OPTIMAL ALLOCATION OF 'BOD' LOADINGS

IN A TIDAL RIVER

By

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ABSTRACT

A methodology is presented in this thesis which addresses the water quality manager's problem of maximizing the biochemical oxygen demand loadings in reaches of a tidal river, subject to dissolved oxygen concentration regulation at various compliance levels. The non-linear tidal dynamics and BOD-DO processes were explicitly accounted for by numerical, finite-difference models incorporated as equality and inequality constraints of a non-linear programming problem solved by a direct search algorithm. This methodology was applied to the Nicomekl River in Surrey B.C., to investigate policy implications and its applicability, using a microcomputer, to the resolution of actual pollution management problems.

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INTRODUCTION

As multiple uses of water resources within a river basin become more competitive, the management of the resource increases in complexity often aggravated by insufficient and uncertain field information and inadequate control mechanisms. Another order of complexity lies in the dynamic nature of the system being managed. The physical, chemical and biological processes need to be well understood if decisions made at various control levels and in various jurisdictions are to be made with increased confidence.

The pollution of downstream river and groundwater by upstream agricultural enterprises has been a well documented, increasing phenomena in intensively developed watersheds. (Tubbs and Haith, 1981)

The river basin manager requires timely an accurate information on the state of the system (Min. of Env., 1984). This information is often obtained from data gathering networks but increasingly from the judicious use of well validated mathematical models which predict the response of the system to management control acting as amplifiers of field data.

The Nicomekl-Serpentine River Basin in Surrey, B.C. is primarily used for agriculture which is a significant contributor of point ans non-point organic pollution. The impact of these animal wastes in greatest on the fishery resource which also uses the river for spawning (Moore, 1985a). This tidal watershed is a complex ecosystem where various water quality parameters could typify its various states. The choice of these state variables is a non-trivial task especially with the multitude of pollutant constituents.

various states and to develop a mathematical model. The choice of state variables is a non-trivial task especially with multiple pollutants (fertilizer, herbicides and animal wastes). Additionally, optimal water quality controls need to be based on the existing jurisdictional mandates in the region for maximum effectiveness and enforcement.

The problem addressed in this study is viewed from a management perpective of determining the to optimal organic loading of a tidal river given its capacity for dilution and transport of pollutants. Specifically, the objective would be to maximize the pollutant loadings along the river subject to water quality regulations. For this study, the basic parameters chosen to describe the state of the system were the organic biochemical oxygen demand (BOD*) and the resulting dissolved oxygen (DO*) concentrations in the river due to waste loadings. This reduces the dimensions of the problem to two manageable variables for the puposes of this study. A more extensive effort based on adequate field data would be required to address the multidimensional nature of the major pollution processes involved. The use of mathematical models in conjunction with specific management function models (as opposed to research models) is gaining greater justification in the literature (Beck, 1985), and is the approach adopted here.

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* Throughout this study, these two parameters will be referred to as BOD and DO respectively.

The specific objectives of this study were:

1) A survey of the literature in the areas of water quality modeling for management purposes;

2) Develop a water quality model to account for the major dynamic processes of hydrodynamics and BOD/DO pollution in a tidal river;

3) Develop a programming model for the puposes of identifying the optimal BOD loadings subject to regulatory constraints for the purpose of management policy formulation.

The literature was surveyed in chapter 2 describing the development and application of models to the BOD/DO water quality problem. Chapter 3 developed the BOD/DO and programming model used to generate optimal policies in this study. The BOD/DO model is validated in chapter 5 and applied in chapter 6, while chapter provides background site information. Discussion and conclusions follow in chapters 7 and 8 including suggestions for further work.

2. LITERATURE SURVEY

2.1 INTRODUCTION

The application of systems engineering techniques to the solution of quality problems of water bodies is an ongoing area of research. Reviews of the field (for example the Journal of the Water Pollution Control Federation annual review) document the large effort in academia, government and industry in the development of suitable techniques and methodologies in resolving user conflicts and improving the management of water bodies. This growing portfolio of techniques can be used to assist a manager rationalize suitable control strategies, at a minimum cost of information and control, based on knowledge of physical processes within the water resource system.

Models of complex systems are both difficult to build and to validate but are necessary if a system is to be multi-variably controlled. In other words the model should have the 'requisite variety' (Beer, 1966). However, in most cases field data are lacking and only the use of simplified ecological models, such as a BOD/DO model, can be justified using parameter estimation and cautious field application. The need also exists to explicitly model the stochastic nature of the water quality problem in sampling, management and enforcement of regulations. Ward and Loftis (1983) state: "Water quality involves a mix of deterministic and stochastic concepts, and it can be effectively managed only when both components are properly balanced".

This chapter surveys methods available in the literature in the optimal management and control of river water quality.

2.2 WATER QUALITY MODELS

Beck (1983) proposed the following classification of model types:

i) research versus management models;

where the former provides indicators for fruitful directions for investigation while the latter are for strategic and tactical water quality management.

ii) distributed versus lumped models;

where the former attempts to model processes of individual constituents in a water quality system while the latter aggregates these variations.

iii) non-linear versus linear models;

where in the former an increase of an input is reflected in a non-linear model response whereas the latter predicts a linear response.

iv) stochastic versus deterministic models;

where the former accounts for the random inputs into the system which are ignored by the latter.

v) dynamic versus steady-state models;

where the former models the time dimension of the system while the latter does not.

vi) internally descriptive versus 'black box' models;

where the former deals with a priori information and deductive reasoning based on knowledge of the processes involved while the latter is posteriori and inductive based on observation.

Within this classification we shall survey lumped parameter (in one dimension), nonlinear and linear, stochastic and deterministic, dynamic and steady-state, and internally descriptive models.
Most field applications of BOD/DO models are deterministic, steady-state and one-dimensional while unsteady three-dimensional stochastic ecological models tend to be research oriented.

2.2.1 Steady-State Models

The most common BOD/DO model is that of Streeter-Phelps which assumes that:

a) the BOD decay rate is proportional to BOD concentration;

b) the deoxygenation and BOD decay rate are equal;

c) the reoxygenation rate is proportional to the oxygen deficit;

d) BOD and DO are sufficient to describe the biochemical process,

resulting in the following non-linear model:

$$\frac{\partial b}{\partial t} + \frac{v\partial b}{\partial l} = -k_1 b \qquad (2.1a)$$

$$\frac{\partial c}{\partial t} + \frac{v\partial c}{\partial l} = -k_1 b + k_2(C_s - c) \qquad (2.1b)$$

with boundary conditions: $b(0,t) = b_o(t)$ and $c(o,t) = c_o(t)$,
and initial conditions: $b(l,0) = b_i(l)$ and $c(l,0) = c_i(l)$,
THe boundary conditions can vary in any manner from impulse functions through sinusoidal functions to steady-state values. This is also true for the initial conditions along the stretch of the river considered.
In equations 2.1:

$b(l,t)$ - the (ultimate) BOD concentration in the river $[M.L^{-3}]$

$c(l,t)$	- the DO concentration	$[M.L^{-3}]$
t	- is the time dimension	$0 \leq t \leq \tau$
l	- is the space dimension	$0 \leq l \leq L$
v	- is the average stream velocity	$[L.T^{-1}]$
k_1	- the deoxygenation coefficient	$[T^{-1}]$
k_2	- the reaeration coefficient	$[T^{-1}]$
C_s	- saturation DO concentration	$[M.L^{-3}]$

The last three parameters are temperature dependent as $k_T = k_{20}\theta^{(T-20)}$. The BOD is normally evaluated as BOD_n, where n is usually taken as five days and $BOD_n = (1- e^{-K_L n})BOD_{ult}$ with K_L the BOD decay coefficient derived from standard laboratory tests. Solution of equations 2.1 with steady-state assumptions ($b_o(t)=b_o, c_o(t)=c_o$, $\partial b/\partial t = 0$, $\partial c/\partial t = 0$) yields the classical Streeter-Phelps equation:

$$b(T) = b_o\, e^{-k1T} \tag{2.2a}$$

$$c(T) = C_s-(C_s-c_o)e^{-k2T} + \frac{b_o(e^{-k1T}-e^{-k2T})}{k_1-k_2} \tag{2.2b}$$

where $dl/dT = v$ and b_o results from both carbonoceous and nitrogeneous BOD loadings and k_1 is a composite deoxygenation coefficient. Alternatively, b(T) could be computed as $b_{carb}(T)$ and $b_{nitr}(T)$ respectively. Equation 2.2 describes the oxygen-sag curve shown in figure 2.1.

Dobbins (1964) proposed modifications based on the following, more explicit, assumptions:

a) the stream flow is steady and uniform;

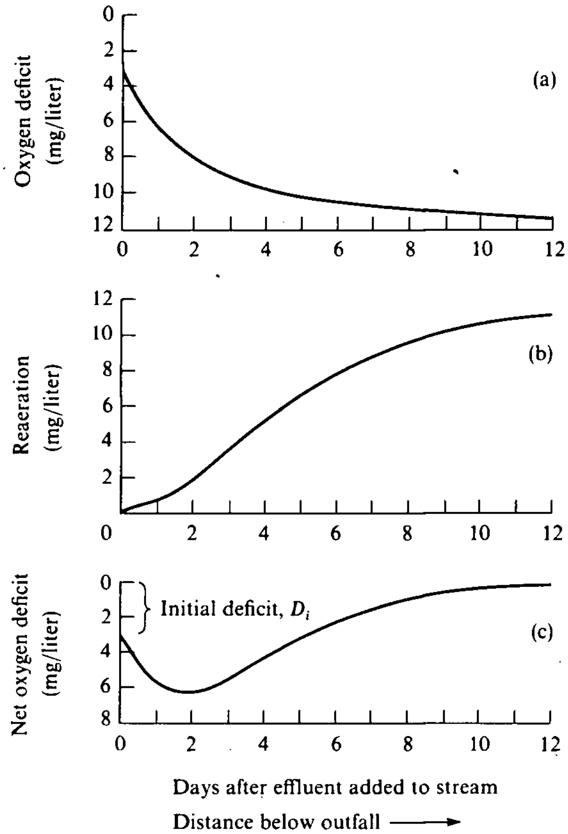


Figure 2.1 Components of the oxygen sag curve: (a) Deoxygenation, the rate of which is proportional to the concentration of remaining BOD in the water, therefore declines as the oxidizable material is consumed. (b) Reaeration, the rate of which is proportional to the oxygen deficit at the time. (c) Net oxygen deficit obtained by subtracting reaeration from deoxygenation. The form of the curve leads to its being called the "oxygen sag."

(Dunne and Leopold, 1978)

b) the process for the stretch or river as a whole is steady-state, the condition at each cross-section being unchanged with time;

c) the removal of BOD by both bacterial oxidation and sedimentation and/or adsorption are first-order reactions, the rates of removal at any cross-section being proportional to the ammount present;

d) the removal of oxygen by benthal demand and plant respiration, the addition of oxygen by photosysnthesis, and the addition of BOD from benthal layers or local runoff are all uniform along the stretch;

e) the BOD and oxygen are uniformly distributed over each cross-section thus permitting the equations to be written in the usual one-dimensional form.

These assumptions suggest the following non-linear steady-state model:

$$D_L\frac{d^2b}{dx^2} - v\frac{db}{dx} + (k_1+k_3)b + b_a = 0 \qquad (2.3a)$$

$$D_L\frac{d^2c}{dx^2} + k_2(C_s-c) - v\frac{dc}{dx} - k_1b - D_B = 0 \qquad (2.3b)$$

where:

D_L - the longitudinal dispersion coefficient $[L^2.T]$

k_3 - coefficient of BOD removal by sedimentation/adsorption $[T^{-1}]$

D_B - net rate of removal of oxygen by benthal demand and photosynthesis $[M.T^{-1}]$

b_a - rate of addition of oxygen along the stretch by artificial aeration [M.T^{-1}]

The solution of equation 2.3 , with negligible longitudinal dispersion, is given as: $b = b_o e^{-(k_1+k_3)} + \frac{b_a}{k_1+k_3}\left(1 - e^{-(k_1+k_3)t}\right)$ (2.4a)

and:

$$C_s - c = k_1 \left(b_o - \frac{b_a}{k_1+k_3}\right)\left[e^{-(k_1+k_3)t} - e^{-k_2 t}\right] + (C_s - c)e^{-k_2 t} + \left(\frac{D_B}{k_2} + \frac{k_1 b_a}{k_2(k_1+k_3)}\right)\left(1 - e^{-k_2 t}\right) \qquad (2.4b)$$

where b_o and c_o are initial BOD and DO concentrations.

A variety of other one-dimensional models have been formulated depending on the assumptions made. For example Shastry et al (1973) proposed a non-linear decay equation as $\frac{db}{dt} = \frac{-k_1 bc}{K_L + b} + k_3 b + b_a$

and $\frac{dc}{dt} = \frac{k_1 bc}{K_L + b} + k_2 (C_s - c) + D_B$

with K_L as a saturation constant. These equations combined never predict negative DO values as does the Streeter-Phelps model.

This class of steady-state models are relatively easy to combine, directly or indirectly, with optimization procedures such as mathematical programming or optimal control and to apply to problems of resource allocation, treatment or regulation where BOD and/or DO loadings could be controlled. The sensitivity of these models and the resulting control policies is dependent on the model parameters chosen. Wen et al (1982) derived sensitivity functions for BOD and DO for a Dobbin's type model to decay rate and dispersion parameters in non-tidal and tidal rivers where the coefficient of longitudinal dispersion was zero and non-zero respectively.

The application of steady-state type models is extensively documented in the literature due to their relative simplicity, and hence greater potential for mis-use, since the correct determination of parameter values is a non-trivial task.

Rickert et al (1976) applied a steady-state model (incorporating nitrogenous BOD demand) to the Willamette River, Oregon to test the impact of management alternatives such as BOD loading allocations, ammonia loadings, low flow augmentation and the reduction of benthos on river DO. Ramm (1976) determined the impact of industrial and domestic wastes of the DO profiles in the navigation channel of the Cuyahoga River. They used a compartment model (where mass balances were conserved over river reaches as opposed to elemental sections) for extensive sensitivity analysis and presented a transfer matrix for use as a loading decision table. Putz et al (1983) used a steady-state transverse mixing model to predict the two-dimensional dispersion and decay of fecal coliforms from a proposed lagoon for Fort Smith on the Slave River, NWT. Their model, validated with steady dye tracer studies, predicted higher than allowed coliform concentrations during winter. Kingscott (1976) used a steady-state model to assess the cost effectiveness of waste load allocations to various treatment unit processes by predicting the resulting river DO due to stochastic (Monte Carlo generated) BOD loadings. Hung et al (1976) presented a modified Streeter-Phelps model used to establish waste load allocations for Indiana rivers by combination with optimization models and probabalistic methods. Yearsley (1976) also discussed Monte Carlo techniques used to generate DO profiles for the Milner reach of the Snake River, Idaho.

The inherent variability in model parameters (recognized and accounted for in the last three references above) has prompted the development of analytic stochastic steady-state models where these parameters are considered to be stochastic variables. Thayer and Krutchkoff (1967) formulated a differential difference equation describing the probabalistic dynamics of BOD and DO interactions which they solved using moment generation functions in deriving joint probability densities. They validated their model by succesfully matching DO variances for the Sacramento River with their predicted values. Padgett and Durham (1976) solved a probabilistic version of Dobbins' model as a set of "random differential equations with random coefficients, random inhomogeneous terms, and random initial conditions' which they solved using the mean square theory approach proposed by Soong (1973). Further work by Padgett et al (1976), Bell and Papadopolous (1978), Padgett and Papadopolous (1978) and Damaskos and Papadopolous (1983) has led to a formulation of a general stochastic BOD/DO model. These models yield stochastic steady-state BOD/DO profiles for various parameter joint distributional assumptions such as normality and independence with stochastic initial and boundary conditions. These models however, do not appear to have been field tested.

2.2.2 Dynamic Models

Deterministic water quality models which describe the physical, chemical and biological processes found in water bodies are widely used in the assessment of impacts of pollutant loadings. However, the application of such models can, as mentioned previously, be severely limited by the lack of field data, the lack of knowledge of the underlying processes, and the lack of advanced modeling techniques. (Mausberger, 1983).

The general mass balance of a constituent in a segment of water is

Rate change of Mass Density	+	Rate of Convection	+	Rate of Diffusion	=	Net Rate of Addition

In one-dimension this reduces to the advection-diffusion equation:

$$\frac{\partial (Ac)}{\partial t} = \frac{\partial}{\partial x}\left(AE\frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial x}(Qc) + AG \tag{2.5}$$

where A - cross-sectional area $[L^2]$

c - concentration of the constituent $[M.L^{-3}]$

Q - river discharge $[L^3.T^{-1}]$

E - effective dispersion coefficient $[L^2.T^{-1}]$

G - source-sink term $[M.T^{-1}]$

t - time $[T]$

x - distance $[L]$

For an organic pollutant (ultimate BOD) to equation takes the form:

$$\frac{\partial (Ab)}{\partial t} = \frac{\partial}{\partial x}\left(AE\frac{\partial b}{\partial x}\right) - \frac{\partial}{\partial x}(Qb) - (k_1+k_3)Ab + Ab_a \tag{2.6}$$

where b - the ultimate first-stage BOD $[M.L^{-3}]$

k_1 - decay coefficient for BOD removal $[T^{-1}]$

k_3 - decay coefficient for BOD removal by sedimentation and adsorption $[T^{-1}]$

L_a - rate of BOD addition along the stream $[M.T^{-1}]$

For dissolved oxygen (DO) the equation is:

$$\frac{\partial(Ac)}{\partial t} = \frac{\partial}{\partial x}\left(AE\frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial x}(Qc) - k_1 Ab + k_2 A(c_s - c) + AD_B \tag{2.7}$$

where: c - DO concentration $[M.L^{-3}]$

c_s - saturated DO concentration $[M.L^{-3}]$

k_2 - reaeration coefficient $[T^{-1}]$

D_B - net rate of DO addition due to photosynthesis, respiration and benthic demand $[M.L^{-3}.T^{-1}]$

(Dresnack and Dobbins, 1968)

Using variations of the above mathematical models, many studies reported in the literature have assumed steady-state hydrodynamics ($Q=Q_o$) while pollutant loadings vary. Pheiffer et al (1976) apply a dynamic BOD model coupled with a quasi-dynamic hydraulic model, which generated stepped streamflow hydrographs, to the Patuxent River Basin to determine the effect of secondary treatment plants on stream DO. Pence et al (1968) apply a dynamic BOD/DO model with steady river flows to the Delaware Estuary to evaluate control schemes to alleviate low DO conditions due to municipal, industrial, tributary and stormwater overflow BOD loadings. Weatherbe (1976) used a similar model in establishing water quality guidelines for the Thames River, England using historical data to generate daily streamflow by 'stochastic' techniques.

Other studies, however, also accounted for the hydrodynamics of the river. Assuming that the interaction between water quality effects and water quantity processes are independent, namely, that constituent concentrations do not affect the density of river water, the model can be decoupled with the hydrodynamics described by the equations of continuity and momentum.

Continuity: (2.8)

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -g\frac{\partial h}{\partial x} - \frac{g|U|U}{c^2 d}$$

where U - stream velocity in direction x $[L.T^{-1}]$

g - acceleration due to gravity $(9.81\ m.sec^{-2})$

c - Chezy coefficient

h - flow level with respect to datum [L]

d - depth of flow [L]

Momentum: (2.9)

$$\frac{\partial (AU)}{\partial x} + b\frac{\partial h}{\partial t} = 0$$

where b - top river width [L]

(Dronkers, 1969).

Tucci and Chen (1981) used the Saint Venant formulation of the equations of continuity and momentum above, coupled with a one-dimensional transport equation in modeling the estuarial river network of the Jacui Delta, Brazil for the purpose of evaluating management options for BOD loadings in the four tributary rivers. Johanson et al (1976) applied a noded coupled unsteady model to the harbour and lower Chahalis River, Washington. They conducted sensitivity analyses of on parameter values while evaluating the DO impact of industrial discharges. Sargent (1976) used plug-flow and completely-mixed coupled models (with zero and non-zero dispersion respectively) in evaluating the impact of water treatment on the Buffalo River. Bansal (1976) applied dynamic and steady-state models to the Big Blue River in Nebraska concluding that the one-dimensional dispersion model was the most satisfactory in verification with field data. Beckers et al (1976) applied coupled models to predict the concentration of phosphorus, the 'nitrogens', coliforms, BOD, DO, chlorophyll and salinity to channels of the Pawtuxet River of Rhode Island where field data was sufficient, concluding that the application of unsteady models was preferred in application to dynamic processes found in estuaries and stormwater runoff.

The parameters in a dynamic model are, as mentioned previously, stochastic in nature either due to the lack of understanding of the phenomena involved or to inherent variability of the natural processes. Since equations 2.6 and 2.9 are solved numerically, Monte Carlo simulation methods are commonly used often with Markovian generation techniques. Ford et al (1980), for example, used such methods in developing 95% confidence envelopes for the prediction of algal concentrations in the proposed Twin Valley lake using the U.S. Corps of Engineers CE-QUAL-RI model as shown in figure 2.2. Coefficient values were independently generated from a variety of probability distributions using recorded mean and variance data. Similar methods were used in a study of the water quality impacts of the proposed B.C. Hydro dams on the Stikine and Liard Rivers of northern British Columbia (Schultz, 1983). Scholl and Wycoff (1981) also applied Markov models in generating coefficient values for their Continuous Stormwater Pollution Simulation System in their analysis of the impact of the Springfield advanced water treatment plant on the James River in Missouri and the identification of the pollutant sources responsible for fish kills.

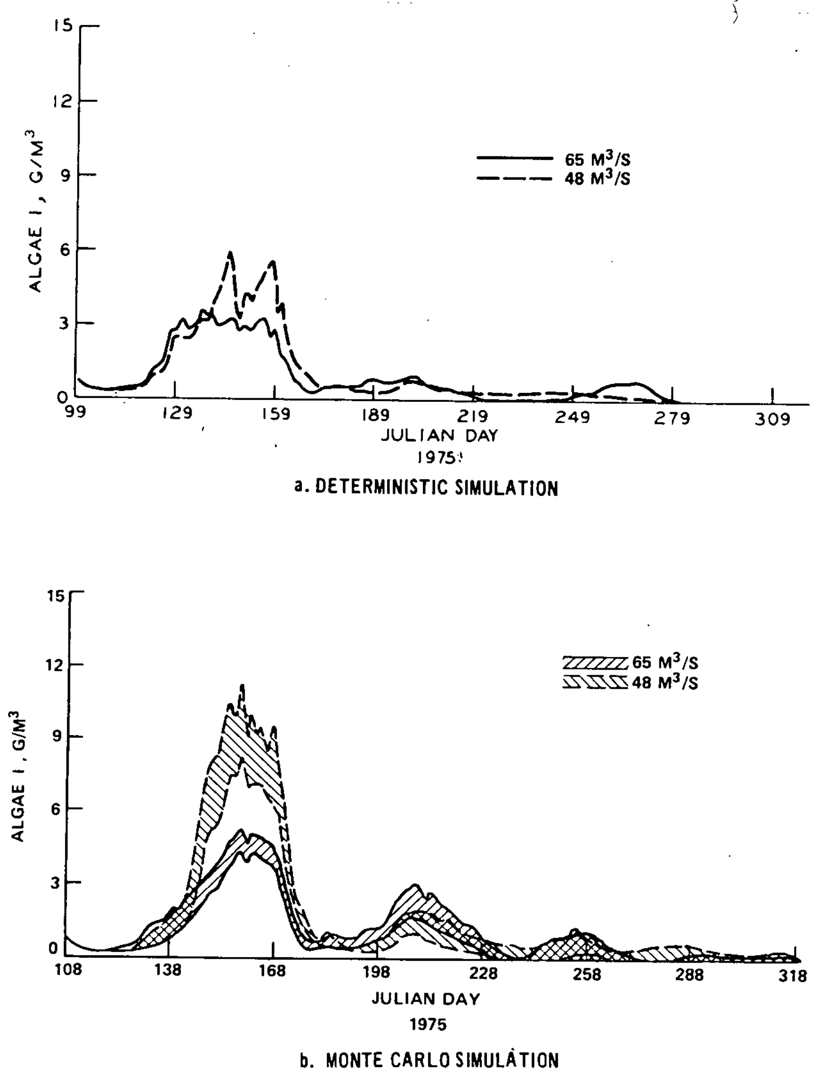


FIG. 2.2 Comparison of Algae Concentrations for Deterministic Simulation and Monte Carlo Generated 95 Percent Confidence Interval for Two Release Rates.

(Ford et al, 1980)

2.2.4 Estimation Techniques

Parameter estimation techniques account for the variability in parameters of correctly identified and verified water quality models (Beck, 1985). The system can be viewed as in figure 2.3 where uncertainty is found in both the processes observed and in the observation process,

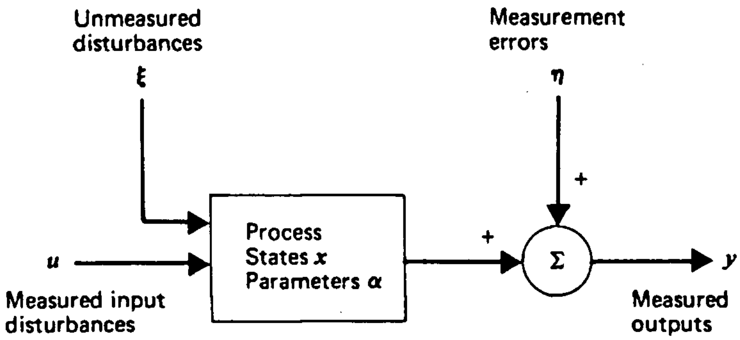


FIGURE 2.3 Definition of the system and associated variables. (Beck, 1985)

where the variations with times of the system state vector are given by

$$\frac{d\underline{x}(t)}{dt} = f\,[\,\underline{x}(t),\,\underline{u}(t),\,\underline{a}(t),\,\underline{e}(t)\,] \qquad (2.10)$$

with sampled discrete-time output observation

$$\underline{y}(t_k) = h\,[\,\underline{x}(t_k),\,\underline{a}(t_k)+\underline{n}(t_k)\,] \qquad (2.11)$$

where $\underline{x}$ = n-dimensional vector of state variables

$\underline{u}$ = m-dimensional vector of measured input disturbances

$\underline{y}$ = l-dimensional vector of (discretely sampled) measured output variables

$\underline{a}$ = p-dimensional vector of model parameters

$\underline{e}$ = s-dimensional vector of random unmeasured (unknown) input disturbances

$\underline{n}$ = l-dimensional vector of random output measurements and f and h are nonlinear, vector-valued functions; t is the independent time dimension and t_k is the k'th discrete sampling instant in time.

(Beck, 1985).

Two major estimation efforts arise: state estimation and parameter estimation whereby optimal estimates of vectors $\underline{x}$ and $\underline{a}$ are made using, for example, maximum liklehood, least squares, Markov and Bayesian estimators. However, a significant problem that arises is that of 'identifiability', whereby a model is chosen which 'best' simulates the output of the system (Rinaldi et al, 1979). Parameter estimation techniques are used both 'off-line' and in 'real-time' to estimate steady as well as varying parameters.

Tamura and Kawaguchi (1980) presented a real-time parameter estimation procedure for BOD parameters in four reaches of the Yomo River, Japan. The procedure was based on a method fitting $\underline{n}$-dimensional time series data to an autoregressive model of finite order, using a steady-state model. Olsen (1980) applied the techniques of dynamic modeling, real-time prediction, estimation and control to the Kappala waste-waster treatment plant, Sweden, concluding that the prediction and estimation methods were probably the most profitable areas for the application of control theory to waste-water treatment systems. Rinaldi et al (1979) reviewed three major estimation applications:

a) parameter estimation of a modified Streeter-Phelps model using a direct search method for a nonlinear programming formulation of minimized squared deviations;

b) state and parameter estimation by the quasi-linearization technique for solving nonlinear multi-point boundary value problems.

Squared deviations, incorporating measurement noise, were minimized using maximum likelihood estimators for the Rhine River;

c) Kalman filtering techniques were applied to state and parameter estimation for the Yomo River, based on a steady-state, distributed-lag model and compared with other filters in terms of efficiency.

These researchers concluded that a majority of water quality studies apply a priori developed models to water quality systems with the purpose of showing their invalidity, rather than using posteriori techniques of estimation to identify the appropriate model. On this same theme Beck (1983, 1985) noted a trend towards posterior analyses as the field of water quality modeling matures. An example of such an analysis is the work of Shastry et al (1973) who used nonlinear estimation techniques to select the most appropriate steady-state model (of three considered) in the simulation of DO in the Sacramento River. Applying least squares and maximum liklehood methods, they estimated both model parameters and initial conditions for the river and noted that the unimodality of the nonlinear response surface was difficult to determine a priori, especially for high dimensioned parameter vectors.

2.3 SYSTEMS OPTIMIZATION TECHNIQUES

The application of optimization methods to the control of water pollution has steadily developed since the early applications of linear programming in the 1960's. These methods can be generally classified as:

1) Linear Programming (LP)

2) Nonlinear Programming (NLP)

3) Dynamic Programming (DP)

4) Stochastic Programming (SP)

5) Optimal Control (OC)

2.3.1 <u>Linear Programming Formulations</u>

A typical linear program is

$$\begin{matrix} (\max) \\ (\min) \end{matrix} \; C'\underline{X} \qquad \text{such that } \underline{A}\underline{X} \leq \underline{b} \text{ and } \underline{X} \geq \underline{0}$$

where the objective function optimizes, for example, treatment costs or pollutant loadings. The constraints describe budget restrictions, capacity limitations or pollutant discharge regulations. This formulation is discretized over space where the constraints could apply to a finite set of monitoring stations. The cost function may also require to be linearized resulting in large LP models. Rinaldi et al (1980) suggest a decomposition technique to reduce the LP to a multistage problem with a reduction in the number of constraints by identifying critical monitoring points and the use of linear approximations to the steady-state DO sag levels as, for example, proposed by Arbabi et al (1974). This approximation results in a DO deficit function of $DO^i_{deficit} = \delta_0 + \delta_1 BOD^i + \delta_2 DO^i \leq DO_{regulation}$ based on the Streeter-Phelps model. Biswas and Reynold (1973) used linearized Streeter-Phelps equations in a screening model in identifying possible minimum-cost treatment operating points in their study of DO regulation of the Saint John River Basin of New Brunswick, Quebec and Maine. They then used a dynamic pollutant model, driven by synthetic river hydrographs, to predict seasonal DO profiles as shown in figure

2.4. Loucks (1983) discussed the following problems for solution by linear programming or mixed integer formulations:

a) regional treatment and transport where total costs are minimized subject to conservation of flow constraints;

b) multiple point source waste reduction to meet water quality standards where costs of treatment were minimized subject to BOD/DO dynamics modeled (linearized similarly to Arbabi et al, 1974);

c) prediction of algal bloom potential by maximizing algal production subject to nutrient and light constraints.

Due to the steady-state assumptions required for the LP, these approximate formulations are often used as screeening models for further detailed simulation models which are then used to explore the optimal solution. The dynamics of the river processes, however, would have been averaged in arriving at this optimum.

An alternative approach which incorporates the pollutant dynamics into an LP formulation was proposed by Gorelick and Remson (1982) and Gorelick (1982). This methodology is applicable to any linear, distributed parameter, solute transport water quality model. Gorelick (1982) used the deterministic steady-state groundwater flow equation:

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial H}{\partial x_i}\right) = W \qquad (2.12)$$

where T_{ij} - the transmissivity tensor $[L^2.T^{-1}]$

H - the hydraulic head $[L]$

W - the volume flux per unit area $[M.L^{-2}.T^{-1}]$

x_i - the two horizontal dimensions $[L]$

(Biswas and Reynold, 1973)

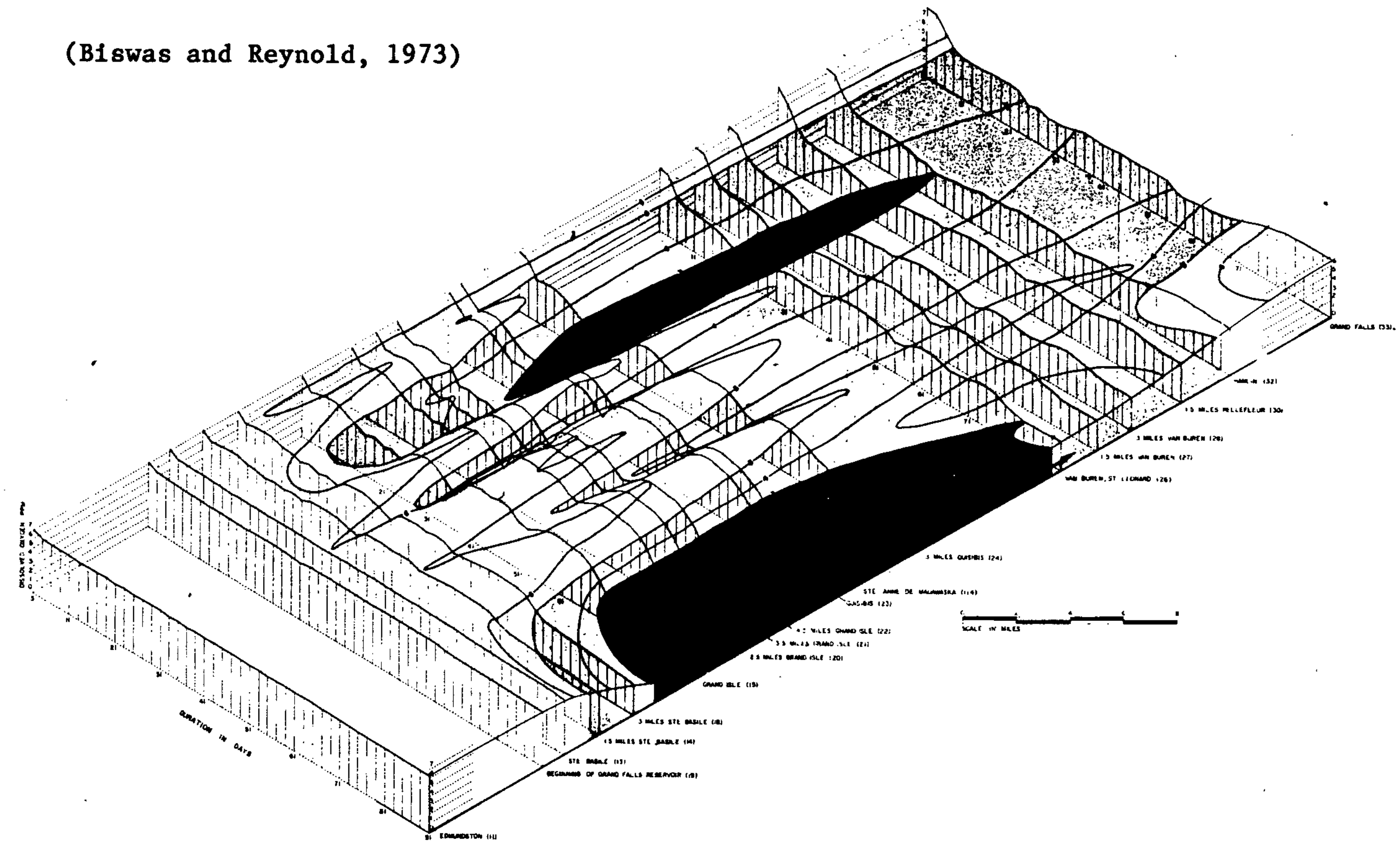


FIGURE 2.4 TEMPORAL AND SPATIAL VARIATION IN DO LEVELS

and the deterministic advective-dispersive equation:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_i}\right) - \frac{\partial}{\partial x_i}(CV_i) - \frac{C'W}{\phi b} \tag{2.13}$$

where C - concentration of dissolved chemical specie $[M.L^{-3}]$

D_{ij} - dispersivity tensor $[L^2.T^{-1}]$

V_i - average pore water velocity in the direction of x_i $[L.T^{-1}]$

b - saturated aquifer thickness $[L]$

ϕ - effective aquifer porosity

C' - solute concentration in a fluid source or sink $[M.L^{-3}]$

t - time $[T]$

These equations describe the movement of a non-conservative pollutant in a two-dimensional horizontal aquifer. Unit pollutant slug loadings at point sources in the aquifer were modeled at observation wells as 'break-through' curves which were assumed to occur annually. These curves were discretized over a management period of five years to form a unit concentration response matrix to be used in the following LP formulation:

$$\max Z = \underline{u}' \, \underline{f}$$

subject to $\underline{R} \, \underline{f} \leq \underline{c}^*$ for each observation well.

and $\underline{f} \geq \underline{0}$

where $\underline{R}$ is the concentration response matrix

$\underline{f}$ is the vector of unit pollutant loading rates

$\underline{c}^*$ is the vector of water quality standards

$\underline{u}'$ is the unit vector

The objective was to maximize the total, equally weighted loading of the aquifer. This formulation was subsequently solved as a dual LP. This methodology was applied by Danskin and Gorelick (1985) to a multi-aquifer groundwater and surface system near Livermore, California where the cost of water allocation was minimized subject to physical and economic constraints. Without this management model, these researchers state that the problem posed would have been impossible or extremely difficult to solve. A similar methodolgy had also been applied by Bishop et al (1976) to a steady-state river with multiple pollutants, where the objective was to minimize treatment costs subject to water quality and treatment contraints. These researchers coupled a linearized steady-state simulation model with an integer programming model via a coefficient matrix and also remarked that iterative solution techniques would have been required for non-linear water quality models.

2.3.2 Non-Linear Programming Models

These programming models can accommodate the non-linearity of water quality processes. These models are usually of the form:

$$\begin{Bmatrix} \max \\ \min \end{Bmatrix} f(\underline{x})$$

subject to: $g(\underline{x}) \leq \underline{b}$

where f(.) and g(.) can be convex and/or concave functions, defining either convex or non-convex solution sets, to be solved by appropriate numerical techniques.

Rinaldi et al (1980) discussed the following examples:

a) the minimization of waste-water treatment and water production costs less recreational and aesthetic benefits subject to dynamics of a discretized Dobbins model and BOD/DO standards using the sequential unconstrained minimization technique (SUMT);

b) the minimization of treatment costs in an estuary described by a discretized Streeter-Phelps model using Rosen's gradient projection method for optimization;

c) the minimization of quadratic aeration costs with a Streeter-Phelps model, using variational approaches for solution;

d) a non-convex minimization of non-linear costs of treatment and flow augmentation subject to flow conservation and quality standards using a dirct search method (ensuring, however only a local optimum).

Bayer (1972) obtained minimum treatment plant, dam and reservoir costs subject to BOD and DO standards, low flow limits and plant efficiency ranges. Both first-order (carbonaceous) and second-order (nitrogenous) BOD uptake demands were modeled with discretized Streeter-Phelps equations applied to the Willamette River, Oregon. Comparisons were made with previously developed LP and DP models for the same river showing high agreement in optimal policies generated. Deininger (1972) discussed a minimum-cost regional pollution control system with the optimal location of treatment plants and a sewer system. The resulting NLP with concave constraints was solved iteratively by a procedure involving the formulation of a related LP and a ranking extreme point approach to avoid local minima. This method of solution was found to be appropriate for 'small to medium' sized problems. Taylor and Judy (1972) used a three phase approach in solving a hypothetical water quality management problem by rent allotment control. An NLP, minimizing discharges plus agency costs subject to regulatory standards, was used to generate Kuhn-Tucker conditions which described the optimal use of treatment and flow augmentation. These conditions were then used as a basis for an agency-discharger game, with bidding, which was 'played out' when a set of agreements on rents and allotments had been reached. These agreements were then used in a rent allotment market model to allow decision makers to adopt more efficient waste abatement measures.

These examples dealt with steady-state approximations to dynamic systems with reasonably simple models. The discretized version of the dynamic model of section 2.2.3 incorporated into a optimization model, though more descriptive, would result in large NLP or LP models in order to meet stabililty requirements and would require large scale optimization techniques, such as those proposed by Lasdon (1970), for solution. Loucks (1983) describes the conversion of the U.S. Environmental Protection Agency's QUAL II model into an economic management model by exploiting finite-difference techniques for the formulation of a multi-constituent optimization. Similar discussions related to lake and reservoir management were also presented.

2.3.3 Dynamic Programming Formulation

Due to the sequential multi-stage nature of the river water quality control problem, it is amenable to a classical dynamic programming formulation as:

$$H_i(\underline{X}_i) = \min_{\underline{u}_i} [C_i(\underline{u}_i) + H_{i+1}(\underline{f}_i(\underline{X}_i,\underline{u}_i))] \tag{2.14}$$

where $C_i(\underline{u}_i)$ is some cost function, such as treatment or pollutant impact on fish.

$\underline{x}_i$ is the state value of the system (for example BOD and DO)

$\underline{u}_i$ is the control action taken, such as treatment, aeration, pollutant loadings.

$\underline{f}_i(\underline{x}_i,\underline{u}_i)$ is the state at the end of each stage (reach) if control u_i is taken with state values x_i which satisfies the constraints. For example, the use of the Streeter-Phelps or input-output models.

$H_i(\underline{x}_i)$ is the (minimum) cost function at state $\underline{x}_i$ and stage i.

Due to the relatively complex nature of the BOD/DO models in the formulation above, a closed form solution is difficult to obtain, suggesting the use of discrete, dynamic programming techniques. The major limitation in the use of this method is the well known 'curse of dimensionality' which limits applicability on even the most powerful computers. This has led to the development of DP techniques such as differential DP, discrete differential DP and state incremental DP which, according to Yakowitz (1982), have the potential to avoid or eliminate the 'curse'. For deterministic problems, forward or backward stage calculation methods can be applied, while stochastic problems require the latter.

The scope of research and application of DP to water resource problems has expanded considerably in the past decades with a primary focus on reservoir management (Yeh, 1985). Applications to water quality problems have also received increased attention. Chang and Yeh (1973) looked at the problem of optimal allocation of artificial aeration along a polluted stream to to meet a dissolved oxygen standard of 5.0 mg/l. They used the discrete DP approach in allocating aerator capacity to equidistant aerators at minimum cost, subject to some total aeration capacity. The formulation used was

$$\min_{V} J(D,V) = \sum_{i=1}^{N} W_1(D_L - D_{ti})^2 + W_2 V_{ti}^2 \qquad (2.15)$$

where W_1 and W_2 are weighting factors and D_L is the regulated DO deficit (from saturation) and D_{ti} is the actual DO level, while V_{ti} are the aerator DO capacities, subject to

1) the limitation of total available aeration capacity;

$$\sum_{i=1}^{N} V_{t_i} \leq Y$$

2) the physical limitation of DO deficit and aeration capacity

$$0 \leq D_{ti} \leq C_s$$
$$0 \leq V_{ti} \leq C_s \qquad i=1,\ldots,N$$

3) the DO deficit at the end of every reach (aerated or not) derived from the Streeter-Phelps equations:

$$D_{t_i} = (D_{t_{i-1}} - V_{t_i})\, e^{-k_2 \Delta t} + \frac{k_1 b_a}{k_2 - k_1} e^{-(i-1)\Delta t k_1} \left(e^{-k_1 \Delta t} - e^{-k_2 \Delta t}\right)$$

Using a Lagrange multiplier technique these researchers added the second constraint to the objective functional while constraints 1 and 3 above defined the admissable region for the problem. Using a range of multipliers for a 24 stage, 11 state formulation, all 11^{24} possible states were searched resulting in control trajectories shown in figure 2.5. The Lagrange multiplier was interpreted as the cost of allocating a unit of aeration capacity to the stream with a zero multiplier value at full capacity.

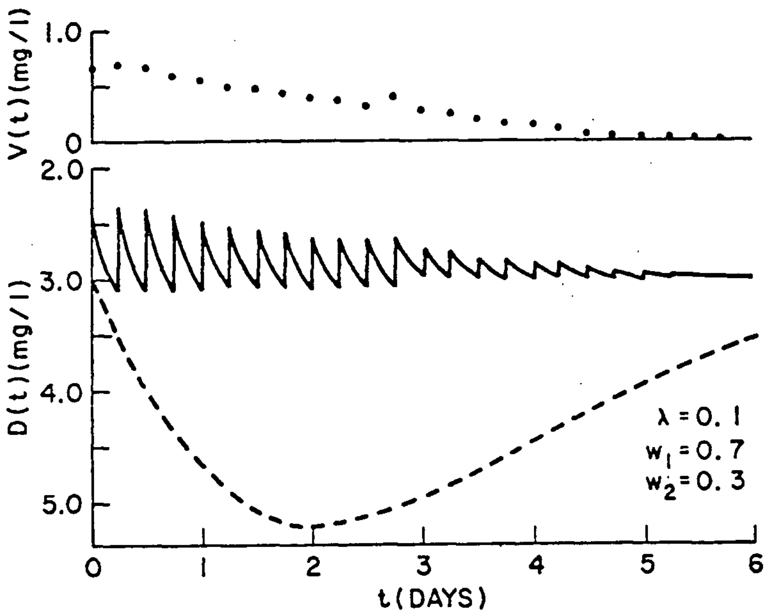


Figure 2.5 Typical Optimal Aeration Capacity Allocation Policy and the Corresponding DO Sag Profile (Chang and Yeh, 1973)

Rinaldi et al (1979) discussed two DP applications: one being the optimal allocation of wastewater treatment to the Rhine River, Germany and the other being the optimal allocation of artificial instream aeration in a reach of the same river. Both applications are formulated as the optimal allocation of complementary pollution control mechanisms using deterministic, steady-state BOD/DO models to bound the admissable solution region. River conditions were established for the most unfavourable water quality conditions during which environmental standards of 4.0 mg/l DO and 20 mg/l BOD were to be enforced. The treatment problem with 6^6 states was reduced to 6^4 states by exploiting the special structure of the DO/BOD model and solved using backward recursion.

The aerator problem was formulated as follows:

$$\min \left[\sum_{i=1}^{N} C_i(u_i, c_i, Q_i) \right]$$

$$\text{s.t.} \quad \frac{dc}{dt} = k_2(\ell)(C_s - c) + g(\underline{z}(\ell), \omega(\ell)) + \sum_{i=1}^{N} u_i \delta(\ell - \ell_i) \tag{2.16}$$

$$\text{and} \quad c(\ell) \geq c_{std} \quad \text{with} \quad C(0) = C_0 ,$$

where C_i is the cost of aeration [$]

Q_i is the river discharge [$L^3.T^{-1}$]

$\underline{z}(l)$ is a general organic decomposition function equivalent to BOD in the Streeter-Phelps model [$M.L^{-3}$]

$w(l)$ is the net source of pollutant [$M.T^{-1}$]

$\delta(\cdot)$ is an impulse function

N is the number of aerators

u_i is the aerator capacity [$M.T^{-1}$]

L_i is the stream reach length [L]

The problem as formulated was simplified by decomposition into independent sub-problems based on segments of the river. The water quality model was solved using a Runge-Kutta integration scheme. An interesting extension made was where river flow was fitted with a piece-wise linear function by distance along the river. This was an attempt to describe the spacial dynamics of the river and would, however, have to be extended into the time dimension to account for unsteady flows along the river, adding a possibly unmanageable level of complexity to the solution method.

The application of a dynamic programming formulation to tidal river water quality problems poses an additional problem. A DP algorithm would be required which would optimize over upstream and downstream flows and BOD/DO interactions (an oscillating scheme). The development of such a technique would be a fruitful area of research. An additional order of complexity would arise in accounting for the stochastic nature of the processes involved.

2.3.4 Stochastic Programming Models

The application of LP, NLP and DP formulations to the water quality problem discussed previously were primarily deterministic. The probablistic nature of the studies reviewed was not specifically addressed. Random unit costs, resource constraints and unit resource usage could significantly complicate the optimizations schemes reviewed.

Yeh (1985) documented the application of stochastic programming techniques in the resolution of reservoir management problems. The application of these techniques to water quality problems is also growing.

Yeh (1985) discussed the application of chance-constrained LP (with and without recourse) and reliabilty-constrained NLP and DP methods.

Chance-constrained programming problems are of the form (as studied by Charnes, Cooper and Symonds, 1958) of:

$$\{\max/\min\} \; \underline{C}' \; \underline{X}$$

subject to $$\Pr[\, \underline{A}\; \underline{X} \leq \underline{B} \,] \geq \underline{e}$$

where $\underline{C}$ is the cost vector

$\underline{X}$ is the decision vector

$\underline{A}$ is a deterministic matrix

$\underline{B}$ is a vector of random variables

$\underline{e}$ is a vector of exceedance probabilities

(Kolbin, 1976)

Assuming normality and independence for the elements of $\underline{B}$, the above constraint can then be converted to a determininstic equivalent as:

$$\sum_j a_{ij}\, x_i \leq b_i^{(1-e_i)} \qquad \text{for all i.}$$

Using this type of formulation Lohani and Thanh (1978) addressed the minimization problem of total operating cost of BOD removal by optimizing the removal fraction required by 'n' treatment plants to meet DO standards in a common receiving river. Their problem was formulated as:

$$\min \sum_{i=1}^{n} c(1-Y_i)$$

$$\text{s.t.} \quad \Pr\left[\frac{L_i}{B_i}\left[Y_i + \sum_{j=1}^{i} \frac{L_j}{B_i} L_{ji} Y_i - \sum_{j=1}^{i} q_j \leq Q_i\right]\right] \geq a_i \quad \text{for } i=1,\dots,n$$

$$\text{and} \quad r_i' \leq Y_i \leq R_i' \;; \quad Y_i \geq 0 \tag{2.17}$$

where L_i is the ultimate BOD waste discharge at reach i.

B_i is the allowable BOD to maintain DO in reach i

Y_i is the unremoved fraction of BOD in reach i

L_{ij} is the decomposition ratio for BOD between reaches i and j

q_i is the waste discharge flow at reach i

Q_i is the stream discharge at i

a_i is the reliability parameter

where the constraints are the allowable risk of violating the performance requirements (namely the DO standard) and the bounds on treatment fraction with stream flow as the only random quantity. They used a steady-state model due to Camp in establishing the values of B_i and L_{ij}. By sensitivity analysis, these researchers obtained the minimum operating cost function for various reliability levels and various DO limits for the Hsintian River in Taiwan.

Jacoby and Loucks (1972) used a stochastic programming model to determine a small set of superior management plans for further evaluation with a simulation model in their study of the Delaware River which focussed on the optimal mix of recreation, hydroelectric and water supply projects. They also used a dynamic program to determine the optimal project completion dates.

Burn and McBean (1985) formulated a program to minimize treatment costs at two effluent sources on the Speed River near Guelph, Ontario which had six monitoring stations. They also assume steady-state conditions using the Streeter-Phelps model. Their problem was:

$$\max \sum_{j} \sum_{i} B_j \, W_i \, d_{ij} \, X_{ij}$$

where B_j is a weighting factor

$W_i\ d_{ij}$ is the random variable which is the combination of random pollutant loading and random transfer coefficients of BOD at site i monitorred at site j

X_{ij} is the removal efficiency (decision variable) of treatment at site i on site j subject to

$$\sum_{i \in \mathrm{II}} \sum_{k} C_i^k X_{ij}^k \leq C_L$$

a budgetary constraint where C_i^k is a piecewise linear cost curve and C_L is the budget. Minimum and maximum treatment level constraints were also used with a range of treatment 'equity'. The programming problem was solved as a dual LP and compared with results from a Monte Carlo simulation model with less than 5% variance in the results. These researchers conclude that a programming model which accounts for the stochastic nature of variables involved would enhance the agreement between screening models and simulations allowing for better optimal solutions.

2.3.5 Control Methods

The optimal allocation of BOD can also be formulated as a classical optimal control problem solvable by continuous or discrete methods. These formulations address the problem of real-time control of BOD and DO in a river by load regulation or artificial aeration.

Tarassov et al (1969) used an algorithm akin to Pontryagin's maximum principle based on the method of characteristics. Using the Streeter-Phelps model as a "quasi-linear, first-order, multivariable, distributed parameter optimal control model" with pertubation techniques, they developed a Hamiltonian function and associated costate equations and boundary conditions. These equations were solved for a quadratic objective function by numerical integration generating various state response surfaces. They examined aerator controls in time and space, space only and time only for an initially polluted river.

Davidson and Bradshaw (1970) studied the same problem using the maximum principle and noted that the solution of the problem was also possible using dynamic programming techniques. However, their objective was to develop a mathematically tractable model for numerical solution which tested the sensitivity of the optimal control policy to stream temperature. Tabak and Kuo (1970) presented extensive discussion on the relationships between mathematical programming methods and solution methods for optimal control problems showing their complementary and interchangeable uses.

These two early works were assumed steady-state BOD/DO processes while later studies surveyed incorporated dynamic BOD/DO models with steady hydrodynamics. Tamura (1974) used the Streeter-Phelps model to develop multi-dimensional, high-order difference equations which accounted for BOD/DO transport delays. This formulation was solved numerically using duality and decomposition techniques in minimizing the deviation between the system output DO, due to BOD loadings, and regulatory DO concentration levels. Gourishankar and Lawson (1978) used a steady-state model to study a multivariable control formulation of the BOD pollution problem. They developed control equations to test a BOD dumping controller and one for both dumping and instream aeration. Ozgaren et al (1975) developed a control strategy for aeration in a river system where observations are time-lagged while Olgac et al (1976) studied the optimal allocation of measurement and control resources by introducing white noise processes into the BOD/DO demand and DO measurements. Jain and Denn (1976) applied control techniques to short-term BOD impulses in the Delaware Estuary and concluded that their optimal policy required rapid control responses that were impossible to achieve in practise. Gourishankar and Ramar (1977) designed a continuous DO controller for a dynamic BOD/DO non-tidal steady discharge river model which was enhanced by Kudva and Gourishankar (1977) who considered the case of an observer of the same system where inputs were inaccessible. Gourishankar and Lawal (1978) presented a digital DO controller design with lower instrumentation costs for equivalent performance compared to previously designed continuous controllers. Spear and Hornberger (1983) discussed the impacts of parameter uncertainty on controller operation showing that the simple controller studied could control DO levels eighty-four percent of the time for the River Cam, England.

Additional to these works on real-time controller design, were studies applying multilevel of hierarchical techniques to real-time and seasonal control strategies. For example, Haimes (1976) presented an analytic two-level formulation of a regional pollution problem using the Streeter-Phelps model. The first level represented local polluters on the river with their respective treatment costs while the second level represented a river basin wide regional authority with respective social costs. The resulting NLP problem was solved using gradient projection methods. A three-level scheme was also proposed where the regional authorities would report to a central one. Singh (1977) used hierarchical decomposition techniques in the optimal control of pollutant loading so as to return the state of the system to 'steady' conditions at minimum cost, also for the River Cam. Chichester (1976) also presented a multi-level formulation based on a compartment dynamic pollutant model of Bella and Dobbins (1968). Three control methods were discussed: aeration, dumping and a combination of the two. The control problem was split into sub-problems based on the solution of the state, control and costate equations derived for a one-dimensional tidal river which were combined hierarchically to a 'co-ordinator'. The numerical solution for the optimal real-time control of DO required computing resources of 9 MB of core and 45 seconds CPU time for a four reach tidal river with spacially controlled aeration and pollutant dumping. Thus the application of multi-level techniques appear restricted by the number of sub-problems that can be managed computationally and conceptually.

2.4 Summary

The management of river systems is difficult and complex process aided by the judicious choice and use of well developed and validated models. Friedman et al (1984) in an investigation of the potential uses of models in the U.S., states that the use "of mathematical models have significantly expanded the nation's ability to understand and manage its water resources" and that "models have the potential to provide even greater benefits for water resource decision making in the future". The stochastic nature of the water quality processes also need to be explicitly addressed by these models (Ward and Loftis, 1983) while the combined use with optimization models would guide managers and users towards more optimal resource use (Loucks, 1983).

3. MODEL DEVELOPMENT

This chapter presents the formulation of BOD control discussed in chapter 1 as a mathematical programming problem, followed by the development of a water quality model used in this study. Specifically, a dynamic BOD/DO model and a direct search algorithm are described.

3.1 Problem Formulation

The problem outlined in chapter 1 is one of maximizing BOD loadings of a river system while maintaining a certain level of DO for fishery requirements. Since the river system is both dynamic and stochastic, an absolute DO standard would be difficult to impose due to its rigidity as compared to one which allows an acceptable frequency of non-compliance. The concept of flexible water quality standards has been used more recently for multi-user watersheds in the United States and the European Economic Community (Beck, 1985).

3.1.1 Objective

A variety of objective function forms are possible for use as surveyed in section 2.3. The objective as stated above could be constructed as:

a) a utility function based on, say, social costs or perhaps agency and user preference curves; or

b) a cumulative function of BOD loadings.

These functions could be either linear or non-linear, convex or concave. The form b) above was chosen primarily for its simplicity and amenability to linear formulations. Form a), once defined, could also be easily formulated. A similar choice allowed Gorelick (1982) to formulate a simple linear program for a groundwater management problem. BOD loadings were assumed to occur over discretized river reaches which were judged to be sufficiently fine so as to generate meaningful management policies. These reaches were assumed to contain both pollutant loadings and monitoring stations. The issue of the optimal level of river discretization was not addressed.

Thus the objective function was simply

$$\max \sum_{i=1}^{N} x_i \, w_i \tag{3.1}$$

where:

x_i is the continuous, steady non-point BOD loading along reach i of the river; $[M.L^{-1}]$

w_i is the length of reach i; $[L]$

N is the number of reaches in the river.

(Note that w_i could be chosen to reflect relative importance or economic value of the different river reaches.)

3.1.2 Constraints

The constraint on the maximization problem was that of adherance to a regulatory DO standard for all reaches of the river over a decision horizon, with some overall level of compliance. Thus individual reach regulations were not assumed nor was the overall risk apportioned by reach. This may have been possible with a priori information on sensitive sites along the river where specific flexible standards may be useful for pollutant control.

The regulatory standard used was the minimum in-stream DO levels for resident and migratory fish species, which are primarily salmonids for Pacific tidal rivers.

The constraints were formulated as nonlinear equality and inequality equations:

$$(1) \quad y_{ijt} = f_{jt}(x_i + x_o{}^i, m_B)$$

$$(2) \quad p_{jt} = \sum_i y_{ijt}$$

$$(3) \quad z_{ij} = g_{jt}(p_{jt}, m_D)$$

$$(4) \quad u_{jt} = \begin{cases} 1 & \text{if } z_{jt} \geq C_{reg} \\ 0 & \text{if } z_{jt} < C_{reg} \end{cases}$$

$$(5) \quad S = (1/N\tau) \sum_j \sum_t u_{jt} \geq \gamma > 0$$

$$(6) \quad x_i \geq 0$$

$$y_{ijt} \geq 0$$

$$z_{jt} \geq 0$$

where $i,j = 1,..,N$ reaches, i being the input reach and j the output reach and $t = 1,..,\tau$, the decision horizon with Δt determined from stability criteria of the finite-difference model used (section 3.2). Of the constraints above:

(1) are the difference equations of the BOD model (section 3.2.2.3);

(2) represent the additivity assumptions whereby the BOD concentrations of simultaneous loadings in multiple reaches are equivalent to the sum of the effects of individual loadings;

(3) are the difference equations of the DO models (section 3.2.2.4);

(4) and (5) comprise the proportionality constraint.

In the above constraints:

y_{ijt} is the BOD concentration in reach j at time t due to loading x_i in reach i at time $t-1$;

p_{jt} is the total BOD concentration in reach j at time t due to the total loadings x_i in reaches $i=1,..,N$ at time $t-1$;

z_{jt} is the DO concentration in reach j at time t due to the BOD concentration in reach j;

u_{jt} is an indicator variable of z_{jt} versus C_{reg} for reach j at time t;

S is the proportion of u_{jt} for $j=1,..,N$ and $t=1,..,\tau$ which meet the C_{reg} regulation limit;

m_B, m_D are matrices of model parameters;

$f_{ijt}(x_i, m_B)$ is the set of equations 3.11-3.13, 3.17-3.18;

$g_{jt}(p_{jt}, m_D)$ is the set of equations 3.11-3.13 and 3.19;

x_o^i is the initial BOD concentrations in reach i at $t=0$;

C_{reg} is the DO limit for all reaches;

is the acceptable compliance level; and

N is the number of reaches.

Constraints (4) and (5) can be viewed in the spirit of the classical joint-chance constraint which describe the probabilities of exceedance of stochastic variables. The problem studied here does not address the stochastic nature of the previously defined variables but deals with the average statistics of a deterministic process. Had the stochastic nature of the pollutant and hydrodynamic processes modeled been included in the non-linear program, analytic facilitation of the solution would have been highly unlikely. Wagner and Miller (1965) discussed the classical joint-chance constraint problem with random linear coefficients:

$$\max \quad z = \underline{C}\,\underline{X} \quad \text{such that} \quad \underline{A}\,\underline{X} \underline{<} b \quad \text{and} \quad \underline{X} \underline{>} \underline{0} \tag{3.3}$$

and
$$\prod_{i \in S} P_r\left[b_i \leq \tilde{a}_i x\right] \geq \beta > 0, \tag{3.4}$$

S being the set of chance constraints) where a_i is a random vector and b_i are known. They, however, considered a for each i as multinormal, with mean and covariance matrix v, and as completely independent.

Analytical solution techniques proposed by Prekopa (Sengupta, 1972) and Sengupta (1972) also require `nice` distributional assumptions. These restrictions are too narrow for the stochastic version of the problem at hand where a_i are non-normal with spacial and temporal auto-correlation. The simplified deterministic approximation method of constraints (4) and (5) above, based on averaged statistics for $j=1,..,N$ and $t=1,..,\tau$ may not be an elegant solution method, with possible analytical simplifications, but appeared viable with the numerical techniques and pollutant models presented later in this chapter.

There is one important difference between functions g_{jt} and f_{ijt} which allows one to simplify the numerical method of solution of the optimization problem as formulated. The BOD function f_{ijt} is based on a linear lumped parameter solute transport model. A linear increase in input loadings results in an equivalent linear increase in output concentrations. This property was exploited by Gorelick (1982) in developing a simulation-based linear program in managing groundwater pollution sources. This would the allow f_{ijt} to be replaced by a linear function of unit loadings or

$f_{ijt}(x_i+x_o{}^i,m_B) = (x_i/u_i)*f_{ijt}(u_i,m_B)+x_o{}^i$ where

u_i are unit loadings in reach i. This also allowed for the additive assumption of constraint (2). The function g_{jt} is a non-linear DO model and thus cannot be conveniently simplified. Thus, within the solution procedure (section 3.3.1) the BOD and DO functions were separated, the latter being directly incorporated into the optimization algorithm of section 3.3.3.

The specification of the acceptable level of compliance is a difficult process and may need to be rationalized using fish population models. The compliance level, river discharge, regulatory DO, BOD decay coefficient and longitudinal dispersion were used parametrically in policy sensitivity analysis.

3.2 Water Quality Model

The water quality model used in this study comprised two components:

I) a hydrodynamic river model;

II) an unsteady BOD/DO model.

Model I simulated river stage and discharge dynamics while model II predicted DO and BOD concentrations due to BOD loadings. The uncoupling of the water quality model into these two components was based on the assumption that the two phenomena were weakly interdependent such that the density of the river waters was not affected by temperature, salinity and hydro-biological effects (Gromiec et al, 1983). This then allowed:

a) simplification of model formulation and development;

b) the incorporation of a hydrodynamic model used in a previous modeling study (Jamal, 1980);

c) the development of a separate BOD/DO model;

d) independent validation of models I and II (which though not as robust a procedure as combined validation, was necessary due to the lack of data).

3.2.1 Model I: Hydrodynamic River Model

3.2.1.1 Basic Assumptions

The model was developed for modeling of the river system and did not include processes related to groundwater recharge and discharge, overland flow, evapotranspiration, small tributary inflows, salinity effects, wind effects, bed load and sediment effects. The impact of licensed irrigation uptakes was reviewed and is discussed in section 4.4. It was also assumed that the hydrodynamics of the river could be adequately described by the set of one-dimensional differential equations of continuity and momentum.

3.2.1.2 Basic Equations

Continuity: (3.6)

$$\frac{\partial Q}{\partial x} + W\frac{\partial h}{\partial t} = 0$$

where Q is the river discharge $[L^3.T^{-1}]$

W is the bank width [L]

h is the river stage to (Geologic Survey of Canada) datum [L]

x is the distance along the river axis [L]

t is the time dimension [T]

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} - g\frac{|u|u}{c^2 d} \qquad (3.7)$$

where u is the river velocity $[L.T^{-1}]$

g is the acceleration due to gravity $[L.T^{-2}]$

c is the Chezy coefficient

d is the depth of flow [L]

A is the cross-sectional area $[L^2]$

(Gromiec et al, 1983)

Substitution of the averaged discharge as Q=vA in equation 3.7 and with in-substitution of equation 3.6 leads to

$$\frac{1}{A}\frac{\partial Q}{\partial t} - \frac{Q}{A^2}\frac{\partial A}{\partial t} + \frac{Q}{A}\left\{-\frac{W}{A}\frac{\partial h}{\partial t} - \frac{Q}{A^2}\frac{\partial A}{\partial x}\right\} = -g\frac{\partial h}{\partial x} - g\frac{Q|Q|}{c^2A^2d} \qquad (3.8)$$

The boundary conditions for equations 3.6 and 3.8 were the upstream inflow discharges into the river system and downstream tidal heights. Initial conditions were the stage and discharge at various cross-sections along the river.

In this study two finite-difference schemes were attempted in the solution of equations 3.6 and 3.8 subject to boundary conditions. The first attempt was the development of an implicit difference scheme based on Dronkers (1969). This scheme required the solution of a set of simultaneous equations for cross-sections along the river at each time step. The 'double sweep' method for matrix solution was used with recorded initial and boundary conditions. The method, however, failed to adequately simulate the dynamics of a tidal gate which existed on the river of concern and had a primary influence on the river flows. The method was abandoned in favour of an explicit scheme presented by Ages (1973). An explicit finite difference formulation of equations 3.6 and 3.8 was presented by Jamal (1980) using a 'leap frog' scheme and was adapted to fit the BOD/DO model discussed in section 3.2.3. This formulation is summarized below.

The river was schematized as shown in figure 3.1 with the x-direction positive upstream. Equations 3.6 and 3.8 were discretized by central differences to yield equations 3.9 and 3.10 in a scheme represented in figure 3.2 showing initial and boundary conditions for the Nicomekl River in Surrey, B.C. Details of the development of equations 3.9 and 3.10 are found in Jamal (1980). The tidal gate was simulated by simply forcing to zero the discharge at the gate for all periods that the river discharge was in the positive upstream direction. The calibration parameter was the Manning's 'n' number which reflects

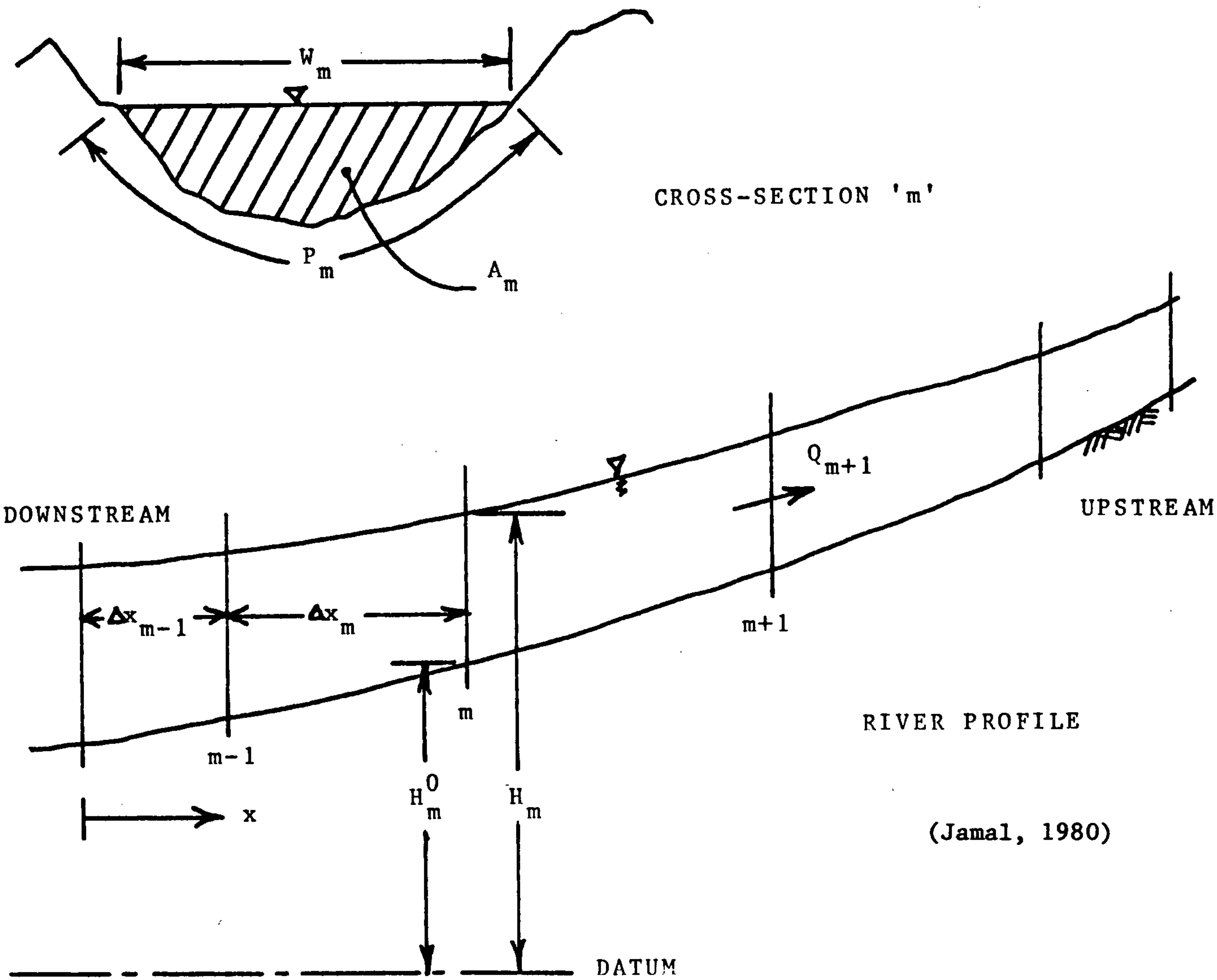


Figure 3.1 : Sketch of Notation used.

$$H_{m+1}^{n+1} = H_{m+1}^{n} - \frac{4\Delta t}{W_m^n + W_{m+1}^n} \cdot \frac{Q_{m+2}^n - Q_m^n}{\Delta x_m + \Delta x_{m+1}} \qquad (3.9)$$

$$Q_m^{n+1} = \left\{ \frac{Q_m^n}{\Delta t} \cdot \frac{1}{A_m^{n+1} + A_{m+1}^{n+1}} - g \frac{H_{m+1}^{n+1} - H_{m-1}^{n+1}}{\Delta x_m + \Delta x_{m-1}} \right\}$$

$$\cdot \left\{ \frac{1}{\Delta t \, (A_m^{n+1} + A_{m+1}^{n+1})} - \frac{A_{m+1}^{n+1} + A_{m-1}^{n+1} - A_{m+1}^{n} - A_{m-1}^{n}}{\Delta t \, (A_m^{n+1} + A_{m+1}^{n+1})^2} \right.$$

$$- \frac{W_m^{n+1} + W_{m+1}^{n+1}}{2} \cdot \frac{H_{m+1}^{n+1} + H_{m-1}^{n+1} - H_{m-1}^{n} - H_{m+1}^{n}}{\Delta t \, (A_m^{n+1} + A_{m+1}^{n+1})^2}$$

$$- \frac{8 Q_m^n \, (A_{m+1}^{n+1} - A_{m-1}^{n+1})}{(A_{m+1}^{n+1} + A_m^{n+1})^3 (\Delta x_m + \Delta x_{m+1})}$$

$$\left. + \frac{4 g \left| Q_m^n \right|}{(A_m^{n+1} + A_{m+1}^{n+1})^2 c_m^2 (H_m^{n+1} - H_m^0)} \right\}^{-1} \qquad (3.10)$$

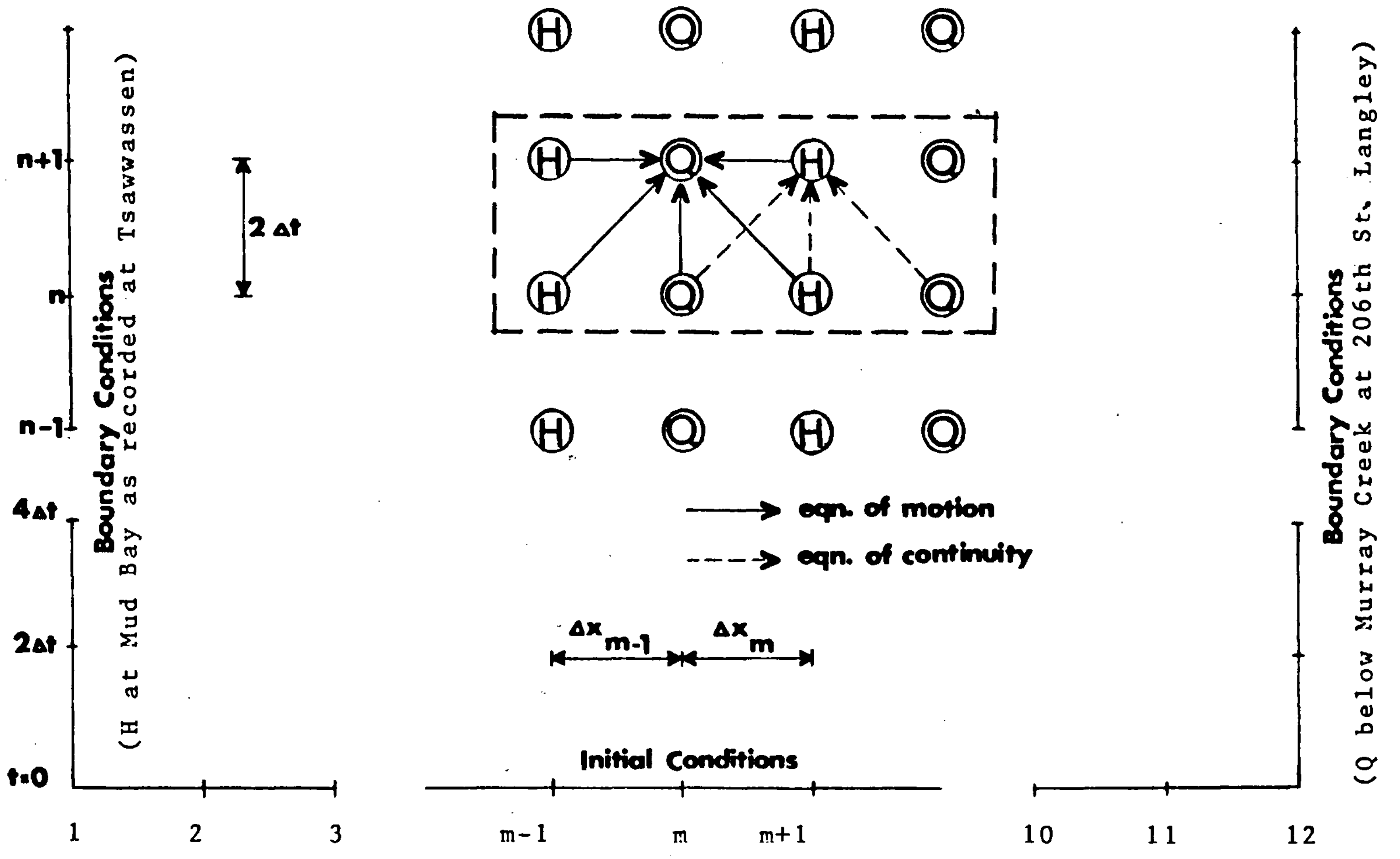


Figure 3.2 : Modified computer scheme (adapted from Ages, 1973)

the roughness of the river channel. The numerical stability of the scheme is based on a 'Δt' chosen from the well known criteria

$$\left(\frac{\Delta x}{c}\right)_{min} \geq \Delta t \; ; \qquad c = \sqrt{gh} \quad .$$

An overview flowchart of this model is presented in figure 3.3 where the stage and discharges are updated at each time step for all river cross-sections.

3.2.3 Model II: The BOD/DO model

The models surveyed in section 2.2.2 were of three types:

a) models based on PDE's and solved by finite difference equations

b) as a) above but solved by characteristic methods;

c) compartment or box models representing mass balances on river reaches.

The development of this model was based of the study of Bella and Dobbins (1968) (type c) above) which, though not state-of-the-art (Ambrose, 1986), was of a simpler formulation than a) or b) and thus easier to incorporate into the numerical hydrodynamic model of section 3.2.2.

3.2.3.1 Basic Assumptions

The water quality model developed was based strictly on river BOD/DO dynamics. No attempt was made to develop ecological model components due to the complexity of such a task especially with such paucity of data and lack of field resources. The model also does not simulate overland BOD loadings due to storm events or fall runoff, such as Tubbs and Haith (1981), although these have been recognized as significant pollutant processes (Moore, 1985). It is assumed that the BOD decay was first-stage carbonaceous process with no second-stage nitrogenous influence. This was a simplifying assumption since

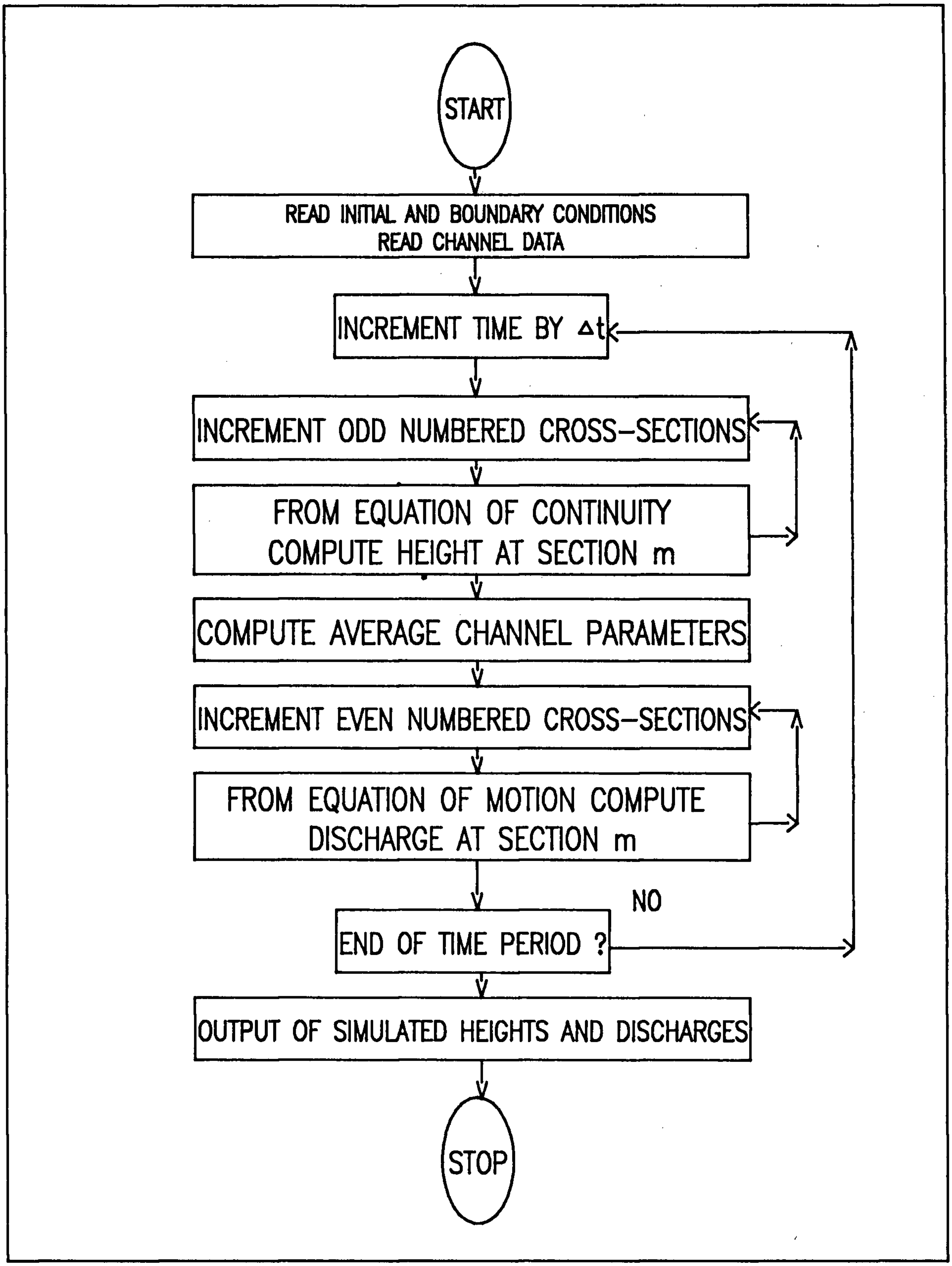


Figure 3.3: Flow Chart of the Hydrodynamic Model
(Jamal, 1980)

second-stage processes are often present (Ambrose, 1986). It was also assumed that oxygen sources and sinks in the form of detritus, benthos and algae could be accounted for by some base level consistent with field experience. Diurnal photosynthetic models are available in the literature (e.g. Jorgensen, 1983) but were not used due to the lack of data for validation. Steady net oxygen rates were used to account for photosysthesis and respiration. The stream temperature was also assumed constant for the time period of interest such that temperature dependent parameters were non-varying. It was assumed that the processes of convection, dispersion and decay (or oxidation) were sufficient to describe the one-dimensional BOD/DO dynamics with homogeneous concentrations in each reach.

3.2.3.2 Basic Equations

Convection: the transport of the pollutant by the movement of the river waters downstream or upstream.

The mass balance of a pollutant over a stream segment for a time interval is described as:

$$\text{mass at end of interval} = \text{mass at start of interval} + \text{mass convected in during interval} - \text{mass convected out during interval}$$

or (for downstream flows),

$$L(n,T+\Delta T)A(n,T+\Delta T)\Delta x_n = L(n,t)A(n,T)\Delta x_n + UA(n+1/2,T)L(n+1,T)\Delta T - UA(n-1/2,T)L(n,T)\Delta T \qquad (3.11)$$

which can be simply solved for $L(n,T+\Delta T)$ where:

L is the pollutant concentration $[M.L^{-3}]$

A is the river cross-sectional area $[L^2]$

Δx_n is the length of reach n (+ve upstream)

where n is located at even numbered sections of figure 3.2. [L]

ΔT is the time interval for T=1,..,T [T]

U is the river velocity (+ve upstream) $[L.T^{-1}]$

UA is the river discharge (+ve upstream) $[L^3.T^{-1}]$

n is the midpoint of a reach Δx_n running from n-1/2 to n+1/2 as in figure 3.4, n=2,4,..,N.

For upstream flows the mass balance equation is:

$$L(n,T+\Delta T)A(n,T+\Delta T)\Delta x_n = L(n,T)A(n,T)\Delta x_n + UA(n-1/2,T)L(n-1,T)\Delta T - UA(n+1/2,T)L(n,T)\Delta T \qquad (3.12)$$

Bella and Dobbins (1968) discussed to some length the definitions of mixing and dispersion to ensure that the convection equations above correctly represent the physical processes involved.

Dispersion: the diffusion of the pollutant due the mixing and exchange of river waters while the river is flowing.

The mass balance on a stream segment is:

$$L(n,T+\Delta T)A(n,T) = \frac{DA(n-1/2,T)\Delta T}{\Delta x^2_n}[L(n-1,T) - L(n,T)] + L(n,T)A(n,T) + \frac{DA(n+1/2,T)\Delta T}{\Delta x^2_n}[L(n+1,T) - L(n,T)] \qquad (3.13)$$

where D is the dispersion coefficient for the river as a whole or by reach. In finite-difference modeling, pseudo- (or numerical) dispersion often results due to the numerical approximations made. This dispersion can be minimized, as suggested by Bella and Dobbins (1968), by reducing the coefficient D by a corresponding amount or by choosing Δx_n and ΔT appropriately. The pseudo-dispersion was estimated as

$$D_pA(n+1/2,T) = \frac{UA(n+1/2,T)}{2}\left[\Delta x_n - \frac{UA(n+1/2,T)\Delta T}{A(n,T+\Delta T)}\right] \qquad (3.14)$$

such that D is corrected as $D' = D - D_p$. Alternatively, Δx_n and ΔT

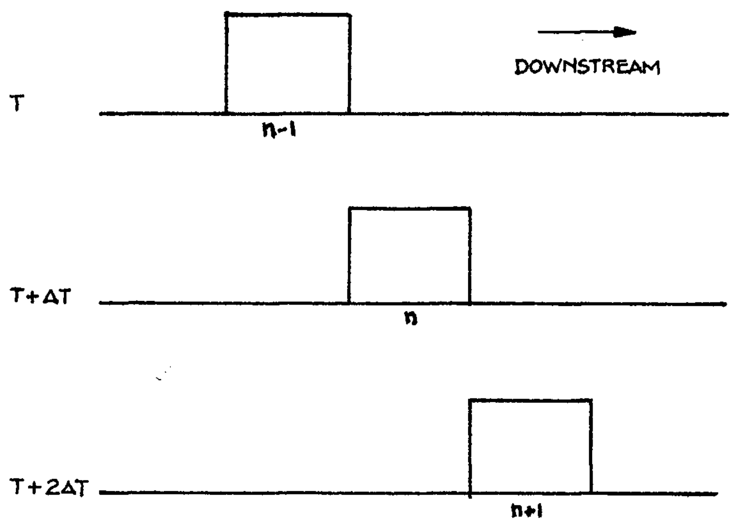


Figure 3.4: Convection of a Slug Load
(Bella and Dobbins, 1968)

can be chosen from $\left(\dfrac{UA(n+1/2,T)}{A(n,T+\Delta T)} \right)_{max} = \dfrac{\Delta x_n}{\Delta T}$ (3.15)

If, however, D_p is 'small' compared to D then it can be neglected. In this study the first method was used.

The stability criteria for the dispersion model is given as

$$\left(\frac{D\ \Delta T}{\Delta x_n^{\ 2}} \right) < \frac{1}{2} \qquad (3.16)$$

to prevent oscillating errors.

Decay: is the process whereby the pollutant is biochemically neutralized within the river. This was represented in a first-order decay equation as:

$$L(n,T+\Delta T) = L(n,T) - K_1(n,T)\Delta T\ [(1-\theta)L(n,T) + \theta L(n,T+\Delta T)] \qquad (3.17)$$

where $K_1(n,T)$ is the decay coefficient which may vary in time and space, though is was considered constant in this study.

θ is a coefficient between 0 and 1 which averages the pollutant over a simulated time interval, ΔT. A value of 0.5 was chosen for simple averaging.

Addition: pollutant sources can be added either as impulse or continuous loads over a reach of the river. A continuous load is represented as a series of impulse loadings over each time interval. This was modeled as

$$L(n,T+\Delta T) = L(n,T) + \frac{m(n,T)\Delta T}{A(n,T)\Delta x_n} \qquad (3.18)$$

where m(n,T) is the average rate of pollutant added to reach n over ΔT and can vary in any reasonable manner.

3.2.3.3 <u>BOD model:</u>

Equations 3.11 to 3.18 above are sufficient to model the fate of BOD loadings in a tidal river and as such could be calibrated and validated separately from the DO model discussed in section 5.2. The model was used in a "multi-step" scheme proposed by Bella and Dobbins (1968) where for every time step, and all reaches, the pollutant's fate is simulated by sequentially modeling the processes of addition, convection, dipersion and decay in an "add-con-disp-dk" procedure. This scheme is flowcharted in figure 3.5.

3.2.3.4 DO model

Equations 3.11 to 3.16 and 3.18 can also be used to describe the processes involved in the addition, convection and dispersion of oxygen. However, a source/sink DO model based on the mass balance concepts used earlier was required and presented as:

$$C(n,T+\Delta T) = C(n,T) - k_1(n,T)[(1-\theta)L(n,T)-\theta L(n,T+\Delta T)]\Delta T$$
$$+ k_2(n,T)[(1-\theta)(C_s(n,T)-C(n,T))+\theta(C(n,T+\Delta T)-C(n,T+\Delta T))]\Delta T$$
$$+ O(n,T)\Delta T \qquad (3.19)$$

where $C(n,T)$ is the DO concentration in reach n at time T $[M.L^{-3}]$

$k_2(n,T)$ is the coefficient of aeration $[T^{-1}]$

$C_s(n,T)$ is the saturated DO in time and space $[M.L^{-3}]$

$O(n,T)$ is the net of oxygen sources and sinks $[M.T^{-1}]$

and the second term on the RHS of equation 3.22 was the effect of BOD concentration on DO levels. It is this term which makes the overall DO model a non-linear lumped parameter model. For this study, k_2 was computed from a reaeration equation due to Dobbins and O'Conner (Covar, 1976) while the saturated DO level was taken as a constant based on field data and O(n,T) was set at some minimum level due to benthic demand and photosynthesis.

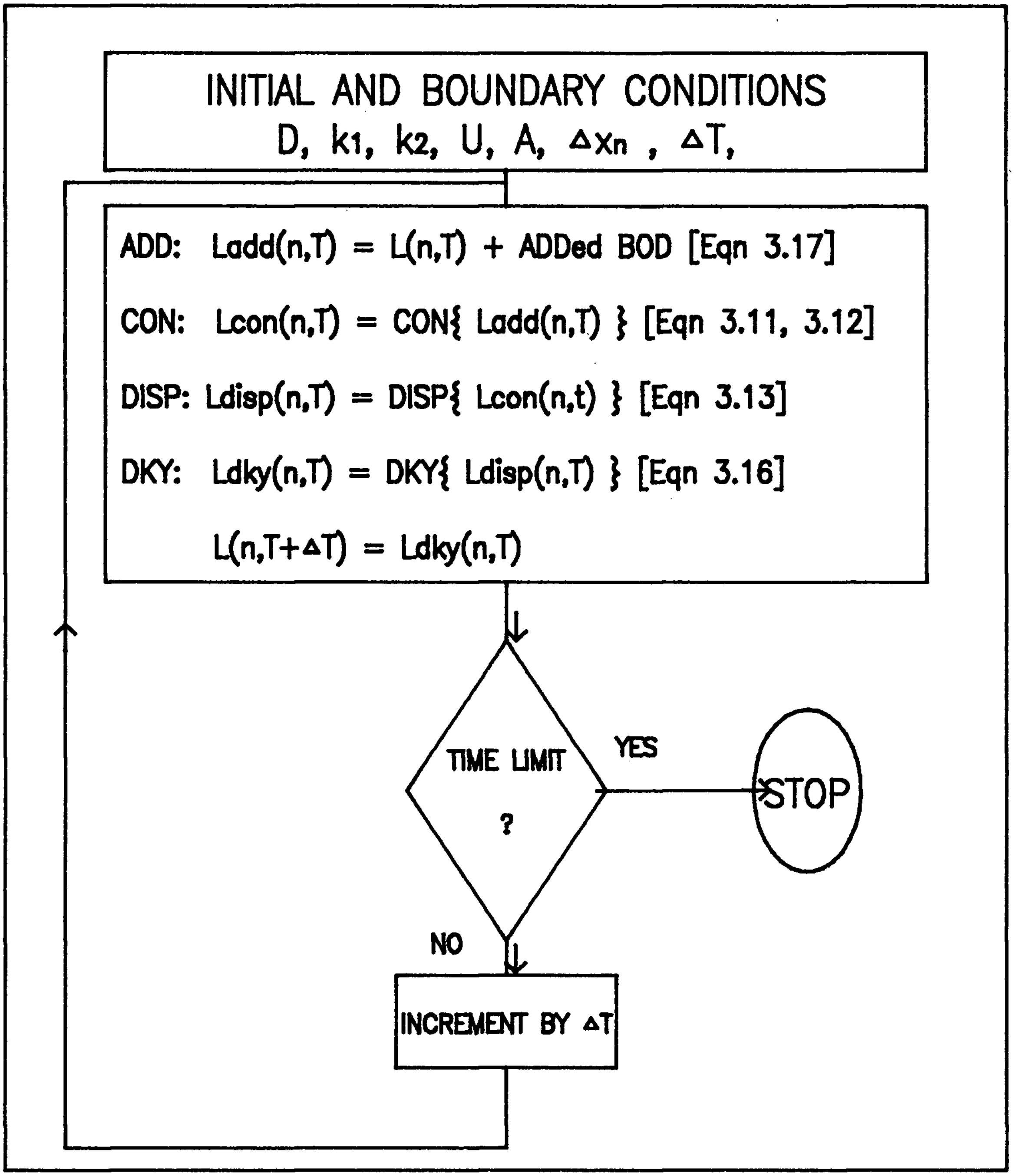


Figure 3.5: Flow Chart of the BOD Multi-Step Procedure

The combined BOD/DO model was used in the multistep procedure in figure 3.5 where each time step iteration was for all reaches in the river. The case of artificial river aeration could easily modeled by 'adding' oxygen in the multistep procedure and would be an interesting area for further research and was not addressed in this study though it is currently of great interest (Mavinic, 1986).

3.2.4 Combination of Water Quantity and Quality Models

Since the interdependence of these two models was assumed to be weak the models developed previously can be coupled as a typical linked water quality model (Beck ,1985). Thus, the predictions of cross-sectional area and river velocity made by using the hydrodynamic model would be inputs to the water quality model for the prediction of BOD and DO with no feedback. This procedure is flowcharted in figure 3.6.

3.2.5 Initial and Boundary Conditions

The initial BOD and DO concentrations were set at pristine river conditions of zero and one-hundred percent saturation respectively in the scheme of figure 3.7. These conditions would likely bias the predictions if a suitable stabilizing period had not been incorporated into the simulation runs and a typical system state not reached, especially for low flow conditions where the tidal gate may remain closed for extended periods of time. The upstream boundary conditions were also set as pristine with no BOD entering the system at the uptream end while DO entered at saturated levels. Thus the upstream BOD loadings are assumed to have negligible impacts on the river reaches of interest. The downstream conditions were more difficult to formulate

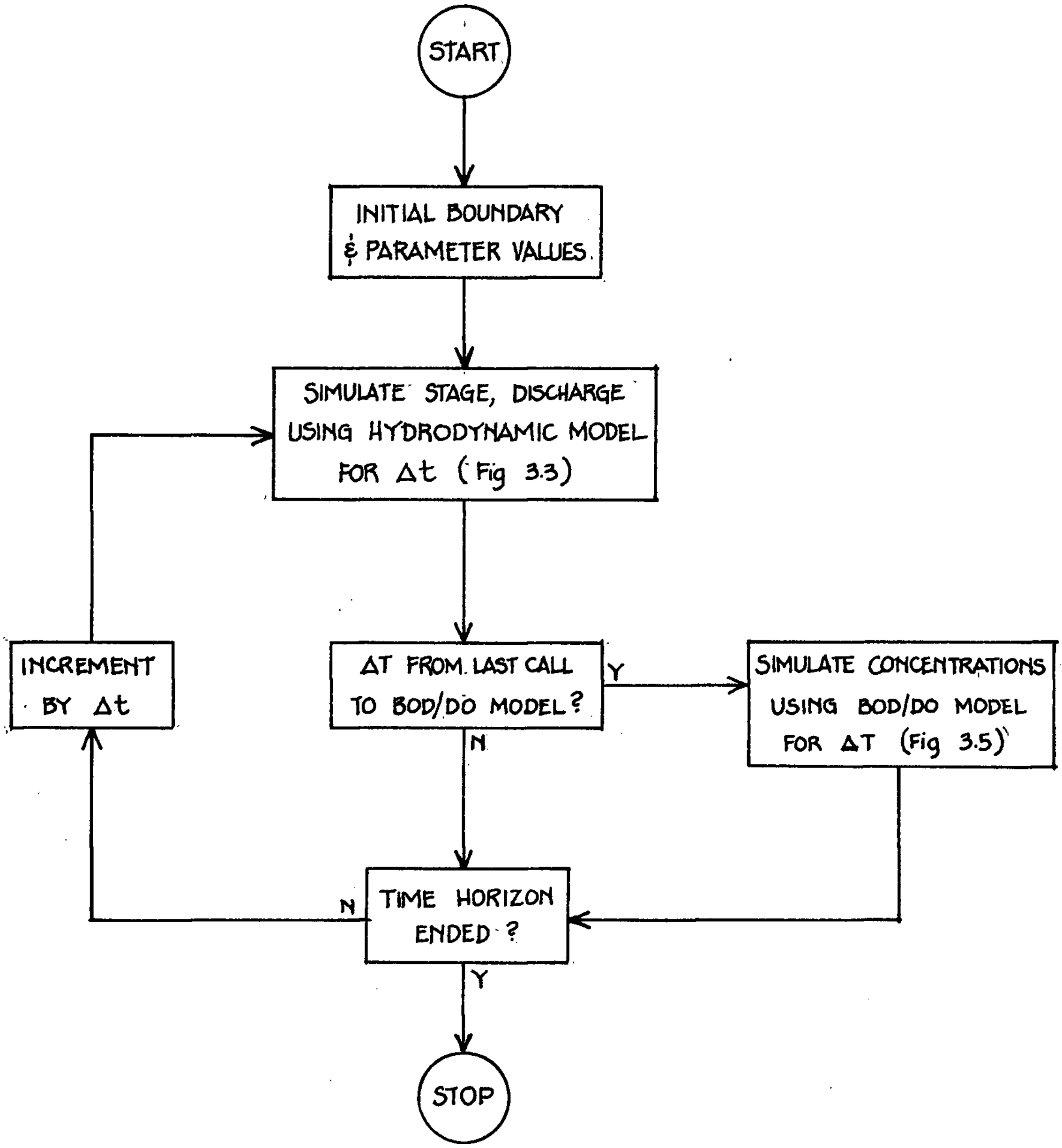
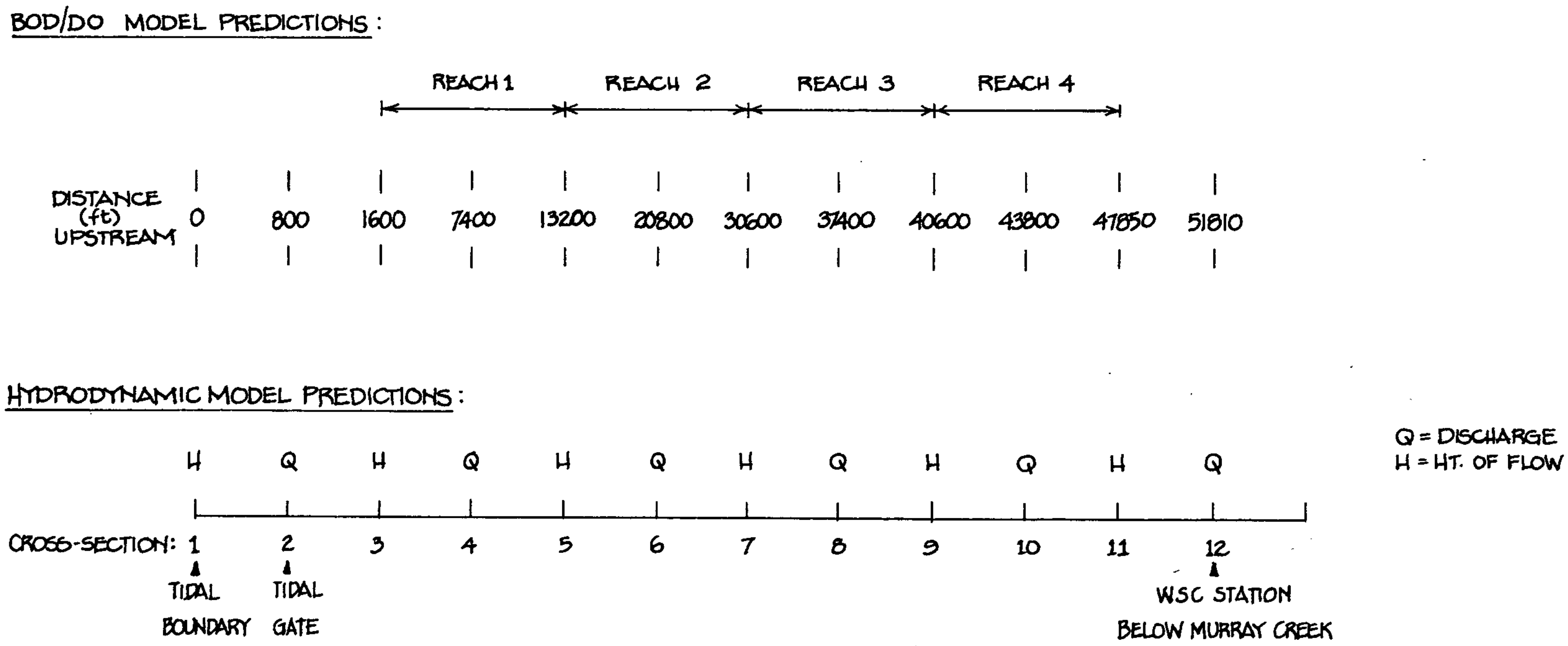


Figure 3.6: Flowchart of the Coupled Hydrodynamic and BOD/DO Models

Figure 3.7: BOD/DO Model Schematic of River Reaches



due to the tidal conditions. Two possible assumptions were available in the absence of estuarial quality data:

a) river and tidal dilution is sufficiently great to dilute the BOD and DO to negligible levels;

b) the dilution is not sufficient to assume negligible levels;

The former was assumed for simplicity of formulation since the latter required field data and estuary modeling to quantify its implications.

3.3 Nonlinear Programming Model

This section describes the methodology of solution of the NLP problem of section 3.1 by the combination of the simulation models of section 3.2 and a nonlinear optimization algorithm.

3.3.1 Methodology

In section 3.2 the water quality model was split into BOD and DO components respectively. The solution methodology exploited this separation and was as follows:

1) identify model parameters, boundary and initial conditions;

2) simulate the BOD concentrations resulting from unit loadings in all reaches over all time steps in the decision horizon

3) use the BOD concentration (time and space) profiles from 2) as input to a non-linear programming algorithm, with an embedded DO model, to identify the optimal BOD loadings subject to DO standards;

4) do 1) through 3) for sensitivity analysis.

Thus, steady (time invariant) unit BOD loadings were used in step 2) above to simulate dynamic BOD profiles which were linearly combined in step 3) as required for input into the NLP optimization algorithm. The objective function values were then determined for solutions in the feasible space defined by the level of compliance estimated from simulated DO profiles, for all reaches over a decision horizon. A flowchart of this methodology is presented in figure 3.8 which combines aspects found in the works of Gorelick (1982) and Bishop et al (1976) applied to a nonlinear programming problem.

3.3.2 Non-Linear Programming Algorithms

The NLP problem formulated in section 3.1 is a constrained optimization problem with linear objective function and nonlinear equality and inequality constraints. An efficient nonlinear optimization algorithm was then sought. Due to the computational intensity of the solution methodology proposed in section 3.3.1, only derivative-free multidimensional optimization algorithms were reviewed. Additionally, since all unconstrained methods can accommodate contraints (Walsh, 1979), the review was restricted to such methods.

Himmelblau (1972) surveyed some 15 different optimization techniques in an objective framework where all techniques were executed on fifteen test problems and judged in terms of:

a) robustness: whether the algorithm could solve most of the problems posed to within a certain precision;

b) effectiveness: the number of evaluations of the objective function in arriving at the optimal solution;

c) computation time: the time to termination within a desired degree of precision.

The derivative-free algorithms included in the test (with the year of publication) were:

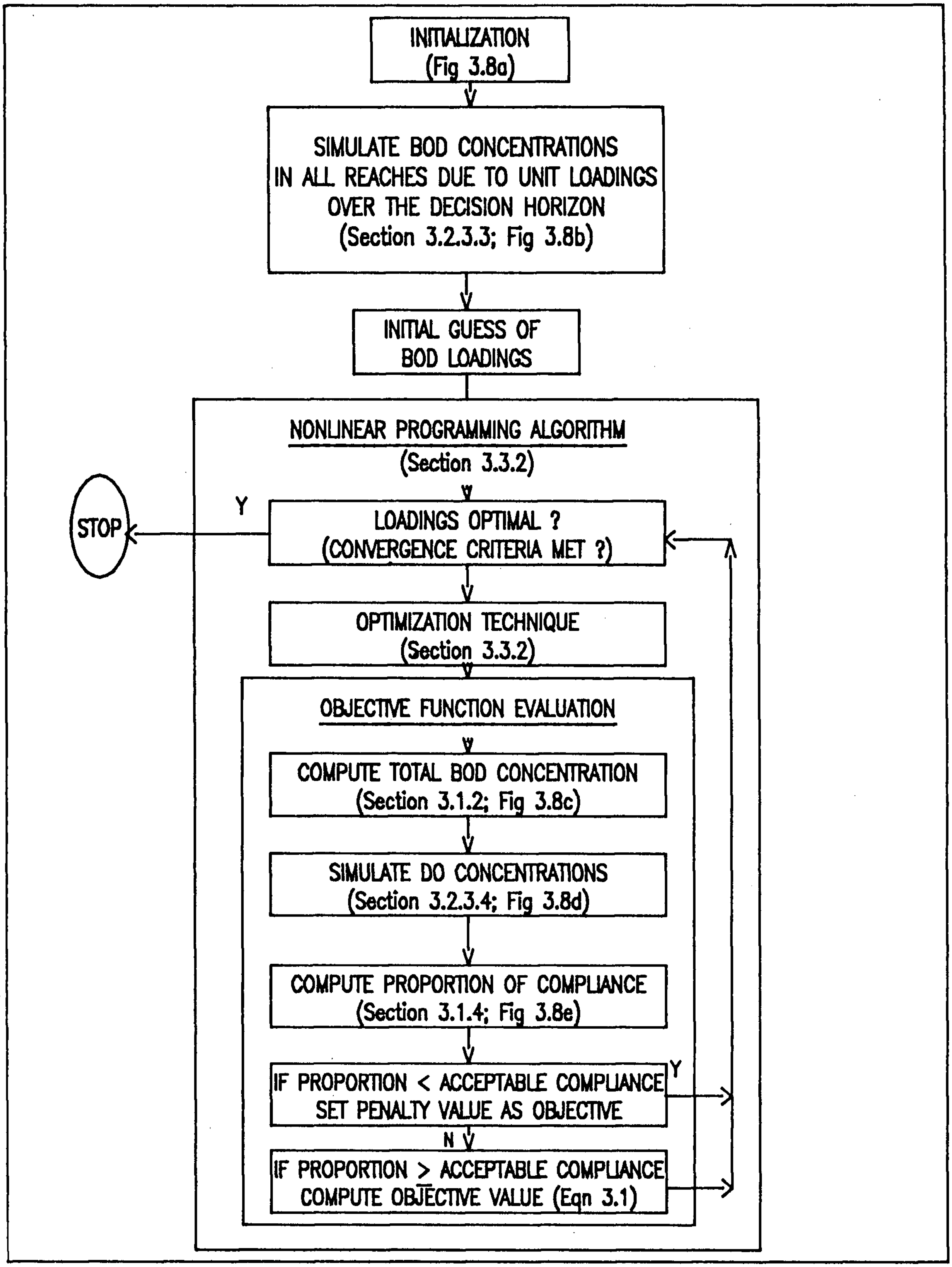


Figure 3.8: Flowchart of the Solution Methodology
for the NLP Programming Problem

FIGURE 3.8a) INITIALIZATION

INITIAL CONDITIONS :	DISCHARGE, STAGE, BOD, DO FOR ALL REACHES
BOUNDARY CONDITIONS:	UPSTREAM DISCHARGES, DOWNSTREAM TIDES, UNIT BOD ADDITIONS
PARAMETERS :	BOD AND DO COEFFICIENTS, LONGITUDINAL DISPERSION
DATA :	RIVER CROSS-SECTIONS, BENTHIC DEMAND, PHOTOSYNTHESIS, TEMPERATURE

FIGURE 3.8b) SIMULATE BOD CONCENTRATIONS

SIMULATE y_{ijt} DUE TO A 200 kg/d UNIT LOADING OF BOD AS $f_{ijt}(u_i,m_B)$ IN FOUR REACHES SEPARATELY FOR 7 DAYS, WHERE i IS THE ´INPUT´ REACH AND j IS THE ´OUTPUT´ REACH (i,j = 1,.,4) AND t IS FOR 10 MINUTE TIME PERIODS (SECTION 3.2.3.3). REACH VELOCITIES ARE DERIVED FROM THE HYDRODYNAMIC RIVER MODEL (SECTION 3.2.1)

FIGURE 3.8c) COMPUTE TOTAL BOD CONCENTRATIONS

SIMULATE p_{jt} AS $\sum_i y_{ijt} \cdot x_i/u_i$
WHERE x_i ARE THE BOD LOADINGS TESTED FOR
OPTIMALITY IN THE NLP ALGORITHM AND u_i
IS 200 kg/d UNIT LOADING.

FIGURE 3.8d) SIMULATE DO CONCENTRATIONS

SIMULATE z_{jt} AS $g_{jt}(p_{jt}, m_D)$
WHERE $g_{jt}(\cdot,\cdot)$ IS THE DO MODEL DEVELOPED
IN SECTION 3.2.3.4.

FIGURE 3.8e) PROPORTION OF COMPLIANCE

ESTIMATE PROPORTION OF THE RATIO OF TOTAL
z_{jt} ($j = 1,.,4$ and $t = 1,.,7$ every 10 min)
WHICH EXCEEDS THE DO REGULATORY LIMIT
(SECTION 3.1.2).

1) Hooke-Jeeves (1961)

2) Nelder-Mead (1965)

3) Powell (1964)

4) Rosenbrock (1960)

5) Stewart (1967)

The last one was not pursued since it required the computation of numerical derivatives which would increase the computational burden. Based on computation time, the testing procedure ranked the algorithms as:

1) Powell (6.1)

2) Hooke-Jeeves (9.6)

3) Nelder-Mead (10.0)

4) Rosenbruck (11.9)

with their averaged ranking over all tests relative to the other fifteen algorithms tested, shown in parentheses. The Powell algorithm appeared very promising for use in this study; however, due to the availability of computer code the Hooke-Jeeves algorithm was selected as the second best and implemented using FORTRAN code available from Kuester and Mize (1973).

3.3.3 Hooke-Jeeves Method

This direct search algorithm seeks the minimum of a multivariable unconstrained nonlinear function $F(x_1, x_2, \ldots x_n)$ which can be easily modified to incorporate constraints. The procedure assumes a unimodel function and thus should be used with multiple starting values if this assumption does not hold.

The algorithm is as follows (Kuester and Mize, 1973):

1) A base point is picked and the objective function evaluated;

2) Local searches are made in each direction by stepping x_i a distance s_i to each side and evaluating the objective function to see if a lower function value is obtained;

3) If there is no function decrease, the step size is reduced and searches made from the previous best point;

4) If the value of the objective function has decreased, a "temporary head" $x_{i,o}(k+1)$, is located using the two previous base points $x_i(k+1)$ and $x_i(k)$:

   $x_{i,o}(k+1) = x_i(k+1) + a(x_i(k+1) - x_i(k))$

   where i is the variable index = 1, 2, 3, ..., N

   o denotes the temporary head

   k is the stage index (a stage is the end of N searches)

   a is an acceleration factor, $a \geq 1$;

5) If the temporary head results in a lower function value, a new local search is performed about the temporary head, a new head is located and the value of F checked. This expansion continues as long as F decreases;

6) If the temporary head does not result in a lower function value a search is made from the previous best point;

7) The procedure terminates when the convergence criterion is satisfied.

A flowchart of the above is presented in figure 3.9. Constraints are incorporated into the algorithm by indicating a constraint violation with a high F value (penalty) which would discourage exploratory and

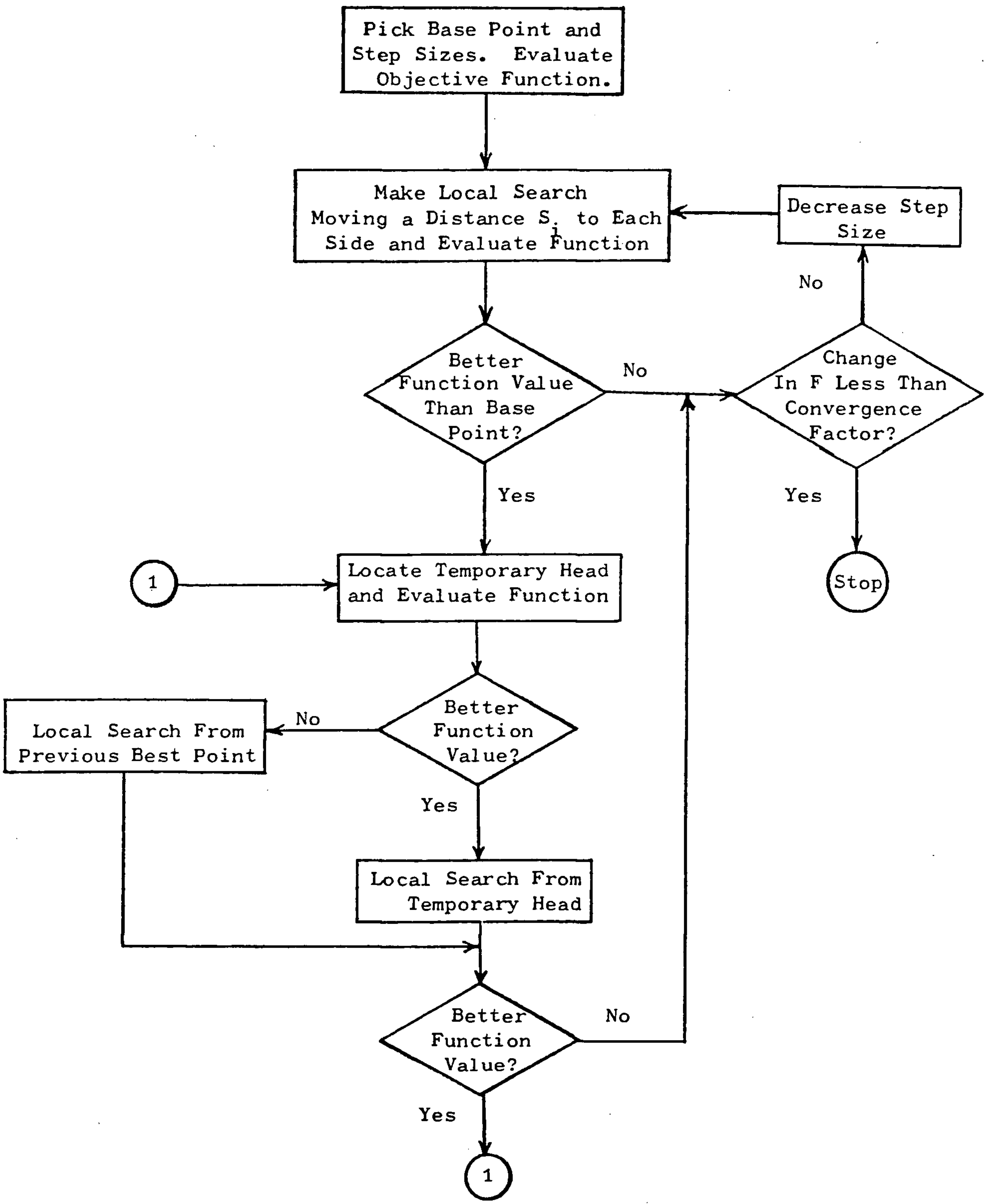


Figure 3.7: Hooke and Jeeves (HOOKE ALGORITHM) Logic Diagram
(Kuester and Mize, 1973)

pattern moves across a constraint boundary (Walsh, 1979). The constraint can be of any complex form as long as it can be accurately determined.

For the optimal BOD allocation problem the evaluation of the joint constraints was done as outlined in figure 3.7 and 3.8e). The DO model was incorporated into the Hooke-Jeeves algorithm simulating DO profiles for all reaches at each objective function evaluation required in the procedure outlined on the previous page. Any negative x_i values generated in the procedure were also discouraged with penalty values. The unimodal assumption made by the Hooke-Jeeves method was tested with the results presented in section 6.2.

The Hooke-Jeeves code was developed and validated with sample problems used by Kuester and Mize (1973).

3.4 Computer Resources and Code

The microcomputer is fast replacing main-frame machines in the modeling and optimization of small- to medium-sized enginnering and management problems. A COMPAQ Deskpro 286 microcomputer (IBM PC-AT compatible at 8MHz) was used throughout this study for the development and execution of computer code written in FORTRAN 77 with Ryan-McFarland's RM/FORTRAN. Programs were limited to a 640 KB size while the scope of the river models was reduced to 4 reaches (from an initial 11) and a decision horizon of one week (reduced from one month) such that execution times were not prohibitive since time step-sizes of 10 minutes were required to maintain stability of the finite difference solution in the NLP formulation of section 3.1. Thus, a total number of 4 reaches and 1008 time periods were used. Throughout the work performed, the microcomputer was evaluated as to its applicability to tactical and strategic management usage in terms of execution time and problem size.

4. FIELD INFORMATION

4.1 Site Description

The application site for this study was the Nicomekl River in the Nicomekl-Serpentine river basin in the Municipality of Surrey in British Columbia (see figure 4.1). The climate and geology was summarized from a 1963 'Drainage report of the Municipality of Surrey' presented below.

4.1.1 Climate

"The climate of the southwestern section of the Municipality has been described as of the 'cool Mediteranean type, characterized by warm temperate rainy winters and cool dry summers'....while the remainder of the Municipality has 'a marine west-coast climate featured by warm temperate rainy winters and cool summers'. Surrey is protected from the direct onslaught of storms from every direction by various mountain ranges....the Coast Range to the north and the Cascades to the east. It is protected from....rainstorms....from the west and southwest over the Pacific Ocean by the mountains of Vancouver Island and the Olympic Mountains of northwest Washington.

The effect of storms....from the southwest and west varies considerably....due to the uplift caused by the Coast Mountains.... consequently the total annual rainfall and snowfall is much greater in the north-eastern section of the Municipality than it is in the south-western section. The minimum to maximum average annual total precipitation is approximately 40 inches (1020 mm) to 70 inches (1780 mm)". The average annual temperature is fairly consistent throughout Surrey at approximately 50 degrees F (11.3 celsius). The

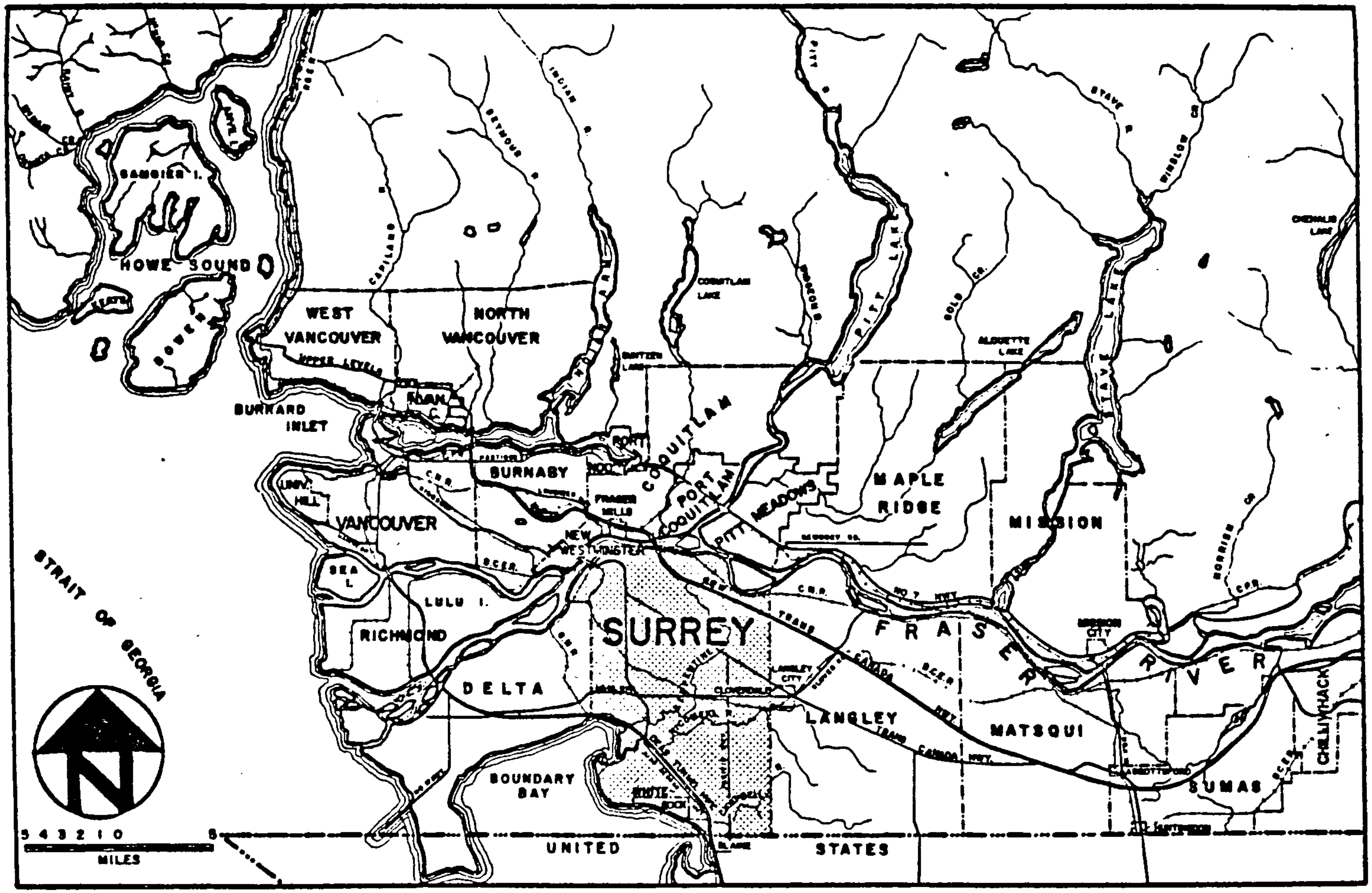


Figure 4.1: Location plan showing Surrey in relation to the Greater Vancouver area (District of Surrey, 1963)

average hours of sunshine vary from 1900 hours in the southwest to 1500 hours in the northwest section.

"During the summer months of most years all the Municipality requires supplemental irrigation, particularly during July to August, for the maximum development and yield of crops." The Department of Agriculture and Food has suggested an irrigation duty of 18 inches (457 mm).

"The worst drainage conditions occur when there are high rainfall intensities after a heavy fall of wet snow has fallen on frozen ground which was previously saturated by rain". Such an event was recorded in 1935.

"Wind effects are not significant in terms of increasing evaporation from water surfaces but may caused extended period of high tide causing the river to back up".

4.1.2 Geology

"Surrey is part of the general area known as the Fraser Lowland which lies between two major geological features called the Coastal Mountains and the Coastal Trough. The Surrey portion of the Fraser Lowland consists of extensive low hills or uplands ranging up to 400 feet (122 m) in elevation and separated by the fairly wide flat bottomed valleys of the Nicomekl and Serpentine Rivers and the narrower valley of the Campbell River. The valleys of the former two rivers were originally enlargements of the sea and were not caused by river erosion. The upper part of the Campbell River Valley was probably created by the river as the land rose after being depressed relative to the sea by the last ice sheet.... between 5000 to 10000 years ago. It was estimated that the thickness of the ice was 7500 feet (2286 m) causing great pressures which account for the very hard nature of the clays and silts which were laid down prior to the ice forming.

"The upland areas of Surrey are of two main types. The first consists of a core of various types of clays, silts, sands and gravels deposited by glacial ice, streams, lakes, swamps and the sea. Thess uplands are usually flat or terraced with sufaces of glacial outwash found in the Campbell River upland. The second type of upland has a similar core but the areas are usually rolling and hummocky with surfaces of glacial till and glacio-marine deposits occurring in the Newton, Clayton and Sunnyside Uplands.

"Bedrock lies several hundred feet below the above deposits and no bedrock is to be seen anywhere in Surrey. In the Boundary Bay area the bedrock is over 2000 feet (610 m) deep.

The tills, clays and silts have low permeabilities discouraging high infiltration rates, The sand and gravel outwash allow higher infiltration ans sub-surface drainage.

"The Nicomekl and Serpentine River Valleys consist mainly of peat underlain by impervious layers of clay and silt deposits. The groundwater body is located near the surface and excess rainfall cannot be absorbed."

4.1.3 River Discharges, Temperature and Irrigation

The Nicomekl River is a low lying meandering tidal river as shown in the photographs of figure 4.2. The river profile in figure 4.3 identifies the break in river slope in the upland reaches. A detailed plan of the watershed in shown in figure 4.4a) with measured cross-sections indicated. Summary data for cross-sections used in this study are shown in figure 4.5b).

Historical monthly discharge data for the Nicomekl are available from Water Survey of Canada (Stn: 08MH105) and are summarized on an annual basis in figure 4.5 showing 5%, mean and 95% discharge envelopes. A frequency analysis indicated that these monthly flows are log-normally distributed (figure 4.6).

Stream temperatures from the Serpentine River for various stations and various sampling periods are summarized in figure 4.7 by Julian day. These temperatures were applied to the Nicomekl in lieu of actual data to assess the temperature dependency of BOD and DO decay parameters. Following techniques used by several researchers surveyed in sections 2.2.1 and 2.2.2, the river discharges and stream temperature could be generated by Monte Carlo or Markov techniques to account for temporal and possibly spacial variability. This was not done in this study but would make an interesting extension.

Detailed though sporadic water quality data was available for the Serpentine River in a report by Moore (1985a). A similar report for the Nicomekl is in preparation (Moore, 1985b). This grab sample data was from a monitoring network of stations on the river, although it is not sufficient for mathematical model development or validation. The use of this data for model 'identification' as discussed in section 2.2.3 where

Figure 4.3a: The Nicomekl River: Downstream Tidal Gate

Figure 4.3b: The Nicomekl River: Nicomekl at 40 Ave.

Figure 4.3c: The Nicomekl River: Nicomekl before entering Mud Bay

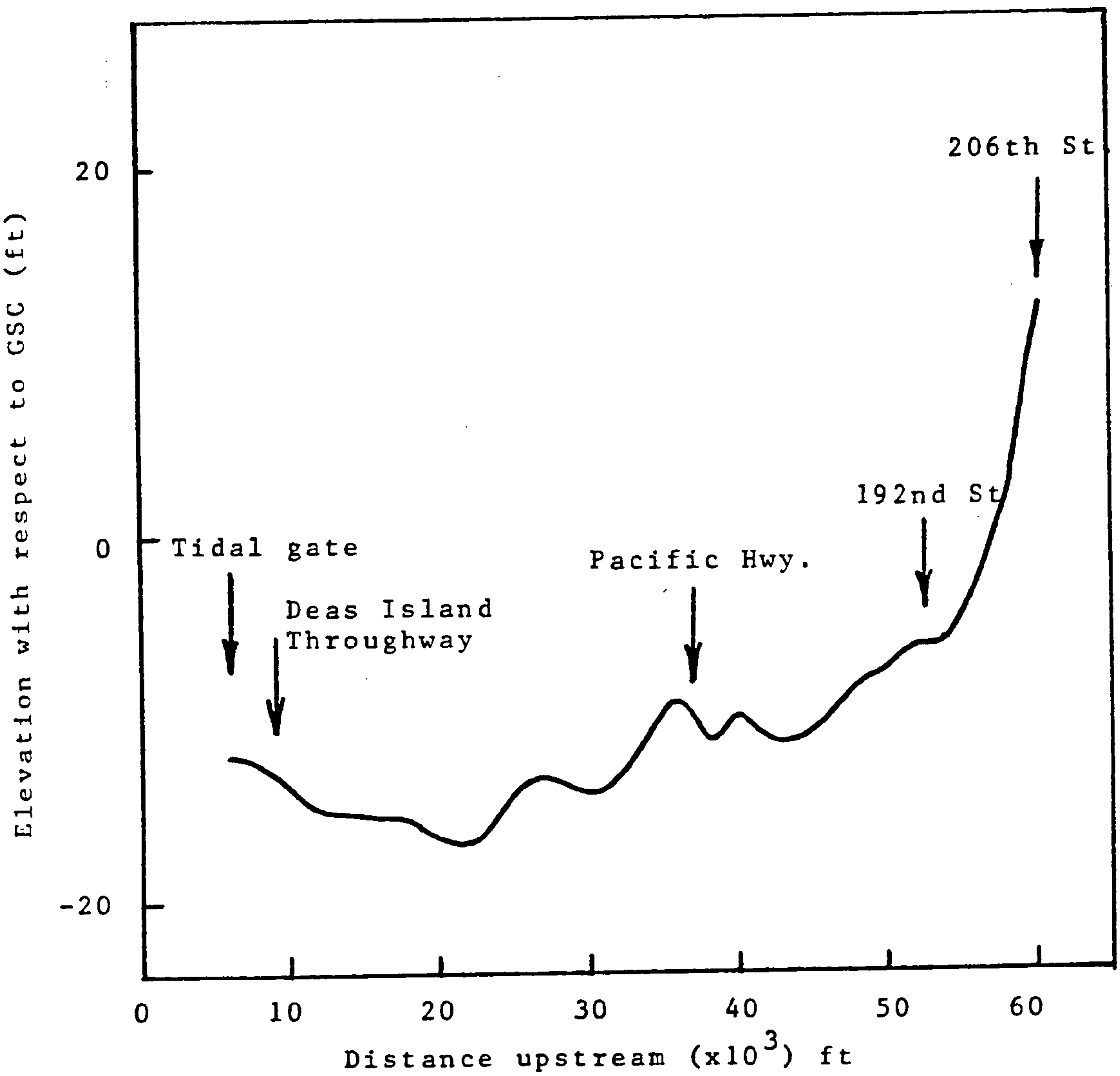


Figure 4.4: Nicomekl River Profile

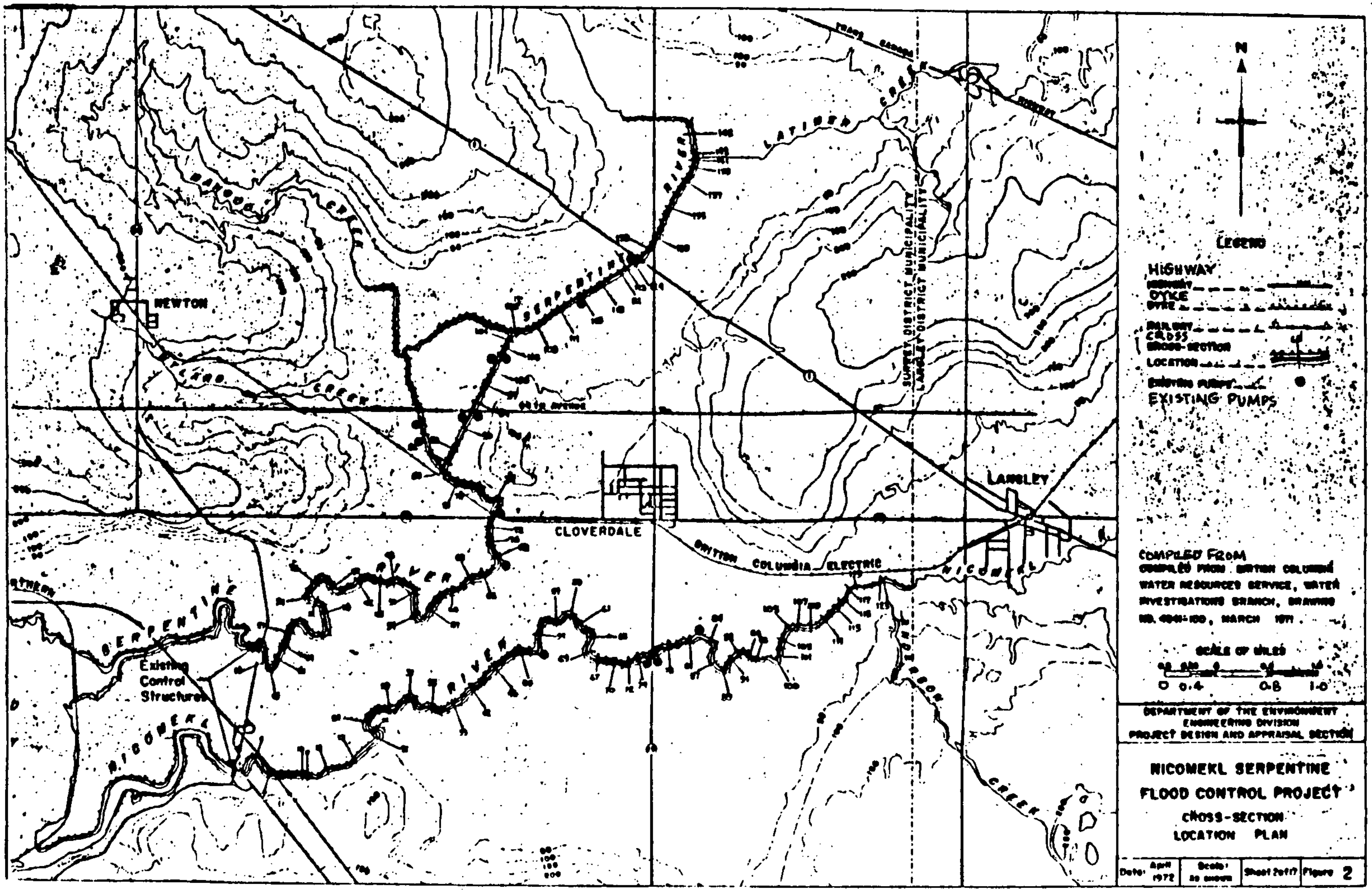


Figure 4.5a): Plan of the Nicomekl-Serpentine floodplain

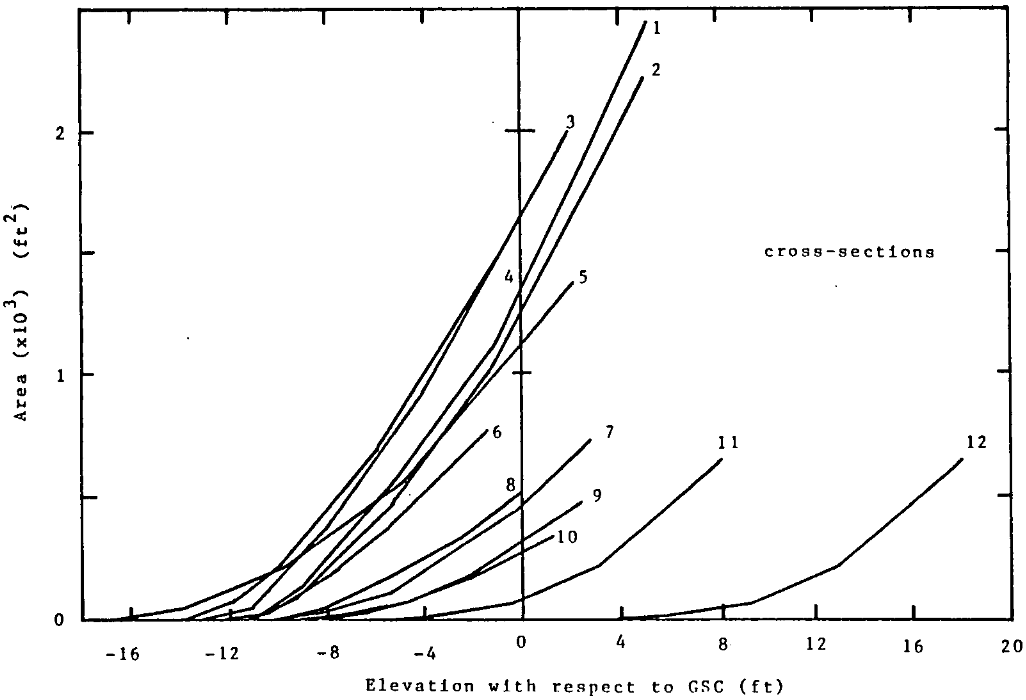


Figure 4.5bi): Graph of cross sectional area versus river water elevation

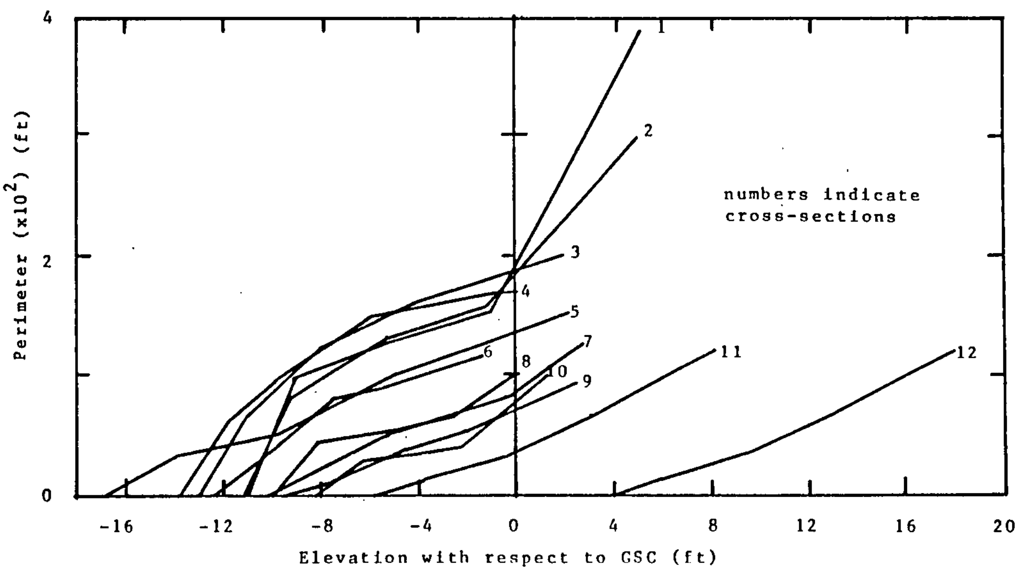


Figure 4.5bii): Graph of perimeter versus river water elevation

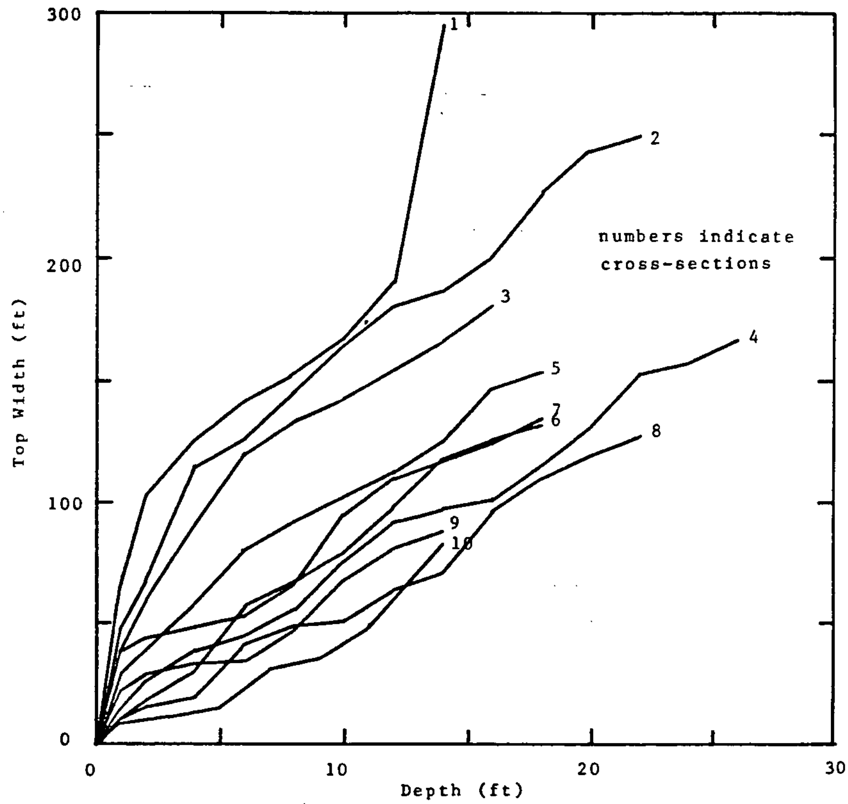


Figure 4.5b(iii): Graph of river top-width versus depth of flow

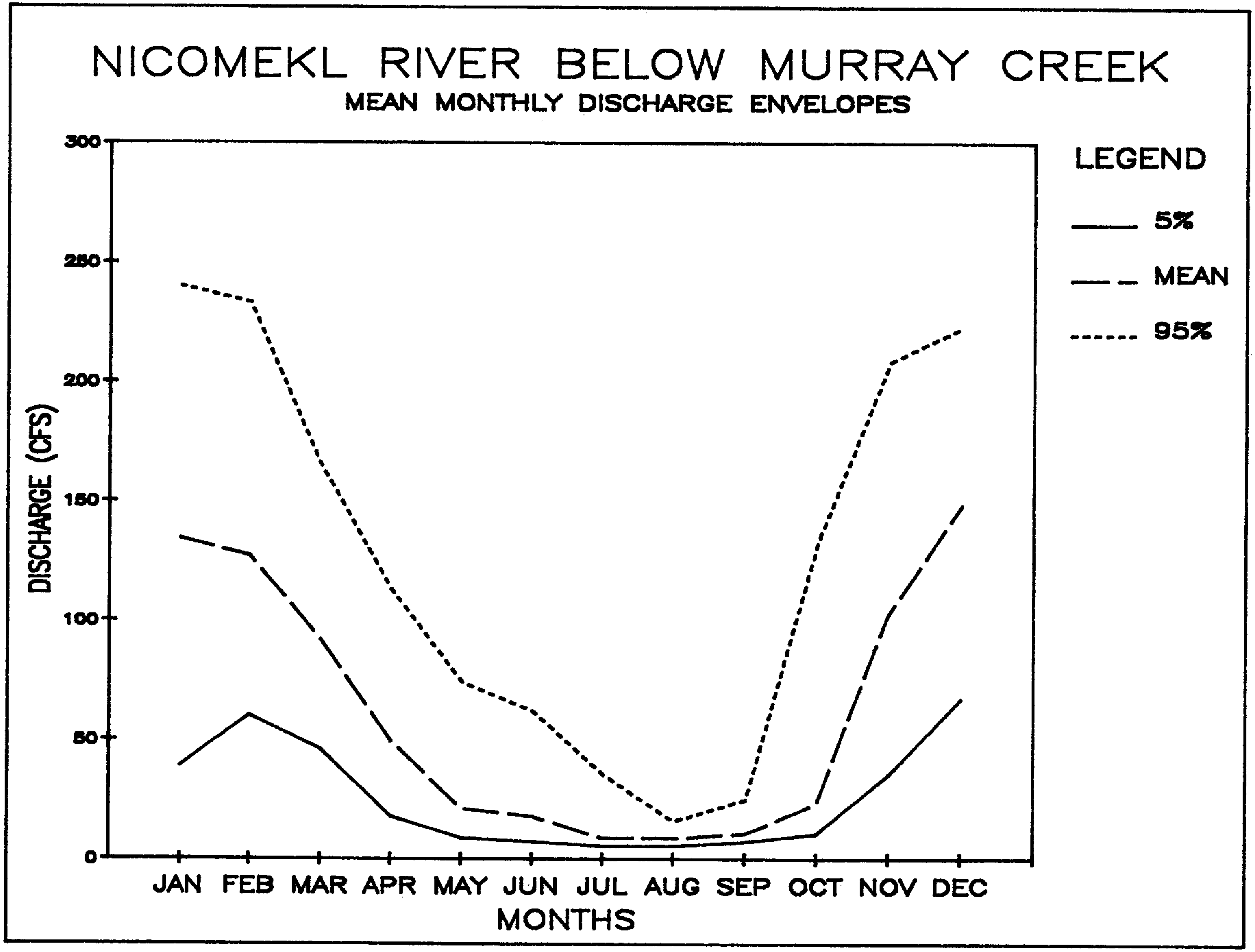

NICOMEKL RIVER BELOW MURRAY CREEK
MEAN MONTHLY DISCHARGE ENVELOPES
DISCHARGE (CFS)
300
250
200
150
100
50
0
JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC
MONTHS
LEGEND
5%
MEAN
95%

Figure 4.6

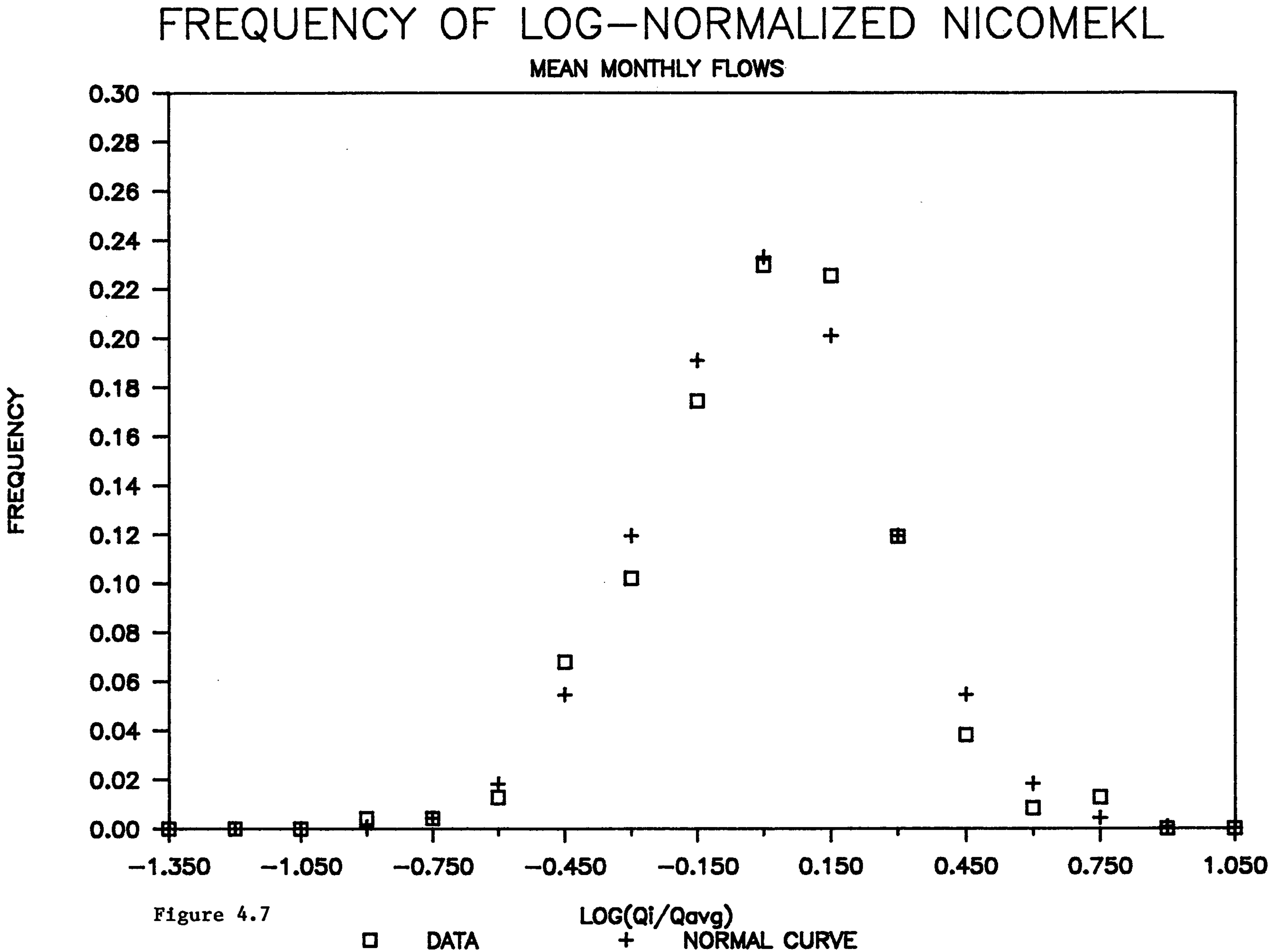
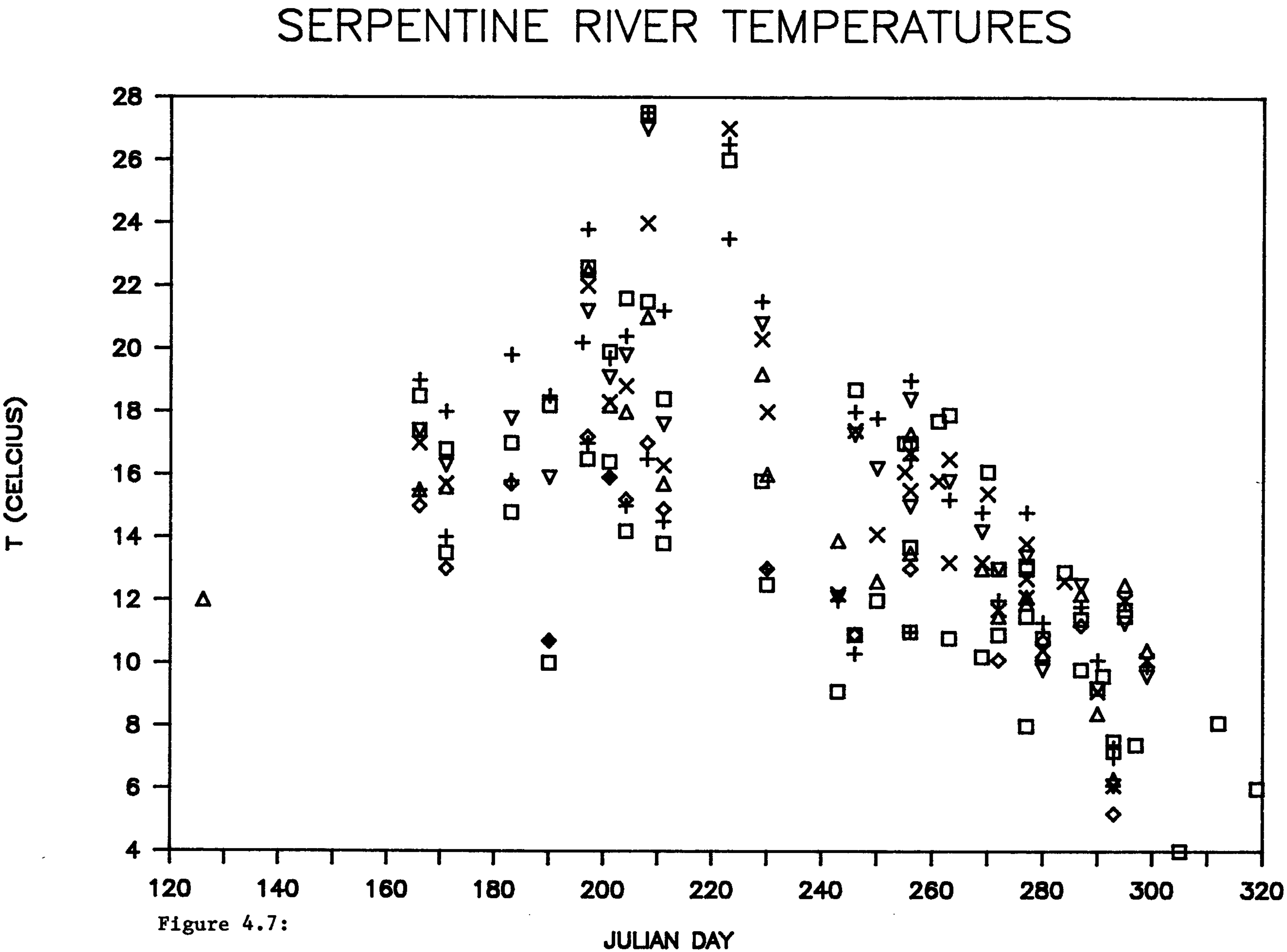

FREQUENCY OF LOG-NORMALIZED NICOMEKL
MEAN MONTHLY FLOWS
FREQUENCY
0.30
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-1.350
-1.050
-0.750
-0.450
-0.150
0.150
0.450
0.750
1.050
LOG(Qi/Qavg)
DATA
NORMAL CURVE

Figure 4.7



SERPENTINE RIVER TEMPERATURES
T (CELCIUS)
28
26
24
22
20
18
16
14
12
10
8
6
4
120
140
160
180
200
220
240
260
280
300
320
JULIAN DAY

Figure 4.7:

model parameters and initial conditions could be estimated, is another interesting extension. DO data for the Nicomekl River was available for a limited sampling in 1974 to 1875 from Bourque and Hebert (1982) and was used to determine initial and boundary conditions for the BOD/DO model.

Irrigation license information was available from the Water Management Branch giving maximum allowable seasonal (April to September) irrigation uptakes. This provided an upper limit of irrigation demands but was too coarse for this study, even when monthly estimates were made (Van der Gulik, 1986). Additionally, actual uptake data was not available to characterize the frequency and volume distributions of withdrawals. Thus, both irrigation (and ditch inflows) were not accounted for. This exclusion would be least valid at low flow periods when irrigation withdrawals may have significant impact on water quality by reducing the dilution and transport capability of the river.

4.1.4 Tides

The tides in Boundary and Mud Bays are mixed and mainly semi-diurnal displaying a major M2 component with amplitudes of 0.8m and 156 to 164 phase (Crean, 1983). Sample tide records are shown in figure 4.8.

4.1.5 Land Use and Principle Jurisdictions

"The Serpentine-Nicomekl lowlands have been reserved for agriculture and the (Surrey) Council has adopted a policy aimed at protecting these farmlands against frequent flooding. On the Upland areas, the rural and forested lands are steadily being occupied by housing and commercial development." (Figure 4.9)

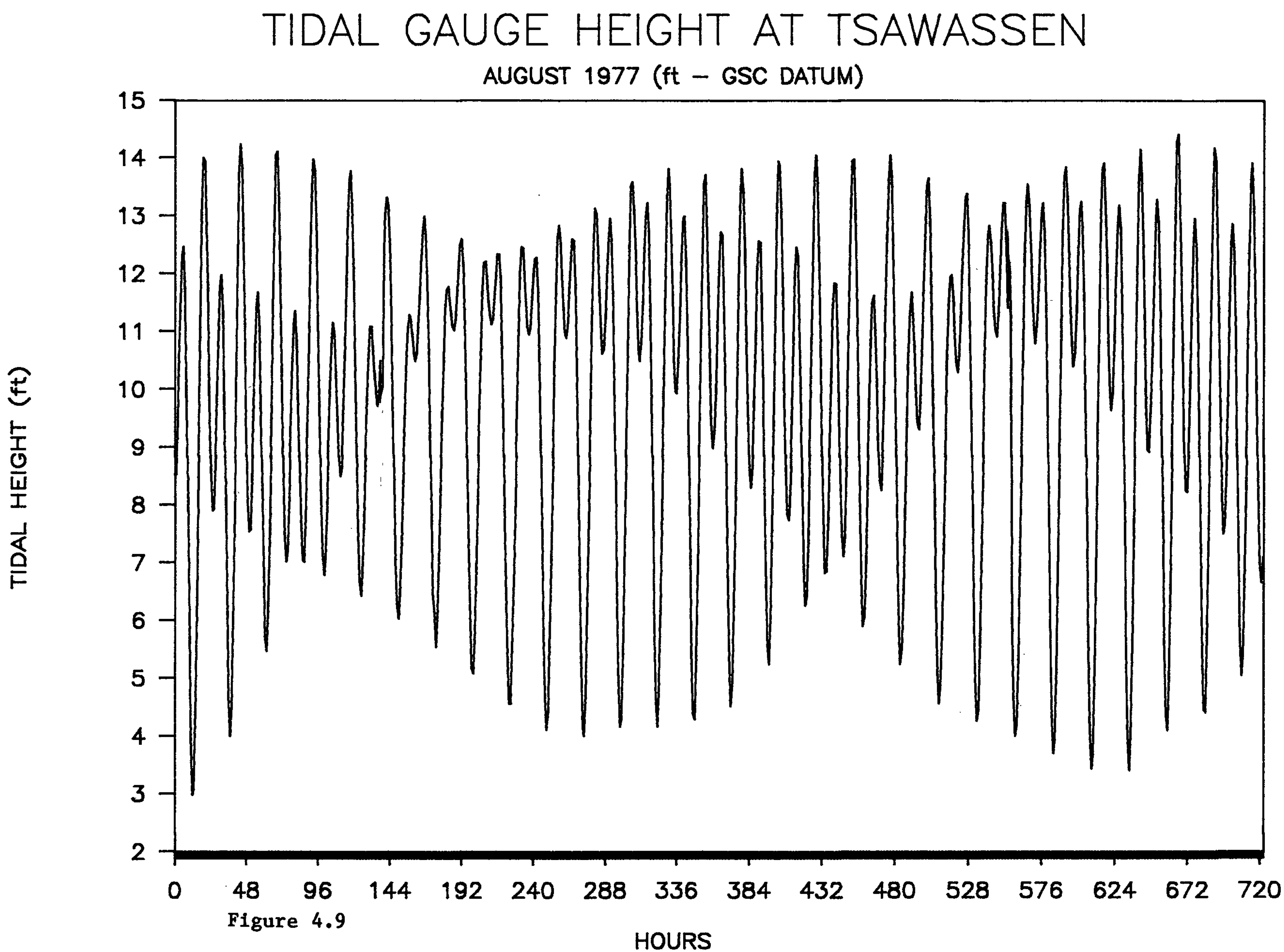

TIDAL GAUGE HEIGHT AT TSAWASSEN
AUGUST 1977 (ft – GSC DATUM)
TIDAL HEIGHT (ft)
15
14
13
12
11
10
9
8
7
6
5
4
3
2
0
48
96
144
192
240
288
336
384
432
480
528
576
624
672
720
HOURS

Figure 4.9

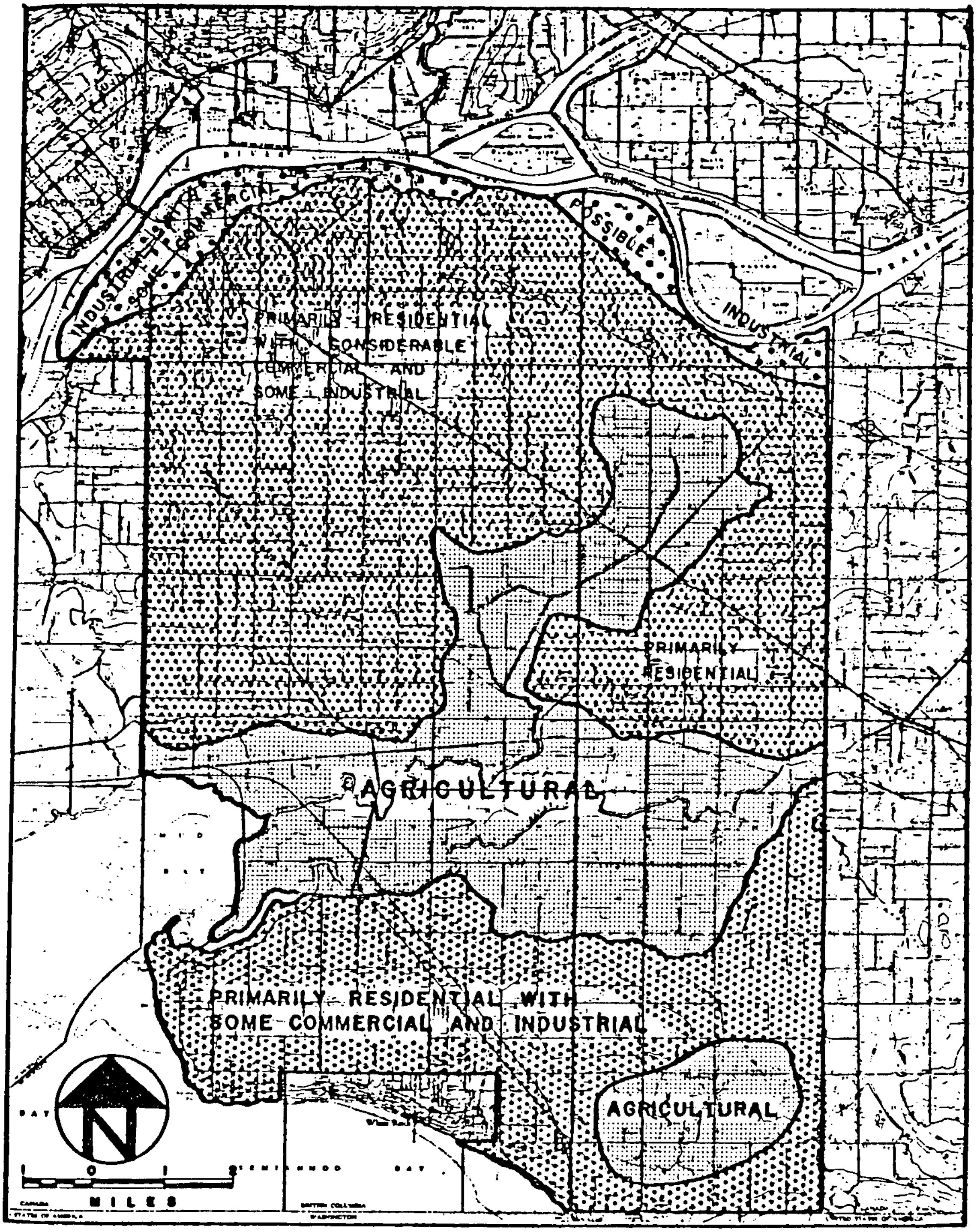


Figure 4.10 Plan showing expected ultimate land use (District of Surrey, 1963)

The agricultural lands are are protected by a dyking system maintained by the New Westminister Dyking Commission. At the downstream ends of both rivers are control structures in the form of tidal gates which minimize salinity intrusion and attenuate tidal fluctuations. The present structures were installed by the federal Ministry of the Environment in 1972. A seasonal irrigation licensing system for the river waters is maintained by the Water Management Branch of the provincial Environment Ministry. This agency grants licenses and has the mandate to control actual irrigation (by license seniority) which may occur towards the end of summer when irrigation demands are high and river flows low. The tidal flats area of the watershed is an ecological sanctuary and is supported by the provincial Fish and Wildlife Branch. Both the Nicomekl and the Serpentine rivers support salmon runs which travel through Boundary Bay to spawn. The federal Fisheries and Oceans is resposible for protecting the fishery resource and in conjunction with the provincial Water Quality Branch is pursuing strategies to alleviate and control the negative impact of poor water quality. It is the mandate of the Municipality of Surrey to formulate land use policies while providing basic services and infrastructure to the residents, businesses and industries. The agricultural land is also part of the provincial Agricultural Land Reserve with land sales and purchase regulated by the Land Use Commission. The provincial Ministry of Agriculture and Food also assists agricultural land use decisions by providing technical support and consultation (Van der Gulik, 1983).

The primary source of poor water quality in the Serpentine-Nicomekl rivers is the long and short term non-point organic waste pollution from agricultural lands bordering the rivers and secondary

short-term commercial and industrial point sources. Currently the Serpentine river has the poorer water quality where significant fish kills have been reported as a result of low dissolved oxygen (Moore, 1985a). The Nicomekl river is estimated to reach this same condition in 'about five years' (Gough, 1985). These organic wastes appear to reach the the rivers primarily by surface runoff processes and groundwater drainage with groundwater seepage playing a secondary role. The low dissolved oxygen problem has reach crisis proportion in the Serpentine river such that a federally funded project is underway to aerate a critical lowland river reach. (Mavinic, 1986).
The Nicomekl which was chosen as the river of interest for this study, due to river data availability, though application of the methodology presented previously may have had more immediate policy implications if applied to the Serpentine.

4.5 Data Summary

The following data sets were available for this study:

a) monthly and annual Nicomekl river flows for the purpose of quantifying the longer-term hydrologic regime;

b) hourly tidal data from the Tsawassen tidal station number 2/4900/12307 for 1972 and 1977;

c) hourly stream flow data from Nicomekl river stations WSC-08MH105 and WSC-08MH050 below Murray Creek and at 192nd Street respectively for 1972 and 1977;

d) cross-sectional data from various sections on the Nicomekl River from 1970 Environment Canada project data;

e) hourly tidal predictions for Tsawassen for twenty years provided by the Institute of Ocean Sciences at Patricia Bay, British Columbia.

f) stream water quality measurements for the Serpentine river from monitoring surveys done by the Water Quality Branch and the Environmental Protection Service.

g) irrigation license information on the maximum seasonal withdrawals allowed from the Water Management Branch.

h) water quality information for the purposes of validating the water quality model developed in section 3.2.2 was available in the report of Pence et al (1968) for the Delaware Estuary.

Data sets b), c) and d) were used to calibrate and validate the water quantity model of section 3.2.1. Data set f) was used to identify critical low dissolved oxygen periods and for stream temperature data. Since sufficient water quality data was not available to validate the BOD/DO model data set h) was used as one method of partial validation while model parameters were obtained from the literature.

5. MODEL CALIBRATION AND VALIDATION

The models developed in section 3.2 were calibrated and validated using data sets described in section 4.2. This chapter summarizes the validation procedure justifying the use of the models, although validation of the water quality model on another river does not validate the model for use on the site of interest (Ambrose, 1986). Transient conditions were used for validation so as to ensure applicability to the dynamic regime in question.

5.1 Hydrodynamic Model

Calibration runs were made using the data sets described in section 4.2a) and b). Specifically, the boundary condtions were established by:

a) tidal gauge height data from Tsawassen;

b) discharge data for the station below Murray Creek which was computed from a nonlinear regression of a stage-discharge curve (Jamal, 1980);

c) initial conditions were taken as steady state flow of 50 cfs and 0.2 ft depth of flow as a simplified assumption.

The transient effect of the initial conditions were allowed to dissipate over the first 2000 iterations of the model. Nicomekl cross-section data of section 4.2d) was used to characterize the river geometry. The Manning's 'n' was used as the calibarating parameter and manipulated during calibration until a 'good' visual fit was obtained with April 1977 data. This was compared to field estimated values of similar rivers of 0.03 to 0.05 (Chow, 1959).

The discrepancies between the simulated and actual river stages may be due to:

a) not accounting for surface water discharge from ditches and minor tributaries and irrigation uptake;

b) groundwater recharge and discharge at different points in the tidal cycle depending on tidal velocity and soil moisture conditions at different locations along the river.

Model validation was made for April 1972 as shown in figure 5.2 and confirmed by statistical testing (t and F tests at the 95% confidence level). An issue not addressed here is the variability of the calibrating parameter which could be determined, both for intrinsic and seasonal variation, using real-time estimation techniques as outlined in section 2.2.4. Calibration of the model was done for April 1977 data for higher dischargess than summer flows but where the dynamics of flood hydrographs would be a good test of the model. Summer flows may indeed be better fit with different values of ´n´. In summary, the hydrodynamic model appears to be a ´well identified´ model.

5.2 BOD/DO Model

The validation of the BOD/DO model for the Nicomekl River was not possible due to the lack of suitable field data. Alternate methods for validation of the model were reviewed:

a) validation of the algorithm by similar studies documented in the literature;

b) validation of the algorithm by comparison with well-known analytic models;

c) validation of the model on a data set from another river system.

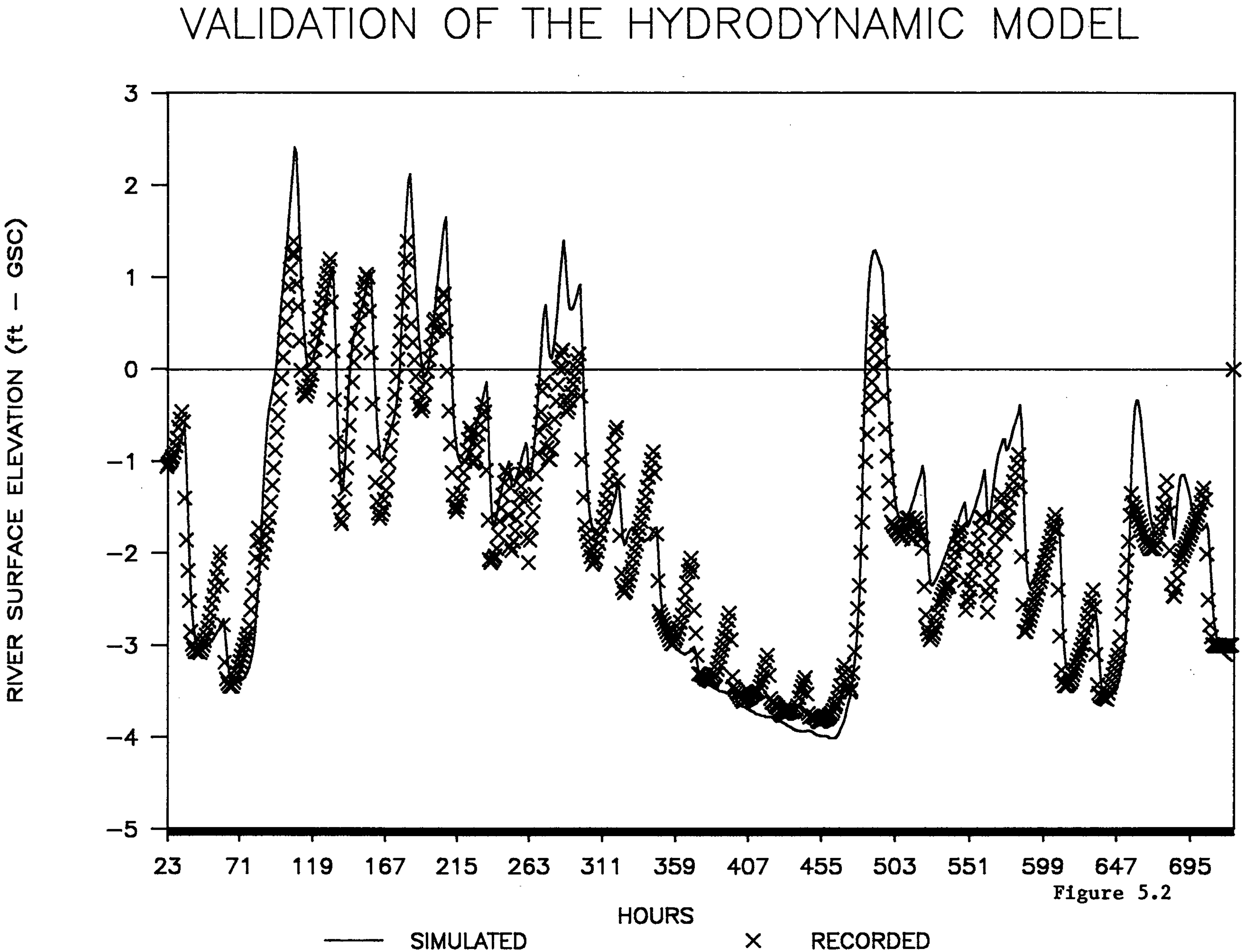

VALIDATION OF THE HYDRODYNAMIC MODEL
RIVER SURFACE ELEVATION (ft – GSC)
3
2
1
0
−1
−2
−3
−4
−5
23
71
119
167
215
263
311
359
407
455
503
551
599
647
695
HOURS
SIMULATED
RECORDED

Figure 5.2

5.2.1 Validation in the Literature

A review of studies using the model described in section 3.2.2 revealed a few studies which have validated the use of this model for field conditions.

Bella (1970) used the model (in two-dimensions) to study the DO profiles in Lake Sammamish, Washington. The processes of reaeration, photosynthetic oxygenation, vertical mixing and oxygen uptake were modeled while accounting for temperature variation. Figure 5.2 shows the results from this study with 5.2b) and 5.2c) as the measured and predicted vertical DO profile at two lake stations. Sensitivity analysis was also performed for the coefficient of atmospheric reaeration, photosynthetic and respirative rates and the vertical dispersion coefficient.

Aiba and Ohtake (1977) used the same modeling procedure in their study of the balance of PO4-P (phosphate) in the shallow polluted Tama-gawa River, Japan. They modeled the processes of convection, dipersion, suspended and river-bed adsorption, hydrolysis, benthic demand, aquatic demand and decomposition of river organics. This sophisticated material balance model was used to predist the PO4-P concentrations at four river stations as validated in figure 5.3.

These two studies address different pollution problems successfully when using the basic Bella and Dobbins (1968) model. Though these studies do not validate the BOD/DO model for the river of concern in this study, they do lend credit and justification to the use of the algorithms proposed.

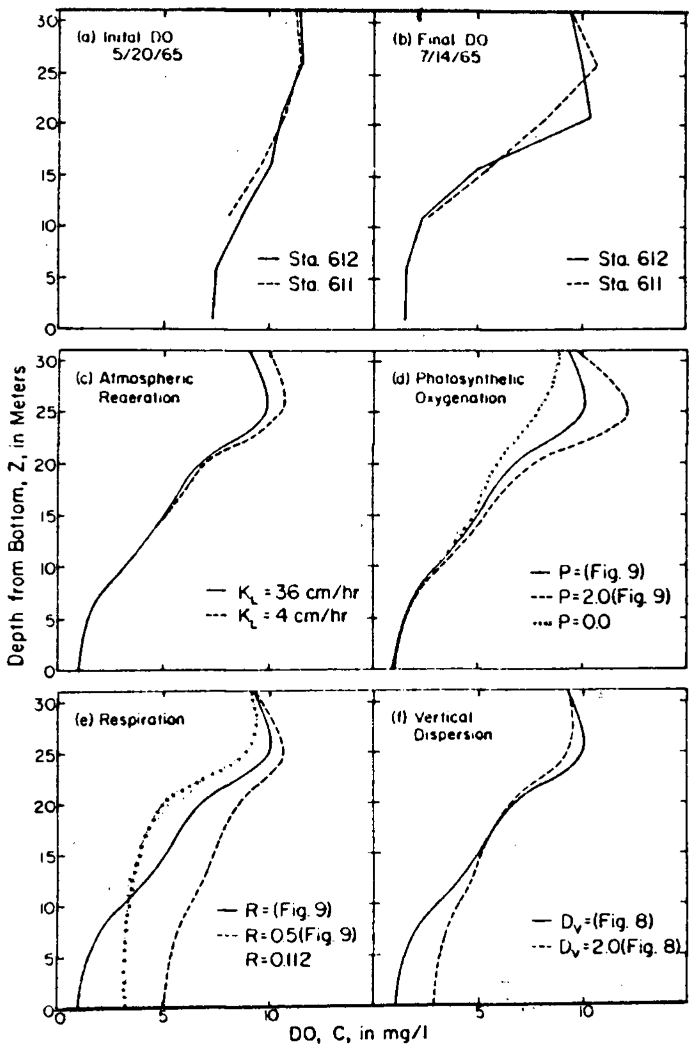


FIG. 5.3.—DO PROFILES IN RESPONSE TO DIFFERENT PARAMETERS

(Bella, 1970)

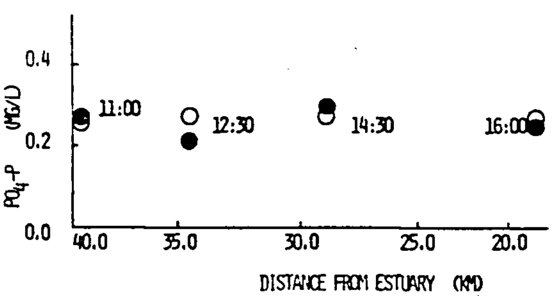


Fig.5.4. PO_4–P concentrations, observed (●) and simulated (O). (August 18, 1972); time of sampling is shown in the figure. (Aiba and Ohtake, 1977)

5.2.2 Validation Against Analytic Models

A similar validation process as presented by Bella and Dobbins (1968) was developed where two scenarios were considered:

i) slug BOD river loading of a tidal river to test the convection, dispersion and decay processes of the BOD model;

ii) continuous BOD estuary loadings to test steady-state BOD and DO profiles.

For both scenarios, results using the model developed in section 3.2.2 were compared with well known analytic models discussed by Bella and Dobbins (1968) and their own numerical results.

5.2.2.1 Slug Loads

An initial load of 100 units in a sinusoidal tidal river was used with parameters summarized in table I .

Table I : PARAMETERS USED FOR ANALYTICAL VALIDATION

Parameter	Value	Unit	Remarks
D	2	$mi^2.day^{-1}$	Dispersion
k_1	0.3	day^{-1}	BOD Decay
Δx_n	0.25	miles	Reach Length
ΔT	1/96	days	Time Step
U	$17.3\sin(2\pi t/P)$	$mi.day^{-1}$	Tidal Velocity
P	12.5	hours	Tidal Period
a	$\Delta x_n/2$	miles	Slug Length

An analytical solution for the instantaneous line source heat conduction problem was presented by Carslaw and Jaeger (1960, as reported by Bella and Dobbins, 1968; and Sageev, 1986) as:

$$L(x,T) = 50\ \left[\ \mathrm{erf}\frac{(a-(x-x'))}{2\ DT} - \mathrm{erf}\frac{(a+(x-x'))}{2\ DT}\ \right]\ e^{-k_1 T}$$

where $x' = \int_0^T U(t)\ dt$.

A favourable comparison with Bella and Dobbins' (1968) results and the analytical model are shown in table II .

Table II : SLUG LOAD COMPARISONS

Distance (mi)	1 Tidal Cycle A*	B+D**	S***	2 Tidal Cycles A	B+D	S	6 Tidal Cycles A	B+D	S
0	5.903	5.900	5.902	3.572	3.570	3.572	1.104	1.103	1.104
1	4.646	4.647	4.648	3.169	3.168	3.169	1.061	1.060	1.061
2	2.266	2.268	2.268	2.212	2.212	2.213	0.941	0.940	0.941
3	0.684	0.683	0.683	1.215	1.215	1.216	0.770	0.771	0.771
5	0.015	0.014	0.014	0.179	0.177	0.178	0.406	0.406	0.407
7	0.000	0.000	0.000	0.010	0.010	0.010	0.155	0.156	0.156

* Analytical Model
** Bella and Dobbins (1968)
*** This Study.

For steady-state BOD-DO profiles, O'Connor developed the following analytical equations: $L(x) = L_0 \exp\left[\pm x \left(\frac{K_1}{D}\right)^{1/2}\right]$, $L_0 = \frac{m}{2A\sqrt{K_1 D}}$;

$$C(x) = C_s - FL_0\left\{\exp\left[\pm x\left(\frac{K_1}{D}\right)^{1/2}\right] - \left(\frac{K_1}{K_2}\right)^{1/2}\exp\left[\pm x\left(\frac{K_1}{D}\right)^{1/2}\right]\right\},\quad F = \frac{K_1}{K_2 - K_1}.$$

These were used for validation of the steady DO and BOD concentrations due to steady loadings. Table III shows a favourable comparison for BOD and DO for steady state after 19 tidal cycles. Longer simulations would have yielded closer results.

Table III : STEADY STATE COMPARISONS

Distance (mi)	BOD (mg/l) A	B+D	S	DO (mg/l) A	B+D	S
1	6.653	6.671	6.674	1.833	1.714	1.724
2	4.516	4.522	4.530	2.538	2.420	2.427
3	3.066	3.065	3.074	3.373	3.255	3.262
4	2.082	2.077	2.086	4.200	4.082	4.088
6	0.959	0.952	0.961	5.592	5.479	5.487
10	0.204	0.200	0.204	7.175	7.087	7.096
16	0.020	0.018	0.020	7.872	7.831	7.837

The two comparisons above resulted in maximum deviations from the analytical solution of 0.6% and 5.9% respectively.

The comparisons above validate the code developed in this study as being equivalent to the analytic slug load equations and that of Bella and Dobbins (1968).

5.2.3 Validation Using Alternate Field Data

Data for the Delaware Estuary was available from the study of Pence et al (1968), who used characteristic methods in solving a finite-difference scheme of BOD and DO dynamics in the estuary. This data was used in an attempt to demonstrate the applicability and validity of the model developed in section 3.2.3. A sucessful validation here does not imply that the BOD/DO is validated for the Nicomekl River. A field data gathering program based on a rationalized experimental design for the Nicomekl would be required to throroughly validate the model, or more appropriately, to "identify" the best of various models available using estimation techniques.

Pence et al (1968) present the following data in their study:

a) daily river flow data at a Delaware gauging station;

b) daily river temperature at another station;

c) mid-tidal river cross-section data for 30 sections;

d) steady-state BOD loadings (carbonaceous and nitrogenous) in each reach;

e) equations describing the temperature dependency of BOD decay and reaeration coefficients;

f) steady benthic oxygen demand for each reach (photosynthesis was negligible); and

g) base (20 deg. celcius) decay and dispersion coefficients.

Initial conditions were not specified while boundary conditions were established by the river discharge. No diurnal tidal data was available so that mid-tidal cross-sectional data was used. There were no calibrating parameters.

Using data a) through g), DO concentrations in the Delaware for 1964 were simulated with time steps of 0.25 days. The three validation cross-sections used by Pence et al (1968) are shown in figures 5.4 a), b), c), with recorded data, model results of these researchers and the simulation performed in this study. It is apparent that the BOD/DO model of section 3.2.3 predicts more conservative profiles. This is likely due to the averaged tidal conditions used (especially for section 22 which the furthest downstream). The upstream section 4 and 7 yield better fits to the data as the tidal influence is reduced. The model reflects a greater sensitivity to river temperature than the Pence et al (1968) model.

This validation indicates the usefulness of the BOD/DO models developed in this study, although full validation was not obtained. As mentioned previously, validation of water quality models with limited ill-designed data programs is a difficult excercise.

5.2.4 <u>Summary</u>

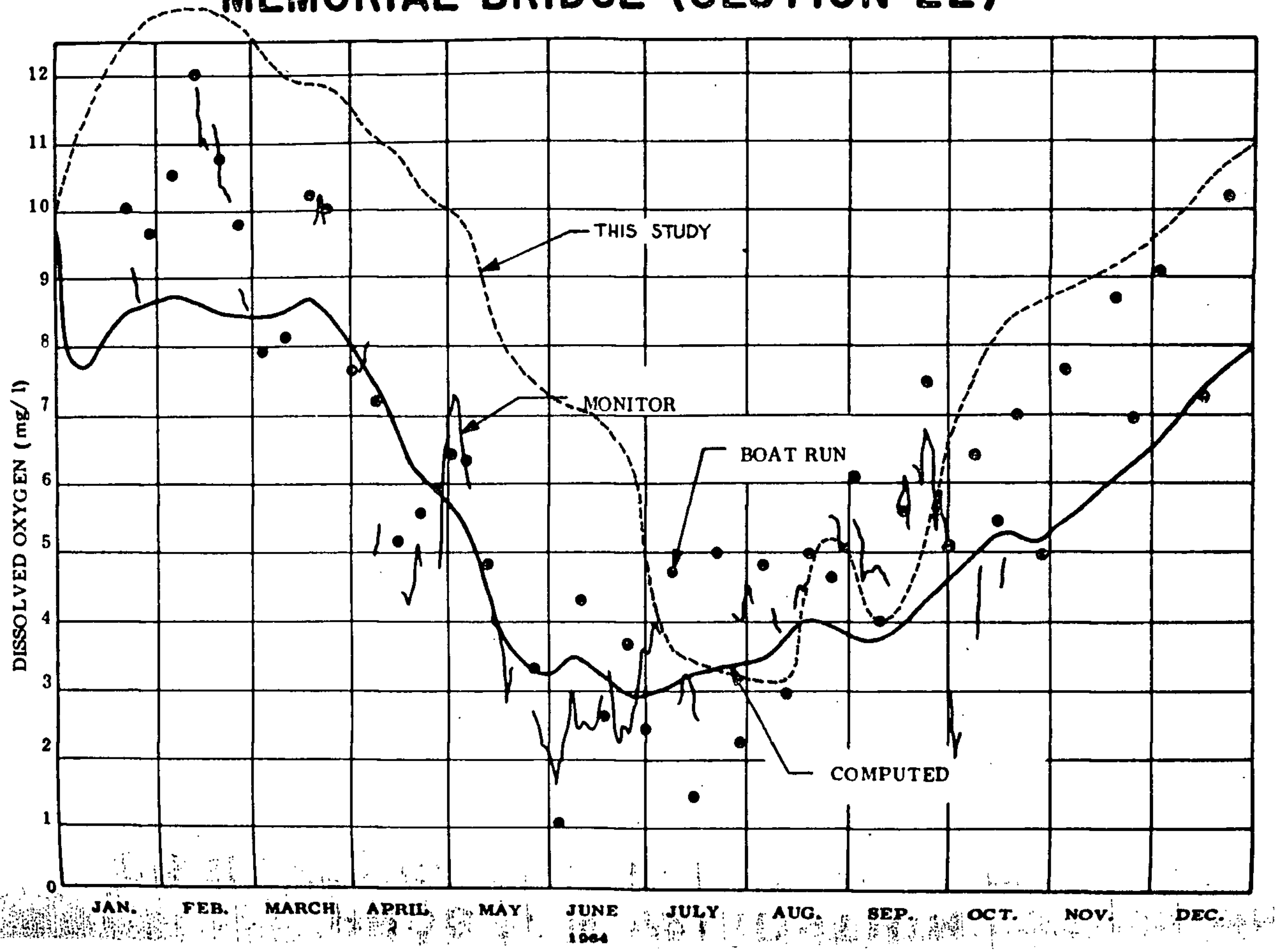
In summary, the BOD/DO model has not been fully validated for the river system under study since:

a) the alternative data set and application studies surveyed dealt with other river systems;

b) the time scales of the phenomena analyzed by the studies in a) are of different orders: seasonal versus diurnal;

c) the analytic models only validate the algorithm used in the BOD/DO model but do not validate its applicability.

PROGRAM VALIDATION
DISSOLVED OXYGEN PROFILE AT DELAWARE MEMORIAL BRIDGE (SECTION 22)



(Pence et al, 1968)

Figure 55a: Dissolved Oxygen at the Delaware Memorial Bridge.

PROGRAM VALIDATION
DISSOLVED OXYGEN PROFILE AT TORRESDALE (SECT. 7)

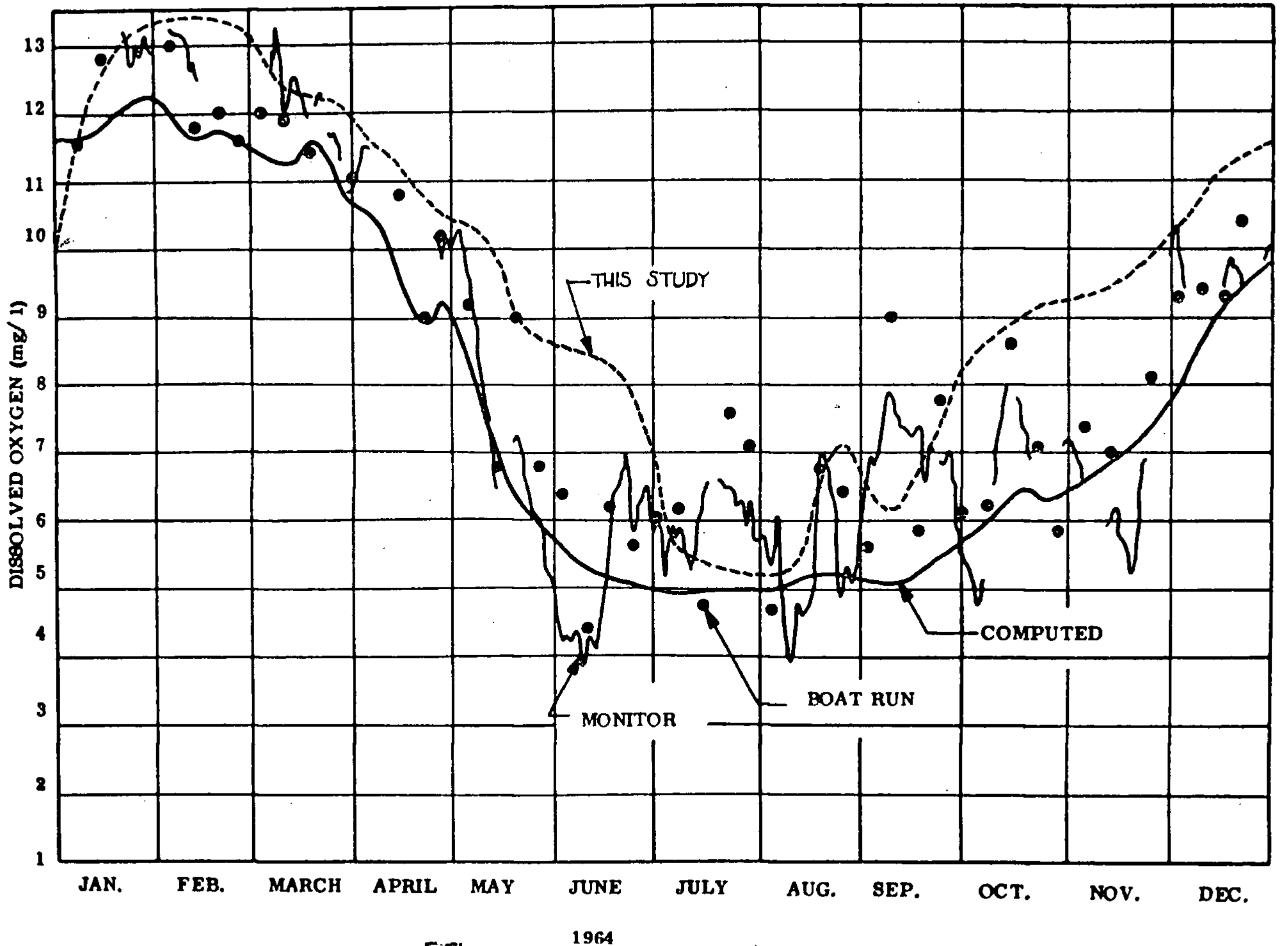


Figure 5.5b: Dissolved Oxygen at Torresdale, Pa.

(Pence et al, 1968)

PROGRAM VALIDATION
DISSOLVED OXYGEN PROFILE AT BRISTOL
(SECTION 4)

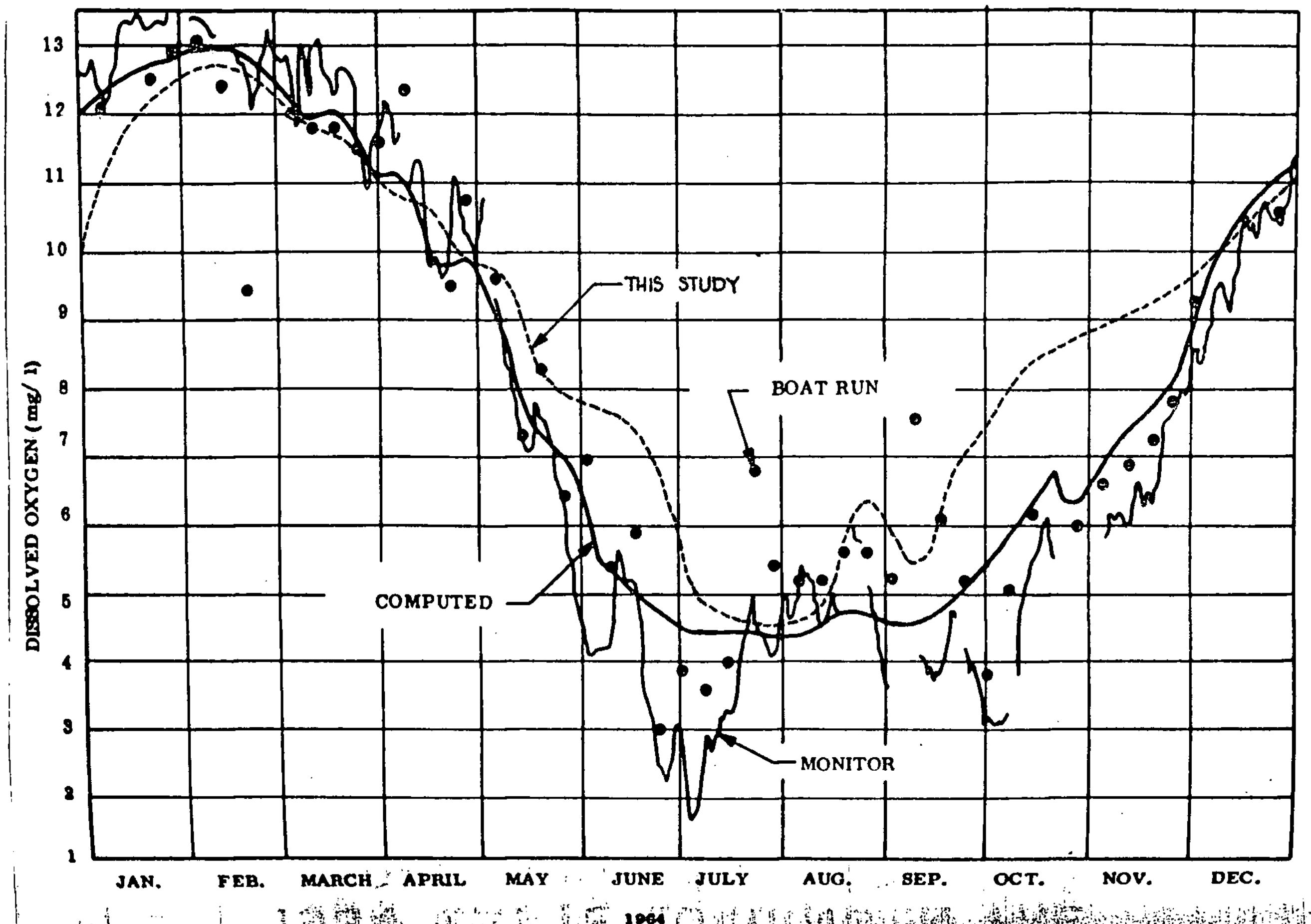


Figure 5.5c: Dissolved Oxygen at Bristol, Pa.

(Pence et al, 1968)

d) there is insufficient field water quality data to validate the BOD/DO model for the Nicomekl river.

The applicability and use of the model must therefore be used with due caution and the results presented in the next chapter, taken as estimated predictions subject to full model validation.

6. MODEL APPLICATION TO THE NICOMEKL RIVER

This chapter discusses the application of the models developed in chapter 3, following the methodology of figure 3.8, to the study area in solving the problem formulated in section 3.1. Specifically the methodology used was as follows:

1) choose model parameters for model application (section 6.1.5) with discharges at the 5% and 95% percentage levels;

2) using the BOD model (section 3.2.2.3) and the hydrodynamic model (section 3.2.1) simulate, with models parameters and discharges above, BOD concentration time profiles resulting from unit BOD loadings, for all reaches in the river for each time period in the decision horizon;

3) using the NLP algorithm (section 3.3.3) and DO model (section 3.2.2.4) obtain the optimal BOD loadings which yield DO profiles which meet the regulatory limits at some risk level;

4) test the sensitivity of the optimal solutions to model parameters (section 6.3).

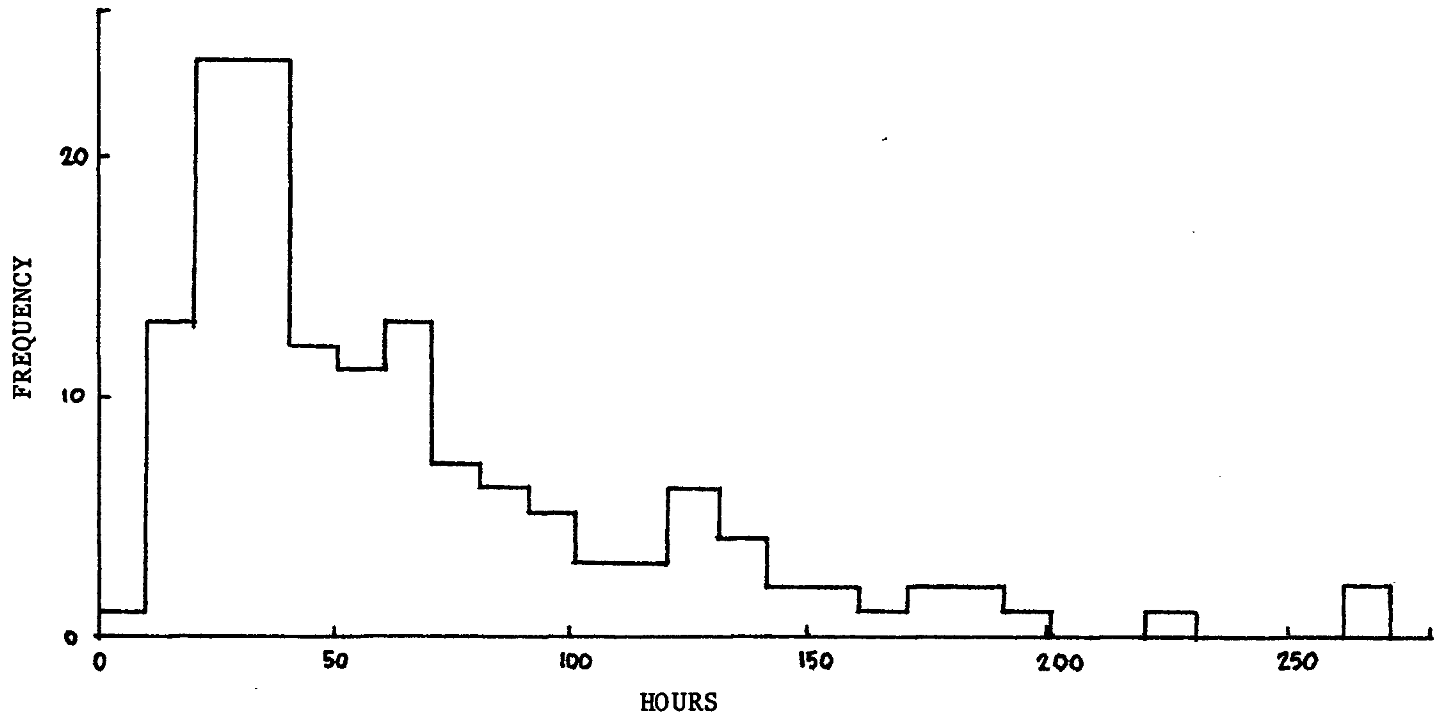
6.1 Factors in BOD Simulation

A variety of factors were considered in the prediction of BOD profiles.

6.1.1 River Discharge

Steady and unsteady discharges were analysed. Using data for 1971 and 1977, the interarrival times between storm events were estimated and summarized in a frequency plot of figure 6.1.

Figure 6.1: Storm Interarrival Time for the Nicomekl River below Murray Creek for 1972 and 1977 data



The mean interarrival time of about 67 hours (2.8 days; in a log-normal distribution) was about forty percent of the one week decision horizon. Thus the use of a synthetic hydrograph generator developed in this study did not appear to be justified for the purposes of simulating upstream inflows since the variability of the synthetic hydrographs over one week would require extensive use of Monte Carlo simulation techniques extending considerably the execution times of 'production' runs of the model beyond the time limits for completion of this study. Additionally, since critical periods occur during the dryer summer months, the use of steady discharges appeared sufficient to describe this boundary condition. These discharges could be modeled either as constants or as some stationary autocorrelated process. The former simplified method was used though the latter could have been easily incorporated as was done by Jamal (1980).

6.1.2 Tides

A one week tidal period was chosen to represent typical conditions at the downstream boundary of the river system. This was required since different tidal periods may reflect different tidal harmonics of long or short frequencies. The use of various tidal periods could have been made to reflect maximum and minimum tidal conditions as sensitivity tests on the optimal decisions. This was not done in this study, again, due to lengthy model execution times required on the microcomputer.

An alternative source of tidal data for the downstream boundary was that available from the Institute of Ocean Sciences. This hourly data was generated by a model based on Gordon's tidal heights analysis by using harmonic constituents for station of interest. Predictions for

Tsawassen were made using 62 constituents including a high astronomical frequency of 18.6 years. This data was initially intended for use in a Monte Carlo simulation framework which was later discarded due to constraints on time and the computing resources available.

Data was also available from coastal models developed by Crean (1983) which simulated the mixed tides of the Straits of Georgia and Juan de Fuca though the primary focus of the study was to model estuarine circulation.

6.1.3 BOD Loadings

Figure 6.1a shows the river reaches considered for BOD loading which were chosen to correspond to the scheme used for the hydrodynamic model. Continuous unit BOD loadings of 200 kg/day were assumed for each of the four reaches although stochastic loadings of the same mean could have been assumed. An initial choice of eleven reaches for consideration was reduced to four due to the necessarily lengthy microcomputer execution times. A stabilization period of 24 simulated hours was used for both the hydrodynamic and the BOD/DO models to dissipate the effect of the initial conditions. From the validations in chapter 5, this period appeared to be adequate.

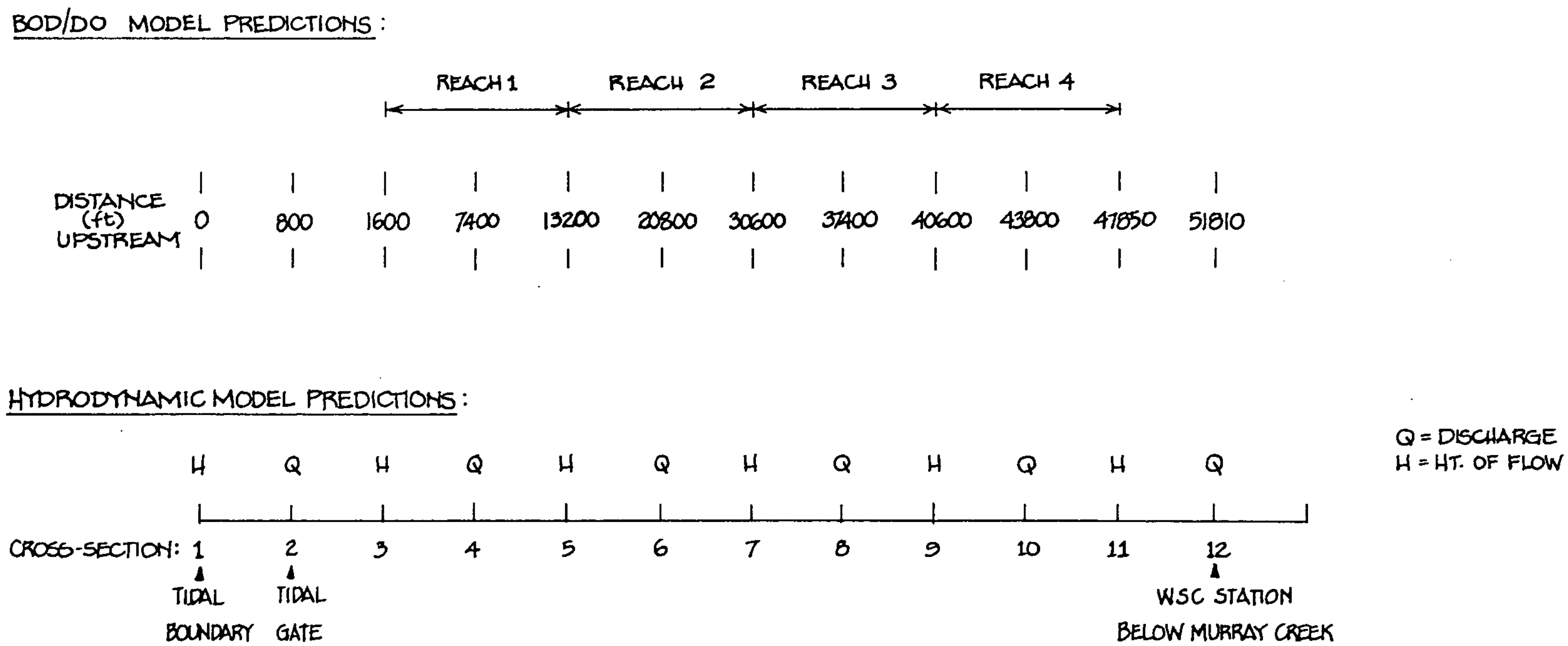
Decay and longitudinal dispersion coefficients could also be considered as stochastic, and generated using means and variances from appropriate statistical distributions and including possible autocorrelations. This was not done in this study.

6.1.4 Photosynthesis

Data from the literature was used to determine an oxygen source level consistent with the observations of Moore (1985a) and Bourque and Hebert (1982). Steady source rates were assumed as simplifying

Figure 6.1a:

BOD/DO Model Schematic of River Reaches



assumptions although more complex models for diurnal variations are available (Jorgensen, 1983). An equivalent rate of 0.09 mg/l/day was used for each river reach.

6.1.5 DO Regulatory Limit

The regulatory limit, C_{reg}, depends on the specie of fish which frequent the river for spawning and feeding. A review of minimum DO requirements for fish species by Davies (1975) identified a level of 5.0 mg/l for the highest protection level 'A' for salmonids. This level of protection allows one standard deviation from the 'mean average incipient oxygen response threshold'. A more exacting protection regime would also account for ranges in river velocity and minimum depths of flow required, say, during spawning. These standards would be more relevant to additional studies of flow augmentation and irrigation where multivariable controls were assessed.

6.1.6 Parameters Used

The parameters used for the simulation and optimization runs made are summarized below:

MODEL	PARAMETER	VALUE	UNITS	REMARKS
Decision Horizon		7	day	reduce from 30 days due to computational burdens.
Hydrodynamic				
	n	0.060	-	from calibration
	Δt	30	sec	from stability criteria
	Q_o	50	cfs	initial condition
	d_o	0.2	ft	initial condition
Water quality				
	k_1	2	1/day	Dobbins(1964), Orlob(1983), Ott (1976)

k_2	$[Du^{1/2}/H^{3/2}]$**	1/day	Orlob(1983), Ott(1976)
D	2	sq mi/d	Dobbins(1964)
D_b	0.5	$gm/d/m^2$	Ambrose(1986)
ΔT	600	sec	stabililty criteria
BOD_o, BOD_i*	0	mg/l	boundary conditions
DO_o, DO_i*	11.5	mg/l	boundary conditions
C_{reg}	5.0	mg/l	Davies (1975)

NLP model

obj. function

coefficients for:	W_i		
X1	11600	ft	length of reach 1
X2	17000	ft	length of reach 2
X3	10000	ft	length of reach 3
X4	7250	ft	length of reach 4

* non-pristine background conditions for BOD and DO could have been easily incorporated.

** This is a reaeration model proposed by O'Connor (1958) where D is the oxygen diffusivity through liquid film (8.1×10^{-5} ft^2/hr) and u is the mean tidal velocity and H is the mean depth.

6.2 Test of the Unimodal Assumption

The key assumptions made in the Hooke-Jeeves algorithm is the unimodality and smoothness of the constraints which bound the feasible region. This ensures the location of a global optimal. For multimodal constraint surfaces only local optima can be guaranteed, requiring the use of multiple start-points (Kuester and Mize, 1973).

In this study, the constraints are nonlinear and need exploration to determine unimodality. For a constant discharge of 25.0 cfs and a regulation limit of 5.0 mg/l (k_1 = 0.3 per day and D= 2 mi^2 per day) the feasible solution space for this problem was explored by grid search. The search was easier to visualize due to the manageable number of variables (four) which would not have been the case for a higher dimensioned problem. For various levels of loadings in reaches 1 and 2 the estimated risk level was determined for two loading levels in reach 3. The loading in reach 4 appeared to consistently violate the constraints except at negligible load levels and was assumed to be zero.

This limited search indicated that the constraints bounding the solution space were apparently unimodal. However, there were portions of the constraint surface which were almost parallel to the objective function, posing possible convergence problems. This was confirmed with further testing such that a reduction in convergence criteria from 1% to 0.001% yielded better optimal points. Figure 6.3 shows the constraints on the solution space for various compliance levels for BOD loadings in reaches 1, 2 and 3 (from cross-section 3 to 5, 5 to 7 and 7 to 9 respectively) resulting from a fuller grid search. The constraints indicated possible local multi-modality suggesting the use of multiple start points. The objective function is also shown being almost parallel to the constraints near the optimal region of the solution space.

6.3 Results and Sensitivity Analysis

The optimization problem formulated was solved using the methodology previously outlined. The sensitivity of the optimal decision to variations in the model parameters were tested as shown in table IV.

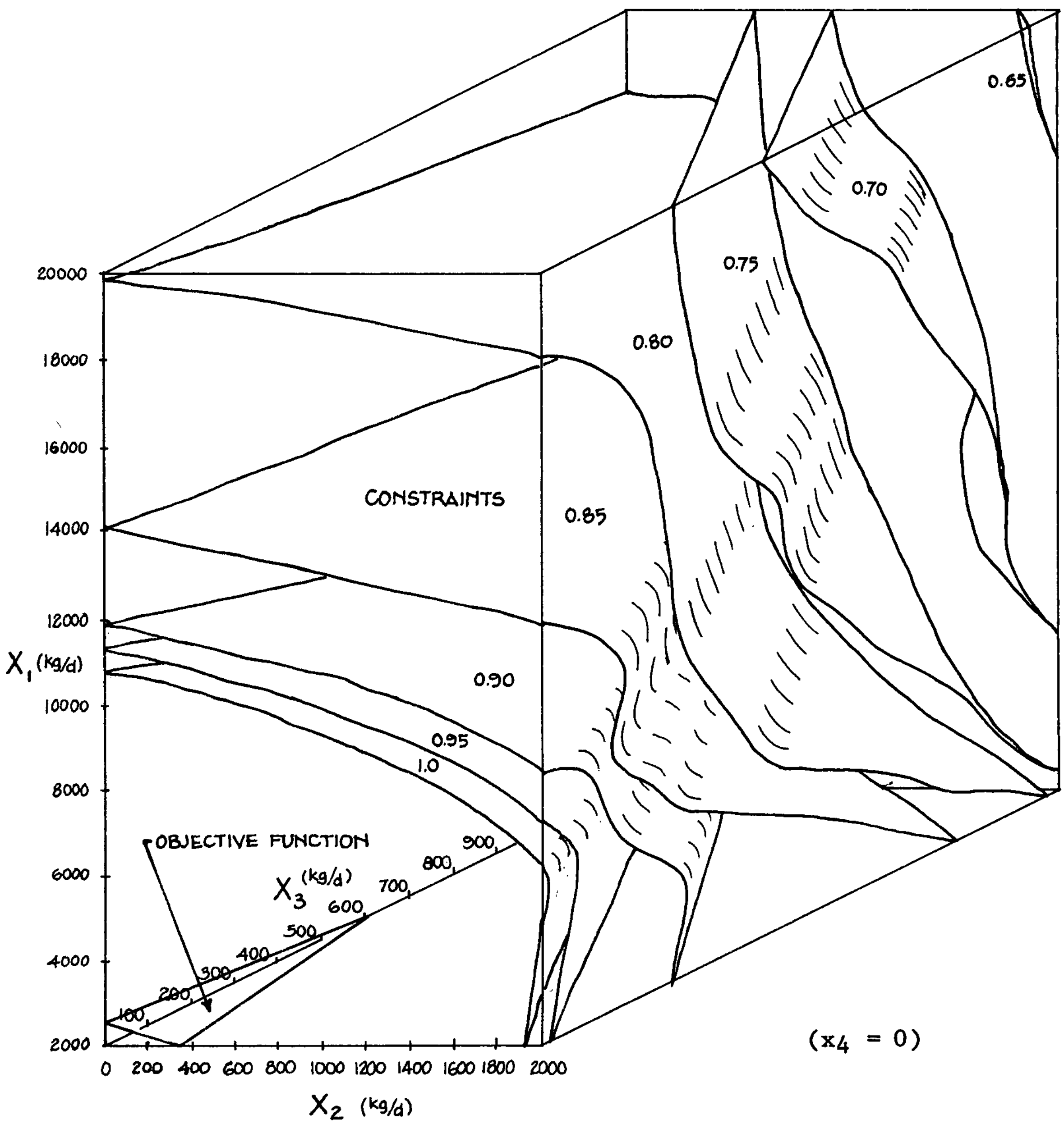


Figure 6.3: Graphical Representation of a Limited Grid Search of the Feasible Region Showing Various Compliance Constraint Surfaces

Table IV: Sensitivity Ranges for Parameters Tested.

PARAMETER	VALUES	UNITS
k_1	0.05, 0.3	day^{-1}
D	0.5 , 2.0	mi^2/day
Q	5, 25, 37.5	ft^3/sec
C_{reg}	3, 5, 7,	mg/l
Compliance Level	.8, .9, 1.0	-

Multiple start points were used to ensure consistency in the optimal solutions and detection of small-scale anomalies in the solution surface possible yielding local optima.

The NLP algorithm took an average of 1 hour when executed on the COMPAQ 286, to generate an optimal solution.

With five sets of multiple start points and convergence criteria, optimal BOD loadings were obtained. These may still, however, reflect local optima, requiring more iterations of the NLP algorithm, to help identify the global optimal solution. The NLP algorithm may also be unstable near the optimal point where the constraint boundary and the objective function are almost parallel. This problem-specific characteristic may well be algorithm independent (of the ones reviewed) and may require incresed number of start points for adequate resolution. However, tentative conclusions were drawn from computer runs for discharge levels of 5, 25 and 37.5 cfs, C_{reg} of 3, 5, and 7 mg/l and compliance levels of 0.8, 0.9 and 1.0.

6.3.1 Discharge Levels

Figures 6.3 to 6.5 present the objective function values for the three steady discharge levels used (5, 25 and 37.5 cfs). The trend in BOD loadings appears consistent with intuition, namely higher loadings allowable at higher discharges due to added dilution. In going

Sensitivity of Optimal Loadings to Discharge Levels

- ⊙ 1.0 COMPLIANCE
- △ 0.9 COMPLIANCE
- ▽ 0.8 COMPLIANCE

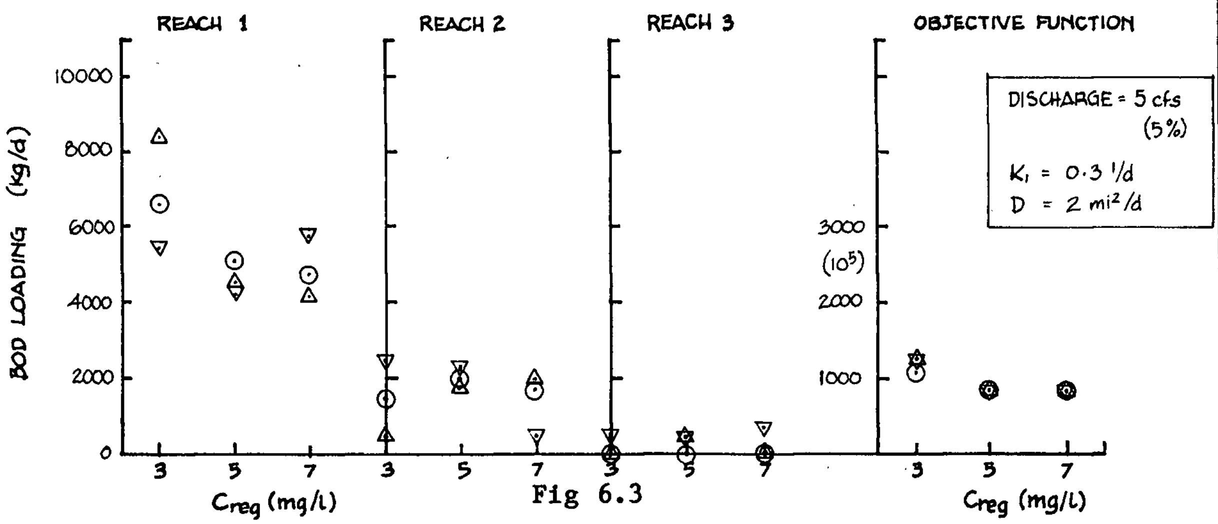


Fig 6.3

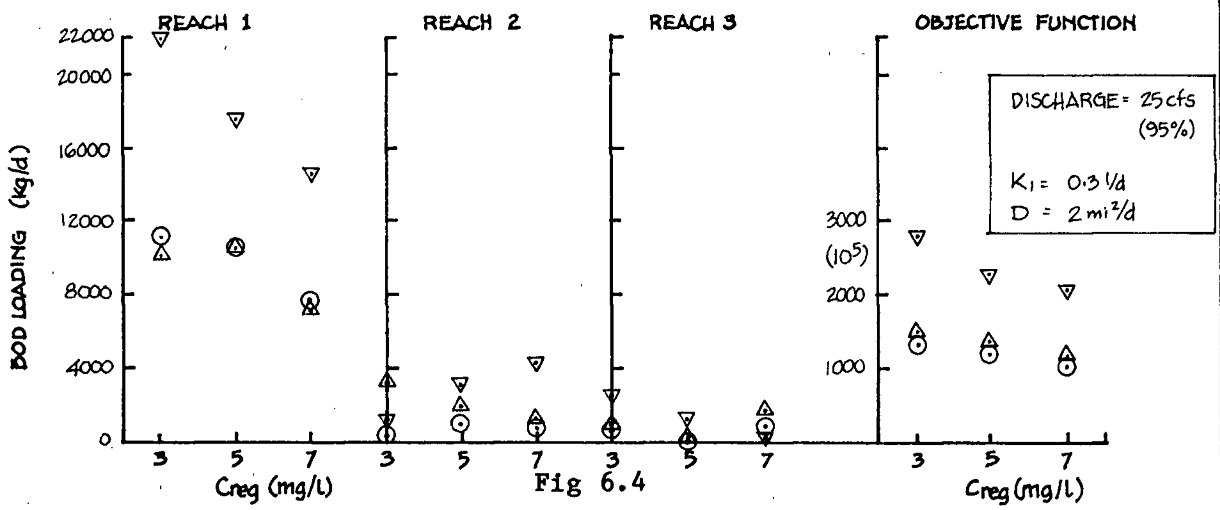


Fig 6.4

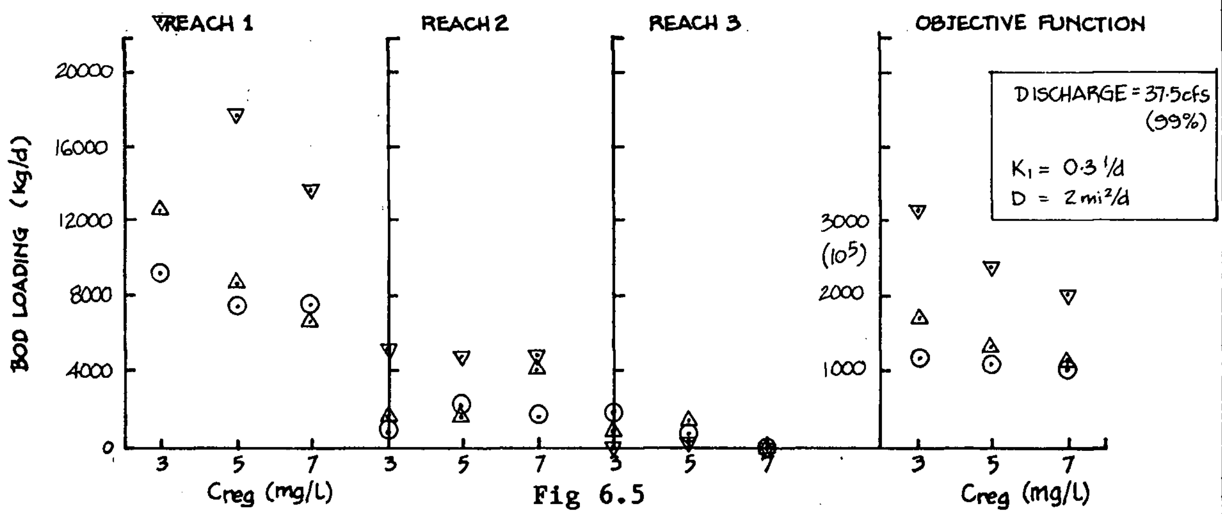


Fig 6.5

from a 5% mean monthly summer discharge (5 cfs) to the 95% discharge (25 cfs), the total optimal BOD loading of the river increased about 100% for a C_{reg} of 5 mg/l and at the 0.9 compliance level. This suggests a relatively narrow range within which optimal summer BOD loadings are permissable due to the low dilution capacity. At 5cfs, loadings in the four reaches were 4000 to 5000, 1900 to 2500, 0 to 500 and 0 kg/d respectively for compliance levels of 0.8 to 1.0 and a C_{reg} of 5 mg/l. This is compared to 10000 to 11000, 600 to 1000, 0 to 500 and 0 kg/d respectively for four reaches at 25 cfs. For reach 2, it appears that at 5 cfs (5% discharge level) and 25 cfs (95% discharge level) respectively the optimal loadings were higher for the former. This result may be due to a higher level of variation in the optimal loadings generated due to trade-offs between reaches made in the NLP while seeking the optimal value or due to premature termination of the NLP algorithm at coarse convergence criteria. This loadings difference may also result from the dynamics of the tidal gate at low flows where the gate remains closed for extended periods in the tidal cycle with infrequent openings for river discharge. The closed gate would cause river waters to back up thus increasing the volume of water in all four reaches. This would provide added dilution capacity for BOD loadings with reduced convection ability. The results suggest that at low flows the dilution effects in reach 2 are greater than the lost convection effects (which would tend to reduce BOD concentrations) allowing for greater loadings. This phenomena is reduced at higher flows where convection effects dominate. Further investigation would be required to confirm this speculation. For all discharge levels the optimal BOD loadings decreased in the upstream direction as dilution capacities became reduced. Reach 4 is particularly sensitive with negligible optimal loadings.

These optimal loading levels would be useful in establishing a BOD permit system whereby agricultural polluters would be allowed prorated (by cultivated area) loadings permits depending on their location along the river. However, such a system is weak at best as an abatement strategy since significant compliance monitoring resources would be required since the pollutant sources are largely non-point and, more importantly, difficult to trace to specific agricultural enterprises. Thus, it would be difficult to relate actual BOD river concentrations to individual agricultural practices making it difficult to identify enterprises for on-farm treatment strategies, unless enterprise specific monitoring schemes were established, at higher costs. Beyond such a permit system, other measures would be necessary as part of an overall abatement strategy based on longer-term policies but with short-term policies to accommodate transient shocks to the system. The impact of these various measures (aeration, better land use, flow augmentation, waste treatment) on water quality, as defined by the BOD/DO models developed, could be assessed by using the methodology developed previously, since it provides a rational framework for determining the optimal loading capacity of the river system which is a pre-requisite to policy formulation.

Flow augmentation strategies, based results of figures 6.3 to 6.5, could be designed to redistribute the dilutive capacity of the river or introduce additional capacity from outside the system. Reach 1 could be a possible source of waters for dilution in a form of a feedback dilution system to reaches 2, 3 and 4. The optimal volume of diversion would be determined such that the increased BOD levels in reach 1, due to upstream loadings, would still meet the DO regulations. This optimal level could be determined by modifying the hydrodynamic and BOD/DO models to include transient feedback so as to determine steady-state (where concentrations changes due to additions have stabilized) DO levels. An inter-basin transfer policy would be required for situations where feedback would generate unacceptably low DO concentrations, based on optimal transfer levels for addition to, or in replacement of, intra-basin diversions.

6.3.2 DO Regulatory Limit

Primarily for reach 1 and the objective function value, a reduction in C_{reg} allows for higher BOD loadings. Reaches 2 and 3 have a high variance with unclear trends with respect to changes in C_{reg} as discussed in the previous section. BOD loadings would increase in an exponential manner with the reduction of C_{reg} to 0 mg/l where infinite optimal loadings would be (theoretically) possible. Within the 3 to 7 mg/l range, the BOD loadings are approximately linear with highly uncertain predictive ability outside this range. At 5 cfs the optimal loading for 5 and 7 mg/l regulatory limits are within 1000 kg/d indicating an equivalence of the two if used as an enforced standard. At higher discharges the sensitivity of the optimal loadings to C_{reg} is higher indicating the need to correctly determine the regulatory limit when used for seasonal BOD control.

6.3.3 Compliance Levels

The total BOD loading of the river increase with decrease in compliance level. In establishing a flexible DO standard two parameters are important:

a) the DO regulatory limit;

b) the acceptable compliance levels.

$\Big\}$ Pr(LEVEL$\geq$LIMIT)>COMPLIANCE

A DO standard which explicitly regulates compliance levels would be almost impossible to enforce due to the high resource level required for field monitoring. However, the compliance level can be a useful concept in the design of standards since assumptions of polluter compliance behaviour (conscious or non-conscious) can be tested. For example, a 5 mg/l standard with an assumed 80% compliance level would result in an effective standard of 4 mg/l (from figure 6.2). Thus, a higher C_{reg} could be set for each month such that the effective DO limit would be 5 mg/l at some assumed compliance level.

6.3.4 <u>Model Parameters</u>

The sensitivity of optimal loadings to BOD decay (k_1) and longitudinal dispersion (D) is shown in figure 6.6 and 6.7. As expected, the loadings increase with lower k_1 and higher D (see equations 3.17 and 3.13) due to the lower impact on DO concentrations of decaying BOD and lower BOD concentrations due to greater BOD dispersion. However, the BOD loadings were more sensitive to changes in the former parameter compared to the latter, for the same percentage change. This has general implications to data gathering and parameter estimation efforts, namely that greater resources should be allocated in the determination of k_1 than of D. This is also likely the case for determination of k_2 although this was not explicitly tested in this study.

From the validation application to the Delaware Estuary in chapter 5 the sensitivity of decay parameters to stream temperatures was evident and, though not explored in this study, would require careful field and laboratory work to establish appropriate temperature

Sensitivity of BOD Loadings to Longitudinal Dispersion, BOD Decay and Oxygen Source Levels

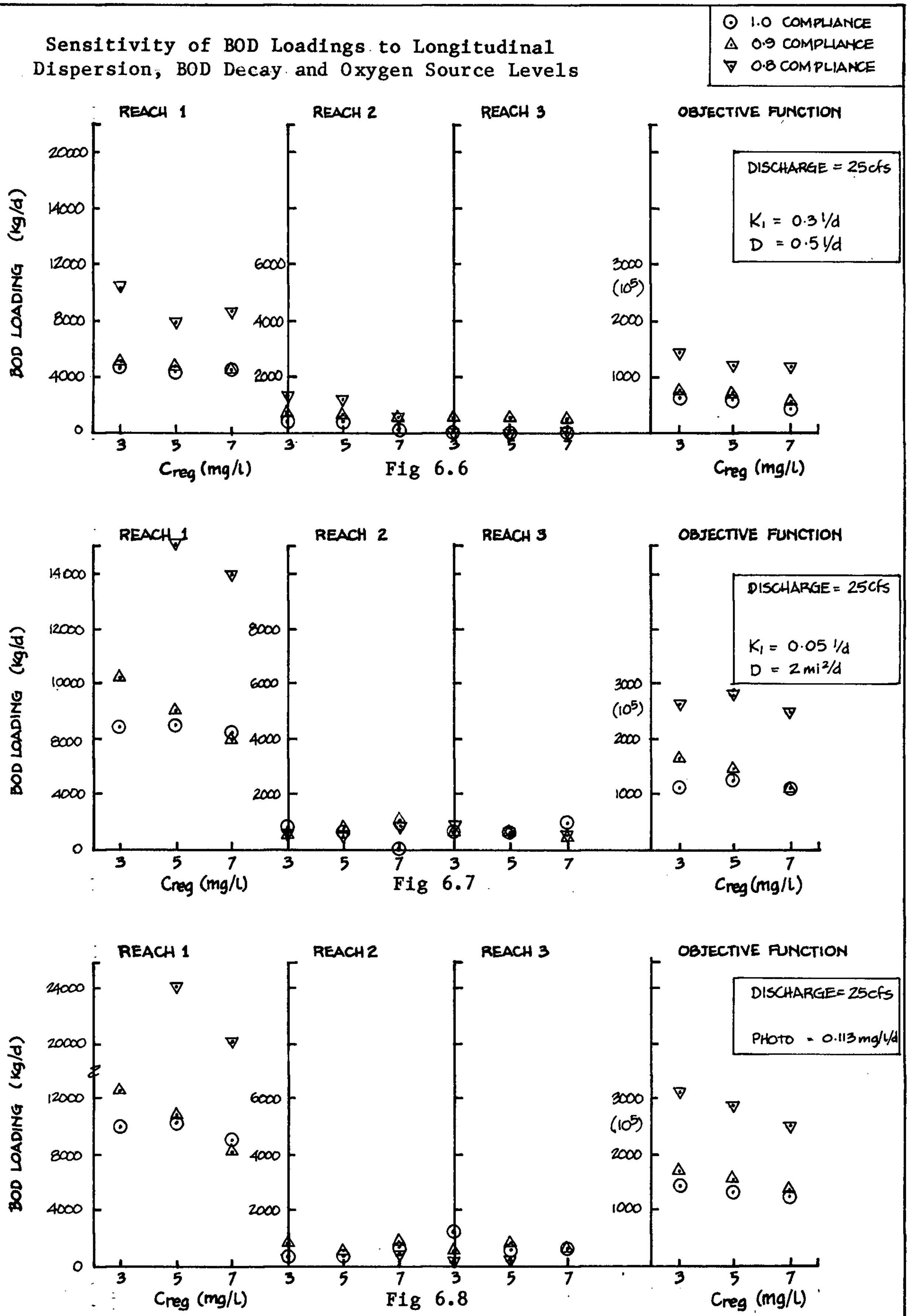


Fig 6.6

Fig 6.7

Fig 6.8

coefficients. This would be of great concern for summer low flow conditions where higher temperatures would increase BOD decay rates and thus reduce DO concentrations further.

The optimal allocation of data gathering resources for maximum information is an issue for further study.

6.3.5 Oxygen Sources

As discussed in section 6.1.4 a net oxygen source level of 0.09 mg/l was assumed for the four reaches of the DO model. This was a steady source approximation to diurnal photosynthetic processes and could also be interpreted as incorporating steady artficial aeration. The optimal solutions were tested for an increase of 25% to determine the sensitivity of this assumed source level and to assess the utility of longer-term oxygen addition (compared with ad-hoc emergency aeration). The 25% increase in net oxygen addition (at 25 cfs) resulted in an average increase of about 10% in BOD loading in reach 1 (figure 6.8). This lower response may be due to the low level of DO addition assumed. Order-of-magnitude changes in photsynthetic DO addition over the diurnal cycle, may be required to represent the great variations noted in field studies (Moore, 1985a). The incorporation of late summer algal decay into the BOD/DO model would further add to describing this variation. However, the impact of the average DO loading used in this study could be used to test designs of steady oxygen delivery systems as opposed to ad-hoc intense aeration with diurnal variations.

6.3.6 Other Abatement Policies

As outlined in section 6.3.1, a multi-level abatement strategy is necessary for broader control of water pollution. The optimal BOD loadings generated in this study are useful in comparison with actual (though difficult to determine) loadings for establishing and combining the components of an overall abatement strategy. Additional to the previous policies outlined are other long-term policies are suggested:

a) On farm treatment and management: Treatment or reduction at source is a better policy requiring additional on-farm expense with likely government subsidy support for an already-supported financially stressed industry. An alternative would be to encourage the optimal land application (temporally and spacially) of animal or other organic wastes, to minimize their impact on water quality through runoff and groundwater seepage. The optimal loadings generated would provide constraints to non-point seepage models, such as Cappelaere (1978), which simulate the impact of pollutants on groundwater quality and river discharges while modeling the effect of waste nutrients on crop growth. The establishment of optimal farm cultivation and application schedules would be an interesting multiobjective optimization problem for further study.

b) Land use policies: Agricultural land use policies for marginal land development based on soil quality and availability of irrigation water should ideally account for water quality impacts. The impact of agricultural development along reaches 2, 3 and 4 would be reduced by the diversion of river inflow in the form of land runoff, ditch flows and seepage towards reach 1 with its greater dilution capacity and/or the use of instream aeration to compensate for oxygen demands in sub-reaches during critical low DO periods such as algal die-off and spawning.

Throughout this study, various model parameters (such as k_1, k_2, D) and inputs (such as discharge, temperature) were assumed to be deterministic. Ward and Loftis (1983) stressed the balance between deterministic and stochastic view of water quality modeling and management. This stochastic nature could have been incorporated into the models developed in this study, based on simple moments of normal and log-normal distributions, by random generation of model parameter values possible using Markov techiques. This would have embelished the joint-chance constraint NLP formulation of section 3.1 and would be an interesting enhancement for further study.

The discussions above were based on results of this study derived from an unvalidated BOD/DO model. Even with sufficient validation the applicability of the model would still be tenuous since policies bases on limited dimensions of a system do not contain the requisite variety to multi-variably control the complex ecosystem processes of organic pollution in dynamic rivers.

7. GENERAL DISCUSSION

7.1 Hydrodynamic Model

A good model of the hydrodynamics is crucial to the accurate simulation of the water quality in a dynamic river. The model should account for the major sources and sinks of water in the river system including tributary flow, groundwater flow (if significant), irrigation (uptake and return flow) and runoff. The model used in this study did not simulate these latter processes but accounted only for upstream inflows and downstream tides, which appears to adequately match historical records.

There are more efficient finite-difference schemes and algorithms for the solution of the equations of continuity and momentum (Mauersberger, 1983), allowing for shorter model execution times, which can be significant on a microcomputer. The extension of the model into two dimensions may be of research interest, especially for the prediction of pollutant plumes. Another extension of great interest would be the modeling of the estuarial conditions of Mud Bay with multiple pollutant loadings from both the Nicomekl and Serpentine rivers. This would help assess the dilution and mixing capacities of the bay.

The use of stochastic hydrographs as upstream boundary condition generators (as done by Biswas and Reynolds, 1973) combined with computed tides (from harmonic equations) could be useful in predicting seasonal or annual stream flow and hence, water quality variations. This would of value for longer-term decision horizons requiring, however, greater computing power. This extension would also facilitate Monte Carlo simulation of various stochastic model parameters.

In reviewing the hydrodynmics of the Nicomekl River during summer low flow conditions, it appears that a steady-state flow model may be applicable without a great loss of information and reduce computing efforts. This is of use if the BOD loading limits were to be regulated based on an inflexible DO standard based on low flow (high temperature) extremes. A flexible regulatory system would necessarily have to accommodate dynamic river processes.

7.2 BOD/DO Model

The BOD/DO model used in this study was based on a compartmentalization of the river for the purposes of pollutant mass balances. More efficient schemes and algorithms also exist for such models with extensions into broader water quality/ecological formulations which provide greater number of dimensions for regulatory control and policy evaluation (Mauersberger, 1983).

The application of estimation techniques could be used to amplify the limited field data available in identifying possible stochastic, steady-state BOD/DO models (Damaskos and Papadopoulos, 1983) with probabilistic variables such as stream flow (to reflect the tidal and hydrograph dynamics); stream temperature (and hence model parameters: k_1, k_2, D); initial and boundary conditions (for dynamic inputs); and measurement (for BOD and DO testing, the former being off-line and more uncertain); (Beck, 1981,1983,1985). For the Serpentine river, this approach would have been interesting, not only in model identification but also in terms of real-time DO control, as is currently underway (Mavinic, 1986).

A simpler steady-state model would also have been easier to incorporate into the NLP formulation and, again, would require less time for model execution on the microcomputer. The results would have to be more carefully interpreted, especially for short-term applications.

7.3 Uncertainty of Measured BOD

As mentioned in section 2.2.1, the use of BOD as a measure of water quality has an inherent measurement problem. Standard tests measure BOD as a 5-day BOD or the level of BOD after a 5-day incubation. The BOD used in the models formulated are based on the ultimate BOD level, or BOD_{ult}, which is often modeled as

$$BOD_n = (1 - e^{-K_L n})BOD_{ult}$$

where K_L is a laboratory determined decay parameter. Thus, the optimal BOD levels discussed further in this chapter have to be converted to BOD_5 so as to be used for actual field control purposes. Pheiffer et al (1976) found that laboratory errors in the measurement of BOD made it a non-ideal water quality parameter without sufficient testing for statistical validity. This variation could be modeled as measurement noise using estimation techniques as outlined in section 2.2.4. Alternatively, more consistent real-time water quality indicators such as total organic carbon (TOC) could be used to estimate organic oxygen demand.

7.4 Data Gathering

Data gathering is an expensive exercise requiring optimization also so as to obtain the maximum amount of information subject to some field budget. The frequency, intensity and quality of information gathered is different for mathematical model building purposes and for real-time forecasting and control, with greater effort required for the former. An interesting methodology for network design based on modern concepts of communication theory was developed by Husain (1980) applicable to watersheds of the size of the Nicomekl/Serpentine basin.

The present data gathering efforts need to be rationalized on the basis of experimental designs for mathematical model building and validation, with field experiments related specifically to convection, dispersion and decay processes of water pollution for more water quality measures than BOD.

7.5 NLP Formulation

There are possible simplifications of the NLP formulation in section 3.1. These are:

i) the decoupling of the joint constraints for derivation of deterministic equivalents while still incorporating the dynamic DO model. This would then reflect separate allowable risk levels for the different reaches considered incorporating reach-specific information.

ii) the NLP formulation could be approximated by an iterative LP method where linear constraints could be derived from the regression of constraint contours in appropriate constraint dimensions (such as reach 2 and reach 3 in section 6.2). The objective function would be evaluated in various discrete nonlinear dimensions and the maximum obtained. This approach, however, would require grid searches in identifying linear dimensions which may be prohibitive for higher dimensioned problems.

7.6 NLP Algorithm

The Hooke-Jeeves algorithm was adequate for the optimization problem formulated, though a faster converging algorithm (perhaps Powell's) may have have marginally reduced the elapsed time for searching for the optimal solution. The computations involved in the DO model were by far the limiting factor in running the NLP algorithm, so that efforts in streamlining the model scheme would be more productive. Additionally, a comparison of the stability of this scheme with other direct search techniques (such as Powell, Nelder-Mead and Rosenbruck) would also have been of great value in identifying the better algorithm for solving the problem formulated in section 3.1.

7.7 Computer Resource

The COMPAQ 286 microcomputer provides limited resources for the implementation of the methodology proposed in section 3.3.1 at no cost to research or hard-dollar mainframe accounts. With the previously mentioned modifications to increase model efficiency, the execution times for a four-reach, seven-day problem may be reduced significantly. Larger problems, less than 640 KB in size, would take longer to run providing greater justification for mainframe or 32-bit microcomputer use.

The microcomputer is a good research and model development tool for small- to medium-sized strategic analyses (monthly, seasonal, annual) of water quality management problems as judged from the limited application to non-linear optimization in this study. However, due to its limited processing capabilities, the machine may not be appropriate for real-time control or analysis where short (diurnal) prediction or optimization may be important for timely decision making.

7.8 Utility of Proposed Methodology to Management Practise

The methodology presented in this study with enhancements (model validation, use of robust NLP algorithms, use of a steady-state pollutant model) could be used by management to rationalize solutions for field problems and formulating short- and long-term abatement policies. The (near) optimal solutions generated by the methodology provide guideline BOD loading levels which in comparison with actual loadings would help identify the nature of pollution control technique required.

8. CONCLUSIONS

1) Using a one-dimensional hydrodynamic model it was possible to correctly predict the stage and discharge at various cross-sections of a tidal river.

2) Using a one-dimensional BOD/DO model coupled with 1) above it was possible to describe and, with less confidence due to the lack of site specific data for validation, predict the resulting concentrations due to the processes of convection, dispersion and decay.

3) Using a non-linear programming algorithm coupled with the DO model in 2), it was possible to maximize the total BOD loadings of a tidal river with various DO regulatory limits and risk levels of violation.

4) A powerful microcomputer is an effective tool for one-dimensional hydrodynamic and water quality modeling. However, execution times may become prohibitive compared with mainframe computers but at zero cost. The methodology proposed in this study is suitable for 'smaller-sized' problems where the limited computing resources are not a serious constraint.

5) The methodology presented appears to be a viable method of determining the optimal pollutant loadings of a tidal river for strategic decision-making. However extensive model identification should be conducted to justify the choice of water quality models.

6) The water quality data gathering on the Nicomekl/Serpentine rivers should be based on a more complete experimental design to allow for

water quality model development and validation, and for longer-term pollutant and DO control requirements.

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