BANKRUPTCY: A PROPORTIONAL HAZARD APPROACH

By

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ABSTRACT

The recent dramatic increase in the corporate bankruptcy rate, coupled with a similar rate of increase in the bank failure rate, has re-awakened investor, lender and government interest in the area of bankruptcy prediction. Bankruptcy prediction models are of particular value to a firm's current and future creditors who often do not have the benefit of an actively traded market in the firm's securities from which to make inferences about the debtor's viability. The models commonly used by many experts in an endeavour to predict the possibility of disaster are outlined in this paper.

The proportional hazard model, pioneered by Cox [1972], assumes that the hazard function, the risk of failure, given failure has not already occurred, is a function of various explanatory variables and estimated coefficients multiplied by an arbitrary and unknown function of time. The Cox Proportional Hazard model is usually applied in medical studies; but, has recently been applied to the bank failure question [Lane, Looney & Wansley, 1986]. The model performed well in the narrowly defined, highly regulated, banking industry. The principal advantage of this approach is that the model incorporates both the survival times observed and any censoring of data thereby using more of the available information in the analysis. Unlike many bankruptcy prediction models, such as logit and probit based regression models, the Cox model estimates the probability distribution...
of survival times. The proportional hazard model would, therefore, appear to offer a useful addition to the more traditional bankruptcy prediction models mentioned above.

This paper evaluates the applicability of the Cox proportional hazard model in the more diverse industrial environment. In order to test this model, a sample of 109 firms was selected from the Compustat Industrial and Research Industrial data tapes. Forty one of these firms filed petitions under the various bankruptcy acts applicable between 1972 and 1985 and were matched to 67 firms which had not filed petitions for bankruptcy during the same period. In view of the dramatic changes in the bankruptcy regulatory environment caused by the Bankruptcy reform act of 1978, the legal framework of the bankruptcy process was also examined.

The performance of the estimated Cox model was then evaluated by comparing its classification and descriptive capabilities to those of an estimated discriminant analysis based model. The results of this study indicate that while the classification capability of the Cox model was less than that of discriminant analysis, the model provides additional information beyond that available from the discriminant analysis.
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I. INTRODUCTION

On average a U.S. business firm failed every forty-nine minutes during 1970. Thirteen years later, in 1983, a firm failed, on average, every sixteen minutes. This startling increase in the corporate bankruptcy rate, coupled with a similar rate of increase in the bank failure rate, has re-awakened investor, lender and government interest in the area of bankruptcy prediction. The early identification of a potential bankrupt would be invaluable to investor, creditor and management alike. Bankruptcy prediction models are of particular value to a firm's current and future creditors who often do not have the benefit of a actively traded market in the firm's securities from which to make inferences about the debtor's viability. One would hope that a successful "Prediction Model" would allow a firm's management to act on the signal of impending doom and by taking corrective measures avoid the filing of petitions for bankruptcy. In this way, a 'good' predictor of bankruptcy would, in effect, become a self-defeating prophesy. The models commonly used by many experts in an endeavour to predict the possibility of disaster will be outlined in this paper.

* On average in 1970, a firm was incorporated every two minutes while on average in 1983, one was incorporated every fifty-two seconds.
Altman's Z Score model, based on discriminant analysis, is one of the most commonly applied bankruptcy prediction models. This type of model has been applied in many different industries and situations and has, on average, performed well. Joy and Tollefson [1975] found that Altman's Z Score performed significantly better than a proportional chance model. Any new model should, therefore, offer similar "general purpose" applicability and reliability.

Many other models have been developed in an effort to improve on the discriminant analysis based Z Score model. While these latter models have added to our understanding of the bankruptcy condition, they have not been as widely used as the Z Score. Altman's model appears to be one of the most generally applicable bankruptcy prediction model and, therefore, acts as the standard by which all others must be judged. Among the latter are Beaver's [1966] Univariate analysis of financial ratios, logit models [Martin, 1977], factor analytic models [West, 1985], and First Passage Time [Santomero and Vinso, 1977].

The Cox Proportional Hazard model is usually applied in medical studies; but, has recently been applied to the bank failure question [Lane, Looney & Wansley, 1986]. The model performed well in the narrowly defined, highly regulated, banking industry. The principal advantage of the Cox model is that more of the available information is used in the analysis. The model incorporates both the survival times
observed and any censoring of data. Censoring of data occurs when the study terminates before every firm has failed. Unlike many bankruptcy prediction models, such as logit and probit based regression models, the Cox model estimates the probability distribution of survival times. The Cox model would, therefore, appear to offer a useful addition to the more traditional bankruptcy prediction models mentioned above.

This paper seeks to evaluate the applicability of the Cox Proportional hazard model in a more diverse industrial environment. In order to test this model, a sample of 109 firms was selected from the Compustat Industrial and Research Industrial data tapes. Forty one of these firms filed petitions under the various bankruptcy acts applicable between 1972 and 1985. In view of the dramatic changes in the bankruptcy regulatory environment caused by the Bankruptcy reform act of 1978, the legal framework of the bankruptcy process was also examined. The Cox model was estimated on a sample of 41 firms which filed for bankruptcy between 1972 and 1985. These failures were matched to 67 firms which had not filed petitions for bankruptcy during the same period. The performance of the estimated Cox model was then evaluated by comparing its classification and descriptive capabilities to those of an estimated discriminant analysis based model. The results of this study indicate that while the classification capability of the Cox
model was less than that of discriminant analysis, the Cox model provides information beyond that provided by the discriminant analysis.
A. DEFINITION

The terms "business failure", "insolvency" and "bankruptcy" are often used interchangeably. However, the situations described by these terms and the consequences of each are very different.

Business failure describes the economic condition in which the return realized on assets is less than the opportunity costs. In such a case the firm's earnings are often minimal but sufficient to cover variable costs. Such a firm is not necessarily going to become bankrupt, it may even become financially profitable in the near future.

Insolvency essentially refers to liquidity problems. Technical insolvency occurs when a firm cannot meet it's current obligations and is not in itself a sufficient condition for bankruptcy as the inability to meet current obligations can be a temporary crisis to an otherwise healthy firm. Such situations are likely to occur in highly seasonal industries.

Bankruptcy is by far the most serious financial crisis a firm has to face. The major difference between this condition and the previously discussed crises is that a third party, the Bankruptcy Courts, has now entered the proceedings. Any firm which becomes embroiled in bankruptcy proceedings is unlikely to emerge unchanged: it will either be reorganized...
and emerge as a new firm or it will be liquidated and disappear. Neither the owners (stock holders) nor the original management retain full control the firm's destiny, control has largely been relinquished to the Courts who will, acting in the best interests of the creditors, decide whether the firm is re-organized or is liquidated.

Of the three "distress" conditions outlined, business failure is the hardest to identify as it involves determining the rate of return possible in an alternative use of a firm's assets and comparing this rate to the actual rate of return being realized. This comparison requires a determination of the true market value and market rate of return on the firm's assets. Such a valuation is very difficult to arrive at practically in the case of a firm without an active market for it's securities. If the firm's securities are actively traded, the market data can be used to compare these relative market rates of return.

The financial "distress" conditions of insolvency and bankruptcy are usually easier to identify. Insolvency requires a comparison of the relative values of the cash flows and current obligations, both of which can often be determined from the firm's financial statements. Bankruptcy is the most easily identifiable condition as it is a legal state and once entered, a firm cannot hide the fact that the petition has been filed.
This paper will, therefore, concentrate on the prediction of the filing of a bankruptcy petition as it is an objectively verifiable condition.

B. LEGAL ENVIRONMENT

BACKGROUND: The need for legal recourse in a case of bankruptcy was first recognized by the European mercantile states of the late middle ages. As early bankruptcy law was primarily aimed at the merchant who was planning to remove either his assets or himself from the area, it was an involuntary procedure and essentially retaliatory in nature. The solvency of the "bankrupt" was rarely if ever questioned, the law's purpose was to ensure that debts were paid before leaving a jurisdiction.\(^3\)

As the economic environment became more complex, the need for a legal mechanism for the management of insolvency cases arose in Britain. These cases, unlike the "bankruptcy" cases, generally involved insolvency rather than flight and consequently, were of a more co-operative nature.

The British insolvency laws were voluntary in nature and were based on the notion of giving a basically honest "unfortunate" a means of distributing his assets equitably among his creditors. Unlike the bankruptcy laws, the insolvency laws allowed and encouraged negotiation between debtor and creditor for their mutual benefit. If an insolvent satisfied the courts, he could be discharged from his debts
and begin again with a clean slate which was not possible under the bankruptcy laws. 4

All modern bankruptcy legislation has developed from these opposing philosophies of retaliation and co-operation.

**THE CHANDLER ACT:** The Constitution of the United States vests the sole jurisdiction over bankruptcy laws with the Federal Government. 5 The aim is to establish uniform and fair treatment of all bankrupts and their creditors.

In 1898, the U.S. Congress passed the Bankruptcy Act, and essentially it served the United States until the 1930's when the Great Depression focussed public and government attention on the functioning of the Bankruptcy Act. In 1938 the U.S. Congress responded to the massive social and economic upheaval caused by the depression by passing an extensive amendment to the Bankruptcy Act. The amendment sponsored by Chandler, although technically only an amendment, made such wide sweeping changes to the original act, that the Bankruptcy Act became known as the Chandler Act.

Several features of the act deserve special mention. Firstly, while any firm may become insolvent, not all firms may declare bankruptcy. The Act does not allow building and loan associations and insurance and banking corporations to either voluntarily or involuntarily file under the Chandler Act, rather their liquidation or rehabilitation is left to state authority. 6 The historical basis of this exemption is a
basic deference to state authority. Involuntary bankruptcy is also denied to the creditors of federal and state governments and farmers.

Railroads receive special treatment under the Act. Prior to 1933, the only course open to a financially distressed railway was to enter either federal or state receivership. Section 77, dealing with railroad reorganization, was enacted in 1933 with the aim of facilitating and streamlining the reorganization process while taking into account "the public interest". It is interesting to note that while the Chandler Act made extensive revisions to the Bankruptcy Act, it made no changes to Section 77.

The Chandler Act can be applied to a debtor either voluntarily or involuntarily. The only requirements for a voluntary petition for bankruptcy are that the petitioner has debts, pay the filing fee, and not be one of the exempt institutions. The solvency of the petitioner is irrelevant, the only relevant matter is that at least one creditor exists. A petition for reorganization can only be entered voluntarily.

Except in certain circumstances an involuntary petition can be filed against any debtor whose total debts exceed $1000. The petition must be signed by at least three creditors with $500 in debts (in aggregate) if more than twelve exist, otherwise only one creditor's signature is required. However, a bankruptcy petition cannot be filed on
a whim; rather, the debtor must have committed an "act of bankruptcy". These "acts" can be summarized as: hiding assets, giving preferential treatment to certain creditors, allowing a lien or assignment of assets to occur while insolvent or admitting an inability to pay. Unlike a voluntary petition which usually results in an immediate adjudication of bankruptcy, an involuntary petition can be contested in the Bankruptcy Courts. If the creditors are unable to prove insolvency and the commission of at least one of the "Acts of Bankruptcy", the petition will be dismissed.

In a straight bankruptcy, under Chapter V, the bankrupt emerges with a "clean slate" financially; but, generally, with no assets as they have been liquidated in order to satisfy creditors' claims. Unfortunately these proceedings impose serious costs on debtor and creditor alike. An Arrangement, under Chapter XI of the Act, attempts to both reduce the cost of bankruptcy and allow the business to continue activity.

An arrangement is a proceeding pursuant to which an embarrassed debtor, by agreement with his creditors and subject to court approval, remains in business but secures either an extension on time for payment or pays them off on a pro rata basis, or both.

Chapter X of the Act allows corporations to refinance and reorganize, change corporate structure, without interruption of business activity. While Reorganization and Arrangement both involve negotiation between debtor and creditors with the courts acting as intermediary, there exist
considerable differences between Chapters X and XI in the areas of control, representation, and fairness.

Control of a Chapter XI proceeding is vested in the debtor. Only the debtor can file the petition and only the debtor can propose a plan. The major weakness in a Chapter XI proceeding is that independent verification of financial information disclosed by the debtor is often not undertaken as the Act makes no provision for investigation. This information problem for unsecured creditors is compounded by two other features of the Chapter: the "friendly" creditors committee and the expense reimbursement provision.\(^{13}\)

The creditors' committee, theoretically, is the unsecured creditors representative. However, in practice, it is made up of major creditors who often have a close association with the debtor. The problems of inadequate representation are compounded by the Chapter's reimbursement provision that the expenses incurred by the committee may not be reimbursed if the debtor's arrangement plan is not confirmed.\(^{14}\) Consequently, most debtors are subject to only a cursory investigation as acceptance of the proposed plan is often better financially than forcing liquidation as a fair plan under Chapter XI is required to provide general creditors with more than would be provided in liquidation.\(^{15}\)

Chapter X, in contrast, vests control in an independent trustee rather than either creditor or equity owners. The court appointed trustee has wide investigative powers and has
the first opportunity to propose a plan of reorganization. All other interested parties, except the debtor, may also file proposals. The debtor may only file after the trustee's time to file has expired. 16

Representation requirements are significantly different between Chapters X and XI. Chapter XI officially deals only with unsecured creditors and has no regulations regarding the appointment or selection of creditors representatives. Chapter X, in contrast, deals with all creditors and requires that the trustee and his attorney be truly "disinterested" parties who are free of any conflicts of interest. The Chapter also requires full disclosure and imposes fiduciary standards on any representatives. There are no fee reimbursement restrictions similar to those in Chapter XI. 17

Fairness of the plan is another area in which Chapters X and XI differ markedly.

As originally enacted in 1938, Chapter XI plans had to be in the best interests of creditors, fair and equitable and feasible. Chapter X plans, on the other hand, had to be fair and equitable and feasible. A fair plan under Chapter XI requires that general creditors receive more than they would in a liquidation. Chapter X in contrast follows the principles of absolute priority. The junior creditors have no interests under the plan until more senior creditors are paid in full. 19

A major problem in using Chapter X and XI is that the Act makes no distinction between who may apply. Chapter XI was intended for use by smaller closely held corporations and
non-business individuals. There have been several attempts by
the Securities and Exchange Commission to forbid the use of
Chapter XI to any public corporation with more than one
hundred stock holders. These moves have not been
successful.

Given a choice between filing under Chapter X or XI, a
debtor, in general, would prefer Chapter XI as he maintains
control of the business and the proceedings. While the
control and representation problems may not be major in a
small firm these weaknesses can offer little protection to
the creditors who aren't "friendly".

Chapter XI case law has evolved to the point that
although the chapter only deals with unsecured creditors,
there is sufficient precedent and procedure to deal with
secured creditors within a Chapter XI bankruptcy action. A
rather disturbing trend of large corporations using the more
informal proceedings of Chapter XI rather than the more
appropriate Chapter X has developed and is illustrated by
Table 1.

THE BANKRUPTCY REFORM ACT OF 1978: By the late 1960's
and early 1970's it had become obvious that the Chandler Act
was no longer functioning in its intended manner. The
bankruptcy courts were faced with an ever increasing number
of bankruptcies as more and more consumers and businesses
made use of the process. The annual case load of a typical
Table 1

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</tr>
<tr>
<td>Harvard MDS</td>
<td>1/72</td>
<td>50.3M</td>
</tr>
<tr>
<td>FAS International</td>
<td>2/72</td>
<td>40.7M</td>
</tr>
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<td>Famous Schools</td>
<td>2/72</td>
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</tr>
<tr>
<td>Federal's Inc.</td>
<td>8/72</td>
<td>37.0M</td>
</tr>
<tr>
<td>King</td>
<td>na</td>
<td>34.8M</td>
</tr>
<tr>
<td>Modular Housing</td>
<td>2/73</td>
<td>33.0M</td>
</tr>
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<td>Topper Toys</td>
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<tr>
<td>Botany Industries</td>
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<td>DCA Development</td>
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<td>20.0M</td>
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from: Report of the Commission on the Bankruptcy Laws
Page 261

referee was 132 cases in 1947; by 1967, it had risen to 1074 cases. 21

... it is clear that in recent years the bankruptcy system has been processing a far heavier case load than was contemplated either at the time of enactment of the Bankruptcy Act in 1898 or at the time of its complete revision by the Chandler Act in 1938.

In an effort to streamline and modernize the bankruptcy procedure, the Bankruptcy Reform Act, or Bankruptcy Code as it is commonly referred to, was enacted in 1978. The Chandler Act was officially repealed on October 1, 1979.

The two chapters of interest in the Code are Chapters 7 (liquidation) and 11 (reorganization). In order to file an involuntary petition, the creditors must show that the debtor is generally not paying his debts as they become due or that within 120 days prior to filing, a custodian took possession
or was appointed. This section eliminates the more cumbersome "Acts of Bankruptcy" and replaces them with a more reasonable solvency type of test. The amount of debt held by filing creditors has been increased to $5000.

Chapter 11 of the Code consolidates and refines the Arrangement and Reorganization chapters (X and XI) of the old Act. Under the revised Chapter, a debtor may file his plan of reorganization after filing his petition. After this period, if a trustee has been appointed by the courts, any interested party may file a plan as well. Confirmation requires that the plan be accepted by two thirds in amount and one half in number of each class of creditors and by two thirds in amount, regardless of number, of shareholders. The court may confirm a plan even if a class of creditors object if the court finds that the interests of that class are not impaired by the plan.

Chapter 11 permits the debtor to continue running his business unless the court finds it in the best interests of creditors and/or the owners or if "cause" can be shown. Cause is generally classed as fraud, incompetence or gross mismanagement.

One of the major goals of the Bankruptcy Reform Act of 1978 was to speed up the process as the longer a firm spends in the bankruptcy system, the higher the costs to debtors, creditors and society in general. According to the Bankruptcy Commission [1973], the average time spent in 1971 by court
officials on Chapter X cases was 243.2 hours. The average
time spent on Chapter XI cases ranged from 29 to 57 hours,
depending on the size of the firm in question.\textsuperscript{26} The average
costs attributable to the courts in a Chapter X case was
$7660 while in a Chapter XI case the costs were $1207.\textsuperscript{27}
These costs are among the direct costs of bankruptcy,
however, a major component in the total costs is likely to be
the indirect costs of bankruptcy such as lost sales and
foregone opportunities.\textsuperscript{28} In an effort to reduce the time
spent in the bankruptcy system, the Act lays down several
time limits and generally makes it easier to enter the
bankruptcy process. However, recently concern has been
voiced that bankruptcy is too easy; firms were using it to
escape rather than as a last resort. There were several
proposals before Congress in 1983 to not only make going into
bankruptcy more hazardous and less easy; but, also to deal
with the problems caused by the recent Supreme Court ruling
that the powers of the Bankruptcy Courts are
unconstitutional.\textsuperscript{29}

It is unclear exactly what the repercussions of the
Congressional proposals and the Supreme Court ruling will be
for the bankruptcy process.
C. THE BANKRUPTCY RECORD

Figure 1 illustrates the dramatic changes which have occurred in the bankruptcy record since the passage of the Bankruptcy Reform act.

The primary area of concern in this situation, is whether the new Bankruptcy Act has caused the decision making
process of a firm approaching bankruptcy to change. In other words, are some firms now filing for bankruptcy which under the same circumstances would not have filed under the old act. A major issue is whether this change is statistically significant, and what are the implications of the change in the Act for Bankruptcy Prediction models which do not take into account this change.
The purpose of bankruptcy prediction models is two fold: to classify or identify financially troubled businesses and to predict their future failure on the basis of current and past data. The data and the techniques used often reflect the differing views of market efficiency.

Models of bankruptcy or failure can typically be identified by the type of data they analyze. The more modern approach of stock price based models seeks to use the efficiency of the market to evaluate the underlying solvency of the firm, while the more traditional models, which implicitly assume that market data is either unavailable or does not fully reflect available information, utilize financial data in one of three ways: comparison to a peer group, static multivariate analysis and time series based regression analysis. A major drawback of the common statistical models of bankruptcy is their largely atheoretic nature. These models may predict an event, or describe the data very well; but, cannot explain the phenomenon under examination as they are not based on any underlying financial theory.

A. STOCK-PRICE BASED MODELS

Stock-price based models of business failure are the most theoretically appealing as, according to the efficient
capital markets hypothesis, the securities of an institution are continually and fairly priced based on all available information. The data required for these models can usually be acquired with minimal effort and time delay.

In general, these models begin by estimating an "expected return" function, usually based on an asset pricing model such as the Capital Asset pricing model. The residuals from such a regression analysis, therefore, reflect any "unusual" results which may be due to the market's perception of the company's long term solvency. These models have generally yielded very promising results. Shick and Sherman [1980] found that from 1967-1974 stock prices began a downward trend before bank examiners recognized an adverse change in the bank's condition.¹

Stock-price based bankruptcy/failure models offer many advantages on both theoretic and practical grounds, over the more traditional financial ratio based models, they do, however, depend on the existence of a reasonably active market for the stock in question. Unfortunately, many cases of bankruptcy and weakness occur in small, closely held institutions which haven't the necessary active market for their stocks.

B. UNIVARIATE RATIO ANALYSIS

One of the oldest technique for predicting failure or financial distress is based on Peer Group comparison. In
this technique financial ratios are calculated and then compared to Peer Group results. The major statistical weaknesses of this procedure arise from the selection of the Peer Group and the ratio comparison techniques.

Peer Group selection is a vital part of this type of analysis as the observed value of a financial ratio depends not only on the underlying solvency of the firm, but also on other factors unique to that type of business. For example, the ratios of a toy manufacturer are not comparable to those of a jewelry retailer.

Peer group models typically make use of several key financial ratios and the comparison of these ratios to the peer group norms causes many statistical problems as one is essentially attempting to do univariate analysis on multivariate data. In fact, seemingly insignificant or unimportant ratios on a univariate basis can become very important in combination with other ratios. This type of failure model can consequently yield very confusing results, especially if the firm in question is above standard in certain areas and substandard in others.

C. STATIC MULTIVARIATE ANALYSIS

This type of failure model seeks to distill the information found in the many financial ratios available into a composite score or value. This approach was pioneered by
Altman who applied discriminant analysis to financial ratios in an effort to predict bankruptcy.  

Altman examined financial ratios from 66 manufacturing firms to determine a function which best discriminated between bankrupt and non-bankrupt firms. The validity of this was evaluated in two ways: first the ability to classify correctly was evaluated using a second data set drawn from the same population as the initial sample. The function was found to classify firms extremely well. A second test was undertaken to determine how well the model classified potential bankrupts by evaluating data from two years prior to bankruptcy. Altman found that the model performed well just prior to failure, and it's classification accuracy was still good two years prior to failure. However, the discriminatory ability declined significantly as the time to failure increased. 

This type of financial ratio analysis has been criticized on several fronts. Statistically, concerns have been raised regarding the validity of the underlying assumptions in the analyses. The primary concerns issued concerning the distributions of the variables, as non-normality can cause significant problems regarding inference, evaluation of the costs of erroneous classification and the estimation of error rates of these models among others. Multivariate analysis of financial ratios has been extended in the hope of developing more useful tools for the early
identification of a troubled firm. Martin [1977] extended the approach by applying a Logit Regression analysis to the problem, while West [1985] has used factor analysis in conjunction with Logit estimation to analyze the bank condition problem. Bankruptcy or failure models which rely on financial ratios and use either discriminant or factor analysis are subject to several major theoretical criticisms.

Any model of a business entity which relies on financial data is suspect, due to the very nature of financial reporting and accounting techniques. Financial data, even if reported honestly and accurately, is generally out of date by the time it has been collected and presented. The data may be accurate in the sense that all the values are independently verifiable, and are correct for the period in question but if the statements are published a considerable time after the period in question, the value of the data is limited for current decision making. The basic tenant of all accounting systems is that all reported values must be independently verifiable, consequently, most publicly disclosed financial data are reported on an historical cost basis. Not only is the timeliness of the financial data highly suspect; but the data itself is also highly susceptible to manipulation.

The prevalence of record manipulation is very evident in the bank failure records. In a study of the 57 Federal Deposit Insurance Corp. insured banks closed due to financial difficulties between 1960 and 1972, it was found that nearly
30% of the failures were due to fraud, embezzlement and manipulation.¹⁰

The ratio based models outlined thus far are also subject to criticism for their essentially static nature.

Rather than obtaining evidence concerning the banks likely exposure to failure in its operations, these ratios question the ability of the bank to avoid present failure with its present asset characteristics.

A major concern with all statistical models is their largely atheoretic nature. In order for a model of bankruptcy or failure to have economic meaning, the variables should be deduced from a theory of the underlying economic and decision processes faced by the company management.¹²

The principal reason for including specific ratios in many of the failure studies is that they appear to discriminate well between failure and non-failure, not for any particular theoretical reason. While the atheoretic nature of a purely statistical model is of little concern if the sole purpose of an analysis is to predict bankruptcy; however, one is often interested in models which can offer insight into why bankruptcies occur as well as predicting their occurrence.

The major drawback of the traditional models of bankruptcy or failure is that they yield a probability that a particular firm will fail within one period. Such a probability is not very useful in either investment analysis or in management decision making. Another drawback of the discriminant analysis based models is that data from one year
prior to failure cannot be mixed with data from two years prior to failure. Consequently, these models are very data specific in that all the available data is not used, rather only specific subsets are analyzed.

D. REGRESSION AND TIME BASED MODELS

In an effort to model the dynamics of failure, Santomero & Vinso [1977] apply time series techniques which had been quite commonly used in modelling insurance risk exposure. Essentially their model is derived from the premise that the firm's capital acts as a buffer against future losses and that variations in this buffer occur stochastically. The two techniques, known as First Passage Time or Gambler's Ruin and Maximum Risk exposure are used to model the bank capital buffer.\textsuperscript{13}

Santomero & Vinso concentrate on applying the maximum risk exposure technique as the Gambler's Ruin technique relies on the unpalatable assumption of a zero or negative drift in the capital account. This assumption would appear to be more appropriate for non-profit or charitable organizations than for profit maximizing institutions.\textsuperscript{14} The maximum risk exposure model of the banking system was analyzed in cross section.\textsuperscript{15}

Santomero & Vinso's results indicate that the traditional capital asset ratio is a useful tool in identifying troubled banks; but, that a bank's capital stock
variability is also a very important component in determining a bank's survival potential. Santomero and Vinso, therefore, conclude that their model, which takes into account this capital stock variability, in conjunction with a proposed screen offers a more exact classification and identification technique than the previously available methods.\textsuperscript{16}

The data requirements of the Santomero and Vinso model are very different from the more traditional "Altman" type of model. In order to estimate the behaviour of a company's capital stock over time, the Maximum Risk Exposure model requires a time series of data. The most desirable data set would be a continuous record of the market valued capital levels; however, the best one can do is the weekly reports filed by approximately three hundred banking institutions with the Federal Reserve. The capital values reported are essentially accounting values as the "Fed." does not require market value restatement in these weekly reports.\textsuperscript{17} While extensive time series data is required, it is required only on the firm of interest. The "Altman" based models, in contrast, require cross-sectional data. While such a requirement is unlikely to concern a regulatory agency who is responsible for monitoring the population of businesses within an industry, it is likely to be of considerable concern to a portfolio manager who is interested in a small subset of an industry.
THE COX MODEL: Recently, Lane, Looney and Wansley [1986] have extended the "Altman" type of cross-sectional analysis through the application of the Cox Proportional Hazard Model to the bank failure problem. This model offers several advantages over the more traditional failure models in that it generates an expected time until failure and has fewer restrictive underlying distributional assumptions. One of the basic assumptions underlying the Proportional Hazards Model, which will be discussed in more detail below, is that the observed values of the variables effect the hazard faced by an average firm. 18

The more traditional models tend to classify companies as either failure or non-failure and do not take into account that all one can say about a non-failed firm is that it has not failed as yet. Proportional Hazard models, in contrast, can analyze such censored data samples. 19

Lane, Looney, and Wansley evaluated the Cox model as part of an early warning system for bank failure in two ways: by assessing the model's accuracy in predicting time until failure and by comparing the model's ability to classify individuals in the failure/non-failure context with the more common multiple discriminant analysis approach.

Twenty-one ratios were used as potential predictors of failure. These ratios were selected to represent the five CAMEL dimensions of bank operations which appear to measure bank solvency, and are used extensively by the bank
examiners. CAMEL is an acronym representing Capital (leverage), Asset (loan) quality, Management (loan composition, efficiency or pricing), Earnings, and Liquidity.

The results of the Lane study indicate that the Cox Hazard model is useful in analyzing bank financial condition. Discriminant analysis and the Proportional Hazard models appear to yield similar classification rates. Therefore, it appears that the Cox model offers the examiner additional information regarding probable time to failure without losing significant classification accuracy.

The hazard function approach, based on survival-theory, offers several advantages over the more traditional approach by providing an estimate of the probable time until failure, and by lessening the burden imposed by the highly parametric nature of the traditional analysis. It would, therefore, appear that this approach merits further examination.
IV. PLAN OF ANALYSIS

In this paper, the Cox proportional hazard model will be applied to the problem of bankruptcy prediction in a diverse industrial setting and the model's performance will be compared to the performance of an estimated linear discriminant function based on Altman [1963].

A. RATIO SELECTION

"Ratios" were selected on the basis of three criteria: popularity, performance and availability. Popularity and performance refer to the frequency of use of the ratio and its performance in other studies. Two studies of major significance for this purpose were Beaver [1966] and Altman [1966]. Availability refers to data limitations as not all the firms reported the necessary data.

The ratios used are found in table 2.¹

Table 2.

<table>
<thead>
<tr>
<th>LIST OF RATIOS USED.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cash flow Ratios:</td>
</tr>
<tr>
<td>1. Cash flow to sales.</td>
</tr>
<tr>
<td>2. Cash flow to Total Assets.</td>
</tr>
<tr>
<td>3. Cash flow to Total Debt.</td>
</tr>
<tr>
<td>1. Net income / Sales.</td>
</tr>
</tbody>
</table>
4. Retained earnings / Total Assets.
5. Earnings before Interest and Tax / total Assets.

3. Debt to Total Assets.
   1. Long term debt / Total Assets.
   2. Current + longterm debt + pref. shares / Total Assets.
   3. Interest / long term debt.

   1. Working capital / Total Assets.
   2. Cash / current liabilities.
   4. Quick assets / Total Assets.
   5. Quick Assets / current liabilities.

5. Turnover Ratios.
   1. Inventory / Sales.
   3. Quick Assets / Sales.
   4. Total Assets / Sales.
   5. No credit interval. (quick assets minus current liabilities / funds for operations)
   6. Defensive Interval. (quick assets / funds for operations).
   8. Change in margin.

   1. Closing market value of equity / total assets.
   2. Change in Market value of equity.
   3. Invested Capital / Total Assets.

* cash flow = income from operations before depreciation plus change in accounts payable minus change in accounts receivable minus change in inventory.
* working capital = current assets minus current liabilities.
* quick assets equals cash plus accounts received.
* funds expenditures for operations equals sales minus operating income before depreciation.
* invested capital equals the sum of longterm debt, preferred stock, minority interests and common stock.
* change in market equals closing market time (t) divided by closing value time (t-1).

B. THE DATA SET

Data Sources: The principal data sources for this study were the Compustat research and industrial tapes for 1986. These tapes contain financial statement data on a large number of firms for the period 1966-1986. The Compustat Research industrial tape contains data on firms removed from the Compustat industrial annual tape. Firms can be eliminated from the industrial tape for a variety of reasons, including mergers, inconsistent reporting, bankruptcy or returning to private ownership.

Compustat data is derived from the published financial statements and the 10K reports filed by firms with the Securities and Exchange Commission in the United States. Most firms listed on the Compustat tapes are the larger publicly traded ones rather than the more common smaller private firms.

Any use of an analysis of bankruptcy based on these data must take into account this potential data bias when applying results to the general population of firms.

Sample Selection: This study excluded railroads, insurance and banking institutions, and savings and loan associations due to their special treatment under the Bankruptcy Acts, both the Chandler and Reform Acts. Real estate, other financial companies, other transportation firms
and service industries such as travel agencies were also excluded as most of the data available on bankruptcy and failure rates appears to either exclude or separate these industries. Therefore, the sample was restricted to commercial and industrial failures including construction, manufacturing and mining, and the retail and wholesale trades.

Bankruptcies were identified by examining the capital structure changes of the remaining firms on the research industrial file, as reported in the Capital Changes Reporter. Of the 618 firms examined, 84 were identified as filing petitions under Chapters X and XI of the Chandler Act or under Chapter 11 of the Reform Act.

In order to develop a non-failure sample with similar characteristics to the bankruptcy sample, bankrupt and non-bankrupt firms were required to have had data reported on the Compustat tape for at least five years prior to failure, six years was the preferred value. This time period was selected for the following reason:

Beaver's study of financial ratios as predictors of failure found that at approximately five years prior to failure it becomes more difficult to distinguish the future failure from the future survivor. In this study bankrupt firms were matched to similar non-bankrupt firms five and six years prior to failure. The aim of this procedure was to begin with a sample of similar firms before the failures
began their financial slide in order to ensure comparability of the financial ratios to be analyzed.

The five to six year prior data requirement eliminated 28 failures from consideration leaving a sample of 56 firms which filed petitions for bankruptcy between 1971 and 1985.

**Data Validation.** According to the Compustat Codebook, the data does not exceed the F10.6 format, in other words, the largest value possible is 999,999,999 million. However, when the data obtained was checked against the format requirement, one firm, W. T. Grant, was rejected.

This result was not unexpected. At the time of bankruptcy in 1975, W. T. Grant had 82,000 employees, 1200 stores and sales of over $1.6 billion. However, when the Compustat sales figures were examined, the data appeared questionable. (See table 3.)

| Table 3 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | W. T. GRANT SALES |                |                |                |                |
|                | ( in millions )    |                |                |                |                |
| Platt (p35)    | 1096             | 1210           | 1254           | 1374           | 1644           | 1849           |
| Compustat      | 1214             | 1254           | 1374           | 1644           | 1849           | 1761           |

Unfortunately, W. T. Grant's financial data were not verifiable through other sources such as Moody's, as the earliest Moody's corporate directory available in the
University of British Columbia Main Library was for 1977, and
due to the above problem the company was eliminated from
consideration.

Data was also examined to ensure that vital values were
not missing. Seventeen firms were identified as missing data
immediately prior to failure. The results of further
examination are presented in table 4.

| Table 4 |
| CHARACTERISTICS OF INCOMPLETE FINANCIAL DATA |
| # of firms | Characteristics |
| 7 | Filed 1st quarter. |
| 7 | Filed 2nd quarter. |
| 2 | Filed 3rd quarter. |
| 1 | Missing 3 years of data. |

The above results are a direct consequence of the
problem of timeliness of financial data. In the majority of
cases, closing stock prices were reported for December 31st
of the year prior to failure.

In an effort to maintain an adequate sample size of
bankrupts, and in view of the slow delivery of financial
data, firms which filed petitions in the first half of the
year and had missing financial data were included in the
sample. In these cases, the last filed statement was treated
as the one year prior data. Therefore, one year prior data is
actually between 1 and 18 months prior to failure.

Comparability. The data used in this study were obtained
by "matching" bankrupt firms with non-bankrupt firms for two
reasons. First, a random sample of firms drawn from the test industries (industrial and commercial firms excluding finance, insurance, railroads and services) is unlikely to provide a sufficient number of bankrupt firms to enable any inferences to be drawn from the results of this study. Between 1972 and 1985, 80 firms within the test industries filed for bankruptcy, however, over 8000 un-failed firms were listed on the Compustat industrial tape in 1986. Therefore, any random sample drawn from a population such as this would be predominantly un-failed firms as the probability of selecting a failure is less than 1%.

The second reason for this "matching" was to ensure comparability of the bankrupt and nonbankrupt samples. The purpose of most studies of bankruptcy is to isolate and identify the unique financial characteristics of firms which will file for bankruptcy in the future in the belief that these unique attributes can then be used to predict failure. While in principle accounting data reflects the firm's "absolute" financial condition, bankruptcy prediction requires an estimation of the "relative" financial condition of the firm. Comparing a future bankrupt to "similar" non-bankrupt firms facing similar economic conditions enables one to isolate the differences due to the pre-bankruptcy state from those due to industry or size differentials.
The sample of un-failed firms was obtained from the Compustat industrial tape. A non-failed firm "matched" a bankrupt if the following conditions were met:

1. The firms must be in the same or similar industry as represented by the SIC codes. An un-failed firm's SIC code was considered "matched" if it was within plus or minus 5 of the units digit of the failure's SIC code.

2. Net income before extraordinary items, sales, and net assets were compared in the same year which was 5 to 6 years prior to failure. For example if a firm failed in 1981, the comparison would be carried out on 1975 and 1976 data. A match was accepted if on at least four of these six comparisons, the un-failed firm's data was within a factor of three (less than 3 times but greater than 1/3 times) of the failed firm's data. The requirement that firms' values be within a factor of three arrived at somewhat arbitrarily. A factor of three was the lowest factor that permitted most of the failures to be matched to at least one non-failure.

Net income before extraordinary items and sales were selected as a basis of comparison in an attempt to identify firms of similar efficiency, as measured by net income, and market power, as indicated by sales. Net assets were included in an effort to match firm's of similar size. Essentially, this paper attempted to match firms of similar 'market power' (sales), 'efficiency' (net income) and 'size' (assets) five
to six years prior to one of the two firms' filing a petition
for bankruptcy.

Each failure was restricted to at most 2 matches in
order to ensure similarity between the industrial composition
of the bankrupt and non-bankrupt samples.

The exchange codes reported by Compustat include
information such as exchange listed on and inclusion or
exclusion in various indices such as the Standard and Poor's
500 (Compustat Codebook). These codes were used as a final
matching criteria in the case of multiple matches to ensure
further comparability as each exchange and each S & P index
imposes different criteria on member firms.

After the "matching" process had been undertaken, forty-
one failures remained and were matched to sixty-seven
survivors for a total sample size of one hundred and eight
firms.
V. SAMPLE CHARACTERISTICS

A. SAMPLE STATISTICS

The sample characteristics are illustrated in table 5.

<table>
<thead>
<tr>
<th></th>
<th>CHANDLER ACT</th>
<th>REFORM ACT</th>
<th>OVERALL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure</td>
<td>Match</td>
<td>Failure</td>
</tr>
<tr>
<td>5 years prior</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Assets:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>36.02</td>
<td>37.22</td>
<td>82.42</td>
</tr>
<tr>
<td>s.t.dev.</td>
<td>31.96</td>
<td>43.00</td>
<td>108.72</td>
</tr>
<tr>
<td>Sales:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>57.11</td>
<td>69.01</td>
<td>132.64</td>
</tr>
<tr>
<td>s.t.dev</td>
<td>71.80</td>
<td>80.15</td>
<td>157.77</td>
</tr>
<tr>
<td>Net Income:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>.65</td>
<td>1.73</td>
<td>2.51</td>
</tr>
<tr>
<td>s.t.dev</td>
<td>1.14</td>
<td>2.17</td>
<td>3.87</td>
</tr>
<tr>
<td>N</td>
<td>25</td>
<td>41</td>
<td>16</td>
</tr>
</tbody>
</table>

In order to describe the characteristics of the overall failure and survival samples, a profile was constructed of eight ratios. The aim of this analysis is descriptive, to present a graphical representation of the changes which occurred in the two groups over time. The profiles are presented in figure 2.

These profiles show that, on a univariate basis, differences exist between bankrupt and non-bankrupt firms. The statistical significance of the apparent differences
between the bankrupt and non-bankrupt firms will be evaluated in section VI(b). However, any further analysis cannot be conducted on a univariate basis as no single financial ratio is capable of completely representing the complex interrelationships which constitute a firm.

Figure 2.

**RATIO PROFILE of BANKRUPT AND SURVIVOR FIRMS**

--- survivor --- bankrupt

Cash flow / Total Debt

Net income / Total Assets

Long term debt / Total Assets

years prior
B. RATIO DISTRIBUTIONS

The normality of the distributions of the financial ratios was evaluated in order to establish the validity of any inferences drawn from the discriminant analysis. The Cox Model, in contrast, does not rely on any such distributional assumptions.

Multivariate normality is not required for the data description and estimation aspects of multivariate theory, it's impact is felt in the area of inference. Unfortunately, practical tests of multivariate normality essentially do not exist and one generally infers multivariate normality from the normality of the underlying variables.

The normality of the distribution of the ratios was evaluated in two ways, first by examining the skewness and kurtosis statistics and secondly, by examining the histograms and normal probability plots of the ratios.

Several characteristics of the ratios become apparent from the normal probability plots which showed on average that the distribution of the variables did not become any less normal the closer one moved toward bankruptcy.

A few exceptions to this were ratios involving net income/sales, (see appendix II.) which showed that the distribution became less normal as the sample moved closer to bankruptcy. This was not surprising as one would expect firms matched 5 to 6 years prior to failure to begin to split into two groups as bankruptcy becomes imminent if there does
exist a difference between the non-bankrupt and bankrupt groups.

It is apparent from the profiles constructed previously that imminently bankrupt and similar non-bankrupt firm's ratios exhibited a very different time path from each other immediately prior to failure.

In general the normal probability plots fall between the extremes of the working capital to total assets plot and the very abnormal plot of cash / sales. (see appendix II).

Due to the nonnormality of these ratios, any inferences drawn from the discriminant analysis undertaken on the un-transformed variables would be very suspect. It is often possible to transform a non-normally distributed variable into a normal variable by using a log or power transformation. Such transformations were not undertaken in this study as the primary area of interest was the Cox model's performance. The discriminant analysis was undertaken merely to provide a comparison model for more descriptive purposes.

C. CHANDLER VS. REFORM ACTS

A second issue to be addressed was: Does a significant difference exist between firms declaring bankruptcy prior to the Reform act and to those declaring bankruptcy at a later date?
As the underlying ratios were not normally distributed, the Kruskal-Wallis test of group similarity was undertaken. The formal null hypothesis of the Kruskal-Wallis test is that the distributions of the two groups are identical, but, not necessarily normal. Unlike the more commonly applied ANOVA, Analysis of Variance, test of distribution similarity, Kruskal-Wallis test is less sensitive to the basic assumption that the populations are normally distributed.

The Kruskal-Wallis test was undertaken to test two hypotheses, firstly, that no differences existed between the distributions of bankrupt and surviving firms, and secondly, that no differences existed between firms filing for bankruptcy before and those filing after the passage of the Bankruptcy Reform Act of 1978. Unfortunately the Kruskal-Wallis tests undertaken were univariate, they tested one variable at a time and can therefore only give a general indication that a difference does, or does not, exist. Table 6 gives a summary of the test results comparing bankrupt and non bankrupt firms one year prior to failure. Similar results were obtained for data four years prior to failure.

From the ratio profile (figure 2), it is apparent that certain variables are capable of discriminating between failures and non-failures when examined on a univariate basis. It would also appear that certain ratios such as inventory/sales and current assets/sales are not good
discriminators on a univariate basis. However, one cannot form any conclusions regarding the performance of these variables when examined in a multivariate setting.

Recently concern has been expressed that the Bankruptcy reform act has made it too easy to declare bankruptcy, and, consequently, the process has been abused. In order to evaluate this concern, the Kruskal-Wallis test was conducted on the hypothesis that at one year prior to failure no differences existed between the distributions of firms filing for bankruptcy under the Chandler and Reform Acts. The results of these tests are summarized in table 6.

At the 0.01 significance level, all the variables, individually evaluated, indicate that there does not appear to be a significant difference between the ratio distributions of firms filing under the Chandler and Reform Acts.

In order to evaluate the overall significance of the Kruskal-Wallis tests undertaken, the Bonferroni approach was used to arrive at an overall significance level. If K tests are to be undertaken, then the Bonferroni approach uses $a' = a / K$ where $a$ is the minimum overall significance level, where $a'$ is the significance level for each test. At an overall significance level of 0.01, the individual significance level was 0.0003. It is apparent from the p-values (probability of the observed value or a more extreme value occurring under the null hypothesis) that many of the
variables indicate that a significant difference exists between the samples of bankrupt and nonbankrupt firms. In contrast, at an overall significance level of 0.01, the Kruskal-Wallis test results indicate that no significant difference exists between the ratios of firms filing for bankruptcy under the Chandler and Reform acts. On the basis of the Kruskal-Wallis tests we can conclude the following:

A) That a statistically significant, at the 0.01 level, difference exists between bankrupt and non-bankrupt firms one year prior to failure on an individual ratio basis. This result is similar to Beaver's finding that financial ratios were a useful tool in the evaluation of firm viability.

B) That not all ratios perform equally well in identifying differences between firms.

C) That although there has been a startling increase in the number and size of bankruptcies since the passage of the Reform Act, there does not appear to be a statistically significant difference between the one year prior ratios of firms which filed under the Chandler Act and those that filed under the Reform Act.
Table 6
TESTS OF GROUP SIMILARITY
(using the Kruskal-Wallis methods)
of

$H_0^1$: no difference between Survivor and Bankrupt.

$H_0^2$: no difference between Bankrupt before and after 1978.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>$p$-value $H^1$</th>
<th>$p$-value $H^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow / sales</td>
<td>0.0399</td>
<td>0.4705</td>
</tr>
<tr>
<td>Cash flow / total assets</td>
<td>0.0236</td>
<td>0.7893</td>
</tr>
<tr>
<td>Cash flow / total debt</td>
<td>0.0001</td>
<td>0.7484</td>
</tr>
<tr>
<td>Net income / sales</td>
<td>0.0</td>
<td>0.9361</td>
</tr>
<tr>
<td>Net income / total assets</td>
<td>0.0</td>
<td>0.3360</td>
</tr>
<tr>
<td>Net income / total debt</td>
<td>0.0</td>
<td>0.2851</td>
</tr>
<tr>
<td>Long term debt / total assets</td>
<td>0.0103</td>
<td>0.6496</td>
</tr>
<tr>
<td>Total claims / total assets</td>
<td>0.0</td>
<td>0.1211</td>
</tr>
<tr>
<td>Working capital / total assets</td>
<td>0.0</td>
<td>0.1416</td>
</tr>
<tr>
<td>Cash / current liabilities</td>
<td>0.0</td>
<td>0.2091</td>
</tr>
<tr>
<td>Current assets / current liabilities</td>
<td>0.0</td>
<td>0.1814</td>
</tr>
<tr>
<td>Inventory / sales</td>
<td>0.5883</td>
<td>0.9361</td>
</tr>
<tr>
<td>Current assets / sales</td>
<td>0.8123</td>
<td>0.6689</td>
</tr>
<tr>
<td>Total assets / sales</td>
<td>0.3875</td>
<td>0.4705</td>
</tr>
<tr>
<td>Quick assets / sales</td>
<td>0.1128</td>
<td>0.8937</td>
</tr>
<tr>
<td>Quick assets / current liabilities</td>
<td>0.0</td>
<td>0.1211</td>
</tr>
<tr>
<td>Quick assets / total assets</td>
<td>0.0193</td>
<td>0.1995</td>
</tr>
<tr>
<td>Quick assets / funds for operations</td>
<td>0.6970</td>
<td>0.9361</td>
</tr>
<tr>
<td>No credit period</td>
<td>0.0</td>
<td>0.1566</td>
</tr>
<tr>
<td>Retained earnings / total assets</td>
<td>0.0</td>
<td>0.0215</td>
</tr>
<tr>
<td>Earnings before interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and tax / total assets</td>
<td>0.0</td>
<td>0.7283</td>
</tr>
<tr>
<td>Market value equity / book val. debt</td>
<td>0.0</td>
<td>0.3924</td>
</tr>
<tr>
<td>Margin</td>
<td>0.0003</td>
<td>0.6689</td>
</tr>
<tr>
<td>Interest / long term debt</td>
<td>0.0001</td>
<td>0.1277</td>
</tr>
<tr>
<td>Invested capital / total assets</td>
<td>0.0</td>
<td>0.9149</td>
</tr>
<tr>
<td>Change in margin</td>
<td>0.0555</td>
<td>1.0</td>
</tr>
<tr>
<td>Change in market</td>
<td>0.0</td>
<td>0.6689</td>
</tr>
</tbody>
</table>
D) At an overall significance level of 0.01, using the Bonferroni approach, there appears to be a significant difference between the sample of bankrupt and non-bankrupt firms. There does not appear to be a significant difference, at an overall significance of 0.01, between the firms filing for bankruptcy under the Chandler and Reform acts (before and after 1978).

In view of these results, the analysis will not be undertaken on the two groups of bankrupt firms, rather the analysis will be carried out on the entire sample of bankrupts.
VI. DISCRIMINANT ANALYSIS

A. THEORY

Discriminant analysis is a multivariate statistical technique which seeks to transform a series of scores into a single value, and will be used in this paper as a "benchmark" by which to judge the performance of the Cox model. Essentially, this type of analysis deals with the case when one has a sample from population A and a sample from population B, and on the basis of these wish to classify individuals drawn from an unknown population. A graphical representation of this situation is found in figure 3.

---

**figure 3.**

DISCRIMINANT ANALYSIS (GRAPHICALLY)\(^1\)
Linear discriminant analysis for classification into two 'a priori' groups results in one discriminant function of the form:

\[ Z = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_m x_m \]

where:  
- \( x_j \) = \( j^{th} \) attribute or individual variable  
- \( b_j \) = discriminant function coefficient for \( j^{th} \) attribute  
- \( Z \) = discriminant score

The discriminant function maps the multidimensional attribute space \((x,y \text{ in figure 3})\) onto a one-dimensional space (line I in figure 3) and maximizes the separation between the two groups.

The output of the BMDP Statistical package is in a somewhat different form than the more common single equation form. In the two group case BMDP produces two functions, one for each group.

\[ Z_i = b_{0i} + b_{1i} x_1 + \ldots + b_{mi} x_m \]

where the subscript \( i \) denotes which group the function is evaluating.

A firm is then classified into which ever group has the largest \( Z_i \). (See BMDP Statistical Software)

The major advantage of this type of analysis is the reduction in dimensions. Rather than having to evaluate twenty-seven ratios, one now only evaluates a single score.

Non-normality of the underlying variables can cause severe problems in discriminant analysis particularly in
quadratic discriminant analysis. The linear discriminant function is an appropriate assignment rule when the following assumptions are satisfied:

A) The underlying populations are multivariate normally distributed.

B) The covariance matrix is the same for the two populations.

C) The 'a priori' probabilities of an individual belonging to either population is known.

D) The mean and covariance matrices are known.

E) The initial classifications were done correctly.

A major concern in discriminant analysis is the estimation of an error rate. The most common approach is to partition the original sample into two groups. The discriminating function is then estimated using one sub-sample and performance is evaluated using the other sub-sample.

This "partitioning" approach has been criticized on several fronts. The basic criticisms are that it wastes data; not all the available data is used to develop the model which can be a severe problem in a small sample; and second, that it does not evaluate the discriminant function which will be used in practice. The discriminant function which will be used to classify firms in practice should use all the sample data available, to partition the data and then use a function
derived on just one partition is not efficient as it does not use all the information available in the total sample.

The "leaving-one-out" approach estimates the error rate in a slightly different manner. In this method, the discriminating function is estimated using N-1 firms. The remaining firm is then classified on the basis of this function. This technique uses most of the sample data, but, can be very time consuming with a large sample size.

In this study, discriminant analysis was carried out using the BMDP:7M package. This package uses stepwise discriminant analysis whereby variables are only included when they significantly improve the performance of the linear discriminant function. One must keep in mind that the stepwise procedure is not necessarily the optimal procedure for identifying variables for inclusion in the analysis. It is not unlikely that another subset of the variables is the optimal set in terms of model performance.

The BMDP:7M package presents a "jackknife" classification matrix. The Jackknife technique is essentially the "leaving-one-out" error classification method. BMDP:7M yields two classification functions, one for each group. Observations are classified into which ever group yields the higher score.
B. ONE-YEAR RESULTS

Results of the discriminant analysis carried out on the one year prior to failure data are summarized in table 7.

All the variables identified by this study were significant on a univariate basis. (See Table 6.) It is interesting to note that only a few of the many individually significant variables were used in this multivariate discriminant analysis. This is partly due to the stepwise procedure used in the discriminant analysis whereby any variables not significantly improving performance were not included in the classification function. It would therefore appear that while a variable may be significant when considered in isolation, it's effect becomes less significant when combined with another variable. In other words the affect of one variable becomes subsumed in the effect of the other.

Most of the variables identified by this analysis are very different to those identified by Altman's Z Score model. Altman's discriminant analysis identified the following variables: working capital to total assets, retained earnings to total assets, earnings before interest and taxes to total assets, market value of equity to book value of total liabilities and sales to total assets.
When discriminant analysis was carried out on the 'four year prior' data, several interesting features emerged. As expected, the classification accuracy declined. The results of the discriminant analysis based on the four year prior data are summarized in table 8. However, of more interest was that the discriminating variables changed. None of the variables used in the one year prior discriminant function appeared in the four year prior model.

This result is not unexpected. One would expect that the predictors of long term viability would not necessarily be the best predictors of short-term survival.
Table 8

**CLASSIFICATION FUNCTION**

Four Years Prior to Failure

<table>
<thead>
<tr>
<th></th>
<th>Survive</th>
<th>Bankrupt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net income /total assets</td>
<td>-4.61612</td>
<td>-26.18338</td>
</tr>
<tr>
<td>Total assets / sales</td>
<td>2.60267</td>
<td>4.95144</td>
</tr>
<tr>
<td>Invested capital / total assets</td>
<td>41.57300</td>
<td>36.40367</td>
</tr>
<tr>
<td>Constant</td>
<td>-15.58192</td>
<td>-13.48379</td>
</tr>
</tbody>
</table>

**CLASSIFICATION ACCURACY (4 YEAR PRIOR)**

(Jackknife technique)

<table>
<thead>
<tr>
<th></th>
<th>Survive</th>
<th>Bankrupt</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survive</td>
<td>52</td>
<td>13</td>
<td>80.0%</td>
</tr>
<tr>
<td>Bankrupt</td>
<td>9</td>
<td>26</td>
<td>74.0%</td>
</tr>
<tr>
<td></td>
<td>61</td>
<td>39</td>
<td>78.0%</td>
</tr>
</tbody>
</table>

It would, therefore, appear that Altman's Z score approach yields useful results. However, any inference based on these results is subject to doubt due to the nonnormality of the distributions of the underlying variables.
VII. THE COX MODEL

A. SURVIVAL THEORY

There are three basic functions which describe the survival experience. The failure density function, \( f(t) \), which defines the pattern of failure by describing the instantaneous risk of financial collapse.

\[
f(t) = \lim_{\Delta t \to 0^+} \frac{Pr(t < T < t + \Delta t)}{\Delta t}
\]

where \( T \) is the time of failure.

The survival function, \( S(t) \), which describes the probability of surviving at least as long as \( t \).

\[
S(t) = Pr(T \geq t) \quad S(t) = 1 \text{ at } t = 0
\]

\[
F(t) = 1 - S(t) \quad \text{distribution of time until failure.}
\]

The third function, the Hazard function, \( H(t) \), describes the instantaneous risk of failure providing failure has not already occurred.\(^1\)

\[
H(t) = \lim_{\Delta t \to 0^+} \frac{Pr(t < T < t + \Delta t | T \geq t)}{\Delta t}
\]

As these three basic functions describe the same survival process, they are closely related.\(^2\)

\[
f(t) = -\frac{dS(t)}{dt}
\]

\[
S(t) = \exp \left[ -\int_0^t H(u) \, du \right]
\]

\[
H(t) = \frac{f(t)}{S(t)}
\]
Consequently, describing one function implies the other two. As the failure and survival functions are very hard to estimate or characterize, one has to concentrate on the hazard function.

Survival analysis faces two major complications: the hazard function may not be constant over time and, secondly, the data may be censored. The time varying hazard function has serious implications for statistical inference and the interpretation of the results of the analysis. In an effort to ensure that the covariates, which determine the hazard function, are constant over time, the analysis was restricted to the very short period of just one year prior to failure. The Cox model, however, can estimate the probability distribution of survival times when the covariates are time dependent. In such an analysis, one must first estimate the relationship between the covariate and time, and then estimate the Cox model. Time dependent covariates were not estimated or analyzed in this paper.

In order to make the constant covariates assumption palatable, a very short time period was analyzed over which it would seem more reasonable to assume that the ratios did not change significantly. The validity of the time independent covariate assumption was evaluated by stratifying the sample on the basis of time, and then observing if the proportional hazards assumptions remained valid. There was
little evidence to suggest that omitted time dependent covariates had a significant impact on the overall model.

Data are censored when the sample yields incomplete survival data, usually because the experiment or study terminates before a desired response or event occurs in every subject in the study. In order to analyze the censored samples one must assume that censoring and failure are generated by independent mechanisms.³

B. THE COX PROPORTIONAL HAZARDS MODEL

The major problem remaining in survival analysis is that the form of the hazard function is unknown. The Cox Proportional Hazard Model presents a technique for estimating the hazard function by allowing a semi-parametric assessment of the relationships among the hazard functions. The proposed model is:⁴

\[ H(t) = H_0(t) \exp \left( X' B \right) \]

where: \( H_0(t) \) is an arbitrary function.
\( X \) is a vector of concomitant information
\( B \) is a vector of parameters

This model is semi-parametric as it depends on the vector \( B \) of regression parameters. However, \( H_0(t) \) is arbitrary and no distributional assumptions are necessary to estimate \( H_0(t) \) or \( B \).⁵
Cox defines the "Relative Hazard Function" as:

\[ \frac{H(t)}{H_0(t)} = \exp \left( X' B \right) \]

The proportional hazard model assumes that the relative hazard function is constant through time. In proportional hazards, the survival distribution for the different X's are related as powers of one another.\(^6\)

\[ S(t) = \exp \left[ - \int_0^t H_0(u) \exp \left( X' B \right) du \right] = \left[ S_0(t) \right] \exp \left( X' B \right) \]

where \(S_0(t)\) is the baseline survival function of the average firm. \(S_0(t)\) corresponds to \(H_0(t)\).

The major advantage of the Cox proportional hazard model approach to survival analysis is that inference can be restricted to the effect of the concomitant information without knowledge of the form of the survival distribution. In other words, we can restrict our analysis to the impact of the observed vector \(X\), the concomitant information, on the hazard or survival functions while not having to estimate the form of the survival distribution.

A problem arising in the use of the Cox model is due to the issue of continuous versus discrete time. The hazard approach is based on continuous time and, therefore, no deaths or failures can occur at the same point in time. However, in practice, one has to use discrete time periods and the problem of "tied" failure time observations arises. The tied observations cause difficulty as the hazard model makes use of the temporal ordering of the observations and,
therefore, cannot be directly applied to the case of tied survival times.

The obvious solution to the problem is to decrease the time period in question in an effort to ensure that only one death occurred per period. However, in many cases this is not feasible or is excessively expensive. Many different techniques have been proposed to deal with the issue of tied observations ranging from evaluating every possible ordering of observations to randomly and arbitrarily breaking the ties.

This study solves the problem of tied observations by measuring survival time in days which, in this sample, results in no tied survival time observations. The time until failure, in this study, was measured, in days, from January 1 of the year in which failure occurred. In other words, if a firm failed on January 5, 19xx, the recorded survival time was 4 days. The impact of the tied observations will be illustrated by estimating the Cox model using a one month time interval which results in many tied observations.

Cox models are usually evaluated in two ways: by testing the reasonability of the proportional hazards assumption and by examining the goodness of fit. The predictive ability of the model is rarely tested.

The assumption of proportional hazards is evaluated by stratifying the overall sample on the basis of a variable, and estimating a hazard function based on each strata. If
proportional hazards is valid, each strata could have its own base line hazard, but the multiplicative portion of the hazard function would be the same across strata. Therefore, if the proportional hazards assumption is supported by the data, one would expect the graph of the log of the underlying cumulative hazard functions or log of -log of the survival function versus time to produce parallel or near parallel lines.\(^\text{10}\)

Goodness-of-fit is evaluated by examining the 'residuals' which for the Cox model are determined by the transformation:\(^\text{11}\)

\[
e_i = \exp \left( \beta' X_i \right) \int_0^{T_i} H_0(u)\,du
\]

The residuals used in the more common regression analysis are defined as the observed value minus the predicted or 'fitted' value. This approach is not valid in the proportional hazard model as the 'fitted' value is a probability distribution while there is no observed distribution. Consequently, one must, in effect, evaluate the 'fitted' cumulative probability of failing before actual death and comparing it to the value of 1, the actual cumulative probability of failing by the observed failure date. These residuals should behave as a random sample drawn from a unit exponential distribution. If the Cox model "fits" the data well, one would expect the graph of the residuals
versus the cumulative hazard of the residuals to yield a straight line with an expected slope of 1.\textsuperscript{12}

The cumulative hazard function was calculated using the formula given in the BMDP Statistical Software appendix.\textsuperscript{13}

\[ H(e) = \sum_{j=1}^{l} \frac{m_i}{#R_j} + (e - e_1) \frac{m_{l+1}}{#R_{l+1}} \]

where:
- \( m_i \) = number of deaths at time \( i \)
- \( #R_j \) = number of survivors at the beginning of period \( j \)
- \( e \) = residuals
- \( e_1 < e < e_{l+1} \)

The Cox proportional hazard model was estimated using the BMDP:2L computer software. This package allows a stepwise procedure using the maximum partial likelihood values to determine entry and exit. Once again, one must bear in mind, the sub-optimality of the step-wise procedure in identifying variables for inclusion.

BMDP:2L estimates the regression coefficients of the Cox model by maximizing the log of the likelihood function using a Newton-Raphson algorithm. The asymptotic covariance matrix is then obtained by inverting \( I(b) \) where \( I(b) \) is the information matrix. The information matrix is the negative of the matrix of second partials of the log likelihood function. (See BMDP Statistical Software, Appendix 31 for a discussion of the estimation techniques used by BMDP:2L) The eigenvalues of the information matrix rather than the more commonly used
correlation matrix should be evaluated in order to determine if multicollinearity is a potential problem. In order to evaluate multicollinearity, both the estimated asymptotic correlation matrix and the common correlation matrix will be presented in order to illustrate the differences between the two.

C. ONE YEAR ESTIMATION

The Proportional Hazard model was estimated using the step-wise procedure available in the BMDP Statistical Software. The overall set of financial ratios evaluated are found in table 2. The results of the Cox estimation are found in table 9. The overall significance of the Cox estimation for the one-year prior data is very good as illustrated by the p-value. The p-value measures the probability of the observed relationship occurring if there is no true relationship. Therefore, we can reject the hypothesis that all the coefficients are zero with some confidence.

As the signs of the coefficients are of great importance, the coefficients were tested to determine if the sign observed was significant by using a one-tailed test. As the degrees of freedom of this t-distribution are 102, approximate p-values will be presented for 60 degrees of freedom due to inadequacies in the t-tables. The results of these tests are presented in table 9. It would appear that at
a significance level of 0.05, each coefficient's sign is significant when evaluated individually.

**Table 9**

**COX MODEL ESTIMATION**

**ONE YEAR PRIOR TO BANKRUPTCY SAMPLE**

LOG LIKELIHOOD = -114.4753
GLOBAL CHI-SQUARE = 99.14 D.F. = 7 P-VALUE = 0.0

<table>
<thead>
<tr>
<th>VARIABLE*</th>
<th>COEFFICIENT</th>
<th>ERROR</th>
<th>COEFF./S.E.</th>
<th>EXP(COEF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAIM1</td>
<td>1.7504</td>
<td>0.8452</td>
<td>2.0710</td>
<td>5.7572</td>
</tr>
<tr>
<td>DELMKT1</td>
<td>-1.5624</td>
<td>0.8917</td>
<td>-1.7522</td>
<td>0.2096</td>
</tr>
<tr>
<td>INTLTD1</td>
<td>0.1738</td>
<td>0.0770</td>
<td>2.2579</td>
<td>1.1899</td>
</tr>
<tr>
<td>NETYSL1</td>
<td>-2.1242</td>
<td>1.3748</td>
<td>-1.5451</td>
<td>0.1195</td>
</tr>
<tr>
<td>CFTD1</td>
<td>-1.7469</td>
<td>0.7184</td>
<td>-2.4315</td>
<td>0.1743</td>
</tr>
<tr>
<td>QKCL1</td>
<td>-1.3441</td>
<td>0.5804</td>
<td>-2.3159</td>
<td>0.2608</td>
</tr>
<tr>
<td>CASLS1</td>
<td>1.5477</td>
<td>0.7804</td>
<td>1.9833</td>
<td>4.7008</td>
</tr>
</tbody>
</table>

**TEST OF SIGN APPROX. P-VALUE.**

<table>
<thead>
<tr>
<th>SIGNIFICANCE</th>
<th>60 DEGREES</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAIM1</td>
<td>.025</td>
</tr>
<tr>
<td>DELMKT1</td>
<td>.05</td>
</tr>
<tr>
<td>INTLTD1</td>
<td>.01</td>
</tr>
<tr>
<td>NETYSL1</td>
<td>.075</td>
</tr>
<tr>
<td>CFTD1</td>
<td>.01</td>
</tr>
<tr>
<td>QKCL1</td>
<td>.01</td>
</tr>
<tr>
<td>CASLS1</td>
<td>.05</td>
</tr>
</tbody>
</table>

* CLAIM1 = Current + longterm debt + pref.shares / Total Assets.
DELMKT1 = Change in Market Value of equity.
INTLTD1 = Interest / long term debt.
NETYSL1 = Net income / Sales.
CFTD1 = Cash flow / Total debt.
QKCL1 = Quick assets / Current liabilities.
CASLS1 = Current Assets / Sales.
Intuitively, the Cox model proposes that a base line hazard level exists for an average firm and any deviation from that average results in a change in the hazard, instantaneous risk of failure, faced by the non-average firm.

\[ H_i(t) = H_0(t) \exp(X_i' B) \]

Consequently, any negative coefficient results in a decrease in the estimated instantaneous risk of failure given failure has not already occurred.

<table>
<thead>
<tr>
<th>Table 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CORRELATION MATRIX</strong> (ONE YEAR PRIOR DATA)</td>
</tr>
<tr>
<td>CLAIM</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>CLAIM</td>
</tr>
<tr>
<td>DELMKT</td>
</tr>
<tr>
<td>INTLTD</td>
</tr>
<tr>
<td>NETYSLS</td>
</tr>
<tr>
<td>CFTD</td>
</tr>
<tr>
<td>QKCL</td>
</tr>
<tr>
<td>CASL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESTIMATED ASYMPTOTIC CORRELATION MATRIX (ONE YEAR PRIOR DATA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLAIM</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>CLAIM</td>
</tr>
<tr>
<td>DELMKT</td>
</tr>
<tr>
<td>INTLTD</td>
</tr>
<tr>
<td>NETYSLS</td>
</tr>
<tr>
<td>CFTD</td>
</tr>
<tr>
<td>QKCL</td>
</tr>
<tr>
<td>CASL</td>
</tr>
</tbody>
</table>

The problem of multicollinearity was evaluated by examining the estimated asymptotic correlation matrix of the
variables. (See table 10) There does not appear to be a problem of excessive collinearity among the variables as most of the correlation coefficients in Table 10 are less than .3 and only two are between .3 and .4. There is quite a remarkable difference between the correlation matrix and the estimated asymptotic correlation matrix. One might have expected multicollinearity on the basis of the correlation matrix as several of the correlation coefficients are above .6, however, when the estimated asymptotic correlation matrix is examined, the apparent collinearity is reduced.

The signs of the coefficients are as expected. One might have expected the sign of the current assets to sales ratio to be negative indicating an improvement in the hazard rate for an increase in the ratio. However, current assets also include the relatively less liquid assets of accounts receivable and inventory. Accounts receivable and inventory are generally less liquid than cash for the simple fact that it takes time to convert these assets into cash.

\[
\frac{\text{current assets}}{\text{sales}} = \frac{\text{cash}}{\text{sales}} + \frac{\text{accounts receivable}}{\text{sales}} + \frac{\text{inventory}}{\text{sales}}
\]

We would expect that as the accounts receivable and inventory to sales ratios increase that the firm's risks of being in financial difficulty would also rise. An increase in these two ratios often reflects an inability to collect on credit sales and or an inability to sell inventory.
According to the efficient markets hypothesis, the firm's stock price, if actively traded, fully reflects the publicly available information. Therefore, we would expect the two market related ratios, market value to debt and change in market value, to be important variables in the identification of bankrupts. As expected, the change in market value is a significant variable in the Cox estimation.

The fact that the change in market value is not the dominant variable is not indicative of a failure on the part of the efficient markets hypothesis. Studies of the stock price behaviour of firms approaching bankruptcy (see Shick and Sherman) indicate that changes in market value significantly precede observable changes in the financial data. As the change in market variable analyzed in this study is the one-year change calculated one year prior to failure, we would expect that the major changes in the stock price would already have occurred or would be masked by the very long time period used.

The proportional hazards assumption was tested using the graphical technique discussed previously. Figure 4 illustrates the plot of log of minus log survival function versus time for the group of firms filing bankruptcy under the Chandler Act and their matches, and the similar group filing under the Reform Act. The results for different strata can be found in Appendix III. These graphs indicate that the proportional hazards assumption is not inappropriate.
The high degree of similarity and overlap in the comparative plot of the Chandler and Reform groups lends support to the results of the univariate test of group differences which found that there did not appear to be a significant difference between the two groups. The stratification of the data in this analysis allows each group to have a different baseline hazard rate; but, the multiplicative part of the hazard function should be the same in each strata. Therefore, if proportional hazards holds and the groups have different base line hazard rates we would expect Kay's parallel lines. As we observe lines that essentially overlap, there is further evidence that there does not exist a significant difference between the Chandler and Reform Act groups.

Goodness-of-fit of the estimated hazard function was evaluated graphically by plotting the residuals against the cumulative hazard of the residuals (see Figure 5). As can be seen in this case the "fit" line was reasonably straight with a slope of approximately 0.30 rather than the expected value of 1. Overall, the goodness-of-fit does not appear to be unreasonable as the "fit" line is relatively straight and the overall regression is significant.

As it is possible that the very high level of significance could be overstated due to an estimation problem and therefore, the model will also be evaluated on the basis of its performance on non-sample data.
Figure 4

**LOG MINUS LOG SURVIVAL FUNCTION**

Cox Proportional Hazard Model
(1 year prior to bankruptcy data)

<table>
<thead>
<tr>
<th>STRATA</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANDLER</td>
<td>A</td>
</tr>
<tr>
<td>REFORM</td>
<td>B</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>70.</td>
</tr>
<tr>
<td>140.</td>
</tr>
<tr>
<td>210.</td>
</tr>
<tr>
<td>280.</td>
</tr>
<tr>
<td>350.</td>
</tr>
</tbody>
</table>
Figure 5

GOODNESS OF FIT
OVERALL SAMPLE, 1 YEAR PRIOR TO FAILURE

R = .9807  
P(R) 0.000

MEAN   ST.DEV.   REGRESSION LINE   RES.MS.
X  .57752   .46155   X= 3.3445*Y-.19973   .00837
Y  .23239   .13534   Y= .28756*X+ .06632  719E-6

XVAR = 2 RESIDU VERSUS YVAR = 1 CUMHAZRD
D. FOUR-YEARS PRIOR ESTIMATION

The Cox proportional hazards model was also estimated on the basis of the four-year prior data set. The results of the estimation are summarized in Table 11 while the goodness-of-fit graphs can be found in figure 6.

<table>
<thead>
<tr>
<th>Table 11.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>COX MODEL ESTIMATION</strong></td>
</tr>
<tr>
<td><strong>4 YEARS PRIOR TO FAILURE</strong></td>
</tr>
<tr>
<td>LOG LIKELIHOOD = -127.9253</td>
</tr>
<tr>
<td>GLOBAL CHI-SQUARE = 42.29 D.F. = 6 P-VALUE = 0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE*</th>
<th>COEFFICIENT</th>
<th>ERROR</th>
<th>COEFF./S.E.</th>
<th>EXP(COEF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NETYTD4</td>
<td>-12.1462</td>
<td>3.0109</td>
<td>-4.0341</td>
<td>0.0000</td>
</tr>
<tr>
<td>INVCTA4</td>
<td>-6.3279</td>
<td>1.4204</td>
<td>-4.4548</td>
<td>0.0018</td>
</tr>
<tr>
<td>NETYSL4</td>
<td>22.4321</td>
<td>6.7332</td>
<td>3.3316</td>
<td>4*10</td>
</tr>
<tr>
<td>TASLS4</td>
<td>1.6977</td>
<td>0.5012</td>
<td>3.3875</td>
<td>5.4615</td>
</tr>
<tr>
<td>DELMKT4</td>
<td>-0.4064</td>
<td>0.2468</td>
<td>-1.6464</td>
<td>0.6660</td>
</tr>
<tr>
<td>QKTA4</td>
<td>-3.0010</td>
<td>1.7569</td>
<td>-1.7082</td>
<td>0.0497</td>
</tr>
</tbody>
</table>

* NETYTD4 = Net income / Total debt.  
  INVCTA4 = Invested capital / Total assets.  
  NETYSL4 = Net income / Sales.  
  TASLS4 = Total assets / Sales.  
  DELMKT4 = Change in market value.  
  QKTA4 = Quick assets / Total assets.

As one would expect, the set of predictors of long term viability are different from those identified by the short term model. However, despite the high significance of the model, several of the coefficients are somewhat unexpected.
It would seem unlikely that increasing net income to total debt would have such a dramatic impact on the risk of failure as the model would suggest. The coefficient of net income to sales is also very surprising. One would normally expect that an increase in net income to sales would indicate increasing efficiency and therefore, should reduce the risk of failure. However, according to the results of the model, this increase net income to sales would actually increase the risk of failure dramatically.

In an effort to identify the source of these "strange" results, the correlation matrix and the estimated asymptotic correlation matrix of these six variables was examined (See table 12). As can be seen from this table there exists considerable correlation between net income to total debt and net income to sales, and to change in market value. However, the largest correlation coefficient values were similar to those in table 10 where there was apparently little multicollinearity. In order to evaluate the possibility of multicollinearity, the estimated asymptotic correlation matrix was also examined.

The estimated asymptotic correlation between net income to sales and net income to total debt is very high, nearly .8 in absolute value. The estimation was, therefore, re-calculated using the five variables remaining after net income to sales was removed. Net income to sales was selected for exclusion because the variable total assets to
sales is included. Therefore, with the new set of five variables, no variables use the same components. The results of this re-estimation are found in table 13.

Table 12.

**CORRELATION MATRIX (FOUR YEARS PRIOR DATA)**

<table>
<thead>
<tr>
<th></th>
<th>NETYTD</th>
<th>INVCTA</th>
<th>NETYSLS</th>
<th>TASLS</th>
<th>DELMKT</th>
<th>QKTA</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
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<tr>
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<td>.154</td>
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<tr>
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<td>.156</td>
<td>.412</td>
<td>.030</td>
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<td>.069</td>
<td>.036</td>
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**ESTIMATED ASYMPTOTIC CORRELATION MATRIX (FOUR YEARS PRIOR DATA)**

<table>
<thead>
<tr>
<th></th>
<th>NETYTD</th>
<th>INVCTA</th>
<th>NETYSLS</th>
<th>TASLS</th>
<th>DELMKT</th>
<th>QKTA</th>
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<td></td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-.783</td>
<td>-.408</td>
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<td></td>
</tr>
<tr>
<td>TASLS</td>
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<td>-.176</td>
<td>.076</td>
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<td></td>
</tr>
<tr>
<td>DELMKT</td>
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<td>.016</td>
<td>-.243</td>
<td>-.213</td>
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<td></td>
</tr>
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<td>QKTA</td>
<td>.050</td>
<td>.332</td>
<td>-.043</td>
<td>-.139</td>
<td>.074</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 13.  

COX MODEL, RE-ESTIMATION  

4 YEARS PRIOR TO FAILURE  

LOG LIKELIHOOD = -153.2074  
GLOBAL CHI-SQUARE = 36.90  D.F. = 5  P-VALUE = 0.0000  

<table>
<thead>
<tr>
<th>VARIABLE*</th>
<th>COEFFICIENT</th>
<th>ERROR</th>
<th>COEFF./S.E.</th>
<th>EXP(COEF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NETYTD4</td>
<td>-3.7497</td>
<td>2.0162</td>
<td>-1.8598</td>
<td>0.0235</td>
</tr>
<tr>
<td>INVCTA4</td>
<td>-3.7199</td>
<td>1.2784</td>
<td>-2.9099</td>
<td>0.0242</td>
</tr>
<tr>
<td>TASLS4</td>
<td>1.3512</td>
<td>0.5041</td>
<td>2.6804</td>
<td>3.8622</td>
</tr>
<tr>
<td>DELMKT4</td>
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<td>0.2248</td>
<td>-1.2453</td>
<td>0.7558</td>
</tr>
<tr>
<td>QKTA4</td>
<td>-2.8003</td>
<td>1.6581</td>
<td>-1.6889</td>
<td>0.0608</td>
</tr>
</tbody>
</table>

NETYTD4 = Net income / Total debt.  
INVCTA4 = Invested capital / Total assets.  
TASLS4 = Total assets / Sales.  
DELMKT4 = Change in market value.  
QKTA4 = Quick assets / Total assets.  

With the variable net income to sales omitted, the variables now have the expected signs and the impact of a change in each variable is more reasonable. When the goodness-of-fit was evaluated graphically, there was little difference from figure 6. Therefore, it would appear that collinearity between the variables has a serious impact on the coefficients of those variables but a relatively minor impact on the "fit" of the model.
Figure 6

GOODNESS OF FIT 4 YEAR PRIOR

+...................XY....................+

.45 +

11

.36 +

1

C

11

M

11

H

11

A

1

Z

2

R

D

.09 +

Y 11

X 1

-11

0.0 +

.0 +

0.0 .30 .60 .90 1.2 1.5

RESIDU

R = .9720
P(R) 0.000

MEAN ST.DEV. REGRESSION LINE RES.MS.
X .46036 .37587 X = 2.7870*Y -.19243 .00802
Y .23423 .13108 Y = .33898*X + .07818 976E-6
E. TIED OBSERVATIONS

The impact of tied observations, firms with identical times until bankruptcy, on the Cox proportional hazards model was examined by estimating the model on the same sample but now using time to failure measured in months. This change in the time period resulted in most of the observed times until failure being tied. The estimation was carried out using the BMDP statistical package with no correction factors for the tied observations. No corrections were used in order to simulate the extreme case caused by the tied observations in order to compare these results to those of the "correct" no-ties model. Any reasonable corrections would, therefore, result in estimates which would fall between these two extremes. The results of this uncorrected tied observations estimation are presented in Table 14.

Once again, the regression is highly significant and the variable coefficients all have the expected sign, however, the variables identified by the stepwise procedure are very different from those identified in the more correct model with out ties in the observations. The proportionality assumption was also evaluated, and showed that the hazard proportionality assumption was not unreasonable.

The problem caused by the many tied observations becomes apparent when one examines the goodness-of-fit plot in figure 7. As can be seen in this graph, the "fit" line is far from
being linear and in fact appears to be two lines rather than one.

Table 14.
"TIED" OBSERVATIONS
RESULTS OF COX ESTIMATION
(1 year prior)
LOG LIKELIHOOD = -141.6433
GLOBAL CHI-SQUARE = 76.09 D.F. = 6 P-VALUE = 0.0000

<table>
<thead>
<tr>
<th>VARIABLE*</th>
<th>COEFFICIENT</th>
<th>STANDARD ERROR</th>
<th>COEFF./S.E.</th>
<th>EXP(COEFF.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFTD1</td>
<td>-1.8776</td>
<td>0.6206</td>
<td>-3.0255</td>
<td>0.1529</td>
</tr>
<tr>
<td>CACL1</td>
<td>-0.9892</td>
<td>0.3075</td>
<td>-3.2172</td>
<td>0.3719</td>
</tr>
<tr>
<td>INVSLS1</td>
<td>4.7322</td>
<td>1.0407</td>
<td>4.5470</td>
<td>113.5419</td>
</tr>
<tr>
<td>EBIT1</td>
<td>-4.6722</td>
<td>1.5361</td>
<td>-3.0417</td>
<td>0.0094</td>
</tr>
<tr>
<td>DELMKTL1</td>
<td>-1.8206</td>
<td>0.9886</td>
<td>-1.8416</td>
<td>0.1619</td>
</tr>
<tr>
<td>DELMRL1</td>
<td>4.4704</td>
<td>2.2057</td>
<td>2.0268</td>
<td>87.3882</td>
</tr>
</tbody>
</table>

CFTD1 = Cash flow / total debt. CACL1 = Current Ratio
INVSLS1 = Inventory / sales EBIT1 = Earnings /assets
DELMKTL1 = Change in market value DELMRL1 = Margin change

When compared to the no-ties model, the tied model has resulted in a very distorted estimation. The variables identified by the stepwise procedure and the goodness-of-fit results indicate that tied observations can cause very distorted estimates and poorly fitting estimated hazard functions.

It would therefore, seem that the Cox model, when corrected for tied observations, appears to model the interactions between different components of the firms financial structure as they relate to bankruptcy. The model
identifies financial variables which one would intuitively expect to have an effect on short term viability in the one year prior case, while also identifying more long-term survival related ratios in the four-year prior case.

The adequacy of the Cox proportional hazards model survival function estimates will be examined and compared to the results of the discriminant analysis.
Figure 7.

GOODNESS OF FIT, TIED OBSERVATIONS

```
5.6 +
-
-
11 12 1 1 1 1 1
-
-
4.9 +
-
-
2 1 1
-
-
4.2 +
-
-
1
-
-
3.5 + 2
-
-
1
C
-
-
U
-
- M
-
-
A 2.8 +
-
-
Z
-
-
R
-
-
D
-
-
2.1 + 4
-
-
1
-
-
1.4 +
-
-
-
0.70 +
-
-
Y
-
-
0.0 +Q77526 2121 1111 11
-
.+X+
0.0 .40 1.2 2.0
RESIDU
```
VIII. MODEL COMPARISON

The performance of the Cox model was evaluated using two criteria: first, accuracy of classification or prediction and secondly, descriptive power. The baseline level of performance was that of the discriminant analysis model.

The classification and prediction capability of the discriminant and Cox models was evaluated by applying the estimated one year prior functions to the data from four, three, two and one year prior to failure. The results are summarized in table 15. As the one year prior data set was used to develop the models, the results from that time period cannot be used to make any inferences about the predictive power of the models. The one year prior results are presented as they help illustrate the improvement in model classification as we move closer to failure.

Unlike discriminant analysis, the Cox model does not lend itself well to the problem of classifying firms as bankrupt or non-bankrupt as one is presented with the problem of how to classify a specific expected time to failure as indicative of bankruptcy or non-bankruptcy. In this paper, a firm was classified as bankrupt if its expected time until failure was less than 350 days. The value of 350 days are arbitrarily chosen as all but one of the bankrupt firms had filed within 350 days of the end of the prior period.
Table 15 (a)

MODEL PERFORMANCE

DISCRIMINANT ANALYSIS:

<table>
<thead>
<tr>
<th>Years prior</th>
<th>Classification</th>
<th>Survivor</th>
<th>Bankrupt</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>66</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Survivor</td>
<td>31</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Bankrupt</td>
<td>97</td>
<td>12</td>
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<tr>
<td>3</td>
<td></td>
<td>62</td>
<td>6</td>
</tr>
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<td></td>
<td>Survivor</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Bankrupt</td>
<td>89</td>
<td>20</td>
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<tr>
<td>2</td>
<td></td>
<td>63</td>
<td>5</td>
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<tr>
<td></td>
<td>Survivor</td>
<td>15</td>
<td>26</td>
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<tr>
<td></td>
<td>Bankrupt</td>
<td>78</td>
<td>31</td>
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<tr>
<td>1</td>
<td></td>
<td>61</td>
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</tr>
<tr>
<td></td>
<td>Survivor</td>
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</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>
Table 15 (b)

**MODEL PERFORMANCE**

**COX MODEL:**

<table>
<thead>
<tr>
<th>Years prior</th>
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<th>Bankrupt</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>57</td>
<td>11</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>78</td>
<td>31</td>
<td>109</td>
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</table>

<table>
<thead>
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<th>Years prior</th>
<th>Survivor</th>
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<th>Total</th>
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<tbody>
<tr>
<td>3</td>
<td>60</td>
<td>8</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>22</td>
<td>41</td>
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<tr>
<td></td>
<td>79</td>
<td>30</td>
<td>109</td>
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<table>
<thead>
<tr>
<th>Years prior</th>
<th>Survivor</th>
<th>Bankrupt</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>58</td>
<td>10</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>27</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>37</td>
<td>109</td>
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<table>
<thead>
<tr>
<th>Years prior</th>
<th>Survivor</th>
<th>Bankrupt</th>
<th>Total</th>
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<tbody>
<tr>
<td>1</td>
<td>57</td>
<td>11</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>38</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>49</td>
<td>109</td>
</tr>
</tbody>
</table>

As can be seen from table 15 (a,b), the ability of both models to foretell bankruptcy declined as the time until failure increased. It would appear that both the discriminant and Cox models predict that bankruptcy will occur before the
actual event occurs. In other words, these models appear to underestimate the time until bankruptcy. For example, discriminant analysis classified 14 future bankrupts as bankrupt, 4 years prior to the petition being filed while the Cox model gave expected times until failure of less than 350 days to 20 firms 4 years before they filed.

The filing of a bankruptcy petition is usually the last stage in a firm's death throes. Therefore, we would expect that the true failure of the firm had occurred at some point before the petition was filed. Consequently, it is not surprising that both the discriminant and Cox models tend to identify bankrupts before they become legal bankrupts.

In order to assess the descriptive power of the Cox model a random sample of four firms (two bankrupt and two survivors) was drawn from the original sample. The survival function of each firm was then plotted against time. In order to see the changes in the survival function over time and to evaluate the validity of the estimated proportional hazard model on data outside the data set used to estimated the model, a function was calculated at three, two and one year prior to failure using the Cox one year prior estimated coefficients.

These graphical descriptions, found in figures 8 through 11 in Appendix I, of the firms' survival functions were obtained by plotting the firms probability of surviving at least as long as 't' against 't' for each year prior to
failure. One would expect the survival functions of firms which were non-failures at the time of analysis to be close to one for any 't' as these firms would have a very high probability of surviving at least as long as 't'. In contrast, one would expect the survival plot of bankrupt firms to decline as 't' increases, and perhaps to decline faster the closer 't' was to the time of actual failure.

In general, these graphs exhibit the expected results. They give one a "picture" of how a firm's probability of surviving changes over time. The discriminant analysis, in contrast, merely gives one a score which indicates probable group membership.

The most unexpected result was that of firm number 297882 (figure 11). As this firm is one of the survivors, one would not expect to see survival functions exhibiting this type of behaviour which is very similar to what one would expect of a failure. ETZ Lavud, firm number 297882, is an Israeli firm which is listed on the American Stock Exchange. The survival functions presented here represent the early to mid 1970's, and it is perhaps significant that ETZ Lavud was only listed on the AMEX in 1972. Further examination failed to reveal any other possible reasons for this somewhat unexpected survival function behaviour.

The results of applying the 1-year prior linear discriminant function to these firms are presented in table 16.
It would therefore appear, in the case of these four firms, that the Cox model and the discriminant model yield very similar results. In both models, the future bankrupt firms did not appear to be in the bankruptcy group until one year prior to failure. The Cox model's graphs show a decline in the probability of survival beginning usually in the second year prior to filing the bankruptcy petition.

Both the Cox and Discriminant models show that firm #297882 was in trouble over the time period in question. Discriminant analysis classifies it as a member of the bankrupt group in all three prior periods, and, the survival function graphs show that the firm was in trouble over those same years.

Table 16

<table>
<thead>
<tr>
<th>Firm:</th>
<th>#042078</th>
<th>#564402</th>
<th>#868168</th>
<th>#297882</th>
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<tbody>
<tr>
<td>Three years prior</td>
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<tr>
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<td>7.26</td>
<td>3.09</td>
<td>2.43</td>
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<tr>
<td>Bankrupt Score</td>
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<td>-3.15</td>
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</tr>
<tr>
<td>Two years prior</td>
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<td></td>
</tr>
<tr>
<td>Survivor Score</td>
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<td>1.46</td>
<td>2.91</td>
<td>10.38</td>
</tr>
<tr>
<td>Bankrupt Score</td>
<td>6.70</td>
<td>1.04</td>
<td>-2.56</td>
<td>11.19</td>
</tr>
<tr>
<td>One year prior</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Survivor Score</td>
<td>4.01</td>
<td>2.64</td>
<td>4.89</td>
<td>11.17</td>
</tr>
<tr>
<td>Bankrupt Score</td>
<td>9.50</td>
<td>6.01</td>
<td>1.33</td>
<td>12.24</td>
</tr>
</tbody>
</table>

Classify observation into whichever group's score is the highest.
Therefore, it would appear that the Cox model is able to provide information that the linear discriminant model cannot. Essentially the discriminant model provides a single benchmark which can be used to classify firms as either bankrupt, non-bankrupt, or indeterminate. The Cox model in contrast, provides a probability distribution of the time until failure from which an expected time until failure can be derived.

For example, ETZ Lavud, a survivor, was identified by both models as a problem. According to the discriminant analysis, the firm is most probably a bankrupt. However, the Cox estimation finds that at the beginning of the one year prior period, the company had a probability of surviving longer than 353 days of over 20%. The company went on to survive for at least ten more years.
IX. SUMMARY AND CONCLUSION

This paper has evaluated the Cox proportional hazard model's applicability to the problem of bankruptcy prediction. The Cox model offers several advantages over the more traditional prediction models. This model does not depend on the very restrictive distributional assumptions of multivariate normality and it produces an estimated probability distribution of times until failure from which an expected time of failure can be determined. In general, most traditional models of failure only estimate a probability of failure within a given time period.

Two major areas of concern in the application of the Cox model are multicollinearity and "tied" survival time observations. Multicollinearity appears to result in severely distorted estimated coefficients, but, appears to have a relatively minor affect on the overall "fit" of the model.

"Tied" observations in contrast cannot be handled by the Cox model as the model makes use of the temporal ordering of events in its estimation procedure. Unlike multicollinearity, an incorrect treatment of "tied" observations results in a severely distorted model. Consequently, any application of the model must take into account these potential problems.

In this paper, the Cox proportional hazard model was applied to a sample of 109 firms, 41 of whom filed petitions for bankruptcy between 1972 and 1985. As this time period included a major change to the bankruptcy act, the two
groups of bankrupt firms were examined to test the hypothesis that no difference existed between firms filing under the two bankruptcy acts. The results of the Kruskal-Wallis tests and the stratified Cox model tests, indicate that there does not appear to be a significant difference between the two groups of bankrupts.

Discriminant analysis was performed on the bankrupt and non-bankrupt groups to establish a base line level of performance. If the Cox model is of value, it must offer something beyond what is already available.

The Cox model was estimated for one year and four years prior to failure. The degree of fit and the proportional hazards assumption appeared to be satisfied. In both the one and four year models, the identified variables had the sign one would intuitively expect. The longer term model (4 year prior) identified different variables from the short term model as being important factors in the survival experience. As one would expect, the proportional hazards model estimated in this more diverse industrial setting did not perform as well as the model estimated by Lane, Looney, and Wansley [1986]. This decline in performance is not surprising as the Lane study evaluated performance in a very narrow industrial definition, only the Federal Deposit Insurance Corporation insured banks were evaluated where the assumption of a 'representative' or average hazard rate implicit in the proportional hazards model would seem more plausible.
The conclusions drawn from this analysis are that if the sole purpose of the model is to classify firms as potential bankrupts, then the discriminant analysis based model is more appropriate. The classification performance of the discriminant analysis based model was shown to be superior in all periods to that of the Cox model. This result is not surprising as certain very restrictive assumptions were made in evaluating the classification ability of the Cox model.

It was shown that the use of the Cox model primarily for classification purposes does not exploit the model's full potential.

Unlike the discriminant analysis model which yields a single score and a classification of either survivor, bankrupt or indeterminate, the estimated Cox model produces the distribution of expected times until failure. These survival functions can give a very interesting and a useful picture of the firm's survival potential over a period of time.

The conclusion drawn from this analysis is that the Cox model is applicable in the case of commercial and industrial bankruptcy. The model provides additional and useful information to that provided by the commonly used discriminant analysis based models.

The intuitive appeal of the Cox model, its performance and applicability to bankruptcy prediction, indicates that further research is warranted.
FOOTNOTES

I. INTRODUCTION


II. BANKRUPTCY


2 Platt, p. 8.


4 Bankruptcy Commission, p 63.


6 Bankruptcy Commission, p. 183.

7 Bankruptcy Commission, p. 262.

8 Mulder, pp. 16-19.

9 Mulder, p. 16.

10 Mulder, p. 21.

11 Mulder, pp. 22-29.

12 Mulder, p. 143.

13 Bankruptcy Commission, pp. 243-244.

14 Bankruptcy Commission, p. 244.

15 Bankruptcy Commission, p. 245.

16 Bankruptcy Commission, p. 244.

17 Bankruptcy Commission, pp. 244-245.
III. MODELS COMMONLY USED


5 Altman, "Financial Ratios. ..".

6 Altman, "Financial Ratios. ..".
IV. PLAN OF ANALYSIS

1 This classification scheme is based on Beaver, p. 78.


V. SAMPLE CHARACTERISTICS


VI. DISCRIMINANT ANALYSIS


3 Joy and Tollefson, 1975.


5 Lachenbruch, p. 33.

6 Lachenbruch, p. 33.

7 Altman, *Distress*, p. 106.

VII. THE COX MODEL


2 Carter et al, p. 40.


4 Carter et al, p. 48.
5 Lane et al., p. 514.

6 Carter et al., p. 49.


8 Elandt-Johnson and Johnson, p. 8.

9 Lane et al., p. 530.

10 Kay, p. 231.

11 Kay, p. 230.

12 Kay, p. 230-231.

BIBLIOGRAPHY


APPENDIX I

Figure 8

PLOT OF SURVIVAL FUNCTION

FIRM: CUSIP = 042078  (BANKRUPT)

<table>
<thead>
<tr>
<th>PATTERN</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
</tr>
</tbody>
</table>

ESTIMATED SURVIVAL FUNCTION

```
1.0 +**********CCCCCCCCCCCCCCCCCCCCCCCCCCCCC +
    - A BBBB
    - A BBBB
    - AA BBB
    - A BB

.80 + A BB
    - A
    - A
    - A
    - A

.60 + A AA
    - A
    - A

.40 + A AA
    - A

.20 + AA
    - A
    - AA

0.0 + AAAAAA
```

DAYS

0.0  70. 140  210  280  350
Figure 9

PLOT OF SURVIVAL FUNCTION
FIRM: CUSIP = 564402 (BANKRUPT)

<table>
<thead>
<tr>
<th>PATTERN</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
</tr>
</tbody>
</table>

ESTIMATED SURVIVAL FUNCTION
**Figure 10**

**PLOT OF SURVIVAL FUNCTION**

**FIRM: CUSIP = 868168 (SURVIVOR)**

<table>
<thead>
<tr>
<th>PATTERN</th>
<th>SYMBOL</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>ONE YEAR PRIOR TO WITHDRAWAL</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>TWO YEARS PRIOR TO WITHDRAWAL</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>THREE YEARS PRIOR TO WITHDRAWAL</td>
</tr>
</tbody>
</table>

**ESTIMATED SURVIVAL FUNCTION**

```
0.0 +.........................................................+..+
    0.210 350

DAYS
```
Figure 11

PLOT OF SURVIVAL FUNCTION

FIRM: CUSIP = 297882 (SURVIVOR)

<table>
<thead>
<tr>
<th>PATTERN</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
</tr>
</tbody>
</table>

ESTIMATED SURVIVAL FUNCTION

```
          +--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+--------+        
```
APPENDIX II.

Figure 12

NORMAL PLOT OF CURRENT ASSETS / SALES (ONE YEAR PRIOR)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COUNT</th>
<th>MEAN</th>
<th>ST.DEV.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>109</td>
<td>0.411</td>
<td>0.192</td>
</tr>
</tbody>
</table>

VALUES FROM NORMAL DISTRIBUTION WOULD LIE ON THE LINE INDICATED BY THE SYMBOL / .
Figure 13

NORMAL PLOT OF WORKING CAPITAL / TOTAL ASSETS
(FOUR YEARS PRIOR)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COUNT</th>
<th>MEAN</th>
<th>ST.DEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>109</td>
<td>0.323</td>
<td>0.138</td>
</tr>
</tbody>
</table>

WCTA4
VALUES FROM NORMAL DISTRIBUTION WOULD LIE ON THE LINE INDICATED BY THE SYMBOL / .
### Figure 14

**NORMAL PLOT OF WORKING CAPITAL / TOTAL ASSETS (ONE YEAR PRIOR)**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COUNT</th>
<th>MEAN</th>
<th>ST.DEV.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>109</td>
<td>0.272</td>
<td>0.226</td>
</tr>
</tbody>
</table>

WCTA1

VALUES FROM NORMAL DISTRIBUTION WOULD LIE ON THE LINE INDICATED BY THE SYMBOL / .
Figure 15

NORMAL PLOT OF NET INCOME / SALES
(FOUR YEARS PRIOR)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COUNT</th>
<th>MEAN</th>
<th>ST.DEV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>109</td>
<td>0.019</td>
<td>0.037</td>
</tr>
</tbody>
</table>

VALUES FROM NORMAL DISTRIBUTION WOULD LIE ON THE LINE INDICATED BY THE SYMBOL / .
NORMAL PLOT OF NET INCOME / SALES
(ONE YEAR PRIOR)

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COUNT</th>
<th>MEAN</th>
<th>ST.DEV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>109</td>
<td>-0.019</td>
<td>0.096</td>
</tr>
</tbody>
</table>

NETYSLL
VALUES FROM NORMAL DISTRIBUTION WOULD LIE
ON THE LINE INDICATED BY THE SYMBOL / .
APPENDIX III

Figure 17

LOG MINUS LOG SURVIVAL FUNCTION

Stratified on: Cash flow / total debt two years prior

```
-2 + ++ AAAA +
- - AAAA AAA BB
- - A AAAA BBB
- - A BB
- - A BBB
- - AAAA BB

-3 + AAAA +
- - AAAAA BB
- - A BBB
- - A BB
- - AAAA BB

-4 + A BB +
- - A B
- - A B
- - A BBB
- - A BB
- - A BB

-5 + AA B +
- - AA BB
- - A B
- - A B
- - A BBB
- - A BB
- - A BB

-6 + A B +
- - A B
- - A B
- - A B
- - A B
- - A B
- - A BB

-7 + A B +
- - A BB
- - A
```

0.0  70.  140  210  280  350  DAYS
Figure 18

LOG MINUS LOG SURVIVAL FUNCTION

Stratified on: Net income / sales (two years prior)

0 + ........ + ........ + ........ + ........ + ........ + ........ + ........ +

-3 + AAAA BBBBBBB +

-6 + ABB +

-9 +

0.0 70. 140 210 280 350

DAYS
Figure 19

LOG MINUS LOG SURVIVAL FUNCTION

Stratified on: Total claims / Assets (two years prior)

-2
-3
-4
-5
-6
-7

+........+........+........+........+........+........+........+........+........+........+........+

-2 + B A +

-3 + B A*

-4 + BBBBBB**B BBBBBB AA

-5 + A**AA

-6 + A B

-7 + * B

+................+................+................+................+................+................+

0.0  70.  140  210  280  350
DAYS