the roll's radius, the projected length of the arc of contact increases which causes the value of elongation in the lateral direction to increase (refer to section 4.4.1). Figure 3-7 also shows that larger rolls' radii have smaller effects on the increase of spread. However, the increase in roll's radius is limited because of its effects on the process constraints and other parameters which are discussed in different places in the ensuing chapters.

3.2 MULTI-PASS DESIGN CONCEPT

It was shown *theoretically*, that a part with a non-uniform rectangular cross-section can be made through two passes of flat rolling. Practically, however, it is not always possible to complete the process within two operations due to some physical process constraints. Satisfying these constraints necessitates to increase the number of operations involved. The procedure of determining the practical number of passes is termed the *multi-pass design*. The multi-pass design is a scheme which is capable of satisfying the practical constraints at all times.

3.2.1 PROCESS CONSTRAINTS

3.2.1.1 Kinematic Constraint

The kinematic constraint refers to the limit of continuous free rolling. In order to draw the material into the roll gap without back pressure or forward tension, the following inequality should be satisfied at the onset of rolling (see section 2.1.3)

$$\Delta h \leq R \cdot f^2 \tag{3-12}$$

It has also been shown that once the roll gap is filled with material, condition (3-12) is relaxed in a way that more absolute draft is permitted. So, the condition of free rolling becomes

$$\Delta h \leq b \cdot R \cdot f^2 \tag{3-13}$$

where 'b' is a coefficient that varies from 1 in the onset of rolling to almost 2 (recommended for hot rolling of steel), when the roll gap is completely filled with material. Normally, the incoming material is tapered at its front head so the condition of the filled gap can be assumed at all times. The value of 'b' can be increased by applying forward tension or back pressure. Rearranging equation (3-13)

$$\Delta h/R \leq (b \cdot f^2 = C1) \tag{3-14}$$

C1 is termed *the kinematic constraint* and its value is related mainly to the frictional conditions.

During rolling, inequality (3-14) should be satisfied; otherwise, the material deformation will stop and the rollers will begin to slip on the surface of the material.

3.2.1.2 Dynamic Constraint

The dynamic constraint refers to the capacity of the machinery in performing the process. Maximum torque available by the rolling machine, T, is a convenient criterion. It was shown in section 2.1.4 that the required torque for hot rolling is approximately

$$M=(0.6)R.\Delta h.W_{mean}.Y$$

which should be less than or equal to the available torque by the machine, i.e.,

$$(0.6)R.\Delta h.W_{mean}.Y \leq T$$

or

$$R.\Delta h.W_{mean} \leq ((T/(0.6Y))=C2) \qquad (3\text{-}15)$$

$C2$ is called *the dynamic constraint*; it is specified according to the maximum nominal torque available by the rolling mill and the composition of the material to be rolled. If condition (3-15) is not satisfied during rolling, the machine can no longer form the material due to the overload, and will stall.

3.2.1.3 Convexity Constraint

Most rolled blocks end up with some form of convexity or *out of flatness* on their sides. Here, the term *convexity constraint* has been adopted to mean an acceptable degree of out of flatness on finished parts. The less the material is reduced on one side, the less the convexity occurs on the other side. It is therefore suitable to have a constraint on the value of absolute drafts for the last pair of the rolling operations. Convexity constraints are then formulated for the finishing passes as below

Draft in the 1st finishing pass=constant=CC1 (3-16)

Draft in the 2nd finishing pass=constant=CC2 (3-17)

Values *CC1* and *CC2* are specified according to the requirements of the desired part. It is clear that the use of high capacity machinery will decrease the number of the required operations but on the other hand , will increase the need for having constraints on the draft during the finishing passes.

3.2.2 MULTI-PASS DESIGN PROCEDURE

The concepts developed so far can be extended to include the determination of the number of operations required as well as their specifications in forming a non-uniform part in general.

The *convexity constraint* is first applied to determine the dimensions of the material which must go through the finishing pair of passes. These pre-finish dimensions are then taken as the final dimensions which have to be gained from the initial blank.

After separating the finishing passes in the manner described above, the process of manufacturing such a shape is explored by means of only two operations. Both the *kinematic* and the *dynamic* constraints are considered to check the feasibility of the assumed process. The control is based on determining the roll gap variation for both passes in such a way that the incremental values of draft do not exceed the maximum allowed by the constraints. This will lead to a new rolled state which may be different, but will be closer, in shape to the desired geometry. The new material is then subjected to the next pair of passes aiming to reach the desired shape. The described procedure continues until all the requirements related to the constraints are satisfied. This eventually leads to the design of a number of passes which is frequently more than two.

3.3 COMPUTER SOFTWARE

A computer program has been developed to calculate the information for the multi-pass rolling of parts with variable rectangular cross-section. The results are then output in different formats for different purposes such as graphical visualization, operating control, user reference, etc. Referring to Fig. 3-9, the computer program consists of four major modules:

- *Data Generator*
- *Multi-Pass Process Planner*
- *Two and Three-Dimensional Graph Generator*
- *Curve-Fit and Micro-Processor Data Generator*

There are also a number of user directed programs which serve to provide organized data as input for the above modules.

This chapter describes very briefly how the solution to a general case, i.e., rolling of a non-uniform part, is determined by the program.

3.3.1 DATA GENERATOR

The data generator provides data necessary for *the multi-pass process planner*. These data are categorized as follows:

(i) Information about the properties of the material:

- coefficient of thermal expansion.
- modulus of elasticity at rolling temperature.
- Yield point at rolling temperature.

(ii) Information about the rolling condition:

- Rolling temperature.
- Ambient temperature.
- Frictional condition between the surface of the material and the rollers.

(iii) Information about the initial and final shapes:

- Variation of width and height of the initial material versus its length.
- Variation of width and height of the desired part versus its length.

(iv) Information about the process constraints:

- Minimum, maximum and incremental values allowed for the rolls' radii.

- Values of convexity constraints.
- Values of kinematic and dynamic constraints.

(v) Information about the format of the output results:

- Precision of calculations.
- Number of discrete data points in the output.

The user inputs the above information according to the condition of the underlying problem. The data generator manipulates, organizes and passes these data to the multi-pass planner's data-read file. For example, the data generator uses the values of the coefficient of thermal expansion, modulus of elasticity and the yield stress to determine the correction factors by which the computed roll gap variation and the corresponding rolled length of the material should be multiplied, in order to consider the effects of the thermal expansion and the elastic deformation of the material.

3.3.2 MULTI-PASS PLANNER

The next phase in the program hierarchy is the procedure of the process design. This is the main part which uses the data generated by the *data generator*, and determines the number of passes needed, the incremental values for the variation of the roll gap and the rolls' radii for each pass. Messages are also provided to be output during the execution in different situations or as a guide to the designed features at the end of the execution.

As was shown earlier, there exist at most four solutions to each problem. The program selects one case at a time and performs the related computations. These computations start with working on the finishing passes. Knowing the values of the convexity constraints, the program first calculates the appropriate rolls' radii. It then calculates the dimensions of the material entering each pass. Recalling inequalities (3-14) and (3-15)

$$R_k . RED_{fk} . W_k \leq C2 \qquad (3\text{-}19)$$

$$RED_{fk} / R_k \leq C1 \qquad (3\text{-}20)$$

- $W_{k(k=1,2)}$, is the largest possible value of the width of the material for each pass,
- $RED_{fk(k=1,2)}$, is the allocated reduction (draft) for each one of the finishing passes,
- $R_{k(k=1,2)}$, is the roll's radius.

Rearranging the above inequalities for R_k gives

$$R_k \geq RED_{fk}/C1$$

$$R_k \leq C2/(RED_{fk} W_k)$$

$R_l = (RED_{fk}/C_1)$ and $R_u = C2/(RED_{fk} W_k)$ are the two boundaries which bracket the roll's radius. It is clear that R_l should be less than or equal to R_u. The roll's radius for each pass is then found as the minimum possible value, i.e.,

$$R_k = RED_{fk}/C1$$

Note that small rollers lead to a decrease in the required torque and load.

The calculated roll's radius is then modified to satisfy the conditions of the availability of the rollers. This is done by rounding up the value of R_k to the nearest available size. A control is necessary to check whether the new value of $R_{k(k=1,2)}$, is still between the stated boundaries. If at any stage of calculations, the applied conditions could not be fully satisfied, the program returns with an appropriate messages and some guidelines.

Knowing the values of the drafts and rolls' radii for the finishing passes, the dimensions of the material entering each pass can be determined using the *inverse calculation method*:

Referring to Fig. 3-8, for the second of the finishing passes the following relationships hold

$$Hmj=Hfj+RED_{f2} \qquad (3\text{-}21a)$$

$$\frac{Ln(Wfj/Wmj)}{Ln(Hmj/Hfj)}=A.e^{-B(Wmj/Hmj)^C(Hmj/R_2)^D((Hmj-Hfj)/Hmj)^E}f.g.j \qquad (3\text{-}21b)$$

$$Lmj.Hmj.Wmj=Lfj.Hfj.Wfj \qquad (3\text{-}21c)$$

$$(j=1,2,3,\ldots,n)$$

This set of equations is solved simultaneously to find the dimensions of the material entering the second pass or leaving the first pass. The same procedure can be followed to find the dimensions of the material entering the first of the finishing passes:

$$Wij=Wmj+RED_{f1} \qquad (3\text{-}22a)$$

$$\frac{Ln(Hmj/Hij)}{Ln(Wij/Wmj)}=A.e^{-B(Hij/Wij)^C(Wij/R_1)^D((Wij-Wmj)/Wij)^E}f.g.j \qquad (3\text{-}22b)$$

$$Lij.Hij.Wij=Lmj.Hmj.Wmj \qquad (3\text{-}22c)$$

$$(j=1,2,3,...,n)$$

The material with the incremental dimensions of Wij, Hij, Lij, is now assumed to be the desired part which is to be shaped within an unknown number of passes from the known initial material. The numerical approach for the determination of the above process within two passes was shown in section 3.1. The rolls' radii necessary for the pair of passes are determined by reapplying the inequalities (3-19) and (3-20)

$$R_k \geq DRAFT_{kj}/C1 \qquad (3\text{-}23)$$

$$R_k \leq C2/(DRAFT_{kj}.W_{kj}) \qquad (3\text{-}24)$$

$$k=(1,2) \quad , \quad j=(1,2,..,n)$$

- $DRAFT_{kj}$ stands for the incremental value of the absolute draft during rolling and is unknown.
- W_{kj} stands for the incremental value of the side of the material under the rollers at each pass.

Combining the two inequalities and having in mind that the smaller radius and the larger draft is desirable, the following inequality is derived

$$R_k^2 \geq C2/(C1.W_k)$$

which gives the lower limit on R_k as

$$R_k = \sqrt{C2/(C1.W_k)} \qquad (3\text{-}25)$$

where $W_{k(k=1,2)}$ is chosen as the maximum value of the width for each pass.

The values of rolls' radii for each pass, $R_{k(k=1,2)}$, are then selected as mentioned earlier.

After the process has been determined (refer to section 3.1.2), it is evaluated with respect to the kinematic and dynamic constraints. This is done by comparing the calculated incremental values for the drafts to those permitted by the kinematic and dynamic constraints. The permissible incremental reduction ($PRED_{kj}$) can then be specified by the following inequalities

$$PRED_{kj} \leq C2/(\mathrm{R}_k.\mathrm{W}_{kj})$$

$$PRED_{kj} \leq C1.\mathrm{R}_k$$

$$k=(1,2) \quad , \quad j=(1,2,\ldots,n)$$

The values of the calculated drafts should be less than or equal to the permissible values. The violating draft values are reduced to meet this constraint.

After the first pass is modified, dimensions of the new intermediate material are then used to re-define both the permissible incremental and calculated drafts for the second pass. Modification is then done on the second pass. If the modified values of drafts for a pass are too small, it may not be worthwhile to allocate a pass for that operation, unless it is one of the last pair of passes. In this case the corresponding pass is deleted and the process is continued to the next major pass. This implies that the total number of the passes is not necessarily an even number as it might be deduced at the first instance.

Direct or forward solution is used to determine the dimensions of the material leaving the modified passes. This is done by solving sets of equations like those in (3-21) and (3-22) repectively , providing that RED_{kj} (k=1,2) be substituted for RED_{f1} and RED_{f2} in equations (3-21a) and (3-22a). The dimensions of the new intermediate material are then interpreted as the dimensions of a new initial material which is to be rolled within a subsequent pair of passes. The procedure is repeated until constraints are fully satisfied, i.e., until there is no need for pass modification. The incremental values of roll gap calculated for each pass are then multiplied by the *correction factor* to take into account the effects of the thermal expansion and the elastic deformation of the material.

For cases in which both the initial and the final dimensions are variable, it may happen during the execution that the program detects a pair of segments for which conditions *(iii)* or *(iv)* of section 3.1.1.3 are not satisfied. In these cases, the program automatically increases the value of the corresponding roll's radius by some allowable increments until those conditions are satisfied. Then, if the new value of the roll's radius is less than the maximum available, the execution starts from the beginning using the new roll's radius; otherwise, it returns back with a message with regard to the situation and some guidelines.

The multi-pass design routine completes its execution by passing the information and the related messages in a proper format for the user's reference. Also, it outputs some relevant data into other files for different uses such as graph generations. The general flow-chart of *the multi-pass design algorithm* is shown in Fig. 3-10.

3.3.3 GRAPH GENERATOR

There are also several routines which help to generate visualized information of the rolled parts after each operation ,*three-dimensional visualization*, and variation of some parameters such as roll gap during the operations ,*two-dimensional visualization*. Three-dimensional visualization is generated by the *three-dimensional graphic generator* which consists of three routines. The first uses the incremental values of width and height of the intermediate material(s) and generates the incremental coordinates of the shape. The next routine finds the projected values of these coordinates. The third routine serves to provide a visual representation of the object either on a graphics terminal or on a plotter. *Two-dimensional visualization* is also possible by using any two dimensional plotting package.

3.3.4 OTHER ROUTINES

Some miscellaneous routines have been provided which are used for the purpose of operation control. The application and the algorithmic procedures of these routines are described in the following section.

3.4 OPERATING ASPECTS

In the previous sections, it was shown how the variation of the roll gap as a function of the rolled length for each pass is determined in the form of discrete data points. Figure 3-11 shows a typical variation of roll gap versus the rolled length. To derive an analytical form for this relationship, curve-fitting has been used. The curve has a continuity up to the second order to be smooth enough and to simulate the actual conditions. Appendix *B* describes the curve-fitting algorithm. The rollers are supposed to follow this curve during the operation (position *a* in Fig. 3-11). Parts with a high degree of geometric changes, may be *over-rolled* while the rolls are following a certain curve. In order to prevent this, the rollers should adjust their path. However, this will result in *under-rolling* (position *b* in Fig. 3-11). It is clear that smaller rolls permit parts with high variation in geometry to be manufactured.

Roll's radius can be decreased in many ways; by either limiting the roll's radii available for the use by the program or by changing the kinematic and/or dynamic constraints. Decreasing the value of the dynamic constraint leads to the selection of a smaller radius but on the other hand, it may increase the number of operations needed. Provision has been made in the program so that this can be done artificially and in an optimized way. Increasing the value of the kinematic constraint will also lead to the use of smaller roll's radius but it may lead to a need for forward tension.

The rolling machine should have a facility for changing the roll gap as accurately and as fast as possible. Roll gap control based on the displacement of the rolled material is a convenient way to control the process. Control strategies based on other parameters such as time will lead to the use of empirical relations which are not suggested (see section 2.1.2, *calculation of rolling speed*). Both values of the roll gap and the rolled length can be detected and read out during the rolling by various digital read out systems such as *linear scales*. The application of two different control systems is briefly discussed here.

(i) Analog Control System

With analog control, each individual output variable (actual roll gap and rolled length) is monitored. Changes are then made in the corresponding input variable (rotation of the servomotor or opening of the hydraulic valve) to maintain the output at a desired level. This is done continuously. The general block diagram of such a system is shown in Fig. 3-12. As is seen, the difference between the desired value and the actual value is used by the analog controller. The mechanism by which the process variables are altered, depends on the particular system and the design of the machine. The desired value is not a set point but variable in itself. It should be continuously followed during the process as a function of the actual rolled length. So, a template with the shape of the desired roll gap variation and a mechanical follower which moves proportionally to the length of the rolled material (Fig.3-12) are needed. The movement of the follower is scaled and read out by a linear scale measuring device.

As it can be deduced, in the analog control, for each process a different template is needed. Besides, analog control is hardwired and can not be easily changed when the design is changed. This strategy can also be called *adaptive model control*, because a template of the desired profile and a mechanical follower are used to control the roll gap.

(ii) Direct Digital Control (D.D.C.)

Due to the increased complexity and the inflexibility of the model control systems, new methods based on micro-computers, namely *Direct Digital Cotrol* have become increasingly relevant. More than one loop may be controlled with a micro-computer. In a digital control strategy a micro-computer samples the variables (actual roll gap and rolled length) periodically with a sufficient sampling frequency. Fig. 3-13 shows the general block diagram of a typical *D.D.C*. The micro-computer is programmed such that at each time interval, the actual values of the rolled length and the roll gap are read out. The recorded variables are then used by the micro-computer to determine the desired roll gap for the next increments. The curve-fitting program is used to organize data in a fashion suitable for the use by the micro-computer.

The control strategy based on *D.D.C* is sometimes called *adaptive data control* because in this method instead of continuous data transmission from the template, discrete data is fed to the control system. By the use of *D.D.C*, programming (*software design*) is substituted for wiring-up and modelling (*hardware design*). Moreover, with a fast computer system, simultaneous control of several machines is possible. Micro-computers can be easily reprogrammed for a new process as the design changes occur[27].

3.5 SUMMARY AND EVALUATION

The method of rolling of parts with variable rectangular cross-section was first described for a simple unconstrained two-pass process. This was followed by numerically solving a resultant set of non-linear equations such as (3-9) and (3-10), to find the incremental values of the dimensions of the intermediate material. Rearranging equations (3-9) and (3-10) as follows

$$\chi=\frac{Ln(Hmj/Hij)}{Ln(Wij/Wmj)}-A.e^{-B(Hij/Wij)^{C}(Wij/RW)^{D}((Wij-Wmj)/Wij)^{E}f.g.j}$$

$$\psi=\frac{Ln(Wfj/Wmj)}{Ln(Hmj/Hfj)}-A.e^{-B(Wmj/Hmj)^{C}(Hmj/RH)^{D}((Hmj-Hfj)/Hmj)^{E}f.g.j}$$

$$(j=1,2,\ldots,n)$$

it is seen that χ and ψ are functions of two unknowns, Hmj and Wmj, and four variable parameters Hij, Wij, Hfj and Wfj. A study of the behavior of the above functions shows that they both are continuous with respect to the two unknowns and the four parameters in the physical range of their existence. Having a solution for each segment (refer to section 3.1.1.3), it is deduced that the set of solutions, i.e, Wmj and Hmj, (j=*1, 2, ..., n*), are continuous versus the rolled length providing that Wij, Hij, Wfj and Hfj, (j=*1, 2, ..., n*), are also continuous with respect to the length (see 26, pp 209 to 213, "*inverse function theorem*"). This implies that the solution of the deformation of a continuous shape to another, i.e., the intermediate material is also continuous in shape.

The method was then extended to include and satisfy the physical constraints. It was discussed how these constraints lead to an increase in the required number of passes from the ideal minimum of two. For example, the inclusion of the convexity constraints, alone, immediately increases this number to four.

Normally, the initial material used would be a uniform block. A suitable initial material may be one whose cross-sectional dimensions are equal to the largest cross-sectional dimensions of the material prior to the finishing passes. Roll's radius plays an important role in the magnitude of spread. It is also a flexible tool in making the deformation of more complex shapes possible.

The last part of the chapter was allocated to a brief discussion of operating aspects. Two control strategies were described. The control strategy based on the rolled length implies the fact that the material should completely leave a pass before entering the next, otherwise the difference between the exit and the entrance velocity, not only causes a variable tension or compression along the length of the block, but also makes the process of measurement considerably difficult (for more details, see 33 pp 9 to 16).

4. EXPERIMENTAL EVALUATION

4.1 INTRODUCTION

The theoretical analysis of rolling parts with variable rectangular cross-section was presented in chapter three. At this stage the need for experimental evaluation of the method is sensed. The purpose of doing the experiments was to examine the physical aspects of the method discussed, and to evaluate its accuracy in predicting the process behaviour. The objectives were:

1. To evaluate the capability of the spread formula, *as used*, in predicting the geometric deformation of the material during unsteady rolling and to determine and implement the necessary corrections.
2. To evaluate the validity of the computer algorithm in determining the process specifications.

The first objective relates to the fact that for most of the operations of the process under investigation, the incoming material is non-uniform and the roll gap is subjected to variation. El-Kalay and Sparling's formula, as stated, is best suited for the estimation of spread in the conventional constant thickness rolling. Thus, it was known that the results of the direct application of this formula to the general situation where the dimensions of the input material and the roll gap are variable would be in certain error, and that some corrective factors could be derived from experiments and incorporated in the formula.

The second objective is basically the test for the validation of the computer program. In this regard, some experimenets were performed on uniform blocks under different rolling conditions.

Most of the experiments were conducted on hot mild steel; although, there were some experiments which were performed on warm aluminum.

4.2 EXPERIMENTAL ARRANGEMENTS

4.2.1 INSTRUMENTATION

The rolling mill used for experiments was a small laboratory unit (see Fig. 4-24). The nominal torque of the motor used, was 4.77Kgf-m. and four angular speeds were available, specifically: 17.2, 34.3, 51.4 and 68.5rev/min. The rolls' radii were 50.8mm and with different surface conditions. The upper roll could be moved vertically for adjusting the roll gap. This was done through a hand-wheel and a power screw. Sixteen rotation of the hand-wheel translated into one inch vertical movement of the upper roll. Due to wear on the sides of the threads, the upper roll had some slack or *backlash*, in the vertical direction. Due to the misalignment of the bearings, a slight horizontal slack also existed. Preliminary experiments with aluminum were performed to measure the amount of the vertical slack. This was found to be 0.6mm in the range of roll gaps between 10mm to 25mm. The existance of the horizontal slack could have caused the rolled material to twist in the horizontal plane; to prevent this, a guide way or jig was built and installed at the entry point. The absolute maximum reduction that a heated steel slab could have been subjected to, in a single pass, was experimented to be about 5mm for a 25mm wide specimen.

The dimensions of the furnace used were "600x400x400mm". The maximum temperature that could have been reached with this furnace was about 1200°C. The distance between the furnace and the rolling mill was about two meters (see Fig. 4-25).

4.2.2 SPECIMENS AND MEASUREMENT

Suitably long specimens were used in order to isolate the effect of the excessive spread at the front and back ends which can be significant for small specimens[23]. Two major modes of deformations were observed during the experiments with hot steel; *double bulged* and *rhombic* (*regular* or *irregular*) cross-sections. Fig. 4-1 shows these two and the procedure for the calculation of the mean value of spread. Preliminary experiments on the constant thickness rolling of steel showed that a good agreement exists between the results of the experiments and those predicted by the formula (2-18) providing that conditions of rough rolls and heavy scales are assumed (see table 2-6). To obtain the roll settings, the required final thickness of the rolled material was modified by adding the thermal expansion of the bar. The thermal expansion from the room temperature to the working temperature of 1100°C was 1.48% of the exit thickness. The effects of elastic deformation of the bar under the rolls, and the mill spring on the exit thickness, was known to be very small for hot steel (less than 0.1% of the exit thickness[22]) and thus was neglected.

4.3 STEADY STATE ROLLING

The rolling of uniform parts when the roll gap is kept constant is referred to as the *steady state rolling*. In this section some typical experiments on steady state rolling are described. The purpose of these experiments were to evaluate the computer algorithm and to study the shapes of the rolled cross-sections.

4.3.1 EFFECTS OF SEQUENTIAL ROLLING ON SPREAD

A heated uniform block "19.1mmx19.1mm", 180mm long, was rolled on one side down to 18mm. The specimen was then re-heated and re-rolled on the same side to a new thickness of 15.55mm. The increased width after these operations was measured 19.70mm (Fig. 4-3a). Next, a similar block was rolled in a single pass to the same thickness of 15.55mm. The increased width was measured 19.78mm (Fig. 4-3b), which was greater than the value obtained in the first experiment.

The same set of experiments were performed with different sizes of blocks and similar results were observed which indicated that, for a given total draft, the magnitude of spread decreases as the number of passes increases. Physically, this is due to the fact that the higher the draft, the higher the spread; also, the smaller the width of the material under the rolls, the larger the spread. Figure 4-3 shows a reproduction of the real cross-sections of the specimens at each stage of the above experiments. It is seen that when the draft is small, the distortion on the spread side is also small. This implies that the rolled parts can have a flatter surfaces *if the drafts in the last pair of passes in the multi-pass process are kept small.*

4.3.2 THE EVALUATION OF THE MULTI-PASS ALGORITHM

A uniform block of cross-section "25.4mmx25.4mm" was rolled within two operations and was reduced in new cross-sectional dimensions to "20mmx20mm". The value of the roll gap for each pass was predicted by the program and the roll gap was set to that value beforehand. The dimensions and the cross-sectional views of the material at each stage are reproduced in Fig. 4-4a. Also, the output of the program is illustrated in Fig. 4-5.

In the second experiment, the multi-pass design concept was applied and two finishing passes with a 2mm of reduction for each, were specified. The cross-sectional shapes of the rolled material after each pass, as well as the computer prediction are provided in figures 4-4b and 4-6, respectively.

It is seen that applying the concept of the convexity constraints to the process improves the appearance of the finished part noticeably.

4.4 UNSTEADY STATE ROLLING

When the material to be rolled is non-uniform and/or the roll gap changes during the rolling, the process is termed *unsteady state rolling*. Most of the rolling operations involved in the production of parts with non-uniform cross-section are of an unsteady state nature. No previous work has been done to evaluate the geometric deformation of the material in this type of rolling.

It was detailed earlier that in the process planning scheme for the deformation of non-uniform parts, each segment is assumed to be a member of a uniform block which is rolled within a certain roll gap. In reality, however, not only the roll gap varies while the segment is being rolled, but also the segment is affected by the succeeding and the preceding segments which are different from the segment in question. In other words, each portion of the material encounters a *history of deformation* which is different from that of the steady state rolling. Some experiments were needed to evaluate the effects of such unsteadiness on the spread. Thus, unsteady state rolling takes place, whenever any one or a combination of the following situations exist

1. The width of the material being rolled is variable.

2. *The height of the material being rolled is variable.*

3. *The roll gap changes during rolling.*

The effects of the above conditions on spread, can be evaluated independently. Thus, in order to evaluate the effect of each, the other two were kept constant in the experiments. These important experiments along with the results obtained are presented in the following sections.

4.4.1 EFFECTS OF HEIGHT VARIATION ON SPREAD

To evaluate the effect of height variation, three types of specimens with the same length of 225mm and the same width width of 12.8mm were used (see Fig. 4-7). The height of each specimen varied from 12mm to 30mm but in different patterns. The variation of height for the three types of specimens were formulated versus the length as follows

- *1st specimen; concave parabolic*

$$HEIGHT=3.55\times10^{-4}(LENGTH)^{2}+12 \qquad (mm)$$

- *2nd specimen; straight taper*

$$HEIGHT=0.08(LENGTH)+12 \qquad (mm)$$

- *3rd specimen; convex parabolic*

$$HEIGHT=1.2\sqrt{(LENGTH)}+12 \qquad (mm)$$

The length is measured from the narrower head.

Sides of the specimens were marked logitudinally and laterally (see Fig. 4-26), with 5mm intervals. The rate of height variation for each specimen is shown in Fig. 4-8. All three specimens were rolled from the narrower head to a constant thickness of 12.9mm. The plots of the experiments and the predicted spread versus the corresponding initial height are shown in figures 4-9, 4-10 and 4-11.

Referring to figures 4-8 and 4-10, for specimen (*ii*) which has a constant rate of height variation (0.08mm/mm), it is seen that when the initial height is small, i.e., when the absolute draft is small, experimental values and the simulated values are in a good agreement, but as the absolute draft increases, the actual values begin to deviate from the values calculated by El-Kalay and Sparling's conventionl (unmodified) spread formula. The maximum error was found to be more than 3%.

It is seen from figures 4-9 and 4-10 that, for the same draft, the specimen (*i*) shows larger deviation from the unmodified predicted values than does the specimen (*ii*), Particularly for the draft values greater than 3mm. This is because the rate of height increase is larger for specimen (*i*) (refer to Fig. 4-8). It is also seen from Fig. 4-11, that even for small drafts, the measured values are larger than the unmodified predicted values. This is due to the fact that for the specimen (*iii*), the rate of height variation is larger on the head end.

It is deduced that for low drafts, less than 2.0mm, and for low rate of height variations, less than 0.05 mm/mm, El-kalay and Sparling's predictions are reasonably accurate to be used, but as the absolute draft and/or the rate of height variation increases, the actual spread increases such that the conventional formula begins to deteriorate in its predictive, accurately. The deviation is seen to be more than 5% in some instances.

An explanation for the effect of *height variation* on spread is that the height variation affects the length of the arc of contact which is an important parameter in determining the value of spread. Hill[18] showed how the lateral spread deponds inversely on the length of arc of contact:

- *(i)*- when $\sqrt{R.\Delta h} \longrightarrow \infty$, the lateral elongation is maximum.
- *(ii)*- when $\sqrt{R.\Delta h} \longrightarrow 0$, condition of plane strain occurs, i.e., the lateral elongation is zero.

Physically, when the length of the arc of contact between the roll's surface and the material increases, due to the increase in friction, the longitudinal elongation of the material becomes difficult, such that material tends to elongate more on the lateral direction. This is why the actual lateral elongation is larger than the predicted value when neglecting the effect of height variation.

Another effect of height variation is on the strain rate which indirectly affects the spread. Sim[12] formulated the value of mean strain rate as follows

$$\textit{MEAN STRAIN RATE} = v(R.\Delta h)^{-0.5} \mathrm{Ln}(1/(1-r))$$

- v is the peripheral velocity of rolls,
- r is the reduction in thickness($\Delta h/h_1$)

A study of the behaviour of the above formula shows that draft is directly related to the mean strain rate; as draft increases, mean strain rate associated with the process also increases. Strain rate causes the material under rolling to be hardened in all directions. The degree of strain hardening in longitudinal and lateral direction depends on the respective strains. Under the conditions which promote more strain in elongation than spread, *which is true for the most rolling purposes*, strain hardening would cause a greater resistance to deformation in elongation than in spread.

A modification of the spread formula was then needed to improve the theoretical prediction by considering the effect of height variation. In doing so, the following points are presented

- *(a)*- Sparling[18] included some coefficients in his proposed formula, (2-18), for the effects of temperature variation, strain rate and material composition (see section 2.3). He suggested that assigning values less than unity for these coefficients, would predict the larger spread and vice versa.
- *(b)*- Modification to the spread formula should be such that, for a uniform block, i.e., a steady state process, it becomes ineffective.
- *(c)*- Rate of height variation should appear in the improved formula. Also its effect on the length of the arc of contact should be included.

- *(d)*- The effect of height variation on spread depends on the frictional condition[22]; the lower the friction between the rolls and the material, the lesser it affects the value of spread and vice versa.

The following formula is then suggested to satisfy the above requirements

$$\frac{Ln(w_m/w_1)}{Ln(h_1/h_2)}=A.EXP[-B(w_1/h_1)^C(h_1/R)^D(\Delta h/h_1)^E\cdot T]$$

T is a parametric coefficient which was defined as follows

$$T=(1+\xi\cdot Hv\sqrt{R.\Delta h}/w_1)^{-D} \qquad (4-1)$$

- Hv is the absolute rate of height variation of the material at the cross-section where the spread is to be predicted.
- ξ is a constant; the value of which was found to be *6.15* for minimum difference between the experimental and the predicted results. The method of least square fitting was used in this regard.

As it is seen, the necessary requirements are satisfied in the improved formula:

- *(i)*- When the block is uniform, i.e., when $Hv=0$, then $T=1$ and the formula converts to its conventional form.
- *(ii)*- For higher rates of height variation, or for larger lengths of the arc of contact, the value of T becomes smaller and consequently, the calculated spread will become larger.

- *(iii)-* The effect of height variation is weighted as in the El-Kalay and Sparling's formula, using parameter D (table 2-6) as the power.
- *(iv)-* For very large values of w_1, where the plain strain condition holds, effect of height variation on spread becomes negligible.

The improved formula was then used to predict the values of spread for the experiments performed. The results are shown in figures 4-9, 4-10 and 4-11. As is seen, the new predictions are in a very good agreement with the experimental results. The mean deviation between the new predictions and the experimental values was found to be within ±0.8% .

4.4.2 EFFECTS OF WIDTH VARIATION ON SPREAD

In the following experiments, all the relevant parameters were kept constant expect the width of the material being rolled. Identical specimens as in Fig. 4-7 were used. The constant dimension this time was interpreted as the height and the non-uniform side was selected as the width. All the three types of specimens were heated to 1150°C and rolled from the narrower head to a thickness of 10.3mm. Figures 4-12, 4-13 and 4-14 show the results. For each specimen, the variation of the increased width versus the initial width has been plotted. The predicted values of the spread width using El-Kalay and Sparling's formula are also superimposed on each graph. It is seen that, there is an agreement between the results obtained from the experiments and those predicted by the simulation, except for a slight constant shift. The deviation was measured and was within 1.7%.

In the second set of experiments, the same type specimens were hot-rolled to a thickness of 10.3mm but this time the wider head was rolled first. The values of width were plotted against their initial values and were compared with those predicted by the formula. Figures 4-15 and 4-16 show the results. Maximum deviation from the prediction was measured and was 1.2%.

These experiments show that both width increase and the decrease cause a reduction in spread. It is seen that for larger values of width, the deviation is slightly greater than the deviation for the smaller widths, however, the relative deviation doesn't change. A comparison between figures 4-13 and 4-15, for example, shows that spread is more sensitive to the width increase than it is to the width decrease. Experiments on aluminum pieces showed the proof of this recent assertion. Figure 4-17 shows an aluminum specimen having a constant thickness and a symmetric linearly variable width. The primary purpose of this rolling experiment was to observe the extent to which the condition of width variation affects the spread. In other words, if the sign of the width variation is to have a significant effect, then the rolled specimen should lose its symmetry after it is rolled. The specimen was rolled to an absolute reduction of about 5mm at a temperature of 450°C. The rolled specimen maintained its symmetry. Fig. 4-18 shows this result graphically. It is seen that the points corresponding to the two symmetric ends overlap except for a slight diviation near the middle region where the aspect ratio is large.

The above experiments show that using the conventional spread formula and applying it for each individual cross-section, overestimates the value of spread by about 1.7%. A parametric correction factor, Λ, similar to the case of height variation, was then introduced in order to reduce this error

$$\Lambda=(1.0+\eta.Wv)^{C} \qquad (4\text{-}2)$$

- Wv is the *absolute* rate of width variation,
- C is chosen from table 2-6,
- η is a constant; value of which was found to be *7.75*

The implementation of Λ in the spread formula decreased the error to less than 0.5%.

4.4.3 EFFECTS OF ROLL GAP VARIATION ON SPREAD

The purpose of the following experiments was to evaluate the effects of the roll gap variation on the spread. As it was mentioned earlier, while developing the method, each lengthwise segment of the material was imagined as if it was an element from a uniform block being rolled through a predetermined and fixed roll gap. In fact this is assumed for all the segments in a sequential manner. Thus, for any element being rolled, although the roll gap is continuously changing, the entry and exit dimensions are known. Two cases can be identified in this regard:

(i) The roll gap is opening

(ii) The roll gap is closing

For either case, the effects of the roll gap variation on spread is still unknown. The experiments were performed with different rates of roll gap variation. Some typical examples are illustrated here.

A long square rod with cross-sectional dimensions "12.7mmx12.7mm" was heated to 1150°C and rolled through a closing roll gap operation which varied from 12.7mm to 9.95mm. Due to the lack of mechanical facility in doing so, the roll gap was changed manually with a speed as uniform as possible. The variation of the roll gap during this process is shown in Fig. 4-19. The spread for some cross-sections along the length were measured and compared to those predicted by El-kalay and Sparling's formula as applied to each segment. Figure 4-20a shows the results, while Fig. 4-20b depicts an alternative representation of the same.

In a complementary experiment, an identical uniform block was hot-rolled while the roll gap was opened from 9.1mm to 12.7mm during the operation (Fig. 4-22). The incremental values of the increased width along the rolled length were measured and are shown in Fig. 4-23 along with the corresponding predicted values. The above experiments indicated that both the opening and the closing of the roll gap have an increasing effect on spread.

The effects of roll gap opening and closing becomes more significant for higher drafts. This was experimentally verified and the findings are shown in figure 4-20b, where the difference between the predicted and the experimental values increase as the draft increases. Also, for the same draft, *higher rate of roll gap changing causes more spread.* Figure 4-21 shows this assesment; referring to this figure, the higher values of spread correspond to the specimen which was rolled with the higher rate of roll gap closing. The comparison was based on measuring the increased width at equal thickness cross-sections.

4.5 SUMMARY AND EVALUATION

This chapter described the experiments which were conducted to evaluate the different aspects of the process of rolling parts with variable rectangular cross-section. The experiments were performed on both uniform and non-uniform blocks. Some experiments on uniform blocks were primarily conducted to confirm the basic concepts. These led to appropriate arrangements to be set up for the subsequent experiments.

Handling the material from the furnace to the rolling mill was done manually. The heating time was about 20 minutes. The specimens which should have been rolled twice or more were immediately re-heated before re-rolling. The re-heating time was then reduced to almost half. Large temperature variations was found to have a significant effect on the magnitude of spread. Although the furnace was placed as close to the rolling mill as possible, the temperature drop during the handling, was postulated to be large enough to fall below the range in which El-kalay and Sparling's formula was accepted to be accurate, i.e., $1100\pm10°C$. This was observed from the results of section 4.3.2 where the computer predicted values were all underestimated. In order to take into account the temperature drop during the handling, the specimens were heated to 1150°C. Good agreement was then observed between the results from the theory and those of the experiments. The strain rate was found to be less effective on spread at higher temperatures (>1100°C). this assertion was experimented with different rolling speed at a fixed temperature. However, for other experiments, the angular speed of the rollers was set to the lowest value which satisfied the condition of strain rate in El-kalay and Sparling's formula. All the *quantitative experiments* were done on hot mild steel, however, some were performed on aluminum, namely, *the qualitative or critical experiments*. Figure 4-27 shows the aluminum specimens which were used in these experiments. The problem with aluminum stocks was their different modes of deformation which quite often happened during the experiments (see Fig. 4-2). Therefore, aluminum was found unsuitable for simulating the hot rolling of steel.

Whenever necessary, the thickness of the scales were also taken into account during the measurements. The thickness of the scales were found to vary from 0.2mm to 0.8mm, depending mostly on the number of times the specimen had been heated in the furnace.

The effects of width, height and the roll gap variation on spread were investigated. It was found that the increase in the height of the incoming material has a significant effect on the spread. A method of correction was proposed in the form of equation (4-1). This correction can also consider the effect of negative rate of height variation *by changing the sign of the coefficient D, in the formula (4-1), to positive.*

Width variation, either positive or negative, was observed to have a decreasing effect on spread, however, its effect was not as much significant as the effect of height variation. A parametric correction factor in the form of equation (4-2) was then proposed to implement these effects.

A qualitative experimental study on the effects of the roll gap changing on spread showed that both the opening and the closing of the roll gap have an increasing effect on the magnitude of spread. This effect is probable to be proportional to the length of the arc of contact, absolute draft and the rolling speed. Due to the lack of access to an appropriate facility for automatic and continuous changing of the roll gap, an adequate quantitative examination was not possible.

The capacity of the rolling mill did not allow to extend the experiments to the rolling of larger specimens with high rates of variation in height and width. An extensive range of experiments in a variety of real cases are needed to enhance the results extracted in this work.

5. SAMPLE ILLUSTRATIONS

5.1 EXAMPLE NO.1

A uniform block with cross-sectional dimensions of 70mmx250mm is to be rolled to a part with linearly variable height (Fig. 5-1). The width of the part is required to remain constant and equal to the width of the initial block. Figure 5-2 shows the spread on the side of the constant width, if only one rolling pass with linearly variable gap is performed. With a single pass, the final dimensions, though predictable, are uncontrollable. So, for a dimensionally controllable process, at least two rolling passes should be used. The rolls' radii of these passes were arbitrary set to 175mm and 180mm, respectively. One run of the computer program produced two solutions for the above problem. The first solution implied that the smaller side of the initial material should be rolled first followed by the rolling of the other side. Figure 5-3a shows the shape of the intermediate material for this case. Figures 5-4 and 5-5 show the variation of the roll gap and the reduction for each pass. Reduction has been defined as the incremental ratio of draft over the initial thickness.

The second solution implied that the larger side of the initial material, i.e., the height should be rolled first, followed by an operation on the lateral dimension. Figure 5-3b shows the shape of the intermediate material for this case. Also, figures 5-6 and 5-7 show the variation of roll gap and the reduction during each pass for this feasible solution.

In the second case, there is only one pass with variable roll gap. On the other hand, in the first case, both passes have variable roll gap; because the second pass has been assigned to form the height of the part, better geometry on this side is obtained and consequently, the barrelling occurs on the uniform side. Thus, depending on which side of the material is preferred to attain a better finish, one or the other sequence of operations may be selected. However, it is seen that since in this example the process constraints were not considered, the values of reduction reach to more than 80%, a proportion which is practically impossible. The second example which follows, illustrates the rolling of a more complex part with the process constraints in effect.

5.2 EXAMPLE NO.2

In this illustrative example, the production of a part the width of which is constant and the height of which varies parabolically is considered. Unlike example No.1, all the process constraints are applied in this case. The following information were supplied to the computer program as the input data:

- Dimensions of the initial block (Fig. 5-8a):
  width, 155mm
  height, 110mm
- Dimensions of the final part(Fig. 5-8b):
  width, *constant*, 150mm
  height, *varies parabolically* (*minimum value*, 50mm/ *maximum value*, 100mm)
  length, 2000mm
- Yield stress of the material at the working temperature, 15Kgf/mm^2
- Modulus of elasticity of the material at the working temperature, 15x10^{3}Kgf/mm^2
- Coefficient of thermal expansion of the material, 3.3x10^6(1/°C)
- Working temperature, 1100°C
- Ambient temperature, 30°C
- Coefficient of friction between the roll's surface and the material, assuming rough rolls, 0.35
- Maximum torque available by the rolling mill, 5000Kgf·m

- roll sizes available:

  Minimum radius, 100mm

  Maximum radius, 500mm

  (with increments of 5mm)

- Applied drafts for the finishing passes:

  First pass, 5mm

  Second pass, 5mm

- Precision of computation, 0.001

- Number of segments that the final part is divided into, 25

Five rolling passes were determined by the program. Figure 5-9 shows the sequence of these passes. Fig. 5-10 shows the variation of the roll gap versus the rolled length. Figure 5-11 provides more information about the particulars of the planned process.

6. CONCLUSIONS AND SCOPE FOR FUTURE WORK

A computer-aided process planning scheme for rolling of parts having variable rectangular cross-section was developed. El-Kalay and Sparling's spread formula was adopted and a number of modifications were incorporated. These were done through analytical and experimental evaluations of the particular characteristics of the process. These included width, height and the roll-gap variation. The results of the experiments were contrasted against those predicted by the modified formula and good agreements were seen.

An important observation arising from this study was that the process constraints and the unsteady nature of the process played the key roles in planning the scheme. A computer algorithm has been developed which determines the number of rolling passes required and the specifications for each pass.

The flat rolling process, where formed-die rolling and die-forging are also applicable, can have a considerable cost advantage as it replaces the *forging hardware* with the *rolling software*. This implying that the same tooling set may be used for the production of an infinite number of different shapes.

This work was a first attempt in developing a computer-aided system for rolling parts with variable rectangular cross-section. More work, both *theoretically* and *experimentally*, in different aspects are suggested for the future work. The study on the geometric deformation of non-uniform blocks through variable roll gaps should be continued extensively. This is essential not only to evaluate further, the accuracy of the formula which was developed during this work, but also to modify and expand it for inclusion of the effects of the roll gap variation which was not completed in this study due to the laboratory constraints.

The limit of part complexity is not yet adequately known. An algorithmic approach may be developed to relate all the parameters in effect and to determine beforehand whether the production of a part with a given shape is feasible by this method.

Finally, the method can be extended to permit continuous operations. This can start with considerations relating to the design of machinery (and controls) and is an important and relatively difficult task to do.

Table 2-1 Value of correction factor as function of peripheral roll velocity in Bachtinov's formula(6)

v_r (m/sec.)	to2	3	4	5	6	7	8	9	10	12	14	16	18	20to22
k	1.	.9	.8	.72	.66	.6	.57	.55	.52	.47	.45	.43	.42	.41

Table 2-2 Summary of the average deviations of predicted spread from the experimental values in Wusatowski's studies

the average deviation from real values			spread formulae
-	+	total deviation	
%	%	%	
1.88	1.24	3.12	Ekelund
3.45	1.14	4.59	Wusatowski
5.32	0.50	5.82	Seible
4.84	2.19	7.03	Tafle & Sedlaczek
6.79	2.90	9.69	Trinks

Table 2-3 Effect of steel composition on spread ratio in Wusatowski's formula(20)

composition of steel						correctoin factor d	type of steel
C%	Si%	Mn%	Ni%	Cr%	W%		
0.06		0.22				1.00000	Basic Bessemer st.
0.20	0.20	0.50				1.02026	0.25% carbon steel
0.30	0.25	0.50				1.02338	0.35% carbon steel
1.04	0.30	0.45				1.00734	Tool steel
1.25	0.20	0.25				1.01454	Tool steel
0.35	0.50	0.60				1.01636	Manganese steel
1.00	0.30	1.50				1.01068	Manganese steel
0.50	1.70	0.70				1.01410	Spring steel
0.50	0.40	24.0				0.99741	Wear resistant st.
1.20	0.35	13.0				1.00887	Wear resistant st.
0.06	0.20	0.25	3.50	0.40		1.01034	Case hardening st.
1.30	0.25	0.30		0.50	1.80	1.00902	Alloy tool steel
0.40	1.90	0.60	2.00	0.30		1.02719	Alloy tool steel

Table 2-4 Values of the constants in Sparling's formula(18)

C	K	A	B	G
0.981	1.615	0.900	0.550	-0.250

Table 2-5 Comparison between three spread formulae in Sparling's experiments

formula	No. of tests	No. of tests with error more than 10%	15%
Hill	26	23	21
Wusatowski	26	13	8
Sparling	26	3	0
Hill	9	9	9
Wusatowski	9	5	2
Sparling	9	3	1

Table 2-6 Values of the constants in El-Kalay and Sparling's formula(18)

condition	A	B	C	D	E
smooth rolls					
light scales	0.851	1.766	0.643	0.386	-0.104
heavy scales	0.955	1.844	0.643	0.386	-0.104
rough rolls					
light scales	0.993	2.186	0.569	0.402	-0.123
heavy scales	0.980	2.105	0.569	0.402	-0.123

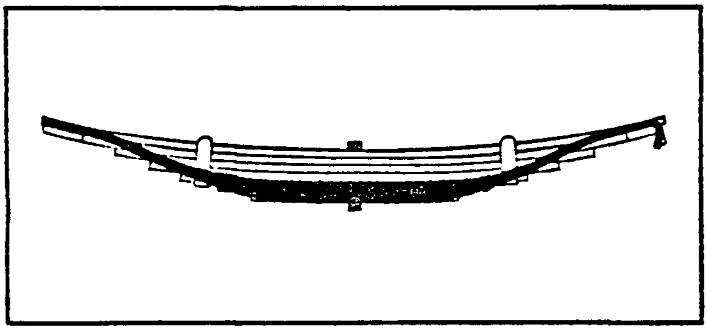


Figure 1-1 Taper leaf spring superimposed on multi-leaf spring of similar load bearing characteristics(32)

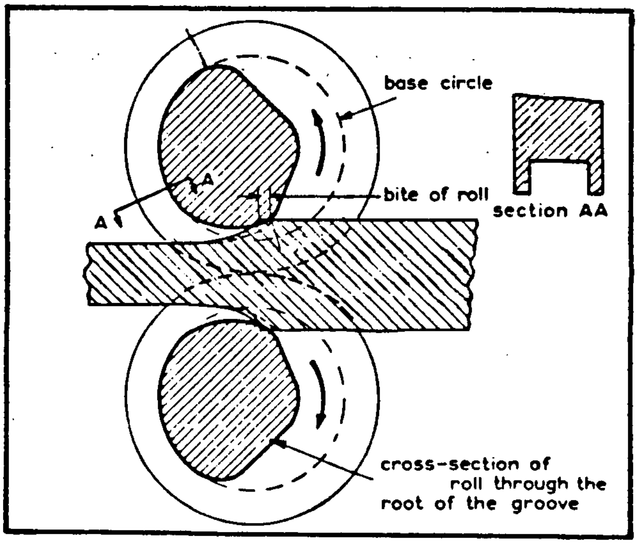
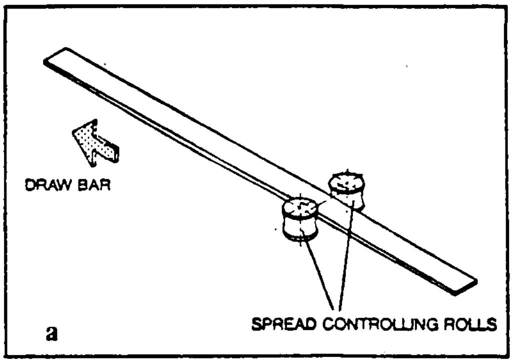


Figure 1-2 Schematic representation of an eccentric-die rolling



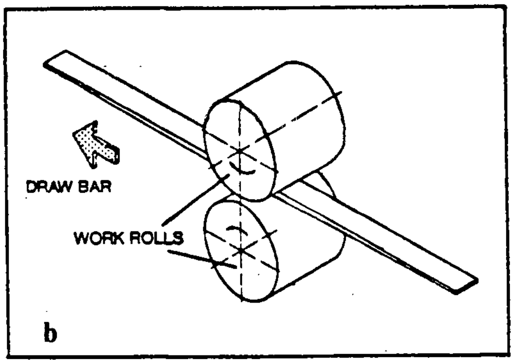


Figure 1-3 Sequence of flat rolling of taper leaf springs; (a) width forming, (b) height forming

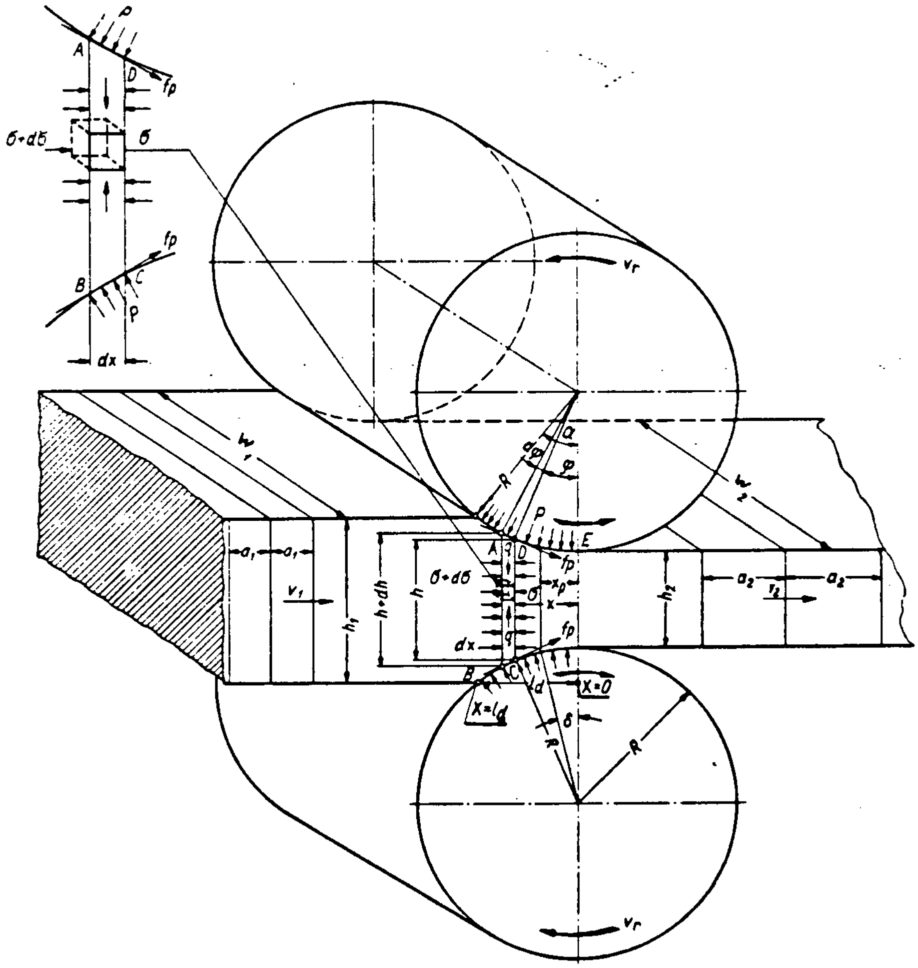


Figure 2-1 Schematic representation of a rolling process(6)

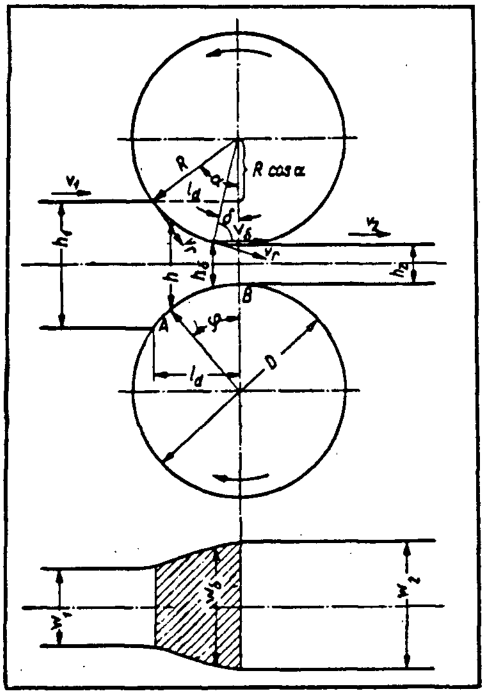


Figure 2-2 Velocity diagram in a rolling process(6)

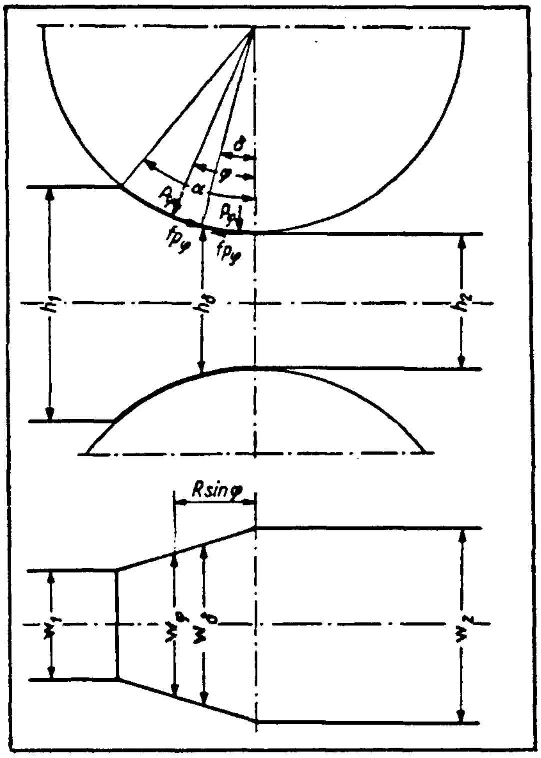


Figure 2-3 Typical variation of width in rolling(6)

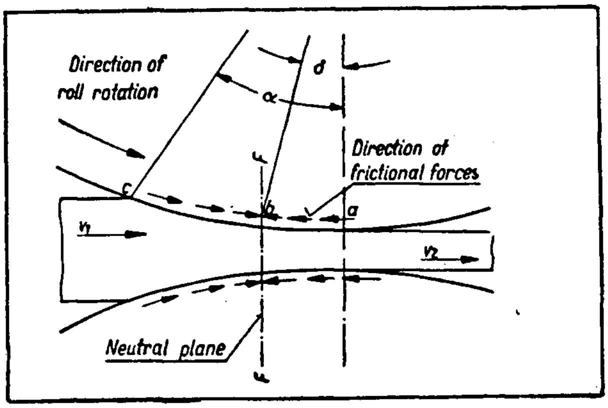


Figure 2-4 Distribution of frictional forces(6)

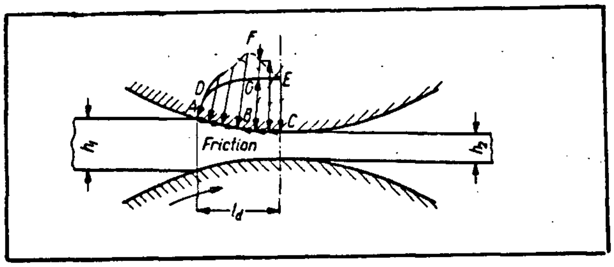


Figure 2-5 Pressure distribution along the arc of contact(6)

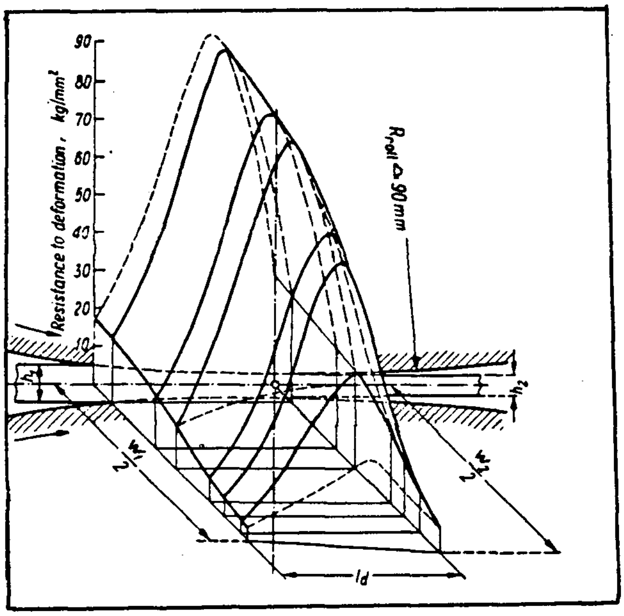


Figure 2-6 A typical example of true pressure distribution in the roll gap over a half width of the rolled stock(6)

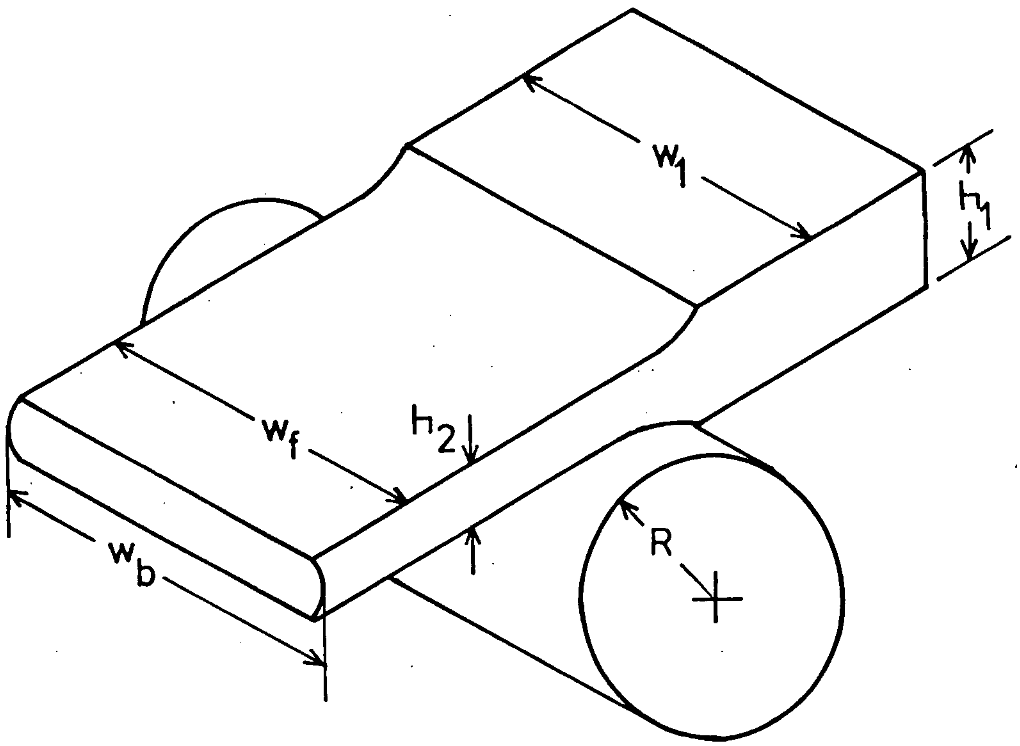


Figure 2-7 Schematic representation of geometric deformation in rolling(32)

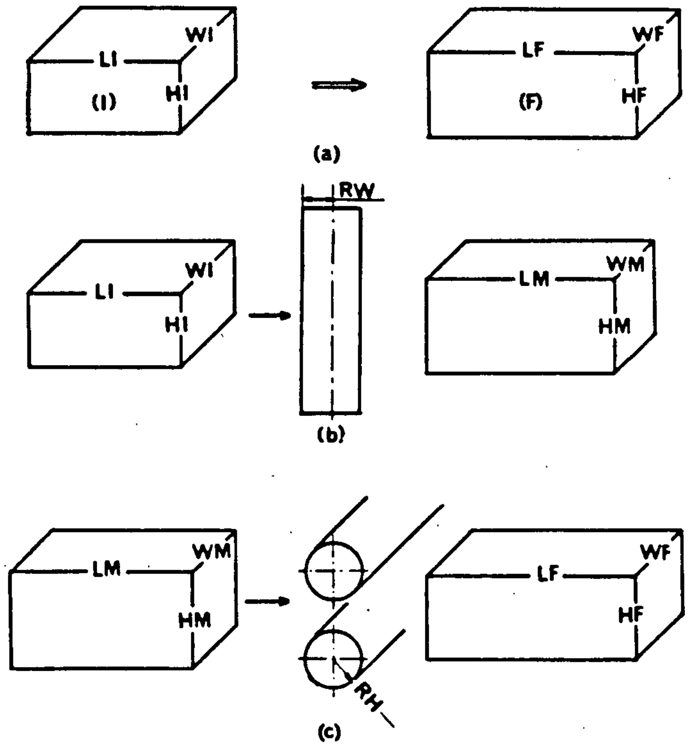


Figure 3-1 Forming a uniform block;
(a) initial and final dimensions
(b) pass one
(c) pass two

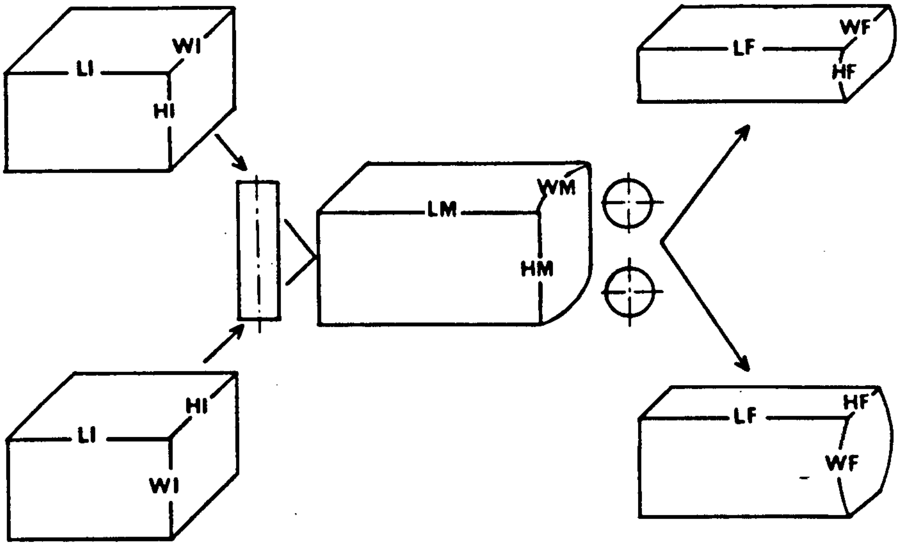


Figure 3-2 Different sequences of rolling

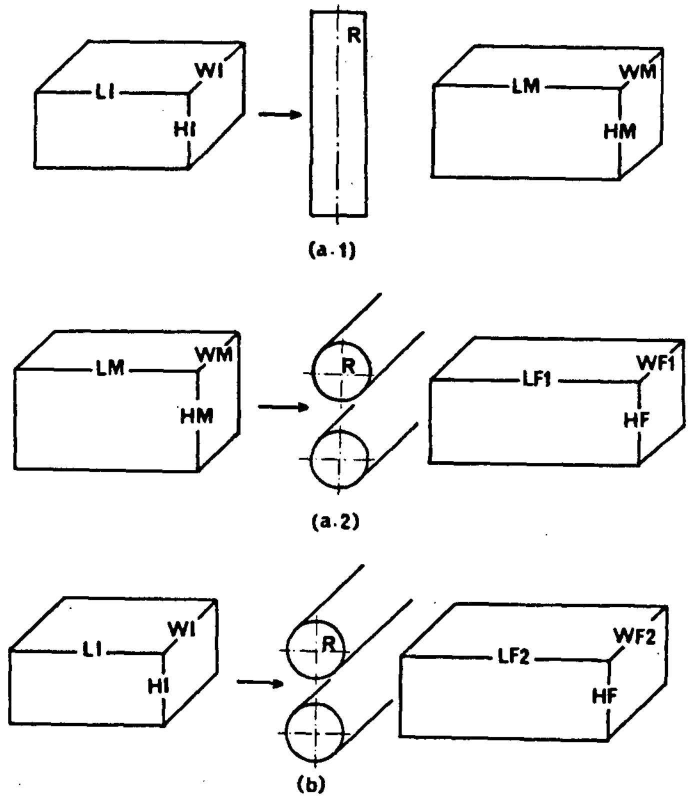


Figure 3-3 Comparison between values of spread
in two types of rolling;
(a) double pass
(b) single pass

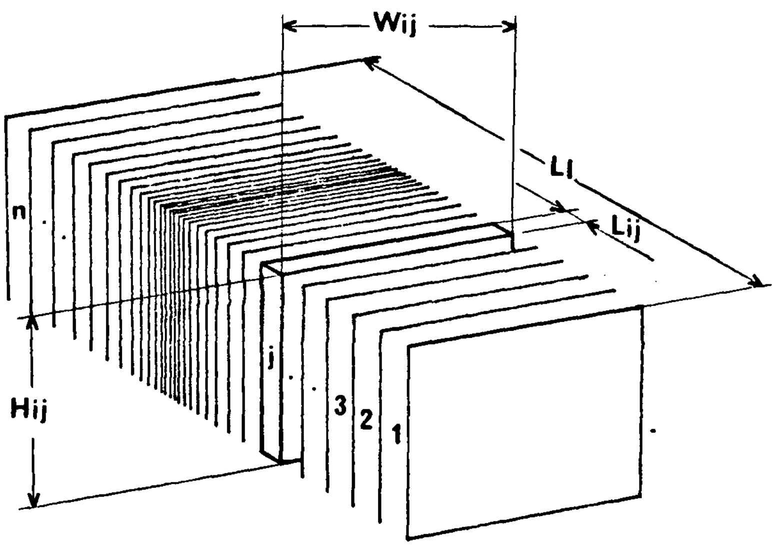


Figure 3-4 Typical example of a uniform initial block

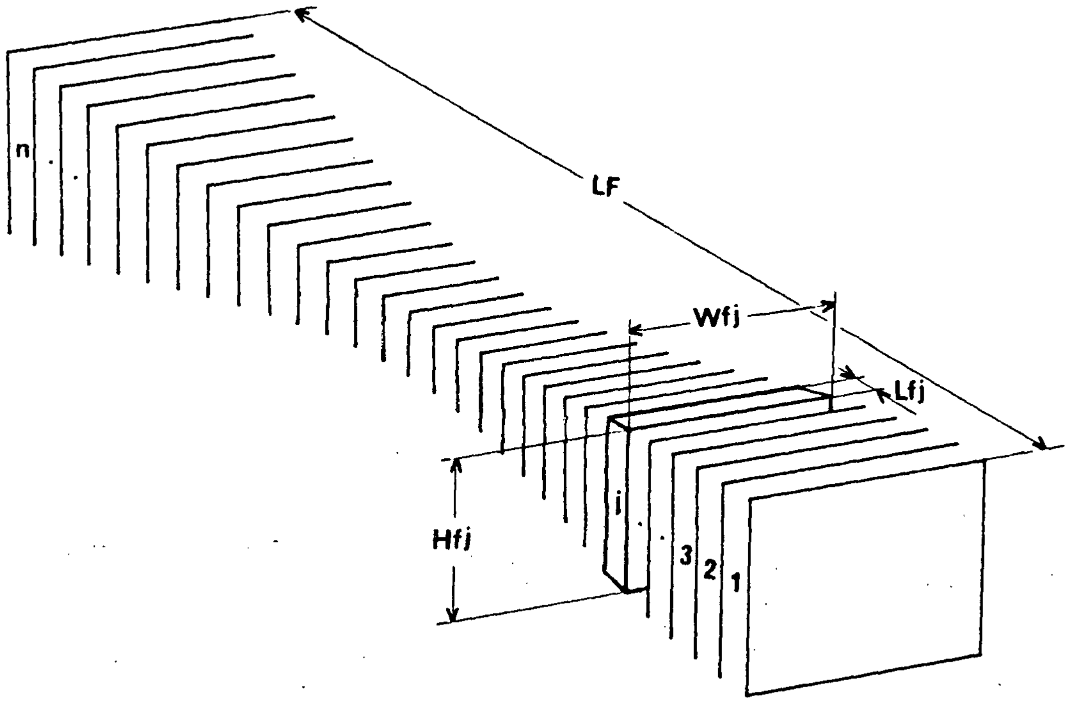


Figure 3-5 Typical example of a non-uniform final part

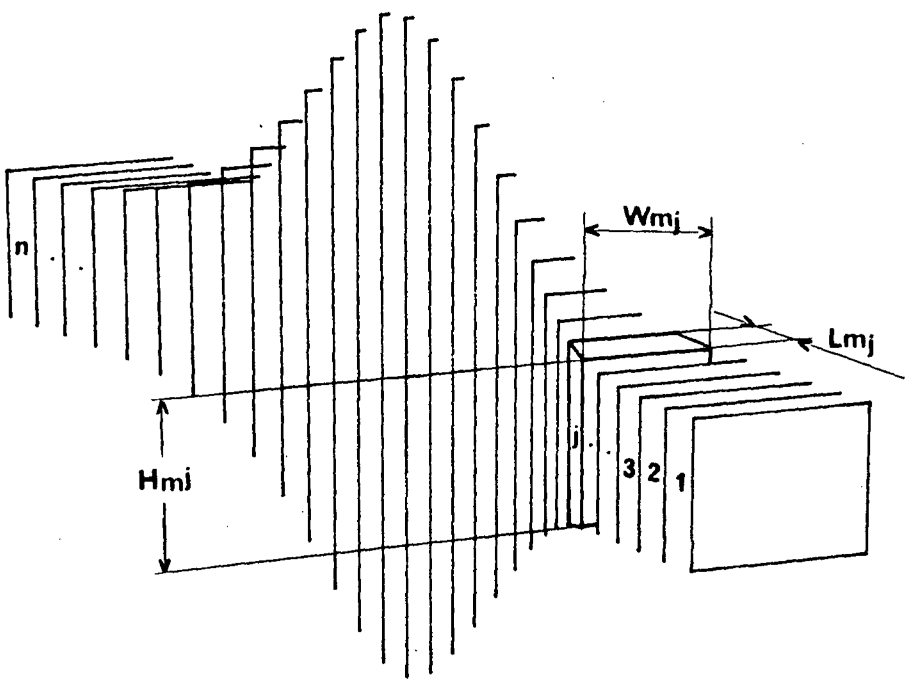


Figure 3-6 Exaggerated represetation of an intermediate state

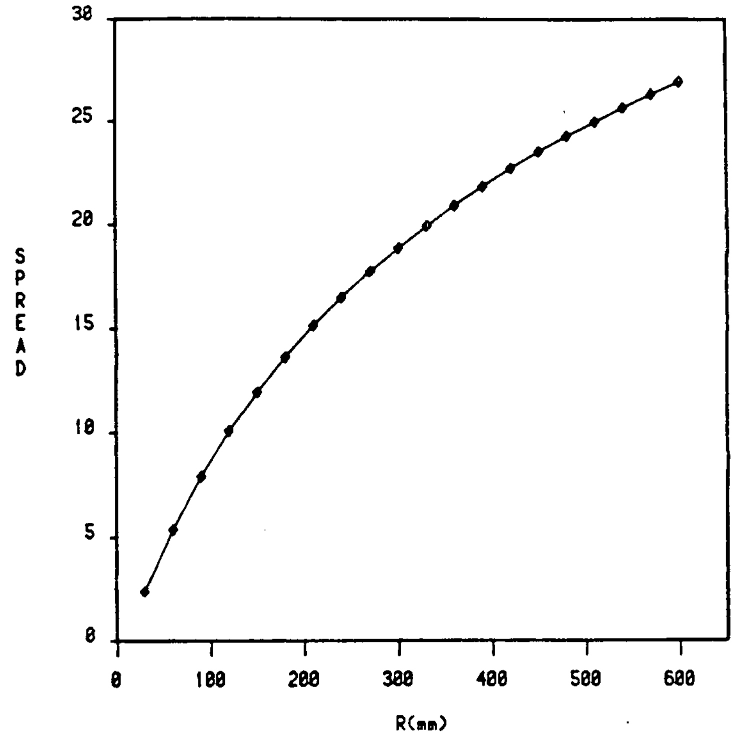


Figure 3-7 Typical variation of spread versus the roll's radius for a constant draft

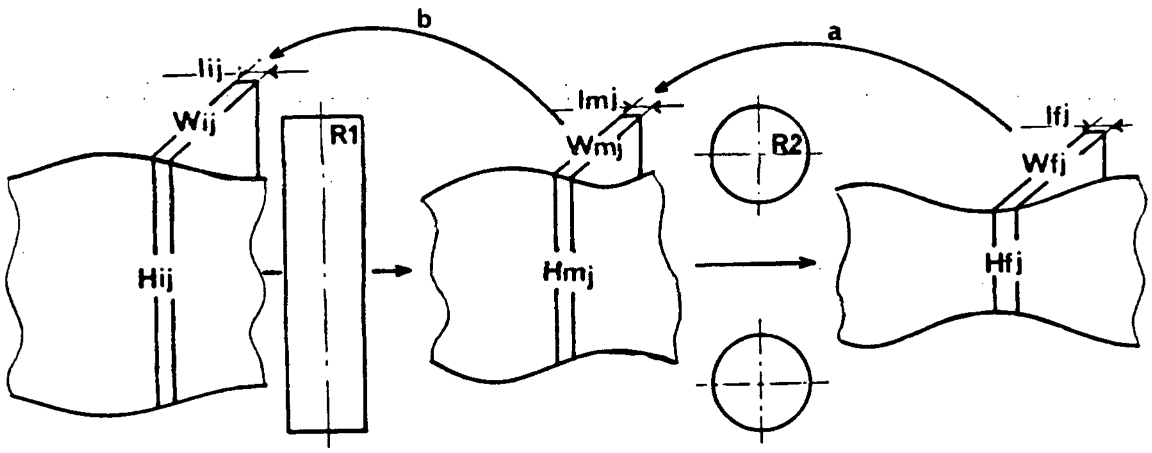


Figure 3-8 Schematic representation of the inverse calculation method

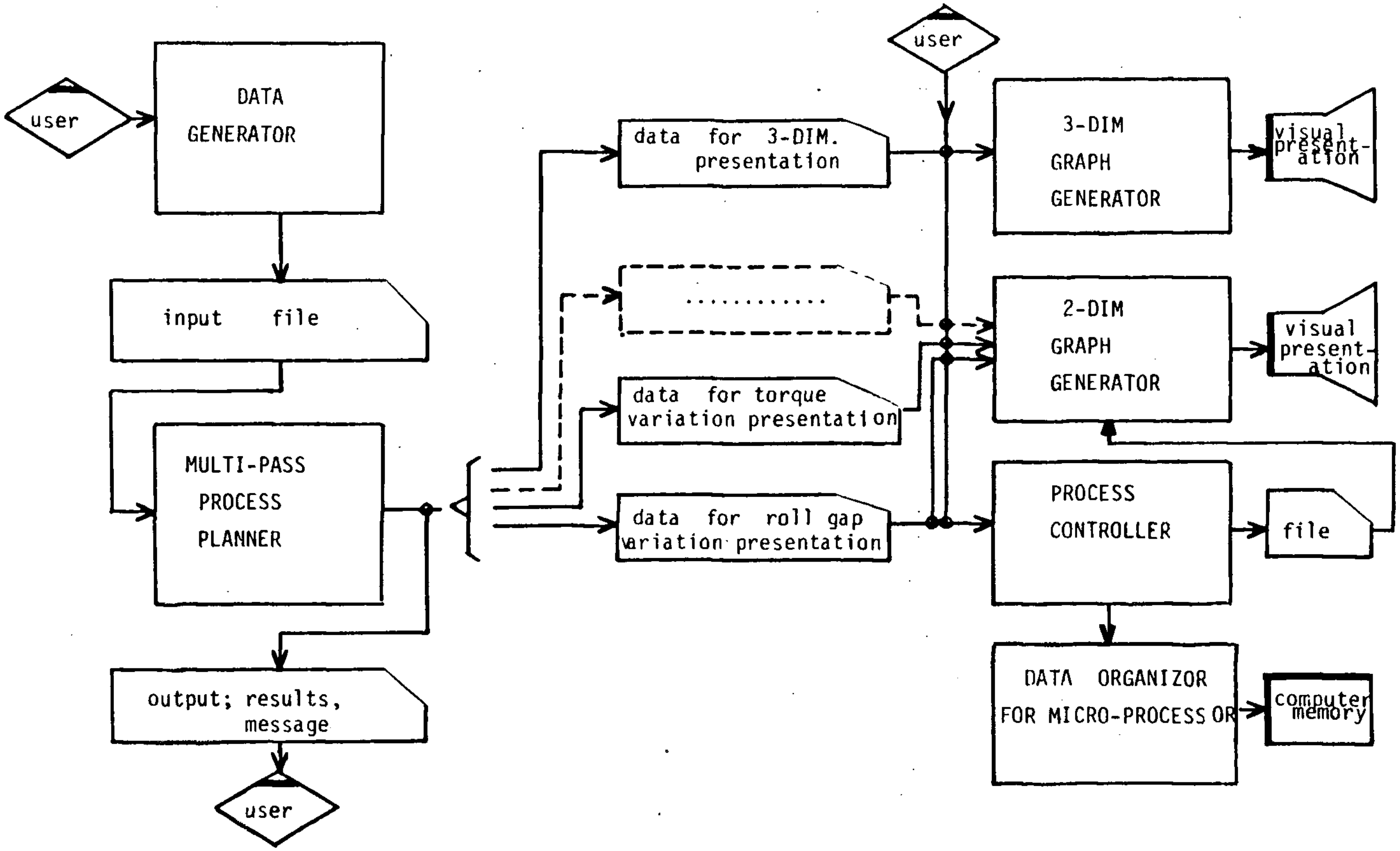


Figure 3-9 General flow-diagram of the computer software

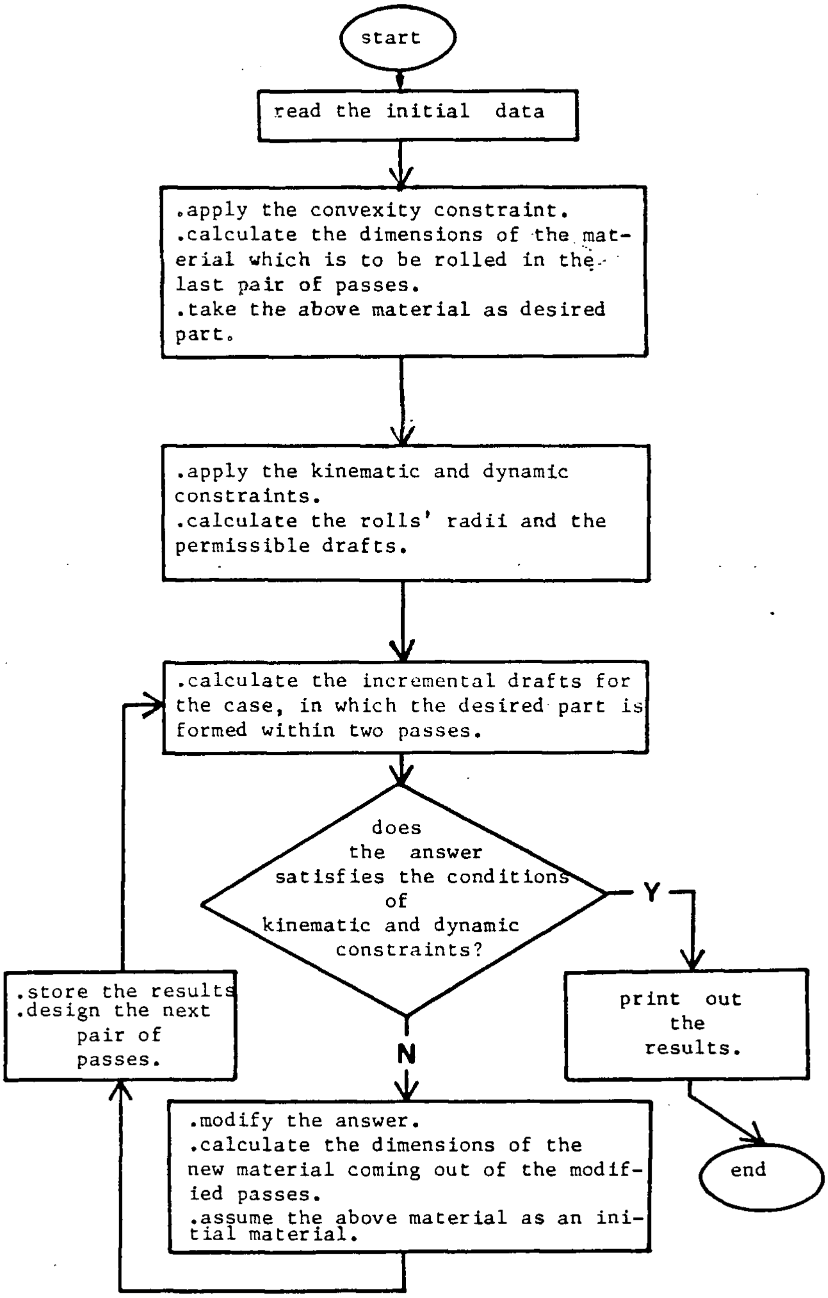


Figure 3-10 General flow-chart of the multi-pass design routine

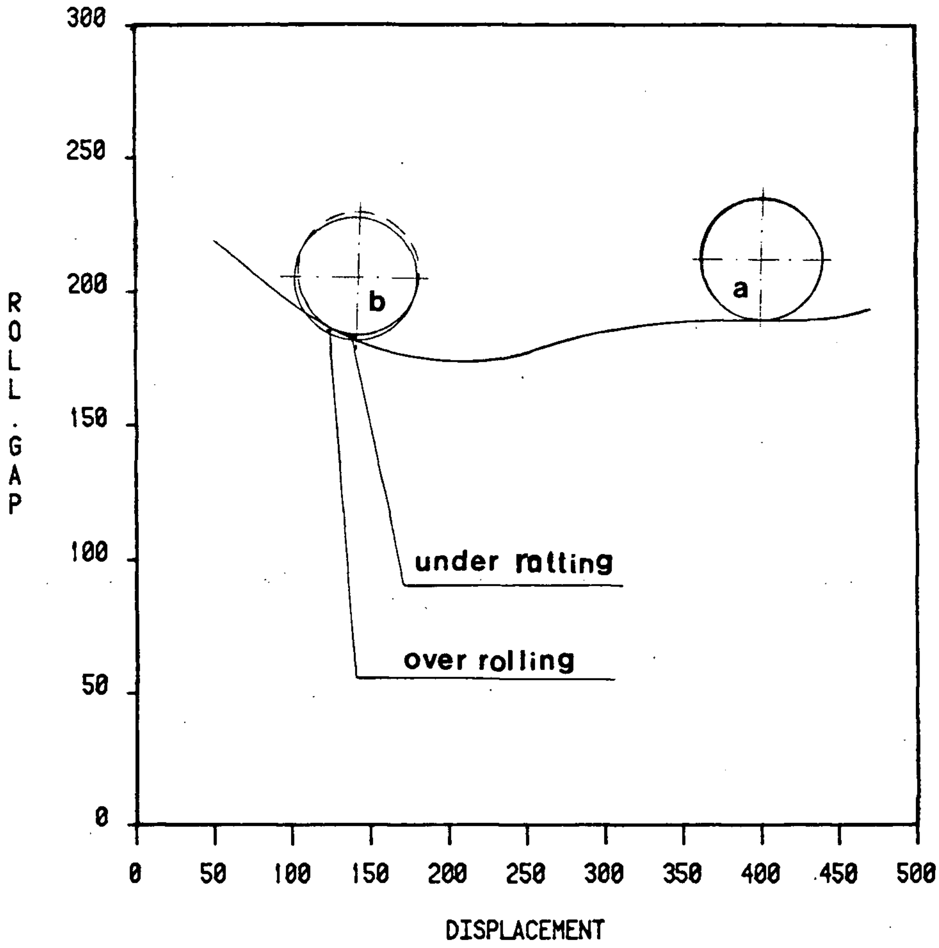


Figure 3-11 An exaggerated representation of roll interference

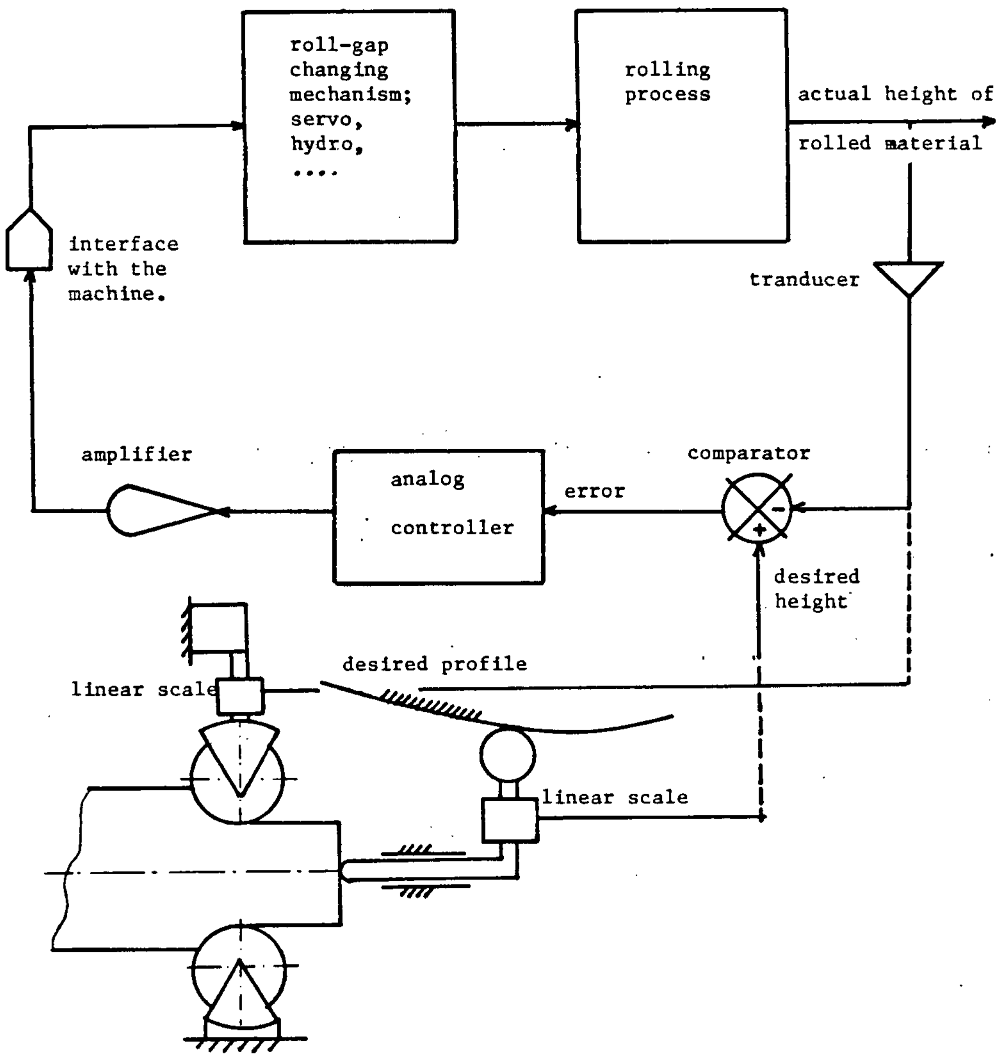


Figure 3-12 Block diagram of an analogue control system

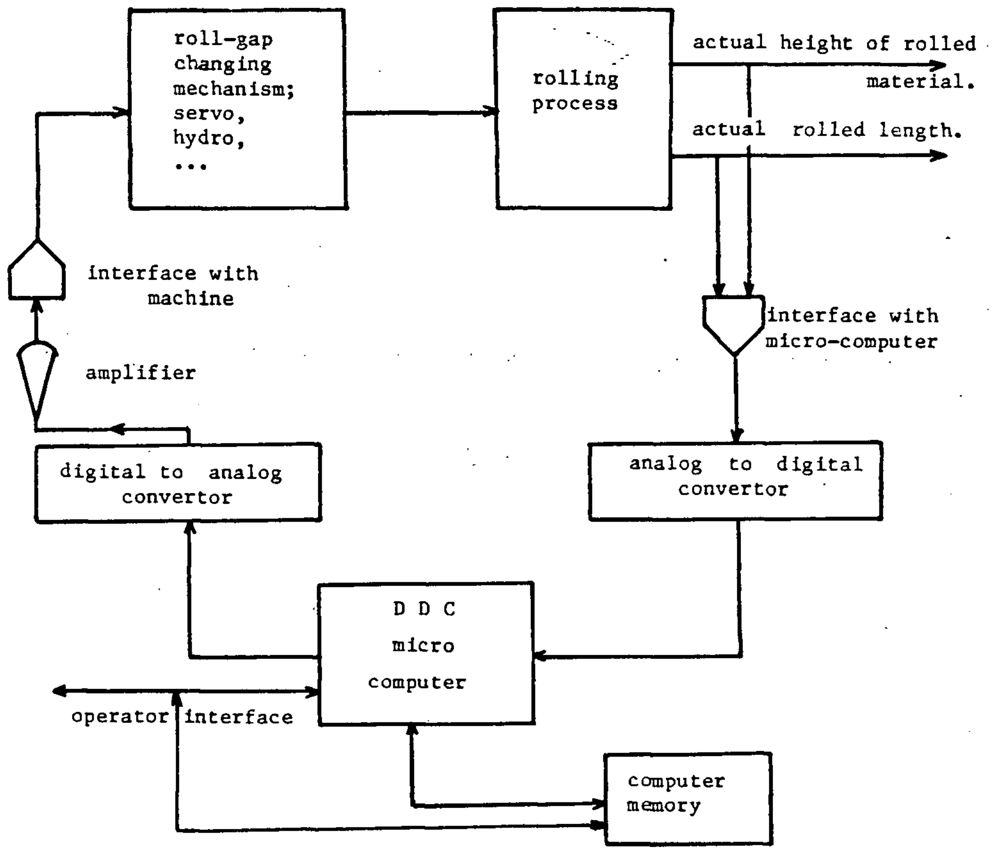
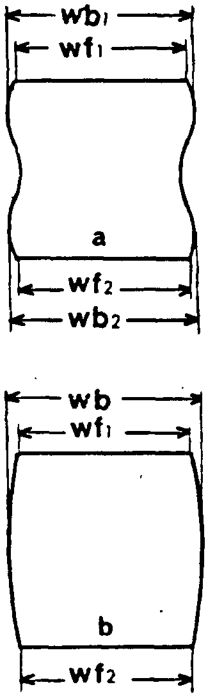


Figure 3-13 Block diagram of a D.D.C. system



$w_{mean}=w_b-1/3(w_b-w_f)$

$w_b=(w_{b1}+w_{b2})/2$

$w_f=(w_{f1}+w_{f2})/2$

$w_{mean}=w_b-1/3(w_b-w_f)$

$w_f=(w_{f1}+w_{f2})/2$

Figure 4-1 Two different modes of deformation for rolled steel stocks;
(a) double bulged cross-section
(b) rhombic cross-section

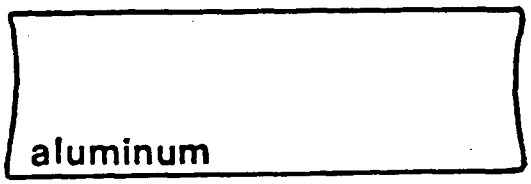


Figure 4-2 A possible mode of deformation for aluminum stocks

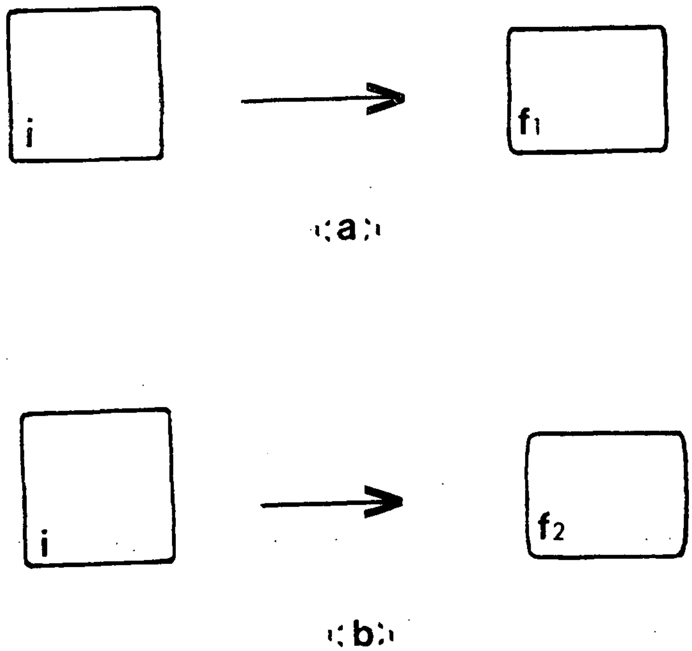
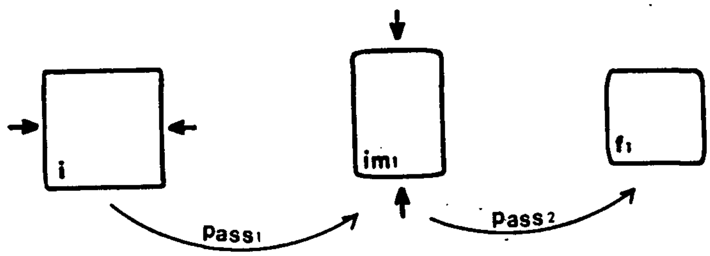
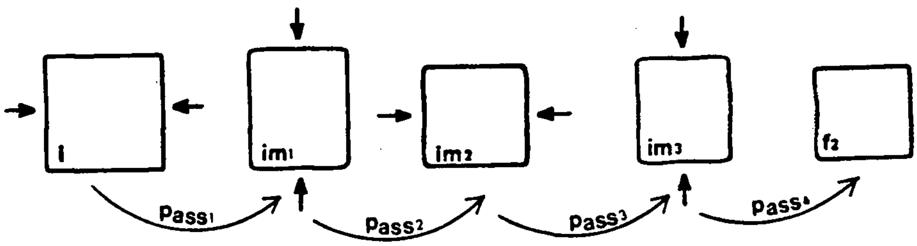


Figure 4-3 Comparison between two cross-sectional views of a material, under different rolling procedures;
(a) two pass process, total draft=3.55mm
(b) one pass process, " " "



"a"

state of material	width,height(mm)
initial	25.40,25.40
intermediate	19.05,26.85
final	20.30,20.05



"b"

state of material	width,height(mm)
initial	25.40,25.40
1st intermediate	21.30,26.18
2nd intermediate	21.80,21.95
3rd intermediate	19.80,22.30
final	20.13,20.05

Figure 4-4 Deformation of a uniform block within:
(a) two passes
(b) four passes

```
INITIAL DIMENSIONS OF MATERIAL:
   WIDTH=25.40000
   HEIGHT=25.40000
   LENGTH=40.00000
.....................................
FINAL DIMENSIONS OF THE PRODUCT:
   WIDTH=20.00000
   HEIGHT=20.00000
   LENGTH=64.51600
.....................................
RADIUS OF ROLLERS:
   IN THE FIRST PASS=50.80000
   IN THE SECOND PASS=50.80000
.....................................
RESULTS;
..................
CASE 1
     FIRST PASS=WIDTH REDUCTION
     SECOND PASS=HEIGHT REDUCTION
     CONVEXITY ON THE WIDTH OF THE FINAL PRODUCT
......................................

     DIMENSIONS OF THE INTERMEDIATE CROSS SECTION:
          WIDTH=18.89746
          HEIGHT=26.54404
          LENGTH=51.44663
     FIRST PASS ROLL GAP=19.17714
     SECOND PASS   ROLL GAP=20.29600
     REDUCTION IN FIRST PASS(%)=25.60055
     REDUCTION INSECOND PASS(%)=24.65351
.............



........................................END OF EXECUTIONS
```

Figure 4-5 Typical output of the computer program, two pass deformation of a uniform block(dimensions in mm)

```
                  INITIAL DATA
                  """"""""""
    INITIAL DIMENSIONS OF MATERIAL:
       WIDTH=25.40000
       HEIGHT=25.40000
       LENGTH=40.00000
.............................
    FINAL DIMENSIONS OF THE PRODUCT:
       WIDTH=20.00000
       HEIGHT=20.00000
       LENGTH=64.51600
.............................
     KINEMATIC AND DYNAMIC CONSTRAINT:C1=0.2000,C2=300000.000
     REDUCTION FOR THE LAST PAIR OF PASSES:1.ST PASS=2.000,2.ND PASS=2.000
.............................
                    RESULTS;
...........................................................................

    CASE 1**********
         VERY FIRST PASS=WIDTH REDUCTION
         NEXT PASS=HEIGHT REDUCTION .etc.

""""""""""""""""""""""""""""""""""""""""""""""""""

  ***DIMENSIONS OF MATR. COMING OUT OF THE 1.ST PASS:
         WIDTH=21.07888
         HEIGHT=26.04848
         LENGTH=46.99996

  ***DIMENSIONS OF MATR. COMING OUT OF THE 2.ND PASS:
         WIDTH=21.72186
         HEIGHT=21.92156
         LENGTH=54.19496

   *FIRST PASS ROLL GAP=21.39084
    SECOND PASS ROLL GAP=22.24599

   *DRAFT IN 1.ST PASS=4.32112
    DRAFT IN 2.ND PASS=4.12692

   *RADIUS OF ROLLERS IN THIS PAIR OF PASSES
       IN THE FIRST PASS=50.80000
       IN THE SECOND PASS=50.80000

      ¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬

  ***DIMENSIONS OF MATR. COMING OUT OF THE 1.ST PASS:
         WIDTH=19.72186
         HEIGHT=22.00000
         LENGTH=59.47808

  ***DIMENSIONS OF MATR. COMING OUT OF THE 2.ND PASS:
         WIDTH=20.00000
         HEIGHT=20.00000
         LENGTH=64.51600

   *FIRST PASS ROLL GAP=20.013740
    SECOND PASS ROLL GAP=20.29600

   *DRAFT IN 1.ST PASS=2.00000
    DRAFT IN 2.ND PASS=2.00000

   *RADIUS OF ROLLERS IN THIS PAIR OF PASSES
       IN THE FIRST PASS=50.80000
       IN THE SECOND PASS=50.80000

      ¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬¬

    :::::::::

    FINAL PRODUCT IS ACHIEVED AT THIS STAGE

    NO; OF PAIR OF PASSES:    2

    aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
    """"""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""""



    ...............................................     END OF THE EXECUTION
```

Figure 4-6 Typical output of the computer program, deformation of a uniform block, considering the process constraints (dimensions in mm)

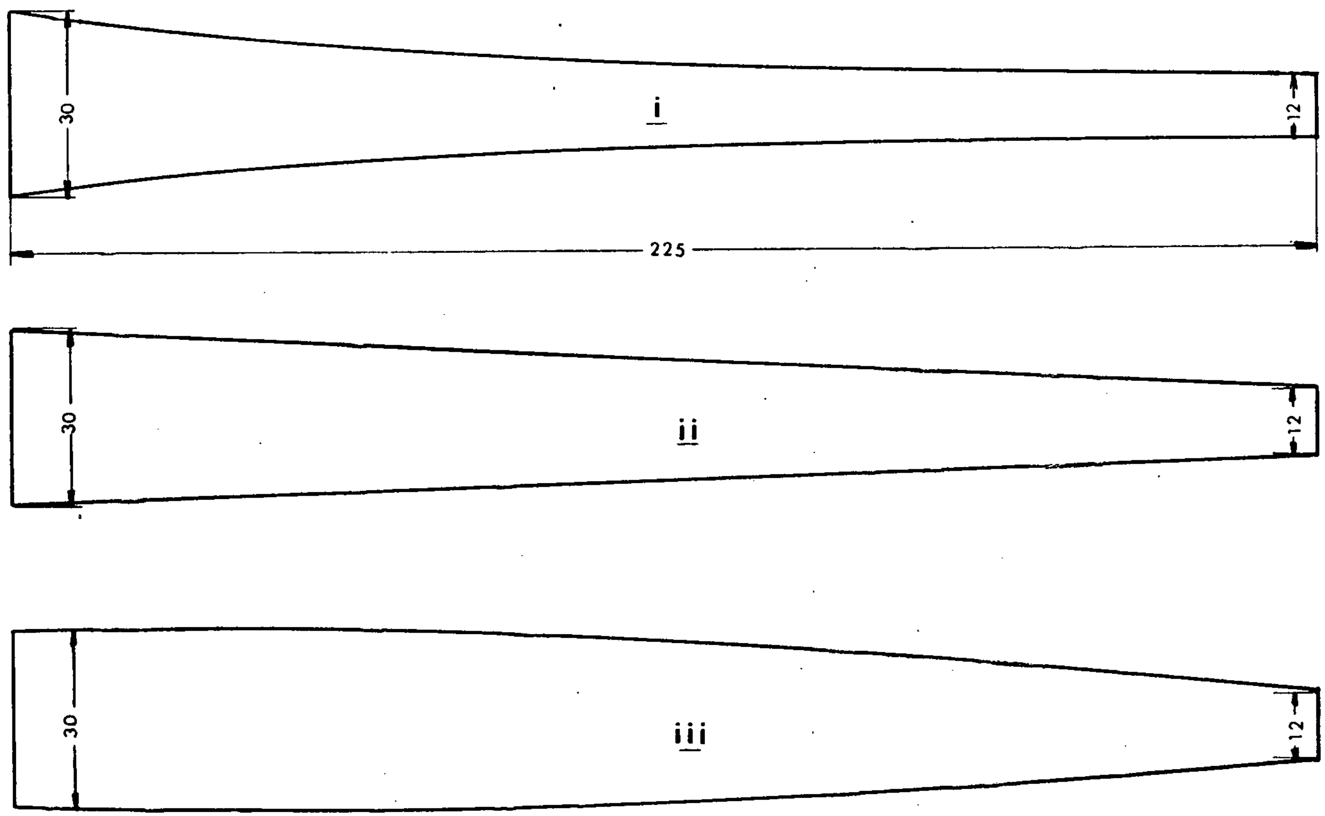


Figure 4-7 Profiles of the three experimental steel specimens(dimensions in mm)

(i) concave

(ii) linear

(iii) convex

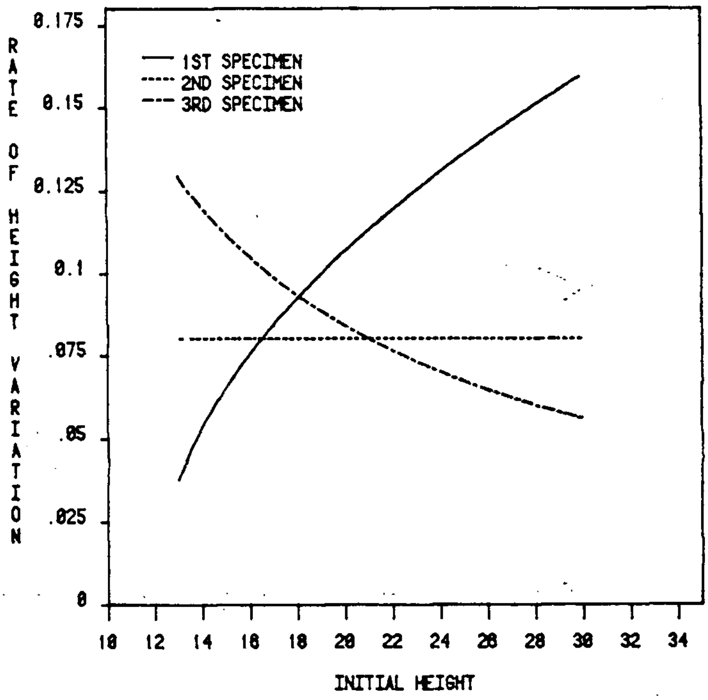


Figure 4-8 Rate of height variation for three different experimental specimens

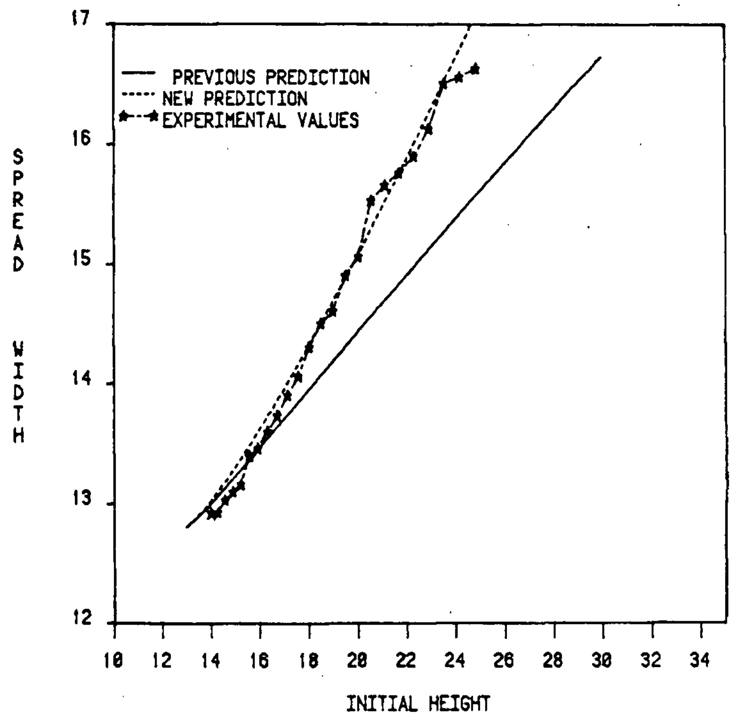


Figure 4-9 Comparison of experimental results, showing both the previous and new prediction of spread (specimen i)

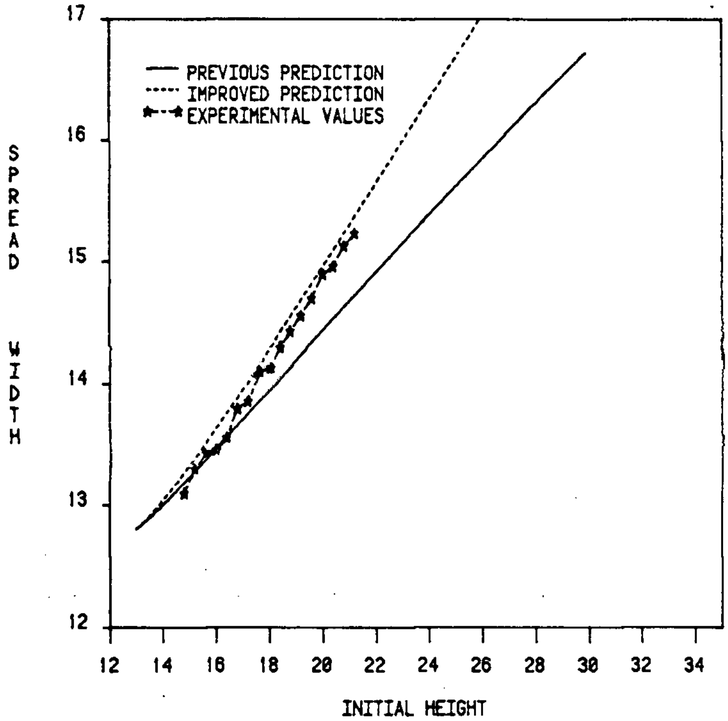


Figure 4-10 Comparison of experimental results, showing both the previous and new prediction of spread (specimen ii)

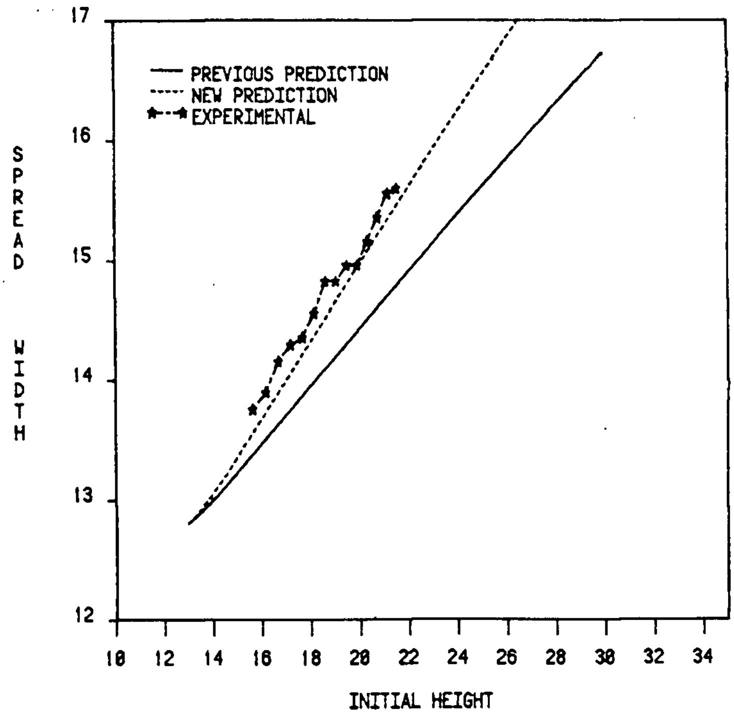


Figure 4-11 Comparison of experimental results, showing both the previous and new prediction of spread (specimen iii)

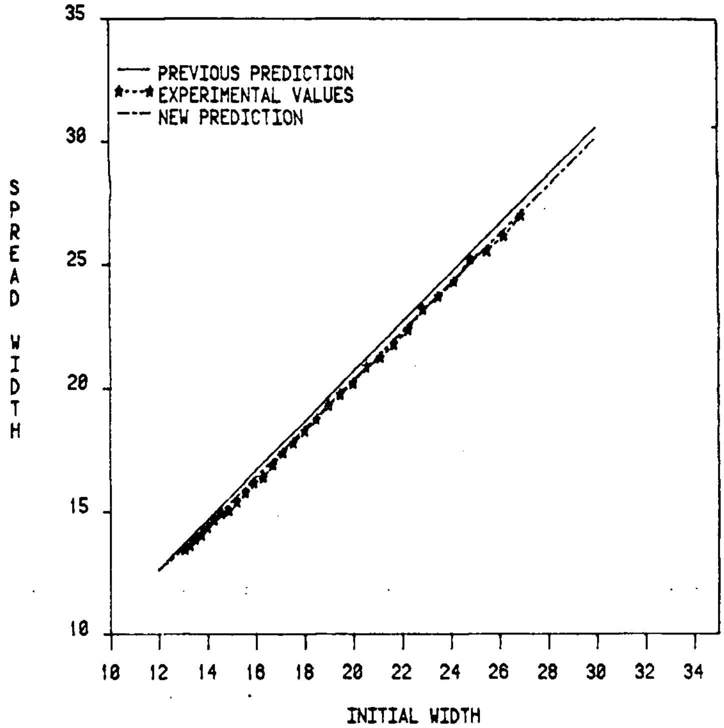


Figure 4-12 Comparison of experimental and predicted values of spread, specimen i, width increasing

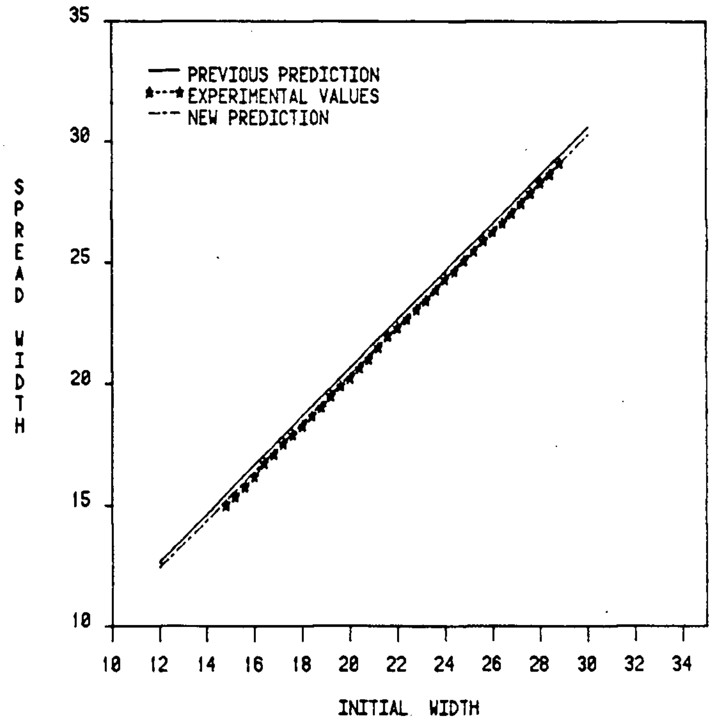


Figure 4-13 Comparison of experimental and predicted values of spread, specimen ii, width increasing

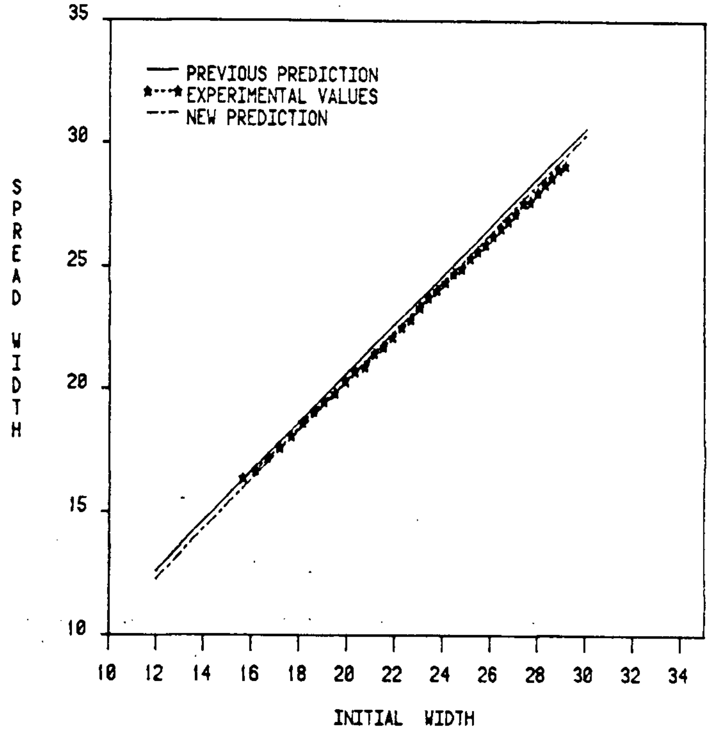


Figure 4-14 Comparison of experimental and calculated values of spread, specimen iii, width increasing

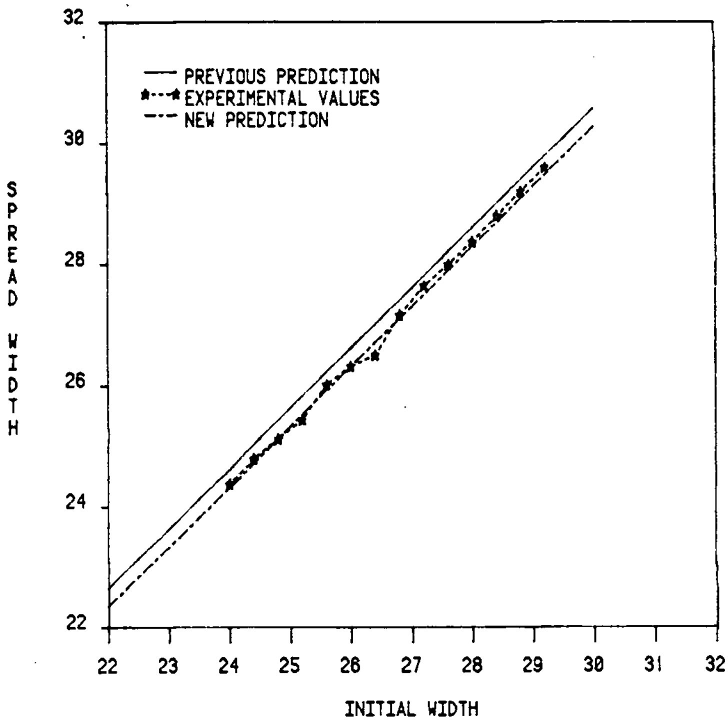


Figure 4-15 Comparison of experimental and calculated values of spread, specimen ii, width decreasing

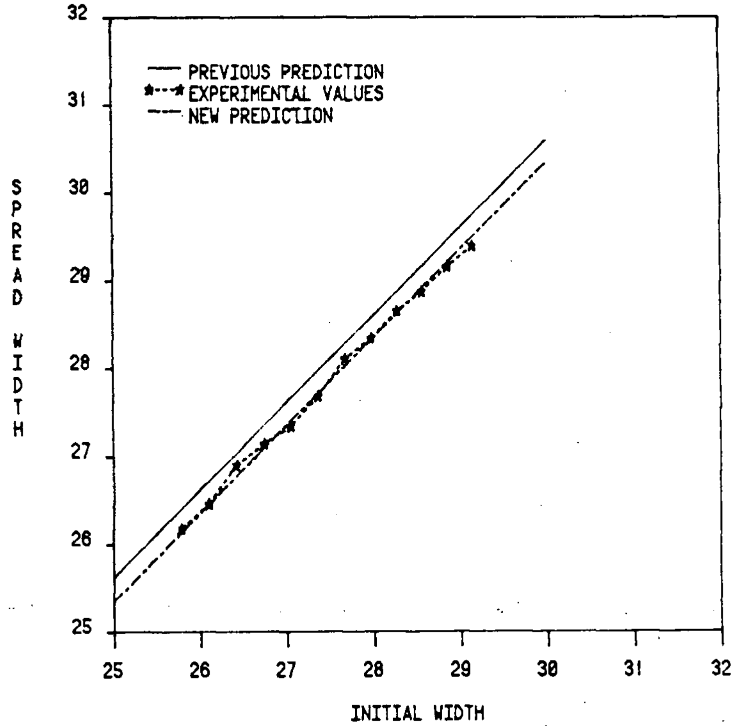


Figure 4-16 Comparison of experimental and calculated values of spread, specimen iii, width decreasing

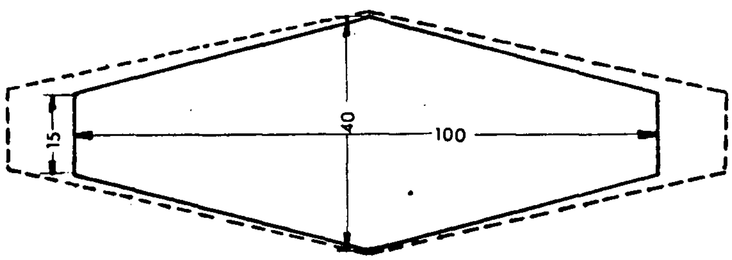


Figure 4-17 The geometry of the constant height, variable width aluminum specimen, before and after rolling (dimensions in mm)

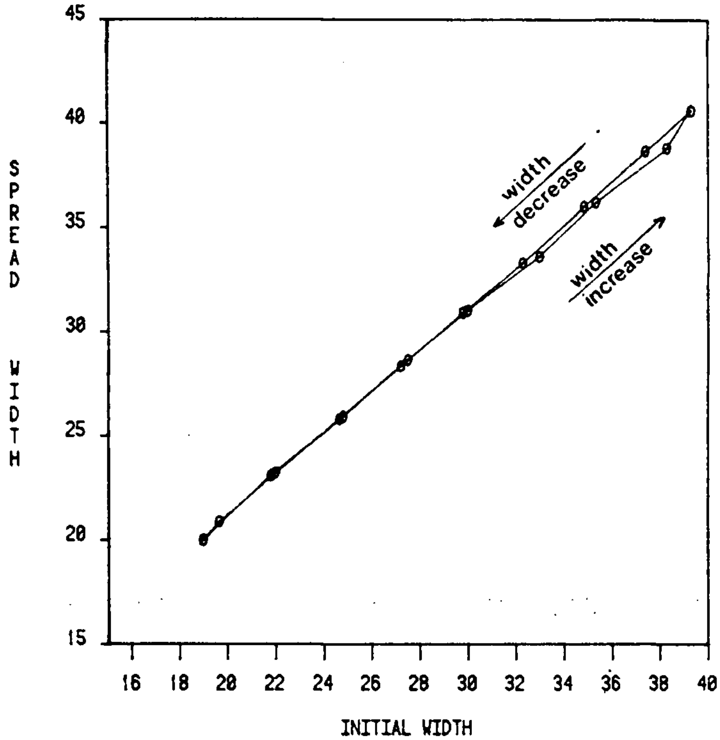


Figure 4-18 Comparison of values of spread for two different modes of width variation in the aluminum specimen

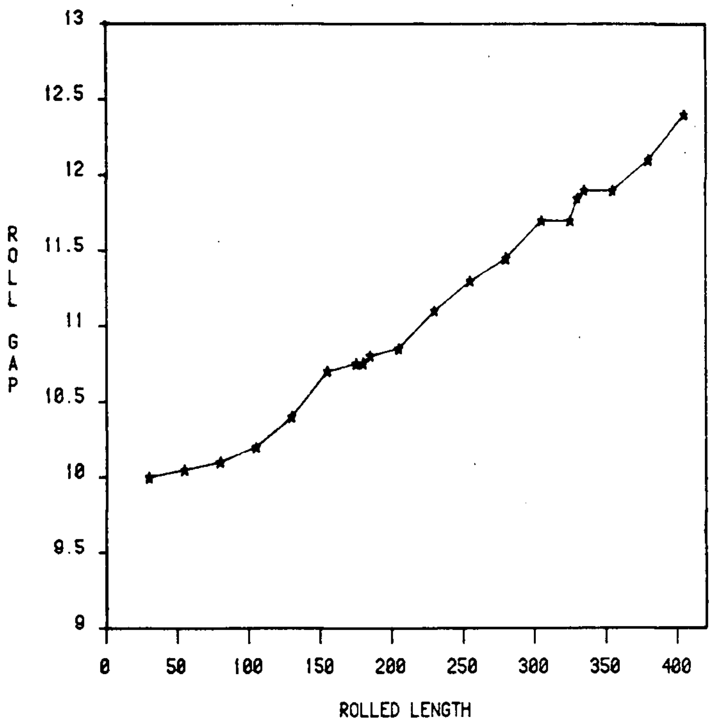


Figure 4-19 Variation of roll gap(mm) versus the rolled length in a typical closing roll gap process

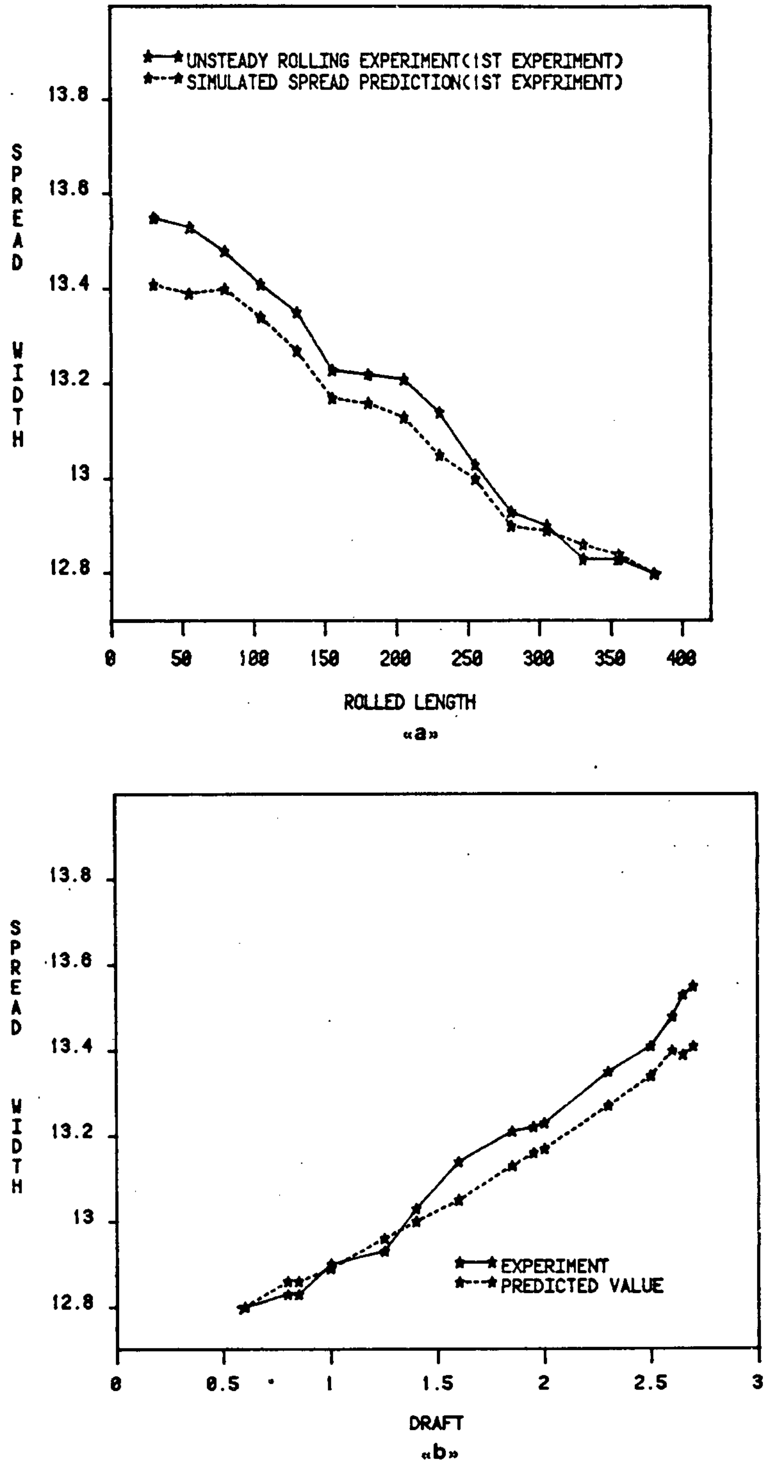


Figure 4-20 Comparison of experimental and calculated values of spread(mm) for a uniform block in a closing roll gap process; (a) versus rolled length
(b) versus draft

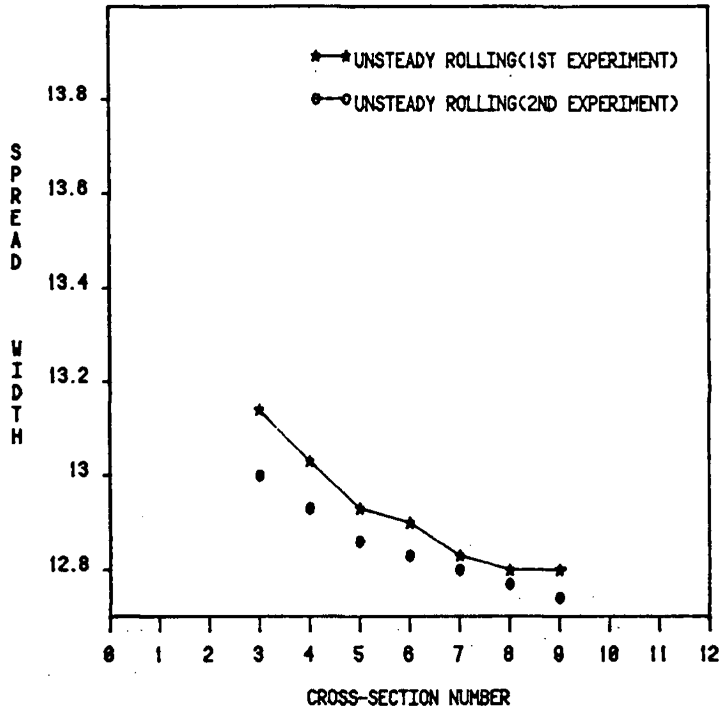


Figure 4-21 Comparison of spread for two similar specimens under different rates of roll gap closing

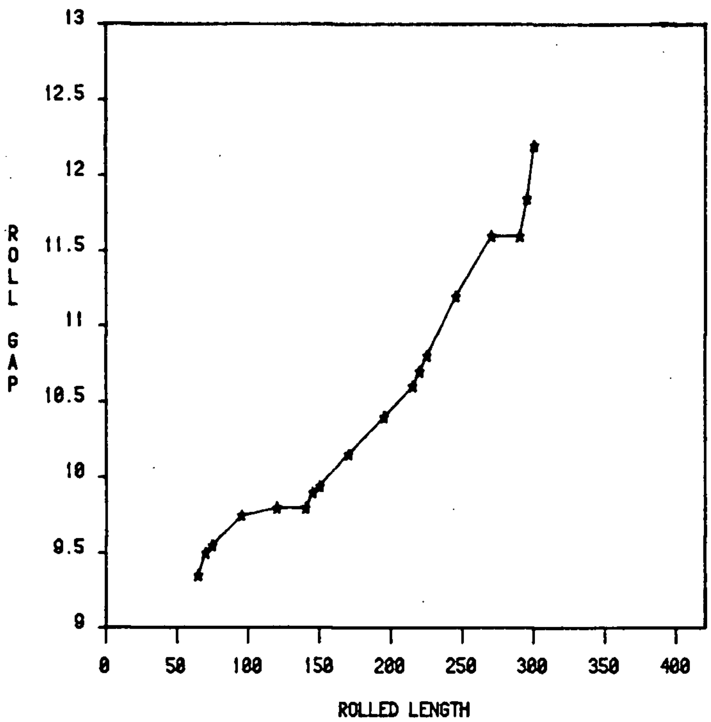


Figure 4-22 Variation of roll gap(mm) versus rolled length(mm) in an opening roll gap process

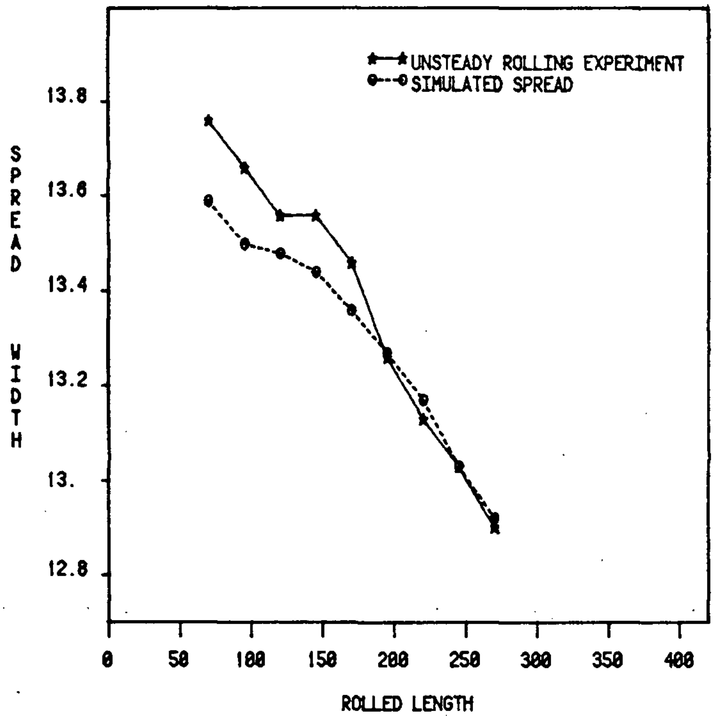


Figure 4-23 Comparison of experimental and calculated spread(mm) for a uniform block in an opening roll gap process

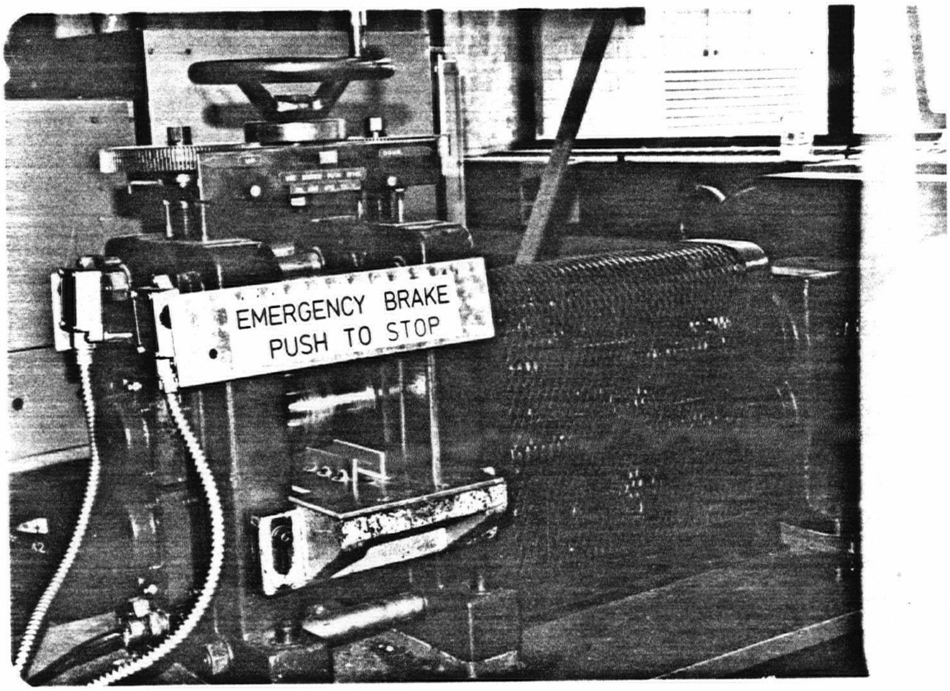


Figure 4-24 Appearance of the experimental rolling mill

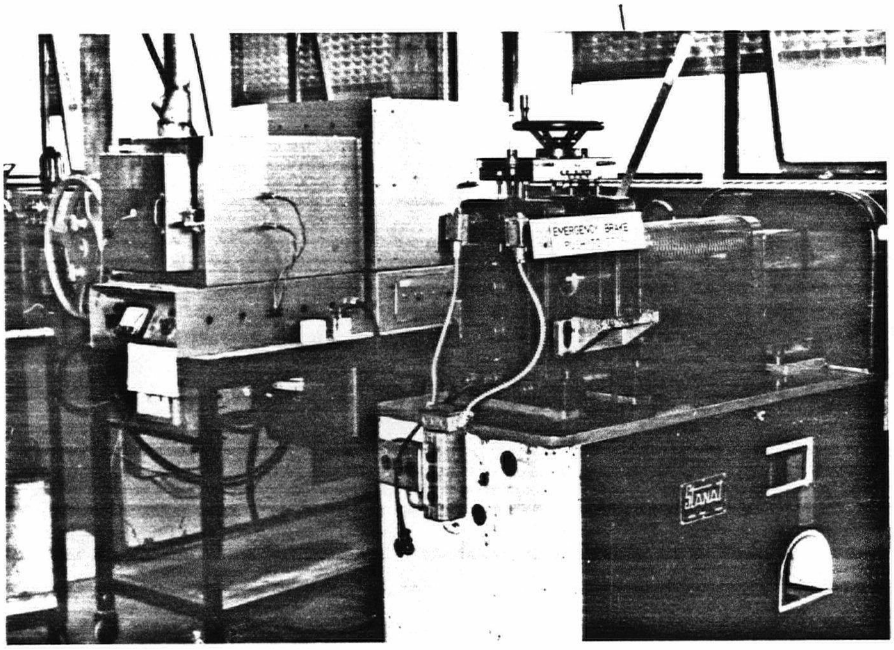


Figure 4-25 Position of the heating furnace with respect to the rolling mill

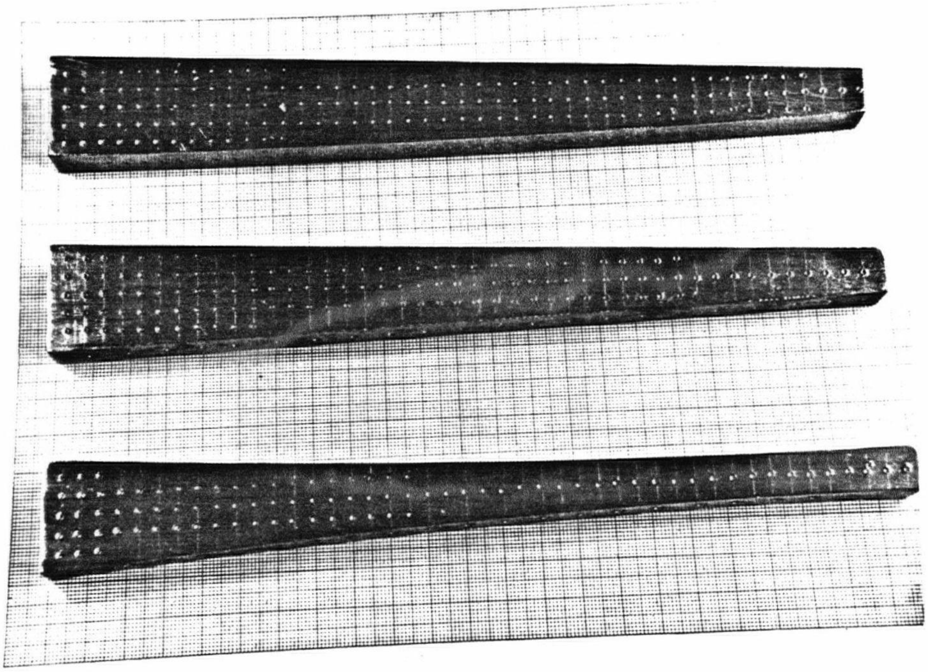


Figure 4-26 The steel specimens used to simulate the conditions of unsteady rolling

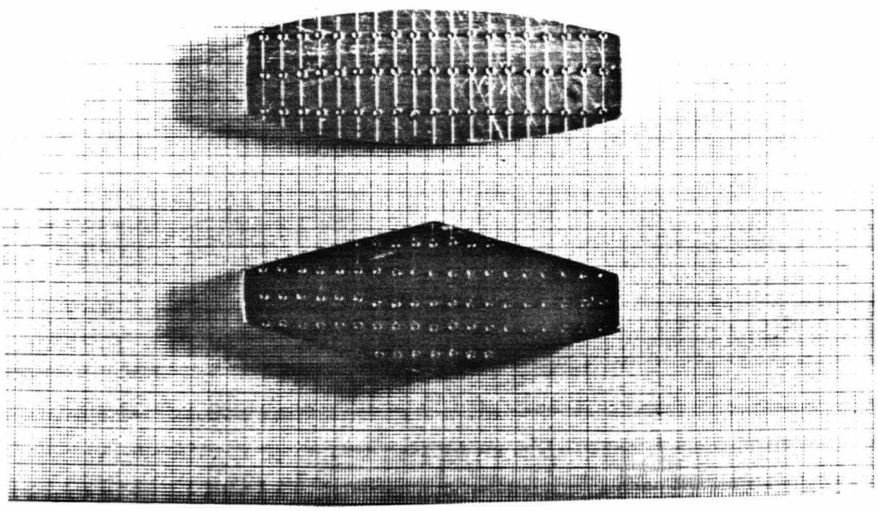


Figure 4-27 The aluminum specimens used for the critical experiments

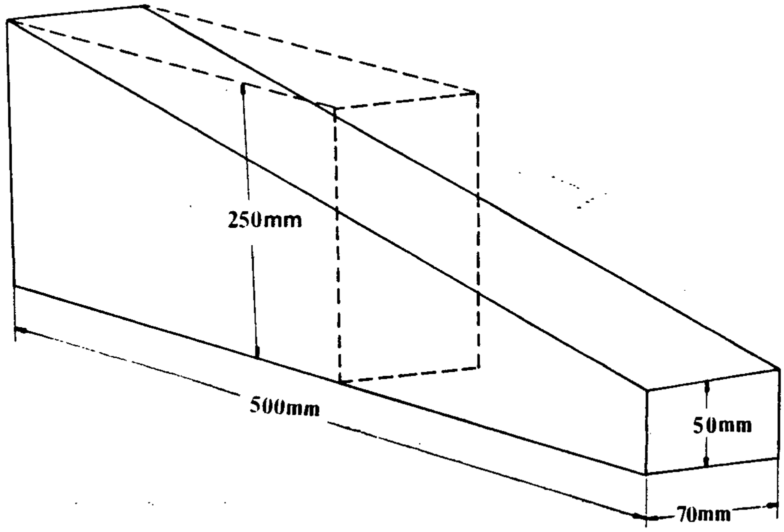


Figure 5-1 A part with constant width and linearly variable height

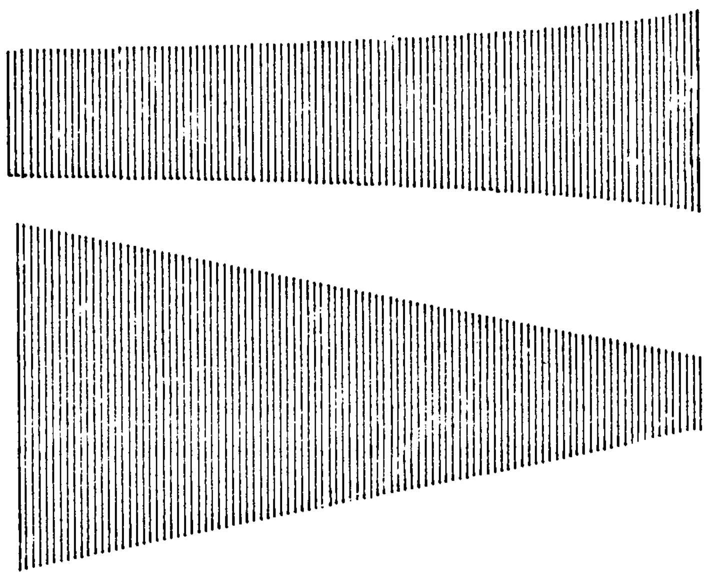


Figure 5-2 Two dimensional views of a uniform material, after being rolled in an operation with linearly variable roll gap

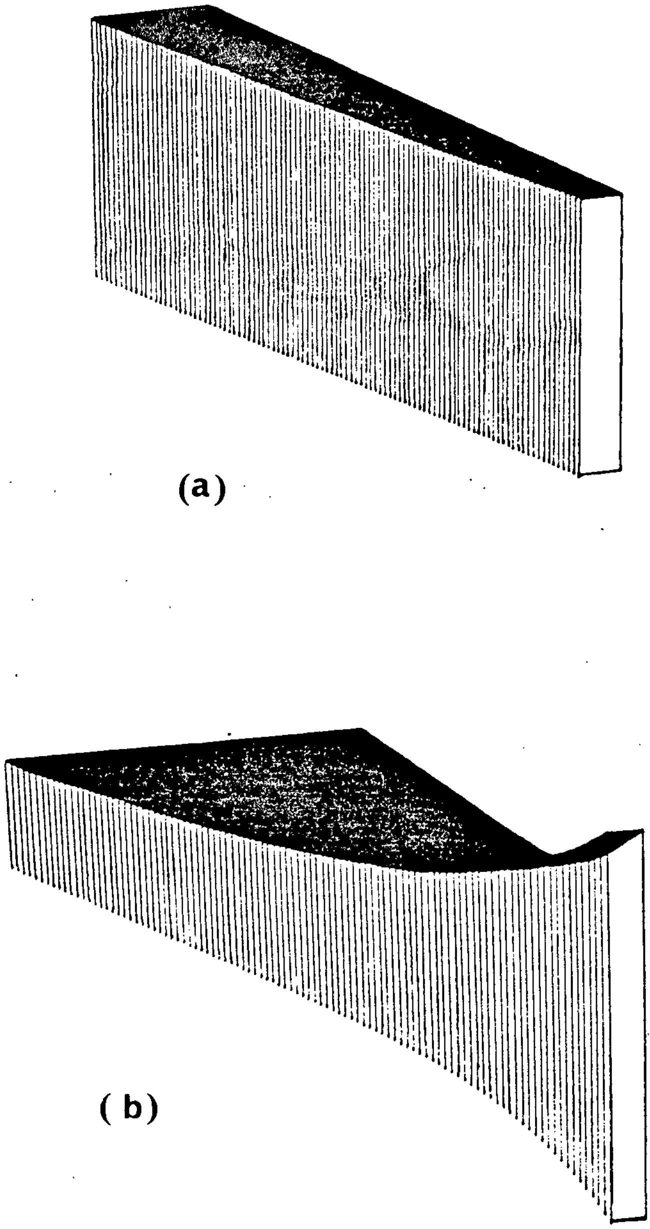


Figure 5-3 Two possible intermediate shapes in rolling a part with linearly variable height, from a uniform block;

(a) first solution

(b) second solution

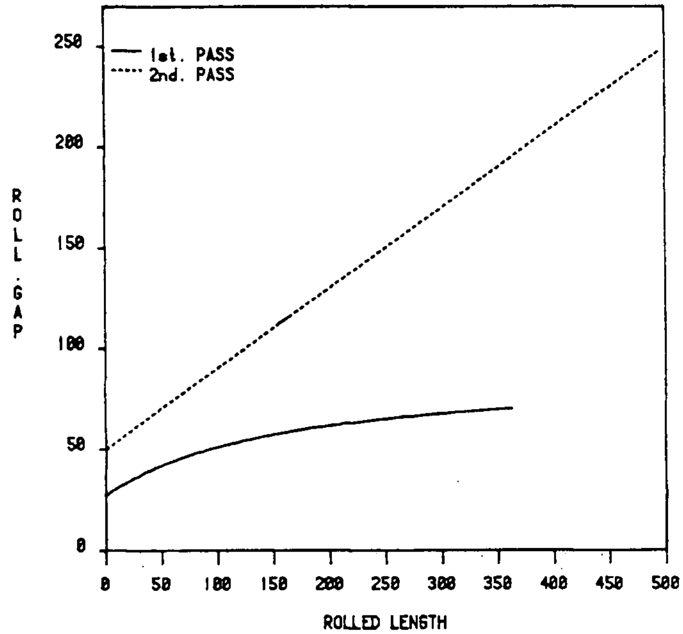


Figure 5-4 Process behaviour of the first possible solution: roll gap(mm) versus rolled length(mm)

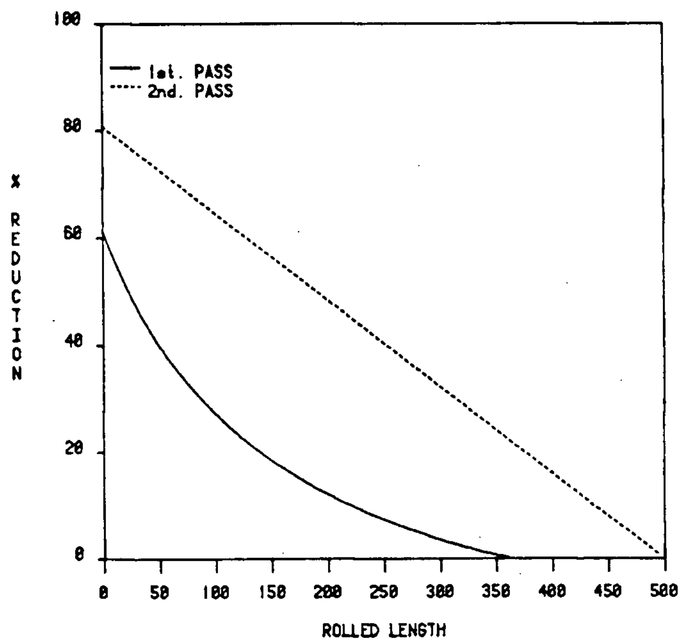


Figure 5-5 Process behaviour of the first possible solution: percentage of reduction versus rolled length(mm)

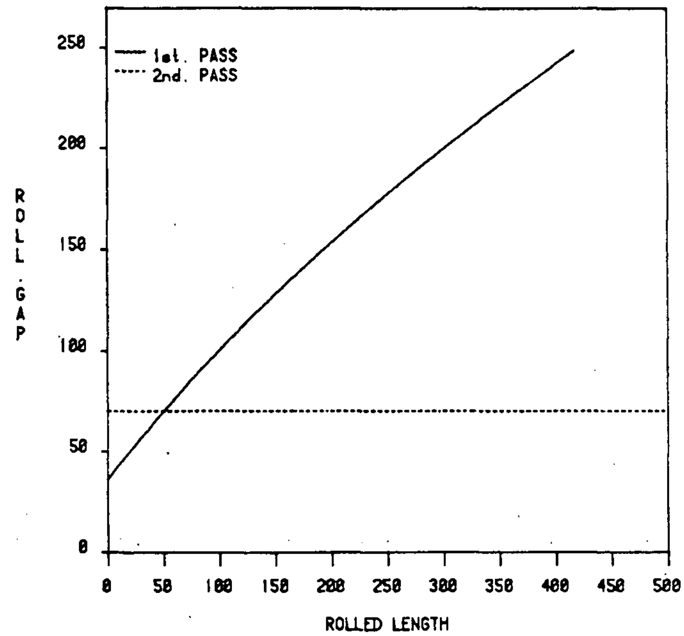


Figure 5-6 Process behaviour of the second possible solution: roll gap(mm) versus rolled length(mm)

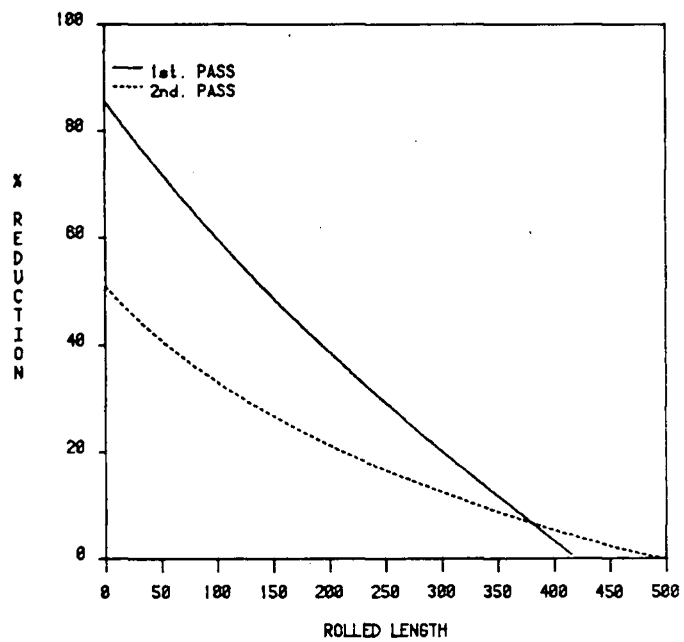


Figure 5-7 Process behaviour of the second possible solution: percentage of reduction versus rolled length(mm)

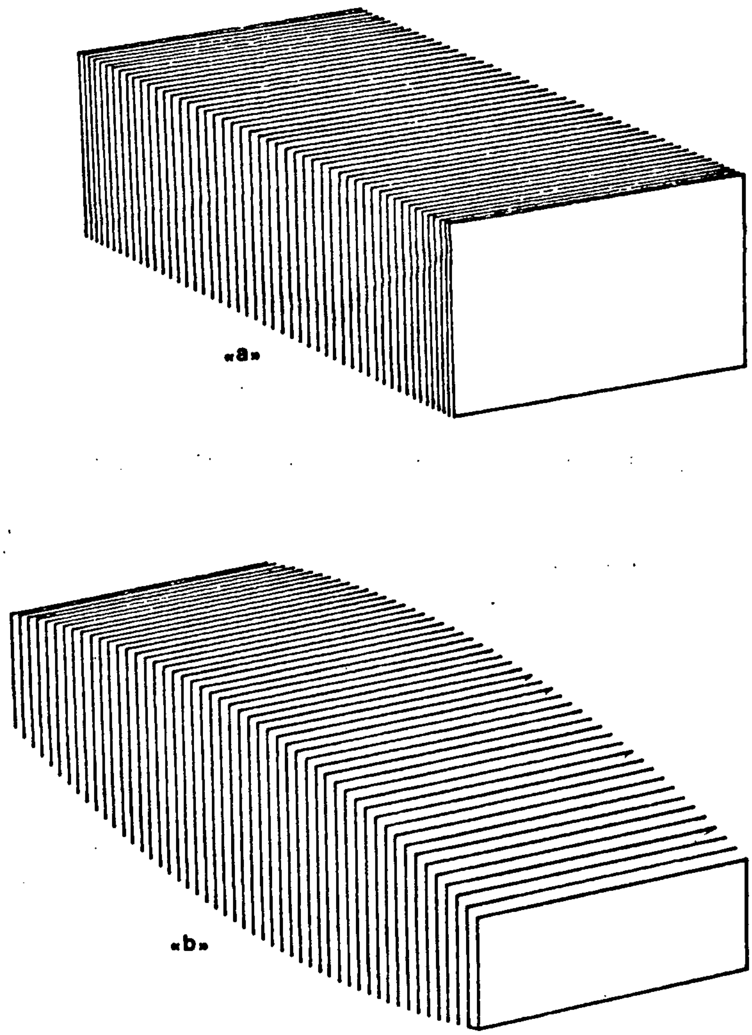


Figure 5-8 Initial and final geometry of the part in example 2

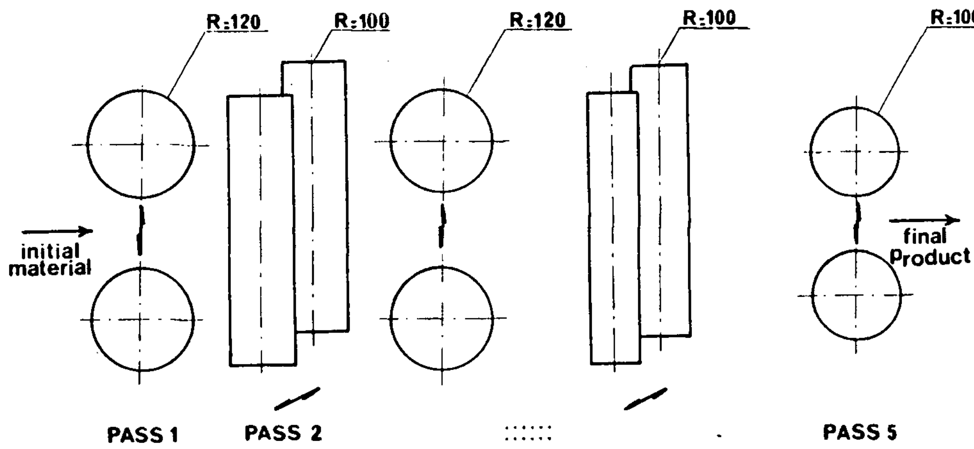


Figure 5-9 Sequence of rolling for a typical example of a multi-pass rolling process(dimensions in mm)

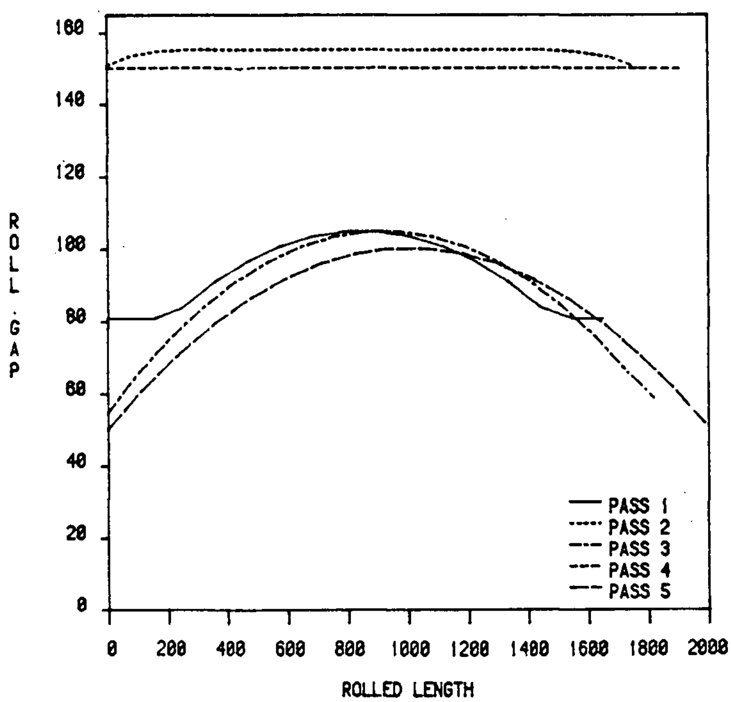


Figure 5-10 Variation of roll gap(mm) versus rolled length for the example of figure 5-9

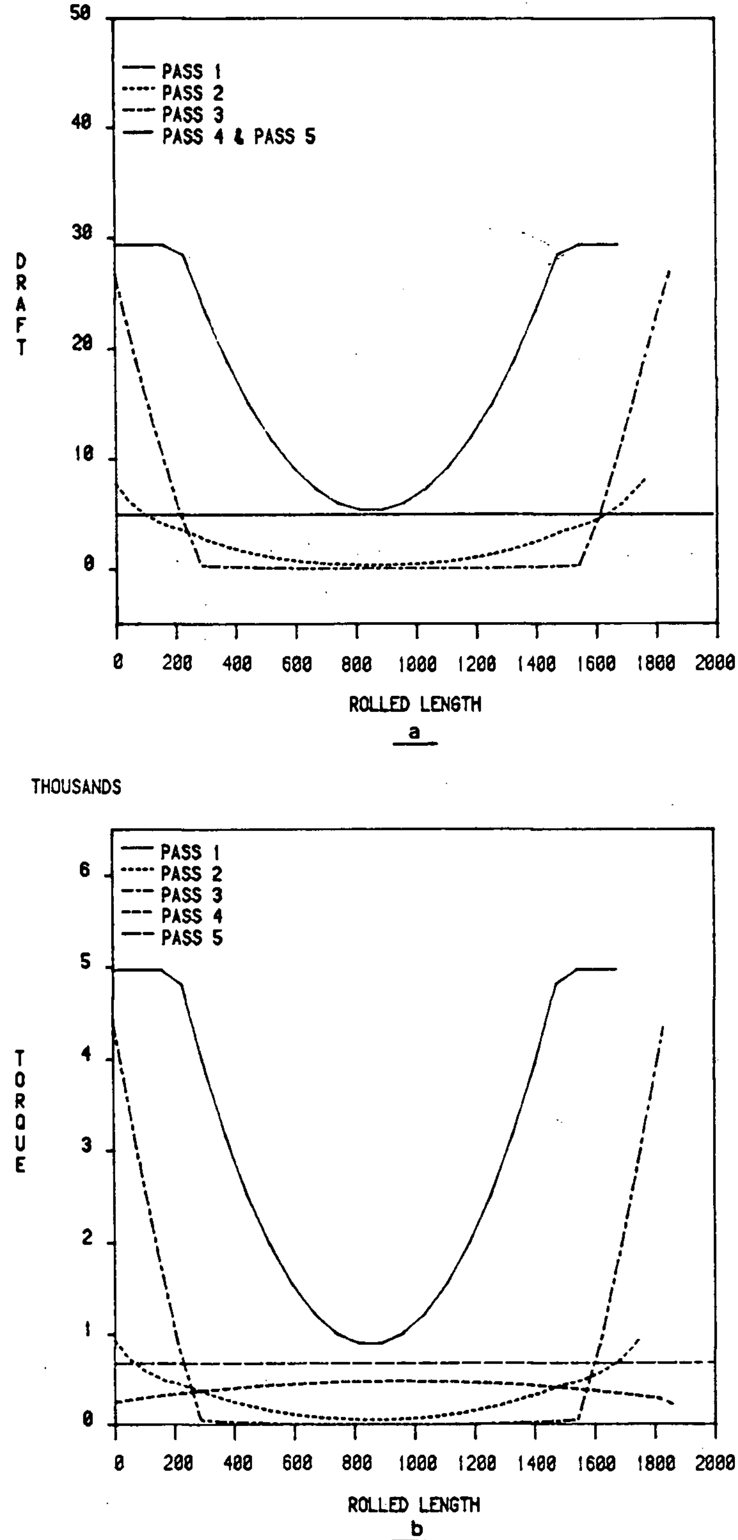


Figure 5-11 Variation of draft(mm),a, and torque(kg.m),b, versus rolled length(mm) for the example of figure 5-9

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APPENDIX A

Method of Incremental Search in Root Finding

General Problem:

Given an algebraic equation of the form $f(x)=0$, find the value(s) of x, i.e., the root(s), that satisfy the equation.

Algorithm[29,30]:

- given starting point, x_0, an increment Δx is chosen.
- Values of f(x) for successive values of x_0, $x_0+\Delta x$, $x_0+2\Delta x, \ldots,$ are determined until a sign change in f(x) occurs, i.e., when $f(x)\cdot f(x+\Delta x)\leq 0$.
- The last value of x, preceding the sign change is reverted back and the incremental search is repeated using a smaller increment (e.g., $\Delta x=\Delta x/10$) until a sign change in f(x) occurs again.
- The above procedure is repeated using progressively smaller increments, until a sufficiently accurate value of the root is obtained.

APPENDIX B

Curve fitting

The main purpose in curve-fitting is to find an analytical form for describing a set of discrete data points. A curve with continuity up to second order was considered to be suitable for application in the present study. *A conical arc interpolation technique* was selected. Conical arc interpolation is an approach which is devised to avoid unwanted oscillation[31]. The general form of a conic curve is

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

with the slope

$$y'=-(2Ax+By+D)/(2Cy+Bx+E)$$

and the second derivative

$$y''=-2(y'^2+By'+A)/(2Cy+Bx+E)$$

The expression for the conic curve may be rewritten as

$$x^2+B'xy+C'y^2+D'x+E'y+F'=0$$

where $A\neq0$, $B'=B/A$, $C'=C/A$,...etc.
The above expression, thus, involves five independent constants which can be found to satisfy five conditions, e.g. two positions, two slopes and one curvature.

To illustrate the algorithm of the curve-fitting routine, consider the given points in Fig. B-1. A conic curve is first passed through the first three points in such a way that the slopes at points *2* and *3* satisfy the conditions of *weighted mean slope* between points *1* , *3* and points *2* , *4* (for details on weighted mean slope concepts, see 31, pp 130 to 135). Values of slope and curvature at point *3* as well as the weighted mean slope at point *4* are then used to find the coefficients of the conical curve which passes through points *3* and *4*. The same procedure is repeated for points *4,5* and *5,6*...,etc. The last conical curve passes through three points $(n-2)$, $(n-1)$ and n, satisfying the conditions of slope and curvature at point $(n-2)$.

Using the weighted mean slope concept guarantees the smoothness of the joined curves. Also, due to the nature of the conical curves, oscillation between the two adjecent points does not occur[31].

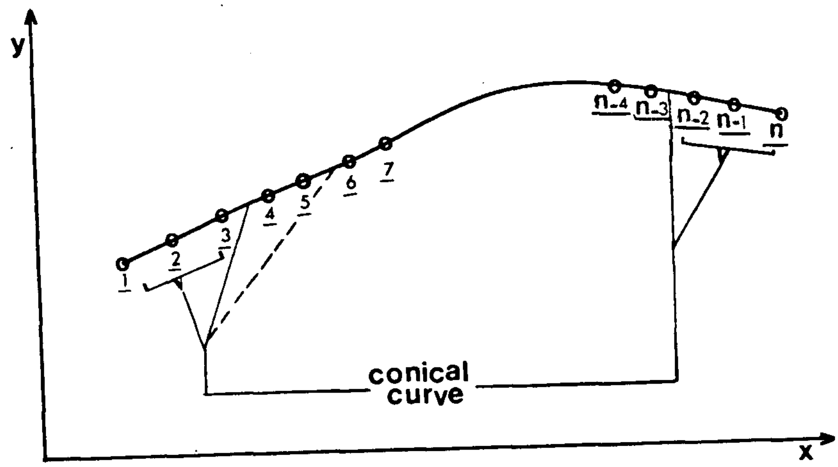

Figure B-1 Curve-fitting through known data points