A QCD-PARTON CALCULATION OF ASSOCIATED HIGGS BOSON PRODUCTION IN HADRON-HADRON COLLISION

by

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Abstract

This thesis contains a study of the reaction proton+proton or proton-antiproton into a Higgs boson and a pair of heavy quarks, in the region of high energy and high momentum transfer. The Higgs boson mass is treated as a free parameter. Numerical results are obtained through a Monte Carlo integration. Several differential cross sections relevant to experiment are given.

Table of Contents

List of Tables

List of Figures

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I. INTRODUCTION

Within the last decade the world has witnessed a total revolution in the understanding of particle physics. Until then, the weak interactions (WI) were described by a phenomenological, non-renormalizable Fermi interaction of four fields at a point. The WI are the weakest, after gravitation, of the basic known forces of nature. They are responsible for the beta decay of the neutron, for example, and other relatively slow processes in nuclear and particle physics.

On the other hand, the newly developed quark model of that time could account for the previously mind boggling hundreds of "elementary" particles produced in strong interactions (hereafter SI), or their decay products. What was still badly needed however, was a theory of SI itself. The four known forces acting on matter in the universe, -gravitation, electromagnetism (EM), WI and SI, -did not seem to have much in common.

Then, at the end of the 60's, EM and WI were "unified" within the framework of a gauge model, the Glashow-Weinberg-Salam model (GWSM)[1]. A few years later, it was the turn of SI to be described by a gauge theory -quantum chromodynamics (QCD)[2]. Now, most models of particle interaction are based on the gauge idea. Among them, are the

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[1] For historical accounts and references, see Nobel lectures of (Glashow, 1980), (Weinberg S. 1980), (Salam 1980)

[2] For a review of QCD, see (Reya, 1981)

grand unified theories (GUTS)[1] whose goal is to unify EM, WI and SI into a single interaction with a non-abelian gauge group. Its main prediction is the instability of the proton, which is being intensively tested in many laboratories. Other gauge models are: technicolor[2], supergravity[3], and several alternatives to the GWSM[4]. What makes the concept of gauge invariance attractive is its inherent elegance. Its main feature is the following. You start with a symmetry you know to be valid, (or hypothesize to be valid), in general in a world where the matter fields are spin 0 bosons or spin 1/2 fermions. You require this symmetry to be conserved locally, i.e. at any point in space-time. To do so, you must introduce a new boson field, which will mediate some new interaction between the matter particles, in such a way that the symmetry remains non-violated. Hence, you have "deduced" a force from the symmetry requirement. It has been known for quite some time that EM can be "deduced" this way from the phase invariance in quantum mechanics (Fock, 1927), (Weyl, 1929). It was seen as merely an elegant way of linking EM and QM. Yang and Mills (1954) broadened the class of symmetries that can be "localized" this way, to include non-abelian symmetries. A non-abelian symmetry can be compared to a rotation in space -the order in which you apply the transformations is important. In this analogy, an abelian theory would be a rotation in some

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[1] For a review of GUTS and their phenomenology, see (Langacker, 1981)
[2] see for example (Susskind, 1979)
[3] For a review of supergravity, see (van Niewenhuizen, 1981)
[4] For example (Georgi and Glashow, 1974), (Pati and Salam, 1973)

plane.

The forces generated by a non-abelian symmetry are much more complicated than those generated by an abelian one, mainly because the particles or fields responsible for carrying the interactions are "charged" themselves. But the Yang-Mills theory did not attract much attention for a while, because the boson particles you must introduce to carry the interactions must be massless, giving rise to long-range forces we do not observe. The forces generated by non-abelian symmetries did not seem to correspond to any of the known forces. One had to solve the problem of giving a mass to the vector bosons if one wants the theory to describe WI which are short range.

The solution to this problem had to wait till 1964, when Higgs (1964) invented the spontaneous symmetry breaking (SSB) scheme. At the price of introducing a elementary scalar field, the vacuum would be made to be non-trivial. Real particles propagating through such a vacuum would interact with it, giving them effectively a mass, in much the same way as the apparent mass the of electron may be greatly affected when it travels through a lattice or a plasma.

A few years later, Weinberg, Salam and Glashow came up independently with a model for the weak and electromagnetic interactions, using a non-abelian symmetry based on isospin, represented by an SU(2) group, and an abelian symmetry U(1). The SU(2) group has three generators, which implies three bosons mediating the interactions; the U(1) group has one. The symmetry is broken by introducing an interacting doublet of complex scalars, endowed with a negative mass-squared. Of the four degrees of freedom brought in by the scalars, three are used to give mass to three of the four bosons. The fourth degree of freedom appears as a physical elementary field, with a real mass. It is called the Higgs boson, symbolically H^0. The GWS model accounted well for what was known at the time of the WI, but it predicted a new component to the weak force; a neutral one. An example of it would be the reaction $\nu q \longrightarrow \nu q$ in which a neutrino, a particle which interacts only through WI, interacts with a quark and remains a neutrino. To see this experimentally, one would send a neutrino beam on a target, and wait to detect a deposition of energy and momentum, with no lepton produced. (The charged WI would produce a charged lepton in the final state). These neutral current interactions have extensively been measured and studied from their discovery in 1973 till now. Since then, the existence of the neutral boson has been comfirmed by its spectacular discovery in proton-antiproton collisions, at the collision beam facilities at CERN (Conseil Européen de Recherche Nucléaire), in the 1983 summer (Arnison et al. 1983). Its discovery had to wait so long because no particle accelerator in the world could reach the center of mass energy necessary to its production, since its mass was predicted to be 91 GeV/c^2. The next very important task facing the experimentalists is to look for the Higgs boson. The discovery of the Higgs boson would be a badly needed confirmation that the mechanism which endows the gauge bosons with masses is the spontaneous symmetry breaking mechanism. This is a corner stone of the GWS model, and indeed, of nearly all unification theories based on the gauge principle. The main obstacle to its discovery, if it exists, is that unlike the intermediate vector boson W^+, W^- and Z°, its mass and decay products are free parameters of the theory. These factors make its production, and especially its identification, very difficult.

Several production mechanisms have already been suggested. Those pertinent to hadron-hadron collisions generally lack a clear signature. However, if the H° is too massive, its production will not yet be possible in the cleaner electron-positron collider rings. For the e^+e^- colliders that are planned now the highest energy of 200 GeV would be reached by LEP II at CERN. On the other hand, a hadron collider of c.m. energy 5 to 40 TeV (1 TeV = 1000 GeV) is being planned.

One more argument may be given in favor of the existence of an elementary scalar, independently of the spontaneous symmetry breaking scheme. It concerns the high-energy behavior of the theory (Halzen and Martin, 1984). The predicted cross-section for any process must not diverge, i.e. the probability of occurence of this process must remain less than one. If one calculates the cross-section for the elastic scattering of a pair of charged W , from the three diagrams of Figure 1.

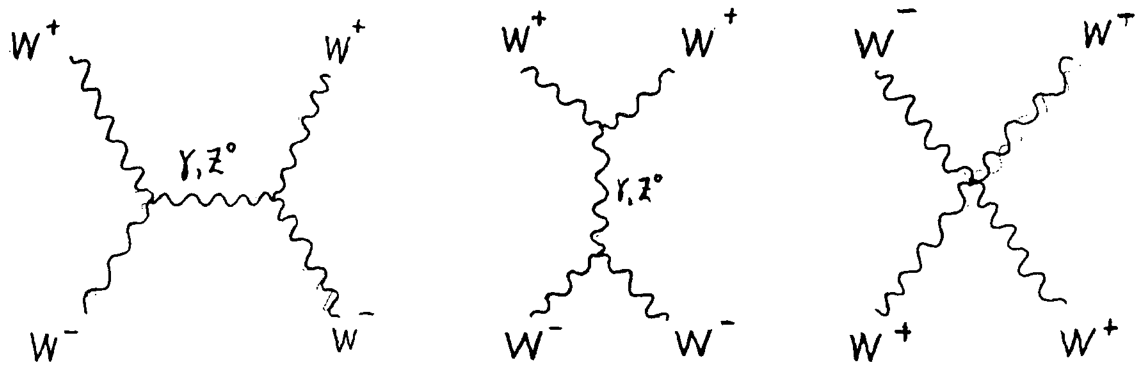

Figure 1 - Feynman diagram for $W^+W^- \rightarrow W^+W^-$, without scalar contribution

one finds that their sum diverges as s/M_W^2 as $s \rightarrow \infty$, (where the square of the total energy is denoted by s). A simple solution is to introduce a scalar particle to cancel this divergence, through the diagram of Figure 2.

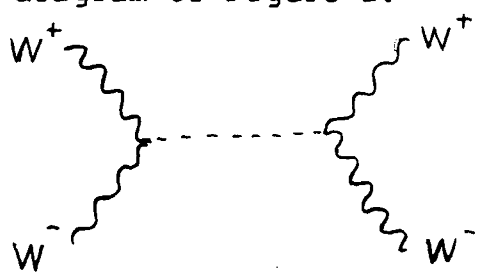

Figure 2 - Scalar contribution to the process $W^+W^- \rightarrow W^+W^-$

The coupling of the h particle must be proportional to the W mass to cancel the divergences of the other diagrams. Therefore, if we had not introduced the Higgs boson to give mass to the gauge bosons, a la SSB mechanism, we would have been forced to invent it to cancel out divergences in other processes!

This thesis is divided into two parts. The first one covers the background material pertinent to Higgs mechanism and phenomenology, and includes the first six chapters. Chapter II gives a general treatment of gauge theories. The third chapter introduces the phenomena of spontaneous symmetry breaking and the important Higgs mechanism. The Glashow-Weinberg-Salam model is developed in chapter IV. Chapter V brings in the hadron contribution. There is presented the extremely useful, yet simple parton model. Using it, one may use perturbative QCD and derive useful predictions for experiments. We get to the core of the subject in chapter VI with the known phenomenology of the "standard" Higgs boson. This is where is rooted any analysis of Higgs boson production. The whole work relies heavily on it.

The second part of the thesis includes chapters seven through nine. The starting point of the calculations is described in chapter VII, and the results are to be found in chapter VIII. The details of the calculations, in particular the matrix element squared, and the Monte-Carlo integration routine developed, have been confined to appendices. I summarize the work and suggest possible routes of extensions in chapter IX.

II. LOCAL GAUGE TRANSFORMATIONS

Because local gauge invariance is at the heart of today's attempts to unify and/or explain fundamental interactions in physics, we will start with a brief account of this important subject.

GENERAL CASE; FERMIONS: We start with the Lagrangian for free fermions.

$$\mathcal{L}_{free} = \bar{\Psi} i \gamma^{\mu} \partial_{\mu} \Psi - m \bar{\Psi} \Psi \qquad \text{(II.1)}$$

We demand that $\mathcal{L}_{free}$ be locally invariant under transformations of a simple Lie group G, and Ψ transforms as a certain representation of G. The generators of G have representation matrices T_a which satisfy

$$[T_a, T_b] = i C_{abc} T_c \qquad \text{(II.2)}$$

where the C_{abc} are the totally antisymmetric structure constants. If the fermion fields, under infinitesimal transformations, transform as

$$\Psi \rightarrow \Psi' = \Psi - i T_a \theta^a(x) \Psi \qquad \text{(II.3)}$$

it is easy to check that the free Lagrangian $\mathcal{L}_{free}$ is not invariant under this transformation. The derivative introduces a term $-i\bar{\Psi}T_a\gamma^\mu[\partial_\mu\theta^a(x)]\Psi$

which spoils the invariance of $\mathcal{L}_{free}$. The local property of the symmetry is expressed by the x-dependence in θ.

To make $\mathcal{L}_{free}$ invariant, one introduces the covariant derivative D_μ;

$$D_\mu\Psi = (\partial_\mu - igA^a_\mu T_a)\Psi \tag{II.4}$$

where a set of new 4-vector "gauge" fields A^a_μ have been introduced. Now, if one demands that the covariant derivative has the same transformation property as Ψ itself, i.e.

$$D_\mu\Psi \rightarrow (D_\mu\Psi)' \simeq (1-iT_a\theta^a(x))(D_\mu\Psi) \tag{II.5}$$

then one must introduce vector gauge fields which transform under infinitesimal transformations as;

$$A^a_\mu \rightarrow A'^a_\mu = A^a_\mu + C_{abc}\theta_b A^c_\mu - \frac{1}{g}\partial_\mu\theta^a(x) \tag{II.6}$$

In this expression, the second term is the transformation law for the adjoint multiplet under G. This implies that the gauge fields A^a_μ carry the non-abelian quantum numbers, i.e. they are "charged".

We need now to introduce in the lagrangian a kinetic term for the vector gauge fields. In analogy with the abelian case (QED), a possible antisymmetric second rank tensor for the fermion field is;

$$(D_\mu D_\nu - D_\nu D_\mu)\Psi = -igT_a\left[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gC_{abc}A^b_\mu A^c_\nu\right]\Psi \tag{II.7}$$

which leads us to define

$$F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gC_{abc}A^b_\mu A^c_\nu \tag{II.8}$$

Under infinitesimal transformation, $F^a_{\mu\nu}$ transforms as a multiplet under G;

$$F'^a_{\mu\nu} = F^a_{\mu\nu} + C_{abc}\,\theta^b F^c_{\mu\nu} \tag{II.9}$$

The combination $F^a_{\mu\nu} F_{\mu\nu a}$ is then invariant under G. Notice that;

1: A mass term for the gauge field would not be invariant (unless the gauge field was invariant under G).

2: The kinetic energy term for the gauge field implies triple and quadruple vertices, since

$$\begin{aligned} F^a_{\mu\nu} F^{a\mu\nu} = &\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gC_{abc}A^b_\mu A^c_\nu\right) \\ &\times\left(\partial^\mu A^\nu - \partial^\mu A^{a\mu} + gC_{abc}A^{b\mu}A^{c\nu}\right) \end{aligned} \tag{II.10}$$

The G-invariant Lagragian is finally;

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \bar{\Psi}(i\gamma^{\mu} D_{\mu} - m)\Psi \tag{II.11}$$

<u>ABELIAN CASE; U(1) SYMMETRY:</u> The U(1) case is simply QED. There is only one generator; therefore the structure constant is 0 and the gauge field tensor is;

$$F_{\mu\nu} = (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}) \tag{II.12}$$

The complete Lagrangian is then just the usual QED Lagrangian, with the field A_{μ} being readily related to the vector potential of electromagnetism.

<u>SU(2) CASE:</u> This is the Yang Mills case where SU(2) is usually taken to be an isospin symmetry, relating to "internal" isospin quantum numbers. Equations (II.2) to (II.11) hold with the identification

$$C_{abc} = \epsilon_{abc} \tag{II.13}$$

and the two-dimensional representation matrices can be chosen to be the usual Pauli matrices τ_i, i=1,2,3.

<u>SU(3) CASE:</u> The local SU(3) symmetry has found an application

in the attempt to develop a fundamental theory of strong interactions (Reya, 1981). The fermions are quarks, the eight gauge particles are called gluons, and the internal quantum number on which the symmetry is based is called color.

QCD is an exactly locally invariant theory, i.e. the Lagrangian (II.11) applies without any modification. There are eight generators of the SU(3) group, usually labelled λ_i, i=1,...,8. The most important properties of QCD are asymptotic freedom and confinement: Its effective coupling constant, at a given momentum transfer squared Q , is given by:

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(q_0^2)} + \frac{33-2n_f}{12\pi}\log\left(\frac{Q^2}{q_0^2}\right) \qquad \text{(II.14)}$$

in the leading approximation, and where n_f is the number of quark flavors. As long as $n_f \leq 16$, $\alpha_s(Q^2)$ grows smaller at large Q^2. This is called asymptotic freedom, and is a most useful feature of QCD, as it permits perturbative treatment of many "hard" scattering processes. In fact, QCD is the only candidate theory which explains this behavior of the SI coupling constant, corresponding to the phenomenon of "scaling" in experiments (see chapter V).

If the coupling constant grows smaller at large Q^2 and correspondingly short distances, the opposite is also true. Lower energy transfer interactions correspond to larger distances and large couplings, which leads to the notion of quark confinement. Quark confinement means that quarks are forever confined within hadrons and cannot appear isolated. Confinement has not been derived from QCD yet, but the behavior of the QCD coupling constant makes it qualitatively plausible. Asymptotic freedom and confinement are the most important reason QCD is now considered the complete theory of strong interactions.

III. HIGGS MECHANISM

The Higgs mechanism can cause the spontaneous symmetry breaking of some locally invariant Lagrangians (Higgs, 1964). But before to studying this case, one has to see the effect of the spontaneous breaking of a globally invariant lagrangian.

SPONTANEOUSLY BROKEN SYMMETRY: Let us consider the case of two real scalar fields and ;

$$\mathcal{L} = \frac{1}{2}\left[(\partial^\mu\phi_1)(\partial_\mu\phi_1)+(\partial_\mu\phi_2)(\partial^\mu\phi_2)\right] - V(\phi_1^2 + \phi_2^2) \qquad \text{(III.1)}$$

which is invariant under rotation U;

$$\phi_i \rightarrow \phi_i' = U\phi_i \qquad U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \text{(III.2)}$$

The effective potential is chosen for illustration to be;

$$V(\phi_1^2+\phi_2^2) = \frac{\mu^2}{2}(\phi_1^2+\phi_2^2) + \frac{|\lambda|}{4}(\phi_1^2+\phi_2^2)^2 \qquad \text{(III.3)}$$

and one can distinguish two cases:

-case 1: $\mu^2 > 0$. The minimum of V occurs at $\phi_1 = \phi_2 = 0$ and this will give simply a degenerate doublet of mass .

-case 2: $\mu^2 < 0$. The minimum occurs at

$$\phi_1^2 + \phi_2^2 = -\frac{\mu^2}{|\lambda|} \equiv v^2 \qquad \text{(III.4)}$$

and there is a continuum of degenerate states at the minimum. The potential for this case is represented in Fig. 3 below.

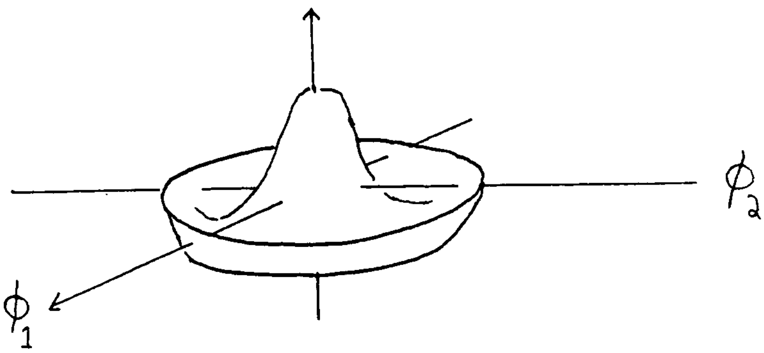

Figure 3 - Potential III.3 for the case $\mu^2 < 0$.

One can always define coordinates so that the physical vacuum is at

$$\phi_1 = v \qquad\qquad \phi_2 = 0$$

in the classical field theory, that is, in the quantum field theory;

$$\langle 0 | \phi_1 | 0 \rangle = v \qquad\qquad \langle 0 | \phi_2 | 0 \rangle = 0 \tag{III.5}$$

To do perturbation theory around the classical minimum, one has to expand in powers of $\phi_1' = \phi_1 - V$ instead of ϕ_1. ϕ_2, of course, is still expanded around the value zero.

$$\mathcal{L} \approx \frac{1}{2}[(\partial_\mu \phi_1')(\partial^\mu \phi_1') + (\partial_\mu \phi_2)(\partial^\mu \phi_2)] + \mu^2 \phi_1'^2 + 0 \cdot \phi_2^2 + \ldots \tag{III.6}$$

The important feature here resides in the mass terms. The field φ_1 has acquired a $(\text{mass})^2 = -2\mu^2 > 0$, while the φ_2 particle is massless.

This is an example of the Goldstone theorem, which states that if a theory has an exact continuous symmetry of the Lagrangian which is not shared by the vacuum, a massless particle must occur.

<u>HIGGS MECHANISM:</u> In the case of a locally invariant gauge theory, there is no massless Goldstone boson when the symmetry is spontaneously broken. The would-be Goldstone boson combines with the massless gauge boson to give a massive vector boson. This is the Higgs mechanism.

To illustrate that point, let us consider the simple case of the Abelian gauge theory with Lagrangian

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \text{(III.7)}$$

where

$$\begin{aligned} F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \\ D_\mu\phi &= (\partial_\mu - igA_\mu)\phi \end{aligned} \qquad \text{(III.8)}$$

The Lagrangian (III.7) describes a charged scalar interacting with itself, and with a gauge field A_μ. If $\mu^2 < 0$, it describes scalar QED.

The Lagrangian is invariant under the local transformations

$$\phi \rightarrow \phi'(x) = e^{-i\theta(x)}\phi(x)$$
$$A_\mu \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu \theta(x) \tag{III.9}$$

When $\mu^2 > 0$, ϕ develops again a vacuum expectation value. It is

$$\langle 0|\phi|0\rangle = \frac{v}{\sqrt{2}} \quad ; \qquad v^2 \equiv -\frac{\mu^2}{|\lambda|} \tag{III.10}$$

Let us use polar variables to parametrize ϕ, and expand about a specific vacuum point. This is done to show more clearly the physical content of the theory. The new set of coordinates is

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x)]\, e^{i\xi(x)/v} \tag{III.11}$$

Consider now the gauge transformation (III.9) with $\theta(x) = \xi(x)/v$. The transformed fields are:

$$\phi \rightarrow \phi' = e^{-i\frac{\xi(x)}{v}}\phi(x) = \frac{v + \eta(x)}{\sqrt{2}} \tag{III.12}$$

and

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{gv}\partial_\mu \xi(x)$$

$$F_{\mu} \rightarrow F'_{\mu} = \partial_{\mu} A'_{\nu} - \partial_{\nu} A'_{\mu} \qquad \text{(III.13)}$$

This is refered to as the U-gauge in the literature (Abers and Lee, 1973).

If one substitutes these new expressions for the fields into the Lagrangian (III.7), and expands, one gets:

$$\begin{aligned}
\mathcal{L} = {} & \tfrac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) - \tfrac{1}{4}F'^{\mu\nu}F'_{\mu\nu} && \Big\} \text{ kinetic terms} \\
& + \tfrac{v^2}{2} g^2 A'_{\mu} A'^{\mu} - \tfrac{1}{2}\eta^2(3\lambda v - \mu^2) && \Big\} \text{ mass terms} \\
& + \tfrac{g^2}{2} A'_{\mu} A'^{\mu}\, \eta(2v + \eta) && \Big\} \text{ mixed interaction terms} \\
& - \lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4 && \Big\} \text{ self-interaction of } \eta \\
& + \text{const.}
\end{aligned} \qquad \text{(III.14)}$$

The scalar meson has acquired a mass $3\lambda v^2 - \mu^2 = 2\mu^2$ and self-interactions represented by the vertices of Fig. 4.

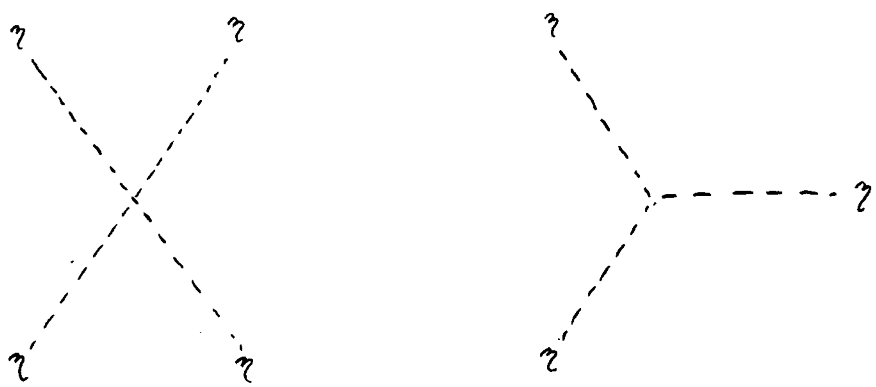

Figure 4 - Vertices of the self-interacting scalar meson

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The interactions between the vector gauge boson and the scalar meson will give rise to the vertices of figure 5.

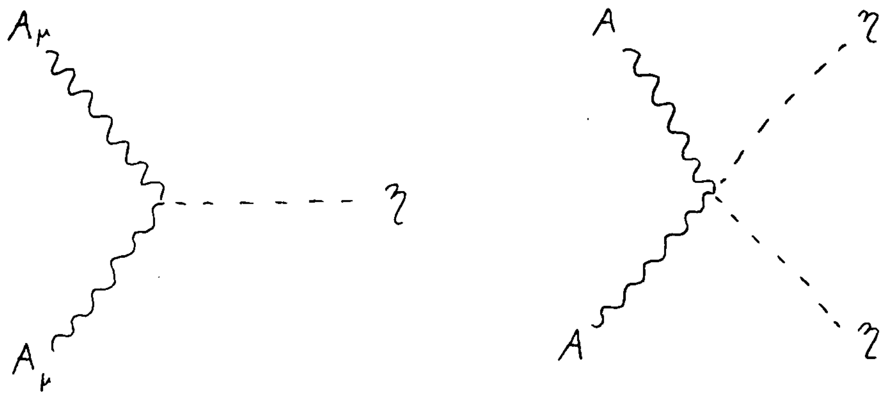

Figure 5 - Vector-scalar vertices

The gauge boson has also acquired a mass, which is the aim of this mechanism. We can henceforth build gauge theories giving rise to short-range interactions. Moreover, the theory, altough having its symmetry explicitely broken, is still renormalizable. This was demonstrated by t'Hofft (1971). The Higgs mechanism finds its best applications in the GWS model, which will be described in the next chapter.

IV. THE GLASHOW-WEINBERG-SALAM MODEL

The GWS model is usually introduced first with one doublet and one singlet of fermions only. The other known fermions can easily be introduced thereafter. This is the path I will follow.

BASIC LAGRANGIAN: One wants to identify the massive vector bosons arising in the Higgs mechanism with the intermediate vector bosons (IVB) carrying the WI. In the phenomenologically successful IVB theory, the lagrangian for weak interactions is given by:

$$\mathcal{L}_{weak} = g\left(J_\lambda W^\lambda + h.c.\right) \qquad \text{(IV.1)}$$

where $J_\lambda = \bar{\nu}_e \gamma_\lambda (1 - \gamma_5) e$ is the leptonic charged current in its so-called V-A form, and h.c. stands for "hermitian conjugate". On the other hand, the lagrangian for the electromagnetic interactions is given by

$$\mathcal{L}_{elec.} = e\, J^\lambda_{elec} A_\lambda \qquad \text{(IV.2)}$$

where $J^\lambda_{elec} = \bar{e}\gamma^\lambda e$ is the electromagnetic current.

Then, to unify EM with WI, one needs at least 3 gauge bosons, W^+, W^-, Z^0, to couple with the currents J_λ, $J^\dagger_\lambda$ and J^λ_{elec}. The simplest group with three such generators is SU(2). However, if ν_e and e^- are to form a doublet under SU(2), as suggested by the form of the current J^{λ}_{elec}, Q_{elec} cannot be a generator of the group, because the electric charges of the doublet do not add up to zero, whereas all 2 X 2 SU(2) representations matrices are traceless.

One is then led to introduce a fourth gauge boson Z^o. The smallest group is now SU(2) $\otimes$ U(1). Assuming that only V-A interactions occur for $W^{\pm}$, one takes the generators of the SU(2) groups to be the isospin operators T^{a}, whose 2-D representation may be taken to be the Pauli matrices divided by two.

$$\vec{T} = \vec{\tau}/2 \tag{IV.3}$$

$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ is then a doublet under SU(2) and $R = e_R$ is a singlet, as wanted. The subscript R or L means that only the right-handed or left-handed component of the lepton wave function is selected. One does it by multiplying the spinor by the projection operator $(1 - \gamma_5)$ to obtain the left-handed component, or $(1 + \gamma_5)$ for the right-handed component, i.e.

$$\Psi_{L,R} \equiv \left(\frac{1 \mp \gamma_5}{2}\right)\Psi \tag{IV.4}$$

The generator of U(1) is chosen such that the electric charge is a linear combination of the U(1) generator and the generator T_3 of SU(2). One can choose

$$Y = 2(Q - T_3) \tag{IV.5}$$

as generator of the U(1) group. The basic Lagrangian L of the GWS model may be split into 4 parts as follow;

$$\mathcal{L}_{GWS} = \mathcal{L}_{GAUGE} + \mathcal{L}_{fermions} + \mathcal{L}_{scalar} + \mathcal{L}_{YUKAWA} \tag{IV.6}$$

The gauge part of the Lagrangian is;

$$\mathcal{L}_{GAUGE} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{IV.7}$$

with

$$\left.F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \epsilon^{abc} A^b_\mu A^c_\nu \quad \right] \text{SU(2) GAUGE FIELD TENSOR}$$

$$\left.B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \right] \text{U(1) GAUGE FIELD TENSOR} \tag{IV.8}$$

The leptonic part of the Lagrangian is;

$$\mathcal{L}_{fermion} = \bar{L}\, i \gamma^\mu \left(\partial_\mu + \frac{i}{2} g' B - \frac{i}{2} g \tau^a A^a_\mu\right) L$$

$$+ \bar{R}\, i \gamma^\mu \left(\partial_\mu + i g' B_\mu\right) R \tag{IV.9}$$

where

g = coupling constant associated with SU(2)

g' = coupling constant associated with U(1)

Notice one cannot have a bare mass term of the form $\bar{e}_R e_L$, which is forbidden by SU(2) invariance. Also, the terms $\bar{e}_R e_R$ and $\bar{e}_L e_L$ vanish, because they contain the products of orthogonal operators $(1 - \gamma_5)$ and $(1 + \gamma_5)$.

To give a mass to the gauge bosons A_μ^a and the electron, let us introduce a doublet of complex Higgs scalars

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{IV.10}$$

They have Y = 1 to satisfy (IV.5), and transform as a doublet under SU(2). The scalar part of the lagrangian is then

$$\begin{aligned} \mathcal{L}_{scalar} &= \left(\partial_\mu + \frac{ig'}{2} B_\mu + \frac{ig}{2} A_\mu \tau^a A_\mu^a \right) \phi^+ \\ &\times \left(\partial^\mu - \frac{ig'}{2} B_\mu - \frac{ig}{2} A_\mu \tau^a A_\mu^a \right) \phi - \mu^2 \phi^+ \phi \\ &- \lambda (\phi^+ \phi)^2 \end{aligned} \tag{IV.11}$$

The most general renormalizable Higgs potential $V(\phi)$ is (Flores and Sher, 1982);

$$V(\phi) = \mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2 \tag{IV.12}$$

Also, one is free to add a coupling between the scalar doublet and the leptons, of the form;

$$\mathcal{L}_{YUKAWA} = - G_e \left[\bar{R} \phi^{\dagger} L + \bar{L} \phi R \right] \tag{IV.13}$$

The coupling of the form $\bar{\Psi}\phi\Psi$ is known as the Yukawa coupling. It has been introduced by Yukawa (1935) to explain the nuclear binding force between nucleons, through the exchange of bosons. There cannot be terms of the form $\bar{L}\phi L$ because, being the product of three SU(2) doublets, they cannot form an SU(2) invariant. One needs to include both a singlet and a doublet, hence the form of (IV.13). One now must spontaneously break the SU(2) ⊗ U(1) symmetry.

<u>SPONTANEOUS SYMMETRY BREAKING (SSB):</u> Assume once more that $\mu^2 < 0$. The two minima are at $|\phi|^2 = v^2/2$ with $v^2 = -\mu^2/|\lambda|$. One now requires the neutral scalar field to develop a vacuum expectation value (VEV). One must let the VEV of the charged scalar vanish, in order not to have a charged vacuum. This leads to

$$\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} / \sqrt{2} \tag{IV.14}$$

and again one expresses the scalar fields in polar coordinates to bring out the physical meaning of the theory. Using the unitary transformation;

$$\mathcal{U}(\xi) = e^{-i\xi^a \tau_a / v} \tag{IV.15}$$

the scalar fields read in the new coordinates;

$$\phi = \mathcal{U}^{-1}(\xi) \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} \tag{IV.16}$$

The four real components of ϕ are now distributed in 3 components for ξ^a and one for the scalar η.

The symmetry breaking scheme that Weinberg and Salam adopted breaks both SU(2) and $U(1)_Y$, but preserves $U(1)_{elec}$. One can check this using the condition for a generator $\mathcal{G}$ to leave the vacuum invariant; where $(\mathcal{G})$ is the 2 X 2 matrix representation of the operator $\mathcal{G}$.

$$(\mathcal{G})\langle\phi\rangle_0 = 0 \tag{IV.17}$$

For the generators of SU(2) X U(1), we find

$$\tau_1 \langle\phi\rangle_0 = \begin{pmatrix} v \\ 0 \end{pmatrix} \neq 0 \qquad \tau_2 \langle\phi\rangle_0 = \begin{pmatrix} -iv \\ 0 \end{pmatrix} \neq 0$$

$$\tau_3 \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0 \qquad Y\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0 \tag{IV.18}$$

But

$$Q\langle\phi\rangle_0 = \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = 0 \tag{IV.19}$$

The photon, and only the photon will then remain massless. Transforming now to the U-gauge (IV.15):

$$\begin{aligned} &\phi \rightarrow \phi' = U(\xi)\phi = \binom{0}{v+\eta}/2 \\ &L \rightarrow L' = UL\,; \qquad e'_R = e_R\,; \quad B'_\mu = B_\mu \\ &T^a A^a_\mu \rightarrow T^a A'^a_\mu = U(\xi)\left[T^a A^a_\mu - \frac{i}{g}U^{-1}(\xi)\partial_\mu U(\xi)\right]U^{-1}(\xi) \end{aligned} \tag{IV.20}$$

and A^a_μ still transforms according to (II.6). In terms of the new fields, the Lagrangians become;

$$\mathcal{L}_{GAUGE} = -\frac{1}{4}F'^a_{\mu\nu}F'^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{IV.21}$$

$$\begin{aligned} \mathcal{L}_{lepton} = {} & \bar{L}' i\gamma^\mu\left(\partial_\mu + \frac{i}{2}g'B_\mu - \frac{ig}{2}\tau^a A'^a_\mu\right)L' \\ & + \bar{R} i\gamma^\mu(\partial_\mu + ig'B_\mu)R \end{aligned} \tag{IV.22}$$

The mass of the electron arises from the Yukawa term;

$$\mathcal{L}_{YUKAWA} = -G_e\left(\bar{R}\phi'^{\dagger}L' + \bar{L}'\phi' R\right)$$

$$= G_e\left[\frac{v}{\sqrt{2}}\left(\bar{e}'_R e'_L + \bar{e}'_L e'_R\right) + \frac{\eta}{\sqrt{2}}\left(\bar{e}'_R e'_L + \bar{e}'_L e'_R\right)\right] \qquad \text{(IV.23)}$$

and is seen to be $m_e = vG_e/\sqrt{2}$.

The coupling of the remaining Higgs boson η to the electron is

$$\frac{G_e}{\sqrt{2}} = \frac{m_e}{v} \qquad \text{(IV.24)}$$

which will be of primary importance to produce and detect it. The scalar field will give rise to gauge boson masses via the term

$$\mathcal{L}_{scalar} = \left(D_\mu\phi'\right)^{\dagger}\left(D^\mu\phi'\right) - \mu^2\phi'^{\dagger}\phi' - \lambda\left(\phi'^{\dagger}\phi'\right)^2 \qquad \text{(IV.25)}$$

with

$$D_\mu\phi' = \left(\partial_\mu - \frac{ig\tau^a}{2}A'^a_\mu - \frac{ig'}{2}B_\mu\right)\phi' \qquad \text{(IV.26)}$$

Let us isolate the vector mass terms, which are those terms quadratic in vector fields, into a Lagrangian $\mathcal{L}_M$, subset of

$\mathcal{L}_{scalar}$:

$$\mathcal{L}_M = \frac{v^2}{8}\left\{ g^2\left[(A_\mu^{\prime 1})^2 + (A_\mu^{\prime 2})^2 \right] + (g' B_\mu - g A_\mu^{\prime 3})^2 \right\} \qquad \text{(IV.27)}$$

If we define $\quad W_\mu^{\pm} \equiv \frac{1}{\sqrt{2}}\left(A_\mu^{\prime 1} \mp i A_\mu^{\prime 2} \right)$

$$Z_\mu \equiv \frac{-g' B_\mu + g A_\mu^{\prime 3}}{\sqrt{g^2 + g'^2}} \qquad \text{(IV.28)}$$

$$A_\mu = \frac{g B_\mu + g' A_\mu^{\prime 3}}{\sqrt{g^2 + g'^2}}$$

The Lagrangian $\mathcal{L}_M$ becomes

$$\mathcal{L}_M = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \qquad \text{(IV.29)}$$

with

$$M_W^2 = \left(\tfrac{1}{2} g v\right)^2 \qquad \text{(IV.30)}$$

$$M_Z^2 = (g^2 + g'^2)\frac{v^2}{4}$$

The massless vector gauge boson A_μ can then be identified with the photon.

For convenience, one usually introduces an angle θ_W (Weinberg angle), which relates the coupling g of the $SU(2)_L$ to the coupling g' of the $U(1)_Y$ group. Explicitly

$$g' = g \tan \theta_W \tag{IV.31}$$

So that

$$\sqrt{g^2 + g'^2} = g / \cos \theta_W \tag{IV.32}$$

The interaction between the leptons and the gauge fields can now be read off from the lepton lagrangian in (IV.22). Re-expressing $\mathcal{L}_{lepton}$ in function of the new fields $W^{\pm}_{\mu}$, Z^{0}_{μ} and A_{μ}, one gets, using (IV.32);

$$\begin{aligned}\mathcal{L}_{lepton} = \mathcal{L}_{free} &+ \frac{Z_\mu \cos\theta_W}{g}\left(\frac{g^2}{2}(1-\tan\theta_W)\,\bar{e}_L \gamma^\mu e_L - g^2 \tan^2\theta_W\, \bar{e}_R \gamma^\mu e_R\right) \\ &- \frac{\cos^2\theta_W}{2g} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L - g \sin\theta_W A_\mu \bar{e} \gamma^\mu e \\ &+ \frac{g}{\sqrt{2}}\left[\bar{\nu}_L \gamma^\mu e_L W^+_\mu + \bar{e}_L \gamma^\mu \nu_L W^-_\mu\right]\end{aligned} \tag{IV.33}$$

Equating the coupling between the photon A_μ and the electron to the electromagnetic coupling e gives the relation:

$$g \sin \theta_W = e \tag{IV.34}$$

The IVB coupling is consistent with low energy phenomenology provided we identify

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} \qquad \text{(IV.35)}$$

The only missing piece is the mass of the Higgs boson which can be worked out from $\mathcal{L}_{scalar}$. This is equal to:

$$m_H = \sqrt{2}\ \mu \qquad \text{(IV.36)}$$

that is, completely undetermined in this model. However, upper and lower limits have been derived, which we will consider in chapter VI.

<u>ADDITION OF QUARKS:</u> Let us introduce the first two families of quarks. The incorporation of the third family follows the same line of argument.

According to Cabbibo's picture of WI (Cabibbo, 1963), the hadronic charged currents are represented by the weak-isospin doublets

$$L_1 = \begin{pmatrix} u \\ d' \end{pmatrix}_L ; \quad L_2 = \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \quad u_R,\ d'_R,\ c_R,\ s'_R \ \text{singlets} \qquad \text{(IV.37)}$$

where
$$\begin{aligned} d' &= d \cos\theta_c + s \sin\theta_c \\ s' &= -d \sin\theta_c + s \cos\theta_c \end{aligned}$$

and θ_c is refered to as the (Cabbibo) mixing angle. The Lagrangian terms corresponding to (IV.22) and (IV.23) are

$$\mathcal{L}_{QUARKS} = \sum_{\alpha} \left[\bar{q}_R^{\alpha} i\gamma^{\mu}(\partial_\mu + ig'B_\mu) q_R^{\alpha} + \bar{q}_L^{\alpha} i\gamma^{\mu}(\partial_\mu + \frac{i}{2}g'B_\mu - \frac{ig}{2}\tau^a A_\mu^a) q_L^{\alpha} \right] \qquad \text{(IV.38)}$$

q^{α} = u, d, s, c

and

$$\mathcal{L}^{QUARK}_{YUKAWA} = G_1 [\bar{L}_1 \Phi u_R + h.c.] + G_2 [\bar{L}_1 \Phi d_R + h.c.]$$
$$+ G_3 [\bar{L}_1 \Phi s_R + h.c.] + G_4 [\bar{L}_2 \bar{\Phi} c_R + h.c.] \qquad \text{(IV.39)}$$
$$+ G_5 [\bar{L}_2 \Phi d_R + h.c.] + G_6 [\bar{L}_2 \Phi s_R + h.c.]$$

respectively. The Lagrangian piece (IV.38) gives rise to exactly the same kind of results as for the lepton case. If we now perform SSB by replacing by its expectation value (IV.14) we obtain a serie of mass terms equivalent to (IV.23).

$$\mathcal{L}^{QUARK}_{YUKAWA} = -\sum_{\alpha} \frac{(v + \eta)}{\sqrt{2}} G_{\alpha} \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \qquad \text{(IV.39)}$$

We must now chose the Yukawa couplings $G_1, \ldots, G_6$ so that u,d,s and c are mass eigenstates;

$$\begin{aligned}
G_1 &= m_{\mu} \sqrt{2} / v \\
G_2 &= m_d \cos\theta_c \sqrt{2} / v \\
G_3 &= m_s \sin\theta_c \sqrt{2} / v \\
G_4 &= m_c \sqrt{2} / v \\
G_5 &= -G_2 \tan\theta_c \\
G_6 &= +G_3 \operatorname{cotan}\theta_c
\end{aligned} \qquad \text{(IV.40)}$$

The generalization to three generations introduces two more quark mixing angles, but the rest of the procedure stays essentially the same.

V. PARTON MODEL AND HADRON-HADRON COLLISION

In order to calculate the production rates in hadron-hadron and lepton-hadron collisions, some simplifying hypotheses are needed about the structure of hadrons. Such a set of hypotheses, well supported by experience, forms the parton model[1] . Let us describe its sources, links with QCD and applications to hadron-hadron collision.

When high energy electrons or neutrinos are scattered from nucleons their angular distributions look as if they were scattering from hard, pointlike constituents inside the nucleons. It is a repetition, at higher energies, of Rutherford's experiment. These pointlike constituents of nucleons have been given the name "partons", and the partons interacting electromagnetically or weakly with the leptons have been identified (theoretically) as quarks. Another class of nucleon constituents, mediating the QCD force between the quarks, are "gluons". The gluons do not interact through WI, and are electrically neutral. They are spin-1 bosons, and carry the "color" charge which gives rise to strong interactions. Therefore, gluons interact with gluons, making quantitative predictions of QCD very difficult. Still, the method of lattice gauge theory[2] managed to produce an acceptable hadron spectrum. The lattice gauge theory is a non-perturbative way of getting predictions from a theory with a large coupling constant, like QCD at low momentum transfer.

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[1] see for example (Close, 1979)
[2] for a review see (Drouffe and Itzykson, 1978)

Also, when experiments reach very high energy, the phenomenon of scaling occurs, and one may use perturbative QCD to calculate production rates.

<u>SCALING:</u> The parton model picture stems from a property of lepton-proton scattering, called scaling. Here are the foundations of it.

When one calculates the amplitude for lepton-lepton scattering, say $e^-\mu^- \rightarrow e^-\mu^-$, one gets from the amplitude corresponding to the diagram of Fig. 6

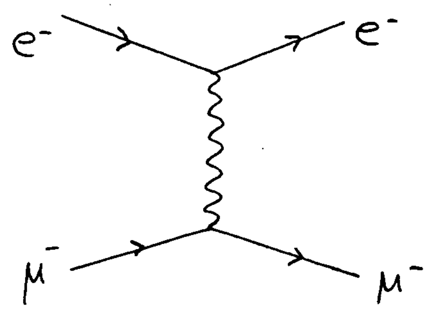

Figure 6 - Feynman diagram for electron-muon scattering

the cross-section:

$$\frac{d^2\sigma}{dE'd\Omega_e} = \frac{4\alpha^2(E')^2}{Q^4}\left\{\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right\}\delta\left(\nu - \frac{Q^2}{2M}\right) \qquad \text{(V.1)}$$

where

E : energy of the incoming electron in the lab frame

E': energy of the scattered electron in the lab frame

ν : energy transfer to the muon in the lab frame

Q^2: minus one times the momentum transfer squared

θ : scattering angle of the electron in the lab frame

M : rest mass of the muon

On the other hand, if one wants to calculate inclusive electron-proton scattering, one is forced to introduce structure functions in the hadronic rate tensors $W^{\mu\nu}$. The hadronic tensor $W^{\mu\nu}$ is the piece which must be introduced in the spin-summed amplitude squared at the location of the photon-proton vertex in evaluating the rate corresponding to the Feynman diagram of Fig. 7

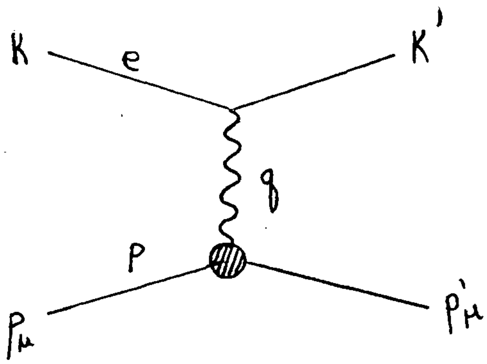

Figure 7 - Feynman diagram for electron-proton scattering.

(See appendix B for introduction to Feynman diagrams). It serves to parametrize our ignorance of the form of the current at the proton end of the photon propagator. The most general form for the proton structure function is

$$W^{\mu\nu} = W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} \left(p^\mu q^\nu + q^\mu p^\nu\right) \qquad (V.2)$$

p^μ being the proton 4-momentum. The $W^{\mu\nu}$ are in general functions of ν and Q^2. Gauge invariance $q_\mu W^{\mu\nu} = 0$ gives us some relations between the coefficients, and one is left with only two independent structure functions;

$$W^{\mu\nu} = W_1(\nu, Q^2)\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2(\nu, Q^2)}{M^2}\left[\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\mu - \frac{p\cdot q}{q^2} q^\nu\right)\right] \quad \text{(V.3)}$$

assuming parity-conserving interactions. Contracting with the leptonic rate tensor $L^{\mu\nu}$ and expressing the cross-section in the laboratory frame where the initial proton is at rest yields;

$$\frac{d^2\sigma}{dE' d\Omega_e} = \frac{4\alpha^2(E')^2}{Q^4}\left\{\cos^2\frac{\theta}{2} W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\sin\frac{\theta}{2}\right\} \quad \text{(V.4)}$$

One can now compare expressions (V.1) and (V.4) and deduce that if the virtual photon scatters off a pointlike Dirac particle, the structure functions reduce to:

$$\nu W_2^{pt}(\nu, Q^2) = \delta\left(\frac{Q^2}{2M\nu} - 1\right)$$

$$2MW_1^{pt}(\nu, Q^2) = \frac{Q^2}{2M\nu}\,\delta\left(\frac{Q^2}{2M\nu} - 1\right) \quad \text{(V.5)}$$

the equations being written this way to form dimensionless ratio $\omega = 2M\nu/Q^2$ only.

For scattering from the proton, in general one expects a dependence of these functions on ν and Q^2 separately. But, at high momentum transfer the phenomena of scaling occurs, i.e. for fixed ω and $Q^2 \gtrsim 1$ GeV;

$$M W_1^{proton}(\omega, Q^2) \longrightarrow F_1(\omega)$$

$$\nu W_2^{proton}(\omega, Q^2) \longrightarrow F_2(\omega) \qquad \text{(V.6)}$$

is observed to hold empirically (Bjorken, 1969). The energy and momentum-transfer dependence of the process behaves exactly as if the electrons were scattering off hard, pointlike constituents inside the protons, i.e. like (V.5). This is why it was said earlier that it is a repetition, at higher energies, of the Rutherford scattering experiment.

Impulse approximation: The parton model contains implicitly the equivalent of the impulse approximation in nuclear physics. The basic assumptions are the following:

1. During the time of interaction one can neglect interactions between the partons.

2. Final state interactions can be ignored.

That is to say the parton is quasi-free in the proton, and can be considered free at very high energy. The effect of confinement acts much later, when the scattered parton has moved a distance of the same order as the size of the proton.

In terms of Feynman diagrams, this means that we consider subprocesses with different initial or final states as being non-interfering. One does not have to worry about the other "spectator" partons to the lowest order. These assumptions are extremely useful for calculations of processes in hadron-hadron scattering. Another element that one needs is the parton momentum distribution, which will be discussed below.

<u>Electromagnetic spin 1/2 structure function:</u> Now we will express (V.1) and (V.4) using the Mandelstam invariants s, t and u:

$$s = (p+q)^2$$
$$t = (p-p')^2$$
$$u = (p-q')^2 \tag{V.7}$$

This set of variables makes explicit the Lorentz invariance of any quantity expressed in terms of it. The relation (V.1) becomes

$$\frac{d^2\sigma}{dt\,du} = \frac{4\pi}{t^2}\,\frac{1}{2}\left(\frac{s^2+u^2}{s^2}\right)\delta(s+t+u) \tag{V.8}$$

It will be useful later to know that

$$\frac{-t}{s+u} = \frac{Q^2}{2M\nu} \equiv \frac{1}{\omega} \tag{V.9}$$

If one wants to compare (V.8) to inelastic electron-proton scattering cross-section (V.4), one uses the parton model, where it is hypothesised that inelastic, electron-proton scattering comes from the sum of incoherent elastic scattering of electrons on the partons in the target. If these partons have spin 1/2 and couple to the photon the same way the μ^- couples to the photon, then one can easily obtain an expression for the cross-section.

Going into a reference frame where the proton has infinite momentum, one effectively "freezes" the slow interactions. One defines

$$P_{parton} \equiv x \; P_{proton} \tag{V.10}$$

The relation (V.8) can be written in terms of the momentum of a parton with the substitutions

s—>xs u—>xu t—>t

$$\left(\frac{d^2\sigma}{dt\,du}\right)_{eq\to eq} \quad \frac{4\pi\alpha^2}{t^2}\;\frac{1}{2}\;\left(\frac{s^2+u^2}{s^2}\right)\; x \;\; \delta(t + x(s+u)) \tag{V.11}$$

The proton being supposedly made of several partons, denoted by the index i, one has to sum over these, and integrate over the probability function f(x) for a parton i to have a momentum fraction between x and x+dx, to get the cross-section for electron-proton scattering:

$$\left(\frac{d^2\sigma}{dt\,du}\right)_{ep\to eX} \quad \frac{4\pi\alpha^2}{t^2}\;\frac{1}{2}\;\frac{s^2+u^2}{s^2}\int dx \sum e_i^2\, x\, f_i(x)\,\frac{1}{s+u}\,\delta(x-\tfrac{1}{\omega}) \tag{V.12}$$

where the relation (V.9) has been used in rewriting the delta function. We are now ready to compare with the general expression for e-p scattering, eq. (V.4) which we recast into the form

$$\left(\frac{d^2\sigma}{dt\,du}\right)_{ep\to eX} = \frac{4\pi\alpha^2}{t^2}\,\frac{1}{2}\,\frac{1}{s^2(s+u)}\left[2x(s+u)^2F_1 - 2usF_2\right] \tag{V.13}$$

Comparing the coefficients in (V.12) and (V.13), one gets

$$2xF_1(x) = F_2(x) = \sum_i e_i^2\, x\, f_i(x) \tag{V.14}$$

This is the Callan-Gross formula (Callan and Gross, 1969) for spin 1/2 parton model. Identifying the spin 1/2 partons with the quarks, and denoting f_{q_i} by the symbol q_i, one puts for the proton and the neutron

$$\begin{aligned}\frac{1}{x}F_2^{ep}(x) &= \frac{4}{9}(u_p+\bar{u}_p) + \frac{1}{9}(d_p+\bar{d}_p+s_p+\bar{s}_p)+\cdots\\ \frac{1}{x}F_2^{eN}(x) &= \frac{4}{9}(d_N+\bar{d}_N) + \frac{1}{9}(u_N+\bar{u}_N+s_N+\bar{s}_N)+\cdots\end{aligned} \tag{V.15}$$

<u>Sum rules and momentum parton distributions:</u> The fundamental relations the quarks distributions u(x) and d(x) must obey come

from the isospin properties and zero net strangeness of the proton and neutron:

$$\begin{aligned} 0 &= \int_0^1 dx\,[s(x) - \bar{s}(x)] \\ 1 &= \int_0^1 dx\,[\tfrac{2}{3}(u-\bar{u}) - \tfrac{1}{3}(d-\bar{d})] \\ 0 &= \int_0^1 dx\,[\tfrac{2}{3}(d-\bar{d}) - \tfrac{1}{3}(u-\bar{u})] \end{aligned} \qquad \text{(V.16a)}$$

Or

$$\begin{aligned} 2 &= \int_0^1 dx\,[u(x) - \bar{u}(x)] \\ 1 &= \int_0^1 dx\,[d(x) - \bar{d}(x)] \end{aligned} \qquad \text{(V.16b)}$$

A similar relation for electrically neutral partons (gluons) comes from momentum conservation

$$\int_0^1 dx\; x(u+\bar{u}+d+\bar{d}+s+\bar{s}) = 1-\mathcal{E} \qquad \text{(V.17)}$$

where $\mathcal{E}$ is the fraction of momentum carried by the gluons. Then the gluon momentum distribution G(x) must obey

$$\int_0^1 dx\; x\, G(x) = \mathcal{E} \qquad \text{(V.18)}$$

The value of $\mathcal{E}$ turned out in experiments to be about

$\varepsilon \simeq 0.5$ (Smith, 1974). That means that half of the proton momentum is carried by the gluons.

The quark momentum distributions can also be deduced from experiments after inverting equations (V.15)

$$u(x) \simeq \frac{1}{x}\frac{9}{15}\left[4F_2^{ep}(x) - F_2^{eN}(x)\right]$$

$$d(x) \simeq \frac{1}{x}\frac{9}{15}\left[4F_2^{eN}(x) - F_2^{ep}(x)\right] \tag{V.19}$$

A few particular parametrizations are given in appendix F. The most useful clues come from the ν-hadron and e-hadron deep inelastic scattering.

<u>Hadron-hadron scattering:</u> The calculation of a process in QCD for hadron-hadron scattering proceeds according to the following scheme.

a) Calculate the subprocess in a perturbative way, using QCD rules, and other models, such as the Weinberg-Salam model. This gives a sub-cross-section $\sigma_{sub}(x_i, \hat{u}, \hat{s}, \hat{t})$ where x_i is the fraction of the momenta $\hat{u}$, $\hat{s}$ and $\hat{t}$ carried by the incoming parton i.

b) Convolute σ_{SUB} with the parton distributions $f_H(x)$ of the incoming hadrons, which reads;

$$\sigma_{HADRON} = \int dx\, f_A(x) \int dy\, f_b(y)\, \sigma_{SUB}(x, y, \hat{u}, \hat{s}, \hat{t}) \tag{V.20}$$

Let us now come back to the subject of Higgs boson, to examine its properties in more detail. This will enable us to calculate the sub-cross-sections related to Higgs production in hadron collisions.

VI. HIGGS BOSON PHENOMENOLOGY

Here, we are coming to the heart of the subject -the Higgs boson properties. The knowledge of these is essential to the development of a strategy in the Higgs boson "hunt". Its mass and couplings to matter are needed to predict the production mechanisms and detection modes.

Mass of the Higgs boson: There exist several arguments giving rise to upper and lower mass bounds on the Higgs boson. Only one, a lower bound, is derived from solid experimental facts. All other ones depend on theoretical expectations.

One upper bound on the mass of the Higgs boson comes from the unitary restriction in the elatic scattering $W^+W^- \longrightarrow W^+W^-$ (Lee et al., 1977). The subscript L denotes a longitudinally polarized particle. As we have seen in chapter I, the process violates unitarity, and this is removed by the scalar contribution. The scattering amplitude for this process is, after cancellation of the divergences

$$T = -\frac{4}{\sqrt{2}} G_F m_H^2 \qquad s \gg m_H^2 \tag{VI.1}$$

The amplitude T has the usual partial-wave expansion

$$T = 16\pi \sum_j (2j+1)\, t_j\, P_j(\cos\theta) \tag{VI.2}$$

Partial wave unitarity requires $|t_j| < 1$. Here, j=0, and

$$m_H^2 \lesssim \frac{4\pi\sqrt{2}}{G_F} \sim 1.5\ TeV/c^2 \qquad \text{(VI.3)}$$

A more refined calculation yields $m_H \lesssim 1\ TeV/c^2$ (Lee, Quigg and Thacker, 1977). If m_H lies above this limit, perturbation expansion breaks down and higher order terms are as important as the lowest order one. It can be shown (Veltman, 1977) that as m_H increases, the parameter of the scalar potential (IV.11) increases, and when $\lambda >> 1$, perturbative theory is meaningless. This happens at around $m_H = 1\ TeV/c^2$.

There is nothing wrong in itself for the perturbative expansion technique to break down. A perturbative theory is merely a desirable condition. It has been shown that a nonperturbative Higgs sector would show very little effect in current phenomenology (Appelquist and Bernard, 1980). The Higgs sector itself could give rise to some new physics, for example with bound states of elementary Higgs bosons.

Another upper bound on the Higgs comes from a study of the triviality of the scalar $\lambda\phi^4$ interaction (Callaway, 1983). There is evidence that the $\lambda\phi^4$ theory is a trivial theory, i.e. the interaction screens itself and is equivalent to a free field theory. The $\lambda\phi^4$ interaction coupled with fermions and/or vector bosons might not be trivial however. This could happen only within certain limits, one of which being a bound on the H^0 mass. For the standard model:

$$\left(\frac{m_H}{m_W}\right)^2 \lesssim 12.8 \qquad m_H \lesssim 290\ GeV/c^2 \tag{VI.4}$$

Lower bounds on the Higgs mass have been proposed from several sources. The decay $K^{\pm} \longrightarrow \pi^{\pm} l^{+} l^{-}$ gives a lower bound of about 325 MeV/c^2 for the mass of the Higgs boson (Willey and Yu, 1982). If the Higgs boson was any lighter, it would appear as a resonance peak in the invariance mass of the lepton pair. Other lower bounds come from studies of the radiatively corrected Higgs potential. However, all conclusions derived from the studies of the Higgs potential are suspect[1] . The technique of the scalar potential is developed in the following way. We introduced in (IV.11) the classical scalar potential $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ which is the most general renormalizable scalar field expression. The "effective" potential for the quantum field can be written in terms of the classical field . ϕ_c (Jona-Lasinio, 1964), (Coleman and Weinberg ,1973). According to quantum field theory, one can calculate effective potentials at the n^{th} order level, corresponding to n-loop graphs. For example, the effective potential corresponding to the zeroth order correction (tree graph) is the classical potential (IV.10). The first order quantum correction includes the sum over all one-loop graphs of the theory, etc.

At the first order, including scalar + vector loops, the effective potential is (Jackiw, 1974)

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[1] Ng, private communication

$$V(\phi_c) = -\tfrac{1}{2}(\lambda + 2B)\sigma^2\phi_c^2 + \tfrac{1}{4}\lambda\phi_c^4 + B\phi_c^4 \ln \phi_c^2/\sigma^2 \qquad \text{(VI.5)}$$

where $B = 3[2g^4 + (g^2 + g'^2)]/1024\pi^2$ and is the minimum of the potential.

With this potential, one finds

$$\mu^2 = \left.\frac{d^2V}{d\phi_c^2}\right|_{\phi_c=\sigma} = (\lambda + 2B)\sigma^2 \qquad \text{(VI.6)}$$

The effective potential for different values of the parameters is plotted in figure 8, extracted from (Flores and Sher, 1982). Notice that one can have μ^2 negative, (non-tachyonic scalar mass) and still achieve spontaneous symmetry breaking. The elegant hypothesis $\mu^2 = 0$, due to Coleman and Weinberg (1973), gives a calculable value $m_{cw} = 10.4$ GeV/c^2. In this hypothesis, no mass scale is introduced at the level of the bare Lagrangian. Also if $\mu^2 < 4B\sigma^2$, the spontaneously broken vacuum is not stable. It could "tunnel through" a lower lying vacuum at $\phi = 0$. The probability of tunnelling increases as m_H decreases, and this brings various limits on m_H, depending on what one assumes about the conditions prevailing at the beginnning of the universe.

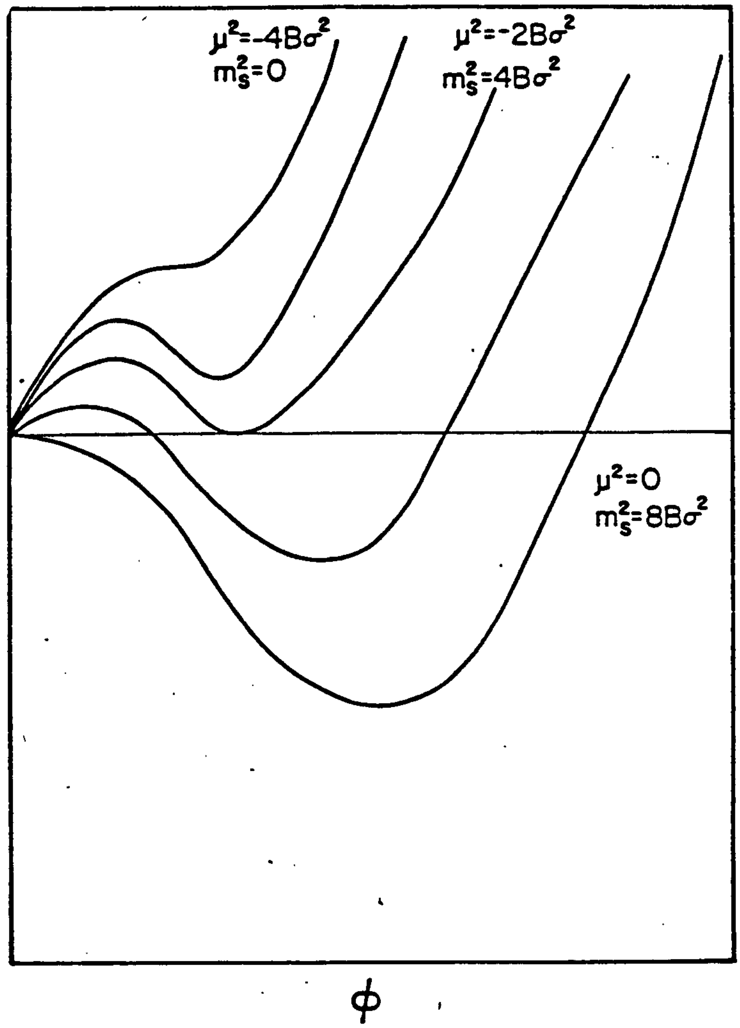

Figure 8 - $V(\phi)$ for different values of μ^2.

If one assumes that the universe has somehow been brought in the asymmetric-state vacuum after its birth, one requires the lifetime of this state to be more than the age of the universe. This leads to a lower bound of $m_H > 260$ MeV/c^2.

If one instead takes the position that the universe was in the symmetric state just after its birth, and underwent a phase transition to the spontaneously symmetry-broken vacuum, then this later must lie below the $\phi = 0$ point. The lower limit on m_H becomes 7 GeV/c^2 (Weinberg, 1976). It was also pointed out by Linde (1976) that the lifetime of this transition must be substantially smaller than the age of the universe, and he got a limit $m_H > 0.99\ m_{cw}$.

The inclusion of fermion loops makes m_{cw} drop by 6 MeV(m_f/15 GeV/c^2) . Therefore, for $m_f <$ 30 GeV/c^2, the fermion contribution is negligible. If the top quark mass (or any other heavy quark mass) is larger than 30 GeV/c^2, m_{cw} will drop significantly. If $m_f >$ 100 GeV/c^2, m_{cw} becomes negative, and more care in the Coleman-Weinberg mechanism is needed to derive meaningful results.

<u>Couplings of the Higgs boson:</u> The coupling (IV.24) was derived in the model with one fermion doublet and singlet. We saw in (IV.39) that many Yukawa coupling terms , each with its constant G_i must be introduced. Adjusting the G_i to reproduce the fermion mass spectrum, one gets for the coupling of the Higgs boson to fermions:

$$-i m_f (G_F \sqrt{2})^{1/2} \tag{VI.7}$$

which means that the probability of a reaction producing a Higgs boson will be proportional to the square of the mass of the fermion it is coupled to. It also means that the Higgs will decay almost exclusively into the heaviest particle kinematically allowed. The coupling of the Higgs to photons and gluons is made only through loop diagrams. The coupling to $W^{\pm}$ and Z° bosons, however, is;

$$-2i M_W^2 \left(G_F \sqrt{2}\right)^{1/2} \tag{VI.8}$$

and will be dominant when the available energy allows it.

<u>Decay of the Higgs:</u> The decay rates for the Higgs boson into leptons are given by (Sudaresan and Watson, 1972);

$$\Gamma(H^{\circ} \rightarrow f\bar{f}) = \frac{G_F\, m_f^2\, m_H}{4\sqrt{2}\,\pi} \left(1 - \frac{4m^2}{m_H}\right)^{3/2} \tag{VI.9}$$

For quarks, simply multiply by three, because of the color degree of freedom. A plot of the branching ratios of the H° is given in fig. 9 extracted from (Ellis, Gaillard and Nanopoulos, 1976).

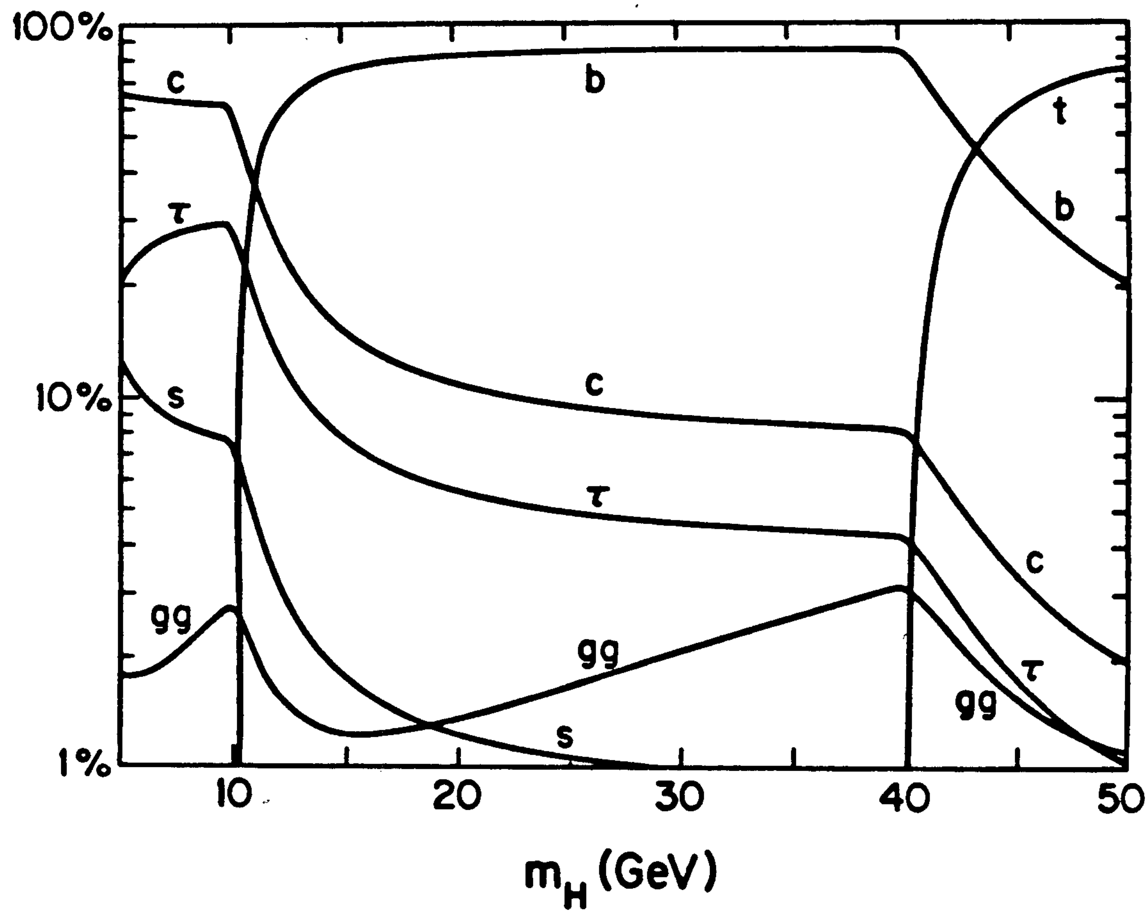

Figure 9 - Branching ratio of the H° in function of m_H. from (Ellis, Gaillard and Nanopoulos, 1976)

The decay rate of the Higgs boson into vector bosons is

$$\Gamma(H^{\circ} \to V\bar{V}) = \frac{G_F M_W^2}{8\pi\sqrt{2}} \, m_H \frac{(1-x)^{1/2}}{x}(3x^2-4x+4) \qquad \text{(VI.10)}$$

where $x = 4\, m_V^2/m_H^2$. This rate becomes rapidly very large as m_H increases. The consequence is that, because of their width, Higgs of mass greater than 700-800 GeV/c may never be observed (Ali, 1981).

<u>Higgs signature:</u> Once one has produced the Higgs boson, how does one know about it? One characteristic of the Higgs boson is its strong tendency to decay into the heaviest particle kinematically allowed. So, even before analysing in detail the spin and angular distributions of its reaction products, one might suspect Higgs bosons had been produced if the final state contained an anomalously large fraction of heavy particles. If the Higgs mass lies above the b-quark threshold, but below the top quark one, then it would decay predominantly into b-quark pairs. If its mass is below twice the W mass, but above t-quark threshold, it would decay mostly into t-quark pairs, which would then decay into b-quarks.

Thus, the observation of events with one or two jets of invariant mass m_H, containing at least two bottom quarks would be the signature for a Higgs of mass $2m_b < m_H < 2m_W$. Moreover, each production mechanism will produce something different along with the Higgs, and can be used to discriminate it from the background.

<u>Production of the Higgs boson:</u> Here will be presented the principal mechanisms that have been proposed up to now, in the search for the Higgs boson. The strategy is to produce particles that have very large couplings to the H^o, and look for the signal of a H^o which could be produced with it, radiated from it, or decay from it, dependent on the process. The first three processes are more pertinent to e^+e^- machines, and the last two are appropriate for hadron-hadron collisions.

1) Decay of the Z^o:

a) The Z^o can decay into a Higgs and photon, through a fermion loop or a $W^\pm$ loop (Cahn et al., 1978) represented by the diagram of figure 10.

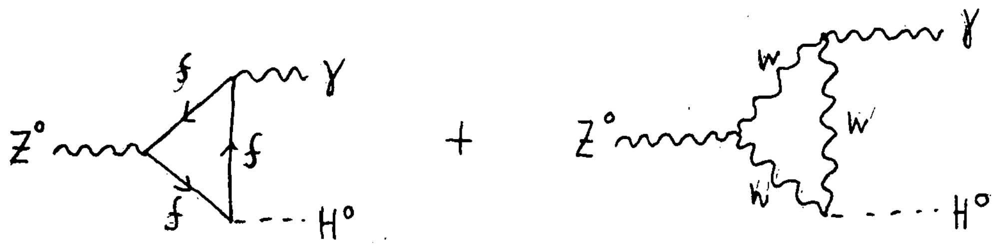

Figure 10 - Feynman diagram for Z^{o} --> γ + H^{o} decay.

with a ratio

$$\frac{\Gamma(Z^{o} \rightarrow H^{o} \gamma)}{\Gamma(Z^{o} \rightarrow \mu^{+}\mu^{-})} \simeq 10^{-6} \times \left(1 + 0.17 \frac{m_{H}^{2}}{m_{Z}^{2}}\right) \qquad \text{(VI.11)}$$

However, the background for this process, Z^{o} -> $l^{-} l^{+} \gamma$ is so large that the process 1a) would be buried in it (Barbiellini et al., 1979)

b) Z^{o} decay along the channel Z^{o} —> H^{o} + $l^{+} l^{-}$ (Bjorken, 1976) represented by the diagram of figure 11,

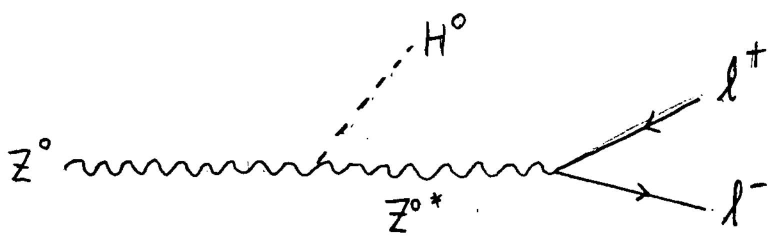

Figure 11 - Z^{o} —> H^{o} + $l^{+}l^{-}$ decay diagram.

where Z^{o} denotes a virtual Z^{o}. The branching ratio for this decay channel is:

$$Br\,(Z^{o} \rightarrow H^{o} + \mu^{+}\mu^{-}) = 10^{-5} \qquad \text{(VI.12)}$$

which is observable for a high-luminosity Z^{o} factory. A Z^{o} factory is an $e^{+}e^{-}$ collider where the center-of-mass collision energy can be tuned to the Z^{o} mass, allowing a very large Z^{o} production rate. The process peaks at large dimuon mass. The angular distribution and dilepton mass distribution may be used to distinguish Higgs bosons and other scalar particles, elementary or not (Kalyniak et al., 1984).

The main drawback to the process 1b) is that it works only if the mass of the Higgs is less than about 60 GeV/c^{2}. To circumvent the problem one needs to produce a Higgs together with a Z^{o}, from a virtual Z^{o}.

2) Bremsstrahlung from a virtual Z^{o}: $e^{+}e^{-} \rightarrow H^{o} + Z^{o}$ illustrated by the diagram of figure 12.

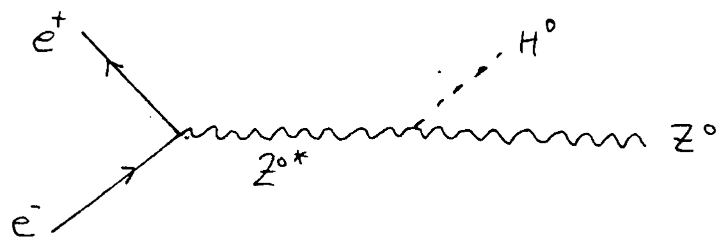

Figure 12 - Feynman diagram for $e^{+}e^{-} \rightarrow H^{o} + Z^{o}$.

The production rate peaks at $\sqrt{s} = m_Z + 2m_H$ (Glashow et al., 1978) and the total rate is encouraging for e^+e^- machines, predicting a cross-section of 4×10^{-35} cm^2 for a Higgs of 10 GeV/c^2 for a center-of-mass energy of 104 GeV. The problem is that one has to wait for the LEP II project to be completed. For the $p\bar{p}$ colliders, the production rates corresponding to this process are below the minimum acceptable (Ellis et al., 1976).

3) Decay of quarkonia: The form of the Higgs coupling to fermions makes it worthwhile to investigate heavy quarkonia decay. For the upsilon particle radiative decay

$$\Upsilon(9.46\ \mathrm{GeV}) \rightarrow H^0 + \gamma \qquad \text{(VI.13)}$$

an upper limit branching ratio

$$BR(\Upsilon \rightarrow H^0 + \gamma) = 2.5 \times 10^{-4} \qquad \text{(VI.14)}$$

has been calculated using a non-relativistic quark model (Wilczek, 1977). However, if the Coleman-Weinberg estimate is correct, this decay is not accessible. One of the best ways to produce a relatively light Higgs is through the decay of the (still unobserved) toponium state: $J_T \rightarrow H^0 + \gamma$

which has a huge branching ratio for $m_{J_T} < m_H$:

$$\frac{\Gamma(J_T \rightarrow H^\circ + \gamma)}{\Gamma(J_T \rightarrow \mu^+ \mu^-)} > 0.13 \qquad \text{(VI.15)}$$

In the case $m_{J_T} > m_Z$, the J_T will decay mostly through weak decay, thus depleting the branching ratio for $J_T \longrightarrow H^\circ + \gamma$.

Also, all of the above processes share the same characteristic that their rates are insignificant in a hadron-hadron collider, because of the impossibility of "sitting on" a resonance, as with e^+e^- colliders. The following processes may lead to more sizeable cross-section at hadron-hadron colliders.

4) Gluon-gluon fusion: The fusion of two gluons into a single Higgs boson through a quark loop as in the Feynman diagram of figure 13 allows the use of the important gluon component of the hadrons (Georgi et al., 1978).

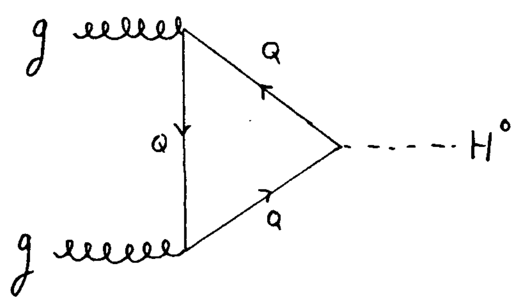

Figure 13 - Feynman diagram for gg —> H°.

It also exibits the interesting feature of counting all possible quark loops, even for quarks that are so heavy they would not be produced in the laboratory. This comes from the particular form of the Higgs coupling to fermions, which being proportional to the fermion mass, cancel out a fermion mass term in the denominator of the phase space integration to yield a cross-section which is not sensitive to the quark mass, but proportional to the square of the number of heavy quarks. This process has been described as a "heavy quark counter" because of this feature.

For Higgs bosons of masses less than $2M_w$, the background is several orders of magnitude larger than the signal (Keung, 1981), and there is no hope to discriminate them. The background process is the creation of a fermion pair through quark-antiquark annihilation. The cross-section for $p\bar{p} \rightarrow H^{o} + X$ through $gg \rightarrow H^{o}$ and the estimated Drell-Yan background are given in Fig. 14, from (Keung, 1981)

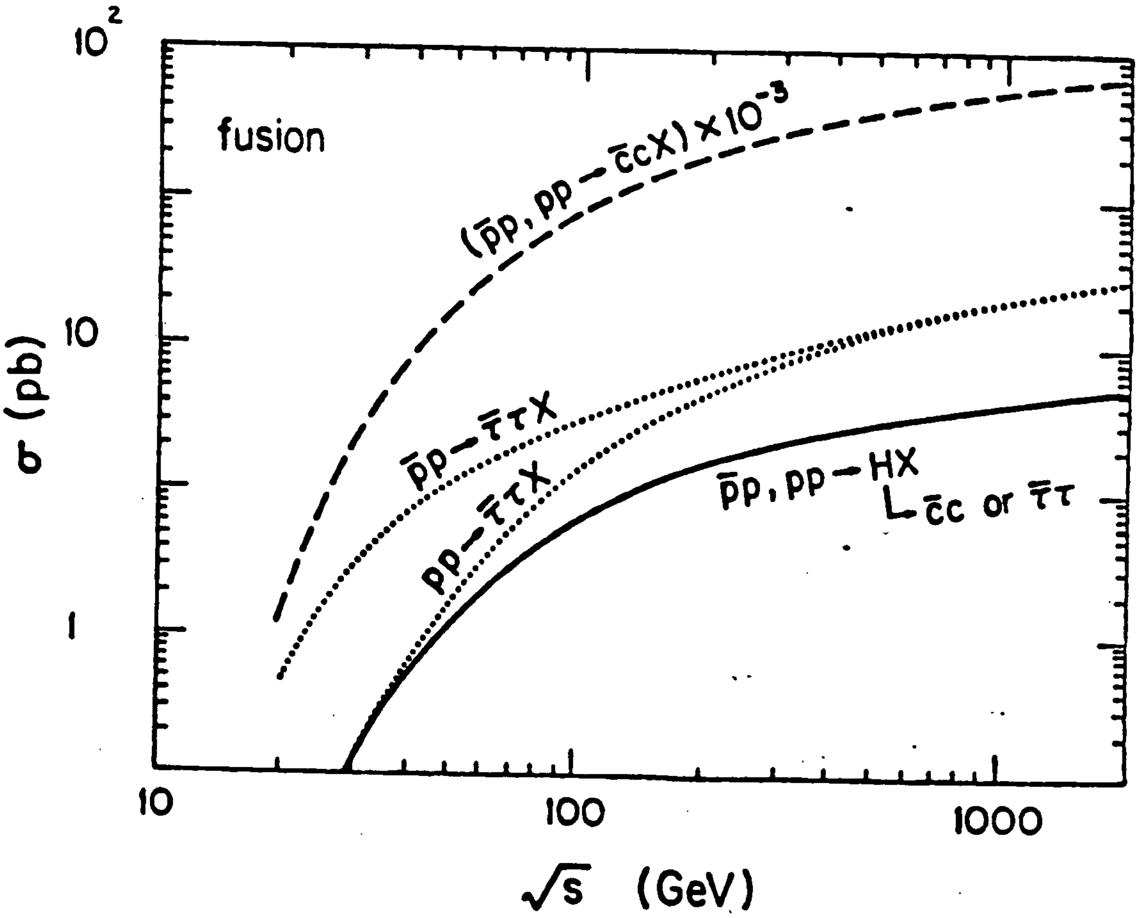

Figure 14 - Cross sections for $p\bar{p} \to H^\circ + X$ through the process $gg \to H^\circ$ (solid curve) and background (dotted curves) for m_H = 10 GeV/c^2 from (Keung, 1981)

However, if the mass of the Higgs is such that it can decay into a pair of vector bosons, the signal may become more important than the background (Cahn and Dawson, 1984). More calculations are needed.

5) Compton-like processes: This is the compton scattering of gluons from heavy sea quarks, illustrated in fig. 15

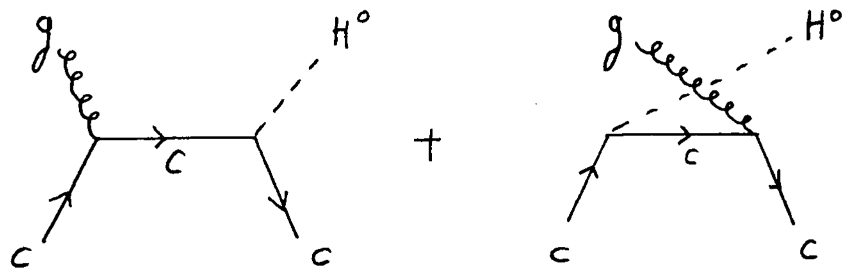

Figure 15 - Feynman diagram g +c --> c + H^o.

The signal and background are estimated in (Barger and al., 1982) and reproduced in fig. 17. The authors claim the final state would produce a dramatic signature. The final state would be the same as the one described and calculated later in this thesis. However, for the reaction 5) the rate is very low, of order 1 picobarn or less, because of the very small c-quark content of the proton and antiproton.

6) Vector-boson fusion: Cahn and Dawson (1984) have proposed another mechanism as part of a study of very massive Higgs boson production. It makes use of the large vector boson coupling to the Higgs boson, according to the process illustrated in Fig. 16

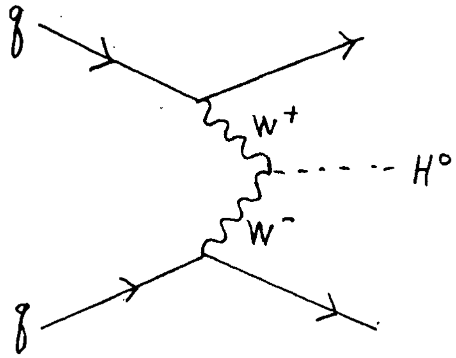

Figure 16 - Feynman diagrams for the process qq --> H^oqq

The total cross section for $m_H = 5$ M_W for this process, together with processes 2) and 5) at SSC energies, are shown in Fig. 18, extracted from (Cahn and Dawson, 1984). After having examined the principal channels suggested up to now for Higgs production, we are now in position to introduce the new mechanism on which this work is based.

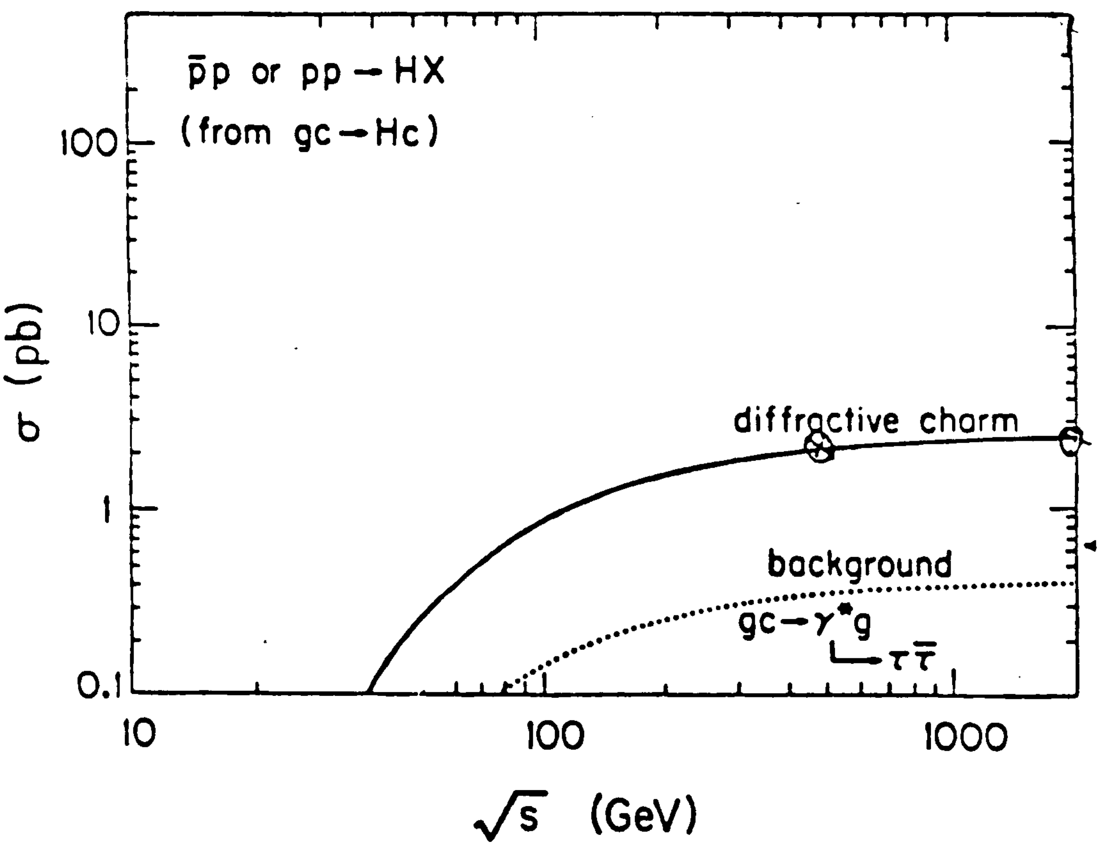

Figure 17 - Cross sections for compton-like process, for m_H = 10 GeV/c^2.
The solid curve represents the signal, the dotted curve is the background for the Higgs decaying into a tau pair.

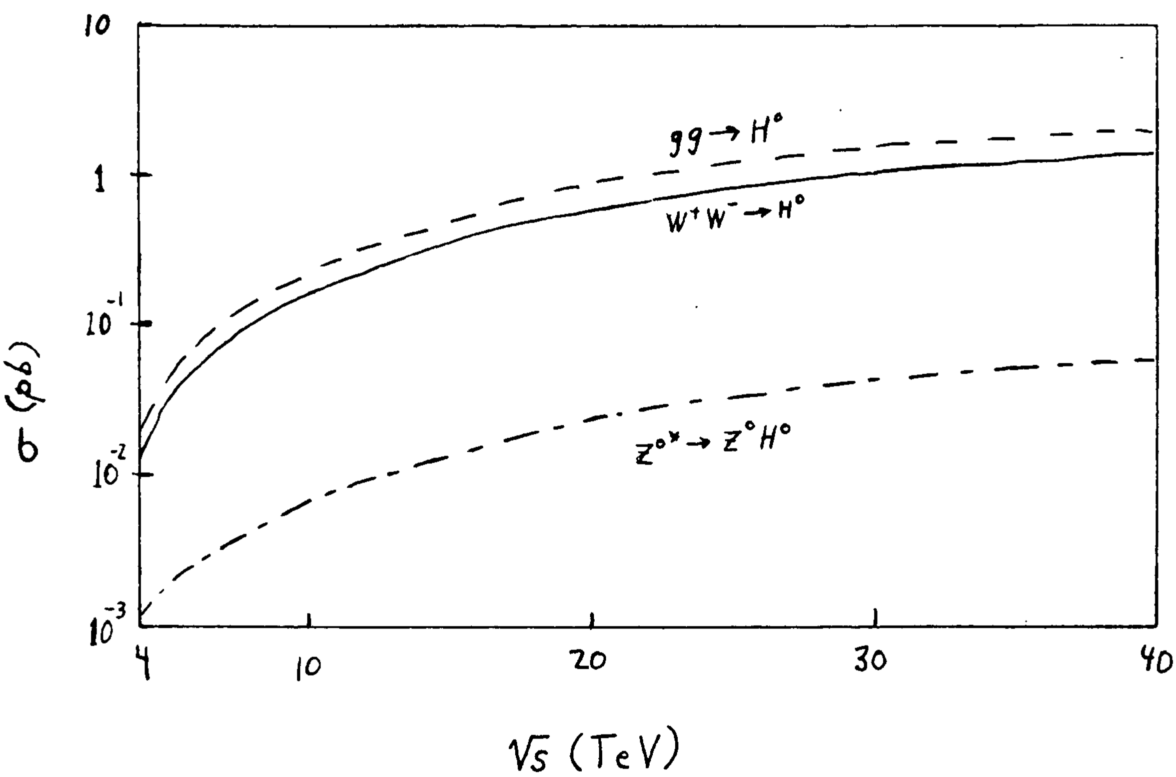

Figure 18 - Total cross section for processes 2), 5) and 6) for m_H = 410 GeV/c^2.

VII. CALCULATION OF ASSOCIATED PRODUCTION OF HIGGS BOSON AND HEAVY FLAVOR IN PROTON-ANTIPROTON COLLIDERS

In the last chapter, we surveyed several mechanisms through which the Higgs boson could be produced in present or planned accelerators. It was suggested that H^0 bremsstrahlung from Z^0 bosons in e^+e^- annihilation at the Z^0 resonance will provide the cleanest signal, if the H^0 mass, m_H, is less than the Z^0 mass. In fact the luminosities of currently planned machines such as LEP and SLC will restrict the detectability to $m_H < 50$ GeV/c^2. It becomes important to know what are the possibilities of producing and detecting the H^0 in proton-antiproton colliders, such as the CERN SPS collider, the FNAL Tevatron or even the SSC (see appendix G for properties of these colliders). These machines will be capable of taking the search for the H^0 up to the mass range of $m_H \sim 1$ TeV/c^2, far beyond the range reached by e^+e^- colliders available in the foreseeable future. This is why estimates of the production cross-sections of the H· in hadron-hadron colliders are now very important for the planning and the designing of colliders experiments. All of the following will thus be concerned with proton-antiproton or proton-proton collision only.

We saw in the last chapter that the H^0 production mechanism with the highest cross-section was the gluon-gluon fusion. If $m_H < 2\ m_W$ one expects to observe two back-to-back jets which are isotropic with respect to the beam direction, each containing at least one heavy flavored particle. Unfortunately, the process illustrated in Fig. 19 can also lead to two heavy quarks and it is estimated to be an overwhelming background.

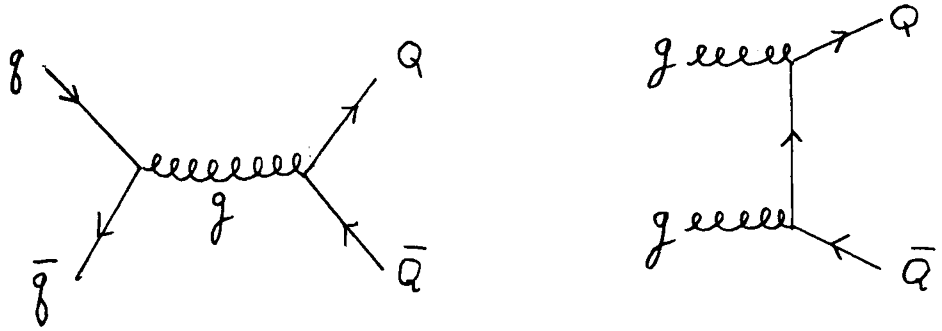

Figure 19 - Feynman diagrams for the background to the process hadron+hadron —> H° + anything.

One can attempt to suppress the background by considering H° production in conjunction with heavy quarks. If one has enough incoming energy a possibility is

$$pp \rightarrow F + \bar{F} + H^{\circ} + \text{hadrons} \qquad \text{(VII.1)}$$

where $F(\bar{F})$ denotes a hadron which contains a heavy quark such as the b- or t-quark. One expects that the remaining hadrons (VII.1) do not contain heavy quarks, as indicated in SPS collider data. Sequential weak decays will then lead to up to twenty c-quarks or 4 b-quarks and 4 c-quarks, or 4 c-quarks plus 8 charged leptons in the final state for the case of t-quarks. Table I gives the number of c-quarks and charged leptons obtainable after $F(\bar{F})$ and H° decays. Other intermediate combinations of c-quarks and charged leptons are all possible. One ends the chain of sequential decays at the c-quarks in anticipation that the tagging of charm or beauty hadrons may become a possibility with rapidly developing vertex detectors (Stone, 1983)

Within the framework of QCD parton model (see appendix A for QCD rules) the production of H^o that will result in accompanying heavy quark final states can proceed via at least three mechanisms:

1) gluon-heavy quark scattering (Barger et al., 1982)

$$g_a + f_i(\bar{f}_i) \rightarrow f_j(\bar{f}_j) + H^o \qquad \text{(VII.2)}$$

2) Higgs bremsstrahlung from heavy quarks in light quarks (q_i) annihilation

$$q_i + \bar{q}_K \rightarrow f_j + \bar{f}_l + H^o \qquad \text{(VII.3)}$$

3) Higgs bremsstrahlung from heavy quarks in gluon-gluon fusion

$$g_a + g_b \rightarrow f_j + \bar{f}_i + H^o \qquad \text{(VII.4)}$$

where the subscripts denote colour indices of the gluons and quarks and $f(\bar{f})$ is a heavy quark (antiquark) such as the b- or t-quark. The Feynman diagrams depicting processes 2) and 3) are given in Figs. 20 and 21. Mechanism 1) has been introduced in the last chapter.

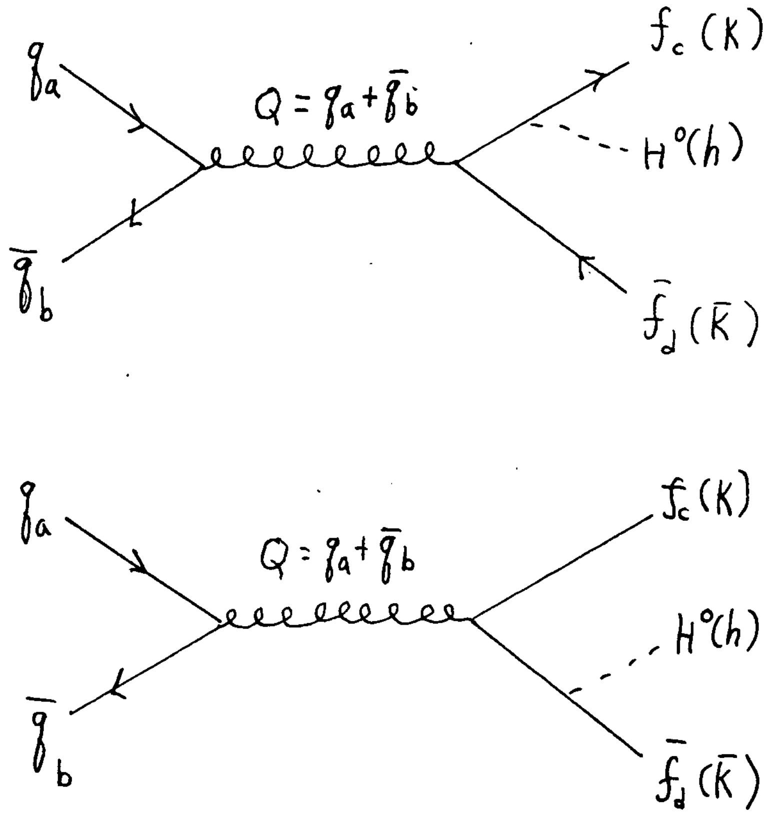

Figure 20 - Feynman diagrams for $q\bar{q} \rightarrow f + \bar{f} + H^\circ$.

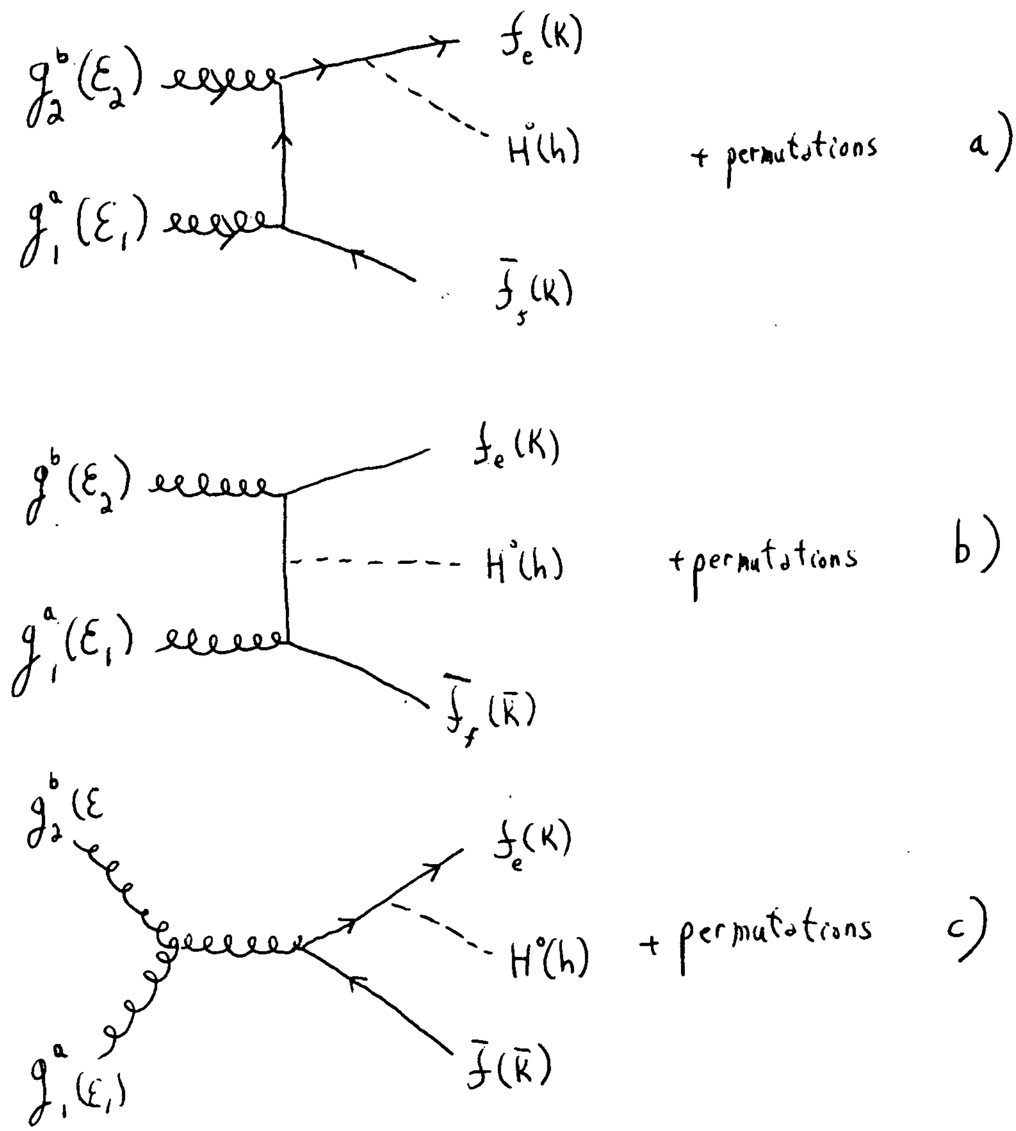

Figure 21 - Feynman diagrams for gg --> f + $\bar{f}$ +H .

It makes essential use of the heavy quark (antiquark) content in the sea component of the hadron wave function. The actual size of this component is not very well known but can be estimated from hadronic charm production data (Field and Feynman, 1977). In general the probability of finding a heavy quark in the proton (antiproton) is expected to be very small: less than a per cent or so. However, it is a two-body final state and hence less suppressed by phase space. Reaction (VII.2) gives a rate which is of order $\alpha_s y^2$ where α_s is the colour fine-structure constant and y denotes the Yukawa coupling of the Higgs boson to the quarks and is given by $y = 2^{3/4} m_f G_F^{1/2}$.

On the other hand reactions (VII.3) and (VII.4) are both of the order $\alpha_s^2 y^2$; hence are down by a factor α_s compared to the previous mechanism. They also have three-body final state phase space suppression. These are compensated by the large probability of finding light quarks or antiquarks and gluons in both the p and $\bar{p}$. Hence, one would expect the mechanisms (VII.2) to (VII.4) to give comparable rates of H^0 production in $p\bar{p}$ annihilations.

In this thesis the results of a complete calculation of the processes (VII.1) using both reactions (VII.3) and (VII.4) as the fundamental subprocesses are given. The parton picture has been assumed to convolute the initial quark and gluon distributions over the fundamental subprocesses. The calculations for both mechanisms are presented in this chapter. In chapter VIII the results of the numerical calculations are given. Distributions in energy of the Higgs boson and the behaviour of the production rate as a function of the center-of-mass energy, and other essential kinematical variables are also given. Chapter IX contains discussions of the results and their experimental implications are given.

Calculation of the subreaction amplitude: As the subreactions (VII.3) and (VII.5) result in the same final state, they are indistinguishable at the macroscopic level. Since they have different initial states at the parton level they add incoherently. Together they give us a first-order QCD estimate for the semi-inclusive process (VII.1). In the final state, the meson F and its charge conjugate contain at least a heavy quark of flavour c,b or t. We will concentrate on the six-quarks model. The case where the $f\bar{f}$ forms a resonance such as toponium (T) results in

$$p + \bar{p} \rightarrow H^{0} + T + \text{hadrons} \qquad \text{(VII.5)}$$

which has been estimated to be small[1] . This is due to the smallness of the wave functions at the origin for this process.

We will now discuss the two mechanisms (VII.3) and (VII.4) separately.

[1] Ng and Zakarauskas, unpublished

Quark-antiquark annihilation mechanism: The u- and d-type quarks are mainly responsible for this process, since they are the dominant quark components of the proton wave function. Also due to the small coupling that the H° has with u- and d-quarks, H° bremsstrahlungs off the initial quarks can be neglected. To lowest order in α_s one needs only to calculate the diagrams depicted in Fig. 20.

Within the framework of perturbative QCD model, the cross section for the reaction (VII.1) is given by first calculating the elementary subprocesses (VII.3), then convoluting with the quark and antiquark distributions in the proton and antiproton.

The amplitude for (VII.4) is (see fig. 20 for kinematics)

$$M_{q\bar{q}\to f\bar{f}H^{\circ}} = (ig_s)^2\, T^a_{ik}\, T^a_{jl}\, \bar{v}^k(\bar{q})\, \gamma^\mu u^i(q)\, \frac{(-ig_{\mu\nu})}{(q+\bar{q})^2} \times \bar{u}^j(k)\, \frac{(\not{h}+\not{K}+m_f)}{[(h+k)^2-m_f]}\, \gamma^\nu \bar{v}\left(-i\, m_f\, G_F^{1/2}\, 2^{1/4}\right) \tag{VII.5}$$

$$+\,\text{permutations } (\bar{k}\leftrightarrow k)$$

where

$i,j,k,l = 1\text{-}3$ (quark color indices)

$a,b = 1\text{-}8$ (gluon color indices)

$\mu,\nu = 1\text{-}4$ (Lorentz indices)

$T^a = \frac{1}{2}\lambda^a$ are the SU(3) matrices, introduced by Gell-Mann (see appendix I).

The cross section $\hat{\sigma}_{q\bar{q}\to f\bar{f}H^{\circ}}$ for this elementary process is given by

$$\hat{\sigma}_{q\bar{q}\to f\bar{f}H^0}(\hat{s}) = \frac{1}{2\hat{s}} \int \frac{d^3k}{2k^0} \int \frac{d^3\bar{k}}{2\bar{k}^0} \int \frac{d^3h}{2h^0} \delta^4(Q-k-\bar{k}-h) |M|^2_{q\bar{q}\to f\bar{f}H^0} \tag{VII.1}$$

$$= \frac{\alpha_s^2 G_F m_f^2}{36\sqrt{2}\pi^3 \hat{s}^3} \int \frac{d^3k}{2k^0} \int \frac{d^3\bar{k}}{2\bar{k}^0} \int \frac{d^3h}{2h^0} \delta^4(Q-k-\bar{k}-h) H^{\mu\nu} g_{\mu\nu}$$

where $\hat{s} \equiv Q^2 \equiv (q_a+\bar{q}_b)^2$.

For the value of α_s we used the running coupling constant (II.14) with n_f the number of quarks flavors equal to 6. The value Λ = 0.2 GeV has been chosen for this QCD parameter. The matrix element element squared is given by

$$\begin{aligned} H^{\mu\nu} g_{\mu\nu} = \frac{32}{(2h\cdot\bar{k}+m_H^2)(2h\cdot k+m_H^2)} \Bigg\{ & Q^2(Q\cdot h)^2 \left[1 + \frac{(4m_f^2-m_H^2)Q^2}{(2h\cdot k+m_H^2)(2h\cdot\bar{k}+m_H^2)}\right] \\ & + \left[Q^2 + m_H^2 - 4m_f^2 + \frac{2Q\cdot h(4m_f^2-m_H^2)}{(2h\cdot\bar{k}+m_H^2)}\right]\left(\frac{Q^2}{2}m_f^2 - 2k\cdot q\, k\cdot\bar{q}\right) \\ & + \left[(Q^2 + m_H^2 - 4m_f^2) + \frac{2Q\cdot h(4m_f^2-m_H^2)}{(2h\cdot k+m_H^2)}\right]\left(\frac{Q^2}{2}m_f^2 - 2\bar{k}\cdot q\, \bar{k}\cdot\bar{q}\right) \\ & - (Q^2 + m_H^2 - 4m_f^2)\left[2k\cdot q\, \bar{k}\cdot\bar{q} + 2(k\cdot\bar{q})(\bar{k}\cdot q) - Q^2\, k\cdot\bar{k}\right] \Bigg\} \end{aligned} \tag{VII.8}$$

The spin and color degrees of freedom have been summed over. Also the Feynman gauge is used for the gluon propagator. The contribution to the cross section of (VII.1) stemming from VII.3 is then:

$$\sigma(s,m_H,m_f) = \int dx_1\, dx_2\, \hat{\sigma}(\hat{s},m_H,m_f)\left[u(x_1)\bar{u}(x_2) + d(x_1)\bar{d}(x_2)\right] \tag{VII.9}$$

with $\hat{s} = x_1 x_2 s$ and x_1 and x_2 being, respectively, the fractions of momenta carried by the quark and antiquark in their parent hadrons. For numerical calculations we have used two different parametrizations of the quark distribution functions, to get an estimate of the uncertainty introduced by the quark distributions. They are written explicitly in equations (F.1a) to (F.2b).

The differences in the results in using one or the other parametrization were no more than a few per cent. The contributions from the sea quarks in the proton or antiproton have also been omitted, since their importance is of the few percent level. The integrations are performed using a Monte-Carlo method described in Appendix C.

<u>Gluon-gluon fusion mechanism:</u> This mechanism takes advantage of the large gluon component in both proton and antiproton wave functions as well as the large coupling of Higgs boson to heavy quarks in order to compensate for phase space and α_s suppression discussed before. As a result it also carries with it the not so well-measured gluon distribution functions, thus leading to uncertainties in the estimates of the production cross sections. We will further discuss these points later and also exhibit quantitatively these uncertainties.

The calculation proceeds by evaluating the Feynman diagrams shown in fig. 21. The amplitudes are given by

$$M^{a}_{gg\to f\bar{f}H^0} = -A\, T^{a}_{ik}\, T^{b}_{kj}\, \bar{u}^{j}(K)\, \frac{(\not{h}+\not{K}+m_f)}{(2h\cdot K+m_H^2)}\, \not{\varepsilon}_2\, \frac{(-\not{\bar{K}}+\not{g}_1+m_f)}{(-2g_1\cdot\bar{K})}\, \not{\varepsilon}_1\, v^{i}(\bar{K})$$

$$+ \text{permutations} \quad \begin{array}{l} (g_1\leftrightarrow g_2,\ \varepsilon_1\leftrightarrow\varepsilon_2) \\ (g_1\leftrightarrow g_2,\ \varepsilon_1\leftrightarrow\varepsilon_2,\ K\leftrightarrow\bar{K}) \\ (K\leftrightarrow\bar{K}) \end{array} \qquad \text{(VII.10)}$$

for the diagram of fig. 21a,

$$M^{b}_{gg\to f\bar{f}H^0} = -A T^{a}_{ik} T^{b}_{kj} \bar{u}_j(K) \not\epsilon_2 \frac{(\not K - \not g_2 + m_f)}{[-K\cdot g_2]} \frac{(-\not{\bar K} + \not g_1 + m_f)}{[-\bar K \cdot g_1]} \not\epsilon_1 v_i(\bar k) \qquad \text{(VII.11)}$$

$$+ \text{permutations}(g_1 \leftrightarrow g_2,\ \epsilon_1 \leftrightarrow \epsilon_2),\ (g_1 \leftrightarrow g_2, \epsilon_1 \leftrightarrow \epsilon_2, K \leftrightarrow \bar K)$$
$$(K \leftrightarrow \bar K)$$

for the diagram of fig. 21b, and

$$M^{c}_{gg\to f\bar{f}H^0} = -iA f^{abc} T^{c}_{ij} \bar{u}_j(K) \frac{\epsilon_1^{\mu} \epsilon_2^{\nu} \gamma^{\lambda}}{\hat{s}}$$

$$\times \left[2 g_1^{\nu} g^{\lambda\mu} + (g_2 - g_1)^{\lambda} g^{\mu\nu} - 2 g_2^{\mu} g^{\nu\lambda}\right] \frac{(-\not{\bar K} - \not K + m_f)}{(2\bar k \cdot h + m_H^2)} v_i(k) \qquad \text{(VII.12)}$$

$$+ \text{permutations}\ (K \leftrightarrow \bar K)$$

for the diagram of fig. 21c, with $A = g_s^2 m_f G_F^{1/2}\ 2^{1/4}$.

Here, ϵ_1^{μ} and ϵ_2^{μ} are the polarization 4-vectors of the incoming gluons. The SU(3) structure constants are given by the f_{abc} . To evaluate the square of the amplitude given by

$$\left| M^{a} + M^{b} + M^{c} \right|^2_{gg\to f\bar{f}H^0} \qquad \text{(VII.13)}$$

the traces are obtained by using the symbol manipulation program REDUCE. The REDUCE program written for this is given in appendix D. The gauge invariance of the result has been checked by making the substitution $\varepsilon_1 \to g_1$ or $\varepsilon_2 \to g_2$. The initial gluon polarization and colour states are then averaged, and the final state spins and colour factors are summed over.

The resulting output for the amplitude squared is given at length in appendix E.

The total cross section $\hat{\sigma}(\hat{s}, m_H, m_f)$ for the subprocess is obtained by integrating over the phase space for the H°, f and $\bar{f}$. Using the parton model assumptions one convolutes over the gluon distributions via

$$\sigma(s) = \int dx_1\, dx_2\, \hat{\sigma}(\hat{s}, m_H, m_f)\, G^{p}(x_1)\, G^{\bar{p}}(x_2) \qquad \text{(VII.14)}$$

to obtain the total production rate. The lower limits of the x_1 and x_2 integrals are given by the kinematical requirements of

$$x_1 x_2 s \geq (m_H + 2m_f)^2 . \qquad \text{(VII.15)}$$

The condition of eq. (VII.15) requires that the events generated in the Monte-Carlo calculations satisfy the kinematics for heavy particle production. The production cross section depends on the gluon distributions. From general CPT arguments it is expected that $G^{p}(x)$ has the same form as $G^{\bar{p}}(x)$; thus any uncertainty in the gluon distributions will be doubled in the cross section $\sigma(s)$. We will study this below.

To this end, two specific parametrizations representing extreme cases (F.3 and F.4) are chosen. The differences in the cross sections coming from the use of one or the other of the gluon momentum distributions gives an estimate of the approximate size of the uncertainty in the results due to an incomplete knowledge of this distribution.

In addition to restricting the generated events to be physical ones, one has to take into account that QCD perturbative calculations have strict validity only in the high energy deep inelastic region. One should therefore avoid the region of phase space where the gluons or quarks become soft and thereby invalidate the use of the parton model.

Hence the integrations have been restricted to take place in the region of high momentum transfer. The event generating routine requests that all scalar products between the 4-momenta of the incoming and outgoing particles be larger than 3 GeV^2. More stringent cuts may be imposed to reproduce experimental configurations.

Several differential cross-sections have also been generated by the Monte-Carlo integration routine. These may be extremely useful in selecting experimental cutoffs and hence reducing the background. The differential cross sections calculated are those relative to the Higgs boson and heavy quark's kinetic energies and transverse momenta.

VIII. RESULTS

In this chapter are presented the results of the Monte-Carlo calculation of the two processes described last chapter. The free parameters of the theory are m_K, the mass of the top quark, and m_H. The variables on which the total cross-section depends are the center-of-mass energy of the proton-antiproton pair, and the lower cutoff on relative transverse momenta of the produced particles.

Fig. 22 to 24 display the production cross section versus the c.m. energy of the $p\bar{p}$ for m_H = 10 GeV/c^2 for two values of m_K, corresponding to the b-quark with m_K = 4.5 GeV/c^2 and a 35 GeV/c^2 t-quark. The cross sections from quark-antiquark annihilation and gluon-gluon fusion are shown separately in fig. 22 and 23, and they are added in fig. 24. The quark-antiquark channel reaches a peak in the picobarn range around $\sqrt{s}$ = 2 TeV. However, it dominates over the gluon fusion mechanism at lower energies in the range $\sqrt{s}$ < 60 GeV. At these relatively low energies one is required to use partons with large x in order to produce the final state particles. The gluon momentum distribution is steeply peaked toward small x as opposed to the quark distributions. This can be uniderstood by noticing that the gluons are rediated from the quarks, and therefore must show a radiative spectrum. As a result there are fewer gluons at large x. On the other hand, the gluon fusion is totally dominating at high energies, where small x still makes the incoming parton very energetic.

There are two types of curves in all of the figs. 22 to

30. The dotted lines represent the results of the calculations done using the scale-violating gluon momentum distribution given by (F.3). The continuous lines are results using the scaling distribution (F.4). The difference between the two is indicative of the effects of scaling versus scale violating gluon distributions. The intersection points are reflections of the particular values of x where the two parametrizations of G(x) cross each other.

Explicitly, for the case of $p\bar{p}$ collider at the Tevatron, the production of 10 GeV/c^2 Higgs boson in conjunction with a t-quark pair of mass 35 Gev/c^2 is well over 100 pb. Interestingly, the production in conjunction with two b-quarks has the same cross section, in spite of the fact that the Yukawa coupling is proportional to the quark mass. This suppression is here overcome by kinematics and quark dynamics. The kinematic reason is that the subenergies of the two gluons must be such that $\sqrt{s}$ > 80 GeV for the t-quark case and and this is hindered by the rapidly falling gluon distribution functions. Then the dynamical enhancement occurs for the cross section via the propagator effect which favours smaller quark masses. This results in the crossing over of the production cross section at $\sqrt{s}$ =2 TeV.

A detailed examination of σ_T as a function of m_f is given in Fig. 25 for m_H = 10 GeV/c^2. The upper curves and points correspond to the expected production rate at FNAL, the lower ones to SPS collider. Here, there is a rise in the cross section which reaches a peak at m_f = m_b for $\sqrt{s}$ = 540 GeV and

m_f =20 GeV/c^2 for $\sqrt{s}$ = 2 TeV.

In table II are given the total cross sections relevant for lower energies ($\sqrt{s}$ = 45 GeV) for different values of m_H and m_K. This will be of relevance for a 1 TeV $\bar{p}$ scattering on fixed target where one can probe much smaller cross sections than possible with colliders, due to the higher luminosity.

The production cross section is also a very sensitive function of m_H and this is depicted in fig.26. Here we have chosen the reference value of m_K= 4.5 GeV/c^2. The behaviour seen as m_H varies is mainly due to the propagator effect of the heavy quark. From equations (VII.7) and (VII.10) to (VII.12) we see that four of six denominators in the amplitude have their minimum values near m_H when either $h \cdot k$ or $h \cdot \bar{k}$ is small. This corresponds to collinear H^o bremsstrahlung from the heavy quark (or antiquark).

We also calculated the cross-section for c.m. energies of 10, 20 and 40 TeV, for a wide range of m_H, up to m_H= 1 TeV/c^2. These energies are relevant to the planned SSC (Super Supraconductor Collider), a pp or $p\bar{p}$ collider which would be built in U.S. before 1995. The results are reproduced in table III, for two values of the cutoff on 4-momenta scalar products, 3 GeV^2 and 100 GeV^2. The former value is the QCD cutoff, introduced last chapter, guaranteeing applicability of perturbative QCD. The 100 GeV^2 value may be more relevant to experimental cutoffs, especially at the SSC.

Coming back to present day energies, in figs. 27 to 30 are plotted several differential cross sections, four different values of m_H, m_K and $\sqrt{s}$. The distributions in energies of the H^o and the heavy quark are compared in figs. 27 and 28. In general, the H^o has an average energy higher than the heavy quarks, the mean value being 40-50 GeV for the H^o, and 10-15 GeV for the fermions, in the case of a 10 Gev/c^2 Higgs produced with a pair of b-quarks at the Fermilab collider.

The transverse momentum of H^o is shown in fig. 29. It is seen that these distributions are peaked at $h_\perp$ = 15 GeV at Fermilab and $h_\perp$ = 5 GeV at CERN, and the peak increases for heavier H^o.

Similarly, the transverse momenta of the heavy quarks produced are given in fig. 30. They have the same features as $h_\perp$ with the peak located at $k_\perp$ = 5 GeV for CERN and $k_\perp$ = 15 GeV for Fermilab, which is still a high value.

The numerical calculations needed to produce these curves have been performed on a VAX-780, and necessitated approximatively 60 hours of CPU time.

Table 1 - Number of charm quarks, n_c, and number of charged leptons, n_ℓ, in the final state particles of reaction (VII.1) after weak decays of the hadronsF, $\bar{F}$ and the Higgs boson.

The first column denotes the heavy quark flavour contained in F. The second, third and fourth column entries give the values of n_c if the heavy quark decays non-leptonically and the values of (n_c, n_ℓ) if they decay semileptonically. The headings of columns give mass ranges of H^o.

F	$2m_c < m_H < 2m_b$	$2m_b < m_H < 2m_t$	$m_H > 2m_t$
c	4	6	12
		(4,2)	(4,4)
b	6	8	14
	(4,2)	(4,4)	(4,6)
t	12	14	20
	(4,4)	(4,6)	(4,8)

Table 2 - Fixed target cross section for reaction (VII.1).

Cross section, in picobarns, for the production of a Higgs boson of mass m_H and a pair of charm quarks (m_c = 1.5 GeV/c^2), or bottom quarks (m_b = 5 GeV/c^2), in proton-antiproton collision, with centre-of-mass energy $\sqrt{s}$ = 45 GeV, corresponding to a fixed target experiment using a 1 TeV antiproton beam.

m_f \ m_H (GeV/c^2)	10	5	2	1	0.5
1.5	1.3×10^{-4}	9.0×10^{-3}	0.4	3.5	20.0
5	2.0×10^{-3}	1.0×10^{-2}	0.18	0.65	1.5

Table 3 - Very high energy cross sections

Total cross section, in Picobarns, for the process pp or $p\bar{p} \longrightarrow H^{o} + F + \bar{F}$, with $m_t = 35$ GeV/c^2, for different values of $\sqrt{s}$ and m_H, and two values of the scalar product cutoff. The gluon distribution used is the scale violating one (F.4).

cutoff (GeV)	s (TeV)	m (GeV/c)					
		10	50	100	250	500	1000
3	2	159	1.1	$7.2\text{X}10^{-2}$	$1.3\text{X}10^{-3}$	$2\text{X}10^{-5}$	
	10	10^{5}	$8\text{X}10^{3}$	800	13	0.18	$3\text{X}10^{-3}$
	20	$4\text{X}10^{5}$	$4\text{X}10^{4}$	10^{3}	1000	8.5	0.2
	40	$5\text{X}10^{6}$	$7\ 10^{5}$	$2\text{X}10^{5}$	$2\text{X}10^{4}$	410	5.4
100	2	18	0.6	$7\text{X}10^{-2}$	10^{-3}	$2\text{X}10^{-5}$	
	10	1000	380	100	8.4	0.17	$3\text{X}10^{-3}$
	20	3000	1000	600	140	4.5	0.18
	40	5500	3700	2500	1000	350	1.7

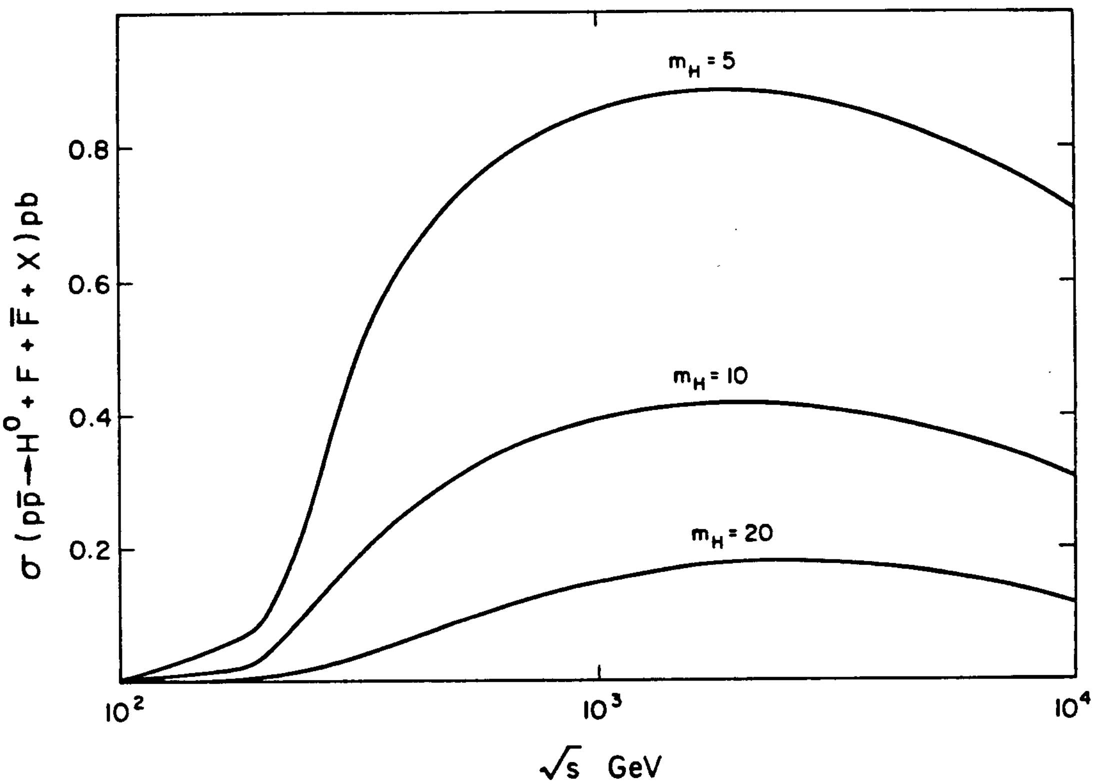

Figure 22 - Total cross section for the process (VII.3) as a function of $\sqrt{s}$, for $p\bar{p}$ collision.

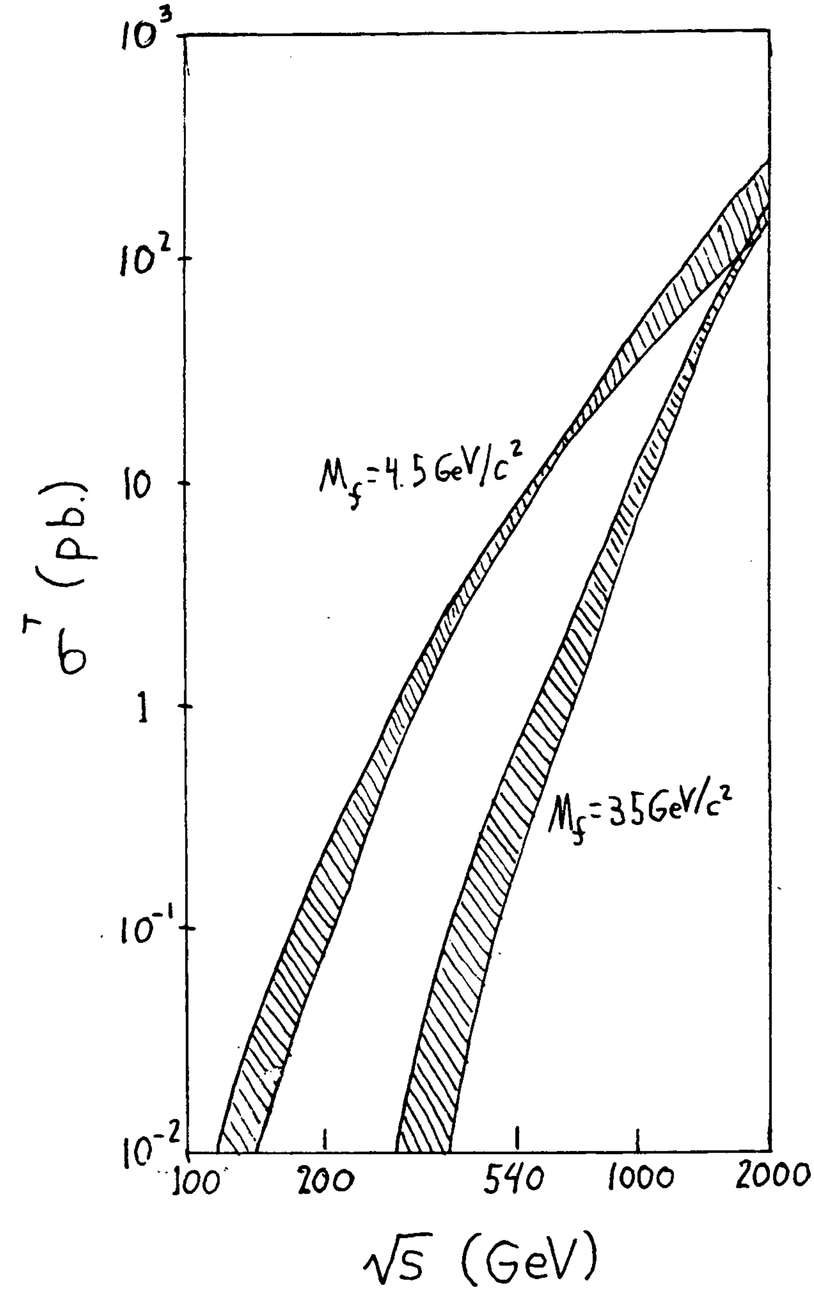

Figure 23 - Total cross section for the process (VII.4) as a function of $\sqrt{s}$, with m_H = 10 GeV/c^2.

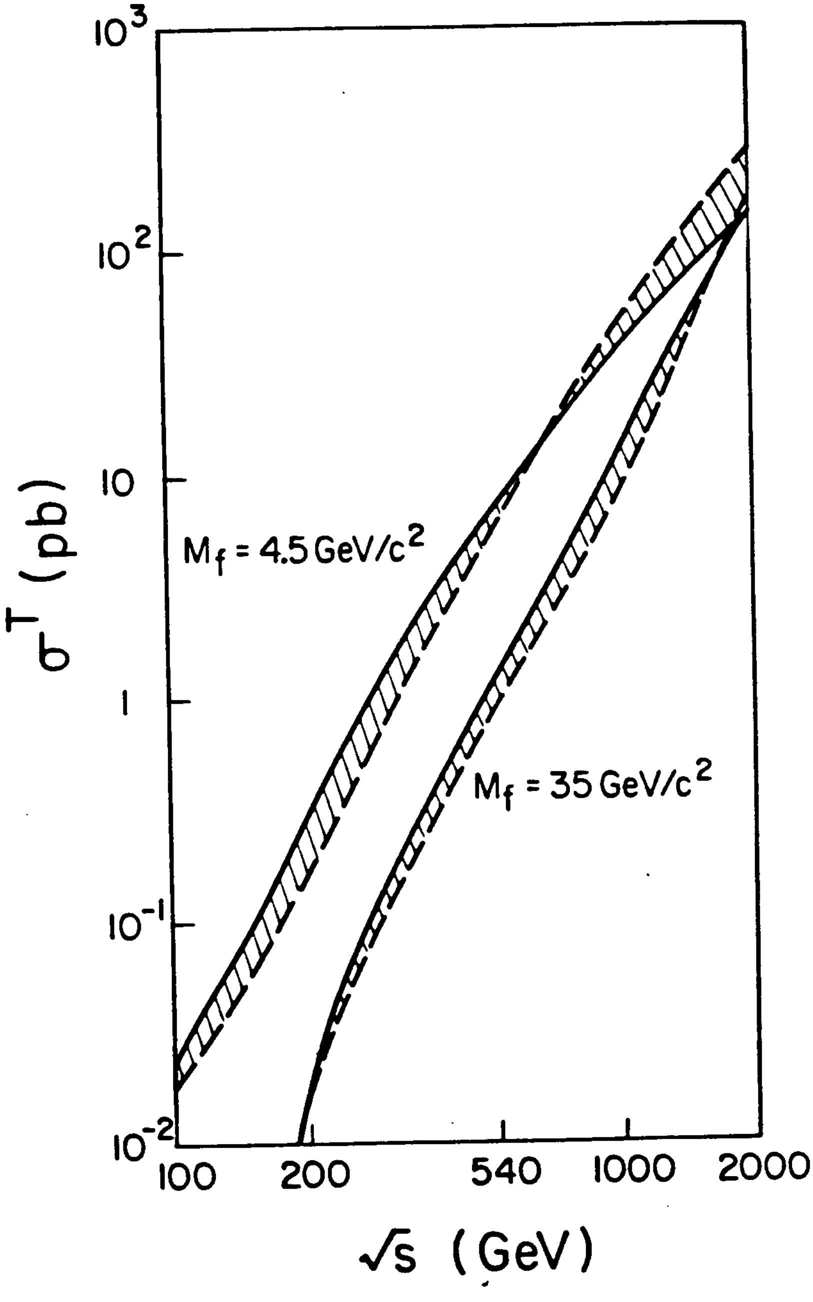

Figure 24 - Total cross section in $p\bar{p}$ from the sum of subreaction (VIII.3)and (VIII.4), as a function of $\sqrt{s}$, with m_H = 10 GeV/c^2.

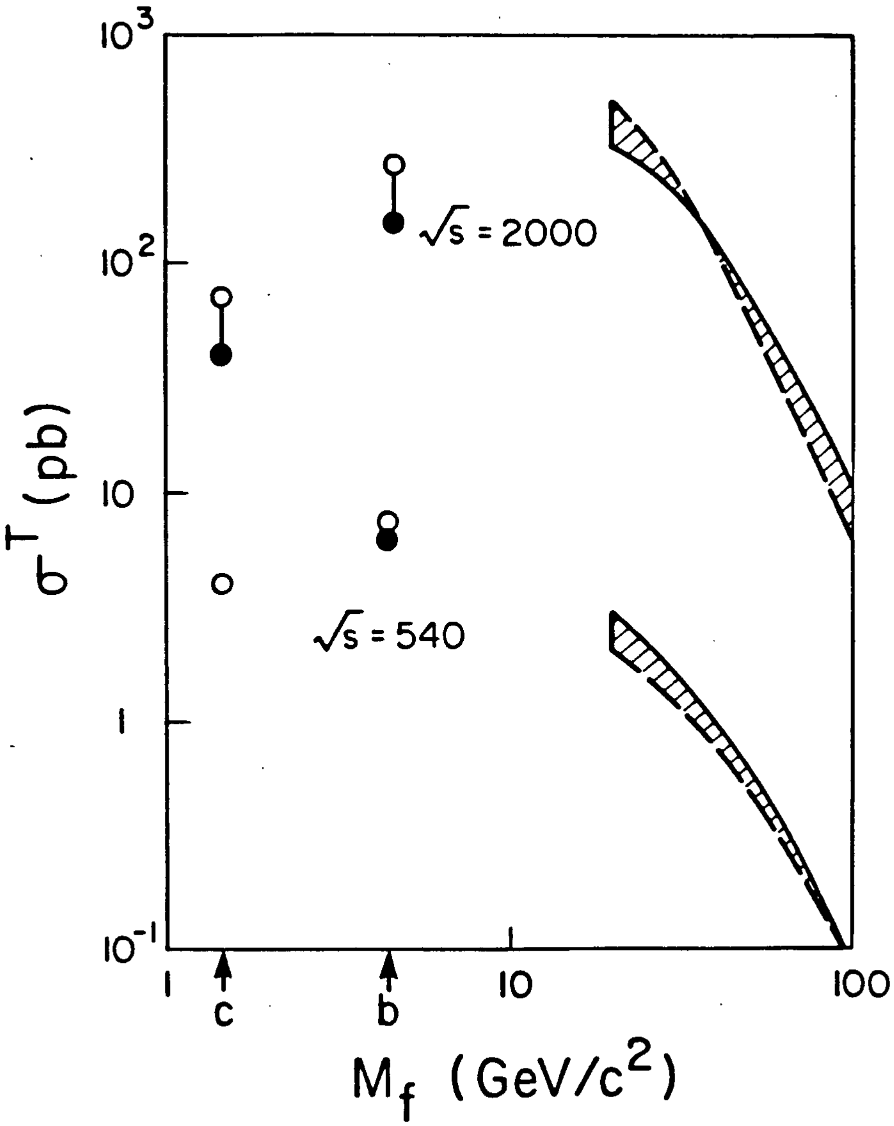

Figure 25 - Total cross section for the process (VII.1) as a function of the mass of the heavy quark produced with the Higgs boson, for m = 10 Gev/c . Discrete points refer to the value of the cross section at the masses of the c- and b-quarks. The continuum portion, starting at m_K = 20 GeV/c^2 corresponds to the t-quark contribution. The dotted line has been added to guide the eye.

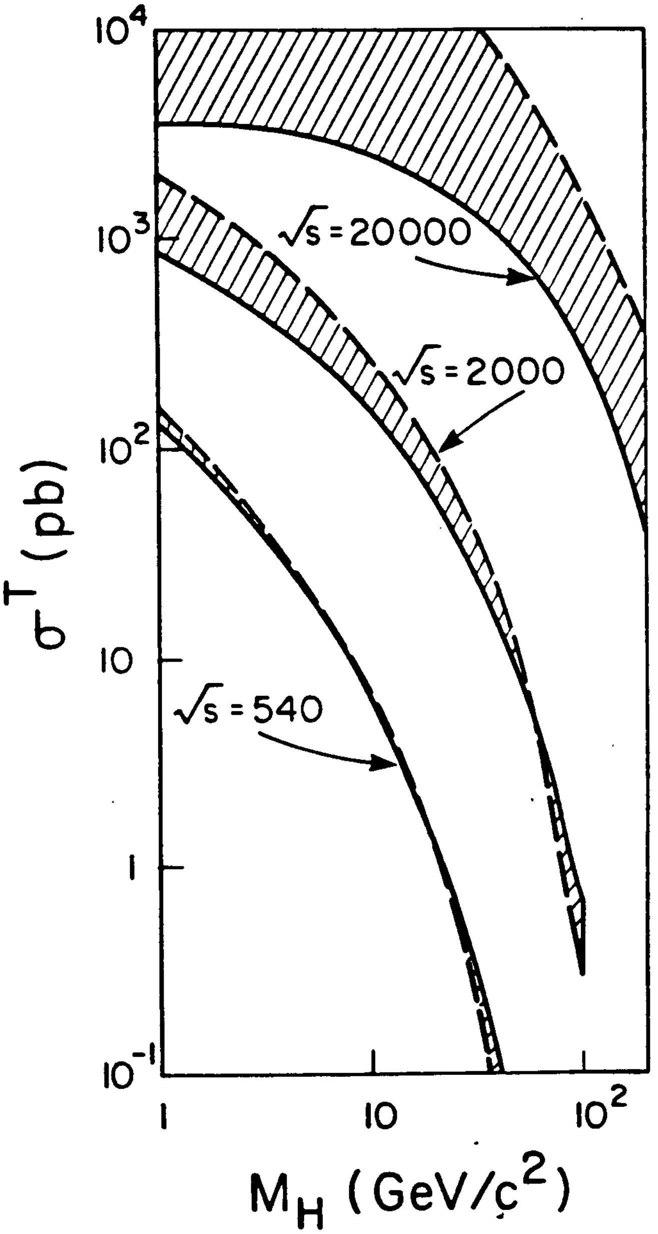

Figure 26 - Total cross section for the process (VII.1) as a function of the mass of the Higgs boson.

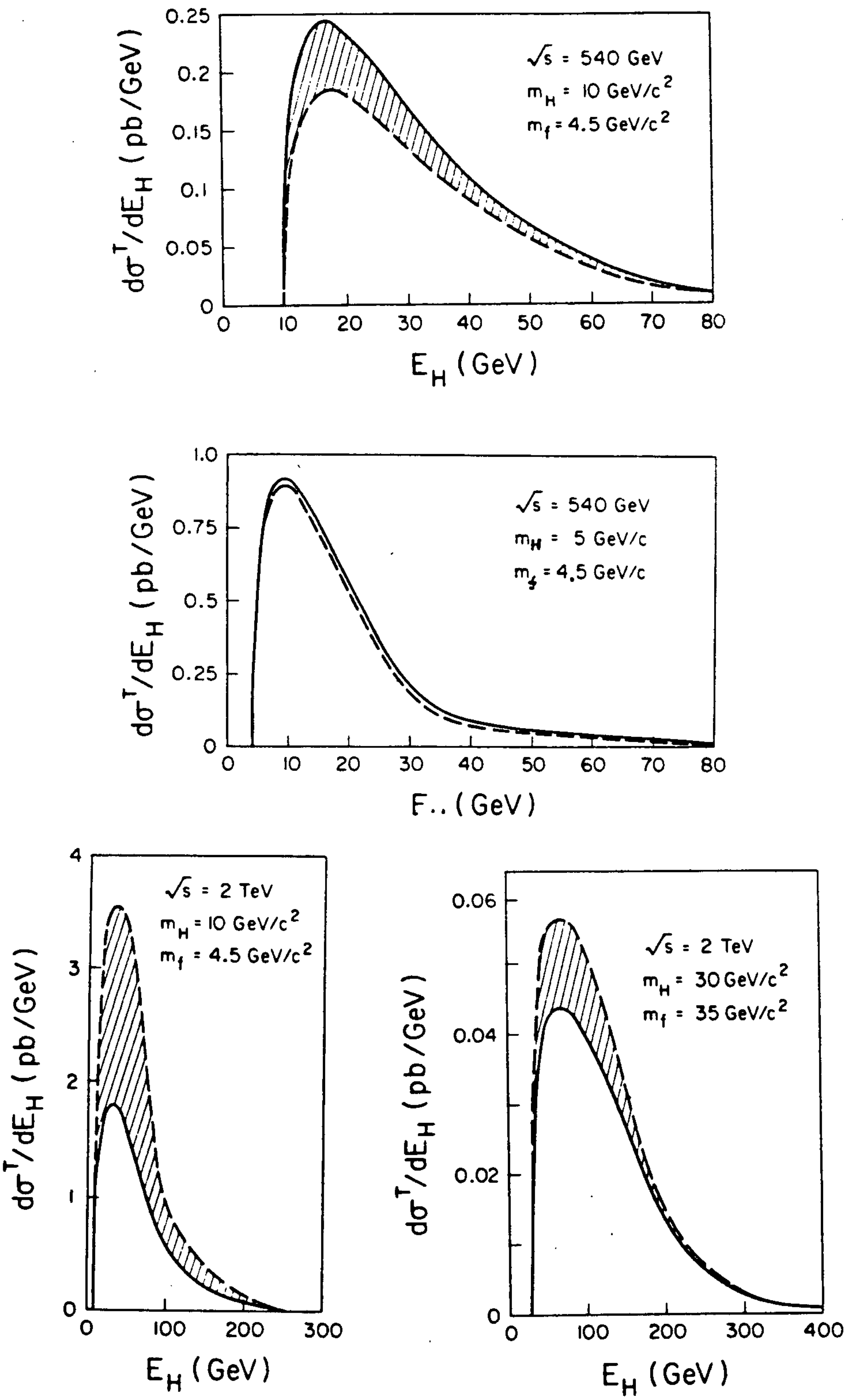

Figure 27 - Differential cross section $d\sigma/dE_{H^\circ}$ for four different sets of the parameters m_H, m_f, and $\sqrt{s}$.

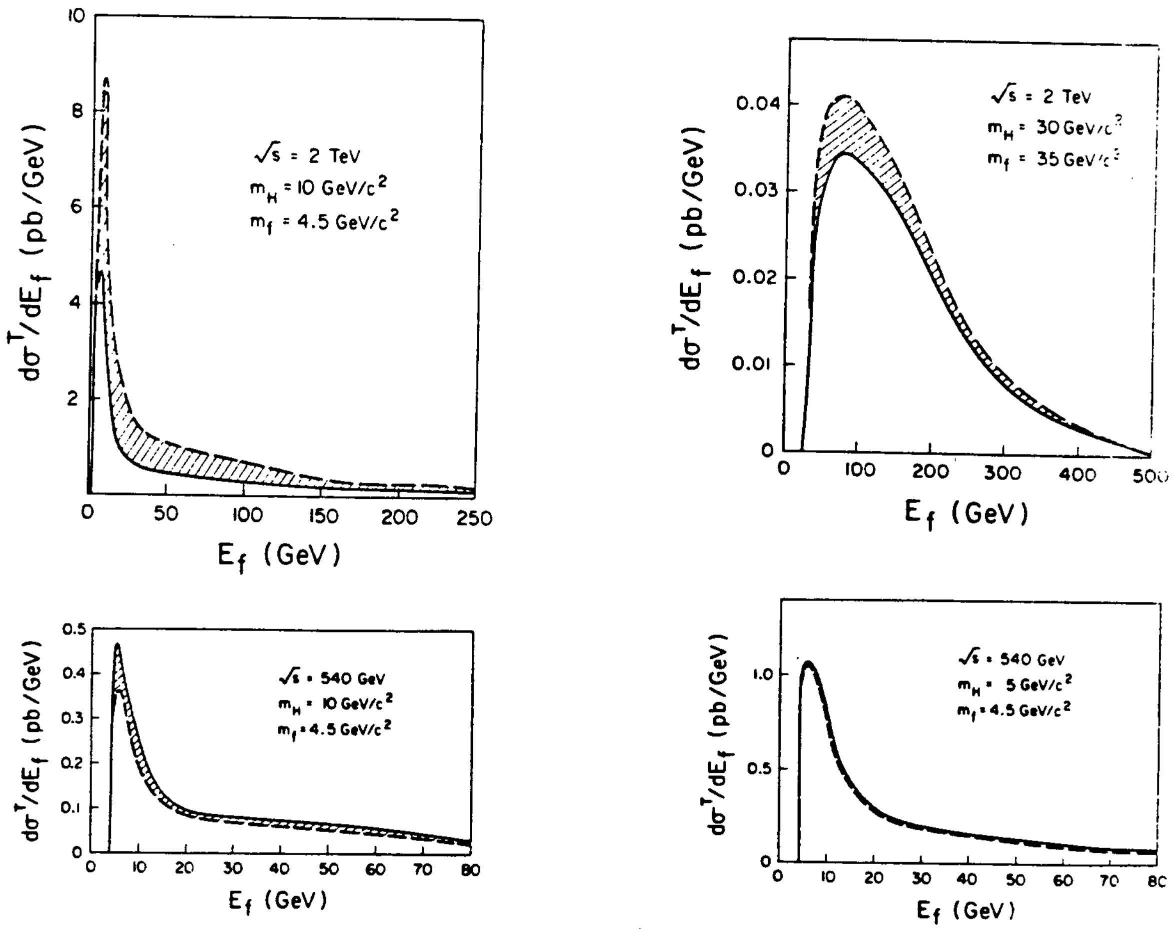

Figure 28 - Differential cross section $d\sigma/dE_K$ for four different set of the parameters m_H, m_K, and $\sqrt{s}$.

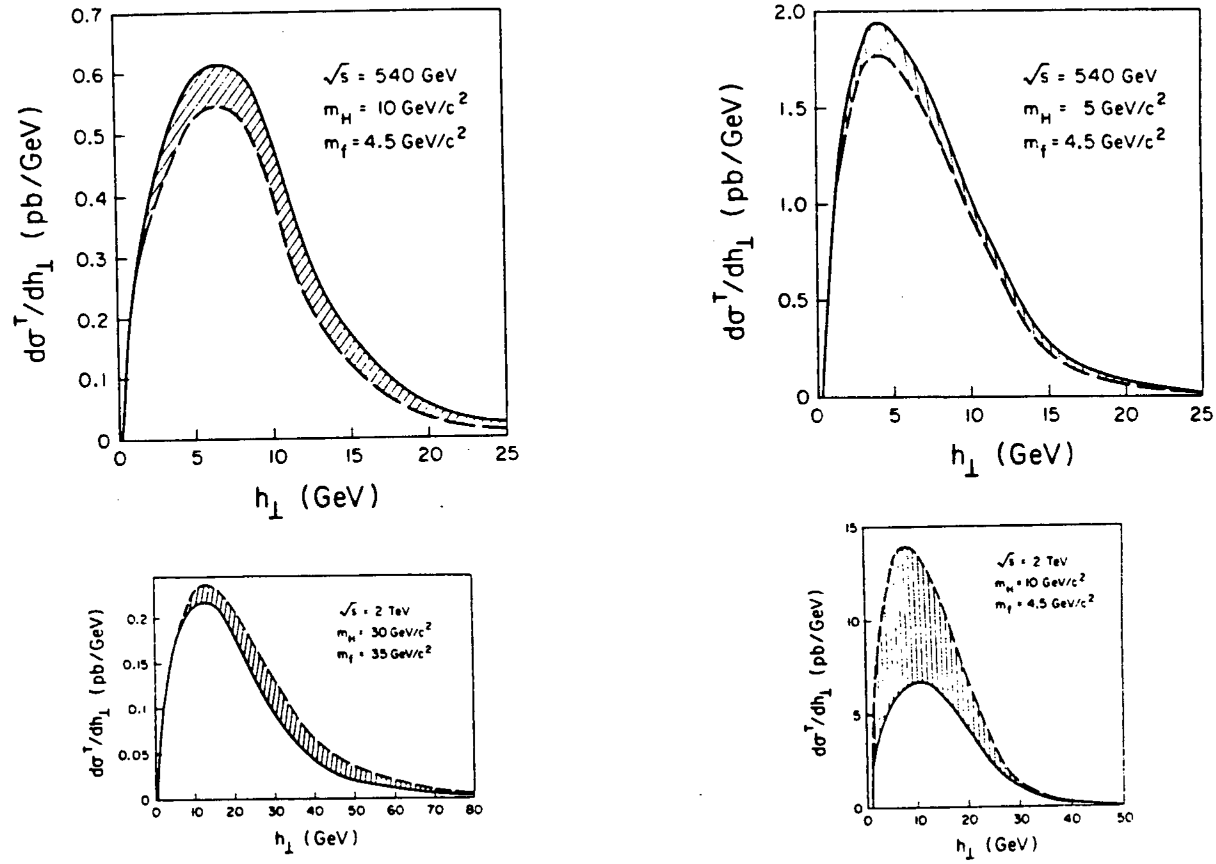

Figure 29 - Differential cross section $d\sigma/dh_\perp$ for four different sets of the parameters m_H, m_K and $\sqrt{s}$.

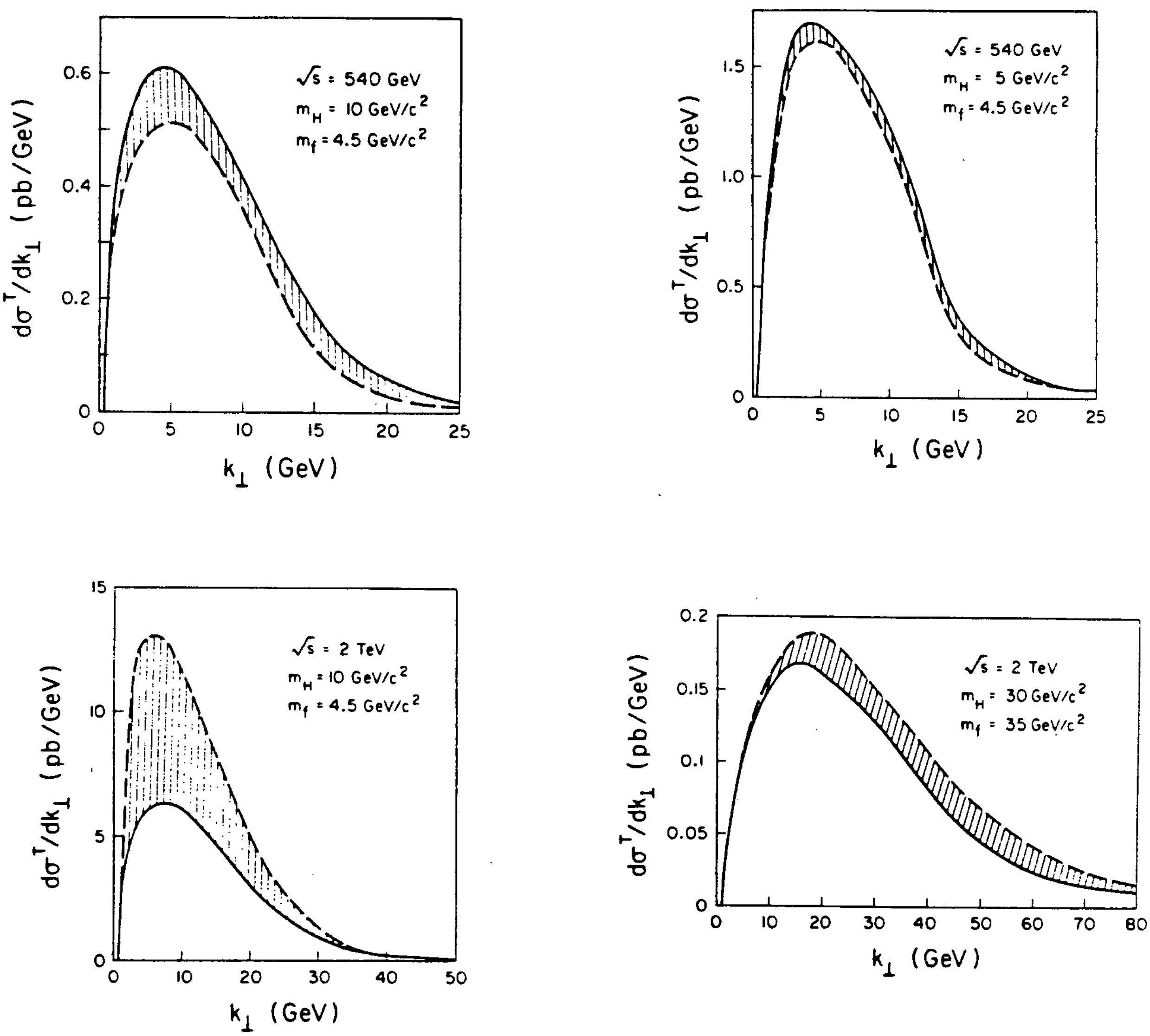

Figure 30 - Differential cross section $d\sigma$ /$dk_\perp$ for four different sets of the parameters m_H, m_K, and $\sqrt{s}$.

IX. DISCUSSION AND CONCLUSION

This thesis presents the QCD-parton model calculation of the production cross section of a Higgs boson plus a heavy quark pair in proton-antiproton or proton-proton collisions. We employed QCD and the Weinberg-Salam model of electroweak interactions, where the $SU(2) \otimes U(1)$ group is broken by a doublet of scalar fields. After the spontaneous symmetry breaking occurs, one is left with one real scalar particle, called the Higgs boson. Now that the $W^{\pm}$ and Z^{0} bosons have been found at CERN, the Higgs boson is the only particle predicted by the Weinberg-Salam model yet to be discovered. Thus, it is the whole concept of spontaneous symmetry breaking which would be confirmed in the event of a positive identification of a Higgs particle. This mechanism gives masses to particles in the popular gauge theories, hence the great importance of getting some experimental evidence supporting or invalidating it.

The total production rate for the reaction (VII.1) has been calculated for center-of-mass energies ranging from 45 GeV to 40 TeV, as well as for a wide range of the Higgs mass, from 1 GeV/c^2 to 1 TeV/c^2. The dependence of the cross section on m_H, $h_\perp$, $k_\perp$, E_h and E_K has been calculated, and should be useful to place experimental cuts or discriminate against the background.

At this point we compare the results of our calculations of reaction (VII.1) with that of the estimate using the bremsstrahlung technique of Ellis et al (1976). There the production cross section is given by

$$d\sigma_H = \frac{\sqrt{2}\,G_F}{4\pi^2}\, m_f^2\, d\sigma_f\, \frac{|\vec{h}|\, dh_0}{h_0^2} \tag{IX.1}$$

in the rest system of the heavy quark. The differential cross section $d\sigma_f$ is that of pair production of heavy quarks without the Higgs boson, i.e.,

$$p + \bar{p} \longrightarrow F + \bar{F} + X \tag{IX.2}$$

If we take this cross section to scale like m_f^{-2}, then we see that σ_H will be governed by the charm-quark pair production. Using $\sigma_c \simeq 10^{-28}$ cm^2, one obtains $\sigma_H \simeq 10^{-34}$ cm^2 for m_H = 10 GeV/c^2 and $\sqrt{s}$ = 540 GeV.This is about two orders of magnitude larger than our calculation. The phenomenological estimate given above includes all possible mechanisms for the physical process to occur,including (VII.2), (VII.3) as well as (VII.4). Our calculations are only good to first order in QCD. Furthermore, eq. (IX.1) gives an overestimate since it does not take into account the transverse momentum of the heavy quark and other kinematic-suppression factors. One can expect the real value of the croos section to lie somewhere in between the two calculations.

The rates for high c.m. energy and high m_H, even with the 100 GeV^2 cutoff, remain quite impressive compared to the three other production rates, calculated by Cahn and Dawson(1984), and reproduced in Fig. 17. A comparison is difficult, because it is not clear what cutoff(s), if any, has been used by Cahn and Dawson. A analysis of these processes and their background is under way[1] . Any production rate larger than 1 pb. is large enough for the corresponding process to be observable in present or planed collider rings (see appendix G). Then one must deduce from it all the cuts needed to detect the signal and discriminate it against the background; certain cuts may depend on the detectors used, like minimum transverse momenta or energies. The background rates must also be calculated and compared to the signal rate. If the former is larger than the total signal rate, there is still a chance that the signal and background have markedly different angular distributions, energy spectrum or some other kinematic variable dependence. A careful analysis of the signal and its background in the different decay channels of the Higgs boson is needed.

This work could also be extended in assuming a different Higgs sector in the symmetry breaking mechanism, leading to several Higgs bosons, of both charged and neutral types. The coupling constants in these alternate models are very nearly free however, in constrast to the minimal scalar field case of the Weinberg-Salam model. This would introduce one or more new free parameters to the calculations.

The main sources of uncertainty on the calculations presented in this thesis are brought by the gluon distribution

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[1] Ng, Bates and Zakarauskas

and the behavior of the amplitude squared in the low momentum transfer region. The uncertainties relative to the gluon distribution have been explicitely calculated in most cases. Only the SSC region calculations (Table III) have been covered using only one gluon distribution (F.4), because the scaling one, (F.3), is no longer appropriate at these energies.

The low momentum transfer region has been completely avoided by using the 3 GeV^3 cutoff on scalar products. All events within this region, which correspond often to larger sub-cross-sections, have been discarded by the Monte-Carlo phase space generator. But because these events have generally low $p_\perp$ or a small opening angle between two of the final state components, they would also be discarded in real experiments.

What has been done in this thesis is the complete first-order calculation of the process (VII.1). It points out the importance of this process in the search for the Higgs boson. Second order QCD corrections are expected to be of order α_s times the calculated rate, or less. The strong coupling constant α_s is approximatively 0.2 in the kinematic region considered.

Of course, a more specific calculation of the signal and bakcground rates, including all kinematic cuts, geometry of detectors, decay and hadronization of reaction products, would have to be done before an experiment looking for the H° proceeds on a particular set of accelerator and detector.

It must be pointed out that other models of electroweak interactions and grand unified theories all include at least one scalar boson which corresponds to the Higgs boson in the Weinberg-Salam model. Thus, the calculation exposed in this thesis is relevant to all these theories.

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APPENDIX A - FEYNMAN DIAGRAMS AND QCD RULES

In this appendix are collected the set of rules and conventions used throughout this work, as well as an introduction to Feynman diagrams and cross-section calculations. Appendix C. gives more details about cross-section integration.

The conventions used in the calculations relative to Dirac matrices and cross-section calculations have been adopted from Bjorken and Drell (1964). The scalar product of two 4-vectors, p and q, is defined as:

$$p \cdot q = p_\mu q^\mu = p_\mu q_\nu g^{\mu\nu} = p_0 q_0 - \vec{p} \cdot \vec{q} \tag{A.1}$$

where $g^{\mu\nu}$ is the metric

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A.2}$$

The gamma matrices must be introduced when solving the problem of describing the motion of spin 1/2 particles (fermions). The equation one must solve is the Dirac equation (for the non-interacting case)

$$i\gamma^\mu \frac{\partial \Psi}{\partial x^\mu} - m\Psi = 0 \tag{A.3}$$

The Ψ are spinors, which are usually represented as 4-component vectors. The γ^{μ} are the so-called gamma matrices, obeying the following anticommutation relations

$$\{\gamma^{\mu}, \gamma^{\nu}\} = \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu} I \qquad \text{(A.4)}$$

I being the 4 X 4 unit matrix.

According to quantum field theory, interactions between particles described by a Lagrangian give rise to scattering amplitudes which may be represented by an expansion in terms of Feynman diagrams, examples of which are given in figs. (VII.1) to (VII.5). To each Lagrangian corresponds a unique set of Feynman diagrams at each order of the expansion parameters g 's, the g 's being constants measuring the strength of an interaction. If the interaction is very weak, the first order expansion will yield a good approximation. If not, one has to calculate the next order, etc.

The Feynman diagrams are constructed in the following way. From each Lagrangian may be derived a set of vertices and propagators. The vertices are nodes in a diagram; the propagators are lines in between the nodes. To each vertex is associated a strength g_i. An expansion at the n^{th} order in g_i is the set of all possible diagrams in which the product of the g_i is g_i^n or less. One looks for all the ways to combine available propagators and vertices from a given Lagrangian, with the same initial state A and final state B. The sum of these constitutes the A —> B process n^{th} order expansion.

The Feynman diagrams really are elegant mnemonic devices, allowing one to represent visually any given process, and find all the allowable ways to combine vertices and propagators into a calculable expression.

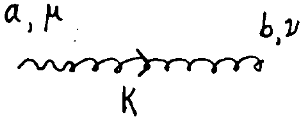
To each vertex or propagator corresponds either a scalar, a $g^{\mu\nu}$ or a gamma matrix times a scalar. To external lines are assigned either spinors, for fermions, or polarization 4-vectors for spin-1 bosons. All the terms are then combined to form the transition amplitude of the process. The matrix element squared expresses the probability for the transition to happen. One then sums over allowed configurations in phase space, multiplies by the appropriate kinematic factors to get the total cross-section σ, or some differential cross section relative to any desired variables x_i, $d^n\sigma/dx_1 dx_2 \ldots dx_n$. The integration over phase space using the Monte-Carlo integration method is explained in appendix C.

The term "on-mass shell" refers to real particles, as opposed to virtual ones; for the former, total energy, momentum and mass obey the relativistic relation

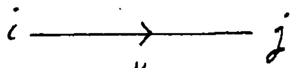
(A.5)

The rules associated with vertices and propagators in QCD are (Politzer, 1974);

Propagators

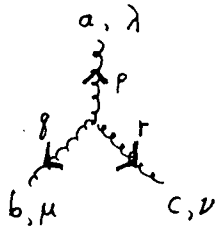
gluon 

$$-i\,\delta^{ab}\left[g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}\right] / K^2 \qquad (A.6)$$

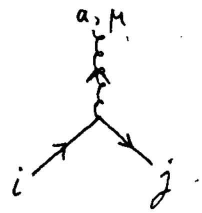
fermion 

$$\frac{i\,\delta^{ij}\,\not{K}}{K^2} \qquad (A.7)$$

vertices



$$-g f^{abc}\left[(p-q)_\nu\, g_{\lambda\mu} + (q-r)_\lambda\, g_{\mu\nu} + (r-p)_\mu\, g_{\nu\lambda}\right] \qquad (A.8)$$



$$-ig\,\gamma^\mu\, T^a_{ij} \qquad (A.9)$$

T^a_{ij} as defined below i,j,k,l = 1-3 (quark color)

a,b = 1-8 (gluon color)

Polarization sum

$$\sum_\lambda \epsilon_1^\mu(K_1,\lambda_1)\, \epsilon_2^\nu(K_2,\lambda_2) = -g_{\mu\nu} + \frac{(K_1^\mu K_2^\nu + K_1^\nu K_2^\mu)}{K^2} \qquad (A.10)$$

Color sum

The qqV vertex involves the factor $T_a = \frac{1}{2}\lambda_a$ where the are the SU(3) matrices. The T_a's obey the commutation rules

$$[T_a , T_b] = i f_{abc} T_c \tag{A.11}$$

$$\{T_a , T_b\} = \frac{4}{3} \delta_{ab} I_{(3)} + 4 d_{abc} T_c \tag{A.12}$$

where f_{abc} are antisymmetric and the d_{abc} are symmmetric under interchange of any two indices. $I_{(3)}$ is the 3 X 3 unit matrix. Some identities that will be used in appendix B involving the matrices T_a and symbols f_{abc} are:

$$\mathrm{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab} \tag{A.13}$$

$$\mathrm{Tr}(T_a T_b T_c) = \frac{1}{4}(\delta_{abc} + i f_{abc}) \tag{A.14}$$

$$\mathrm{Tr}(T_a T_b T_a T_c) = -\frac{1}{12} \delta_{bc} \tag{A.15}$$

$$f_{acd} d_{bcd} = 0 \tag{A.16}$$

$$f_{acd}\, f_{bcd} = 3\, \delta_{ab} \tag{A.17}$$

APPENDIX B - COLOR SUMMATION CALCULATION

Having introduced the QCD rules in appendix A, we are now in a position to calculate the color factors for different terms of the amplitudes. For the amplitude (VI.6), using the relations (A.13) to (A.17), the color factor of the matrix element squared is

$$\begin{aligned} T^a_{ik}\, T^a_{jl}\, T^b_{ik}\, T^b_{jl} &= Tr\left(T^a T^b\right) Tr\left(T^a T^b\right) \\ &= \left(\tfrac{1}{2}\delta^{ab}\right)\left(\tfrac{1}{2}\delta^{ba}\right) \qquad \text{(B.1)} \\ &= 2 \end{aligned}$$

Because we are averaging over initial color, we must also divide by a factor 9 for quarks. This is the total number of different color states for two incoming quarks. Hence, the factor 2/9 is obtained after averaging over quark colors.

In the process (VII.4), there are two gluons in the initial state. Therefore, to average over color, we must divide by an overall factor of 64, which is the number of different color combinations for two incoming gluons.

For the calculation of the color factors, the different terms arising from the squaring of the amplitude (VII.10) to (VII.12) are divided into five classes. For the squares of M_1 to M_6, and the cross terms in between M_1 to M_3, or M_4 to M_6, the color factor is:

$$T^a_{ik}\, T^b_{kj}\, T^a_{k'i}\, T^b_{jk'} = Tr\left(T^a T^b T^b T^a\right) \qquad \text{(B.2)}$$

use eq. (A.12)

$$= Tr(T^a T^b T^a T^b) - i f_{abc} Tr(T^a T^b T^c)$$

use eqs. (A.14) and(A.15)

$$= -\frac{1}{4\cdot 3}\delta_{bb} - i f_{abc}\frac{1}{4}(\delta_{abc} + i f_{abc})$$

use eq. (A.16)

$$= -\frac{8}{12} + \frac{f_{abc} f_{abc}}{4}$$

we get with eq. (A.17)

$$= 16/3$$

For the cross terms between one of M_1, M_2 or M_3 on one hand, and M_5, M_6 or M_7 on the other, the color factor is:

$$Tr(T^a T^b T^a T^b) = -\frac{1}{12}\delta_{ab} = -2/3 \tag{B.3}$$

For M_7 and M_8 squared:

$$\begin{aligned} f^{abc} f^{abd} T^c_{ij} T^d_{ij} &= f^{abc} f^{abd} Tr(T^c T^d) \\ &= f^{abc} f^{abc} = \frac{1}{2} 3\,\delta_{aa} = 12 \end{aligned} \tag{B.4}$$

For cross terms between one of M_4, M_5, or M_6 on one hand, and M_7 or M_8 on the other, the color factor is:

$$\begin{aligned} -i f^{abc} T^a_{ik} T^b_{kj} T^c_{ji} &= -i f^{abc} Tr(T^a T^b T^c) \\ &= \frac{i f^{abc}}{4}(d_{acb} + i f_{abc}) \end{aligned} \tag{B.5}$$

And finaly, for cross terms in between M_4, M_5 or M_6 on one hand, and M_7 or M_8 on the other:

$$
\begin{aligned}
-i f^{abc} T^b_{iK} T^a_{Kj} T^c_{ji} &= -i f^{abc} Tr(T^a T^c T^b) \\
&= \frac{i f^{abc}}{4}(d_{abc} + i f_{acb}) = \frac{i \cdot i}{4} N \delta_{aa} = -6
\end{aligned}
\tag{B.6}
$$

These color factors are substituted when summing the different terms of the amplitude squared. They are essential to get the correct gauge invariance.

APPENDIX C - THE MONTE-CARLO INTEGRATION ROUTINE

In this appendix is given a brief outline of the Monte-Carlo method, followed by a listing of the program used in this work to integrate over phase space and parton momenta. For a complete description of the Monte-Carlo method in particle physics, see (Byckling and Kajantie, 1973). What we want to do is to integrate the amplitude squared over phase space to get the sub cross section $\hat{\sigma}$, and then over x_1 and x_2 to get the total cross section. The Monte-Carlo method consists of generating (simulating) events, and then calculate the probability of it to happen, through the amplitude squared. The average of these for a large number of events converges toward the total cross section faster than standard integration method when the dimensionality D of the integral is large. In our case D = 7. To get differential cross sections is as easy. Suppose you want to calculate the derivative of the cross section relative to some angle θ. You create a vector V of dimension 100 or so, define θ from the 4-vectors generated by the simulation, and calculate for each event in which position in V the event falls. That is, if for one event the angle θ is in between 0 and 1.8 , you add the probability corresponding to it into the first element of V. The probability acts here as a weight to the event.

The cross section of a process $Q + \bar{Q} \rightarrow k + \bar{k} + h$ where k, $\bar{k}$ and h are the heavy quark, antiquark and Higgs boson 4-momenta respectively is:

$$\hat{\sigma} = \frac{1}{F}\int |M|^2 \frac{d^3K}{(2\pi)^3 E_K}\,\frac{d^3\bar{K}}{(2\pi)^3 E_{\bar{K}}}\,\frac{d^3h}{(2\pi)^3 E_h}\,(2\pi)^4 \delta^4(G-h-K-\bar{K}) \tag{C.1}$$

where F = 2s is the flux and $|M|^2$ is the amplitude squared. The delta function insures that the conservation of energy momentum is respected. What the Monte-Carlo process does is generate 4-momenta of real particles in the center of mass of decaying or virtual particles, and then boost them back into the laboratory c.m. The simulation happens as if the reaction was taking place as in the diagram of figure 31.

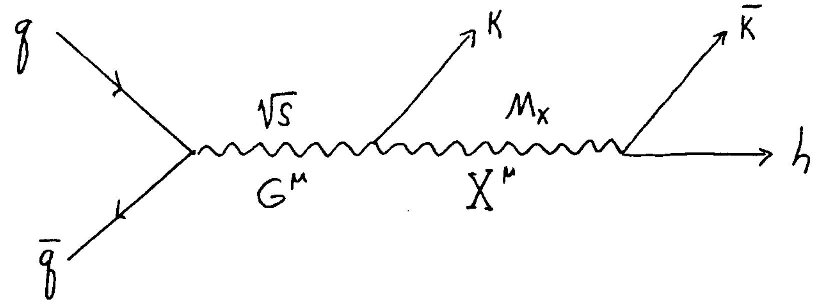


Figure 31 - Order of particle generation in the Monte-Carlo method in particle physics.

Therefore, in our case, it is as if the incoming quark and antiquark were producing a fictitious particle G, which would decay into a real heavy quark k and a fictitious particle X. The 4-momentum of k in the G center of mass is stored, as well as a boost matrix to pass from the G c.m to the laboratory c.m. We then go in the X c.m. and let it decay into a heavy antiquark and a real Higgs boson. Their 4-momenta in the X c.m. are calculated, then boosted back to the G c.m. All three particles' 4-momenta are finally boosted back to the laboratory c.m. where they are used to calculate the cross section. To this end, all possible scalar products between the 4-momenta are formed and substituted into the amplitude squared. The result is then stored and the whole process is repeated N times.

To cast the integral (C.1) into a form compatible with the Monte-Carlo technique , one integrates it over the δ function and introduces a fictitious mass $M_X = (\bar{k} + h)^2$, corresponding to a virtual particle with 4-momentum $X = \bar{k} + h$. One must also make another change of variables to spherical coordinates of the k in the G c.m. and the h in the X c.m. What is left is an integral over this M_X variable and the angular coordinates of the two real particles, as in

$$\hat{\sigma} = \frac{1}{F} \quad \frac{|M|^2}{2\sqrt{s}} \frac{1}{(2\pi)^5} \, dM_x \, d\Omega_{xcm} \, d\Omega_{scm} \, \frac{|\vec{\bar{k}}|}{2} \frac{|\vec{k}|}{2} \tag{C.2}$$

where $|\vec{\bar{k}}|$ is the $\bar{k}$ momentum in the X center of mass, and $|\vec{k}|$ is the k momentum in the G c.m. To produce only physical events, the X particle is restricted to have a mass of at least $m_H + m_K$, but at most $\sqrt{s} - m_K$. The Monte-Carlo program generates X's masses and real particles angular distributions randomly, using a random number generator called GGUBFS out of the IMSL library. GGUBFS acts on an argument SEED, which is changed every time it is used. The Jacobian for the variable change from M_X and the Ω's to the random numbers generated between 0 and 1 is $(4\pi)^2 [E - m_H - 2m_K]$. The Monte-Carlo routine has been tested on the phase space ($|M|^2 \equiv 1$), where any mistake in the program would show up as a suspect momentum distribution, or an asymmetry between particles. The total phase space integration has been compared to analytical calculation and found to agree within 1 %.

```
      REAL*8 P, XI, V1, THETA1, PHI1,SEED,V2,X1,X2,Q1,Q2,Q1P
      REAL*8 Q1M,Q2P,Q2M,CUTOFF,CUT2,VOLUME,ST
      REAL*8 DCADRE,F,A,B,AERR,RERR,ERROR,SC
      REAL*8 B1(4,4), E2, MH, MK, MH2, MK2, MX, MX2, S, PI
      REAL*8 K(4), AK(4), EX2, PX2,B2(4,4),XI2, THETA2, PHI2,H(4)
      REAL*8 K2(4),AK2(4),H2(4),AK3(4),H3(4),SUPX
      REAL*8 COSTHETA1,COSTHETA2,W1,FLUX,C,CONSTANTE
      REAL*8 D1,D2,D3,D4,D5,D6,SUMW
      REAL*8 HSP1,HSP2,Q1SH,Q2SH,Q1SP1,Q1SP2,Q2SP1,Q2SP2,P1SP2
      REAL*8 HG, LAMBDA1, LAMBDA2, W, SUMW2, INTEGRALE
      REAL*8 EH(100),EK(100),EAK(100),ETH(100),ETK(100),ETAK(100)
      REAL*8 XFE(100),RAP(100), Y,XF
      REAL*8 ENERGY,TRANSH, TRANSK, TRANSAK,BINR
      REAL*8 R1(4),R2(4),RT1,RT2,UPS1,UPS2,PHIT1,PHIT2,PG
      REAL*8 DISTRIBUTION, DENSITY,STOT
      REAL*8 MA1,MA2,MA3,MA4,MA5,
     C M11A,M12A,M13A,M14A,M15A,M16A,M17A,M18A,M22A,M23A,M24A,M25A,
     C M26A,M27A,M28A,M33A,M34A,M35A,M36A,M37A,M38A,M44A,M45A,M46A,
     C M47A,M48A,M55A,M56A,M57A,M58A,M66A,M67A,M68A,M77A,M78A,M88A
      INTEGER START,DATA,IER
      EXTERNAL F
      COMMON C,STOT,ST
      COMMON EH,EK,EAK,ETH,ETK,ETAK
C-----------------------------------------------------------------
C     RECEIVE PARAMETERS
C-----------------------------------------------------------------
C                 GIVE THE CHOICE OF THE FORM OF ENTRY
C-----------------------------------------------------------------
16    WRITE(6,19)
19    FORMAT(' DO YOU WANT TO START A NEW CALCULATION (TYPE 0) OR
     C CONTINUE A PREVIOUS ONE (TYPE 1)')
      READ(5,18) DATA
18    FORMAT (I1)
      IF (DATA .NE. 0 .AND. DATA .NE. 1) WRITE(6,17), STOP
17    FORMAT(' YOU MUST ENTER 0 OR 1')
C-----------------------------------------------------------------
C                    READ ENTRIES THROUGH TERMINAL
C-----------------------------------------------------------------
      WRITE(6,20)
20    FORMAT(' ENTER SEPERATELY CUTOFF,P, MH AND MK')
      READ(5,10) CUTOFF,P, MH, MK
      WRITE(6,29)
29    FORMAT(' ENTER STEEPNESS OF GLUON DISTRIBUTION')
      READ(5,10)ST
      WRITE (6,30)
30    FORMAT(' ENTER NUMBER OF EVENTS DESIRED')
      READ (5,12)N
      IF (DATA .EQ. 0) THEN
      START = 1
      SUMW = 0.
      SEED = 12345.0
      END IF
C-----------------------------------------------------------------
C             READ ENTRIES THROUGH FILES 20 AND 21
C-----------------------------------------------------------------
      IF (DATA .EQ. 1) THEN
      WRITE (6,11)
```

```
11    FORMAT(' ENTER IJ, SEED AND SUMW')
      READ(5,12) START
12    FORMAT (I7)
      READ(5,9)SEED,SUMW
9     FORMAT(D18.10)
10    FORMAT(F15.8)
      DO 15 I=1,100
15    READ(27,96)EH(I),EK(I),ETH(I),ETK(I),RAP(I),XFE(I)
      END IF
C-----------------------------------------------------------------------
C                      INITIALIZE VARIABLES                            1
C-----------------------------------------------------------------------
      WRITE(20,98)
98    FORMAT('       P          MK           MH          CUTOFF     ST    ')
      WRITE(20,99) P,MK,MH,CUTOFF,ST
99    FORMAT(4D12.5,D8.3)
      WRITE(20,97)
97    FORMAT('    IJ          SEED
     C                   SUMW                 X-SECTION ')
      MH2 = MH * MH
      MK2 = MK * MK
      C = MH + 2.* MK
      STOT = 4.  * P*P
      RT1=0.
      RT2=0.
C-----------------------------------------------------------------------
C     ESTABLISH 4-VECTOR OF QUARKS IN LAB SYSTEM                       1
C-----------------------------------------------------------------------
100   DO 1000 IJ=START,N
150   X1 = GGUBFS(SEED)
      G1M=1.
      G1P=EXP(-ST)
      X1 = -LOG(G1M+X1*(G1P-G1M))/ST
      G1 = ST* EXP(-ST*X1)
      X2 = GGUBFS(SEED)
      CUT2 = C*C/(STOT*X1)
      G2M=EXP(-ST*CUT2)
      G2P = G1P
      X2= -LOG(G2M+X2*(G2P-G2M))/ST
      G2 = ST*EXP(-ST*X2)
      UPS1 = X1 - (RT2**2.)/(X2 * 2.*P)
      UPS2 = X2 - (RT1**2.)/(X1 * 2.*P)
      IF(UPS1 .LE. 0.) GO TO 150
      IF(UPS2 .LE. 0.) GO TO 150
C-----------------------------------------------------------------------
C     PICK-UP ANGLE FOR TRANS. MOMENTUM OF PARTONS                     1
C-----------------------------------------------------------------------
      PI = 3.141592654
      PHIT1 = 2. * PI * GGUBFS(SEED)
      PHIT2 = 2. * PI * GGUBFS(SEED)
C-----------------------------------------------------------------------
C     CALCULATE PARTONS 4-MOMENTA IN LAB SYSTEM                        1
C-----------------------------------------------------------------------
      R1(1) = UPS1*P + RT1**2./(4.*UPS1*P)
      R1(2) = RT1 * COS(PHIT1)
      R1(3) = RT1 * SIN(PHIT1)
      R1(4) = UPS1*P - RT1**2./(4.*UPS1*P)
```

```
      R2(1) = UPS2*P + RT2**2./(4.*UPS2*P)
      R2(2) = RT2 * COS(PHIT2)
      R2(3) = RT2 * SIN(PHIT2)
      R2(4) = -UPS2*P + RT2**2./(4.*UPS2*P)
C-----------------------------------------------------------------------
C     CALCULATE VELOCITY V1 AND RAPIDITY XI OF CM IN LAB FRAME         1
C-----------------------------------------------------------------------
      PG = SQRT((R1(2)+R2(2))**2.+(R2(3)+R1(3))**2.+(R2(4)+R1(4))**2.)
      V1 = PG / (R1(1) + R2(1))
      XI = LOG((1. + V1)/(1. - V1))/2.
C-----------------------------------------------------------------------
C                        E2 IS ENERGY IN CM                            1
C-----------------------------------------------------------------------
      S = X1*X2*STOT
      E2 = SQRT(S)
C-----------------------------------------------------------------------
C     CHECK IF ENOUGH ENERGY IS AVAILABLE FOR REACTION TO OCCUR        1
C-----------------------------------------------------------------------
      IF(S .LE. (MH + 2*MK)**2.)GO TO 150
C-----------------------------------------------------------------------
C                         PICK-UP ANGLES OF ROTATION                   1
C -----------------------------------------------------------------------
      PI = 3.141592654
      COSTHETA1 = 2.* GGUBFS(SEED) -1.
      THETA1 = ACOS(COSTHETA1)
      PHI1 = 2. * PI * GGUBFS(SEED)
C-----------------------------------------------------------------------
C                          COMPOSE ROTATION MATRIX                     1
C-----------------------------------------------------------------------
      CALL BOOST(B1,XI,THETA1,PHI1)
C-----------------------------------------------------------------------
C            PICK UP INVARIANT MASS OF THE OFF MASS-SHELL QUARK        1
C-----------------------------------------------------------------------
      SUPX= E2 - MK
      MX = SUPX - GGUBFS(SEED)* (SUPX - (MH+MK))
      MX2=MX*MX
C-----------------------------------------------------------------------
C          CALCULATE 4-MOMENTA OF X AND K QUARKS IN QUARKS CM          1
C-----------------------------------------------------------------------
      K2(1) = (S + MK2 - MX2)/ (2.*E2)
      K2(2) = 0.
      K2(3) = 0.
      K2(4) = SQRT(K2(1) *K2(1) - MK2)
      EX2 = E2 - K2(1)
      PX2 = -K2(4)
C-----------------------------------------------------------------------
C            COMPOSE BOOST MATRIX BETWEEN QUARKS CM AND X CM           1
C-----------------------------------------------------------------------
      V2 = PX2/EX2
      XI2= LOG((1. + V2)/(1. - V2))/2.
      COSTHETA2 = 2.* GGUBFS(SEED)-1.
      THETA2 = ACOS(COSTHETA2)
      PHI2 = 2. * PI* GGUBFS(SEED)
      CALL BOOST(B2, XI2, THETA2, PHI2)
C-----------------------------------------------------------------------
C           CALCULATE 4-MOMENTA OF HIGGS AND ANTI-QUARK IN X CM        1
C-----------------------------------------------------------------------
```

```
      H3(1) = (MX2+ MH2 - MK2)/(2.*MX)
      H3(2) =0.
      H3(3) = 0.
      H3(4) = SQRT(H3(1) * H3(1) - MH2)
      AK3(1) = MX - H3(1)
      AK3(2) = 0.
      AK3(3) = 0.
      AK3(4) = -H3(4)
C-----------------------------------------------------------------------
C             TRANSFORM THE 4-MOMENTA BACK INTO FRAME 2                 1
C-----------------------------------------------------------------------
      CALL MULT(B2,H3,H2)
      CALL MULT(B2,AK3,AK2)
C-----------------------------------------------------------------------
C               TRANSFORM THE 4-MOMENTA BACK INTO LAB FRAME             1
C-----------------------------------------------------------------------
      CALL MULT(B1,H2,H)
      CALL MULT(B1,AK2,AK)
      CALL MULT(B1,K2,K)
C-----------------------------------------------------------------------
C                         CALCULATE THE SCALAR PRODUCTS                 1
C-----------------------------------------------------------------------
      CALL SCALP(AK, R1, Q2SP2)
      CALL SCALP(AK, R2, Q1SP2)
      CALL SCALP(K, R1, Q2SP1)
      CALL SCALP(K, R2, Q1SP1)
      CALL SCALP(R1, H, Q2SH)
      CALL SCALP(R2,H,Q1SH)
      CALL SCALP(K, AK, P1SP2)
      CALL SCALP(K, H, HSP1)
      CALL SCALP(AK, H, HSP2)
C-----------------------------------------------------------------------
C                    PUT CONDITIONS ON VALIDITY OF CALCULATION          1
C                          IN PERTURBATIVE QCD                          1
C-----------------------------------------------------------------------
      IF (Q1SP1 .LT. CUTOFF) GO TO 150
      IF (Q1SP2 .LE. CUTOFF) GO TO 150
      IF (Q2SP1 .LE. CUTOFF) GO TO 150
      IF (Q2SP2 .LE. CUTOFF) GO TO 150
      IF (P1SP2 .LE. CUTOFF) GO TO 150
      IF (Q1SH .LE. CUTOFF) GO TO 150
      IF (Q2SH .LE. CUTOFF) GO TO 150
      IF (HSP2 .LE. CUTOFF) GO TO 150
      IF (HSP1 .LE. CUTOFF) GO TO 150
C-----------------------------------------------------------------------
C               CALCULATE THE DENOMINATORS OF AMPLITUDE                 1
C-----------------------------------------------------------------------
      D1 = MH2 + 2.*HSP2
      D2 = MH2 + 2.*HSP1
      D3 = -2.*Q1SP1
      D4 = -2.*Q2SP2
      D5 = -2.*Q1SP2
      D6 = -2.*Q2SP1
      CALL AMPL(S,D1,D2,D3,D4,D5,D6,
     C MK2,MH2,HSP1,HSP2,Q1SH,Q2SH,Q1SP1,
     C Q1SP2,Q2SP1,Q2SP2,P1SP2,HC,MA1,MA2,MA3,MA4,MA5,M11A,M12A,M13A,
     C M14A,M15A,M16A,M17A,M18A,M22A,M23A,M24A,M25A,M26A,M27A,M28A,
```

```
     C M33A, M34A, M35A, M36A, M37A, M38A, M44A, M45A, M46A, M47A, M48A, M55A,
     C M56A, M57A, M58A, M66A, M67A, M68A, M77A, M78A, M88A)
C-----------------------------------------------------------------------
C    CALCULATE THE SCALE VIOLATING GLUON DISTRIBUTION                  1
C-----------------------------------------------------------------------
      SC = LOG(25.*S)
      SC = SC / LOG(125.0)
      SC = LOG(SC)
      CALL DIST(X1, X2, SC, DISTRIBUTION)
C-----------------------------------------------------------------------
C     CALCULATE PHASE SPACE DENSITY AND ELEMENT OF INTEGRALE           1
C-----------------------------------------------------------------------
      LAMBDA1 = SQRT((S - MK2 - MX2)**2-4.*MX2*MK2)
      LAMBDA2 = SQRT((MX2 - MH2 - MK2)**2 - 4.*MK2*MH2)
      VOLUME = (Q1P-Q1M)*(Q2P-Q2M)
      DENSITY = VOLUME/(Q1*Q2)
      CONSTANTE = 4.488D+04 * MK2/(LOG(25.*S))**2
      FLUX = 2.*S
      W1 = ((4.*PI)**2*(E2 - 2*MK - MH)/(32.*S*MX)) * LAMBDA1 *LAMBDA2
      W = W1 *HG * DISTRIBUTION * CONSTANTE * DENSITY /((2.*PI)**5*FLUX)
      SUMW = SUMW + W
88    FORMAT(' IJ=',I10)
C-----------------------------------------------------------------------
C                       WRITE OUT designed EVENTS                      1
C-----------------------------------------------------------------------
      IF (IJ .LE. 10) THEN
      WRITE(17,89)
89    FORMAT('  IJ     H(I)           AK(I)              K(I)')
      DO 90 I = 1,4
      WRITE(17,91) IJ, H(I), AK(I), K(I)
91    FORMAT(I9, 3D20.8)
90    CONTINUE
      WRITE(17,92)
92    FORMAT('   ')
      WRITE(17,93)
93    FORMAT('  IJ              HG             W1
     C          E2       DISTRIBUTION          W                X1
     C           X2')
      WRITE(17,94) IJ, HG,W1, E2, DISTRIBUTION, W, X1, X2
94    FORMAT(I7,7D15.5)
      WRITE(17,92)
      END IF

      IF (X1 .LE. -.01) THEN
      WRITE(17,89)
      DO 85 I = 1,4
      WRITE(17,91) IJ, H(I), AK(I), K(I)
85    CONTINUE
      WRITE(17,92)
      WRITE(17,93)
      WRITE(17,94) IJ, HG,W1, E2, DISTRIBUTION, W, X1, X2
      WRITE(17,92)
      END IF

      IF (W .LE. 0. ) THEN
      WRITE(17,93)
      WRITE(17,94) IJ,HG,W1,E2,DISTRIBUTION,W,X1,X2
```

```
0286          END IF
0287
0288          IF (W .LE. 0) THEN
0289          WRITE(6,8)
0290          WRITE(17,7)IJ,HC
0291          WRITE(17,9)MA1,MA2,MA3,MA4,MA5,M11A,M12A,M13A,M14A,M15A,M16A,M17A,
0292         C M18A,M22A,M23A,M24A,M25A,M26A,M27A,M28A,M33A,M34A,M35A,M36A,M37A,
0293         C M38A,M44A,M45A,M46A,M47A,M48A,M55A,M56A,M57A,M58A,M66A,M67A,M68A,
0294         C M77A,M78A,M88A
0295    8     FORMAT(' THERE IS A NEGATIVE CROSS-SECTION')
0296    7     FORMAT(I10,D20.8)
0297          END IF
0298    C----------------------------------------------------------------------
0299    C     BIN THE ENERGY AND TRANSVERSE ENERGY OF H,K,AK                   1
0300    C----------------------------------------------------------------------
0301          ENERGY =( 2*P - MH - 2*MK)/3.
0302          TRANSH = SQRT(H(2)*H(2) + H(3)*H(3))
0303          TRANSK = SQRT(K(2)*K(2) + K(3)*K(3))
0304          CALL BIN(H(1), EH, O., ENERGY, W)
0305          CALL BIN(K(1), EK, O., ENERGY, W)
0306          CALL BIN(TRANSH, ETH, O., ENERGY, W)
0307          CALL BIN(TRANSK, ETK, O., ENERGY, W)
0308    C----------------------------------------------------------------------
0309    C       BIN THE RAPIDITY Y AND FEYNMAN SCALING VARIABLE XF OF HIGGS    1
0310    C----------------------------------------------------------------------
0311          Y = ABS(0.5*LOG((H(1)+H(4))/(H(1)-H(4))))
0312          XF = ABS(H(4)/P)
0313          CALL BIN(Y,RAP,O.,4.,W)
0314          CALL BIN(XF,XFE,O.,1.,W)
0315    C----------------------------------------------------------------------
0316    C             WRITE DOWN ANSWER EVERY 1000 EVENTS, IN CASE SYSTEM      1
0317    C                            BREAKS DOWN                               1
0318    C----------------------------------------------------------------------
0319          RAT = MOD(IJ,1000)
0320          IF (RAT .EQ. 0) THEN
0321          WRITE(20,95)IJ,SEED,SUMW,SUMW/IJ
0322          OPEN (UNIT = 27)
%FORT-I-DEFSTAUNK, Default STATUS= 'UNKNOWN' used in OPEN statement
        [PEN (UNIT = 27)] in module GLUON$MAIN at line 322

0323          DO 112 I=1,100
0324          WRITE(27,96)EH(I),EK(I),ETH(I),ETK(I),RAP(I),XFE(I)
0325    95    FORMAT(I7,D25.10,D20.10,D15.6)
0326    96    FORMAT(6D15.8)
0327    112   CONTINUE
0328          CLOSE (UNIT = 27)
0329          END IF
0330    1000  CONTINUE
0331    C----------------------------------------------------------------------
0332    C                      CALCULATE THE INTEGRALE                         1
0333    C---------------------------------------------------------------------
0334          INTEGRALE= SUMW/N
0335          WRITE(17,43)N
0336    43    FORMAT(' N = ',I7)
0337          WRITE(17,42)CUTOFF
0338    42    FORMAT(' CTOFF='D15.6)
0339          WRITE(17,44)P
```

```
44     FORMAT(' P = ',D15.6)
       WRITE(17,45)MH
45     FORMAT(' MH = ',D15.6)
       WRITE(17,46)MK
46     FORMAT(' MK = ',D15.6)
       WRITE(17,55)INTEGRALE
55     FORMAT(' X-SECTION = ',D15.6,' PICOBARN')
       BINR = 0.
       DO 111 I= 1,100
       WRITE(18,56) BINR*ENERGY/100., EH(I)/N, EK(I)/N
       WRITE(19,56) BINR*ENERGY/100., ETH(I)/N,ETK(I)/N
       WRITE(22,57) BINR*4./100.,RAP(I)/N
       WRITE(23,57) BINR/100.,XFE(I)/N
57     FORMAT(2D15.4)
       BINR = BINR + 1
56     FORMAT(3D15.4)
111    CONTINUE
       END
```

```

       SUBROUTINE DIST(X1,X2,SC,DISTRIBUTION)
C---------------------------------------------------------------------
C      CALCULATES THE SCALE VIOLATING GLUON DISTRIBUTION FROM        1
C      PARAMETERS X1, X2 AND THE SCALE PARAMETER SC                  1
C---------------------------------------------------------------------
       REAL*8 X1,X2,SC,DISTRIBUTION,E1,E2
       E1 = -0.93*SC + 0.36*SC**2
       E2 = 2.9 + 1.83*SC
       DISTRIBUTION = X1**E1 * X2**E1
       DISTRIBUTION = DISTRIBUTION * (1-X1)**E2 * (1-X2)**E2
       DISTRIBUTION = DISTRIBUTION * (2.01 - 2.73*SC + 1.29*SC**2)**2
       DISTRIBUTION = DISTRIBUTION / (X1*X2)
       RETURN
       END
```

```

       SUBROUTINE BOOST(B,XI,THETA,PHI)
       REAL *8 B(4,4), XI, THETA, PHI
       B(1,1)= COSH(XI)
       B(1,2)= -SINH(XI) *  SIN(THETA)
       B(1,3)= 0
       B(1,4)= SINH(XI)*COS(THETA)
       B(2,1)= 0.
       B(2,2)= COS(PHI)*COS(THETA)
       B(2,3)= -SIN(PHI)
       B(2,4)= SIN(THETA)*COS(PHI)
       B(3,1)= 0.
       B(3,2)= SIN(PHI)*COS(THETA)
       B(3,3)= COS(PHI)
       B(3,4)= SIN(THETA)*SIN(PHI)
       B(4,1)= SINH(XI)
       B(4,2)= -COSH(XI)*SIN(THETA)
       B(4,3)= 0.
       B(4,4)= COSH(XI)*COS(THETA)
       RETURN
       END
```

```
      SUBROUTINE BIN(F,AR,INF,SUP, W)
C-----------------------------------------------------------------------
C        CLASSES F INTO ONE OF 100 BINS BETWEEN INF AND SUP AND PUT     1
C                               IT INTO ARRAY AR                        1
C-----------------------------------------------------------------------
      REAL*8 F,AR(100),INF,SUP,POS
      COMMON EH,EK,EAK,ETH,ETK,ETAK
      POS = INT(100. * (F - INF)/(SUP - INF)) + 1.
      IF (POS .GT. 100.) POS = 100.
      AR(POS) = AR(POS) + W
      RETURN
      END
```

```
      SUBROUTINE SCALP(V1,V2,S)
C-----------------------------------------------------------------------
C     TAKE THE SCALAR PRODUCT OF THE TWO 4-VECTORS V1                   1
C     AND V2 AND PUT THE RESULT INTO S                                  1
C-----------------------------------------------------------------------
      REAL*8 V1(4),V2(4),S
      S = V1(1)*V2(1) - V1(2)*V2(2) - V1(3)*V2(3) - V1(4)*V2(4)
      RETURN
      END
```

```
      SUBROUTINE MULT(B,V1,V2)
C-----------------------------------------------------------------------
C             CALCULATES THE PRODUCT BETWEEN THE MATRIX B AND           1
C                VECTOR M1 AND PUTS RESULT INTO V2                      1
C-----------------------------------------------------------------------
      REAL*8 B(4,4), V1(4), V2(4), PH
      DO 300 I=1,4
      PH=0.
      DO 301 J=1,4
301   PH = B(I,J) * V1(J) + PH
      V2(I) = PH
300   CONTINUE
      RETURN
      END
```

APPENDIX D - CALCULATION OF THE TRACE

Trace calculations come in evaluating Feynman diagrams involving fermions. Standard methods for calculating the traces are given in Bjorken and Drell (1964).

When the number or length of traces to evaluate become too large to manage, one may now use one of a few number of programs designed to this end. One of them is REDUCE (see UBC REDUCE), which was used in a program for evaluating the amplitude squared of the process (VII.4). In this appendix is given a listing of this REDUCE program. The input is included in the program, and consists of the numerators of the amplitude (VII.10) to (VII.12). The output is a FORTRAN code, in term of the scalar products of the outgoing particles 4-momenta.

```
$SIGNON OUCH     T=3M PAGES=80 PROUTE=PHYS
$$$$
$SOURCE *REDUCE
OFF ECHO;
%----------------------------------------------------------------------
1                                                                     1
1              THIS PROGRAM CALCULATES THE  AMPLITUDE                1
1                    SQUARED FOR THE PROCESS                          1
1           GLUON+GLUON --> QUARK + ANTI-QUARK + HIGGS                1
1                                                                     1
1                    ----------------------                           1
1                                                                     1
1           DEFINE VECTORS AND MASSES OF COMPONENTS                   1
1---------------------------------------------------------------------;
MASS G1=0, G2=0, P1=MK, P2=MK, H=MH;
MSHELL G1,G2,P1,P2,H;
VECTOR E1,E2;
LET G1.E1=0, G2.E2=0;
LET G1.G2=S/2;
OPERATOR V2,U2,GM,GMH;
%----------------------------------------------------------------------
1                  GIVE THE RULES FOR SUMMATION                       1
1                  OVER POLARIZATION OF GLUONS                        1
1---------------------------------------------------------------------;
LET E1.E1 = -2;
FOR ALL P LET E1.P * E1.P = -P.P + 2 *( P.G1*Q.G2+P.G2*Q.G1)/S;
FOR ALL R,Q LET E1.R * E1.Q = -R.Q + 2 * (R.G1*Q.G2 + R.G2+Q.G1)/S;
LET E2.E2 = -2;
FOR ALL P LET E2.P * E2.P = -P.P + 2 *( P.G1*Q.G2+P.G2*Q.G1)/S;
FOR ALL P,Q LET E2.P * E2.Q = -P.Q + 2 * (P.G1*Q.G2 + P.G2+Q.G1)/S;
%OFF MCD;
FACTOR S, P1.G1, P1.G2, P2.G1, P2.G2, P1.P2, H.G1, H.G2, H.P1, H.P2;
%----------------------------------------------------------------------
1               DEFINE NEW OPERATORS TO SIMPLIFY THE                  1
1               TYPING OF THE AMPLITUDE                               1
1---------------------------------------------------------------------;
FOR ALL T,U LET GM(T+U) = G(L,T)+G(L,U) + MK;
FOR ALL H LET GMH(H) = G(L,H) + 2*MK;
LET V2 = G(L,P1) - MK;
LET U2 =  G(L,P2) + MK;
LET P1G1 = GM(G1 - P1);
LET P2G1 = GM(P2 - G1);
LET P2G2 = GM(P2 - G2);
LET P1G2 = GM(G2 - P1);
LET GE2 = G(L,E2);
LET GE1 = G(L,E1);
LET VERTEX = 2*G1.E2*GE1 + E1.E2*(G(L,G2)-G(L,G1))
 -2*G2.E1*GE2;
LET MH**2 = MH2;
LET MK**2 = MK2;
OFF NAT;
%----------------------------------------------------------------------
1              WRITE THE  AMPLITUDE COMPONENTS                        1
1              AND THE SUM OVER U AND V SPINORS                       1
1---------------------------------------------------------------------;
LET M1 = U2 * GMH(H) * GE2 * P1G1 * GE1 * V2;
LET M2 = U2 * GE2 * P2G2 * P1G1 * GE1 * V2;
LET M3 = U2 * GE2 * P2G2 * GE1 * GMH(-H) * V2;
```

```
LET M4 = U2 * GE1 * P2G1 * GE2 * GMH(-H) * V2;
LET M5 = U2 * GE1 * P2G1 * P1G2 * GE2 * V2;
LET M6 = U2 * GMH(H) * GE1 * P1G2 * GE2 *V2;
LET M7 = U2 * VERTEX * GMH(-H) * V2;
LET M8 = U2 * GMH(H) * VERTEX * V2;
%-----------------------------------------------------------------------
1               WRITE THE COMPLEX CONJUGATE OF THE AMPLITUDE          1
1---------------------------------------------------------------------;
LET M1R = GE1 * P1G1 * GE2 * GMH(H);
LET M2R = GE1 * P1G1 * P2G2 * GE2;
LET M3R = GMH(-H) * GE1 * P2G2 * GE2;
LET M4R = GMH(-H) * GE2 * P2G1 * GE1;
LET M5R = GE2 * P1G2 * P2G1 * GE1;
LET M6R = GE2 * P1G2 * GE1 * GMH(H);
LET M7R = GMH(-H) * VERTEX;
LET M8R = VERTEX * GMH(H);
%----------------------------------------------------------------------
1                    WRITE THE SQUARE OF THE AMPLITUDE               1
1                       (WITHOUT THE DENOMINATORS)                   1
1--------------------------------------------------------------------;
OFF NAT;
OFF ECHO;
%WRITE "M23H = ", M2*M3R;
%WRITE "M33H = ", M3*M3R;
%WRITE "M34H = ", M3*M4R;
%WRITE "M35H = ", M3*M5R;
%WRITE "M36H = ", M3*M6R;
%WRITE "M37H = ", M3*M7R;
%WRITE "M38H = ", M3*M8R;
%WRITE "M44H = ", M4*M4R;
%WRITE "M45H = ", M4*M5R;
%WRITE "M46H = ", M4*M6R;
%WRITE "M47H = ", M4*M7R;
%WRITE "M48H = ", M4*M8R;
%WRITE "M55H = ", M5*M5R;
%WRITE "M56H = ", M5*M6R;
%WRITE "M57H = ", M5*M7R;
%WRITE "M58H = ", M5*M8R;
%WRITE "M66H = ", M6*M6R;
%WRITE "M67H = ", M6*M7R;
%WRITE "M68H = ", M6*M8R;
%WRITE "M77H = ", M7*M7R;
%WRITE "M88H = ", M8*M8R;
%SHUT ZGLUONRES23;
%ON NAT;
%-----------------------------------------------------------------------
1                  REWRITE THE AMPLITUDE IN FILE GLUONRES2            1
1                         IN A FORM READABLE BY FORTRAN               1
1---------------------------------------------------------------------;
%ON FORT;
%OUT GLUONRES2;
%WRITE "LET M46H =",M46H;
%WRITE "LET M47H =",M47H;
%WRITE "LET M48H =",M48H;
%WRITE "LET M55H =",M55H;
%WRITE "LET M56H =",M56H;
%WRITE "LET M57H =",M57H;
%WRITE "LET M58H =",M58H;
```

```
%WRITE "LET M66H =",M66H;
%WRITE "LET M67H =",M67H;
%WRITE "LET M68H =",M68H;
%WRITE "LET M77H =",M77H;
%WRITE "LET M78H =",M78H;
%WRITE "LET M88H =",M88H;
%SHUT GLUONRES2;
%ON ECHO;
%-----------------------------------------------------------------
1                        DEFINE DENOMINATORS                      1
1                    AND DENOMINATORS SQUARED                     1
1----------------------------------------------------------------;
LET D1 = S - 2*P1.(G1+G2);
LET D2 = S - 2*P2.(G1+G2);
LET D3 = -2*G1.P1;
LET D4 = -2*P2.G2;
LET D5 = -2*P2.G1;
LET D6 = -2*P1.G2;
LET D12 = D1*D1;
LET D22 = D2*D2;
LET D32 = D3*D3;
LET D42 = D4*D4;
LET D52 = D5*D5;
LET D62 = D6*D6;
%IN GLUONRESULT2;
LET H = G1 + G2 - P1 - P2;
LET P1.P2 = (G1 + G2).(P1 + P2) - (S + 2*MK**2 - MH**2)/2;
LET MH**2 = MH2;
LET MK**2 = MK2;
%OUT GLUONRESULT2;
%WRITE "M11 = ";
%M11H;
%-----------------------------------------------------------------
1             PUT ALL COMPONENTS OF AMPLITUDE SQUARED             1
1                    OVER SAME DENOMINATOR                        1
1----------------------------------------------------------------;
LET M11A = M11 * D22 * D42 * D52 * D62 * S2;
LET M12A = M12 * D1 * D22 * D4 * D52 * D62 * S2;
LET M13A = M13 * D1 * D2 * D3 * D4 * D52 * D62 * S2;
LET M14A = M14 * D1 * D2 * D3 * D42 * D5 * D62 * S2;
LET M15A = M15 * D1 * D22 * D3 * D42 * D5 * D6 * S2;
LET M16A = M16 * D22 * D3 * D42 * D52 * D6 * S2;
LET M17A = M17 * D1 * D2 * D3 * D42 * D52 * D62 * S;
LET M18A = M18 * D22 * D3 * D42 * D52 * D62 * S;
LET M22A = M22 * D12 * D22 * D52 * D62 * S2;
LET M23A = M23 * D12 * D2 * D3 * D52 * D62 * S2;
LET M24A = M24 * D12 * D2 * D3 * D4 * D5 * D62 * S2;
LET M25A = M25 * D12 * D22 * D3 * D4 * D5 * D6 * S2;
LET M26A = M26 * D1 * D22 * D3 * D4 * D52 * D6 * S2;
LET M27A = M27 * D12 * D2 * D3 * D4 * D52 * D62 *S;
LET M28A = M28 * D1 * D22 * D3 * D4 * D52 * D62 *S;
LET M33A = M33 * D12 * D32 * D52 * D62 * S2;
LET M34A = M34 * D12 * D32 * D4 * D5 * D62 * S2;
LET M35A = M35 * D12 * D2 * D32 * D4 * D5 * D6 * S2;
LET M36A = M36 * D1 * D2 * D32 * D4 * D52 * D6 * S2;
LET M37A = M37 * D12 * D32 * D4 * D52 * D62 *S;
LET M38A = M38 * D1 * D2 * D32 * D4 * D52 * D62 *S;
LET M44A = M44 * D12 * D32 * D42 * D62 * S2;
```

```
LET M45A = M45 * D12 * D2 * D32 * D42 * D6 * S2;
LET M46A = M46 * D1 * D2 * D32 * D42 * D5 * D6 * S2;
LET M47A = M47 * D12 *D32 * D42 * D5 * D62 *S;
LET M48A = M48 * D1 * D2 * D32 * D42 * D5 * D62 *S;
LET M55A = M55 * D12 * D22 * D32 * D42 * S2;
LET M56A = M56 * D1 * D22 * D32 * D42 * D5 * S2;
LET M57A = M57 * D12 * D2 * D32 * D42 * D5 * D6 *S;
LET M58A = M58 * D1 * D22 * D32 * D42 * D5 * D6 *S;
LET M66A = M66 * D22 * D32 * D42 * D52 * S2;
LET M67A = M67 * D1 * D2 * D32 * D42 * D52 * D6 *S;
LET M68A = M68 * D22 * D32 * D42 * D52 * D6 *S;
LET M77A = M77 * D12 * D32 * D42 * D52 * D62;
LET M78A = M78 * D1 * D2 * D32 * D42 * D52 * D62;
LET M88A = M88 * D22 * D32 * D42 * D52 * D62;
LET M88A = M88 * D22 * D32 * D42 * D52 * D62;
%-----------------------------------------------------------------
1                    REGROUP THE TERMS ACCORDING TO               1
1                             COLOR FACTOR                        1
1----------------------------------------------------------------;
LET MA1 = M11A + M22A + M33A + M44A + M55A + M66A +
2*(M12A + M13A + M23A + M45A + M46A + M56A);
LET MA2 = 2*(M14A + M15A + M16A + M24A + M25A + M26A
+ M34A + M35A + M36A);
LET MA3 = M77A + M88A + 2*M78A;
LET MA4 = 2*(M17A + M18A + M27A + M28A + M37A + M38A);
LET MA5 = 2*(M47A + M48A + M57A + M58A + M67A + M68A);
%OUT GLUONRESULT3;
%WRITE "MA1 =",MA1;
%WRITE "MA2 =",MA2;
%WRITE "MA3 =",MA3;
%WRITE "MA4 =",MA4;
%WRITE "MA5 =",MA5;
%WRITE "M77A =",M77A;
%WRITE "M11 =",M11;
%WRITE "D12 =",D12;
%16*MA1/3 -2*MA2/3 + 12*MA3 + 6*MA4 - 6*MA5;
%SHUT GLUONRESULT3;
MTS;
SIG
```

APPENDIX E - PRINTOUT OF THE AMPLITUDE SQUARED OF THE PROCESS

Here is given the amplitude squared of the process (VII.4), called by the subroutine AMPL of the routine GLUON, whose listing appears in appendix D. The denominators inserted in lines 852 to 887 come from the propagators in the amplitude (V .10) to (VII.12). The color factors in line 896 have been calculated in appendix C. The variables in the numerator are defined as follow:

S = $\hat{s}$

MK2 = m_K^2

MH2 = m_H^2

G1SP1 = $g_1 \cdot p_1$

HSP1 = $h \cdot p_1$

etc.

```
      SUBROUTINE AMPL(S,D1,D2,D3,D4,D5,D6,
     C MK2,MH2,HSP1,HSP2,Q1SH,Q2SH,Q1SP1,
     C Q1SP2,Q2SP1,Q2SP2,P1SP2,HC)
C-----------------------------------------------------------------------
C                 CALCULATES THE AMPLITUDE OF THE PROCESS              1
C-----------------------------------------------------------------------

      REAL*8 S,MK2,MH2,HSP1,HSP2,Q1SH,Q2SH,Q1SP1,Q1SP2,Q2SP1,Q2SP2,P1SP2
      REAL*8 M11,M12,M13,M14,M15,M16,M17,M18,M22,M23,M24,M25,M26,M27,M28
      REAL*8 M33,M34,M35,M36,M37,M38,M44,M45,M46,M47,M48,M55,M56,M57,M58
      REAL*8 M66,M67,M68,M77,M78,M88,D1,D2,D3,D4,D5,D6,HC
      REAL*8 M11A,M12A,M13A,M14A,M15A,M16A,M17A,M18A
      REAL*8 M22A,M23A,M24A,M25A,M26A,M27A,M28A
      REAL*8 M33A,M34A,M35A,M36A,M37A,M38A
      REAL*8 M44A,M45A,M46A,M47A,M48A
      REAL*8 M55A,M56A,M57A,M58A
      REAL*8 M66A,M67A,M68A
      REAL*8 M77A,M78A,M88A
      REAL*8 MA1,MA2,MA3,MA4,MA5

      M11 =(-32.*S**2*MK2**3
     C -8.*S**2*MK2**2*MH2+32.*S**2*MK2**2*(-HSP2+Q1SH+
     C Q1SP2)-8.*S**2*MK2*MH2*Q1SP2+16.*S**2*MK2*(HSP2*Q1SH+2.*Q1SH*
     C Q1SP1+2.*Q1SP1*Q1SP2)-8.*S**2*MH2*Q1SP1*Q1SP2+16.*S**2*HSP2*Q1SH
     C *Q1SP1+64.*S*MK2**2*(-Q1SH*Q2SP1-Q2SH*Q1SP1+2.*Q1SP1*Q2SP1-Q1SP1
     C *Q2SP2-Q1SP2*Q2SP1)+16.*S*MK2*MH2*(2.*Q1SP1*Q2SP1+Q1SP1*Q2SP2+
     C Q1SP2*Q2SP1)+32.*S*MK2*(-HSP2*Q1SH*Q2SP1-HSP2*Q2SH*Q1SP1+4.*HSP2
     C *Q1SP1*Q2SP1-4.*Q1SH*Q1SP1*Q2SP1-4.*Q1SP1*Q1SP2*Q2SP1)+32.*S*MH2
     C *Q1SP1*Q1SP2*Q2SP1-64.*S*HSP2*Q1SH*Q1SP1*Q2SP1+256.*MK2*Q1SP1*
     C Q2SP1*(Q1SH*Q2SP1+Q2SH*Q1SP1+Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*
     C Q1SP1*Q2SP1)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+128.*HSP2*Q1SP1*Q2SP1*(
     C Q1SH*Q2SP1+Q2SH*Q1SP1))/S**2
      M12 =8.*S**3*MK2**2+4.*S**3*MK2*(HSP2+2.*Q1SP1)
     C +4.*S**3*HSP2*Q1SP1+
     C 16.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(HSP2+Q1SP1)+16.*S**2*
     C P1SP2*HSP2*Q1SP1-16.*S**2*MK2**3+8.*S**2*MK2**2*(-HSP2-HSP1+Q1SH
     C -Q2SH-2.*Q2SP1-2.*Q2SP2)+8.*S**2*MK2*(-HSP2*Q1SP2-HSP2*Q2SP1-
     C HSP1*Q1SP1+Q1SH*Q1SP1+Q1SH*Q2SP2-Q2SH*Q1SP2-4.*Q1SP1*Q2SP1)+8.*S
     C **2*Q1SP1*(-HSP2*Q1SP2-2.*HSP2*Q2SP1+Q1SH*Q2SP2-Q2SH*Q1SP2)+16.*
     C S*P1SP2*MK2*(-Q1SH*Q2SP2-Q2SH*Q1SP2-4.*Q1SP1*Q2SP1)+16.*S*P1SP2*
     C Q1SP1*(-4.*HSP2*Q2SP1-Q1SH*Q2SP2-Q2SH*Q1SP2)+16.*S*MK2**2*(Q1SH*
     C Q2SP2+Q2SH*Q1SP2+4.*Q1SP1*Q2SP1-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.
     C *Q1SP2*Q2SP2)
      M12=M12+16.*S*MK2*(2.*HSP2*Q1SP1*Q2SP1-HSP2*Q1SP1*Q2SP2-
     C HSP2*Q1SP2*Q2SP1+2.*HSP1*Q1SP1*Q2SP1+2.*HSP1*Q1SP2*Q2SP2-Q1SH*
     C Q1SP1*Q2SP1-Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1**2+2.*
     C Q2SH*Q1SP1*Q2SP1+Q2SH*Q1SP2**2+Q2SH*Q1SP2*Q2SP1-2.*Q1SP1**2*
     C Q2SP2-2.*Q1SP1*Q1SP2*Q2SP1+4.*Q1SP1*Q2SP1**2+4.*Q1SP1*Q2SP1*
     C Q2SP2)+16.*S*Q1SP1*(-2.*HSP2*Q1SP1*Q2SP2+2.*HSP2*Q2SP1**2+2.*
     C HSP1*Q1SP2*Q2SP2-Q1SH*Q1SP2*Q2SP2-2.*Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP2
     C **2+2.*Q2SH*Q1SP2*Q2SP1)+64.*P1SP2*Q1SP1*Q2SP1*(Q1SH*Q2SP2+Q2SH*
     C Q1SP2)+64.*MK2*Q1SP1*Q2SP1*(-Q1SH*Q2SP2-Q2SH*Q1SP2+2.*Q1SP1*
     C Q2SP2+2.*Q1SP2*Q2SP1-4.*Q1SP2*Q2SP2)+32.*Q1SP1*(2.*HSP2*Q1SP1*
     C Q2SP1*Q2SP2+2.*HSP2*Q1SP2*Q2SP1**2-4.*HSP1*Q1SP2*Q2SP1*Q2SP2+
     C Q1SH*Q1SP1*Q2SP2**2+Q1SH*Q1SP2*Q2SP1*Q2SP2+2.*Q1SH*Q2SP1**2*
     C Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP2-Q2SH*Q1SP2**2*Q2SP1-2.*Q2SH*Q1SP2*
     C Q2SP1**2)
```

```
      M12=M12/S**2
      M13 =-2.*S**3*P1SP2*MH2
     C +8.*S**3*MK2**2+4.*S**3*MK2*(HSP2+HSP1)+4.*S**
     C 3*HSP2*HSP1+16.*S**2*P1SP2*MK2**2+4.*S**2*P1SP2*MK2*MH2+16.*S**2
     C *P1SP2*MK2*(HSP2+HSP1)+4.*S**2*P1SP2*MH2*(Q1SP2+Q2SP1)+8.*S**2*
     C P1SP2*(2.*HSP2*HSP1+Q1SH*Q2SH)-16.*S**2*MK2**3+4.*S**2*MK2**2*
     C MH2-(16.*S**2*MK2**2)*(HSP2+HSP1+Q1SP1+Q2SP2)+4.*S**2*MK2*(-2.*
     C HSP2**2-2.*HSP2*HSP1-HSP2*Q1SH+HSP2*Q2SH-2.*HSP2*Q1SP1-2.*HSP2*
     C Q1SP2-2.*HSP1**2+HSP1*Q1SH-HSP1*Q2SH-2.*HSP1*Q2SP1-2.*HSP1*Q2SP2
     C -2.*Q1SH*Q2SP1+2.*Q1SH*Q2SP2+2.*Q2SH*Q1SP1-2.*Q2SH*Q1SP2)+4.*S**
     C 2*MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(8.*S**2)*(HSP2*HSP1*Q1SP2+HSP2*
     C HSP1*Q2SP1+HSP2*Q1SH*Q2SP1+HSP1*Q2SH*Q1SP2)+32.*S*P1SP2*MK2*(
     C Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(8.*S*P1SP2*MH2)*(Q1SP1*Q2SP2+Q1SP2*
     C Q2SP1)
      M13=M13-(16.*S*P1SP2)*(HSP2*Q1SH*Q2SP1+HSP2*Q2SH*Q1SP1+HSP1*Q1SH*
     C Q2SP2+HSP1*Q2SH*Q1SP2+Q1SH*Q2SH*Q1SP2+Q1SH*Q2SH*Q2SP1)+32.*S*MK2
     C **2*(Q1SH*Q2SP1+Q1SH*Q2SP2+Q2SH*Q1SP1+Q2SH*Q1SP2+2.*Q1SP1*Q2SP1-
     C Q1SP1*Q2SP2-Q1SP2*Q2SP1+2.*Q1SP2*Q2SP2)+8.*S*MK2*MH2*(-2.*Q1SP1*
     C Q2SP1-Q1SP1*Q2SP2-Q1SP2*Q2SP1-2.*Q1SP2*Q2SP2)+8.*S*MK2*(2.*HSP2*
     C Q1SH*Q2SP1+2.*HSP2*Q1SH*Q2SP2+2.*HSP2*Q2SH*Q1SP1+2.*HSP2*Q2SH*
     C Q1SP2+8.*HSP2*Q1SP1*Q2SP1-4.*HSP2*Q1SP1*Q2SP2-4.*HSP2*Q1SP2*
     C Q2SP1+2.*HSP1*Q1SH*Q2SP1+2.*HSP1*Q1SH*Q2SP2+2.*HSP1*Q2SH*Q1SP1+
     C 2.*HSP1*Q2SH*Q1SP2-4.*HSP1*Q1SP1*Q2SP2-4.*HSP1*Q1SP2*Q2SP1+8.*
     C HSP1*Q1SP2*Q2SP2-Q1SH**2*Q2SP1+Q1SH**2*Q2SP2+Q1SH*Q2SH*Q1SP1+
     C Q1SH*Q2SH*Q1SP2+Q1SH*Q2SH*Q2SP1+Q1SH*Q2SH*Q2SP2+2.*Q1SH*Q1SP1*
     C Q2SP2+2.*Q1SH*Q1SP2*Q2SP2+2.*Q1SH*Q2SP1**2+2.*Q1SH*Q2SP1*Q2SP2+
     C Q2SH**2*Q1SP1-Q2SH**2*Q1SP2+2.*Q2SH*Q1SP1*Q1SP2+2.*Q2SH*Q1SP1*
     C Q2SP1+2.*Q2SH*Q1SP1*Q2SP2+2.*Q2SH*Q1SP2**2+8.*Q1SP1*Q1SP2*Q2SP2+
     C 8.*Q1SP1*Q2SP1*Q2SP2)
      M13=M13-(8.*S*MH2)*(Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP1*
     C Q2SP2+Q1SP2**2*Q2SP1+Q1SP2*Q2SP1**2)+16.*S*(2.*HSP2**2*Q1SP1*
     C Q2SP1-HSP2*HSP1*Q1SP1*Q2SP2-HSP2*HSP1*Q1SP2*Q2SP1+HSP2*Q1SH*
     C Q1SP2*Q2SP1+HSP2*Q1SH*Q2SP1**2+2.*HSP1**2*Q1SP2*Q2SP2+HSP1*Q2SH*
     C Q1SP2**2+HSP1*Q2SH*Q1SP2*Q2SP1)+32.*P1SP2*(Q1SH**2*Q2SP1*Q2SP2+
     C Q1SH*Q2SH*Q1SP1*Q2SP2+Q1SH*Q2SH*Q1SP2*Q2SP1+Q2SH**2*Q1SP1*Q1SP2)
     C +32.*MK2*(-Q1SH**2*Q2SP1*Q2SP2-Q1SH*Q2SH*Q1SP1*Q2SP2-Q1SH*Q2SH*
     C Q1SP2*Q2SP1-4.*Q1SH*Q1SP1*Q2SP1*Q2SP2-4.*Q1SH*Q1SP2*Q2SP1*Q2SP2-
     C Q2SH**2*Q1SP1*Q1SP2-4.*Q2SH*Q1SP1*Q1SP2*Q2SP1-4.*Q2SH*Q1SP1*
     C Q1SP2*Q2SP2-2.*Q1SP1**2*Q2SP2**2-12.*Q1SP1*Q1SP2*Q2SP1*Q2SP2-2.*
     C Q1SP2**2*Q2SP1**2)+16.*MH2*(Q1SP1**2*Q2SP2**2+6.*Q1SP1*Q1SP2*
     C Q2SP1*Q2SP2+Q1SP2**2*Q2SP1**2)-64.*(HSP2*Q1SH*Q1SP1*Q2SP1*Q2SP2+
     C HSP2*Q2SH*Q1SP1*Q1SP2*Q2SP1+HSP1*Q1SH*Q1SP2*Q2SP1*Q2SP2+HSP1*
     C Q2SH*Q1SP1*Q1SP2*Q2SP2)
      M13=M13/S**2
      M14 =8.*S**2*P1SP2**2*MH2
     C +32.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(
     C HSP2+HSP1-2.*Q1SH-Q1SP1-Q1SP2)+8.*S**2*P1SP2*Q1SH**2+4.*S**2*MK2
     C *MH2*(Q1SP1+Q1SP2)+8.*S**2*MK2*(-HSP2*Q1SH+HSP2*Q1SP1-HSP2*Q1SP2
     C -HSP1*Q1SH-HSP1*Q1SP1+HSP1*Q1SP2)+8.*S**2*MH2*Q1SP1*Q1SP2-(8.*S
     C **2*Q1SH)*(HSP2*Q1SP1+HSP1*Q1SP2)-32.*S*P1SP2**2*Q1SH*Q2SH+32.*S
     C *P1SP2*MK2*(Q1SH*Q2SH+Q1SH*Q2SP1+Q1SH*Q2SP2+Q2SH*Q1SP1+Q2SH*
     C Q1SP2+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)-(32.*S*P1SP2*MH2)*(Q1SP1*
     C Q2SP2+Q1SP2*Q2SP1)+16.*S*P1SP2*(HSP2*Q1SH*Q2SP1+HSP2*Q2SH*Q1SP1+
     C HSP1*Q1SH*Q2SP2+HSP1*Q2SH*Q1SP2-Q1SH**2*Q2SP1-Q1SH**2*Q2SP2)-(
     C 64.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)
      M14=M14+16.*S*MK2*(-2.*HSP2*Q1SP1
     C *Q2SP2-2.*HSP2*Q1SP2*Q2SP1-2.*HSP1*Q1SP1*Q2SP2-2.*HSP1*Q1SP2*
```

```
C Q2SP1+Q1SH**2*Q2SP1+Q1SH**2*Q2SP2+Q1SH*Q1SP1*Q2SP1+3.*Q1SH*Q1SP1
C *Q2SP2+3.*Q1SH*Q1SP2*Q2SP1+Q1SH*Q1SP2*Q2SP2+Q2SH*Q1SP1**2-2.*
C Q2SH*Q1SP1*Q1SP2+Q2SH*Q1SP2**2+2.*Q1SP1**2*Q2SP2+2.*Q1SP1*Q1SP2*
C Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP2**2*Q2SP1)-(16.*S*MH2*Q1SP1*
C Q1SP2)*(Q2SP1+Q2SP2)+16.*S*Q1SH*(HSP2*Q1SP1*Q2SP1+HSP2*Q1SP1*
C Q2SP2+HSP1*Q1SP2*Q2SP1+HSP1*Q1SP2*Q2SP2)+64.*P1SP2*Q1SH*Q2SH*(
C Q1SP1*Q2SP2+Q1SP2*Q2SP1)+64.*MK2*(-Q1SH*Q2SH*Q1SP1*Q2SP2-Q1SH*
C Q2SH*Q1SP2*Q2SP1-Q1SH*Q1SP1*Q2SP1*Q2SP2-Q1SH*Q1SP1*Q2SP2**2-Q1SH
C *Q1SP2*Q2SP1**2-Q1SH*Q1SP2*Q2SP1*Q2SP2-Q2SH*Q1SP1**2*Q2SP2-Q2SH*
C Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q1SP2*Q2SP2-Q2SH*Q1SP2**2*Q2SP1-2.*
C Q1SP1**2*Q2SP2**2-4.*Q1SP1*Q1SP2*Q2SP1*Q2SP2-2.*Q1SP2**2*Q2SP1**
C 2)+32.*MH2*(Q1SP1**2*Q2SP2**2+2.*Q1SP1*Q1SP2*Q2SP1*Q2SP2+Q1SP2**
C 2*Q2SP1**2)-32.*(HSP2*Q1SH*Q1SP1*Q2SP1*Q2SP2+HSP2*Q1SH*Q1SP2*
C Q2SP1**2+HSP2*Q2SH*Q1SP1**2*Q2SP2+HSP2*Q2SH*Q1SP1*Q1SP2*Q2SP1+
C HSP1*Q1SH*Q1SP1*Q2SP2**2+HSP1*Q1SH*Q1SP2*Q2SP1*Q2SP2+HSP1*Q2SH*
C Q1SP1*Q1SP2*Q2SP2+HSP1*Q2SH*Q1SP2**2*Q2SP1)
 M14=M14/S**2
 M15 =-8.*S**3*P1SP2*MK2
C +4.*S**3*P1SP2*Q1SH-4.*S**3*MK2*HSP1-4.*S**3*
C HSP1*Q1SP2-16.*S**2*P1SP2**2*MK2+8.*S**2*P1SP2**2*Q1SH+16.*S**2*
C P1SP2*MK2**2+8.*S**2*P1SP2*MK2*(HSP2-HSP1-Q1SH+Q2SH+Q1SP1-Q1SP2+
C 2.*Q2SP1+2.*Q2SP2)+8.*S**2*P1SP2*(HSP2*Q2SP1-HSP1*Q1SP2-Q1SH*
C Q1SP1-Q1SH*Q2SP1-Q1SH*Q2SP2)-(8.*S**2*MK2**2)*(Q1SH+Q1SP1+Q1SP2)
C +4.*S**2*MK2*(-HSP2*Q1SP1+HSP2*Q1SP2+HSP1*Q1SP1+HSP1*Q1SP2+2.*
C Q1SH*Q2SP1+2.*Q2SH*Q1SP1+4.*Q1SP1*Q2SP2+4.*Q1SP2*Q2SP1)+8.*S**2*
C (HSP1*Q1SP1*Q1SP2+HSP1*Q1SP2*Q2SP1+HSP1*Q1SP2*Q2SP2-Q1SH*Q1SP1*
C Q2SP2+Q2SH*Q1SP1*Q1SP2)-(16.*S*P1SP2**2)*(Q1SH*Q2SP1+Q2SH*Q1SP1)
C +16.*S*P1SP2*MK2*(Q1SH*Q2SP1+Q2SH*Q1SP1-4.*Q1SP1*Q2SP1+4.*Q1SP1*
C Q2SP2+4.*Q1SP2*Q2SP1)+16.*S*P1SP2*(-2.*HSP2*Q1SP1*Q2SP1+HSP1*
C Q1SP1*Q2SP2+HSP1*Q1SP2*Q2SP1+Q1SH*Q1SP1*Q2SP1-Q1SH*Q1SP1*Q2SP2-
C Q1SH*Q1SP2*Q2SP1+Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1*Q1SP2-Q2SH*Q1SP2*
C Q2SP1)
 M15=M15-(32.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+8.*S*MK2*(-2.*
C HSP2*Q1SP1*Q2SP2-2.*HSP2*Q1SP2*Q2SP1+2.*HSP1*Q1SP1*Q2SP2+2.*HSP1
C *Q1SP2*Q2SP1+3.*Q1SH*Q1SP1*Q2SP1+3.*Q1SH*Q1SP1*Q2SP2+Q1SH*Q1SP2*
C Q2SP1+Q1SH*Q1SP2*Q2SP2-Q2SH*Q1SP1**2-2.*Q2SH*Q1SP1*Q1SP2-2.*Q2SH
C *Q1SP1*Q2SP2-Q2SH*Q1SP2**2-2.*Q2SH*Q1SP2*Q2SP1+4.*Q1SP1**2*Q2SP1
C -2.*Q1SP1**2*Q2SP2+2.*Q1SP1*Q1SP2*Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2-4.*
C Q1SP1*Q2SP1*Q2SP2-4.*Q1SP1*Q2SP2**2+2.*Q1SP2**2*Q2SP1-4.*Q1SP2*
C Q2SP1**2-4.*Q1SP2*Q2SP1*Q2SP2)+16.*S*(HSP2*Q1SP1**2*Q2SP1-HSP2*
C Q1SP1*Q1SP2*Q2SP1-HSP2*Q1SP1*Q2SP1*Q2SP2-HSP2*Q1SP2*Q2SP1**2-
C HSP1*Q1SP1*Q1SP2*Q2SP1+HSP1*Q1SP2**2*Q2SP1+Q1SH*Q1SP1**2*Q2SP2+
C Q1SH*Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP1*Q2SP2**2-Q2SH*Q1SP1**2*Q1SP2-
C Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q1SP2*Q2SP2)+32.*P1SP2*(Q1SH*
C Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1**2+Q2SH*Q1SP1**2*Q2SP2+Q2SH*
C Q1SP1*Q1SP2*Q2SP1)
 M15=M15+32.*MK2*(-Q1SH*Q1SP1*Q2SP1*Q2SP2-Q1SH*Q1SP2*
C Q2SP1**2-Q2SH*Q1SP1**2*Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP1+4.*Q1SP1**2*
C Q2SP1*Q2SP2-2.*Q1SP1**2*Q2SP2**2+4.*Q1SP1*Q1SP2*Q2SP1**2-4.*
C Q1SP1*Q1SP2*Q2SP1*Q2SP2-2.*Q1SP2**2*Q2SP1**2)+32.*(2.*HSP2*Q1SP1
C **2*Q2SP1*Q2SP2+2.*HSP2*Q1SP1*Q1SP2*Q2SP1**2-HSP1*Q1SP1**2*Q2SP2
C **2-2.*HSP1*Q1SP1*Q1SP2*Q2SP1*Q2SP2-HSP1*Q1SP2**2*Q2SP1**2-2.*
C Q1SH*Q1SP1**2*Q2SP1*Q2SP2-Q1SH*Q1SP1*Q2SP1*Q2SP2**2-Q1SH*Q1SP2*
C Q2SP1**2*Q2SP2+2.*Q2SH*Q1SP1**2*Q1SP2*Q2SP1+Q2SH*Q1SP1*Q1SP2*
C Q2SP1*Q2SP2+Q2SH*Q1SP2**2*Q2SP1**2)
 M15=M15/S**2
 M16 =-8.*S**3*P1SP2*MK2
```

```
      C +2.*S**3*P1SP2*MH2-8.*S**3*MK2*HSP1-4.*S**3*
      C HSP2*HSP1+16.*S**2*P1SP2*MK2**2-4.*S**2*P1SP2*MK2*MH2+16.*S**2*
      C P1SP2*MK2*(Q1SP1+Q2SP1)-(4.*S**2*P1SP2*MH2)*(Q1SP1+Q2SP1)-16.*S
      C **2*MK2**3-4.*S**2*MK2**2*MH2+16.*S**2*MK2**2*(-HSP2+HSP1)+8.*S
      C **2*MK2*(HSP2*HSP1+2.*HSP1*Q1SP1+2.*HSP1*Q2SP1+2.*Q1SH*Q2SP1+2.*
      C Q2SH*Q1SP1+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)-(4.*S**2*MH2)*(Q1SP1*
      C Q2SP2+Q1SP2*Q2SP1)+8.*S**2*HSP2*(HSP1*Q1SP1+HSP1*Q2SP1+Q1SH*
      C Q2SP1+Q2SH*Q1SP1)-64.*S*P1SP2*MK2*Q1SP1*Q2SP1+16.*S*P1SP2*MH2*
      C Q1SP1*Q2SP1+64.*S*MK2**2*(-Q1SH*Q2SP1-Q2SH*Q1SP1+Q1SP1*Q2SP1-
      C Q1SP1*Q2SP2-Q1SP2*Q2SP1)+16.*S*MK2*MH2*(Q1SP1*Q2SP1+Q1SP1*Q2SP2+
      C Q1SP2*Q2SP1)
       M16=M16+32.*S*MK2*(-HSP2*Q1SH*Q2SP1-HSP2*Q2SH*Q1SP1+2.*HSP2
      C *Q1SP1*Q2SP1-2.*HSP1*Q1SP1*Q2SP1-Q1SH*Q1SP1*Q2SP1-Q1SH*Q2SP1**2-
      C Q2SH*Q1SP1**2-Q2SH*Q1SP1*Q2SP1-Q1SP1**2*Q2SP2-Q1SP1*Q1SP2*Q2SP1-
      C Q1SP1*Q2SP1*Q2SP2-Q1SP2*Q2SP1**2)+8.*S*MH2*(Q1SP1**2*Q2SP2+Q1SP1
      C *Q1SP2*Q2SP1+Q1SP1*Q2SP1*Q2SP2+Q1SP2*Q2SP1**2)+16.*S*HSP2*(-2.*
      C HSP1*Q1SP1*Q2SP1-Q1SH*Q1SP1*Q2SP1-Q1SH*Q2SP1**2-Q2SH*Q1SP1**2-
      C Q2SH*Q1SP1*Q2SP1)+256.*MK2*Q1SP1*Q2SP1*(Q1SH*Q2SP1+Q2SH*Q1SP1+
      C Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*Q1SP1*Q2SP1)*(Q1SP1*Q2SP2+
      C Q1SP2*Q2SP1)+128.*HSP2*Q1SP1*Q2SP1*(Q1SH*Q2SP1+Q2SH*Q1SP1)
       M16=M16/S**2
       M17 =4.*S**2*P1SP2*MK2
      C -S**2*P1SP2*MH2+4.*S**2*MK2**2+S**2*MK2*MH2+4.*
      C S**2*MK2*(HSP2+HSP1)+2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(Q1SH-Q2SH
      C +Q1SP1-Q2SP1)+2.*S*P1SP2*MH2*(-Q1SP1+Q2SP1)+4.*S*P1SP2*Q1SH*(-
      C Q1SH+Q2SH)+8.*S*MK2**2*(Q1SP2-Q2SP2)+2.*S*MK2*MH2*(-Q1SP2+Q2SP2)
      C +4.*S*MK2*(HSP1*Q1SH-HSP1*Q2SH+2.*HSP1*Q1SP1-2.*HSP1*Q2SP1+Q1SH
      C **2-Q1SH*Q2SH+2.*Q1SH*Q1SP1+2.*Q1SH*Q1SP2-2.*Q1SH*Q2SP1-2.*Q2SH*
      C Q1SP2+4.*Q1SP1*Q1SP2-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1)+2.*S*MH2*(-
      C 2.*Q1SP1*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1)+4.*S*(HSP2*HSP1*Q1SP1-
      C HSP2*HSP1*Q2SP1+HSP2*Q1SH*Q1SP1-HSP2*Q1SH*Q2SP1+HSP1*Q1SH*Q1SP2-
      C HSP1*Q2SH*Q1SP2)+8.*P1SP2*(Q1SH**2*Q2SP1+Q1SH*Q2SH*Q1SP1-Q1SH*
      C Q2SH*Q2SP1-Q2SH**2*Q1SP1)
       M17=M17+8.*MK2*(-Q1SH**2*Q2SP1-Q1SH*Q2SH*Q1SP1
      C +Q1SH*Q2SH*Q2SP1-2.*Q1SH*Q1SP1*Q2SP1-2.*Q1SH*Q1SP1*Q2SP2-2.*Q1SH
      C *Q1SP2*Q2SP1+2.*Q1SH*Q2SP1**2+Q2SH**2*Q1SP1-2.*Q2SH*Q1SP1**2+2.*
      C Q2SH*Q1SP1*Q2SP1+2.*Q2SH*Q1SP1*Q2SP2+2.*Q2SH*Q1SP2*Q2SP1-2.*
      C Q1SP1**2*Q2SP2-6.*Q1SP1*Q1SP2*Q2SP1+6.*Q1SP1*Q2SP1*Q2SP2+2.*
      C Q1SP2*Q2SP1**2)+4.*MH2*(Q1SP1**2*Q2SP2+3.*Q1SP1*Q1SP2*Q2SP1-3.*
      C Q1SP1*Q2SP1*Q2SP2-Q1SP2*Q2SP1**2)+8.*(-HSP2*Q1SH*Q1SP1*Q2SP1+
      C HSP2*Q1SH*Q2SP1**2-HSP2*Q2SH*Q1SP1**2+HSP2*Q2SH*Q1SP1*Q2SP1-HSP1
      C *Q1SH*Q1SP1*Q2SP2-HSP1*Q1SH*Q1SP2*Q2SP1+HSP1*Q2SH*Q1SP1*Q2SP2+
      C HSP1*Q2SH*Q1SP2*Q2SP1)
       M17=M17/S
       M18 =(4.*S**2*P1SP2*MK2
      C -S**2*P1SP2*MH2+4.*S**2*MK2**2+S**2*MK2*MH2+4.*
      C S**2*MK2*(HSP2+HSP1)+2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(Q1SP1-
      C Q2SP1)+2.*S*P1SP2*MH2*(-Q1SP1+Q2SP1)+8.*S*MK2**2*(Q1SH-Q2SH+
      C Q1SP2-Q2SP2)+2.*S*MK2*MH2*(-Q1SP2+Q2SP2)+4.*S*MK2*(HSP2*Q1SH-
      C HSP2*Q2SH+2.*HSP1*Q1SP1-2.*HSP1*Q2SP1+4.*Q1SH*Q1SP1-2.*Q1SH*
      C Q2SP1-2.*Q2SH*Q1SP1+4.*Q1SP1*Q1SP2-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1
      C )+2.*S*MH2*(-2.*Q1SP1*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1)+4.*S*HSP2*(
      C HSP1*Q1SP1-HSP1*Q2SP1+2.*Q1SH*Q1SP1-Q1SH*Q2SP1-Q2SH*Q1SP1)+16.*
      C MK2*(-3.*Q1SH*Q1SP1*Q2SP1+Q1SH*Q2SP1**2-Q2SH*Q1SP1**2+3.*Q2SH*
      C Q1SP1*Q2SP1-Q1SP1**2*Q2SP2-3.*Q1SP1*Q1SP2*Q2SP1+3.*Q1SP1*Q2SP1*
      C Q2SP2+Q1SP2*Q2SP1**2)+4.*MH2*(Q1SP1**2*Q2SP2+3.*Q1SP1*Q1SP2*
      C Q2SP1-3.*Q1SP1*Q2SP1*Q2SP2-Q1SP2*Q2SP1**2)+8.*HSP2*(-3.*Q1SH*
```

```
     C Q1SP1*Q2SP1+Q1SH*Q2SP1**2-Q2SH*Q1SP1**2+3.*Q2SH*Q1SP1*Q2SP1))/S
      M22 =(8.*S**3*MK2**2
     C +8.*S**3*MK2*(Q1SP1+Q2SP2)+8.*S**3*Q1SP1*Q2SP2+16.
     C *S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(Q1SP1+Q2SP2)+16.*S**2*
     C P1SP2*Q1SP1*Q2SP2-16.*S**2*MK2**3-(16.*S**2*MK2**2)*(Q1SP2+Q2SP1
     C )+16.*S**2*MK2*(-Q1SP1*Q1SP2-2.*Q1SP1*Q2SP1-2.*Q1SP2*Q2SP2-Q2SP1
     C *Q2SP2)-(32.*S**2*Q1SP1*Q2SP2)*(Q1SP2+Q2SP1)-(64.*S*P1SP2*MK2)*(
     C Q1SP1*Q2SP1+Q1SP2*Q2SP2)-(64.*S*P1SP2*Q1SP1*Q2SP2)*(Q1SP2+Q2SP1)
     C +64.*S*MK2**2*(Q1SP1*Q2SP1+Q1SP2*Q2SP2)+32.*S*MK2*(-Q1SP1**2*
     C Q2SP2+Q1SP1*Q1SP2*Q2SP1+2.*Q1SP1*Q2SP1**2-Q1SP1*Q2SP2**2+2.*
     C Q1SP2**2*Q2SP2+Q1SP2*Q2SP1*Q2SP2)+32.*S*Q1SP1*Q2SP2*(-Q1SP1*
     C Q2SP2+2.*Q1SP2**2+3.*Q1SP2*Q2SP1+2.*Q2SP1**2)+256.*P1SP2*Q1SP1*
     C Q1SP2*Q2SP1*Q2SP2-256.*MK2*Q1SP1*Q1SP2*Q2SP1*Q2SP2+128.*Q1SP1*
     C Q2SP2*(Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP1*Q2SP2-Q1SP2**2*Q2SP1-Q1SP2*
     C Q2SP1**2))/S**2
      M23 =8.*S**3*MK2**2
     C +4.*S**3*MK2*(HSP1+2.*Q2SP2)+4.*S**3*HSP1*Q2SP2+
     C 16.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(HSP1+Q2SP2)+16.*S**2*
     C P1SP2*HSP1*Q2SP2-16.*S**2*MK2**3+8.*S**2*MK2**2*(-HSP2-HSP1-Q1SH
     C +Q2SH-2.*Q1SP1-2.*Q1SP2)+8.*S**2*MK2*(-HSP2*Q2SP2-HSP1*Q1SP2-
     C HSP1*Q2SP1-Q1SH*Q2SP1+Q2SH*Q1SP1+Q2SH*Q2SP2-4.*Q1SP2*Q2SP2)+8.*S
     C **2*Q2SP2*(-2.*HSP1*Q1SP2-HSP1*Q2SP1-Q1SH*Q2SP1+Q2SH*Q1SP1)+16.*
     C S*P1SP2*MK2*(-Q1SH*Q2SP1-Q2SH*Q1SP1-4.*Q1SP2*Q2SP2)+16.*S*P1SP2*
     C Q2SP2*(-4.*HSP1*Q1SP2-Q1SH*Q2SP1-Q2SH*Q1SP1)+16.*S*MK2**2*(Q1SH*
     C Q2SP1+Q2SH*Q1SP1+4.*Q1SP1*Q2SP1-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.
     C *Q1SP2*Q2SP2)+16.*S*MK2*(2.*HSP2*Q1SP1*Q2SP1+2.*HSP2*Q1SP2*Q2SP2
     C -HSP1*Q1SP1*Q2SP2-HSP1*Q1SP2*Q2SP1+2.*HSP1*Q1SP2*Q2SP2+Q1SH*
     C Q1SP2*Q2SP1+2.*Q1SH*Q1SP2*Q2SP2+Q1SH*Q2SP1**2+Q1SH*Q2SP2**2-Q2SH
     C *Q1SP1*Q1SP2-Q2SH*Q1SP1*Q2SP1-Q2SH*Q1SP2*Q2SP2+4.*Q1SP1*Q1SP2*
     C Q2SP2-2.*Q1SP1*Q2SP2**2+4.*Q1SP2**2*Q2SP2-2.*Q1SP2*Q2SP1*Q2SP2)+
     C 16.*S*Q2SP2*(2.*HSP2*Q1SP1*Q2SP1-2.*HSP1*Q1SP1*Q2SP2+2.*HSP1*
     C Q1SP2**2+2.*Q1SH*Q1SP2*Q2SP1+Q1SH*Q2SP1**2-2.*Q2SH*Q1SP1*Q1SP2-
     C Q2SH*Q1SP1*Q2SP1)
      M23=M23+64.*P1SP2*Q1SP2*Q2SP2*(Q1SH*Q2SP1+Q2SH*Q1SP1)+
     C 64.*MK2*Q1SP2*Q2SP2*(-Q1SH*Q2SP1-Q2SH*Q1SP1-4.*Q1SP1*Q2SP1+2.*
     C Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)+32.*Q2SP2*(-4.*HSP2*Q1SP1*Q1SP2*
     C Q2SP1+2.*HSP1*Q1SP1*Q1SP2*Q2SP2+2.*HSP1*Q1SP2**2*Q2SP1-Q1SH*
     C Q1SP1*Q2SP1*Q2SP2-2.*Q1SH*Q1SP2**2*Q2SP1-Q1SH*Q1SP2*Q2SP1**2+
     C Q2SH*Q1SP1**2*Q2SP2+2.*Q2SH*Q1SP1*Q1SP2**2+Q2SH*Q1SP1*Q1SP2*
     C Q2SP1)
      M23=M23/S**2
      M24 =-8.*S**3*P1SP2*MK2
     C +4.*S**3*P1SP2*Q1SH-4.*S**3*MK2*HSP2-4.*S**3*
     C HSP2*Q1SP1-16.*S**2*P1SP2**2*MK2+8.*S**2*P1SP2**2*Q1SH+16.*S**2*
     C P1SP2*MK2**2+8.*S**2*P1SP2*MK2*(-HSP2+HSP1-Q1SH+Q2SH-Q1SP1+Q1SP2
     C +2.*Q2SP1+2.*Q2SP2)+8.*S**2*P1SP2*(-HSP2*Q1SP1+HSP1*Q2SP2-Q1SH*
     C Q1SP2-Q1SH*Q2SP1-Q1SH*Q2SP2)-(8.*S**2*MK2**2)*(Q1SH+Q1SP1+Q1SP2)
     C +4.*S**2*MK2*(HSP2*Q1SP1+HSP2*Q1SP2+HSP1*Q1SP1-HSP1*Q1SP2+2.*
     C Q1SH*Q2SP2+2.*Q2SH*Q1SP2+4.*Q1SP1*Q2SP2+4.*Q1SP2*Q2SP1)+8.*S**2*
     C (HSP2*Q1SP1*Q1SP2+HSP2*Q1SP1*Q2SP1+HSP2*Q1SP1*Q2SP2-Q1SH*Q1SP2*
     C Q2SP1+Q2SH*Q1SP1*Q1SP2)-(16.*S*P1SP2**2)*(Q1SH*Q2SP2+Q2SH*Q1SP2)
     C +16.*S*P1SP2*MK2*(Q1SH*Q2SP2+Q2SH*Q1SP2+4.*Q1SP1*Q2SP2+4.*Q1SP2*
     C Q2SP1-4.*Q1SP2*Q2SP2)+16.*S*P1SP2*(HSP2*Q1SP1*Q2SP2+HSP2*Q1SP2*
     C Q2SP1-2.*HSP1*Q1SP2*Q2SP2-Q1SH*Q1SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q1SH
     C *Q1SP2*Q2SP2+Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1*Q1SP2-Q2SH*Q1SP1*Q2SP2)
     C -(32.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)
      M24=M24+8.*S*MK2*(2.*HSP2*
```

```
     C Q1SP1*Q2SP2+2.*HSP2*Q1SP2*Q2SP1-2.*HSP1*Q1SP1*Q2SP2-2.*HSP1*
     C Q1SP2*Q2SP1+Q1SH*Q1SP1*Q2SP1+Q1SH*Q1SP1*Q2SP2+3.*Q1SH*Q1SP2*
     C Q2SP1+3.*Q1SH*Q1SP2*Q2SP2-Q2SH*Q1SP1**2-2.*Q2SH*Q1SP1*Q1SP2-2.*
     C Q2SH*Q1SP1*Q2SP2-Q2SH*Q1SP2**2-2.*Q2SH*Q1SP2*Q2SP1+2.*Q1SP1**2*
     C Q2SP2+2.*Q1SP1*Q1SP2*Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2-4.*Q1SP1*Q2SP1*
     C Q2SP2-4.*Q1SP1*Q2SP2**2-2.*Q1SP2**2*Q2SP1+4.*Q1SP2**2*Q2SP2-4.*
     C Q1SP2*Q2SP1**2-4.*Q1SP2*Q2SP1*Q2SP2)+16.*S*(HSP2*Q1SP1**2*Q2SP2-
     C HSP2*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*Q2SP2**
     C 2+HSP1*Q1SP2**2*Q2SP2-HSP1*Q1SP2*Q2SP1*Q2SP2+Q1SH*Q1SP2**2*Q2SP1
     C +Q1SH*Q1SP2*Q2SP1**2+Q1SH*Q1SP2*Q2SP1*Q2SP2-Q2SH*Q1SP1*Q1SP2**2-
     C Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q1SP2*Q2SP2)+32.*P1SP2*(Q1SH*
     C Q1SP1*Q2SP2**2+Q1SH*Q1SP2*Q2SP1*Q2SP2+Q2SH*Q1SP1*Q1SP2*Q2SP2+
     C Q2SH*Q1SP2**2*Q2SP1)
      M24=M24+32.*MK2*(-Q1SH*Q1SP1*Q2SP2**2-Q1SH*Q1SP2*
     C Q2SP1*Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP2-Q2SH*Q1SP2**2*Q2SP1-2.*Q1SP1
     C **2*Q2SP2**2-4.*Q1SP1*Q1SP2*Q2SP1*Q2SP2+4.*Q1SP1*Q1SP2*Q2SP2**2-
     C 2.*Q1SP2**2*Q2SP1**2+4.*Q1SP2**2*Q2SP1*Q2SP2)+32.*(-HSP2
     C *Q1SP1**2
     C *Q2SP2**2-2.*HSP2*Q1SP1*Q1SP2*Q2SP1*Q2SP2-HSP2*Q1SP2**2*Q2SP1**2
     C +2.*HSP1*Q1SP1*Q1SP2*Q2SP2**2+2.*HSP1*Q1SP2**2*Q2SP1*Q2SP2-Q1SH*
     C Q1SP1*Q2SP1*Q2SP2**2-2.*Q1SH*Q1SP2**2*Q2SP1*Q2SP2-Q1SH*Q1SP2*
     C Q2SP1**2*Q2SP2+Q2SH*Q1SP1**2*Q2SP2**2+2.*Q2SH*Q1SP1*Q1SP2**2*
     C Q2SP2+Q2SH*Q1SP1*Q1SP2*Q2SP1*Q2SP2)
      M24=M24/S**2
      M25 =2.*S**4*P1SP2+8.*S**3*P1SP2**2
     C -8.*S**3*P1SP2*MK2-(4.*S**3*P1SP2)
     C *(Q1SP1+Q1SP2+Q2SP1+Q2SP2)-8.*S**3*MK2**2-(4.*S**3)*(Q1SP1*Q2SP2
     C +Q1SP2*Q2SP1)+16.*S**2*P1SP2**3-16.*S**2*P1SP2**2*MK2-(8.*S**2*
     C P1SP2**2)*(Q1SP1+Q1SP2+Q2SP1+Q2SP2)+8.*S**2*P1SP2*MK2*(Q1SP1+
     C Q1SP2+Q2SP1+Q2SP2)+8.*S**2*P1SP2*(2.*Q1SP1*Q1SP2+Q1SP1*Q2SP1-3.*
     C Q1SP1*Q2SP2-3.*Q1SP2*Q2SP1+Q1SP2*Q2SP2+2.*Q2SP1*Q2SP2)+8.*S**2*
     C MK2*(3.*Q1SP1*Q2SP1+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1+3.*Q1SP2*Q2SP2
     C )+8.*S**2*(Q1SP1**2*Q2SP2+Q1SP1*Q1SP2*Q2SP1+Q1SP1*Q1SP2*Q2SP2+
     C Q1SP1*Q2SP1*Q2SP2+Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1+Q1SP2*Q2SP1**2+
     C Q1SP2*Q2SP1*Q2SP2)-(64.*S*P1SP2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+
     C 64.*S*P1SP2*MK2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+32.*S*P1SP2*(Q1SP1**2*
     C Q2SP2+Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1+Q1SP2*Q2SP1**2)-(16.*S*MK2)*
     C (Q1SP1**2*Q2SP2+Q1SP1*Q1SP2*Q2SP1+Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP1*
     C Q2SP2+Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1+Q1SP2*Q2SP1**2+Q1SP2*Q2SP1*
     C Q2SP2)
      M25=M25+16.*S*(-2.*Q1SP1**2*Q1SP2*Q2SP2-Q1SP1**2*Q2SP1*Q2SP2+
     C Q1SP1**2*Q2SP2**2-2.*Q1SP1*Q1SP2**2*Q2SP1-Q1SP1*Q1SP2*Q2SP1**2-
     C 2.*Q1SP1*Q1SP2*Q2SP1*Q2SP2-Q1SP1*Q1SP2*Q2SP2**2-2.*Q1SP1*Q2SP1*
     C Q2SP2**2+Q1SP2**2*Q2SP1**2-Q1SP2**2*Q2SP1*Q2SP2-2.*Q1SP2*Q2SP1**
     C 2*Q2SP2)+64.*P1SP2*(Q1SP1**2*Q2SP2**2+2.*Q1SP1*Q1SP2*Q2SP1*Q2SP2
     C +Q1SP2**2*Q2SP1**2)+64.*MK2*(-Q1SP1**2*Q2SP2**2-2.*Q1SP1*Q1SP2*
     C Q2SP1*Q2SP2-Q1SP2**2*Q2SP1**2)+32.*(-Q1SP1**3*Q2SP2**2+Q1SP1**2*
     C Q1SP2*Q2SP2**2+Q1SP1**2*Q2SP1*Q2SP2**2-Q1SP1**2*Q2SP2**3+Q1SP1*
     C Q1SP2**2*Q2SP1**2-Q1SP2**3*Q2SP1**2-Q1SP2**2*Q2SP1**3+Q1SP2**2*
     C Q2SP1**2*Q2SP2)
      M25=M25/S**2
      M26 =-8.*S**3*P1SP2*MK2
     C +4.*S**3*P1SP2*Q2SH-4.*S**3*MK2*HSP1-4.*S**3*
     C HSP1*Q2SP2-16.*S**2*P1SP2**2*MK2+8.*S**2*P1SP2**2*Q2SH+16.*S**2*
     C P1SP2*MK2**2+8.*S**2*P1SP2*MK2*(HSP2-HSP1+Q1SH-Q2SH+2.*Q1SP1+2.*
     C Q1SP2+Q2SP1-Q2SP2)+8.*S**2*P1SP2*(HSP2*Q1SP1-HSP1*Q2SP2-Q2SH*
     C Q1SP1-Q2SH*Q1SP2-Q2SH*Q2SP1)-(8.*S**2*MK2**2)*(Q2SH+Q2SP1+Q2SP2)
```

```
     C +4.*S**2*MK2*(-HSP2*Q2SP1+HSP2*Q2SP2+HSP1*Q2SP1+HSP1*Q2SP2+2.*
     C Q1SH*Q2SP1+2.*Q2SH*Q1SP1+4.*Q1SP1*Q2SP2+4.*Q1SP2*Q2SP1)+8.*S**2*
     C (HSP1*Q1SP1*Q2SP2+HSP1*Q1SP2*Q2SP2+HSP1*Q2SP1*Q2SP2+Q1SH*Q2SP1*
     C Q2SP2-Q2SH*Q1SP2*Q2SP1)-(16.*S*P1SP2**2)*(Q1SH*Q2SP1+Q2SH*Q1SP1)
     C +16.*S*P1SP2*MK2*(Q1SH*Q2SP1+Q2SH*Q1SP1-4.*Q1SP1*Q2SP1+4.*Q1SP1*
     C Q2SP2+4.*Q1SP2*Q2SP1)+16.*S*P1SP2*(-2.*HSP2*Q1SP1*Q2SP1+HSP1*
     C Q1SP1*Q2SP2+HSP1*Q1SP2*Q2SP1-Q1SH*Q1SP1*Q2SP2+Q1SH*Q2SP1*Q2SP2+
     C Q2SH*Q1SP1*Q1SP2+Q2SH*Q1SP1*Q2SP1-Q2SH*Q1SP1*Q2SP2-Q2SH*Q1SP2*
     C Q2SP1)
      M26=M26-(32.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+8.*S*MK2*(-2.*
     C HSP2*Q1SP1*Q2SP2-2.*HSP2*Q1SP2*Q2SP1+2.*HSP1*Q1SP1*Q2SP2+2.*HSP1
     C *Q1SP2*Q2SP1-2.*Q1SH*Q1SP1*Q2SP2-2.*Q1SH*Q1SP2*Q2SP1-Q1SH*Q2SP1
     C **2-2.*Q1SH*Q2SP1*Q2SP2-Q1SH*Q2SP2**2+3.*Q2SH*Q1SP1*Q2SP1+Q2SH*
     C Q1SP1*Q2SP2+3.*Q2SH*Q1SP2*Q2SP1+Q2SH*Q1SP2*Q2SP2-4.*Q1SP1**2*
     C Q2SP2-4.*Q1SP1*Q1SP2*Q2SP1-4.*Q1SP1*Q1SP2*Q2SP2+4.*Q1SP1*Q2SP1**
     C 2+2.*Q1SP1*Q2SP1*Q2SP2+2.*Q1SP1*Q2SP2**2-4.*Q1SP2**2*Q2SP1-2.*
     C Q1SP2*Q2SP1**2+2.*Q1SP2*Q2SP1*Q2SP2)+16.*S*(-HSP2*Q1SP1**2*Q2SP2
     C -HSP2*Q1SP1*Q1SP2*Q2SP1+HSP2*Q1SP1*Q2SP1**2-HSP2*Q1SP1*Q2SP1*
     C Q2SP2-HSP1*Q1SP1*Q2SP1*Q2SP2+HSP1*Q1SP1*Q2SP2**2-Q1SH*Q1SP1*
     C Q2SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1*Q2SP2-Q1SH*Q2SP1**2*Q2SP2+Q2SH*
     C Q1SP1*Q1SP2*Q2SP1+Q2SH*Q1SP2**2*Q2SP1+Q2SH*Q1SP2*Q2SP1**2)+32.*
     C P1SP2*(Q1SH*Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1**2+Q2SH*Q1SP1**2*
     C Q2SP2+Q2SH*Q1SP1*Q1SP2*Q2SP1)
      M26=M26+32.*MK2*(-Q1SH*Q1SP1*Q2SP1*Q2SP2-
     C Q1SH*Q1SP2*Q2SP1**2-Q2SH*Q1SP1**2*Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP1+
     C 4.*Q1SP1**2*Q2SP1*Q2SP2-2.*Q1SP1**2*Q2SP2**2+4.*Q1SP1*Q1SP2
     C *Q2SP1
     C **2-4.*Q1SP1*Q1SP2*Q2SP1*Q2SP2-2.*Q1SP2**2*Q2SP1**2)+32.*(2.*
     C HSP2*Q1SP1**2*Q2SP1*Q2SP2+2.*HSP2*Q1SP1*Q1SP2*Q2SP1**2-HSP1*
     C Q1SP1**2*Q2SP2**2-2.*HSP1*Q1SP1*Q1SP2*Q2SP1*Q2SP2-HSP1*Q1SP2**2*
     C Q2SP1**2+Q1SH*Q1SP1**2*Q2SP2**2+Q1SH*Q1SP1*Q1SP2*Q2SP1*Q2SP2+2.*
     C Q1SH*Q1SP1*Q2SP1**2*Q2SP2-Q2SH*Q1SP1**2*Q1SP2*Q2SP2-Q2SH*Q1SP1*
     C Q1SP2**2*Q2SP1-2.*Q2SH*Q1SP1*Q1SP2*Q2SP1**2)
      M26=M26/S**2
      M27 =4.*S**2*P1SP2*MK2
     C -2.*S**2*P1SP2*Q1SH+4.*S**2*MK2**2+2.*S**2*MK2*
     C (HSP2+HSP1+Q1SH+2.*Q1SP1+2.*Q2SP2)+2.*S**2*(HSP2*Q1SP1+HSP1*
     C Q2SP2)+4.*S*P1SP2**2*(-Q1SH+Q2SH)+4.*S*P1SP2*MK2*(Q1SH-Q2SH+2.*
     C Q1SP1-2.*Q1SP2-2.*Q2SP1+2.*Q2SP2)+4.*S*P1SP2*(HSP2*Q1SP1-HSP2*
     C Q2SP1-HSP1*Q1SP2+HSP1*Q2SP2+Q1SH*Q1SP2+Q1SH*Q2SP1)+4.*S*MK2*(-
     C Q1SH*Q2SP1-Q1SH*Q2SP2-Q2SH*Q1SP2+Q2SH*Q2SP2-2.*Q1SP1*Q2SP1-2.*
     C Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1-2.*Q1SP2*Q2SP2)+4.*S*(-HSP2*Q1SP1*
     C Q1SP2-HSP2*Q1SP1*Q2SP1-HSP1*Q1SP2*Q2SP2-HSP1*Q2SP1*Q2SP2+Q1SH*
     C Q1SP2*Q2SP1-Q1SH*Q2SP1*Q2SP2-Q2SH*Q1SP1*Q1SP2+Q2SH*Q1SP1*Q2SP2)+
     C 8.*P1SP2*(Q1SH*Q1SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q2SP2
     C -Q2SH
     C *Q1SP2*Q2SP1)
      M27=M27+8.*MK2*(-Q1SH*Q1SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q2SH*
     C Q1SP1*Q2SP2+Q2SH*Q1SP2*Q2SP1-2.*Q1SP1**2*Q2SP2-2.*Q1SP1*Q1SP2*
     C Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*Q2SP2-2.*Q1SP1*Q2SP2**
     C 2+2.*Q1SP2**2*Q2SP1+2.*Q1SP2*Q2SP1**2-2.*Q1SP2*Q2SP1*Q2SP2)+8.*(
     C -HSP2*Q1SP1**2*Q2SP2-HSP2*Q1SP1*Q1SP2*Q2SP1+HSP2*Q1SP1*Q2SP1*
     C Q2SP2+HSP2*Q1SP2*Q2SP1**2+HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*
     C Q2SP2**2+HSP1*Q1SP2**2*Q2SP1-HSP1*Q1SP2*Q2SP1*Q2SP2-Q1SH*Q1SP2**
     C 2*Q2SP1-Q1SH*Q1SP2*Q2SP1**2+Q1SH*Q1SP2*Q2SP1*Q2SP2+Q1SH*Q2SP1**2
     C *Q2SP2+Q2SH*Q1SP1*Q1SP2**2+Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*
     C Q1SP2*Q2SP2-Q2SH*Q1SP1*Q2SP1*Q2SP2)
```

```
      M27=M27/S
      M28 =4.*S**2*P1SP2*MK2
     C -2.*S**2*P1SP2*Q2SH+4.*S**2*MK2**2+2.*S**2*MK2*
     C (HSP2+HSP1+Q2SH+2.*Q1SP1+2.*Q2SP2)+2.*S**2*(HSP2*Q1SP1+HSP1*
     C Q2SP2)+4.*S*P1SP2**2*(Q1SH-Q2SH)+4.*S*P1SP2*MK2*(-Q1SH+Q2SH+2.*
     C Q1SP1-2.*Q1SP2-2.*Q2SP1+2.*Q2SP2)+4.*S*P1SP2*(HSP2*Q1SP1-HSP2*
     C Q2SP1-HSP1*Q1SP2+HSP1*Q2SP2+Q2SH*Q1SP2+Q2SH*Q2SP1)+4.*S*MK2*(
     C Q1SH*Q1SP1-Q1SH*Q2SP1-Q2SH*Q1SP1-Q2SH*Q1SP2-2.*Q1SP1*Q2SP1-2.*
     C Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1-2.*Q1SP2*Q2SP2)+4.*S*(-HSP2*Q1SP1*
     C Q1SP2-HSP2*Q1SP1*Q2SP1-HSP1*Q1SP2*Q2SP2-HSP1*Q2SP1*Q2SP2+Q1SH*
     C Q1SP1*Q2SP2-Q1SH*Q2SP1*Q2SP2-Q2SH*Q1SP1*Q1SP2+Q2SH*Q1SP2*Q2SP1)+
     C 8.*P1SP2*(-Q1SH*Q1SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q2SH*Q1SP1*Q2SP2+
     C Q2SH*Q1SP2*Q2SP1)
      M28=M28+8.*MK2*(Q1SH*Q1SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1-Q2SH
     C *Q1SP1*Q2SP2-Q2SH*Q1SP2*Q2SP1-2.*Q1SP1**2*Q2SP2-2.*Q1SP1*Q1SP2*
     C Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*Q2SP2-2.*Q1SP1*Q2SP2**
     C 2+2.*Q1SP2**2*Q2SP1+2.*Q1SP2*Q2SP1**2-2.*Q1SP2*Q2SP1*Q2SP2)+8.*(
     C -HSP2*Q1SP1**2*Q2SP2-HSP2*Q1SP1*Q1SP2*Q2SP1+HSP2*Q1SP1*Q2SP1*
     C Q2SP2+HSP2*Q1SP2*Q2SP1**2+HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*
     C Q2SP2**2+HSP1*Q1SP2**2*Q2SP1-HSP1*Q1SP2*Q2SP1*Q2SP2-Q1SH*Q1SP1*
     C Q1SP2*Q2SP2-Q1SH*Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1*Q2SP2+Q1SH*
     C Q2SP1**2*Q2SP2+Q2SH*Q1SP1*Q1SP2**2+Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*
     C Q1SP2**2*Q2SP1-Q2SH*Q1SP2*Q2SP1**2)
      M28=M28/S
      M33 =(-32.*S**2*MK2**3
     C -8.*S**2*MK2**2*MH2+32.*S**2*MK2**2*(-HSP1+Q2SH+
     C Q2SP1)-8.*S**2*MK2*MH2*Q2SP1+16.*S**2*MK2*(HSP1*Q2SH+2.*Q2SH*
     C Q2SP2+2.*Q2SP1*Q2SP2)-8.*S**2*MH2*Q2SP1*Q2SP2+16.*S**2*HSP1*Q2SH
     C *Q2SP2+64.*S*MK2**2*(-Q1SH*Q2SP2-Q2SH*Q1SP2-Q1SP1*Q2SP2-Q1SP2*
     C Q2SP1+2.*Q1SP2*Q2SP2)+16.*S*MK2*MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1+2.*
     C Q1SP2*Q2SP2)+32.*S*MK2*(-HSP1*Q1SH*Q2SP2-HSP1*Q2SH*Q1SP2+4.*HSP1
     C *Q1SP2*Q2SP2-4.*Q2SH*Q1SP2*Q2SP2-4.*Q1SP2*Q2SP1*Q2SP2)+32.*S*MH2
     C  *Q1SP2*Q2SP1*Q2SP2-64.*S*HSP1*Q2SH*Q1SP2*Q2SP2+256.*MK2*Q1SP2*
     C  Q2SP2*(Q1SH*Q2SP2+Q2SH*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*
     C  Q1SP2*Q2SP2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+128.*HSP1*Q1SP2*Q2SP2*(
     C  Q1SH*Q2SP2+Q2SH*Q1SP2))/S**2
      M34 =-8.*S**3*P1SP2*MK2
     C +2.*S**3*P1SP2*MH2-8.*S**3*MK2*HSP2-4.*S**3*
     C HSP2*HSP1+16.*S**2*P1SP2*MK2**2-4.*S**2*P1SP2*MK2*MH2+16.*S**2*
     C P1SP2*MK2*(Q1SP2+Q2SP2)-(4.*S**2*P1SP2*MH2)*(Q1SP2+Q2SP2)-16.*S
     C **2*MK2**3-4.*S**2*MK2**2*MH2+16.*S**2*MK2**2*(HSP2-HSP1)+8.*S**
     C  2*MK2*(HSP2*HSP1+2.*HSP2*Q1SP2+2.*HSP2*Q2SP2+2.*Q1SH*Q2SP2+2.*
     C  Q2SH*Q1SP2+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)
      M34=M34-(4.*S**2*MH2)*(Q1SP1*
     C Q2SP2+Q1SP2*Q2SP1)+8.*S**2*HSP1*(HSP2*Q1SP2+HSP2*Q2SP2+Q1SH*
     C Q2SP2+Q2SH*Q1SP2)-64.*S*P1SP2*MK2*Q1SP2*Q2SP2+16.*S*P1SP2*MH2*
     C Q1SP2*Q2SP2+64.*S*MK2**2*(-Q1SH*Q2SP2-Q2SH*Q1SP2-Q1SP1*Q2SP2-
     C Q1SP2*Q2SP1+Q1SP2*Q2SP2)+16.*S*MK2*MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1+
     C Q1SP2*Q2SP2)+32.*S*MK2*(-2.*HSP2*Q1SP2*Q2SP2-HSP1*Q1SH*Q2SP2-
     C HSP1*Q2SH*Q1SP2+2.*HSP1*Q1SP2*Q2SP2-Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP2
     C **2-Q2SH*Q1SP2**2-Q2SH*Q1SP2*Q2SP2-Q1SP1*Q1SP2*Q2SP2-Q1SP1*Q2SP2
     C **2-Q1SP2**2*Q2SP1-Q1SP2*Q2SP1*Q2SP2)+8.*S*MH2*(Q1SP1*Q1SP2*
     C Q2SP2+Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1+Q1SP2*Q2SP1*Q2SP2)+16.*S*
     C HSP1*(-2.*HSP2*Q1SP2*Q2SP2-Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP2**2-Q2SH*
     C Q1SP2**2-Q2SH*Q1SP2*Q2SP2)+256.*MK2*Q1SP2*Q2SP2*(Q1SH*Q2SP2+Q2SH
     C *Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*Q1SP2*Q2SP2)*(Q1SP1*
     C Q2SP2+Q1SP2*Q2SP1)+128.*HSP1*Q1SP2*Q2SP2*(Q1SH*Q2SP2+Q2SH*Q1SP2)
```

```
      M34=M34 /S**2
      M35 =-8.*S**3*P1SP2*MK2
     C +4.*S**3*P1SP2*Q2SH-4.*S**3*MK2*HSP2-4.*S**3*
     C HSP2*Q2SP1-16.*S**2*P1SP2**2*MK2+8.*S**2*P1SP2**2*Q2SH+16.*S**2*
     C P1SP2*MK2**2+8.*S**2*P1SP2*MK2*(-HSP2+HSP1+Q1SH-Q2SH+2.*Q1SP1+2.
     C *Q1SP2-Q2SP1+Q2SP2)+8.*S**2*P1SP2*(-HSP2*Q2SP1+HSP1*Q1SP2-Q2SH*
     C Q1SP1-Q2SH*Q1SP2-Q2SH*Q2SP2)-(8.*S**2*MK2**2)*(Q2SH+Q2SP1+Q2SP2)
     C +4.*S**2*MK2*(HSP2*Q2SP1+HSP2*Q2SP2+HSP1*Q2SP1-HSP1*Q2SP2+2.*
     C Q1SH*Q2SP2+2.*Q2SH*Q1SP2+4.*Q1SP1*Q2SP2+4.*Q1SP2*Q2SP1)+8.*S**2*
     C (HSP2*Q1SP1*Q2SP1+HSP2*Q1SP2*Q2SP1+HSP2*Q2SP1*Q2SP2+Q1SH*Q2SP1*
     C Q2SP2-Q2SH*Q1SP1*Q2SP2)-(16.*S*P1SP2**2)*(Q1SH*Q2SP2+Q2SH*Q1SP2)
     C +16.*S*P1SP2*MK2*(Q1SH*Q2SP2+Q2SH*Q1SP2+4.*Q1SP1*Q2SP2+4.*Q1SP2*
     C Q2SP1-4.*Q1SP2*Q2SP2)+16.*S*P1SP2*(HSP2*Q1SP1*Q2SP2+HSP2*Q1SP2*
     C Q2SP1-2.*HSP1*Q1SP2*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q1SH*Q2SP1*Q2SP2+Q2SH
     C *Q1SP1*Q1SP2-Q2SH*Q1SP1*Q2SP2-Q2SH*Q1SP2*Q2SP1+Q2SH*Q1SP2*Q2SP2)
     C -(32.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)
      M35=M35+8.*S*MK2*(2.*HSP2*
     C Q1SP1*Q2SP2+2.*HSP2*Q1SP2*Q2SP1-2.*HSP1*Q1SP1*Q2SP2-2.*HSP1*
     C Q1SP2*Q2SP1-2.*Q1SH*Q1SP1*Q2SP2-2.*Q1SH*Q1SP2*Q2SP1-Q1SH*Q2SP1**
     C 2-2.*Q1SH*Q2SP1*Q2SP2-Q1SH*Q2SP2**2+Q2SH*Q1SP1*Q2SP1+3.*Q2SH*
     C Q1SP1*Q2SP2+Q2SH*Q1SP2*Q2SP1+3.*Q2SH*Q1SP2*Q2SP2-4.*Q1SP1**2*
     C Q2SP2-4.*Q1SP1*Q1SP2*Q2SP1-4.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*
     C Q2SP2-2.*Q1SP1*Q2SP2**2-4.*Q1SP2**2*Q2SP1+2.*Q1SP2*Q2SP1**2+2.*
     C Q1SP2*Q2SP1*Q2SP2+4.*Q1SP2*Q2SP2**2)+16.*S*(HSP2*Q1SP2*Q2SP1**2-
     C HSP2*Q1SP2*Q2SP1*Q2SP2-HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP2**2*
     C Q2SP1-HSP1*Q1SP2*Q2SP1*Q2SP2+HSP1*Q1SP2*Q2SP2**2-Q1SH*Q1SP1*
     C Q2SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1*Q2SP2-Q1SH*Q2SP1*Q2SP2**2+Q2SH*
     C Q1SP1**2*Q2SP2+Q2SH*Q1SP1*Q1SP2*Q2SP2+Q2SH*Q1SP1*Q2SP2**2)+32.*
     C P1SP2*(Q1SH*Q1SP1*Q2SP2**2+Q1SH*Q1SP2*Q2SP1*Q2SP2+Q2SH*Q1SP1*
     C Q1SP2*Q2SP2+Q2SH*Q1SP2**2*Q2SP1)
      M35=M35+32.*MK2*(-Q1SH*Q1SP1*Q2SP2**2-
     C Q1SH*Q1SP2*Q2SP1*Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP2-Q2SH*Q1SP2**2*
     C Q2SP1-2.*Q1SP1**2*Q2SP2**2-4.*Q1SP1*Q1SP2*Q2SP1*Q2SP2+4.*Q1SP1*
     C Q1SP2*Q2SP2**2-2.*Q1SP2**2*Q2SP1**2+4.*Q1SP2**2*Q2SP1*Q2SP2)+32.
     C *(-HSP2*Q1SP1**2*Q2SP2**2-2.*HSP2*Q1SP1*Q1SP2*Q2SP1*Q2SP2-HSP2*
     C Q1SP2**2*Q2SP1**2+2.*HSP1*Q1SP1*Q1SP2*Q2SP2**2+2.*HSP1*Q1SP2**2*
     C Q2SP1*Q2SP2+Q1SH*Q1SP1*Q1SP2*Q2SP1*Q2SP2+Q1SH*Q1SP2**2*Q2SP1**2+
     C 2.*Q1SH*Q1SP2*Q2SP1*Q2SP2**2-Q2SH*Q1SP1**2*Q1SP2*Q2SP2-Q2SH
     C *Q1SP1
     C  *Q1SP2**2*Q2SP1-2.*Q2SH*Q1SP1*Q1SP2*Q2SP2**2)
      M35=M35/S**2
      M36 =8.*S**2*P1SP2**2*MH2
     C +32.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(
     C HSP2+HSP1-2.*Q2SH-Q2SP1-Q2SP2)+8.*S**2*P1SP2*Q2SH**2+4.*S**2*MK2
     C *MH2*(Q2SP1+Q2SP2)+8.*S**2*MK2*(-HSP2*Q2SH+HSP2*Q2SP1-HSP2*Q2SP2
     C -HSP1*Q2SH-HSP1*Q2SP1+HSP1*Q2SP2)+8.*S**2*MH2*Q2SP1*Q2SP2-(8.*S
     C **2*Q2SH)*(HSP2*Q2SP1+HSP1*Q2SP2)-32.*S*P1SP2**2*Q1SH*Q2SH+32.*S
     C *P1SP2*MK2*(Q1SH*Q2SH+Q1SH*Q2SP1+Q1SH*Q2SP2+Q2SH*Q1SP1+Q2SH*
     C Q1SP2+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)-(32.*S*P1SP2*MH2)*(Q1SP1*
     C Q2SP2+Q1SP2*Q2SP1)+16.*S*P1SP2*(HSP2*Q1SH*Q2SP1+HSP2*Q2SH*Q1SP1+
     C HSP1*Q1SH*Q2SP2+HSP1*Q2SH*Q1SP2-Q2SH**2*Q1SP1-Q2SH**2*Q1SP2)-(
     C 64.*S*MK2**2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+16.*S*MK2*(-2.*HSP2
     C *Q1SP1
     C *Q2SP2-2.*HSP2*Q1SP2*Q2SP1-2.*HSP1*Q1SP1*Q2SP2-2.*HSP1*Q1SP2*
     C Q2SP1+Q1SH*Q2SP1**2-2.*Q1SH*Q2SP1*Q2SP2+Q1SH*Q2SP2**2+Q2SH**2*
     C Q1SP1+Q2SH**2*Q1SP2+Q2SH*Q1SP1*Q2SP1+3.*Q2SH*Q1SP1*Q2SP2+3.*Q2SH
     C *Q1SP2*Q2SP1+Q2SH*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*Q2SP2+2.*Q1SP1*
```

```
C Q2SP2**2+2.*Q1SP2*Q2SP1**2+2.*Q1SP2*Q2SP1*Q2SP2)
 M36=M36-(16.*S*MH2*
C Q2SP1*Q2SP2)*(Q1SP1+Q1SP2)+16.*S*Q2SH*(HSP2*Q1SP1*Q2SP1+HSP2*
C Q1SP2*Q2SP1+HSP1*Q1SP1*Q2SP2+HSP1*Q1SP2*Q2SP2)+64.*P1SP2*Q1SH*
C Q2SH*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+64.*MK2*(-Q1SH*Q2SH*Q1SP1*Q2SP2-
C  Q1SH*Q2SH*Q1SP2*Q2SP1-Q1SH*Q1SP1*Q2SP1*Q2SP2-Q1SH*Q1SP1*Q2SP2**2
C -Q1SH*Q1SP2*Q2SP1**2-Q1SH*Q1SP2*Q2SP1*Q2SP2-Q2SH*Q1SP1**2*Q2SP2-
C Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q1SP2*Q2SP2-Q2SH*Q1SP2**2*
C  Q2SP1-2.*Q1SP1**2*Q2SP2**2-4.*Q1SP1*Q1SP2*Q2SP1*Q2SP2-2.*Q1SP2**
C 2*Q2SP1**2)+32.*MH2*(Q1SP1**2*Q2SP2**2+2.*Q1SP1*Q1SP2*Q2SP1*
C Q2SP2+Q1SP2**2*Q2SP1**2)-32.*(HSP2*Q1SH*Q1SP1*Q2SP1*Q2SP2+HSP2*
C Q1SH*Q1SP2*Q2SP1**2+HSP2*Q2SH*Q1SP1**2*Q2SP2+HSP2*Q2SH*Q1SP1*
C Q1SP2*Q2SP1+HSP1*Q1SH*Q1SP1*Q2SP2**2+HSP1*Q1SH*Q1SP2*Q2SP1*Q2SP2
C +HSP1*Q2SH*Q1SP1*Q1SP2*Q2SP2+HSP1*Q2SH*Q1SP2**2*Q2SP1)
 M36=M36/S**2
 M37 =(4.*S**2*P1SP2*MK2-S**2*P1SP2*MH2
C +4.*S**2*MK2**2+S**2*MK2*MH2+4.*
C S**2*MK2*(HSP2+HSP1)+2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(-Q1SP2+
C Q2SP2)+2.*S*P1SP2*MH2*(Q1SP2-Q2SP2)+8.*S*MK2**2*(-Q1SH+Q2SH-
C Q1SP1+Q2SP1)+2.*S*MK2*MH2*(Q1SP1-Q2SP1)+4.*S*MK2*(-2.*HSP2*Q1SP2
C +2.*HSP2*Q2SP2-HSP1*Q1SH+HSP1*Q2SH-2.*Q1SH*Q2SP2-2.*Q2SH*Q1SP2+
C 4.*Q2SH*Q2SP2-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.*Q2SP1*Q2SP2)
C +2.*S*
C MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1-2.*Q2SP1*Q2SP2)+4.*S*HSP1*(-HSP2*
C Q1SP2+HSP2*Q2SP2-Q1SH*Q2SP2-Q2SH*Q1SP2+2.*Q2SH*Q2SP2)+16.*MK2*(
C 3.*Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP2**2+Q2SH*Q1SP2**2-3.*Q2SH*Q1SP2*
C Q2SP2+3.*Q1SP1*Q1SP2*Q2SP2-Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1-3.*
C Q1SP2*Q2SP1*Q2SP2)+4.*MH2*(-3.*Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP2**2-
C Q1SP2**2*Q2SP1+3.*Q1SP2*Q2SP1*Q2SP2)+8.*HSP1*(3.*Q1SH*Q1SP2*
C Q2SP2-Q1SH*Q2SP2**2+Q2SH*Q1SP2**2-3.*Q2SH*Q1SP2*Q2SP2))/S
 M38 =4.*S**2*P1SP2*MK2-S**2*P1SP2*MH2
C +4.*S**2*MK2**2+S**2*MK2*MH2+4.*
C  S**2*MK2*(HSP2+HSP1)+2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(-Q1SH+
C Q2SH-Q1SP2+Q2SP2)+2.*S*P1SP2*MH2*(Q1SP2-Q2SP2)+4.*S*P1SP2*Q2SH*(
C Q1SH-Q2SH)+8.*S*MK2**2*(-Q1SP1+Q2SP1)+2.*S*MK2*MH2*(Q1SP1-Q2SP1)
C  +4.*S*MK2*(-HSP2*Q1SH+HSP2*Q2SH-2.*HSP2*Q1SP2+2.*HSP2*Q2SP2-Q1SH
C *Q2SH-2.*Q1SH*Q2SP1+Q2SH**2-2.*Q2SH*Q1SP2+2.*Q2SH*Q2SP1+2.*Q2SH*
C  Q2SP2-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.*Q2SP1*Q2SP2)+2.*S*MH2*(
C Q1SP1*Q2SP2+Q1SP2*Q2SP1-2.*Q2SP1*Q2SP2)+4.*S*(-HSP2*HSP1*Q1SP2+
C HSP2*HSP1*Q2SP2-HSP2*Q1SH*Q2SP1+HSP2*Q2SH*Q2SP1-HSP1*Q2SH*Q1SP2+
C HSP1*Q2SH*Q2SP2)+8.*P1SP2*(-Q1SH**2*Q2SP2-Q1SH*Q2SH*Q1SP2+Q1SH*
C Q2SH*Q2SP2+Q2SH**2*Q1SP2)
 M38=M38+8.*MK2*(Q1SH**2*Q2SP2+Q1SH*Q2SH*Q1SP2-
C Q1SH*Q2SH*Q2SP2+2.*Q1SH*Q1SP1*Q2SP2+2.*Q1SH*Q1SP2*Q2SP1+2.*Q1SH*
C Q1SP2*Q2SP2-2.*Q1SH*Q2SP2**2-Q2SH**2*Q1SP2-2.*Q2SH*Q1SP1*Q2SP2+
C 2.*Q2SH*Q1SP2**2-2.*Q2SH*Q1SP2*Q2SP1-2.*Q2SH*Q1SP2*Q2SP2
C +6.*Q1SP1
C  *Q1SP2*Q2SP2-2.*Q1SP1*Q2SP2**2+2.*Q1SP2**2*Q2SP1-6.*Q1SP2*Q2SP1*
C Q2SP2)+4.*MH2*(-3.*Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP2**2-Q1SP2**2*
C Q2SP1+3.*Q1SP2*Q2SP1*Q2SP2)+8.*(HSP2*Q1SH*Q1SP1*Q2SP2+HSP2*Q1SH*
C Q1SP2*Q2SP1-HSP2*Q2SH*Q1SP1*Q2SP2-HSP2*Q2SH*Q1SP2*Q2SP1+HSP1*
C Q1SH*Q1SP2*Q2SP2-HSP1*Q1SH*Q2SP2**2+HSP1*Q2SH*Q1SP2**2-HSP1*Q2SH
C *Q1SP2*Q2SP2)
 M38=M38/S
 M44 =(-32.*S**2*MK2**3
C -8.*S**2*MK2**2*MH2+32.*S**2*MK2**2*(-HSP1+Q1SH+
C Q1SP1)-8.*S**2*MK2*MH2*Q1SP1+16.*S**2*MK2*(HSP1*Q1SH+2.*Q1SH*
```

```
     C Q1SP2+2.*Q1SP1*Q1SP2)-8.*S**2*MH2*Q1SP1*Q1SP2+16.*S**2*HSP1*Q1SH
     C *Q1SP2+64.*S*MK2**2*(-Q1SH*Q2SP2-Q2SH*Q1SP2-Q1SP1*Q2SP2-Q1SP2*
     C Q2SP1+2.*Q1SP2*Q2SP2)+16.*S*MK2*MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1+2.*
     C Q1SP2*Q2SP2)+32.*S*MK2*(-HSP1*Q1SH*Q2SP2-HSP1*Q2SH*Q1SP2+4.*HSP1
     C *Q1SP2*Q2SP2-4.*Q1SH*Q1SP2*Q2SP2-4.*Q1SP1*Q1SP2*Q2SP2)+32.*S*MH2
     C *Q1SP1*Q1SP2*Q2SP2-64.*S*HSP1*Q1SH*Q1SP2*Q2SP2+256.*MK2*Q1SP2*
     C Q2SP2*(Q1SH*Q2SP2+Q2SH*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*
     C Q1SP2*Q2SP2)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+128.*HSP1*Q1SP2*Q2SP2*(
     C  Q1SH*Q2SP2+Q2SH*Q1SP2))/S**2
      M45 =8.*S**3*MK2**2
     C +4.*S**3*MK2*(HSP1+2.*Q1SP2)+4.*S**3*HSP1*Q1SP2+
     C 16.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(HSP1+Q1SP2)+16.*S**2*
     C P1SP2*HSP1*Q1SP2-16.*S**2*MK2**3+8.*S**2*MK2**2*(-HSP2-HSP1+Q1SH
     C -Q2SH-2.*Q2SP1-2.*Q2SP2)+8.*S**2*MK2*(-HSP2*Q1SP2-HSP1*Q1SP1-
     C HSP1*Q2SP2+Q1SH*Q1SP2+Q1SH*Q2SP1-Q2SH*Q1SP1-4.*Q1SP2*Q2SP2)+8.*S
     C **2*Q1SP2*(-HSP1*Q1SP1-2.*HSP1*Q2SP2+Q1SH*Q2SP1-Q2SH*Q1SP1)+16.*
     C S*P1SP2*MK2*(-Q1SH*Q2SP1-Q2SH*Q1SP1-4.*Q1SP2*Q2SP2)+16.*S*P1SP2*
     C Q1SP2*(-4.*HSP1*Q2SP2-Q1SH*Q2SP1-Q2SH*Q1SP1)+16.*S*MK2**2*(Q1SH*
     C Q2SP1+Q2SH*Q1SP1+4.*Q1SP1*Q2SP1-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.
     C *Q1SP2*Q2SP2)
      M45=M45+16.*S*MK2*(2.*HSP2*Q1SP1*Q2SP1+2.*HSP2*Q1SP2*Q2SP2
     C -HSP1*Q1SP1*Q2SP2-HSP1*Q1SP2*Q2SP1+2.*HSP1*Q1SP2*Q2SP2-Q1SH*
     C Q1SP1*Q2SP1-Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1**2+Q2SH
     C *Q1SP1*Q2SP2+Q2SH*Q1SP2**2+2.*Q2SH*Q1SP2*Q2SP2-2.*Q1SP1*Q1SP2*
     C Q2SP2-2.*Q1SP2**2*Q2SP1+4.*Q1SP2*Q2SP1*Q2SP2+4.*Q1SP2*Q2SP2**2)+
     C 16.*S*Q1SP2*(2.*HSP2*Q1SP1*Q2SP1-2.*HSP1*Q1SP2*Q2SP1+2.*HSP1*
     C  Q2SP2**2-Q1SH*Q1SP1*Q2SP1-2.*Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1**2+2.*
     C Q2SH*Q1SP1*Q2SP2)+64.*P1SP2*Q1SP2*Q2SP2*(Q1SH*Q2SP1+Q2SH*Q1SP1)+
     C 64.*MK2*Q1SP2*Q2SP2*(-Q1SH*Q2SP1-Q2SH*Q1SP1-4.*Q1SP1*Q2SP1+2.*
     C Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1)+32.*Q1SP2*(-4.*HSP2*Q1SP1*Q2SP1*
     C Q2SP2+2.*HSP1*Q1SP1*Q2SP2**2+2.*HSP1*Q1SP2*Q2SP1*Q2SP2+Q1SH*
     C Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1**2+2.*Q1SH*Q2SP1*Q2SP2**2-
     C Q2SH*Q1SP1**2*Q2SP2-Q2SH*Q1SP1*Q1SP2*Q2SP1-2.*Q2SH*Q1SP1*Q2SP2**
     C  2)
      M45=M45/S**2
      M46 =-2.*S**3*P1SP2*MH2
     C +8.*S**3*MK2**2+4.*S**3*MK2*(HSP2+HSP1)+4.*S**
     C  3*HSP2*HSP1+16.*S**2*P1SP2*MK2**2+4.*S**2*P1SP2*MK2*MH2+16.*S**2
     C *P1SP2*MK2*(HSP2+HSP1)+4.*S**2*P1SP2*MH2*(Q1SP1+Q2SP2)+8.*S**2*
     C P1SP2*(2.*HSP2*HSP1+Q1SH*Q2SH)-16.*S**2*MK2**3+4.*S**2*MK2**2*
     C MH2-(16.*S**2*MK2**2)*(HSP2+HSP1+Q1SP2+Q2SP1)+4.*S**2*MK2*(-2.*
     C HSP2**2-2.*HSP2*HSP1+HSP2*Q1SH-HSP2*Q2SH-2.*HSP2*Q2SP1-2.*HSP2*
     C Q2SP2-2.*HSP1**2-HSP1*Q1SH+HSP1*Q2SH-2.*HSP1*Q1SP1-2.*HSP1*Q1SP2
     C +2.*Q1SH*Q2SP1-2.*Q1SH*Q2SP2-2.*Q2SH*Q1SP1+2.*Q2SH*Q1SP2)+4.*S**
     C 2*MH2*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(8.*S**2)*(HSP2*HSP1*Q1SP1+HSP2*
     C HSP1*Q2SP2+HSP2*Q2SH*Q1SP1+HSP1*Q1SH*Q2SP2)+32.*S*P1SP2*MK2*(
     C Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(8.*S*P1SP2*MH2)*(Q1SP1*Q2SP2+Q1SP2*
     C Q2SP1)-(16.*S*P1SP2)*(HSP2*Q1SH*Q2SP1+HSP2*Q2SH*Q1SP1+HSP1*Q1SH*
     C Q2SP2+HSP1*Q2SH*Q1SP2+Q1SH*Q2SH*Q1SP1+Q1SH*Q2SH*Q2SP2)+32.*S*MK2
     C **2*(Q1SH*Q2SP1+Q1SH*Q2SP2+Q2SH*Q1SP1+Q2SH*Q1SP2+2.*Q1SP1*Q2SP1-
     C Q1SP1*Q2SP2-Q1SP2*Q2SP1+2.*Q1SP2*Q2SP2)
      M46=M46+8.*S*MK2*MH2*(-2.*Q1SP1*
     C Q2SP1-Q1SP1*Q2SP2-Q1SP2*Q2SP1-2.*Q1SP2*Q2SP2)+8.*S*MK2*(2.*HSP2*
     C Q1SH*Q2SP1+2.*HSP2*Q1SH*Q2SP2+2.*HSP2*Q2SH*Q1SP1+2.*HSP2*Q2SH*
     C Q1SP2+8.*HSP2*Q1SP1*Q2SP1-4.*HSP2*Q1SP1*Q2SP2-4.*HSP2*Q1SP2*
     C Q2SP1+2.*HSP1*Q1SH*Q2SP1+2.*HSP1*Q1SH*Q2SP2+2.*HSP1*Q2SH*Q1SP1+
     C 2.*HSP1*Q2SH*Q1SP2-4.*HSP1*Q1SP1*Q2SP2-4.*HSP1*Q1SP2*Q2SP1+8.*
```

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     C HSP1*Q1SP2*Q2SP2+Q1SH**2*Q2SP1-Q1SH**2*Q2SP2+Q1SH*Q2SH*Q1SP1+
     C Q1SH*Q2SH*Q1SP2+Q1SH*Q2SH*Q2SP1+Q1SH*Q2SH*Q2SP2+2. *Q1SH*Q1SP1*
     C Q2SP1+2. *Q1SH*Q1SP2*Q2SP1+2. *Q1SH*Q2SP1*Q2SP2+2. *Q1SH*Q2SP2**2-
     C Q2SH**2*Q1SP1+Q2SH**2*Q1SP2+2. *Q2SH*Q1SP1**2+2. *Q2SH*Q1SP1*Q1SP2
     C +2. *Q2SH*Q1SP2*Q2SP1+2. *Q2SH*Q1SP2*Q2SP2+8. *Q1SP1*Q1SP2*Q2SP1+8.
     C *Q1SP2*Q2SP1*Q2SP2)-(8. *S*MH2)*(Q1SP1**2*Q2SP2+Q1SP1*Q1SP2*Q2SP1
     C +Q1SP1*Q2SP2**2+Q1SP2*Q2SP1*Q2SP2)+16. *S*(2. *HSP2**2*Q1SP1*Q2SP1
     C -HSP2*HSP1*Q1SP1*Q2SP2-HSP2*HSP1*Q1SP2*Q2SP1+HSP2*Q2SH*Q1SP1**2+
     C HSP2*Q2SH*Q1SP1*Q2SP2+2. *HSP1**2*Q1SP2*Q2SP2+HSP1*Q1SH*Q1SP1*
     C  Q2SP2+HSP1*Q1SH*Q2SP2**2)
      M46=M46+32. *P1SP2*(Q1SH**2*Q2SP1*Q2SP2+Q1SH*
     C Q2SH*Q1SP1*Q2SP2+Q1SH*Q2SH*Q1SP2*Q2SP1+Q2SH**2*Q1SP1*Q1SP2)+32. *
     C MK2*(-Q1SH**2*Q2SP1*Q2SP2-Q1SH*Q2SH*Q1SP1*Q2SP2-Q1SH*Q2SH*Q1SP2*
     C Q2SP1-4. *Q1SH*Q1SP1*Q2SP1*Q2SP2-4. *Q1SH*Q1SP2*Q2SP1*Q2SP2-Q2SH**
     C  2*Q1SP1*Q1SP2-4. *Q2SH*Q1SP1*Q1SP2*Q2SP1-4. *Q2SH*Q1SP1*Q1SP2*
     C Q2SP2-2. *Q1SP1**2*Q2SP2**2-12. *Q1SP1*Q1SP2*Q2SP1*Q2SP2-2. *Q1SP2
     C **2*Q2SP1**2)+16. *MH2*(Q1SP1**2*Q2SP2**2+6. *Q1SP1*Q1SP2*Q2SP1*
     C Q2SP2+Q1SP2**2*Q2SP1**2)-64. *(HSP2*Q1SH*Q1SP1*Q2SP1*Q2SP2+HSP2*
     C Q2SH*Q1SP1*Q1SP2*Q2SP1+HSP1*Q1SH*Q1SP2*Q2SP1*Q2SP2+HSP1*Q2SH*
     C Q1SP1*Q1SP2*Q2SP2)
      M46=M46/S**2
      M47 =(-4. *S**2*P1SP2*MK2+S**2*P1SP2*MH2
     C -4. *S**2*MK2**2-S**2*MK2*MH2-(
     C 4. *S**2*MK2)*(HSP2+HSP1)-2. *S**2*HSP2*HSP1+8. *S*P1SP2*MK2*(
     C -Q1SP2
     C +Q2SP2)+2. *S*P1SP2*MH2*(Q1SP2-Q2SP2)+8. *S*MK2**2*(-Q1SH+Q2SH-
     C Q1SP1+Q2SP1)+2. *S*MK2*MH2*(Q1SP1-Q2SP1)+4. *S*MK2*(-2. *HSP2*Q1SP2
     C +2. *HSP2*Q2SP2-HSP1*Q1SH+HSP1*Q2SH-4. *Q1SH*Q1SP2+2. *Q1SH*Q2SP2+
     C 2. *Q2SH*Q1SP2-4. *Q1SP1*Q1SP2+2. *Q1SP1*Q2SP2+2. *Q1SP2*Q2SP1)
     C +2. *S*
     C  MH2*(2. *Q1SP1*Q1SP2-Q1SP1*Q2SP2-Q1SP2*Q2SP1)+4. *S*HSP1*(-HSP2*
     C Q1SP2+HSP2*Q2SP2-2. *Q1SH*Q1SP2+Q1SH*Q2SP2+Q2SH*Q1SP2)+16. *MK2*(
     C 3. *Q1SH*Q1SP2*Q2SP2-Q1SH*Q2SP2**2+Q2SH*Q1SP2**2-3. *Q2SH*Q1SP2*
     C Q2SP2+3. *Q1SP1*Q1SP2*Q2SP2-Q1SP1*Q2SP2**2+Q1SP2**2*Q2SP1-3. *
     C Q1SP2*Q2SP1*Q2SP2)+4. *MH2*(-3. *Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP2**2-
     C Q1SP2**2*Q2SP1+3. *Q1SP2*Q2SP1*Q2SP2)+8. *HSP1*(3. *Q1SH*Q1SP2*
     C Q2SP2-Q1SH*Q2SP2**2+Q2SH*Q1SP2**2-3. *Q2SH*Q1SP2*Q2SP2))/S
      M48 =-4. *S**2*P1SP2*MK2
     C +S**2*P1SP2*MH2-4. *S**2*MK2**2-S**2*MK2*MH2-(
     C 4. *S**2*MK2)*(HSP2+HSP1)-2. *S**2*HSP2*HSP1+8. *S*P1SP2*MK2*(
     C -Q1SH+
     C Q2SH-Q1SP2+Q2SP2)+2. *S*P1SP2*MH2*(Q1SP2-Q2SP2)+4. *S*P1SP2*Q1SH*(
     C Q1SH-Q2SH)+8. *S*MK2**2*(-Q1SP1+Q2SP1)+2. *S*MK2*MH2*(Q1SP1-Q2SP1)
     C +4. *S*MK2*(-HSP2*Q1SH+HSP2*Q2SH-2. *HSP2*Q1SP2+2. *HSP2*Q2SP2-Q1SH
     C  **2+Q1SH*Q2SH-2. *Q1SH*Q1SP1-2. *Q1SH*Q1SP2+2. *Q1SH*Q2SP2+2. *Q2SH*
     C Q1SP1-4. *Q1SP1*Q1SP2+2. *Q1SP1*Q2SP2+2. *Q1SP2*Q2SP1)+2. *S*MH2*(2.
     C *Q1SP1*Q1SP2-Q1SP1*Q2SP2-Q1SP2*Q2SP1)+4. *S*(-HSP2*HSP1*Q1SP2+
     C HSP2*HSP1*Q2SP2-HSP2*Q1SH*Q1SP1+HSP2*Q2SH*Q1SP1-HSP1*Q1SH*Q1SP2+
     C HSP1*Q1SH*Q2SP2)
      M48=M48+8. *P1SP2*(-Q1SH**2*Q2SP2-Q1SH*Q2SH*Q1SP2+Q1SH*
     C Q2SH*Q2SP2+Q2SH**2*Q1SP2)+8. *MK2*(Q1SH**2*Q2SP2+Q1SH*Q2SH*Q1SP2-
     C Q1SH*Q2SH*Q2SP2+2. *Q1SH*Q1SP1*Q2SP2+2. *Q1SH*Q1SP2*Q2SP1+2. *Q1SH*
     C Q1SP2*Q2SP2-2. *Q1SH*Q2SP2**2-Q2SH**2*Q1SP2-2. *Q2SH*Q1SP1*Q2SP2+
     C 2. *Q2SH*Q1SP2**2-2. *Q2SH*Q1SP2*Q2SP1-2. *Q2SH*Q1SP2*Q2SP2
     C+6. *Q1SP1
     C *Q1SP2*Q2SP2-2. *Q1SP1*Q2SP2**2+2. *Q1SP2**2*Q2SP1-6. *Q1SP2*Q2SP1*
     C Q2SP2)+4. *MH2*(-3. *Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP2**2-Q1SP2**2*
```

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     C Q2SP1+3.*Q1SP2*Q2SP1*Q2SP2)+8.*(HSP2*Q1SH*Q1SP1*Q2SP2+HSP2*Q1SH*
     C Q1SP2*Q2SP1-HSP2*Q2SH*Q1SP1*Q2SP2-HSP2*Q2SH*Q1SP2*Q2SP1+HSP1*
     C Q1SH*Q1SP2*Q2SP2-HSP1*Q1SH*Q2SP2**2+HSP1*Q2SH*Q1SP2**2-HSP1*Q2SH
     C *Q1SP2*Q2SP2)
      M48=M48/S
      M55 =(8.*S**3*MK2**2
     C +8.*S**3*MK2*(Q1SP2+Q2SP1)+8.*S**3*Q1SP2*Q2SP1+16.
     C *S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(Q1SP2+Q2SP1)+16.*S**2*
     C P1SP2*Q1SP2*Q2SP1-16.*S**2*MK2**3-(16.*S**2*MK2**2)*(Q1SP1+Q2SP2
     C )+16.*S**2*MK2*(-Q1SP1*Q1SP2-2.*Q1SP1*Q2SP1-2.*Q1SP2*Q2SP2-Q2SP1
     C *Q2SP2)-(32.*S**2*Q1SP2*Q2SP1)*(Q1SP1+Q2SP2)-(64.*S*P1SP2*MK2)*(
     C Q1SP1*Q2SP1+Q1SP2*Q2SP2)-(64.*S*P1SP2*Q1SP2*Q2SP1)*(Q1SP1+Q2SP2)
     C +64.*S*MK2**2*(Q1SP1*Q2SP1+Q1SP2*Q2SP2)+32.*S*MK2*(2.*Q1SP1**2*
     C Q2SP1+Q1SP1*Q1SP2*Q2SP2+Q1SP1*Q2SP1*Q2SP2-Q1SP2**2*Q2SP1-Q1SP2*
     C Q2SP1**2+2.*Q1SP2*Q2SP2**2)+32.*S*Q1SP2*Q2SP1*(2.*Q1SP1**2+3.*
     C Q1SP1*Q2SP2-Q1SP2*Q2SP1+2.*Q2SP2**2)+256.*P1SP2*Q1SP1*Q1SP2*
     C Q2SP1*Q2SP2-256.*MK2*Q1SP1*Q1SP2*Q2SP1*Q2SP2+128.*Q1SP2*Q2SP1*(-
     C Q1SP1**2*Q2SP2+Q1SP1*Q1SP2*Q2SP1-Q1SP1*Q2SP2**2+Q1SP2*Q2SP1*
     C Q2SP2))/S**2
      M56 =8.*S**3*MK2**2
     C +4.*S**3*MK2*(HSP2+2.*Q2SP1)+4.*S**3*HSP2*Q2SP1+
     C  16.*S**2*P1SP2*MK2**2+16.*S**2*P1SP2*MK2*(HSP2+Q2SP1)+16.*S**2*
     C P1SP2*HSP2*Q2SP1-16.*S**2*MK2**3+8.*S**2*MK2**2*(-HSP2-HSP1-Q1SH
     C +Q2SH-2.*Q1SP1-2.*Q1SP2)+8.*S**2*MK2*(-HSP2*Q1SP1-HSP2*Q2SP2-
     C HSP1*Q2SP1-Q1SH*Q2SP2+Q2SH*Q1SP2+Q2SH*Q2SP1-4.*Q1SP1*Q2SP1)+8.*S
     C **2*Q2SP1*(-2.*HSP2*Q1SP1-HSP2*Q2SP2-Q1SH*Q2SP2+Q2SH*Q1SP2)+16.*
     C S*P1SP2*MK2*(-Q1SH*Q2SP2-Q2SH*Q1SP2-4.*Q1SP1*Q2SP1)+16.*S*P1SP2*
     C Q2SP1*(-4.*HSP2*Q1SP1-Q1SH*Q2SP2-Q2SH*Q1SP2)+16.*S*MK2**2*(Q1SH*
     C Q2SP2+Q2SH*Q1SP2+4.*Q1SP1*Q2SP1-2.*Q1SP1*Q2SP2-2.*Q1SP2*Q2SP1+4.
     C *Q1SP2*Q2SP2)
      M56=M56+16.*S*MK2*(2.*HSP2*Q1SP1*Q2SP1-HSP2*Q1SP1*Q2SP2-
     C HSP2*Q1SP2*Q2SP1+2.*HSP1*Q1SP1*Q2SP1+2.*HSP1*Q1SP2*Q2SP2+2.*Q1SH
     C *Q1SP1*Q2SP1+Q1SH*Q1SP1*Q2SP2+Q1SH*Q2SP1**2+Q1SH*Q2SP2**2-Q2SH*
     C Q1SP1*Q1SP2-Q2SH*Q1SP1*Q2SP1-Q2SH*Q1SP2*Q2SP2+4.*Q1SP1**2*Q2SP1+
     C 4.*Q1SP1*Q1SP2*Q2SP1-2.*Q1SP1*Q2SP1*Q2SP2-2.*Q1SP2*Q2SP1**2)
     C +16.*
     C S*Q2SP1*(2.*HSP2*Q1SP1**2-2.*HSP2*Q1SP2*Q2SP1+2.*HSP1*Q1SP2*
     C Q2SP2+2.*Q1SH*Q1SP1*Q2SP2+Q1SH*Q2SP2**2-2.*Q2SH*Q1SP1*Q1SP2-Q2SH
     C *Q1SP2*Q2SP2)+64.*P1SP2*Q1SP1*Q2SP1*(Q1SH*Q2SP2+Q2SH*Q1SP2)+64.*
     C MK2*Q1SP1*Q2SP1*(-Q1SH*Q2SP2-Q2SH*Q1SP2+2.*Q1SP1*Q2SP2+2.*Q1SP2*
     C Q2SP1-4.*Q1SP2*Q2SP2)+32.*Q2SP1*(2.*HSP2*Q1SP1**2*Q2SP2+2.*HSP2*
     C Q1SP1*Q1SP2*Q2SP1-4.*HSP1*Q1SP1*Q1SP2*Q2SP2-2.*Q1SH*Q1SP1**2*
     C Q2SP2-Q1SH*Q1SP1*Q2SP2**2-Q1SH*Q1SP2*Q2SP1*Q2SP2+2.*Q2SH*Q1SP1**
     C 2*Q1SP2+Q2SH*Q1SP1*Q1SP2*Q2SP2+Q2SH*Q1SP2**2*Q2SP1)
      M56=M56/S**2
      M57 =-4.*S**2*P1SP2*MK2
     C +2.*S**2*P1SP2*Q2SH-4.*S**2*MK2**2+2.*S**2*MK2
     C *(-HSP2-HSP1-Q2SH-2.*Q1SP2-2.*Q2SP1)-(2.*S**2)*(HSP2*Q2SP1+HSP1*
     C Q1SP2)+4.*S*P1SP2**2*(-Q1SH+Q2SH)+4.*S*P1SP2*MK2*(Q1SH-Q2SH+2.*
     C  Q1SP1-2.*Q1SP2-2.*Q2SP1+2.*Q2SP2)+4.*S*P1SP2*(HSP2*Q1SP1-HSP2*
     C Q2SP1-HSP1*Q1SP2+HSP1*Q2SP2-Q2SH*Q1SP1-Q2SH*Q2SP2)+4.*S*MK2*(-
     C Q1SH*Q1SP2+Q1SH*Q2SP2+Q2SH*Q1SP1+Q2SH*Q1SP2+2.*Q1SP1*Q2SP1+2.*
     C Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1+2.*Q1SP2*Q2SP2)
      M57=M57+4.*S*(HSP2*Q1SP1*
     C Q2SP1+HSP2*Q2SP1*Q2SP2+HSP1*Q1SP1*Q1SP2+HSP1*Q1SP2*Q2SP2-Q1SH*
     C Q1SP2*Q2SP1+Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1*Q1SP2-Q2SH*Q1SP1*Q2SP2)+
     C 8.*P1SP2*(Q1SH*Q1SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1-Q2SH*Q1SP1*Q2SP2
```

```
      C -Q2SH
      C *Q1SP2*Q2SP1)+8.*MK2*(-Q1SH*Q1SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q2SH*
      C Q1SP1*Q2SP2+Q2SH*Q1SP2*Q2SP1-2.*Q1SP1**2*Q2SP2-2.*Q1SP1*Q1SP2*
      C Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*Q2SP2-2.*Q1SP1*Q2SP2**
      C 2+2.*Q1SP2**2*Q2SP1+2.*Q1SP2*Q2SP1**2-2.*Q1SP2*Q2SP1*Q2SP2)+8.*(
      C -HSP2*Q1SP1**2*Q2SP2-HSP2*Q1SP1*Q1SP2*Q2SP1+HSP2*Q1SP1*Q2SP1*
      C Q2SP2+HSP2*Q1SP2*Q2SP1**2+HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*
      C Q2SP2**2+HSP1*Q1SP2**2*Q2SP1-HSP1*Q1SP2*Q2SP1*Q2SP2+Q1SH*Q1SP1*
      C Q1SP2*Q2SP1-Q1SH*Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1*Q2SP2-Q1SH*
      C Q2SP1*Q2SP2**2-Q2SH*Q1SP1**2*Q1SP2+Q2SH*Q1SP1**2*Q2SP2-Q2SH*
      C Q1SP1*Q1SP2*Q2SP2+Q2SH*Q1SP1*Q2SP2**2)
       M57=M57/S
       M58 =-4.*S**2*P1SP2*MK2
      C +2.*S**2*P1SP2*Q1SH-4.*S**2*MK2**2+2.*S**2*MK2
      C *(-HSP2-HSP1-Q1SH-2.*Q1SP2-2.*Q2SP1)-(2.*S**2)*(HSP2*Q2SP1+HSP1*
      C Q1SP2)+4.*S*P1SP2**2*(Q1SH-Q2SH)+4.*S*P1SP2*MK2*(-Q1SH+Q2SH+2.*
      C Q1SP1-2.*Q1SP2-2.*Q2SP1+2.*Q2SP2)+4.*S*P1SP2*(HSP2*Q1SP1-HSP2*
      C Q2SP1-HSP1*Q1SP2+HSP1*Q2SP2-Q1SH*Q1SP1-Q1SH*Q2SP2)+4.*S*MK2*(
      C Q1SH*Q2SP1+Q1SH*Q2SP2+Q2SH*Q1SP1-Q2SH*Q2SP1+2.*Q1SP1*Q2SP1+2.*
      C Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1+2.*Q1SP2*Q2SP2)+4.*S*(HSP2*Q1SP1*
      C Q2SP1+HSP2*Q2SP1*Q2SP2+HSP1*Q1SP1*Q1SP2+HSP1*Q1SP2*Q2SP2-Q1SH*
      C Q1SP1*Q2SP2+Q1SH*Q2SP1*Q2SP2+Q2SH*Q1SP1*Q1SP2-Q2SH*Q1SP2*Q2SP1)+
      C 8.*P1SP2*(-Q1SH*Q1SP1*Q2SP2-Q1SH*Q1SP2*Q2SP1+Q2SH*Q1SP1*Q2SP2+
      C Q2SH*Q1SP2*Q2SP1)
       M58=M58+8.*MK2*(Q1SH*Q1SP1*Q2SP2+Q1SH*Q1SP2*Q2SP1-Q2SH
      C *Q1SP1*Q2SP2-Q2SH*Q1SP2*Q2SP1-2.*Q1SP1**2*Q2SP2-2.*Q1SP1*Q1SP2*
      C Q2SP1+2.*Q1SP1*Q1SP2*Q2SP2+2.*Q1SP1*Q2SP1*Q2SP2-2.*Q1SP1*Q2SP2**
      C 2+2.*Q1SP2**2*Q2SP1+2.*Q1SP2*Q2SP1**2-2.*Q1SP2*Q2SP1*Q2SP2)+8.*(
      C -HSP2*Q1SP1**2*Q2SP2-HSP2*Q1SP1*Q1SP2*Q2SP1+HSP2*Q1SP1*Q2SP1*
      C Q2SP2+HSP2*Q1SP2*Q2SP1**2+HSP1*Q1SP1*Q1SP2*Q2SP2-HSP1*Q1SP1*
      C Q2SP2**2+HSP1*Q1SP2**2*Q2SP1-HSP1*Q1SP2*Q2SP1*Q2SP2+Q1SH*Q1SP1**
      C 2*Q2SP2-Q1SH*Q1SP1*Q2SP1*Q2SP2+Q1SH*Q1SP1*Q2SP2**2-Q1SH*Q2SP1*
      C Q2SP2**2-Q2SH*Q1SP1**2*Q1SP2+Q2SH*Q1SP1*Q1SP2*Q2SP1-Q2SH*Q1SP1*
      C Q1SP2*Q2SP2+Q2SH*Q1SP2*Q2SP1*Q2SP2)
       M58=M58/S
       M66 =(-32.*S**2*MK2**3
      C -8.*S**2*MK2**2*MH2+32.*S**2*MK2**2*(-HSP2+Q2SH+
      C Q2SP2)-8.*S**2*MK2*MH2*Q2SP2+16.*S**2*MK2*(HSP2*Q2SH+2.*Q2SH*
      C Q2SP1+2.*Q2SP1*Q2SP2)-8.*S**2*MH2*Q2SP1*Q2SP2+16.*S**2*HSP2*Q2SH
      C *Q2SP1+64.*S*MK2**2*(-Q1SH*Q2SP1-Q2SH*Q1SP1+2.*Q1SP1*Q2SP1-Q1SP1
      C *Q2SP2-Q1SP2*Q2SP1)+16.*S*MK2*MH2*(2.*Q1SP1*Q2SP1+Q1SP1*Q2SP2+
      C Q1SP2*Q2SP1)+32.*S*MK2*(-HSP2*Q1SH*Q2SP1-HSP2*Q2SH*Q1SP1+4.*HSP2
      C *Q1SP1*Q2SP1-4.*Q2SH*Q1SP1*Q2SP1-4.*Q1SP1*Q2SP1*Q2SP2)+32.*S*MH2
      C *Q1SP1*Q2SP1*Q2SP2-64.*S*HSP2*Q2SH*Q1SP1*Q2SP1+256.*MK2*Q1SP1*
      C Q2SP1*(Q1SH*Q2SP1+Q2SH*Q1SP1+Q1SP1*Q2SP2+Q1SP2*Q2SP1)-(64.*MH2*
      C Q1SP1*Q2SP1)*(Q1SP1*Q2SP2+Q1SP2*Q2SP1)+128.*HSP2*Q1SP1*Q2SP1*(
      C Q1SH*Q2SP1+Q2SH*Q1SP1))/S**2
       M67 =-4.*S**2*P1SP2*MK2+S**2*P1SP2*MH2
      C -4.*S**2*MK2**2-S**2*MK2*MH2-(
      C 4.*S**2*MK2)*(HSP2+HSP1)-2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(Q1SH-
      C Q2SH+Q1SP1-Q2SP1)+2.*S*P1SP2*MH2*(-Q1SP1+Q2SP1)+4.*S*P1SP2*Q2SH*
      C (-Q1SH+Q2SH)+8.*S*MK2**2*(Q1SP2-Q2SP2)+2.*S*MK2*MH2*(-Q1SP2+
      C Q2SP2)+4.*S*MK2*(HSP1*Q1SH-HSP1*Q2SH+2.*HSP1*Q1SP1-2.*HSP1*Q2SP1
      C +Q1SH*Q2SH+2.*Q1SH*Q2SP2-Q2SH**2+2.*Q2SH*Q1SP1-2.*Q2SH*Q2SP1-2.*
      C Q2SH*Q2SP2+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1-4.*Q2SP1*Q2SP2)+2.*S*
      C MH2*(-Q1SP1*Q2SP2-Q1SP2*Q2SP1+2.*Q2SP1*Q2SP2)
       M67=M67+4.*S*(HSP2*HSP1*
```

```
     C Q1SP1-HSP2*HSP1*Q2SP1+HSP2*Q2SH*Q1SP1-HSP2*Q2SH*Q2SP1+HSP1*Q1SH*
     C Q2SP2-HSP1*Q2SH*Q2SP2)+8.*P1SP2*(Q1SH**2*Q2SP1+Q1SH*Q2SH*Q1SP1-
     C Q1SH*Q2SH*Q2SP1-Q2SH**2*Q1SP1)+8.*MK2*(-Q1SH**2*Q2SP1-Q1SH*Q2SH*
     C Q1SP1+Q1SH*Q2SH*Q2SP1-2.*Q1SH*Q1SP1*Q2SP1-2.*Q1SH*Q1SP1*Q2SP2-2.
     C *Q1SH*Q1SP2*Q2SP1+2.*Q1SH*Q2SP1**2+Q2SH**2*Q1SP1-2.*Q2SH*Q1SP1**
     C 2+2.*Q2SH*Q1SP1*Q2SP1+2.*Q2SH*Q1SP1*Q2SP2+2.*Q2SH*Q1SP2*Q2SP1-2.
     C *Q1SP1**2*Q2SP2-6.*Q1SP1*Q1SP2*Q2SP1+6.*Q1SP1*Q2SP1*Q2SP2+2.*
     C Q1SP2*Q2SP1**2)+4.*MH2*(Q1SP1**2*Q2SP2+3.*Q1SP1*Q1SP2*Q2SP1-3.*
     C Q1SP1*Q2SP1*Q2SP2-Q1SP2*Q2SP1**2)+8.*(-HSP2*Q1SH*Q1SP1*Q2SP1+
     C HSP2*Q1SH*Q2SP1**2-HSP2*Q2SH*Q1SP1**2+HSP2*Q2SH*Q1SP1*Q2SP1-HSP1
     C *Q1SH*Q1SP1*Q2SP2-HSP1*Q1SH*Q1SP2*Q2SP1+HSP1*Q2SH*Q1SP1*Q2SP2+
     C HSP1*Q2SH*Q1SP2*Q2SP1)
      M67=M67/S
      M68 =(-4.*S**2*P1SP2*MK2
     C +S**2*P1SP2*MH2-4.*S**2*MK2**2-S**2*MK2*MH2-(
     C  4.*S**2*MK2)*(HSP2+HSP1)
     C -2.*S**2*HSP2*HSP1+8.*S*P1SP2*MK2*(Q1SP1-
     C  Q2SP1)+2.*S*P1SP2*MH2*(-Q1SP1+Q2SP1)+8.*S*MK2**2*(Q1SH-Q2SH+
     C Q1SP2-Q2SP2)+2.*S*MK2*MH2*(-Q1SP2+Q2SP2)+4.*S*MK2*(HSP2*Q1SH-
     C HSP2*Q2SH+2.*HSP1*Q1SP1-2.*HSP1*Q2SP1+2.*Q1SH*Q2SP1+2.*Q2SH*
     C Q1SP1-4.*Q2SH*Q2SP1+2.*Q1SP1*Q2SP2+2.*Q1SP2*Q2SP1-4.*Q2SP1*Q2SP2
     C )+2.*S*MH2*(-Q1SP1*Q2SP2-Q1SP2*Q2SP1+2.*Q2SP1*Q2SP2)+4.*S*HSP2*(
     C HSP1*Q1SP1-HSP1*Q2SP1+Q1SH*Q2SP1+Q2SH*Q1SP1-2.*Q2SH*Q2SP1)+16.*
     C MK2*(-3.*Q1SH*Q1SP1*Q2SP1+Q1SH*Q2SP1**2-Q2SH*Q1SP1**2+3.*Q2SH*
     C Q1SP1*Q2SP1-Q1SP1**2*Q2SP2-3.*Q1SP1*Q1SP2*Q2SP1+3.*Q1SP1*Q2SP1*
     C Q2SP2+Q1SP2*Q2SP1**2)+4.*MH2*(Q1SP1**2*Q2SP2+3.*Q1SP1*Q1SP2*
     C Q2SP1-3.*Q1SP1*Q2SP1*Q2SP2-Q1SP2*Q2SP1**2)+8.*HSP2*(-3.*Q1SH*
     C Q1SP1*Q2SP1+Q1SH*Q2SP1**2-Q2SH*Q1SP1**2+3.*Q2SH*Q1SP1*Q2SP1))/S
      M77 =8.*S*P1SP2*MK2
     C -2.*S*P1SP2*MH2+8.*S*MK2**2+2.*S*MK2*MH2+8.*S*MK2*(
     C HSP2+HSP1)+4.*S*HSP2*HSP1+16.*MK2*(Q1SH*Q1SP2-Q1SH*Q2SP2-Q2SH*
     C Q1SP2+Q2SH*Q2SP2+Q1SP1*Q1SP2-Q1SP1*Q2SP2-Q1SP2*Q2SP1+Q2SP1*Q2SP2
     C )+4.*MH2*(-Q1SP1*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1-Q2SP1*Q2SP2)+8.*
     C HSP1*(Q1SH*Q1SP2-Q1SH*Q2SP2-Q2SH*Q1SP2+Q2SH*Q2SP2)
      M78 =8.*S*P1SP2*MK2
     C -2.*S*P1SP2*MH2+8.*S*MK2**2+2.*S*MK2*MH2+8.*S*MK2*(
     C HSP2+HSP1)+4.*S*HSP2*HSP1+4.*P1SP2*(-Q1SH**2+2.*Q1SH*Q2SH-Q2SH**
     C 2)+4.*MK2*(Q1SH**2-2.*Q1SH*Q2SH+2.*Q1SH*Q1SP1+2.*Q1SH*Q1SP2-2.*
     C Q1SH*Q2SP1-2.*Q1SH*Q2SP2+Q2SH**2-2.*Q2SH*Q1SP1-2.*Q2SH*Q1SP2+2.*
     C Q2SH*Q2SP1+2.*Q2SH*Q2SP2+4.*Q1SP1*Q1SP2-4.*Q1SP1*Q2SP2-4.*Q1SP2*
     C Q2SP1+4.*Q2SP1*Q2SP2)+4.*MH2*(-Q1SP1*Q1SP2+Q1SP1*Q2SP2+Q1SP2*
     C Q2SP1-Q2SP1*Q2SP2)+4.*(HSP2*Q1SH*Q1SP1-HSP2*Q1SH*Q2SP1-HSP2*Q2SH
     C *Q1SP1+HSP2*Q2SH*Q2SP1+HSP1*Q1SH*Q1SP2-HSP1*Q1SH*Q2SP2-HSP1*Q2SH
     C *Q1SP2+HSP1*Q2SH*Q2SP2)
      M88 =8.*S*P1SP2*MK2
     C -2.*S*P1SP2*MH2+8.*S*MK2**2+2.*S*MK2*MH2+8.*S*MK2*(
     C HSP2+HSP1)+4.*S*HSP2*HSP1+16.*MK2*(Q1SH*Q1SP1-Q1SH*Q2SP1-Q2SH*
     C Q1SP1+Q2SH*Q2SP1+Q1SP1*Q1SP2-Q1SP1*Q2SP2-Q1SP2*Q2SP1+Q2SP1*Q2SP2
     C )+4.*MH2*(-Q1SP1*Q1SP2+Q1SP1*Q2SP2+Q1SP2*Q2SP1-Q2SP1*Q2SP2)+8.*
     C HSP2*(Q1SH*Q1SP1-Q1SH*Q2SP1-Q2SH*Q1SP1+Q2SH*Q2SP1)
C-----------------------------------------------------------------------
C           DO THE DIVISION BY THE PROPAGATORS
C-----------------------------------------------------------------------
      M11A = M11/(D1*D3*D1*D3)
      M12A = M12/(D1*D3*D3*D4)
      M13A = M13/(D1*D3*D2*D4)
      M14A = M14/(D1*D3*D2*D5)
```

```
      M15A = M15/(D1*D3*D5*D6)
      M16A = M16/(D1*D3*D1*D6)
      M17A = M17/(D1*D3*D2*S)
      M18A = M18/(D1*D3*D1*S)
      M22A = M22/(D3*D4*D3*D4)
      M23A = M23/(D3*D4*D2*D4)
      M24A = M24/(D3*D4*D2*D5)
      M25A = M25/(D3*D4*D5*D6)
      M26A = M26/(D3*D4*D1*D6)
      M27A = M27/(D3*D4*D2*S)
      M28A = M28/(D3*D4*D1*S)
      M33A = M33/(D2*D4*D2*D4)
      M34A = M34/(D2*D4*D2*D5)
      M35A = M35/(D2*D4*D5*D6)
      M36A = M36/(D2*D4*D1*D6)
      M37A = M37/(D2*D4*D2*S)
      M38A = M38/(D2*D4*D1*S)
      M44A = M44/(D2*D5*D2*D5)
      M45A = M45/(D2*D5*D5*D6)
      M46A = M46/(D2*D5*D1*D6)
      M47A = M47/(D2*D5*D2*S)
      M48A = M48/(D2*D5*D1*S)
      M55A = M55/(D5*D6*D5*D6)
      M56A = M56/(D5*D6*D1*D6)
      M57A = M57/(D5*D6*D2*S)
      M58A = M58/(D5*D6*D1*S)
      M66A = M66/(D1*D6*D1*D6)
      M67A = M67/(D1*D6*D2*S)
      M68A = M68/(D1*D6*D1*S)
      M77A = M77/(D2*S*D2*S)
      M78A = M78/(D2*S*D1*S)
      M88A = M88/(D1*S*D1*S)

      MA1 = M11A + M22A + M33A + M44A + M55A + M66A +
     C 2.*(M12A + M13A + M23A + M45A + M46A + M56A)
      MA2 = 2.*(M14A + M15A + M16A + M24A + M25A + M26A
     C + M34A + M35A + M36A)
      MA3 = M77A + M88A + 2.*M78A
      MA4 = 2.*(M17A + M18A + M27A + M28A + M37A + M38A)
      MA5 = 2.*(M47A + M48A + M57A + M58A + M67A + M68A)
      HQ = 16.*MA1/3. -2.*MA2/3. + 12.*MA3 + 6.*MA4 -6.*MA5
      RETURN
      END
```

APPENDIX F - QUARK AND GLUON DISTRIBUTION PARAMETRIZATIONS

One of the sources of uncertainty of rate predictions in $p\bar{p}$ or pp interactions is the shape of the parton distribution employed to convolve over parton momenta. The quark distribution can be directly measured from lepton-proton interactions (Field and Feynman, 1977), because they can interact through the electromagnetic force. The discrepancy between the possible quark distributions arises from experimental uncertainty, and the different parametrizations used to fit data. Gluons distributions on the other hand, are only indirectly probed in hadron-hadron interactions. The uncertainty on them is much larger, as is reflected in the differences in cross sections they give rise to (Fig. 22 and 23).

For quarks, we used the following distributions (Peierls et al., 1977)

$$\sqrt{x}\, u(x) = 2.19\,(1-x)^3 \tag{F.1a}$$

$$\sqrt{x}\, d(x) = 1.14\,(1-x)^4 \tag{F.1b}$$

and (Barger and Phillips, 1974)

$$\sqrt{x}\, u(x) = 0.594\,(1-x^2)^3 + 0.461\,(1-x^2)^5 + 0.621\,(1-x^2)^7 \tag{F.2a}$$

$$\sqrt{x}\, d(x) = 0.072\,(1-x^2)^3 + 0.206\,(1-x^2)^5 + 0.621\,(1-x^2)^7 \tag{F.2b}$$

For gluons, we used the simple ansatz (Brodsky and Farrar, 1973)

$$xG(x) = 3(1-x)^5 \tag{F.3}$$

and the scale-violating distribution (Baier et al., 1980)

$$xG(x) = (2.01 - 2.73\rho + 1.29\rho)\, x^{-.93\rho + 0.36\rho^2} (1-x)^{2.9 + 1.83\rho} \tag{F.4}$$

with $\rho = \ln \left\{ \dfrac{\ln \hat{s}/\Lambda^2}{\ln Q_o^2/\Lambda} \right\}$

and $Q_o = 5$ GeV .

The choices for the gluon distributions are motivated by the fact that they represent two extreme expectations on the actual gluon distributions.

APPENDIX G - HADRON-HADRON COLLIDERS

Here is a table of the three planned or existing hadron-hadron colliders, and the estimated values of their c.m. energy, luminosity and corresponding event rates for a reference cross sectin of 1 Picobarn. Also listed is their starting year of operation. There are two values of luminosity listed for the SSC, as it is not decided as yet if it will be a proton-proton or proton-antiproton collider.

Collider	year	c.m. energy (TeV)	Luminosity $cm^{-2}\ s^{-1}$	rate/1 Pb.
SPS	1980	0.54	10^{29}	1 event/115 day
FERMILAB	1986	2	10^{30}	1 event/11.5day
SSC	1995?	10 to 40		
$p\bar{p}$			10^{31}	1 event/11.5day
pp			10^{33}	1 event/17 min