AN EMPIRICAL EXAMINATION:
MANAGING MORTGAGE PAYMENT RISK WITH OPTIONS

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ABSTRACT

Canadian real estate investors who use variable rate mortgage financing assume a great deal of risk with regard to the cash flows resulting from the mortgage. These investors may wish to reduce the risk of rising mortgage payments without giving up the opportunity to benefit from lower mortgage payments. The introduction in the U.S. of trading in T-bond futures options and Canadian dollar futures options may allow investors to do this. The hypothesis of this thesis is that a real estate investor using variable rate mortgage financing can use T-bond futures options, possibly in conjunction with Canadian dollar futures options, to effectively hedge against the risk of rising mortgage payments.

Chapters 2 and 3 examine the basics about futures and options, respectively. In Chapter 4, the duration based approach to hedging (i.e. risk reduction) with T-bond futures (and options) is explained. The rationale for the use of Canadian dollar futures (and options) is detailed in Chapter 5. Their inclusion in the hedge portfolio is based upon the interest rate parity theorem.

A thorough literature review was performed. Chapter 6 contains a summary of the relevant theories exploring reasons why hedging may be beneficial. Empirical studies have been confined to the use of futures, rather than options. These are summarized in Chapter 7.

To determine the risk reduction potential of T-bond and Canadian dollar futures options, I examined the relationship between changes in the price of these options and the present value of changes in mortgage payments over the term of the mortgage. An indirect test was used, based on regressions of changes in the price of the futures underlying the T-bond and Canadian dollar futures options. The regressions used data from 1979 to 1986. They showed that the use of T-bond futures would have significantly reduced risk over the period tested. The inclusion of Canadian dollar futures in the hedged portfolio was beneficial in all cases. However, even with their addition, mortgage payment risk cannot be completely eliminated using futures (or options on these futures). The hedge was shown to perform much better when there are large movements in mortgage payments than when these movements are small.

In Chapter 10, I examined how well an option hedging strategy using T-bond futures options would have done for the period of rising mortgage rates in 1984. This has been the only period of rising rates since these options began trading in late 1982. The results indicate that these options would have reduced mortgage payment risk significantly. The amount of risk reduction is greater than that evidenced by the regressions of futures prices.

The costs and benefits of an options hedging strategy are explored in Chapter 11. The interest savings of a variable rate mortgage over a 5 year fixed rate mortgage greatly outweigh transaction costs.

In the concluding chapter, the hypothesis is rejected. Although T-bond and Canadian dollar futures options do reduce a portion of a real estate investor's mortgage payment risk, they do not reduce it sufficiently to be a viable alternative to fixed rate mortgage financing.

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CHAPTER 1: INTRODUCTION

In the last decade new tools have been created which may be of great help to risk managers. One of the most promising developments in this area was the introduction of options on Treasury bond (T-bond) futures by the Chicago Board Of Trade (CBOT). The prices of these options vary systematically with long term interest rates in the United States. Thus they may provide the opportunity to offset adverse interest rate movements. How they do so will be explained later in the paper.

This study addresses the risk reduction potential of the above mentioned options from the point of view of a Canadian real estate investor holding a mortgage. To illustrate, consider the investor facing two financing alternatives. His first alternative is to take out a five year, fixed rate, interest only mortgage from a chartered bank. The second is to take out a three or six month, variable rate, interest only mortgage from a chartered bank. At the end of the term, the variable rate loan is refinanced with a five year mortgage with the same terms as that in the first alternative. The key difference between these alternatives is interest rate risk.

Interest Rate Risk:

Interest rate risk is the risk that a change in interest rates will create a change in an investor's cash flows over time. In this study, it is assumed that the chief source of interest rate risk for a real estate investor is the risk that mortgage payments will fluctuate over the term of the mortgage. Of course, changes in interest rates may affect an investor's cash flow in other ways, but these are beyond the scope of this paper. Thus there is no interest rate risk with the fixed rate loan. However, with the variable rate loan, the investor assumes all the interest rate risk. If rates rise significantly over the term of the variable rate loan, mortgage payments will increase and the investor's cash flows will suffer. He will have to pay higher mortgage payments than planned for, and may risk insolvency if the rise in payments is sufficiently large. Of course, if rates fall, the investor stands to gain as a result of lower mortgage payments. In addition, mortgage rates are usually lower for variable rate mortgages under normal conditions of a positively sloped yield curve. This is discussed further in chapter 11.

Hedging with T-bond Options:

The introduction of trading in options on U.S. T-bond futures may allow investors to guard against the risk of rising mortgage payments when a variable rate mortgage is taken out. With options, the investor can enjoy lower mortgage payments should rates fall. This is in sharp contrast to hedging with futures, where mortgage payment increases and decreases are offset by gains and losses on the futures contract(s). This results in a constant portfolio value, eliminating the possibility of increased profits should mortgage rates fall. Thus for some real estate investors an option hedging strategy will be preferred to one using futures.

The use of options to protect real estate investors against increases in mortgage rates is one example of hedging. This thesis will examine what hedging is and why it may be beneficial to real estate investors. The strategy to be examined involves the purchase of T-bond futures options, and possibly Canadian dollar futures options (see Chapter 4). The focus of this paper is the hedged portfolio containing these options and the variable rate mortgage. To properly use such a strategy the optimal number of option contracts to be included in the portfolio must be determined.

The best way to determine both the effectiveness of such a strategy and the optimal number of option contracts is to examine a portfolio containing the futures contracts upon which the T-bond and Canadian dollar futures options are based (the underlying futures) instead of the options themselves. A perfect hedge will result in a constant portfolio value. This is because gains or losses on the mortgage are perfectly offset by losses and gains on the futures contracts. A portfolio with futures is examined rather than one with options because of the statistical difficulty of testing the options directly. In addition, these options began trading very recently, while the futures have been traded for a number of years. Finally, the large volume of trading ensures that the options will be fairly priced relative to the underlying future.

The empirical tests are based upon data beginning in October, 1979 and ending in July, 1986. A case study will also be included. This will detail how the option strategy would have worked during the period of rising mortgage rates between February and July of 1984. Finally, there is a discussion of the costs and benefits of such a strategy.

Hypothesis of Thesis:

A real estate investor using variable rate mortgage financing can use T-bond futures options, possibly in conjunction with Canadian dollar futures options, to effectively hedge against the risk of rising mortgage payments.

CHAPTER 2: FINANCIAL FUTURES:

A financial futures contract is a legally binding commitment to make or take delivery of a given quantity of financial instruments (such as fixed income securities or foreign currencies) at a given price. For T-bond futures each contract represents US T-bonds with a face value of $100,000 and a coupon rate of 8%. The trading unit for the Canadian dollar futures contract is 100,000 Canadian dollars. If the commitment is to take delivery, then the investor gains when the futures price rises, since the contract entitles him to buy the financial instrument at less than the market price. This is known as buying the future or taking a long position. Conversely, if the commitment is to make delivery, the investor will gain if the futures price falls, since he can sell the financial instrument for more than the market price. This is known as selling the future or taking a short position.

Delivery of the financial instruments rarely takes place, as most financial futures market participants simply make an offsetting transaction to close out their position. Thus an investor who shorts a futures can close out his position by buying back the future. If he buys it back at a lower price than when he took the short position, he will have made money, and vice versa.

Margin Requirements:

A futures contract is just that, a contract. The exchange does not take place until the maturity date of the futures contract. However, cash inflows and outflows will occur as a result of the margin requirement put forth by the exchange. The futures exchange guarantees that all futures commitments will be honoured. In return for this guarantee, the exchange requires that participants deposit a certain amount of money per contract with the securities dealer (who is a member of the exchange). This money is known as the initial margin. There is an opportunity cost to this margin, but this may be reduced by substituting Treasury bills for cash (so that interest is earned on the margin).

Futures contracts are settled at the end of each day. This means that if the value of the contract (i.e. the investor's gain or loss on it) entered into by an investor has decreased (i.e. if the price falls for those with a long position or rises for those with a short position), the investor must make additional deposits of cash. If the contract increases in value, the investor is given this increase in cash. The funds used are those deposited by those who suffered losses that day.

The value of any interest rate future will fall when rates rise and vice versa, since the future is written on the price of the financial instrument. Thus a real estate investor concerned over rising interest rates should take a short position in the financial future whose price changes are most highly correlated with changes in the value of mortgage payments.

Hedging and Risk

There are several sources of risk in hedging using futures. The first is the risk that the difference between the spot price of the financial instrument underlying the future, and the futures price itself, will vary over time. This is known as basis risk. The basis is the difference in price between the spot instrument and the future. If the future is held until expiry then basis risk is eliminated, since the futures price at expiry will be equal to the spot price.

Another source of risk arises from the fact that the Canadian mortgage is not the instrument underlying the future. Thus there exists instrument risk. This is the risk that proportional changes in the value of the mortgage will not equal those in the price of the bond underlying the future. Instrument risk may be quite large in the case of hedging Canadian mortgage values with U.S. traded T-bond futures. This is because mortgage rates are not only a function of the general level of interest rates, but also of default risk and supply and demand conditions in the mortgage market.

There is an additional source of risk, but it should be very small in most hedges. The T-bond future contract does not have an actual underlying bond, only a theoretical 15 year, 8% coupon bond. The exchange then specifies which bonds are deliverable, and how they are to be valued. Deliverable bonds must have a maturity date (or call date if callable) at least 15 years from the contract date. The value of each bond is determined using duration (see chapter 3) calculated for an 8% yield. Because changes in yield affect duration, and because of the non-zero slope of the yield curve, some bonds become cheaper than others to deliver. Usually there are one or two issues which are cheapest to deliver. Thus the yield on the deliverable bonds is typically less than the yield would be on a 15 year 8% coupon bond. However, changes in this discount over time should not be large enough to significantly affect risk.

CHAPTER 3: OPTION THEORY

An option is the right to buy or sell some asset at/by a specified date at a specified price, called the strike price. A call option creates the right to buy and a put option the right to sell the asset (known as the "underlying" asset). Note that an option confers a right and not an obligation. Thus an option can never go below zero in value.

An option's exerciseable value is positive when it allows you to sell above the market price or buy below it. When an option has such intrinsic value it is said to be "in-the-money". Such is the case when a put option carries a strike price above the market price of the underlying asset or when a call option carries a strike price below it. The option has value as long as there is some positive probability of it being in-the- money. When exerciseable value is negative it is said to be "out-of-the-money", and when it is zero, the option is "at-the-money" (i.e. the strike price equals the market price of the underlying asset).

The ability to exercise an option prior to its maturity date defines the American option, as opposed to the European option, which can only be exercised at maturity. Thus American options are always worth at least as much as their European counterparts, as they also contain the 'option' to exercise at maturity. However, because in most cases an option is worth more if not exercised, American options are only worth more in a few situations. These include put options which are deep in-the-money and in-the-money call options on coupon or dividend paying securities. All option contracts examined in this thesis are American options, as are most of the exchange traded options in North America.

There are many financial futures options available in the highly active U.S. market. There are three interest rate futures options: eurodollars (like T-bills but backed by European banks), 10 year T-notes, and T-bonds. The strike prices for the T-bond futures options are in dollars per hundred dollar par value, and are spaced in 2 dollar increments. There are also five foreign currency futures options: Canadian dollars, British pounds, West German marks, Swiss francs, and Japanese Yen. The strike prices of the Canadian dollar futures options are in U.S. dollars per Canadian dollar, and are spaced in 1 cent increments.

Using options as a hedging strategy is analogous to buying an insurance policy (if the hedge is effective). The strategy pays off if there is an adverse move in the hedged asset, while there is no penalty if the asset price moves favourably. By choosing the appropriate strike price you can set the price at which the option 'pays off'. For example, look at the case of hedging the risk of increased cash outflows resulting from increases in mortgage rates. It is a two state world (time t_0 and t_1) with the options expiring at t_1. Assume that proportional changes in the value of the increased mortgage payments are perfectly negatively correlated with proportional changes in the T-bond future price. Suppose that if the value of the mortgage payments rises from zero at t_0 to $30,000 at t_1, the T-bond price falls from 100 (its price at t_0) to 94. You purchase 5 put options on the T-bond. If you purchased the '100' put options you would be fully protected from the rise in mortgage payments, as the options would be worth $30,000. The '98' puts would be worth $20,000 and the '96' puts would be worth $10,000. Puts with a strike price of 94 or lower would be worth zero. If the mortgage payments had risen even more, these puts might offer some payoff. Thus the hedger can choose the appropriate level of protection. Of course, the '100' puts would cost far more than say the '96', just as a car insurance policy with a zero deductible would cost more than one with a $1000 deductible.

CHAPTER 4: CONSTRUCTING THE HEDGE WITH FUTURES

One possible solution to the problems caused by volatile mortgage rates is to hedge against adverse rate changes by using the futures markets (13, 18, 19, 20, 21, 29, 35). The basic premise underlying such a hedge is that increases (decreases) in mortgage payments (because of an increase in mortgage rates) will be offset by gains (losses) in the futures market. This assumes a linear relationship between changes in the value of the futures contract (i.e. gains and losses) and changes in the present value of mortgage payments.

In order to construct a good hedge, the optimal hedge ratios must be determined. There are two basic approaches to solving this problem (19): the portfolio approach and the price-sensitivity (duration-based) approach. The portfolio approach is based on a linear regression of spot price changes against futures price changes. The portfolio approach incorporates risk and is based upon empirical data, while the duration based approach is based on theory and does not incorporate risk. I shall examine the price-sensitivity approach in the following paragraphs. The portfolio approach will be examined later.

The Price Sensitivity Approach:

The price sensitivity approach says that the hedge should be constructed so that any increase (decrease) in the value of the mortgage is exactly offset by a gain (loss) on the futures contract. Thus the investor's financial position will not change. Of course, for the hedge to perform so well both the instrument relationship and the basis relationship must be perfect.

The first step in constructing the hedge is to determine the duration of the mortgage and of the T-bond future (11). Macauly defined duration as the weighted average of the times of the various payments, where each weight is the proportion of the total present value of the mortgage or bond represented by the payment at that date. Hicks derived the same formula, but went on to show how duration (D) measures the interest elasticity of the financial instrument, i.e. the responsiveness of the instrument's price to movements in interest rates.

Let T = years to maturity of the instrument

C_t = payment promised at date t

P = current price of the instrument

r = annual yield to maturity

Then:

$$D = \Sigma\, t\, (C_t/(1+r)^t)/P \quad \text{(where } P = \Sigma\, C_t/(1+r)^t \text{)} \tag{1}$$

$$PD = \Sigma\, t\, C_t/ (1+r)^t \tag{2}$$

But if we differentiate P with respect r:

$$dP/dr = \Sigma - t\, C_t/(1+r)^{t+1}$$

$$dP/dr = -PD/(1+r) \tag{3}$$

Rearranging, we obtain

$$dP/P = -D\, dr/(1+r) \tag{4}$$

Thus the percentage change in the price of any financial instrument for any marginal change in yield is simply the negative of duration multiplied by the yield change divided by one plus the yield. This assumes a flat yield curve, i.e. that interest rates are constant over all terms.

The value of the variable rate mortgage at any point in time is defined as the present value of all future mortgage payments evaluated at the mortgage rate prevailing at the beginning of the hedge period. Changes in mortgage rates affect mortgage value by changing the mortgage payments, not the discount rate. Thus duration, as previously defined, is not an appropriate measure of the sensitivity of the mortgage value to changes in mortgage rates. However, if we define duration as the interest elasticity of the financial instrument (as did Hicks), then we may still use duration to determine optimal hedge ratios.

The interest elasticity of the value of the variable rate mortgage is positive, since increased mortgage rates imply increased mortgage values as a result of higher mortgage payments being evaluated at the same discount rate. For instance, if mortgage rates rise from 14.00% to 14.01%, a mortgage originally valued at $100,000 will rise in value by $35.80. Thus a 1/100th percentage rate increase in the mortgage rate resulted in a 3.58/100th percent increase in mortgage value. To solve for duration we simply rearrange equation 4:

$$D = -\,(dP/P)\,(1+r)\,/\,dr \tag{4 A}$$

If we substitute the above values into equation 4 A , we find that the duration of a variable rate mortgage with a beginning mortgage rate of 14% is -4.08.

To account for differences in the dollar value of the mortgage versus the T-bond, we look at the dollar change in the instrument. To get this we simply multiply both sides of equation 4 by P:

$$dP = -D\,P\,dr/(1+r) \tag{5}$$

We can set dP_b (the change in the value of the T-bond) equal to $-dP_m$ (the change in the value of the mortgage) so that changes in the value of the mortgage will be exactly offset by changes in the value of the T-bond.

$$D_b\,P_b\,dr_b/(1+r_b) = -D_m\,P_m\,dr_m/(1+r_m)$$

$$P_b = -\,(D_m/D_b)\,[dr_m/(1+r_m)]/[dr_b/(1+r_b)]\,(P_m) \tag{6}$$

If we set the mortgage price, P_m, equal to \$100,000 CDN, we can solve for the dollar amount of futures, P_b. If we divide P_b by the current price of one T-bond futures contract, F_b, and multiply equation 6 by -1 to account for the fact that the mortgage represents a liability and not an asset, we arrive at the optimal number of futures contracts to hold:

$$num = (D_m/D_b)\,[dr_m/(1+r_m)]/[dr_b/(1+r_b)]\,(P_m/F_b) \qquad (7)$$

Using equation 7 we can solve for the optimal number of T-bond futures contracts which should be entered into. This indicates that we should short T-bond futures (since D_m is negative).

Over the period 1979 to 1986, 5 year mortgage rates in Canada have ranged from about 10% p.a. to over 22% p.a. Duration of the 5 year mortgage over this range of rates is between -4.31 years (@ 10%) and -3.68 years (@ 22%). Over the same range the duration of the T-bonds (15 year maturity, 8% coupon) is between 8.85 and 6.11 years. The ratio D_m/D_b thus ranges from -0.49 to -0.60. However the long term nature of the T-bonds has prevented the range of yields from being as volatile as the more short term mortgage. As noted by Eckbo (11), "short-term bond yields are usually more volatile than long-term bond yields". In addition, bond rates in the U.S. have generally been lower than mortgage rates in Canada. The T-bond yield never climbed above 15% (duration of 7.64 years), but it fell as low as 7.1% (duration of 9.54 years). Thus at very high interest rates D_m/D_b will be about -0.48, compared to the ratio of -0.45 obtained at low rates. I therefore propose that the range of duration ratios is quite small and can be best represented by a point estimate of -0.46.

As previously noted, the yield change of the mortgage will tend to be greater than that of the T-bond. Standard deviations were calculated for the percentage interest rate changes over a 3 month period for both Canadian mortgages and U.S. T-bond futures (using the near contract) over the period from October 1979 to July 1986. Mortgage rates had a standard deviation of 1.68% while that for T-bond futures was 1.30%. Thus the yield change ratio of the mortgage is 1.25 times greater than that of the T-bond ([.168/1.16]/[.13/1.125]) (the medians of the ranges of rates are used in the denominator).

Each T-bond future represents $100,000 U.S. in par value, but over most of the range it traded well under par, ranging from about $60,000 to $100,000. Let us use the average value of $80,000 U.S. A mortgage value of $100,000 CDN is the benchmark initial mortgage value. Exchange rates over the period ranged from 0.69 to 0.87. Let us use the average value of 0.78. Thus the average ratio of mortgage value to T-bond price is $78,000 U.S./$80,000 U.S. which equals 0.975. However, this value has taken on a fairly wide range over the sample period. When interest rates were at their peak in 1981 this ratio was 1.42 ($85,000/$60,000), while the ratio in July 1986 was 0.74 ($72,000/$97,000).

To calculate the optimal number of T-bond futures to be shorted per $100,000 CDN of mortgage value, we simply substitute the above values into equation 7:

$$num = (D_m/D_b)\,[dr_m/(1+r_m)]/[dr_b/(1+r_b)]\,(P_m/P_b)$$

$= (-0.46)\,(1.25)\,(0.975)$

$num = -0.56$

Thus for every $100,000 CDN of mortgage value, you should short approximately 0.56 T-bond futures contracts to effectively hedge interest rate risk. If we regress changes in U.S. T-bond futures prices against changes in mortgage value, the coefficient for the T-bonds should be -0.56. However, this ratio is materially affected by the price relationship between the futures contract and mortgage value. It can take on values between -0.43 and -0.82. It is more appropriate to write the hedge ratio as:

$$num = -0.58\,(P_m/P_b).$$

The above hedge ratio forms the basis of the price adjusted regressions which will be discussed.

CHAPTER 5: INTEREST RATE PARITY

A Canadian investor may encounter difficulty hedging in the U.S. interest rate markets for two reasons. First, the value of the futures contract (or option) when the hedge period ends is influenced by the value of the Canadian dollar relative to its U.S. counterpart. This is known as currency risk. Currency risk is not a great concern because it will only cause the hedge to vary in value by 5% or more about 5% of the time, based upon data for 3 month hedging periods from October 1979 to July 1986 (i.e. the standard deviation of 3 month percentage changes in the exchange rate is 2.5%). The second and more important source of risk is the risk that the amount to be exchanged (i.e. the gain or loss from the futures), given constant exchange rates (i.e. no currency risk), will not perfectly offset gains or losses on the mortgage. This would be the case if interest rate movements in Canada do not parallel interest rate movements in the U.S. This will cause the hedge using T-bond futures (or options) to break down, as movements in T-bond futures prices will not be proportional to changes in mortgage value.

In the past, there has been a tendency for U.S. and Canadian interest rates to move together. In order for the hedge using only the T-bond to work they must continue to move together. The higher the degree of correlation between the two, the better will be the hedge's performance. Kneeshaw and Van den Bergh (25) regressed Canadian bond yields and U.S. short term rates against U.S. bond yields for the period April 1979 to June 1984. The coefficient for the Canadian bond yield was 0.90 with a t-statistic of 6.72. The partial R^2 for this variable was 0.47. Thus there exists an a priori expectation that a hedge using only T-bond futures will reduce risk, but will only lessen changes in portfolio value by about 47% in the case of a Canadian bond. It is expected that a hedge of mortgage value would not perform quite as well as one using bonds, since there may be different supply and demand relationships in the two interest rate markets. Kneeshaw and Van den Bergh's Canadian bond coefficient of 0.90 would lead us to believe that interest rates are about 11% more volatile in the U.S., given the same terms and interest rate markets (in this case the bond market).

The interest rate parity theory may help explain why interest rates in Canada and the U.S. have been highly correlated in the past. In addition interest rate parity can explain why the addition of Canadian dollar futures (or options) to the hedge portfolio may improve the hedge's performance. It is unclear, however, whether interest rate parity exists for long term interest rates such as bonds or mortgages.

The interest rate parity theorem, simply stated, says that an investor in any one country should be able to earn the same risk free interest rate no matter which country he invests in. If the investment is in a foreign interest rate market, currency futures must be purchased to eliminate exchange rate risk. These futures will lock in the Canadian dollar value of the receipts from the foreign security. The futures contract will result in a foreign currency gain or loss, because of the difference between the rates at which the foreign currency will be purchased and sold (predetermined by currency future prices). This will either increase or decrease the return earned in the foreign financial market. If the purchase price of the foreign currency is less (greater) than the sales price then the return earned by a Canadian investor will be higher (lower) than the return in the foreign financial market. The relationship between the foreign currency purchase and sales prices is determined in the futures market.

When the futures price is greater than (less than) the spot price, there is said to be forwardation (backwardation) in the futures contract. Forwardation (backwardation) implies that futures with further maturity dates will have progressively higher (lower) prices. The interest rate parity theorem says that the degree of forwardation or backwardation in the foreign currency futures must be exactly enough so that a Canadian investor will earn the same risk free return, regardless of the country in which the capital is invested. The causes of backwardation or forwardation are beyond the scope of this paper.

Arbitrage in the Spot and Futures Markets:

If a significantly different risk free return can be earned in another country then an arbitrage opportunity exists. If, for examply, a higher interest rate can be earned by purchasing U.S. T-bills (and Canadian dollar futures) than by purchasing

Canadian T-bills, the investor simply sells Canadian T-bills and uses the proceeds to purchase U.S. T-bills. He also buys enough Canadian dollar futures to lock in the Canadian dollar value of the proceeds of the U.S. T-bill. Since the investor earns a greater return by investing in the U.S., the proceeds of the U.S. T-bill (converted to Canadian dollars at the predetermined rate) will exceed the funds required to pay off the Canadian T-bill. For a zero investment the investor earns risk free profits. As investors engage in this arbitrage strategy, they drive up U.S. T-bill prices and Canadian dollar futures prices, and drive down Canadian T-bill prices (prices and interest rates are inversely related). This process continues until there is interest rate parity. Thus this arbitratge relationship ensures that interest rate parity will hold in the short term. Of course, transaction costs and taxes reduce the precision of relationship, but only by allowing small differences to exist between the risk free returns earned by a Canadian investor in the U.S. versus Canada. Once the difference increases beyond some given level, arbitrage profits can be earned.

An example may be helpful. Say that a Canadian one year T-bill is priced at $94 per $100 par value. A U.S. one year T-bill is priced at $94.50 per $100 par value. The Canadian dollar spot rate (in U.S. dollars) is $0.7518 while the one year futures price is $0.7441. The interest rate earned in Canada is 6.38%, while that earned in the U.S. is 6.92%. Thus the investor buys U.S. T-bills and sells Canadian T-bills. If he sells $100 par value in Canadian T-bills, he will have $94 Canadian to invest. At the spot rate of 0.7518 this is worth $70.67 in U.S. dollars. If he invests this $70.67 in U.S. T-bills he will receive $74.78 at maturity (70.67/.945). Since he locked in the exchange rate by using Canadian dollar futures, this will be exchanged for $100.50 (74.78/.7441). But only $100.00 is needed to retire the Canadian T-bill. Thus the investor has earned arbitrage profits of $0.50. If transaction costs are below $0.50 per $100 par value, an arbitrage opportunity exists. Notice that although U.S. investors earn a lower rate of return on their T-bills than Canadians do on theirs, the Canadian investor can earn a higher rate of return in the U.S. than in Canada. This is due to the backwardation of the Canadian dollar futures contracts (forwardation of foreign currency prices), which raises the return for Canadian investors.

Arbitrage Using Only the Futures Markets:

The arbitrage portfolio described above can be generalized and we can remove the constraint that the purchase of the T-bills be made in the spot market. Thus we must consider the prices of four different instruments: U.S. Treasury bill futures (USTB), Canadian Treasury bill futures (CTB), and two Canadian dollar futures (in U.S.$), one with the same contract date as the two T-bill futures (CD[0]), and one with a contract date when the T-bills mature (CD[1]). For ease of exposition, let's assume the T-bills have a maturity of one year and that the par value of all futures contracts is one dollar in the appropriate currency. I shall use the convention that the price of each instument is denoted by P with each instrument's abbreviation as the subscript.

Consider three points in time; the current time, t_{now}, the contract date for the T-bill futures, t_{start}, and the contract date for CD[1], t_{unwind}. Now let's look at the arbitrage strategy. You must make four simultaneous transactions:

1) sell **one** CTB future (receive P_{CTB} \$C at t_{start}, pay 1 \$C at t_{unwind}).

2) sell $\mathbf{P_{CTB}}$ \$C worth of CD0 futures (receive $P_{CTB}{}^*P_{CD[0]}$ \$US at t_{start}).

3) buy $\mathbf{P_{CTB}{}^*P_{CD[0]}}$ \$US worth of USTB futures (contract date t_{start}, receive $P_{CTB}{}^*P_{CD[0]}/P_{USTB}$ \$US at t_{unwind}).

4) buy $\mathbf{(P_{CTB}{}^*P_{CD[0]}/P_{USTB})}$ \$US CD[1] futures (receive $[(P_{CTB}{}^*P_{CD[0]})/(P_{USTB}{}^*P_{CD[1]})]$ \$C at t_{unwind}).

Thus there is no net investment and no net cash flows until t_{unwind}. At time t_{unwind} you receive $[(P_{CTB}{}^*P_{CD[0]})/(P_{USTB}{}^*P_{CD[1]})]$ Canadian dollars by exchanging the receipts of the US T-bill at the predetermined rate of $P_{CD[1]}$. With this you must pay one Canadian dollar to retire the Canadian T-bill that was sold. Thus to prevent arbitrage:

$$P_{CTB} = P_{USTB}{}^* \ (P_{CD[1]}/P_{CD[0]}) \tag{8}$$

This relationship can only be expected to hold for short term rates. Longer term rates are not bound by this arbitrage relationship because of the lack of trading in Canadian dollar futures beyond 30 months into the futures. This is because the returns earned in foreign financial markets cannot be made risk free unless the exchange rate at which the investment will be repatriated can be locked in using futures.

Interest Rate Parity and Hedging:

Consider the variable rate mortgage hedge using only T-bond futures. The biggest problem with this strategy is that movements in T-bond prices may not be proportional to movements in mortgage value. But interest rate parity states that if, for example, T-bond prices fall while mortgage value remains constant, then the increase in U.S. T-bond rates relative to Canadian mortgage rates must be offset in the Canadian dollar futures market. In this case we can predict that the future Canadian dollar price will increase relative to the spot price. This will increase exchange gains (or reduce losses) and allow the difference between returns earned in U.S. T-bonds versus Canadian mortgages by a Canadian investor to be constant. Thus the addition of Canadian dollar futures (or options) should improve hedging performance.

In order to apply interest rate parity to the relationship between U.S. T-bond yields and Canadian mortgage rates two assumptions must be made. The first is that changes in 5 year Canadian mortgage rates are equal to changes in Canadian T-bill rates. The second is that changes in 15 year, 8% coupon U.S. T-bond yields are equal to changes in U.S. T-bill rates. This is equivalent to saying that the slope of the yield curves in both countries must be constant. Because of the lack of forward or futures contracts that extend so far into the future, long term interest rate parity must at best be an approximation. Thus the value to the exchange rate hedge must be determined empirically.

Because only three options contracts are traded, the far Canadian dollar future (CD[1]) can never have a contract date beyond 6 - 9 months. In addition the short contract must not expire before the end of the hedge period, and must have a contract date at least 3 months before CD[1]. Thus the only hedge period which can be properly tested is the 3 month hedge, using Canadian dollar future prices for the 3-6 month contract (CD[0]) and the 6-9 month contract (CD[1]).

Using equation 8, and the two above assumptions:

$$FRMV = a + b \; \{ P_{TBF}{}^{*} \, [(P_{CD[1]}/P_{CD[0]})^{4}] \} \qquad (9)$$

where CD[1] and CD[0] are only 3 months apart rather than the only year separation assumed in equation 8. P_{TBF} represents the U.S. T-bond future price. The coefficient 'a' represents the differences between long and short rates in both countries. FRMV represents the value of a fixed rate mortgage. The coefficient 'b' simply equates the initial value of the mortgage with that of the T-bond future. However for a variable rate mortgage the present value of payments rises when mortgage rates rise, using a constant discount rate. Thus we would expect mortgage value to increase with decreases in P_{TBF} or $P_{CD[1]}$ and with increases in $P_{CD[0]}$. Thus the expected sign for changes in T-bond futures prices and changes in far Canadian dollar futures prices is negative, while that for near Canadian dollar futures prices is positive.

Equation 9 does not show a linear relationship, but over a three or six month period the relationship may approximate linearity. This must be determined empirically. Non-linear relationships imply optimal hedge ratios which are constantly changing. This study assumes that the hedger will not change the hedge ratio over the hedge period. Thus only linear relationships will be empirically tested, as only linear relationships are easily applied in the marketplace. A good proxy for the ratio of far to near Canadian dollar futures prices is the difference between them. Thus the addition of the variable representing changes in this difference should improve hedging performance. It is expected that this variable will have a negative coefficient, and that it will be about four times the magnitude of the T-bond coefficient (because the difference between them represents differences in quarterly returns, not annual ones).

CHAPTER 6: THEORIES OF HEDGING POLICY

Risk aversion is usually given as the reason why firms (or their management) hedge. However Smith and Stulz (36) looked for other reasons why a firm would hedge, namely to maximize the value of the firm. If marginal tax rates rise with the level of income, or if negative taxable incomes do not elicit tax credits to the full extent of previous or future positive incomes, then taxes will be minimized by smoothing a firm's taxable income over time. This is because the after-tax value of a firm is a concave function of its pre-tax value. This concavity is increased by such things as excess profit taxes (such as were levied against oil companies in the late 1970's), and reduced by the allowance of trading of tax credits (so that negative taxable incomes will yield tax benefits regardless of when the firm returns to profitability).

The costs of bankruptcy also serve to create an incentive to hedge. These costs include legal costs and the liquidation of assets at below market prices. If hedging can reduce the expected value of these costs by more than the costs of hedging, then firms should hedge.

If a firm borrows money, the costs of borrowing are positively related to the probability of the firm becoming insolvent. Since hedging reduces this probability, it should lower borrowing costs. However, for this to work, bondholders must be convinced that the firm will hedge, and that the hedge will reduce their risk.

The firm may be subject to restrictive covenants which constrain a firm's operating policy as it approaches insolvency. Since these constraints impose a cost on the firm, the firm will seek to avoid triggering such covenants.

A final reason why firms hedge is because of agency issues. Managers are often fired because of poor performance, and thus an incentive is created for them to minimize the risk of being fired. At the same time, management compensation is often levered so that superior performance yields huge rewards (in the form of bonuses and stock options), while anything less than that means the managers will simply be entitled to their base salary. Thus over time a smooth income stream will mean less managerial compensation and a higher firm value.

Several authors have looked at hedging effectiveness from a risk/return perspective, rather than simple risk minimization. Both Nelson and Collins (29) and Howard and D'Antonio (23) use a measure similiar to Sharpe's measure, i.e. the slope of the security market line. Thus to maximize hedging effectiveness a firm should seek to maximize the ratio of expected return less the risk free rate, divided by the standard deviation of the expected return. If use of a hedge increases this ratio, the authors state that the firm should engage in hedging. If the hedge is based on entering a futures contract, and if the expected return from entering the futures contract is zero (as it should be if futures prices represent unbiased expectations), then the authors conclude that hedging effectiveness will increase with the correlation between the futures price and the hedged portfolio, while the expected return will be unaffected. Thus their results are no different from simple risk minimization if the expected value of the hedge is zero.

Kolb, Timme, and Gay (25) compare macro versus micro futures hedges. A macro hedge is any hedge that seeks to reduce the interest rate risk of the entire firm, while a micro hedge only reduces the interest rate risk of some portion of the firm's balance sheet, such as a mortgage. The authors point out that many firms engage in micro hedging because the decision to hedge is not made by top management, but by managers in charge of some specific group of assets and/or liabilities. Although this may reduce the risk of some portion of the company, care must be taken to ensure that the risk of the entire firm is not increased. For example, suppose profits from business unit A are inversely related to profits from all other business units. If profits from the other units dominate the profits from unit A, then business unit A effectively hedges the profits from the other units. Thus if the manager of unit A engages in hedging, he will reduce the variability of unit A's profits and thus reduce the value of unit A as a hedge to the rest of the firm. Thus a macro hedging approach is better than a micro one. However, if top management approves all micro hedges to ensure they are in the best interests of the company as a whole, then micro hedging will be effective, as it will effectively be macro hedging.

CHAPTER 7: EMPIRICAL STUDIES OF HEDGING WITH FUTURES

Many researcher have examined the effectiveness of hedging interest rate risk with futures. They include Gau and Goldberg (13), Hegde and Branch (18), Hegde and Nunn (19,20), and Overdahl and Starleaf (31).

Gau and Goldberg:

Gau and Goldberg look at the ability of both Canadian and U.S. organizations with exogenously fixed spot positions in a single asset (for developers this is usually a negative asset, i.e. a liability) to hedge against moves in the asset's price using American futures markets. In particular, both the GNMA (1976-80) and T-bond (1977-80) futures traded by the Chicago Board of Trade are examined. The single asset used for the U.S. is a $100 mortgage with a 30 year amortization period and prepayment after 12 years. The yield is set at bi-weekly FNMA auctions. The Canadian asset is a $100 mortgage with a 5 year term, 25 year amortization, and semi-annual compounding. The yield changes weekly according to rates offered on originating mortgage loans with similiar terms. To shift the viewpoint from an American to a Canadian firm, the authors include the effect of supplementing their futures position with one in the Canadian dollar futures market of the Chicago Mercantile Exchange(76-80).

Rather than looking at offsetting gains and losses, the authors choose to look at R^2s and correlation coefficients. These are based on regressions where the dependent variable is the asset price and the various futures prices (both alone and in combination) are the explanatory variables. Regressions are based on periods of time ranging from 2 to 52 weeks. In general the longer the hedge, the better it worked. Based on R^2s the U.S. hedges worked fairly well (.435 to .793) but the Canadian ones did poorly (.005 to .656). Because currency futures were only widely traded out to the 6-9 month contract, the maximum hedge period looked at was 26 weeks. The added futures position helped, but it did not allow Canadian investors to hedge as well as their U.S. counterparts. The authors concluded that risk can be significantly reduced for all the above mentioned investors except for those with short term positions in Canadian mortgages.

Hegde and Branch:

Hegde and Branch look at the use of the T-bond futures contract to hedge the value of twelve bond portfolios over both a two and a four year period. Risk minimization is the only goal of hedging in this article. The bond portfolios each have $1 million face value of bonds chosen randomly from all NYSE listed bonds. Some portfolios are constructed of bonds grouped according to coupon rate, maturity, and investment quality rating. Hedge ratios are based on the duration of the bonds in each portfolio and the duration of the T-bond. These ratios are then rounded off to arrive at an integer number of contracts (between 6 and 13). Reinvestment of coupon payments was taken into account, and this improved performance significantly. The hedge worked very well. For the two year hedge, portfolio value was held to within an average of 3.8% of its original value, where unhedged portfolios varied in value by an average of 16.9%. The corresponding numbers for the four year hedge were 4.9% and 20.9%. The T-bond hedge worked much better for the high quality bond portfolio than the low quality one. This is because of the additional risk of default, a risk which is effectively zero for T-bond futures. Intra-period adjustment of hedge ratios was looked at by the authors. They conclude that although some transactions costs are involved the benefits exceed the costs in the case of the longer hedge. Adjusting hedge ratios seven times over the four year period rather than twice resulted in reducing deviations in portfolio value to just 0.4%.

Hegde and Nunn:

Hegde and Nunn look the the cross-hedging performance of the T-bond and GNMA futures contracts in hedging corporate and municipal bonds. They compare the portfolio approach with the price-sensitivity approach and find that "the measures of cross-hedging effectiveness and cross-hedge ratio are generally comparable" (ref 19, p159). In addition, term to maturity was found to have the most explanatory power in terms of hedging performance. Performance improves when the maturities of the bond underlying the future and the bond being hedged are closer together. Also hedges using longer bonds performed better than those using medium term ones. Default risk was not found to be a significant factor. Finally, the authors found that the use of the nearby futures contracts was superior to the more distant ones. This is explained by the fact that futures prices are not necessarily unbiased with respect to forward prices. It has been found that this bias increases with increasingly distant futures contracts. Hegde and Nunn also looked at the effects of increased volatility and trading volume on the hedging performance of T-bond and GNMA futures. They found that the increased volume has improved their hedging performance significantly.

Overdahl and Starleaf:

Overdahl and Starleaf look at hedging the value of a portfolio of short term CD's with T-bill and CD futures. They find that the CD futures outperform the T-bill futures. This is to be expected, given that the use of CD futures is a straight hedge while use of the T-bill futures is a cross hedge. Thus no instrument risk is involved in the CD futures hedge. This study illustrates that instrument risk can harm a hedge's performance.

CHAPTER 8: METHODOLOGY

The regressions performed in this study are based on the portfolio approach to hedging. In the case of a simple regression, the optimal hedge ratio will be equal to the covariance of changes in the value of mortgage payments and futures price changes, divided by the variance of the futures price changes. For a multiple regression, the optimal hedge ratio also depends on the covariance of the futures prices.

The portfolio approach assumes the investor has an investment portfolio consisting of the liability for the mortgage and whatever futures positions are used for hedging. Since the mortgage is a negative asset, increases in its value must be offset by increases in the value of the futures contract(s). A perfect hedge ensures that the increase in the value of the futures contract(s) will always be equal to increases in mortgage value. Thus the value of the entire portfolio will not vary.

<u>Data:</u>

The data used for the portfolio approach consists of weekly observations of changes in mortgage value and futures prices (converted to Canadian dollars) from October 1979 to July 1986. These observations are derived from the raw data as detailed in the next few paragraphs.

The first variable that must be calculated is the change in mortgage value. First the annual rates must be converted to equivalent monthly rates:

$$mrate_{beg} = [((1+ (\text{annual rate}_{beg}/2))^{1/6}) -1]$$

$$mrate_{end} = [((1+ (\text{annual rate}_{end}/2))^{1/6}) -1]$$

The beginning payment (assuming an unamortized mortgage) must be calculated:

$$begpay = 100{,}000 * \text{monthly rate}_{beg}$$

Then the ending payment must be calculated:

$$endpay = 100{,}000 * \text{monthly rate}_{end}$$

The change in mortgage value can now be calculated. Since a fixed rate loan would have matured 5 years (60 months) from the beginning of the hedge period, the change in mortgage value is calculated by taking the present value of the change in mortgage payments for the next 57 months (60 less 3) for the 3 month hedge, and 54 months (60 less 6) for the 6 month hedge. The discount rate used is the beginning mortgage rate, since this rate was used when the investor made the decision to enter into a mortgage. The formula used to calculate DELMOR, the change in mortgage value, is the formula for a simple annuity:

$$DELMOR = [(endpay - begpay)/mrate_{end}] + \frac{(endpay-begpay)/mrate_{end}}{(1+mrate_{end})^{54 \text{ or } 57}}$$

The gain on the T-bond future (DELTB) is:

$$DELTB = (\text{T-bond}_{end} - \text{T-bond}_{beg}) * 1{,}000 / endspot$$

The gain on the near C$ future (DELCD[0]) is:

$DELCD[0] = (NEARC\$_{end} - NEARC\$_{beg}) * 100{,}000 / endspot$

The gain on the far C$ future (DELCD[1]) is:

$DELCD[1] = (FARC\$_{end} - FARC\$_{beg}) * 100{,}000 / endspot$

Once these variables have been obtained, the following regressions can be run using the change in mortgage value as the dependent variable:

$DELMOR = a + b\ DELTB$ and

$DELMOR = a + b_1\{ DELTB + [4 * (DELCD[1] - DELCD[0])] \}$

<u>Measures of Hedging Effectiveness:</u>

The key measures of hedging effectiveness are the correlation coefficients and the R^2s. The correlation coefficients measure the degree of co-movement between variables, while the R^2 gives the most precise measure of hedging effectiveness, the proportion of price variations which would have been reduced if the hedge had been in place over the time period of the regression. In addition, the coefficients of each independent variable represent the empirically determined optimal hedge ratios. These must be compared to the theoretically obtained hedge ratios, and disparities should be analysed.

<u>Optimal Hedge Ratios:</u>

The empirically obtained hedge ratios will most likely not be integers. This may present problems to hedgers, especially those only purchasing a few contracts. The more that actual (integer) hedge ratios deviate from the optimal the greater will be the loss in hedging effectiveness. However, given the uncertainty present in the empirically determined hedge ratios (because these are ex post optimal hedge ratios), the loss may not be that great in subsequent runs.

The stability of the obtained regression statistics will be tested by running the same regressions over different time periods. In addition, the predictive ability of these regressions will be tested. Regressions will be run on the entire data set excluding the last year of data. Optimal hedge ratios from these regressions will be used to predict changes in mortgage value for the last year of data, based upon actual changes in futures prices. These predicted values can then be compared to actual values to determine the regressions' predictive ability.

CHAPTER 9: EMPIRICAL RESULTS

The regression results were checked to ensure that all the assumptions of linear regression were met. A significant degree of autocorrelation between error terms was found, however further tests showed that the results were not affected. For a further discussion see Appendix I.

No Exchange Hedge:

The results of the regressions with no exchange hedge are given in Tables I and II. First, we shall look at the regressions for the entire period from October 1979 to July 1986. The results are similiar for both the 3 and 6 month hedging periods. The T-bond coefficient has the anticipated negative sign. The magnitude of the T-bond coefficient is slightly greater for the 6 month hedge (-0.337) than for the 3 month (-0.322). Both these coefficients are significantly different from zero at the 0.001 confidence level. The R^2s are .2767 for the 3 month hedge and .3302 for the six. Thus, on average, 28% of the variation in mortgage value is eliminated by the 3 month hedge, and 33% by the six.

The same regressions were run for three different subperiods; the period of rising rates (10/79 - 10/81), falling rates (11/81 - 7/85), and the last year (7/85 - 7/86). For the three month hedge, the T-bond coefficients for the first two periods are significant at the 0.001 confidence level, while that for the last period is significant at the 0.1 level. The magnitude of the coefficient is unstable, falling from 0.454 in the second period to 0.032 during the last period. The hedge performs best during the period of falling rates (R^2 of .38) and worst during the last period (R^2 of .04), when interest rates fell and then remained fairly steady. An additional regression was run for the entire range excluding the last year. The coefficient rose to -0.456 while the R^2 increased to 0.37.

The magnitude of the coefficient is dependent upon the variability in mortgage rates. This is because of the discrete nature of changes in mortgage rates as opposed to the continuous changes in T-bond rates. Consider a theoretical mortgage rate which would be charged by banks if they adjusted their mortgage rates daily. Now assume that T-bond rates in the U.S. and theoretical mortgage rates in Canada move in tandem. Since mortgage rates move in discrete intervals of at least 1/4%, it takes a rate change of at least 1/4% for movement in both T-bond and mortgage rates. Any lesser move will most likely not affect stated mortgage rates. When rate movements over the hedge period are this small, the T-bond future hedge will tend to overcompensate since all or part of the move in interest rates will not be reflected in the stated mortgage rate. In fact, for the 3 month hedge period, the last 53 observations (the period of poorest hedge performance) contained 19 observations where the mortgage value (and mortgage rates) did not change over the hedge period. Obviously hedge performance will be poor in these cases, since the hedge will always increase portfolio variability. Now remove the assumption of identical changes in T-bond rates versus theoretical mortgage rates, but add the condition that the difference between the two rates cannot exceed some number. This has been the case over the range of data tested. Thus large movements in mortgage rates will always be accompanied by T-bond rate movements in the same direction, while for small movements in mortgage rates the relationship may not hold. Within this interest rate environment the correlation between movements in the two rates will be much higher for large changes in interest rates. Thus we would expect hedging performance to be far better for periods of large interest rate movements (all periods tested except the last year). In addition, we can disregard the optimal hedge ratios derived for the last year, since they will tend to greatly understate the appropriate ratios for periods of large interest rate movements.

The six month hedge yields a wider range of results than the three. The T-bond coefficient is -0.604 during the period of rising rates, -0.508 during the next, and -0.004 for the last year. The first two coefficients are significant at the 0.001 confidence level. That for the last year is not significant. The last period also has a very low R^2 of .0007, compared to R^2s of .323 and .498 for the first and second periods, respectively. The results for the last year were so bad that another regression was run to see how far back this poor performance went. This covered the last 18 months of the data, and showed a far better hedging ability (R^2 of .32).

Overall, the six month hedges performed better than the three month hedges.

With Exchange Hedge:

The interest rate parity theorem yields the conclusion that the best exchange hedge requires both a near and a far Canadian dollar futures contract. This is because the appropriate hedging variable is the amount of backwardation or forwardation in the futures prices. This is represented by the spread between the near and the far Canadian dollar contracts. As previously noted, the interest rate parity theorem is only applicable to short term interest rates, in the absence of longer term currency futures. Thus interest rate parity for long term rates is dependent upon the yield curve having a constant slope (i.e. the spread between short and long term interest rates is constant).

The addition of the Canadian dollar spread to the 3 month regressions increased explanatory power for every period tested (see Table III). The regression using the entire data set yielded an R^2 of .3129. The coefficient of -4.55 has the expected negative sign , and is significantly different from zero at the 0.0001 confidence level. It's magnitude is over 12 times that of the T-bond coefficient (-0.361), much greater than the four times as large predicted by interest rate parity. The T-bond coefficient is also significant at the 0.0001 level.

The addition of the spread variable gave a modest boost to the R^2 for the regression covering the period from October 1979 to October 1981. It increased from

0.1871 to 0.1990. The spread coefficient is negative but not significant. However, it's magnitude is closer to being 4 times that of the T-bond.

For the period between November 1981 and July 1985, the spread variable increases the R^2 to .4247. The spread coefficient is -5.09 and is significant at the 0.0001 level. It is over 10 times as large as the T-bond coefficient of -0.483.

The regression covering the final year of data is much improved by the addition of the Canadian dollar spread. The R^2 increases from 0.0359 to 0.1755. The spread coefficient is -3.72 and is significant at the 0.005 level. The T-bond coefficient of -0.059 is too low to make a useful comparison.

The spread coefficient modestly increases the explanatory power of the regression. It improves the R^2 by between 6.4% and 13.1%, except for the final year of data, where the R^2 is improved by 480% (albeit from .0359 to .1755). The spread coefficient ranges in value from -2.43 to -5.12. It does, however, tend to rise concurrently with increases in the T-bond coefficient. T-bond coefficients computed for portfolios with the exchange spread hedge are all close to those computed for portfolios without it. It appears the addition of the spread between the near and the far Canadian dollar futures contracts is a worthwhile addition to the hedge portfolio.

The addition of a single Canadian dollar futures contract to the portfolio may be useful (13). Thus the far contract was omitted from the regressions (they both yielded equivalent results, and the near contract can be used in both the 3 and the 6 month hedges). The results are given in Tables IV and V.

The addition of a single Canadian dollar futures contract improved the hedge's performance significantly better than did the addition of an exchange spread. For the three month hedge, all coefficients were significant at the 0.001 level, except for the T-bond coefficient for the last subperiod. The regression covering the entire range of data yielded a T-bond coefficient of -0.26 and a Canadian dollar coefficient of -1.08. The R^2 is 0.43, indicating that such a hedge offers worthwhile risk reduction opportunities.

For the different subperiods, both the T-bond and Canadian dollar coefficients fall with more recent time periods. All R^2s increased to between 0.32 and 0.53. The regression covering all but the last year yielded a T-bond coefficient of -.371, a Canadian dollar coefficient of -1.10, and an R^2 of .5111.

As with the three month hedge, the exchange hedge improves the performance of every regression tested using the six month hedge. Coefficients for both the T-bond and Canadian dollar contracts are all significant at the 0.001 level except for the T-bond coefficient for the last year's data, which is insignificant.

For the six month hedge, in every period tested in which the T-bond coefficient is significant, its magnitude is smaller than for the corresponding regressions with no exchange hedge. Not including the insignificant coefficient for the last year, the percentage decrease is between 15% and 28%, a fairly narrow range. This is to be expected, given the positive correlation between the two (rho = 0.25). The positive correlation reflects the fact that over the period tested, the Canadian dollar tended to rise (fall) relative to the U.S. dollar when T-bond prices increased (decreased), i.e. when T-bond rates fell (rose). According to the interest rate parity theorem, this can be explained by the fact that interest rates in the U.S. were higher than in Canada when rates peaked, and are now lower than in Canada, as rates have fallen. The interest rate parity theorem would lead us to believe that the currency of the country with the higher interest rates would tend to rise in value.

The six month hedge for the entire period has a T-bond coefficient of -0.266, a Canadian dollar coefficient of -1.17, and an R^2 of 0.51. For the period of rising rates, the T-bond coefficient is -0.435, the Canadian dollar coefficient is -1.75, and the R^2 is 0.46. From October 1981 to July 1985, the corresponding numbers are -0.384, -1.11, and 0.76, respectively. The regression using the last year's data yields a T-bond coefficient of -0.016, a Canadian dollar coefficient of -0.51, and an R^2 of 0.45. The regression run on the whole range of data excluding the last year performs also very well. Both the T-bond coefficient and the R^2 increase significantly (to -0.523 and 0.68 respectively) compared to the portfolio with no exchange hedge. The Canadian dollar coefficient does not change much, falling only slightly in magnitude to -1.08.

According to interest rate parity, the Canadian dollar coefficient should be 4 times the magnitude of that for the T-bond. Regressions were run using this constraint for both the three and six month hedge periods. The results are terrible. The R^2s are close to zero (0.0062 and 0.0037) and the coefficients were far too small (T-bond coefficients of -0.004 and -0.003). Thus another reason is given to reject interest rate parity for long term rates.

Results both with and without the exchange hedge indicate the same thing. The hedge appears to have have reduced quite a bit of the variation in portfolio value. However the hedge's ability to reduce portfolio variation falls during the last year, especially in the case of a three month hedge. It appears the hedge works best when rates are more volatile. The addition of a single Canadian dollar futures contracts to the hedge portfolio yielded the best results, followed by the addition of an exchange spread hedge (i.e. a near and a far Canadian dollar futures contract). The portfolio with only the T-bond futures contract did not perform as well.

PREDICTIVE ABILITY:

Forecasts were made based upon both the portfolio and price sensitivity approaches. The construction of these forecasts is as follows. The optimal hedge ratios are derived using the above two approaches. A prediction of the change in mortgage value is then calculated by multiplying each optimal hedge ratio by the actual changes in the value of each futures contract, and summing. These predictions are then compared to the observed changes in mortgage value and the R^2 between observed and predicted values is calculated. Predictions were made for

53 observations from August, 1983 to August, 1984 and for 53 observations from July, 1985 to July, 1986 (the last observations in the data set). The 1983-84 predictions were made because of the poor performance of the hedge during the last year of data.

For the portfolio approach, optimal hedge ratios were obtained as follows. For each of these time periods optimal hedge ratios were obtained by using the coefficients derived in regressions run from October, 1979 (the first observation in the data set) to the last observation before the period of prediction.

Optimal T-bond hedge ratios were obtained for the price sensitivity approach by substituting the values that were observed during the week before the period of prediction into the formula derived in Chapter 4. Recall this ratio is equal to the product of the duration ratio, the yield change ratio, and the price ratio between the mortgage and the T-bond future. There is no negative sign because the mortgage represents a liability to the real estate investor. The optimal hedge ratio used for the Canadian dollar contract was simply 4 times the optimal hedge ratio for the T-bond future.

Predictions were made for both the 3 and 6 month hedge periods and both with and without the Canadian dollar futures contract. Results are given in Table V.

First, we shall examine the results of the period from August, 1983 to August, 1984. The price sensitivity approach yielded optimal hedge ratios of -0.614 for the

T-bond future and -2.46 for the Canadian dollar future. These are based on durations of -3.58 years for the mortgage and 8.31 years for the T-bond. The ratio of mortgage value to T-bond futures value was 1.14 The results are most comparable to those obtained in the regressions covering the entire data set excluding the last year.

For the 3 month hedge period, for the strategy without an exchange hedge, both the portfolio and the price sensitivity approaches yielded R^2s of .42. This is even better than the R^2 of .37 obtained in the comparable regression. The addition of the Canadian dollar futures contract increased the R^2 between observed and predicted to .45 using the price sensitivity approach and .46 using the portfolio approach.

For the six month hedge period with no exchange hedge the R^2 between observed and predicted is .42 for both approaches. This compares to an R^2 of .56 for the results of the regression covering the period of prediction. The addition of the Canadian dollar contract improved performance for the six month hedge for both the portfolio approach (R^2 of .64) and the price sensitivity approach (R^2 of .62). These R^2 are almost as high as for the comparable regression results (R^2 of .67).

We shall now examine the results of the period from July, 1985 to July, 1986. The price sensitivity approach yielded optimal hedge ratios of -0.547 for the T-bond future and -2.19 for the Canadian dollar future. These are based on durations of -3.77 years for the mortgage and 8.48 years for the T-bond. The ratio of mortgage value to T-bond futures value was 0.985. The results are comparable to those obtained in the regressions covering the last year of data.

For the three month hedge period, for the strategy without an exchange hedge, both the portfolio and the price sensitivity approaches yielded R^2s of .036. This is exactly the same as the comparable regression's R^2. The addition of the Canadian dollar future improved performance quite a bit. The portfolio approach yielded an R^2 of .18 while the price sensitivity approach yielded an R^2 of .24. However, these are not quite as high as the R^2 .32 for the comparable regression.

For the six month hedge period, the portfolio with no exchange hedge yielded an R^2 of .0004 for both approaches. The comparable regression produced a similiarly poor R^2 of .0007. The addition of the Canadian dollar future improved the R^2 to .07 using the portfolio approach. The price sensitivity approach yielded much better results (R^2 of .22). However, neither approach produced results as good as the comparable regression (R^2 of .45).

The predictive ability of both approaches to hedging is much better for the first test period than for the later one. However, this is to be expected, given the poor performance of regressions covering the last year of data.

The portfolio approach performed slightly better for the period from August, 1983 to August, 1984, while the price sensitivity approach greatly outperformed the portfolio approach for the later time period, especially in the case of the six month hedge. It appears that when the hedge breaks down, as it does for the last year of data, that the price sensitivity approach is not harmed nearly as much as the portfolio approach. However, this was only evidenced for hedges containing the Canadian dollar futures contract. The results with no exchange hedge yielded similiarly poor results for both approaches.

In all cases the exchange hedge significantly improved hedging performance. The empirical evidence strongly argues for the inclusion of the Canadian dollar futures contract in the hedge portfolio.

For the last year, the poor predictive ability of both approaches can be explained by looking at the relative variability of the changes in the portfolio components on a yearly basis (see Figure 1 and 2). Based upon standard deviations, there is a large divergence in 1986. While changes in the T-bond futures were about twice as volatile in 1986 as they were in 1985, changes in mortgage value were about 25% less volatile, for both the three and six month periods. The standard deviation of Canadian dollar futures value does not change appreciably. In the past, increased volatility of the changes in one future was often offset by decreased volatility of the changes in the other.

For the last year's data, nearly all of the model's predictions of changes in mortgage value were far in excess of actual changes. For example, let's look at the forecasts based upon a three month hedge period, using data beginning in Ocober, 1979, with no exchange hedge. In 10 of 53 cases the change in T-bond futures prices have the same sign as changes in mortgage value (for the hedge to work they must have opposite signs). Also, in 11 cases the hedge overcompensated by more than twice as much (i.e. a CDN$2000 decrease in mortgage value overcompensated by a CDN$4000 increase in T-bond futures value). When this happens, variability of the portfolio increases rather than decreases. A final reason why the hedge is so ineffective during this time period is that in 19 of the 53 observations, the change in morgage value was zero. In these cases, the hedge could only increase the variability of portfolio value.

Theoretically, options would have done far better. Options can only act to significantly increase variability when they rise in value. Thus the large rises in T-bond futures would not have more than offset the benefit of lower mortgage values enjoyed by real estate investors. The put options on T-bond futures never drop below zero in value. Thus the benefit of lower mortgage value is only reduced by the cost of the options. If these options were out-of-the-money when they were purchased, this cost would not have been too great. If mortgage values had risen, and if T-bond futures prices had risen even more, then the investor would gain more from his put options than he would lose on his mortgage. Thus increased relative volatility of futures prices is not a problem for hedgers using options. The cost of the options would, of course, be greater when T-bond futures prices are more volatile. It is unlikely, however, that the market is able to predict future price volatility well enough to entirely offset this benefit. However, reduced relative volatility of futures prices is a problem, since reduced volatility means undercompensation when mortgage values rise.

<u>EXTENSIONS:</u>

The price sensitivity approach tells us that as the prices of the T-bond and Canadian dollar contracts change, the optimal hedge ratio will also change, given that the price of the mortgage (CDN$100,000) is constant. This result is derived from the equation for calculating optimal hedge ratios using the price sensitivity approach. The optimal hedge ratio is equal to the product of the duration ratio, the yield change ratio, and the price ratio between the mortgage and the T-bond future. Thus movements in the T-bond futures price and in the spot exchange rate cause the price ratio to change. As previously noted, the duration ratio and the yield change ratio do not vary much over time. The greatest source of volatility is the price ratio. To eliminate this source of volatility, we can express the hedge ratio in terms of a constant multiplied by the ratio of the price of the futures contract to that of the mortgage. Since beginning mortgage value is always CDN$100,000, this ratio will simply be the futures contract price (in Canadian dollars) divided by 100,000. Thus a 20% decrease in the price of the T-bond futures contract will result in a 20% decrease in the optimal hedge ratio.

The results using this price adjustment are contained in Tables VII to X. First, we shall examine the results with no exchange hedge. For the three month hedge period, the adjustment improves hedging performance (as measure by the R^2) by 17% over the entire data set. It improves performance in each of the subsets tested except for the last year of data, when the R^2s are very close to zero anyway. The proportionate improvement is greater for the entire data set than for any of the subsets. This is to be expected, since the range of futures prices is greater over longer periods of time. This greater range results in wider swings in the ratios of mortgage value to futures prices. The adjustment also tends to stabilize the coefficients over time, although only a slight improvement in stability is evidenced. The results are similiar for the six month hedge period. The R^2 for the entire data set was improved by 20%.

With the addition of an exchange hedge, the effect of the price adjustment is not quite as significant. However hedging performance is still improved. For the entire data set the improvement is 7.2% for the three month hedge, and 9.2% for the six month hedge. There is only a negligible effect on the stability of coefficients.

The improvement in predictive ability was also tested, based on regressions run on the entire data set excluding the last year, and using the portfolio approach. With no exchange hedge, the three month regression still has almost no predictive ability ($R^2 = 0.0312$). However the six month regression yields an R^2 between the observed and predicted values of 0.2158. This represents a 21% improvement over the regression with no price adjustment (i.e. using the portfolio approach).

The addition of an exchange hedge acted to reduce the predictive ability in the absence of a price adjustment. With the price adjustment, the predictive ability is still less, but it is much improved. Although the R^2 for the three month regression is an insignificant 0.0009, it is 125% greater than without the price adjustment. The R^2 for the six month regression is 0.1082, an 52% improvement.

All in all it appears that the price adjustment is worthwhile. It improves the hedging performance in almost every case. In addition, it allows hedge ratios derived from the past to be more easily applied to the future.

TABLE I
3 Month Hedge Period
No Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	T-Bond Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	337	96.1 (0.39)	-0.322 (-11.3)	.2767	.2745
10/79 - 10/81	92	1989.1 (3.10)	-0.313 (-4.55)	.1871	.1781
11/81 - 7/85	192	-1012 (-3.73)	-0.454 (-10.9)	.3846	.3814
7/85 - 7/86	53	-754.0 (-2.55)	-0.032 (-1.38)	.0359	.0170
10/79 - 7/85	284	-239.1 (-0.90)	-0.456 (-13.0)	.3745	.3723

[a]Figures in parentheses under estimated constant and coefficients are t-ratios.

TABLE II
6 Month Hedge Period
No Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	T-Bond Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	323	278.3 (0.89)	-0.337 (-12.6)	.3302	.3281
10/79 - 10/81	79	1522 (1.67)	-0.604 (-6.06)	.3229	.3141
11/81 - 7/85	191	-1610 (-5.16)	-0.508 (-13.7)	.4980	.4953
7/85 - 7/86	53	-2047 (-4.58)	-0.004 (-0.18)	.0007	-.0185
8/84 - 7/86	78	1504 (1.64)	-0.602 (-6.00)	.3214	.3125
10/79 - 7/85	271	-519.8 (-1.78)	-0.620 (-18.3)	.5558	.5541

[a]Figures in parentheses under estimated constant and coefficients are t-ratios.

TABLE III
3 Month Period
With Exchange Spread Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	T-Bond Coefficient[a]	Cdn. Spread Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	337	-57.6 (-0.24)	-0.361 (-12.3)	-4.55 (-4.19)	.3129	.3088
10/79- 10/81	92	1586 (2.17)	-0.365 (-4.44)	-2.43 (-1.15)	.1990	.1810
11/81- 7/85	192	-1062 (-4.03)	-0.483 (-11.7)	-5.09 (-3.63)	.4247	.4186
7/85 - 7/86	53	-848.3 (-3.09)	-0.059 (-2.53)	-3.72 (-2.96)	.1755	.1432
10/79 - 7/85	284	-438.5 (-1.69)	-0.506 (-14.2)	-5.12 (-4.53)	.4171	.4129

[a]Figures in parentheses under estimated constant and coefficients are t-ratios.

TABLE IV
3 Month Hedge Period
With Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	T-Bond Coefficient[a]	Cdn. Dollar Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	337	-572.8 (-2.52)	-0.258 (-9.91)	-1.081 (-9.65)	.4343	.4309
10/79- 10/81	92	109.4 (0.20)	-0.398 (-7.43)	-2.23 (-8.10)	.5321	.5216
11/81- 7/85	192	-2001 (-7.31)	-0.260 (-5.78)	-0.979 (-7.45)	.5243	.5193
7/85 - 7/86	53	-664.1 (-2.65)	-0.040 (-2.05)	-0.459 (-4.61)	.3236	.2965
10/79 - 7/85	284	-944.1 (-3.81)	-0.371 (-11.4)	-1.098 (-8.86)	.5111	.5077

[a]Figures in parentheses under estimated constant and coefficients are t-ratios.

TABLE V
6 Month Hedge Period
With Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	T-Bond Coefficient[a]	Cdn. Dollar Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	323	-1388 (-4.47)	-0.266 (-11.1)	-1.172 (-10.7)	.5066	.5035
10/79- 10/81	79	-132.9 (-0.15)	-0.435 (-4.46)	-1.75 (-4.36)	.4582	.4439
11/81- 7/85	191	3401 (13.7)	-0.384 (-14.2)	-1.106 (-14.4)	.7612	.7587
7/85 - 7/86	53	-2051 (-6.12)	-0.016 (-1.00)	-0.507 (-6.42)	.4474	.4257
8/84 - 7/86	78	-174.9 (-0.19)	-0.431 (-4.38)	-1.77 (-4.38)	.4595	.4451
10/79 - 7/85	271	-2021 (-7.08)	-0.523 (-17.4)	-1.078 (-10.5)	.6848	.6824

[a]Figures in parentheses are t-ratios.

TABLE VI
Predictive Ability
3 and 6 Month Hedge Periods
With and Without Exchange Hedge

Explanatory Variables	Hedge Period	R^2s: 8/83 - 8/84 Price	Portfolio	R^2s: 7/85 - 7/86 Price	Portfolio
T-bond	3 mos.	.4216	.4216	.0359	.0359
T-bond, Cdn Dollar	3 mos.	.4547	.4581	.2406	.1779
T-bond	6 mos.	.4208	.4208	.0004	.0004
T-bond, Cdn Dollar	6 mos.	.6213	.6369	.2162	.0710

R^2's are between observed and predicted values
Price - using the Price Sensitivity (Duration based) approach
Portfolio - using the Portfolio (Regression based) approach

FIGURE 1
Relative Variability of Portfolio Components
3 Month Hedge Period

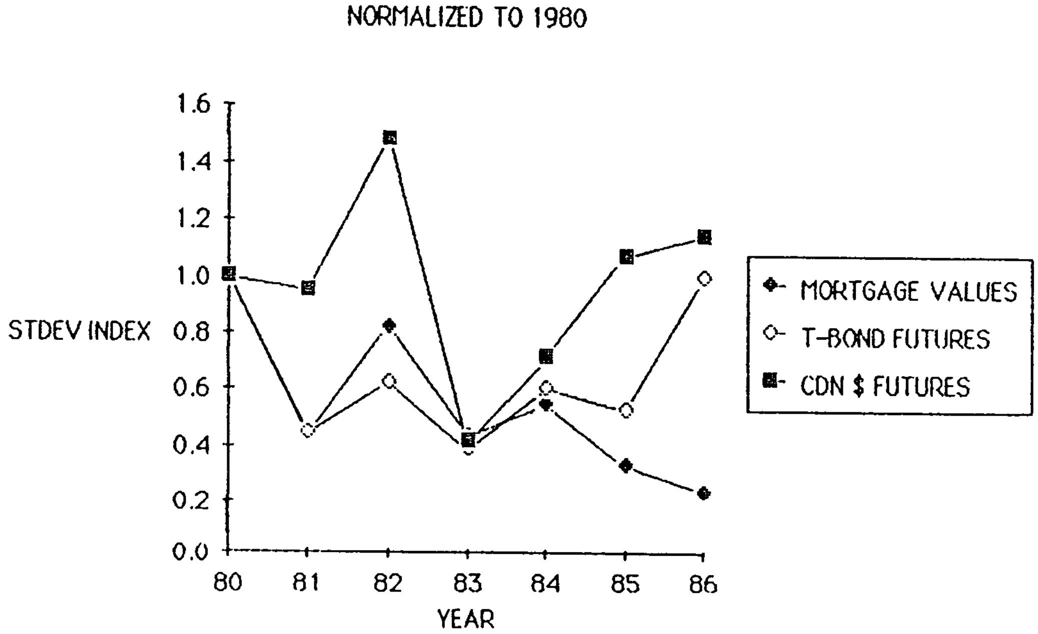

FIGURE 2
Relative Variability of Portfolio Components
6 Month Hedge Period

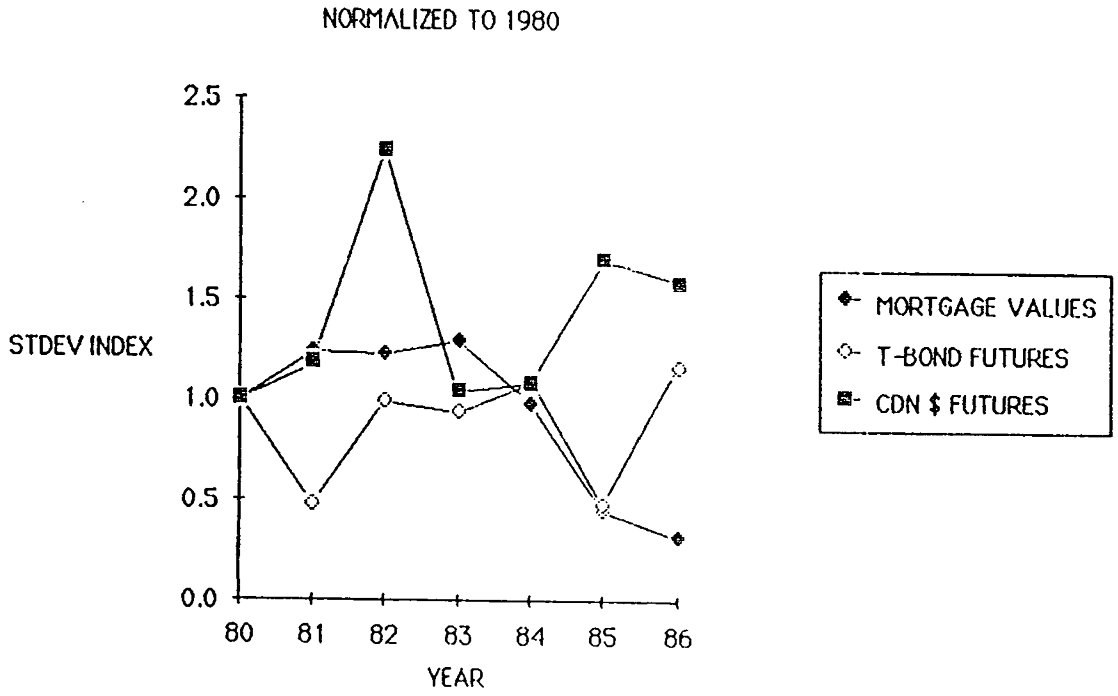

TABLE VII
3 Month Hedge Period
No Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	Adj T-Bond Coefficient[a]	$\bar{R}^2$	R^2
10/79 - 7/86	337	114.8 (0.49)	-0.322 (-12.7)	.3240*	.3220*
10/79 - 10/81	92	2002.9 (3.20)	-0.284 (-4.87)	.2084*	.1996*
11/81 - 7/85	192	-950.5 (-3.50)	-0.385 (-11.0)	.3906*	.3874*
7/85 - 7/86	53	-788.6 (-2.61)	-0.337 (-1.27)	.0303	.0117
10/79 - 7/85	284	-148.9 (-0.57)	-0.395 (-13.3)	.3865*	.3844*

[a]Figures in parentheses are t-ratios.
*higher than without the price adjustment

TABLE VIII
6 Month Hedge Period
No Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	Adj T-Bond Coefficient[a]	$\bar{R}^2$	R^2
10/79 - 7/86	323	360.6 (1.17)	-0.352 (-14.5)	.3962*	.3943*
10/79 - 10/81	79	1245 (1.37)	-0.593 (-6.86)	.3793*	.3712*
11/81 - 7/85	191	-1569 (-4.82)	-0.431 (-13.7)	.4975	.4949
7/85 - 7/86	53	-2099 (-4.47)	-0.006 (-0.26)	.0013*	-0.018
8/84 - 7/86	78	-734.5 (-1.54)	-0.111 (-3.92)	.1330	.1244
10/79 - 7/85	271	-358.4 (-1.24)	-0.520 (-18.7)	.5656*	.5640*

[a]Figures in parentheses are t-ratios.
*higher than without the price adjustment

TABLE IX
3 Month Hedge Period
With Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	Adj T-Bond Coefficient[a]	Adj Cdn. D Coefficient[a]	$\bar{R}^2$	R^2
10/79 - 7/86	337	-531.2 (-2.40)	-0.261 (-11.1)	-1.034 (-9.41)	.4657*	.4625*
10/79- 10/81	92	455.7 (0.82)	-0.361 (-7.35)	-1.84 (-6.10)	.5324*	.5201
11/81- 7/85	192	-1962 (-7.07)	-0.221 (-5.78)	-0.973 (-7.30)	.5245*	.5195*
7/85 - 7/86	53	-637.2 (-2.50)	-0.049 (-2.20)	-0.480 (-4.83)	.3349*	.3088*
10/79 - 7/85	284	-857.7 (-3.46)	-0.321 (-11.5)	-1.074 (-8.63)	.5151*	.5116*

[a]Figures in parentheses are t-ratios.
*higher than without the price adjustment

TABLE X
6 Month Hedge Period
With Exchange Hedge
Present Value of Change in Mortgage Payments

Time Period	No. of Observs	Constant[a]	Adj T-Bond Coefficient[a]	Adj Cdn. D Coefficient[a]	R^2	$\bar{R}^2$
10/79 - 7/86	323	-1254 (-4.23)	-0.274 (-13.0)	-1.12 (-10.6)	.5534*	.5506*
10/79- 10/81	79	-337.1 (-0.37)	-0.448 (-5.24)	-1.68 (-4.19)	.4956*	.4823*
11/81- 7/85	191	-3337 (-13.1)	-0.312 (-13.9)	-1.107 (-14.1)	.7565	.7539
7/85 - 7/86	53	-2012 (-5.76)	-0.024 (-1.41)	-0.54 (-6.55)	.4576*	.4363*
10/79 - 7/85	271	-1869 (-6.56)	-0.438 (-17.7)	-1.065 (-10.3)	.6896*	.6873*

[a]Figures in parentheses are t-ratios.
*higher than without the price adjustment

CHAPTER 10: OPTIONS STUDY

This is an examination of how effective put options on T-bond futures would have been in protecting against rising mortgage values over all three month hedge periods between February and July of 1984. During this time Canadian mortgage rates rose between 1.25 and 2.00 percentage points. This translates to an increase in the value of mortgage debt of between 4.1% and 6.5%. This period contains the only significant increases in mortgage rates since the options began trading in 1983. Canadian dollar futures options did not begin trading until June 1986 and thus cannot be directly tested.

There are 118 observations in all. Each observation consists of the change in mortgage value, the change in option value (converted to Canadian dollars using the appropriate spot rate), and the position of the option's strike price relative to the future (i.e. in-the-money, etc). The position of the option's strike price relative to the future will hereafter be referred to as "relative position". The observations are grouped according to relative position when the hedge began. "Far in-the-money" options have a strike price three to five U.S. dollars above the futures price. "In-the-money" options have a strike price one to three dollars above it. If the strike price is within one dollar of the futures price, it is "at-the-money". "Out-of-the-money" and "Far out-of-the-money" options are the same amount out of the money as the in-the-money options are in. The options are grouped in (U.S.) two dollar increments because the option strike prices are in two dollar increments.

For each test, the change in portfolio value is equal to the change in option value times the number of option contracts held, less the change in mortgage value.

The options strategy will not exactly offset changes in mortgage value, even if the futures hedge is perfect. This is because the price of each option contains a premium over exerciseable value. This premium erodes over time, so that, all other things being equal, the passage of three months will result in a decrease in this premium. In fact, this premium will erode simply by the option moving more and more into the money. Thus we would expect the gain on the option to be something less than the gain on the future. But as the option moves more deeply into the money, the ratio of the marginal increase in the option price, to the marginal decrease in the T-bond futures price, will approach one. Thus for very large increases in interest rates the optimal hedge ratio for options approaches that for futures. Since the main reason for employing an options hedging strategy is to protect against large rises in mortgage value (in particular those large enough to cause insolvency), the hedge ratios tested for the options are equal to those derived for the futures.

The further in the money is the option employed in the hedging strategy, the greater will be the reduction in portfolio value variability. However, there are disadvantages. Options which are further in the money cost more, and thus require a greater cash outlay. In-the-money put options lose a greater amount when rates fall, and this exposure is increased as you get more and more in the money. At the limit, a very deep in-the-money option will behave similiarly to a future (except that the entire option value must be paid up front). Thus the investor is faced with a trade off. The greater the protection against increases in mortgage value, the greater the initial cost, and the greater the exposure to losses on the options when mortgage value falls. If the main reason for pursuing an options hedging strategy is to prevent insolvency, it is recommended that deep out-of-the-money options be purchased. The cost will be relatively small (about 0.17% of mortgage value per 3 month period or about half a percent per year if the option value goes to zero).

Test of Price Sensitivity Approach:

The first test uses an optimal hedge ratio based on the price sensitivity approach adjusted for relative prices. Over the period tested, the median ratio of mortgage value to T-bond futures contract value was 1.15. This yields 0.64 as the optimal number of put option contracts (0.56 X 1.15). During the period tested, the average increase in mortgage value was $5447 (representing approximately a 1.75% increase in mortgage rates at these levels). The option strategy reduced the average change in portfolio value for all relative positions (see Table XI). The results of this test, as with all others, verify the conjecture that the more in the money the options are, the greater the proportion of the change in mortgage value which will be offset. In this case, the average percentage reduction ranged from 40% for the far out-of-the-money options to 75% for the far in-the-money options. A 100% reduction would mean that the average change in portfolio value has been reduced to zero. A greater than 100% reduction implies that the gain on the options exceeded the increase in mortgage value. A negative reduction (which was never encountered) indicates that there was a loss on the options in addition to the increase in mortgage value.

Out-of-the-money put options are designed to increase significantly in value when the futures price falls below the strike price. If a real estate investor purchases out-of-the-money options he is essentially opting for an increased "deductible" in his mortgage rate "insurance". Thus we should not measure the hedging ability of out-of-the-money options against a zero variability in portfolio value standard. We must deduct the amount the T-bond futures price must decrease before the option is at-the-money. Thus for the out-of-the-money options 2000 U.S. dollars were subtracted from the change in mortgage value. The average decrease in T-bond futures prices were too small to adequately measure the effectiveness of the far out-of-the-money options. For the first test, the adjusted average reduction in the variability of portfolio value is 94.2% for out-of-the-money options and 492% for far out-of-the-money options. The adjusted figures for far out-of-the- money options are highly subject to distortion due to the small size of the denominator ($5447 - $5000 = $447).

Tests of Portfolio Approach:

For the second test, the optimal number of option contacts was derived using the portfolio approach. The regression used was based upon the entire data set from October 1979 to July 1986, and used the price adjusted change in T-bond price as the explanatory variable. The magnitude of the coefficient (0.322) is multiplied by the relative price adjustment of 1.15 used in the previous test, to arrive at an optimal hedge ratio of 0.37. Average variability reduction was between 24% and 44%. The average adjusted reduction was 55% for out-of-the-money options and 286% for far out-of-the-money options.

The third test is also based upon the portfolio approach, but uses the second data subset (November 1981 to July 1985). The coefficient of -0.385 results in an optimal hedge ratio of 0.443. The average reductions range from 28% to 52%. Average adjusted deductions are 65% for out-of-the-money options and 342% for far out-of-the-money options.

The option strategy appears to work quite well. A significant portion of the risk was reduced. In addition, in no case was there a loss on the option. Thus at least from February to July of 1984, T-bond prices moved in proportion to mortgage values.

The optimal hedge ratio decreases for bigger increases in mortgage rates and bigger decreases in T-bond futures prices. This can be seen by the apparent overcompensation for far out-of-the-money options. If the increase in mortgage values had been greater, the denominator in the adjusted measure would be much larger, and the percent reduction would be greatly reduced. Since a perfect hedge would give a 100% reduction in the variability in porfolio value, this is the target percentage reduction. Thus if the increase in mortgage value is greater than that witnessed in 1984, the adjusted percentage reduction for far out-of-the-money options would be much closer to 100%. To hedge against a very large increase in mortgage rates, the optimal hedge ratio for options will be the same as that for futures. However for anything but very large increases, applying the futures hedge ratio to options will result in less than full protection. Thus it is advised that a number 30 to 50 percent greater be used. If the increase is large, the number of option contracts held can be reduced by selling them during the hedge period. Even if they are not sold, the worst that will occur is a larger than expected gain on the options. Of course, a greater number of contracts will require a larger initial payment, and if mortgage rates do not rise much, or if they fall, then the cost of this strategy will rise by 30 to 50 percent.

The reader must be careful to remember that the results of this option study are applicable only to periods of moderate mortgage rate increases. The hedging performance of T-bond futures options for changes in mortgage rates greater than 2 percentage points or less than 1.25 percentage points was not documented. The findings support the use of these options as an effective hedging tool. However, this tool can only reduce interest rate risk, not eliminate it.

TABLE XI
Results of Option Study
February to July 1984

Relative Position (No. of Obs.)	Far In The Money (19)	In The Money (24)	At The Money (26)	Out of The Money (26)	Far Out of The Money (23)
Test #1 (price sensitivity approach) No. of Contracts = 0.64					
Average Reduction*	75.4%	71.1%	63.1%	51.0%	40.4%
Adjusted**	-	-	-	94.2%	492%
Test #2 (portfolio approach, whole data set) Number = 0.37					
Average Reduction*	43.7%	41.8%	36.5%	29.5%	23.5%
Adjusted**	-	-	-	54.5%	286%
Test #3 (portfolio approach, 11/81 to 7/85) Number = 0.443					
Average Reduction*	52.3%	49.7%	43.7%	35.3%	28.1%
Adjusted**	-	-	-	65.2%	342%

* $\dfrac{\text{change in portfolio value without hedge - change with hedge}}{\text{change in portfolio value without hedge}}$

** $\dfrac{\text{change in portfolio value without hedge - change with hedge}}{\text{change without hedge - (distance out of the money X 1000)}}$

CHAPTER 11: COSTS AND BENEFITS OF HEDGING WITH OPTIONS

The largest cost of an options hedging strategy is the cost of the options themselves. However, this is not simply an expense, since the expected return on the option is the risk free rate. Thus money spent on purchasing options can be expected to generate some returns, and the magnitude of these returns may in fact exceed that of the option costs.

The largest expense of an options hedging strategy is transaction costs. Using a full service broker creates round turn commissions (sum of buy and sell costs) of between 6% and 25%. The more in-the-money an option is and the larger the number of contracts purchased, the lower will be the commission costs, on a percentage basis. Average commissions for at-the-money options are about 8% round turn. Savings with discount brokers can be up to 80% (i.e between 1% and 10%, averaging about 2%). If option purchase costs represent 2% of the value of a mortgage, and transaction costs represent 2% of the option price, then transaction costs amount to approximately 0.04% of mortgage value.

The greatest benefit of an options hedging strategy is the interest rate savings over the hedge period. This will be the case unless rates rise rather dramatically. Over the period November 4, 1981 to July 30, 1986 (since the Canadian Imperial Bank of Commerce offered variable rate mortgages), 5 year fixed rate mortgages carried an annual interest rate averaging 1.52% more than the corresponding variable rate mortgage. Thus over a 3 month hedge, average interest savings amount to 0.38% of the outstanding morgtgage balance, while for a six month hedge savings average 0.76%. Interest savings greatly outweigh transaction costs (by at least 9 times).

CHAPTER 12: CONCLUSION

The results of the regressions indicate that there is a statistically significant relationship between changes in mortgage value and the explanatory variables. However, this relationship is far from perfect; probably too far to be of interest to real estate investors. As previously stated, the theoretical basis for this relationship rests upon two major assumptions. First, 5 year mortgage rates in Canada must move in step with Canadian T-bill rates. Second, 15 year Treasury bond rates must move in step with American T-bill rates. A priori, it was known that yield curves in both the U.S. and Canada are not always flat. However it was not known how greatly this would affect the results. Apparently this had a tremendous effect. We cannot expect this problem to be solved until the scope of futures markets is greatly expanded. Contract dates must be extended far into the future. Then, and only then, will the hedge be completely effective, due to the applicability of interest rate parity.

Currently, T-bond futures options offer some mortgage rate risk reduction for holders of variable rate mortgages in Canada, although the size of this reduction appears to be quite limited. Canadian dollar futures improved hedging performance in the past, despite the fact that interest rate parity does not hold. The inclusion of Canadian dollar futures or futures options in the hedge portfolio is recommmended.

The option study points to the conclusion that the T-bond futures option hedge works best in protecting against large rises in mortgage rates. Thus a strategy of using out-of-the-money put options is recommended.

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GLOSSARY OF VARIABLE NAMES

T	years to maturity of financial instrument
C_t	payment promised at date t
P	current price of a financial instrument
r	annual yield to maturity
D	duration of a financial instrument
b	subscript denoting that variable pertains to a T-bond
m	subscript denoting that variable pertains to a mortgage
num	optimal number of futures (or option) contracts
USTB	U.S. T-bill(s)
CTB	Canadian T-bill(s)
CD(0)	near Canadian dollar future(s)
CD(1)	far Canadian dollar future(s)
t_{now}	date when riskless investment in foreign financial instrument is contracted for
t_{start}	date of purchase of foreign financial instrument
t_{unwind}	maturity date of foreign financial instrument
FRMV	fixed rate mortgage value
TBF	U.S. traded T-bond futures price
DELMOR	change in present value of mortgage payments
DELTB	gain on the T-bond future(s)
DELCD[0]	gain on the near Canadian dollar future(s)
DELCD[1]	gain on the far Canadian dollar future(s)

APPENDIX

PROBLEMS WITH AUTOCORRELATION

Every regression run in this study suffered from the problem of positive autocorrelation of error terms. This can cause misspecification of coefficients, overestimation of t-ratios, and R^2s which are too high.

Autocorrelation was detected by Durbin-Watson statistics, which ranged from 0.07 to 1.13. They were all below d_L at a 1% level of significance.

The most widely used method to correct for autocorrelation is the iterative Cochran-Orcutt technique. This transforms all observations by subtracting a certain proportion of the previous observation, p (rho), to arrive at the transformed variables ($0 \leq p \leq 1$). The Cochran-Orcutt method estimates p, runs the regression on the transformed data, re-estimates p, and runs the regression again. Each time the sum of squared errors (SSE) is noted, and the technique continues until the reduction in SSE is below some (small) number. However, this method is only capable of correcting for first order autocorrelation. As I shall explain, the autocorrelation encountered in this thesis is of a higher order. Because of this, the Cochran-Orcutt method failed to increase the Durbin-Watson statistic sufficiently. In addition, the consistently high estimates of p yielded coefficients which were ridiculously low (magnitude less than 0.1) and R^2s which are far too high (often exceeding 0.9). Thus the Cochran-Orcutt method did not work.

The most probable cause of autocorrelation is information sharing between observations. Each observation represents the difference between two values. These two values are based upon the same security, but at different points in time (either 3 or 6 months apart). For instance, in the case of a three month hedge, one of the variables in the first observation is the difference between mortgage values at week 1 and week 14. The next observation contains the difference between mortgage values at week 2 and week 15. It is obvious that if the first observation is, for example, strongly positive, the next one is also likely to be strongly positive. This is because each observation is composed of 13 weekly differences. Twelve of these differences are contained in each adjacent observation. Thus the first and second observations will both contain the difference in value between week 2 and week 13. The first observation will also contain the difference between week 1 and week 2, while the second observation also contains the difference between week 13 and week 14.

To eliminate the bias caused by this "information sharing", the data was divided into 13 groups, each containing observations with mutually exclusive differences (see Table A-1). Thus group 1 contained the differences between weeks 1, 14, 27 etc., group 2 contained the differences between weeks 2,15, 28 etc., and so on. Regressions were run on each group, for the entire data set, for both 3 and 6 month hedges, and with and without an exhange hedge. In all cases the Durbin-Watson improved sufficiently to reject the hypothesis of autocorrelation. Also, coefficients were not affected, with the median coefficient always coming very close to those estimated previously. R^2s were not affected either. Thus we can conclude that the "information sharing" prevalent in the data does not cause misspecification of regression results. Although autocorrelation is present in the primary results, the similiar results using mutually exclusive data, and the lack of autocorrelation in the regressions using mutually exclusive data, confirm that the autocorrelation in the primary results do not affect the coefficients or R^2s.

TABLE A-1
Durbin Watson (D-W) Statistics
For Mutually Exclusive 3 Month Hedge Periods
and All 3 Month Hedge Periods
No Exchange Hedge
October, 1979 to July, 1985

Group No	No of Observs	Constant[a]	T-Bond Coefficient[a]	R^2	D-W
1	26	296 (0.32)	-0.313 (-3.01)	.2741	2.48
2	26	240 (0.22)	-0.296 (-2.41)	.1942	2.67
3	26	325 (0.32)	-0.373 (-3.16)	.2941*	2.59
4	26	415 (0.48)	-0.468 (-4.73)	.4824	2.37
5	26	226 (0.27)	-0.366 (-4.01)	.4009	2.45
6	26	156 (0.19)	-0.390 (-4.19)	.4223	2.21
7	26	25 (0.03)	-0.350* (-3.61)	.3524	2.24
8	26	59 (0.07)	-0.383 (-3.90)	.3884	2.23
9	23	-112 (-0.12)	-0.379 (-3.32)	.3448	2.16
10	26	-113 (-0.13)	-0.279 (-2.61)	.2205	1.96
11	26	-170 (-0.20)	-0.192 (-1.95)	.1373	1.82
12	27	-222 (-0.25)	-0.133 (-1.32)	.0659	2.16
13	27	46 (0.05)	-0.258 (-2.13)	.1548	2.59
All	337	96 (0.39)	-0.322 (-11.3)	.2767	0.17

* denotes the median value
a figures in parentheses represent t-ratios