A COMPARISON BETWEEN TWO METHODS OF FATIGUE LIFETIME PREDICTIONS FOR RANDOM LOADS

By

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Abstract

The purpose of this thesis is to compare the results obtained from two different methods to account for fatigue, the Root Mean Square (RMS) and the Histogram, to determine which method better represents reality.

The test procedure used subjected compact tension specimens to randomly selected block loads, then compared the actual lifetimes obtained by experiment to the lifetimes predicted by the methods. A statistical analysis was attempted to determine which method was superior.

The results of the analysis suggest that the RMS model is superior. However, no firm conclusions can be drawn, since the data obtained suggest that the Paris Law parameters used in the analysis are possibly biased.

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a	One-half or whole crack length or one-half of the major axis of an elipse.
a_c, a_i	Critical, initial crack lengths.
a_{TH}	Threshold crack length.
A	Paris Law parameter.
A_I	Initial cross sectional area.
b	One-half of the minor axis of an elipse or a dimension on a compact test specimen.
c	Critical value.
c,d,D	Dimensions on a compact tension specimen.
F_N, F_{LN}	Normal, Log-Normal cumulative distribution frequency (cdf).
F_1	Function of compact tension specimen dimensions for determining a stress intensity factor.
E	Young's Modulus.
h	Dimension on compact tension specimen.
H_0, H_1	Hypothesis.
i	Complex number $\sqrt{-1}$ or a sample number.
k	Number of different loads.
K	Stress intensity factor.
K_I	Mode I stress intensity factor.
K_{IC}, K_{1C}	Plane strain, plane stress critical stress intensity factor.
K_{max}, K_{min}	Maximum, minimum stress intensity factor.
K_{rms}	Root Mean Square (RMS) stress intensity factor.
K_{TH}	Threshold stress intensity factor.
L_I, L_F	Initial, final tension specimen lengths.

n	Paris Law parameter.
n_i	Cycles per pseudo-histogram or observations.
n_j	Cumulative frequency.
N	Number of cycles.
N_i	Number of cycles per load.
P	Applied load.
P_Q	Test load for determining K or K .
P_f	Fatigue load.
P_u, P_y	Ultimate, yield load.
r	Polar coordinate.
r_p	Radius of plastic zone.
s	Standard deviation.
S_x, S_y	Standard deviation of a particular population.
t	Value obtained from a t-distribution.
u	A random number.
U	elastic energy.
W	Resistance to crack growth.
x, y	Rectangular coordinates or observed value.
$\bar{x}, \bar{y}$	Sample average.
z	Normal value or a complex number.
Z	Complex function.
α	Significance level for one-sided tests and type I errors.
β	Significance level for type II errors.
γ	Specific surface energy.
Θ	Polar coordinate.

μ_0, μ_1	Value for hypothesis H_0, H_1.
ρ	Crack tip radius or sample correlation coefficient.
ρ_c	Critical value of sample correlation.
σ	Applied stress.
σ_{rms}	RMS stress.
σ_{tip}	Stress at crack tip.
σ_u	Ultimate stress.
σ_x, σ_y	Stress in the x-direction, y-direction or yield stress.
σ_{ys}	Yield stress
τ_{xy}	Shear stress in x-y plane.
Ψ	Stress function.

Acknowledgements

I would like to thank Dr. Nadeau, Ph.D. and my supervisor, Dr. Vaughan, Ph.D., for their helpful suggestions throughout this work; all the technicians in both the Mechanical and Metallurgy Engineering Departments of the University of British Columbia, who assisted me in all phases of the experimental work; Peter and John Somoya for their help in the preparation of the manuscript; and finally Jim Fraser, who spent a great deal of time proof reading this thesis.

Introduction

During the past fifty years, engineers have become increasingly aware of the problem of crack growth caused by fatigue. (Fatigue occurs when a structure or part wears out due to cyclic loading.) Regrettably, this enlightenment has only arisen as a result of the sudden failure of some structures, examples of which include the American Liberty and T2-tankers during the Second World War[1] and the bridge at Point Pleasant, West Virginia in 1967[2]. From these and other similar failures, engineers have been able to establish the principles of a new science called fracture mechanics.

The purpose of fracture mechanics is to determine the effect that a crack has on the strength of a structure. This is done by determining the crack's "stress intensity factor", a measurement of stress intensity at a crack tip. Stress intensity is governed by the geometry of a structure and the load applied to it. When an applied load reaches a critical level, called the "critical load", the structure to which it is applied to will fail. The value of stress intensity at the critical load is known as the "critical stress intensity" and it is assumed to be a material constant.

Most structures are subjected to some form of cyclic loading. This requires consideration of the effects of fatigue, using a proven design procedure, the most common

of which is the Paris Law equation[3], which states that the rate of crack growth is directly proportional to the change of stress intensity caused by cyclic loading. The Paris Law has been confirmed experimentally for constant amplitude cyclic loading, but still needs modification to take into account the effects of random amplitude loading. One such modification consists of calculating the Root Mean Square (RMS) value of the cyclic loads, which is then used to calculate stress intensity (also a RMS value) at the crack tip, which in turn is used in the Paris Law. The method has been shown to work well[4] but it is not complete; the RMS method incorrectly assumes that all loads cause crack growth[5]. However, crack growth does not occur in some materials (such as steel) in the presence of stresses below a level known as a "stress intensity threshold".

Using this knowledge, an alternative approach known as the Histogram method[6] has been devised. This paper will test the Histogram method to determine if it can produce any statistical improvement over the standard RMS method.

Theoretical Background

The development of fracture mechanics began in the late 19th and early 20th centuries when engineers noticed that many so-called brittle failures seemed to start at a crack. (A brittle failure occurs when a structure or part fails suddenly due to rapid crack growth.) Suspecting that such cracks could weaken structures, various attempts were made to analyse the effects that they could have on structural strength.

The first attempt in calculating the stress around a crack tip was made by Inglis[7] in 1913. Representing the crack as an ellipse, Inglis came up with the following equation for the stress at a crack tip (see figure 1).

$$\sigma_{tip} \approx 2\sigma(a/\rho)^{1/2} \qquad 1$$

where σ_{tip} is the stress at the crack tip

σ is the applied stress

a is half the crack length

ρ is the crack tip radius

Obviously, the radius of the crack tip must be known in order to determine the stress around it using this equation, but that figure is generally not available, so an assumption is made. Since an assumption of a zero radius produces the useless result of infinite stress at a crack tip, the usual

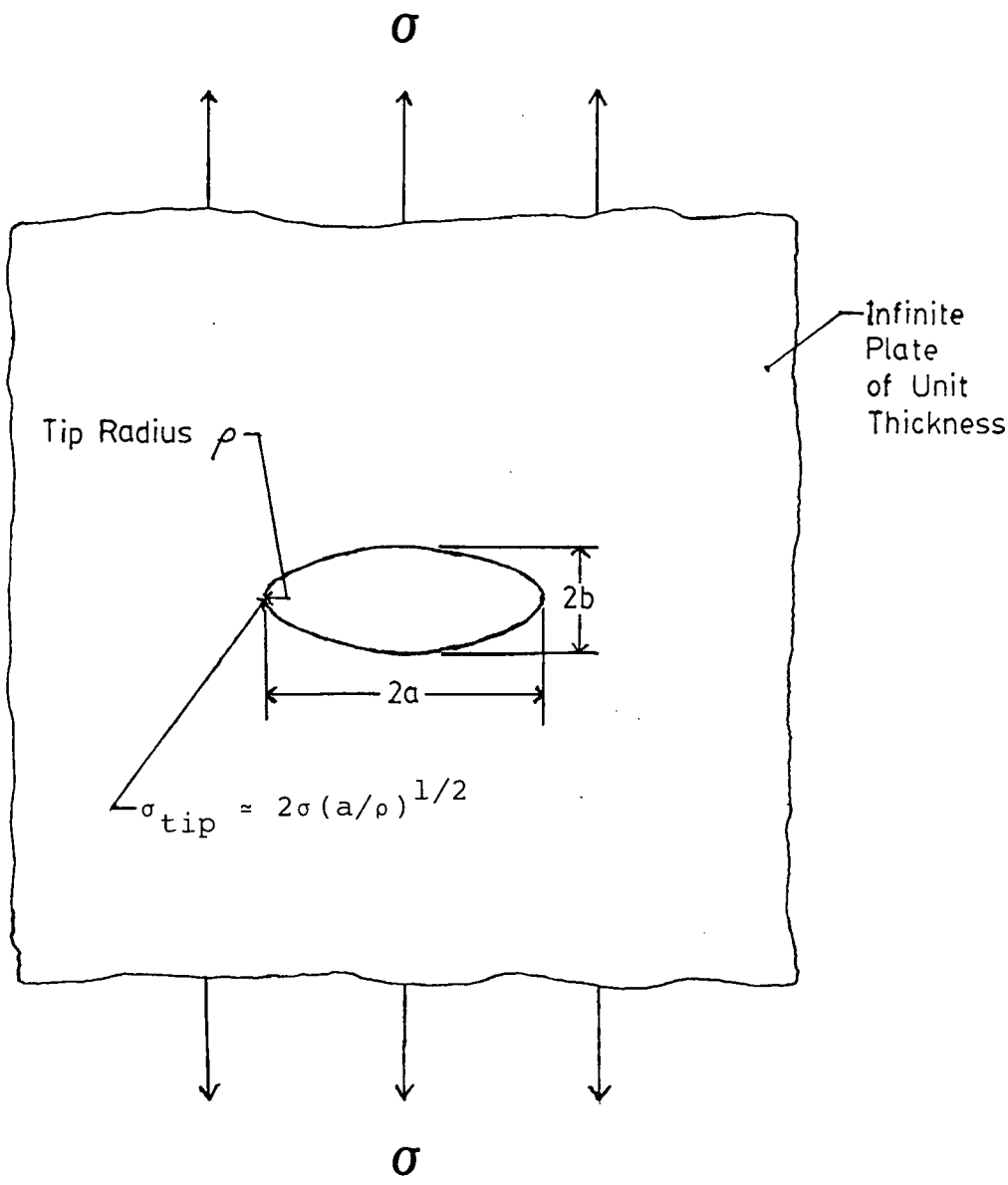


FIGURE 1 Elliptical Crack in Infinite Plate

assumption is that the radius is equal to the atomic spacing in the subject material. This produces a finite answer, but one which is equally useless, since it yields a product stress which is much higher than the ultimate strength of the material.

Griffith[8] subsequently developed another approach, which used an energy method based on Inglis's results. Griffith's equation is:

$$\frac{dU}{da} = \frac{2\pi\sigma^2 a}{E} \qquad 2$$

where U is the elastic energy

a is half the crack length

σ is the applied stress

E is Young's Modulus

The elastic energy represents the amount of energy available to increase the length of a crack by amount "da". Griffith reasoned that where a crack is produced, elastic energy had to be equal to the amount of energy resisting the growth of a crack dW/da. He further assumed that the energy resisting the growth of a crack did not change with crack length and that when the elastic energy release rate equalled the energy resisting the growth of a crack, a structure would break. Griffith reasoned that the critical energy resisting crack growth should be equal to the energy required to

create new surface, known as surface energy.

$$\frac{dU}{da} = \frac{dW}{da} = 4a\gamma \qquad 3$$

where W is the resistance to crack growth

γ is the specific surface energy

Hence,

$$\frac{2\pi\sigma_F^2 a}{E} = 4a\gamma \qquad 4$$

which gives

$$\sigma_F = (2E\gamma/\pi a)^{\frac{1}{2}} \qquad 5$$

where σ_F is the stress at which failure occurs

Surface energy is not an easy material property to measure, but it can be measured for certain materials like glass. After measuring the surface energy of the glass used in his experiments, Griffith found that his assumptions about critical energy were reasonably accurate.

Engineers finally had a reasonable method of determining when cracks in a structure would lead to failure but problems still remained. One shortcoming of the Griffith method was the difficulty of determining critical resistance to crack growth. In brittle materials, critical resistance is equal to surface energy, but the latter property cannot

be measured accurately. Resistance in ductile materials consists not only of surface energy but also the amount of energy required to form a plastic zone in front of a crack. This is usually so large that surface energy can be neglected. The easiest way to determine resistance to crack growth is to measure it indirectly by measuring the stress required to break a specimen with a certain crack length.

Another shortcoming of the solution was that it was only correct for a crack in an infinite plate. Griffith solved this problem in his experiments by using glass bulbs. (The surface of a sphere has no edges, just like an infinite plate.) Solutions to practical problems were not available until the work of Westergaard was published[9].

Westergaard's solution begins with the assumption that a crack can be represented as either a plane strain or plane stress elasticity problem. This assumption permits the use of Airy's stress function[10]:

$$\Delta^2\phi = \nabla^2(\nabla^2\phi) = 0 \qquad 6$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

ϕ is a stress function that satisfies the above equation and all boundary conditions

Westergaard's contribution was to define the following function that satisfies Airy's function.

$$\phi = \mathrm{Re}\ \bar{\bar{Z}} + y\,\mathrm{Im}\ \bar{Z} \qquad 7$$

where $\frac{d\bar{\bar{Z}}}{dz} = \bar{Z}$, $\frac{d\bar{Z}}{dz} = Z$, $\frac{dZ}{dz} = Z'$

$$Z(z) = \mathrm{Re}\ Z + i\,\mathrm{Im}\ Z \qquad 8$$

with $z = x + iy$

Substituting Westergaard's function in the solution for the equilibrium equations[11] gives:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = \mathrm{Re}\ Z - y\,\mathrm{Im}\ Z'$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} = \mathrm{Re}\ Z + y\,\mathrm{Im}\ Z' \qquad 9a\text{-}c$$

$$\tau_{xy} = \frac{\partial^2 \phi}{\partial x \partial y} = -y\,\mathrm{Re}\ Z'$$

where σ_x, σ_y, τ_{xy}, are the stresses in the directions shown in figure 2

For the problem shown in the figure 2, the following equation for "Z" meets all the boundary conditions:

$$Z = \frac{\sigma(z + a)}{\sqrt{z(z + 2a)}} \qquad 10$$

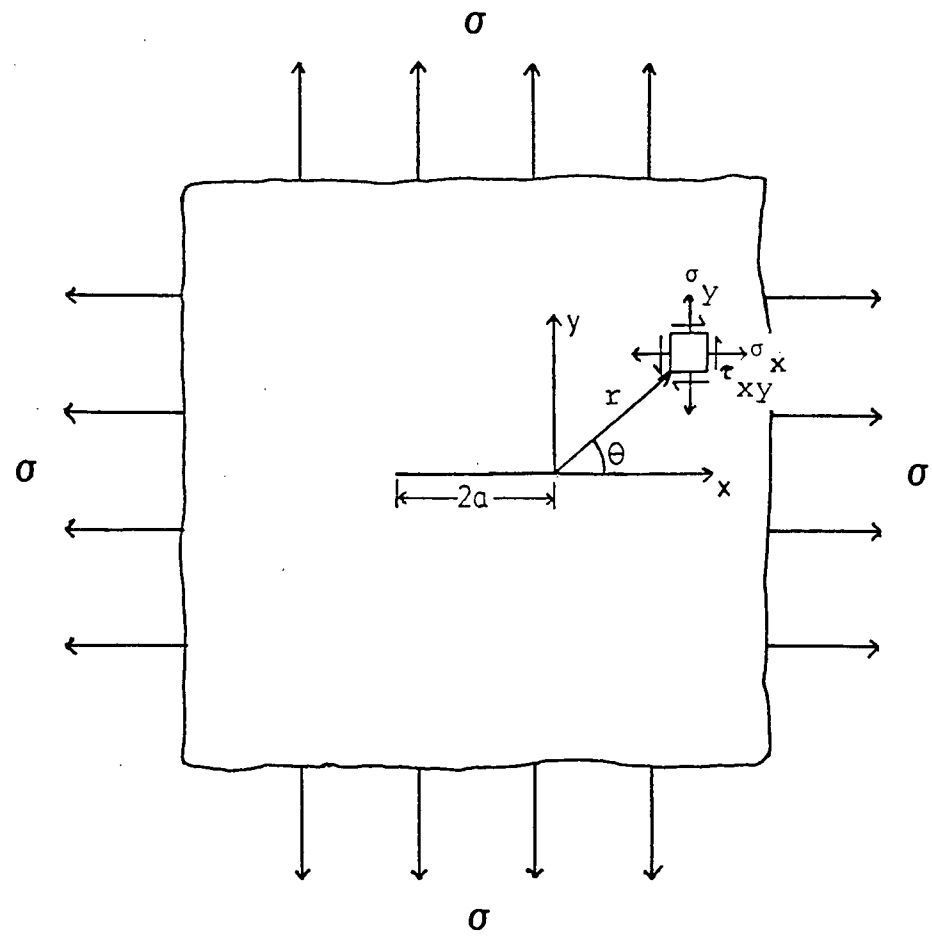


FIGURE 2 Crack in Infinite Plate

Letting the value of "a" be much larger than the value of "z", i.e. letting the solution only be valid near a crack tip, gives:

$$Z = \sigma a/\sqrt{2az} \qquad 11$$

Changing this equation to polar coordinates gives:

$$Z = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \exp(-i\theta/2) \qquad 12$$

Finally, substituting the above into equations 9a-c, yields the solution for the stresses around a crack tip.

$$\sigma_x = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)]$$

$$\sigma_y = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos(\theta/2)[1 + \sin(\theta/2)\sin(3\theta/2)] \qquad 13a\text{-}c$$

$$\tau_{xy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)$$

The solution can be considered to consist of two parts. The geometric part yields stress distribution around a crack, including the singularity at the crack tip. The second part consists of a scaling term, $\sigma\sqrt{\pi a}$, a simple function of applied stress. This function is defined as the "Mode I Stress Intensity Factor" and is given the symbol K_I. The different types of stress intensity modes are illus-

trated in figure 3.

Rewriting the crack tip stress equations using the stress intensity factor produces the equations:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2)[1 + \sin(\theta/2)\sin(3\theta/2)] \qquad 14a\text{-}c$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)$$

It is this stress intensity factor that is relevant, because when it reaches a critical value, a structure will break. It is assumed that this critical value will be constant for any given material. The critical stress intensity criterion is similar to the critical energy criterion Griffith proposed, but unlike Griffith's method, Westergaard's can be used to solve practical crack problems[12].

Experimental results confirm the assumption of a critical stress intensity as long as a structure meets certain thickness and plane strain requirements which assure that the plastic zone, which formed at the crack tip, remains small compared to the crack length. Since the whole of Westergaard's analysis is based on a linear elastic approach, the plastic zone must be small compared to the dimensions used in the analysis if fracture mechanics is to

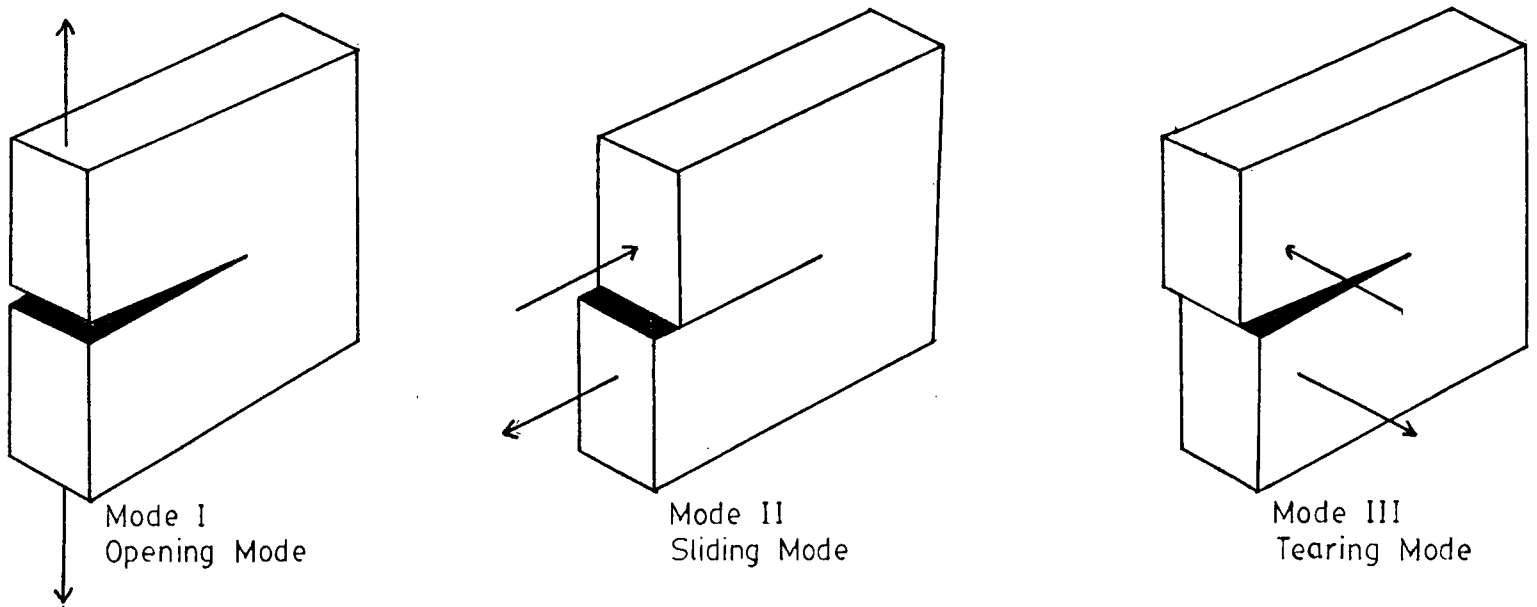


FIGURE 3 The Three Loading Modes for a Crack

be valid. If a structure does not meet these requirements, then each thickness of a material proposed for the structure has its own unique critical stress intensity.

Westergaard's solution is useful if failure is the result of a single load. Most failures are not. Structures are usually subjected to cyclic loading and in many cases cracks grow as a result of fatigue. A design procedure for fatigue is helpful and the one suggested by Paris[13] is the most accepted method of determining the life of a structure under cyclic loading. The life is defined as the number of cycles necessary to enlarge a crack from its initial size to its critical size. Paris suggested that cracks experiencing the same variations of stress intensity will grow at the same rate.

This solution is expressed as follows:

$$\frac{da}{dN} = f(\Delta K) \qquad 15$$

where da is the incremental crack growth

dN is the incremental life in cycles

ΔK is the variation of stress intensity

Generally it is invalid to assume that fatigue crack growth is due to the variation of stress intensity alone. There are other influences such as the maximum stress intensity experienced by a crack and the constraint on a crack.

Other factors are relevant, but for two crack growth cases to be considered similar, all the factors that affect crack growth must be similar.

In most cases, the procedure proposed by Paris is accurate enough to be useful for practical design. For the purposes of this paper, the Paris relationship is assumed. All the tests hereinafter described were designed so that change in stress intensity is the dominant factor in crack growth.

Experimental data show that the function required to relate crack growth rate to the range of stress intensities is a simple power function of the following form:

$$\frac{da}{dN} = A(\Delta K)^n \qquad 16$$

where 'A' and 'n' are material properties

This equation is known as the Paris Law. The Paris Law is useful in many practical situations, but is not valid for stress intensity at either end of the stress intensity range (see figure 4). However, if threshold stress intensity and critical stress intensity are used as limits on the range of the Paris Law, the error in using it is small.

The Paris law in the above form can only be used to calculate the life of a structure undergoing a constant variation of loads. It must be modified to calculate the

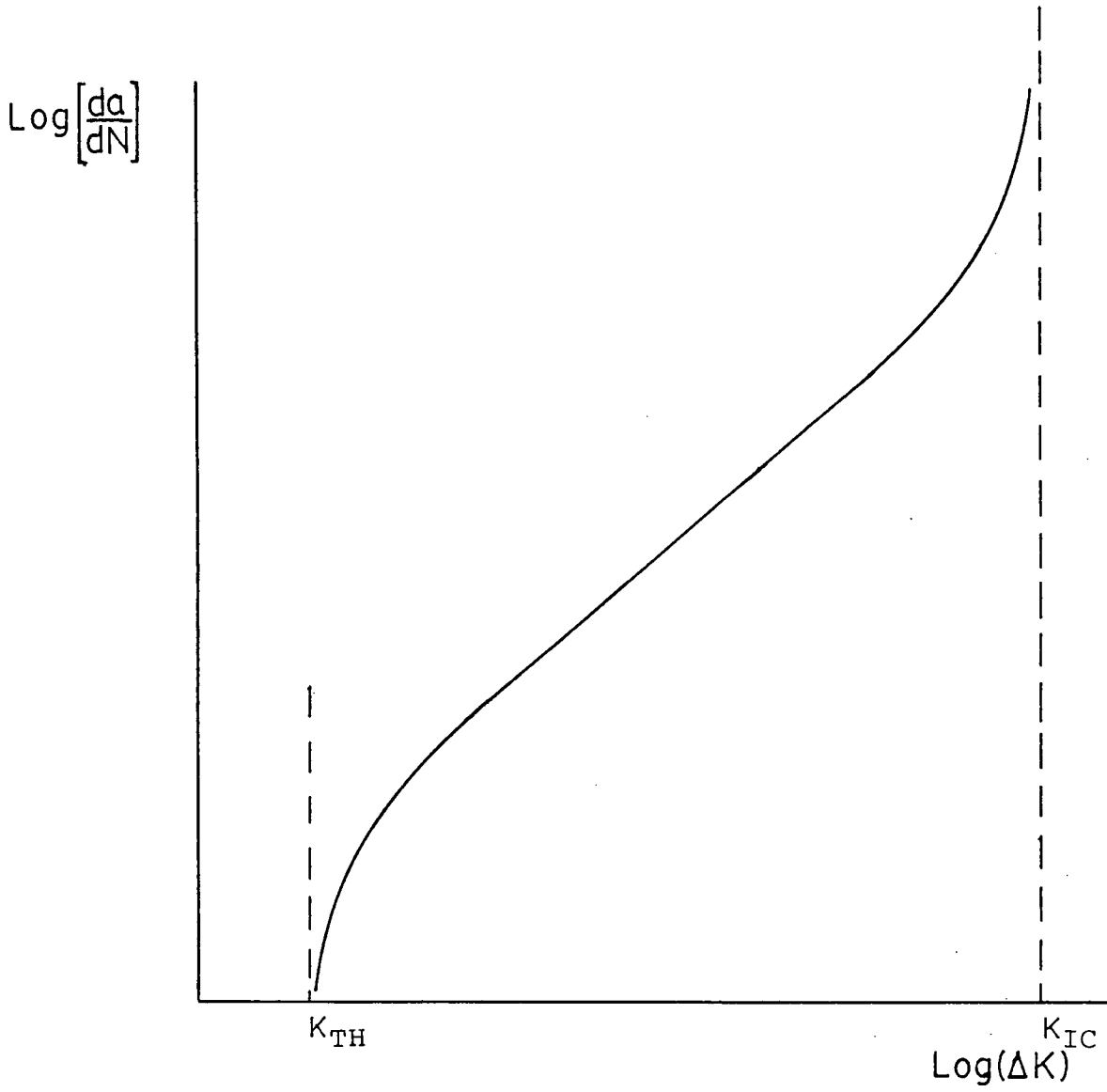


FIGURE 4 A Typical Paris Plot

effect of random loads.

One method is to calculate the Root Mean Square (RMS) value of the loads applied to a structure and use the RMS value in the Paris Law[14]. The consequent changes to the Paris Law are as follows:

$$\frac{da}{dN} = A(\Delta K_{rms})^n \qquad 17$$

where $$\Delta K_{rms} = \sqrt{\frac{\sum_{i=1}^{k} N_i (\Delta K_i)^2}{\sum_{i=1}^{k} N}}$$

k is the number of different loads

N is the number of cycles for each load

Barson and Rolfe[15] have shown that this relationship works well. However, it fails to recognize that crack growth does not occur in certain materials if the stress experienced by a structure is less than a critical value known as the "stress intensity threshold". By including this value in a "Histogram Method", Vaughan[16] has developed a method for calculating fatigue crack growth that should be better than the RMS Method. A histogram is a block representation of a load distribution on a graph showing load vs. number of cycles that a load is applied to a structure (see figure 5). The use of the histogram in

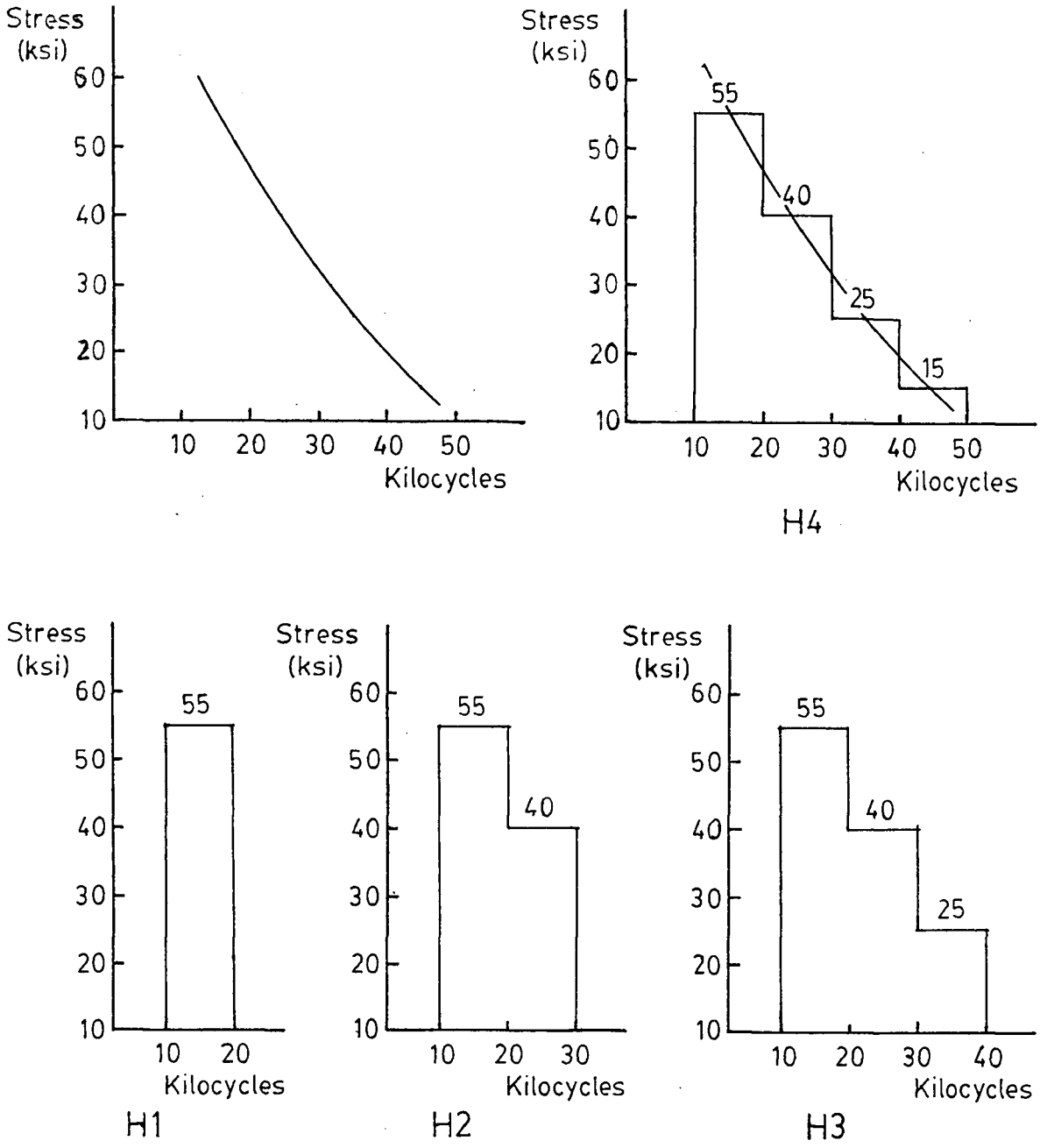


FIGURE 5 Sample Histogram

calculating the life of a structure is shown as follows.

First the threshold crack length is calculated. It is the crack length that will cause stress intensity at the crack tip to be equal to the stress intensity threshold. Threshold crack lengths are required for each load level on the histogram. Next, for the largest stress level, critical crack size is calculated. Critical crack size is reached when the stress intensity at a crack tip is equal to the critical stress intensity.

If it is assumed that there is initially a crack of a certain size in a structure and that crack is smaller than the the smallest threshold crack length calculated, there can be no fatigue crack growth. If the initial crack is larger than the critical crack size, failure of a structure can be expected almost immediately. If a crack is larger than some of the threshold crack lengths, but smaller than the critical crack size, crack growth occurs.

The latter case is studied by constructing a sub-category of the histogram known as a pseudo-histogram (for example see histograms H1-3, figure 5), which is used to calculate the RMS stress intensity value used in the Paris Law. This value is calculated for currently effective loads, which make up the pseudo-histogram, and is used until the crack grows so that the next lowest load in the main histogram has an effect on crack growth. Then an additional section of the histogram is used to construct a new pseudo-

histogram, which is used to calculate a new RMS stress intensity factor. This procedure is continued until the crack reaches critical crack length. The structure is then assumed to fail and its lifetime is calculated from the sum of all the pseudo-histogram lifetimes.[1]

1. See figure 6 for an example of how the Histogram Method is used.

Given: $K_I = \sigma\sqrt{\pi a}$

$K_{TH} = 6.0$ ksi$\sqrt{in}$

$K_{IC} = 100.$ ksi$\sqrt{in}$

$a_I = .005$ in,

$\sigma_{min} = 0$ ksi

σ_{max} is obtained from histograms in figure 5

Paris Law: $\frac{da}{dn} = 1.44 \times 10^{-9} (\Delta K_{rms})^3$

	σ_{rms} (ksi)	a_{TH} (in)	a_{CR} 2 (in)	n_i (kc)	da 3 (in)	dN 4 (cyc.)	dN*Ω 5 (cyc.)
H1	55.0	.004[1]	1.052	15	.002	3,283	26,261
H2	46.2	.007	1.052	40	.011	11,378	34,134
H3	37.8	.018	1.052	75	.033	13,971	22,353
H4	31.3	.051	1.052	120	1.001	28,085	28,085
						N =	110,830

Footnotes:
1. The initial crack length 'a_I' is greater than 'a_{TH}' for Histogram H1.
2. Critical crack length is the same for all histograms because the maximum load can occur at any time.
3. da = $a_{TH1} - a_I$ or $a_{TH(i+1)} - a_{THi}$ or $a_{CR} - a_{TH}$
4. Number of cycles experienced by each Histogram.
5. Total amount of cycles experienced by the structure during the time that histogram is applied; N.B. $\Omega = 120/\Sigma n_i$

FIGURE 6 Histogram Method

Experimental Procedure

This paper will attempt to determine which method, the RMS or the Histogram, more accurately represents crack growth due to fatigue. The experiments which form the basis for the study require the subjection of a number of specimens to random loads and the recording of the number of cycles that each specimen survives. This number is compared to the number of cycles predicted by each method for that specimen. Using statistics, a comparison is then made to determine which method comes closer to the actual results. The details of the procedure are as follows.

Since it is a common material used for engineering purposes, it was decided that ASTM-36 type steel would be used in the experiments. Once it was selected, various tests were conducted on the metal to determine its material properties (see Appendix I).

The next step was to select the geometry of the test specimen; the writer chose a compact tension specimen commonly used in determining fracture toughness and other kinds of fracture experiments. Its size was made as large as possible to produce a satisfactory range for crack growth. The apparatus used for the experiments was the small frame MTS testing machine (see figure 7) located in the Metallurgical Engineering building at the University of British Columbia (UBC). Within the limits imposed by this machine, the final specimen size was determined and the

result is shown in figure 8.

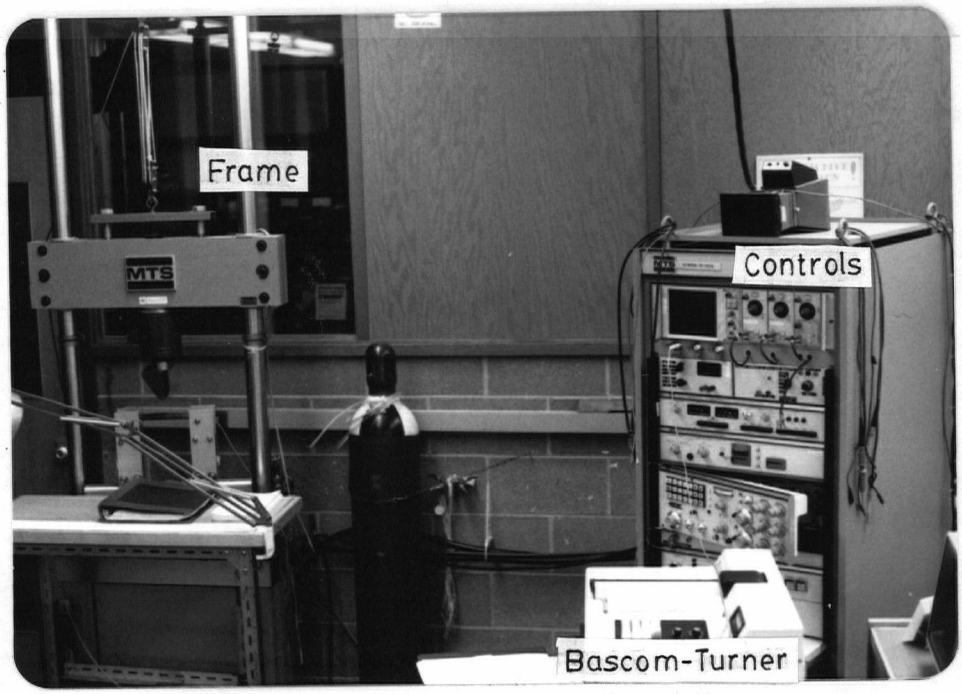
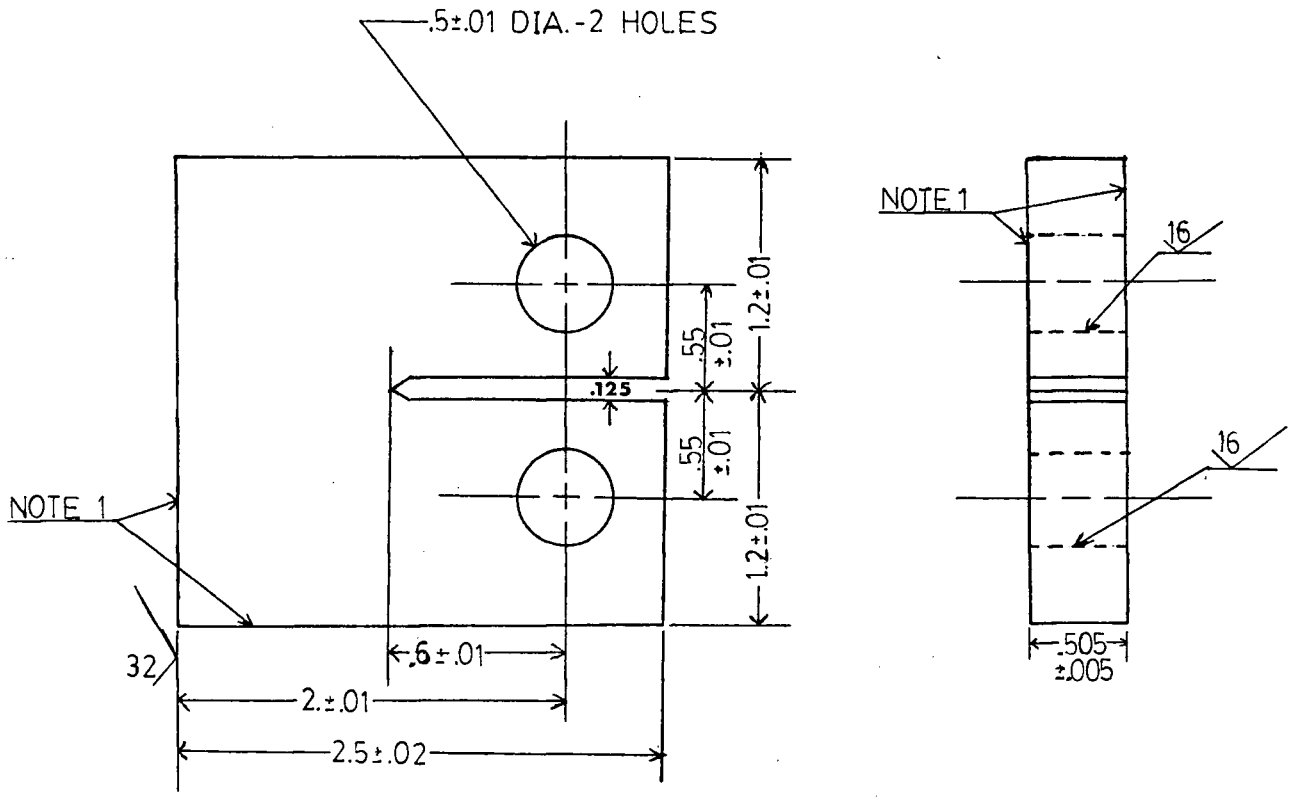


FIGURE 7 Small Frame MTS

The specimens were prepared as shown in figure 8 and polished using a wet sanding process with the following grit sizes; 180,320,400,600. Polishing a specimen made it easier to measure the crack length, as was necessary to do for determining the Paris Law parameters (see Appendix I). It also makes it easier to measure the initial crack length of about .7 inches that each specimen was precracked to. Precrack length was measured on both sides of the specimen before fatigue tests were begun. Since the experiment measures the number of cycles applied to a specimen from the

ALL DIMENSIONS IN INCHES

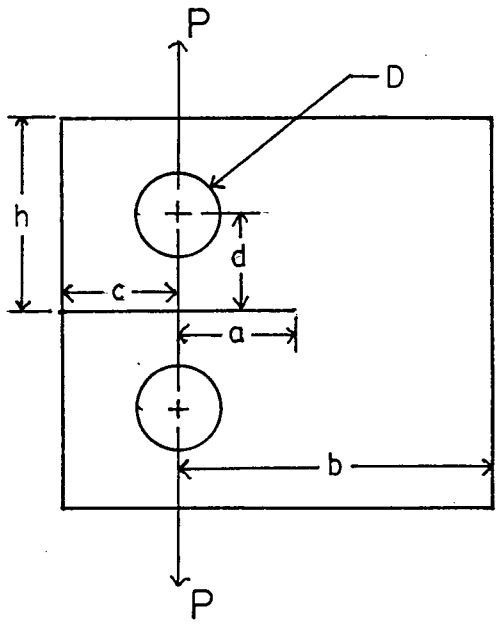


NOTE 1 - Surfaces shall be perpendicular & parallel as applicable to within .004 TIR.

FIGURE 8 Compact Test Specimen

initial crack length to the critical crack length, the initial crack created by precracking should be affected by the first load applied experimentally to the specimen. The only way to ensure that this is so is to require the plastic zone size resulting from the applied load to be greater than that created by the precrack load, thus the load used for precracking was less than the first load applied in the test.

The next problem was to construct a random sequence of block loads. The first step was to select a loading distribution that would produce a significant difference between specimen lifetime predictions made by the RMS and the Histogram methods. The loading distribution selection was restricted by time and operating constraints on both the MTS machine and its operator. Various loading distributions were tried, usually based on some polynomial relationship between stress and cycles, until two loading distributions met the constraints. This was done by constructing a histogram for each distribution from which the life of the specimen for both the RMS and Histogram methods could be calculated and compared. A computer program was used to determine these lifetimes (see Appendix II), since the stress intensity function "K_I" used is not a simple function to invert (see figure 9). The general procedure used by the computer program to calculate these lifetimes is the same as the example shown in the Theoretical Background, with the



Standard Specimen

h = .6b
d = .275b
D = .25b
c = .25b

For $.3 \leq a/b \leq .7$

$$K_1 = \sigma\sqrt{a}F_1(a/b, h/b, d/b)$$

where $\sigma = P/b$

$$F_1 = 29.6 - 185(a/b) + 655.7(a/b)^2 - 1017(a/b)^3 + 638.9(a/b)^4$$

FIGURE 9 Stress Intensity Factor for a Standard Specimen [12]

material properties, initial crack length and a histogram of the loading distribution being the inputs to the program.

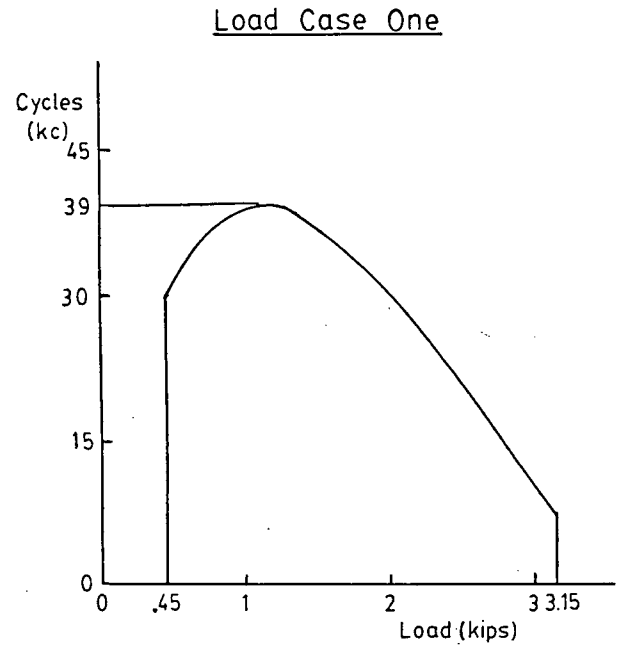
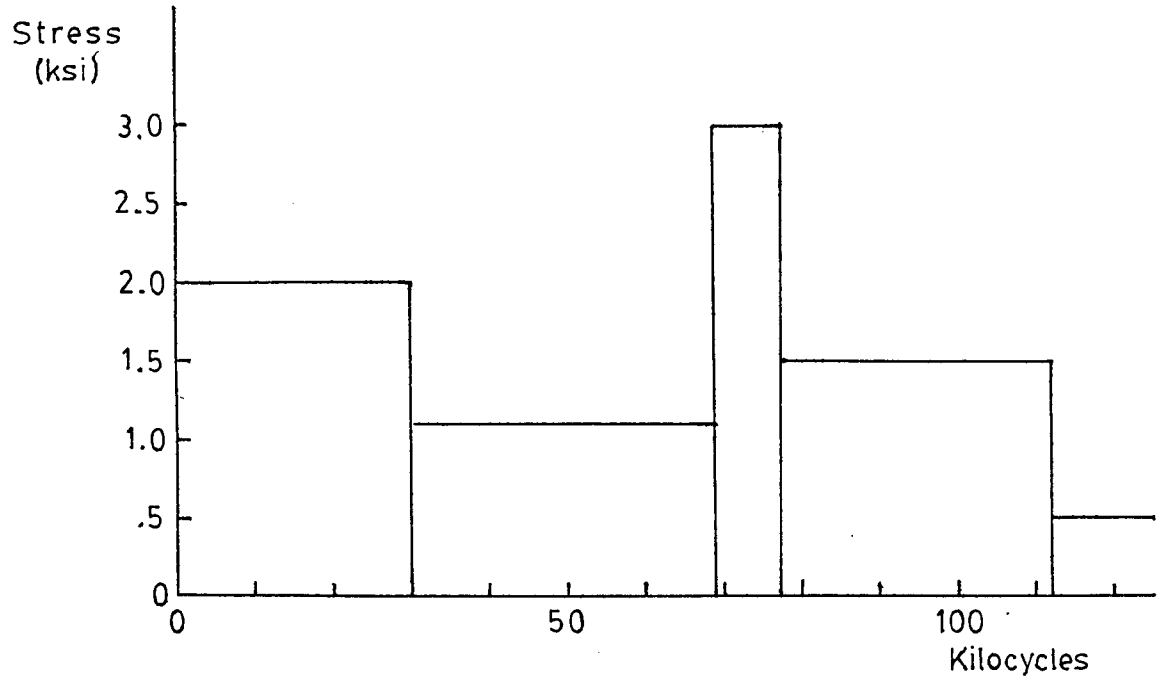
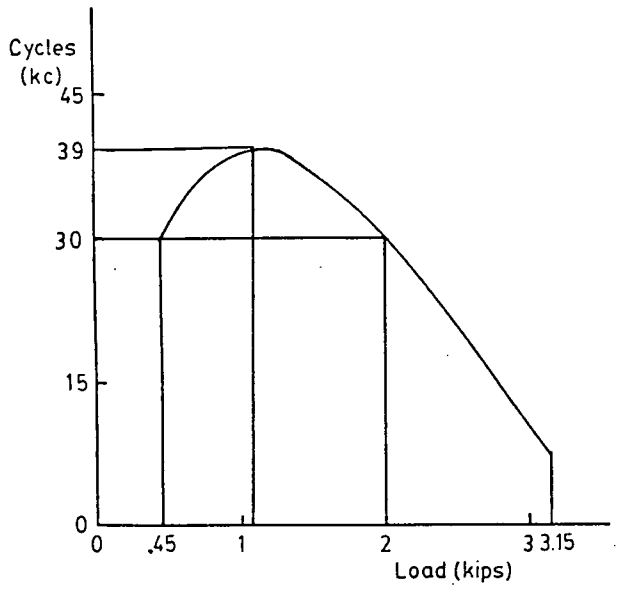


FIGURE 10 A Loading Distribution

Once a load distribution is selected (see figure 10), a means must be devised to create a random sequence of loads. The easiest way of doing this is to choose a random number between 0 and 1 and transform the random number into the equivalent random load. The frequency with which any load occurs can be obtained from the loading distribution. The remainder of the sequence is created in the same way with the selection of more random loads. (See figure 11 for an example of the procedure.)



Load Case One



A random number
$(0 \leq u \leq 1)$

If $u = .56$ — Linear mapping

$\sigma_{max} = .45 + u(3.15 - .45)$
$= 2.0$ ksi

%No. of cycles = 30 kc

If $u = .24$

$\sigma_{max} = 1.1$ ksi

%No. of cycles = 39 kc

FIGURE II Creation of a Random Sequence of Loads

It would have been tedious to calculate a random load sequence of 20 to 30 loads, so a computer program (see Appendix II) was written to generate the sequences to be applied to a prepared specimen. The same load sequence was ran on a number of test specimens in order to observe the random variations of the experiment. With more than one result for each sequence, a statistical analysis could be attempted.

The final step of course is the test itself. The test is simply a matter of applying loads to a specimen for the required number of cycles per load. The MTS was programmed to shut itself off when the crack opening reached a critical value, which was predetermined by visual inspection and set so that a test would end before any great amount of plastic work was done in fatiguing the specimen.

Observations

The load distributions selected for this work are shown in figure 12. Each load distribution was tested on a group of specimens; known as test 1 for load case one and test 2 for load case two.

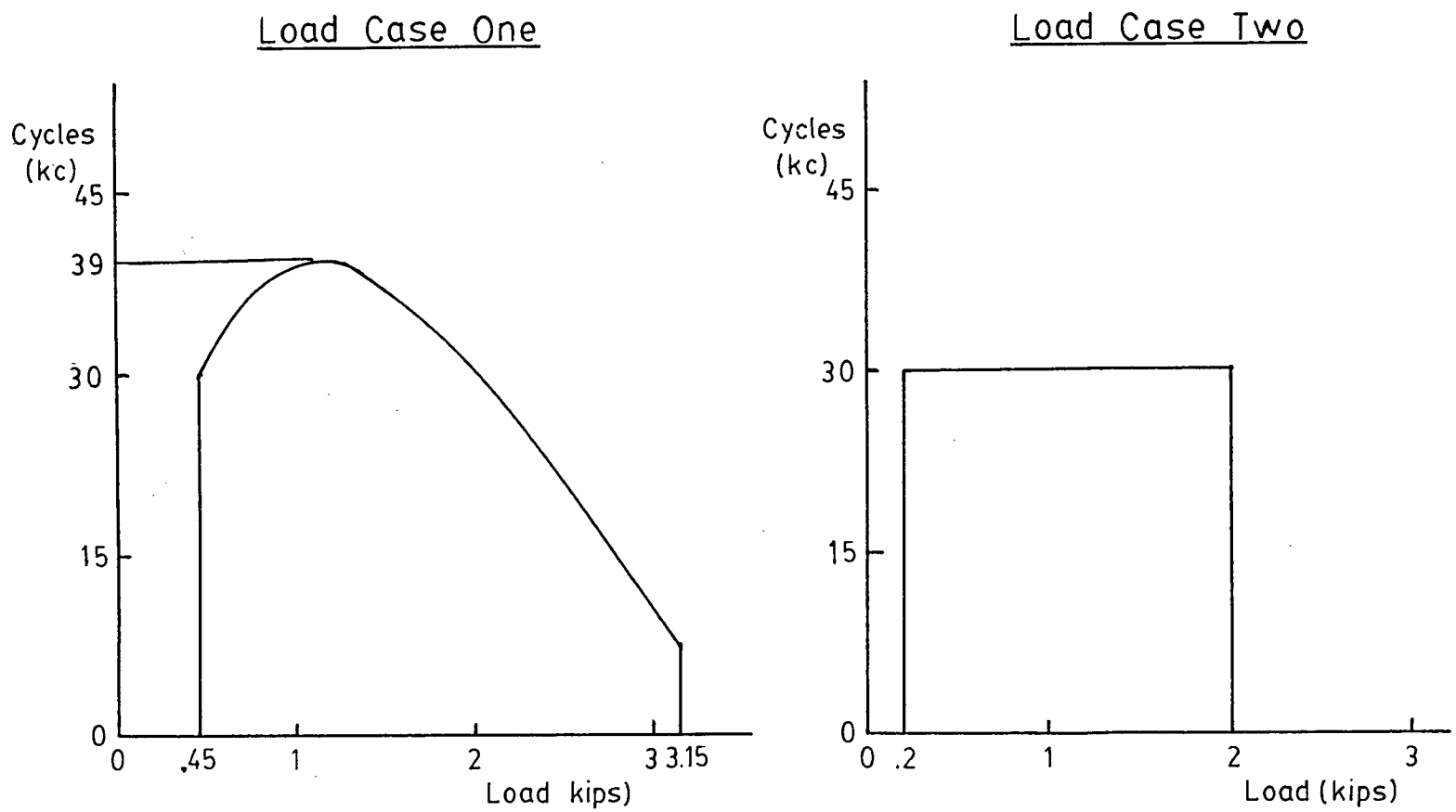


FIGURE 12 Load Distributions for Load Cases One and Two

The observations are shown in Table I. The first column shows the specimen number, a number system used to keep track of data on each individual specimen; the second shows the test group to which the specimen belongs (i.e. the load distribution); the third column shows the initial crack length in the specimen and the fourth shows the number of cycles a specimen endured to the point of failure.

Specimen	Test	Initial Crack (in)	Cycles
10	1	.710	399,780
11	1	.701	420,040
12	1	.700	391,810
13	1	.704	401,290
15	1	.699	401,170
42	1	.698	422,000
43	1	.699	398,000
44	1	.690	420,520
45	1	.695	399,000
46	1	.712	396,700
48	2	.697	681,820
49	2	.706	751,010
50	2	.722	749,000
51	2	.702	848,570
52	2	.691	737,930
53	2	.689	846,000
54	2	.708	747,600
55	2	.691	785,390
56A	2	.716	594,640
56B	2	.690	741,600
57	1	.707	281,500
58	1	.686	244,000
59	1	.695	367,000
60	1	.695	401,730
61	1	.715	405,800

TABLE I Observations

Treatment of Data

Once the experiments were completed, a statistical analysis of the data was performed to determine which method fit the data better. First the expected lifetime of each specimen was calculated for both the methods using the same computer program used to select loading distributions, then divided by actual lifetimes recorded in the experiments, yielding a set of ratios to be used in the analysis. If the method used to predict the lifetime of a specimen is perfectly accurate, the ratio should have a value of one; the method that predicts the life of a specimen more accurately should yield a ratio closer to one than that yielded by the rival method. Figure 13 shows these effects and Table II illustrates the results obtained for the first test run, group 1.

Specimen No.	Actual Cycles	RMS Cycles Predicted	Ratio RMS / Actual	Histogram Cycles Predicted	Ratio Histogram / Actual
10	399,780	265,970	.665	252,560	.632
11	420,040	280,600	.668	265,890	.633
12	391,810	282,250	.720	267,390	.682
13	401,290	275,670	.687	261,390	.651
15	401,170	283,920	.708	268,910	.670
42	422,000	285,580	.677	270,430	.641
43	398,000	283,920	.713	268,910	.676
44	420,520	299,160	.711	282,800	.673
45	399,000	290,630	.728	275,030	.689
46	396,700	262,790	.662	249,660	.629
	Mean		.694		.658
	Standard Deviation		.025		.023

TABLE II Test 1 Results

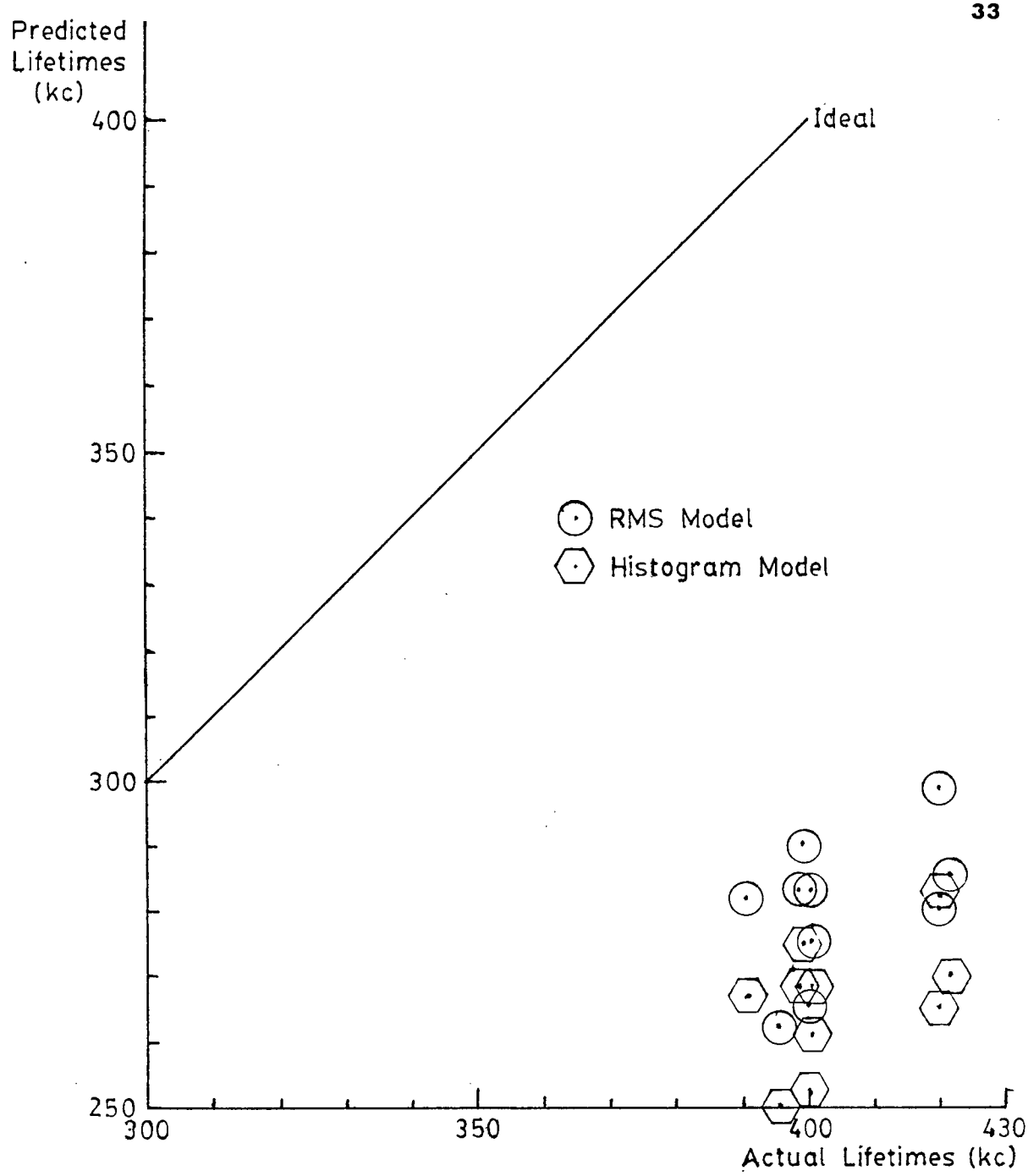


FIGURE 13 A Set of Observations (Test I)

The last five observations for test 1 were not used in any of the following analysis. The reason for this is that a statistical test, known as a sample test, shows that the dispersion and mean of these five observations is different than that of the first ten observations. The sample test used was a rank sum[18], using significance levels (the meaning of which is discussed below) of 0.05 and 0.1. The reason for this rejection is unknown, but may be attributed to some improper control settings on the MTS.

The next statistical test performed was designed to determine whether the observations are independent of one another. The test used was the Label test[19], which is performed by considering the sequence of results as a variable and applying a test of independence to the sequence. This produces a sample correlation coefficient which is used to analyze whether there is a relationship between the set of numbers. The Label test can only "reject" or "not reject" (that is, show that the data does not conflict with) the hypothesis that the data is random. This is known as a significance type of test, since no alternative hypothesis can be selected.

For all statistical tests, a significance level " α " must be selected. The higher the significance, the more significant is the test, with values of .01, .05 and .1 being typical values for "α". No sample is rejected for a value for " α " equal to zero. The value of "α", along with

the sample size, is used to select a critical value of a parameter that is used to "reject" or "not reject" a hypothesis. For the Label test, this is the sample correlation coefficient. If it is less than the critical value, then the hypothesis that the data is not random cannot be rejected. An example of a Label test is shown below for the data from group test 1.

Label (x)	Ratio (y) $\frac{RMS}{Actual}$
1	.665
2	.668
3	.720
4	.687
5	.708
6	.677
7	.713
8	.711
9	.728
10	.662

	Label	Ratio
Mean	5.5	.694
Standard Deviation	3.03	.025
Σ(Label)(Ratio)	38.37	

$$\rho = \frac{\Sigma xy - \bar{x}\bar{y}n}{(n-1)S_xS_y} = \frac{38.37 - 5.5(.694)(10)}{(10-1)(3.03)(.025)}$$

$$= .295$$

For $\alpha = .20$ and $n = 10$: $\rho_c = .443$

Hence, $\rho < \rho_c$

Therefore, the hypothesis that each reading is independent of the previous results cannot be rejected.

TABLE III Test 1 Independence Test

The group of ratios were then fitted to the Normal distribution model selected for its ease of use in hypothesis testing. The Kolmogoroff "goodness-of-fit" test[20] was carried out to see how well the distribution fitted. The purpose of this test is to find the maximum absolute difference between sample cumulative distribution

and the cumulative distribution predicted by a distribution model (see Table IV). This test is also a significance test and a critical value must be selected for a given significance and sample size. If the largest difference is less than the critical value, the hypothesis that the data fits a Normal distribution cannot be rejected.

The final analytical procedure was a test of which fatigue method fitted the data better. It was performed on the hypothesis that the two distributions from the ratios resulting from the RMS and Histogram methods could be distinguished from one another, as opposed to the hypothesis that no difference could be detected.

This is a hypothesis test and it differed from the significance tests previously described in that two hypotheses are proposed. The charactistic behind the test is the acceptance of one hypothesis and the rejection of the other. Again, a critical value must be selected to determine which hypothesis is correct. The critical value is determined according to sample size and the possibility of error in determining the result. For the hypothesis test however, there are two possible types of error. One is called type I error and its probability is assigned the symbol "α". Type I error occurs when the first hypothesis should be accepted but is not. The second type of error, type II, occurs when the first hypothesis should be rejected but is not. Type II error is given the symbol "β" and it ,

Class i	Observed Value x_i	Observed Frequency n_i	Cumulative Frequency Σn_j	Sample Dis. $\Sigma n_j / n$	Normal Value z_i	Normal c.d.f. F_N	Absolute Difference $\lvert \Sigma n_j/n - F_N \rvert$
1	.6624	1	1	.1	-1.270	.1020	.0020
2	.6653	1	2	.2	-1.555	.1240	.0760
3	.6680	1	3	.3	-1.045	.1480	.1520
4	.6767	1	4	.4	- .696	.2432	.1568
5	.6870	1	5	.5	- .285	.3859	.1141
6	.7078	1	6	.6	.553	.7098	.1098
7	.7114	1	7	.7	.695	.7564	.0564
8	.7134	1	8	.8	.774	.7806	.0194
9	.7204	1	9	.9	1.055	.8542	.0458
10	.7284	1	10	1.0	1.377	.9158	.0842

TABLE IV Normal Model Fit Test for Test 1

like type I, should be as small as possible.

For type I error, the critical value is calculated on sample size and the desired magnitude of the error. Type II error depends on sample size, magnitude of type I error and the nature of the hypothesis. Type II errors are difficult to determine and are calculated only for those tests for which the diagram showing the amount of type II error in reference 21 applies. An example of a two-sided hypothesis test for the data obtained in test 1 follows:

Hypothesis H_0: $\mu_x - \mu_y = 0$

Hypothesis H_1: $\mu_x - \mu_y \neq 0$

Let type I error be no greater than 0.05

Let type II error be no greater than 0.05

With every value used to estimate the mean "μ_x" having a corresponding value used to estimate the mean "μ_y", obtained from the same test piece, the method of correlated pairs can be used to select one of the hypothesis[22].

For test 1:

For $\alpha = 0.05$, $\beta = 0.05$, and n = 10 (d.o.f.; $\upsilon = 9$)

From reference 21: $\Delta/\sigma_d\sqrt{2} = 2.87$

where "Δ" is the critical value for test statistics.

Test statistic: $\bar{d} = \bar{x} - \bar{y} = 0.03643$

$\sigma_d = s_d = 0.00213$

Hence, $\Delta = 0.009$ and is less than the test statistic "$\bar{d}$".

Therefore, hypothesis H_0 is rejected in favor of

hypothesis H_1, that is; $\mu_x - \mu_y \neq 0$

Results

The first results (see Tables II, V) show the ratio obtained by dividing the expected lifetime by the actual lifetime for each of the specimens. The mean and the standard deviation are also given.

Specimen No.	Actual Cycles	RMS Cycles Predicted	Ratio RMS / Actual	Histogram Cycles Predicted	Ratio Histogram / Actual
10	399,780	265,970	.665	252,560	.632
11	420,040	280,600	.668	265,890	.633
12	391,810	282,250	.720	267,390	.682
13	401,290	275,670	.687	261,390	.651
15	401,170	283,920	.708	268,910	.670
42	422,000	285,580	.677	270,430	.641
43	398,000	283,920	.713	268,910	.676
44	420,520	299,160	.711	282,800	.673
45	399,000	290,630	.728	275,030	.689
46	396,700	262,790	.662	249,660	.629
	Mean		.694		.658
	Standard Deviation		.025		.023

TABLE II (repeated) Test 1 Results

Specimen No.	Actual Cycles	RMS Cycles Predicted	Ratio RMS / Actual	Histogram Cycles Predicted	Ratio Histogram / Actual
48	681,820	587,850	.862	530,370	.778
49	751,010	563,130	.750	508,900	.678
50	749,000	521,230	.696	472,070	.630
51	848,570	574,020	.676	518,350	.611
52	737,930	604,800	.820	545,080	.739
53	846,000	610,530	.722	550,060	.650
54	747,600	557,760	.746	504,230	.674
55	785,390	604,800	.770	545,080	.694
56A	594,640	536,640	.902	485,740	.817
56B	741,600	607,660	.819	547,570	.738
	Mean		.776		.700
	Standard Deviation		.073		.066

TABLE V Test 2 Results

The results are far from the ideal ratio of one. To discover why, a sensitivity analysis was conducted to show the effects that changes in the input and material parameters have on the results. The results are shown in Table VII (following page) and it is submitted that they demonstrate that changes in the Paris Law parameters cause the greatest changes in the results. To see how such changes would change the predicted lifetimes, the data for test two was recalculated, but using Paris Law parameters obtained from literature[23] (see Table VI).

Specimen No.	Actual Cycles	RMS Cycles Predicted	Ratio RMS / Actual	Histogram Cycles Predicted	Ratio Histogram / Actual
48	681,820	763,850	1.120	695,990	1.021
49	751,010	732,410	.975	668,310	.890
50	749,000	746,260	.996	680,500	.909
51	848,570	785,380	.926	714,960	.843
52	737,930	679,040	.920	620,800	.841
53	846,000	792,670	.937	721,380	.853
54	747,600	725,570	.971	662,280	.886
55	785,390	785,380	1.000	714,960	.910
56A	594,640	698,680	1.175	638,420	1.074
56B	741,600	789,020	1.064	718,160	.968
	Mean		1.008		.919
	Standard Deviation		.086		.078

TABLE VI Test 2 Results (using published Paris Law parameters)

Using the published Paris Law parameters results in ratios that are closer to the ideal ratio of one. Though the conclusions about which method produces better lifetime predictions does not change for these values (see Table X,

n Δ%	Cycles Δ%
-3.03	29.98
-1.52	13.90
0.	0.
1.52	-12.28
3.03	-23.04

Paris Law Parameter 'n'

Log(A) Δ%	Cycles Δ%
1.01	30.00
0.51	18.18
0.	0.
-0.51	- 7.14
-1.01	-18.75

Paris Law Parameter 'Log(A)'

a_i Δ%	Cycles Δ%
-1.14	3.87
-0.57	1.92
0.	0.
0.57	-1.89
1.14	-3.75

Initial Crack Length 'a_i'

K_{1c} Δ%	Cycles Δ%
-10.	-2.03
- 5.	-0.88
0.	0.
5.	0.69
10.	1.23

Fracture Toughtness 'K_{1c}'

K_{TH} Δ%	Cycles Δ%
-16.67	2.53
- 8.33	1.06
0.	0.
8.33	-1.16
16.67	-2.10

Fracture Threshold 'K_{TH}'

Footnotes: 1. $\Delta\% = \frac{\text{Changed Parameter - Base Parameter}}{\text{Base Parameter}} \times 100$

Base Parameters: $a_i = 0.700"$
$K_{1C} = 40.\ \text{ksi}.\sqrt{\text{in.}}$
$K_{TH} = 6.0\ \text{ksi}.\sqrt{\text{in.}}$
$n = 3.3$
$\text{Log}(A) = -9.9$

TABLE VII Results from the Sensitivity Analysis

page 46), it does show how critical it is to use accurate values for the Paris Law parameters in order to obtain valid conclusions about method accuracy.

The next set of tables (page 44) shows goodness-of-fit results for the Normal distributions used at a significance level of 0.20. It should be noted that the test was based only on the results for one set of Paris Law parameters. It is submitted that changing the parameters will not change the shape of the distribution curve, since shape would depend more on the variation of the experimental results.

Table IX (page 45) illustrates that the observations obtained are independent of one another.

The last set of results, (table 19, page 46) reveals which method produced better results in each set of experiments.

Data in table X summarizes the differences observed between the histogram and RMS lifetime ratios. It also contains the results of the hypothesis tests (see treatment of data) that were conducted to determine whether or not these differences are significant. The differences are shown to be significant, using limits of 0.05 for type I and II error. For test one and two this means that the method that produces a lifetime ratio closer to the ideal ratio of one is more accurate. For these cases, this method is the RMS one.

Class i	Observed Value x_i	Observed Frequency n_i	Cumulative Frequency Σn_j	Sample Dis. $\Sigma n_j / n$	Normal Value z_i	Normal c.d.f. F_N	Absolute Difference $\lvert \Sigma n_j/n - F_N \rvert$
1	.6624	1	1	.1	-1.270	.1020	.0020
2	.6653	1	2	.2	-1.555	.1240	.0760
3	.6680	1	3	.3	-1.045	.1480	.1520
4	.6767	1	4	.4	- .696	.2432	.1568
5	.6870	1	5	.5	- .285	.3859	.1141
6	.7078	1	6	.6	.553	.7098	.1098
7	.7114	1	7	.7	.695	.7564	.0564
8	.7134	1	8	.8	.774	.7806	.0194
9	.7204	1	9	.9	1.055	.8542	.0458
10	.7284	1	10	1.0	1.377	.9158	.0842

TABLE IV (repeated) Normal Model Fit Test for Test 1

Class i	Observed Value x_i	Observed Frequency n_i	Cumulative Frequency Σn_j	Sample Dis. $\Sigma n_j / n$	Normal Value z_i	Normal c.d.f. F_N	Absolute Difference $\lvert \Sigma n_j/n - F_N \rvert$
1	.6765	1	1	.1	-1.367	.0858	.0142
2	.6959	1	2	.2	-1.101	.1355	.0645
3	.7217	1	3	.3	- .748	.2272	.0728
4	.7461	1	4	.4	- .414	.3394	.0606
5	.7498	1	5	.5	- .363	.3583	.1417
6	.7701	1	6	.6	- .008	.4960	.1040
7	.8194	1	7	.7	.589	.7221	.0221
8	.8196	1	8	.8	.591	.7227	.0773
9	.8622	1	9	.9	1.174	.8798	.0202
10	.9025	1	10	1.0	1.725	.9591	.0409

TABLE VIII Normal Model Fit Test for Test 2

	No. of Samples n	Significant Level α	Sample Correlation Coefficient ρ	Critical Correlation Coefficient ρ	Is $\rho < \rho$?	Comments
Test 1	10	.20	.295	.433	Yes	H_0 cannot be rejected
Test 2	10	.20	.292	.433	Yes	H_0 cannot be rejected

H_0: That each reading is independent of previous results.

Table IX Data Independence Test Results

Test	Type I Error α	Type II Error β	n	$\bar{x}_o$	$\bar{x}_1$	$\bar{x}_d = \bar{x}_o - \bar{x}_1$	s_d	Critical Point $\mu_o - \mu_1$	Is $\bar{x}_d$ > Critical Point ?	Comments
1	.05	.05	10	.694	.658	.036	.002	.009	Yes	Accept H_1
2	.05	.05	10	.776	.701	.075	.007	.030	Yes	Accept H_1
2Pub	.05	.05	10	1.008	.919	.089	.007	.030	Yes	Accept H_1

Hypothesis: $H_0: \mu_0 - \mu_1 = 0$

$H_1: \mu_0 - \mu_1 = 0$

Notes: The variables with subscribt '0' represent the lifetimes for the RMS method. The variables with subscript '1' represent the lifetimes for the Histogram method. The variables with subscript 'd' represent the differences in the lifetimes between the two methods.

Table X Results from the Statistical Analysis

Discussion

A brief comment should be made on the results obtained from the significance tests that were conducted to test the quality of the experiment. For example, the significance test for independence of observations shows that the experiment has little or no bias, since the order of the data appears independent for a reasonable given significance level. The selection of the Normal distribution also appears to be a good choice, as confirmed by the significance level used for the Kolmogoroff test.

The results from tests of fatigue method superiority indicates that the RMS method is superior for the load distributions tested. However, the lifetime prediction produced by either method is quite inconsistent with actual lifetimes measured in the experiment. The sensitivity analysis suggests that the most likely source of error is the estimate of the Paris Law parameters.

Other possible sources of error do not have such a large effect on the lifetime prediction of the specimen. For example, Table VII (page 42) shows that changing the fracture toughness of the material has little effect on the lifetime prediction of the specimen. This is to be expected, since most of the lifetime of the specimen occurs when the crack is short and crack propagation speed is low. That means that initial crack length can have a considerable effect on the life of a specimen, and this is confirmed by

the sensitivity analysis. The accuracy in the measurement of initial crack length in the experiment is high, so that this is an unlikely source of error. Sensitivity analysis also shows that small changes in the stress intensity threshold have little effect on the predicted life of a specimen.

In summary, the bias seen in the results probably results from error in the estimation of the Paris Law parameters. The source of this error is probably errors incidental to determining the change in crack length that has occurred in a given number of cycles. In the procedure used to determine the Paris Law parameters, changes in crack length "da" were quite small, usually about a hundredth of an inch or less, but error in the measurement of change in crack length could be as high as two thousandths of an inch, that is, an error of as much as twenty percent. Error of this magnitude would possibly produce bias in parameter estimation[24]. However, if enough measurements were taken of the crack growth rate, the error in determining "da" could be reduced. The bias observed in the lifetime predictions made the Paris Law parameters obtained for this experiment, suggest that too few measurements were taken.

The error in the Paris Law parameters is probably not great, since by changing one of the parameters obtained in this work by its 80% confidence limit, the parameters match some of the results found in the literature[25]. The result

is that the new predicted lifetimes are much closer to the actual lifetimes measured. However, the RMS method still gives better results.

The results obtained in this work show that the RMS method gives a higher accuracy than the Histogram method. This result conflicts with the basic notion that methods that incorporate more influential parameters than similar rival methods should be more accurate. For this case, the Histogram method incorprates a "stress intensity threshold", that the RMS method does not. The net result is a RMS method that neglects stresses that do not cause crack growth. Therefore, it would be expected that the Histogram method would give better results. The fact that it does not can be explained by the following.

The most reasonable explaination is that the Paris Law parameters used are biased. It has already been shown that a small change in a Paris Law parameter can produce large variations in the resulting lifetime predictions. It also has been demonstrated that the lifetime predictions obtained were not highly accurate. It is therefore submitted that the lifetime predictions made in this thesis would possibly support the Histogram method if "correct" values for the Paris Law parameters were used.

Conclusion

The various statistical methods used to test the experimental procedure suggest that it was sound and that its results should be accurate. The lifetimes predicted by both methods for any specimen were considerably different from the actual lifetimes measured, due to suspected error in the Paris Law parameters. The use of Paris Law parameters published in literature confirmed this suspicion.

The results show that the RMS method is superior to the Histogram method. This result was not expected, since by including an extra parameter, the "stress intensity threshold", the Histogram method should model reality more accurately. The most likely reason given for it not doing so was that the Paris Law parameters used in the lifetime predictions were biased.

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Appendix I

Material Properties

In order to obtain an accurate analysis of the data presented in this thesis, a detailed knowledge of the material properties of the metal being tested was necessary. These had to be determined from seperate tests rather than using published values, which are for a large number of specimens taken from many different plates. The specimens used in the experiments came from one small section of a large plate, so the material property values obtained and used herein should be more specific than those found in literature. Test results were also useful for comparison with published values as a check on the techniques used by the author.

The material properties required for the analysis of the experiment are fracture toughness, threshold stress intensity and the constants "n" and "A" for the Paris Law equation. Other material properties measured were yield strength, ultimate strength and Young's Modulus. A microstructure analysis was also carried out. Details of the measurements and the results obtained are described below.

Yield Strength, Ultimate Strength and Young's Modulus

A tension test was performed on the test specimens machined to the specifications shown in figure 14 to determine yield strength, ultimate strength, and Young's Modulus. The tests were performed with the Tinus Olsen materials testing machine at the Department of Mechanical Engineering.

Procedure

Before tension tests were began, it was necessary to select a strain rate that would produce consistent results. It is known[26] that a rate of between .01/s and .0001/s has little or no effect on the stress-strain curve at room temperature. Accordingly, a rate of approximately .001/s was selected for these tests.

Each specimen was then measured and loaded into the machine. Testing was carried out in two stages. The first stage consisted of recording the load vs. the displacement of the specimen using the Tinus Olsen model S-1000-2A strain gauge, which is accurate to .0001 inch, giving an accurate measure of Young's Modulus. The Tinus Olsen was then stopped and the strain gauge was removed. The second stage consisted of the same recording procedure but utilized the Tinus Olsen type D-2 Deflectometer, which measures the entire load vs. the displacement curve. Both curves were recorded on the same sheet of graph paper and were used to determine the various loads and displacements required to

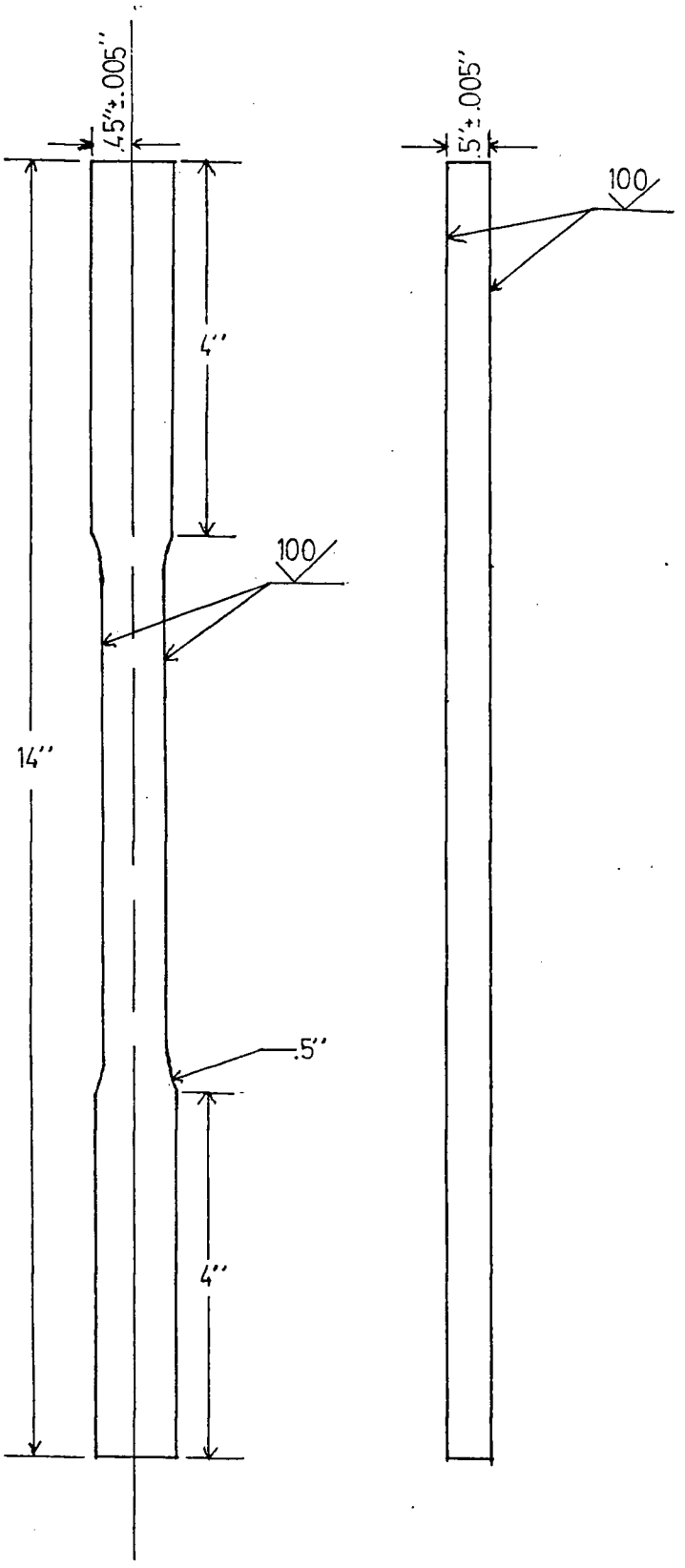


FIGURE 14 Tension Test Specimen

calculate the material properties.

Analysis

For a simple tension test, Young's modulus can be determined quite easily as the slope of the line shown in figure 15. This simple calculation is shown in figure 15, as are the yield and the ultimate loads, the initial and final dimensions of the specimen and the calculations required to determine the yield and ultimate strength of the material.

Results

The results from six tests are summarized in Table XIV. An analysis was performed on the data assuming it fitted a Log-Normal distribution[27]. The results are summarized in Table XIV.

Determining Fracture Toughness

The fracture toughness of the steel used for the experiments was determined using a standard compact tension specimen (see figure 8). The test machine used was the Materials Testing System MTS 810 (referred to as just the MTS) operated by U.B.C.'s Department of Metallurgy Engineering.

Procedure

The test procedure followed was that described in the ASTM E399-72 standard "Plane-Strain Fracture Toughness of Metallic Materials"[28], using the MTS to apply the loads

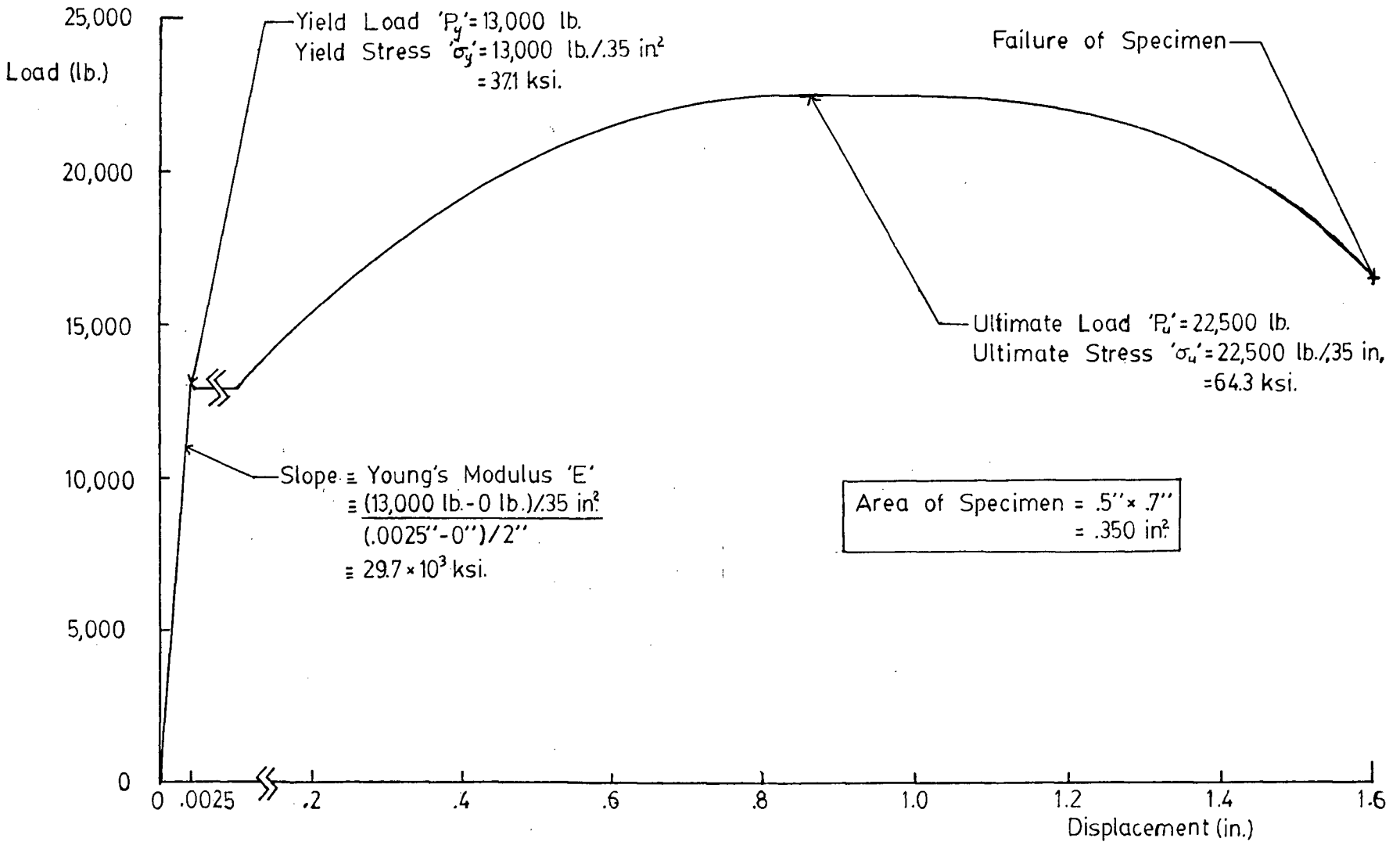


FIGURE 15 Load vs. Displacement Record for a Tension Test

Test Specimen	Initial Area A_i (in**2)	Initial Length L_i (in)	Final Length L_f (in)	Percent Elongation	Yield Load P_y (lb)	Ulimate Load P_u (lb)	Yield Stress σ_y (ksi)	Ulimate Stress σ_u (ksi)	Young's Modulus (ksi) E
A	.351	6.	7.60	21 %	13,000	22,200	37.0	63.2	N.M.[1]
B	.350	6.	7.60	21 %	12,750	22,000	36.4	62.9	N.M.
C	.357	6.	7.65	22 %	15,000	22,900	42.0	64.1	31,700
D	.361	6.	7.65	22 %	13,800	22,400	38.2	62.0	31,600
E	.362	6.	7.50	20 %	15,000	22,800	41.4	61.9	28,900
F	.362	6.	7.50	20 %	15,400	22,800	42.5	63.0	32,200
					Expected Value:		39.6	62.9	31,100
					Standard Deviation:		2.7	0.8	1,500

Footnotes: 1. Not Measured (N.M.)

TABLE XI Results From the Tension Tests

and a Barson-Turner Instruments series 8000 DataCenters Program Version 4 to record the load vs. crack displacement. Crack displacement was measured with a Instrom Crack Opening 2670-004.

One non-standard modification was made, reducing specimen thickness by a factor of about one-half the standard, so that specimens would have a larger crack growth region for use in the experiment. The chances of the plane strain conditions of the standards being met were slim, so these conditions were ignored. Thus, instead of measuring the plane strain fracture toughness, which is a material constant, the fracture toughness value obtained was a function of the test specimen geometry, which remained the same throughout the experiment, except for a change in crack length during the test. The results of the fracture toughness tests suggest that there was little change in fracture toughness due to different crack lengths.

Analysis

The analysis required to determine the fracture toughness of the specimen is described clearly in the ASTM standards. However, a brief discussion of the procedure follows for convenience.

The first step is to determine crack length by measuring it directly with a vernier caliper at three locations along the crack front. These locations are the quarter, the half and the three-quarter points along the

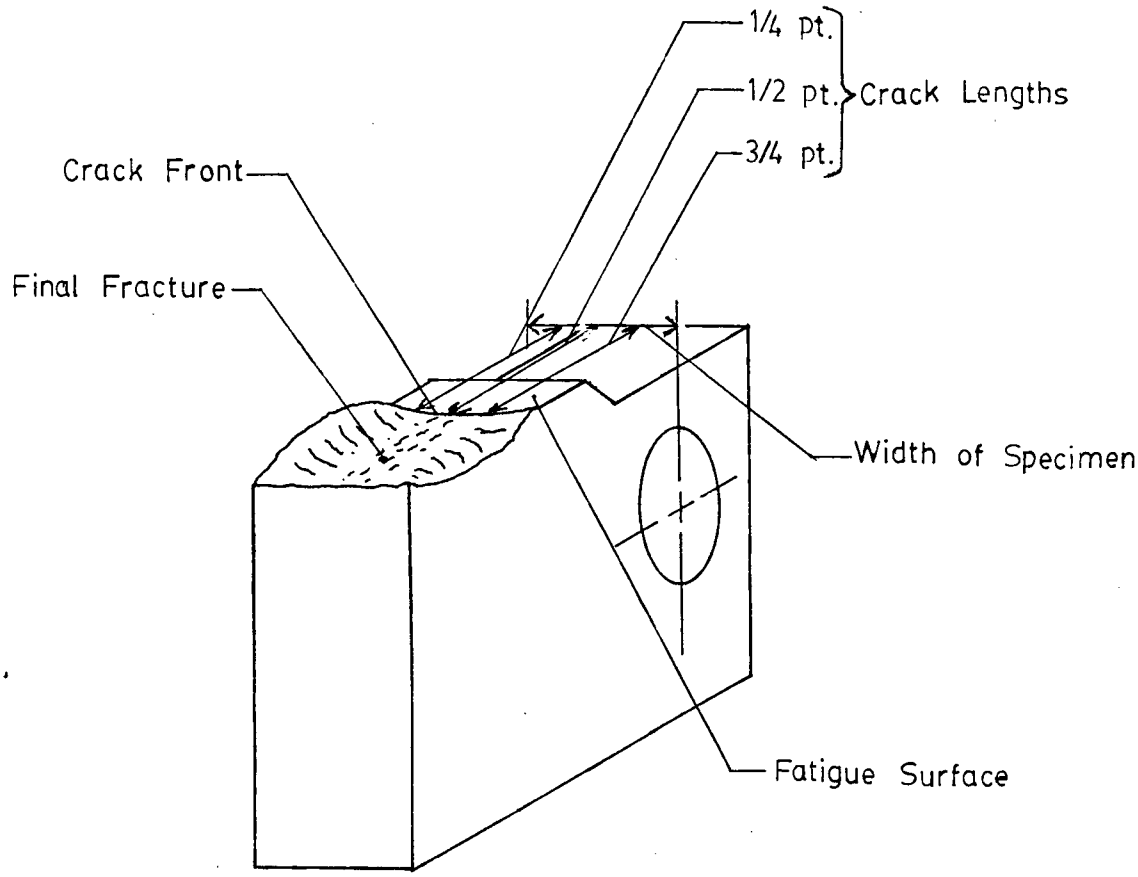


FIGURE 16 Definition of Crack Length

length of the crack front (see figure 16).

The average of these crack length values is nominated the crack length for the specimen. However, if the length of the crack deviates more than five percent along the crack front, as compared to the average length of the crack, the specimen cannot be used to determine fracture toughness.

Once crack length has been determined, the next step is to calculate the fracture load by determining the initial slope of the load vs. the crack displacement plot shown in figure 17, determining the 95 percent slope and drawing it on the plot. The fracture load of the specimen is that value at the intersection of this slope and the load vs. the crack displacement curve. If the fatigue load used to create the initial crack is less than or equal to 60 percent of the fracture load, then the fracture test meets the load criteria of the standard and is acceptable.

The last step in the analysis is to calculate fracture toughness, which can be done by employing the equation shown in figure 9. For the load and crack length values determined by the above procedure: (see page 63)

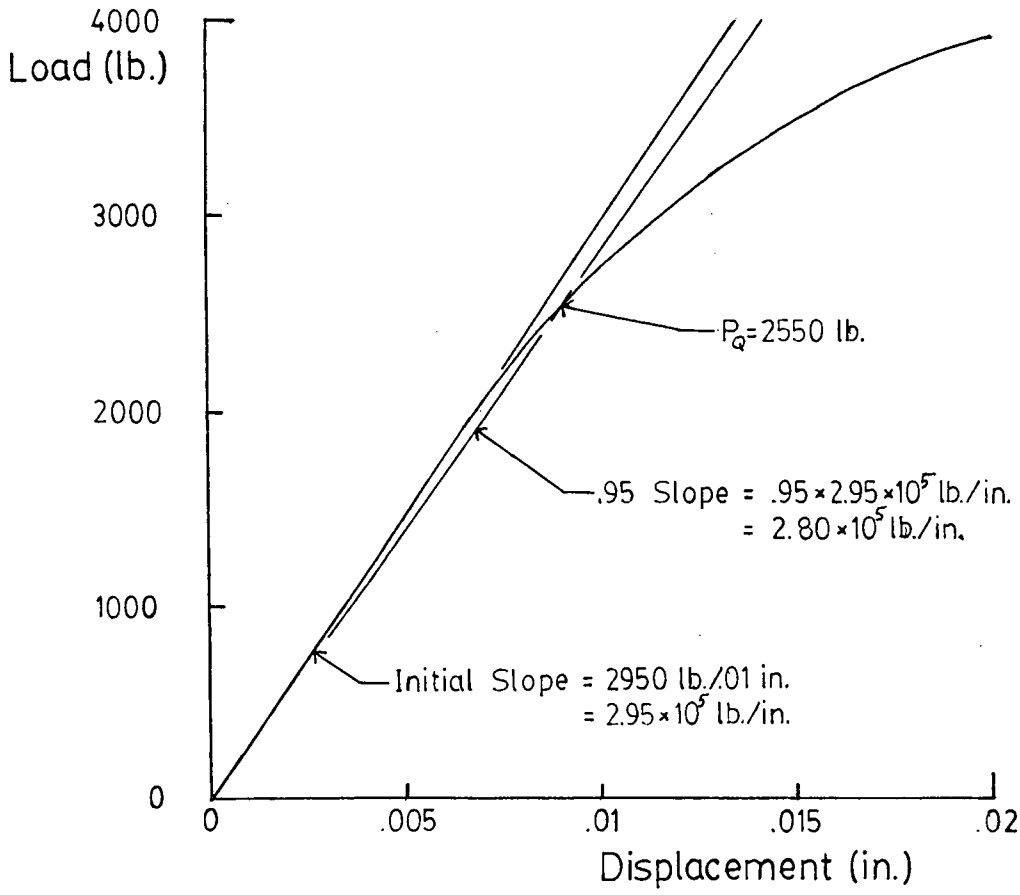


FIGURE 17 Load vs. Displacement Record for a Fracture Test

$$K_I = \sigma\sqrt{a}\, F_1(a/b)$$

$$F_1(a/b) = 29.6 - 185(a/b) + 655.7(a/b)^2 - 1017(a/b)^3 + 638.9(a/b)^4$$

for a = .735"; b = 2." $\longrightarrow$ a/b = .368

$F_1 = 11.3$

for P = 4.25 kips; t = .510"

$\sigma = P/bt = 4.25 \text{ kips}/((2")(.510")) = 4.167 \text{ ksi}$

Hence, $K_{IC} = 4.167 \text{ ksi}\sqrt{.735"}(11.3) = 40.3 \text{ ksi}\sqrt{\text{in}}$

Results

The results from the five tests are summarized in Table XV.

The analysis was carried out by assuming that the data fits a Log-Normal distribution[29]. The results of the analysis are also shown in Table XV.

Threshold Stress Intensity

Threshold stress intensity is critical in testing the Histogram and the RMS methods for predicting specimen life; unfortunately, it is also one of the hardest material properties to measure.

Procedure

The MTS testing machine was used to carry out this test as well and was set to the constant stroke mode, which causes the MTS to decrease the load as the crack grows, so that the crack displacement (stroke) on the specimen remains constant.

Test No.	Crack Length (in) 1/4 pt. (on the specimen)	1/2 pt.	3/4 pt.	Ave. Crack Length (in)	Crack Criterion Met ?	Slope (kips/in)	95% Slope (kips/in)	Fracture Load P_f (kips)	Maximum Load P_F (kips)	Is P_f / P_F < .6 ?	Fracture Toughness (ksi$\sqrt{in}$)
16	.740	.734	.725	.735	Yes	372	354	4.25	2.50	Yes	40.3
19	.720	.717	.716	.718	Yes	453	430	4.20	2.50	Yes	39.0
20	.771	.766	.758	.765	Yes	457	430	4.10	2.50	ok	40.3
34	.972	.967	.961	.967	Yes	260	247	3.10	1.68	Yes	39.7
35	1.009	1.020	1.014	1.014	Yes	254	241	2.85	1.68	Yes	39.1
										Expected Value:	39.7
										Standard Deviation:	0.6

TABLE XII Results From the Fracture Tests

Since load is decreased as crack length increases, there is an instant in the growth of a crack where stress intensity, upon which the crack tip becomes equal to threshold stress intensity, and the crack stops growing. By recording the load and the length of the crack, threshold stress intensity can be determined. An accurate means of measuring crack length can be obtained by breaking the specimen in half with a large force and measuring the fatigue crack using the same technique used in the fracture toughness test.

Unfortunately in this case, this procedure simply did not work. The MTS was not stable enough to keep the displacement at the crack opening constant at the low loads required for the test, so the procedure was changed to the simple alternative of measuring average grain size in the composition of the metal. A picture of the grain structure was made according to the instructions laid out in the section of this appendix treating the subject of grain structure. A section of the picture was marked off and the number of grains within the section were counted. The average area of each grain was calculable once the area of the section and the number of grains per section were determined. If it is assumed that the average grain is round, an average grain radius can be calculated. According to reference 30, the plastic zone has a diameter of the size of the average grain when the threshold stress intensity is

reached. Assuming the plastic zone is also round, threshold stress intensity may be calculated as follows.

Analysis

From the photograph in figure 21:

Average diameter of grain: .0067"

Radius of plastic zone: $r_p = K_I^2/\pi\sigma_{ys}^2$

Hence, $K_{TH} = \sqrt{r_p \pi \sigma_{ys}^2} = \sqrt{.0067" \times \pi \times (39.6ksi)^2} = 5.7$ ksi$\sqrt{in}$

Results

The result of 5.7 ksi in is within five percent of the value published in reference 31 for mild steels of the type used in the experiments.

The Paris Law Parameters

The Paris Law states that the rate of crack growth is directly proportional to the nth power of the stress intensity factor. This means that two parameters have to be determined in order to calculate the rate of crack growth. The parameters are the proportional factor "A" and the power factor "n", and are constants for a given material.

Procedure

To determine the parameters, a standard test specimen was precracked and the crack length was measured on both sides of the specimen with an optical crack measuring device made by Grertner Scientific Corporation and having an accuracy of approximately one thousandth of an inch. After the crack had been measured, the specimen was placed back

into the MTS, the crack measuring device was set up to measure relative crack growth on the specimen and the specimen was then placed under a constant cyclic load. When cyclic loading had begun, a reading was taken from the crack measuring device and the cycle counter on the MTS machine. After sometime, the crack in the specimen was measured to record how much it had grown, and the number of cycles applied to the specimen to that point was recorded. These measurements were taken intermittenly during crack growth, from precrack length until the crack was growing in a unstable manner.

Analysis

The first task in determining the Paris Law parameters is to transform raw data into stress intensity factors and crack growth rate values. The latter is the easiest of the two values to calculate. The difference between two crack lengths gives the change in length of the crack: "da". Dividing "da" by the number of cycles recorded to grow the crack by "da" produces the crack growth rate "da/dN". Examples of these calculations are shown in figure 18.

To calculate the related stress intensity, the total crack length to that instant must be determined from the readings obtained from the crack measuring device. Once crack length is known, the stress intensity factor can be calculated by using the equation shown in figure 9. Examples of these calculations are also shown in figure 18.

CRACK LENGTH READING	NO. OF CYCLES COUNTER
58.370	0.
59.173	21800.
59.559	31800.
59.695	37300.
59.897	44200.
60.165	50600.
60.428	57800.
60.628	63800.
61.141	72900.
61.395	78900.
61.625	84000.
61.973	90200.
62.140	95400.
62.442	100400.
62.873	105000.
63.007	106800.
63.133	107600.
63.207	109100.
63.496	111300.
63.534	113000.
63.649	114600.
63.761	116100.
.	.
.	.
.	.

Initial crack length 'a_0' = .636"
Load 'P' = 1.88 kips

$da = (59.173\,mm - 58.370\,mm)/(25.4\,mm/in) = .032"$

$dN = 21{,}800 - 0 = 21{,}800$ cycles

$da/dN = .032"/21{,}800\ cycles = 1.450 \times 10^{-6}\ in./cycle$

$\therefore Log(da/dN) = -5.8386$

$a = .636" + .032"/2 = .652$ inches

From Figure $b = 2"$

$$K_1 = P\sqrt{a}\,[29.6 - 185.5(a/b) + 655.7(a/b)^2 - 1017.(a/b)^3 + 638.9(a/b)^4]$$
$$= 16.372\ ksi\sqrt{in}$$

$\therefore Log(K_1) = 1.2144$

$LOG(K_1)$	$LOG\left(\frac{da}{dN}\right)$
1.2144	-5.8386
1.2254	-5.8182
1.2303	-6.0117
1.2336	-5.9383
1.2381	-5.7829
1.2433	-5.8422
1.2479	-5.8820
1.2551	-5.6538
1.2629	-5.7781
1.2679	-5.7507
1.2739	-5.6556
1.2793	-5.8981
1.2842	-5.6238
1.2920	-5.4331
1.2981	-5.5330
1.3009	-5.2076
1.3031	-5.7117
1.3070	-5.2864
1.3123	-5.5483
1.3148	-5.5317
1.3182	-5.4833
.	.
.	.
.	.

FIGURE 18 Raw Data Transformation to Stress Intensity & Crack Growth Rate Data

Once the stress intensity factor and crack growth rate are known, the Paris Law parameters can be determined by plotting the crack growth rate against the stress intensity factor on log paper. A straight line should be produced and by using linear regression techniques[32], the Paris Law parameters "n" and "A" can be determined.

Results

A plot of all the points used in the linear regression and the results of the analysis are shown in figure 19.

Grain Structure

It was decided that the metal used for the specimens should be subjected to a optical grain structure analysis to determine the rolling direction of the plate, since the orientation of a specimen to the rolling direction would effect the values obtained for the material properties.

Procedure

An approximately half inch by three quarter inch section of steel was cut from one specimen used for the tests and polished on one side to a one-micron surface finish. A sequence of rough polishing using wet sandpaper of 180,320,400, and 600 grits was followed by fine polishing using diamond powder of six and then one-micron size. All polishing was done in the Metallurgy polishing lab using motorized polishing wheels. At each stage, the specimen was polished in alternate directions to minimize grain

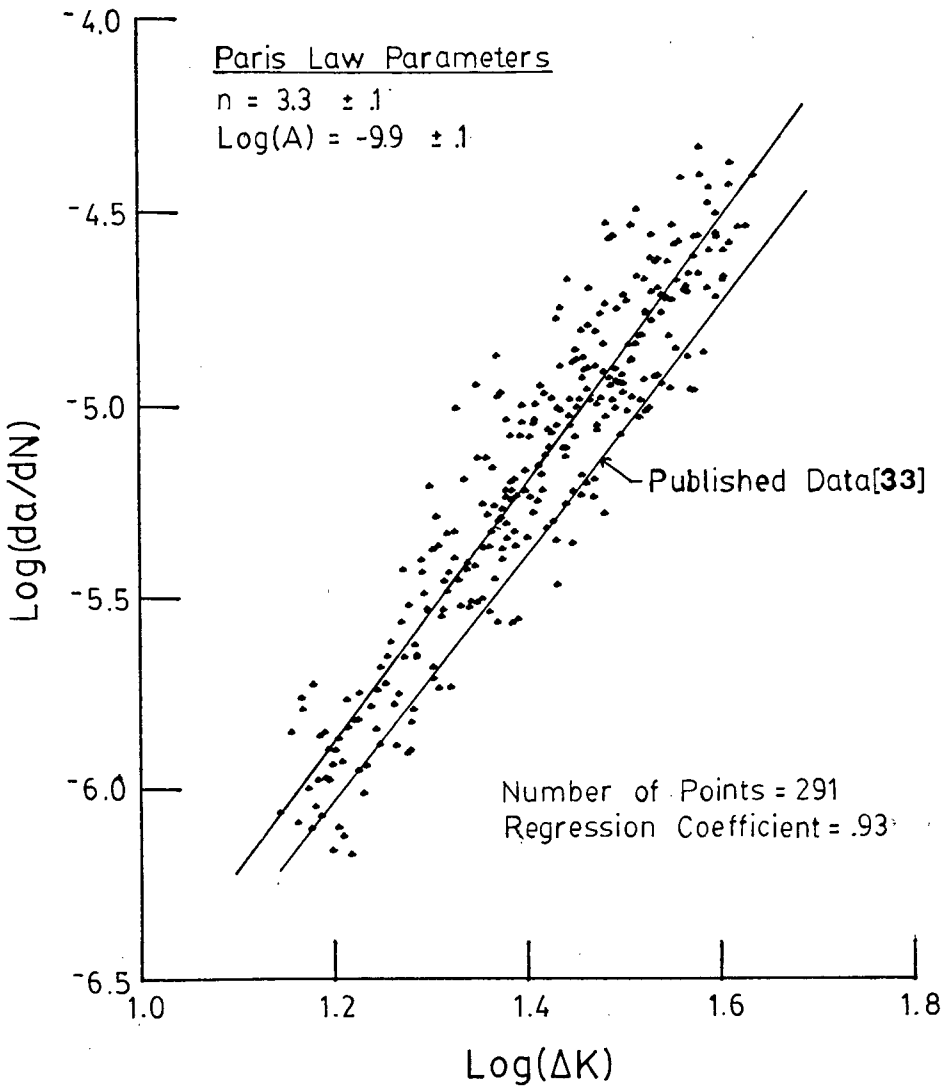


FIGURE 19 Paris Plot for A-36 Steel

distortion.

The polished surface was etched in two percent nital, a solution of two percent nitric acid in alcohol, for ten seconds, examined and photographed using a Carl Zeiss "ULTRAPHOT" Camera Microscope.

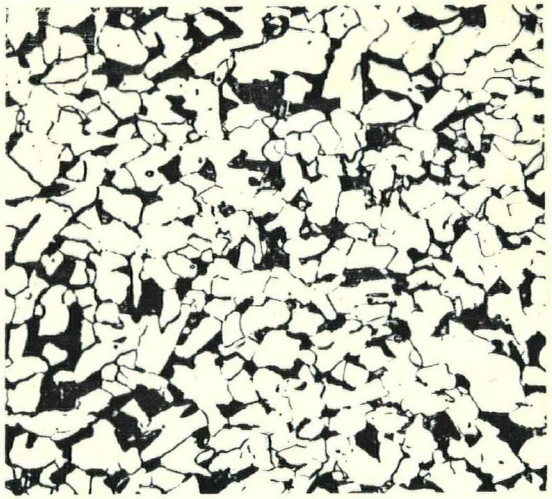
Results

The picture in figure 20 shows the grain structure of the steel used. As is obvious from the photograph, the rolling direction of the plate , which would be shown by the enlongation of the grains in a predominant direction, cannot be determined. This does not invalidate the experiments, since as long as each test specimen has the same orientation to the plate as all the others, the results will be consistent and valid.

A comparison of the photograph of the grain structure of the steel used in the experiment and a photograph of the same kind of steel taken from the Metals Handbook[34] (figure 20), reveals that the grain structure of each are similar.

2% nital 265×

Grain Structure From A-36 Steel Tested (1/2" Plate)



1% nital 250×

116 **ASTM A36** steel plate, ⅜ in. thick, as rolled. Structure consists of equiaxed ferrite (white areas) and pearlite (black areas).

From the Metals Handbook [**34**]

Figure 20 A-36 Steel Grain Structure

APPENDIX II

```
C       FORTRAN FATIGUE PROGRAM
C
C       NO. OF CYCLES TO FAILURE DUE TO
C       VARIABLE LOADIND
C
C       INPUT PARAMETERS
C
C       C1K=CRITICAL FRACTURE TOUGHNESS
C       HTH=THRESHOLD FRACTURE TOUGHNESS
C       AP=PARIS LAW CONSTANT
C       AN=PARIS LAW EXPONENT
C       INOL=NO. OF LOADS
C       ANIC=NO. OF CYCLES PER LOAD
C       XP=LOAD P
C       AI=INITIAL CRACK SIZE
C       NO=NO. OF LOAD CASES
C
        IMPLICIT REAL*8(A-H,O-Y)
        DIMENSION ANIC(30),ANN(2),XP(30),AT(30),AC(30)
5       FORMAT(I2)
7       FORMAT(F7.0,F15.4)
10      FORMAT(F15.4)
        CALL ASSIGN(1,'FATIN.DAT')
        CALL ASSIGN(2,'FATOUT.DAT')
        READ(1,5) NO
        WRITE(2,1500)NO
        DO 230 II=1,NO
        READ(1,10)C1K
        READ(1,10)HTH
        READ(1,10)AP
        READ(1,10)AN
        READ(1,10)AI
        READ(1,5)INOL
C
C       ECHO INPUT DATA
C
        WRITE(2,1510) II,C1K,HTH,AP,AN,AI,INOL
        WRITE(2,1520)
        DO 20 I=1,INOL
        READ(1,7) ANIC(I),XP(I)
        WRITE(2,1530) I,ANIC(I),XP(I)
20      CONTINUE
C
C       CALULATE CRITICAL CRACK LENGTHS
C
        A=AI
        ATMI=1000.
        DO 22 I=1,INOL
```

```
      CALL CRAC(HTH,XP(I),A,AT(I))
      IF(AT(I).LT.ATMI) ATMI=AT(I)
      CALL CRAC(C1K,XP(I),A,AC(I))
      WRITE(5,21) AT(I),AC(I)
21    FORMAT(2(1X,F7.4))
22    CONTINUE
      IF(AT.LT.ATMI) GOTO 500
C
C     HISTOGRAM METHOD
C
70    XN=0.
      J=0
      AT1=AI
      ATM=AT(1)
      ACM=AC(1)
      SC=ANIC(1)
      DO 85 I=2,INOL
      SC=SC+ANIC(I)
      IF(AT(I).GT.ATM) ATM=AT(I)
      IF(AC(I).LT.ACM) ACM=AC(I)
85    CONTINUE
      IF(ATM.GT.ACM) ATM=ACM
      AT2=ACM
      AT3=ACM
90    AL=0.
      TC=0.
      DO 100 I=1,INOL
      IF(AT(I).GT.AT1) GOTO 95
      AL=AL+ANIC(I)*XP(I)**2
      TC=TC+ANIC(I)
      IF(AT(I).LE.AT1) GOTO 100
95    IF(AT(I).LT.AT2) AT2=AT(I)
100   CONTINUE
      AL=(AL/TC)**.5
      IF((AT2.EQ.ATM).AND.(J.EQ.2)) AT2=ACM
      IF(AT1.EQ.AT2) GOTO 110
      N=20
      WRITE(5,105) AT1,AT2
105   FORMAT(2(1X,F8.4))
      CALL CRAF(AT1,AT2,AL,XN1,N,AP,AN)
      XN=XN+XN1*(SC/TC)
      IF(AT2.EQ.ACM) GOTO 110
      AT1=AT2
      IF(AT2.EQ.ATM) J=2
      AT2=ATM
      GOTO 90
110   WRITE(2,1110) XN
C
C     RMS METHOD
C
      XN=0.
      AT1=AI
```

```
      AL=0.
      TC=0.
      DO 115 I=1,INOL
      AL=AL+ANIC(I)*XP(I)**2
115   TC=TC+ANIC(I)
      AL=(AL/TC)**.5
      N=20
      CALL CRAF(AT1,AT3,AL,XN1,N,AP,AN)
      XN=XN+XN1
      WRITE(2,1120) XN
      GOTO 230
500   WRITE(2,1000)
1000  FORMAT(1X,//6X,'INITIAL CRACK LENGTH IS BELOW THE
     1THRESHOLD'/1X,'CRACK LENGTH. THEREFORE, THE LIFE
     2OF THE SPECIMEN IS'/1X,'INFINITE FOR THE HISTO-
     3GRAM AND RMS METHODS.'//)
230   CONTINUE
      CALL CLOSE(1)
      CALL CLOSE(2)
C
1110  FORMAT(1X,//6X,'THE LIFE OF THE SPECIMEN AS
     1DETERMINED BY THE',/1X,'HISTOGRAM METHOD IS:'
     2,D12.5,' CYCLES.'/)
1120  FORMAT(6X,'THE LIFE OF THE SPECIMEN AS DETERMINED
     1BY THE',/1X,'RMS METHOD IS:',D12.5,' CYCLES.'/)
1500  FORMAT(1X,//11X,'RESULTS FROM FATIGUE PROGRAM',
     1///6X,'THE NUMBER OF LOAD CASES IS: ',I2,/)
1510  FORMAT(1X,/1X,'THE INPUT DATA FOR LOAD CASE ',I2,
     1' IS:',//6X,'CRITICAL FRACTURE TOUGHNESS:',D12.5,
     2/6X,'THRESHOLD FRACTURE TOUGHNESS:',D12.5,
     3/6X,'PARIS LAW CONSTANT:',D12.5,/6X,'PARIS LAW
     4EXPONENT:',D12.5,/6X,'INITIAL CRACK SIZE:',
     5D12.5,/6X'NO. OF LOADS: ',I2,//)
1520  FORMAT(1X,'LOAD   NO. OF',7X,'LOAD',/2X,
     1'NO.   CYCLES',9X,'P',/)
1530  FORMAT(2X,I2,2D12.5)
C
      STOP
      END
C
      SUBROUTINE CRAC(FT,AL,AI,A)
C
C     SUBROUTINE TO CALCULATE CRITICAL CRACK SIZE
C     GIVEN FRACTURE TOUGHNESS 'FT' AND LOAD 'AL'
C
      IMPLICIT REAL*8(A-H,O-Y)
      CALL FRAC(AI,AL,FT1,IOS)
      IF(IOS.EQ.0) GOTO 5
      WRITE(2,7)
7     FORMAT(1X,//1X,'***WARNING*** AI IS OUT OF
     1BOUNDS'//)
5     A1=AI
```

```
      I=0
      J=0
      B=FT1-FT
      A2=2.*AI
10    CALL FRAC(A2,AL,FT2,IOS)
      IF((IOS.EQ.1).AND.(I.EQ.1)) GOTO 32
      IF(IOS.EQ.1) I=1
      C=FT2-FT
12    IF((B.GE.0.).AND.(C.LE.0.)) GOTO 32
      IF((B.LT.0.).AND.(C.GT.0.)) GOTO 32
      IF(J.EQ.1) GOTO 15
      IF(J.EQ.2) GOTO 20
      B1=DABS(B)
      C1=DABS(C)
      IF(B1.LT.C1) GOTO 20
15    A1=A2
      A2=2.*A2
      B=C
      J=1
      GOTO 10
20    A2=A1
      A1=.5*A1
      C=B
      J=2
      CALL FRAC(A1,AL,FT1,IOS)
      IF((IOS.EQ.1).AND.(I.EQ.1)) GOTO 32
      IF(IOS.EQ.1) I=1
      B=FT1-FT
      GOTO 12
32    A=(A1+A2)/2.
      T=DABS((A-A2)/A)
      IF(T.LT.5.D-06) GOTO 40
      CALL FRAC(A,AL,FT1,IOS)
      B=FT1-FT
      IF(B.LE.0.) A1=A
      IF(B.GE.0.) A2=A
      GOTO 32
40    RETURN
      END
C
      SUBROUTINE CRAF(A,A2,AL,XN,N,AP,AN)
C
C     SUBROUTINE TO CALCULATE THE NO. OF CYCLES 'DELTA
C     N' TO GROW CRACK FROM CRACK LENGTH A TO A2
C
      IMPLICIT REAL*8(A-H,O-Y)
      DIMENSION ANN(2)
      IM=1
3     CALL FRAC(A,AL,C1K,IOS)
      X0=1./(C1K**AN)
      CALL FRAC(A2,AL,C1K,IOS)
      IF(IOS.EQ.1) GOTO 40
```

```
      X0=X0+1./(C1K**AN)
      DO 25 K=IM,2
      M=K*N
      X1=0.
      X2=0.
      A1=A
      H=(A2-A)/(2.*M)
      J=2*M-1
      DO 20 I=1,J
      X=A1+I*H
      ZE1=FLOAT(I)/2.
      I1=I/2
      ZE2=FLOAT(I1)
      IF(ZE1.EQ.ZE2) GOTO 10
      CALL FRAC(X,AL,C1K,IOS)
      X1=X1+1./(C1K**AN)
      GOTO 20
10    CALL FRAC(X,AL,C1K,IOS)
      X2=X2+1./(C1K**AN)
20    CONTINUE
25    ANN(K)=H*(X0+2.*X2+4.*X1)/(3.*AP)
      T=DABS(ANN(2)-ANN(1))/ANN(2)
      IF(T.LT.3.125D-07) N=N/2+1
      WRITE(5,27) ANN(2)
27    FORMAT(1X,F10.2)
      IF(T.LT.5.D-06) GOTO 30
      ANN(1)=ANN(2)
      IM=2
      N=2*N
      GOTO 3
30    XN=ANN(2)
40    RETURN
      END
C
      SUBROUTINE FRAC(A,AL,C1K,IOS)
C
C     SUBROUTINE TO CALCULATE FRCTURE TOUGHNESS K1
C     GIVEN CRACK SIZE 'A', LOAD 'AL': 'OS' IS A LIMIT
C     FLAG
C
      IMPLICIT REAL*8(A-H,O-Y)
      IOS=0
      IF((A.LT.0.6).OR.(A.GT.2.)) IOS=1
      IF(A.LT.0.6) A=0.6
      IF(A.GT.2.) A=2.
      AB=A/2.
      IF(AB.GT.0.7) GOTO 10
      F1=29.6-185.5*AB+655.7*AB**2-1017.*AB**3+638.9*
     1AB**4
      GOTO 20
10    F2=0.03651067*(AB-.7)+.6520468
      IF(AB.EQ.1.) AB=0.9999D 00
```

```
      F1=2.*(2.+AB)*F2/((1.-AB)**1.5*(AB**.5))
20    C1K=F1*AL*(A**.5)/1.01
      RETURN
      END
```

```
C
C       PROGRAM TO GENERATE RANDOM LOAD SEQUENCE
C
        IMPLICIT REAL*8(A-H,O-Y)
        DIMENSION F(30)
        WRITE(5,100)
        READ(5,120) I1
        WRITE(5,110)
        READ(5,120) I2
        WRITE(5,130)
        READ(5,125) ICL
        WRITE(5,160)
        READ(5,170) B,S,SF
        DO 1 I=1,30
1       F(I)=0.
        A=1.
        C1=1./(1.-DEXP(-(B-A)/S))
        CALL ASSIGN(1,'RAN.DAT')
        DO 5 I=1,ICL
        U=RAN(I1,I2)
        X=A-S*DLOG(1.-U/C1)
        DO 5 J=1,29
        IF((X.GT.J).AND.(X.LE.(J+1))) F(J)=F(J)+1.
5       CONTINUE
        DO 7 I=1,29
7       WRITE(1,180) I,F(I)/F(29)
10      CALL CLOSE(1)
C
100     FORMAT(1X,'I1=?')
110     FORMAT(1X,'I2=?')
120     FORMAT(I5)
125     FORMAT(I7)
130     FORMAT(1X,'ICL=?')
160     FORMAT(1X,'B=?,S=?,SF=?')
170     FORMAT(E12.5,/E12.5,/E12.5)
180     FORMAT(1X,I2,1X,E12.5)
C
        STOP
        END
```