THE DESIGN OF INDUSTRY-SPECIFIC TRADE POLICIES AND A SEQUENTIAL ENTRY-EXIT MODEL OF INTERNATIONAL TRADE

by

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ABSTRACT

Conventional trade theory assumes perfect competition among firms and makes on balance a strong case for free trade. An important observation in the modern international economy is that competition among firms in many industries is imperfectly competitive. These firms, usually few and large, strategically interact with each other and may earn supernormal profits. As shown by the recently growing literature on trade with imperfect competition, allowing for the importance of imperfect competition leads to new insights about causes, effects, and patterns of trade, and has major implications for the analysis of trade policy as well. This study investigates the effects of firms' imperfect competition on trade policy designs and on trade patterns, product variety, and specialization. The thesis consists of two parts. The first part is entitled "The design of industry-specific trade policies" and the second part "A sequential entry-exit model of international trade".

The first part of the thesis addresses the following two questions: (1) Whether government intervention can raise the national welfare and how important the effect of intervention would be in raising welfare; and (2) Whether, or when, trade restrictions are first-best policies, and when other policy instruments would achieve the same aims more efficiently.

Dixit (1985) has recently undertaken an empirical study of strategic trade policy for a specific industry. The rivalry between the U.S. firms and Japanese firms in the U.S. passenger car market is examined. It is noticed that in Dixit's work, only the numerical (simulation) results are given and evaluated, and only the U.S. government is assumed to be active in policy-making. The purpose of the first part of the thesis is to provide a theoretical treatment of Dixit' s model, to discuss the role of policy intervention and compare the importance and efficiency of tariffs vis-à-vis domestic production subsidies under different market structures, and to examine the consequences of allowing Japan to be active in policy-making.

The basic results of this part are as follows. First, when the domestic (foreign) firms' conduct has rather significant effects on the market equilibrium relative to the foreign (domestic) firms', policy is usually directed by the domestic (foreign) firms' monopoly in the market, and a domestic production subsidy (a tariff) is usually more important and more efficient than a tariff (a domestic production subsidy). Secondly, allowing Japan to simultaneously pursue its optimal policy can reverse the result of positive U.S. welfare gains from the optimal policies, a result obtained under the condition that Japan adopts complete laissez-faire. Furthermore, Japan does have an incentive to pursue the optimal policy. Thus, The U.S. policy gains are not at all automatic or riskless. This result is obtained by examined the non-cooperative Nash equilibrium in tariff/subsidy for the U.S. and Japan. Thirdly, both countries would nonetheless be better off if they could cooperatively choose policy parameters to maximize the joint welfare rather than non-cooperatively pursue their own optimal policies. The two countries may play a bargaining game.

The purpose of the second part is to examine firms' strategic behaviour in international rivalry, and its effects on trade pattens and on product variety by using a sequential entry-exit model of trade. The paper models an industry consisting of two firms, each in a different country. The two firms are assumed to be able to potentially produce and export two imperfectly substitutable products, to be able to make their entry, exit, and production (quantity, price, etc.) decisions sequentially, and to be able to choose these strategy variables for each country separately. Two four-stage games are constructed and examined. The paper intends to do an exploration of models of international trade. The new feature of our model is that the fixed cost of withdrawing a product is considered as a variable and firms are allowed to exit in response to entry.

Three basic results emerge from the second paper. First, firms' strategic behaviour can give rise to two-way trade in identical products which are produced only for trade. The kind of two-way trade can introduce products which would otherwise not be produced in autarky. The non-cooperative solution to the firm's profit-maximizing problem involves such a two-way trade, but each firm may nonetheless be better off if the two firms could agree not to invade each other's home markets. This result is more likely to hold as exit costs are low, as transport costs are small, as products are better substitutes, as competition in identical products is more intense, and as firms are more likely to treat different countries as different markets.

Secondly, our model gives mixed results on the issue of whether trade, through intra-industry trade, makes a greater variety of products available to consumers. Whether trade increases or reduces variety depends on the firms' payoffs of various market structures and on the level of entry, exit, and transport costs. Firms' strategic interaction through trade in order to maximize profits can increase or reduce product variety. In the case of Cournot or Bertrand conduct with linear demand, trade would increase product variety. Moreover, changes in variety can be brought about by either an actual flow of trade or a potential for trade. Finally, instead of producing all substitutable products and monopolizing their home markets, firms may specialize in some products and invade each other's countries. So the third result is in favour for intra-industry trade, and it also shows specialization can be independently caused by the rivalry of oligopolistic firms.

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PART 1

THE DESIGN OF INDUSTRY-SPECIFIC TRADE POLICIES

1. INTRODUCTION

Conventional trade theory with perfect competition among firms makes on balance a strong case for free trade. In the modern international business environment, we observe that competition among firms in many world markets is imperfectly competitive. These firms, usually few and large, may thus earn supernormal profits. From a purely national perspective, policies that could shift this profit from foreign to domestic firms would be attractive to a government. Current research on trade policy in oligopolistic international markets has indicated that intervention can be beneficial to a country (early work including Brander and Spencer (1984) and (1985), Krugman (1984)). For example, Brander and Spencer (1985) finds in a Cournot duopoly model that an export subsidy can help domestic firms capture a large share of any supernormal profit in the industry. Dixit (1984) extends this result to cases with more than two firms, and finds in a Cournot oligopoly that an export subsidy is optimal as long as the number of domestic firms is not too large. Eaton and Grossman (1986) finds in a Bertrand duopoly that an export tax is optimal.

For an importing country, the government would affect not only the competitiveness of domestic firms vis-à-vis foreign firms, but also the welfare of domestic consumers. Thus, policy instruments may also include domestic production subsidies or taxes, often referred to as industrial policies. Eaton and Grossman (1986) theoretically classifies policy intervention under three circumstances: (1) When foreign firms earn pure profits and firms are competing in the home market, the home country can capture some of this profit with a tariff; (2) When prices of home products remain above marginal costs in the home market, the home country can achieve a welfare gain with a domestic antitrust policy or a production subsidy; and (3) When the home firms compete too much with each other in their exporting activities, the home country can gain with an export tax.

In the design of industry-specific trade policies, the following two questions are important: (1) Whether intervention with free trade can raise the national welfare; and (2) If it can, whether or when it is the first-best policy, and when other policy instruments would achieve the same aims more efficiently.

Dixit (1985) has recently undertaken an empirical study on strategic trade policy for a specific industry. The U.S. passenger car market is chosen and the rivalry between U.S. and Japanese firms is examined. Only the U.S. government is assumed to be active in policy-making. The policy instruments are a tariff on Japanese firms and/or a production subsidy to U.S. firms, and the latter is considered as an imperfect proxy for an antitrust policy. The policy perspective is that of U.S. social welfare. The model is constructed in the following way: in the first stage, governments simultaneously choose policies that are credible to their firms; in the second stage, firms simultaneously choose their output levels and the industrial equilibrium is then determined.

One helpful aspect of the U.S. car market is that exports of U.S. cars are negligible, and hence we can simply assume that the entire market for U.S. firms' outputs is at home. In this simple case, the production subsidy (tax) and the export subsidy (tax) are identical. We can further avoid the complexity introduced by the possible two-way trade in an oligopolistic world market: a country simultaneously exports and imports the same good. The second part of my M.Sc. thesis discusses the two-way trade using a sequential entry-exit model.

Dixit then applies his model to the U.S. car market for 1979 and 1980, which are the most recent years with reasonably free trade for which data are available. The broad findings are : (1) The market was much more competitive in 1980 than in 1979, and the case for strategic trade and antitrust policies was therefore weaker in 1980; and (2) The aggregate economic gains to the U.S. are quite small with tariffs, and the role of domestic antitrust policies or production subsidies is more significant than that of tariffs.

In Dixit (1985), only the numerical results for the particular cases of 1979 and 1980 are given and evaluated. Consequently, for the demand and cost figures used, the paper provides examples in which

interventionist trade policy for the U.S. automobile industry can raise national welfare, and suitable antitrust measures to bring domestic prices closer to marginal costs are quantitatively more important and more efficient than tariffs in raising U.S. social welfare. It is of some interest, however, to see whether we can extract some general conclusions from the established model. This also seems necessary in view of probably unreliable data. The purpose of this paper is to provide a theoretical treatment to the basic model, to discuss the role of policy intervention and compare the importance and efficiency of tariffs vis-à-vis domestic production subsidies under different market structures, and to examine the consequences of allowing Japan to be active in policy-making.

The paper is organized as follows. Section 2 briefly describes the basic model established in Dixit (1985). Section 3 shows a number of properties of this model. Section 4 is based on a slightly different model through which we discuss the role of tariffs and domestic production subsidies and compare the relative importance and efficiency of tariffs and production subsidies under different market structures. Section 5 extends the basic model, where U.S. is active in policy-making while Japan takes a laissez-faire position, into the environment where Japan is active in making policies. Two cases are examined there. One of them is the case where only Japan is active in pursuing the profit-shifting policy, and the other is the case where both the U.S. and Japan are active in policy-making. In the second case, the Nash equilibria in tariff/subsidy for U.S. and Japan are examined, and the results are compared with those from the basic model. Section 6 provides concluding remarks.

2. THE BASIC MODEL

For computational simplicity, demand functions are assumed to be linear. U.S. and Japanese cars are assumed to be imperfect substitutes for each other, but perfect substitutes within each country. Let the

subscripts 1 and 2 denote U.S. and Japanese cars respectively, and P_1, P_2 and Q_1, Q_2 denote the prices and the total quantities respectively. Then the following demand functions are considered,

$$Q_1 = \alpha_1 - \beta_1 P_1 + \gamma P_2 \tag{1}$$

$$Q_2 = \alpha_2 + \gamma P_1 - \beta_2 P_2 \tag{2}$$

where all the parameters are positive, and $\beta_1\beta_2 - \gamma^2 > 0$.

The corresponding inverse demand functions are,

$$P_1 = a_1 - b_1 Q_1 - kQ_2 \tag{3}$$

$$P_2 = a_2 - kQ_1 - b_2 Q_2 \tag{4}$$

Again, all the parameters are positive and $b_1 b_2 - k^2 > 0$.

The demand parameters in (1) and (2) or (3) and (4) are estimated using each year's prices and quantities as well as assumed elasticities. We can show (see Appendix 1) that,

$$a_1 = (e_1 + 1)P_{10} \tag{5}$$

$$a_2 = (e_1 + 1)P_{20} \tag{6}$$

$$b_1 = \frac{P_{10}(e_2 P_{10} Q_{10} + e_1 P_{20} Q_{20})}{e_1 e_2 Q_{10} P Q_0} \tag{7}$$

$$b_2 = \frac{P_{20}(e_1 P_{10} Q_{10} + e_2 P_{20} Q_{20})}{e_1 e_2 Q_{20} P Q_0} \tag{8}$$

$$k = \frac{(e_2 - e_1)P_{10}P_{20}}{e_1 e_2 PQ_0} \tag{9}$$

where

$PQ_0 = P_{10}Q_{10} + P_{20}Q_{20}$;

P_{10}, P_{20} and Q_{10}, Q_{20} are the actual prices and quantities for the year under consideration;

e_1 is the overall price elasticity of demand for passenger cars in the U.S.; and

e_2 is the elasticity of substitution between U.S. and Japanese cars, and $e_2 > e_1 > 0$.

For the central case in which $e_1 = 1$ and $e_2 = 2$, system (5)-(9) becomes,

$$a_1 = 2P_{10} \tag{10}$$

$$a_2 = 2P_{20} \tag{11}$$

$$b_1 = \frac{P_{10}(2P_{10}Q_{10} + P_{20}Q_{20})}{2Q_{10}PQ_0} \tag{12}$$

$$b_2 = \frac{P_{20}(P_{10}Q_{10} + 2P_{20}Q_{20})}{2Q_{20}PQ_0} \tag{13}$$

$$k = \frac{P_{10}P_{20}}{2PQ_0} \tag{14}$$

As shown in Appendix 2, the variables in the model are not sensitive to the changes of elasticities e_1 and e_2: the percent change of the variables from base are less than those of e_1 or e_2. We shall assume $e_1 = 1$ and $e_2 = 2$ when system (5)-(9) is used.

From (10) and (11), we have,

$$da_1 = 2dP_{10} \qquad \text{and} \qquad da_2 = 2dP_{20}$$

Therefore, an error in price data will double the estimated error in a_1 and a_2. This instability could be one of shortcomings of the current method of estimating demand parameters.

The inverse demand functions (3) and (4) can be regarded as coming from an aggregate utility function,

$$U(Q_1, Q_2) = a_1 Q_1 + a_2 Q_2 - \frac{1}{2}(b_1 Q_1^2 + 2kQ_1 Q_2 + b_2 Q_2^2)$$

with $\frac{\partial U}{\partial Q_1} = P_1$, and $\frac{\partial U}{\partial Q_2} = P_2$.

The relevant policies for the U.S. are a tariff, t, on Japanese firms and a production subsidy, s, to U.S. firms. Denote c_1 to be the unit production cost of U.S. firms, and c_2 the Japanese unit production cost plus the unit transport cost to the U.S. market. The marginal costs c_1 and c_2 are assumed as constants.

Consider one U.S. firm. Its profit, given the credible government subsidy, is $(P_1 - c_1 + s)q_1^1$ (fixed cost is not considered). The firm chooses output level q_1^1 to maximize its profit, giving the following first-order condition,

$$P_1 - c_1 + s + q_1^1 \frac{dP_1}{dq_1^1} = 0 \tag{15}$$

and

$$\frac{dP_1}{dq_1^1} = \frac{\partial P_1}{\partial Q_1}(1 + \sum_{j=2}^{n_1} \frac{dq_j^1}{dq_1^1}) + \frac{\partial P_1}{\partial Q_2} \sum_{j=1}^{n_2} \frac{dq_j^2}{dq_1^1}$$

where $q_j^1 (j = 2, 3, ..., n_1)$ are the other U.S. firms' outputs, and $q_j^2 (j = 1, 2, ..., n_2)$ are foreign (Japanese) firms' outputs (imports). The term $\frac{dP_1}{dq_1^1}$ represents the firm's belief about the effect on its price if it sells another unit, taking into account its conjecture about the output responses of the other home firms and foreign firms. $\frac{dP_1}{dq_1^1} = \frac{\partial P_1}{\partial Q_1} = -b_1$ if the firm adopts the Cournot strategy: it assumes that the output of other firms remains unchanged; $\frac{dP_1}{dq_1^1} = 0$ if the firm adopts the competitive strategy: it believes that it can sell as much output as it likes at the going price.

Aggregate the first-order conditions over all n_1 U.S. firms, we have,

$$P_1 - c_1 + s - Q_1 V_1 = 0 \tag{16}$$

where $V_1 = -(\sum_{j=1}^{n_1} q_j^1 \frac{dP_1}{dq_j^1})/n_1 Q_1$ is the aggregate version of the conjectural variation parameter. It equals b_1/n_1 in the case of Cournot conduct. It is numerically smaller if the oligopoly is more competitive than that, and zero in the case of perfect competition. The effects of the U.S. firms' oligopolistic conduct on the market equilibrium are thus channelled through V_1.

Similarly, for the Japanese firms, we have,

$$P_2 - c_2 - t - Q_2 V_2 = 0 \tag{17}$$

The parameter V_2 can be similarly interpreted. We shall assume V_1 and V_2 are positive in this paper.

For each year, V_1 and V_2 are determined by eqs. (16)-(17) using the observed prices, quantities, costs, tariff and subsidy. The degree of competition implied would thus be different for different observed figures used. For example, it may vary from one year to another.

As long as V_1 and V_2 have been determined, the equilibrium prices P_1, P_2 and quantities Q_1, Q_2 can be obtained from eqs. (3), (4), (16), and (17) for any given policy configuration (t,s) (suppose c_1 and c_2 have been already given). The U.S. social welfare can then be calculated, which is the sum of the U.S. consumer surplus, the U.S. firms' profits, and the government tariff revenue and subsidy cost,

$$W = (U(Q_1, Q_2) - P_1 Q_1 - P_2 Q_2) + (P_1 - c_1 + s)Q_1 + (tQ_2 - sQ_1) \tag{18}$$

By perfectly anticipating firms' behaviour, the U.S. government, before firms choose their optimal output levels, chooses the optimal tariff and/or the optimal subsidy rates to maximize the national welfare.

3. PROPERTIES OF THE BASIC MODEL

As was mentioned, the equilibrium quantities Q_1, Q_2 and the equilibrium prices P_1, P_2 can be obtained from (3), (4), (16), and (17) for any given policy configuration (t,s). They turn out to be,

$$Q_1 = \frac{1}{D}\big((b_2 + V_2)(a_1 - c_1 + s) - k(a_2 - c_2 - t)\big) \tag{19}$$

$$Q_2 = \frac{1}{D}\big((b_1 + V_1)(a_2 - c_2 - t) - k(a_1 - c_1 + s)\big) \tag{20}$$

$$P_1 = a_1 - b_1 Q_1 - kQ_2 \tag{21}$$

$$P_2 = a_2 - kQ_1 - b_2 Q_2 \tag{22}$$

where $D \equiv (b_1 + V_1)(b_2 + V_2) - k^2 > 0$.

From (19)-(22), we can carry out the comparative statics of the equilibrium quantities and prices with respect to changes in tariffs and subsidies. To the tariff, we have,

$$\begin{aligned}
\frac{\partial Q_1}{\partial t} &= \frac{k}{D} > 0 \\
\frac{\partial Q_2}{\partial t} &= -\frac{b_1 + V_1}{D} < 0 \\
\frac{\partial P_1}{\partial t} &= \frac{kV_1}{D} > 0 \\
\frac{\partial P_2}{\partial t} &= \frac{b_2(b_1 + V_1) - k^2}{D} > 0
\end{aligned}$$

Thus, a tariff has the following effects: (1) shifting profits away from the Japanese firms (as $Q_1 \uparrow$, $Q_2 \downarrow$), (2) improving the U.S. terms-of-trade (as $\frac{\partial P_2}{\partial t} - 1 = -\frac{(b_1+V_1)V_2}{D} < 0$), (3) indirectly reducing the domestic distortion (as $Q_1 \uparrow$), (4) maybe decreasing the U.S. consumer surplus (as $P_1 \uparrow$), and (5) further distorting

trade (as $Q_2 \downarrow$, $P_2 \uparrow$). The first three effects are welfare improving for the U.S. while the last two are welfare worsening. In our case where demand functions are assumed to be linear, we shall see shortly that the net effect on the U.S. welfare is positive if the tariff is optimally chosen. Eaton and Grossman (1986) discusses the general case.

Similarly, to the subsidies, we have,

$$\frac{\partial Q_1}{\partial s} = \frac{b_2 + V_2}{D} > 0$$
$$\frac{\partial Q_2}{\partial s} = -\frac{k}{D} < 0$$
$$\frac{\partial P_1}{\partial s} = -\frac{b_1(b_2 + V_2) - k^2}{D} < 0$$
$$\frac{\partial P_2}{\partial s} = -\frac{kV_2}{D} < 0$$

As we can see, a subsidy has the direct role of reducing the domestic distortion (as $Q_1 \uparrow$, $P_1 \downarrow$) and indirect role of shifting profits from Japanese firms (as $Q_2 \downarrow$). But meanwhile, trade may be distorted (as $Q_2 \downarrow$). Again, as is to be shown, the net effect on the U.S. welfare is positive for the case under consideration if the subsidy is optimally chosen.

We turn now to the discussions of optimal policies. Eqs. (19)-(22) indicate that the equilibrium quantities and prices are functions of (t,s). Hence, according to (18), the total U.S. welfare is also a function of (t,s). Calculations show,

$$W(t,s) = \frac{1}{D^2}(-K_1 t^2 - K_2 ts - K_3 s^2 + K_4 t + K_5 s) + W(0,0) \tag{23}$$

where

$$K_1 = \frac{1}{2}\big((b_1 + 2V_1)(b_1 b_2 - k^2) + b_2 V_1^2 + 2V_2(b_1 + V_1)^2\big) \tag{24}$$

$$K_2 = k\big((b_1 b_2 - k^2) + (2b_1 + V_1)V_2\big) \tag{25}$$

$$K_3 = \frac{1}{2}\big(b_2(b_1 b_2 - k^2) + b_1 V_2(2b_2 + V_2)\big) \tag{26}$$

$$K_4 = k(b_2 V_1 - b_1 V_2)(a_1 - c_1) + \big((b_1 + V_1)^2 V_2 - k^2 V_1\big)(a_2 - c_2) \tag{27}$$

$$K_5 = \big((b_2 + V_2)^2 V_1 - k^2 V_2\big)(a_1 - c_1) + k(b_1 V_2 - b_2 V_1)(a_2 - c_2) \tag{28}$$

and W(0,0) is the total welfare when both t and s are set at zero, that is, U.S. adopts complete laissez-faire. That $K_1 > 0$, $K_2 > 0$, and $K_3 > 0$ can be readily seen from (24), (25), and (26) respectively. Further, we can discuss different optimal policy configurations (t,s) by using (23). In particular, the following three policy configurations are of interest and are to be analysed.

1) The optimal tariff (t^*,0)

When one of the policy instruments, the subsidy, is not available but the tariff can be optimally chosen, for any t the total welfare is,

$$W(t,s) \equiv W(t,0) = \frac{1}{D^2}(-K_1 t^2 + K_4 t) + W(0,0) \tag{29}$$

with

$$\frac{\partial W(t,0)}{\partial t} = \frac{1}{D^2}(-2K_1 t + K_4),$$
$$\frac{\partial^2 W(t,0)}{\partial t^2} = -\frac{2K_1}{D^2} < 0.$$

The optimal tariff, t^*, can be obtained from the first-order condition, giving

$$t^* = \frac{K_4}{2K_1} \tag{30}$$

where K_1 and K_4 are given by (24) and (27) respectively. Substituting (30) into (29), we obtain the welfare level $W(t^*,0)$ under the optimal tariff. $W(t^*,0)$ is the maximal welfare when only the tariff instrument is

available.

$$W(t^*,0) = \frac{1}{D^2}K_1(t^*)^2 + W(0,0) \tag{31}$$

It can be easily seen from (31) that

$$W(t^*,0) > W(0,0), \qquad \text{and}$$

$$W(t^*,0) = W(0,0) \quad \text{iff} \quad t^* = 0.$$

where "iff" means "if and only if".

2) The optimal subsidy $(0, s^*)$

When the tariff instrument is not available but the subsidy can be optimally chosen, we can similarly calculate the optimal subsidy as well as the corresponding maximal welfare. They are,

$$s^* = \frac{K_5}{2K_3} \tag{32}$$

and

$$W(0,s^*) = \frac{1}{D^2}K_3(s^*)^2 + W(0,0) \tag{33}$$

respectively. Therefore,

$$W(0,s^*) > W(0,0), \qquad \text{and}$$

$$W(0,s^*) = W(0,0) \quad \text{iff} \quad s^* = 0.$$

3) The optimal tariff and subsidy (t^{**}, s^{**})

When both the tariff and the subsidy instruments are available, the welfare function is that of (23). The first-order conditions are,

$$\frac{\partial W(t,s)}{\partial t} = \frac{1}{D^2}(-2K_1 t - K_2 s + K_4) = 0,$$
$$\frac{\partial W(t,s)}{\partial s} = \frac{1}{D^2}(-K_2 t - 2K_3 s + K_5) = 0.$$

The second-order conditions are,

$$\frac{\partial^2 W(t,s)}{\partial t^2} = -\frac{2K_1}{D^2} < 0,$$
$$\frac{\partial^2 W(t,s)}{\partial s^2} = -\frac{2K_3}{D^2} < 0,$$
$$\frac{\partial^2 W(t,s)}{\partial t^2}\frac{\partial^2 W(t,s)}{\partial s^2} - \frac{\partial^2 W(t,s)}{\partial s \partial t}\frac{\partial^2 W(t,s)}{\partial t \partial s} = \frac{1}{D^4}(4K_1K_3 - K_2^2) > 0.$$

The inequality $4K_1K_3 - K_2^2 > 0$ is proved in Appendix 3. From the second-order conditions, we see that W(t,s) is strictly concave. Therefore, the point (t^{**}, s^{**}) which satisfies the first-order conditions is the unique maximizing solution to W(t,s), and

$$t^{**} = \frac{2K_3K_4 - K_2K_5}{4K_1K_3 - K_2^2} \tag{34}$$

$$s^{**} = \frac{2K_1K_5 - K_2K_4}{4K_1K_3 - K_2^2} \tag{35}$$

In particular, it can be shown that

$$W(t^{**}, s^{**}) > W(t^*, 0), \qquad \text{and}$$

$$W(t^{**}, s^{**}) = W(t^*, 0) \qquad \text{iff} \qquad s^{**} = 0.$$

The proof is not hard. If $s^{**} \neq 0$, $\Rightarrow (t^{**}, s^{**}) \neq (t^*, 0)$, $\Rightarrow W(t^{**}, s^{**}) > W(t^*, 0)$ because (t^{**}, s^{**}) is the unique maximizing point; if $s^{**} = 0$, $\Rightarrow K_5/K_4 = K_2/2K_1$, $\Rightarrow t^{**} = \frac{2K_3 - K_2K_5}{4K_1K_3 - K_2^2} = \frac{K_4}{2K_1}\frac{2K_3 - K_2K_5/K_4}{2K_3 - K_2^2/2K_1} = \frac{K_4}{2K_1} = t^*$, $\Rightarrow (t^{**}, s^{**}) = (t^*, 0)$, $\Rightarrow W(t^{**}, s^{**}) = W(t^*, 0)$. Similarly, we have that,

$$W(t^{**}, s^{**}) > W(0, s^*), \qquad \text{and}$$

$$W(t^{**}, s^{**}) = W(0, t^*) \qquad \text{iff} \qquad t^{**} = 0.$$

The foregoing discussions have proved the following result,

Result 1. For the basic model of section 2, the optimal tariff and the optimal production subsidy are each welfare improving (relative to complete laissez-faire) if implemented separately; but an optimal policy package involves both tariffs and production subsidies.

In the optimal tariff and the optimal subsidy formulas,

$$t^* = \frac{K_4}{2K_1} \qquad \text{and} \qquad s^* = \frac{K_5}{2K_3}$$

the signs of t^* and s^* depend, respectively, on the signs of K_4 and K_5 because $K_1 > 0$, $K_3 > 0$. We have,

Result 2. If the demand parameters are estimated using eqs. (10)-(14), and $2Q_{10} > P_{10}$, $2Q_{20} > P_{20}$, then

$$t^* > 0 \qquad \text{and} \qquad s^* > 0.$$

Proof Appendix 4 shows that if the conditions are satisfied, then $K_4 > 0$ and $K_5 > 0$. Therefore, $t^* > 0$ and $s^* > 0$.

Q.E.D.

It is noted that if both V_1 and V_2 are zero, then from (27) and (28) both K_4 and K_5 are zero, and thus $t^* = 0$, $s^* = 0$, and $t^{**} = s^{**} = 0$. In words, if competition is perfect, laissez-faire is the optimal policy for U.S.. But in general, as Result 2 indicates, the optimal policy includes a positive tariff or a positive production subsidy if only one policy instrument is available.

It is interesting to examine how large the welfare gains that are available from the pursuit of an optimal tariff. When a domestic production subsidy or antitrust policy is not available, the gain from using the optimal tariff over free trade is $\Delta_{t,0} \equiv W(t^*, 0) - W(0,0)$. When the subsidy or antitrust policy is available, the contribution of the tariff over what is possible with the optimal subsidy is given by $\Delta_{ts,s} \equiv W(t^{**}, s^{**}) - W(0, s^*)$. We have,

Result 3. If $t^{**} > 0$ and $s^{**} > 0$, then

(i) $0 < \Delta_{ts,s} < \Delta_{t,0}$,

(ii) $\Delta_{s,0}/\alpha < \Delta_{t,0} < \alpha\Delta s, 0$, where $\Delta_{s,0} \equiv W(0, s^*) - W(0,0)$, $\alpha = 4K_1K_3/K_2^2 > 1$.

Proof (i) Rearranging (34), (35) and substituting for t^* and s^*, we have,

$$t^{**} = \frac{4K_1K_3t^* - 2K_2K_3s^*}{4K_1K_3 - K_2^2} \tag{36}$$

$$s^{**} = \frac{4K_1K_3s^* - 2K_1K_2t^*}{4K_1K_3 - K_2^2} \tag{37}$$

Solve for t^* and s^*,

$$t^* = t^{**} + \frac{K_2}{2K_1}s^{**} \tag{38}$$

$$s^* = s^{**} + \frac{K_2}{2K_3}t^{**} \tag{39}$$

If $t^{**} > 0$ and $s^{**} > 0$, then from (38) and (39),

$$t^* > t^{**} > 0 \qquad \text{and} \qquad s^* > s^{**} > 0.$$

$$\begin{aligned}
\Delta_{ts,s} &= W(t^{**}, s^{**}) - W(0, s^*) \\
&= \frac{1}{D^2}\Big(-K_1(t^{**})^2 - K_2 t^{**} s^{**} - K_3\big((s^{**})^2 - (s^*)^2\big) + K_4 t^{**} + K_5(s^{**} - s^*)\Big) \\
&= \frac{1}{D^2}\left(-\frac{4K_1K_3 - K_2^2}{4K_3}(t^{**})^2 + \frac{2K_3K_4 - K_2K_5}{2K_3}t^{**}\right) \\
&= \frac{1}{D^2}\frac{4K_1K_3 - K_2^2}{4K_3}(t^{**})^2
\end{aligned}$$

where the second from the last equality follows by using (39) and the last equality follows by using (34). Therefore, $\Delta_{ts,s} > 0$.

From (31), $\Delta_{t,0} = \frac{1}{D^2}K_1(t^*)^2$. Thus,

$$\frac{\Delta_{ts,s}}{\Delta_{t,0}} = \frac{4k_1K_3 - K_2^2}{4K_1K_3}\left(\frac{t^{**}}{t^*}\right)^2 < \left(\frac{t^{**}}{t^*}\right)^2 < 1 \qquad \text{if} \quad t^{**} > 0, s^{**} > 0.$$

That is, $\Delta_{ts,s} < \Delta_{t,0}$.

(ii) If $t^{**} > 0$ and $s^{**} > 0$, then from (36) and (37),

$$4K_1K_3t^* - 2K_2K_3s^* > 0$$

$$4K_1K_3s^* - 2K_1K_2t^* > 0$$

that is,

$$\frac{K_2}{2K_3} < \frac{s^*}{t^*} < \frac{2K_1}{K_2} \tag{40}$$

From (31) and (33), we have,

$$\frac{\Delta_{s,0}}{\Delta_{t,0}} = \frac{K_3}{K_1}\left(\frac{s^*}{t^*}\right)^2$$

Then use (40),

$$\frac{K_2^2}{4K_1K_3} < \frac{\Delta_{s,0}}{\Delta_{t,0}} < \frac{4K_1K_3}{K_2^2}$$

Therefore, $\Delta_{s,0}/\alpha < \Delta_{t,0} < \alpha\Delta_{s,0}$.

Q.E.D.

In the remaining of this section, we consider the effects of changing the marginal costs c_1 and c_2 on the optimal tariff and the optimal subsidy.

In the basic model, the aggregate conjectural variations are related to the marginal costs,

$$V_1 = \frac{P_{10} - c_1 + s_0}{Q_{10}} \qquad \text{and} \qquad V_2 = \frac{P_{20} - c_2 - t_0}{Q_{20}}$$

where P_{10}, P_{20}, and Q_{10}, Q_{20}, as before, are the actual prices and quantities in a year; s_0 and t_0 are the actual subsidy and tariff. Thus,

$$\frac{dV_1}{dc_1} = -\frac{1}{Q_{10}} \qquad \text{and} \qquad \frac{dV_2}{dc_2} = -\frac{1}{Q_{20}},$$

indicating that the lower the marginal costs, the higher degree of implicit monopoly in the market.

The following result arises from the basic model,

Result 4. If the demand parameters are estimated using eqs. (10)-(14), then

$$\frac{\partial s^*}{\partial c_1} < 0.$$

Proof $\frac{\partial s^*}{\partial c_1} = \frac{\partial}{\partial c_1}\left(\frac{K_5}{2K_3}\right) = \frac{1}{2K_3}\frac{\partial K_5}{\partial c_1} < 0$ if $\frac{\partial K_5}{\partial c_1} < 0$ (as $K_3 > 0$). That $\frac{\partial K_5}{\partial c_1} < 0$ is proved in Appendix 5.

Q.E.D.

Result 4 shows that for a lower domestic unit cost, corresponding to a higher degree of implicit collusion among the domestic firms, a higher subsidy level can be justified when only the subsidy policy is implemented.

In $\frac{\partial t^*}{\partial c_2} = \frac{\partial}{\partial c_2}\left(\frac{K_4}{2K_1}\right)$, that $\frac{\partial K_4}{\partial c_2} < 0$ can be similarly shown. But now $\frac{\partial(2K_1)}{\partial c_2} = 2(b_1 + V_1)^2\frac{dV_2}{dc_2} < 0$. So a fall of c_2 would cause both K_4 and $2K_1$ rising, and whether the optimal tariff t^* rises or falls would depends on the sign of $\left(\frac{\partial K_4/\partial c_2}{K_4} - \frac{\partial K_1/\partial c_2}{K_1}\right)$, an expression which is computationally hard to sign. Similar situations are also met in signing $\frac{\partial t^*}{\partial c_1}$ and $\frac{\partial s^*}{\partial c_2}$. Thus, we instead carry out a sensitivity analysis for c_1 and c_2 using 1979 data of the U.S. car market. The results are contained in Table 1.

Table 1 is inserted here.

Three observations are obvious from Table 1.

1) Both t^* and s^* are negatively related to c_1, c_2, suggesting that for lower marginal costs, corresponding to a higher degree of implicit monopoly in the market, greater government intervention can be justified.

2) Both t^* and s^* are sensitive to the domestic unit cost c_1. Of them, the optimal subsidy is particularly sensitive to c_1. For instance, a 5% decrease in c_1 from base would cause a 50% increase in s^* from base. On the other hand, the optimal subsidy s^* is not sensitive to the change of the foreign marginal cost c_2, while the optimal tariff t^* is very sensitive to the change of c_2: a 5% decrease in c_2 from base would cause a 20% increase in t^* from base. In view of the relationship between c_1 and V_1, c_2 and V_2, the tariff seems more closely related to the competitiveness of foreign firms whereas the optimal subsidy seems more closely related to the competitiveness of domestic firms. This point is also supported by the following third observation from Table 1.

3) In the column OPT-TS of Table 1 where tariff and subsidy are jointly chosen to maximize the welfare, a fall of c_1, corresponding to an increase in implicit collusion of U.S. firms, would cause the subsidy component of the optimal policy package, s^{**}, to rise and the tariff component t^{**} to fall. Thus, in the

TABLE 1
SENSITIVITY ANALYSIS FOR C1 AND C2 IN THE CENTRAL CASE, 1979
(PERCENT CHANGE FROM BASE)

	OPT-TARI	OPT-SUBS	OPT-TS
C1	-5	-5	-5
T	10.13		-3.359
S		50.51	55.67

	OPT-TARI	OPT-SUBS	OPT-TS
C1	-2.500	-2.500	-2.500
T	5.174		-1.680
S		24.76	27.29

	OPT-TARI	OPT-SUBS	OPT-TS
C1	2.500	2.500	2.500
T	-5.400		1.680
S		-23.76	-26.19

	OPT-TARI	OPT-SUBS	OPT-TS
C1	5	5	5
T	-11.04		3.359
S		-46.52	-51.28

	OPT-TARI	OPT-SUBS	OPT-TS
C2	-5	-5	-5
T	20.79		28.90
S		2.503	.1642

	OPT-TARI	OPT-SUBS	OPT-TS
C2	-2.500	-2.500	-2.500
T	10.55		14.66
S		1.289	.0864

	OPT-TARI	OPT-SUBS	OPT-TS
C2	2.500	2.500	2.500
T	-10.91		-15.15
S		-1.371	-.0963

	OPT-TARI	OPT-SUBS	OPT-TS
C2	5	5	5
T	-22.24		-30.87
S		-2.833	-.2044

TABLE 2
SENSITIVITY ANALYSIS FOR P10 AND P20 IN THE CENTRAL CASE, 1979
(PERCENT CHANGE FROM BASE)

	OPT-TARI	OPT-SUBS	OPT-TS
P10	-5	-5	-5
T	-11.66		3.581
S		-51.03	-56.15

	OPT-TARI	OPT-SUBS	OPT-TS
P10	-2.500	-2.500	-2.500
T	-5.553		1.749
S		-26.04	-28.66

	OPT-TARI	OPT-SUBS	OPT-TS
P10	2.500	2.500	2.500
T	5.073		-1.672
S		27.02	29.73

	OPT-TARI	OPT-SUBS	OPT-TS
P10	5	5	5
T	9.727		-3.271
S		54.95	60.45

	OPT-TARI	OPT-SUBS	OPT-TS
P20	-5	-5	-5
T	-27.82		-36.78
S		-3.379	-.2896

	OPT-TARI	OPT-SUBS	OPT-TS
P20	-2.500	-2.500	-2.500
T	-13.66		-18.05
S		-1.628	-.1374

	OPT-TARI	OPT-SUBS	OPT-TS
P20	2.500	2.500	2.500
T	13.27		17.52
S		1.525	.1258

	OPT-TARI	OPT-SUBS	OPT-TS
P20	5	5	5
T	26.23		34.61
S		2.964	.2425

TABLE 3
SENSITIVITY ANALYSIS FOR Q10 AND Q20 IN THE CENTRAL CASE, 1979
(PERCENT CHANGE FROM BASE)

	OPT-TARI	OPT-SUBS	OPT-TS
Q10	-5	-5	-5
T	-.0369		.0629
S		.4404	.0695

	OPT-TARI	OPT-SUBS	OPT-TS
Q10	-2.500	-2.500	-2.500
T	-.0180		.0307
S		.2155	.0339

	OPT-TARI	OPT-SUBS	OPT-TS
Q10	2.500	2.500	2.500
T	.0171		-.0294
S		-.2067	-.0324

	OPT-TARI	OPT-SUBS	OPT-TS
Q10	5	5	5
T	.0333		-.0575
S		-.4052	-.0634

	OPT-TARI	OPT-SUBS	OPT-TS
Q20	-5	-5	-5
T	.0350		-.0604
S		-.4257	-.0666

	OPT-TARI	OPT-SUBS	OPT-TS
Q20	-2.500	-2.500	-2.500
T	.0175		-.0301
S		-.2119	-.0332

	OPT-TARI	OPT-SUBS	OPT-TS
Q20	2.500	2.500	2.500
T	-.0175		.0300
S		.2102	.0331

	OPT-TARI	OPT-SUBS	OPT-TS
Q20	5	5	5
T	-.0350		.0598
S		.4186	.0660

optimal policy package, the subsidy becomes quantitatively more significant while the tariff less. The similar situation happens as c_2 falls. When foreign firms become more collusive, t^{**} would rise rapidly while s^{**} would remain almost unchanged.

Similar observations can be found in Table 2 where the sensitivity analysis for the actual prices is conducted. Furthermore, it is noted that,

$$\frac{dV_i}{V_i} = \left(\frac{P_{i0}}{Q_{i0}V_i}\right)\frac{dP_{i0}}{P_{i0}} = \left(-\frac{c_i}{Q_{i0}V_i}\right)\frac{dc_i}{c_i}, \qquad i = 1,2$$

Using 1979 data, we calculate,

$$\frac{P_{10}}{Q_{10}V_1} = 10.8, \qquad \frac{c_1}{Q_{10}V_1} = 9.8, \qquad \frac{P_{20}}{Q_{20}V_2} = 8.0, \qquad \frac{c_2}{Q_{20}V_2} = 6.8.$$

That is, a 1% change of P_{10} or c_1 (P_{20} or c_2) from base would, approximately, cause a 10% (7.5%) change in V_1 (V_2) from base. Hence the aggregate conjectural variations are very sensitive to the price or cost data.

The practical matter of the above sensitivity analysis is in the design of good policies. The difficulty in finding accurate cost and price information would make the profit-shifting policy less attractive in practice.

Finally, the sensitivity analysis for actual quantities (see Table 3) shows that neither the tariff nor the subsidy is sensitive to the quantity variables. Moreover, the quantity data are relatively easy to collect and are much less volatile than the data for either costs or prices.

4. THE ROLE OF TARIFFS AND PRODUCTION SUBSIDIES

In section 3 we have demonstrated several properties of the basic model. (1) A tariff has a direct role of shifting pure profit away from foreign frims to domestic firms and an indirect role of reducing domestic distortion, whereas a production subsidy has a direct role of reducing domestic distortion and an indirect

role of shifting profit. (2) The optimal tariff and the optimal subsidy are each welfare improving (Result 1). (3) The higher the degree of implicit collusion of domestic firms is, the larger the production subsidy is required (Result 4). The simulation results also show the positive relations between the optimal tariff level and the degree of firms' implicit monopoly in the market; moreover, the subsidy (tariff) seems more closely related to how collusive the domestic (foreign) firms are (Table 1).

It is noted that the formulas for the optimal tariff t^* and the optimal subsidy s^* involve both the marginal costs c_i and the aggregate conjectures V_i. In the basic model V_i is related with c_i through

$$V_1 = \frac{P_{10} - c_1 + s_0}{Q_{10}}, \qquad V_2 = \frac{P_{20} - c_2 - t_0}{Q_{20}}.$$

This would make the comparative statics of t^* and s^* with respect to changes in c_1 and c_2 computationally too complex to do. Partly for this reason, we shall in this section slightly change the basic model by assuming that the conjecture V_1 (V_2) is independent of the cost c_1 (c_2). So the effect of the domestitic (foreign) firms' conduct on the market equilibrium is assumed to be independent of the change in the marginal cost of domestic (foreign) firms. In the case of Cournot conduct, $V_i = b_i/n_i$ (i=1,2), and hence our assumption means that the exogenous number of firms n_i would not be affected by changes in the marginal cost c_i which may be brought by an exogenous technology advance. This is similar to the model of Brander (1981), Brander and Spencer (1983), and Dixit (1984), in that firms in each country follow Cournot behaviour and the number of firms in each economy is arbitrarily fixed.

The equilibrium quantities and prices are given by (19)–(22) and are functions of c_1, c_2 for given t, s, and V_1, V_2. We consider the following three different policy configurations (t,s):

(i) complete laissez-faire (0,0), $W = W_0(c_1, c_2)$;

(ii) the optimal tariff $(t^*, 0)$, $t^* = t^*(c_1, c_2)$, and $W = W_t(c_1, c_2)$;

(iii) the optimal subsidy $(0, s^*)$, $s^* = s^*(c_1, c_2)$, and $W = W_s(c_1, c_2)$.

When marginal costs are exogenously changed, it would be expected that the monopoly power that is collectively possessed by domestic (foreign) firms in the market might also be changed. We define,

$$\phi_1 = \phi_1(c_1, c_2) = \frac{P_1 - c_1}{P_1} \tag{41}$$

$$\phi_2 = \phi_2(c_1, c_2) = \frac{P_2 - c_2}{P_2} \tag{42}$$

ϕ_1 (ϕ_2) is an index which is assumed to measure the monopoly power collectively possessed by domestic (foreign) firms in the market. It is noted that in a single product and monopolized industry, $\phi = \frac{P-c}{P}$ is the Lerner Index of monopoly power.

In the following analysis we shall variate the domestic marginal cost c_1 while hold the foreign marginal cost c_2 fixed (Note 1).

Result 5. In complete laissez-faire,

(i) $\partial\phi_1/\partial c_1 < 0$;

(ii) $\partial\phi_2/\partial c_2 > 0$.

Proof (i)

$$\begin{aligned}\frac{\partial P_1}{\partial c_1} &= -b_1\frac{\partial Q_1}{\partial c_1} - k\frac{\partial Q_2}{\partial c_1}\\ &= \frac{b_1}{D}(b_2 + V_2) - \frac{k}{D}k\\ &= \frac{1}{D}(b_1V_2 + b_1b_2 - k^2) > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial\phi_1}{\partial c_1} &= \frac{\partial}{\partial c_1}\left(\frac{P_1 - c_1}{P_1}\right)\\ &= -\frac{P_1 - c_1\partial P_1/\partial c_1}{P_1^2}\\ &= -\frac{(P_1 - c_1)(b_1V_2 + b_1b_2 - k^2) + P_1V_1(b_2 + V_2)}{P_1^2D} < 0\end{aligned}$$

(ii) Similarly, we have

$$\frac{\partial P_2}{\partial c_1} = \frac{kV_2}{D} > 0$$

$$\frac{\partial \phi_2}{\partial c_1} = \frac{kV_2 c_2}{P_2^2 D} > 0$$

Q.E.D.

Thus, as the domestic marginal cost falls, the domestic firms would increase their monopoly power while the monopoly power possessed by the foreign firms would be reduced. We do not yet know whether, and when, the overall market becomes less or more competitive. For this purpose we would like to compare the rate of increase in ϕ_1 with the rate of decrease in ϕ_2, that is, compare $|\partial\phi_1/\partial c_1|$ with $|\partial\phi_2/\partial c_2|$. To avoid the algebraic complexity, we shall instead compare $|\partial\bar{\phi}_1/\partial c_1|$ with $|\partial\bar{\phi}_2/\partial c_1|$, where $\bar{\phi}_1 = P_1 - c_1 = P_1\phi_1$ and $\bar{\phi}_2 = P_2 - c_2 = P_2\phi_2$. It is easy to show that

$$\frac{\partial\bar{\phi}_1}{\partial c_1} = -\frac{(b_2 + V_2)V_1}{D} < 0$$

$$\frac{\partial\bar{\phi}_2}{\partial c_1} = \frac{kV_2}{D} > 0$$

Therefore,

$$\left|\frac{\partial\bar{\phi}_1}{\partial c_1}\right| > \left|\frac{\partial\bar{\phi}_2}{\partial c_1}\right| \quad \text{iff}$$

$$\frac{V_1}{V_2} > \frac{k}{b_2 + V_2} \tag{43}$$

Condition (43) holds when the effect of the domestic firms' collusive conduct on the market outcome is sufficiently large. If the market structure is such that condition (43) holds, the rate of increase in the monopoly power possessed by the domestic firms firms, as their marginal cost falls, would be greater than the rate of decrease in the monopoly power of foreign firms. It is suspected in this case that intervention

would be more important and, since the domestic monopolistic distortion becomes a more serious problem, the role of subsidies would be more significant than that of tariffs. We now examine this suspicion.

From the optimal tariff (30) and the optimal subsidy (32), we have

$$\frac{\partial t^*}{\partial c_1} = \frac{\partial}{\partial c_1}\left(\frac{K_4}{2K_1}\right) = -\frac{k}{2K_1}(b_2V_1 - b_1V_2) \tag{44}$$

and

$$\frac{\partial s^*}{\partial c_1} = \frac{\partial}{\partial c_1}\left(\frac{K_5}{2K_3}\right) = -\frac{1}{2K_3}\left((b_2 + V_2)^2V_1 - k^2V_2\right). \tag{45}$$

Thus,

$$\frac{\partial t^*}{\partial c_1} < 0 \qquad \text{iff} \qquad b_2V_1 - b_1V_2 > 0 \quad \text{iff}$$

$$\frac{V_1}{V_2} > \frac{b_1}{b_2} \tag{46}$$

and

$$\frac{\partial s^*}{\partial c_1} < 0 \qquad \text{iff} \qquad (b_2 + V_2)^2V_1 - k^2V_2 > 0 \quad \text{iff}$$

$$\frac{V_1}{V_2} > \frac{k^2}{(b_2 + V_2)^2} \tag{47}$$

It can be easily shown that if $b_1 > k$ (Note 2), then

$$\frac{b_1}{b_2} > \frac{k}{b_2 + V_2} > \frac{k^2}{(b_2 + V_2)^2} \tag{48}$$

Therefore, when condition (46) holds, i.e.,$V_1/V_2 > b_1/b_2$, both conditions (43) and (47) will hold, given $b_1 > k$.

Further, let

$$\Delta_{t,0} \equiv \Delta_{t,0}(c_1, c_2) \equiv W_t(c_1, c_2) - W_0(c_1, c_2)$$

$$\Delta_{s,0} \equiv \Delta_{s,0}(c_1, c_2) \equiv W_s(c_1, c_2) - W_0(c_1, c_2)$$

that is, $\Delta_{t,0}$ and $\Delta_{s,0}$ are welfare gains from the optimal tariff and the optimal subsidy over complete laissez-faire, respectively. From (31) and (33), we have

$$\Delta_{t,0} = \frac{K_1}{D^2}(t^*)^2 \qquad \text{and} \qquad \Delta_{s,0} = \frac{K_3}{D^2}(s^*)^2$$

Then

$$\frac{\partial \Delta_{t,0}}{\partial c_1} = \frac{K_1}{D^2} 2t^* \frac{\partial t^*}{\partial c_1} = \frac{K_1}{D^2} 2 \frac{K_4}{2K_1} \frac{\partial t^*}{\partial c_1} = \frac{K_4}{D^2} \frac{\partial t^*}{\partial c_1}$$

Similarly,

$$\frac{\partial \Delta s, 0}{\partial c_1} = \frac{K_5}{D^2} \frac{\partial s^*}{\partial c_1}$$

We know from Result 2 that if the demand parameters are estimated using eqs. (10)-(14), and $2Q_{10} > P_{10}, 2Q_{20} > P_{20}$, then $K_4 > 0$ ($t^* > 0$) and $K_5 > 0$ ($s^* > 0$). Thus $\partial \Delta_{t,0}/\partial c_1$ and $\partial \Delta_{s,0}/\partial c_1$ have the same sign as $\partial t^*/\partial c_1$ and $\partial s^*/\partial c_1$, respectively.

The foregoing discussions have shown the following result,

Result 6. If $V_1/V_2 > b_1/b_2$ and $b_1 > k$, then

(i) $|\partial \bar{\phi}_1/\partial c_1| > |\partial \bar{\phi}_2/\partial c_1|$;

(ii) $\partial t^*/\partial c_1 < 0$ and $\partial s^*/\partial c_1 < 0$;

(iii) if $t^* > 0$ and $s^* > 0$, then $\partial \Delta_{t,0}/\partial c_1 < 0$ and $\partial \Delta_{s,0}/\partial c_1 < 0$.

The crucial condition of Result 6 is that $V_1/V_2 > b_1/b_2$ or $V_1/b_1 > V_2/b_2$, which may be interpreted as the domestic firms' collusive conduct exerts more influences on the market equilibrium than the foreign firms'. In the case of Cournot conduct, the condition $V_1/b_1 > V_2/b_2$ is equivalent to the condition $n_1 < n_2$, that is, there are fewer domestic firms in the market.

Result 6 says that if the market structure is such that $V_1/b_1 > V_2/b_2$, a fall of the domestic marginal cost would increase the monopoly power possessed by the domestic firms. This is more than offset by the decrease of the foreign firms' monopoly power in the market (Result 6, (i)). As was suspected, higher tariff and subsidy rates are required in the optimal policy (Result 6, (ii)). As a result, greater welfare gains over complete laissez-faire can usually be achieved (Result 6, (iii)).

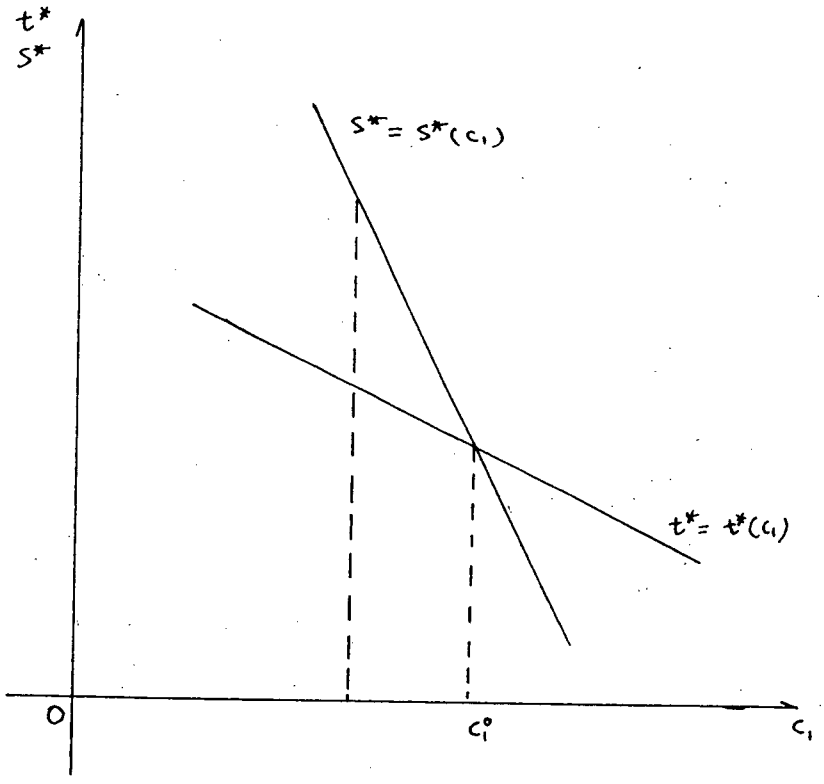
As for the relative importance and efficiency of the optimal tariff vis-à-vis the optimal subsidy, we have Result 7,

Result 7. If $V_1/V_2 > b_1/b_2$ and $b_1 > k$, then

$$\left|\frac{\partial s^*}{\partial c_1}\right| > \left|\frac{\partial t^*}{\partial c_1}\right|.$$

The proof is given by Appendix 6. When the monopoly power gained by the domestic firms is more than that lost by the foreign firms (as the result of a fall of c_1), Result 6 shows that both the optimal tariff t^* and the optimal subsidy s^* would rise. Now Result 7 further indicates that the rate of increase in s^* is greater than the rate of increase in t^*. In other words, when domestic monopoly becomes a major problem, the value of an optimal production subsidy would rise faster than the value of an optimal tariff if each policy instrument is implemented separately, suggesting that in a sense, the subsidy becomes more important than the tariff.

Because both t^* and s^* are linear functions of c_1 (from (30) and (32)), the situation can be depicted using Figure 1 where both t^* and s^* are assumed to be positive.



where $\frac{V_1}{b_1} > \frac{V_2}{b_2}$ $t^*>0$, $s^*>0$

Figure 1
The optimal tariff and optimal subsidy as functions of c_1

Figure 1 is inserted here.

From Results 6 and 7, the following corollary can be derived,

Corollary 1. If $V_1/V_2 > b_1/b_2$, $b_1 > k$, $t^* > 0$, and $s^* > 0$, then for any given c_2,

(i) if c_1^0 is such a point that $s^*(c_1^0) = t^*(c_1^0)$, then for any c_1, $0 < c_1 < \min(P_{10}, c_1^0)$,

$$s^*(c_1) > t^*(c_1);$$

(ii) if $b_2 > k$ (Note 2), for any c_1, $0 < c_1 < \min(P_{10}, c_1^0)$,

$$\left|\frac{\partial \Delta_{s,0}}{\partial c_1}\right| > \left|\frac{\partial \Delta_{t,0}}{\partial c_1}\right|.$$

Proof (i) It can be seen from Figure 1.

(ii) Form (i), $s^*(c_1) > t^*(c_1)$ for c_1, $0 < c_1 < \min(P_{10}, c_1^0)$. Hence

$$\begin{aligned}\frac{|\partial \Delta_s/\partial c_1|}{|\partial \Delta_t/\partial c_1|} &= \frac{(b_2+V_2)^2 - k^2V_2}{k(b_2V_1 - b_1V_2)}\frac{s^*(c_1)}{t^*(c_1)} \\ &> \frac{(b_2+V_2)^2 - k^2V_2}{k(b_2V_1 - b_1V_2)} \\ &> 1\end{aligned}$$

the last inequality holds as long as $b_1 > k$ and $b_2 > k$.

Q.E.D.

Therefore, when domestic monopoly becomes a major problem, the optimal subsidy rate tends to be greater than the optimal tariff rate. Moreover, the rate of increase in the welfare gain from the optimal subsidy over laissez-faire tends to be greater than that of increase from the optimal tariff, suggesting that in this case a production subsidy is more efficient in raising the welfare than a tariff.

It is noted that the condition $V_1/V_2 > b_1/b_2$ (in Cournot, $n_1 < n_2$) plays a crucial role in deriving Results 6, 7 and Corollary 1. As was mentioned earlier, if $b_1 > k$, then $b_1/b_2 > k/(b_2 + V_2) > k^2/(b_2 + V_2)^2$. The case where $V_1/V_2 > b_1/b_2$ has been analysed. It can be similarly shown that in the case where $0 < V_1/V_2 < k^2/(b_2 + V_2)^2$ or $V_2/V_1 > (b_2 + V_2)^2/k^2$, the results just reverse those reported in Results 6, 7 and Corollary 1. The proofs are similar. To save space, we shall omit the proofs and only state the results.

Result 8. If $V_2/V_1 > (b_2 + V_2)^2/k^2$ and $b_1 > k$, then

(i) $|\partial\bar{\phi}_1/\partial c_1| < |\partial\bar{\phi}_2/\partial c_1|$;

(ii) $\partial t^*/\partial c_1 > 0$, $\partial s^*/\partial c_1 > 0$;

(iii) if $t^* > 0$ and $s^* > 0$, then $\partial\Delta_{t,0}/\partial c_1 > 0$ and $\partial\Delta_{s,0}/\partial c_1 > 0$.

Result 9. If $V_2/V_1 > (b_2 + V_2)^2/k^2$, $b_1 > k$, and $b_2 > k$, then

$$\left|\frac{\partial t^*}{\partial c_1}\right| > \left|\frac{\partial s^*}{\partial c_1}\right|.$$

Corollary 2. If $V_2/V_1 > (b_2 + V_2)^2/k^2$, $b_1 > k$, $t^* > 0$, and $s^* > 0$, then for any given c_2,

(i) if c_1^0 is such a point that $s^*(c_1^0) = t^*(c_1^0)$, then for any c_1, $c_1^0 < c_1 < P_{10}$,

$$t^*(c_1) > s^*(c_1);$$

(ii) if $b_2 > k$, then for any c_1, $c_1^0 < c_1 < P_{10}$,

$$\left|\frac{\partial\Delta_{t,0}}{\partial c_1}\right| > \left|\frac{\partial\Delta_{s,0}}{\partial c_1}\right|.$$

The interpretation of these results can be similarly given except that the tariff and the subsidy have reversed their positions. When the market structure is such that the effect of the foreign firms' conduct on

the market outcome is rather significant relative to that of the domestic firms', a fall of domestic marginal cost would reduce the monopoly power possessed by the foreign firms, a reduction which is more significant than the increment of monopoly power gained by the domestic firms. In a sense, the overall market would become more competitive. Both the optimal tariff and subsidy rates would thus fall, so as the welfare gains from using the optimal policy. On the other hand, a rise of domestic marginal cost would increase the foreign firms' monopoly power, an increase which is more than offset by the reduction of the domestic firms' monopoly power. So the overall market would become less competitive and the foreign firms' monopoly would be the major problem. In this case more intervention is called for and by doing so, more welfare gains can be achieved. Furthermore, the rate of increase in the optimal tariff t^* is greater than that in the optimal subsidy s^*, and t^* tends to be greater than s^*. Moreover, the rate of increase in welfare gains from t^* tends to be greater than that from s^*. These suggest that the tariff instrument is now more important and more efficient than the subsidy.

The case in which V_1/V_2 is between b_1/b_2 and $k^2/(b_2+V_2)^2$ is briefly discussed as following.

(i) $k/(b_2+V_2) < V_1/V_2 < b_1/b_2$

From conditions (43), (44), and (45), we have

$$\left|\frac{\partial\bar{\phi}_1}{\partial c_1}\right| > \left|\frac{\partial\bar{\phi}_2}{\partial c_1}\right| \tag{49}$$

$$\frac{\partial t^*}{\partial c_1} > 0 \tag{50}$$

$$\frac{\partial s^*}{\partial c_1} < 0 \tag{51}$$

It may be interesting to compare this case with the case where $V_1/V_2 > b_1/b_2$. In both cases, the increase in the domestic firms' monopoly power outweighs the decrease in the foreign firms' monopoly power as the domestic marginal cost falls. In the case of $V_1/V_2 > b_1/b_2$, both the optimal subsidy and tariff rates would

rise. In the current case where $V_1/V_2 < b_1/b_2$, the subsidy would still rise but the tariff would instead fall. Thus as the ratio of V_1/V_2 is falling, the foreign firms' conduct would be exerting more and more influences on the market outcome relative to the domestic firms'. This is first reflected in the change of tariff rates in that the tariff rate falls as the foreign firms become competitive despite that the overall market becomes less competitive, suggesting that tariffs are more closely related to how foreign firms behave in the market.

(ii) $k^2/(b_2+V_2)^2 < V_1/V_2 < k/(b_2+V_2)$

We have,

$$\left|\frac{\partial \bar{\phi}_1}{\partial c_1}\right| < \left|\frac{\partial \bar{\phi}_2}{\partial c_1}\right| \tag{52}$$

$$\frac{\partial t^*}{\partial c_1} > 0 \tag{53}$$

$$\frac{\partial s^*}{\partial c_1} < 0 \tag{54}$$

Similarly, a comparison of this case with the case where $0 < V_1/V_2 < k^2/(b_2+V_2)^2$ suggests that production subsidies are more closely related to how domestic firms behave in the market.

By conducting the sensitivity exercises of ϕ_i, t^*, s^*, $\Delta_{t,0}$, and $\Delta_{s,0}$ with respect to c_1, we have shown in this section several interesting results concerning the role of tariffs and production subsidies. Whether the tariff and subsidy rates should be raised or lowered depends on whether the overall market becomes less or more competitive. This is in turn closely related to the initial market structure: whether the domestic firms' conduct exerts more or less effects on the market equilibrium than the foreign firms'. When the domestic firms' conduct has rather significant effects on the market equilibrium, the domestic firms' monopoly is usually the major issue, and a production subsidy is usually more important and more efficient than a tariff. On the other hand, when the foreign firms' conduct has rather significant effects on the market equilibrium, the foreign firms' monopoly is usually the major issue, and a tariff usually becomes more important and more efficient than a production subsidy.

5. JAPANESE POLICY RESPONSE

In the analysis up to this point, it has been assumed that the foreign (i.e., Japanese) government pursues a laissez-faire policy and only the domestic (i.e., U.S.) government is active in policy-making. What happens if we depart from this assumption? Here we consider two cases separately. One is the case where the U.S. government pursues the status quo policy, with the MFN tariff $100 per car on Japanese imports and a zero production subsidy to U.S. firms, while Japan is active in policy-making. The other is the case where both governments are active in making policy. We assume that the policy instrument for Japan in this context is an export subsidy or an export tax, and the policy perspective is the Japanese welfare which is the sum of Japanese firms' profit from exporting to the U.S. market and Japanese government's subsidy cost (or tax revenue).

Since part of Japanese firms' output is in the domestic (Japanese) market, we must assume a segmented-markets perception adopted by the Japanese firms in order to separate our U.S. market from the Japanese market and focus on the U.S. market. Segmented markets arise when firms treat different countries as different markets in that they choose their strategy variables for each market separately. Consider one Japanese (exporting) firm. Its profit in the U.S. market is $(P_2 - c_2 + s_2 - t_1)q_1^2$, where s_2 is the export subsidy from the Japanese government (if s_2 turns out to be negative, it is interpreted as the export tax), t_1 is the tariff imposed by the U.S. government. The firm chooses its export level q_1^2 to maximize its profit from exporting,

$$P_2 - c_2 + s_2 - t_1 + q_1^2 \frac{dP_2}{dq_1^2} = 0$$

Suppose that there are n_2 such Japanese firms. Aggregate over them,

$$P_2 - c_2 + s_2 - t_1 - Q_2 V_2 = 0 \tag{55}$$

where V_2 has the same meaning as that in previous sections.

By combining (55) with previous eqs. (3), (4), and (16),

$$P_1 = a_1 - b_1 Q_1 - kQ_2 \tag{3}$$

$$P_2 = a_2 - kQ_1 - b_2 Q_2 \tag{4}$$

$$P_1 - c_1 + s_1 - Q_1 V_1 = 0 \tag{16}$$

where s_1 is the U.S. domestic production subsidy, V_1 and V_2 are the aggregate version of conjectural variation parameters, we can determine the equilibrium quantities and prices, given policy parameters t_1, s_1, and s_2.

1) Only Japan is active in policy-making

In the first case where only Japan is active in policy-making, $t_1 \equiv 100$ and $s_1 \equiv 0$. The Japanese welfare W^{JAP} is,

$$W^{JAP} = (P_2 - c_2 - 100 + s_2)Q_2 - s_2 Q_2 \tag{56}$$

where $W^{JAP} \equiv W^{JAP}(s_2)$. The comparative statics of quantities and prices with respect to the Japanese export subsidy (tax) s_2 are,

$$\frac{dQ_2}{ds_2} = \frac{b_1 + V_1}{D} > 0 \tag{57}$$

$$\frac{dP_2}{ds_2} = -\frac{(b_1 + V_1)b_2 - k^2}{D} < 0 \tag{58}$$

$$\frac{dQ_1}{ds_2} = -\frac{k}{D} < 0 \tag{59}$$

$$\frac{dP_1}{ds_2} = -\frac{kV_1}{D} < 0 \tag{60}$$

a) Effects on Japanese welfare

The comparative static effect of the Japanese export subsidy (tax) on Japanese welfare is,

$$\frac{dW^{JAP}}{ds_2} = (P_2 - c_2 - 100)\frac{dQ_2}{ds_2} + Q_2\frac{dP_2}{ds_2} \tag{61}$$

The first term in dW^{JAP}/ds_2 indicates that an export subsidy is desirable to Japan because in equilibrium $P_2 > (c_2 + 100)$ and a subsidy would expand Japanese exports (from (57)). But the second term is negative: a subsidy would lower the price charged by Japanese producers (from (58)) and might thus lower their profits. An export tax works in just an opposite way: it is harmful from viewpoint of the first term but desirable from viewpoint of the second term. In this example, because

$$\frac{d^2W^{JAP}}{ds_2^2} = 2\frac{dQ_2}{ds_2}\frac{dP_2}{ds_2} < 0,$$

a point, s_2^*, at which the first-order condition is satisfied, is the unique welfare-maximizing point,

$$W^{JAP}(s_2^*) > W^{JAP}(0) \qquad \text{and} \qquad W^{JAP}(s_2^*) = W^{JAP}(0) \quad \text{iff} \quad s_2^* = 0.$$

In words, we have

Result 10. The optimal export subsidy (tax) s_2^* is welfare-improving (relative to complete laissez-faire) for Japan.

Solving $dW^{JAP}/ds_2 = 0$ for s_2^*, we have

$$s_2^* = \frac{\big((b_1 + V_1)(a_2 - c_2 - 100) - k(a_1 - c_1)\big)\big((b_1 + V_1)(V_2 - b_2) + k^2\big)}{2(b_1 + V_1)(b_2V_1 + b_1b_2 - k^2)} \tag{62}$$

Because $2(b_1 + V_1)(b_2V_1 + b_1b_2 - k^2) > 0$ and $(b_1 + V_1)(a_2 - c_2 - 100) - k(a_1 - c_1) = DQ_{20} > 0$ (from (20)),

we have

$$s_2^* < 0 \qquad \text{iff} \qquad (b_1 + V_1)(V_2 - b_2) + k^2 < 0 \qquad \text{iff}$$

$$V_2 < \frac{(b_1 + V_1)b_2 - k^2}{b_1 + V_1} \tag{63}$$

and we thus obtain the following result,

Result 11. The necessary and sufficient condition for Japan to adopt an optimal export tax, rather than an optimal export subsidy, is that the domestic firms behave sufficiently competitively in exporting activities such that condition (63) holds.

In the Cournot case, $V_i = b_i/n_i$, and condition (63) is thus equivalent to

$$n_2 > \frac{(n_1 + 1)b_1 b_2}{(n_1 + 1)b_1 b_2 - n_1 k^2} \tag{64}$$

Obviously,

$$\frac{(n_1 + 1)b_1 b_2}{(n_1 + 1)b_1 b_2 - n_1 k^2} > 1.$$

Therefore, we derive the following corollary from Result 11,

Corollary 3. In the case of Cournot conduct, the necessary and sufficient condition for Japan to adopt an export tax is that the number of domestic firms is so large that condition (64) holds. In particular, a necessary condition for Japan to adopt an export tax is that the number of the domestic firms is greater than one.

When there is only one domestic firm, an export subsidy is often desirable for an exporting country (Brander and Spencer (1985)). So if firms' rivalry is along the Cournot line, the optimal trade policy for an

exporting country changes from an export subsidy to an export tax as the number of domestic firms increase from one to such a level that condition (64) holds. Similar results have been provided by Dixit (1984).

If the demand parameters are estimated using eqs. (10)-(14), it can be easily shown that if $P_{20} < 2(c_2 + 100)$, or $P_{20} - c_2 < c_2 + 200$, then condition (63) holds and therefore $s_2^* < 0$. So if the price charged by the domestic firms does not deviate from their marginal cost significantly, an export tax would be optimal for Japan. This is certainly the case for the U.S. car market in 1979 and 1980.

Result 11 also confirms the analysis of Eaton and Grossman (1986) in that when home firms fail to collude among themselves in exporting, the home country can gain by restricting exports, and can do so by means of an export tax.

b) Effects on the U.S. welfare

Next consider the effects of an export subsidy (tax) by Japan on the U.S. welfare,

$$W^{US} \equiv W^{US}(s_2) = \big(U(Q_1, Q_2) - P_1 Q_1 - P_2 Q_2\big) + (P_1 - c_1)Q_1 + 100 Q_2$$

and

$$\frac{dW^{US}}{ds_2} = (P_1 - c_1)\frac{dQ_1}{ds_2} - Q_2 \frac{dP_2}{ds_2} + 100 \frac{dQ_2}{ds_2} \tag{65}$$

The first term implies that an export subsidy is harmful to U.S. because the U.S. firms' output level would be lowered (from (59)) and the U.S. domestic distortion would therefore be worsened. The other two terms show the benefit of an export subidy to U.S. due to lower price of imports (from (58)) and higher tariff revenues. Once again, the effects of a Japanese export tax on the U.S. welfare are opposite to those of a Japanese export subsidy.

Substituting s_2^* of (62) for s_2 in $W^{US}(s_2)$, we have

$$W^{US}(s_2^*) - W^{US}(0) = \frac{s_2^*}{D}(K_6 s_2^* + K_7) \tag{66}$$

where

$$K_6 = \frac{1}{2}\big(b_2(b_1 + V_1)^2 - b_1 k^2\big) > 0,$$

$$K_7 = \big(b_2(b_1 + V_1)^2 - b_1 k^2\big)(a_2 - c_2 - 100) - k\big((b_1 b_2 - k^2) + V_1(2b_2 + V_2)\big)(a_1 - c_1)$$
$$+ 100(b_1 + V_1)\big((b_1 + V_1)(b_2 + V_2) - k^2\big).$$

Appendix 7 shows that $K_7 = D\big(Q_{20}(b_1 b_2 - k^2) + Q_{20} b_2 V_1 - Q_{10} k + 100(b_1 + V_1)\big)$, and if the demand parameters are estimated using eqs. (10)-(14), then $K_7 > 0$. Thus, both K_6 and K_7 are positive in this case.

As was shown in (63) or (64), a positive s_2^*, i.e., an optimal export subsidy, could be justified if the domestic (Japanese) firms are able to behave rather collusive. If this is the case, $W^{US}(s_2^*)$ would be greater than $W^{US}(0)$ (from (66)). In other wards, both countries can gain from the Japanese subsidy policy.

In most situations, however, an export tax would be expected. This is because, as has been shown, if there are several home firms engaging in exporting, it is generally desirable to increase their tacit collusion with one another. When s_2^* is negative, we have

$$W^{US}(s_2^*) - W^{US}(0) = \frac{s_2^*}{D}(K_6 s_2^* + K_7) > 0 \qquad \text{iff} \quad (-s_2^*) > \frac{K_7}{K_6}.$$

In words, both countries would gain from the Japanese export tax if and only if $(-s_2^*) > K_7/K_6$.

Table 4 is inserted here.

Table 4 shows the results when U.S. takes the status quo while Japan pursues the optimal export policy, using data for 1979 and 1980. As was predicted, an optimal tax is implied. The tax rates are quite sizable:

Table 4
U.S. holds the status quo policy and Japan pursues an optimal export policy

	1979		1980	
	Laissez-faire	Optimal export subsidy	Laissez-faire	Optimal export subsidy
S_2^* $/car	0	-1005	0	-1109
P_1 $	5951	5962	6407	6417
P_2 $	4000	4813	4130	5135
Q_1 million	8.341	8.511	6.581	6.811
Q_2 million	1.546	0.955	1.819	1.003
Japan profit $billion	0.773	0.295	0.418	0.127
U.S. profit $billion	4.596	4.786	2.020	2.164
U.S. Consu. Surpl. $billion	27.91	26.80	24.84	23.35
U.S. Tariff Rev. $million	155	95	182	100
Japan Subsidy Cost $billion	0	-0.959	0	-1.112
U.S. Welfare $billion	32.66	31.68	27.04	25.61
Japan Welfare $billion	0.773	1.254	0.418	1.239
Gain over Laissez-faire $million				
U.S.	0	-980	0	-1430
Japan	0	481	0	821

about 30% expressed relative to the marginal cost of Japanese firms. By levying the tax Japan could achieve a significant welfare gain: a 62% and a 200% increase in its welfare for 1979 and 1980 respectively. Most of the gains is attributed to the tax revenues. On the other hand, U.S. could suffer losses as the result of the Japanese export tax: about one billion dollar for 1979 and 1.4 billion for 1980. Most of the losses is in the form of the depressing U.S. consumer surplus. Finally, a comparison of the results between the two years suggests that the more competitive the market is and in particular, the more competitive the Japanese firms are among themselves, the more effective the export tax would be for Japan.

2) Both U.S. and Japan are active in policy-making

What happens if both governments are active in policy-making? We would examine the non-cooperative Nash equilibrium in tariff and/or subsidy for U.S. and Japan in which each country is assumed to choose its policy parameters given those of the other country. As discussed earlier, the equilibrium quantities and prices can be determined by eqs. (3), (4), (16), and (55),

$$P_1 = a_1 - b_1 Q_1 - kQ_2 \tag{3}$$

$$P_2 = a_2 - kQ_1 - b_2 Q_2 \tag{4}$$

$$P_1 - c_1 + s_1 - Q_1 V_1 = 0 \tag{16}$$

$$P_2 - c_2 - t_1 + s_2 + Q_2 V_2 = 0 \tag{55}$$

where the aggregate versions of conjectural variations are determined using the actual quantites (Q_{10}, Q_{20}), prices (P_{10}, P_{20}), and tariff and subsidy figures $(t_1 = 100, s_1 = 0, s_2 = 0)$. $(t_1, s_1; s_2) = (100, 0, 0)$ is referred to as the status quo and is to be compared with the Nash equilibrium tariff and subsidy. The U.S. welfare

function is

$$W^{US} \equiv W^{US}(t_1, s_1; s_2)$$

$$= \big(U(Q_1, Q_2) - P_1 Q_1 - P_2 Q_2\big) + (P_1 - c_1 + s_1)Q_1 + (t_1 Q_2 - s_1 Q_1).$$

When both t_1 and s_1 can be optimally chosen by the U.S. government, the first-order conditions are,

$$\frac{\partial W^{US}}{\partial t_1} = 0: \qquad H_1 t_1 + H_2 s_1 + H_3 s_2 = G_1 \tag{67}$$

$$\frac{\partial W^{US}}{\partial s_1} = 0: \qquad H_4 t_1 + H_5 s_1 + H_6 s_2 = G_2 \tag{68}$$

Since the expressions of parameters H_i and G_i are not very informative in the current context, they will be given in Appendix 8. Eqs. (67) and (68) define the U.S. best-response functions for each export tax s_2 chosen by Japan.

The Japanese welfare function is

$$\frac{\partial W^{JAP}}{\partial s_2} = 0: \qquad H_7 t_1 + H_8 s_1 + H_9 s_2 = G_3 \tag{69}$$

H_i and G_3 are given in Appendix 8. Eq. (69) defines the Japanese best-response function for each pair of tariff and subsidy (t_1, s_1) chosen by the U.S. government.

Solving eqs. (67), (68), and (69) simultaneously using Cramer's rule gives the non-cooperative Nash equilibrium in tariff and subsidy for U.S. and Japan.

The cases, where only one of the two U.S. policy instruments is available, can be similarly analysed.

(i) If only the tariff instrument is available, then eq. (67) with $s_1 \equiv 0$ gives the U.S. best-response function,

$$H_1 t_1 + H_3 s_2 = G_1 \tag{70}$$

whereas eq. (69) with $s_1 \equiv 0$ gives the Japanese best-response function,

$$H_7 t_1 + H_9 s_2 = G_3 \tag{71}$$

Figure 2 is inserted here.

Figure 2 is depicted using 1979 data for the U.S. car market. It can be seen from the figure that both best-response functions are upward sloping, suggesting certain relations between tariffs and export subsidies (for example, countervailing is usually desirable to a country). As Japan switches from subsidizing to taxing exports, U.S. should reduce the tariff rates. As U.S. raises the tariff rates, Japan should reduce its tax rates on exports. With different slopes, the intersection indicates the Nash equilibrium in tariff by U.S. and export subsidy (tax) by Japan.

(ii) If only the production subsidy instrument is available for U.S., then eq. (68) with $t_1 \equiv 0$ gives the U.S. best-response function,

$$H_5 s_1 + H_6 s_2 = G_2 \tag{72}$$

whereas eq. (69) with $t_1 \equiv 0$ gives the Japanese best-response function,

$$H_8 s_1 + H_9 s_2 = G_3 \tag{73}$$

The Nash equilibrium in subsidy/subsidy for U.S. and Japan is the solution of eqs. (72) and (73).

Figure 3 is inserted here.

Figure 3 is depicted using the same data as in Figure 2. It can be seen from Figure 3 that s_1 and s_2 are almost independent with each other. The U.S. government can choose its optimal production subsidies

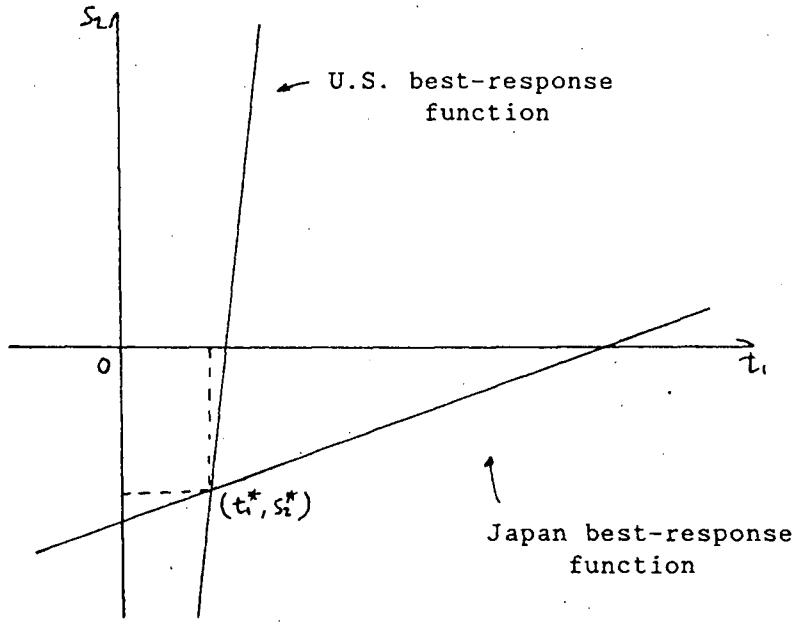


Figure 2

The best-response function of U.S. using tarriffs (t_1) and the best-response function of Japan using export subsidies (s_2).

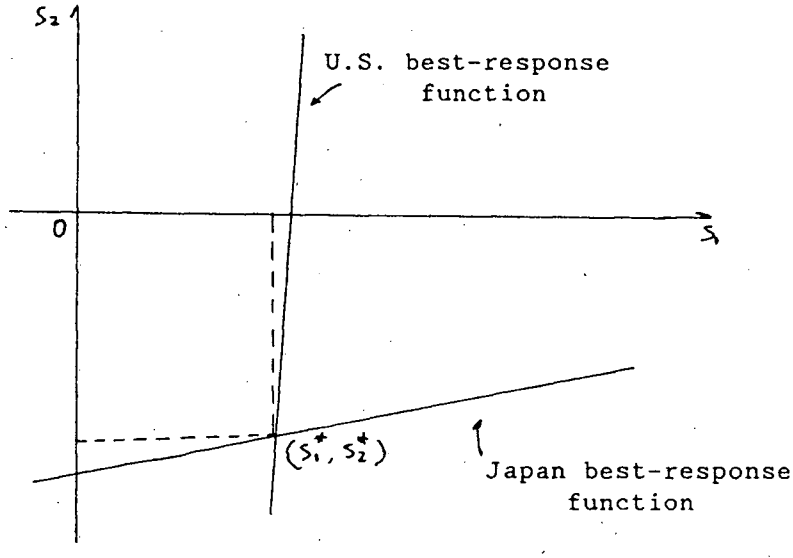


Figure 4

The best-response function of U.S. using production subsidies (s_1) and the best-response function of Japan using export subsidies (s_2).

without referring very much to the Japanese export subsidy levels. In this example the U.S. optimal production subsidy is rather stable in the sense that it won't change much under the Japanese pursuit of the optimal policy.

Next, we examine the numerical results from this two-active-player model, called Model 2, and compare them with the results from the model in which only U.S. is active in policy-making. The latter model is to be referred to as Model 2. Table 5 and 6 are the reproductions of Table 5 and 4, respectively, of Dixit (1985) using Model 1 (Note 3). Table 7 and 8 are the corresponding results from Model 2.

The following Table 9 are taken from Table 5 and Table 7 where 1979 data are used.

Table 9 is inserted here.

From Model 1 to Model 2, the optimal tariff rate falls by 24% while the optimal subsidy falls by only 2%. In the optimal tariff and subsidy case, the tariff component falls from $t^{**} = 407.9$ to $t_1^{**} = 268.0$ while the subsidy component rises slightly. As the result, the ratio of two components, tariff vs. subsidy, in an optimal policy package falls significantly: from $t^{**}/s^{**} = 67\%$ in Model 1 to $t_1^{**}/s_1^{**} = 43\%$ in Model 2. This comparison demonstrates that if Japan is also assumed to be active in policy-making, the tariff rate may be even further less significant than the production subsidy level for the U.S. auto industry.

Table 10 is inserted here.

One obvious observation from Table 10 is that the U.S. welfare in the Nash equilibria would be below the level in the status quo where Japan is assumed to take a laissez-faire position. The U.S. losses are sizable. Thus, allowing Japan to pursue profit-shifting policy reverses the previous result of positive U.S. welfare gains from the optimal policies over complete laissez-faire or status quo shown in Model 1. On the Japanese side, however, Japan would gain from an optimal export tax over the status quo no matter what the U.S. policies are. It would gain most if U.S. takes complete laissez-faire and least if U.S. chooses both tariff and subsidy optimally. Thus, Japan clearly has an incentive to pursue this welfare-improving policy. To reduce losses, U.S. also wants to use tariff and/or subsidy.

(Part 1)

TABLE 5
POLICY CALCULATIONS FOR THE CENTRAL CASE, 1979
(ONLY US IS ACTIVE IN POLICY-MAKING)

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
T	100	570.7	0	407.9
S	0	0	670.3	611.0
P1	5951	5956	5342	5400
P2	4000	4381	3882	4216
Q1	8341000	8420823	9262589	9248688
Q2	1546000	1268960	1491184	1261168
JAPAN PROFIT	7.730E8	5.208E8	7.192E8	5.144E8
US PROFIT	4.596E9	4.684E9	5.668E9	5.651E9
US CONS SURPL	2.791E10	2.733E10	3.345E10	3.245E10
TARI REV	1.546E8	7.242E8	0	5.144E8
SUBS COST	0	0	6.209E9	5.651E9
US WELFARE	3.266E10	3.274E10	3.291E10	3.297E10
GAIN OVER MFN ($million)				
U.S.	0	78	251	307
JAPAN	0	-252	-54	-259

TABLE 6
POLICY CALCULATIONS FOR THE CENTRAL CASE, 1980
(ONLY US IS ACTIVE IN POLICY-MAKING)

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
T	100	298.4	0	211.1
S	0	0	367.8	325.0
P1	6407	6409	6057	6100
P2	4130	4310	4030	4222
Q1	6581000	6622102	6970312	6966264
Q2	1819000	1672916	1816429	1669874
JAPAN PROFIT	4.184E8	3.539E8	4.172E8	3.526E8
US PROFIT	2.020E9	2.046E9	2.266E9	2.264E9
US CONS SURPL	2.484E10	2.451E10	2.739E10	2.676E10
TARI REV	1.819E8	4.993E8	0	3.526E8
SUBS COST	0	0	2.564E9	2.264E9
US WELFARE	2.704E10	2.706E10	2.709E10	2.711E10
GAIN OVER MFN ($million)				
U.S.	0	16	52	69
JAPAN	0	-65	-1	-66

TABLE 7
NASH EQUILIBRIA IN TARIFF / SUBSIDY FOR THE CENTRAL CASE, 1979
(BOTH US AND JAPAN ARE ACTIVE IN POLICY-MAKING)

	STATUS-QUO	OPT-TARI	OPT-SUBS	OPT-TS
T1	100	432.4	0	268.0
S1	0	0	657.7	620.0
S2	0	-877.4	-970.3	-871.9
P1	5951	5965	5364	5400
P2	4000	4979	4669	4808
Q1	8341000	8546167	9409525	9385496
Q2	1546000	833932	922212	828736
JAPAN PROFIT	7.730E8	2.249E8	2.751E8	2.221E8
US PROFIT	4.596E9	4.825E9	5.849E9	5.819E9
US CONS SURPL	2.791E10	2.663E10	3.230E10	3.183E10
TARI REV	1.546E8	3.606E8	0	2.221E8
US SUBS COST	0	0	6.189E9	5.819E9
JAPAN SUBS COST	0	-7.317E8	-8.948E8	-7.226E8
US WELFARE	3.266E10	3.182E10	3.196E10	3.206E10
JAPAN WELFARE	7.730E8	9.566E8	1.170E9	9.447E8
GAIN OVER STATUS QUO ($million)				
US	0	-840	-700	-600
JAPAN	0	184	397	172

TABLE 8
NASH EQUILIBRIA IN TARIFF / SUBSIDY FOR THE CENTRAL CASE, 1980
(BOTH US AND JAPAN ARE ACTIVE IN POLICY-MAKING)

	STATUS-QUO	OPT-TARI	OPT-SUBS	OPT-TS
T1	100	210.7	0	120.9
S1	0	0	359.0	334.8
S2	0	-1059	-1108	-1057
P1	6407	6418	6076	6100
P2	4130	5191	5035	5099
Q1	6581000	6823268	7189954	7177489
Q2	1819000	957926	1002442	956132
JAPAN PROFIT	4.184E8	1.160E8	1.271E8	1.156E8
US PROFIT	2.020E9	2.172E9	2.412E9	2.403E9
US CONS SURPL	2.484E10	2.329E10	2.584E10	2.561E10
TARI REV	1.819E8	2.018E8	0	1.156E8
US SUBS COST	0	0	2.581E9	2.403E9
JAPAN SUBS COST	0	-1.014E9	-1.111E9	-1.011E9
US WELFARE	2.704E10	2.566E10	2.567E10	2.572E10
JAPAN WELFARE	4.184E8	1.130E9	1.238E9	1.126E9
GAIN OVER STATUS QUO ($million)				
US	0	-1380	-1370	-1320
JAPAN	0	712	820	708

Table 9

The optimal tariff and subsidy rates in the two models

	Model 1	Model 2
the optimal tariff	$t^* = 570.7$	$t_1^* = 432.4$
the optimal subsidy	$s^* = 670.3$	$s_1^* = 657.7$
the optimal tariff and subsidy	$t^{**} = 407.9$ $s^{**} = 661.0$	$t_1^{**} = 268.0$ $s_1^{**} = 620.0$

Table 10

Welfare gains in Model 2, 1979

U.S. policy Configuration (t_1, s_1) / Gains over status quo	Laissez-faire (0,0)	MFN-TARI (100,0)	the optimal subsidy $(0, s_1^*)$	the optimal tariff $(t_1^*, 0)$	the optimal tariff and subsidy (t_1^{**}, S_1^{**})
U.S. $million	-1030	-980	-700	-840	-600
Japan $million	578	481	397	184	172

It is also found in Table 10 that the U.S. optimal subsidy policy would be preferred by both countries to the U.S. optimal tariff policy if only one of them can be implemented. Compared to the tariff, the subsidy would bring less losses to U.S. whereas more gains to Japan.

Finally, the welfare comparisons in Model 2 between 1979 and 1980 (Table 7 and 8) clearly show that the more competitive the market is and in particular, the more competitive the Japanese firms are with one another, the more Japan would gain by taxing exports, and the more U.S. would lose. Further, the Japanese gain is at the expense of the U.S. welfare, especially of the U.S. cousumer surplus. So for the two countries as whole, it is possible that only loss is left.

If policies could be chosen to maximize the joint welfare, both countries may be better off. In other words, t_1, s_1, and s_2 are to be chosen to maximize the joint welfare $W^J \equiv W^J(t_1, s_1, s_2)$,

$$W^J = W^{US} + W^{JAP} \tag{74}$$

Since t_1 is the tariff on Japanese firms by U.S. and s_2 is the export subsidy to Japanese firms by Japan, $(t_1 - s_2)$ is the total tax on the Japanese firms, and t_1 and s_2 are actually one variable in this jointly maximizing framework. Setting $\partial W^J/\partial t_1 = 0$ and $\partial W^J/\partial s_1 = 0$ gives,

$$H_{10}(t_1 - s_2) + H_{11}s_1 = G_4 \tag{75}$$

and

$$H_{12}(t_1 - s_2) + H_{13}s_1 = G_5 \tag{76}$$

Again, the expressions of H_i and G_i are given in Appendix 8. The optimal tax on the Japanese firms by both governments, $(t_1 - s_2)^J$, and the optimal subsidy to the U.S. firms by the U.S. government, s_1^J, can be

found by solving eqs. (75) and (76) simultaneously. For our particular cases at hand, we have

$$1979: \qquad (t_1 - s_2)^J = -600.5, \quad s_1^J = 598.5;$$

$$1980: \qquad (t_1 - s_2)^J = -254.6, \quad s_1^J = 320.2.$$

That is, the jointly optimal policy involves subsidies to both U.S. and Japanese firms. The U.S. firms are subsidized by the U.S. government while the Japanese firms are either subsidized by both governments, or subsidized by one government and taxed by the other. As the result of this jointly optimal policy, $P_1 = c_1$ and $P_2 = c_2$. Therefore, the firms are subsidized sufficiently to bring their prices in line with the true marginal costs to solve both domestic and foreign monopoly problems. It is also worth noting that complete laissez-faire is normally not jointly optimal in the oligopolistic market.

Table 11 is inserted here.

Changing t_1 and s_2 while keeping their difference at the optimal level $(t_1 - s_2)^J$ would leave the equilibrium quantities and prices, and therefore the optimal value of joint welfare, unchanged, and would merely transfer revenues from one country to the other. In particular, a fall (rise) of s_2 will raise (lower) the Japanese welfare and lower (raise) the U.S. welfare. In our framework, the less Japan subsidizes its firms, the more U.S. would subsidize the Japanese firms in this framework, and hence the higher (lower) the Japanese (U.S.) welfare.

Consider the case in which U.S. can use both a tariff and a production subsidy and Japan can use an export tax. Because Japan can secure itself a payoff of \$.945 billion by playing the game non-cooperatively (see Table 11), to be willing to play the game cooperatively, Japan must at least obtain a \$.945 billion payoff. In our cooperative game, the most Japan is willing to subsidize its firms is $s_2 = \$91.75$ per car, corresponding to a $t_1 = -\$508.8$ per car U.S. subsidy on the Japanese firms. In this case Japan gains nothing over the non-cooperative outcome. This happens at point B in Figure 4. Similar analysis applies to the U.S. side:

Table 11

Welfare effects of jointly optimal tariff and subsidy schedule, 1979

Policy State / Welfare State	Status Quo	The Opt'l tariff	The Opt'l Subsidy	The Opt'l tariff and Subsidy (OPT-TS)	Jointly optimal tariff and subsidy			
					t_1= -508.8 s_2= 91.75	t_1= -897.0 s_2= -296.5	t_1= -600.5 s_2= 0	t_1= -703.1 s_2= -102.6 Nash Solution
U.S. Welfare $billion	32.66	31.82	31.96	32.06	32.78	32.06	32.61	32.42
Jap. Welfare $billion	.773	.957	1.170	.945	.945	1.665	1.115	1.305
Joint Welfare $billion	33.43	32.77	33.13	33.01	33.73	33.73	33.73	33.73

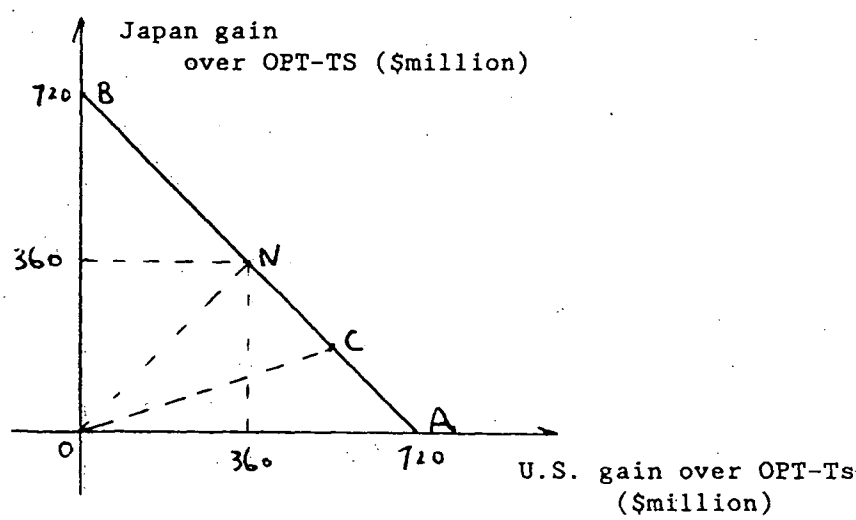


Figure 4

A bargaining game

the most U.S. is willing to subsidize the Japanese firms is $t_1 = -\$897$ per car, corresponding to a Japanese export tax $s_2 = -\$296.5$ per car on the Japanese firms. Of the \$897 U.S. subsidy for each car, \$296.5 will be indeed accrued to the Japanese government. This occurs at point A in Figure 4.

Figure 4 is inserted here.

It is noted that when $t_1 = -600.5$ and $s_2 = 0$, both countries would gain over the non-cooperative solution: U.S. gains a total of \$550 million and Japan \$170 million (point C in Figure 4). But the Nash solution to this bargaining game is at point N in which both countries achieve the same amount of gains. In this Nash solution, U.S. would subsidize the Japanese firms \$703.1 per car and Japan would tax its firms \$102.6 per car. In other words, that part of subsidies beyond \$600.5 per car would go to the Japanese government, which is a kind of side payments.

Even though both countries can be better off by cooperation, each has an incentive to pursue its own optimal policies. If the game is played only once, then the self-interest pursuit by each country would lead to a poor outcome for both.

6. CONCLUDING REMARKS

Current research on trade policy in the oligopolistic international markets has indicated that intervention with free trade can be beneficial to a country. Many theoretical models have been developed in the literature, but the amount of empirical work is rather small. Empirical work is needed in the design of industry-specific trade policies. In practice, we are interested in not only whether government intervention can raise the national welfare, but also how large the policy gains would be. Dixit's work (1985) is essentially an analysis of simulation models and is important in its attempt to apply new trade theories to a specific industry. The technique used in Dixit's paper may also have implications in other similar projects. The possible shortcomings of the analytical method has been noted in Dixit (1985). To try to improve the method is considered as an important research topic.

It is shown in Dixit (1985) that U.S. can achieve, although not very impressive, welfare gains by using optimal interventionist policies in the U.S. automobile industry, given that Japan takes laissez-faire. This paper shows that allowing Japan to simultaneously pursue optimal policies can change the result and U.S. may suffer sizable welfare losses. Moreover, Japan does have an incentive to pursue the profit-shifting policy. Thus, the U.S. policy gains are not at all automatic. Nevertheless, U.S. would suffer more losses if unilaterally adopting laissez-faire. It is shown in the paper that both countries would nonetheless be better off if they could cooperatively choose the policy parameters to maximize the joint welfare rather than non-cooperatively pursue their own optimal policies. The cooperative approach may, however, meet difficulties in real world. For instance, the countries in a bargaining game may be unable to agree on a schedule of revenue division between them. That agreements between nations are enforceable is often a question. The third party may be not sufficiently authoritative or may simply not exist. For these reasons, it may be more interesting to seek alternative models in which cooperation emerges in a tacit fashion between countries. For example, the use of repetition or incomplete information may be considered.

Another question which is of interest in the design of industry-specific trade policies is whether interventionist trade policies are the first-best policies, and when other policy instruments would achieve the same aims more efficiently. By working on the U.S. auto industry, Dixit finds that the role of domestic antitrust policies or production subsidies is quantitatively more significant and more efficient than the role of tariffs. By using a slightly different model in this paper, we find that the relative importance and efficiency of tariffs and production subsidies are related to the market structure, that is, whether the domestic firms' conduct exerts more effects on the market equilibrium than the foreign firms'. When the domestic (foreign) firms' conduct has more significant effects than the foreign (foreign) firms', the policy is usually directed by the domestic (foreign) firms' monopoly in the market, and a production subsidy (a tariff) is usually more important and more efficient than a tariff (a production subsidy).

Besides inaccurate information discussed earlier, information asymmetry may also cast problems in the design of good policies. This is in turn related to modeling methods in the strategic trade policy literature.

The basic problem in strategic trade policy concerns strategic interactions in international trade. Conventional trade theory treats all individual firms in an industry as price-takers. The new trade theory introduces the strategic interaction among firms in an imperfectly competitive international market. This new approach to trade policy usually assumes a sequential structure of the trade game: governments interact among themselves in the first stage whereas oligopolistic firms, taking governments' policies as given, interact among themselves in the second stage. In other words, the interaction between governments is at one level while that between firms at another. It is possible, however, that one country's firms may strategically interact with another country's government and/or their own government. In Dixit (1985), the U.S. government's objective is to maximize the U.S. social welfare whereas the objective of the group of Japanese firms is to maximize their own profit. When the Japanese government adopts complete laissez-faire and the U.S. government pursues the optimal policies, U.S. would achieve a welfare gain over the status quo and the Japanese firms would suffer a loss, as was shown in Table 5 and 6. But the results in Table 5 and 6 are based on one of informational assumption, namely the U.S. government has complete information about the Japanese firms' cost or at least has the same information about cost as do the firms. This assumption is unlikely to be met in reality, since the Japanese firms would be expected to have better information about their cost than would the U.S. government. As a result of information asymmetry, the Japanese firms may collectively have an incentive to conceal cost information or reveal wrong information in order to pursue their self-interests. The strategic use of information by the Japanese firms may significantly change the U.S. optimal policies as well as the design of policies itself. As a practical matter this may presumably make the design of good policies more difficult. It should be noted that the type of interaction between the Japanese firms and the U.S. government is normally different from that between a multinational enterprise and the host government. The latter is the interaction conducted in the host country with foreign direct investment, while the former is the interaction conducted in an international environment with trade. Similarly, the U.S. firms may also have an incentive to strategically use their cost information in order to receive a more favorable policy from their government. Examining this incomplete information game, in which interaction between firms, between

governments, and between firms and governments are considered, may be an interesting area needing further research.

PART 2

A SEQUENTIAL ENTRY-EXIT MODEL

OF INTERNATIONAL TRADE

1. INTRODUCTION

The purpose of this paper is to examine firms' strategic behaviour in international market share rivalry as well as its effects on trade patterns and product veriety by using a sequential entry-exit model of trade. It is observed in an industry that an incumbent firm may withdraw some products to prevent competition with an actual entrant from reducing profits on other products. In other words, a multiproduct incumbent may exit in response to entry. Such a reaction would make entry through trade more attractive to a pontential foreign entrant, and invasion is hence more likely to succeed. As trade is opened, the kind of interaction between firms in different countries may thus have an impact on the type of trade pattern emerging as well as product variety of consumption. In the recently growing literature on trade with imperfect competition, however, there seems no model which deals with the possible firms' exit in an international rivalry. This paper intends to do an exploration of models of international trade by allowing firms to exit in response to entry.

The paper models an industry consisting of two firms, each in a different country. The two firms are assumed to be able to potentially produce and export two imperfectly substitutable products, and may exit from either the home market or the foreign market in response to an entry. These firms are also assumed to make their entry, exit, and production (quantity, price, etc.) decisions sequentially, and to be able to choose separately these strategy variables for each country. We examine two four-stage games. The first of them is the basic case in which the two firms are in a symmetric position: at the first stage, they have equal opportunity to enter their home markets; at the second stage, they have equal opportunity to invade the foreign markets; at the third stage, they make exit choices simultaneously; and at the last stage, they engage in competition in the final international environment. The second game differs from the first one in that one of the firms is now able to move first and has the option to enter both products in both home and foreign markets, and at the second stage, the other firm makes entry decisions in both home and foreign markets.

The constructed model is based on the recent work by Judd (1985) where spatial preemption in a closed economy is examined. The sequential equilibrium concept used by Judd is similar to that in Prescott and Visscher (1977) and Brander and Eaton (1984). But in Judd's model, a firm is allowed to exit in response to a rival's entry while in the other two papers, an irreversable location or product line decision constitutes a commitment. Judd thus demonstrates that credible preemption by a multiproduct incumbent may be impossible unless exit costs are high. This is in contrast to other papers such as, for example, Eaton and Lipsey (1979), where exit costs are assumed prohibitively high and where an incumbent firm may deter entry into substitutes by being the first firm to produce the products and by crowding the product spectrum sufficiently to leave no niche for potential entrants.

The work of this paper is also related to the recent literature on trade under imperfect competition. Krugman (1979, 1980, 1981), and Helpman (1981), among others, examine trade using a monopolistic competition model which incorporates an increasing returns-to-scale technology. They assume that in equilibrium a number of differentiated products are produced by firms which possess monopoly power but earn no monopoly profits. As pointed out by Eaton and Kierzkowski (1984), using a zero-profit condition to determine equilibrium requires two assumptions about the nature of the process whereby entry is determined: first, firms enter taking the prices of existing firms as given; second, any fixed cost of entry is not a *sunk* cost that is incurred sequentially and irreversibly *before* the pricing decision takes place. Eaton and Kierzkowski (1984) develop a model of industrial structure where entry and price decisions are taken sequentially. This sequential decision-making is considered particularly appropriate in the international context and it follows in the tradition of Linder (1961) and Vernon (1966). Pure profits can now exist in full equilibrium. Since firms select products at the first stage and take price decision at the second stage, free entry no longer leads to average-cost pricing. Firms use product selection as a means of entry deterrence and an artificial monopoly may be established by an appropriate product choice. Opening of trade may have an impact on the structure of an economy even if actual trade does not materialize. It is noted, however, that in their model, once a firm has chosen a product, there is no later chance to exit that product. Finally, Brander (1981) constructs

a model in which there are two firms, each located in a different country. He shows that when the firms choose separately their deliveries for each country, a Cournot duopoly can give rise to two-way trade even in identical products. But there is no attempt to explain the firms' choice of product in Brander (1981).

By considering exit cost as a variable and allowing firms to exit in response to entry, we examine in this paper firms' strategic behavior in international market share rivalry and its effects on trade pattern and variety of consumption. Three basic results of the paper are as follows. The first of them is that firms' strategic behavior can give rise to two-way trade in identical products which are, perhaps surprisingly, produced only for trading to each other's countries. This result is more likely to hold as exit costs are low, as transport costs are small, as the products are better substitutes, and as competition in identical products is intense. The non-cooperative solution to the profit-maxmizing problem involves such a two-way trade, but each firm may be better off if they could agree not to invade each other's home markets. The second issue analysed concerns whether trade, through intra-industry trade, makes a greater variety of products available to consumers. Our model gives mixed results on this issue. Whether trade increases or decreases variety depends on the firms' payoffs of various final market structures as well as the level of entry and exit costs. A change in product variety can be brought about by either an actual flow of trade or a potential for trade. For a specific case, opening of trade would unambiguously either increase or decrease variety. Finally, when one of the firms has the advantage of moving first, it may only enter more profitable products and specialize in them for both domestic and foreign markets, leaving other substitutable products to the foreign entrant. Thus specialization can be independently caused by oligopolistic interaction between firms.

The paper is organized as follows. Section 2 presents the basic model which is a four-stage game between a domestic incumbent and a foreign entrant. The game is modified from Judd (1985). The role exit costs play in the model is addressed. The unique subgame-perfect Nash equilibrium of the game is determined under a set of assumptions on firms' payoffs, entry costs, and exit costs. Section 3 then extends the basic model into a two-country international environment where the firms are assumed to have equal opportunity. The rivalry of firms may give rise to two-way trade. By changing the assumptions on firms' payoffs, section

4 moves to the discussion of the issue concerning whether trade will bring about greater product variety. Section 5 examines the implications of the model when one firm is able to move first. The specialization issue is addressed there. Finally, section 6 provides concluding remarks.

It is noted that the results of the paper depends crucially on several different sets of assumptions concerning firms' payoffs under various market structures as well as the level of entry and exit costs. The nature of post-entry rivalry, whether it is in quantity, price, or something else, is not essential. We show in Appendix, however, that these sets of assumptions are consistent with two common duopoly models, namely Cournot and Bertrand rivalry with linear demand.

2. A DOMESTIC INCUMBENT AND A FOREIGN ENTRANT

We first examine the case of a domestic incumbent and a foreign entrant in the home market. Both firms can potentially produce two imperfectly substitutable goods, bearing the same fixed and unit production costs. Since the foreign firm bears extra unit transport costs, its marginal costs are higher than those of the domestic firm producing idendical products by the positive amount of unit transport costs. Consequently, the foreign firm is in a disadvantaged position in terms of payoffs. For instance, if both firms end up selling the same product in the home market, then the home firm would earn higher profit than its foreign rival.

In such an environment, it seems that entry could be effectively deterred, as argued by several previous studies. If post-entry competition is in price and there are no limitations on firms' production capacity, then the Bertrand equilibrium would yield a price which is equal to or slightly less than the foreign entrant's marginal cost, namely unit production cost (m) plus unit transport cost (t). If the equilibrium price is equal to m+c, the two firms will divide the market evenly and the foreign entrant earns zero profit; if the equilibrium price is less than m+c but greater than m, the domestic incumbent captures the whole market whereas its foreign rival sells nothing. In the presence of fixed production costs, the foreign firm would even suffer losses. On the other hand, the home firm earns positive profit in either case because price exceeds its

marginal cost m, and it would have no incentive to leave. Foreign entry would be irrational since post-entry profit could not at least cover the fixed entry cost. The incumbent could therefore commit itself to stay by threatening the entrant with intense post-entry competition and deter potential entry by being first to enter both products.

However, it could be argued that in the case of a differentiated product market, it may not be credible for a multiproduct incumbent to threaten an entrant with intense post-entry competition. In other words, foreign entry would actually happen under suitable conditions. We still take the Bertrand price competition as an example to illustrate the basic argument. But here we make an assumption of zero fixed prodution costs. Our previous discussion suggests that post-entry profits would be zero for the foreign firm and positive for the home firm. Suppose that the home firm initially is a two-product monopolist. Suppose also that the foreign firm does then enter one of products, say product 1. Then the post-entry price competition would yield smaller, though positive, profit from product 1 for the incumbent and zero profit for the entrant. Furthermore, the lower duopoly price of product 1 would depress demand for its substitute, product 2. The depressed demand for product 2 would result in a smaller profit from product 2 for the incumbent.

At this stage, the two firms are not in a symmetric position in terms of the likelihood of exiting product 1. The foreign entrant has no reason to exit since it earns nonnegative profit and exiting is not costless, while the domestic incumbent might exit. This is because after the incumbent exits product 1, the higher monopolistic price of product 1 charged by the foreign entrant would bring about a higher demand for product 2, provided that the entrant would not further enter product 2. The incumbent's choice between "stay in both products" and "exit product 1", therefore, is actually between multiproduct profits facing head-to-head competition on product 1 with the entrant and product 2-only profit in differentiated duopoly subtracting costs of exiting product 1. The fact that the home firm earns positive profit of product 1 whereas the foreign firm earns zero profit is not essential to firms' exit decisions. Moreover, sunk entry costs at this stage are irrelevant to firms' choices.

It is thus conceivable that the domestic incumbent may exit product 1 when the exit cost is small, when the transport cost is sufficiently low, and when products in question are better substitutes. If this is the case, foreign entry is very likely to occur. In anticipating that the incumbent would exit product 1 in response to its entry, the foreign entrant would enter that product if the entry cost could be covered by the rent that accrues to a differentiated rather than an undifferentiated duopolist. Therefore, crowding the product spectrum may not credibly deter foreign entry if the level of exit costs is low and the level of transport costs is small.

It is found in the foregoing argument that when entry costs are not so high as to blockade entry, it could be exit costs (as well as transport costs), not sunk entry costs, that deter foreign entry against a multiproduct domestic incumbent. Therefore, when transport costs are sufficiently small, the argument makes a critical distinction between exit costs on the one hand and irretrievable entry costs on the other. It is also noted that the nature of post-entry rivalry, whether it is Bertrand, Cournot, or others, is not essential to the foregoing argument for possible foreign entry. The crucial elements are firms' profits under various domestic market structures (e.g. the assumption of firms' nonnegative post-entry profits made in the argument, etc.), and the level of entry and exit costs, as well as the level of transport costs.

We set the following notation. N, I, II, and I&II represent the state of the domestic incumbent's being in no market, being in product 1 only, in product 2 only, and in both products, respectively. N^*, I^*, II^*, and $I\&II^*$ represent the foreign entrant's being in no market (no invasion), being in product 1 only, in product 2 only, and in both products, respectively. The market structure in the home market is given by the states of the two firms.

A firm in any of the four states must have made entry, exit, and production (quantity, price, etc.) decisions. In this paper these decisions are assumed to be made sequentially. Further, the home firm is assumed to make entry decisions before the foreign firm does. The basic model is a four-stage game:

Stage 1. The home firm decides how many products to produce and which particular products to produce, and correspondingly pays entry costs. (the entry decision)

Stage 2. The foreign firm decides how many products to produce and which particular products to produce, and correspondingly pays entry costs. (the entry decision)

Stage 3. Both firms simultaneously decide how many products to exit and which particular products to exit, and correspondingly pay exit costs. (the exit decision)

Stage 4. Firms play the duopoly game of the final market structure, and correspondingly bear production costs (fixed production costs and marginal costs) as well as earn sales revenue.

This basic model is very similar to the model in Judd (1985) except one feature. In Judd, as both the incumbent and the entrant are in one country, they can be assumed to be perfectly symmetric in costs. Since the entrant from the foreign country now bears extra unit transport costs, we must make the assumptions on firms' payoffs for both firms rather than for just one firm as Judd does.

The equilibrium is subgame perfect in the four-stage game where each firm anticipates the other firm will act in its own best interests when it chooses its strategy variables. To find the equilibrium, we solve each subgame. We start with the last stage, taking firms' entry and exit decisions as given. Then we analyse, one stage by another, how a firm makes exit as well as entry decisions in the previous stages, correctly taking into account subsequent decisions and their impact on profits.

1) The fourth-stage subgame

In the last stage, firms play the duopoly game of the final market structure. There are 16 possible market structures of entry and exit decisions with which firms enter the final stage. In this paper, the final stage is not modeled as a specific form of rivalry such as a Cournot or a Bertrand game. Instead, we give a number of assumptions concerning firms' payoffs under various market structures as well as the level of entry and exit costs. Once these assumptions are given, some duopoly models can be found to be consistent

with these assumptions. Moreover, these assumptions have summarized the possible outcomes at the final stage.

In what follows, we present a set of assumptions, namely Assumptions 1–10, and then detailly derive the unique equilibrium of the game under these assumptions. Assumptions 1–10 are regarded as appropriate and interesting for the discussions of credible multiproduct preemption. Moreover, by using these basic assumptions, we can analyse production line rivalry, specialization, and two-way trade in later sections. By changing some of them, we can also examine the effects of trade on product variety of consumptions. These fairly intuitive assumptions are checked in Appendix 9 and 10 to be consistent with Cournot and Bertrand duopoly model, respectively.

Before presenting assumptions, we state the following notation:

P(S1,S2) is the profit (the revenue net of production costs) to a firm in state S1 if the other firm in state S2, Si = I, II, I&II, or N for the home firm; Si = $I^*, II^*, I\&II^*$, or N^* for the foreign firm, i=1,2;

A.i stands for Assumption i, i=1,2,3,.....;

F(E,i) is the nonnegative fixed cost of entering product i, i=1,2;

F(X,i) is the nonnegative fixed cost of exiting product i, i=1,2; and

F(E,1&2) and F(X,1&2) are the nonnegative fixed costs of entering and exiting both products, respectively.

Assumption 1.

$$F(E, 1\&2) = F(E, 1) + F(E, 2)$$

$$F(X, 1\&2) = F(X, 1) + F(X, 2)$$

This assumption is not stated in Judd (1985), but has been implicitly used. The assumption says that the two products, though imperfectly substitutable, are independent in incuring entry and exit costs. So there are no economies of scope in terms of entry and exit costs. It may not be met in reality. For instance, it may

well be $F(E,1) + F(E,2) > F(E,1\&2)$. Nevertheless, a slght deviation from it would not affect the results of this paper.

Assumption 2.

$$P \geq P(I\&II^*, I\&II) \geq 0.$$

Assumption 2 says that post-entry profits are always nonnegative. This holds as long as post-entry economies of scale are not so severe that competition forces profits to be negative. In the Bertrand example, our earlier discussion has indicated that the foreign firm would earn nonnegative profit facing head-to-head competition with the home firm if and only if the fixed production costs are zero. If post-entry competition is not so intense, such as in a Cournot industry where low-cost firms do not drive out high-cost firms, we may expect A.2 more likely to hold. Moreover, the smaller the transport costs, the more likely the post-entry profits of foreign firms to be positive. This assumption is necessary in order to avoid inessential complications.

Assumption 3.

$$F(E,1) > P(I,I^*), P(I,I\&II^*)$$

$$F(E,2) > P(II,II^*), P(II,I\&II^*)$$

Because of the relations:

$$P(I,I^*) > P(I^*,I) \qquad \text{and} \qquad P(II,II^*) > P(II^*,II),$$

the parallel assumptions for the foreign firm automatically hold if the above inequalities hold for the domestic firm.

A.3 says that entry costs are sufficiently high that it does not pay a firm to enter a product if it will not eventually become a monopolist in that product. This assumption is made in order to focus on preemption issues and it is not essential for the results.

Assumption 4.

$$P(I\&II, I^*) > P(I, I^*) \geq P(I, I\&II^*)$$

$$P(I\&II, II^*) > P(II, II^*) \geq P(II, I\&II^*)$$

$$P(I\&II^*, I) > P(I^*, I) \geq P(I^*, I\&II)$$

$$P(I\&II^*, II) > P(II^*, II) \geq P(II^*, I\&II)$$

A.4 states that a single-product firm facing competition in that product will receive more post-entry profits by introducing the other product (Note 4), and may lose post-entry profits if its rival expands into the other product. This assumption reflects that the products are imperfect substitutes.

Assumption 5.

$$P(I, II^*) - F(E, 1) > P(II, I^*) - F(E, 2)$$

$$P(I, N^*) - F(E, 1) > P(II, N^*) - F(E, 2), 0$$

$$P(I^*, II) - F(E, 1) > P(II^*, I) - F(E, 2)$$

$$P(I^*, N) - F(E, 1) > P(II^*, N) - F(E, 2), 0$$

According to A.5, product 1 is chosen to be more profitable than product 2 even taking into account entry costs.

Assumption 6.

$$P(I^*, II) - F(E, 1) > 0$$

and

Assumption 7.

$$P(II^*, I) - F(E, 2) > 0$$

Because $P(I, II^*) > P(I^*, II)$ and $P(II, I^*) > P(II^*, I)$, the parallel assumptions for the domestic firm automatically hold as long as the above inequalities hold for the foreign firm.

A.6 and A.7 say that if one firm is selling one product, the other can profitably enter the other product.

Assumption 8.

$$P(I, II^*) - F(X, 2) > P(I\&II, II^*)$$

$$P(I^*, II) - F(X, 2) > P(I\&II^*, II)$$

and

Assumption 9.

$$P(II, I^*) - F(X, 1) > P(I\&II, I^*)$$

$$P(II^*, I) - F(X, 1) > P(I\&II^*, I)$$

A.8 and A.9 state that it is better to be a differentiated duopolist than a multiproduct firm competing head-to-head in one of the products even if costs of exiting that competing product are included. A.8 and A.9 are crucial for the purpose of demonstrating the importance of considering exit costs on entry issue. They have implicitly assumed that exit costs are small, the products are good substitutes, competition is intense, and transport costs are small.

We shall see that under Assumptions 1-9, we can solve for the unique subgame perfect equilibrium in our four-stage game. But, to focus on the spatial preemption issue, we add one more assumption on the domestic firm's payoff,

Assumption 10.

$$P(I\&II, N^*) - F(E,1) - F(E,2) > P(I, N^*) - F(E,1), P(II, N^*) - F(E,2)$$

Under A.10, the domestic incumbent will enter both products if there is no threat of foreign entry because it is more profitable to be a monopolist in both products than in either one alone.

Thus, the possible outcomes and associated payoffs at the fourth stage have been summarized in the P(S1,S2)'s defined above (Note 5).

2) The third-stage subgame

Table 12 lists the possible states in the domestic country just before Stage 3 and the resulting states in equilibrium at the end of the third-stage game.

In cases 1,2,3,4,5,9 and 13, there is only one monopoly firm in the market. In cases 6 and 11, both firms enter the same product. In cases 7 and 10, each enters one different product. In all these cases, firms have no incentive to exit their products because Assumption 2 implies nonnegative profits for both firms and exiting is not costless, given that entry costs are sunk.

The more interesting situations arise when at least one firm has entered two products and the two firms compete at least in one product. These are case 8,12,14,15 and 16. In case 14, the domestic firm entered both products at the first stage and then the foreign firm entered one of products, product 1, in the domestic market at the second stage. In case 8, home firm entered product 1 domestically and then foreign newcomer

Table 12

Stage 3 equilibria under Assumptions 1-9 in the domestic country

	Initial States		Final States	
Case	domestic firm	foreign firm	domestic firm	foreign firm
1	N	N*	N	N*
2	N	I*	N	I*
3	N	II*	N	II*
4	N	I+II*	N	I+II*
5	I	N*	I	N*
6	I	I*	I	I*
7	I	II*	I	II*
8	I	I+II*	I	II*
9	II	N*	II	N*
10	II	I*	II	I*
11	II	II*	II	II*
12	II	I+II*	II	I*
13	I+II	N*	I+II	N*
14	I+II	I*	II	I*
15	I+II	II*	I	II*
16	I+II	I+II*	(I+II,I,II)	(I+II*,II*,I*)

Table 13

Stage 3 in the domestic country. Case 14

foreign firm / domestic firm	Exit I	Stay in I
Exit I	P(II,N*)-F(X,1), -F(x,1)	P(II,I*)-F(x,1), P(I*,II)
Stay in I+II	P(I+II,N*), -F(x,1)	P(I+II,I*), P(I*,I+II)
Exit II	P(I,N*)-F(x,2), -F(x,1)	P(I,I*)-F(x,2), P(I*,I)
Exit I+II	-F(x,1)-F(x,2), -F(x,1)	-F(x,1)-F(x,2), P(I*,N)

Table 14

Stage 3 in the domestic country, case 8

foreign firm / domestic firm	Exit I	Stay in I
Exit I	P(II*,N)-F(X,1), -F(x,1)	P(II*,I)-F(x,1), P(I,II*)
Stay in I+II	P(I+II*,N), -F(x,1)	P(I+II*,I), P(I,I+II*)
Exit II	P(I*,N)-F(x,2), -F(x,1)	P(I*,I)-F(x,2), P(I,I*)
Exit I+II	-F(x,1)-F(x,2), -F(x,1)	-F(x,1)-F(x,2), P(I,N*)

Table 15

Stage 3 in the domestic country, Case 16

foreign firm / domestic firm	Stay in I+II*	Exit I*	Exit II*	Exit I+II*
Stay in I+II	P(I+II,I+II*), P(I+II*,I+II)	P(I+II,II*), P(II*,I+II)-F(x,1)	P(I+II,I*), P(I*,I+II)-F(x,2)	P(I+II,N*), -F(x,1) -F(x,2)
Exit I	P(II,I+II*)-F(x,1), P(I+II*,II)	P(I,II*)-F(x,1), P(II*,II) -F(x,1)	P(II,I*) -F(x,1), P(I*,II) -F(x,2)	P(II,N*) -F(X,1), -F(x,1) -F(x,2)
Exit II	P(I,I+II*)-F(x,2), P(I+II*,I)	P(I,II*)-F(x,2), P(II*,I) -F(x,1)	P(I,I*) -F(x,2), P(I*,I) -F(x,2)	P(I,N*) -F(x,2), -F(x,1) -F(x,2)
Exit I+II	-F(x,1) -F(x,2), P(I+II*,N)	F(x,1)-F(x,2), P(II*,N) -F(x,1)	F(x,1) -F(x,2), P(I*,N) -F(x,2)	-F(x,1) -F(X,2), -F(x,1) -F(x,2)

entered both products. The payoff matrixes of the stage-three game for case 14 and case 8 are displayed in Table 13 and Table 14 respectively.

From Table 13, the foreign firm will not exit product 1 since strategy "stay in I" dominates "exit I" (A.2). In response to this credible threat to stay by the foreign firm, the domestic firm has four choices. It won't withdraw both products since staying in both is better than exiting both (A.2). As the products are imperfect substitutes, staying in both is better than exiting non-competing good, product 2 (A.4). However, staying in both is inferior to exiting the competing good, product 1, according to our crucial Assumption 9. Therefore, "exit I" is the best strategy for the domestic firm. Thus we find that the Nash equilibrium in case 14 is "exit I" for home firm and "stay in I" for foreign home. It is noted that large exit costs could make it less likely for the incumbent to exit. Similar analysis can be applied to case 8, and case 12,15.

The last case, case 16, is one where both firms are in both products at the beginning of Stage 3. The payoffs of this stage-three subgame is displayed in Table 15.

First of all, Assumption 2 implies that "stay in both" dominates "exit both" for both firms. After strategies "exit both" are dropped from the payoff matrix, the same reasoning as we did in case 14 will give two pure strategy Nash equilibria, namely each firm exiting one different product at the same time (A.4, A.8, A.9). Further, there is a third possible equilibrium, "stay in both" for both firms. But our assumptions (A.2, A.8, A.9) ensure that this potential equilibrium is worse in terms of payoffs than the first two eqilibria. Thus the first two equilibria provide an upper bound for firms' payoffs.

3) The second-stage subgame

In this stage the foreign entrant is faced with only four possible situations (see Table 16).

Suppose that the domestic incumbent has entered one of the products, say product 1. The entrant won't enter that product again because its postentry profit can not cover its entry cost (A.3), while it may enter product 2 (A.7). But it won't enter both products because it anticipates that it will be forced to exit

Table 16

Stage 2 Subgame under Assumptions 1-9

Domestic firm choices / Foreign firm choices	I	II	I + II	N
I*	P(I*,I)-F(E,1), P(I,I*)-F(E,1)	P(I*,II)-F(E,1), P(II,I*)-F(E,2)	P(I*,II)-F(E,1), P(II,I*)-F(E,1) -F(E,2)-F(X,2)	P(I*,N)-F(E,1), 0
II*	P(II*,I)-F(E,2), P(I,II*)-F(E,1)	P(II*,II)-F(E,2), P(II,II*)-F(E,2)	P(II*,I)-F(E,2), P(I,II*)-F(E,1) -F(E,2)-F(X,2)	P(II*,N)-F(E,2), 0
I+II*	P(II*,I)-F(E,1), -F(E,2)-F(X,1), P(I,II*)-F(E,1)	P(I*,II)-F(E,1), -F(E,2)-F(X,2), P(II,I*)-F(E,2)	<P(I*,II)-F(E,1), -F(E,2)-F(X,2), <P(I,II*)-F(E,2) -F(E,2)-F(X,2)	P(I+II*,N)-F(E,1) -F(E,2), 0
N*	0, P(I,N*)-F(E,1)	0, P(II,N*)-F(E,2)	0, P(I+II,N*)-F(E,1) -F(E,2)	0, 0

Table 17

Stage 1 Subgame under Assumptions 1-9

Domestic firm choices			
I	II	I + II	N
P(I,II*)-F(E,1)	P(II,I*)-F(E,2)	P(I,II*)-F(E,1) -F(E,2)-F(X,2)	0

product 1 at the third stage and hence bear extra entry and exit costs. Therefore after taking into account the outcomes of later stages, the foreign firm will decide to enter product 2.

Suppose that the domestic incumbent has entered both products. As shown in Table 16, the foreign entrant will enter one and only one product in anticipating that any multiproduct producer will eventually be forced to exit one of the products. That it enters which particular product will depend only on the relative profitability of the products in question.

4) The first-stage subgame

It is easily seen from Table 17 that the domestic firm should enter one and only one product (A.6, A.7) after taking into account all the outcomes in later stages. Since Assumption 5 has chosen product 1 to be more profitable, the domestic incumbent will enter product 1 at the first stage.

Under Assumption 1–9, we have determined a unique subgame perfect Nash equilibrium for the case of one domestic incumbent and one foreign entrant in the domestic country. Adding Assumption 10, we obtain the following result:

Result 1. Under Assumptions 1–10, in domestic equilibrium, the domestic incumbent produces only product 1 and the foreign entrant produces product 2. Hence, if (1) the domestic market is sufficient for differentiated duopoly to be profitable net of entry costs, (2) exit costs are small, (3) transport costs are low, (4) the products are better substitutes, (5) competition in homogeneous products is intense, and (6) post-entry economies of scale are not so severe that competition forces firms' post-entry profits to be negative, then at the equilibrium of the domestic market, the home incumbent only produces more profitable goods even though it would have earned more profit by producing all available goods if there were no threat of foreign entry, and allows the foreign entrant to enter less profitable goods.

We offer two examples in the Appendix in which Assumptions 1–10 are consistent with each other. Consequently, Result 1 holds for a certain range of parameters (demand, the level of entry and exit costs, fixed production costs, transport costs, etc.).

3. TWO COUNTRIES

Suppose now there are two countries, A and B, and each country has only one firm, called firm A and firm B respectively. We assume that country A and B have the same pattern of factor endowments and identical technology. Consumers in the two countries have identical tastes and have the same preference to the two products under consideration. Further, firms can potentially produce and export two imperfectly substitutable products. A firm which exports a product will bear extra unit transport cost.

A firm needs to make entry, exit and production decisions for both home and foreign markets. We assume that firms make these decisions sequentially and a firm has the advantage of entering its home market before its foreign rival. This seems an appropriate assumption to make in models dealing with international trade, as Eaton and Kierzkowski (1984) point out. A number of authors, most notably Linder and Vernon, have argued that production is typically first developed for a domestic market. Trade takes place at a later stage of the product cycle, long after firms selected their products and incurred fixed costs. What is new in our model is that there is an exit stage. Firms may exit in response to entry.

More formally, our model is a four-stage game:

Stage 1. Both firms simultaneously decide how many products to produce and which particular products to produce in their domestic markets, and correspondingly pay entry costs. (the home entry decision)

Stage 2. Both firms simultaneously decide how many products to produce and which particular products to produce for the foreign markets, and correspondingly pay entry costs. (the foreign invasion decision)

Stage 3. Both firms simultaneously decide how many products to exit and which particular products to exit in both home and foreign markets, and correspondingly pay exit costs. (the exit decision)

Stage 4. Firms play the duopoly game of the final market structure in a two-country world, and correspondingly bear production and transport costs as well as earn sales revenue.

It is noted that the two firms play this four-stage game with equal opportunity in a two-country world. Each firm is an incumbent at home and a potential entrant abroad. As an incumbent, it would be faced with the threat of foreign entry by trade; as an entrant, it would potentially invade foreign country through trade.

Brander and Spencer (1984) point out that in considering a two-country world one important consideration is whether markets are united or segmented. Segmented markets arise when firms treat different countries as different markets in that they choose their strategy variables for each market separately. Thus, if the rivalry of firms is along the Cournot (Bertrand) line, the segmented-markets perception will lead firms to choose separately their output quantities (prices) for each country. The assumption of segmented markets implies that oligopolistic firms would face distinct country-specific downward sloping demand curves. So there are no cross-effects between the products produced for one country and the products produced for the other country even if the products may be identical. We shall adopt the segmented-markets assumption in our two-country world. In particular, we make the following assumption:

Assumption 11. A production line for domestic country and a production line for foreign country are considered by both firms as two independent lines even though they may produce identical products.

Under Assumption 11, each firm chooses its strategy variables, namely entry and exit decisions, post-entry production (output level, price, particular forms of advertising, etc.) decisions, not only for each product separately, but also for each market separately, and also assumes the other firm acts in the same way. Recall that in Assumption 1, the two products are unrelated in incuring entry as well as exit costs.

Without changing any result of the model, we can also make similar assumptions concerning production costs. Apparently, Assumption 11 is consistent with these assumptions. Moreover, A.11 rules out both economies of scope and economies of scale in our model, and thus allows us to examine trade under neither economies of scope nor economies of scale.

We show in Appendix 11 that in Cournot model with constant marginal costs, firms' maximizing overall profits for both countries is equivalent to firms' separately maximizing profits for each country, provided that final equilibrium market structures are symmetric for both firms and countries. The symmetry of our model has implied symmetrical equilibria. So as long as marginal costs are constant and the equilibria exist, the two countries can be separated. Generally, A.11 implies that our four-stage game in a two-country world may be separated into two parts corresponding to two countries. Since the two parts are perfectly symmetric, the equilibria for one are also the equilibria to the other. Therefore, we only need solve one part in which a home incumbent and a foreign entrant strategicly interact in the home market. This is exactly the game we have discussed in the last section. Based on the obtained results for one country, we may correspondingly derive the results for two countries. This section and next section are concerned with cases where the two countries can be separated.

Result 1 of section 2 says that at the equilibrium of each country, the home incumbent only produces the more profitable good even though it would have earned more profit by producing both goods if there were no threat of foreign entry, and the foreign entrant produces the other good. With segmented markets, we have

Result 2. Under Assumptions 1–11, at the equilibium of our two-country world, each firm produces both products, the more profitable product for the domestic market and the less profitable one for the foreign market. Hence, two-way trade arises in the less profitable product which is produced only for trading.

In addition to the segmented-markets assumptions, the following conditions are either necessary or suitable for the emergence of the kind of two-way trade: (1) post-entry economies of scale are not so severe

that competition forces firms' post-entry profits to be negative, (2) the products are better substitutes, (3) competition in identical products is intense, (4) demand in each country is sufficient for differentiated duopoly to be profitable net of entry costs, (5) exit costs are small, and (6) transport costs are sufficiently low.

The cause of this type of two-way trade need investigating. For this purpose, we consider the game in which firms' choices are to invade or not to invade foreign market. The firms' choices are reflcted in the second stage of our four-stage game. If both firms invade foreign markets, both choose I^*, II^* or $I\&II^*$ in Satge 2. If both firms do not invade foreign markets, both choose N^* in Stage 2. If either firm unilaterally does not invade the foreign market, it chooses N^* whereas its rival chooses I^*, II^*, or $I\&II^*$ in Stage 2. From the determination of equilibrium in the last section, we can obtain Table 18 which summerizes the final equilibrium market structures corresponding to firms' choices in our two-country world.

Suppose initially there were no trade (in both products). Each firm would produce both products domestically and act as a monopolist. Nevertheless, each would then have an incentive to invade the foreign market. As is assumed, it is better for each firm in each country to be a differentiated duopolist than to be a multiproduct duopolist competing head-to-head in one of the products even after taking into account exit costs. When facing foreign entry into one of the products, the home incumbent would thus be better off by exiting that product. Anticipating this, the entrant would take an invasion position as long as its post-entry profit, which would be the rent that accrues to a differentiated rather than an undifferentiated duopolist in the foreign market, can cover the entry cost. If either firm, say firm A, unilaterally did not invade its rival's country, it would produce only one product in country A after foreign invasion, while its rival, firm B, would produce both products in its home market and invade the other product in country A. Consequently, firm A would lose market shares while firm B would expand market shares. It is the firms' incentive to maintain and increase market shares as well as to protect their positions in more profitable products that causes and sustains our two-way trade. The two multiproduct monopolists, each in a different country, invade each

Table 18

Firm A / Firm B	Not Invade	Invade
Not Invade	Each firm is a two-product monopolist in its home market.	Firm B produces only product 1 for B; firm A produces both products for A as well as product 2 for B.
Invade	Firm B produces both products for B as well as product 2 for A; firm A produces only product 1 for A.	Each firm produces both products, product 1 for its home market and product 2 for the foreign market.

Table 19

Firm A / Firm B	Not Invade	Invade
Not Invade	R,R	S,T
Invade	T,S	P,P

other's home markets and become a binational duopolists producing two imperfectly substitutable products, one for each country.

More formally, we can show that strategy "Invade" dominates strategy "Not Invade". The payoff matrix of the game is shown in Table 19. Since the players are in a symmetric position, we observe a symmetric payoff structure.

By referring to Table 18, we can calculate R, T, P, and S. For example, suppose firm A chooses "Not Invade" and firm B "Invade". Then according to Table 18, firm A ends up selling product 1 in A and its payoff S is

$$S = P(I, II^*) - F(E, 1)$$

Firm B ends up with selling both products in B and product 2 in A and its payoff T is

$$T = (P(I\&II, N^*) - F(E, 1) - F(E, 2)) + (P(II, I) - F(E, 2))$$

Similarly, we have

$$R = P(I\&II, N^*) - F(E, 1) - F(E, 2)$$

$$P = (P(I, II^*) - F(E, 1)) + (P(II^*, I) - F(E, 2))$$

From R, T, P, and S, we calculate

$$T - R = P(II^*, I) - F(E, 2)$$

$$P - S = P(II^*, I) - F(E, 2) = T - R$$

According to Assumption 7, one of assumptions made in Result 2, $P(II^*, I) - F(E, 2) > 0$. Therefore, $T - R > 0$ and $P - S > 0$, that is, strategy "Invade" dominates "Not Invade" for both firms.

The level of exit costs plays an important role in the rise of our two-way trade. Lower exit costs make the incumbents more likely to exit in response to foreign entry and thus give the foreign entrants more incentives to increase market shares by invading and driving out the incumbents.

The two-way traded product is less profitable than the non-traded product. Being a domestic incumbent, each could not enter both products when facing potential foreign entry but would enter the more profitable product. By staying out of the more profitable product, the foreign entrant gives the incumbent an acceptable retreat.

We have seen in this model that intra-industry trade may arise due to firms' strategic interactions through trade. Our analysis follows Brander (1981) where the possibility of intra-industry trade in identical goods due to firms' strategic interactions is first examined in the trade literature. The model proposed by Brander considers a single-product industry consisting of two firms, each in a different country. With segmented markets and a Cournot setting, Brander shows that intra-industry trade may take place even in identical goods despite the existence of transport costs. As transport costs fall, goods produced abroad make up a greater and greater share of domestic consumption, with the share approaching a fifty percent as transport costs approach to zero. The cause of this two-way trade comes from the firms' motivation of price discriminating, or "dumping", into each other's markets, so called "reciprocal dumping" by Brander and Krugman (1983).

Among many similarities between our model and Brander's model, several differences are worth noting. First, our model is concerned with a multiproduct industry where firms' entry and exit decisions are made prior to firms' production decisions. Thus firms may use the entry and exit decisions for strategic purposes. In other words, firms understand, before anything is actually produced, how the noncooperative output game will work out. Secondly, the role of exit costs has been introduced in our model, and lower exit costs may make it possible for an incumbent to exit in response to foreign entry. Consequently, lower exit costs may give a firm an oppotunity to expand market shares by entering the foreign market. As the result of firms' noncooperative foreign invasions, intra-industry trade might arise. Thirdly, since the exit stage is

added in our model, a different type of two-way trade is derived. In Brander's model, two-way traded good produced abroad makes up a smaller share of domestic consumption of that good than the good produced at home. In our model, the two-way traded good produced abroad captures the whole domestic market of that good, while the good produced by the domestic firm is entirely delivered to the foreign market despite the existence of transport costs. So a country entirely exports a good and simultaneously entirely imports it. This equilibrium outcome may be viewed as an extreme case of intra-industry trade, and seems not realistic.

This outcome could arise in our model due to firms' noncooperative profit maximization. In the home market, a firm is a multiproduct incumbent and it would be better off by exiting a product when the foreign firm invades that product; in the foreign market, the firm becomes a potential invader and it would pay the firm to invade a product in the foreign market. With segmented markets, firms' maximizing overall profits can be equivalent to firms' separately maximizing each country's profits. It is the noncooperative solution to this profit-maximizing problem faced by firms in a sequential entry-exit-production game that gives rise to our two-way trade. We shall show in the next section that our two-way trade can introduce products which will otherwise not be produced in autarky, and thus bring about greater variety of consumption.

What our model has added to Brander (1981) is that intra-industry trade may be caused by the firms' motivation to drive the foreign firms out of some products in which their positions are vulnerable. What is more, the nature of post-entry rivalry, whether it is Cournot, or Bertrand, or something else, is not essetianl in our model. The crucial elements here, in addtion to segmented markets, are firms' payoffs under various market structures as well as the level of entry, exit and transport costs. Both Cournot and Bertrand rivalry with linear demand can be consistent with the assumptions made, suggesting that two-way trade due to firms' strategic behavior may arise not only in Cournot duopoly model but in others as well. In particular, two-way trade in identical products discussed by Brander (1981) would not arise in Bertrand model. This is because in a homogeneous product industry, only one firm with lower marginal cost can survive if competition is in price. In the presence of unit transport cost, the foreign firm's marginal cost is higher than the domestic

firm's. However, two-way trade in identical products may still arise in our model with Bertrand rivalry where a multiproduct industry is under consideration.

Finally, we show that the two firms may be engaged in a Prisoner's Dilemma game. As defined previously, each firm's choices are to invade or not to invade the foreign market. We shall make the following assumption:

Assumption 12.

$$P(I\&II, N^*) > P(I, II^*) + P(II^*, I).$$

A.12 implies that it is better to be a multiproduct monopolist in the domestic market than to be a single-product duopolist in both domestic and foreign markets. This assumption can be checked in both Cournot and Bertrand models to be consistent with Assumptions 1–10 (Appendix 12, 13).

The definition of the Prisoner's Dilemma requires that two relationships hold among the four different potential outcomes. The first relationship specifies the order of the four payoffs: $T > R > P > S$. Because $T - R = P - S > 0$ (A.7) and $R - P = P(I\&II, N^*) - P(I, II^*) - P(II^*, I) > 0$ (A.12), $T > R > P > S$ is satisfied.

The second part of the definition of the Prisoner's Dilemma is $2R > R + S$. That is, the players cannot get out of their dilemma by taking turns exploiting each other. This condition holds in this game because $2R - (T + S) = (R - P) + (P - S) - (T - R) = R - P > 0$ (since $P - S = T - R$ and A.12). Thus, we have shown that the game in question is a Prisoner's Dilemma game.

As the result, if the game is played only once, both firms would invade foreign markets and two-way trade would take place in the same good which is produced only for trading. Since foreign entry into a particular product usually lasts for several years, the short-run gains from such entry seem very attractive to firms.

4. TRADE AND PRODUCT VARIETY

An important aspect of international trade is that there is a substantial intra-industry tarde: trade with similar products. The greater variety of consumption brought about by trade becomes an important source of gains from trade. Therefore, the issue concerning whether trade, through intra-industry trade, will make a greater variety of consumption is important in the analysis of gains from trade. Jacquemin (1982) notes that both theory and empirical evidence give mixed results on this issue.

First, there is a strong presumption that the diversity of products will be larger after trade than before. When there are economies of scale, there will always be products for which demand is not sufficient to make production profitable. By expanding the market, trade will lessen the importance of scale economies and hence leads to an increase in product variety. Krugman (1979, 1980, 1981), Dixit and Norman (1980), and Helpman (1981), among others, prove that trade can, in addition to improving resouce allocation, bring about greater variety. They use a Chamberlinian monopolistic competition model which incorporates an increasing returns-to-scale technology. In their equilibrium, each firm in different countries ends up producing a single variety of a differentiated product but earns no monopoly profits.

Nonetheless, with different assumptions, different results could be derived. Dixit and Norman (1980) demonstrate through an example that with imperfect competition, some products, although produced with increasing returns to scale, could also disappear and product selection could be altered by a larger economy made possible by trade. It is noted that their example arises in a framework in which there is a single monopolistic firm before as well as after trade. Eaton and Kierzkowski (1984) develop a model in which firms make entry and price decisions sequentially and firms can credibly threaten entrants with intense post-entry competition. Eaton and Kierzkowski show an example where trade reduces the variety of products in the world economy. It does so by eliminating firms serving a small market with idiosyncratic tastes. In the new equilibrium the consumers in this market do not necessarily buy a less desirable product but may cease consuming altogether.

In this section we examine the effects of trade on product variety using the model developed in the previous sections. In our model neither economies of scope nor economies of scale is assumed. Consumers in different countries have identical tastes and consumers have the same preference to the products. Firms, each in a different country, strategically interact with each other through trade and may exit a produt in response to an entry. Therefore, our approach is different from the previous studies just cited. It seems that the model provides a flexible tool of analysis for the variety issue in our two-country world. By changing the firms' payoffs under various market structures as well as the levels of entry and exit costs, we can compute corresponding market equilibria. Since the equilibrium analysis is similar to that in section 2 and 3, we shall not go into details.

a) Trade increases product variety

Suppose that Assumption 10 fails to hold. The following Assumption 13 is the opposite of A.10:

Assumption 13.

$$P(I, N^*) > P(I\&II, N^*) - F(E, 2)$$

Consequently when facing no threat of foreign entry, the domestic incumbent which produces the more profitable good, product 1, will not expand to product 2 since a multiproduct monopoly is not as valuable as a single-product monopoly. In this case, each firm will only produce product 1 in autarky. We want to show that opening of trade will bring about product 2 into the markets.

First, assume that the other assumptions remain true. As was noted earlier, whether A.10 holds or not will have no impact on the determination of the unique equilibrium in section 2 as long as Assumptions 1-9 hold. Thus we have

Result 3. Under Assumptions 1–9, 13 and 11, at the equilibrium of our two-country world, each firm produces both products: product 1 for the home market and product 2 for the foreign market. Hence the actual flow of trade introduces product 2 into the markets that will otherwise not be produced in autarky.

Result 3 is interesting in that our seemingly pointless two-way trade, where trade takes place in identical products which is produced only for trading, can involve the products which would otherwise disappear from consumption without trade and can thus bring about greater variety available to consumers.

Next, consider the case where exit costs are so large that both A.8 and A.9 fail to hold. This is reflected in the following Aassumption 14:

Assumption 14.

$$P(I\&II, II^*) > P(I, II^*) - F(X, 2)$$

$$P(I\&II, I^*) > P(II, I^*) - F(X, 1)$$

In this situation, then, the threat to stay in both products by the incumbent is credible and deterrence is possible. However, whether the incumbent will actually deter foreign entry into product 2 will depend on the relative profitability in the home market between its being a multiproduct monopolist and its being a single-product duopolist. This is reflected in the following Assumption 15:

Assumption 15.

$$P(I\&II, N^*) - F(E, 2) > P(I, II^*)$$

if A.15 holds, then being a multiproduct monopolist is more valuable than being a single-product duopolist in the home market; otherwise, less. Thus, if A.15 holds, it pays for the incumbent to deter foreign entry into product 2 by introducing product 2 itself; otherwise, it is not and the incumbent will allow foreign entry. We therefore obtain the following result:

Result 4. Under Assumptions 1–7, 14, 15, 13, and 11, at the equilibrium of our two-country world, each firm produces both products only for its home market if Assumption 15 holds; and each firm produces both products, product 1 for the home market and product 2 for the foreign market, if Assumption 15 does not hold. Hence, opening of trade introduces product 2 into the markets and thus increases product variety.

When A.15 does not hold, the actual flow of trade makes greater variety possible; when A.15 holds, the potential for trade does the job.

The new feature arising from Result 4 is that the ability to trade, even if no trade actually occurs, can affect the final market structure in an international environment. A greater variety made possible by trade could be associated with either actual trade or potential trade.

b) Trade reduces product variety

No matter what the level of exit costs is, both Result 3 and Result 4 suggest that trade, either actual or potential, can increase product variety available to consumers. But they are not conclusive. Suppose that Assumption 10 holds now, that is, each firm will produce both products in autarky. Here, we want to show that opening of trade may lead product 2 to disappear from the markets.

Suppose also that Assumptions 1-9 hold except Assumption 7. The opposite of A.7 is the following A.16:

Assumption 16.

$$0 > P(II^*, I) - F(E, 2)$$

Either lower post-entry profit of product 2 (due to the high transport cost, for instance) or higher cost of entry into product 2 will make A.16 more likely hold. A.16 implies that from a domestic point of view, it does not pay the foreign firm to invade product 2 when the home firm has entered product 1. As A.10 holds now, the domestic firm producing product 1 would be profitable for its expansion to product 2 if there were no threat of foreign invasion. In the presence of potential entry, however, if it did such an expansion, then A.6 and A.9 imply that the foreign firm would invade product 1 in the domestic market and force the home firm to exit product 1. As the home firm relizes this consequence, it stays out of product 2 to protect its

position in the more profitable good, product 1. On the other side, the foreign firm does not invade product 2 either since A.16 implies that the invasion is not worthwhile. Therefore, we obtain the following Result 5:

Result 5. Under Assumptions 1–11 except Assumption 7, at the equilibrium of our two-country world, each firm produces only product 1 for only the home market. Each firm would also produce product 2 for the home market in autarky but would not do so because of potential foreign invasion through trade. Each firm does not introduce product 2 by invasion either because doing so is not profitable. Therefore, opening of trade, although no actual trade occurs, leads product 2 to disappear from the markets and thus reduces product variety available to consumers.

As has been demonstrated, our model gives mixed answers to the question of whether trade, through intra-industry trade, makes a greater product variety available to consumers. However, we can show that for a specific case, the answer to the variety question will be unambiguous. We prove this by showing that for a specific case, Result 3 (Result 4) and Result 5 can not hold at the same time. Note that in deriving both Result 3 and Result 4 we made the assumtions, among others, that A.10 fails (i.e. A.13 holds) and A.7 holds,

$$A.13: \qquad P(I, N^*) > P(I\&II, N^*) - F(E, 2)$$

$$A.7: \qquad P(II, I^*) - F(E, 2) > 0$$

That A.13 and A.7 hold simultaneously means that there is a range for F(E,2) such that

$$P(II, I^*) > F(E, 2) > P(I\&II, N^*) - P(I, N^*)$$

leading to

$$P(II, I^*) > P(I\&II, N^*) - P(I, N^*) \tag{1}$$

(1) is necessary for Result 3 and Result 4.

Note also that in deriving Result 5, we assumed that A.10 holds and A.7 fails (i.e. A.16 holds). Similarly, the following (2) is necessary for Result 5:

$$P(I\&II, N^*) - P(I, N^*) > P(II, I^*) \tag{2}$$

Obviously, (1) and (2) can not hold simultaneously. Therefore, Result 3 (Result 4) and Result 5 can not be true at the same time. In other words, we have the following result:

Result 6. Opening of trade may increase or reduce product variety available to consumers, but for a specific case trade would unambiguously either increase or reduce product variety.

We show in Appendix 14, 15 that inequality (1) and all assumptions made either in Result 3 or in the first part of Result 4 are consistent with both Cournot and Bertrand models with linear demand functions. Consequently, both Result 3 and Result 4 (the first part) will hold for a certain range of parameters in these two examples. This is contained in the following result:

Result 7. In the case of either Cournot or Bertrand rivalry with linear demand, trade, either potential or actual, would increase product variety available to consumers.

We have seen that firms' strategic interaction through trade in order to maximize their profits can increase or decrease product variety. Opening of trade can have an impact on the final market structure in which firms play oligopolistic games even if there is no actual flow of trade. Furthermore, whether trade, through intra-industry trade, increases or reduces variety, can depend on, among others, the level of entry costs as well as the level of exit costs.

5. SPECIALIZATION

An alternative extension of the basic model of section 2 is that one of the firms, say firm A, is able to act not only in the home market but also in the foreign market prior to firm B. This could arise when firm A is a technology innovating firm, while the new technology can be accessed by firm B at a later stage using the sunk nature of investment costs (buying patent, spending R&D, etc.). This may happen in cases where the legal protection for innovations will be ended soon, or imitation is possible. In this section we examine specialization and trade by using this sequential game in our two-country world.

The game to be analysed is very similar to the game of section 3 where firms have equal opportunity to first enter their home markets and then enter the foreign markets. At the first stage, firm A makes entry decision in both home and foreign markets. At the second stage, firm B makes entry decision in both home and foreign markets. At the third stage, firms simultaneously make exit decisions in both home and foreign markets. At the fourth stage, firms engage in a duopoly game of the final market structure in an international environment. Denote 1 and 2 to be respectively product 1 and 2 firm A (firm B) produces for its home (foreign) market, and 1^* and $2*$ to be respectively product 1 and 2 firm A (firm B) produces for its foreign (home) market.

The Assumptions 1-11 remain to hold in this game. The segmented-markets assumption, A.11, implies that there are no cross-effects between products in one country and products in the other country even though the products are identical or imperfectly substitutable within one country. So commodity pairs (1, 2) and $(1^*, 2^*)$ are imperfectly substitutable while pairs $(1, 1^*)$, $(1, 2^*)$, $(2, 1^*)$, and $(2, 2^*)$ are unrelated. We for simplicity assume that there are no transport costs in exporting. Thus within a country, the two firms incur the same costs in all aspects. Further, since consumers in the two countries have identical tastes, 1 (2) and $1^*(2^*)$ will be equally profitable for a firm. Finally, as has been assumed, product 1 is more profitable than product 2, so 1 (1^*) is more profitable than 2 (2^*) for a firm.

The above model specification may be viewed as a direct extention of the basic model of section 2 (from two-product case to four-product case), but is concerned with the case of two segmented markets rather than just one united market. The unique equilibrium of the basic model of section 2 involves the incumbent's producing product 1 and the entrant's producing product 2. By using the equilibrium analysis there, we can easily find the unique equilibrium for the current game played in our two-country world. Obviously, firm A would enter all the available product markets if there were no potential invasion. But firm A won't enter more than one product in each country, anticipating that if entering both in any of the countries, it will withdraw one of the two substitutable products in response to an invasion in that product. Thus, firm A will enter one and only one product in each country and let firm B enter the other product. Because product 1 is more profitable than product 2, firm A will choose product 1 to enter. We therefore reach the following Result 8:

Result 8. Under the assumption that firm A is an incumbent in both home and foreign markets and Assumptions 1-11, at the equilibrium of our two-country world, firm A produces product 1 for both countries, and firm B produces product 2 for both countries. Hence, countries specialize in production and trade with each other.

Eaton and Lipsey (1979), among others, have the idea that a foresighted monopolist would introduce a new product in a growing market before a rival. According to their model, firm A would crowd into both products in both countries. This is contrast to our result in which firm A would not crowd the product spectrum and allow the entrant to produce the substitutable products in both countries. Brander and Eaton (1984) develop a model to examine production line rivalry. In Brander and Eaton's model, firms make three decisions (scope, line, and output quantity) sequentially. They find that this sequential decision-making can naturally give rise to equilibrium in which a single firm monopolizes close substitutable products, called market segmentation by them. Suppose that firm A, a first mover, can now only choose two out of 1, 2, 1^*, and 2^*. This constraint has no impact on the equilibrium outcome of our model: firm A produces $(1, 1^*)$ and

firm B $(2, 2^*)$. Applying Brander and Eaton's model to the game, however, firm A in equilibrium produces (1, 2) and firm B $(1^*, 2^*)$. So firms monopolize their home markets and a market segmentation based on different countries can be expected. Trade does not actually occur even though it has opened.

It is noted that in both Eaton and Lipsey (1979) and Brander and Eaton (1984), there is no possibility for firms to exit in response to entry. If exit is allowed, we have found that firms would specialize in products and trade to each other. Thus a "market segmentation" based on substitutable products can emerge with trade. In some real cases the story may go like this. Firms initially produce all products for their domestic markets. At later stage of the product cycle when, for instance, the products become better substitutes as the products are more finely differentiated, they invade each other's home markets in order to expand their market shares. Meanwhile, anticipating potential foreign invasion, they withdraw some products in order to protect their positions in the other products. Unfortunitely, by invading one another, they may achieve an inefficient outcome, while both might have been better off by agreeing not to do so.

Trade can be explained as being due to the combined effects of two motives for specialization: differences between countries (as conventional trade theory shows), and economies of scale (as Krugman (1979, 1980, 1981), Lancaster (1980), among others, analyse). Brander (1981) and Brander and Krugman (1983) show in a single-product industry that the rivalry of oligopolistic firms can serve an independent cause of trade. Our model is in a similar spirit, but is concerned with a multiproduct industry where the issue of specialization is explicitly addressed.

6. CONCLUDING REMARKS

This paper has shown that the type of trade pattern which will emerge is closely related to the cost conditions (entry and exit costs, fixed and marginal production costs, transport cost). In particular, the level of exit costs as a variable seems largely ignored in the trade literature. By considering exit costs, we have shown in this paper several interesting and significant results. Two-way trade might arise in identical products

which are produced only for trading in the presence of transport costs. This might happan purely because oligopolistic firms have an incentive to try to gain market shares by invading the foreign markets and driving the foreign firms out of some products. Further, this kind of two-way trade can introduce products which would otherwise not be produced in autarky and thus bring about greater variety of consumption. Furthermore, opening of trade may also reduce product variety available to consumers. Moreover, instead of producing all substitutable products and monopolizing the home markets, firms may specialize in some of the products and invade each other's home markets. Since the costs of withdrawing a product can be small and the firms' strategy of raising exit costs is often not viable, the analysis of considering exit costs may have some suggestive power.

The kind of two-way trade where trade takes place in identical products which are produced only for trading is hardly found in reality. It is possible to construct alternative models based on the model of this paper to explain this empirical aspect. Among them, we briefly discuss the following two which use repetition and incomplete information. First, applying Kreps et al. (1982) to the Prisoners' Dilemma game of section 3 which is now played finite times, we expect that each firm would take a "Not Invade" position until the last few stages, provided that each firm initially assigns a positive probability that the other will not invade. Second, the basic game of section 2 can be varied as an example of the chain-store game. There are two possible equilibria in the chain-store game (see Selten (1978) or Kreps and Wilson (1982)). One of them is a perfect equilibrium involving the entrant's entry, the one discussed in this paper. The second is an imperfect equilibrium involving the entrant's staying out. In an environment of perfect information, the first equilibrium will prevail even if the game is played several times. Suppose, however, that the entrant initially assesses some positive probability, p, that the incumbent will "irrationally" fight, rather than exit, in response to entry. Since having a reputation for being tough is advantegeous to the incumbent, the incumbent would try to develop this reputation early in the game even though by doing so it would suffer short-run losses. Kreps and Wilson (1982) show that even for very small p, the reputation effect soon predominates and it can give rise to credible threat even in a finitely repeated game. Under this model, therefore, foreign entry

may be effectively deterred, especially in earlier stages of the game, and the kind of two-way trade may not occur.

The paper is an attempt to conduct an exploration of the trade models, if not any attempt to prescribe the reality of trade. The kind of two-way trade may be viewed as non-realistic. Furthermore, except the analysis for the effects of trade on variety, we have not doen a systematic welfare analysis incorporating the consumer surplus. Nevertheless, we do show a number of interesting theoretical possibilities concerning intra-industry trade, specialization, and product variety.

Finally, the model in this paper has been described and interpreted as a model of trade. Instead of exporting, invasion may also take the form of foreign direct invesment (establishing production facilities in the foreign countries, for instance). We note that the basic results obtained with trade in this paper would continue to hold in the presence of foreign direct investment.

NOTES

1. The case of changing c_2 given c_1 can be similarly analysed, and most results reported in the paper are simply reversed in this case.

2. If the demand parameters are estimated using eqs. (10)-(14) in the paper, then $b_1 > k$ as long as $2P_{10} > P_{20}$; $b_2 > k$ as long as $Q_{10} > Q_{20}$. Q_{i0} and P_{i0} are respectively the actual quantities and prices for a year.

3. There are small errors in Table 4 and 5 of Dixit (1985). In Table 4, the Japanese profit under the MFN-tariff should be \$.7730 billion rather than \$.928 billion which is given in Dixit's paper. This is because the tariff revenue $100 \times Q_2 = \$100 \times .001546 = \$.1546$ billion must be subtracted from the total Japanese profits of \$.928 billion. The same error is also found in Table 5. Another error in Table 4 is in the calculation of the Japanese profit under the optimal subsidy. In this case the Japanese profit should be \$.7192 billion rather than \$.574 billion. The latter is the difference between \$.7192 billion and $\$.1491 = 100 \times Q_2$ which may be considered as the U.S. tariff revenue. But in the optimal subsidy case, the tariff is set at zero, and hence there are no tariff revenues to be considered.

4. A similar assumption is made in Judd (1985) which says that a single-product firm will receive more profits (gross of entry costs) by introducing the other product. We find the condition of "gross of entry costs" is not necessary for the determination of equilibria. So we exclude this condition from our assumption.

5. The other part of Assumption 10 which concerns the foreign entrant:

$$P(I\&II^*, N) - F(E,1) - F(E,2) > P(I^*, N) - F(E,1), P(II^*, N) - F(E,2)$$

may be added for completeness, even though it is not needed in deriving the results.

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Appendix 1

Consider the linear demand functions:

$$Q_1 = \alpha_1 - \beta_1 P_1 + \gamma P_2 \tag{1}$$

$$Q_2 = \alpha_2 + \gamma P_1 - \beta_2 P_2 \tag{2}$$

To find the parameters α_1, α_2, β_1, β_2, and γ, three seperate sets of conditions are used.

(i) The system should be compatible with the actual prices P_{10}, P_{20} and quantities Q_{10}, Q_{20} for the year under consideration:

$$Q_{10} = \alpha_1 - \beta_1 P_{10} + \gamma P_{20} \tag{3}$$

$$Q_{20} = \alpha_2 + \gamma P_{10} - \beta_2 P_{20} \tag{4}$$

(ii) The overall price elasticity of demand for automobiles in the U.S. is to be e_1. Since U.S. and Japanese cars are being treated as imperfect substitutes, this elasticity is interpreted as the effect of an equiproportionate rise in the price of the two on the corresponding (dual) quantity aggregate. Let

$$P_1 = \pi_1 P, \qquad P_2 = \pi_2 P$$

and change P while holding π_1, π_2 fixed. The dual quantity is

$$Q = \pi_1 Q_1 + \pi_2 Q_2.$$

Use (1) and (2),

$$Q = (\pi_1\alpha_1 + \pi_2\alpha_2) - (\beta_1\pi_1^2 - 2\gamma\pi_1\pi_2 + \beta_2^2)P \tag{5}$$

Then

$$\frac{dQ}{dP} = -(\beta_1\pi_1^2 - 2\gamma\pi_1\pi_2 + \beta_2\pi_2^2)$$

Set $-\frac{dQ}{dP}\frac{P}{Q} = e_1$,

$$\frac{(\beta_1\pi_1^2 - 2\gamma\pi_1\pi_2 + \beta_2\pi_2^2)P}{(\pi_1\alpha_1 + \pi_2\alpha_2) - (\beta_1\pi_1^2 - 2\gamma\pi_1\pi_2 + \beta_2\pi_2^2)P} = e_1$$

$$e_1(\pi_1\alpha_1 + \pi_2\alpha_2) - (e_1 + 1)(\beta_1\pi_1^2 - 2\gamma\pi_1\pi_2 + \beta_2\pi_2^2)P = 0$$

Multiply P on both sides and note $\pi_1 P = P_1, \pi_2 P = P_2$, we get

$$e_1(P_1\alpha_1 + P_2\alpha_2) - (e_1 + 1)(\beta_1 P_1^2 - 2\gamma P_1 P_2 + \beta_2 P_2^2) = 0. \tag{6}$$

Eq.(6) holds at the observed point, i.e.,

$$e_1(P_{10}\alpha_1 + P_{20}\alpha_2) - (e_1 + 1)(\beta_1 P_{10}^2 - 2\gamma P_{10}P_{20} + \beta_2 P_{20}^2) = 0. \tag{7}$$

(iii) The elasticity of substitution between U.S. and Japanese cars is to be e_2. That is, at the observed point,

$$-\frac{d\log(Q_1/Q_2)}{d\log(P_1/P_2)} = e_2 \tag{8}$$

To have Q_1/Q_2 as a function of P_1/P_2 and thus the substitution elasticity defined, the parameters must be at least at the observerd point satisfied an additional condition,

$$P_{10}(\alpha_1\gamma + \alpha_2\beta_1) = P_{20}(\alpha_2\gamma + \alpha_1\beta_2) \tag{9}$$

We can show eqs. (8) and (9) are equivalent to the following two equations,

$$\gamma Q_{10} + \beta_1 Q_{20} = \frac{e_2 Q_{10} Q_{20}}{P_{10}} \tag{10}$$

$$\beta Q_{10} + \gamma Q_{20} = \frac{e_2 Q_{10} Q_{20}}{P_{20}} \tag{11}$$

This is because from (9), and (3), (4),

$$P_{10}(\alpha_1\gamma + \alpha_2\beta_1) = P_{20}(\alpha_2\gamma + \alpha_1\beta_2)$$

$$\Rightarrow \quad P_{10}(\beta_1 Q_{20} + \gamma Q_{10}) = P_{20}(\gamma Q_{20} + \beta_2 Q_{10})$$

$$\Rightarrow \quad \frac{P_{20}}{P_{10}} = \frac{\beta_1 Q_{20} + \gamma Q_{10}}{\gamma Q_{20} + \beta_2 Q_{10}} \tag{12}$$

From (12),

$$\begin{aligned}
\frac{d\log(Q_1/Q_2)}{d\log(P_1/P2)} &= \frac{d\log Q_1 - d\log Q_2}{d\log P_1 - d\log P_2} \\
&= \frac{dQ_1/Q_{10} - dQ_{20}/Q_{20}}{dP_1/P_{10} - dP_2/P_{20}} \\
&= \frac{(\gamma/Q_{10} + \beta_2/Q_{20})dP_2 - (\beta_1/Q_{10} + \gamma/Q_{20})dP_1}{dP_1/P_{10} - dP_2/P_{20}} \\
&= -\frac{\gamma Q_{20} + \beta_2 Q_{10}}{Q_{10}Q_{20}} P_{20}
\end{aligned}$$

From (8), we have

$$\gamma Q_{20} + \beta_2 Q_{10} = \frac{e_2 Q_{10} Q_{20}}{P_{20}}.$$

Similarly, we have

$$\beta_1 Q_{20} + \gamma Q_{10} = \frac{e_2 Q_{10} Q_{20}}{P_{10}}.$$

Thus, eqs. (3), (4), (7), (10) and (11) are five independent and linear equations. We can solve for α_1, α_2, β_1, β_2 and γ using Crammer's rule, yielding

$$\alpha_1 = (e_1 + 1)Q_{10}$$

$$\alpha_2 = (e_1 + 1)Q_{20}$$

$$\beta_1 = \frac{Q_{10}(e_1 P_{10} Q_{10} + e_2 P_{20} Q_{20})}{P_{10}(P_{10} Q_{10} + P_{20} Q_{20})}$$

$$\beta_2 = \frac{Q_{20}(e_2 P_{10} Q_{10} + e_1 P_{20} Q_{20})}{P_{20}(P_{10} Q_{10} + P_{20} Q_{20})}$$

$$\gamma = \frac{(e_2 - e_1) Q_{10} Q_{20}}{P_{10} Q_{10} + P_{20} Q_{20}}.$$

The corresponding inverse demand functions are

$$P_{10} = a_1 - b_1 Q_{10} + k Q_{20}$$

$$P_{20} = a_2 + k Q_{10} - b_2 Q_{20}$$

The parameters a_1, a_2, b_1, b_2 and k can be easily estimated using α_1, α_2, β_1, β_2 and γ, and they turn out to be those given in section 2.

APPENDIX 2
SENSITIVITY ANALYSIS FOR ELASTICITIES E1 AND E2, 1979
(PERCENT CHANGE FROM BASE)

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E1	-5	-5	-5	-5
T	0	1.412		-.3182
S			.4761	-.3511
P1	0	.0064	-.0993	0
P2	0	.1547	-.0578	-.0616
Q1	0	.0689	-.3450	-.3511
Q2	0	-.3123	-.4656	-.3182
JAPAN PROFIT	0	-.6237	-.9290	-.6353
US PROFIT	0	.1378	-.6888	-.7010
US CONS SURPL	5.263	5.331	4.510	4.511
TARI REV	0	1.095		-.6353
SUBS COST			.1294	-.7010
US WELFARE	4.498	4.494	4.441	4.431

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E1	-2.500	-2.500	-2.500	-2.500
T	0	.7031		-.1590
S			.2303	-.1755
P1	0	.0032	-.0487	0
P2	0	.0770	-.0287	-.0308
Q1	0	.0341	-.1724	-.1755
Q2	0	-.1557	-.2312	-.1590
JAPAN PROFIT	0	-.3112	-.4619	-.3178
US PROFIT	0	.0683	-.3445	-.3506
US CONS SURPL	2.564	2.597	2.197	2.198
TARI REV	0	.5462		-.3178
SUBS COST			.0576	-.3506
US WELFARE	2.191	2.190	2.163	2.158

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E1	2.500	2.500	2.500	2.500
T	0	-.6973		.1589
S			-.2159	.1753
P1	0	-.0031	.0468	0
P2	0	-.0763	.0284	.0307
Q1	0	-.0335	.1721	.1753
Q2	0	.1548	.2283	.1589
JAPAN PROFIT	0	.3099	.4570	.3180
US PROFIT	0	-.0670	.3446	.3509
US CONS SURPL	-2.439	-2.469	-2.091	-2.090
TARI REV	0	-.5435		.3180
SUBS COST			-.0441	.3509
US WELFARE	-2.084	-2.083	-2.057	-2.053

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E1	5	5	5	5
T	0	-1.389		.3176
S			-.4182	.3505
P1	0	-.0062	.0919	0
P2	0	-.1520	.0564	.0615
Q1	0	-.0664	.3441	.3505
Q2	0	.3088	.4537	.3176
JAPAN PROFIT	0	.6185	.9095	.6362
US PROFIT	0	-.1328	.6893	.7022
US CONS SURPL	-4.762	-4.820	-4.082	-4.080
TARI REV	0	-1.084		.6362
SUBS COST			-.0756	.7022
US WELFARE	-4.069	-4.067	-4.016	-4.007

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E2	-5	-5	-5	-5
T	0	-.8303		1.101
S			-.8154	-.1262
P1	0	-.0089	.0860	0
P2	0	-.0108	.0747	.2131
Q1	0	-.0951	-.1427	-.1262
Q2	0	1.040	.6014	1.101
JAPAN PROFIT	0	2.090	1.206	2.215
US PROFIT	0	-.1900	-.2852	-.2522
US CONS SURPL	0	.0095	-.1444	-.0536
TARI REV	0	.2008		2.215
SUBS COST			-.9570	-.2522
US WELFARE	0	-.0148	-.0154	-.0182

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E2	-2.500	-2.500	-2.500	-2.500
T	0	-.3995		.5466
S			-.4030	-.0626
P1	0	-.0044	.0424	0
P2	0	-.0037	.0373	.1058
Q1	0	-.0474	-.0709	-.0626
Q2	0	.5159	.3005	.5466
JAPAN PROFIT	0	1.034	.6018	1.096
US PROFIT	0	-.0947	-.1417	-.1252
US CONS SURPL	0	-.0044	-.0714	-.0265
TARI REV	0	.1143		1.096
SUBS COST			-.4736	-.1252
US WELFARE	0	-.0074	-.0077	-.0090

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E2	2.500	2.500	2.500	2.500
T	0	.3707		-.5384
S			.3938	.0617
P1	0	.0044	-.0414	0
P2	0	.0007	-.0373	-.1042
Q1	0	.0470	.0700	.0617
Q2	0	-.5081	-.2999	-.5384
JAPAN PROFIT	0	-1.014	-.5988	-1.074
US PROFIT	0	.0940	.1400	.1234
US CONS SURPL	0	-.0037	.0699	.0260
TARI REV	0	-.1392		-1.074
SUBS COST			.4640	.1234
US WELFARE	0	.0073	.0076	.0089

	MFN-TARI	OPT-TARI	OPT-SUBS	OPT-TS
E2	5	5	5	5
T	0	.7148		-1.069
S			.7786	.1224
P1	0	.0087	-.0818	0
P2	0	-.0014	-.0744	-.2068
Q1	0	.0937	.1390	.1224
Q2	0	-1.008	-.5991	-1.069
JAPAN PROFIT	0	-2.007	-1.195	-2.126
US PROFIT	0	.1874	.2783	.2450
US CONS SURPL	0	-.0068	.1383	.0516
TARI REV	0	-.3009		-2.126
SUBS COST			.9188	.2450
US WELFARE	0	.0145	.0151	.0176
US WELFARE	0	-.0148	-.0154	-.0182

Appendix 3

The Proof of $4K_1K_3 - K_2^2 > 0$

Let $A \equiv b_1 b_2 - k^2 > 0$,

$$4K_1K_3 - K_2^2 = \big(b_2A + b_1V_2(2b_2 + V_2)\big)\big((b_1 + 2V_1)A + V_1^2 b_2 + 2V_2(b_1 + V_1)^2\big) - k^2\big(A + (2b_1 + V_1)V_2\big)^2$$

$$\equiv I_1A^2 + I_2A + I_3$$

where

$$I_1 = b_2(2V_1 + b_1) - k^2 > 0,$$

$$I_2 = b_2(2(V_1 + b_1)^2V_2 + V_1^2b_2) + (2V_1 + b_1)b_1V_2(V_2 + 2b_2) - 2k^2(2b_1 + V_1)V_2$$

$$> V_2\big(2b_2(V_1 + b_1)^2 + b_1(2V_1 + b_1)(V_2 + 2b_2) - 4k^2b_1 - 2k^2V_1\big) \qquad \big(V_1^2b_2 > 0\big)$$

$$> V_2\big(2b_2(2b_1V_1 + b_1^2) + 2b_1^2b_2 - 4k^2b_1 - 2k^2V_1\big) \qquad \big(V_1, V_2 > 0\big)$$

$$= V_2\big(4b_1(b_1b_2 - k^2) + 2b_1b_2V_1 + 2V_1(b_1b_2 - k^2)\big)$$

$$> 0,$$

$$I_3 = b_1V_2(2b_2 + V_2)(V_1^2b_2 + 2V_2(b_1 + V_1)^2) - k^2(2b_1 + V_1)^2V_2^2$$

$$> V_2\big(2b_1b_2 2V_2(b_1 + V_1)^2 - k^2(2b_1 + V_1)^2V_2\big)$$

$$= V_2^2\big(4b_1b_2(b_1^2 + 2b_1V_1 + V_1^2) - k^2(4b_1^2 + 4b_1V_1 + V_1^2)\big)$$

$$= V_2^2\big(4b_1^2(b_1b_2 - k^2) + 4b_1V_1(b_1b_2 - k^2) + V_1^2(4b_1b_2 - k^2) + 4b_1^2b_2V_1\big)$$

$$> 0.$$

Therefore, $4K_1K_3 - K_2^2 > 0$. *Q.E.D.*

Appendix 4

The Proof of $K_4 > 0$ and $K_5 > 0$

$$K_4 = k(b_2V_1 - b_1V_2)(a_1 - c_1) + \big((b_1 + V_1)^2V_2 - k^2V_1\big)(a_2 - c_2)$$

If a_i, b_i, and k are estimated using eqs.(10)-(14) in the paper, and $V_1 = (P_{10} - c_1)/Q_{10}$, $V_2 = (P_{20} - c_2)/Q_{20}$, by subsituting them into the expression of K_4, we have

$$\begin{aligned} K_4 = \frac{1}{4Q_1^2Q_2PQ^2}\Big(&P_1P_2Q_1(2P_1 - c_1)\big(P_2(P_1 - c_1)(P_1Q_1 + 2P_2Q_2) - P_1(P_2 - c_2)(2P_1Q_1 + P_2Q_2)\big) \\ &+ \big((P_1(2P_1Q_1 + P_2Q_2) + 2(P_1Q_1 + P_2Q_2)(P_1 - c_1))^2 - P_1^2P_2^2Q_1(P_1 - c_1)\big)Q_2(2P_2 - c_2)\Big) \\ \equiv \frac{1}{4Q_1^2Q_2PQ_2}&\Big(\Big) \end{aligned}$$

where P_1, P_2 and Q_1,Q_2 are the actual prices and quantities, i.e., $P_i \equiv P_{i0}$, $Q_i \equiv Q_{i0}$, and $PQ \equiv P_1Q_1 +$

$$P_2Q_2 \equiv P_{10}Q_{10} + P_{20}Q_{20} \equiv PQ_0.$$

$$\begin{aligned}
\Big(\Big) &\equiv P_1P_2Q_1\big(P_1 + (P_1 - c_1)\big)P_2(P_1 - c_1)(P_1Q_1 + 2P_2Q_2) \\
&- P_1^2P_2Q_1(2P_1Q_1 + P_2Q_2)\big(P_1 + (P_1 - c_1)\big)(P_2 - c_2) \\
&+ \big(P_1(2P_1Q_1 + P_2Q_2) + 2(P_1Q_1 + P_2Q_2)(P_1 - c_1)\big)^2Q_2(2P_2 - c_2) - P_1^2P_2^2Q_1Q_2(P_1 - c_1)(2P_2 - c_2) \\
&> P_1^2P_2^2Q_1(P_1Q_1 + 2P_2Q_2)(P_1 - c_1) - P_1^3P_2Q_1(2P_1Q_1 + P_2Q_2)(P_2 - c_2) \\
&- P_1^2P_2Q_1(2P_1Q_1 + P_2Q_2)(P_1 - c_1)(P_2 - c_2) + P_1^2Q_1(2P_1Q_1 + P_2Q_2)^2(P_2 - c_2) \\
&+ 4P_1Q_2(P_1Q_1 + P_2Q_2)(2P_1Q_1 + P_2Q_2)(P_1 - c_1)(P_2 - c_2) - P_1^2P_2^3Q_1Q_2(P_1 - c_1) \\
&- P_1^2P_2^2Q_1Q_2(P_1 - c_1)(P_2 - c_2) \\
&= P_1^2P_2^2Q_1(P_1Q_1 + P_2Q_2)(P_1 - c_1) + P_1^2(2P_2Q_1 + P_2Q_2)(P_2 - c_2)\big(P_1Q_1(2Q_2 - P_2) + P_2Q_2^2\big) \\
&+ (P_1 - c_1)(P_2 - c_2)2P_1(P_1Q_1 + P_2Q_2)\big(P_1Q_1(4Q_2 - P_2) + 2P_2Q_2^2\big) \\
&> 0 \qquad \text{if} \qquad 2Q_2 - P_2 > 0
\end{aligned}$$

Therefore, $K_4 > 0$ as long as $2Q_{20} > P_{20}$.

That $K_5 > 0$ as long as $2Q_{10} > P_{10}$ can be similarly shown.

Q.E.D

Appendix 5

The Proof of $\partial K_5/\partial c_1 < 0$

$$
\begin{aligned}
\frac{\partial K_5}{\partial c_1} &= \frac{\partial}{\partial c_1}\Big(\big((b_2+V_2)^2V_1 - k^2V_2\big)(a_1-c_1) + k(b_1V_2 - b_2V_1)(a_2-c_2)\Big) \\
&= \frac{dV_1}{dc_1}\big((b_2+V_2)^2(a_1-c_1) - b_2k(a_2-c_2)\big) - \big((b_2+V_2)^2V_1 - k^2V_2\big) \\
&= \frac{dV_1}{dc_1}b_2\big((b_2+V_2)(a_1-c_1) - k(a_2-c_2)\big) + \frac{dV_1}{dc_1}V_2(b_2+V_2)(a_1-c_1) - (b_2+V_2)^2V_1 + k^2V_2 \\
&= \frac{dV_1}{dc_1}b_2\big((b_2+V_2)(a_1-c_1) - k(a_2-c_2)\big) - (b_2+V_2)^2V_1 - \big((-\frac{dV_1}{dc_1})V_2(b_2+V_2)(a_1-c_1) - k^2V_2\big) \\
&\equiv I_1 - I_2 - I_3.
\end{aligned}
$$

Using (19) in the paper and the fact that $dV_1/dc_1 < 0$,

$$
\begin{aligned}
I_1 &= \frac{dV_1}{dc_1}b_2\big((b_2+V_2)(a_1-c_1) - k(a_2-c_2)\big) \\
&= \frac{dV_1}{dc_1}b_2Q_{10}D < 0, \\
I_2 &= (b_2+V_2)^2V_1 > 0, \\
I_3 &> \left(-\frac{dV_1}{dc_1}\right)V_2k(a_2-c_2) - k^2V_2 \\
&= kV_2\left(\left(-\frac{dV_1}{dc_1}\right)(a_2-c_2) - k\right) \\
&= kV_2\left(\frac{2P_{20}-c_2}{Q_{10}} - \frac{P_{10}P_{20}}{2(P_{10}Q_{10}+P_{20}Q_{20})}\right) \\
&= kV_2\frac{P_{20}-c_2}{Q_{10}} + kV_2\left(\frac{P_{20}}{Q_10} - \frac{P_{10}P_{20}}{2(P_{10}Q_{10}+P_{20}Q_{20})}\right) \\
&= kV_2\frac{P_{20}-c_2}{Q_{10}} + \frac{kV_2P_{20}}{2Q_{10}(P_{10}Q_{10}+P_{20}Q_{20})}(P_{10}Q_{10}+2P_{20}Q_{20}) \\
&> 0.
\end{aligned}
$$

Therefore, $\partial K_5/\partial c_1 < 0$.

Q.E.D.

Appendix 6

The Proof of Result 7

Since $V_1/V_2 > b_1/b_2$, using Result 6,

$$\left|\frac{\partial t^*}{\partial c_1}\right| = \frac{k(b_2V_1 - b_1V_2)}{2k_1}$$
$$\left|\frac{\partial s^*}{\partial c_1}\right| = \frac{(b_2+V_2)^2V_1 - k^2V_2}{2K_3}.$$

$$\begin{aligned}
\left|\frac{\partial s^*}{\partial c_1}\right| > \left|\frac{\partial t^*}{\partial c_1}\right| &\Longleftrightarrow \frac{(b_2+V_2)^2V_1 - k^2V_2}{2K_3} > \frac{k(b_2V_1 - b_1V_2)}{2K_1} \\
&\Longleftrightarrow \big((b_2+V_2)^2V_1 - k^2V_2\big)\big((b_1+2V_1)(b_1b_2-k^2) + V_1^2b_2 + 2V_2(b_1+V_1)^2\big) \\
&- k(b_2V_1 - b_1V_2)\big(b_2(b_1b_2-k^2) + b_1V_2(2b_2+V_2)\big) > 0 \\
&\Longleftrightarrow (b_1b_2-k^2)\Big(\big((b_2+V_2)^2V_1 - k^2V_2\big)(b_1+2V_1) - k(b_2V_1-b_1V_2)b_2\Big) \\
&+ \Big(\big((b_2+V_2)^2V_1 - k^2V_2\big)\big(V_1^2b_2 + 2V_2(b_1+V_1)^2\big) - k(b_2V_1-b_1V_2)b_1V_2(2b_2+V_2)\Big) > 0 \\
&\Longleftrightarrow (b_1b_2-k^2)I_1 + I_2 > 0
\end{aligned}$$

where

$$I_1 = \big((b_2+V_2)^2V_1 - k^2V_2\big)(b_1+2V_1) - k(b_2V_1-b_1V_2)b_2,$$

$$I_2 = \big((b_2+V_2)^2V_1 - k^2V_2\big)\big(V_1^2b_2 + 2V_2(b_1+V_1)^2\big) - k(b_2V_1-b_1V_2)b_1V_2(2b_2+V_2).$$

By adding the term $b_1 b_2 V_2(b_1 + 2V_1)$ into I_1 while at the same time subtracting it from I_1, we have,

$$
\begin{aligned}
I_1 &= \big((b_2 + V_2)^2 V_1(b_1 + 2V_1) - b_1 b_2 V_2(b_1 + 2V_1)\big) + \big(b_1 b_2 V_2(b_1 + 2V_1) - k^2 V_2(b_1 + 2V_1)\big) \\
&\quad - k(b_2 V_1 - b_1 V_2) b_2 \\
&> \big(b_2^2 V_1(b_1 + 2V_1) - b_1 b_2 V_2(b_1 + 2V_1)\big) + V_2(b_1 + 2V_1)(b_1 b_2 - k^2) \\
&\quad - k(b_2 V_1 - b_1 V_2) b_2 \qquad\qquad \big((b_2 + V_2)^2 > b_2^2\big) \\
&= b_2(b_1 + 2V_1)(b_2 V_1 - b_1 V_2) + V_2(b_1 + 2V_1)(b_1 b_2 - k^2) - k(b_2 V_1 - b_1 V_2) b_2 \\
&= b_2(b_2 V_1 - b_1 V_2)(b_1 + 2V_1 - k) + V_2(b_1 + 2V_1)(b_1 b_2 - k^2) \\
&> 0 \qquad\qquad \text{if} \quad b_1 > k.
\end{aligned}
$$

Once again, by adding the term $b_1 b_2 V_2\big(V_1^2 b_2 + (b_1 + V_1)^2 2V_2\big)$ into I_2 while at the same time subtracting

it from I_2,we have,

$$
\begin{aligned}
I_2 = {} & \big((b_2+V_2)^2V_1(V_1^2b_2+2V_2(b_1+V_1)^2) - b_1b_2V_2(V_1^2b_2+2V_2(b_1+V_1)^2)\big) \\
& + \big(b_1b_2V_2(V_1^2b_2+2V_2(b_1+V_1)^2) - k^2V_2(V_1^2b_2+2V_2(b_1+V_1)^2)\big) \\
& - k(b_2V_1-b_1V_2)b_1V_2(2b_2+V_2) \\
= {} & \big(V_1^2b_2+2V_2(b_1+V_1)^2\big)\big((b_2+V_2)^2V_1-b_1b_2V_2\big) + V_2\big(V_1^2b_2+2V_2(b_1+V_1)^2\big)(b_1b_2-k^2) \\
& - k(b_2V_1-b_1V_2)b_1V_2(2b_2+V_2) \\
> {} & \big(V_1^2b_2+2V_2(b_1+V_1)^2\big)b_2(b_2V_1-b_1V_2) + V_2\big(V_1^2b_2+2V_2(b_1+V_1)^2\big)(b_1b_2-k^2) \\
& - k(b_2V_1-b_1V_2)b_1V_2(2b_2+V_1) \qquad \big((b_2+V_2)^2>b_2^2\big) \\
= {} & (b_2V_1-b_1V_2)\big(V_1^2b_2^2+2V_2(b_1+V_1)^2b_2-kb_1V_2(2b_2+V_2)\big) + V_2\big(V_1^2b_2+2V_2(b_1+V_1)^2\big)(b_1b_2-k^2) \\
> {} & (b_2V_1-b_1V_2)\big(V_2^2b_1+2V_2b_1^2b_2-kb_1V_2(2b_2+V_2)\big) \\
& + V_2\big(V_1^2b_2+2V_2(b_1+V_1)^2\big)(b_1b_2-k^2) \qquad \big((V_1b_2)^2>(V_2b_1)^2,(b_1+V_1)^2>b_1^2\big) \\
= {} & (b_2V_1-b_1V_2)(2b_2+V_2)V_2b_1(b_1-k) + V_2\big(V_1^2b_2+2V_2(b_1+V_1)^2\big)(b_1b_2-k^2) \\
> {} & 0 \qquad \text{if} \quad b_1>k.
\end{aligned}
$$

Therefore,

$$\left|\frac{\partial s^*}{\partial c_1}\right| > \left|\frac{\partial t^*}{\partial c_1}\right|.$$

Q.E.D.

Appendix 7

The Proof of $K_7 > 0$

From (19) and (20) in the paper, we have

$$Q_{10}D = (b_2 + V_2)(a_1 - c_1) - k(a_2 - c_2 - 100)$$

$$Q_{20}D = (b_1 + V_1)(a_2 - c_2 - 100) - k(a_1 - c_1)$$

$$\begin{aligned}
K_7 &= (b_1 b_2 - k^2)\big((b_1 + V_1)(a_2 - c_2 - 100) - k(a_1 - c_1)\big) + b_2 V_1 (2b_1 + V_1)(a_2 - c_2 - 100) \\
&\quad - kV_1(2b_2 + V_2)(a_1 - c_1) - V_1(b_1 b_2 - k^2)(a_2 - c_2 - 100) + 100(b_1 + V_1)D \\
&= (b_1 b_2 - k^2)Q_{20}D + V_1\big(b_2(b_1 + V_1) + k^2\big)(a_2 - c_2 - 100) - kV_1 b_2(a_1 - c_1) \\
&\quad - kV_1(b_2 + V_2)(a_1 - c_1) + 100(b_1 + V_1)D \\
&= (b_1 b_2 - k^2)Q_{20}D + V_1 b_2\big((b_1 + V_1)(a_2 - c_2 - 100) - k(a_1 - c_1)\big) \\
&\quad - kV_1\big((b_2 + V_2)(a_1 - c_1) - k(a_2 - c_2 - 100)\big) + 100(b_1 + V_1)D \\
&= (b_1 b_2 - k^2)Q_{20}D + V_1 b_2 Q_{20}D - kV_1 Q_{10}D + 100(b_1 + V_1)D
\end{aligned}$$

Using eqs. (10)-(14),

$$\begin{aligned}
&(b_1 b_2 - k^2)Q_{20} + V_1 b_2 Q_{20} - kV_1 Q_{10} \\
&= \left(\frac{P_1 P_2}{2Q_1 Q_2} + \frac{P_1(P_1 Q_2 + 2P_2 Q_2)}{2Q_2 PQ}\frac{P_1 - c_1}{Q_1}\right)Q_2 - \frac{P_1 P_2}{2PQ}\frac{P_1 - c_1}{Q_1}Q_1 \\
&= \frac{P_2}{2Q_1 PQ}\big(P_1 PQ + 2P_2 Q_2(P_1 - c_1)\big) \\
&> 0
\end{aligned}$$

where $P_i \equiv P_{i0}$, $Q_i \equiv Q_{i0}$, $PQ \equiv PQ_0 \equiv P_{10}Q_{10} + P_{20}Q_{20}$. Therefore, $K_7 = \big((b_1 b_2 - k^2)Q_{20} + V_1 b_2 Q_{20} - kV_1 Q_{10}\big)D + 100(b_1 + V_1)D > 0$.

Q.E.D.

Appendix 8

The List of Parameters H_i and G_i

$$H_1 = (b_1 + V_1)^2(b_2 + 2V_2) - k^2(b_1 + 2V_1)$$

$$H_2 = kb_1(b_2 + V_2) + k\big((b_1 + V_1)V_2 - k^2\big)$$

$$H_3 = k^2V_1 - (b_1 + V_1)^2V_2$$

$$G_1 = k(b_2V_1 - b_1V_2)(a_1 - c_1) + \big((b_1 + V_1)^2V_2 - k^2V_1\big)(a_2 - c_2)$$

$$H_4 = kb_1(b_2 + V_2) + kV_2(b_1 + V_1) - k^3$$

$$H_5 = k^2b_2 - (b_2 + V_2)^2b_1$$

$$H_6 = k(b_2V_1 - b_1V_2)$$

$$G_2 = k(b_1V_2 - b_2V_1)(a_2 - c_2) + \big((b_2 + V_2)^2V_1 - k^2V_2\big)(a_1 - c_1)$$

$$H_7 = (b_1 + V_1)\big((b_1 + V_1)(V_2 - b_2) + k^2\big)$$

$$H_8 = k(b_1 + V_1)(V_2 - b_2) + k^3$$

$$H_9 = 2(b_1 + V_1)\big((b_1 + V_1)b_2 - k^2\big)$$

$$G_3 = \big((b_1 + V_1)(a_2 - c_2) - k(a_1 - c_1)\big)\big((b_1 + V_1)(V_2 - b_2) + k^2\big)$$

$$H_{10} = (b_1 + V_1)^2 b_2 - k^2(b_1 + 2V_1)$$

$$H_{11} = k(b_1 b_2 - k^2 - V_1 V_2)$$

$$H_{12} = H_{11}$$

$$H_{13} = (b_2 + V_2)^2 b_1 - k^2(b_2 + 2V_2)$$

$$G_4 = k(b_1 V_2 + b_2 V_1 + 2V_1 V_2)(a_1 - c_1) - \big((b_1 + V_1)^2 V_2 + k^2 V_1\big)(a_2 - c_2)$$

$$G_5 = \big((b_2 + V_2)^2 V_1 + k^2 V_2\big)(a_1 - c_1) - k(b_1 V_2 + b_2 V_1 + 2V_1 V_2)(a_2 - c_2)$$

Appendix 9

Assumptions 1–10 Are Consistent With Each Other in a Cournot Duopoly Model

Suppose that there are two countries, A and B; each country has only one firm. The firm located in country A is called Firm A and the firm in B is called Firm B. In this Appendix, we examine the case of country A.

The inverse demand functions are linear,

$$P_1 = d - bQ_1 - aQ_2 \tag{1}$$

$$P_2 = d - aQ_1 - bQ_2 \tag{2}$$

where all parameters are positive, and $b > a$. Q_1 and Q_2 are respectively the total output of product 1 and 2 in country A. P_1 and P_2 are corresponding prices.

The cost functions are,

$$forFirmA: \qquad c_i = F(p,i) + mq_i \tag{3}$$

$$forFirmB: \qquad c_i = F(p,i) + (m+t)q_i \tag{4}$$

where

F(p,i) is the fixed production cost of product i, i=1,2;

m and t are respectively the unit production cost and the unit transport cost, which are assumed as constants; and

F(E,i) and F(X,i) are respectively denoted as entry and exit costs of product i, i=1,2.

1) First, we check Assumption 2:

$$P \geq P(I\&II^*, I\&II) \geq 0.$$

The inverse demand functions are,

$$P_1 = d - b(q_1^A + q_1^B) - a(q_2^A + q_2^B)$$

$$P_2 = d - a(q_1^A + q_1^B) - b(q_2^A + q_2^B)$$

where q_i^A is the output level of product i produced by Firm A, and q_i^B by Firm B, i=1,2. The firms' profit functions are,

$$FirmA: \qquad P(I\&II, I\&II^*) = (P_1 - m)q_1^A + (P_2 - m)q_2^A - F(p,1) - F(p,2),$$

$$FirmB: \qquad P(I\&II^*, I\&II) = (P_1 - m - t)q_1^B + (P_2 - m - t)q_2^B - F(p,1) - F(p,2).$$

The first-order conditions are ,

$$\frac{\partial P_A}{\partial q_1^A} = 0, \quad \frac{\partial P_A}{\partial q_2^A} = 0; \quad \frac{\partial P_B}{\partial q_1^B} = 0, \quad \frac{\partial P_B}{\partial q_2^B} = 0.$$

that is,

$$2bq_1^A + 2aq_2^A + bq_1^B + aq_2^B = d - m \tag{5}$$

$$2aq_1^A + 2bq_2^A + aq_1^B + bq_2^B = d - m \tag{6}$$

$$bq_1^A + aq_2^A + 2bq_1^B + 2aq_2^B = d - m - t \tag{7}$$

$$aq_1^A + bq_2^A + 2bq_1^B + 2aq_2^B = d - m - t \tag{8}$$

In eqs. (5)-(8), we assume $d > m + t$, that is, the marginal cost (including transport cost) is lower than the demand.

Solving these simultaneous equastions, we obtain the equilibrium output levels,

$$q_1^A = q_2^A = \frac{d - m + t}{3(b + a)} \tag{9}$$

$$q_1^B = q_2^B = \frac{d - m - 2t}{3(b + a)} \tag{10}$$

It can be seen from (9) and (10) that as the transport cost rises, the domestic firm, Firm A, increses its output level whereas the foreign firm, Firm B, reduces its output level. Furthermore, Firm A produces more outputs in equilibrium than Firm B. Substituting the equilibrium output levels into $P(I\&II^*, I\&II)$, we have

$$P(I\&II^*, I\&II) = \frac{2(d - m - 2t)^2}{9(b + a)} - F(p, 1) - F(p, 2) \tag{11}$$

So Assumption 2 imposes an upper bound on the fixed production costs. A.2 is more likely to hold as transport costs are small, as demand is high, and as the fixed production costs are low.

2) Check Assumption 8:

$$P(I, II^*) - F(X, 2) > P(I\&II, II^*) \qquad \text{for Firm A,}$$

$$P(I^*, II) - F(X, 2) > P(I\&II^*, II) \qquad \text{for Firm B.}$$

Using the similar procedure in 1), we obtain,

$$P(I\&II, II^*) = \frac{(d-m)^2(13b-5a)}{36b(b+a)} + \frac{2(d-m)t}{9b} + \frac{t^2}{9b} - F(p,1) - F(p,2)$$

$$P(I, II^*) = \frac{(d-m)^2 b}{(2b+a)^2} + \frac{2ab(d-m)t}{(2b+a)^2(2b-a)} + \frac{a^2bt^2}{(2b+a)^2(2b-a)^2} - F(p,1)$$

Thus, for Firm A,

$$P(I\&II, II^*) - P(I, II^*) + F(X,2) = \frac{(d-m)^2(b-a)(16b^2+12ab+5a^2)}{(2b+a)^2 36b(b+a)}$$

$$+\frac{(b+a)(b-a)(4b+a)(4b-a)}{9b(2b+a)^2(2b-a)^2}t^2 + \frac{(b-a)(8b^2+3ab+a^2)(d-m)t}{(2b+a)^2(2b-a)} - F(p,2) + F(X,2) \qquad (12)$$

must be negative. There is only one item $(-F(p,2))$ in (12) which is possible to be negative. Therefore, when F(p,2)=0, A.8 cannot hold and the domestic firm will stay in product 2 even if facing competition from the foreign entrant. For $F(p,2) > 0$, A.8 may hold. The following conditions are suitable for A.8: (1) smaller transport costs; (2) smaller $(b-a)$, i.e., the products are better substitutes; (3) smaller F(X,2), i.e., exit costs are small; and (4) larger F(p,2), i.e, the fixed production costs are larger.

It is noted that smaller F(p,2) makes A.2 more likely hold while larger F(p,2) makes A.8 more likely hold. More specificly,

from A.2: $\quad F(p,2) \leq V_1(t) - F(p,1),$

from A.8: $\quad F(p,2) > V_2(t) + F(X,2),$

where

$$V_1(t) \equiv \frac{2(d-m-2t)^2}{9(a+b)},$$

$$V_2(t) \equiv \frac{(d-m)^2(b-a)(16b^2+12ab+5a^2)}{(2b+a)^2 36b(b+a)} + \frac{(b+a)(b-a)(4b+a)(4b-a)t^2}{9b(2b+a)^2(2b-a)^2}$$
$$+ \frac{(b-a)(8b^2+3ab+a^2)(d-m)t}{(2b+a)^2(2b-a)}.$$

Thus, F(p,2) must satisfy the following condition,

$$V_2(t) + F(X,1) < F(p,2) + F(p,1) \leq V_1(t) \tag{13}$$

in order for A.2 and A.8 to be consistent with each other. Similarly, from A.2 and A.9, we have,

$$V_2(t) + F(X,2) < F(p,2) + F(p,1) \leq V_1(t) \tag{14}$$

Because at $t = 0$,

$$\frac{V_1(t)}{V_2(t)} = \frac{8b(2b+a)^2}{(b-a)(16b^2+12ab+5a^2)} > 1 \qquad \text{for any} \quad b > a > 0,$$

Further, because $V_1(t)/V_2(t)$ is a continuous function of t, $\Rightarrow \exists t_0 > 0 \quad \ni \quad \forall t \in [0, t_0), \quad \Rightarrow V_1(t)/V_2(t) > 1$. Because $V_2(t) > 0$, $\Rightarrow V_1(t) > V_2(t)$. Therefore, when t, F(X,1) and F(X,2) are sufficiently small, there exist appropriate F(p,1) and F(p,2) such that conditions (13) and (14) hold simultaneously. The similar analysis can be done for Firm B.

We have so far checked for A.2, A.8 and A.9, and the consistency among them . Next we examine the other assumptions. In what follows, we shall assume that the transport costs are zero. As we have seen, an infinitesimal increase in t will preserve the results obtained under $t = 0$.

3) Check Assumption 4 :

$$P(I\&II, I^*) > P(I, I^*) \geq P(I, I\&II^*)$$

Tedious calculations show

$$P(I\&II, I^*) = \frac{(d-m)^2(13b-5a)}{36b(b+a)} + \frac{t^2}{9b} + \frac{2(d-m)t}{9b} - F(p,1) - F(p,2)$$
$$P(I, I^*) = \frac{(d-m+t)^2}{9b} - F(p,1)$$

Therefore, at t=0,

$$P(I\&II, I^*) - P(I, I^*) = \frac{(d-m)^2(b-a)}{4b(a+b)} - F(p,2)$$

Hence, A.4, $P(I\&II, I^*) - P(I, I^*) > 0$, sets an upper bound on F(p,2). Recall that A.8 sets a lower bound on F(p,2). Thus, if both A.4 and A.8 hold, an appropriate F(p,2) must be found to satisfy

$$\frac{(d-m)^2(b-a)(16b^2+12ab+5a^2)}{36(2b+a)^2(b+a)b} + F(X,2) < F(p,2) < \frac{(d-m)^2(b-a)}{4b(a+b)} \tag{15}$$

It can be shown that

$$\frac{(d-m)^2(b-a)(16b^2+12ab+5a^2)}{36(2b+a)^2(b+a)b} < \frac{(d-m)^2(b-a)}{4b(a+b)}$$

Therefore, for small F(X,2), such an F(p,2) exists. The other part of A.4, $P(I, I^*) \geq P(I, I\&II^*)$ can be similarly checked.

4) Check A.3, A.6, A.7, and A.10

A.3 imposes a lower bound on entry costs, whereas A.6, A.7 and A.10 impose upper bounds. More specificly, for product 1, we have

$$P(I, I\&II^*),\ P(I, I^*) < F(E,1) < P(I\&II, N^*) - P(II, N^*),\ P(I, II^*). \tag{16}$$

$P(I, II^*)$ obviously exceeds both $P(I, I^*)$ and $P(I, I\&II^*)$. Here we only show

$$P(I\&II, N^*) - P(II, N^*) > P(I, I^*).$$

Because

$$P(I\&II, N^*) = \frac{(d-m)^2}{2(a+b)} - F(p,1) - F(p,2),$$
$$P(II, N^*) = \frac{(d-m)^2}{4b} - F(p,2),$$
$$P(I, I^*)\big|_{t=0} = \frac{(d-m)^2}{9b} - F(p,1),$$

$$\Longrightarrow P(I\&II, N^*) - P(II, N^*) - P(I, I^*) = \frac{(d-m)^2(5b-13a)}{36b(a+b)} > 0 \qquad \text{if} \quad 5b > 13a,$$

a condition consistent with other assumptions.

5) In the foregoing analysis, the two products are assumed to be equally profitable, but an infinitesimal decrease in the marginal cost of product 1 will yield Assumption 5 without affecting any of the other assumptions. Finally, Assumption 1 can be independently made without affecting the other assumptions. Thus, our exercises in this Appendix show that Assumptions 1–10 are consistent with each other in Cournot competition with linear demand.

Appendix 10

Assumptions 1-10 Are Consistent With Each Other in a Bertrand Duopoly Model

In Appendix 1, the inverse demand functions are assumed to be linear,

$$P_1 = d - bQ_1 - aQ_2$$

$$P_2 = d - aQ_1 - bQ_2$$

where $d > 0, b > a > 0$.

The corresponding demand functions can be derived as,

$$Q_1 = \alpha - \beta P_1 + \gamma P_2$$

$$Q_2 = \alpha + \gamma P_1 - \beta P_2$$

where $\alpha = \frac{d}{b+a} > 0, \beta = \frac{b}{(b+a)(b-a)} > 0$, $\gamma = \frac{a}{(b+a)(b-a)} > 0$, and $\beta > \gamma$.

1) Note first that the neccessary condition for Assumption 2, $P(I\&II^*, I\&II) \geq 0$, is that the fixed production costs must be zero; otherwise, by setting the price below the marginal cost of the foreign firm (m+t), the domestic firm can make the foreign firm suffer losses. In price competition, we thus assume that the fixed production costs are zero.

2) We now show that $P(I, II^*) > P(I\&II, II^*) > 0$ in price competition.

i) In the case where the domesitc firm, Firm A, produces both products and the foreign firm, Firm B, produces one of the products, say, product 2. The firms' porfit functions are,

$$forFirmA: \qquad P_A(I\&II, II^*) = (P_1^A - m)q_1^A + (P_2^A - m)q_2^A$$

$$for firm B: \qquad P_B(II^*, I\&II) = \big(P_2^B - (m+t)\big)q_2^B$$

Suppose that Firm A chooses $P_2^A = (m+t) - \epsilon, 0 < \epsilon < t$, so as to force Firm B out of the market . Thus, $q_2^B = 0$. Then, at given $P_2^A = (m+t) - \epsilon$, Firm A chooses P_1^A to maximize its profit,

$$\frac{\partial P_A}{\partial P_1^A} = 0: \qquad P_1^A = \frac{\alpha + \beta m + \gamma m + 2\gamma(t-\epsilon)}{2\beta}$$

and the maximal profit is,

$$P(I\&II, II^*) = \frac{(\alpha - \beta m + \gamma m)^2 + 4(\beta+\gamma)(t-\epsilon)\big(\alpha - (\beta-\gamma)(m+t-\epsilon)\big)}{4\beta}.$$

As $t \to 0, \epsilon \to 0$ since $0 < \epsilon < t$, hence $(t-\epsilon) \to 0$ and

$$P(I\&II, II^*) \longrightarrow \frac{(\alpha - \beta m + \gamma m)^2}{4\beta} > 0.$$

Therefore, for small transport costs, t, $P(I\&II, II^*) > 0$.

ii) In the case where Firm A produces product 1 and Firm B product 2, the firms' profit functions are,

$$for Firm A: \qquad P_A(I, II^*) = (P_1^A - m)q_1^A$$

$$for Firm B: \qquad P_B(II^*, I) = \big(P_2^B - (m+t)\big)q_2^B$$

and demand functions are,

$$q_1^A = \alpha - \beta P_1^A + \gamma P_2^B$$

$$q_2^B = \alpha + \gamma P_1^A - \beta P_2^B$$

Firm A and B choose, respectively, P_1^A and P_2^B to maximize their profits. The first-order conditions are,

$$\frac{dP_A}{dP_1^A} = 0: \qquad 2\beta P_1^A - \gamma P_2^B = \alpha + \beta m$$

$$\frac{dP_B}{dP_2^B} = 0: \qquad -\gamma P_1^A + 2\beta P_2^B = \alpha + \beta(m+t)$$

Solve for the equilibrium prices,

$$P_1^A = \frac{(\alpha + \beta m)(2\beta + \gamma) + \gamma\beta t}{(2\beta + \gamma)(2\beta - \gamma)} \tag{1}$$

$$P_2^B = \frac{(\alpha + \beta m)(2\beta + \gamma) + 2\beta^2 t}{(2\beta + \gamma)(2\beta - \gamma)} \tag{2}$$

Thus, the equilibrium prices P_2^B and P_1^A will rise as the transport cost rises, and $P_2^B > P_1^A$. Further,

$$P_A(I, II^*) = \frac{\beta(\alpha - \beta m + \gamma m)^2}{(2\beta - \gamma)^2} + \frac{\beta^2(3\gamma - 2\beta)(\alpha - \beta m + \gamma m)t}{(2\beta + \gamma)(2\beta - \gamma)^2} - \frac{2\beta^3\gamma(\beta - \gamma)t^2}{(2\beta + \gamma)^2(2\beta - \gamma)^2}$$

as $t \to 0$,

$$P_A(I, II^*) \longrightarrow \frac{\beta(\alpha - \beta m + \gamma m)^2}{(2\beta - \gamma)^2} > 0$$

Therefore, as $t \to 0$,

$$\begin{aligned} P_A(I, II^*) - P_A(I\&II, II^*) &\longrightarrow \frac{\beta(\alpha - \beta m + \gamma m)^2}{(2\beta - \gamma)^2} - \frac{(\alpha - \beta m + \gamma m)^2}{4\beta} \\ &= \frac{(\alpha - \beta m + \gamma m)^2}{4\beta(2\beta - \gamma)^2}\gamma(4\beta - \gamma) > 0. \end{aligned}$$

When the transport cost , t, is sufficently small, $P(I, II^*) > P(I\&II, II^*) > 0$. The crucial Assumption 8 therefore holds for small exit cost F(X,2), as does Assumption 9.

3) In the absence of transport costs, if both firms produce a common product, its price will be driven to marginal cost in price competition. This implies that

$$forFirmA: \qquad P(I, I^*) = P(I, I\&II^*) = P(II, II^*) = P(II, I\&II^*) = 0$$

$$forFirmB: \qquad P(I^*, I) = P(I^*, I\&II) = P(II^*, II) = P(II^*, I\&II) = 0$$

if fixed production costs are zero. Thus, for a sufficiently small transport cost, these profits will be close to zero. This property assures other assumptions hold in price competition.

4) Finally, in the foregoing analysis, the two products are assumed to be equally profitable, but an infinitesimal decrese in the marginal cost of product 1 will yield Assumption 5 without affecting any of the other assumptions. Assumption 1 can always be independently made. Therefore, Assumptions 1–10 are consistent with each other in Bertrand competition with linear demand.

Appendix 11

The Two Contries Can Be Seperated

Suppose that there are two countries, A and B. Each contry has only one firm. The firm in country A is called Firm A (FA), and the firm in B is called Firm B (FB). The inverse demand functions are assumed to be linear,

$$P_1 = d - bQ_1 - aQ_2$$

$$P_2 = d - aQ_1 - bQ_2$$

where Q_1 and Q_2 are respectively the total output of product 1 and 2 in a country , P_1 and P_2 are the corresponding prices. Under Assumption 11, each firm perceives the distinct country-specific demand curve, and therefore the demand system is corresponding to only one country, i.e., the prices in one country depend only on the quantities produced for that country. Moerover, the two countries have the same structure of demand functions.

The cost functions are the same as those given in Appendix 1. The firms' profit functions are,

for Firm A:

$$\begin{aligned} R_{FA}^{AB} &= R_{FA}^{A} + R_{FA}^{B} \\ &= \left(q_{1,FA}^{A}(P_1^A - m) + q_{2,FA}^{A}(P_2^A - m) - F(P,1) - F(P,2)\right) \\ &+ \left(q_{1,FA}^{B}(P_1^B - m - t) + q_{2,FA}^{B}(P_2^B - m - t) - F(P,1) - F(P,2)\right), \end{aligned}$$

for Firm B:

$$\begin{aligned} R_{FB}^{AB} &= R_{FB}^{A} + R_{FB}^{B} \\ &= \left(q_{1,FB}^{B}(P_1^B - m) + q_{2,FB}^{B}(P_2^B - m) - F(P,1) - F(P,2)\right) \\ &+ \left(q_{1,FB}^{A}(P_1^A - m - t) + q_{2,FB}^{A}(P_2^A - m - t) - F(P,1) - F(P,2)\right) \end{aligned}$$

where R^A_{FA} represents the profit of Firm A in contry A, $q^A_{1,FA}$ is the output level of product 1 produced for contry A by Firm A, and the other notations can be similarly interpreted. Note that if Firm A does not produce product 1 for country A in the market equilibrium, $q^A_{1,FA}$ is set to be zero; otherwise, it is chosen optimally by Firm A. Since there are eight quantity variables, there are 256 possibilities of the final market structures in our two-country world. Because both the countries and the firms come into the model of section 3 with perfect symmetry, it is neccessary that the equilibria, if they exist, of the four-stage game of section 3 are symmetric with respect to both firms and countries. We try to show that in the symmetric market equilibria, the two countries can be separated if the marginal costs are constant.

We illustrate this by using one of the symmetric cases in which both firms produce both products for both countries. Since the firms are assumed to be able to choose separately their output levels for each country, eight first-order conditions are ensued ,

for Firm A:

$$\frac{\partial R^{AB}_{FA}}{\partial q^A_{1,FA}} = 0, \quad \frac{\partial R^{AB}_{FA}}{\partial q^A_{2,FA}} = 0, \quad \frac{\partial R^{AB}_{FA}}{\partial q^B_{1,FA}} = 0, \quad \frac{\partial R^{AB}_{FA}}{\partial q^B_{2,FA}} = 0. \tag{1}$$

for Firm B:

$$\frac{\partial R^{AB}_{FB}}{\partial q^B_{1,FB}} = 0, \quad \frac{\partial R^{AB}_{FB}}{\partial q^B_{2,FB}} = 0, \quad \frac{\partial R^{AB}_{FB}}{\partial q^A_{1,FB}} = 0, \quad \frac{\partial R^{AB}_{FB}}{\partial q^A_{2,FB}} = 0. \tag{2}$$

Look at

$$\frac{\partial R^{AB}_{FA}}{\partial q^A_{1,FA}} = \frac{\partial R^A_{FA}}{\partial q^A_{1,FA}} + \frac{\partial R^B_{FA}}{\partial q^A_{1,FA}}$$

Because the four quantity variables in country B are independent of $q^A_{1,FA}$ and (m+t) are assumed as constants,

$$\frac{\partial R^B_{FA}}{\partial q^A_{1,FA}} = 0$$

Thus

$$\frac{\partial R^{AB}_{FA}}{\partial q^A_{1,FA}} = \frac{\partial R^A_{FA}}{\partial q^A_{1,FA}}$$

Therefore, conditions (1) and (2) are equivalent to the following conditions (3) and (4),

for Firm A:

$$\frac{\partial R^A_{FA}}{\partial q^A_{1,FA}} = 0, \quad \frac{\partial R^A_{FA}}{\partial q^A_{2,FA}} = 0, \quad \frac{\partial R^B_{FA}}{\partial q^B_{1,FA}} = 0, \quad \frac{\partial R^B_{FA}}{\partial q^B_{2,FA}} = 0. \tag{3}$$

for Firm B:

$$\frac{\partial R^B_{FB}}{\partial q^B_{1,FB}} = 0, \quad \frac{\partial R^B_{FB}}{\partial q^B_{2,FB}} = 0, \quad \frac{\partial R^A_{FB}}{\partial q^A_{1,FB}} = 0, \quad \frac{\partial R^A_{FB}}{\partial q^A_{2,FB}} = 0. \tag{4}$$

That is, the firms' maximizing overall profit is equivalent to the firms's separately maximizing profit of each product in each country. In particular, there are four first-order conditions in each country, each firm taking the other firm's output levels to each country as given. In country A:

$$P^A_1 - m + q^A_{1,FA}(-b) + q^A_{2,FA}(-a) = 0 \tag{5}$$

$$P^A_2 - m + q^A_{2,FA}(-b) + q^A_{1,FA}(-a) = 0 \tag{6}$$

$$P^A_1 - m - t + q^A_{1,FB}(-b) + q^A_{2,FB}(-a) = 0 \tag{7}$$

$$P^A_2 - m - t + q^A_{2,FB}(-b) + q^A_{1,FB}(-a) = 0 \tag{8}$$

where

$$P^A_1 = d - b(q^A_{1,FA} + q^A_{1,FB}) - a(q^A_{2,FA} + q^A_{2,FB}) \tag{9}$$

$$P^A_2 = d - a(q^A_{1,FA} + q^A_{1,FB}) - b(q^A_{2,FA} + q^A_{2,FB}) \tag{10}$$

The conditions (5)-(8) contain four unknowns which are the four quantity variables in country A. These four unknowns can be determined by eqs. (5)-(8). Similarly, we can write dowm the four conditions in country B which contains the other four unknowns, and we can solve those four quantity variables in country B

without refering to eqs.(5)-(8). Therefore, conditions (3) and (4) in eight unknowns can be partitioned into two separable sets based on different countries. Moreover, the two sets are perfect symmetric. In other words, the two countries can be separated and only one country need considering.

Appendix 12

Assumption 12 and Assumptions 1-10 Are Consistent With One Another in a Cournot Duopoly Model

In Appendix 1, we have demonstrated that Assumptions 1–10 are consistent with each other in a Cournot duopoly model with linear demand. In this Appendix, we show that Assumption 12 made in section 3 is also consistent with the Cournot model.

$$A.12: \qquad P(I\&II, N^*) > P(I, II^*) + P(II^*, I).$$

According to the results obtained in Appendix 1, as $t \to 0$,

$$\begin{aligned}
P(I\&II, N^*) - P(I, II^*) - P(II^*, I) &\longrightarrow \left(\frac{(d-m)^2}{2(a+b)} - F(p,1) - F(p,2)\right) \\
&- \left(\frac{b(d-m)^2}{(2b+a)^2} + F(p,1)\right) - \left(\frac{b(d-m)^2}{(2b+a)^2} - F(p,2)\right) \\
&= (d-m)^2\left(\frac{1}{2(a+b)} - \frac{2b}{(2b+a)^2}\right) \\
&= (d-m)^2 \frac{a^2}{2(a+b)(2b+a)^2} \\
&> 0.
\end{aligned}$$

Therefore, for small transport costs, t, A.12 holds.

Appendix 13

Assumption 12 and Assumptions 1-10 Are Consistent With One Another in a Bertrand Duopoly Model

In Appendix 2, we have demonstrated that Assumptions 1–10 are consistent with one another in a Bertrand duopoly model with linear demand. Here we show that Assumption 12 made in section 3 is also consistent with the Bertrand model.

$$A.12: \qquad P(I\&II, N^*) > P(I, II^*) + P(II^*, I).$$

According to the results obtained in Appendix 2, as $t \to 0$,

$$\begin{aligned} P(I\&II, N^*) - P(I, II^*) - P(II^*, I) \longrightarrow & \frac{(\alpha - \beta m + \gamma m)^2}{2(\beta - \gamma)} \\ & - \frac{(\alpha - \beta m + \gamma m)^2 \beta}{(2\beta - \gamma)^2} - \frac{(\alpha - \beta m + \gamma m)^2 \beta}{(2\beta - \gamma)^2} \\ & = (\alpha - \beta m + \gamma m)^2 \frac{\gamma^2}{2(\beta - \gamma)(2\beta - \gamma)} \\ & > 0. \end{aligned}$$

Therefore, for small transport costs, t, A.12 holds.

Appendix 14

Result 3 and Result 4 Hold in the Cournot Duopoly Model

1. Result 3 holds

In Appendix 1, we have demonstrated that Assumptions 1–10 are consistent with a Cournot model with linear demand. In Result 3 of section 4, Assumptions 1–9 hold, while A.10 fails. The opposite of A.10 is A.13,

$$A.13: \qquad P(I, N^*) > P(I\&II, N^*) - F(E, 2)$$

A.13 puts a lower bound on F(E,2). Because only A.3 and A.7 among Assumptions 1-9 concern the level of costs of entry into product 2, we need only examining the consistency of A.13, A.3, and A.7. The other assumptions won't be affected by A.13.

First, since A.7 sets an upper bound on F(E,2), A.13 and A.7 must be consistent with each other, that is,

$$P(I\&II, N^*) - P(I, N^*) < F(E, 2) < P(II, I^*) \tag{1}$$

Thus the following condition must hold,

$$P(I\&II, N^*) - P(I, N^*) < P(II, I^*) \tag{2}$$

which is condition (1) in section 4.

Based on the results in Appendix 1, as $t \to 0$,

$$
\begin{aligned}
P(II, I^*) - \big(P(I\&II, N^*) - P(I, N^*)\big) & \\
& \longrightarrow \left(\frac{(d-m)^2 b}{(2b+a)^2} - F(p,1)\right) - \left(\frac{(d-m)^2(b-a)}{4b(b+a)} - F(p,1)\right) \\
& = (d-m)^2 \frac{4ab^2 + 3a^2 b + a^3}{(2b+a)^2 4b(a+b)} \\
& > 0
\end{aligned}
$$

Thus , for small transport costs, t, condition (2) holds and the entry cost F(E,2) can be chosen to satisfy (1).

Second, as A.3 also puts a lower bnound on F(E,2), it won't be affected by A.13. This shows that Assumptions made in Result 3 are consistent with the Cournot duopoly model with linear demand.

2. Result 4 holds

In Result 4, besides A.10, both A.8 and A.9 fail. If the failure of A.8 and A.9 can be assumed to be due to high exit costs, it won't affect other assumptions. The new assumption made in Result 4 is thus Assumption 15 or its opposite. In the first part of Result 4, A.15 holds,

$$A.15: \qquad P(I\&II, N^*) - F(E,2) > P(I, II^*).$$

So A.15 imposes an upper bound on F(E,2), and it will affect both A.3 and A.13 which impose lower bounds on F(E,2).

First, we examine the consistency between A.15 and A.3, i.e.,

$$P(II, II^*) < F(E,2) < P(I\&II, N^*) - P(I, II^*) \tag{3}$$

Because as $t \to 0$,

$$
\begin{aligned}
& P(II, II^*) - P(I\&II, N^*) + P(I, I^*) \\
& \longrightarrow \left(\frac{(d-m)^2}{9b} - F(p,2)\right) - \left(\frac{(d-m)^2}{2(a+b)} - F(p,1) - F(p,2)\right) + \left(\frac{(d-m)^2 b}{(2b+a)^2} - F(p,1)\right) \\
& = (d-m)^2 \frac{2a^3 + a^2 b - 10b^3 - 2ab^2}{9b(2b+a)2(a+b)} \\
& < (d-m)^2 \frac{2b^3 + b^3 - 10b^3 - 2ab^2}{9b(2b+a)2(a+b)} \qquad (a < b) \\
& < 0.
\end{aligned}
$$

Therefore, for small transport costs, t, F(E,2) can be chosen to satisfy condition (3).

Next, we examine the consistency between A.15 and A.13,

$$P(I\&II, N^*) - P(I, N^*) < F(E,2) < P(I\&II, N^*) - P(I, II^*) \tag{4}$$

Because as $t \to 0$,

$$P(I, II^*) - P(I, N^*) \longrightarrow \frac{(d-m)^2(-4ab - a^2)}{4b(a+2b)^2} < 0$$

So

$$P(I\&II, N^*) - P(I, N^*) < P(I\&II, N^*) - P(I, II^*)$$

Hence, for small transport costs, t, F(E,2) can be chosen to satisfy condition (4).

Therefore,the first part of Result 4 in which A.15 holds can be checked to be consistent with the Cournot model.

It is noted that the second part of Result 4 in which A.15 fails cannot be consistent with our Cournot model. This is because when A.15 fails, it imposes a lower bound on F(E,2), whereas A.7 puts an upper

bound on F(E,2), and the two must be consistent with each other, i.e.,

$$P(I\&II, N^*)_P(I, II^*) < F(E,2) < P(II, I^*) \tag{5}$$

But as $t \to 0$,

$$P(I\&II, N^*) - P(I, II^*) - P(II, I^*) \longrightarrow \frac{(d-m)^2 a^2}{2(a+b)(2b+a)^2} > 0.$$

Therefore, for small transport costs, t, no positive F(E,2) can be found to satisfy condition (5).

Appendix 15

Result 3 and Result 4 Hold in the Bertrand Duopoly Model

1. Result 3 holds

As have been analysed in Appendix 6, we only need examining whether the following inequality holds for small transport costs,

$$P(I\&II, N^*) - P(I, N^*) < P(II, I^*) \tag{1}$$

Since the profit of an monopolist will be the same in cases where either the quantity or the price as the decision variable, from Appendix 1,

$$P(I\&II, N^*) - P(I, N^*) = \frac{(d-m)^2}{2(a+b)} - \frac{(d-m)^2}{4b}.$$

From Appendix 2, when t=0,

$$P(II, I^*) = \frac{(\alpha - \beta m + \gamma m)^2 \beta}{(2\beta - \gamma)^2} \tag{2}$$

Also from Appendix 2,

$$\alpha = \frac{d}{b+a}, \quad \beta = \frac{b}{(b+a)(b-a)}, \quad \gamma = \frac{a}{(b+a)(b-a)}.$$

Substituting into (2), and (2) becomes

$$P(II, I^*) = \frac{b(b-a)(d-m)^2}{(2b-a)^2(b+a)} \tag{3}$$

Hence as $t \to 0$,

$$\begin{aligned} &\big(P(I\&II, N^*) - P(I, N^*)\big) - P(II, I^*) \\ &\longrightarrow \frac{(d-m)^2 a(b-a)(-4b+a)}{4b(b+a)(2b-a)^2} < 0 \qquad (b > a) \end{aligned}$$

that is, (1) holds for small t. Result 3, therefore, holds for a certain range of parameters in the Bertrand Model.

2. Result 4 holds

As has been shown in Appendix 6, when A.15 holds, we only need examining the consistency between A.15 and A.3, and the consistency between A.15 and A.13.

i) A.15 and A.3:

$$P(II, II^*) < F(E, 2) < P(I\&II, N^*) - P(I, II^*) \tag{4}$$

As $t \to 0$, $P(II, II^*) \to 0$ and

$$\begin{aligned} P(I\&II, N^*) - P(I, II^*) &\longrightarrow \frac{(\alpha - \beta m + \gamma m)^2}{2(\beta - \gamma)} - \frac{\beta(\alpha - \beta m + \gamma m)^2}{2(\beta - \gamma)^2} \\ &= \frac{(\alpha - \beta m + \gamma m)^2(2\beta^2 - 2\beta\gamma + \gamma^2)}{2(\beta - \gamma)(2\beta - \gamma)^2} \\ &> 0 \qquad (\beta > \gamma) \end{aligned}$$

Therefore, for small t, F(E,2) can be chosen to satisfy (4).

ii) A.15 and A.13:

$$P(I\&II, N^*) - P(I, N^*) < F(E,2) < P(I\&II, N^*) - P(I, II^*) \tag{5}$$

As $t \to 0$,

$$\begin{aligned}
&\big(P(I\&II, N^*) - P(I, II^*)\big) - \big(P(I\&II, N^*) - P(I, N^*)\big) \\
&\qquad = -P(I, II^*) + P(I, N^*) \\
&\qquad \longrightarrow -\frac{(d-m)^2 b(b-a)}{(2b-a)^2(b+a)} + \frac{(d-m)^2}{4b} \\
&\qquad = \frac{(d-m)^2\big(4ab(b-a) + a^2 b + a^3\big)}{(2b-a)^2(b+a)4b} \\
&\qquad > 0 \qquad (b > a)
\end{aligned}$$

for small t, F(E,2) can be chosen to satisfy (5). Therefore, the first part of Result 4 holds for a certain range of parameters in the Bertrand model.

It is noted that the second part of Result 4, in which A.15 does not hold, can not be consistent with our Bertrand model. This is because

$$P(I\&II, N^*) - P(I, II^*) > P(II, I^*)$$

holds for small t. Consequently, no positive F(E,2) can be found to simultaneously satisfy both A.7 and the opposite of A.15 for small transport costs t:

$$P(I\&II, N^*) - P(I, II^*) < F(E,2) < P(II, I^*)$$