INFLUENCE OF CUTTING BLOCK SIZE ON THE EFFICIENCY OF SEVERAL 3-P SAMPLING VARIATIONS AS COMPARED TO POINT SAMPLING

by

ERIC CARL TURNBLOM

B.Sc., The University of Illinois, 1983

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

THE FACULTY OF FORESTRY

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

November 1986

Department of Forest Resources Management

The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date 8-27-1986

ABSTRACT

For 3-P sampling, an ocular prediction of volume must be made on every tree in a cutting block, which casts doubt on its efficiency in larger blocks. However, it has three main advantages over the standard BCMF prism cruising procedures. First, individual trees are the sampling units, not plots of trees, so greater precision may result for a given sample size, second, no height/dbh regressions need be calculated, third, it potentially avoids volume equation biases when it is used in conjunction with dendrometry. A simpler sampling procedure, ratio estimation, differing from 3-P in that sample trees are selected with equal probability, may also prove advantageous. A study was conducted in the UBC research forest in which several blocks were cruised using all three of these methods. A volume equation, the wide-scale relaskop, and the Barr and Stroud dendrometer type FP-12 were tested as tree measurement methods for use with 3-P and ratio estimation. Results indicate that 3-P is more efficient than point sampling in blocks ranging in size up to between 5 and 14 hectares, depending on the tree measurement method chosen and the height/dbh regressions deemed necessary for use in the standard cruise. Ratio estimation is more efficient than point sampling in block sizes ranging up to 14 hectares depending on the same factors as 3-P.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

The author wishes to acknowledge the help and advice of his principal advisor, Dr. Julien P. Demaerschalk, whose guidance was invaluable and greatly appreciated. The advice and opinions of Dr. Kim Iles, biometrician at Macmillan Bloedel Limited, were weighted heavily, and appreciated. Consultations with Dr. A. Kozak were also quite valuable. Finally, thanks to Dr. Peter L. Marshall for the advice and feedback.

INTRODUCTION

In any forest inventory, there are three widely recognized sources of error: 1) error due to the chance location of sample plots, or sampling error, 2) error due to the volume tables or the method used to predict or obtain individual tree volumes, and 3) measurement error (Cunia 1965). In cut block volume determination, performed to the standards for point sampling set forth by the British Columbia Ministry of Forests (BCMF), an additional source of error must be recognized: 4) error due to the height/dbh regression constructed for each individual species in each differing stand type (BCMF 1982).

The first source of error can easily be determined. It is chiefly a function of sampling design and sample size and it is always calculated. According to the Manual of Cruising Procedures and Cruise Compilation (BCMF 1982), when stumpage assessment is based on the scale, the cruise should be designed to yield a sampling error of + or - 15% to the close utilization standard, 19 times out of 20. However, when point sampling is considered, and the cutting block is less than or equal to 10 ha, the statistical formula for estimating the sample size needed to meet this error objective frequently gives a sample size that seems absurdly high. The sampling intensity is either much too great from a logical, intuitive standpoint, or it will cause the sampler to consider a 100% cruise (a 100% cruise is considered, according to the cruise manual, when greater than 50% of the area must be sampled to achieve the error objective). The sampler may choose to "default" to the minimum requirement, which is one plot per ha, provided there are at least 4 plots per type, but he may run a greater risk of not meeting the cruise objective. A similar situation exists when planning for cruise-based stumpage assessment, although the sampling error requirements are a little different. This can be a problem in that a 100% cruise may be performed unnecessarily, or the required error objective may not be met.

Error component 3) is often assumed to be negligible if great care is taken in obtaining tree measurements.

As regards source 2), Brickell (1976) asserted that proper use of volume tables usually requires checking them against a sample of trees measured by direct methods (volumes computed from a series of stem measurements on standing trees, or water displacement techniques) and adjusted if necessary. This is because most volume tables are not based on a sample drawn from the populations to which they will be applied. There may be no reason, in some cases, to suppose that the populations are greatly different, however, biased volumes may result if the assumption of equality is not tested. Usually, volume tables predict volumes only for undeformed, single stemmed (or "normal") trees, therefore, estimates may be reliable for well managed stands, where there are few abnormal trees, but perhaps not for more wild or untended stands.

Grosenbaugh (1964) introduced 3-P (Probability Proportional to Prediction) sampling to forestry which eliminates the use of volume tables and height/dbh regressions, thus removing these sources of error from the total volume per hectare inventory estimate. In actual practice, the error contribution of these sources is seldom calculated, but knowing for certain that they are not present will give more credence to the sampling error that is calculated. A sample of direct tree measurements over a forest area is required by 3-P with dendrometry. This sampling system is also statistically very efficient, in that it can produce estimates that are extremely precise (low variance). The implications of this fact are clear. Fewer samples are required to meet the cruise objectives, which eliminates the need for consideration of 100% enumeration in smaller block sizes, and the execution of much too intensive surveys.

Direct measurements of tree volume are usually obtained by measuring felled trees. However, there are many instances when felling trees is not practical. For example, if the felled trees are a valuable species and remain in the woods too long after the cruise so that potential for decay or stain increases, or if they are crop trees under rotation age, they might not be utilized. In addition to loss of value and growing stock, the felled trees provide breeding places for certain insects and pests, and increase forest fire hazard (Brickell 1976). Sample trees might not be the property of the party or agency conducting the survey, hence felling them may be prohibited. In some cases, the trees may be located on permanent sample plots and cannot be destroyed. Climbing trees seems to be an alternative to felling, but has disadvantages. For example, spurs may injure trees and provide pathways for disease entry. Often, long ladders are too unwieldly and heavy to carry through the woods (Brickell 1976). Also, James and Kozak (1984) hypothesized that it is very likely that standing tree measurement with instruments such as the Barr and Stroud optical dendrometer will be efficient and cost effective when compared with felled tree measurement. For these reasons, much attention has turned to the use of optical dendrometry which enables the measurement of diameters at various heights on the standing tree while the operator remains on the ground.

Much literature exists on 3-P sampling and dendrometry. The method is known to be a highly efficient technique because it employs variable sample tree selection probabilities as does point sampling. However, unlike point sampling, to effect the 3-P method, an estimate (ocular prediction) of volume must be made on every tree in the stand, which has logically led to the belief that it becomes impractical in larger stands. A study which establishes a practical upper limit to stand size has never been published.

This study was conducted to determine whether 3-P sampling, or ratio estimation, with any of three differing sample tree measurement techniques, is more efficient than point sampling in Forest Inventory Zone C (South Coast of British Columbia). Three sample tree measurement methods were considered for use with 3-P and ratio estimation. They are: 1) measurement of diameter and height for entry into an appropriate volume table (volume equation, taper equation), 2) measurement of several diameters and associated heights along the stem of the tree with the wide-scale relaskop, and 3) measurement of several diameters and associated heights with the Barr and Stroud optical dendrometer. Ratio estimation has not widely been used or tested as an on-the-ground method of timber cruising, where the auxiliary variable is an ocular prediction, but has been previously described in other forestry applications (e.g., Freese 1962).

Specifically, the objectives in this study are as follows: 1) quantify the effects of cutting block size on the cost (measured as total cruise time) to obtain a given precision (percent error of total), 2) compare the cost of 3-P, ratio estimation, and point sampling, and 3) make recommendations as to which method is most efficient under conditions likely to be encountered in Forest Inventory Zone C.

SAMPLING TECHNIQUES USED

The three sampling techniques relevant to this study are:

1) Point sampling
2) 3-P sampling
3) Ratio estimation.

A brief description of the fundamental working concepts of each technique will be given along with formulas for sample size estimation, for T_S, the sample based estimator of the population total, for $v(T_S)$, the sample based estimator of the variance of T_S, for E_{TS}, the probable error of the estimate of the total, and for confidence limits. For further information and rigorous mathematical proofs of their statistical validity, the reader is referred to the appropriate literature.

1. 3-P SAMPLING

The first step in conducting a 3-P cruise, as with any survey, is deciding what size sample, n, should be drawn. This is usually done with statistical considerations in mind and the appropriate formula for sample size when sampling from a finite population is:

$$n = \frac{t^2 * C^2 * N}{t^2C^2 + ND^2} \qquad (1)$$

where,

N = number of individuals in population,

D = desired sampling error limit expressed as a decimal fraction of the mean, m: $t*s_m/m$, where s_m is standard error of the mean, t as below,

t = a value of Student's t statistic with appropriate type I error probability and degrees of freedom, and

C = coefficient of variation which expresses the standard deviation of observations as a decimal fraction of the mean: s/m, where s is the standard deviation of observations.

Often, N is not known in advance of sampling, or is assumed to be very large, and the formula becomes:

$$n = \frac{t^2 * C^2}{D^2} \quad . \tag{2}$$

The only consequence of assuming N large when it really isn't is the possibility of greater precision than planned with a corresponding increase in cost. It is noteworthy that C in most sampling plans refers to the dispersion of individual tree volumes around the average volume per tree or the dispersion of plot volumes around the average volume per plot, expressed as a percentage of mean tree volume or mean plot volume. With 3-P, C refers to the dispersion of the ratios of measured volume to predicted volume for individual trees around the average ratio. So, with most conventional sampling techniques, the coefficient of variation is subject to the volume dispersion as it happens to exist. This variation is usually large even with stratification by diameter classes, generally not much lower than 35%. With 3-P, C depends on the predictor's skill. It is a measure of the consistency with which he can predict volumes. It is of no consequence whether he can predict "on-the-nose" or not. All that is required is that he predict in a manner such that approximately the same ratio of actual to predicted volume will result for every sample tree measured. Beginners may be expected to achieve around 25 to 30%, whereas trained cruisers can readily attain 15 to 20% (Mesavage 1971).

The second step is to make a rough estimate of the total sale volume, denoted by X, which stands for the sum of the cruiser's predictions of each tree's volume. The object is to select one sample tree for each accumulation of predictions that totals to

$$L = \frac{X}{n} \quad . \qquad (3)$$

Third, this quantity L is the largest random number that will be generated for the list that is taken into the field.

The writer has found it extremely difficult to predict tree volumes to greater precision than 0.1 m^3 so numbers should be generated from 0.1 up to L. As many numbers should be generated as there are trees expected in the sale (extra, unused random numbers are cheap).

Fourth, when in the field, for each tree:

a. Predict gross total volume, x_i, directly or indirectly using a volume table.

b. Record the estimate.

c. Draw a number from the random number list.

If the prediction, x_i, for the tree is greater than or equal to the random number drawn, the tree should be measured accurately for volume and is considered "selected" as a sample tree. If the random number drawn is greater than the prediction, x_i, for that tree, nothing more is required, i.e., it is "rejected" and not sampled. This procedure selects larger trees with greater probability in the following manner. Suppose the random number list has integers ranging from 1 to 20. With the selection system just described, a tree with a prediction of 1 would have only one chance in 20 to qualify as a sample (to be carefully measured for volume). A tree with a predicted volume of 15, however, would have 15 chances in twenty of being selected, because its predicted volume is greater than or equal to 15 numbers appearing on the list. Thus, the probability of selection is clearly proportional to prediction.

Fifth, cruise statistics are calculated. The sample size actually obtained when the cruise is finished may be slightly different than planned with the 3-P method for two reasons. The sample tree selection probabilities are approximate (recall the estimate of the sum of predictions, or total volume, necessary prior to field procedures). So, when the cruise is completed, it may turn out that some trees may have received a slightly greater selection probability or a slightly reduced probability. Also, sample size varies randomly due to the nature of the selection probabilities themselves. An event may still occur even though it has an extremely low probability, or vice versa. For this reason, the sample size given by (1) or (2) is called the "expected" sample size, or n_E. A given sample size can be insured, however, in a statistically correct way if this objective is absolutely important. Simply inflate n_E over what the formula gives, and after all the sample trees have been selected, randomly elminate the number of sample trees in excess of the desired n_E. The estimated total volume is given by what is called the adjusted estimator:

$$T_S = X_T * (\sum_{i}^{n} (y_i/x_i)/n) \qquad (4)$$

where, $X_T = \sum x_i$, summing over i=1,N or the total of all the cruiser's predictions after the cruise is completed,

y_i= actual measured volume on tree "i"

x_i= ocular prediction of volume on tree "i".

n = sample size actually obtained.

This estimator is called adjusted because it makes use of all the ocular predictions. Although there is an unadjusted estimator, which does not require knowledge of the sum of all the ocular predictions, the adjusted estimator is always used in practice because it yields more precise estimates. Further discussion appears in the Literature Review section. There seems to be some argument as to the best approximation for calculating the variance of this estimate from a sample. A very good, stable, reliable estimator is:

$$v(T_S) = \frac{X_T^2(1-n_E/N)}{n(n-1)}\left\{ \sum_i^n (y_i/x_i)^2 - 1/n(\sum_i^n y_i/x_i)^2 \right\} \qquad (5)$$

where n, as above, is the actual sample size obtained after sampling is completed. A detailed discussion of statistical theory can be found in Grosenbaugh (1965) or Schreuder, et al. (1968). The probable sampling error for the estimate T_S, is given by:

$$E_{Ts} = t * s_{Ts} \qquad (5A)$$

where:

$s_{Ts} = \sqrt{v(T_S)}$ and t is as before.

To find the upper and lower confidence limits with the same probability associated with t in (5A), simply find the solution to the following equations:

$$U = T_S + E_{Ts}$$

$$L = T_S - E_{Ts}$$

where:

U = upper confidence limit

L = lower confidence limit.

The range of values from L to U is known as the confidence interval.

2. RATIO ESTIMATION

Ratio estimators can increase the precision of a sample by making use of supplementary information about the population being studied. For example, if the basal area of a stand of timber is exactly known (the supplementary information), the relationship between volume and basal area may help to improve the estimate of volume. A ratio estimator will increase the precision of this volume estimate by reducing the variability of tree volumes as they exist in the forest, to a small residual variance that is unexplained by the relationship between volume and basal area. In this study, a direct ocular prediction of volume is the auxiliary information. As with 3-P, the auxiliary variable, x_i, will be positively correlated with y_i, the variable of interest (volume in this case), but unlike 3-P, trees will not be selected with varying probabilities. As a result, the variance of this estimator may be larger than the 3-P estimator.

Again, the first step in planning a cruise in which a ratio estimator will be used is to select an appropriate sample size. Equation (1) should be used. This is naturally preceded by making an estimate of the total number of trees in the cruise, N.

Step two involves deciding how the sample trees should be selected. Often it is most easy and convenient to select the trees systematically, in which case, every k^{th} tree will be selected for accurate measurement, where

$$k = N/n \quad . \qquad (6)$$

There is much concern over the reliability of estimates from a single systematic sample (Cochran 1977, chpt.8). In general it is true, however, that as long as there is no cyclic pattern that the attribute of interest displays, from the first individual observed to the last, a systematic sample can be treated as if it were random. The mean (or total) will be unbiased, and the estimate of variance will be conservative. A systematic sample more closely approximates a truly random one if multiple random starts are used. For example, selecting one individual at random from every group of k individuals would be the extreme case of multiple random starts. This can be handled with a list of random numbers again, but selecting the individuals will be a bit more time consuming.

Third, when in the field, for each tree:

a. Predict gross total tree volume x_i, directly or indirectly with the use of a volume table.

b. record the estimate.

If the tree is the next k^{th} individual, the tree should be measured accurately for volume and is considered "selected" as a sample tree. If it is not the next k^{th} tree, nothing more is required, i.e., it is "rejected" and not sampled.

Fourth, cruise statistics are calculated. Estimated total volume is given by the mean-of-ratios estimator:

$$T_S = X_T*(\sum_{i}^{n} (yi/xi)/n) \qquad (7)$$

where all variables are defined as before. Similarity with the 3-P estimator is evident. The appropriateness of the mean-of-ratios estimator will be discussed shortly. The sample based estimate of variance for the total is:

$$v(T_S) = \frac{X_T^2(1-n/N)}{n(n-1)} \{ \sum_i^n (y_i/x_i)^2 - 1/n(\sum_i^n y_i/x_i)^2 \} \quad (7)$$

where all variables are defined as before.

The probable error of the total estimate and the confidence limits are computed exactly the same as for the 3-P estimator. Merely substitute the formulas given in this section for T_S and $v(T_S)$ wherever these symbols appear in the equations for E, U and L presented in the 3-P sampling section.

The mean-of-ratios estimator is appropriate in situations where the following conditions hold:

i) the regression of y on x is linear and through the origin

ii) the variance of y_i about the regression line is proportional to x_i^2.

An alternate ratio estimator is called the ratio-of-means. Conditions most appropriate for its use are different from the mean-of-ratios only in

ii) the variance of y_i about the regression line is proportional to x_i (instead of x_i^2).

A Monte Carlo study by Ek (1971) suggested that the ratio-of-means estimator may be more robust and contain less bias than the mean-of-ratios estimator when the conditions appropriate for the use of each are not met. However, as Cochran (1977, chpt.6) suggested, a plotting of sample y_i values against those of x_i is helpful in examining which, if any, of these conditions hold. It is the writer's experience that, when x_i is a direct ocular estimate of volume, the variance of y_i is usually proportional to x_i^2, making the appropriate estimator the mean-of-ratios.

3. POINT SAMPLING

This sampling technique was first described in North America by Grosenbaugh (1952). Since then, it has been described further with numerous examples (e.g., Dilworth and Bell 1967), and has been described in standard mensuration texts (e.g. Husch, et al. 1972).

The first step, again, is estimation of sample size with formula (1). Many foresters feel that point sampling is congruent to sampling from an infinite population, or equivalently, sampling from a finite population with replacement. This belief has followed from the way samples are selected with this technique. A point is chosen to locate the "plot" center, which has led to the belief that the technique selects "points" as samples, and of course there are an infinite number of points in any tract of land. However, trees, or groups of them, not points, are still the sample units and there is a finite number of trees in any tract of land. Cunia (1959) gave proof that the expected cruise intensity (n/N) for a given critical angle can be computed from 3 things:

1) number of sample points,
2) average basal area per tree,
3) total area to be cruised.

This can be generalized for any critical angle to arrive at the following formula for estimating sample size:

$$n = \frac{t^2 \; C^2 \; A \; F}{AFD^2 + gt^2C^2} \qquad , \qquad (8)$$

where C = the dispersion of plot volumes around the mean plot volume expressed as a decimal fraction of the mean (instead of the dispersion of individual ratios around the mean ratio as before),

F = basal area factor to be used (m^2/ha),

A = stand (cruise) area in ha,

g = average basal area per tree,

all other variables as before (proof in appendix 1).

Second, in the field, a fixed horizontal angle is projected subtending the plot center and rotated 360 degrees, on each of the n points. Points are usually located on the intersections of lines belonging to an imaginary grid pattern, the spacing of the lines depending on desired cruise intensity. Trees whose diameters appear larger than this critical angle are tallied and measured for dbh, and sometimes height, and these values are recorded.

Third, cruise statistics are calculated. The actual volume of the trees selected at each plot is unknown but is usually estimated by appropriate volume tables. The estimate of total volume is:

$$T_S = \frac{A\ F}{n}\{\sum_{i}^{n} \sum_{j}^{m_i} (y_{ij}/g_{ij})\} \qquad (9)$$

where m_i = Number of "in" trees on plot "i", and

g_{ij}= basal area of tree j on plot i, j = 1,...,m_i

with other variables as before. The sample based estimate of variance is:

$$v(T_S)= \frac{A^2F^2(1-gn/AF)}{n(n-1)}\{\sum_{i}^{n} (\sum_{j}^{m_i} y_{ij}/g_{ij})^2-1/n(\sum_{i}^{n}\sum_{j}^{m_i} y_{ij}/g_{ij})^2\} \qquad (10)$$

Proofs of the lack of bias and validity of these point sampling estimators can be found in Palley and Horwitz (1961).

The probable error of the estimated total is calculated the same way as was described in the section on 3-P sampling, as are the confidence limits.

LITERATURE REVIEW

Foresters were introduced for the first time to 3-P sampling by Grosenbaugh (1964), who claimed that 3-P can make objective use of subjective judgement. In the old days, ocular estimating was quite popular, but assessing probable errors for this technique was impossible. The one well known valid theory for using ocular estimates to improve sampling efficiency was through stratification. For example, a large forest can be broken down into separate stands, exhibiting fairly homogenous conditions within themselves, but differing from contiguous stands in the numbers of stems per hectare, species composition, or in any other way that may cause the stands to have differing volume dispersions. By treating the stands as strata, each strata will have a smaller variability because of homogenous stand conditions, thus the error of the estimate will be lower than if stratification were not used. Grosenbaugh observed that very intensive stratification tended to stabilize the coefficient of variation at about the same level for all strata, so that the optimum allocation of samples would be in proportion to stratum volume. This can be extended to the special case where each individual tree is regarded as a stratum. Grosenbaugh discussed how 3-P sampling is similar to list sampling and pps sampling, but has fundamental differences from each. First, with 3-P sampling no list of population units is available until after sampling is completed. Second, the selection probability assigned to each individual in the population is purely arbitrary. This probability must be positively correlated with the variable of interest if any gain in sampling efficiency is to be expected, though the form of the relationship is immaterial and need not be postulated. Subjective prediction of these probabilities, stated Grosenbaugh, is so much cheaper than measurement of some simple geometric dimension (such as diameter or height), that it can usually be applied to every individual in the population. Grosenbaugh described the mechanics of selecting a 3-P sample which have been described in this work in the section titled "Sampling Techniques Used". Grosenbaugh described two 3-P estimators which he called the "unadjusted" and "adjusted" estimates. The adjusted estimate requires knowledge of the sum of the predicted arbitrary probabilities for all population individuals, but should have a much smaller variance than the unadjusted estimate. Consequently, it is the one that is most frequently, if not always, used. Formulas of both unadjusted and adjusted estimates for the population total are given, as well as a formula for the relative variance of the adjusted estimator.

Sharpnack (1964) claimed that computer simulation is the appropriate starting point for testing a new sampling method, in lieu of time consuming field tests which may also confuse field procedures with operation and performance of the sampling technique itself. Sharpnack used a total of 101 trials in a Monte Carlo study of the 3-P sampling technique. The trials included 8 different sample designs to simulate different coefficients of variation for different predictors of selection probabilities. He constructed 67% confidence intervals around the means within each sample. The true mean fell in these intervals 64% of the time over all samples. Sharpnack considered this as an indication that error estimates are reliable using the 3-P method.

Hartman (1966) reported the results of a field trial conducted by the Bureau of Land Management (BLM) in the U.S. The BLM sells timber on a lump sum basis mainly due to intermingled land ownership patterns which causes problems in the location and numbers of scaling facilities. The BLM expects that greater tree utilization is fostered through lump sum sales. Traditionally, the BLM is committed to a 100% cruise policy as the variability encountered in old growth Douglas-fir is generally too great to effect economy, or gain any accuracy, through sampling. A volume estimate is made for every tree in the sale using an average 16-foot form class for each species and entering an appropriate form class volume table. Since each tree is visited anyway it was little extra effort to make a quick estimate of volume at each tree. The standard error of total volume achieved with a sampling intensity of only 4% of the 3,056 trees in the sale, using the 3-P method was 3.9%. Hartman concluded that 3-P is every bit as precise and accurate as the standard technique, but the costs of using 3-P seemed to be just as much as the standard technique. He admitted that this is only one trial and as more experience is gained with the method, this may change.

Johnson, _et_ _al_. (1967) reported another trial of 3-P sampling. They compared 3-P to the accepted sampling method for use in selectively cut sales on most U.S. national forests in eastern Oregon and Washington. The standard method is to select one tree at random from every 5, 10, 20 or other number of trees, depending on desired cruise intensity. Board foot volumes for sample trees are taken from form class volume tables. The quick ocular predictions made in the study were aided by the use of a "crutch" table", _i.e._, a local volume table derived from a similar type sale. The results led them to hypothesize that 3-P may be sufficiently accurate to justify selling on the cruise instead of on the scale. However, they were cautious not to make a blanket endorsement, as this was only a single trial, and felt that additional field tests are justified.

Grosenbaugh (1967) gave a "simplified" explanation as to how several different variable probability sampling methods can all be more efficient than equal probability sampling techniques. Of course, deciding on the optimal design, he stated, will require local studies of empirical variances and costs associated with the particular techniques.

Schreuder, _et al_. (1968), discussed at length the 3-P sampling theory, and gave rigorous, yet simple proofs of its statistical validity. Their proofs showed that Grosenbaugh's (1964) "unadjusted" estimate was unbiased. They also proved that the "adjusted" estimate contained bias, but was likely to have a much smaller variance than the unadjusted estimate, which favored the adjusted estimator's use. This was supported by a Monte Carlo study they performed. The bias of the adjusted estimate was found to be negligable. However, they were concerned with three potential weaknesses of 3-P: 1) there is a possibility of observing a sample less precise or more costly _but more precise_ than planned, because of the random sample size (underlined portion mine), 2) there is a possible loss of efficiency by not making optimal use of information we are fortunate enough to have about the relationship between x (the auxilliary variable, as previously mentioned, an ocular prediction of the variable of interest) and y (the variable of interest), and 3) the apparent lack of a reliable sample based estimate of the adjusted estimator's variance. Schreuder *et al*. thought their Monte Carlo study indicated that the sample based estimate of the unconditional variance originally suggested by Grosenbaugh (1965) was somewhat unstable, but their results were slightly ambiguous. Schreuder, *et al*. (1968) went on to discuss the conditional properties of these estimators, *i.e.*, the properties *after* n observations have been selected. These conditional properties, they stated, are the most relevant, for they are related to the actual sample observed. For example, if the expected value of the total estimate and its variance vary significantly with n, doubt would be cast on the sampling procedure. This was examined also in their Monte Carlo study. It was found that the sample based estimate of variance provided reasonable estimates of true conditional variance. However, subsequent studies (Schreuder, *et al*. 1971) cast further doubt on the utility of these estimators. But, Grosenbaugh (1976) showed that a stable, reliable estimate of the variance based on a single sample does exist. It is obtained simply by multiplying the conditional variance formula (which is also the formula used if sampling is with replacement) by the expected finite population correction (fpc) or $(1-n_E/N)$. This is a crude approximation to an unbiased estimate of the true variance for this type of sampling without replacement but it behaves as would be expected under Monte Carlo simulation. This is formula (5) given in the previous section. Schreuder, *et al*. (1968) concluded by saying that to fully determine whether potential difficulties of 3-P sampling are important relative to possible weaknesses of alternatives will require refinement and extension of existing sampling theory, *and* an accumulation of experience with all the alternatives.

In Finland, Seppälä (1971) tested 3-P sampling in which the variable selection probabilities were based on dbh. A Monte Carlo study was performed on previously collected data from 6 different stands. Optimum powers of dbh were between 2.1 and 2.4 for the different stands. The adjusted 3-P estimator was found to be most efficient. Variances were calculated for expected and actual sample sizes and differences were found to be small. Seppälä concluded that 3-P sampling is a statistically efficient method when the population has not been previously enumerated (listed, or framed).

Johnson and Hartman (1972) proposed falling, bucking, and scaling 3-P sample trees as a logical alternative to dendrometry of standing trees when the opportunity exists. This was proposed because they found that volume estimates from dendrometers can be erratic and/or biased. The errors may vary in magnitude according to instrument operator. Johnson and Hartman were concerned with effects of its cost on its practicality, however, as their method was more expensive than measuring the standing trees.

Bonnor (1972) tested 3-P sampling in a two-stage forest inventory in a white spruce forest type in northern B.C. The first stage was sampling photo plots, which was not described in detail. The second stage, or field stage, was locating some of the photo plots on the ground, and sampling them. Ground plots (0.9 ac) were sampled using three methods: fixed area plots, point sampling, and 3-P. For the plot method, each ground plot was divided into 9 tenth-acre subplots, two of which were sampled. A deliberate effort was made to select the same number of trees in the point and 3-P samples as were selected in the plot sample, to facilitate comparisons. The 3-P method was judged to be vastly superior to the point and plot methods when used in the sampling plan tested. According to Bonnor, this indicated that 3-P would be more efficient in sampling most forest populations. More definite conclusions were deferred until more practical experience could be accumulated.

Hussain (1974) and Tariq (1974) described the 3-P technique and reported on field trials in Pakistan. It was recognized as an efficient timber cruising technique, especially in instances where applicable volume tables are not available (Tariq 1974).

In Mexico, 3-P sampling gained attention. Caballero Deloya (1974) described the principles of 3-P and discussed its application to forest inventory work in that country.

Iles (1978) noted that the suggestion to field crews that predicting volumes directly is preferable has largely been ignored for several reasons. For one, crews may be very inexperienced at this. Two, cruises often use the combined estimates of several people who may possess differing degrees of skill at predicting. Though stratification could be employed, it is often convenient to combine the raw data, thus standard prediction procedures are necessary. It is much easier to predict diameters at breast height than total tree volume. Iles showed that it is a simple matter to incorporate real powers of dbh, local volume tables or much more complicated functions to improve estimation efficiency. Briefly, real random numbers (vs. integers) are generated in terms of volume, and the appropriate volume equation is inverted to yield the diameter which generated that volume, and it is this list which is taken into the field. The actual selection probabilities are then computed after the cruise in the office by computer. This technique seems to eliminate the objection by Schreuder et al. (1968) that 3-P may suffer a loss of efficiency by not making optimal use of information we may have about the relationship between x and y.

Pelz (1980) cited an additional benefit regarding the 3-P method. The stand area need not be determined because the sampling elements are individual trees and are not area related. In doing so, an additional source of error is being eliminated which may be important in irregularly shaped stands in which conventional area based estimates may be biased. Further research is needed to test 3-P's relative efficiency compared to other sampling methods (Pelz 1980).

A comparison was made of 3-P sampling with the tarif system used in the U.K. (Hetherington 1982a). The 3-P method seemed to out perform the tarif system. The methods were compared by the fairly common procedure which provides an "index of efficiency". The standard error of the total in percent of the total is multiplied by the time (or cost) it takes to achieve that precision and then relative efficiencies are computed in the usual manner using these indexes. Hetherington stressed that 3-P sampling has applications world-wide, particularly where volume tables are poor or not available, where access is difficult but aerial photography available, etc. Hetherington later found an error in a computer file. An erratum was published (Hetherington 1982b) in which it is concluded that the two methods actually performed comparably. It should be noted though, that in Hetherington's trial of 3-P, all the trees were actually taped for dbh to assign the probabilities of selection, which would slow down the method considerably.

One example of where access is difficult is in tropical forests where generally heavy undergrowth, multi-layered stories, and heavily buttressed trees prevent easy application of conventional inventory sampling schemes. Crude and in many cases expensive methods have been developed for inventorying the growing stock volume in these forests. Omule (1981) proposed a method employing 3-P sampling to improve the efficiency of the 'stockmapping' technique (one such crude and expensive method). However, the technique was not tested in this application.

Another computer simulation study was conducted by Takata (1982) in Japan. Comparison was made among four volume estimating methods: 3-P sampling, "equal probabilty" sampling, 3-P point, and "Beers method". Data from the study came from four natural stands and four plantations. The straight 3-P method using d^2h as the predicted value was the most accurate and precise.

Another field test was conducted in Germany, and reported by Süss (1982). 3-P was used to select the primary sampling units in a two stage inventory. The total area of the forest was 901 ha, with 221 stands. Only 15 of the 221 stands were selected by 3-P, and were sampled on-the-ground using conventional angle count sampling. The 3-P predictions came from an inventory 7 years earlier. Conventional sampling, i.e., using only the angle count method for the whole forest would have taken three times as long for the same 7.8% precision.

Steber and Space (1972) described a two phase sampling scheme then gaining wide popularity in the Southern U.S. The first phase is the prism plot and the "in" trees are sampled using the 3-P technique (second phase). They inventoried one million acres in cooperation with the St. Joe Paper Co. in

Northern Florida. The combined sampling error of the entire inventory based on cubic foot volume was + or - 7.1%, two times out of three. Considerable time and money were saved using the combined techniques.

Kasile (1983) proposed what he calls "3-P sampling for dollars". He believed that since timber is bought and sold using dollars, direct estimation of dollars should be the major goal of any timber sale inventory. His method requires an a priori probability distribution of frequency of trees by dollar class. Kasile pointed out that this method has great advantages in a mixed hardwood stand in the hardwood region of the U.S. where sawlog size black walnut is worth seven times more than elm and three times more than hard maple. Therefore, volume alone would not adequately reflect differences in value found in these mixed hardwood stands. Kasile described an interesting method to optimize the 3-P sampling scheme, based on the a priori probability distribution of trees by dollar class which seems valid also for frequency of trees by dbh class, volume class or any other classification scheme.

The preceding citations show that many Monte Carlo studies have proven 3-P to be a statisically efficient estimation system. The limited field trials that have been published, give a favorable view of the technique overall, but point to the need for further testing of the method under local conditions. It is evident also that there is a dearth of information as to how 3-P compares with prism cruising in larger stands where clear felling is the objective.

MATERIALS AND METHODS

Pfeiffer (1967) related how the B.C. Work Study School defines work study. It is the systematic, objective, and critical examination of all factors governing the operational efficiency of any specific activity in order to effect improvement. Pfeiffer gave a thorough review of many of the most important work study techniques, and explained that continuous time study is the favored technique for comparison of alternatives. The present study was, of course, designed to compare various alternatives for cruising timber, so the continuous time study technique was adopted.

With this method, a well defined beginning and end to each activity are necessary. The stop watch is started as the activity begins, and stopped when the defined end is reached. Elapsed time is, naturally the time necessary to do the activity. It should be mentioned at this point, that hand in hand with timing goes the "rating" of the workers' performance. "Normal pace" is defined by the International Labor Organization as "the rate of working which the operator could sustain over long periods of time without resulting in buildup of cumulative fatigue" (Pfeiffer 1967), and is given a rating of 100%. The observed time multiplied by the rating as a decimal fraction results in normal time.

This fact is mentioned because no attempt was made to estimate a normal time for each activity. Various organizations or companies may already have established normal times which their crews display when angle count sampling. So, interested parties can "rate" the times reported here for a specific two man crew (same crew for all times) and adjust them accordingly to their organization's "normal pace". Also, the resultant times from a time study are applicable to the particular job (in this case angle count sampling, 3-P sampling, or ratio estimation) but may be used for the same job at any other place under similar conditions.

To facilitate the timing of each cruising technique, each technique was broken down into components or activities.

The 3-P variations were broken down into 4 activities, which shall be called predicting, marking, locating, and dendrometry. The predicting activity included walking through the block and directly predicting ocular estimates of individual tree volumes. The stand was traversed in successive 10-12 m strips which were bounded by trailing a string. Recording and testing the predictions against the two selection criteria, namely, the random numbers, or checking to see if it was the k^{th} tree, were also included in the predicting activity. Timing started with both men positioned at the edge of the tract to be cruised and ended when the volume for the last tree had been ocularly predicted and checked for inclusion in the sample. Since two selection criteria were being employed simultaneously, the time it took to mark each tree so it could be located later for measuring, was recorded separately under the activity called marking. This is necessary because different trees could be selected under each criteria and because differing numbers of trees may be selected in different cruises using the same cruising method. The locating activity consisted of just that, the time it took to locate each marked tree during the sample tree measurement phase. Last, the dendrometry activity consisted of finding a suitable location or locations in which to set up the dendrometer or take heights with clinometer and chain, actually setting up and using the instruments, recording necessary information on pathology and quality needed for the application of appropriate decay, waste and breakage factors, and then taking the instruments down and storing them properly for transit.

Three different methods were tested to "accurately measure" the sample trees. In the first method, a diameter tape, clinometer, and nylon chain were used to measure dbh and height for entry into the appropriate volume table published by the BCMF (1976). This method for obtaining tree volumes is quite appealing because it requires very few measurements, is quick, will be accurate if the volume table truly represents the local conditions, and may be adequate for certain objectives. However, three sources of error were identified by Hazard and Berger (1972) that may cause problems when an existing volume table is applied to a new stand of trees or to a small portion of the stand for which the table was developed. They are: 1) changes in the relationship between volume and the tree characteristics for the two stands, 2) different standards of utilization in the volume table than those desired, and 3) systematic measurement error incurred in measuring the trees for input into the table. When the volume table predicts gross total tree volume, only error sources 1) and 3) apply. It was found that volume tables (based on Behre's (1927) expression for tree taper) produce substantial differences from tree volumes derived from dendrometer estimates. Hazard and Berger set an arbitrary "acceptable" index of variation between the volume table and dendrometer at 10% , so the Chi-Square test expostulated by Freese (1960) could be used. In their study the volume tables failed resoundingly at the 95% probability level to produce estimates acceptably similar to those obtained from the dendrometer.

The second method of sample tree measurement tested was using the wide-scale Relaskop as a dendrometer. The aim here, as described in the introduction, is calculating volumes of individual "logs" in the tree and summing them to obtain total tree volume, thus eliminating the volume table errors mentioned previously. When using any instrument such as this one, encountering low underbrush in the woods does not pose a measurement problem, because dbh and lower diameters can be measured with a diameter tape or mechanical calipers. The real problems encountered are high brush, trunks and branches of other trees, branches of the subject tree itself, and wind. Sometimes a large amount of time can be spent waiting for the wind to die down, so that the tree no longer appears to travel back and forth over several relaskop bands. Considerable time can also be spent finding an instrument location that offers the best possible view of the stem. It is possible, however, to use more than one instrument location. For example, one location may seem ideal for upper stem measurements and another for lower stem measurements. All that is required is a new observation of the horizontal distance at each new location. Also, with the relaskop it is particularly easy to spot "suspicious" readings that are probably in error, while still in the field. Relaskop units are proportional to diameter so these readings can be spotted through comparison with other readings at the same instrument location.

Relaskop units should generally decrease as readings progress up the stem. When comparing the relaskop to the Barr & Stroud optical dendrometer, Yocom and Bower (1975) found that the relaskop gave estimates 1.6% higher, on the average, than those based on the Barr & Stroud.

The third method used to measure the sample trees was the Barr and Stroud optical dendrometer type FP-12. Use of this instrument certainly gives the same advantages as using the relaskop, but in addition may give even more accurate and precise results because the instrument has 5.5x magnification, and also because the lens "gathers" light which aids viewing in stands with dense canopies or in cloudy weather. There is a newer model available, the FP-15, but design changes from the FP-12 are not related to accuracy (Mesavage 1969). Brickell (1976) found that the Barr and Stroud exhibits + 2.39% bias in basal area estimates and no bias in height measurements. Consequently, one would expect the same magnitude in volume bias. However, +5.4% bias in dendrometered volume was found over felled tree volume. He could not attribute this discrepancy to any specific cause. He hypothesized that human vision itself may have caused this error. Stage (1962) has demonstrated this phenomenon with the wedge prism and Ross (1968) has demonstrated it with the Wheeler penta prism. Brickell (1976) found, and the writer corroborates, that it is very cumbersome to use more than two instrument locations when using the Barr and Stroud. The instrument plus tripod and carrying case can be unwieldly at times, and dismantling and setting up the instrument are time consuming. Brickell (1976) also found that the random error in volume measurements was 3% of tree volume, much less than 10 - 15 % usually associated with volume tables or equations. The BCMF (1976) logarithmic volume equations give standard errors of estimate ranging from 6.3% to 16.8% depending on species. An attempt was made to space measurements at intervals no greater than 5m along the stem for both the Barr and Stroud and the relaskop.

In the field, each of these measurement techniques was applied consecutively to each sample tree as it was visited in the forest. The order in which the methods were applied were rotated systematically after a random start. It was recognized that mobility in the forest depended to a certain extent on the bulkiness and weight of instruments for dendrometry that must be carried. The time necessary for locating trees when carrying a tripod and either the relaskop or the Barr and Stroud was estimated by subtracting the sum of all the times spent actually measuring sample trees from the total time spent in the field. The time necessary for locating trees when carrying only a clinometer, chain, and diameter tape was estimated separately, simply by recording the time necessary to walk to each marked sample tree carrying only those instruments.

The choice among these three methods of sample tree measurement will depend on two things: 1) level of accuracy desired, the number of measurements desired, or the purpose for which they are taken, and 2) relative productivity of the measurement technique (number of trees per hour or per day). It was not possible to fell the sample trees or otherwise obtain "true" volumes, consequently no statements can be made about the bias, or accuracy of any of these tree measurement methods. However, strong statements can be made regarding criteria 2), and will be discussed more fully in the Results and Discussion section.

The prism cruise was broken down into three activities which are referred to as sweeping, sampling trees, and chaining. The sweeping activity included the prism "sweep", in which the prism is rotated around the plot, all "in" trees are marked, measured for species, dbh, quality, pathology, crown class, etc., and all borderline trees are checked. The sampling trees activity included measuring the necessary number of heights for the height/dbh regression. Finally, the chaining activity included the time spent chaining between points, point or "plot" center establishment, and miscellaneous necessary activites.

Before actual timing of the cruising techniques was started, practice cruises using all the methods were performed on contiguous areas to familiarize the crew members with all the methods. Estimates of total volume and number of trees per hectare for each block, necessary for 3-P and ratio estimation respectively, were obtained from one or two quickly executed prism plots. These estimates need not be performed in this manner, aerial photos can be used, or perhaps experience in the particular forest type may be the better judge. The estimate of total block volume needed for 3-P estimation may be only a "ball park figure". It is generally agreed that a slight overestimation is preferable to an underestimation (Grosenbaugh 1965). Aproximately a day spent practicing the ocular prediction of tree volumes was used to obtain an idea of the predictor's (crewman's) variability. The appropriate sample size formula showed that 18 sample trees were required to meet the cruise objective of + or - 15% 19 times out of 20. In this study it was found that the auxiliary variable, ocular prediction of volume, was highly correlated with "actual" or measured volume, overall. The simple correlation coefficient, r, between volumes obtained from the volume table and predictions was 0.9370. The correlation between volumes obtained by the relaskop and predicted volume was 0.9073, while volumes obtained by the Barr and Stroud dendrometer and predicted volume gave a correlation coefficient of 0.9214. The correlations actually experienced in each separate block ranged from 0.8151 to 0.9895. As was outlined in the introduction, difficulty was experienced in arriving at an appropriate sample size for the prism cruise. It was decided that since in the final analysis, the time required to achieve a given precision using each method would be compared, regardless of the precision actually achieved in the field trials, the sample size was arbitrarily set at six for the half hectare blocks and eight for the one hectare blocks.

The study was conducted in the University of British Columbia Research Forest near Haney, B.C. This area was chosen because it represents well the conditions found in Forest Inventory Zone (FIZ) C. The availibility of funds and time, and logistical constraints limited the study to the examination of two replications of two block sizes, 1/2 and one hectare. Two blocks each of one hectare were delineated with yellow plastic flagging using a Silva hand held compass, a nylon chain, and a Suunto clinometer to make slope corrections. Extreme accuracy in the block areas is not necessary, as each cruising method was executed in the identical block and comparisons are valid. Care was taken, however, to ensure that these two blocks were similar in density as measured in number of trees per hectare, and as similar as possible in individual tree dimensions, as these factors were hypothesized to affect overall cruise time. These two blocks were further divided using the compass, chain, and clinometer into 0.5 hectare blocks. Nesting the smaller blocks in this fashion further ensured that these important factors would remain fairly constant. Block M, consisted of old growth hemlock, Douglas-fir, and cedar. Height of dominant and codominant trees was about 50.5m, there were 985 cubic meters gross total volume per ha, 390 stems per ha, 40% slope, west aspect and a significant understory of western hemlock which inhibited work somewhat. Block W consisted of second growth Douglas-fir, had dominant and codominant height of about 51.1 m, about 740 cubic meters gross total volume per hectare, 431 stems per hectare, 35% slope, south west aspect and very low salal ground cover, making travel fairly easy. The areas are summarized in table 1.

TABLE 1. STAND CHARACTERISTICS SUMMARY.

BLOCK	SLOPE	ASPECT	NO./HA	BA/HA	VOL/HA	HEIGHT	DIAM	%F	%C	%H
M	40%	270	392	$68m^2$	$985m^3$	50.5m	47.0cm	32	32	36
W	35	225	431	55	739	51.1	40.3	70	23	7

The data carefully obtained from sampling these block sizes enabled prediction of the time it takes to cruise larger blocks for any given precision. The methods used to make these predictions will be briefly outlined here but discussed in more detail in the next chapter.

Each separate cruising activity was analyzed and the relationship each has with increasing area sampled was hypothesized and developed on strong theoretical considerations.

Then, to arrive at total cruise time to achieve a given precision (+ or - 15% 19 times out of 20), the predicted times for each activity's completion were summed.

A computer program written in BASIC programming language was developed for use on an IBM-PC to calculate standing tree volumes from relaskop and Barr and Stroud readings. An algorithm using Smalian's formula to compute log volumes was adopted as Martin (1984) has shown that it is among the best mathematical formulas available for middle logs on the tree when measurements are not equally spaced along the stem. Tree top volumes were assumed to be cones, and stumps cylinders. No correction for bark was made in this computer algorithm, so tree volumes are output in gross total volume, outside bark, which may increase the estimate of each tree's total volume over the trees whose volumes were obtained from the volume table which predicts gross total volume inside bark. So, the point estimates of the total volume are reported as gross total volume under bark for tree measurement methods involving the volume equations and gross total volume outside bark for methods involving the dendrometers. However, this will not affect the precision of the estimates, that is, the sampling error obtained. It is only the sampling error that is used to estimate sample sizes, not the point estimate itself, so this has no effect on the cruising time necessary for any cruise objective.

RESULTS AND DISCUSSION

The total time it took to point sample the half hectare blocks (HM and HW) ranged from 12.15 to 15.35 hours, and it took from 16.90 to 26.82 hours to cruise the hectare blocks (M and W). By examining column 5 in tables 2 through 5, the reader can see these cruise times, and also the total cruise times for each of the remaining cruising methods and their variations for each block. Column 4 of these tables gives the component times that make up each total time. Table 6 gives a summary of the total volume estimates for each block by method, along with the coefficient of variation, 95% confidence limits, and percent error of the total. An interesting development is that in all eight of the cruises, using the Barr and Stroud as the tree measurement method resulted in cruise volumes greater than when the relaskop is used to measure the trees. In all but one case, using the Barr and Stroud gave total volume estimates greater than when the volume table was used. When the relaskop was used, every cruise gave an estimate of the total less than when the volume table was used to obtain tree volumes. However, upon examining the confidence limits presented in this table it will be seen that in all but one case, a considerable amount of overlap is present between them. Thus, it is difficult to assess whether these differences in the point estimates represent real biases or just chance. In addition, no opportunity was available to actually fell these sample trees to determine a better answer to this question.

TABLE 2. BLOCK HM SUMMARY OF ACTIVITY TIMES

METHOD	SAMP. SIZE	ACTIVITY		TIME(HRS)	TOTAL(HRS)
PRISM	6	SWEEPING		7.7808	
		SAMPLING TREES(33)		6.9025	
		CHAINING		.6667	15.3500
3-P	18	PREDICTING		1.9000	
		MARKING[a]		.7633	
		LOCATING[b]	1)	.8260	
			2)	1.9162	
		DENDROMETRY[c]	1)	4.0967	7.5860
			2)	8.7684	13.3479
			3)	14.5434	19.1229
RATIO EST.[d]	17	MARKING		.2867	
		LOCATING[b]	1)	.6397	
			2)	2.1014	
		DENDROMETRY[c]	1)	3.6833	6.5097
			2)	7.2628	11.5509
			3)	12.8633	17.1514

a. This time includes 12 trees that were randomly discarded to achieve the desired cruise intensity.
b. 1) gives time when carrying only a clinometer and chain
   2) gives time when carrying tripod plus dendrometer and chain.
c. 1) when measured volume is from a table.
   2) when measured volume is from wide-scale relaskop readings.
   3) when measured volume is from Barr & Stroud readings
d. Prediction time is the same as for 3-P so is not listed twice.

TABLE 3. BLOCK HW SUMMARY OF ACTIVITY TIMES

METHOD	SAMP. SIZE	ACTIVITY		TIME(HRS)	TOTAL(HRS)
PRISM	6	SWEEPING		5.5833	
		SAMPLING TREES(41)		5.0333	
		CHAINING		1.5333	12.1499
3-P	13	PREDICTING		1.3017	
		MARKING		.3942	
		LOCATING[b]	1)	.4758	
			2)	1.4083	
		DENDROMETRY[c]	1)	2.3075	4.4792
			2)	7.4894	10.5936
			3)	10.5481	13.6523
RATIO EST.[d]	12	MARKING		.2542	
		LOCATING[b]	1)	.3032	
			2)	.9197	
		DENDROMETRY[c]	1)	1.8994	3.8075
			2)	3.6256	6.1502
			3)	6.4350	8.9596

b,c,d - for explanation see foot of table 2.

TABLE 4. BLOCK M SUMMARY OF ACTIVITY TIMES

METHOD	SAMPLE SIZE	ACTIVITY		TIMES(HRS)	TOTAL(HRS)
PRISM	8	SWEEPING		11.5167	
		SAMPLING TREES(67)		12.1833	
		CHAINING		3.1167	26.8167
3-P	25	PREDICTING		3.4350	
		MARKING		.6217	
		LOCATING[b]	1)	1.2500	
			2)	3.6271	
		DENDROMETRY[c]	1)	4.6111	9.9178
			2)	10.7917	18.4755
			3)	17.5347	25.2185
RATIO EST.[d]	18	MARKING		.4100	
		LOCATING[b]	1)	.9300	
			2)	3.0550	
		DENDROMETRY[c]	1)	4.2200	8.9950
			2)	9.9300	16.8300
			3)	14.7850	21.6850

b,c,d - as described under table 2.

TABLE 5. BLOCK W ACTIVITY TIMES SUMMARY

METHOD	SAMPLE SIZE	ACTIVITY		TIME(HRS)	TOTAL(HRS)
PRISM	8	SWEEPING		7.8667	
		SAMPLING TREES(41)		7.7000	
		CHAINING		1.3333	16.9000
3-P	24	PREDICTING		2.8100	
		MARKING		.5552	
		LOCATING[b]	1)	1.0000	
			2)	2.9785	
		DENDROMETRY[c]	1)	3.4150	7.7802
			2)	11.4933	17.8370
			3)	18.2567	24.6004
RATIO EST.[d]	18	MARKING		.3550	
		LOCATING[b]	1)	.8400	
			2)	2.2750	
		DENDROMETRY[c]	1)	2.9067	6.9117
			2)	7.3100	10.4750
			3)	11.6367	14.8017

b,c,d - same as in table 2.

TABLE 6. SUMMARY OF CRUISE RESULTS

BLOCK	METHOD[a]		TOT VOL(m^3)	C	95% C. LIMITS LOWER	UPPER	ERROR (%)
HW	PRISM		404.	36	309.	499.	23.5
	3-P	1)	412.	19	367.	457.	10.9
		2)	374.	27	316.	432.	15.4
		3)	431.	20	380.	481.	11.7
	R.EST	1)	403.	31	316.	489.	21.6
		2)	322.	23	277.	367.	14.0
		3)	400.	40	303.	497.	24.2
W	PRISM		739.	33	556.	921.	24.7
	3-P	1)	835.	25	749.	919.	10.2
		2)	797.	34	686.	908.	13.9
		3)	930.	30	815.	1044.	12.4
	R.EST	1)	806.	31	686.	925.	14.8
		2)	784.	38	640.	927.	18.3
		3)	976.	42	775.	1176.	20.0
HM	PRISM		389.	38	256.	522.	34.1
	3-P	1)	559.	31	476.	642.	14.8
		2)	476.	29	410.	542.	13.8
		3)	539.	26	473.	605.	12.3
	R.EST	1)	434.	41	348.	517.	19.4
		2)	387.	45	302.	472.	22.0
		3)	461.	37	377.	544.	18.1
M	PRISM		985.	31	752.	1218.	23.6
	3-P	1)	1231.	34	1061.	1401.	13.8
		2)	1150.	36	984.	1315.	14.4
		3)	1304.	33	1129.	1479.	13.4
	R.EST	1)	1168.	34	976.	1359.	16.4
		2)	1087.	33	911.	1263.	16.2
		3)	1266.	31	1074.	1459.	15.2

a. 1) represents the volume table tree measurement procedure.
2) represents the relaskop tree measurement procedure.
3) represents the Barr & Stroud tree measurement procedure.

As can be seen from column 2 of tables 2 through 5, sample sizes varied between cruising methods. Particular note should be made of the variation in the sample sizes of the 3-P cruise. The actual sample size obtained at the termination of the cruise cannot be controlled rigidly in the field, as mentioned earlier. Again, however, if the block is to be returned to for dendrometry later, it is a simple five minute office procedure to randomly eliminate extra sample trees as was done for block HM (see table 2, the marking activity). It is difficult to assess the probability of obtaining a sample size smaller or larger than expected from this study, because the sample size was so small and because the study was not designed to yield such information. The possibility of this happening has much greater implications if the dendrometry activity is to be done concurrently with predictions (another common variation in 3-P field procedures), and should be considered in such cases. The varying sample sizes also affected the precision obtained by each method in this study. Errors range from 10.2 to 34.1%. This obviously prohibits direct comparison of the raw cruise times to discover which sampling technique is the most efficient for a given block size.

A very practical method to compare the efficiency of different sampling techniques is to examine the total cost (or time) it takes to achieve a given precision. To do so requires three basic steps.

The first step is to average the coefficients of variation for each method in each block, to obtain an expected coefficient of variation, C, for the given stand conditions (in the case of point sampling) and the given predictor (in the case of 3-P and ratio estimation). For point sampling the average C was 34%. For 3-P with volume table tree mesurement C was 28%, for relaskop measurement, C was 33%, and for Barr and Stroud measurement 29%. For ratio estimation with volume table tree measurement C was 34%, for relaskop tree measurement, 35%, and for Barr and Stroud tree measurement C was 37%.

Second, the sample sizes needed to obtain the desired precision given the variability (measured as C) encountered in the field are estimated separately for each method.

Third, average values for the time to complete each of the separate activities must be modeled. The time to complete some of the activities will be a function of sample size, others will not.

In point sampling, the sweeping activity is adequately described with a simple average, provided the same mean number of "in" trees per plot is obtained. In this study the average number of "in" trees was eight, and the average time per plot was 1.1105 hours for this activity. The time to complete the sampling trees activity will be a function of the number of trees that must be measured for height. This is dependent on the number of species in differing forest types encountered in the cruise for which height/dbh relationships must be developed. The cruising manual of the B.C. Forest Service recommends 30 height/dbh points for each species in each forest type. For simplicity, it will initially be assumed that only one species in one forest type is encountered, or equivalently, that this one species displays the same height/dbh relationship in all the forest types encountered. The average time to sample a tree for height is 0.1655 hours. The time necessary to complete the chaining activity is largely a function of distance (or cut block size) and the shape of the block, and to a lesser degree, a function of the number of points or plots that must be installed. The block shapes used in this study were square, so this shape is used to extrapolate times to larger block sizes. In essence, after the necessary sample size is estimated, the plots are arranged on paper, in a systematic fashion to provide good coverage of a square block. The total distance to be chained is calculated in meters and multiplied by the average chaining time per meter. An average chaining time per meter obtained in this study, 0.0074 hours, was used.

The components for 3-P sampling and ratio estimation are essentially the same and will be described together. The time to complete the predicting activity is largely a function of block size and the number of trees per hectare. Density was kept fairly constant so the time to predict volumes will vary directly with block size. The relationship is hypothesized to be linear on strong theoretical grounds. Since the block is traversed in strips of a given, more or less constant width (too wide and the predictions go wild, too narrow is inefficient), linearly increasing the block size will produce a corresponding linear increase in the effective strip length, thus producing a linear increase in the amount of time to walk along the strip. The fitted regression line predicting the time necessary to estimate volumes for all trees on a block is based upon four observations. The time to complete the marking activity is a function of the number of trees to mark, and will vary directly with sample size. The time necessary to locate sample trees, as with chaining, is largely a function of the distance traveled, and not the number of stops made, so it will vary chiefly with block size. The relationship is hypothesized to be linear because the easiest method of relocating the sample trees is to walk along the strips marked while predicting, so that no tree is missed. By way of the argument presented earlier for the predicting activity, when the block size is increased linearly, the effective strip length is also increased linearly, thus, the time it takes to walk along the strip is increased linearly. It should be noted that trees selected with 3-P may occur in clumps, which might affect the location time. However, it is further assumed that on the average, the sample trees will occur in a well distributed fashion as with trees selected for ratio estimation. Thus, the fitted regressions used to predict locating times for the differing tree measurement methods are based upon eight observations, i.e., four observations from 3-P, and four observations from ratio estimation. Finally, the time to complete the dendrometry activity is largely a function of the number of trees that must be measured. For the relaskop and the Barr and Stroud, the number of stem measurements per tree ranged from 5 to 14, averaging 8.7. It was found that the time to complete this activity varied with tree size (i.e., dbh and height) much more than with the number of measurements per tree. Attempts were made to relate the time spent per tree simply to dbh and height. The best relationship accounted for less than 50% of the variation. This is the result of failing to account for topography, high brush, boles and branches of neighboring trees, etc. Indeed, the time required to measure a tree with a dendrometer is much dependent on its surroundings (a phenomenon also reported by Brickell 1976; Gregoire, et al 1986). Thus, a simple average time per tree was used for each method. It was found that in general, when using ratio estimation, smaller trees were selected as samples when compared to 3-P. We would expect this because large trees in 3-P are selected with greater probability than small trees. In one measurement method this did appear to have an effect on the amount of time spent measuring the trees selected for ratio estimation.

The pertinent relationships developed for 3-P and ratio estimation are as follows:

$LT_C = .11735 + .88765\ A$ $\quad r^2 = .6184 \quad$ p-value = .02

$LT_D = 0.1889 + 2.7950\ A$ $\quad r^2 = .6879 \quad$ p-value = .01

$PT = 0.0792 + 3.0433\ A$ $\quad r^2 = .8608 \quad$ p-value = .10

$MT_{3P} = .02589$ hours/tree $\quad MT_R = .02094$ hours/tree

$VT_{3P} = .1782$ " $\quad VT_R = .1906$ "

$RT_{3P} = .4175$ " $\quad RT_R = .4615$ "

$BT_{3P} = .7619$ " $\quad BT_R = .7395$ "

where,

LT_C = time necessary to locate all sample trees on a given size block, A in ha, when carrying only clinometer and chain.

A = block size in hectares

r^2 = coefficient of determination which can be interpreted as the decimal fraction of the total variation that is explained by the relationship.

p-value = the probability of observing a more extreme value of r, under the hypothesis of no correlation.

LT_D = time necessary to locate all sample trees on a given block size A, in ha, when carrying a tripod and either of the relaskop or Barr & Stroud.

PT = the time necessary to ocularly predict the volumes of all the trees on a block of A hectares in size.

MT_{3P}, MT_R = the average time necessary to mark a sample tree during a 3-P cruise, or during ratio estimation respectively, for easy location later

VT_{3P}, VT_R = The average time necessary to "measure" a tree during 3-P or ratio estimation respectively, for entry into a volume table

RT_{3P}, RT_R = the time necessary to measure a tree during 3-P or ratio estimation respectively, with the wide-scale relaskop

BT_{3P}, BT_R = the time necessary to measure a tree with the Barr and Stroud dendrometer during 3-P or ratio estimation, respectively,

Fourth, estimate the time necessary to complete each activity given the sample size previously calculated, and derive the total cruise time by summing all the times for the activities. This was done to estimate the required time to complete a cruise for + or - 15% precision 19 times out of twenty, on block sizes ranging from 0.5 to 12 hectares for each method and its variations. The results are displayed graphically in figure 1 for point sampling and variations of 3P.

Note that the curves representing the time it takes to complete a 3-P cruise are below the curve representing the time it takes to complete a point sampling cruise for block sizes below five hectares. This indicates that 3P sampling with any of its variations is more efficient in these block sizes. Figure 2 shows a similar graph comparing ratio estimation with point sampling. Ratio estimation was found to be slightly less efficient than 3-P because it is seen that with all its variations it is more efficient than point sampling only up to block sizes of three hectares. In this study it was found that it is more difficult to ocularly predict volumes of small trees consistently than it is to predict volumes of large trees. When using ratio estimation, trees are selected with equal probability resulting in the selection of more smaller trees than with 3-P. Hence, the variability of predictions when employing a ratio estimator is greater than when a 3-P estimator is used. This results in a larger sample size necessary to meet cruise objectives, thus decreasing its efficiency. It is interesting to note that using the Barr and Stroud as the tree measurement technique is slightly less efficient than point sampling in one hectare block sizes, but becomes more efficient for two hectare block sizes, then less efficient again for block sizes greater than that.

Figure 1. Total cruise time for point sampling compared to 3P using three tree measurement techniques, given the observed variability.

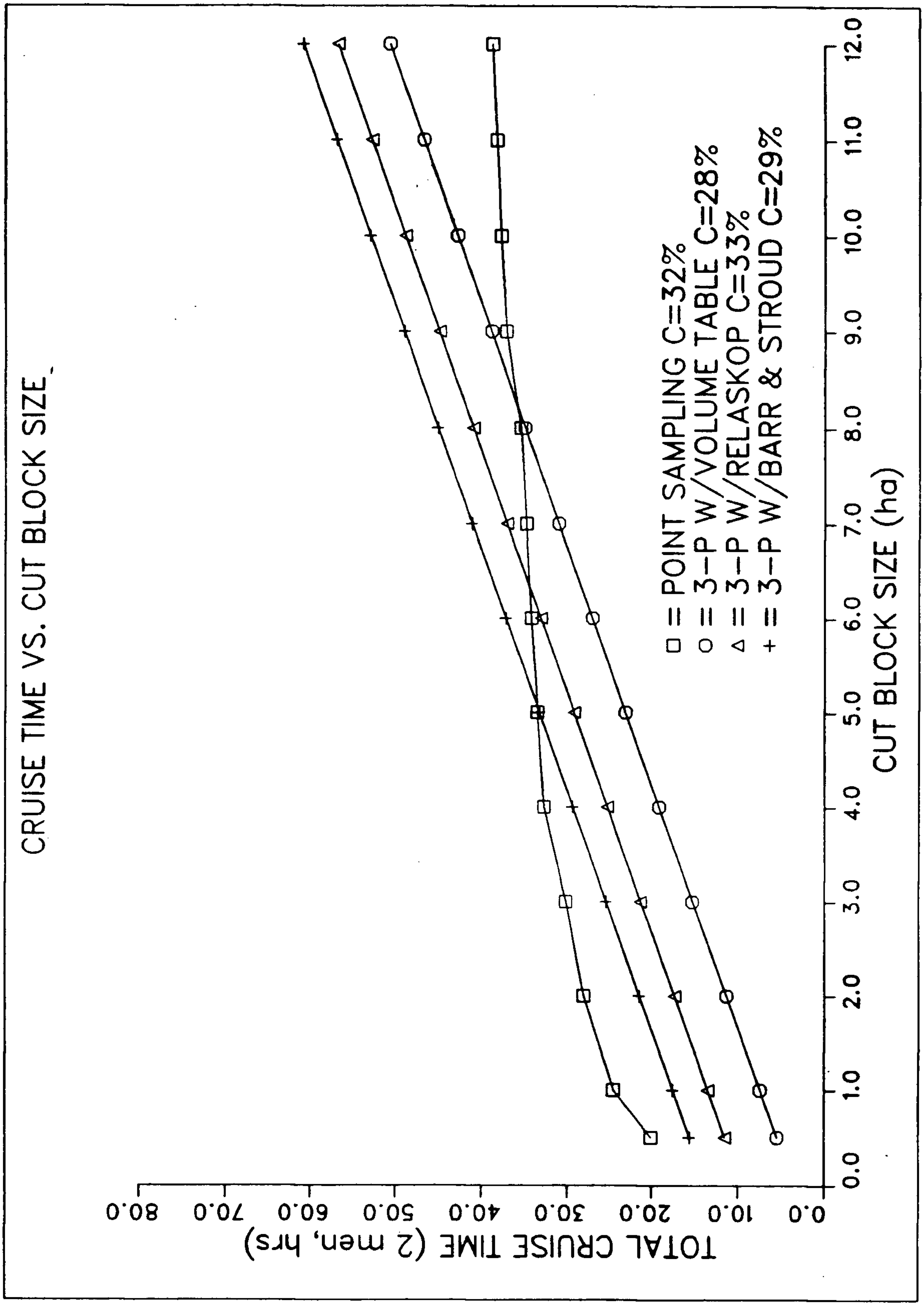
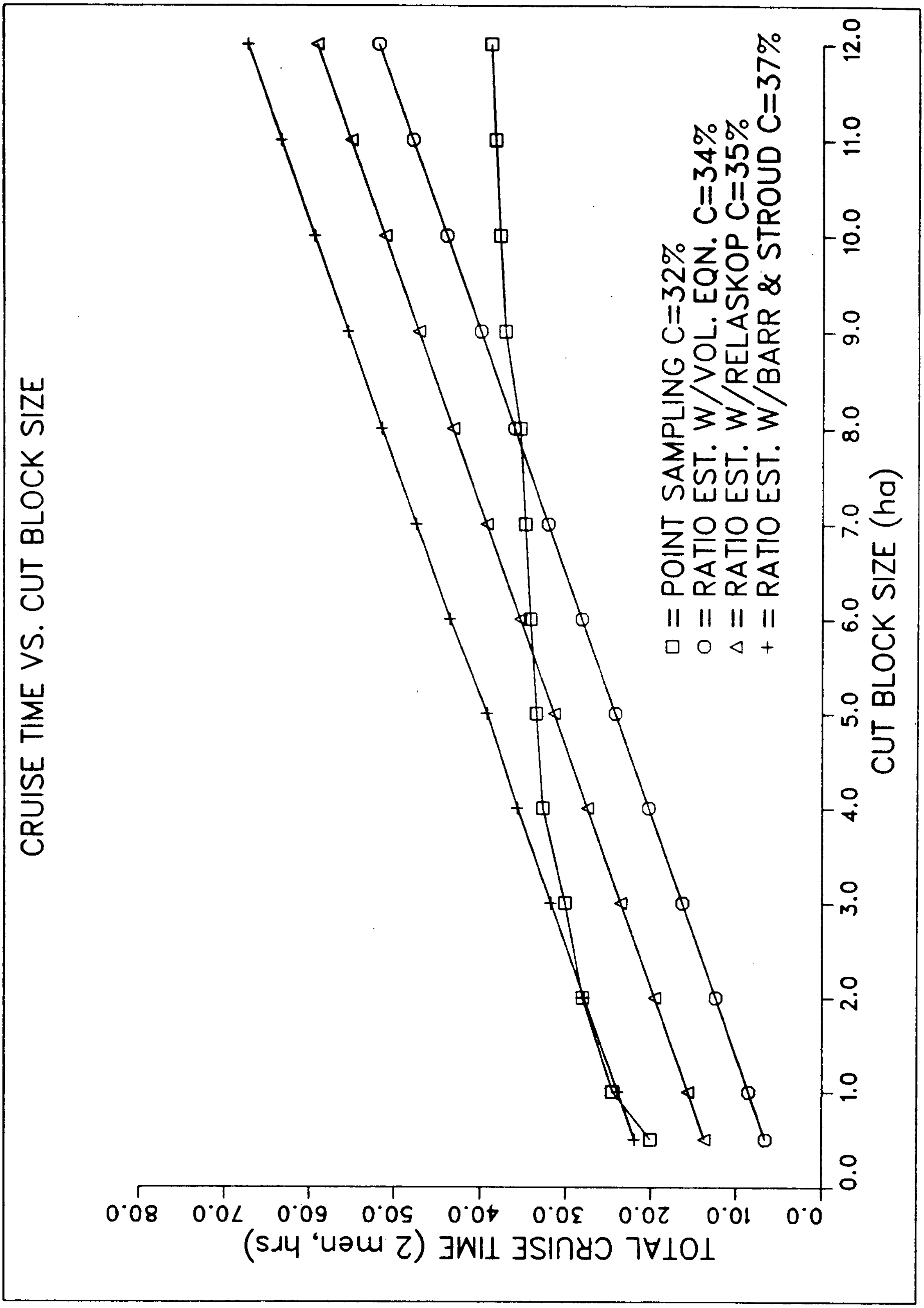


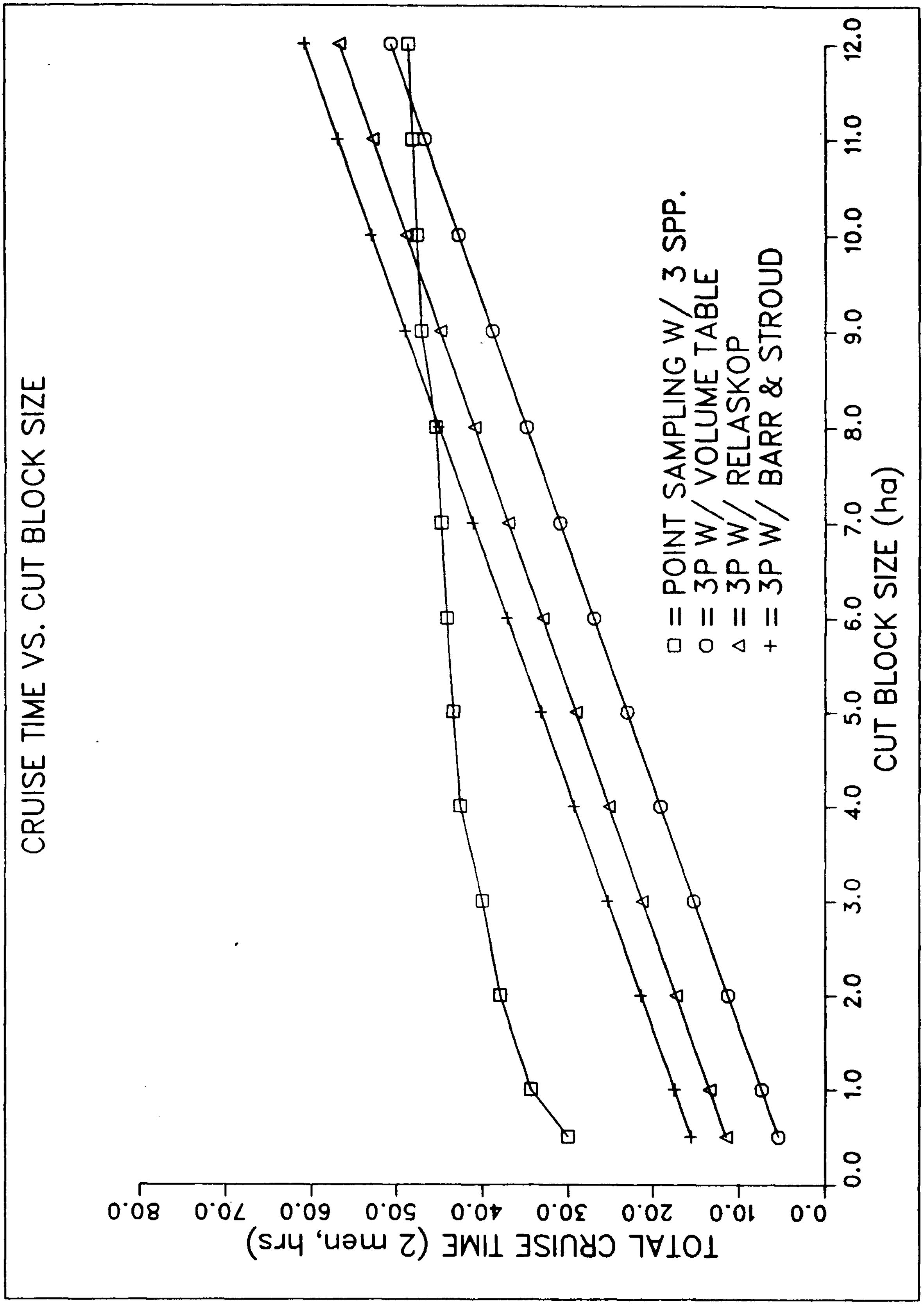
Figure 2. Total cruise times for point sampling compared to ratio estimation using three tree measurement techniques, given the observed variability.



This is the result of the fpc (finite population correction) making a smaller number of samples needed for point sampling in the smallest block size. As can be seen from figure 2 the curve displaying times to complete a point sample levels out quickly, because as cut block size increases, the characteristics of an infinite population are approached rapidly. If 3-P sampling using a volume table only (volume equations or taper equations as well may be used) to obtain tree measurements is considered, then 3-P is more efficient up to approximately eight hectares. If ratio estimation using volume equations to obtain tree volumes is considered, then a ratio estimator is more efficient than a point sampling estimator up to block sizes of just under eight hectares. Use of volume tables or volume (taper) equations may be completely satisfactory in cases where accurate and precise equations have been developed, such as those by Demaerschalk and Kozak (1977), or Ormerod (1986).

The times reported so far for point sampling assume that there is only one species encountered in the block for which a height/dbh relationship must be developed. This necessitates the measurement of approximately 30 trees for height to form a reliable height/dbh curve. When two or three species are encountered in the field, then logically 60 or 90 trees must be measured for height. The impact of this development on the time necessary to complete a point sampling cruise is illustrated in figure 3 in comparison with 3-P using the different tree measurement techniques.

Figure 3. Total cruise times for point sampling in stands with three species compared with 3P using three tree measurement methods, given the observed variability.



It will be seen that 3-P with any of the tree measurement methods is now more efficient in block sizes up to eight hectares. Figure 4 illustrates this situation for ratio estimation with all the different tree measurement methods. Ratio estimation is now more efficient than point sampling up to block sizes of six hectares.

Additionally, one may consider the effects of different coefficients of variation on the amount of time necessary to complete each cruise. The coefficient of variation achieved by the predictor in this study was around 30 percent. This is considered to be somewhat high, and indicative of a beginner (Mesavage 1970). Typically, experienced cruisers can achieve coefficients of variation as low as 20 percent. Also, the coefficient of variation of prism plot volumes was approximately 32 percent. More typically, coefficients of variation around 50 percent or more are not uncommon. The implications of these facts and their effect on total cruising time can be seen in figure 5 for point sampling and 3-P using the volume equation measurement procedure. Note that the size of block in which 3-P is more efficient than point sampling has increased to nearly 14 hectares as compared to only 11, when the observed variability is considered (see figure 3). There is little to choose between a ratio or a 3-P estimator for a given variability of predictions. The time required to complete a cruise using ratio estimation is actually slightly less than the time needed for a 3-P cruise, because less time is spent employing the sample tree selection criteria. The difference is so slight however, that if the curves of total cruise time for ratio estimation and 3-P using the same tree measurement technique were drawn on the same axes they would virtually coincide.

Figure 4. Total cruise times for point sampling in stands with three species compared with ratio estimation using three tree measurement methods, given the observed variablity.

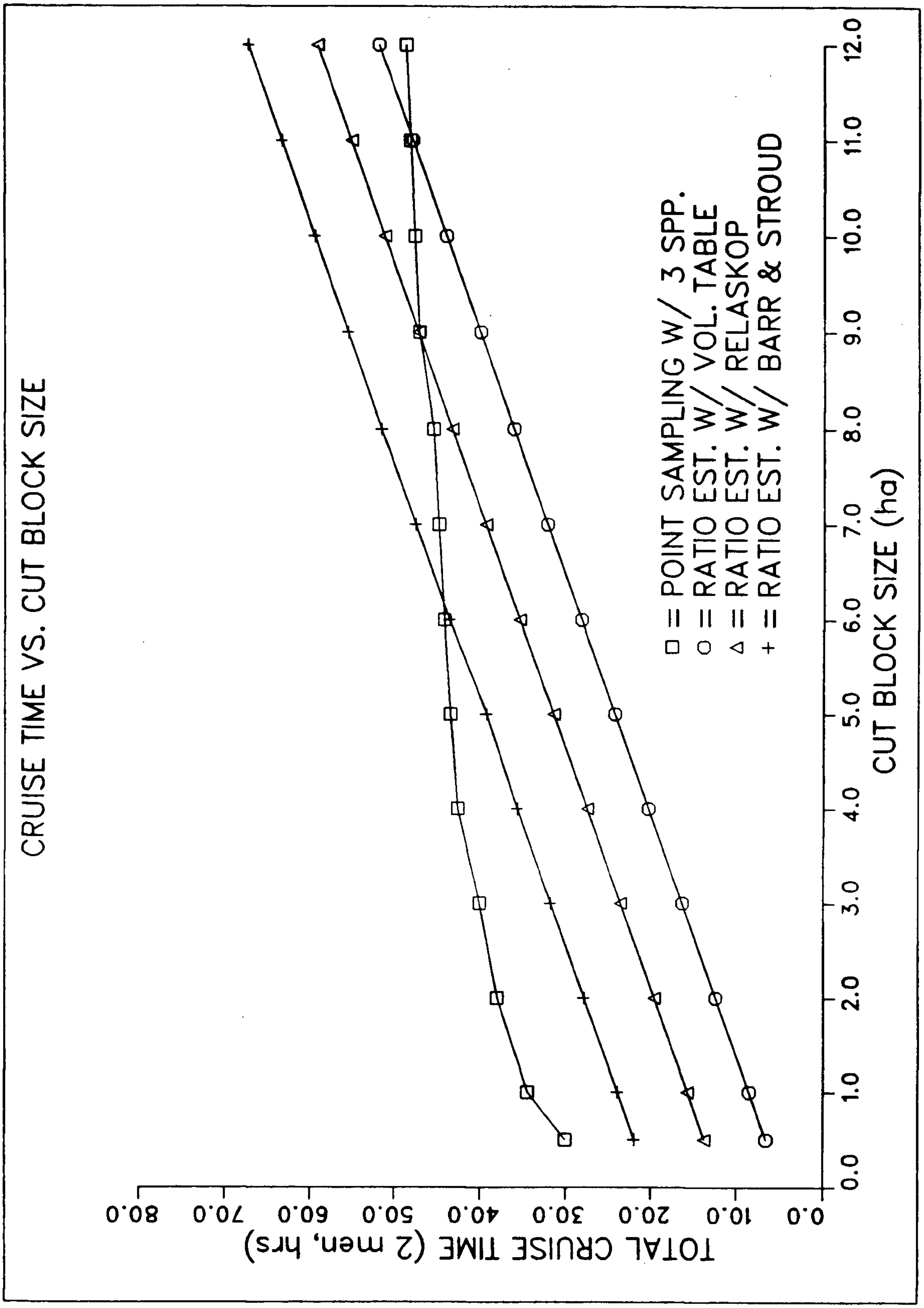
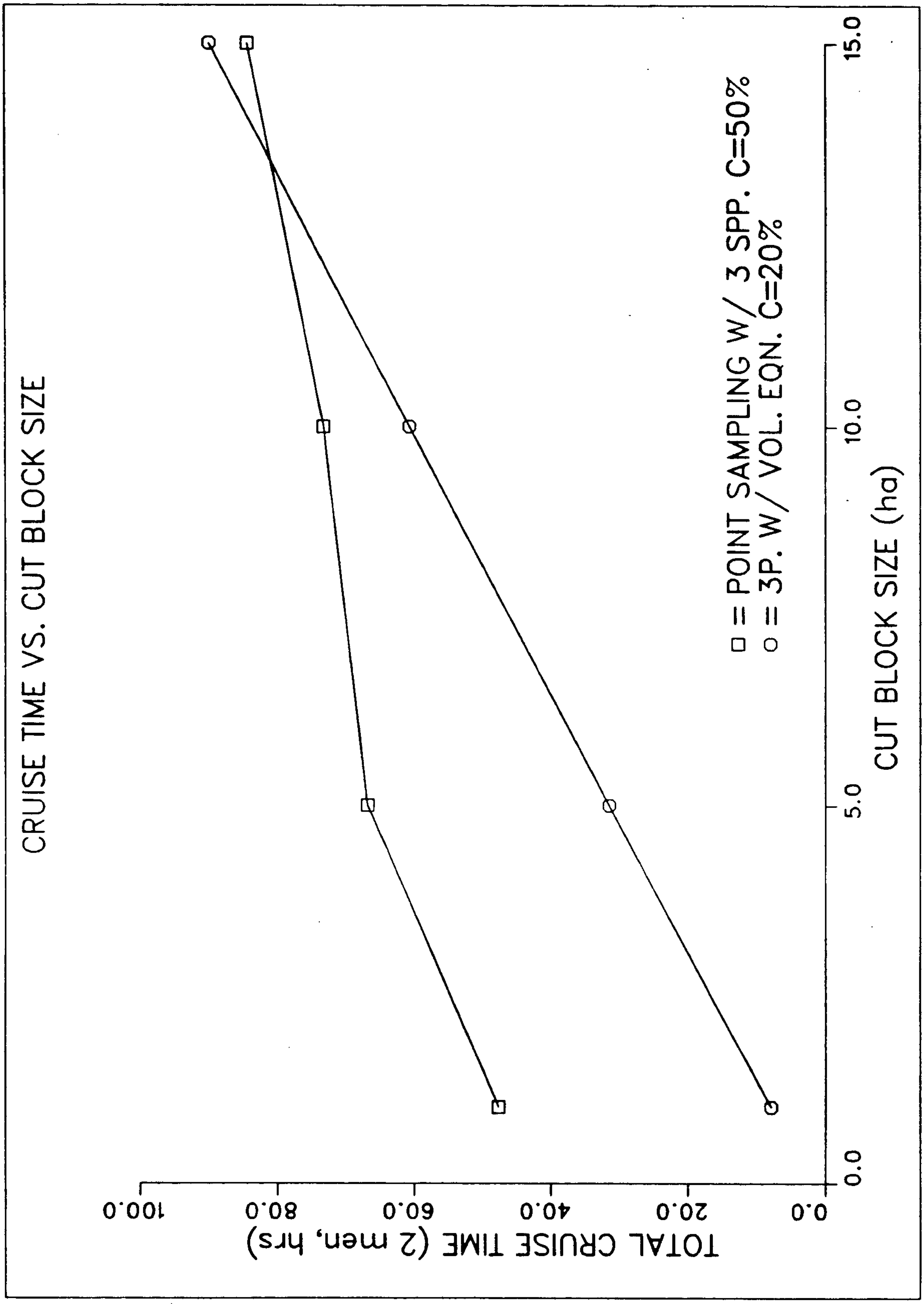


Figure 5. Total cruise times for point sampling in stands with three species compared with 3P using the volume equation tree measurement method, assuming typical variability.

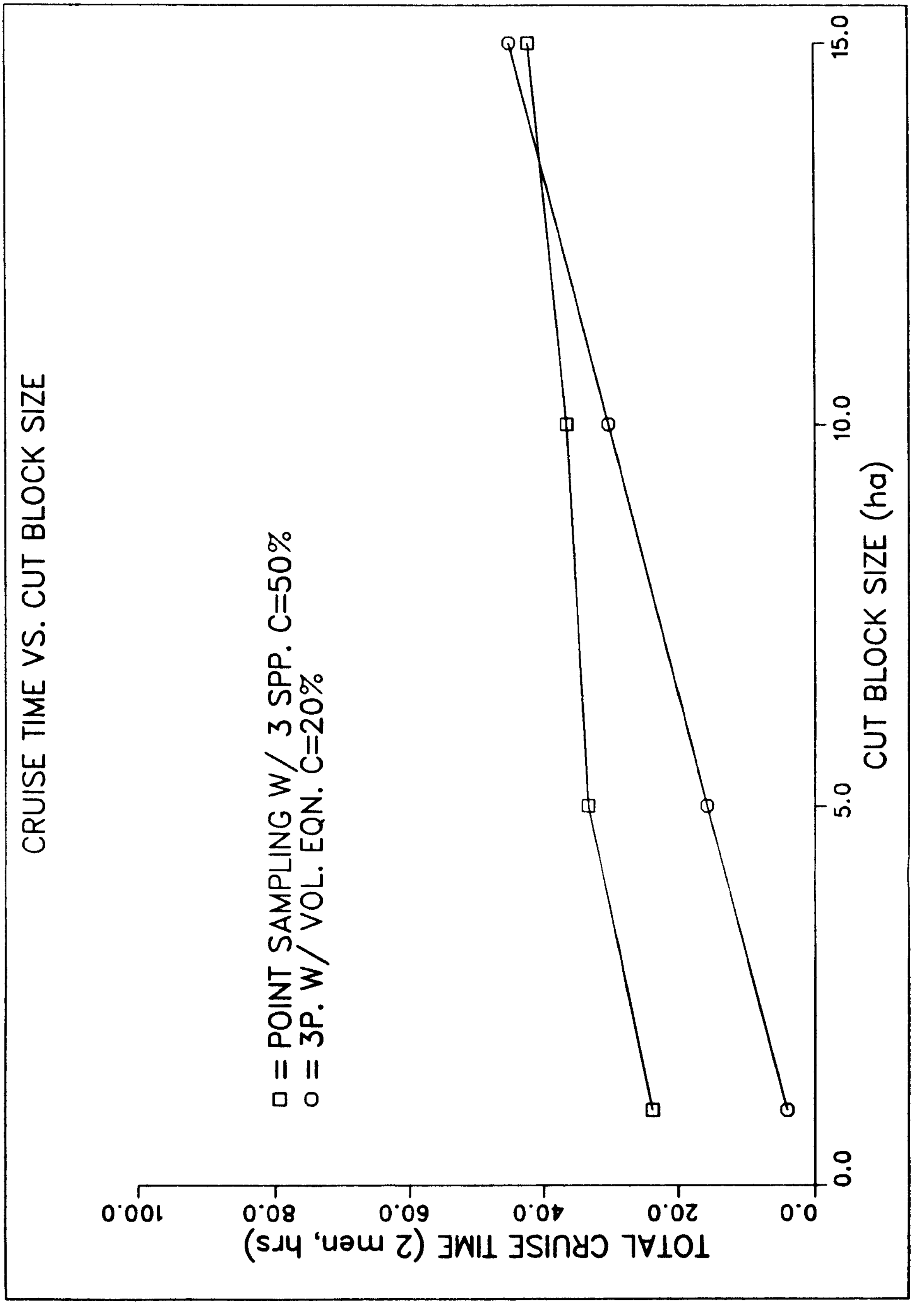


Therefore, a graph comparing point sampling with ratio estimation, given these hypothetical coefficients of variation is not given.

Although the times reported here are for a specific two man crew and may not represent what some would consider realistic times in terms of absolute value, it must be reiterated that the relative differences existing between the methods will remain valid no matter what rating the work is given to arrive at a "normal" time. To illustrate, suppose the estimated cruise times for point sampling that appear in figure 5 are judged to be twice as long as the amount of time that is necessary to cruise the given block sizes. Then, the work in this study would be judged to have been performed at a rate of 50% of normal pace. When all the times are corrected to normal pace by multiplying them by 0.5 (the rating expressed as a decimal fraction), the relationship between cruise time and cut block size is as appears in figure 6. Note that although the slopes of the curves have changed and the absolute differences have changed, the relative differences have not and the 3-P estimator is still more efficient than point sampling up to the same size block as in figure 5.

The foregoing discussion shows that 3-P certainly appears to be an efficient sampling technique, as does ratio estimation, when compared to point sampling. However, as was suspected, they lose their superiority in the larger block sizes. What has not been known before is that this practical upper limit to block size is around 12 to 15 hectares, given typical variabilities likely to be encountered in the field.

Figure 6. Total criuse times for point sampling in stands with three species compared with 3P using the volume equation method, assuming typical variability, and ajusted to normal pace.



CONCLUSIONS

From the results of this study, it can be said that 3-P sampling is unconditionally more efficient than point sampling in cutting block sizes up to five hectares for all the tree measurement techniques tested. If one considers tree measurement by means of the wide-scale relaskop, then it is more efficient in block sizes up to six hectares. If one considers obtaining tree volumes by means of a volume equation (or taper equation), then 3-P is more efficient than point sampling in block sizes up to eight hectares. In general, it was discovered that 3-P is slightly more precise than ratio estimation (mean-of-ratios estimator).

Ratio estimation using the Barr and Stroud optical dendrometer to obtain tree volumes was found to be less efficient than point sampling, given the observed coefficient of variation. If one considers the relaskop as the tree measurement technique, ratio estimation was found to be more efficient than point sampling up to block sizes of six hectares. Considering the volume equation method of obtaining tree volumes, ratio estimation was more efficient than point sampling in block sizes up to approximately eight hectares.

When one considers the possibility of having to develop height/dbh relationships for more than one species in the cruise, then 3-P sampling is more efficient than point sampling in block sizes up to between six and eleven hectares, depending on the tree measurement method chosen. Under this hypothesis, ratio estimation is also more efficient than point sampling in the same block sizes as 3-P, depending again, on the tree measurement method chosen.

Under the hypothesis of typical coefficients of variation likely to be encountered in the field, then 3-P and ratio estimation can be more efficient in block sizes up to a maximum of 14 hectares.

These conclusions apply broadly to any area exhibiting similar stand conditions to those where the methods were tested.

LITERATURE CITED

BCMF. 1976. Whole stem cubic metre volume equations and tables - centimetre diameter class merchantable volume factors. For. Inv. Div. British Columbia Ministry of Forests. Dept. of Forests. 4 p., 64 tables, 13 figs.

BCMF. 1982. Forest service cruising procedures and cruise compilation. Province of British Columbia Ministry of Forests. 117 p., appendices.

Behre, C.E. 1927. Form class taper curves and volume tables and their application. J. Agric. Res. 35:673-744.

Bonnor, G.M. 1972. A test of 3-P sampling in forest inventories. For. Sci. 18(3):198-202.

Brickell, J.E. 1976. Bias and precision of the Barr and Stroud dendrometer under field conditions. USDA For. Serv. Intermountain forest and Range Exp. Sta. Res. Pap. INT-186, 46 p.

Cochran, W.G. 1977. Sampling techniques, 3rd ed. John Wiley & Sons, New York, 428 p.

Cunia, T. 1959. Notes on Cruising Intensity by the Bitterlich method. J. Forestry. 57(11):849-850.

Cunia, T. 1965. Some Theory on reliability of volume estimates in a forest inventory sample. For. Sci. 11(1):115-128.

Demaerschalk, J.P., and A. Kozak. 1977. The whole-bole system: a conditioned dual equation system for precise prediction of tree profiles. Can. J. For. Res. 7:488-497.

Dilworth, J. R., and J.F. Bell. 1967. Variable plot cruising. O.S.U. Bookstores, Inc. Corvallis, OR. USA. 117 p.

Ek, A.R. 1971. A comparison of some estimators in forest sampling. For. Sci. 17(1):2-13.

Freese, F. 1960. Testing accuracy. For. Sci. 6:139-145.

Freese, F. 1962. Elementary forest sampling. USDA For. Serv. Ag. Handbook No. 232, 91 p.

Gregoire, T.G., H.T. Valentine, and G.M. Furnival. 1986. Esimation of bole volume by importance sampling. Can. J. For. Res. 16:554-557.

Grosenbaugh, L.R. 1952. Plotless timber estimates - New, fast, easy. J. For. 50(1):32-37

Grosenbaugh, L.R. 1963. Optical dendrometers for out-of-reach diameters: a conspectus and some new theory. For. Sci. Mon. no. 4, 47 p.

Grosenbaugh, L.R. 1964. Some suggestions for better sample tree measurement. Proceedings. Soc. Am. For. Meeting 1963. p36-42.

Grosenbaugh, L.R. 1965. 3-P sampling theory and program THRP for computer generation of selection criteria. USDA For. Serv. Res. Pap. PSW-21, 53 p.

Grosenbaugh, L.R. 1967. The gains from sample tree selection with unequal probabilities. J. For. 65(3):203-206.

Grosenbaugh, L.R. 1976. Approximate sampling variance of adjusted 3-P estimates. For. Sci. 22(2):173-176.

Hartman, G.B. 1966. Some practical experience with 3-P sampling and the Barr and Stroud dendrometer in timber sales. SAF Proceedings, 1965, p126-130.

Hazard, J.W., and J.M. Berger. 1972. Volume tables vs. dendrometers for forest surveys. J. For. 70(4):216-219.

Hetherington, J.C. 1982a. 3-P sampling - the devil you don't know. Scottish Forestry. 36(1):25-35.

Hetherington, J.C. 1982b. Erratum. Better the devil you know! Scottish Forestry. 36(3):219.

Husch, B., C.I. Miller, and T.W. Beers. 1972. Forest Mensuration, 2nd ed. John Wiley & Sons, New York, 210 p.

Iles, K. 1978. Increasing estimation efficiency in 3-P cruises. For. Chron. 54(1):42-43.

James, C.A., and A. Kozak. 1984. Fitting taper equations from standing trees. For. Chron. 60(6):157-161.

Jeffers, J.N.R. 1956. Barr and Stroud dendrometer type FP-7. For. Comm. Rep. on Forest Research for the year ended March, 1955:127-136. IN Brickell, J.E. 1976. (see above).

Johnson, F.A., W.G. Dahms, and P.E. Hightree. 1967. A field test of 3-P cruising. J. For. 65:722-726.

Kasile, J.D. 1983. 3-P sampling for dollars. IN Renewable resource inventories for monotoring changes and trends. Proceedings of an International conference. Aug. 15-19, 1983. Corvalis, OR. USA. Eds John F. Bell, Toby Atterbury, p703-706.

Martin, A.J. 1984. Testing volume equation accuracy with water displacement techniques. For. Sci. 30(1):41-50.

Mesavage, C. 1969. New Barr and Stroud dendrometer Model FP-15. J. For. 67(1):40-41.

Mesavage, C. 1971. STX timber estimating with 3-P sampling and dendrometry. USDA For. Serv. Ag. Handbook No. 415., 135 p.

Omule, S.A.Y. 1981. Volume estimates in tropical forests using 3-P sampling. Commonw. For. Rev. 60(3):207-209.

Ormerod, D.W. 1986. The diameter-point method for tree taper description. Can. J. For. Res. 16:484-490.

Palley, M.N., and Horwitz, L.G. 1961. Properties of some random and systematic point sampling estimators. For. Sci. 7(1):52-65.

Pelz, D.R. 1980. [Sampling methods with variable probabilities of selection] Stichproben mit variablen auswahlwahrschweinlichkeiten. Allgemeine Forst- und Jagdzeitung. 51(2):37-41.

Pfeiffer, K. 1967. Analysis of methods of studying operational efficiency in forestry. MF thesis, University of British Columbia., 93 p.

Ross, R.D. 1968. On the accuracy of penta-prism calipers. J. For. 66:932-933. IN Brickell, J.E. 1976. (see above).

Schreuder, H.T., J. Sedransk, and K.D. Ware, 1968. 3-P sampling and some alternatives, I. For. Sci. 14(4):429-453.

Schreuder, H.T., J. Sedransk, K.D. Ware, and D.A. Hamilton. 1971. 3-P sampling and some alternatives, II. For. Sci. 17(1):103-118.

Seppälä, R. 1971. Variable probabilities in sample-tree selection. Commun. Inst. For. Fenn. 74(4):29 p. IN For. Abs.

Sharpnack, D.A. 1964. A computer trial of 3-P sampling. Proceedings, Soc. Am. Foresters, Denver, CO. USA, p225-226.

Stage, A.R. 1962. A field test of point sample cruising. USFS Intermountain For. and Range Exp. Sta. Res. Pap. 67., 17p. IN Brickell, J.E. 1976. (see above).

Süss, H. 1982. [Inventory of a ranger district in four days using data from a previous inventory and 3-P sampling.] Allgemeine Forstzeitung. 3(12):335. IN For. Abs.

Takata, K. 1982. [A test of 3-P sampling by computer simulation]. Bulletin, Nigata Universtiy Forests, No. 15., 99-103. IN For. Abs.

Yocom, H.A., and D.A. Bower. 1975. Estimating individual tree volumes with Spiegel Relaskop and Barr and Stroud dendrometer. J. For. 73(9):581-582,605.

APPENDIX

Let, A = total cruising area in ha,

G = total basal area of all trees in stand (m^2),

r = radius of tree (in m),

N = total number of trees in stand,

R = plot radius for tree of radius r in meters.

Then $$G = \sum_{i}^{N} \pi r_i^2 \quad .$$

It is known (Husch, Miller, Beers 1972, p.263) that

$$F = 2500k^2$$

where F = basal area factor of prism in m^2 per ha and k is the "gauge constant" ($k = 2r/R$).

Solving for R in terms of F:

$$F = 2500k^2$$
$$\sqrt{F}/50 = k$$
$$\sqrt{F}/50 = 2r/R$$
$$100r/\sqrt{F} = R \quad .$$

Now, let us define Q as the sum of the areas of the circles generated around each tree by any F factor prism (these areas overlap so each point in the forest is covered by one or more of these circles, sometimes none).

$$Q = \sum_{i}^{N} \pi R_i^2$$
$$= \sum \pi 10000 r_i^2/F$$
$$= 10000G/F.$$

To put this on a hectare basis: $1\ \text{ha}/10000\text{m}^2 * 10000G/F = G/F$.

And, let us define m as the average number of circles falling on any point,

$$m = Q/A = G/FA.$$

So, as is known, basal area per hectare (G/A) = Fm.

If we call q the total area of tree circles in the sample,

$$E(q) = E\left(\frac{Q}{N}\sum_{j}^{n} K_j\right)$$

$$= \frac{Q}{N} E(nK_M)$$

$$= \frac{Q}{N} n\, E(K_M) = \frac{Qnm}{N}$$

where E denotes expectation, n is the sample size, K_j is the number of trees on point j, and K_M is the mean number of trees per point in the sample. Let p = q/Q , or cruise intensity expressed as a decimal. Then,

$$E(p) = E(q/Q) = E(q)/Q$$

$$= nm/N$$

$$= \frac{nG}{NFA}$$

Now, call sampling error of the mean, E.

$$E = t\, S_M$$

where t = an appropriate value of Student's t-statistic

S_M= Standard error of average volume/ha estimate

$$E = \frac{t\, S \sqrt{(1-n/N)}}{\sqrt{n}}$$

where S = standard deviation of plot volume/ha estimates.

But, n/N = p w ng/FA or the average cruise intensity.

Substituting this into the previous expression gives:

$$E = \frac{t\ S\ \sqrt{(1-ng/FA)}}{\sqrt{n}}$$

$$\sqrt{n} = \frac{t\ S\ \sqrt{(1-ng/FA)}}{E}$$

$$n = \frac{t^2 S^2 (1-ng/FA)}{E^2}$$

$$n = \frac{t^2 S^2}{E^2} - \frac{t^2 S^2 ng}{E^2 FA}$$

$$n\ (1 + \frac{t^2 S^2 g}{E^2}) = \frac{t^2 S^2}{E^2}$$

$$n = \frac{\frac{t^2 S^2}{E^2}}{(1 + \frac{t^2 S^2 g)}{E^2}}$$

$$n = \frac{t^2\ S^2\ A\ F}{AFE^2 + gt^2 S^2}$$

Finally, multiply by $\frac{1/V_M{}^2}{1/V_M{}^2}$,

where $V_M{}^2$ = the mean volume per hectare squared,

which gives :

$$n = \frac{t^2\ C^2\ A\ F}{AF\ D^2 + gt^2 C^2} \quad . \qquad (9)$$

which completes the proof.