INCENTIVE COMPATIBLE COMPENSATION MECHANISM FOR CENTRALLY PLANNED INDUSTRY WITH MULTIPLE AGENTS AND COMMUNICATION

by

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B.A.Sc., Shanghai Jiao Tong University, 1983

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE
IN BUSINESS ADMINISTRATION

in

THE FACULTY OF GRADUATE STUDIES

The Faculty of Commerce and Business Administration

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

December 1986

32

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Date Dec. 12, 1986

Abstract

This thesis applies the existing agency theory into the problem of production planning in a centrally planned industry. The planner's objective is to maximize social welfare contributed by the industry, while the firms individually want to maximize utility over money compensation minus disutility over effort. The problem contains both moral hazard and adverse selection because each agent privately observes a predecision information about the production process. A model is built for determining the optimal incentive compatible scheme. The analysis starts with the problem of fixed proportions production. An optimal incentive compatible scheme is first derived in single agent settings. It is then extended to multiple agent settings. Under the optimal incentive scheme, the principal is able to derive all the rent. The solution is the first-best when the agents are all risk neutral, and strictly second-best otherwise. The subgaming issues amongst the agents are investigated. When the agents are not cooperative, a sufficient condition is given for the incentive scheme to be effective, i.e., the equilibrium induced by the scheme is implementable. It is also concluded that, if the agents are able to cooperate, there always exist some state realizations under which the scheme is not effective. Finally, a different type of production problem, namely, production with substitutable inputs, are studied. And an incentive compatible compensation scheme is again proposed.

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Acknowledgement

I would like to express my sincere thanks to Professor Peter Cheng, Professor John Hughes and Professor Asha Sadanand for their enlightening advice and helpful suggestions. The time and effort that they have individually devoted are truly invaluable to every part of this thesis. My principal advisor, Professor Peter Cheng, has guided me to the domain of agency theory and stimulated my initial interest in this area. I would also like to thank many others for their help and contributions relating to this thesis and to my master's program.

Chapter 1. INTRODUCTION

An agency exists when some people (agents) perform a task on behalf of other people (principals). This is a typical form of modern economic organizations. A problem of incentive conflicts between these groups is widely seen in these organizations, and is perhaps one of the most important issues to resolve. Incentive conflicts and information asymmetry together make this problem nontrival. Theoretically, if the principal has perfect information, i.e., the principal knows the action choice of the agent and the underlying condition, he or she can force the agent to take the most desirable action by imposing a potentially severe penalty. However, the existence of information asymmetry makes this forcing contract a practical impossibility. Accordingly, the principal seeks to design an appropriate incentive compensation scheme in order to induce the agent, at his or her own interest, to take actions that the principal prefers.

This paper studies the incentive problem in the context of a centrally planned industry. There are a number of previous studies on planning and regulating industrial production. Domar (1974) has presented an incentive compensation scheme for a single firm industry. The scheme bases the compensation on a weighted average of sales and profit. Baron and Myerson (1982) have derived the optimal policy of prices and subsidies to regulate a monopolistic firm whose cost coefficient is unknown to the regulator. In their setting, the authors assumed that the cost function is linear. More recently, Donaldson and Neary (1984) have proposed an incentive scheme, in which the bonus or penalty a firm gets is based on its profit relative to the industrial average. This scheme is effective when each firm's cost function is identical to that of at least one of the other firms in the industry. These studies assumed that firms are reward-maximizers. Therefore, they only considered the problem of adverse selection, not moral hazard. In this paper, we assume that there are disutilities associated with effort levels. So, we consider the joint problems of moral hazard and adverse selection. The analysis is conducted in both single firm settings and multiple firm settings in which firms' cost functions are unrelated.

Chapter 2 presents a general model of the problem. The model is a special case of the generalized principal-agent model in Myerson (1982). Chapter 3 and Chapter 4 present the analysis. We study two types of production process in Chapter 3 and Chapter 4 respectively. In Chapter 3, the production is of fixed proportions, that is, different types of input are not substitutable. An incentive compatible compensation scheme is first derived for a single firm industry. The scheme is then extended to a multiple firm industry. The subgaming issues amongst the agents are discussed, particularlly on the issue of whether the incentive scheme is collusion immune. In Chapter 4, we assume that the inputs of the production are substitutable. Using a similar approach, we derive an incentive compatible scheme. Chapter 5 provides concluding remarks.

Chapter 2 THE GENERAL MODEL

2.1 Basic Information

The economic structure of the industry is shown in Figure 1. There is one central planner (the principal) and n firms (agents). The principal is delegated the responsibility of planning and controlling the industrial production for the benefit of the society. The principal makes policy on production and compensation, requests local information, and finally decides the actual compensation for each agent based on the agent's performance. Each agent, on the other hand, works in his or her own interest. He or she acquires material from the society and provides effort in order to operate the production process. It should be pointed out that it is the society, not the agents, who bears the cost of material input. All firms produce a homogeneous product.

We denote $I = \{1, \dots, n\}$ as the set of all agents, and agent $i, i \in I$, as any individual agent.

For agent i, the production output, $q_i \in Q_i$, is a function of material input m_i, $m_i \in M_i$, labor input e_i, $e_i \in E_i$, and the state of production efficiency s_i, $s_i \in S_i$,

$$f_i: \ M_i \times E_i \times S_i \rightarrow R, \qquad i \in I.$$

For the industry as a whole, the production function is the following mapping,

$$f: \ M \times E \times S \rightarrow R^n,$$

where $M = \prod_{i=1}^{n} M_i$ is the set of material input combinations, $m = (m_1, \ldots, m_n)$; $E = \prod_{i=1}^{n} E_i$ is the set of labor (or effort) input combinations, $e = (e_1, \ldots, e_n)$; $S = \prod_{i=1}^{n} S_i$ is the set of state combinations, $s = (s_1, \ldots, s_n)$; and $f = \prod_{i=1}^{n} f_i$.

The total output of the industry is $q = \sum_{i=1}^{n} q_i, \quad q \in Q \subset R$. The total material input of the industry is $m = \sum_{i=1}^{n} m_i, \quad m \in M$.

The following information is common knowledge:

— Product demand function, $P = P(q)$, with $P'(q) \leq 0$.

— Material cost function, $c = C(m)$, and marginal material cost function, $MC(m) = C'(m)$, with $MC'(m) \geq 0$.

— Agents' utility functions over money z and effort e, $u_i^A = U_i(z) - v_i(e)$, with $U_i'(z) > 0$, $U_i''(z) \leq 0$, $v_i'(e) > 0$, and $v_i''(e) \geq 0, \forall i \in I$.

— Material set, M_i, effort set, E_i, state set, S_i, and its probability distribution, $p_i(s_i), \forall i \in I$.

— Realization of material input, m_i, and production output, $q_i, \forall i \in I$. These are known ex post.

The sequence of the events is

1. The principal offers a contract, which specifies the rules to determine the production level, $\hat{q} = (\hat{q}_1, \ldots, \hat{q}_n)$, material usage, $\hat{m} = (\hat{m}_1, \ldots, \hat{m}_n)$, and compensation, $z = (z_1, \ldots, z_n)$. These rules are functions of the reported state, $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$, and other commonly known variables. Once an agent accepts the contract, he or she can not quit.
2. Agent i observes the local state of production , $s_i \in S_i, \forall i \in I$.
3. Agent i reports a message to the principal, $\hat{s}_i, \forall i \in I$. Consequently, agent i is required to produce $\hat{q}_i(\hat{s})$ and allowed to use material $\hat{m}_i(\hat{s}), \forall i \in I$.
4. Agent i chooses an input combination $(m_i, e_i), \forall i \in I$.
5. The actual output, $q = (q_1, \ldots, q_n)$, and the actual material input, $m = (m_1, \ldots, m_n)$, are realized and observed by all the parties.
6. Agent i gets monetary compensation $z_i(\hat{s}, \hat{q}_i, \hat{m}_i, q_i, m_i), \forall i \in I$.

2.2 The Principal's Objective Function

The principal is responsible for increasing social welfare through industrial production. We assume that the principal is committed to fulfill this task to the maximum extent.

If output q is produced, material m is used, and compensation z is made, the realized social welfare is measured by

$$w = \int_0^{\sum_{i=1}^n q_i} P(q_o)\, dq_o - \int_0^{\sum_{i=1}^n m_i} MC(m_o)\, dm_o - \sum_{i=1}^n z_i. \tag{1}$$

The principal seeks to design an incentive scheme, $\pi = (\hat{q}, \hat{m}, z)$, to maximize the expected social welfare,

$$Max_{(\hat{q},\hat{m},z)} \quad E_s \left[\int_0^{\sum_{i=1}^n q_i} P(q_o)\, dq_o - \int_0^{\sum_{i=1}^n m_i} MC(m_o)\, dm_o - \sum_{i=1}^n z_i \right]. \tag{2}$$

Where E_s means the expectation taken over the set S.

2.3 Incentive Compatibility Condition

For the production function $q_i = f_i(m_i, e_i, s_i)$, if we substitute v_i (the disutility) for e_i, it becomes $q_i = f_i(m_i, v_i^{-1}(v_i), s_i) = f_i(m_i, v_i, s_i)$. (Here, function f_i is slightly redefined.) Assume it is invertible with respect to v_i, then agent i's disutility of effort, if q_i is produced, m_i is used, and s_i is realized, is

$$v_i = f_i^{-1}(q_i, m_i, s_i) = V_i(q_i, m_i, s_i).$$

If agent i observes s_i, and reports $\hat{s}_i$, with all the other agents observing s^i and reporting $\hat{s}^i$, then agent i's utility is

$$u_i^A(\hat{z}_i(\hat{s}_i, \hat{s}^i, \cdot), s_i, s^i) = U_i[z_i(\hat{s}, \hat{q}_i(\hat{s}), \hat{m}_i(\hat{s}), q_i, m_i)] - V_i(q_i, m_i, s_i), \tag{3}$$

where $s^i = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, and $\hat{s}^i = (\hat{s}_1, \ldots, \hat{s}_{i-1}, \hat{s}_{i+1}, \ldots, \hat{s}_n)$.

If an incentive scheme $\pi = (\hat{q}, \hat{m}, z)$ is incentive compatible, i.e., it induces all the agents to report the true state and to take an action preferred by the principal, it must meet the following Nash condition,

$$(s_i, \hat{m}_i(s_i), \hat{q}_i(s_i)) \in Argmax_{(\hat{s}_i, m_i, q_i)} \quad u_i^A(\hat{z}_i(\hat{s}_i, s^i, \cdot), s_i, s^i), \forall i \in I. \tag{4}$$

where $(\hat{s}_i, m_i, q_i)$ represents agent i's possible strategies.

Let $\hat{z}_i(\hat{s}) = z_i(\hat{s}, \hat{q}_i(\hat{s}), \hat{m}_i(\hat{s}), \hat{q}_i(\hat{s}), \hat{m}_i(\hat{s}))$ be the compensation to agent i if agent i is obedient, i.e., $m_i = \hat{m}_i(\hat{s})$ and $q_i = \hat{q}_i(\hat{s})$. The principal can force agents to be obedient by using the following form of compensation,

$$z_i(\hat{s}, \hat{q}_i(\hat{s}), \hat{m}_i(\hat{s}), q_i, m_i) = \begin{cases} \hat{z}_i(\hat{s}), & \text{if } q_i = \hat{q}_i(\hat{s}) \text{ and } m_i = \hat{m}_i(\hat{s}); \\ penalty, & \text{otherwise.} \end{cases} \tag{5}$$

In this paper, we only focus on the truth-inducing aspect of incentive compatible schemes. It is implicit that obedience aspect is eliminated by using compensation of the form given by (5).

From (3), we see that agent i's utility is affected by other agents' message, $\hat{s}^i$, but not by other agents' state realization, s^i, i.e.,

$$u_i^A(\hat{z}_i(\hat{s}_i, \hat{s}^i), s_i, s^i) = u_i^A(\hat{z}_i(\hat{s}_i, \hat{s}^i), s_i, \hat{s}^i). \tag{6}$$

This implies that a Nash equilibrium in our problem is automatically a dominant strategy equilibrium. Therefore, (4) is also the necessary and sufficient condition of a dominant strategy equilibrium.

2.4 Reservation Utility Constraint

At the time of contracting, every agent has to be offered no less than his or her minimum utility level, $\overline{u}_i$, which may represent either the utility from alternative employment or some welfare standard specified by the goverment.

$$E_s\left[U_i(z_i) - V_i(q_i, m_i, s_i)\right] \geq \overline{u}_i, \quad \forall i \in I. \tag{7}$$

If the principal restricts the incentive scheme to be incentive compatible, then (7) becomes

$$E_s\left[U_i(\hat{z}_i(s_i)) - V_i(\hat{q}(s), \hat{m}_i(s), s_i)\right] \geq \overline{u}_i, \forall i \in I. \tag{8}$$

The complete model of the problem consists of (2), (4) and (8), if the incentive scheme is restricted to be incentive compatible. According to Myerson (1982), this restriction on the incentive scheme does not affect the optimality of the solution with communication.

Proposition 1 (Myerson (1982), Proposition 2). *The optimal incentive schemes which are incentive compatible are also optimal in the class of all incentive schemes with communication.*

In his paper, Myerson has also shown that when the state set and decision sets are all finite, a linear program is sufficient to solve this type of problem. We do not show how this works in our problem since it merely requires some mathematical transformations. In this paper, we mainly focus on the type of problem with continuous variables.

Chapter 3 PRODUCTION WITH FIXED PROPORTION INPUTS

In a fixed proportions production process, one type of input usage will not be reduced by using more of other types of input. In other words, there is no substitution among different types of input. Later, in Chapter 4, we shall discuss the problem in which there are substitutions among inputs.

To be more general, let the state of production be two dimensional, i.e., let $s_i = (\theta_i, \delta_i), \forall i \in I$, where θ_i is the state of material efficiency, and δ_i is the state of labor efficiency. Assume these states are independent.

Since the production is of a fixed proportions type, the production function can be decomposed into two parts,

$$m_i = M_i(q_i, \theta_i) \quad \text{and} \quad v_i = V_i(q_i, \delta_i), \forall i \in I.$$

Since both the material realization, m_i, and the output realization, q_i, are observed ex post by the principal, θ_i can be inferred. Therefore, the principal is able to force agents to report the true state of material efficiency by imposing a penalty if they do not do so. Without loss of generality, we use a deterministic function $m_i = M_i(q_i), \forall i \in I$, for material usage. Hence, the state degenerates to one dimension, $s_i = \delta_i$.

We make additional assumptions as follows:

A1. Material input is strictly increasing in production output,

$$M_i'(q_i) > 0, \quad M_i(0) = 0, \quad \forall i \in I.$$

Moreover, the incremental material cost is increasing in production output,

$$\partial[MC(\sum_{i=1}^{n} M_i(q_i))M_i'(q_i)]/\partial q_i \geq 0, \forall i \in I.$$

A2. The state of production takes positive values on the real line,

$$s_i \in S_i \subset [\underline{s}_i, \infty) \quad \forall i \in I.$$

Finite state sets are also used for illustrations.

A3. Distility function is strictly increasing and weakly convex in output,

$$\partial V_i(q_i, s_i)/\partial q_i > 0, \quad \partial^2 V_i(q_i, s_i)/\partial q_i^2 \geq 0,$$

$$V_i(0, s_i) = 0, \quad \forall s_i \in S_i, \forall i \in I.$$

The larger the value of s_i, the less efficient the production, which reflects that the greater the incremental disutility. In other words,

$$\frac{\partial^2 V_i(q_i, s_i)}{\partial q_i \partial s_i} \geq 0, \quad \forall q_i \in Q_i, \forall i \in I,$$

with strict inequality for some s_i.

It is not within our interest to consider the problem with $\partial^2 V_i/\partial q_i \partial s_i = 0$ for all s_i, since no information asymmetry exists in this case.

3.1 The Principal Has Perfect Information

When the principal has perfect information, he or she observes the state realization and the agents' action choice. The principal can make an optimal central planning decision and force all agents to take the specified action. Therefore, the first-best solution is achieved. Let $\hat{q}^*$ be the optimal production and z^* be the optimal compensation. We then have the following model:

P1:

$$Max_{(\hat{q}^*,z^*)} \int_S \left[\int_0^{\sum_{i=1}^n \hat{q}_i} P(q_o)\, dq_o - \int_0^{\sum_{i=1}^n M_i(\hat{q}_i)} MC(m_o)\, dm_o - \sum_{i=1}^n z_i \right] p(s)\, ds. \tag{9}$$

$$St. \qquad \int_S [U_i(z_i) - V_i(\hat{q}_i(s), s_i)] p(s)\, ds \geq \overline{u}_i. \tag{9A}$$

Compared with the general model in Chapter 2, this model does not have the incentive compatibility constraint. This is because the principal observes state realization, so, he or she is able to force the agents to produce what is desired.

By using the Lagrange formula, we characterize the sufficient conditions to determine $(\hat{q}^*, z^*)$.

$$P(\sum_{i=1}^n \hat{q}_i) - MC(\sum_{i=1}^n M_i(\hat{q}_i)) M_i'(\hat{q}_i) - \frac{1}{U_i'(z_i)} \frac{\partial V_i(\hat{q}_i, s_i)}{\partial \hat{q}_i} = 0, \tag{10}$$

$$z_i = U_i^{-1} \big[\overline{u}_i + \int_S V_i(\hat{q}_i(s), s_i) p(s)\, ds \big], \tag{11}$$

$$\forall s \in S \quad \text{and} \quad \forall i \in I.$$

When all the agents are risk neutral, we set $U_i'(z_i) \equiv 1, \forall i \in I$. Then $\hat{q}^* = (\hat{q}_1^*, \ldots, \hat{q}_n^*)$ and $z^* = (z_1^*, \ldots, z_n^*)$ can be easily solved from (10) and (11). However, when some or all the agents are risk averse, it is difficult to determine $\hat{q}^*$ and z^*. Nevertheless, following properties can be derived from (10) and (11).

Proposition 2. *If the principal has perfect information, given the assumptions on common knowledge, then (1) the optimal production level for agent i increases as his or her state becomes more efficient, $\partial \hat{q}_i^* / \partial s_i \leq 0$, and decreases as any other agent's state becomes more efficient, $\partial \hat{q}_i^* / \partial s_j \geq 0$; $i \neq j, i, j \in I$; (2) the compensation functions of all agents are constants, regardless of the state realization.*

Proof: The proposition follows directly from (10) and (11), given common knowledge and assumptions A1, A2, and A3. Q.E.D.

3.2 Single Agent Problem With Information Asymmetry

Here, we assume that the principal does not observe either state realizations or agent effort. Assume that there is but one firm (agent) in the industry. This situation may be uncommon, however the analysis on single agent problem provides an essential guide towards the solution of multiple agent problem.

P2:

$$Max_{(\hat{q}^*, \hat{z}^*)} \quad E_s \left[\int_0^{\hat{q}(s)} P(q_o)\, dq_o - \int_0^{M(\hat{q}(s))} MC(m_o)\, dm_o - \hat{z}(s) \right] \tag{12}$$

$$St. \qquad U(\hat{z}(s)) - V(\hat{q}(s), s) \geq U(\hat{z}(\hat{s})) - V(\hat{q}(\hat{s}), s) \tag{12A}$$

$$\forall \hat{s} \in S, \forall s \in S,$$

$$E_s[U(\hat{z}(s)) - V(\hat{q}(s), s) \geq \overline{u}. \tag{12B}$$

It should be pointed out that both $\hat{q}^*$ and $\hat{z}^*$ are functions of the reported state.

3.2.1 The State Set Is Finite

In Chapter 2, we used Myerson (1982)'s result and stated that when the sets $Q, Z, M, E,$ and S are all finite, a linear programming technique can be used to solve the problem. In fact, when just S is finite, constraint (12A) contains a finite number of equations, and the problem becomes solvable without requiring the finiteness of $Q, Z, M,$ and E. A simple example will serve as an illustration.

Example 1. Suppose there is the following information, $P(q) = 10 - 2q, MC(m) = 1 + 0.2m, M(q) = 2q, U(z) = z, V(q,s) = sq, \bar{u} = 1, S = \{1,2\}$, with $p(1) = 0.5$, and $p(2) = 0.5$. Then

$$
\begin{aligned}
Max_{(\hat{q}^*,\hat{z}^*)}\ & 0.5\left[\int_0^{\hat{q}(1)} (10-2q)\,dq - \int_0^{2\hat{q}(1)} (1+0.2m)\,dm - \hat{z}(1)\right] \\
& + 0.5\left[\int_0^{\hat{q}(2)} (10-2q)\,dq - \int_0^{2\hat{q}(2)} (1+0.2m)\,dm - \hat{z}(2)\right] \\
St.\quad & \hat{z}(1) - \hat{q}(1) \geq \hat{z}(2) - \hat{q}(2), \\
& \hat{z}(2) - 2\hat{q}(2) \geq \hat{z}(1) - 2\hat{q}(1), \\
& 0.5[\hat{z}(1) - \hat{q}(1)] + 0.5[\hat{z}(2) - 2\hat{q}(2)] \geq 1.
\end{aligned}
$$

Applying the Lagrange formula, we get the optimal solution, $\hat{q}^*(1) = 2.5, \hat{q}^*(2) = 2.143, \hat{z}^*(1) = 4.661, \hat{z}^*(2) = 4.125$, and $EW = 6.589$, where EW stands for the expected social welfare.

3.2.2 The Agent Is Risk Neutral And The State Set Is Infinite

In the usual one principal-one agent setting described in Holmström (1979), the principal's objective is to maximize the monetary return (or profit) from the operation. When the agent is risk neutral, the first-best solution is achieved by renting the production facility to the agent, regardless of the existence of an information asymmetry. This is because there is no risk-sharing aspect when the agent is risk neutral. Moreover, the agent automatically chooses the production which maximizes the net utility—profit minus disutility of effort since the agent keeps the residual. This is exactly the same production level as the principal would choose.

In our problem, because the principal wants to maximize social welfare, not profit, the rental scheme is not directly applicable. If the principal rents the production process to the agent, the agent again chooses the producton to maximize net utility, but this is not the production level the principal wants. Nonetheless, we can design an incentive scheme which is equivalent to the rental scheme and achieve the first-best solution of the problem.

Proposition 3. *For the problem P2, if the agent is risk neutral, then the following compensation scheme induces optimal production, and achieves the first-best solution,*

$$\hat{z} = \int_{\underline{q}}^{q} P(q_o)\, dq_o - \int_{M(\underline{q})}^{M(q)} MC(m_o)\, dm_o. \tag{13}$$

where q is the actual output, and $\underline{q}$ is some constant predetermined by the following condition (denote $\hat{q}^*(s)$ as the optimal production rule),

$$\int_S \left[\int_{\underline{q}}^{\hat{q}^*(s)} P(q_o)\, dq_o - \int_{M(\underline{q})}^{M(\hat{q}^*(s))} MC(m_o)\, dm_o \right] p(s)\, ds - \int_S V(\hat{q}^*(s), s) p(s)\, ds = \overline{u} \tag{14}$$

Proof. If $s \in S$ is observed and q is produced, then the agent's utility is

$$u^A = \int_{\underline{q}}^{q} P(q_o)\, dq_o - \int_{M(\underline{q})}^{M(q)} MC(m_o)\, dm_o - V(q,s).$$

The agent chooses the production by satisfying the following condition,

$$\frac{\partial u^A}{\partial q} = P(q) - MC(M(q))M'(q) - \frac{\partial V(q,s)}{\partial q} = 0.$$

This is the same as (10), the condition to determine the optimal production when the principal has perfect information. Therefore, the production induced by (13) is optimal.

Since $\underline{q}$ is chosen such that the minimun utility constraint is binding, the solution is the first-best. Q.E.D.

It is immediate that by applying (13) to Example 1, we get the same result. From

$$\frac{\partial u^A}{\partial q} = 10 - 2q - (2 + 0.8q) - s = 0,$$

we have $\hat{q}^*(s = 1) = 2.5, \hat{q}^*(s = 2) = 2.134$. From (14), we get $\underline{q} = 0.9979$ given $\overline{u} = 1$. So $EW = 6.589$.

Proposition 3 says that for the single agent problem, if the agent is risk neutral, then the first-best solution can always be achieved.

3.2.3 The Agent Is Risk Averse And The State Set Is Infinite

When the agent is risk averse, the first-best solution is generally not achievable if there exists information asymmetry. First, the forcing contract can no longer be used because the principal does not observe either state realization or agent's action choice. Second, there are both risk-sharing aspects and motivational aspects to consider. It is usually not optimal to let the agent absorb all the risk.

The following numerical example is given to illustrate that only a second-best solution is achieved when the agent is risk averse with an information asymmetry. For simplicity, we use a finite state set.

Example 2. The information is the same as Example 1 except that the agent is risk averse, $U(z) = \sqrt{z}$.

(1). The principal has perfect information.

$$\begin{aligned} Max_{(\hat{q}^*, \hat{z}^*)}\, 0.5 & \left[\int_0^{\hat{q}(1)} (10 - 2q)\, dq - \int_0^{2\hat{q}(1)} (1 + 0.2m)\, dm \right] \\ & + 0.5 \left[\int_0^{\hat{q}(2)} (10 - 2q)\, dq - \int_0^{2\hat{q}(2)} (1 + 0.2m)\, dm \right] - 0.5[\hat{z}(1) + \hat{z}(2)] \\ St.\ & 0.5[\sqrt{\hat{z}(1)} - \hat{q}(1)] + 0.5[\sqrt{\hat{z}(2)} - 2\hat{q}(2)] \geq 1. \end{aligned}$$

The solution is $\hat{q}^*(1) = 1.502, \hat{q}^*(2) = 0.1456, \hat{z}^*(1) = \hat{z}^*(2) = 3.60$, and $EW = 1.40$. This is the first-best solution.

(2). The principal does not observe the state realization.

$$\begin{aligned} Max_{(\hat{q}^*, \hat{z}^*)}\, 0.5 & \left[\int_0^{\hat{q}(1)} (10 - 2q)\, dq - \int_0^{2\hat{q}(1)} (1 + 0.2m)\, dm \right] \\ & + 0.5 \left[\int_0^{\hat{q}(2)} (10 - 2q)\, dq - \int_0^{2\hat{q}(2)} (1 + 0.2m)\, dm \right] - 0.5[\hat{z}(1) + \hat{z}(2)] \\ St.\ & \sqrt{\hat{z}(1)} - \hat{q}(1) \geq \sqrt{\hat{z}(2)} - \hat{q}(2), \\ & \sqrt{\hat{z}(2)} - 2\hat{q}(2) \geq \sqrt{\hat{z}(1)} - 2\hat{q}(1), \\ & 0.5[\sqrt{\hat{z}(1)} - \hat{q}(1)] + 0.5[\sqrt{\hat{z}(2)} - 2\hat{q}(2)] \geq 1. \end{aligned}$$

The solution is $\hat{q}^*(1) = 1.1745, \hat{q}^*(2) = 0.3624, \hat{z}^*(1) = 5.549, \hat{z}^*(2) = 2.3827$, and $EW = 1.2404$. We see that, from the principal's point of view, this solution is strictly inferior to that in (1).

Now, we start to analyse the general problem of P2. We assume that the set S is infinite, with common prior probability density function $p(s)$.

Since the constraint (12A) contains an infinite number of equations, it is impossible to solve the numerical values of $\hat{z}$ and $\hat{q}$ under each state. Rather, we have to look for functions $\hat{z}(\hat{s})$ and $\hat{q}(\hat{s})$. When all externally determined functions are continuous and differentiable, and S is infinite, then we are able to concentrate on those $\hat{z}(\hat{s})$ and $\hat{q}(\hat{s})$ which are continuous and differentiable.

The necessary and sufficient condition for a scheme , $(\hat{z}(\hat{s}), \hat{q}(\hat{s}))$, to be incentive compatible, i.e., it satisfies (12A), is,

$$\frac{\partial u^A(\hat{z}(\hat{s}), s)}{\partial \hat{s}} = \frac{dU(\hat{z}(\hat{s}))}{d\hat{z}} \cdot \frac{d\hat{z}}{d\hat{s}} - \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} \cdot \frac{d\hat{q}}{d\hat{s}} \quad \begin{array}{ll} \geq 0, & \text{if } \hat{s} < s; \\ = 0, & \text{if } \hat{s} = s; \\ \leq 0, & \text{if } \hat{s} > s; \end{array} \tag{15}$$

$$\forall s \in S.$$

The next step is to search for the optimal scheme $(\hat{q}^*(s), \hat{z}^*(s))$ which satisfies the necessary and sufficient condition (15). The result is presented in proposition 4.

Proposition 4. *For the single agent problem P2, if assumptions A1, A2, and A3 hold, let*

$$\hat{z}(\hat{s}) = U^{-1}\left[\int_{\underline{s}}^{\hat{s}} \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} \cdot \frac{d\hat{q}(s)}{ds}\, ds + h\right], \tag{16}$$

where $\hat{s}$ is the reported state, and h is some constant determined by the minimum utility constraint (12B), and $\hat{q}(\hat{s})$ is chosen such that

$$P(\hat{q}) - MC(M(\hat{q})) \cdot M'(\hat{q}) - \frac{1}{U'(\hat{z}(\hat{s}))} \cdot \frac{\partial V(\hat{q}, \hat{s})}{\partial \hat{q}} = 0, \tag{17}$$

$$\forall \hat{s} \in S,$$

then,

1. *the incentive scheme is the optimal incentive compatible scheme;*
2. *the optimal incentive compatible scheme is unique;*
3. *the optimal production $\hat{q}^*$ increases as the state becomes more efficient;*
4. *the principal derives all the rent.*

Proof: see Appendix A.

Corollary 4.1 *For the single agent problem P2, if assumptions A1 to A3 hold, and if the incremental disutility function, $\partial V(q, s)/\partial q$ is strictly increasing in s, then the optimal production $\hat{q}^*$ is strictly decreasing in s.*

Proof: This follows directly from the proof of proposition 4.

Proposition 4 says that there is no other incentive compatible scheme which is superior to the one determined by (16) and (17). Moreover, by Myerson's revelation principle, there is no other incentive scheme of any type which is superior to the one determined by (16) and (17).

When the agent is risk neutral, (let $U'(z) = 1$), then it is straight forward to solve $\hat{q}(\hat{s})$ from (17), and then solve $\hat{z}(\hat{s})$ from (16). Since the optimal incentive compatible scheme is unique, the scheme from (16) and (17) should be the same as (13) if the agent is risk neutral. We show this in the following.

From (16), we have

$$\frac{d\hat{z}(\hat{s})}{d\hat{s}} = \frac{\partial V(\hat{q}, \hat{s})}{\partial \hat{q}} \cdot \frac{d\hat{q}(\hat{s})}{d\hat{s}}.$$

Since $(\hat{q}, \hat{z})$ is incentive compatible, then, $\hat{s} = s$, implying that

$$\frac{d\hat{z}(s)}{ds} = \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} \cdot \frac{d\hat{q}(s)}{ds},$$

and, hence

$$\frac{d\hat{z}}{d\hat{q}} = \frac{d\hat{z}}{ds} \cdot \frac{1}{\frac{d\hat{q}}{ds}} = \frac{\partial V(\hat{q}, s)}{\partial \hat{q}}.$$

Therefore, (17) becomes

$$\frac{d\hat{z}}{d\hat{q}} = P(\hat{q}) - MC(M(\hat{q})) \cdot M'(\hat{q}),$$

which yields

$$\hat{z} = \int_{\underline{q}}^{\hat{q}} P(q_o)\, dq_o - \int_{M(\underline{q})}^{M(\hat{q})} MC(m_o)\, dm_o + h. \tag{18}$$

This compensation is the same as the one derived in Proposition 3.

When the agent is risk averse, determining the function form of $\hat{q}(s)$ and $\hat{z}(s)$ is very complex. We may even be required to take a numerical solution approach.

Part (3) of Porposition 4 is quite intuitive. When the state is more efficient, the incremental cost of production is smaller. Since the incremental value stays the same, then we should increase the production to maximize the welfare.

Since the minimum utility constraint (12A) is binding, it is the principal who derives all the rent from the private information of the agent. In order to have this result, the assumption that the agent can not quit once he or she accepts the contract is crucial.

3.3 Multiple Agent Problem With Information Asymmetry

In this section, we extend our analysis to the multiple agent problem. We still assume that the set S is infinite, where $S = \prod_{i=1}^{n} S_i$. Let $p(s)$ be the joint probability density function over S. We repeat the model for convenience.

P3

$$Max_{(\hat{q}^*(s),\hat{z}^*(s))} \int_S \left[\int_0^{\sum_{i=1}^n \hat{q}_i(s)} P(q_o)\, dq_o - \int_0^{\sum_{i=1}^n M_i(\hat{q}_i(s))} MC(m_o)\, dm_o - \sum_{i=1}^{n} \hat{z}_i(s) \right] p(s)\, ds \quad (19)$$

$$St. \quad U_i(\hat{z}_i(s_i, \hat{s}^i) - V_i(\hat{q}_i(s_i, \hat{s}^i), s_i) \geq U_i(\hat{z}_i(\hat{s}_i, \hat{s}^i)) - V_i(\hat{q}_i(\hat{s}_i, \hat{s}^i), s_i) \quad (19A)$$

$$\forall (\hat{s}_i, \hat{s}^i) \in S, \forall s_i \in S_i \quad \text{and} \quad \forall i \in I,$$

$$\int_S \left[U_i(\hat{z}_i(s)) - V_i(\hat{q}_i(s), s_i) \right] p(s)\, ds \geq \overline{u}_i \quad (19B)$$

$$\forall i \in I,$$

where $\hat{q}(\hat{s}) = (\hat{q}_1(\hat{s}), \ldots, \hat{q}_n(\hat{s}))$ and $\hat{z}(\hat{s}) = (\hat{z}_1(\hat{s}), \ldots, \hat{z}_n(\hat{s}))$.

When there is more than one agent, the optimal production has to be determined by the overall reported state. Hence, $\hat{q}_i$ is a function of both $\hat{s}_i$ and $\hat{s}^i = (\hat{s}_1, \ldots, \hat{s}_{i-1}, \hat{s}_{i+1}, \ldots, \hat{s}_n)$.

By extending the optimal incentive compatible scheme of the single agent problem, we have the following result.

Proposition 5. *For the multiple agent problem P3, if assumptions A1 to A3 hold, if we let the compensation be*

$$\hat{z}_i(\hat{s}_i, \hat{s}^i) = U_i^{-1} \left[\int_{\underline{s}_i}^{\hat{s}_i} \frac{\partial V_i(\hat{q}_i, s_i)}{\partial \hat{q}_i} \cdot \frac{\partial \hat{q}_i(s_i, \hat{s}^i)}{\partial s_i}\, ds_i + h_i \right] \quad (20)$$

$$\forall \hat{s}^i \in S^i \quad \textit{and} \quad \forall i \in I,$$

where h_i is some constant determined by the minimum utility constraint, and if we choose $\hat{q}_i$ such that

$$P(\sum_{i=1}^{n} \hat{q}_i) - MC(\sum_{i=1}^{n} M_i(\hat{q}_i)) \cdot M_i'(\hat{q}_i)) - \frac{1}{U_i'(\hat{z}_i)} \cdot \frac{\partial V_i(\hat{q}_i, \hat{s}_i)}{\partial \hat{q}_i} = 0, \tag{21}$$

$$\forall \hat{s}_i \in S_i, \quad and \quad \forall i \in I.$$

then

1. *equations (20) and (21) give the optimal incentive compatible scheme, implying that truth-telling is a dominant strategy for every agent;*
2. *the optimal production for agent i increases as agent i's state becomes more efficient, and decreases as any other agent's state becomes more efficient;*
3. *the principal derives all the rent from the private information of the agents;*
4. *the solution is the first-best when all the agents are risk neutral, and strictly second-best otherwise.*

Proof: see Appendix B.

The economic intuition behind Proposition 5 is the same as Proposition 4. In (21), the first term is the incremental value of production, and the sum of the second and the third term is the total incremental cost of the production. At the optimum, the total incremental cost of production for each agent is identical, and the total incremental cost for any agent equals the incremental value of production. However, another issue which arises in the multiple agent case is that the principal's incentive scheme creates a subgame among the agents. This increases the dimensions of the problem.

The following numerical example is given to show the calculation of $\hat{q}$ and $\hat{z}$.

Example 3. Suppose for a one principal-two agent problem, there is the following information,

$$P(q) = 10 - 2q, \qquad MC(m) = 2m,$$
$$M_1(q_1) = q_1, \quad V_1(q_1, s_1) = (1 + s_1)q_1^2,$$
$$M_2(q_2) = 2q_2, \quad V_2(q_2, s_2) = (1 + s_2)q_2^2,$$
$$s_1 \in [1, 2], \ p_1(s_1) = 1, \quad s_2 \in [1, 2], \ p_2(s_2) = 1.$$

Let $(\hat{s}_1, \hat{s}_2)$ be the reported state, the optimal production is determined by

$$\begin{cases} 10 - 2(\hat{q}_1 + \hat{q}_2) - 2(\hat{q}_1 + 2\hat{q}_2) - 2(1 + \hat{s}_1)\hat{q}_1 = 0, \\ 10 - 2(\hat{q}_1 + \hat{q}_2) - 4(\hat{q}_1 + 2\hat{q}_2) - 2(1 + \hat{s}_2)\hat{q}_2 = 0. \end{cases}$$

Therefore,

$$\hat{q}_1 = \frac{5(\hat{s}_2 + 3)}{\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9}, \quad \text{and} \quad \hat{q}_2 = \frac{5\hat{s}_1}{\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9}.$$

Since $\underline{s}_1 = 1$ and $\underline{s}_2 = 1$, then

$$\begin{aligned} \hat{z}_1(\hat{s}_1, \hat{s}_2) &= \int_1^{\hat{s}_1} \frac{(-50)(\hat{s}_2 + 3)^2(\hat{s}_2 + 6)(s_1)}{(s_1\hat{s}_2 + 6s_1 + 3\hat{s}_2 + 9)^3}\, ds_1 + h_1 \\ &= \frac{25(\hat{s}_2 + 3)^2(\hat{s}_1 + 1)}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)^2} + \frac{25(\hat{s}_2 + 3)^2}{(\hat{s}_1\hat{s}_2 + 6s_1 + 3\hat{s}_2 + 9)(\hat{s}_2 + 6)} \\ &\quad - \frac{50(\hat{s}_2 + 3)^2}{(4\hat{s}_2 + 15)^2} - \frac{25(\hat{s}_2 + 3)^2}{(4\hat{s}_2 + 15)(\hat{s}_2 + 6)} + h_1. \end{aligned}$$

And

$$\begin{aligned} \hat{z}_2(\hat{s}_1, \hat{s}_2) =& \frac{25\hat{s}_1^2(\hat{s}_2)}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)^2} + \frac{25\hat{s}_1^2}{(\hat{s}_1\hat{s}^2 + 6\hat{s}_1 + 3s_2 + 9)(\hat{s}_1 + 3)} \\ & - \frac{50\hat{s}_1^2}{(7\hat{s}_1 + 12)^2} - \frac{25\hat{s}_1^2}{(7\hat{s}_1 + 12)(\hat{s}_1 + 3)} + h_2. \end{aligned}$$

It is easy to verify that $u_1^A(\hat{s}_1, \hat{s}_2)$ is maximized at $\hat{s}_1 = s_1$, and $u_2^A(\hat{s}_1, \hat{s}_2)$ is maximized at $\hat{s}_2 = s_2$.

3.4 The Subgame Of The Agents

We have proposed an incentive compatible compensation scheme which induces a dominant strategy equilibrium. However, as has been pointed out by Amershi and Cheng (1986), a dominant strategy equilibrium is not always implementable. It is possible that the agents can be made better off, with at least one agent being strictly better off and all others weakly better off, through implicit or explicit collusion. Therefore, we need to study the subgame of the agents. Specifically, we would like to see under what conditions the equilibrium preferred by the principal can be implemented.

The following terms are defined to facilitate our discussion.

Definition 1. *A <u>non-cooperative subgame</u> is the one in which the agents do not play non-Nash strategies and do not use side payments.*

Definition 2. *A subgame induced by an incentive scheme is a <u>prisoners' dilemma</u> if there exists at least one non-Nash strategy combination which, without side payments, strictly Pareto-dominates the equilibrium specificed by the scheme.*

Definition 3. *An equilibrium is <u>non-cooperatively implementable</u> if, in the subgame, the equilibrium at least weekly Pareto-dominates all the other Nash strategies in the non-cooperative subgame.*

Definition 4. *An equilibrium is <u>implementable</u> if, in the subgame, there exists no strategy combination, either cooperative or non-cooperative, which strictly Pareto-dominates this equilibrium.*

Definition 5. *An incentive scheme is <u>collusion immune</u> if the equilibrium induced by the scheme is implementable under every state realization.*

3.4.1 The Agents Do Not Cooperate

When the agents do not cooperate, i.e., they neither use side payments nor play non-Nash strategies, then the following result is obtained.

Proposition 6. *If the agents do not cooperate, and if every agent's incremental disutility is strictly increasing in his or her state realization,* $\partial^2 V_i(q_i, s_i)/\partial q_i \partial s_i > 0, \forall q_i \in Q_i$ *and* $\forall i \in I$*, then the dominant strategy equilibrium induced by the incentive scheme (20) and (21) is non-cooperatively implementable under every state realization.*

Proof: If $\partial V_i/\partial q_i$ is strictly increasing in s_i, then from (21), we know that $\hat{q}_i$ is strictly decreasing in $\hat{s}_i$, i.e., $\partial \hat{q}_i(\hat{s}_i, \hat{s}^i)/\partial \hat{s}_i < 0, \forall \hat{s} \in S$, and $\forall i \in I$. Since

$$\frac{\partial u_i^A}{\partial \hat{s}_i} = [\frac{\partial V_i(\hat{q}_i, \hat{s}_i)}{\partial \hat{q}_i} - \frac{\partial V_i(\hat{q}_i, s_i)}{\partial \hat{q}_i}] \cdot \frac{\partial \hat{q}_i(\hat{s}_i, \hat{s}^i)}{\partial \hat{s}_i},$$

then u_i^A is strictly increasing when $\hat{s}_i < s_i$, and strictly decreasing when $\hat{s}_i > s_i$. Therefore, agent i is strictly worse off by not reporting the true state. This, together with the fact that telling the truth is a dominant strategy for every agent, implies that the equilibrium induced by the scheme is the unique Nash equilibrium. Since the agents do not cooperate, this implies that the equilibrium is non-cooperatively implementable under every state realization. Q.E.D.

When we allow $\partial V_i/\partial q_i$ to be weakly increasing in s_i, then Proposition 6 may not be true. Suppose there are two agents, agent 1 and agent 2. If $\partial V_1/\partial \hat{q}_1$ is weakly increasing in s_1, then it is possible that reporting $\hat{s}_1$, an untruthful state, instead of s_1, the true state, does not change $\hat{q}_1$, nor u_1, but increases agent 2's utility, u_2. Moreover, it is possible that $(\hat{s}_1, s_2)$ is also a Nash equilibrium. (Amershi and Cheng (June 1986) have shown this situation.) Therefore, $(\hat{s}_1, s_2)$ is implemented rather than (s_1, s_2).

3.4.2 The Agents May Cooperate

Without loss of generality, we assume there are two agents in 3.4.2. If the agents coooperate, there appear to be many obstacles to implementing the equilibrium induced by the scheme. The following example shows that a collective strategy of not reporting truthfully may be strictly preferred by the agents.

Example 3 Continued. The utilities of the agents, if (s_1, s_2) is realized and $(\hat{s}_1, \hat{s}_2)$ is reported are

$$\begin{aligned} u_1^A(\hat{z}_1(\hat{s}_1, \hat{s}_2), s_1, s_2) = & \frac{25(\hat{s}_2+3)^2(\hat{s}_1 - s_1)}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)^2} \\ & + \frac{25(\hat{s}_2+3)^2}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)(\hat{s}_2+6)} - \frac{50(\hat{s}_2+3)^2}{(4\hat{s}_2+15)^2} \\ & - \frac{25(\hat{s}_2+3)^2}{(4\hat{s}_2+15)(\hat{s}_2+6)} + h_1 \end{aligned}$$

and

$$\begin{aligned} u_2^A(\hat{z}_2(\hat{s}_1, \hat{s}_2), s_1, s_2) = & \frac{25\hat{s}_1^2(\hat{s}_2 - s_2)}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)^2} \\ & + \frac{25\hat{s}_1^2}{(\hat{s}_1\hat{s}_2 + 6\hat{s}_1 + 3\hat{s}_2 + 9)(\hat{s}_1+3)} \\ & - \frac{50\hat{s}_1^2}{(7\hat{s}_1+12)^2} - \frac{25\hat{s}_1^2}{(7\hat{s}_1+12)(\hat{s}_1+3)} + h_2. \end{aligned}$$

Suppose that the state realization is $(2,2)$. For simplicity, we consider four reporting strategies, $(\hat{s}_1, \hat{s}_2) = (2,2), (2,1), (1,2),$ and $(1,1)$. The result is shown in Figure 2. We see that reporting $(2,1)$ with any side payment within the range of $(0.024, 0.214)$ from agent 1 to agent 2 makes both agents strictly better off.

There are two possible types of collusion when the agents cooperate. One, as is shown in the numerical example, is that a side payment is necessary to make no one worse off. The other is the prisoners' dilemma situation.

The following proposition states that the incentive scheme we have derived is not immune against these types of collusion when considered simultaneously.

Proposition 7. *If the agents cooperate, then the incentive scheme determined by (20) and (21) is not collusion immune.*

Proof: see appendix C.

Proposition 7 says there exists at least one state realization under which the equilibrium induced by (20) and (21) is not implementable, because the agents are better off through collusion.

Even though the incentive scheme is not collusion immune, we are able to say something about the agents' possible dishonest behavior.

Proposition 8. *Under the incentive scheme determined by (20) and (21), if the incremental disutility* $\partial V_i / \partial q_i$ *is strictly increasing in* s_i*, then the agents' possible collusion is either (1)* $\hat{s}_i \leq s_i, \forall i \in I$ *with at least one strict inequality, or (2)* $\hat{s}_i > s_i, \forall i \in I$*. In other words, every agent lies in the same direction.*

Proof: see Appendix D.

Intuitivly, we think that agents tend only to report a state worse than the actual one so they can reduce the amount of effort. This is true when the problem is solely moral hazard. In the problem we are studying, there are both moral hazard and adverse selection present. We try to solve both by a scheme which induces the agents to report the true state. The reason for the result in Proposition 8 is that the principal pays very dearly to induce the truthful reporting in good states. Nevertheless, by getting the true state, the principal gets the best overall.

Finally, we want to point out one more issue. When the agents cooperate, they can collude with a side payment. However, in our single period setting, this side payment is not credible, because there is nothing to enforce its payment. This, of course, reduces the possibility of collusion.

Chapter 4 PRODUCTION WITH SUBSTITUTABLE INPUTS

In Chapter 3, we assumed that the production is of the fixed proportions type. Now, we are going to study the problem with substitutable inputs. The following additional assumptions are made for this chapter. These assumptions hold for every agent.

A4. The production function is strictly increasing and weakly concave in both material input and labor input,

$$\frac{\partial f(m,v,s)}{\partial m} > 0, \qquad \frac{\partial f(m,v,s)}{\partial v} > 0,$$

$$\frac{\partial^2 f(m,v,s)}{\partial m^2} \leq 0, \qquad \frac{\partial^2 f(m,v,s)}{\partial v^2} \leq 0,$$

$$\text{and} \quad \frac{\partial^2 f}{\partial m^2} \cdot \frac{\partial^2 f}{\partial v^2} - \left(\frac{\partial^2 f}{\partial m \partial v}\right)^2 \geq 0.$$

A5. The marginal productivity of either input is increasing in the level of the other input,

$$\frac{\partial^2 f}{\partial m \partial v} \geq 0 \quad \forall s \in S.$$

A6. The state of production, s, is some positive value on real line, $s \in S \subset R, \quad s > 0$, with $\underline{s}$ being the smallest member of the set. Also, marginal production is decreasing in s,

$$\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial m}\right) \leq 0, \quad \text{and} \quad \frac{\partial}{\partial s}\left(\frac{\partial f}{\partial v}\right) \leq 0,$$

with at least one strict inequality for every (m, v, s) combination.

A7. The output is zero when either input is zero,

$$f(0, v, s) = f(m, 0, s) = 0, \forall m \in M, \forall v \in V, \forall s \in S.$$

This, together with A5, implies that $\frac{\partial f}{\partial s} < 0$.

A8. The marginal substitution between inputs is decreasing, i.e., $-\partial v/\partial m$ increases in v and decreases in m, given (q, s).

In addition, all the common knowledge stated in Chapter 2 applies here.

Again, it is implicit that the agents are all obedient under the type of contract given in (5). Agent i's disutility, when $s = (s_1, \ldots, s_n)$ is realized and $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$ is reported, is

$$v_i(\hat{s}_i, \hat{s}^i, s_i, s^i) = f^{-1}(\hat{q}_i(\hat{s}), \hat{m}_i(\hat{s}), s_i). \tag{22}$$

So agent i's utility is

$$u_i^A(\hat{z}_i(\hat{s}_i, \hat{s}^i), s_i, s^i) = U_i(\hat{z}_i(\hat{s})) - v_i(\hat{s}_i, \hat{s}^i, s_i, s^i). \tag{23}$$

The analysis in this section is parallel to that in Chapter 3. We focus on the truth revealing aspect of the incentive compatible scheme.

4.1 Single Agent Problem

When the principal restricts the incentive scheme to be incentive compatible, the problem is then described by the following model:

P4:

$$Max_{(\hat{q}^*, \hat{m}^*, \hat{z}^*)} \quad \int_S \left[\int_0^{\hat{q}(s)} P(q_o)\, dq_o - \int_0^{\hat{m}(s)} MC(m_o)\, dm_o - \hat{z}(s) \right] p(s)\, ds \tag{24}$$

$$St. \quad U(\hat{z}(s)) - v(s,s) \geq U(\hat{z}(\hat{s})) - v(\hat{s},s) \tag{24A}$$

$$\forall \hat{s} \in S \quad \text{and} \quad \forall s \in S,$$

$$\int_S [U(\hat{z}(s)) - v(s,s)]p(s)\, ds \geq \overline{u}, \tag{24B}$$

where $\hat{q}(s) = f[\hat{m}(s), v(s,s), s], \forall s \in S$.

Lemma 1. *The optimal incentive compatible scheme* $(\hat{q}^*, \hat{m}^*, \hat{z}^*)$ *satisfies the following conditions,*

$$P(\hat{q}) \cdot \frac{\partial f}{\partial \hat{m}} - MC(\hat{m}) = 0, \tag{25}$$

$$P(\hat{q}) \cdot \frac{\partial f}{\partial v} - \frac{1}{U'(\hat{z})} = 0, \tag{26}$$

$$\forall s \in S.$$

Proof: The result follows directly by taking the first order conditions of the Lagrange formula.

It is easy to see from (25) and (26) that at the optimum, the incremental costs of production in material input and labor input are identical. Q.E.D.

Lemma 2. *For the single agent problem P4, if assumptions A4 to A8 hold, then the agent's actual disutility,* $v(\hat{s}, s)$, *has the following properties,*

$$v_1'(\hat{s},s) = \frac{\partial v}{\partial \hat{s}} < 0, \quad v_2'(\hat{s},s) = \frac{\partial v}{\partial s} > 0, \quad and \quad v_{12}'' = \frac{\partial^2 v}{\partial \hat{s} \partial s} < 0.$$

Proof: see Appendix E.

Lemma 2 says that the agent can reduce the disutility of effort by reporting a less efficient state, given the state realization; and the more efficient the state realization, the less the disutility of effort is, given the reported state. The agent has incentive to mispresent the state of production. Therefore, it is important for the principal to design an incentive scheme which induces the agent to behave properly.

Proposition 9. *For the problem P4, if assumptions A4 to A8 hold, then the following compensation scheme is truth inducing,*

$$\hat{z}(\hat{s}) = U^{-1}\,[\int_{\underline{s}}^{\hat{s}} v_1'(s,s)\,ds + h], \tag{27}$$

where h is some constant chosen to make the minimum utility constraint binding.

Proof: The agent's utility , under the compensation (27), is

$$\begin{aligned} u^A(\hat{z}(\hat{s}),s) &= U(\hat{z}(\hat{s})) - v(\hat{s},s) \\ &= \int_{\underline{s}}^{\hat{s}} v_1'(s,s)\,ds + h - v(\hat{s},s) \end{aligned}$$

$$\frac{\partial u^A}{\partial \hat{s}} = v_1'(\hat{s},\hat{s}) - v_1'(\hat{s},s).$$

From Lemma 2, $v_{12}'' < 0$, therefore,

$$\begin{cases} \frac{\partial u^A}{\partial \hat{s}} > 0, & \text{if } \hat{s} < s; \\ \frac{\partial u^A}{\partial \hat{s}} = 0, & \text{if } \hat{s} = s; \\ \frac{\partial u^A}{\partial \hat{s}} < 0, & \text{if } \hat{s} > s. \end{cases}$$

So the agent's utility is maximized at $\hat{s} = s$. Q.E.D.

Following is a numerical example to show the calculation of the incentive scheme.

Example 4. Suppose $P(q) = 10$, $MC(m) = 0.1m$, $U(z) = z$, $v(e) = e$, $q = \frac{1}{s}\sqrt{me} = \frac{1}{s}\sqrt{mv}$. Then the optimal incentive scheme is determined by (25), (26) and (27).

$$\begin{cases} 10 - 2\hat{s}^2\hat{q}\frac{1}{\hat{m}} = 0, \\ 10 - 2\hat{s}^2\hat{q}\frac{1}{v}(0.1m) = 0, \\ \hat{q} = \frac{1}{\hat{s}}\sqrt{\hat{m}v}. \end{cases}$$

Therefore,

$$\hat{q}^*(\hat{s}) = \frac{1250}{\hat{s}^4}, \quad v(\hat{s},\hat{s}) = \frac{6250}{\hat{s}^4},$$

$$\text{and} \quad \hat{m}^*(\hat{s}) = \frac{250}{\hat{s}^2}.$$

The actual disutility, if s is observed and $\hat{s}$ is reported, is

$$v(\hat{s}, s) = \frac{6250s^2}{\hat{s}^6}.$$

From (27), we get the compensation

$$\hat{z}(\hat{s}) = 9375(\frac{1}{\hat{s}^4} - \frac{1}{\underline{s}^4}) + h.$$

So the actual utility the agent gets is

$$u^A(\hat{z}(\hat{s}), s) = 9375(\frac{1}{\hat{s}^4} - \frac{1}{\underline{s}^4}) + h - \frac{6250s^2}{\hat{s}^6}.$$

Since

$$\frac{\partial u^A}{\partial \hat{s}} = \frac{37500}{\hat{s}^5}(\frac{s^2}{\hat{s}^2} - 1),$$

therefore u^A is maximized at $\hat{s} = s$, i.e., the agent is truth telling. Suppose that $\overline{u} = 10$, $s \in S = [1,2]$, with $p(s) = 1$, then from the minimum utility constraint, we get $h = 8473.5$. Therefore, the compensation function is

$$\hat{z}^*(\hat{s}) = \frac{9375}{\hat{s}^4} - 901.5.$$

In the above example, we see that the agent's disutility, $v(\hat{s}, \hat{s})$, decreases as the reported state, $\hat{s}$, increases (less efficient). Also, compensation $\hat{z}(\hat{s})$ decreases as $\hat{s}$ increases. Hence, $d\hat{z}/d\hat{v} > 0$. If this condition holds, then we have the following proposition.

Proposition 10. *Under the incentive compensation scheme determined by (25),(26), and (27), if $d\hat{z}/d\hat{v} > 0$, then, the optimal production, $\hat{q}^*$, increases as the state becomes more efficient.*

Proof: see Appendix F

4.2 Multiple Agent Problem

Analogous to the analysis in Chapter 3, the incentive compatible scheme for the single agent problem can be extended to multiple agent problem. It is given below.

$$\hat{z}_i(\hat{s}_i, \hat{s}^i) = U_i^{-1}\left[\int_{\underline{s}_i}^{\hat{s}_i} \frac{\partial v_i(\hat{s}_i, \hat{s}^i, s_i)}{\partial \hat{s}_i}\Big|_{\hat{s}_i = s_i} ds_i + h_i\right], \tag{28}$$

$$P\left(\sum_{i=1}^{n} \hat{q}_i\right) \cdot \frac{\partial f_i}{\partial \hat{m}_i} - MC\left(\sum_{i=1}^{n} \hat{m}_i\right) = 0, \tag{29}$$

$$P\left(\sum_{i=1}^{n} \hat{q}_i\right) \cdot \frac{\partial f_i}{\partial v_i} - \frac{1}{U_i'(\hat{z}_i)} = 0, \tag{30}$$

$$\forall \hat{s} \in S, \quad \forall i \in I.$$

Again, under the incentive compatible scheme determined by (28), (29) and (30), truth telling is a dominant strategy equilibrium.

The subgame of the agents is essentially of the same nature as the one described in section 3.4. We do not expect much to change in the result. The procedure for analysing the subgame is not repeated here.

Chapter 5. CONCLUDING REMARKS

We have developed an incentive compatible compensation scheme for a social planning problem. The restriction of the scheme to be incentive compatible does not change the optimality of the solution. In the problem we haved studied, both moral hazard and adverse selection are present. It is interesting to see that inducing truthful reporting of the state is able to deal with both aspects simultaneously.

It is assumed in the analysis that the agents can not quit once they accept the contract. Relaxing this assumption does not significantly change the anslysis, since the basic incentive problem does not change—making the agents worse off if they do not report the true state. However, the principal is no longer able to derive the rent from the private information of the agents.

Even though the incentive scheme is derived in the context of a planned economy, it may be applicable to other types of problems, such as the planning of a multi-divisional company, as long as they contain the same incentive problem. Technically, this means that the form of the objective function, whether maximizing social welfare or profit, is not crucial in the analysis.

At this point, we would like to make a comparison of the incentive scheme derived in this study with the well-known Groves' scheme. The schemes are derived in different types of social planning problems. In this study, the principal wants to maximize the welfare of the society which is a separate entity from the agents; Groves' scheme applies to the situation where the principal wants to maxmize the sum of all the agents' welfare. The similarities in these two schemes are (1) the compensation to any agent is not only based on this agent's reporting, but on all the other agents' reporting as well; (2) telling the truth is a dominant strategy equilibrium under both schemes; (3) neither scheme is collusion immune.

This study is preliminary, and much further research is necessary. First, the incentive scheme does not provide an obvious economic interpretation. Second, solving for the optimal production and compensation scheme is very complicated when the agents are not all risk neutral. Third, a more thorough exploration of the agents' subgame is possible. For example, it would be interesting to see under what conditions the prisoners' dilemma does not occur.

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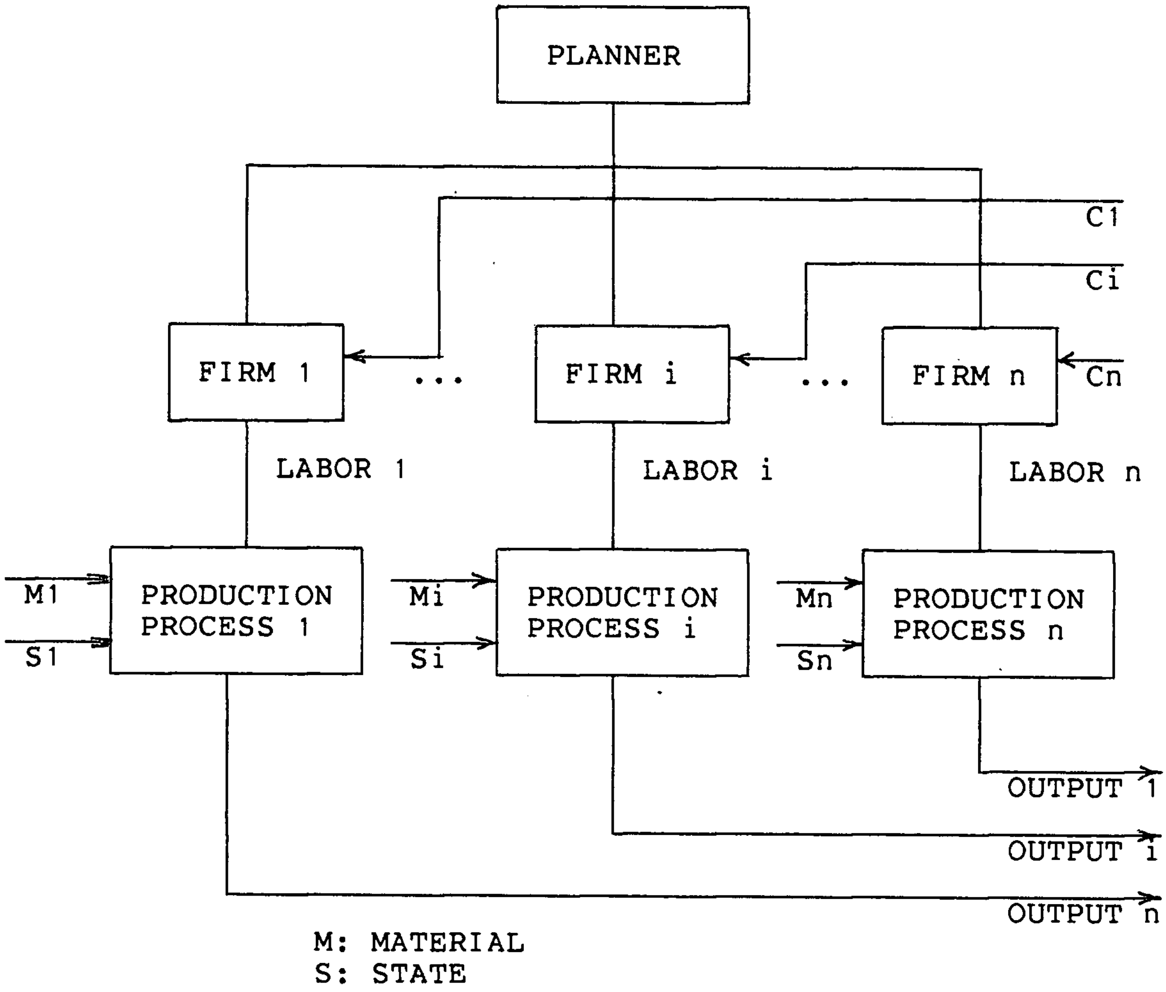

Figure 1 Economic Structure

		Agent 2	
		2	1
Agent 1	2	-3.240 -0.420	-3.026 -0.444
	1	-3.544 -0.196	-3.324 -0.208

* Constants h1 and h2 are ignored.

The true state is (2,2), but the agents are better off by reporting (2,1).

Figure 2 Collusion With Side Payment

Appendix A.

Proof Of Proposition 4:

(1) *The scheme is incentive compatible*

Suppose part (3) of the theorem is true for the moment. (We will verify this later on.) The agent's utility, if s is observed and $\hat{s}$ is reported, is

$$\begin{aligned} u^A(\hat{z}(\hat{s}), s) &= U(\hat{z}(\hat{s})) - V(\hat{q}(\hat{s}), s) \\ &= \int_{\underline{s}}^{\hat{s}} \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} \cdot \frac{d\hat{q}}{ds} \, ds + h - V(\hat{q}(\hat{s}), s). \end{aligned} \tag{A1}$$

$$\frac{\partial u^A}{\partial \hat{s}} = \left(\frac{\partial V(\hat{q}, \hat{s})}{\partial \hat{q}} - \frac{\partial V(\hat{q}, s)}{\partial \hat{q}}\right) \cdot \frac{d\hat{q}}{d\hat{s}}. \tag{A2}$$

Since $d\hat{q}/d\hat{s} \leq 0$, and $\partial^2 V/\partial \hat{q} \partial s \geq 0$, then

$$\begin{cases} \frac{\partial u^A}{\partial \hat{s}} \geq 0, & \text{if } \hat{s} < s; \\ \frac{\partial u^A}{\partial \hat{s}} = 0, & \text{if } \hat{s} = s; \\ \frac{\partial u^A}{\partial \hat{s}} \leq 0, & \text{if } \hat{s} > s. \end{cases}$$

Therefore, u^A is maximized at $\hat{s} = s, \forall s \in S$. The compensation $\hat{z}(\hat{s})$ is truth-inducing. By applying the form of compensation specified in (5), the agent is obedient. So we have a incentive compatible compensation.

The production rule $\hat{q}(s)$ is optimal given $\hat{z}(\hat{s})$

Given any incentive compatible compensation $\hat{z}(\hat{s})$, we can write the model as

$$Max_{(\hat{q}^*(s),\hat{z}^*(s))} \quad \int_S \left[\int_0^{\hat{q}(s)} P(q_o)\, dq_o - \int_0^{M(\hat{q})} MC(m_o)\, dm_o - \hat{z}(s) \right] p(s)\, ds \qquad (A3)$$

$$St. \quad \int_S [U(\hat{z}(s)) - V(\hat{q}(s), s)] p(s)\, ds \geq \overline{u}. \qquad (A4)$$

So

$$L(\hat{q}(s), \hat{z}(s), \lambda) = (A3) + \lambda(A4).$$

The first order conditions are

$$\frac{\partial L}{\partial \hat{q}} = P(\hat{q}) - MC(M(\hat{q})) \cdot M'(\hat{q}) - \lambda \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} = 0,$$

and

$$\frac{\partial L}{\partial \hat{z}} = \quad -1 + \lambda U'(\hat{z}) = 0.$$

We see that λ is positive, so the solution is optimal. Therefore, the optimal production rule, $\hat{q}(s)$, given $\hat{z}(s)$, is determined by

$$P(\hat{q}) - MC(M(\hat{q})) \cdot M'(\hat{q}) - \frac{1}{U'(\hat{z})} \frac{\partial V(\hat{q}, s)}{\partial \hat{q}} = 0. \qquad (A5)$$

Since $\hat{z}(\hat{s})$ is incentive compatible, so $\hat{s} = s$, therefore, (A5) is the same as (17).

(2) Suppose there exists another incentive compatible scheme, say $(q_o(\hat{s}), z_o(\hat{s}))$, such that (17) holds. Since both scheme are incentive compatible, the state reported is the same as the state realized. Since $z_o(\hat{s})$ is different from $\hat{z}(\hat{s})$, there must exist at least one point, $s_1 \in S$, where $z_o(s_1) \neq \hat{z}(s_1)$. Suppose $z_o(s_1) > \hat{z}(s_1)$, then from (A5), the corresponding production outputs must satisfy $q_o(s_1) \leq \hat{q}(s_1)$, and the corresponding utilities satisfy

$$u_o^A(s_1) = U(z_o) - V(q_o, s_1) > u^A(s_1) = U(\hat{z}) - V(\hat{q}, s_1).$$

At the optimum, both schemes should make the minimum utility constraint (A4) binding, i.e.,

$$\int_S u_o^A(s)p(s)\, ds = \overline{u} = \int_S u^A(s)p(s)\, ds. \tag{A6}$$

Since $u_o^A(s_1) > u^A(s_1)$, for (A6) to hold, there must exist another point $s_2 \in S$, where $u_o^A(s_2) < u^A(s_2)$. From (A5), this implies that

$$z_o(s_2) < \hat{z}(s_2), \quad \text{and} \quad q_o(s_2) \geq \hat{q}(s_2).$$

Without loss of generality, we assume that $s_1 < s_2$. Let

$$\Delta z(s) = z_o(s) - \hat{z}(s), \quad \Delta q(s) = q_o(s) - \hat{q}(s).$$

Because all the functions are continuous and differentiable, there must exist one point, $s_0, s_1 < s_0 < s_2$, where

$$\Delta z(s_0) = z_o(s_0) - \hat{z}(s_0) = 0, \tag{A7}$$

$$\frac{d\Delta z(s)}{ds}\Big|_{s=s_0} = z_o'(s_0) - \hat{z}'(s_0) < 0, \tag{A8}$$

$$\Delta q(s_0) = q_o(s_0) - \hat{q}(s_0) = 0, \tag{A9}$$

$$\frac{d\Delta q(s)}{ds}\mid_{s=s_0} = q_o'(s_0) - \hat{q}'(s_0) \geq 0. \qquad (A10)$$

Since both schemes are incentive compatible, they must meet the necessary condition at every state realization. So

$$U'(z_o(s_0))z_o'(s_0) - \frac{\partial V(q_o, s_0)}{\partial q_o} \cdot q_o'(s_0) = 0, \qquad (A11)$$

$$U'(\hat{z}(s_0))\hat{z}'(s_0) - \frac{\partial V(\hat{q}, s_0)}{\partial \hat{q}} \cdot \hat{q}'(s_0) = 0. \qquad (A12)$$

But clearly, if (A7) to (A10) hold, (A11) and (A12) can not be hold simultaneously. Therefore, we conclude that the optimal incentive compatible scheme determined by (16) and (17) is unique.

(3) Verify that $\hat{q}'(\hat{s}) \leq 0$

Suppose this is not true in some small area $\hat{s} \in [a, b]$, that is $\hat{q}'(\hat{s}) > 0$. Then for any two points, s_1, s_2, with $a < s_1 < s_2 < b$, there is

$$\hat{q}(s_2) > \hat{q}(s_1). \qquad (A13)$$

From the common knowledge, this implies that

$$P(\hat{q}(s_2)) - MC(M(\hat{q}(s_2))) \cdot M'(\hat{q}(s_2)) \leq P(\hat{q}(s_1)) - MC(M(\hat{q}(s_1))) \cdot M'(\hat{q}(s_1)). \qquad (A14)$$

From (16), (A13) also implies that

$$\hat{z}(s_2) > \hat{z}(s_1). \qquad (A15)$$

When $s_2 > s_1$, and $\hat{q}(s_2) > \hat{q}(s_1)$, we have

$$\frac{\partial V(\hat{q}(s_2), s_2)}{\partial \hat{q}} > \frac{\partial V(\hat{q}(s_2), s_1)}{\partial \hat{q}} > \frac{\partial V(\hat{q}(s_1), s_1)}{\partial \hat{q}}. \tag{A16}$$

However, equations (A14), (A15) and (A16) upset the optimal condition (17) either at s_1 or at s_2. Therefore, it must be $\hat{q}'(\hat{s}) \leq 0$ for all $\hat{s} \in S$.

(4) Since the minimum utility constraint is binding, the principal derives all the rent. Q.E.D.

Appendix B

Proof Of Proposition 5

(1) The same procedure can be used, as in part (1) of Appendix A, to prove that agent i's utility is maximized at $\hat{s}_i = s_i, \forall s_i \in S_i, \forall i \in I$, given $\hat{s}^i \in S^i$, $\forall \hat{s}^i \in S^i$. So, the scheme is incentive compatible. Since this is true for every agent, then truth-telling becomes a dominant strategy equilibrium. From the proof of Proposition 4 in Appendix A, we know that, the optimal compensation to agent i must be

$$\hat{z}_i = U_i^{-1}\left[\int_{\underline{s}_i}^{\hat{s}_i} \frac{\partial V_i}{\partial \hat{q}_i} \cdot \frac{\partial \hat{q}_i(s_i, s^i)}{\partial s_i}\, ds_i + h_i\right],$$

where h_i is some constant chosen to make the expected utility of agent i over the set S to be $\overline{u}_i$.

The optimal production condition (21) is again derived by taking the first order conditions of the Lagrange formula.

(2) First, we assume there are two agents. Then (21) yields

$$P(\hat{q}_1 + \hat{q}_2) - MC(M_1(\hat{q}_1) + M_2(\hat{q}_2))M_1'(\hat{q}_1) - \frac{1}{U_1'(\hat{z}_1)} \frac{\partial V_1(\hat{q}_1, \hat{s}_1)}{\partial \hat{q}_1} = 0, \tag{B3}$$

$$P(\hat{q}_1 + \hat{q}_2) - MC(M_1(\hat{q}_1) + M_2(\hat{q}_2))M_2'(\hat{q}_2) - \frac{1}{U_2'(\hat{z}_2)} \frac{\partial V_2(\hat{q}_2, \hat{s}_2)}{\partial \hat{q}_2} = 0. \tag{B4}$$

Suppose that $\hat{s}_1$ increases while $\hat{s}_2$ stays the same. Then $\partial V_1/\partial \hat{q}_1$ increases, so equation (B3) is unbalanced. We look at all the possible effects on $\hat{q}_1$ and $\hat{q}_2$ to restore the equality of both (B3) and (B4). Notice that $P - MC \cdot M_i'$ decreases in output, and the incremental disutilities increase in corresponding outputs. If $\hat{q}_1$ increases as $\hat{s}$ increases, then in all cases, (B3) is unbalanced. If

$\hat{q}_1$ and $\hat{q}_2$ both decrease as $\hat{s}_1$ increases, then (B4) is unbalanced. The only possible way to restore the equalities of both (B3) and (B4) simultaneously is to decrease $\hat{q}_1$ and increase $\hat{q}_2$. Therefore, we have

$$\partial \hat{q}_1 / \partial \hat{s}_1 \leq 0, \quad \text{and} \quad \partial \hat{q}_2 / \partial \hat{s}_1 \geq 0.$$

In the same way, we can prove

$$\partial \hat{q}_1 / \partial \hat{s}_2 \geq 0, \quad \text{and} \quad \partial \hat{q}_2 / \partial \hat{s}_2 \leq 0.$$

This conclusion can be extended to the problems with more than two agents. This is because a change of s_i should have a symmetrical effect on all the other agents.

(3) The principal derives all the rent because every agent gets the minimum utility ex ante.

(4) When all the agents are risk neutral, the optimal production is the same as when the principal has perfect information. Moreover, the principal does not have to pay any risk premium. Therefore, the expected social welfare is the same as when the principal has perfect information. The solution is then the first-best. When at least one agent is risk averse, the principal has to pay some risk premium since the amounts of compensation are not constant. Therefore, only a second-best solution is achieved. Q.E.D.

Appendix C

Proof Of Proposition 7

Assume there are two agents, agent 1 and agent 2. Under the incentive scheme determined by (20) and (21), their utilities are

$$u_1^A(\hat{z}_1(\hat{s}_1,\hat{s}_2),s_1,s_2) = \int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1(\hat{q}_1,s_1)}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,\hat{s}_2)}{\partial s_1}\, ds_1 + h_1 - V_1(\hat{q}_1(\hat{s}_1,\hat{s}_2),s_1), \qquad (C1)$$

$$u_2^A(\hat{z}_2(\hat{s}_1,\hat{s}_2),s_1,s_2) = \int_{\underline{s}_2}^{\hat{s}_2} \frac{\partial V_2(\hat{q}_2,s_2)}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1,s_2)}{\partial s_2}\, ds_2 + h_2 - V_2(\hat{q}_2(\hat{s}_2,\hat{s}_2),s_2). \qquad (C2)$$

where $\hat{q}_1 = \hat{q}_1(\hat{s}_1,\hat{s}_2)$ and $\hat{q}_2 = \hat{q}_2(\hat{s}_1,\hat{s}_2)$.

Let $t_1(\hat{s}_1,\hat{s}_2)$ be the amount of money has to be subtracted from or added to $u^A(\hat{z}_1(\hat{s}_1,\hat{s}_2),s_1,s_2)$ in order to make agent 1's utility the same as $u_1^A(\hat{z}_1(s_1,s_2),s_1,s_2)$. Let $t_2(\hat{s}_1,\hat{s}_2)$ be the same term with respect to agent 2. And let $t(\hat{s}_1,\hat{s}_2) = t_1 + t_2$ be the amount of extra money that the agents get by reporting $(\hat{s}_1,\hat{s}_2)$ instead of the true state (s_1,s_2). Let $u_i^A(\hat{z}_i(\hat{s}_1,\hat{s}_2),t_i,s_1,s_2)$ be agent i's utility if $(\hat{s}_1,\hat{s}_2)$ is reported and t_i is transferred given (s_1,s_2). Then,

$$\begin{aligned}
u_1^A(\hat{z}_1(s_1,s_2),s_1,s_2) &= \int_{\underline{s}_1}^{s_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,s_2)}{\partial s_1}\, ds_1 + h_1 - V_1(\hat{q}_1(s_1,s_2),s_1) \\
&= U_1\left[U_1^{-1}\left(\int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,\hat{s}_2)}{\partial s_1}\, ds_1 + h_1\right) - t_1(\hat{s}_1,\hat{s}_2)\right] \\
&- V_1(\hat{q}_1(\hat{s}_1,\hat{s}_2),s_1) \\
&= u_1^A(\hat{z}_1(\hat{s}_1,\hat{s}_2),t_1,s_1,s_2). \qquad (C3)
\end{aligned}$$

$$
\begin{aligned}
u_2^A(\hat{z}_2(s_1,s_2),s_1,s_2) &= \int_{\underline{s}_2}^{s_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(s_1,s_2)}{\partial s_2}\, ds_2 + h_2 - V_2(\hat{q}_2(s_1,s_2),s_2) \\
&= U_2\Big[U_2^{-1}\Big(\int_{\underline{s}_2}^{\hat{s}_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1,s_2)}{\partial s_2}\, ds_2 + h_2\Big) - t_2(\hat{s}_1,\hat{s}_2)\Big] \\
&\quad - V_2(\hat{q}_2(\hat{s}_1\hat{s}_2),s_2) \\
&= u_2^A(\hat{z}_2(\hat{s}_1,\hat{s}_2),t_2,s_1,s_2). \qquad (C4)
\end{aligned}
$$

From (C3) and (C4), we have

$$
\begin{aligned}
t(\hat{s}_1,\hat{s}_2) &= U_1^{-1}\Big[\int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,\hat{s}_2)}{\partial s_1}\, ds_1 + h_1\Big] \\
&\quad - U_1^{-1}\Big[\int_{\underline{s}_1}^{s_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,s_2)}{\partial s_1}\, ds_1 + h_1 - V_1(\hat{q}_1(s_1,s_2),s_1) + V_1(\hat{q}_1(\hat{s}_1,\hat{s}_2),s_1)\Big] \\
&\quad + U_2^{-1}\Big[\int_{\underline{s}_2}^{\hat{s}_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1,s_2)}{\partial s_2}\, ds_2 + h_2\Big] \\
&\quad - U_2^{-1}\Big[\int_{\underline{s}_2}^{s_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(s_1,s_2)}{\partial s_2}\, ds_2 + h_2 - V_2(\hat{q}_2(s_1,s_2),s_2) + V_2(\hat{q}_2(\hat{s}_1,\hat{s}_2),s_2)\Big] \\
&= U_1^{-1}(A) - U_1^{-1}(B) + U_2^{-1}(C) - U_2^{-1}(D), \qquad (C5)
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,\hat{s}_2)}{\partial s_1}\, ds_1 + h_1, \\
B &= \int_{\underline{s}_1}^{s_1} \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(s_1,s_2)}{\partial s_1}\, ds_1 + h_1 - V_1(\hat{q}_1(s_1,s_2),s_1) + V_1(\hat{q}_1(\hat{s}_1,\hat{s}_2),s_1), \\
C &= \int_{\underline{s}_2}^{\hat{s}_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1,s_2)}{\partial s_2}\, ds_2 + h_2, \\
D &= \int_{\underline{s}_2}^{s_2} \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(s_1,s_2)}{\partial s_2}\, ds_2 + h_2 - V_2(\hat{q}_2(s_1,s_2),s_2) + V_2(\hat{q}_2(\hat{s}_1,\hat{s}_2),s_2).
\end{aligned}
$$

So

$$\frac{\partial t}{\partial \hat{s}_1} = \frac{1}{U_1'(U_1^{-1}(A))} \cdot \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_1}$$
$$- \frac{1}{U_1'(U_1^{-1}(B))} \cdot \frac{\partial V_1}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_1}$$
$$+ \frac{1}{U_2'(U_2^{-1}(C))} \left[\int_{\underline{s}_2}^{\hat{s}_2} \left[\frac{\partial^2 V_2}{\partial \hat{q}_2^2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1, s_2)}{\partial \hat{s}_1} \cdot \frac{\partial \hat{q}_2(\hat{s}_1, s_2)}{\partial s_2} + \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial^2 \hat{q}_2(\hat{s}_1, s_2)}{\partial \hat{s}_1 \partial s_2} \right] ds_2 \right]$$
$$- \frac{1}{U_2'(U_2^{-1}(D))} \cdot \frac{\partial V_2(\hat{q}_2(\hat{s}_1, \hat{s}_2), s_2)}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_1}. \qquad (C6)$$

The necessary and sufficient condition for the incentive scheme to be collusion immune is

$$t(s_1, s_2) \geq t(\hat{s}_1, \hat{s}_2), \qquad (C7)$$

$$\forall (\hat{s}_1, \hat{s}_2) \in S_1 \times S_2, \quad \text{and} \quad \forall (s_1, s_2) \in S_1 \times S_2.$$

Since all the functions are continuous and differentiable with respect to $\hat{s}_1$ and $\hat{s}_2$, then (C7) requires, at $(\hat{s}_1, \hat{s}_2) = (s_1, s_2)$,

$$\frac{\partial t(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_1} = 0 \quad \text{and} \quad \frac{\partial t(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_2} = 0. \qquad (C8)$$

Using (C6), (C8) becomes

$$\int_{\underline{s}_2}^{s_2} \left[\frac{\partial^2 V_2}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(s_1, s_2)}{\partial s_1} \cdot \frac{\partial \hat{q}_2(s_1, s_2)}{\partial s_2} + \frac{\partial V_2}{\partial \hat{q}_2} \cdot \frac{\partial^2 \hat{q}_2(s_1, s_2)}{\partial s_1 \partial s_2} \right] ds$$
$$= \frac{\partial V_2(\hat{q}_2, s_2)}{\partial \hat{q}_2} \cdot \frac{\partial \hat{q}_2(s_1, s_2)}{\partial s_1}. \qquad (C9)$$

In order for the scheme to be collusion immune, condition (C9) has to hold for every $(s_1, s_2) \in S_1 \times S_2$. Differentiating both sides of (C9) with respect to s_2, we get

$$\frac{\partial^2 V_2}{\partial \hat{q}_2 \partial s_2} \cdot \frac{\partial \hat{q}_2(s_1, s_2)}{\partial s_1} = 0, \quad \forall (s_1, s_2) \in S_1 \times S_2. \tag{C10}$$

That means either $\frac{\partial^2 V_2}{\partial q_2 \partial s_2} = 0$ or $\frac{\partial \hat{q}_2(s_1, s_2)}{\partial s_1} = 0, \quad \forall (s_1, s_2) \in S_1 \times S_2$. In fact, the latter condition implies the former one. However, this means that the incremental disutility $\partial V_2 / \partial s_2$ is not affected by the state of production s_2, which contradicts assumption A3. Therefore, for the scheme to be collusion immune, it requires that there is no uncertainty on production function. So the scheme can not be collusion immune. Q.E.D.

Appendix D

Proof Of Proposition 8

Agent 1's utility is

$$
\begin{aligned}
u_1^A(\hat{z}_1(\hat{s}_1, \hat{s}_2), s_1, s_2) &= \int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1(\hat{q}_1, s_1)}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1}{\partial s_1} \, ds_1 + h_1 - V_1(\hat{q}_1(\hat{s}_1, \hat{s}_2), s_1) \\
&= V_1(\hat{q}_1(\hat{s}_1, \hat{s}_2), \hat{s}_1) - V_1(\hat{q}_1(\underline{s}_1, \hat{s}_2), \underline{s}_1) \\
&\quad - \int_{\underline{s}_1}^{\hat{s}_1} \frac{\partial V_1(\hat{q}_1, s_1)}{\partial s_1} \, ds_1 + h_1 - V_1(\hat{q}_1(\hat{s}_1, \hat{s}_2), s_1). \qquad (D1)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial u_1^A}{\partial \hat{s}_2} &= \frac{\partial V_1(\hat{q}_1(\hat{s}_1, \hat{s}_2), \hat{s}_1)}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_2} - \frac{\partial V_1(\hat{q}_1(\underline{s}_1, \hat{s}_2), \underline{s}_1)}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(\underline{s}_1, \hat{s}_2)}{\partial \hat{s}_2} \\
&\quad - \int_{\underline{s}_1}^{\hat{s}_2} \frac{\partial^2 V_1(\hat{q}_1, s_1)}{\partial s_1 \partial \hat{q}_2} \cdot \frac{\partial \hat{q}_1}{\partial s_2} \, ds_1 - \frac{\partial V_1(\hat{q}_1(\hat{s}_1, \hat{s}_2), s_1)}{\partial \hat{q}_1} \cdot \frac{\partial \hat{q}_1(\hat{s}_1, \hat{s}_2)}{\partial \hat{s}_2}. \qquad (D2)
\end{aligned}
$$

If $\partial V_1 / \partial q_1$ is strictly decreasing in s_1, then $\partial \hat{q}_1 / \partial s_1$ is strictly negative. Therefore, according to (D2), we can conclude that

$$
\frac{\partial u_1^A}{\partial \hat{s}_2} < 0 \quad \text{when} \quad \hat{s}_1 \leq s_1. \qquad (D3)
$$

For the same reason, we have

$$
\frac{\partial u_2^A}{\partial \hat{s}_1} < 0 \quad \text{when} \quad \hat{s}_2 \leq s_2. \qquad (D4)
$$

Now, look at all the possible ways of colluding.

A. If $\hat{s}_2 \downarrow$ while $\hat{s}_1$ stays the same, so $u_1^A \uparrow$ (according to (D3)) and $u_2^A \downarrow$ (truth-telling is a strictly dominant strategy for agent 2). Therefore, A is a possible means of collusion (but not necessary).

B. If $\hat{s}_1 \downarrow$, and $\hat{s}_2$ stays the same, then $u_1^A \downarrow$ and $u_2^A \uparrow$. So, B is a possible means of collusion.

C. If $\hat{s}_1 \downarrow$ and $\hat{s}_2 \downarrow$, then the outcome can not determined, C is also a possible means of collusion.

D. If $\hat{s}_1 \uparrow$ while $\hat{s}_2 \downarrow$, then the outcome is strictly dominated by A. So, D is not likely to be a possible means of collusion.

E. If $\hat{s}_1 \downarrow$ while $\hat{s}_2 \uparrow$, then the outcome is dominated by B. So, E is not likely to be a possible means of collusion.

F. If either $\hat{s}_1$ or $\hat{s}_2$ increases, and the other stays the same, then the solution is strictly dominated by truth-telling. So F is not a possible means of collusion.

G. If both $\hat{s}_1$ and $\hat{s}_2$ increase, then the solution can not be determined or ordered. So, G may be a possible means of collusion.

Therefore, we have the conclusion in Proposition 8. Q.E.D.

Appendix E

Proof Of Lemma 2

Suppose s realized and $\hat{s}$ is reported, then the principal makes optimal decision on output and material input according to $\hat{s}$, and the agent makes the production as is specified by the principal, but with the real state s. That is

$$\hat{q}(\hat{s}) = f(\hat{m}, \hat{v}, \hat{s}) = f(\hat{m}, v, s). \tag{E1}$$

where $v = v(\hat{s}, s)$ and $\hat{v} = v(\hat{s}, \hat{s})$. Let

$$F = f(\hat{m}, \hat{v}, \hat{s}) - f(\hat{m}, v, s) = 0. \tag{E2}$$

then

$$v_1'(\hat{s}, s) = \frac{\partial v}{\partial \hat{s}} = -\frac{\frac{\partial F}{\partial \hat{s}}}{\frac{\partial F}{\partial v}} = -\frac{f_{\hat{s}}'}{-f_v'} = \frac{f_{\hat{s}}'}{f_v'} < 0. \tag{E3}$$

$$v_2' = \frac{\partial v}{\partial s} = -\frac{\frac{\partial F}{\partial s}}{\frac{\partial F}{\partial v}} = -\frac{-f_s'}{-f_v'} > 0. \tag{E4}$$

$$\frac{dF}{ds} = -f_v' \cdot v_2' - f_s' = 0, \tag{E5}$$

$$\frac{\partial}{\partial \hat{m}}\left(\frac{dF}{ds}\right) = -f''_{v\hat{m}} \cdot v'_2 - f''_{s\hat{m}} = 0, \tag{E6}$$

$$\begin{aligned}\frac{\partial}{\partial \hat{s}}\left(\frac{dF}{ds}\right) = &- f''_{v\hat{m}} \cdot v'_2 \cdot \hat{m}'(\hat{s}) - f''_{vv} \cdot v'_1 \cdot v'_2 \\ &- f'_v \cdot v''_{12} - f''_{s\hat{m}} \cdot \hat{m}'(\hat{s}) - f''_{sv} \cdot v'_1 = 0.\end{aligned} \tag{E7}$$

From (E6) and (E7), we have

$$f''_{vv} \cdot v'_1 \cdot v'_2 + f'_v \cdot v''_{12} + f''_{sv} \cdot v'_1 = 0. \tag{E8}$$

In (E8), the first term and the third term are both positive, and also f'_v is positive, therefore we have

$$v''_{12} = \frac{\partial^2 v}{\partial \hat{s} \partial s} < 0.$$

Q.E.D.

Appendix F

Proof Of Proposition 10

Under the conditions given in the proposition, $\hat{v}$ decreases as $\hat{s}$ increases. Suppose that $\hat{q}^*$ increases as $\hat{s}$ increases. Then $\hat{m}$ must increase. From $P'(\hat{q}) \leq 0$, and $MC'(m) \geq 0$, $\partial f/\partial \hat{m}$ must increases in order to maintain the optimal condition (21), Which means

$$\frac{d}{d\hat{s}}(\frac{\partial f}{\partial \hat{m}}) = f''_{\hat{m}\hat{m}} \cdot \hat{m}'(\hat{s}) + f''_{\hat{m}v} \cdot v'(\hat{s}) + f''_{\hat{m}\hat{s}} > 0. \qquad (F1)$$

The only way to hold (F1) is $v'(\hat{s}) > 0$, which contradicts the condition of the proposition. Therefore, $\hat{q}(\hat{s})/d\hat{s} < 0$. Q.E.D.