PRICE-SENSITIVE INEQUALITY MEASUREMENT

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Abstract

The existing inequality indexes in the economics literature (including the more sophisticated indexes of Muellbauer (1974) and Jorgenson-Slesnick (1984)), are found to be insensitive to relative price changes or are unjustifiable in terms of social evaluation ethics or both. The present research fills this gap in the literature by proposing a new index, named the Individual Equivalent Income (IEI) index.

A household indirect utility function is hypothesized which incorporates certain attribute parameters in the form of equivalence scales. These attributes are demographic and environmental characteristics specific to a given household. This indirect utility function gives a number which represents the utility of each member of the household. A particular level of interpersonal comparison of utilities is assumed which gives rise to an exact individual utility indicator named equivalent income. A distribution of these equivalent incomes forms the basis of a price-sensitive relative inequality index.

This index can be implemented in the Canadian context. Preferences are assumed to be nonhomothetic translog and demand data are derived from cross-section surveys and time-series aggregates.

Based on demand data, the translog equivalent income function can be estimated and equivalent incomes imputed to all individuals in society. An Atkinson index of equivalent incomes is then computed to indicate the actual degree of inequality in Canada.

The new IEI index is compared with other indexes based on a common data set. The main findings are: conventional indexes give bad estimates of the true extent of inequality and the IEI index, while providing a more accurate estimate, indicates distributive price impact in a predictable manner, i.e., food price inflation aggravates while transportation price inflation ameliorates the inequality problem.

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CHAPTER 1 INTRODUCTION

The issue of inequality — the divergence in well-being among the individuals in a society — has traditionally been of great concern to economists. This is hardly surprising because a basic theme in economics is the allocation of society's resources and the distribution of society's wealth. Indeed, systematic study of inequality can be found as early as Cannan (1914) and Dalton (1920). Putting aside the question of what causes inequality and the more controversial issue of what ideology justifies inequality, on a practical level, the measurement of inequality is important for at least two reasons. First, the government may want to know the inequality implications of alternative policies. Second, it is interesting to compare the degree of inequality between different societies contemporaneously and in time-series for a particular society. The objective of the present research is to develop an inequality index which is a vast improvement over existing ones in that it brings into sharp focus the notion of individual welfare in the measurement of inequality. In particular, based on revealed behavioural data, the impact of price changes on individual welfare is incorporated into inequality measurement.

Echoing the idea of Dalton (1920), recent research developments (Atkinson (1970) and Blackorby and Donaldson (1978)) have re-emphasized the fact that underlying every inequality index is a set of ethics.

It is clear that inequality measurement is a normative endeavor rather than a positive one. The claim that one distribution of welfare (however defined and measured) is more unequal than another distribution is contingent on a set of ethics. It is therefore important that the particular set of ethics is made explicit.

The main task of the present research is not in disputing the particular ethics that one should choose to measure inequality. This is an ideological question. The basic line of attack is: what should be the basic entities that we use to measure individual well-being?

Let us look at a common example. The Atkinson (1970) index is, for N households (for a recent application and some ad hoc variants, see Beach, Card and Flatters (1981)),

$$(1.1) \qquad I_A := 1 - \left(\frac{1}{N}\sum_{i=1}^{N}(y_i/\mu)^r\right)^{1/r} \qquad r \leq 1, \quad r \neq 0$$

$$(1.2) \qquad := 1 - \prod_{i=1}^{N}(y_i/\mu)^{1/N} \qquad r = 0 .$$

where $(y_1, \ldots, y_N)$ is a distribution of household incomes and μ is the mean of the distribution. Justifying the inequality index (1.1) (1.2) is the mean of order r social welfare function (provided that the household incomes are restricted to be positive) which is completely ethically characterized in Blackorby and Donaldson (1982).

There are two objections to the index (1.1), (1.2) that arise from using household income as a measure of individual well-being. First, this index is insensitive to price changes while, even intuitively, a change in relative prices should have distributional impacts. For example, an increase in the price of necessities relative to luxuries affects the poor more than the rich. Such a relative price increase must aggravate the inequality situation and the index should increase to reflect this change.[1]

The second objection is, in (1.1) and (1.2), that the distribution was originally taken to be a distribution of household incomes. This is clearly inconsistent with the social welfare view of inequality where individuals are viewed as the basic entities in society, not the collective units — households. Various ad hoc modifications have been made in the literature though none of them is satisfactory (see Chapter 2 for details). The basic question "How should we adjust household income or expenditure so that an individual in a family of say, four members, can be reasonably compared in welfare terms with an individual in a family of one?" has not been adequately dealt with. This is referred to below as the problem of interpersonal comparison of utility.

The present research attempts to fill this gap in the literature. A new approach to inequality measurement is developed which deals with these problems explicitly and systematically. Since the ultimate test of this new index is in its practical usefulness, the approach has been implemented for Canada and the results are in general very appealing.

The new approach can be summarized as follows. Two lines of research are merged together, namely, welfare measurement and evaluation, and demand system estimation. In utilising both techniques, analysis is extended from the micro to the macro. As a first step, household utility is measured by means of an indirect utility function which maps prices and household expenditure to an ordinal utility number. This utility function's novel feature is that household expenditure instead of individual expenditure enters the function. The reason for this specification is that in practice individual expenditure data are not easily obtained. However, as the objective of finding a numerical utility representation is to measure inequality in the aggregate, the utility number must be capable of being interpreted as the utility of each member in the household. Whether this interpretation is acceptable or not depends on the form and the parameter estimates of the utility function. Barten equivalence scales provide one such form and they are estimated together with other parameters from demand data. Thus, by incorporating family size and other attributes into the utility function, each household is endowed with a household-specific utility function.

The ordinal nature of the utility number gives rise to further problems. Subjecting a utility function to an arbitrary individual-specific monotonic transform yields the same set of demand equations. Even if all the parameters in the utility function are accurately estimated from demand data, the utility number is still arbitrary. This problem is not serious if only the utility ranking of a single individual is concerned. But inequality measurement implies utility measurement and comparison for at least two individuals. Consequently, a numerical representation of utility is obtained by using a reference individual and assuming a particular level of interpersonal utility comparison. This representation, named the equivalent income of each member in a specified household is the total expenditure that a reference household needs at reference prices in order that each member in it is just as well off as each member in the household with specified attributes and prices. Equivalent income is a function of prices, expenditure, attributes, reference prices and reference attributes, and it is estimable empirically using demand data. Subject to the reasonableness of the parameter estimates, this approach offers a partial solution to the second problem mentioned above, and to the extent that equivalent income is sensitive to prices, it offers a solution to the first problem.

A distribution of individual equivalent incomes is then aggregated by means of a social welfare function. A mean of order r function is used which has been characterized in terms of ethical axioms in Blackorby and Donaldson (1982). Adopting the Atkinson-Kolm-Sen (AKS) procedure, an inequality index is calculated which is actually an Atkinson index of equivalent incomes.

The estimation phase also involves extension from the micro to the macro. Expenditure share equations are derived from the household indirect utility functions with the equivalence scales incorporated. Estimation is carried out in two stages: the first stage involves only micro household commodity-expenditure share equations which are regressed using cross-section household expenditure survey data. Since some parameters in the equivalent income function are not yet identified, the micro equations are summed together to obtain aggregate commodity-expenditure share equations which allow the utilization of time-series aggregate demand data to estimate the remaining parameters in the equivalent income function.

The implementation and the results of applying this new index in the Canadian context are described later. It might be helpful, nevertheless, to mention some of the main contributions of this research here.

1. Unlike a lot of other empirical demand studies, the present approach does not assume the existence of an aggregate consumer. Instead, households are specific to the extent that their characteristics are captured by attribute vectors incorporated into the utility function. The sum total of all household demands yields aggregate

demand which enables the utilization of time-series aggregate data. This is not only a theoretically exact approach, it is also empirically superior, as the estimation results show that, based on behavioural demand data, meaningful welfare information can be inferred. The equivalence scale estimates appear very reasonable.

2. The new index truly captures distributive price effects, despite the margin of error that we might suspect in this type of demand system studies. Indeed, results show that commodities commonly regarded as luxuries have an inequality-reducing price effect while the opposite is true for necessities.

3. When we compare indexes that vary from 0 to 1, the discrepancy between two indexes is expected to be small. Nevertheless, the new index turns out to be substantially different from all commonly used indexes. We may conclude that these indexes give a distorted picture of the true inequality situation.

4. Although this methodology is developed for inequality measurement, it can be applied with some modifications to other kinds of welfare analyses. The framework is quite general. For example, in cost-benefit analyses, one frequently looks for a social welfare measure as a judgment criterion when alternative states are being compared. This is easily handled within the present framework, given price and expenditure information in each state.

This dissertation is organized as follows. Chapter 2 surveys critically the common inequality indexes with special emphasis on the various measures of utility used. The works of Muellbauer (1974 a, b, c) and Jorgenson and Slesnick (1982 a, b)(1984), which are preliminary attempts to capture price effects, are explained and criticized in relation to the present study.

Chapter 3 is the core chapter. It describes the theoretical rationale of the social evaluation framework and how an inequality index is constructed in this framework.

The empirical specification of the model is presented in Chapter 4. Preferences are assumed to be non-homothetic translog (with Barten equivalence scales incorporated) from which expenditure equations (household and aggregate) are derived.

Chapter 5 explains how the estimation model of household and aggregate expenditure shares can be estimated using cross-section and time-series data sequentially.

Canadian data are used for estimation. Chapter 6 explains how publicly available data can be utilized to estimate the model set out in Chapter 5 and the estimated Barten Equivalence Scales are presented and interpreted. The results are in general very appealing, lending further support to the credibility of the new index.

Applications of the estimates to inequality measurement are presented in Chapter 7. Firstly, various individual welfare measures are used to calculate inequality using the same data set and the same formula for the inequality index. It turns out that the new index gives significantly different answers from other commonly used indexes. Secondly, to demonstrate quantitatively the distributive impacts of relative price changes, inequality is calculated using the new index under hypothetical price increases and the results conform well with intuition. Finally, inequality in Canada in 1975, 1979 and 1981 is estimated to reflect on the inequality trend in the last decade. Chapter 8 concludes the dissertation.

Conclusion

There is no satisfactory price-sensitive inequality index in the literature and the need for filling this gap is evidently urgent. Since preferences must be involved in the evaluation process, a logical way to proceed is to estimate hypothesized preferences from behavioural demand data. Various problems arise, however. There being no objective measure of welfare, no data on individual expenditure, no a priori dominating rule of interpersonal comparison are just some of the problems to which the present research has offered solutions.

A new price-sensitive inequality index is successfully constructed. Implementation results show that the approach is practical and reasonable. It is significantly superior to the other indexes in both its theoretical foundation and empirical usefulness.

Chapter 1 Footnote

1. Jorgenson and Slesnick (1984) have attempted to construct a price-sensitive index, but for reasons that will be made clear in Chapter 2, their approach is not completely satisfactory.

CHAPTER 2 SURVEY OF THE LITERATURE

From a general perspective, inequality measurement is a statistical exercise that is not confined within the realm of welfare economics. Given a distribution of numbers (could be incomes, wealths, or size of firms) a statistician typically applies an inequality formula to map this distribution to an index number. Typical examples of such indexes are the Gini coefficient, the coefficient of variation and the Atkinson index. Per se, the index number does not have any significance besides reflecting certain mathematical characteristics of the distribution.

The present thesis, on the other hand, are mainly concerned with inequality in the distribution of welfare among individuals in society. In general, three considerations are central in any approach to economic inequality measurement. Firstly, what should be the basic entity that reflects individual well-being and how is it obtained empirically? Secondly, since inequality measurement rests on a foundation of social welfare evaluation, what framework should one adopt to summarize the distribution to obtain a social welfare measure? Thirdly, what social welfare function (characterized by a set of ethical axioms) should be used to aggregate the distribution and what inequality index (relative, absolute or others) should be employed?

Interestingly, the methods in the literature do not follow this logical procedure. Section 1 below describes the common entities used. Not only are they inappropriate as measures of individual welfare, they are also price-insensitive, which explains why the traditional indexes are all incapable of indicating distributive price effects. Section 2 cites some evidence of distributive price effects and describes the essence of the Muellbauer (1974) method and the Jorgenson-Slesnick (1984) method which are unsatisfactory attempts to capture these price effects.

Section 1 Some simple indexes

The most commonly used entity in the measurement of economic inequality is household income. The reason for its widespread utilization is probably that income data are easily available. Household incomes are easily extracted from tax returns. Besides, relatively speaking, they are quite reliable in accuracy terms. However, household income as a measure of individual utility is subject to a number of serious objections.

(1) Individual utility, in relation to household income, depends very much on household size and to a lesser extent on household composition, i.e., the number of male and female adults, male and female children in the household. Using household income as a measure of individual utility practically means regardless of

household size, household income indicates the ranking between any two households in welfare terms.

(2) Consumers derive utility from consumption rather than income receipts. While income stream may be uneven over time, consumers tend to smooth out consumption by saving and dissaving. Therefore, utility variations come closer to consumption variations than income variations. Furthermore, as obtained from a cross-section sample, income values are often negative (particularly for old consumers) arising from capital losses. These negative numbers create difficulties when aggregate social welfare is computed from individual incomes.

In view of the second objection, the first inequality measure to be computed in Chapter 7 for comparative purposes is the household expenditure index (HEI). To each household is imputed its total expenditure and inequality is calculated based on the distribution of household expenditures. Because of the first objection, HEI is not justifiable in terms of normal social ethics, but it is worthwhile to check if in practice HEI differs significantly from other measures.

One simple and natural way to improve on the HEI is by denominating household expenditure by household size to arrive at per capita expenditure. The logical way to proceed then is to impute per capita expenditure to each individual of the household as a measure of individual utility. Curiously, this is not what is usually done. A typical example is Beach, Card and Flatters (1981). Although they use income instead of expenditure, what they would have done with expenditure would be to impute per capita expenditure to each household rather than each individual, which is again unjustiable in terms of social ethics. To the extent that individuals constitute society, all individual welfares should have identical weights in social welfare aggregation and not weights that vary with household size. For example, if the social welfare function is additive, such as mean of order r, each person in an n-person household bears a weight of 1/n as opposed to 1.

Therefore the acceptable way of imputing per capita expenditure as a measure of utility is to impute it to each individual in society. This gives rise to the per capita expenditure (PCE) index which is the second inequality index computed for comparative purposes in Chapter 7.

The per-capita approach, as a method of approximating individual utility using household expenditure, has been subject to criticisms. Wolfson (1979) points out that this method ignores economies of scale in the consumption of capital services. A better way is, he suggests, to use "adult equivalents" in place of family size to denominate household expenditure.

His method can be illustrated as follows. Let y_h be the total expenditure of household h. Let L_h be the "low income cut-off" line for household h.[1] The welfare ratio, w_h, is the ratio, y_h/L_h which is imputed to household h as a measure of utility. If the inequality index is relative (i.e., it is homogeneous of degree 0 in the arguments), then this welfare ratio approach is identical to using $y_h/\frac{L_h}{L_o}$, where L_o is the "low income cut-off" line for a reference, say, one-adult-male, household. L_h/L_o can be regarded as the number of equivalent-adults in household h and $y_h/\frac{L_h}{L_o}$ is named "inflated welfare ratio". Since the cut-off values typically exhibits economies of scale, this method is an easy way to capture these scale effects.

Wolfson's method is also subject to the criticism that walfare ratios are imputed to households rather than individuals. In Chapter 7, it will be demonstrated that this mis-specification does make a significant difference in inequality measurement. In that Chapter the third index, which uses inflated welfare-ratios imputed to households (HIWR), is significantly different from the fourth index which imputes to individuals (IIWR).[2]

Furthermore, using welfare-ratio to represent individual utility is unsatisfactory for three reasons even though it is already an improvement over the "per capita" method. Firstly, while "economies of scale" are incorporated, it has been assumed that the degree of economies of scale is the same for all goods and services. This is unrealistic because, intuitively, capital services such as housing and transportation should exhibit higher degree of economies of scale than consumption goods like food and clothing. Secondly, the definition of poverty is controversial. Based on different definitions of poverty, there exist three sets of low-income cut-off lines in Canada and there is no dominating ethical reason for preferring any set over the other two. (see Osberg (1981)). Thirdly, this welfare-ratio method (as normally used) does not give rise to a price-sensitive inequality measure which aims at capturing distributive price effects. In Canada, the commonly used low-income cut-offs are those published by Statistics Canada. This set is revised annually only for inflation which does not affect the final inequality measure if the index is relative, i.e., mean-independent.[3] On the other hand, if the index is non-relative, it is not clear why inflation should affect inequality that is calculated based on expenditure.

Section 2 Distributive price effects

It is somewhat obvious that relative price changes have distributive price effects. In the Canadian context, a recent attempt to study the welfare effects of price changes is contained in Roberts (1982). The main objective of his study is to investigate the effect of food price changes on cost-of-living indexes of households in five income quintiles. He uses family expenditure survey data for five separate years to estimate a linear expenditure system of eight goods and services for each income quintile. In the regression, per-capita expenditure instead of household expenditure is used to adjust for family size. Exact cost-of-living indexes are then computed (being ratios of the minimum expenditure to attain a given utility level under two price situations) for each of the five income quintiles. The basic finding is that, food price inflation tends to increase the cost-of-living index for the lowest income quintile more than the highest income quintile. Since food accounts for a higher percentage of total budget in the poor, relative to the rich, this finding is not surprising at all. What one notes with interest is: the corollary of this result is that relative price changes have a definite impact on inequality. Unfortunately, none of the indexes described so far is capable of measuring this impact because they are all calculated based on income and expenditure or simple adjustments of income and expenditure.

There are two studies in the literature which attempt to capture price effects, namely, Muellbauer (1974, a,b,c) and Jorgenson and Slesnick (1984). However, as explained in the following, both attempts are unsatisfactory.

Muellbauer's method

Muellbauer (1974 a,b,c) attempted to capture relative price effects and adjust income for family size and economies of scale in one coherent model. His method is summarized as follows. He specifies a household utility function which has the image

$$(2.1) \qquad u = U(x_1/m, \ldots\ldots, x_n/m)$$

where $(x_1, \ldots\ldots, x_n)$ are household consumption and m is the number of equivalent adults, taken arbitrarily from Prest and Stark (1967) and Stark (1972). The numbers are

Family size	1	2	3	4	5	6	7	8
m	1	1.6	2.1	2.5	2.8	3.2	3.6	4.0

which as a sequence, shows economies of scale in consumption. It follows from (2.1) that the image of the indirect utility function is

$$(2.2) \qquad u = V(p_1, \ldots\ldots, p_n, \; y/m)$$

and that of the cost function is

$$(2.3) \qquad y = C(u, mp_1, \ldots, mp_n)$$

Muellbauer (1974 a,b,c) employs an adaptation of the money-metric utility of Samuelson (1974) to represent household utility. This concept is further discussed in Chapter 3, but briefly, money-metric utility is the income that enables an individual (a household in the present context) to arrive at a given level of utility at reference prices. Muellbauer's version of money-metric utility, however, is represented, for household h, by

$$(2.4) \qquad \tilde{y}^h = C(u^h,\ m^o p_1^o,\ \ldots\ldots,\ m^o p_n^o)$$

$$= C\big(V(p_1,\ \ldots,\ p_n,\ y^h/m^h),\ p_1^o,\ \ldots,\ p_n^o\big)$$

where y^h is income of household h, m^h is the number of equivalent adults and m^o is taken as unity. (the number of equivalent adults of a one-person household).[4] Given a society of H households, Muellbauer suggests calculating money-metric utility for each household and computing inequality based on the distribution

$$(2.5) \qquad (\tilde{y}^1,\ \ldots\ldots,\ \tilde{y}^H).$$

This method is not entirely satisfactory, although the index based on (2.5) is price-sensitive. Firstly, the scale of equivalent adults is taken from a separate study. Since utility is given a representation (2.1), the scale numbers should be estimated in one pass together with other parameters in the utility function.

Furthermore, as argued above, assuming the same degree of economies of scale for all goods and services is not very realistic.

Secondly, it is not clear what (2.4) means. In particular, Muellbauer seems to suggest the utility number in (2.2) is a measure of the utility of each individual in the household. But if this is his intention, he should impute money-metric utility of a household to each member in the household in (2.5), and expand the dimension of (2.5) to the total number of individuals in society. A related issue is the problem of interpersonal comparison of utility is completely ignored. An objective measure of utility does not exist. Muellbauer's money-metric utility (2.4) represents one particular numerical representation of each individual's utility which must imply a certain underlying rule of interpersonal comparison. This assumption must be made clearly known in any inequality measurement model.[5] In fact, Samuelson's money-metric utility is applicable directly to the case of a single individual only. Extending this concept to a multi-person situation needs more justification.

Jorgenson-Slesnick method

A recent attempt to construct a price-sensitive inequality index can be found in Jorgenson and Slesnick (1984). (see also (1982 a,b,c), (1983 a,b)). Their basic strategy is: they specify a translog household utility function which incorporates commodity-specific equivalence scales to account for attribute differences among households and estimate the parameters using demand data. Based on these estimates, they attempt to use utility numbers in a non-welfarist social welfare framework to arrive at a measure of inequality.

The direct household utility function has the image

$$u = U\left(x_1/m_1(A),\ \ldots\ldots,\ x_n/m_n(A)\right) \tag{2.6}$$

where $m_1(A), \ldots, m_n(A)$ are the commodity-specific equivalence scales, which are functions of attributes A. It follows from (2.6) that the indirect utility function has the image

$$u = V\left(m_1(A)p_1,\ \ldots,\ m_n(A)p_n,\ y\right) \tag{2.7}$$

Assuming translog preferences they claim that given the parameters involved in (2.7), a utility number is obtainable for each household given its attributes, prices and total expenditure. This utility number is taken as a measure of household utility, such that for household h, utility is

$$u^h = V\left(m_1(A^h)p_1,\ \ldots,\ m_n(A^h)p_n,\ y^h\right). \tag{2.8}$$

In the aggregate, the social welfare framework is unorthodox. Social welfare w is given by a non-welfarist social evaluation functional i.e.,

$$(2.9) \qquad w = \sum_h a_h(x)U^h(x) - \gamma(x)\left(\sum_h a_h(x)\,|U^h(x) - \bar{u}_x|^\rho\right)^{1/\rho}$$

where x is a state variable and

$$(2.10) \qquad \bar{u}_x := \sum_h a_h(x)U^h(x)$$

It should be emphasized that (2.9) is non-welfarist because a_h, $h = 1, \ldots, H$ and γ are functions of x.[6] $U^h(x)$ in (2.9) and (2.10) is taken as (2.8) even though the number u^h has no cardinal significance.[7] The form for $a_h(x)$ is assumed to be

$$(2.11) \qquad a_h(x) = m_o(p, A^h)/\sum_h m_o(p, A^h), \text{ where}$$

$$(2.12) \qquad m_o(p, A^h) = \frac{C\left(u^h, m_1(A^h)p_1, \ldots\ldots, m_n(A^h)p_n\right)}{C(u^h, p_1, \ldots\ldots, p_n)}$$

One may recall that $m_i(A^h)$, $i = 1, \ldots, n$ is the equivalence scale factor of household h for good i. If one normalizes the factors for a reference household (having attribute A^o) to be unity, i.e.,

$$(2.13) \qquad m_i(A^o) = 1 \qquad\qquad i = 1, \ldots, n$$

then $m_o(p, A^h)$ can be interpreted as a general market equivalence scale which measures the fraction of total expenditure required to keep household h and the reference household at the same utility level. These scales differ from those in (2.6) in that they are functions of prices as well as attributes.

$\gamma(x)$ in (2.9) is assumed to have the form,

$$(2.14) \quad \gamma(x) = \left[1 + \left(\{\Sigma a_h(x)\}/a_j(x)\right)^{\rho-1}\right]^{-1/\rho}$$

$$= \left(1 + a_j(x)^{1-\rho}\right)^{-1/\rho}$$

where $\quad a_j(x) = \min_k a_k(x)$.

Substituting (2.11) (2.14) into (2.9), the final social welfare function is

$$(2.15) \qquad w = \bar{u}_x - \left[1 + a_j^{\,1-\rho}\right]^{-1/\rho}\left[\left(\Sigma m_o(p, A^h)\,|U^h(x) - \bar{u}_x|^{\rho}\right) \Big/ \Sigma m_o(p, A^h)^{1/\rho}\right]$$

Given fixed society expenditure, one may find the distribution of household expenditure which maximizes w in (2.15). Because of translog preferences and the assumption (2.11), the first order conditions imply each household is endowed with the same "equivalent income", i.e., for household h and k

(2.16) $$\frac{y^h}{m_o(p, A^h)} = \frac{y^k}{m_o(p, A^k)} \qquad h,k = 1, \ldots, H$$ so that

(2.17) $$y^h/y^k = m_o(p, A^h)/m_o(p, A^k).$$

At the maximum, two households with different attributes will be given different incomes according to the general market equivalence scales and given same income if they have the same attributes. At this point, the second term in (2.15) vanishes so that maximum social welfare equals $\bar{u}_x$, and all households are regarded as equally well-off.

Letting w be the actual social welfare given in (2.15), an inequality index is proposed, i.e.,

(2.18) $$I_{JS} = 1 - w/\bar{u}_x \quad \text{and} \quad 0 \leq I_{JS} \leq 1$$

This approach to inequality measurement is not acceptable. The social welfare framework described above is open to criticisms. The ethical reason for adopting a non-welfarist framework is not clear, although they cite Sen's argument (1979) to support their procedure. Sen's argument against welfarism is based on the lack of consideration of absolute rights in a welfarist social welfare function. These absolute rights refer to equal-work-for-equal-pay, freedom from exploitation and social liberty. Sen does not imply, nor is it reasonable to assume that any function that depends explicitly on state characteristics (hence non-welfarist) is an improvement over a welfarist function. Furthermore, the way the social welfare function (2.9) captures these characteristics, through $\gamma(x)$ and $a_h(x)$, is ad hoc and far away from Sen's original intention. It does not capture what welfarism misses.

By the definition of $a_h(x)$, (2.11), the social welfare function does not satisfy anonymity, i.e., each individual is not equally important in the social ranking. Each household's utility is assigned a weight proportional to the estimated number of "equivalent adults" according to the estimated general market equivalence scales (2.12). In case of significant economies of scale in consumption the number of "equivalent adults" is much smaller than family size. But it is unjustifiable to assign smaller weights to members in large households relative to members in small households. The ethical basis is not clear. The authors fail to give a full set of axioms that completely characterize (2.15). This is serious because, as emphasized in Chapter 1, inequality measurement is a normative judgemental exercise that is contingent on an underlying set of ethical axioms. Consequently, this approach has left some room for improvement.

To summarize this chapter, we have seen that all the indexes developed in the literature are unsatisfactory. They are not consistent with the social welfare view of economic inequality. In addition, there is no index that can demonstrate the distributive impact of price changes. Therefore, a new approach is urgently required to fill this gap.

Chapter 2 Footnotes

1. Statistics Canada publishes low-income cut-off lines that are specific to household size and size of area of residence. Cat. No. 13-207.

2. Inflated welfare-ratio is used instead of Wolfson's welfare-ratio to make comparison with other measures more immediate. As the index is relative, inequality is not affected.

3. The Statistics Canada cut-offs are derived from expenditure surveys conducted once every five years. They are estimated by taking the average household income of those households that spend 20% of the budget more than the average household of the same size and area on the necessities — food, clothing and shelter. The "mark-up" of 20% is arbitrary.

4. As explained above it would be more appropriate to use expenditure instead of income for y^h.

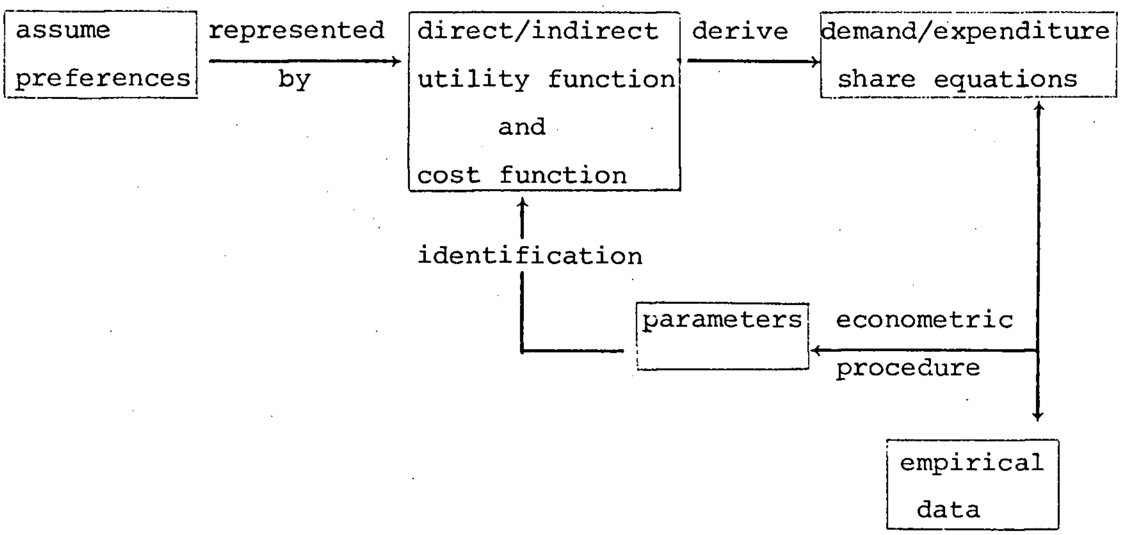
5. By contrast, the interpersonal comparison assumption in the per-capita expenditure method and welfare-ratio method is easier to see.

6. A social evaluation functional is welfarist if the states affect social ranking only through their effects on individual utilities.

7. Any monotonic transformation of a utility function yields the same demand equations. Empirical estimation only yields enough information to allow a ranking of alternative price-income situations. (2.7) does not establish an objective scale of utility measurement. J-S fail to point out that the choice of the particular numerical representation (2.7) is somewhat arbitrary.

CHAPTER 3 A NEW APPROACH

This chapter describes the theoretical background of this new approach to inequality measurement. The basic strategy can be illustrated by the chart below. Based on hypothesized preferences,



one can obtain a functional representation by the direct and indirect utility functions or the cost function. From any one of these functions, demand and expenditure share equations can be derived. A suitable econometric procedure can then be devised to obtain empirical estimates for the parameters in the demand and expenditure share equations. These estimates can also be used to identify the original functions that represent preferences. If these information are available for all consumers, a casual observer might contemplate measuring inequality using, say, the image of each consumer's indirect utility function, given prices and nominal total expenditure distribution.

However, there are several problems involved in this general scheme.

1. The specification of individual preferences need to allow for taste differences arising from various demographic characteristics. Furthermore, only household expenditure data are available, as opposed to individual expenditure data. Therefore, assumed preferences have to (1) incorporate these characteristics and (2) employ household expenditures in a reasonable manner in order that meaningful welfare information about the individuals in the household can be revealed. Section 1 of this chapter suggests an equivalence scales method that deals with these problems directly.

2. It is well-known that any arbitrary monotonic transform of the direct or indirect utility function yields the same demand equations. Consequently, even if perfect parameter estimates are obtainable following the procedure described above, the images of the direct or indirect utility function are still arbitrary as utility numbers. In the context of inequality measurement, this arbitrariness cannot be allowed and the problem of interpersonal comparison of utilities has to be dealt with explicitly. Section 2 suggests that "equivalent income" as defined later, is an acceptable measure of utility for this purpose.

Given a distribution of acceptable measures of utility, one can attempt to measure aggregate social welfare. A social welfare evaluation framework can be constructed based on a set of ethical and informational assumptions. Described later in Section 3 is a welfarist framework that is argued to be appropriate in the present context. In this framework, a social welfare function can be utilized to aggregate individual utilities to a measure of social welfare which then leads to the construction of a relative inequality index in Section 4.

Section 1 Equivalence Scales

Formally, the preferences of a society of H households can be represented by the utility functions U^1, ..., U^H, with the interpretation that

$$(3.1) \qquad u_h = U^h(x^h)$$

is the utility of each member of household h and x^h is the consumption vector of household h.[1] It is assumed that strict equality of utility exists in all the households. It is also assumed in the following that differences in preferences among households can be captured by a vector A which describes household attributes. Formally, this means that the preferences of society can be represented by

$$(3.2) \qquad U(x^1, A^1), \ldots\ldots, U(x^H, A^H).$$

The utility of an individual i, who belongs to household h is therefore

$$(3.3) \qquad u_i = U(x^h, A^h)$$

i.e., the utility common to all members in household h. Note that x^h is household consumption and therefore the search for an appropriate form for $U(x^h, A^h)$, $h = 1, \ldots\ldots, H$ is crucial in order to justify the interpretation (3.3). Subject to this reservation, one utility function can now be applied to all individuals of all households in society.[2]

In order to interpret u_i in (3.3) as individual utility, the present approach adopts Barten (1964) commodity-specific equivalence scales. With n goods, the utility of each member of household h is given by, from (3.3),

$$(3.4) \qquad u_h = U\left(x_1^h / m_1(A^h), \ldots\ldots, x_n^h / m_n(A^h)\right)$$

where $m_1(A^h), \ldots\ldots, m_n(A^h)$ are the commodity-specific equivalence scales for household h, so that $x_i^h / m_i(A^h)$ is the equivalent consumption of good i for household h, relative to a reference household whose scale factors $m_1(A^o), \ldots\ldots, m_n(A^o)$ are normalized to be 1. For example, let family size be the only demographic characteristic described by A. Suppose

(3.5) $\qquad m_j(A^o) = m_k(A^o) = 1, \qquad j,k = 1, \ldots, n$

where A^o describes a one-person-household. Then these equivalence scales can be regarded as factors that deflate household consumption to arrive at effective individual consumption. As family size increases, the scale factors should increase to reflect the increasing need for each good. The rate of increase, being specific to the good, depends on the capability of securing economies of scale in consumption. By way of example, clothing should have smaller economies of scale than housing.

Thus, (3.4) is a general specification that maps household consumption and attributes (through $m_1, \ldots, m_n$) to a utility number which can reasonably be regarded as the utility of each member of the household. Special cases of (3.4) include the "head-counting" method and Engel's method adopted by Muellbauer (1974 a, b, c). The head-counting case is,

(3.6) $\qquad m_j = \text{family size}, \qquad j = 1, \ldots, n$

which does not allow for economies of scale in consumption,

while Engel's case is, letting A represent family size;

(3.7) $\qquad m_j(A) = m_k(A) \qquad j,k = 1, \ldots\ldots, n,$

i.e., common equivalence scale across goods, which does not allow for differing degrees of economies of scale among goods. Whether these special cases are good approximations of the general case can be checked by looking at actual empirical estimates.

Obviously, these Barten equivalence scales can accommodate household characteristics other than family size. Consequently, in the implementation of this model, four characteristics are isolated: the size of the area of residence, the sex of the household head, family size and the age of the household head.

However, what the scales mean now is not as clear. If family size is the only relevant attribute, the structure of these scales reflects the different degrees of economies of scale of different goods. But what does it mean if the scale factor for, say, transportation for rural households is higher than that for urban households? It means that keeping effective consumption (relative to some reference household) of other goods the same, a household moving from an urban area to a rural area needs more transportation in order to be just as well off as before.

Given the specification of the direct utility function (3.4), it follows that the indirect utility function must incorporate the scales by mark-ups in prices, whose image is[3]

$$(3.8) \qquad u_h = V\left(m_1(A^h)p_1, \ldots, m_n(A^h)p_n, y^h\right)$$

where y^h is household h's total expenditure. The cost function, C is obtained by inverting the indirect utility functions (3.8) and solving for y^h.

(3.9) $$y^h = C\left(u_h, m_1(A^h)p_1, \ldots\ldots, m_n(A^h)p_n\right)$$

Section 2 Equivalent Income

In the literature, money-metric utility was introduced in Samuelson (1974) and Varian (1980) to indicate the direction of change in an individual's utility, and is very close to the concepts of compensating variation and equivalent variation in the consumer surplus literature.

Let $\hat{U}$ be a direct utility function satisfying the usual regularity conditions — continuity, positive strict monotonicity (to eliminate satiation) and quasi-concavity.

(3.10) $$u = \hat{U}(x)$$

The corresponding indirect utility function $\hat{V}$ and cost function $\hat{C}$ will have images

(3.11) $$u = \hat{V}(p, y), \text{ and}$$

(3.12) $$y = \hat{C}(u, p)$$

Money-metric utility is defined in Samuelson (1974) as

$$(3.13) \qquad M(x,p^o) := \hat{C}\left(\hat{U}(x),p^o\right)$$

where p^o is a reference price vector. Since the cost function $\hat{C}$ is increasing in u, money-metric utility establishes a scale that measures utility as an income concept. This is justifiable because the same reference price p^o is used for all states, so that M is ordinally equivalent to $\hat{U}$, regardless of the choice of p^o,

i.e.,

$$(3.14) \qquad \hat{U}(x^1) \geq \hat{U}(x^2) \longleftrightarrow M(x^1,p^o) \geq M(x^2,p^o)$$

Empirically, (3.13) is difficult to handle. Alternatively, as King (1983) suggests, (3.11) instead of (3.10) can be substituted into (3.12) which gives rise to a so-called real income function,

$$(3.15) \qquad \tilde{C}(p,y,p^o) := \hat{C}\left(\hat{V}(p,y),p^o\right)$$

$\tilde{C}$ maps prices and expenditure, given a reference price vector, to a real number, the real income. Its nature is made clear by regarding it as a solution to

$$(3.16) \qquad \hat{V}(p^o,\hat{y}^e) = \hat{V}(p,y)$$

so that

$$(3.17) \qquad \hat{y}^e = \hat{C}\left(\hat{V}(p,y),p^o\right)$$

$\hat{y}^e$ is the amount of expenditure at p^o that will keep the consumer just as well off as in the state (p,y). Since p, y are readily observable variables, $\hat{y}^e$ is a practical measure of utility. However, although $\hat{y}^e$ is exact in showing the direction of utility change, it is arbitrary in absolute quantity as a result of p^o in (3.15).[4]

So far, only one person is involved. In the present model, the idea of money-metric utility is adapted to take into consideration households that are identified by an attribute vector A. Equivalent income, y^e will be used to measure utility which can be viewed as a solution to

$$(3.18) \qquad V(p^o,y^e,A^o) = V(p,y,A)$$

so that,

$$(3.19) \qquad y^e = C\left(V(p,y,A),p^oA^o\right)$$

$$=: E(p,y,A,p^o,A^o)$$

where p^o, A^o are reference prices and attribute vector of a reference household. Notice that (3.19) is not a straightforward extension of (3.17). While (3.16) just compares utilities of one person, (3.18) represents explicit interpersonal comparison of utilities between two households. y^e is the household expenditure that will make each individual in the household described by A^o at p^o just as well off as each individual in the household described by A at p.

Notice also that (3.18) represents one particular level of interpersonal comparison. Empirically, there is no objective measure of utility. If V is found to be consistent with demand behaviour, so is any household-specific monotonic transform of V. Indeed, consumers could "announce" their own levels of utility according to their own scales of measurement so that "announced" utilities cannot be compared interpersonally. y^e is not immune to this arbitrariness in utility measurement. By allowing household-specific monotonic transforms on V, a different equivalent income measure could be obtained by solving for $\hat{y}^e$ in

$$(3.20) \qquad \Phi\left(V(p^o,\tilde{y}^e,A^o),A^o\right) = \Phi\left(V(p,y^h,A^h),A^h\right)$$

so that

$$(3.21) \qquad \tilde{y}^e = C\left\{\Phi^{-1}\left(\Phi\left[V(p,y^h,A^h),A^h\right],A^o\right),p^o,A^o\right\}$$

$\tilde{y}^e$ is equal to y^e if and only if A^o and A^h are identical. Therefore, the interpersonal comparison (3.18) is a key assumption in this approach to inequality measurement. Is this assumption justifiable and realistic? It depends on the values of the commodity-specific equivalence scales, which are estimated from demand data. For example, consider family size as the only characteristic. The equivalence scale for size 1 is normalized to be 1. Then the equivalence scales for size 2 should fall between 1 and 2. In other words, the indirect utility function (3.8) has to play fully the role of making interpersonal comparison (3.18) possible.

In the subsequent model of inequality measurement, equivalent income, being a result of the interpersonal comparison (3.18), will be used as a measure of individual utility. This is possible because by the definition of the equivalent income function (3.19) and the fact that C is increasing in utility,[5]

$$(3.22) \qquad E(p^1,y^1,A^1,p^o,A^o) \geqq E(p^2,y^2,A^2,p^oA^o)$$

$$\longleftrightarrow \qquad V(p^1,y^1,A^1) \geqq V(p^2,y^2,A^2)$$

where households A^1 and A^2 face (p^1,y^1) and (p^2,y^2) respectively and $V(p^i,y^i,A^i)$ is the utility of each member in household i.

A corollary is if A^1 is set equal to A^2, then (3.22) implies E preserves each individual's utility ranking. One can readily verify that E is in fact a monotonic transform of V and applying Roy's Identity will yield the same set of demand functions.

More insight in y^e can be gained by referring to Deaton's (1980) interpretation. y^e can be expressed as

$$(3.23) \qquad y^e = \frac{C(u,p,A)}{\dfrac{C(u,p,A^o)}{C(u,p^o,A^o)} \cdot \dfrac{C(u,p,A)}{C(u,p,A^o)}}$$

$$= \frac{y}{\Pi(u,p,p^o,A^o)S(u,A,p,A^o)}$$

where

$$(3.24) \qquad \Pi(u,p,p^o,A^o) := C(u,p,A^o)/C(u,p^o,A^o)$$

is a price index evaluated at u and A^o, and

$$(3.25) \qquad S(u,A,p,A^o) := C(u,p,A)/C(u,p,A^o)$$

is a market equivalence scale factor evaluated at u and p. It is now clear that y^e is sensitive to p because both Π and S are functions of p. Π is specific to a household (and to each individual therein) only to the extent that it is a function of utility. Two households that are equally well-off will have the same price index regardless of attributes. However, the market equivalence scale S is in general a function of both utility and attributes.[6] Therefore, both Π and S capture some distributive price effects.

To conclude: equivalent income y^e is used as a utility measure in social welfare evaluation. The utilities of all individuals in all households are measured by a common yardstick, namely, the total expenditure that will keep each member of a reference household just as well off, at reference prices. Therefore, an equivalent income should be imputed to each individual in society. In a society of H households and N individuals, $H \leqq N$, the distribution of utilities for welfare evaluation purposes will be

$$(3.26) \qquad (y_1^e, \ldots\ldots\ldots\ldots, y_N^e)$$

Since y^e is price-sensitive, the social welfare indicator will be also.

Section 3 <u>Social Choice</u>

This section introduces the social welfare evaluation framework that maps the distribution of individual equivalent incomes to a social welfare number. This framework forms the basis of inequality measurement.

Social welfare evaluation can be looked at as an aggregation problem. A profile is a vector of individual utility functions defined over a set of social states. A social evaluation functional is then a mapping from such a profile to a social ordering over the same set of states. In deriving such a social ordering, two sets of assumptions are usually involved. The first set involves the ethical axioms that are argued as justifiable and acceptable. For example, the weak Pareto rule is commonly assumed, i.e., if every individual prefers state A to state B, the social ordering must rank state A above state B. The second set of assumptions are the assumptions on measurability and interpersonal comparability of utilities. When one searches for social welfare rules, forms that can be completely characterized by axioms are favoured. In welfare evaluation, the ethical basis should be clear, otherwise, no matter how valid the information is about individual welfare, the evaluation procedure in the aggregate is mechanical and unjustifiable.

Let T be the set of all possible profiles of individual utility functions and D the domain of f - the social evaluation functional (D being a subset of T), RR the set of all possible orderings over the set of alternatives, X. Then

(3.27) $$f: D \longrightarrow RR$$

i.e., $$R_u = f(U^1, \ldots\ldots, U^N),$$

where U^k is individual k's utility function and $U^k(x)$ is his or her utility in a particular state x in X. R_u is the social ordering associated with the profile $(U^1, \ldots, U^N)$, through f.

The following three axioms on f are commonly called the "welfarism" axioms,

(1) Unrestricted Domain

$T = D$

(2) Pareto Indifference

Let I_u be the symmetric factor of R_u. If

$U(x) = U(y)$,

where $U(x) := \left(U^1(x), \ldots\ldots\ldots\ldots, U^N(x)\right)$,

then $x I_u y$, for all x, y in X and all U in D.

(3) Binary Independence of Irrelevant Alternatives

For all x, y in X; U', U" in D, if

$U'(x) = U''(x)$ and $U'(y) = U''(y)$, then

$R_{u'}$ and $R_{u''}$ must coincide on (x, y).

These welfarism axioms (Blackorby, Donaldson and Weymark (1983)) are important because they imply strong neutrality (SN), defined as follows:

Strong Neutrality

For all w, x, y, z in X and U', U" in D, if

$U'(x) = U''(w)$, $U'(y) = U''(z)$, then

$xR_{u'}y \longleftrightarrow wR_{u''}z$ and $yR_{u'}x \longleftrightarrow zR_{u''}w$

This property is very strong. In itself, it means individuals' utilities are the only determinants of social welfare. Anything that can affect social ordering has to "pass through" utilities. It is this interpretation that gives the name "welfarism" to the three axioms. (See Sen (1977)). This framework contrasts sharply with the non-welfarist framework of Jorgenson and Slesnick (1982 a, b, c) (1983 a, b) where the social welfare evaluation functional involves parameters a and γ that are both functions of state x.

It can be shown that a welfarist f implies and is implied by the existence of an ordering R on the real Euclidean space R^N such that

(3.28) $xR_{u}y \longleftrightarrow U(x)R\,U(y)$

It is now possible to partition the set of alternatives, X into socially indifferent sets by referring only to utility numbers. An additional continuity assumption on social preferences, (namely that the "socially at least as good as" and "socially at most as good as" sets are closed in R_N) will provide for the existence of a representing function W, generating the same ordering as R.[7] W is commonly referred to as the Bergson-Samuelson social welfare function.

Sen has raised objections against welfarism as an evaluation framework. If welfarism is assumed, it is natural to assume weak Pareto as well since only utilities determine social ordering. In some cases, this denies individuals absolute rights associated with such things as freedom from exploitation and "equal work for equal pay" which refer to state characteristics not captured by utilities. Sen called these non-welfare characteristics. However, under welfarism and weak Pareto, non-welfare characteristics have no role to play in determining the social ordering.[8]

It is clear that the choice between welfarism and non-welfarism depends on the type of analysis. In policy questions where prices and income are partly policy variables, welfarism is adequate. Questions like the impact of tax and tariff changes on economic inequality can be sensibly asked within this framework. Welfarism in practice allows easy estimation of welfare indicators since all that is required is computing individual measures of utility from measurable price and income quantities. In this type of analyses, incorporation of non-welfare characteristics is not relevant.[9]

Let $\bar{W}$ be a social welfare function defined on individual equivalent incomes; we measure social welfare as

$$(3.29) \qquad \bar{w} = \bar{W}(y_1^e, \ldots\ldots\ldots, y_N^e)$$

where N is the number of individuals in society. It should be noted that $\bar{W}$ is not a Bergson-Samuelson social welfare function. The ordering generated by $\bar{W}$ depends on p^o and in general on A^o as well. The Bergson-Samuelson function does not allow this arbitrariness. Note also that the equivalent incomes in (3.29) are positive real numbers. As they are functions of prices, the social ordering depends on prices, and this forms the basis of a price-sensitive inequality index.

Section 4 Inequality Measurement

This section describes how a summary inequality measure is computed using a distribution of individual equivalent incomes, (3.26). The social welfare function defined on (3.26) is assumed to be anonymous mean of order r.[10] An inequality index is then constructed from this social welfare function, following the Atkinson-Kolm-Sen (AKS) procedure. The AKS index is actually an Atkinson index of equivalent income inequality.

Given a social welfare function $\bar{W}$ defined on R^N_{++}, a typical element being a vector of individual equivalent incomes $(y^e_1, \ldots, y^e_N)$, ethically-indifferent-evenly-distributed equivalent income ξ can be implicitly defined as in

$$(3.30) \qquad \bar{W}(\xi i) = \bar{W}(y^e_1, \ldots\ldots, y^e_N)$$

where $i := (1, \ldots\ldots, 1)$, an N-vector. Explicitly, ξ is hence defined as

$$(3.31) \qquad \xi := \Xi(y^e_1, \ldots\ldots, y^e_N)$$

ξ is that level of equivalent income which if commanded by every individual will be ethically indifferent to the actual distribution. Following the AKS procedure, an inequality index is then defined as

$$(3.32) \qquad I := 1 - \xi/\mu$$

where $\mu = (1/N) \sum_{k=1}^{N} y^e_k$,

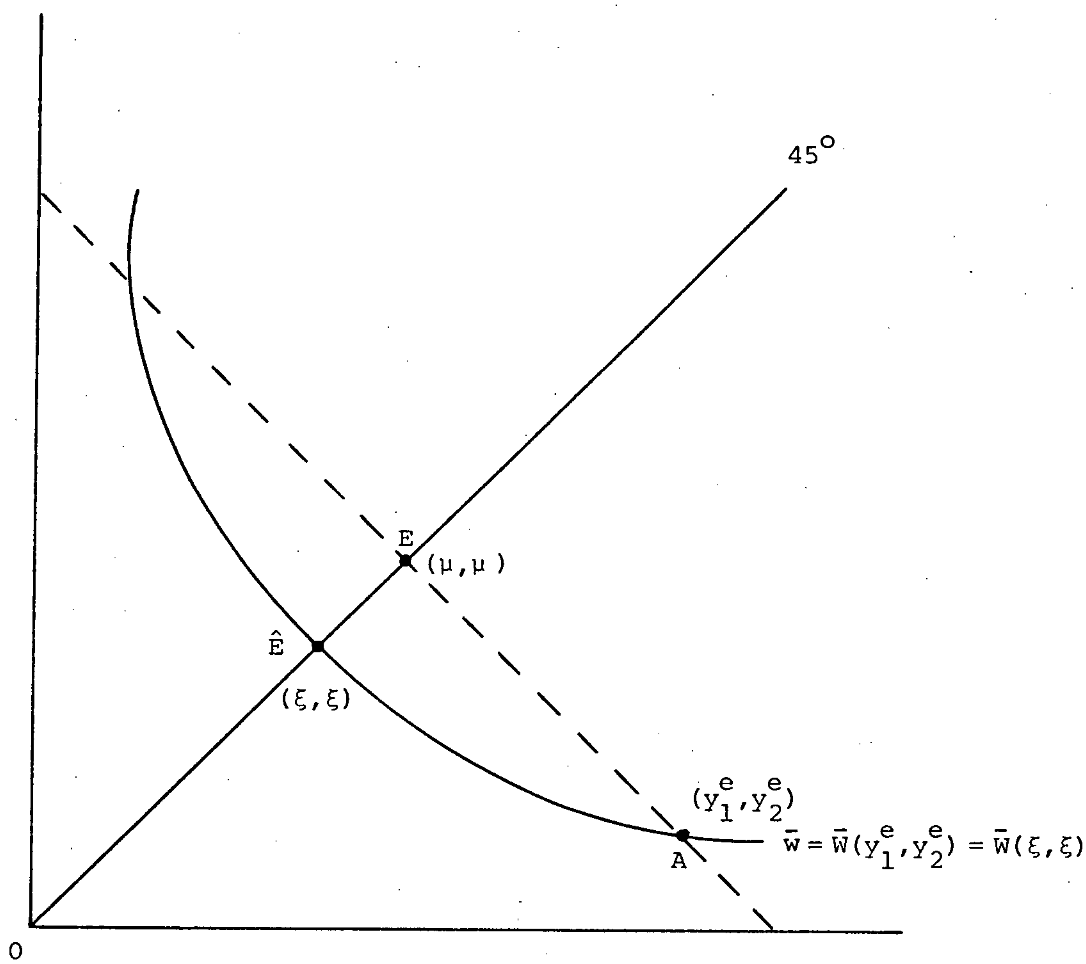
is mean equivalent income. An inequality index is a relative index if it is mean-independent, i.e., homogeneous of degree 0. It is easy to verify that the AKS index is relative if and only if $\bar{W}$ is homothetic.[11]

For N = 2, the present procedure is easily depicted in the equivalent income space. In the following diagram, the actual distribution is A where individual 1 enjoys a higher level of equivalent income. E is the egalitarian situation where each individual enjoys the mean of the distribution. If $\bar{W}(\cdot)$ is assumed to be symmetric quasi-concave (as drawn), then E is unambiguously ethically preferred to A. In fact, the same level of social welfare $\bar{W}$ can be attained by a lower combined equivalent income at point $\hat{E}$. $\hat{E}$, being ethically indifferent to A, is characterized by ξ as in (3.30). It is easy to see that ξ can be regarded as a measure of social welfare and $\Xi(\cdot)$ in (3.31) is ordinally equivalent to $\bar{W}(\cdot)$ in (3.30). The inequality measurement procedure adopted here, similar to the AKS procedure, makes use of the discrepancy between E and $\hat{E}$. The inequality index is defined as the shortfall of ξ relative to μ expressed as a percentage of μ as in (3.32). Geometrically, I can be expressed in terms of distances,

$$I = \frac{d(0,E) - d(0,\hat{E})}{d(0,E)}$$

To implement this procedure, a specific form for $\bar{W}(\cdot)$ is necessary. It is assumed that $\bar{W}$ is a symmetric mean of order r function, i.e.,

$$(3.33) \qquad \bar{W}(y_1^e, \ldots, y_N^e) = \Phi\left(W^*(y_1^e, \ldots, y_N^e)\right)$$


45°
E
(μ,μ)
$\hat{E}$
(ξ,ξ)
(y_1^e, y_2^e)
A
$\bar{w} = \bar{W}(y_1^e, y_2^e) = \bar{W}(\xi,\xi)$
0

where

$$(3.34) \qquad \overset{*}{W}(y_1^e, \ldots, y_N^e) = \Big((1/N)\sum_{k=1}^{N}(y_k^e)^r\Big)^{1/r}, \qquad r \neq 0$$

$$= \prod_{k=1}^{N}(y_k^e)^{1/N} \qquad r = 0$$

and $\Phi'(\cdot) > 0$.

Since $y_1, \ldots, y_N$ are defined as total expenditure, they are positive as obtained from a survey sample. By (3.23), it follows that $y_1^e, \ldots, y_N^e$ are also positive. The mean of order r function (3.33), (3.34) has desirable properties on R_{++}^N, namely, it is a continuous, additively separable, homothetic and symmetric function. Continuity is an obvious requirement for any social welfare function in the present context. Additive separability is ethically desirable because it implies "elimination of (the influence of) indifferent individuals", i.e., the ranking of any two states should be independent of the utility levels enjoyed by the individuals who are indifferent between the two states [see d'Aspremont and Gevers (1977), Blackorby and Donaldson (1982)]. Homotheticity ensures that the index (3.32) is relative, the importance of which will be explained later. Finally, symmetry implies "anonymity" which is an essential ethical requirement in inequality measurement.

Historically, the Lorenz criterion has a profound influence on inequality measurement. (See Sen (1973)). One would like the present inequality index to be consistent with it, i.e., if the distribution y^{eA} is Lorenz-superior to the distribution y^{eB}, then $I(y^{eA})$ should be no greater than $I(y^{eB})$.[12] A sufficient condition is that the inequality index is S-convex, which requires $r \leq 1$ in (3.34). Based on the mean of order r function (3.33) and (3.34), ethically-indifferent equivalent income is easily computed,[13] i.e.,

$$\text{(3.35)} \qquad \xi = \left((1/N)\, \Sigma\, (y_k^e)^r\right)^{1/r}, \qquad r \leq 1, \quad r \neq 0$$

$$= \prod_{k=1}^{N} (y_k^e)^{1/N}, \qquad r = 0$$

Substituting (3.35) into (3.32) yields a relative inequality index,

$$\text{(3.36)} \qquad I_r := 1 - \left((1/N)\, \Sigma\, (y_k^e/\mu)^r\right)^{1/r} \qquad r \leq 1, \quad r \neq 0$$

$$:= 1 - \prod_{k=1}^{N} (y_k^e/\mu)^{1/N} \qquad r = 0 .$$

where $\mu = (1/N) \sum_{k=1}^{N} y_k^e$, and N is the number of individuals. I_r is actually an Atkinson index (see Atkinson (1976)) on individual equivalent incomes. In Chapter 6 and 7, this inequality index, which is sensitive to prices, is estimated for Canada.

Before closing this chapter, it is important to emphasize the difference between relative and absolute indices, and the justification for adopting the former rather than the latter in the present context. In contrast with a relative index which is invariant to a common ratio-scale transform on a distribution, an absolute index is invariant to a common translation-scale transform, meaning that adding the same quantity to each individual's equivalent income does not affect an absolute index. For example, the per-capita index

$$(3.37) \qquad A(y^e) := \mu(y^e) - \Xi(y^e)$$

is an absolute index if $\bar{W}$ is translatable.[14] One could adopt the procedure introduced to compute $A(y^e)$. But there is one serious drawback. One can recall that equivalent income as defined in (3.19) is sensitive to p^o. Since the function E is HD 1 in p,y,p^o, it follows that measuring p,y,p^o in a different currency constitutes a rescaling of equivalent income by an exchange rate factor. However, since the multiple is common among individuals a relative index, (such as I_r in (3.36)), is immune to this type of rescaling, which by contrast, affects an absolute index.

Chapter 3 Footnotes

1. The social choice problem of Samuelson (1956) in aggregating individual preferences to household preferences is ignored. However, one may always assume that within each household there exists a "planner" who allocates consumption to equalize utilities.

2. Pollak and Wales (1979) are skeptical on this specification. They argued that this ignores direct contribution of attributes to utility. This aspect of utility is difficult to reveal empirically and is ignored here.

3. This is readily seen by writing the budget constraint of each household as:

$$m_1(A)p_1\left(x_1/m_1(A)\right) + \ldots\ldots + m_n(A)p_n\left(x_n/m_n(A)\right) = y$$

Maximization of (3.4) over $\left(x_1/m_1(A), \ldots\ldots, x_n/m_n(A)\right)$ subject to the constraint would yield the following set of first order conditions,

$$U_i\left(x_1/m_1(A), \ldots\ldots, x_n/m_n(A)\right) + \lambda m_i p_i = 0 \qquad i = 1, \ldots\ldots, n$$

$$\Sigma m_i(A)p_i\left(x_i/m_i(A)\right) = y$$

Substituting the solutions $\left(x_1^*/m_1(A), \ldots\ldots, x_n^*/m_n(A)\right)$ into (3.4) gives (3.8).

4. The common consumer surpluses, CV and EV are changes in $\hat{y}^e$ evaluated at final and initial prices. In general, CV and EV are not equal, although always take the same sign.

5. The special case of homothetic preferences may illustrate this proposition:

$$V(p^1,y^1,A^1) = \Phi\left(y^1/\Pi(p^1,A^1)\right), \Phi'(\cdot) > 0$$

$$V(p^2,y^2,A^2) = \Phi\left(y^2/\Pi(p^2,A^2)\right), \Phi'(\cdot) > 0$$

then

$$E(p^1,y^1,A^1,p^o,A^o) = y^1\Pi(p^o,A^o)/\Pi(p^1,A^1)$$

$$E(p^2,y^2,A^2,p^o,A^o) = y^2\Pi(p^o,A^o)/\Pi(p^2,A^2)$$

therefore

$$E(p^1,y^1,A^1,p^o,A^o) \gtreqless E(p^2,y^2,A^2,p^o,A^o)$$

$$\longleftrightarrow y^1/\Pi(p^1,A^1) \gtreqless y^2/\Pi(p^2,A^2)$$

$$\longleftrightarrow V(p^1,y^1,A^1) \gtreqless V(p^2,y^2,A^2)$$

6. If preferences are translog (see Chapter 4), then S is not a function of utility.

7. This is an application of Debreu (1959) theorem of utility function representation. See Debreu (1959). Sec. 4.6.

8. Sen (1970) proves a Libertarian theorem saying that unconditional Libertarian rules are inconsistent with weak Pareto and unlimited domain in generating a social ordering. If all are adopted as axioms, a preference cycle results. See also Roberts (1980).

9. The reader may recall that in Chapter 2, it has been argued that Jorgenson and Slesnick (1984) have made use of a non-welfarist framework without making it clear why such a framework is necessary and justifiable.

10. A social welfare function W is anonymous if and only if, for any two distributions of utilities

$$u = (u_1 ,\ldots,u_N) \quad \text{and } u' = (u_1' ,\ldots,u_N')$$

where one is a permutation of the other,

$$W(u) = W(u').$$

11. If $\bar{W}$ is homothetic, then $\bar{W}(y) = \Pi\left(\overset{*}{W}(y^e)\right)$, where Π is a monotonic transform, and $\overset{*}{W}$ is HD 1. It then follows that $\overset{*}{W}(\xi i) =$

$\overset{*}{W}(y_1^e, \ldots\ldots, y_N^e)$ defines ξ, so that $\Xi(y^e)$ is also HD 1. Since μ is HD 1, the index is relative. The converse is now easily verified. See also Sen (1973).

12. One should consult Berge (1962), Dasgupta, Sen and Starett (1973), Sen (1973) and Blackorby, Donaldson and Auersperg (1981). Briefly, suppose there are two distributions X_a, $X_b \varepsilon R^N$ with the same mean μ, then X_a is said to be Lorenz superior to X_b if the Lorenz curve for X_a lies completely inside that of X_b. In this case X_a = BX_b where B is a bistochastic matrix and is not a permutation matrix, and X_a can be obtained from X_b by a finite number of transfers. Then $S : R^N \longrightarrow R$ is an S-concave function if $S(X_a) \geq S(X_b)$ so that if S is a social welfare function, then the AKS index I_s will be S-convex, where

$$I_s(X_a) \leq I_s(X_b)$$

since

$$1 - S(X_a)/\mu \leq 1 - S(X_B)/\mu .$$

13. One can obtain (3.35) by an alternative route. Since $\bar{W}$ in (3.33) is homothetic (as $\overset{*}{W}$ is HD 1 and $\Phi'(\cdot) > 0$), it follows that Ξ is HD 1 and has to be identical to $\overset{*}{W}$.

14. For a full discussion, see Blackorby and Donaldson (1980).

CHAPTER 4 SPECIFICATION

In order to apply this new approach, specification is necessary. This chapter describes translog household preferences and the parameter restrictions necessary to make estimation feasible. Because a large number of parameters are involved, in addition to cross-section data, time-series aggregate data have to be used. An aggregation structure will be described which allows aggregate data to be utilized.

Preferences are assumed to be non-homothetic translog, following Jorgenson, Lau and Stoker (1982). A translog indirect utility function is a second-order approximation of any indirect utility function at a single point. Incorporating commodity-specific equivalence scales, it has the form, for n goods,

$$(4.1) \qquad \ln V\,(p, y, A) = \alpha_o + \sum_{i=1}^{n} \alpha_i \ln(m_i p_i / y) + \tfrac{1}{2}\sum_{ij}\sum b_{ij} \ln(m_i p_i / y) \ln(m_j p_j / y), \qquad b_{ij} = b_{ji}$$

or in matrix form,

$$(4.2) \qquad \ln V\,(p, y, A) = \alpha_o + (\ln mp/y)^T \alpha_p + \tfrac{1}{2}(\ln mp/y)^T B_{pp} (\ln mp/y)$$

where α_o is a scalar, α_p an n-vector, and B_{pp} an $n \times n$ symmetric matrix. By Roy's Identity, expenditure shares are

$$(4.3) \qquad e = \frac{\partial \ln V / \partial \ell n (p/y)}{i^T \left(\partial \ell n V / \partial \ell n (p/y) \right)}$$

where $i^T = (1, \ldots\ldots, 1)$, is an n-vector. Applying this to the translog, the expenditure shares take the form,

$$(4.4) \qquad e = \frac{\alpha_p + B_{pp}(\ell n\, m) + B_{pp}(\ell n\, p) - B_{pp} i (\ell n\, y)}{i^T \alpha_p + i^T B_{pp}(\ell n\, m) + i^T B_{pp}(\ell n\, p) - i^T B_{pp} i (\ell n\, y)} .$$

Since e is homogeneous of degree 0 in the parameters α_p, B_{pp}, it is usual, as in Jorgenson, Lau and Stoker (1982), to normalize as follows,

$$(4.5) \qquad i^T \alpha_p = -1$$

To obtain the translog cost function, the indirect utility function may be inverted to solve for y. (4.2) can be rewritten as

$$(4.6) \qquad \ell n\, V(p,y,A) = \alpha_o + (\ell n\, mp)^T \alpha_p + \ell n\, y + \tfrac{1}{2}(\ell n\, mp)^T B_{pp}(\ell n\, mp)$$
$$- (\ell n\, mp)^T B_{pp}(i\, \ell n\, Y) + \tfrac{1}{2}(i\, \ell n\, y)^T B_{pp}(i\, \ell n\, y)$$

which is a quadratic equation in $\ell n\, y$. In order to obtain an explicit form for the cost function, it is assumed that, (4.7) $$i^T B_{pp} i = 0$$

so that the second degree term in (4.6) vanishes.[1] The cost function will then have the form

(4.8) $$\ell n\, C\,(u,p,A) = \frac{\ell n\, u - \left(\alpha_o + (\ell n\, mp)^T \alpha_p + \tfrac{1}{2} (\ell n\, mp)^T B_{pp} (\ell n\, mp)\right)}{1 - (\ell n\, mp)^T (B_{pp}\, i)}$$

where u is a utility number.

The equivalence scales have so far been left unspecified. Following Jorgenson, Lau and Stoker (1982) and Jorgenson and Slesnick (1982 a, b, c)(1983 a, b), the attribute vector A is assumed to be a vector of dummy variables, i.e., the elements in A are either 0 or 1. Four household attributes are used to describe each household, namely, the area of residence, the sex of household head, the family size and the age of the household head, so that A is an eleven-vector, to be assigned to each household according to Table 1.

These four attributes are thought to be significant determinants of household consumption pattern. Indeed, there are relevant factors which have been ignored here, for example, household composition, education level, race and climate. The binding constraint is data availability. As will be seen in Chapter 6, statistics of expenditure distribution over attribute groups are essential for the

Table 1 : Vector A

		1	0
<u>Size of Area of Residence</u> *			
	A_1	non-metropolitan	metropolitan
<u>Sex of Household Head</u>			
	A_2	female	male
<u>Family Size</u>			
	A_3	two persons	otherwise
	A_4	three persons	otherwise
	A_5	four persons	otherwise
	A_6	five or more persons	otherwise
<u>Age of Household Head</u>			
	A_7	above 24 but 34 or below	otherwise
	A_8	above 34 but 44 or below	otherwise
	A_9	above 44 but 54 or below	otherwise
	A_{10}	above 54 but 64 or below	otherwise
	A_{11}	above 64	otherwise

* Cities with population above 30,000 are classified as metropolitan.

successful estimation of this model, and publicly available Statistics Canada data do not allow incorporation of more attributes. In particular, household composition (i.e., the number of adults, as opposed to children in the household) should be an important attribute. Here, the impact of household composition is reflected in part by the effect of sex of household head. By convention, a household with a female head means either an unattached female or a single female-parent household. Given any two multi-member households with all other attributes being the same, the household with a female head implies in most cases substituting a child for an adult. Hence, intuitively speaking, households with female heads should need more clothing and less food, and this difference should be reflected in estimated equivalence scales. This has indeed been confirmed by the estimation results, (as discussed in Chapter 6 below) which partially justifies the whole approach.

A reference household is defined as the household which has all elements in A equal 0, i.e., an unattached male, of age 24 or below, living in a metropolitan area. Following Jorgenson and Slesnick (1982 a, b, c)(1983 a, b), the equivalence scales are specified in a way that enables simple linear estimation, that is,

$$B_{pp} \ln m(A) = B_{pA} A \tag{4.9}$$

where B_{pA} is an n x 11 matrix, which satisfies

$$i^T B_{pA} = 0 \tag{4.10}$$

This last assumption is necessary in making aggregation across individual expenditure share equations simple, because e in (4.4) will now be linear in all household-specific variables. Furthermore, it should be pointed out that, by (4.9), the equivalence scale factors of the reference household are all equal to unity.

Incorporating the four assumptions (4.5) (4.7) (4.9) and (4.10), the expenditure shares can be expressed as

$$(4.11) \qquad e = \frac{\alpha_p + B_{pA}A + B_{pp}(\ell n\, p) - B_{pp}i(\ell n\, y)}{-1 + i^T B_{pp}(\ell n\, p)}$$

which are suitable for estimation using cross-section data. Note that B_{pA} represents the incremental effects on e as attributes change. Given y, the reference household (whose A equals 0) establishes a central level for e. This effect on expenditure shares through B_{pA} is then translated to a price effect through B_{pp} in (4.9) which depends on demand elasticities.

The aggregate expenditure share equation is obtained by summing individual share equations across the entire population. Let H be the total number of households and A^h, y^h be the attribute vector and spending of household h. The aggregate expenditure shares, E, are

$$(4.12) \qquad E = \sum_{h=1}^{H} \left((y^h e^h)/\Sigma y^h\right)$$

$$= \frac{\alpha_p + B_{pA}\Sigma(y^h A^h)/Y + B_{pp}(\ell n\, p) - (B_{pp} i)\Sigma\left(y^h(\ell n\, y^h)\right)/Y}{-1 + i^T B_{pp}(\ell n\, p)}$$

where $Y = \sum_{h=1}^{H} y^h$, the total spending.[2] These aggregate share equations are suitable for estimation using time-series aggregate data. Notice also that these equations have the same form and involve the same coefficients, α_p, B_{pA}, B_{pp}, as in the individual share equations, so that data from both sources can be combined, and identification of all parameters is obtained.[3]

Using the definition (3.19), translog equivalent income can be expressed as,

$$(4.13) \qquad y^e = \exp\left(\frac{(\omega-\omega^o)^T \alpha_p + \tfrac{1}{2}(\omega^T B_{pp}\omega - \omega^{oT} B_{pp}\omega^o) + (1-\omega^T B_{pp} i)(\ell n\, y)}{1-\omega^{oT} B_{pp} i}\right)$$

where,

$$(4.14) \qquad \omega = \ell n\, m(A)\, p$$

$$(4.15) \qquad \omega^o = \ell n\, m(A^o)\, p^o$$

To estimate the equivalent income of each individual, estimates for α_p, B_{pp} and B_{pA} are required. It will be shown in Chapter 6 that, by using both cross-section and time-series aggregate data, these estimates are obtainable.

It is evident that translog equivalent income (4.13) is sensitive to p^o and the choice of A^o. For example, defining an unattached female, of age 24 or below, living in a metropolitan area as the reference household will give rise to different equivalent income values. However, if the inequality index is relative, inequality is not sensitive to A^o. In order to verify this claim, (4.13) is rewritten as

$$(4.16) \qquad y^e = \exp\left(\frac{\omega^T \alpha_p + \tfrac{1}{2}\omega^T B_{pp}\,\omega + (1 - \omega^T B_{pp}\, i)(\ln y)}{1 - \omega^{oT} B_{pp}\, i}\right).$$

$$\exp\left(\frac{-\omega^{oT}\alpha_p - \tfrac{1}{2}\omega^{oT} B_{pp}\,\omega^o}{1 - \omega^{oT} B_{pp}\, i}\right)$$

but by (4.15), and subsequently, (4.9) and (4.10),

$$\begin{aligned}
\omega^{oT} B_{pp}\, i &= \left(\ln m(A^o) p^o\right)^T B_{pp}\, i \\
&= \left(\ln m(A^o)\right)^T B_{pp}\, i + (\ln p^o)^T B_{pp}\, i \\
&= (\ln p^o)^T B_{pp}\, i
\end{aligned}$$

so that (4.16) can be written as

$$(4.17) \qquad y^e = \exp\left(\frac{\omega^T \alpha_p + \tfrac{1}{2}\omega^T B_{pp}\,\omega + (1-\omega^T B_{pp}\, i)(\ln y)}{1-(\ln p^o)^T B_{pp}\, i}\right) \cdot$$

$$\exp\left(\frac{-\omega^{oT}\alpha_p - \tfrac{1}{2}\omega^{oT} B_{pp}\,\omega^o}{1-(\ln p^o)^T B_{pp}\, i}\right)$$

Note that only the first exponential term is individual-specific but it does not involve A^o. The second exponential term, which involves A^o, is a common scalar multiple on all individual equivalent incomes. Therefore, the choice of A^o cannot affect the relative inequality index.

Adopting the translog specification, it would be interesting to compute empirically the market equivalence scales (3.25). These scales can be compared with the ratios of poverty lines published by Statistics Canada. By definition, the poverty income for household h is,

$$(4.18) \qquad P(p,A^h) = C(\bar{u},p,A^h)$$

where $\bar{u}$ is the level of subsistence utility. Using A^o as a reference household, the poverty-line ratio for household h is

$$(4.19) \qquad P(p,A^h)/P(p,A^o) = C(\bar{u},p,A^h)/C(\bar{u},p,A^o)$$

which is different from the market equivalence scales only in that (4.19) is evaluated at $\bar{u}$. However, it can be shown that the translog market equivalence scales are actually independent of u so that they are identical with translog poverty-line ratios. To see this, notice that the denominator of (4.8) can be written as

$$(4.20) \qquad 1 - \left((\ln m)^T B_{pp}\, i + (\ln p)^T B_{pp}\, i\right)$$

$$= 1 - (\ln p)^T B_{pp}\, i$$

because of assumptions (4.9) and (4.10), so that it is independent of A, and $(\ln u)$ in the numerator will vanish when the difference $(\ln y^h - \ln y^o)$ is taken.

Consequently, the translog market equivalent scales and translog poverty-line ratios can be expressed as

$$(4.21) \qquad \exp\left(\frac{-(\ln m)^T \alpha_p - \frac{1}{2}(\ln m)^T B_{pp}(\ln m) - (\ln p)^T B_{pp}(\ln m)}{1 - (\ln p)^T (B_{pp}\, i)}\right)$$

These scales are estimated from demand data and compared with published numbers in Chapter 7. They provide valuable insights in a comparative study of different inequality indexes.

Chapter 4 Footnotes

1. Incidentally, this assumption is necessary for a linear expenditure share equation. See (4.4). See also Diewert (1974)

2. Notice the advantage of linearity in deriving (4.12) from (4.11), as a result of assumptions (4.7) and (4.10). In theory, any functional form can be aggregated. However, linearity ensures a "complete" aggregation structure regardless of distributions of individual-specific variables. If these were known, linearity would no longer be essential. See Stoker (1983).

3. In Gorman's (1953) aggregation framework, translog preferences do not allow the existence of a rational aggregate consumer. E represents aggregate shares of a "consumer" whose preferences change with income distribution. However, for empirical purposes, Gorman preferences are too restrictive in forcing parallel and linear Engel curves.

CHAPTER 5 ESTIMATION METHOD

Section 1 Introduction

This section describes the stochastic structure of the estimation model and explains how the parameters involved in the equivalent income function (4.13) can be estimated. Six composite goods will be defined. Since 11 dummy variables are utilized to describe demographic attributes, there are totally 108 parameters to be estimated: 6 parameters in α_p, 36 parameters in B_{pp} and 66 parameters in B_{pA}. Using these estimates, the equivalence scales can be estimated which indicate the actual relationship between utility, consumption and the four attributes.

There are two ways to estimate this model. The first approach, named the pooled estimator, has been used in Jorgenson, Lau and Stoker (1982). The technical details are not described here. Basically, they formulate a constrained minimization problem with an objective function being made up of the sum of squared residuals in the cross-section model (i.e., composed of the individual expenditure share equations) and in the time-series model (i.e., composed of the aggregate share equations). The solution δ^* to this minimization problem is the set of parameter estimates provided that they also satisfy the symmetry and monotonicity constraints.[1] δ^* is found by iteration as follows. The combined cross-section and time-series model and the constraints are first-order approximated initially around an arbitrary point δ_o. Then Liew's (1976) inequality constrained three stage least squares method is applied to generate δ_1. δ_1 is substituted into the objective function of residual sum of squares to obtain an objective value. This process is then repeated until the objective value converges. Although the estimator is believed to be consistent, small sample properties are unknown. Furthermore, the procedure is costly and because the start-up value δ_o is arbitrary, an accuracy problem might arise.

A different approach is adopted here which estimates the cross-section model and the time-series model sequentially. Estimates obtained in the cross-section are substituted into the time-series equations as if they were true values. This procedure has the advantage that it involves only linear estimation and no iterations are required. On the other hand, however, relative to the first approach, it is less efficient. For the pooled estimation, since information from both sources is pooled together and estimates generated in a single pass, even the parameters that are estimable using only cross-section data are estimated using additional time-series information. This constitutes some efficiency gain. However, because the size of the sample is large in the cross-section and small in the time-series, the efficiency gain is likely to be small. The estimates will be dominated by the cross-section data. The results in Jorgenson, Lau and Stoker (1982) substantiate this claim, namely, except for the price coefficients which do not enter the cross-section model, the estimates for B_{pA} and α_p obtained from cross-section alone are very close to the pooled estimates.

The procedure adopted here is a sequential one. The first step involves estimating those parameters that are identified in the individual expenditure share equations, i.e., α_p and B_{pA}, using cross-section data only. Because of the lack of price variation in the cross-section data, the price coefficients, B_{pp} are not identified. The second step involves estimating B_{pp} using the aggregate expenditure share equations and time-series data only, proceeding as if the estimates obtained in the first step for α_p and B_{pA} were true values. In other words, the aggregate equations are estimated subject to α_p and B_{pA} being equal to their cross-section estimated values, as well as the usual symmetry conditions on B_{pp}.[2]

Section 2 Cross-section Estimation

The individual share equation (4.11) is non-linear in ($\ln p$). In a family expenditure survey, there is no information on the price each household faces. It will be assumed that prices are uniform across the households. The survey year is taken as the reference year for the price series so that the price vector is (1, ..., 1), an n-vector.[3] The purpose is to avoid ($\ln p$) in (4.11). The cross-section share equation is therefore,

$$e^h = -\alpha_p - B_{pA}A^h + B_{pp}i(\ln y^h) \qquad h = 1, \ldots, H \tag{5.1}$$

where y^h, is taken as the total expenditure of household h in the survey. It then follows that the ith regression equation is,

$$e_i^h = -\alpha_i - \sum_{k=1}^{K} \beta_{ik}A_k^h + \theta_i(\ln y^h) + \varepsilon_i^h \qquad i = 1, \ldots, n \quad h = 1, \ldots, H \tag{5.2}$$

where e_i^h is the expenditure share of the ith good for household h, α_i is the ith component in α_p, β_{ik} is the ikth element in B_{pA}, A_k^h is the kth element in A^h, θ_i is the ith component in $B_{pp}i$, or the sum of the ith row (or ith column) of B_{pp}, ε_i^h is a disturbance term of the ith expenditure share equation for household h.

As is true for any consumption allocation model, one equation is redundant in (5.1). Summing up the expenditure shares in the left-hand side gives 1 identically, and so must the right-hand side, by the restrictions on α_p, B_{pA}, B_{pp}. This implies that, in (5.2), the disturbances are linearly dependent because

$$\sum_{i=1}^{n} \varepsilon_i^h = 0 \qquad h = 1, \ldots, H \tag{5.3}$$

so that the covariance matrix

$$\operatorname{var}(\varepsilon^h) = \operatorname{var}(\varepsilon_1^h, \ldots, \varepsilon_n^h)^T = \Sigma \qquad h = 1, \ldots, H \tag{5.4}$$

must be singular. It is assumed that the disturbance term satisfies the following assumptions, for all h = 1, ..., H,

(5.5) $E(\varepsilon^h) = 0_n$

(5.6) $\mathrm{var}(\varepsilon^h) = \Sigma$

(5.7) $\mathrm{var}(\varepsilon^1, \ldots., \varepsilon^H)^T = \Sigma \otimes I_H$

where Σ is of rank n-1 and disturbances are independent across households.

Actual estimation involves n-1 equations. Since Σ is of rank n-1, and each regression equation involves the same explanatory variables, the Joint Generalized Least Squares estimator (Zellner, Theil) is identical to equation-by-equation OLS estimator which is best linear unbiased.[4] By restrictions, (4.5), (4.10) and (4.7), the estimates for the omitted equation, say the nth one, are obtained as follows,

(5.8) $\hat{\alpha}_n = 1 - \sum_{i=1}^{n-1} \hat{\alpha}_i$

(5.9) $\hat{\beta}_{nk} = - \sum_{i=1}^{n-1} \hat{\beta}_{ik}$ k = 1, ..., K

(5.10) $\hat{\theta}_n = - \sum_{i=1}^{n-1} \hat{\theta}_i$

where a superscript ^ indicate an estimate.[5] Because the cross-section sample is usually very large, the estimates obtained for α_p, B_{pA} and $B_{pp}i$ should be very accurate. However, B_{pp} is not estimable because of the lack of price variation in the sample.

Section 3 Time-series Estimation

The sequential approach adopted here requires time-series data only in the second step. The aggregate share equations are estimated subject to the estimated values for α_p, B_{pA} and $B_{pp}i$ obtained in the cross-section and the symmetry conditions on B_{pp}. The main concern here is to estimate the individual elements in B_{pp}.

The time-series model is derived from summing up (4.11) across all the households in society. Let subscript t denote time period, so that y_t^h is the total expenditure of household h in period t. In addition, the following short-hand notations are adopted, for $t = 1, \ldots, T$,

(5.11) $\quad D(p_t) = -1 + i^T B_{pp}(\ln p_t)$, a scalar,

(5.12) $\quad S_{yAt} = \sum_h y_t^h A^h / Y_t$, a K-vector, where K = 11 from Table 1,

(5.13) $\quad S_{yyt} = \sum_h y_t^h (\ln y_t^h) / Y_t$, a scalar

$$(5.14) \qquad \varepsilon_t = \Sigma_h y_t^h \varepsilon_t^h / Y_t, \text{ an n-vector}$$

where $\quad Y_t = \sum_{h=1}^{H} y_t^h .$

The time-series regression equations can then be expressed as, in matrix form,

$$(5.15) \qquad E_t = D(p_t)^{-1}\left(\alpha_p + B_{pA}S_{yAt} + B_{pp}(\ell n\, p_t) - (B_{pp}i)S_{yyt}\right) + \varepsilon_t$$

$$t = 1, \ldots, T$$

where ε_t is the disturbance term. The ith equation will be

$$(5.16) \qquad E_{it} = D(p_t)^{-1}\left(\alpha_i + \sum_{k=1}^{K} \beta_{ik}S_{yAkt} + \sum_{j=1}^{n} b_{ij}(\ell n\, p_{jt}) - \theta_i S_{yyt}\right) + \varepsilon_{it}, \qquad i = 1, \ldots, n \quad t = 1, \ldots, T$$

where S_{yAkt} is the kth element in S_{yAt}, $(\ell n\, p_{jt})$ is the jth element in $(\ell n\, p_t)$ and ε_{it} is the ith element in ε_t. It is assumed that the covariance structure of ε_t^h is stationary through time, i.e.,

$$(5.17) \qquad E(\varepsilon_t^h) = E(\varepsilon_s^h) = 0 \qquad s,t = 1, \ldots, T$$

$$(5.18) \qquad \mathrm{var}(\varepsilon_t^h) = \mathrm{var}(\varepsilon_s^h) = \Sigma \qquad s,t = 1, \ldots, T$$

where $\ddagger$ is of rank n-1. It then follows that

$$(5.19) \qquad E(\varepsilon_t) = E\left(\Sigma_h \, y_t^h \, \varepsilon_t^h / Y_t\right) = 0 \qquad\qquad t = 1, \ldots, T$$

and,

$$(5.20) \qquad \mathrm{var}(\varepsilon_t) = \left(\Sigma (y_t^h)^2 \, \mathrm{var}\,(\varepsilon_t^h)\right) / Y_t^2$$

$$= \left(Y_t^{-2} \, \Sigma (y_t^h)^2\right) \ddagger$$

$$=: \Omega_t \qquad\qquad t = 1, \ldots, T$$

since household disturbances are not correlated. It is further assumed that ε_t is not serially correlated, i.e.,

$$(5.21) \qquad \mathrm{var}\,(\varepsilon_1, \ldots, \varepsilon_T) = \begin{pmatrix} \Omega_1 & & \\ & \ddots & \\ & & \Omega_T \end{pmatrix}.$$

Since $\ddagger$ is of rank n-1, Ω_t is also of rank n-1. One equation is redundant and should be omitted because of cross-equation symmetry constraints, despite the fact that all equations have the same explanatory variables.

In the actual estimation, the following parameter restrictions are imposed,

$$(5.22) \qquad \alpha_p = \hat{\alpha}_p$$

$$(5.23) \qquad B_{pA} = \hat{B}_{pA}$$

$$(5.24) \qquad (B_{pp} i) = \hat{\theta}$$

and by symmetry,

$$(5.25) \qquad i^T B_{pp} = \hat{\theta}^T$$

where a superscript ^ denotes an estimated value from the cross-section.

These restrictions can be substituted into (5.15) to obtain

$$(5.26) \qquad \tilde{E}_t = B_{pp} (\ell n\, p_t) + \hat{D}\,(p_t) \varepsilon_t$$

where $\tilde{E}_t = \hat{D}(p_t) E_t - \left(\hat{\alpha}_p + \hat{B}_{pA}\, S_{yAt} - \hat{\theta}\, S_{yyt}\right)$

$$\hat{D}(p_t) = -1 + \hat{\theta}^T (\ell n\, p_t)$$

As defined earlier, b_{ij} is the ijth element in B_{pp} and $\hat{\theta}_i$ is the estimated sum of the ith column (or ith row) of B_{pp}. The ith equation in (5.26) is

$$(5.27) \qquad \tilde{E}_{it} = \sum_{j=1}^{n} b_{ij}(\ell n\, p_{jt}) + \hat{D}(p_t)\,\varepsilon_{it}$$

By restriction (5.24), however,

$$(5.28) \qquad b_{in} = \hat{\theta}_i - \sum_{j=1}^{n-1} b_{ij}$$

Therefore, b_{in}, for all i, should be substituted out of the system. In doing so, the ith equation becomes,

$$(5.29) \qquad Q_{it} = \sum_{j=1}^{n-1} b_{ij}\left((\ell n\, p_{jt}) - (\ell n\, p_{nt})\right) + \hat{D}(p_t)\varepsilon_{it}$$

where $\qquad Q_{it} = \tilde{E}_{it} - \hat{\theta}_i(\ell n\, p_{nt})$

It should be noted that the disturbance term in (5.29), $\hat{D}(p_t)\varepsilon_{it}$ is not classical in structure. It varies with time. Given (5.20), it can easily be verified that heteroscedasticity can be corrected for by the factor

$$(5.30) \qquad \rho_t = \hat{D}(p_t)^{-1}\left(Y_t^{\,2} / \sum_{h=1}^{H} (y_t^h)^2\right)^{\frac{1}{2}}$$

so that

$$(5.31) \qquad E\left(\rho_t(\hat{D}(p_t)\varepsilon_t)\right) = 0 \qquad\qquad t = 1, \ldots, T$$

(5.32) $$\operatorname{var}\left(\rho_t (\hat{D}(p_t)\varepsilon_t)\right) = \ddagger \qquad t = 1, \ldots, T$$

(5.33) $$\operatorname{var}\left(\rho_1 (\hat{D}(p_1)\varepsilon_1), \ldots\ldots, \rho_T (\hat{D}(p_T)\varepsilon_T)\right) = \ddagger \otimes I_T$$

After all these manipulations, the time-series estimation problem is to estimate n-1 linear equations, each containing the same explanatory variables. The ith equation of the system is, for period t,

(5.34) $$\rho_t Q_{it} = \rho_t \sum_{j=1}^{n-1} b_{ij} \left((\ell n\, p_{jt}) - (\ell n\, p_{nt})\right) + \rho_t \hat{D}(p_t)\varepsilon_{it}$$

where $$Q_{it} = \hat{D}(p_t)E_{it} - \left(\hat{\alpha}_i + \sum_{k=1}^{K} \beta_{ik} S_{yAkt} - \hat{\theta}_i S_{yyt}\right) - \hat{\theta}_i (\ell n\, p_{nt})$$

To estimate (5.34), Joint Generalized Least Squares can be applied subject to the symmetry constraints on B_{pp}, i.e.,

(5.35) $$b_{ij} = b_{ji} \quad i, j = 1, \ldots, n-1$$

The number of parameter estimates obtained directly is (n-1) X (n-1). From these estimates, the nth coefficient in each of the n-1 equations and all the coefficients in the nth equation can be derived as follows.

By (5.28),

$$\hat{b}_{in} = \hat{\theta}_i - \sum_{j=1}^{n-1} \hat{b}_{ij} \tag{5.36}$$

By the symmetry constraints, one obtains

$$\hat{b}_{ni} = \hat{b}_{in} \quad i = 1, \ldots, n-1 \tag{5.37}$$

and finally,

$$\hat{b}_{nn} = \hat{\theta}_n - \sum_{j=1}^{n-1} \hat{b}_{nj}. \tag{5.38}$$

To summarize this chapter, by using a two-step approach which involves estimating the cross-section and time-series models sequentially, all the parameters in the equivalent income function can be estimated. This means that, given A^h, y^h and (p, p^o), equivalent income can be imputed to each individual in household h. Of independent interest is the commodity-specific equivalence scales m. These scales can be expressed as, from (4.9),

$$m = \exp\left(B_{pp}{}^{-1} B_{pA} A\right) \tag{5.39}$$

A complete set of equivalence scales for different configurations of A can then be estimated using estimated values for B_{pp} and B_{pA}.

Chapter 5 Footnotes

1. Since α_p and B_{pA} enter both cross-section and time-series models, the two terms in the objective function should be minimized together.

2. The monotonicity conditions in Jorgenson, Lau and Stoker (1982) being implied by integrability of demand functions are not imposed. These conditions ensure negative semi-definiteness of the Jacobian matrix of the cost function only if expenditure shares are restricted to be non-negative. The focus at present is on estimating the parameters in the equivalent income function rather than recovering unknown preferences from hypothesized demand functions. However, the rest of the integrability conditions: summability, homogeneity and symmetry are imposed.

3. Rigorously speaking, this involves redefining the physical units in measuring quantities of commodities.

4. See Theil (1970) Chapter 7.

5. Since there are no cross-equation constraints, an equivalent procedure will be to estimate all n equations independently.

CHAPTER 6 IMPLEMENTATION

This chapter describes how the estimation model in Chapter 5 can be implemented in the Canadian context, using publicly available data. The cross-section model requires expenditure survey micro data, whereas the time-series model requires aggregate time-series statistics that are not readily available from the publications of Statistics Canada. These statistics have to be specially computed. The sequential estimation approach has been carried out and the estimation results can be found in Appendix A and Appendix D. Of special interest are the estimated equivalence scales, while being a governing factor in making interpersonal comparison possible, play a crucial role in the determination of equivalent incomes. These scales can be found in Appendix E and are intuitively very appealing.

Section 1 Cross-section Estimation

The sequential approach adopted here calls for, in the first step, estimation of the micro individual household expenditure share equation,

$$(6.1) \qquad e_i^h = -\alpha_i - \Sigma_{k=1}^{11} \beta_{ik} A_k^h + \theta_i (\ell n\, y^h) + \varepsilon_i^h \qquad i = 1, \ldots, n \quad h = 1, \ldots, H$$

where e_i^h is the expenditure share of the ith good for household h, α_i is the ith element in α_p, β_{ik} is the ikth element in B_{pA}. A_k^h is the kth component in A^h (as already defined in Table 1), θ_i is the ith component in $B_{pp}i$, and ε_i^h is the disturbance term. (6.1) is a regression of the expenditure share of good i on an intercept term, a set of 11 dummy variables and the logarithm of total expenditure. In so far as expenditure shares differ across households of different attributes, these differences will be accounted for by the coefficients β_{ik}'s.

The data set for this regression is derived from the Family Expenditure Survey 1978. This survey provides micro data for 1978 on the expenditure patterns and household characteristics of a representative sample of approximately 10,000 households. The information available allows a classification of 6 composite goods, defined as follows,

(6.2)	"Food"	=	food prepared at home and outside + tobacco and alcoholic beverages
(6.3)	"Clothing"	=	all clothing and footwear
(6.4)	"Recreation"	=	recreation and entertainment + reading materials + education + gifts and contributions

(6.5)	"Personal and medical care"	=	all personal maintenance needs + medical treatment
(6.6)	"Shelter"	=	rent + payment for housing mortgages + water + fuel and electricity + household operations + household furnishings and equipment
(6.7)	"Transportation"	=	automobile and truck services + purchased transportation

The survey provides information on the amount of money each household spends in each of these 6 consumption categories. The sum of these expenditures for household h is taken as y^h in the regression (6.1).

Not all the records contained in the survey enter the data set for regression. Seventy households have been excluded because they are classified in the survey as roomers and they did not pay any rent in 1978. These households might exhibit spending behaviour that deviates from the norm and should be discarded. Consequently, after this screening, the sample size for the cross-section regression is 9285.

Since expenditure shares always sum to 1, the transportation equation, or indeed any one of the 6 equations, can be omitted. Parameters in the transportation equation can be derived from the estimates of the other 5 equations using assumptions (4.5), (4.7) and (4.10). Since the Zellner and Theil Generalized Least Squares method reduces to the equation-by-equation ordinary least squares method, OLS can be applied on each of the remaining 5 equations independently using the data set of 9285 households. The results can be found in Appendix A. These coefficients should be very accurate because the sample size is so large. They are also intuitively appealing. Out of the total 78 coefficients, 66 of them are significant at the 95% level. The coefficient of (log y) is very significant in all equations, implying that homotheticity is an unreasonable restriction. Of all the attributes, family size seems to be most important in affecting expenditure shares. It also displays reasonable trends. For example, increasing family size leads to increasing food share and decreasing transportation share. This is consistent with the notion that food is a necessity and transportation is a luxury.

Thus, using cross-section data enables identification of α_p, $B_{pp}i$ and B_{pA}, i.e., in Appendix A, the first row is $(\hat{B}_{pp}\, i)^T$, the second to the second last rows form $(-\hat{B}_{pA})^T$ and the last row is $-\hat{\alpha}_p^T$.

Section 2 Time-series Estimation

In order to estimate equivalent income in (4.13), estimates for B_{pp} are required in addition to the estimates in Appendix A. However, these estimates can be substituted into the aggregate share equations as if they were true values to generate estimates for B_{pp}. In this second step of the sequential approach, only time-series aggregate data are used. The ith regression equation in the time-series model is, after substitution of $\hat{\alpha}_p$, $\hat{B}_{pA}$ and $\hat{\theta}$ (being a vector of estimates for $B_{pp}i$),

$$(6.8) \qquad \rho_t Q_{it} = \rho_t \sum_{j=1}^{n-1} b_{ij} \big((\ln p_{jt}) - (\ln p_{nt}) \big) + \rho_t \hat{D}(p_t)\, \varepsilon_{it}$$

$$t = 1, \ldots\ldots, T$$

where,

$$(6.9) \qquad Q_{it} = \hat{D}(p_t) E_{it} - \Big(\hat{\alpha}_i + \sum_{k=1}^{k} \beta_{ik} S_{yAkt} - \hat{\theta}_i S_{yyt}\Big) - \hat{\partial}_i (\ln p_{nt})$$

$$(6.10) \qquad \hat{D}(p_t) = -1 + \hat{\theta}^T (\ln p_t)$$

The basic problem then is to compute ρ_t, Q_{it}, $(\ln p_{1t}, \ldots\ldots, \ln p_{nt})$ and $\hat{D}(p_t)$ in (6.8) using published data from Statistics Canada. The usable time-series runs from 1971 to 1981, i.e., 11 observations. The procedures that generate the aggregate statistics for successful estimation of (6.8) are described below.

1. Aggregate expenditure shares, E_t

The aggregate share data are derived from Personal Expenditure on Consumer Goods and Services in Current Dollars, in National Income and Expenditure Accounts Catalogue No. 13-201. Some minor adjustments are necessary to regroup those expenditure items so that the classification in the time-series is consistent with that in the cross-section. The groupings are shown as follows, the numbers in parentheses being the account numbers.

(6.11)	Food	=	(1) Food, beverages and tobacco + ½ x (45) Expenditures on restaurants and hotels
(6.12)	Clothing	=	(5) Clothing and footwear
(6.13)	Recreation	=	(36) Recreation, entertainment, education and cultural services + ½ x (45) Expenditures on restaurants and hotels + (48) Net expenditure abroad
(6.14)	Personal and Medical Care	=	(24) Medical care and health services + (43) Toilet articles, cosmetics + (44) Personal care

(6.15) Shelter $= \Big($(9) Gross rent, fuel and power - (10) Gross imputed rent $\times \dfrac{\text{No. of households without mortgage in 1981}}{\text{No. of households with owned accommodation in 1981}}\Big)$ + (16) Furniture, furnishings, etc. + (42) Jewellery, watches and repairs + (46) Financial, legal and other services.[1]

(6.16) Transportation = (29) Transportation and Communication

As in the cross-section estimation, total aggregate expenditure, Y_t, is taken as the sum of the expenditures on the 6 goods in year t.

2. Expenditure/attribute distribution statistic, S_{yAt}

This statistic is a summary statistic reflecting the distribution of aggregate expenditure over the specified attribute groups. Formally,

$$(6.17) \quad S_{yAt} = \sum_h y_t^h A^h / Y_t$$

A^h is an eleven-vector, and so is S_{yAt}. For example, it is easy to see that the first component is simply the total expenditure of all non-metropolitan households (which have $A_1 = 1$) divided by total population expenditure. In other words, each component in S_{yAt} is the proportion of expenditure in year t that is accounted for by a group of households having a common attribute. Unfortunately, expenditure information of this sort, as opposed to its after-tax income counterpart, is not available on a time-series basis. An acceptable approximation, however, is to use available after-tax income distribution data and derive expenditure distributions by observing the relationship between after-tax income and expenditure in the cross-section sample, allowing the relationship to be attribute-specific. More specifically, the following four cross-section consumption functions corresponding to the four attributes are estimated using the cross-section data set.

Area of Residence

$$(6.18) \quad y^h = a_1 + b_1 z^h + \gamma_1 A_1 + v_1^h$$

Sex of Household Head

$$(6.19) \quad y^h = a_2 + b_2 z^h + \gamma_2 A_2 + v_2^h$$

Household Size

$$(6.20) \quad y^h = a_3 + b_3 z^h + \gamma_{31} A_3 + \gamma_{32} A_4 + \gamma_{33} A_5 + \gamma_{34} A_6 + v_3^h$$

Age of Household Head

$$(6.21) \quad y^h = a_4 + b_4 z^h + \gamma_{41} A_7 + \gamma_{42} A_8 + \gamma_{43} A_9 + \gamma_{44} A_{10} + \gamma_{45} A_{11} + v_4^h$$

y^h and z^h are the total expenditure and after-tax income of household h respectively. A_1 to A_{11} are the dummy variables defined in Table 1. v_1^h, ..., v_4^h are the disturbance terms. It is assumed that in each of the 4 equations there is no contemporaneous covariances in the disturbance terms so that OLS is best linear unbiased. The estimated coefficients for these 4 regressions can be found in Appendix B.

The estimated coefficients for (6.18) to (6.21) are utilized to map the after-tax income distribution series, 1971-1981 to a corresponding S_{yAt} expenditure distribution series. The mapping procedure is as follows. Take the first component of S_{yAt} as an example. It is required to estimate the proportion of total population expenditure that is accounted for by non-metropolitan households. Suppose the estimated consumption function for non-metropolitan households according to (6.18) is

$$(6.22) \quad \hat{y}^k = (\hat{a}_1 + \hat{\gamma}_1) + \hat{b}_1 z^k$$

where k is a non-metropolitan household and a superscript ^ indicates an estimated value. (6.22) represents the relationship between after-tax income and expenditure for a non-metropolitan household. Summing (6.22) across all non-metropolitan households gives the estimated total expenditure of non-metropolitan households

$$(6.23) \quad \sum_{k=1}^{K} \hat{y}^k = K(\hat{a}_1 + \hat{\gamma}_1) + \hat{b}_1 \sum_{k=1}^{K} z^k$$

where K is the number of non-metropolitan households. This quantity can be found if K and Σz^k are available. These can be found in "Income After-tax, Distributions by Size in Canada" Cat. No. 13-210, 1971-81. The total expenditure of metropolitan households can be computed using a similar procedure. For a metropolitan household j, the estimated consumption function is, corresponding to (6.22),

$$(6.24) \quad \hat{y}^j = \hat{a}_1 + \hat{b}_1 z^j$$

because $A_1 = 0$. Accordingly, total expenditure is, for J metropolitan households,

$$(6.25) \quad \sum_{j=1}^{J} \hat{y}^j = J\hat{a}_1 + \hat{b}_1 \sum_{j=1}^{J} z^j$$

which is obtainable given J and Σz^j from the same data source.

Therefore, the first component of S_{yAt} is just the share of non-metropolitan expenditure (6.23) in the sum of non-metropolitan expenditure (6.23) and metropolitan expenditure (6.25), in year t. Other components of S_{yAt} can be calculated by a similar procedure.

3. Expenditure distribution statistics S_{yyt}

This is a statistic that depends on both the distribution and the magnitude of expenditure. Formally,

$$(6.26) \quad S_{yyt} = \sum_h y_t^h (\ln y_t^h)/Y_t$$

Since expenditure is used for y_t^h, as opposed to income, a similar procedure of mapping after-tax income distribution to expenditure distribution is required. However, the cross-section consumption function formulated for this purpose does not include any dummy variables since S_{yyt} ignores household attributes. Consequently, the regression equation is simply

$$(6.27) \quad y^h = a_5 + b_5 z^h + v_5^h$$

where y^h, z^h and v_5^h are the expenditure, after-tax income and disturbance respectively of household h, h = 1, ..., H. It is assumed that there is no contemporaneous covariances in the disturbance term so that OLS is best linear unbiased and this regression equation is estimated using the cross-section data set. The results are found in Appendix B, column 5.

The appropriate income data can be found in "Income After-tax, Distributions by Size in Canada" No. 13-210, 1971-81. In each year, the income spectrum is divided into income brackets and the number of households in each bracket is provided. The mapping procedure is therefore: assume that every household in a particular bracket receives the mid-point income, and estimate its expenditure by using the estimated equation, from (6.27)

$$(6.28) \quad \hat{y}^h = \hat{a}_5 + \hat{b}_5 z^h$$

Assuming that all households in that bracket have the same expenditure, a number for the sum of $y^h \log y^h$ over all households in that bracket is calculated. The procedure is then repeated for other income brackets, and $\Sigma y^h \log y^h$ is obtained by summing over the partial sums in all brackets. Finally, S_{yyt} is obtained by dividing the overall sum by the total expenditure in all brackets. This process is admittedly a rough approximation but since the range of expenditure is rather small and S_{yyt} not very sensitive to expenditure distribution, the margin of error involved is not likely to be significant.

4. Price indexes, p_{it}

The 6 composite goods classified in the present model are similar to those classified in the price series published in "The Consumer Price Index" No. 62-001. Almost no recompilation is required for the price series, although one exception is that in that price series, "food" and "tobacco and alcohol" are classified as separate goods. To combine them into one composite good, the price index for food is obtained as a weighted-average of the price indexes for "food" and "tobacco and alcohol", the weights being the expenditures on the two goods divided by the sum, for a particular year.

A final note about the price series used here is that all price indexes have been normalized so that the price indexes for the survey year, 1978, are all unity, as mentioned in Chapter 4.

5. Heteroscedasticity correction factor ρ_t^2

As explained in Chapter 5, the following factor is necessary to correct for heteroscedasticity in the time-series model,

$$(6.29) \quad \rho_t = \hat{D}(p_t)^{-1}\left((Y_t)^2 / \sum_h (y_t^h)^2\right)^{\frac{1}{2}}$$

The computation of $\hat{D}(p_t)^{-1}$ and $(Y_t)^2$ is straight forward. The computation of $\Sigma(y_t^h)^2$ is performed by using the expenditure information generated in the course of computing S_{yyt}. Again, each household within an income bracket is assumed to receive the mid-point income. Using the estimated consumption function (6.28), the corresponding expenditure is obtained which is then squared and multiplied by the estimated number of households in that bracket. $\Sigma(y_t^h)^2$ is then obtained by summing "the sum of squares" over all brackets. This completes the discussion of time-series data generation. A complete set of time-series data can be found in Appendix C.

Since the covariance matrix in the time-series model is singular, the transportation equation is dropped. Time-series estimation is performed by applying Joint Generalized Least Squares on a system of 5 equations, (5.34) or (6.8), each having the same explanatory variables, subject to the following symmetry constraints,

(6.30) $b_{12} = b_{21},\ b_{13} = b_{31},\ b_{14} = b_{41},\ b_{15} = b_{51}$

(6.31) $b_{23} = b_{32},\ b_{24} = b_{42},\ b_{25} = b_{52}$

(6.32) $b_{34} = b_{43},\ b_{35} = b_{53}$

(6.33) $b_{45} = b_{54}$

where 11 observations 1971 - 1981, are used. The estimation results, namely B_{pp}, can be found in Appendix D.

Section 3 Estimated Equivalence Scales

Using (5.39) and estimated values for B_{pp} and B_{pA}, the commodity-specific equivalence scales can be estimated. Since there are 11 dummy variables describing 4 attributes, there are 120 possible configurations for A. The corresponding 120 sets of equivalence scales can be found in Appendix E. They are intuitively very appealing.

One can readily notice that, as a function of A, the estimated scales display certain general trends.

Comparing the scales for metropolitan area of residence with that for non-metropolitan area of residence, the metropolitan scales are higher for every good except transportation. The apparent reason is that, in urban centres, the cost-of-living is higher and, in addition, certain goods and services in the rural areas are home-produced. However, facilities in rural areas are less concentrated so that rural households need more transportation services.

Comparing the scales for male head with that for female head shows that households with male heads have higher needs in food, shelter and transportation but lower needs in clothing, recreation and personal/medical care. These differences are probably due to the way "sex of household head" is defined. Households with female heads usually imply single-parent families, so that other members in these families are probably children. Therefore, other attributes being equal, compared with a male-headed household, a female-headed household involves substituting a child for an adult.

Comparing the scales for different family sizes is most interesting. As an informal test of the structure of the present model, the scales not only have to be increasing with family size but must fall between certain ranges in absolute magnitude. The scales estimated here are, in general increasing with family size, showing strong economies of scale and are all intuitively appealing in magnitude.[3] In the order of decreasing degree of economies of scale, the 6 goods can be ordered as follows: transportation (strongest), food, shelter, recreation, personal/medical care and clothing (weakest).

Age of the household does not affect the scales very much, although there is a noticeable increase from the first age group (below 24) to the fourth age group (between 44 and 54) and a decline thereafter. Although not very significant, this trend could be explained by the differing levels of activity and needs associated with different age groups.

To summarize this chapter: it has been shown that the trans-log equivalent income, being the measure of utility used in the proposed new inequality index, is estimable using a micro-macro sequential approach. Only survey and aggregate time-series demand data are required for successful estimation. These data are either directly obtainable in public files or indirectly after some simple computations. The estimation results are however very encouraging judging from the estimated equivalence scales in Appendix E. This finding definitely supports the entire approach to inequality measurement.

Chapter 6 Footnotes

1. Information on the proportion of households without mortgage is taken from Household Facilities and Equipment 1977-81, Cat. No. 64-202 (occasional). This statistic is only available for 1981. The adjustment is necessary because (9) includes imputed rent as owned housing.

2. Estimated total expenditures calculated according to different attributes may deviate within 5% which would not affect time-series estimation results significantly.

3. The only exception is in the transportation scales and where family size changes from 3 to 4 — causing a slight decrease. But since transportation shows the strongest economies of scale, this is not entirely surprising and contradictory to common sense.

CHAPTER 7 APPLICATIONS

Section 1 Introduction

It might be useful to recall the development through the previous chapters here. In Chapter 2, it has been established that a new inequality index which is able to measure distributive price effects within a rigorous social welfare evaluation framework is urgently needed. The failure of conventional indexes has also been pointed out. A new index is introduced in Chapter 3 which in theory fills the gap in the literature and should be a significant improvement over the existing indexes. Chapter 4 and 5 provide specification for preferences and estimation algorithms to make implementation of the index possible. Chapter 6 describes the actual estimation process, the handling of data and the intermediate results of estimated equivalence scales. Although the estimation results are satisfactory and intuitively reasonable, whether the new index performs well in practice has yet to be seen. In particular, we want to know if in practice, it really differs significantly from the other indexes and captures price effects in a predictable way. This chapter provides the necessary tests. It is organized as follows. Section 2 discusses the estimated market equivalence scales (3.25) using the estimates obtained in Chapter 6. Due to the translog preferences specified in Chapter 4, these scales are independent of utility and hence equal to the poverty-line ratios (4.21). These scales are compared with the low-income cut-off ratios of Statistics Canada to provide insights to the subsequent comparative studies. Section 3 attempts to show that the new index is not very sensitive to changes in p^o empirically, thus the arbitrariness problem is not serious. Section 4 is a comparative study of the new index with other major indexes, using the 1978 expenditure survey sample as the common data set. Section 5 studies distributive price effects on inequality. A hypothetical increase of 10% in the price of each good is considered in turn. Finally, Section 6 applies this new index to measure inequality in Canada for 1975, 1979 and 1981, using income survey data. For comparison, the same data sets are employed using the welfare-ratio approach with Statistics Canada low-income cut-off values.

Section 2 Estimated Market Equivalence Scales

As explained in Chapter 4, the translog specification of this model implies that market equivalence scales (3.25) and poverty line ratios (4.19) are equal and can be expressed as (4.21). Using the parameter estimates obtained, the estimated scales are presented in Table 2 for all attribute configurations from 1971 to 1981.

In Table 2, the reference household is an unattached male, of age below 24, living in a metropolitan area, and his income is used as the denominator in (3.25) to generate these ratios. According to (3.25) the ratio for this reference household is 1 for all years.

TABLE 2: TRANSLOG MARKET EQUIVALENCE SCALES

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4043	1.4039	1.4030	1.4032	1.4028	1.4029	1.4028	1.4024	1.4033	1.4045	1.4051
3	1.7458	1.7447	1.7429	1.7434	1.7424	1.7423	1.7421	1.7409	1.7429	1.7456	1.7468
4	1.9525	1.9508	1.9484	1.9485	1.9467	1.9456	1.9446	1.9421	1.9439	1.9469	1.9465
5	2.6712	2.6671	2.6617	2.6625	2.6594	2.6554	2.6519	2.6463	2.6503	2.6571	2.6567

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.0469	1.0477	1.0486	1.0480	1.0478	1.0492	1.0504	1.0511	1.0500	1.0484	1.0479
2	1.4709	1.4716	1.4719	1.4713	1.4705	1.4726	1.4743	1.4747	1.4742	1.4733	1.4731
3	1.8292	1.8294	1.8291	1.8286	1.8272	1.8295	1.8314	1.8314	1.8315	1.8317	1.8319
4	2.0437	2.0434	2.0427	2.0417	2.0393	2.0409	2.0422	2.0409	2.0407	2.0408	2.0393
5	2.7931	2.7909	2.7877	2.7871	2.7832	2.7827	2.7823	2.7782	2.7796	2.7826	2.7806

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.1821	1.1825	1.1829	1.1824	1.1820	1.1826	1.1832	1.1832	1.1824	1.1813	1.1803
2	1.6617	1.6618	1.6614	1.6608	1.6599	1.6608	1.6615	1.6610	1.6609	1.6609	1.6601
3	2.0666	2.0660	2.0647	2.0643	2.0625	2.0634	2.0641	2.0628	2.0636	2.0650	2.0646
4	2.3204	2.3192	2.3170	2.3161	2.3132	2.3131	2.3130	2.3100	2.3104	2.3119	2.3095
5	3.1811	3.1773	3.1718	3.1713	3.1666	3.1635	3.1607	3.1539	3.1564	3.1617	3.1585

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.3162	1.3158	1.3152	1.3154	1.3156	1.3145	1.3136	1.3131	1.3135	1.3142	1.3140
2	1.8506	1.8495	1.8475	1.8479	1.8477	1.8463	1.8450	1.8436	1.8455	1.8481	1.8484
3	2.3012	2.2991	2.2957	2.2966	2.2958	2.2937	2.2917	2.2893	2.2927	2.2974	2.2986
4	2.5835	2.5805	2.5760	2.5764	2.5744	2.5710	2.5678	2.5633	2.5665	2.5718	2.5709
5	3.5438	3.5373	3.5282	3.5296	3.5261	3.5180	3.5107	3.5016	3.5082	3.5190	3.5178

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.1320	1.1319	1.1316	1.1319	1.1327	1.1319	1.1312	1.1313	1.1318	1.1324	1.1329
2	1.5917	1.5910	1.5896	1.5902	1.5909	1.5899	1.5888	1.5884	1.5902	1.5924	1.5937
3	1.9796	1.9781	1.9755	1.9766	1.9769	1.9754	1.9738	1.9727	1.9759	1.9800	1.9821
4	2.2107	2.2086	2.2052	2.2060	2.2054	2.2027	2.2001	2.1975	2.2006	2.2051	2.2056
5	3.0244	3.0194	3.0124	3.0142	3.0128	3.0062	3.0002	2.9942	3.0002	3.0095	3.0102

URBAN MALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.9635	0.9638	0.9641	0.9642	0.9649	0.9647	0.9645	0.9650	0.9649	0.9646	0.9648
2	1.3546	1.3545	1.3541	1.3543	1.3549	1.3547	1.3545	1.3547	1.3554	1.3562	1.3569
3	1.6852	1.6846	1.6834	1.6840	1.6842	1.6838	1.6833	1.6830	1.6847	1.6868	1.6882
4	1.8793	1.8783	1.8765	1.8768	1.8763	1.8750	1.8737	1.8722	1.8737	1.8760	1.8759
5	2.5692	2.5660	2.5616	2.5626	2.5615	2.5572	2.5533	2.5493	2.5529	2.5586	2.5585

TABLE 2 (CONTINUED)

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8885	0.8887	0.8888	0.8877	0.8862	0.8870	0.8877	0.8868	0.8854	0.8841	0.8818
2	1.2500	1.2499	1.2492	1.2477	1.2453	1.2465	1.2475	1.2458	1.2447	1.2439	1.2412
3	1.5542	1.5535	1.5520	1.5504	1.5470	1.5483	1.5494	1.5467	1.5461	1.5462	1.5433
4	1.7681	1.7669	1.7644	1.7621	1.7574	1.7583	1.7588	1.7543	1.7532	1.7532	1.7486
5	2.4394	2.4359	2.4305	2.4278	2.4208	2.4197	2.4183	2.4100	2.4098	2.4125	2.4061

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.9301	0.9310	0.9320	0.9302	0.9284	0.9305	0.9324	0.9320	0.9297	0.9269	0.9240
2	1.3092	1.3101	1.3105	1.3082	1.3053	1.3084	1.3110	1.3100	1.3075	1.3048	1.3013
3	1.6283	1.6288	1.6287	1.6262	1.6221	1.6257	1.6288	1.6270	1.6246	1.6223	1.6184
4	1.8505	1.8506	1.8497	1.8463	1.8409	1.8443	1.8470	1.8435	1.8403	1.8377	1.8318
5	2.5505	2.5488	2.5454	2.5412	2.5332	2.5355	2.5371	2.5300	2.5272	2.5263	2.5183

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.0616	1.0622	1.0626	1.0607	1.0586	1.0601	1.0615	1.0603	1.0579	1.0554	1.0518
2	1.4951	1.4954	1.4950	1.4925	1.4891	1.4913	1.4932	1.4910	1.4886	1.4864	1.4820
3	1.8596	1.8594	1.8582	1.8553	1.8506	1.8532	1.8553	1.8520	1.8498	1.8483	1.8433
4	2.1240	2.1231	2.1207	2.1168	2.1104	2.1126	2.1141	2.1085	2.1055	2.1037	2.0965
5	2.9365	2.9331	2.9271	2.9223	2.9129	2.9133	2.9129	2.9024	2.9000	2.9006	2.8908

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.1791	1.1790	1.1785	1.1771	1.1753	1.1754	1.1756	1.1738	1.1724	1.1712	1.1680
2	1.6609	1.6602	1.6584	1.6565	1.6536	1.6539	1.6540	1.6509	1.6500	1.6499	1.6460
3	2.0656	2.0641	2.0610	2.0590	2.0548	2.0550	2.0549	2.0503	2.0501	2.0513	2.0472
4	2.3589	2.3564	2.3518	2.3489	2.3430	2.3423	2.3412	2.3341	2.3332	2.3346	2.3280
5	3.2630	3.2573	3.2480	3.2445	3.2356	3.2318	3.2276	3.2146	3.2153	3.2206	3.2117

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.0014	1.0015	1.0014	1.0004	0.9994	0.9996	0.9998	0.9989	0.9979	0.9969	0.9947
2	1.4105	1.4103	1.4092	1.4079	1.4061	1.4065	1.4068	1.4050	1.4045	1.4043	1.4018
3	1.7545	1.7536	1.7515	1.7503	1.7475	1.7478	1.7479	1.7451	1.7453	1.7463	1.7437
4	1.9931	1.9915	1.9883	1.9863	1.9824	1.9819	1.9812	1.9765	1.9762	1.9772	1.9727
5	2.7497	2.7455	2.7388	2.7366	2.7305	2.7274	2.7241	2.7151	2.7163	2.7206	2.7146

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8500	0.8504	0.8508	0.8498	0.8490	0.8496	0.8502	0.8498	0.8484	0.8468	0.8448
2	1.1970	1.1973	1.1971	1.1958	1.1943	1.1952	1.1960	1.1950	1.1939	1.1927	1.1903
3	1.4894	1.4893	1.4884	1.4870	1.4848	1.4858	1.4866	1.4848	1.4841	1.4836	1.4811
4	1.6896	1.6889	1.6873	1.6852	1.6820	1.6824	1.6826	1.6794	1.6781	1.6776	1.6733
5	2.3293	2.3268	2.3225	2.3202	2.3151	2.3136	2.3120	2.3054	2.3050	2.3067	2.3010

TABLE 2 (CONTINUED)

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8648	0.8646	0.8643	0.8647	0.8651	0.8650	0.8647	0.8649	0.8656	0.8664	0.8674
2	1.2133	1.2126	1.2115	1.2122	1.2124	1.2124	1.2120	1.2119	1.2136	1.2158	1.2178
3	1.5077	1.5063	1.5043	1.5055	1.5053	1.5051	1.5044	1.5038	1.5066	1.5103	1.5132
4	1.6743	1.6725	1.6700	1.6710	1.6702	1.6690	1.6678	1.6661	1.6690	1.6730	1.6747
5	2.2815	2.2776	2.2725	2.2744	2.2729	2.2690	2.2655	2.2615	2.2667	2.2746	2.2769

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.9058	0.9062	0.9067	0.9067	0.9068	0.9080	0.9088	0.9095	0.9093	0.9088	0.9094
2	1.2715	1.2717	1.2716	1.2717	1.2716	1.2733	1.2744	1.2750	1.2755	1.2760	1.2773
3	1.5805	1.5803	1.5795	1.5799	1.5793	1.5812	1.5824	1.5827	1.5840	1.5856	1.5878
4	1.7534	1.7528	1.7517	1.7518	1.7505	1.7516	1.7524	1.7517	1.7529	1.7546	1.7554
5	2.3869	2.3846	2.3813	2.3820	2.3798	2.3790	2.3781	2.3754	2.3785	2.3832	2.3843

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.0181	1.0182	1.0183	1.0184	1.0185	1.0189	1.0191	1.0194	1.0195	1.0195	1.0198
2	1.4299	1.4297	1.4289	1.4292	1.4290	1.4295	1.4298	1.4298	1.4308	1.4321	1.4332
3	1.7775	1.7766	1.7750	1.7756	1.7749	1.7754	1.7755	1.7749	1.7770	1.7797	1.7816
4	1.9818	1.9804	1.9781	1.9784	1.9768	1.9764	1.9759	1.9740	1.9759	1.9789	1.9792
5	2.7062	2.7025	2.6973	2.6983	2.6957	2.6925	2.6895	2.6848	2.6891	2.6959	2.6964

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	1.1345	1.1340	1.1331	1.1338	1.1345	1.1334	1.1323	1.1322	1.1334	1.1350	1.1362
2	1.5938	1.5924	1.5902	1.5914	1.5920	1.5905	1.5890	1.5882	1.5910	1.5947	1.5970
3	1.9810	1.9787	1.9751	1.9769	1.9771	1.9751	1.9729	1.9713	1.9757	1.9816	1.9850
4	2.2083	2.2053	2.2009	2.2024	2.2018	2.1984	2.1952	2.1921	2.1966	2.2031	2.2049
5	3.0171	3.0111	3.0027	3.0056	3.0041	2.9966	2.9897	2.9831	2.9911	3.0030	3.0055

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.9810	0.9807	0.9801	0.9808	0.9819	0.9811	0.9802	0.9805	0.9817	0.9831	0.9848
2	1.3782	1.3772	1.3755	1.3767	1.3779	1.3768	1.3756	1.3755	1.3781	1.3813	1.3841
3	1.7133	1.7115	1.7087	1.7105	1.7115	1.7100	1.7082	1.7075	1.7116	1.7167	1.7207
4	1.8998	1.8975	1.8941	1.8957	1.8962	1.8935	1.8909	1.8891	1.8933	1.8988	1.9015
5	2.5887	2.5840	2.5774	2.5803	2.5803	2.5742	2.5685	2.5641	2.5713	2.5815	2.5852

RURAL MALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8361	0.8362	0.8361	0.8366	0.8375	0.8373	0.8369	0.8375	0.8380	0.8385	0.8397
2	1.1744	1.1740	1.1732	1.1740	1.1750	1.1748	1.1742	1.1746	1.1762	1.1779	1.1800
3	1.4604	1.4595	1.4579	1.4591	1.4600	1.4595	1.4586	1.4587	1.4612	1.4644	1.4674
4	1.6172	1.6159	1.6139	1.6149	1.6153	1.6139	1.6124	1.6116	1.6141	1.6175	1.6194
5	2.2021	2.1989	2.1945	2.1966	2.1966	2.1925	2.1888	2.1859	2.1908	2.1976	2.2002

TABLE 2 (CONTINUED)

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.7549	0.7549	0.7549	0.7543	0.7534	0.7540	0.7545	0.7539	0.7534	0.7529	0.7519
2	1.0610	1.0608	1.0600	1.0594	1.0578	1.0586	1.0593	1.0582	1.0582	1.0584	1.0574
3	1.3186	1.3179	1.3164	1.3158	1.3135	1.3144	1.3150	1.3133	1.3138	1.3151	1.3141
4	1.4896	1.4883	1.4862	1.4852	1.4820	1.4823	1.4824	1.4794	1.4796	1.4810	1.4787
5	2.0470	2.0439	2.0392	2.0383	2.0334	2.0319	2.0304	2.0245	2.0261	2.0300	2.0269

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.7906	0.7913	0.7919	0.7909	0.7898	0.7914	0.7928	0.7928	0.7914	0.7898	0.7882
2	1.1119	1.1124	1.1126	1.1113	1.1094	1.1118	1.1137	1.1133	1.1121	1.1108	1.1091
3	1.3823	1.3825	1.3822	1.3808	1.3780	1.3808	1.3831	1.3821	1.3813	1.3805	1.3788
4	1.5598	1.5597	1.5588	1.5569	1.5531	1.5555	1.5575	1.5553	1.5540	1.5531	1.5499
5	2.1414	2.1397	2.1368	2.1346	2.1289	2.1303	2.1312	2.1263	2.1258	2.1268	2.1225

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8983	0.8987	0.8989	0.8978	0.8965	0.8976	0.8986	0.8979	0.8967	0.8953	0.8933
2	1.2640	1.2641	1.2636	1.2622	1.2599	1.2616	1.2629	1.2616	1.2606	1.2598	1.2575
3	1.5715	1.5711	1.5698	1.5684	1.5651	1.5670	1.5684	1.5663	1.5658	1.5658	1.5634
4	1.7822	1.7813	1.7792	1.7770	1.7726	1.7739	1.7749	1.7711	1.7701	1.7701	1.7660
5	2.4542	2.4512	2.4462	2.4438	2.4371	2.4368	2.4360	2.4286	2.4287	2.4313	2.4257

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.9986	0.9983	0.9978	0.9971	0.9961	0.9960	0.9959	0.9948	0.9945	0.9943	0.9928
2	1.4053	1.4045	1.4028	1.4020	1.4002	1.4002	1.4000	1.3980	1.3984	1.3995	1.3978
3	1.7469	1.7454	1.7426	1.7419	1.7392	1.7389	1.7385	1.7354	1.7367	1.7392	1.7377
4	1.9809	1.9786	1.9747	1.9734	1.9695	1.9684	1.9671	1.9621	1.9631	1.9658	1.9625
5	2.7293	2.7243	2.7165	2.7153	2.7093	2.7053	2.7013	2.6919	2.6949	2.7016	2.6971

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.8526	0.8526	0.8523	0.8519	0.8515	0.8515	0.8515	0.8510	0.8509	0.8507	0.8499
2	1.1999	1.1995	1.1983	1.1979	1.1970	1.1970	1.1970	1.1959	1.1965	1.1974	1.1967
3	1.4918	1.4908	1.4888	1.4885	1.4870	1.4869	1.4867	1.4849	1.4862	1.4883	1.4879
4	1.6827	1.6811	1.6783	1.6776	1.6751	1.6743	1.6734	1.6702	1.6713	1.6736	1.6718
5	2.3123	2.3086	2.3028	2.3023	2.2984	2.2952	2.2920	2.2855	2.2885	2.2941	2.2916

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
1	0.7246	0.7249	0.7251	0.7246	0.7243	0.7247	0.7250	0.7249	0.7243	0.7236	0.7227
2	1.0196	1.0196	1.0193	1.0187	1.0180	1.0185	1.0190	1.0185	1.0184	1.0183	1.0174
3	1.2681	1.2677	1.2668	1.2663	1.2650	1.2656	1.2660	1.2650	1.2654	1.2661	1.2654
4	1.4284	1.4276	1.4261	1.4252	1.4231	1.4231	1.4231	1.4209	1.4210	1.4218	1.4199
5	1.9614	1.9591	1.9553	1.9546	1.9513	1.9495	1.9478	1.9431	1.9445	1.9476	1.9449

Table 2 shows that, in general, for each attribute configuration, market equivalence scales are not very sensitive to the price changes experienced in 1971 - 1981. However, they do change with attributes. The scales are increasing with family size although by decreasing increments (except for the change from size 4 to size 5 because the latter category includes all larger family sizes). Furthermore, the scales are in general slightly higher for households with male heads than those with female heads. This means that, all goods considered, families with female heads have slightly lower consumption requirements. This is a reasonable result as families with female heads are usually single-adult families and children normally consume less. One also notes with interest that the scales are higher for metropolitan households than non-metropolitan households (reflecting the higher "cost-of-living" — broadly defined — in the urban cities). Finally, the scales vary with age of head in the same manner as the commodity-specific equivalence scales do, i.e., increase slightly with age up to the 44-54 bracket and decrease thereafter.

Because of translog preferences, market equivalence scales and poverty-line ratios are identical as given by (4.21). Thus Table 2 can be regarded as the estimated translog poverty-line ratios. Intuitively, these ratios indicate the number of equivalent male-adults (of age below 24 in a metropolitan area) for each household.

There are other data sources from which one can derive these equivalent-adults ratios. As suggested in Wolfson (1979), one can use the low-income cut-off values published by Statistics Canada. These cut-off values are classified by family size and size of area of residence. Using a metropolitan unattached individual as the reference household, one can compute similar poverty-line ratios. Table 3 gives the results using cut-off values published for 1975 and 1981.[1] These ratios show significant economies of scale and that non-metropolitan households have overall lower-consumption needs.

The ratios in Table 2 and Table 3 can be compared, although the difference in the classification of attributes precludes comparison on a one-to-one basis. However, the general impression is the ratios derived from the two distinctly different approaches are rather close. This finding has significant bearing on the following comparative study of various inequality indexes.

Section 3 Sensitivity of the IEI index to p^o

Before comparing the various inequality indexes, it is useful to first examine the sensitivity of the new index, named the Individual Equivalent-Income (IEI) inequality index, to changes in the arbitrary price vector p^o empirically. The sensitivity analysis is undertaken by using the Family Expenditure Survey 1978 sample which contains 9285 households and 27651 individuals. Following the procedure

Table 3

Statistics Canada Low-Income Cut-off Ratios

Family Size	1975		1981	
	Metropolitan	Non-Metropolitan	Metropolitan	Non-Metropolitan
1	1.0000	0.8241	1.0000	0.8255
2	1.4496	1.1951	1.3165	1.0828
3	1.8496	1.5250	1.7604	1.4514
4	2.1997	1.8131	2.0326	1.6776
5	2.4588	2.0275	2.3610	1.9483
6	2.6996	2.2252	2.5770	2.1258
7	2.9697	2.4395	2.8403	2.3433

Source: Income Distribution by Size in Canada 1975, 1981. Catalogue No. 13-207 (annual)

Notes: (1) Given family size and area of residence, a low-income cut-off value is derived arbitrarily by setting it equal to the average observed income of those households who spend 20% more than the average Canadian household does on food, clothing and shelter.

(2) The cut-off values for metropolitan households are taken as the means of the cut-off values for the 3 population brackets: over 500,000, 100,000-499,999 and 30,000-99,9999. The cut-off values for non-metropolitan households are taken as the means of the cut-off values for the remaining 2 population brackets: less than 30,000 and rural.

explained in Chapter 3, equivalent income is imputed to each individual taking 1978 prices as p. Atkinson index is then computed for r set equal to 0.5, -1 and $-\infty$. Computation is repeated for p^o set equal to the actual prices experienced in various years from 1971 to 1981.[2]

The results are presented in Table 4. Although there is no benchmark by which assessment of sensitivity can be made, the general impression is that the new IEI index is not very sensitive to p^o. It ranges from, for r = 0.5, 0.046909 for 1981 prices to a maximum of 0.049461 for 1971 prices, i.e., a deviation range of ±2.6%.

Section 4 Comparative Study of Various Measures

As described in Chapter 2, various measures of utility have been used for inequality measurement and because they are all unsatisfactory, a new index, the IEI index, is developed in Chapter 3. It is interesting, however, to compare empirically these indexes and see if they are really significantly different.

To carry out this comparative study, a common data set, namely the Family Expenditure Survey 1978 sample, is used which contains 9285 households and 27651 individuals. p is taken as 1978 prices and p^o, if applicable is taken as 1971 prices. The sample contains information on household expenditures and attributes for each household, so that equivalent income (3.19) or (4.13) can be imputed to each of the 27651 individuals. The results are presented in Table 5.

Table 4: Sensitivity of IEI measure to p°

		r=.5		r=-1		r=-∞	
p°	μ	ξ	I	ξ	I	ξ	I
1971	4915.56	4672.43	.049461	4009.63	.184299	565.24	.885011
1973	5469.58	5207.04	.048000	4489.12	.179256	650.93	.880991
1975	6688.20	6371.17	.047401	5503.17	.177182	807.35	.879288
1977	7775.47	7405.16	.047626	6391.74	.177962	933.59	.879932
1978	8411.36	8019.14	.046629	6943.50	.174507	1034.23	.877044
1979	9155.78	8731.40	.046351	7566.92	.173536	1133.31	.876220
1981	11315.12	10784.35	.046909	9329.60	.175475	1382.00	.877863

p : 1978 prices

No. of entries : 27651

Data Set:Family Expenditure Survey 1978

Table 5: Comparison of Different Measures

"Income" definition	No. of entries	μ	r=.5 ξ	r=.5 I	r=0 ξ	r=0 I	r=-1 ξ	r=-1 I	r=-2 ξ	r=-2 I	r=-∞ ξ	r=-∞ I
1. Household Expenditure (HE)	9285	15159.73	14055.78	.072821	12920.24	.147726	10602.41	.300620	8451.11	.442529	2155.00	.857847
2. Per Capita Expenditure (PCE)	27651	5100.91	4765.96	.065700	4461.80	.125295	3923.42	.230839	3454.99	.322673	808.20	.841558
3. Household Inflated Welfare-Ratio(HIWR) (Stat.Can.)	9285	9119.93	8622.06	.054591	8146.76	.106707	9119.93	.204757	6433.84	.294529	1170.00	.871710
4. Individual Inflated Welfare-Ratio(IIWR) (Stat.Can.)	27651	9006.28	8586.14	.046650	8183.37	.091371	7417.71	.176385	6698.10	.256285	1170.00	.870091
5. Individual Inflated Welfare-Ratio(IIWR) (estimated)	27651	8411.33	8019.11	.046630	7645.73	.091020	6943.50	.174507	6289.10	.252306	1034.23	.877044
6. Household Equivalent Income(HEI)	9285	5081.52	4797.56	.055880	4529.06	.108719	4029.71	.206986	3573.10	.296844	565.24	.888767
7. Individual Equivalent Income(IEI)	27651	4915.56	4672.43	.049461	4441.74	.096392	4009.63	.184299	3608.94	.265813	565.24	.885011

p° : 1971 prices

Data Set:Family Expenditure Survey 1978

In each case, inequality is calculated using the Atkinson index with r set equal to 0.5, 0, -1, -2 and $-\infty$. μ is the mean of the distribution and ξ is the evenly-distributed equivalent of the distribution while I is the Atkinson index defined in (3.32) or (3.36) (3.37) with the corresponding utility measure in each case.

In Table 5, the first inequality measure computed is the Household Expenditure (HE) index which neglects family size, even distribution of social weights and other household attributes. The mean of the distributions is 15159.73. For $r = 0.5$, ξ is 14055.78 giving an inequality measure of 0.072821. An improved index is the second index computed, namely the Per Capita Expenditure (PCE) index. This method imputes per-capita expenditure to each individual giving rise to a significant decrease of the mean. Inequality also drops suggesting that larger households have higher incomes while the HE index has ignored this correlation.

The PCE index ignores economies of scale and other relevant attributes that might affect preferences. Wolfson (1979) and Beach, Card and Flatters (1981) employ welfare ratios as utility measures to capture the scale effects. Although they use income data, what they would have done with expenditure data would be to divide household expenditure by the poverty income for the appropriate attributes and inequality is computed for a distribution of these welfare-ratios.

To make comparisons with other indexes more immediate, "inflated welfare-ratios" are used here in place of welfare-ratios. "Inflated welfare-ratios" are welfare ratios multiplied by the poverty income of a reference household, namely, an unattached individual in a metropolitan area. Since the Atkinson index is relative, the index is not affected by this modification. If one uses the cut-off values published by Statistics Canada, then the "inflated welfare-ratios" will be just household expenditure divided by the appropriate ratios in Table 3.

In Table 5, the third index, namely Household Inflated Welfare-Ratio (HIWR) (Stat. Can.) index, is computed by imputing "inflated welfare ratios" to each of the 9285 households based on the 1975 ratios (identical with those published for 1978) in Table 3. This is the method according to Wolfson (1979) and Beach et. al. (1981). It shows a further decrease in inequality. I is 0.054591 at r = 0.5. However, as explained in Chapter 3, a more acceptable procedure is to impute to all 27651 individuals. Making this alteration, the fourth index named the Individual Inflated Welfare Ratio (IIWR) (Stat. Can.) index is computed which interestingly, is appreciably smaller than the HIWR (Stat. Can.) index. I is 0.046650 at r = 0.5. This suggests that the conventional welfare ratio index offers a distorted picture of the actual inequality situation, hence should be avoided.

The "inflated welfare ratios" are actually expenditures denominated by the poverty-line ratios. The Statistics Canada low-income cut-offs are arbitrarily derived. It is therefore interesting to employ the estimated translog poverty-line ratios for 1978, as given in Table 2 and repeat the above inequality computation. This gives rise to the Individual Inflated Welfare-Ratio (IIWR) (estimated) index in Table 5. As shown, the two IIWR indexes are actually very close empirically. This comes as no surprise because comparing Table 2 and Table 3, the poverty-line ratios are not significantly different. In choosing between the two indexes, the IIWR (est.) is preferred because Statistics Canada low-income cut-offs are not regularly updated for relative price changes.

The last two indexes computed make use of the equivalent income measure of utility explained in Chapter 3. The sixth index is erroneous. Equivalent incomes are imputed to households, as opposed to individuals. It is presented here just to show that empirically it does make a difference if one imputes to households rather than individuals, regardless of the utility measurement concept — equivalent income or inflated welfare ratio. The last index, the Individual Equivalent Income (IEI) index is the new index proposed in the present thesis. Its social welfare foundation is explained in Chapter 3. For r = 0.5, the IEI index is 0.049461 which is appreciably different from the HEI index. However, one cannot conclude that IEI and IIWR (estimated) are empirically different. The reason is the IEI index depends on p^o which is somewhat arbitrary. If p^o is taken as 1978 prices, then it is easy to see that, in (3.23), $\pi(\cdot)$ in the denominator will vanish so that equivalent income is identical to an inflated welfare ratio. In that case, the IEI index and the IIWR (estimated) indexes will coincide. Therefore, based on Table 5, one can assert that, the IIWR (Stat. Can.) index, the IIWR (est.) index and the proposed IEI index are similar empirically but as a group are different from the other four indexes.

However, the proposed IEI index has two advantages over the two IIWR indexes. Firstly, it is justifiable in terms of social welfare evaluation.[3] Secondly, it captures quantitatively distributive price effects on inequality. Referring again to (3.23), since both $S(\cdot)$ and $\pi(\cdot)$ are sensitive to prices, the IEI index is more price-sensitive than the other two indexes.

Section 5 Distributive Price effects

The poor households, relative to the rich households, spend a larger proportion of their household budgets on the necessities. Roberts (1982) is an attempt to show that increases in food price affect the cost-of-living indexes of the poor more than that of the rich. It seems reasonable to conjecture that food price inflation might have a negative impact on the inequality situation. The new IEI index is a valuable tool to demonstrate this impact empirically.

To examine the price-sensitivity of inequality, the price of each good is raised in turn by 10% from the 1978 level. The data set is again the Family Expenditure Survey 1978 sample and p^o is taken as 1971 prices. The results are shown in Table 6. The most interesting result is that an increase in food price actually increases inequality but increases in recreation price and transportation price actually decrease inequality. In the latter two cases, the rich are hurt more by the price increases than the poor, although everyone in society is inevitably worse off. For example, a 10% increase in food, recreation and transportation prices causes respectively, for $r = 0.5$, 2.3% increase, 0.9% decrease, 1.6% decrease in inequality. This is broadly consistent with the usual classification of necessities and luxuries. On the other hand, price changes in clothing, personal/ medical care and shelter have negligible effects on inequality. This is not surprising because these consumption items are highly aggregated and cannot reasonably be classified as necessities or luxuries.

Section 6 Inequality Trend

It has been shown in Table 5 that the IIWR (Stat. Can.) and IIWR (est.) indexes are empirically very close to the proposed IEI index, the reason being that p^o is arbitrary and when set equal to 1978 prices, $\pi(\cdot)$ in the denominator in (3.23) vanishes rendering the IIWR (est.) and IEI index identical. However, it is plausible that if one fixes p^o and examines inequality trend on a time-series

Table 6: Sensitivity of IEI measure to p

			r=.5		r=0		r=-1		r=-2		r=-∞	
	Commodity	μ	ξ	I	ξ	I	ξ	I	ξ	I		I
0%	-	4915.56	4672.43	.049461	4441.74	.096392	4009.63	.184299	3608.94	.265813	565.24	.885011
	Food	4795.16	4552.59	.050587	4322.73	.098523	3892.88	.188165	3495.09	.271121	535.52	.888112
	Clothing	4877.85	4636.89	.049399	4408.28	.096266	3980.17	.184031	3583.32	.265390	564.27	.884320
10%	Recreation	4863.29	4624.65	.049069	4398.10	.095654	3973.49	.182962	3579.50	.263976	563.56	.884120
	P/M Care	4893.39	4651.13	.049507	4421.26	.096482	3990.73	.184464	3591.54	.266042	562.26	.885098
	Shelter	4725.69	4491.60	.049537	4269.41	.096553	3853.02	.184664	3466.64	.266427	539.96	.885740
	Trans.	4838.22	4602.69	.048682	4379.06	.094905	3959.84	.181551	3570.75	.261971	567.98	.882605
20%	Food	4683.66	4441.87	.051625	4213.02	.100486	3785.72	.191718	3391.02	.275990	511.12	.890870

p^0 : 1971 prices

No. of entries : 27651

basis, the two methods might suggest mutually conflicting trends. This would most likely happen if the expenditure distribution is stable over time while relative prices are widely fluctuating, because $\pi(\cdot)$ makes the IEI index more price-sensitive.

An informal test can be carried out to ascertain if this conjecture holds in Canada. Since no additional expenditure survey data are available in the time-series period, income survey data have to be used for trend analyses. Used here are the income survey samples of 1975, 1979 and 1981.[4] After-tax income are treated as if they were expenditures. Furthermore, only positive after-tax incomes are brought into computation. Since after-tax income has a much higher variance than expenditure, inequality measures computed here are much higher than those computed using expenditures. The IEI indexes can be found in Table 7 and the IIWR (Stat. Can.) measures in Table 8.

As evident in Table 7 and Table 8, both the IEI index and the IIWR (Stat. Can.) index suggest that inequality is highest in 1979, followed by 1981 and lowest in 1975. The trends suggested by the two indexes are consistent with one another. The probable reason is: although the IEI index is more sensitive to p, the changes in relative prices experienced in Canada in the last decade are not drastic enough to allow the price index $\pi(\cdot)$ in (3.23) to affect the inequality measure so much that it contradicts the simplistic IIWR (Stat. Can.) index in a trend comparison.

Table 7:Inequality trend (IEI measure)

Year	size of sample	No.of Entries	μ	r=.5		r=0		r=-1		r=-2	
				ξ	I	ξ	I	ξ	I	ξ	I
1975	26569	78640	5006.92	4653.22	.070642	4274.72	.146238	3146.69	.371532	920.48	.816159
1979	39906	105785	5385.98	4978.82	.075600	4532.98	.158374	3074.36	.429192	639.43	.881279
1981	40308	103961	5510.25	5115.82	.071581	4695.11	.147932	3416.67	.379943	801.54	.854537

p° :1971 prices

Table 8:Inequality trend (IIWR(Stat. Can.) measure)

Year	size of sample	No.of Entries	μ	r=.5 ξ	r=.5 I	r=0 ξ	r=0 I	r=-1 ξ	r=-1 I	r=-2 ξ	r=-2 I
1975	26569	78640	7386.01	6869.75	.069897	6315.03	.145001	4707.72	.362617	1677.28	.772911
1979	39906	105785	10747.35	9949.89	.074201	9072.94	.155798	6264.40	.417122	1566.59	.854235
1981	40308	103961	14452.25	13406.39	.072366	12287.10	.149814	8927.17	.382299	2248.46	.844422

Conclusion

In Chapter 3, it has been established that the proposed IEI index is superior to all other existing indexes in its social evaluation foundation. In this chapter, it is shown that this superiority extends to the empirical scene. The empirical evidence indicates that empirically, the IEI index exhibits a different inequality scenario from those by the other major indexes. More importantly, it shows convincingly the distributive price effects on inequality. Food price inflation aggravates inequality while transportation price inflation ameliorates inequality!

Other results are not as clear-cut. The IIWR (Stat. Can.) and IIWR (est.) indexes (both being improvements over the Wolfson (1979) index) approximate the IEI index closely. Even in the dynamic sense where the inequality trends indicated by the IEI index and the IIWR (Stat. Can.) index are compared, the price changes are not drastic enough to cause a conflict in trend although such a conflict is likely if price changes are large enough. One cannot rule out such price changes in the future. Therefore, considering price-sensitivity and the justifiability of the social-evaluation procedure, the IEI index is still the preferred index.

One can conclude that, judging from empirical evidence and the social welfare foundation, the IEI index proposed in Chapter 3 should be adopted in place of all other existing indexes.

Chapter 7 Footnotes

1. Statistics Canada update the cut-off values every year for inflation only so that the poverty-line ratios are constant. However, major revisions are done after each family expenditure survey. The 1969 survey implies ratios for 1971 - 1979, while the 1978 survey implies ratios for 1980 - 1982.

2. Since the Atkinson index is relative, only relative prices in p^o matter. Therefore choosing actual prices is not too restrictive in studying sensitivity.

3. It can easily be verified that the inflated welfare ratio is not ordinally equivalent to the indirect utility function. It can be obtained by solving for y* in,

$$V(y, p, A) = V(y^*, p, A^o)$$

so that

$$y^* = C\left(V(y, p, A), p, A^o\right)$$

However, letting,

$$y_1^* = C\left(V(y_1, p_1, A), p_1, A^o\right)$$
$$y_2^* = C\left(V(y_2, p_2, A), p_2, A^o\right)$$

it is not true that

$$y_1^* \geqq y_2^*$$

if and only if

$$V(y_1, p_1, A) \geqq V(y_2, p_2, A)$$

unless $p_1 = p_2$. This fails y* as an exact utility indicator.

4. The data files are known as Economic Family Incomes, 1975; Census Family Incomes, 1979; Census Family Incomes, 1981. The difference in family definitions is not believed to affect measured inequality significantly.

CHAPTER 8 CONCLUSION

Every economics student knows that, through the budget constraint, attainable utility depends on prices. Since the rich consume more luxuries relative to necessities than the poor, changes in relative prices will affect persons on different utility levels differently. It then follows intuitively that relative price changes have distributive effects, hence affect inequality. However, it is somewhat surprising that despite the existence of empirical evidence substantiating this claim, a satisfactory inequality index that is able to capture these effects is absent in the literature. Most of the existing indexes are calculated based on distributions of incomes or expenditures or some simple adjustments of the two. Two notable exceptions are the Muellbauer (1974) approach and the Jorgenson-Slesnick (1984) approach which, while being worthwhile attempts, are not completely satisfactory in their somewhat ad hoc social evaluation frameworks. By contrast, the present research results in the establishment of a new index that is not subject to these criticisms.

The inequality implication of social choice theories is clear. As a result, the IEI index proposed in this thesis is based on an explicit social welfare evaluation foundation. What is required to generate a new index is the following: a social evaluation framework, a price-sensitive numerical measure of utility, an appropriate social welfare function and a formula for inequality measurement. The second requirement above poses the greatest challenge because of the absence of an objective scale of utility measurement and the absence of behavioral data such as individual demand data and the fact that certain human and environmental characteristics affect the relationship between consumption and utility. To cope with these difficulties, the present model incorporates attribute parameters into the utility function and assumes a particular level of interpersonal comparison of utilities, which result in a numerical representation of utility. This measure is named equivalent income. An equivalent income measure is imputed to each individual in society. A distribution of equivalent incomes then form the basis of inequality measurement in a welfarist social welfare framework.

Besides the theoretical contribution of providing an inequality index that is based on a rigorous social welfare framework, the present research is also marked by its impressive empirical results. Numerically, it indicates a different inequality scenario from those indicated by the major existing indexes. Apparently, the theoretical mis-specification problem that plagues these indexes has turned them into unworthy empirical tools. Furthermore, the proposed IEI index successfully measures distributive price effects. Food price increases do have an aggravating impact on inequality while the opposite is true for transportation. This finding conforms reasonably with the common classification of luxuries and necessities. In addition, one should realize that these are not artificial mechanical results. Neither the translog equivalent scales specification nor any structural assumption in the estimation model necessarily drives these results.

However, this approach does have its limitations. The most fundamental one is that p^o in the equivalent income function is arbitrary. In inequality measurement, it becomes an extra parameter in addition to r, the degree of inequality aversion. While measured inequality is not very sensitive to the choice of p^o empirically, nevertheless, it is impossible to pinpoint precisely the degree of inequality which creates some vagueness in exercises such as inter-country comparisons. Furthermore, the estimation model, while being quite apart from the inequality measurement framework, could be improved in several directions. Firstly, to avoid simultaneous equation bias, the production side of the economy could be incorporated. The model adopted here is a limited-information model which does not make use of supply data. Secondly, in the specification of preferences, assumptions are imposed to arrive at linear expenditure share equations. While they simplify estimation, these assumptions might not be consistent with actual consumer behavior. Thirdly, data availability imposes severe constraints on the number of attributes that can be incorporated into the specification of preferences. The present research, for the sake of credibility, only makes use of publicly available data. But the estimation results will definitely benefit from additional micro data concerning other relevant attributes. For example, household composition is an attribute that plausibly affects the relationship between consumption and utility.

One should notice the implementation advantages of the IEI index. It might seem that this index is very costly to implement, in view of the complexities of the model. This is not true. When implementing this index, the additional work that it requires is in improving the estimates as new demand data become available. This is not costly because all the procedures involved can be executed by computer programs, the feasibility of which have been demonstrated in Chapter 6. However, the small increase in cost gains in return a much improved measure of inequality — in its social welfare foundation, ethical significance and price-sensitivity.

Finally, the scope of application of this research is extremely wide. On one hand, as already explained, it gives rise to an inequality index that can indicate the effects of tax and tariff changes on inequality. This is a valuable policy tool. On the other hand, the approach to utility measurement and social welfare aggregation is applicable to other disciplines such as cost-benefit analyses and social planning. This research represents an important step towards integrating social choice theories with practical policy evaluation. It definitely opens up new research areas that have yet to be explored.

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Appendix A

Cross-section Results

	FOOD		CLOTHING		RECREATION	
Variable	coeff.	t-ratio	coeff.	t-ratio	coeff.	t-ratio
log y	-0.1209	-59.02	0.0140	13.23	0.0472	25.28
A_1	-0.0119	-6.28	0.0021	2.12	0.0009	0.52
A_2	-0.0364	-14.02	0.0169	12.54	0.0006	0.24
A_3	0.0382	12.66	0.0086	5.51	-0.0321	-11.67
A_4	0.0636	18.29	0.0161	8.95	-0.0502	-15.83
A_5	0.0731	19.79	0.0207	10.84	-0.0464	-13.80
A_6	0.1109	28.30	0.0293	14.42	-0.0560	-15.69
A_7	0.0002	0.04	-0.0095	-4.90	-0.0110	-3.20
A_8	0.0148	3.64	-0.0052	-2.45	-0.0078	-2.10
A_9	0.0343	8.53	-0.0023	-1.09	-0.0036	-0.98
A_{10}	0.0205	5.18	-0.0085	-4.17	0.0030	0.84
A_{11}	0.0006	0.15	-0.0145	-7.05	0.0124	3.45
Intercept	1.3593	70.84	-0.0653	-6.57	-0.3076	-17.60
R^2	.3165		.1203		.0756	

Appendix A (continued)

	P/M CARE		SHELTER		TRANSPORTATION	
Variable	coeff.	t-ratio	coeff.	t-ratio	coeff.*	t-ratio
log y	-0.0037	-5.63	-0.0290	-10.53	0.0924	35.55
A_1	-0.0030	-4.88	-0.0229	-8.98	0.0348	14.47
A_2	0.0072	8.60	0.0386	11.03	-0.0268	-8.12
A_3	0.0078	8.00	-0.0059	-1.45	-0.0166	-4.35
A_4	0.0065	5.81	-0.0110	-2.36	-0.0251	-5.68
A_5	0.0085	7.16	-0.0136	-2.73	-0.0424	-9.10
A_6	0.0088	6.92	-0.0357	-6.76	-0.0573	-11.52
A_7	0.0002	0.18	0.0473	9.37	-0.0272	-5.71
A_8	0.0048	3.63	0.0274	5.01	-0.0341	-6.60
A_9	0.0097	7.45	-0.0168	-3.10	-0.0213	-4.18
A_{10}	0.0094	7.37	-0.0211	-3.98	-0.0032	-0.64
A_{11}	0.0046	3.61	-0.0022	-0.41	-0.0010	-0.20
Intercept	0.0674	10.88	0.6451	25.00	-0.6990	-28.70
R^2	.0369		.1123		.1727	

* These are derived estimates.

Appendix B

Cross-section Consumption Function

	1		2		3		4		5	
	coeff.	t-ratio	coeff.	t-ratio	coeff.	t-ratio	coeff.	t-ratio	coeff.	t-ratio
y	0.7461	131.81	0.7294	123.34	0.6820	110.42	0.6986	116.11	0.7461	133.28
A_1	-2.5252	-0.02								
A_2			-1148.3	-8.50						
A_3					819.90	5.36				
A_4					2054.70	11.71				
A_5					2983.20	16.78				
A_6					3581.50	19.23				
A_7							350.55	1.69		
A_8							801.64	3.69		
A_9							24.42	0.11		
A_{10}							-1605.1	-7.26		
A_{11}							-2550.7	-11.85		
Intercept	3126.3	26.95	3620.5	30.55	2410.2	18.82	4344.3	22.33	3125.1	30.17

Appendix C: Time-series Data

Expenditure Shares

Year	Food	Clothing	Recreation	P/M Care	Shelter	Trans.
1971	0.2660592	0.0794667	0.1377769	0.0490649	0.3139157	0.1537163
1972	0.2636354	0.0778922	0.1434587	0.0486184	0.3118088	0.1545862
1973	0.2638876	0.0763336	0.1452276	0.0480812	0.3091659	0.1573038
1974	0.2603992	0.0816908	0.1461441	0.0496107	0.3072199	0.1549349
1975	0.2620369	0.0783482	0.1497651	0.0492756	0.3040745	0.1564994
1976	0.2501935	0.0777230	0.1568799	0.0503369	0.3078621	0.1570042
1977	0.2482974	0.0759179	0.1593904	0.0504417	0.3100061	0.1559463
1978	0.2500862	0.0745530	0.1579281	0.0511057	0.3110649	0.1552619
1979	0.2504032	0.0748948	0.1522763	0.0507470	0.3130199	0.1586584
1980	0.2502917	0.0732464	0.1523522	0.0514989	0.3158874	0.1567232
1981	0.2504500	0.0723800	0.1489500	0.0524800	0.3152200	0.1605200

Appendix C (continued)

Price Indexes

Year	Food	Cloth.	Recrn.	P/M C.	Shelt.	Trans.
1971	0.5149	0.6831	0.6748	0.6017	0.5744	0.6165
1972	0.5469	0.7008	0.6937	0.6306	0.6014	0.6326
1973	0.6102	0.7357	0.7227	0.6607	0.6399	0.6492
1974	0.6941	0.8060	0.7861	0.7184	0.6962	0.7139
1975	0.7832	0.8545	0.8671	0.8002	0.7651	0.7978
1976	0.8120	0.9016	0.9190	0.8682	0.8501	0.8835
1977	0.8769	0.9631	0.9629	0.9326	0.9299	0.9451
1978	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1979	1.1205	1.0922	1.0688	1.0903	1.0695	1.0974
1980	1.2400	1.2206	1.1707	1.1992	1.1568	1.2374
1981	1.3841	1.3074	1.2888	1.3297	1.3004	1.4649

Appendix C (continued)

Expenditure/Attribute Statistics

Year	SAR	SOH	Family Size				Age of Head				
1971	0.3337079	0.1096137	0.2013677	0.1716406	0.1992498	0.2926372	0.2268651	0.2486509	0.2168110	0.1303190	0.0861047
1972	0.3371326	0.1165177	0.2053771	0.1700943	0.2018333	0.2831293	0.2304947	0.2449641	0.2156311	0.1304716	0.0920845
1973	0.3430373	0.1214114	0.2113054	0.1695585	0.2094422	0.2683040	0.2366654	0.2382742	0.2150136	0.1317976	0.0935028
1974	0.3299644	0.1206901	0.2194015	0.1669182	0.2127494	0.2539154	0.2459219	0.2354508	0.2082962	0.1292976	0.0962181
1975	0.3091979	0.1215964	0.2292620	0.1672548	0.2191977	0.2366318	0.2579635	0.2311310	0.2051606	0.1266780	0.0940368
1976	0.3124135	0.1249326	0.2238119	0.1755751	0.2237598	0.2287237	0.2566222	0.2266403	0.2093409	0.1308274	0.0926589
1977	0.3269907	0.1304930	0.2257125	0.1762792	0.2271487	0.2219230	0.2550149	0.2211759	0.2144736	0.1338624	0.0899095
1978	0.3203550	0.1281574	0.2306620	0.1818301	0.2249767	0.2080390	0.2540106	0.2258760	0.2106112	0.1355060	0.0953015
1979	0.3125654	0.1360894	0.2364785	0.1832780	0.2251669	0.1951508	0.2561977	0.2275612	0.2032423	0.1364625	0.0983135
1980	0.3056573	0.1338735	0.2473163	0.1871624	0.2209041	0.1843005	0.2515903	0.2305676	0.1990146	0.1420278	0.1054092
1981	0.3029503	0.1384588	0.2466721	0.1899575	0.2323494	0.1750336	0.2479388	0.2345871	0.2008202	0.1432102	0.1045268

Appendix C (continued)

Year	S_{yyt}	ρ^2
1971	9.16783810	5646258
1972	9.22327710	5767065
1973	9.31293580	5813489
1974	9.42433930	6076686
1975	9.51679800	6227154
1976	9.61170770	6303170
1977	9.68260670	6442341
1978	9.75351240	6617835
1979	9.80900670	6813257
1980	9.92726140	6632325
1981	10.02926450	6932605

Appendix D

Time-series Results, B_{pp}

	Food	Clothing	Recreation	P/M Care	Shelter	Trans*
Food	-0.11148	-0.02224	-0.04682	0.00026	0.14293	-0.08358
	(-4.6197)	(-0.9363)	(-1.7898)	(0.0069)	(5.5466)	
Clothing	-0.02224	0.16707	-0.15309	-0.04365	-0.11494	0.18089
	(-0.6298)	(1.9605)	(-1.8222)	(-0.3949)	(-1.7470)	
Recreation	-0.04682	-0.15309	-0.03234	0.06491	0.48122	-0.26672
	(-0.3139)	(-0.4752)	(-0.0853)	(0.1506)	(1.9837)	
P/M Care	0.00026	-0.04365	0.06491	-0.06829	0.11340	-0.07037
	(0.0066)	(-0.5615)	(0.8211)	(-0.3026)	(1.0731)	
Shelter	0.14293	-0.11494	0.48122	0.11340	-0.98008	0.32847
	(0.9977)	(-0.4672)	(2.0343)	(0.2018)	(-2.9720)	
Trans.	-0.08358	0.18089	-0.26672	-0.07037	0.32847	0.00374

* Estimates in the last row and last column are derived estimates. (See Chapter 5)

Numbers in parentheses are the t-ratios

Time-series: 1971 - 1981

Appendix E : Barten Equivalence Scales

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3836	1.3831	1.4226	1.6080	1.4035	1.3787
3	1.7147	1.6748	1.8084	2.0387	1.7459	1.6960
4	1.8045	2.9199	2.4953	2.6513	1.8894	1.4648
5	2.3943	4.9348	3.9310	3.9552	2.5028	1.7235

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.0388	1.0250	0.9884	1.0196	1.0756	1.0786
2	1.4374	1.4176	1.4061	1.6396	1.5095	1.4871
3	1.7813	1.7166	1.7874	2.0787	1.8778	1.8293
4	1.8745	2.9928	2.4664	2.7033	2.0322	1.5799
5	2.4873	5.0580	3.8854	4.0329	2.6919	1.8590

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.1342	1.5474	1.2937	1.3196	1.1777	1.0299
2	1.5693	2.1403	1.8403	2.1220	1.6529	1.4200
3	1.9448	2.5916	2.3395	2.6903	2.0561	1.7468
4	2.0466	4.5183	3.2281	3.4987	2.2252	1.5086
5	2.7156	7.6364	5.0854	5.2193	2.9475	1.7751

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.2838	1.6544	1.4541	1.5298	1.2771	1.1655
2	1.7763	2.2882	2.0686	2.4599	1.7925	1.6068
3	2.2012	2.7708	2.6296	3.1187	2.2297	1.9767
4	2.3165	4.8306	3.6284	4.0558	2.4131	1.7071
5	3.0738	8.1642	5.7160	6.0506	3.1964	2.0087

URBAN MALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.1565	1.0419	1.0682	1.1733	1.1161	1.2108
2	1.6003	1.4410	1.5195	1.8868	1.5664	1.6693
3	1.9831	1.7449	1.9317	2.3921	1.9485	2.0535
4	2.0869	3.0421	2.6654	3.1109	2.1088	1.7735
5	2.7692	5.1414	4.1989	4.6408	2.7933	2.0868

URBAN MALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.9949	0.8382	0.8682	0.9103	0.9639	1.0799
2	1.3765	1.1593	1.2351	1.4638	1.3529	1.4888
3	1.7059	1.4037	1.5701	1.8559	1.6829	1.8315
4	1.7952	2.4473	2.1665	2.4135	1.8213	1.5818
5	2.3820	4.1362	3.4130	3.6005	2.4125	1.8612

Appendix E (continued)

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7374	2.5273	1.3832	1.3608	0.8447	0.4746
2	1.0203	3.4955	1.9677	2.1882	1.1856	0.6543
3	1.2644	4.2327	2.5013	2.7742	1.4748	0.8049
4	1.3306	7.3794	3.4515	3.6078	1.5961	0.6951
5	1.7656	12.4718	5.4372	5.3822	2.1142	0.8179

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7660	2.5904	1.3672	1.3875	0.9086	0.5119
2	1.0599	3.5828	1.9449	2.2312	1.2752	0.7057
3	1.3135	4.3384	2.4724	2.8287	1.5863	0.8681
4	1.3823	7.5636	3.4115	3.6787	1.7167	0.7498
5	1.8341	12.7832	5.3742	5.4879	2.2740	0.8822

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.8363	3.9109	1.7894	1.7957	0.9948	0.4888
2	1.1572	5.4091	2.5455	2.8875	1.3962	0.6739
3	1.4340	6.5499	3.2359	3.6609	1.7369	0.8290
4	1.5091	11.4192	4.4651	4.7609	1.8797	0.7159
5	2.0025	19.2994	7.0340	7.1024	2.4899	0.8424

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.9466	4.1812	2.0113	2.0817	1.0789	0.5531
2	1.3098	5.7830	2.8612	3.3474	1.5142	0.7625
3	1.6232	7.0026	3.6372	4.2439	1.8836	0.9381
4	1.7082	12.2085	5.0188	5.5191	2.0384	0.8101
5	2.2666	20.6334	7.9063	8.2335	2.7002	0.9533

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.8528	2.6331	1.4775	1.5967	0.9428	0.5746
2	1.1800	3.6418	2.1018	2.5675	1.3232	0.7922
3	1.4623	4.4099	2.6718	3.2551	1.6460	0.9745
4	1.5389	7.6883	3.6867	4.2332	1.7814	0.8416
5	2.0419	12.9939	5.8078	6.3151	2.3596	0.9903

URBAN FEMALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7336	2.1183	1.2009	1.2387	0.8143	0.5125
2	1.0150	2.9298	1.7084	1.9919	1.1428	0.7065
3	1.2579	3.5477	2.1718	2.5254	1.4216	0.8692
4	1.3237	6.1851	2.9967	3.2843	1.5385	0.7506
5	1.7565	10.4534	4.7208	4.8995	2.0380	0.8833

Appendix E (continued)

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.9165	0.5437	0.7126	0.7425	0.8758	1.1027
2	1.2681	0.7520	1.0138	1.1940	1.2291	1.5203
3	1.5715	0.9105	1.2887	1.5138	1.5290	1.8702
4	1.6538	1.5874	1.7782	1.9687	1.6547	1.6152
5	2.1944	2.6829	2.8013	2.9369	2.1919	1.9005

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.9521	0.5572	0.7044	0.7571	0.9420	1.1894
2	1.3174	0.7707	1.0020	1.2175	1.3220	1.6398
3	1.6325	0.9333	1.2738	1.5435	1.6445	2.0172
4	1.7180	1.6271	1.7576	2.0073	1.7798	1.7422
5	2.2796	2.7499	2.7689	2.9946	2.3575	2.0499

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.0395	0.8413	0.9219	0.9799	1.0314	1.1357
2	1.4383	1.1636	1.3115	1.5756	1.4475	1.5658
3	1.7824	1.4090	1.6672	1.9976	1.8007	1.9262
4	1.8757	2.4565	2.3005	2.5979	1.9487	1.6636
5	2.4889	4.1517	3.6240	3.8755	2.5814	1.9574

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.1766	0.8995	1.0362	1.1359	1.1185	1.2852
2	1.6280	1.2440	1.4741	1.8266	1.5698	1.7719
3	2.0175	1.5064	1.8739	2.3158	1.9528	2.1797
4	2.1231	2.6263	2.5857	3.0116	2.1133	1.8825
5	2.8171	4.4386	4.0734	4.4928	2.7994	2.2150

RURAL MALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	1.0600	0.5664	0.7612	0.8712	0.9774	1.3351
2	1.4666	0.7834	1.0829	1.4010	1.3718	1.8408
3	1.8175	0.9486	1.3766	1.7762	1.7065	2.2644
4	1.9127	1.6539	1.8994	2.3099	1.8468	1.9557
5	2.5380	2.7952	2.9923	3.4460	2.4463	2.3012

RURAL MALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.9118	0.4557	0.6187	0.6759	0.8442	1.1908
2	1.2616	0.6303	0.8802	1.0869	1.1848	1.6418
3	1.5634	0.7632	1.1189	1.3780	1.4739	2.0196
4	1.6453	1.3305	1.5439	1.7921	1.5951	1.7442
5	2.1832	2.2487	2.4322	2.6735	2.1128	2.0524

Appendix E (continued)

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 24 OR BELOW

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.6758	1.3740	0.9857	1.0104	0.7398	0.5233
2	0.9351	1.9004	1.4022	1.6248	1.0383	0.7215
3	1.1588	2.3012	1.7825	2.0600	1.2916	0.8875
4	1.2195	4.0120	2.4596	2.6790	1.3978	0.7665
5	1.6181	6.7806	3.8747	3.9965	1.8516	0.9019

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 24-34

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7021	1.4083	0.9743	1.0303	0.7957	0.5644
2	0.9714	1.9479	1.3860	1.6567	1.1168	0.7782
3	1.2038	2.3586	1.7619	2.1004	1.3892	0.9573
4	1.2668	4.1121	2.4311	2.7315	1.5035	0.8268
5	1.6810	6.9499	3.8299	4.0749	1.9915	0.9728

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 34-44

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7665	2.1262	1.2752	1.3334	0.8713	0.5390
2	1.0606	2.9408	1.8140	2.1441	1.2228	0.7431
3	1.3143	3.5610	2.3060	2.7183	1.5211	0.9141
4	1.3831	6.2083	3.1819	3.5352	1.6462	0.7895
5	1.8353	10.4926	5.0126	5.2738	2.1806	0.9289

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 44-54

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.8676	2.2732	1.4333	1.5457	0.9448	0.6099
2	1.2004	3.1440	2.0390	2.4856	1.3261	0.8409
3	1.4876	3.8071	2.5920	3.1513	1.6496	1.0344
4	1.5655	6.6374	3.5766	4.0982	1.7852	0.8934
5	2.0773	11.2178	5.6343	6.1137	2.3648	1.0512

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED 54-64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.7816	1.4315	1.0529	1.1856	0.8257	0.6336
2	1.0815	1.9800	1.4978	1.9065	1.1588	0.8736
3	1.3402	2.3975	1.9040	2.4170	1.4415	1.0746
4	1.4104	4.1799	2.6273	3.1433	1.5601	0.9281
5	1.8715	7.0644	4.1389	4.6892	2.0665	1.0920

RURAL FEMALE HEAD
HOUSEHOLD HEAD AGED OVER 64

FAM. SIZE	FOOD	CLTH	RCRN	MEDC	SHTR	TRAN
1	0.6723	1.1517	0.8558	0.9198	0.7131	0.5651
2	0.9303	1.5928	1.2175	1.4791	1.0009	0.7791
3	1.1529	1.9288	1.5477	1.8752	1.2450	0.9584
4	1.2132	3.3627	2.1356	2.4387	1.3474	0.8277
5	1.6098	5.6832	3.3642	3.6381	1.7848	0.9740