DISCLOSURE, RISK SHARING, AND VALUATION UNDER ASYMMETRIC INFORMATION

by

PATRICIA JANE HUGHES

B.A., Duke University , 1965
M.B.A., The University of Pittsburgh, 1966

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

THE FACULTY OF GRADUATE STUDIES

(Faculty of Commerce and Business Administration)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

JUNE, 1984

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Commerce and Business Administration

The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date August 27, 1984

ABSTRACT

This dissertation analyzes risk sharing between an entrepreneur of a firm and its investors, and valuation of the firm when there is an informational asymmetry between the entrepreneur and investors. Two types of informational asymmetries are examined: the adverse selection problem and the moral hazard problem.

In the adverse selection problem, the entrepreneur knows an exogenous parameter of value which is unobservable by investors. While maximizing his own welfare, he selects a level of direct disclosure in order to communicate his information to investors. A verification role for a third party is developed such that the verified disclosure is credible due to a costly penalty which is imposed if the disclosure is false. An equilibrium is derived in which investors correctly value the firm after observing the disclosure.

In the moral hazard problem, firm value is dependent upon the behavior of the entrepreneur where that behavior is unobservable. The entrepreneur selects costly ownership in his own firm to credibly communicate his behavior to investors. An equilibrium is derived in which investors correctly value the firm after observing the entrepreneur's investment portfolio.

In the conclusion, the two informational problems are integrated in order to indicate the similar nature of the problems.

TABLE OF CONTENTS

LIST OF FIGURES

ACKNOWLEDGEMENTS

I want to thank the individual members of my dissertation committee who advised me: Jerry Feltham, whose comments, and questions about my work helped me to think more intuitively about the problems I was addressing and to reach a deeper level of understanding; Rob Heinkel, who spend many hours with me listening to and discussing my ideas and work in the earlier stages of my research; and Rex Thompson, who asked me questions which made me think about my work from a different perspective. I could not have solved the bivariate signaling model of Chapter Four without the mathematical help of Eduardo Schwartz. I am very grateful to the accounting firm of Deloitte, Haskins, & Sells for financial support while I wrote this dissertation. Finally and most importantly, I thank my husband and three children who never stopped encouraging me during these past years.

CHAPTER ONE

INTRODUCTION

The securities market provides the link between the consumption and investment decisions of individual consumers and the production and investment decisions of firms. A fundamental separation result [Fama and Miller, Ch. 4] in economics and finance is that in a perfect and complete market the objective of the firm should be to maximize the wealth of its shareholders. The firm (i.e., its managers) should disregard the individual preferences of its diverse shareholders by making production, investment, and financing decisions which maximize the market value of the firm's securities. Security holders then will make their individual consumption and investment decisions according to their individual preferences.

Therefore sufficient conditions for maximal efficiency (or maximum social welfare) in the economy in terms of allocations of resources among firms and of wealth among individuals are:

1. Perfect and complete markets
2. Firms maximize market value.

When these sufficient conditions are satisfied, there is no demand or role for information. In order for there to exist any 'social value' to information, it is necessary that there exist a violation of the perfect markets or complete markets assumption. If full allocational efficiency is possible in the absence of information, then that information cannot be used to make any individual better off without impairing the welfare of another individual. If, however, there does exist a market imperfection or incompleteness, it is possible that

the addition of information to the economy will lead to a Pareto-improvement in allocations of wealth and resources.

In a perfect capital market, information is costlessly and simultaneously available to all market participants. One possible market imperfection is that information is an economic good which is not costlessly available to all market participants because its acquisition or revelation is costly. Such an imperfection can result in an asymmetry of information between individuals; where information differences can be about an exogenous characteristic of product quality (the adverse selection problem), or about an unobservable action on the part of one individual which affects product quality (the moral hazard problem). Both the adverse selection and the moral hazard problems can exist between the manager of a firm and its shareholders, and either information problem may generate a demand from shareholders for information.

Stiglitz (1982) has summarized the relationship between information and prices:

"It is clear that central to an understanding of the capital market is an analysis of

1. the incentives, within a market economy, for an individual to acquire information;
2. the extent to which market prices reflect the information of informed individuals; and
3. the role that market prices play in determining the behavior of managers of firms." (p.120)

The typically-defined role of accounting as the measurement and communication of economic information relevant to decision making implies that there is a market imperfection which generates a demand for information. In order to determine whether financial reporting will serve as a viable supply of information demanded by investors, the nature of accounting reporting can be related to the issues defined by Stiglitz. An ex ante informational asymmetry can be assumed to exist which obviates the first issue. Such informational differences arise due to the manager being an insider and the shareholders outsiders to the firm's operations. The second and third can be addressed by analyzing whether market prices will motivate the manager to communicate inside information through financial reporting and if such communication is perceived by investors as credible so that market prices adjust to reflect the information.

The environment in which financial reporting occurs is described by Beaver (1981) as consisting of five constituencies: investors, management, information intermediaries, regulators, and auditors. Due to the existence of regulation, financial reporting is manadatory. However, it is possible that incentives exist in the market for a manager to communicate voluntarily through financial reports while at the same time acting in his own self interest. The purpose of this dissertation is to analyze such incentives for disclosure and the resulting impact on security prices when the manager has information about the value of the firm which investors do not have. The manager's informational advantage arises because he can observe a characteristic of the firm which is unobservable by investors due to his position in the firm, or because he can affect the value of the

firm through his actions which are unobservable by investors. The first three parties described by Beaver have major roles in the analysis in the following chapters. Auditors are assumed to be an information intermediary and regulators are tangentially included in that their economic role is that of enforcing contracts.

Chapter Two describes the formulation of and solution of the adverse selection problem in the economics literature; and a survey of finance and accounting literature addressing the problem is presented. In Chapter Three, the role of an information intermediary under asymmetric information is developed. The results of Chapter Three are used to formulate and solve a bivariate signaling problem in Chapter Four. Chapter Five analyzes the problem of an informational asymmetry about unobservable managerial behavior in the capital market. A synthesis of the adverse selection problem of Chapter Four and the moral hazard problem of Chapter Five is presented in Chapter Six.

CHAPTER TWO

THE ADVERSE SELECTION PROBLEM: A LITERATURE REVIEW

INTRODUCTION

This chapter reviews the formulation of the adverse selection problem and solutions to the problem in the economics, finance, and accounting literature. In Section 1, the problem and several solutions to resolve the problem are discussed. Literature on the adverse selection problem in capital markets is reviewed in Section 2. Section 3 discusses a role for accounting under asymmetric information.

1. THE ADVERSE SELECTION PROBLEM

The adverse selection problem was formulated by Akerlof (1970) as an informational asymmetry between buyers and sellers in a product market. The seller knows the exogenously determined quality of the product he brings to the market, while buyers are unable to observe the quality. Akerlof presents the example of the used car market to illustrate this information problem. If buyers know only the distribution of used car quality, they will not be willing to pay more than the price of an average-quality car for any particular car. Sellers of cars of above-average quality will therefore withdraw their cars from the market due to their inability to reveal quality and receive an above-average price. As a consequence, the average quality of the cars remaining in the market will decline, which leads to the withdrawal of cars with quality above the new average. This process continues until the only cars remaining in the market are those of

the lowest quality. The product market fails. Akerlof suggests that institutions such as guarantees, licensing, and reputation exist so as to eliminate the market failure which can result from uncertainty as to product quality.

As Akerlof suggests, sellers of products of above-average quality have incentives to develop institutions which will enable them to sell their products at above-average prices. Reputation is a characteristic which a seller can earn over time after repeated market transactions. If a seller enters the market for the first time, or does not intend to enter the product market again, then reputation cannot exist. In the used car example, reputation cannot reduce the adverse selection problem for an individual selling his personal used car, or a newly-formed used car dealership offering its first used car for sale. For such an initial or one-time entry into the market, mechanisms such as guarantees and licensing may appear as viable means of eliminating the adverse selection problem.

There are two general types of mechanisms which may prevent Akerlof's extreme market failure. The first general solution is to increase average product quality so that the lemons are not the sole survivors. A second mechanism is one which results in disclosure of product quality to buyers. Examples discussed below of the first type are Leland (1979) and Heinkel (1981), and of the second are Grossman (1981) and Spence (1974)

Leland (1979) examines licensing and the imposition of minimum quality standards in markets characterized by adverse selection. He looks at the effects of the information asymmetry on social welfare, where social welfare is defined relative to a single product and is

maximized when there is no consumer or producer surplus. He shows that uncertainty about product quality serves as a hinderance to trade with the result that product quality supplied to the market is less than that which is socially optimal. He then shows that social welfare can be improved if minimum quality standards are imposed such that the supply of poor quality products is eliminated, thereby raising average quality. The unraveling seen in Akerlof's used car market will also occur in Leland's market such that all products will have the same quality which is that prescribed as the minimum by the regulatory group. It is necessary to Leland's model that the regulatory agent be able to observe quality ex ante in order to enforce the regulation. Consequently the set of markets for which licensing will mitigate the adverse selection problem is restricted.

In Heinkel's (1981) model, product quality is exogenous, but can be increased by the seller through costly maintenance. Buyers are unable to observe quality or maintenance. Sellers of poor quality products are induced to invest in costly maintenance, thereby increasing average quality in the market, due to the imposition of an imperfect ex post test of quality and a penalty if the test reveals that quality is below an acceptable level. Like Leland, Heinkel finds that social welfare is increased through the testing and penalty mechanism because of the increase in average quality of the product supplied to the market. Although Heinkel does not state it as an assumption, it is essential to his model that buyers know total maintenance expenditures in order to know average post-maintenance quality. If they do not know such expenditures, the market will fail so that even the lowest quality product is removed from the market.

In Heinkel's discussion of his numerical solutions, he does not refer to two unusual and interesting results: with the imperfect testing technology, different product qualities exist in the market and consumers are better off than they would be under perfect information. Presumably the first result holds because of the noise in the testing technology, and the second because consumer welfare is increasing in quality and poor quality has been eliminated. The set of markets for which the ex post testing will mitigate the adverse selection problem also is restricted because the seller must be able to improve the quality of his product and ex post testing must be feasible.

The elimination of low quality products may reduce the adverse selection problem in particular markets. A second type of solution involves communication of product quality. Grossman (1981) analyzes the information provided by direct disclosure and product warranties. He first looks at the market for products for which quality is costlessly verifiable ex post and finds that a seller will disclose the exact quality of his product and offer a full guarantee. Overstatement of product quality is eliminated because a seller will not guarantee such a disclosure, and such lack of guarantee will be interpreted by buyers as misrepresentation. Understatement of product quality also will not occur because a seller is always better off by a truthful disclosure (e.g., a statement such as "this box contains at least six oranges" will be interpreted by buyers as there being six oranges in the box). For such products, buyers will know product quality, and there is no role for disclosure laws. However, ex post verification of quality may be prohibitively costly for some

products. For these products, there may occur a future observable event with a probability of occurrence dependent only on product quality, such as automobile breakdown. The seller then can offer a warranty under which a payment is made to the buyer if the event occurs, where the seller knows the probabiltiy of breakdowns for his product. When buyers are risk averse, the resulting equilibrium price and warranty offered provide full insurance to consumers. While consumers cannot infer quality or probability of breakdown from the offered contracts, they know that they are fully insured as to product quality and therefore market failure is avoided.

In Spence's (1973, 1974) job-signaling model, the adverse selection problem is eliminated through the seller's investment in a costly observable signal which is used by the buyer in valuation of the product such that the buyer's inference is self confirming. The setting is a job market in which workers (sellers) have different endowments of an unobservable attribute such as ability (quality) which determines their productivity. In the absence of asymmetric information, profit-maximizing employers (buyers) pay each worker a wage equal to the worker's productivity. Market failure is avoided because workers can invest in a costly signal correlated with ability, such as education, when the level of education selected is observable and thus can serve as a substitute for unobservable quality in the wage schedule offered by employers. A more precise description of Spence's model follows.

Let n = underlying ability

$s(n)$ = productivity

y = education

$w(y)$ = wage schedule

$c(y,n)$ = cost of education

Spence proves that education will serve in the market as a signal for productivity such that:

i) Workers select education to maximize net income,

$$w' = c_y; \text{ and}$$

ii) employers correctly infer productivity from the level of education and offer a wage schedule dependent on y alone,

$$w(y) = s(y(n)) = s(n)$$

when certain cost conditions are satisfied. The necessary cost conditions are

1) $c_y > 0$, the cost of education is increasing in the level selected, and

2) $c_{yn} < 0$, the marginal cost of education is decreasing in ability.

The second condition is the critical condition which prevents misrepresentation. The existence of the signaling mechanism permits buyers to perfectly infer quality. All sellers except that with the lowest quality product invest in the costly signal because there are benefits to signaling. For the lowest quality seller there are no benefits to be gained from costly signaling. The adverse selection problem is fully resolved. However, Spence shows that the resulting allocation of resources may be suboptimal compared to the allocation under perfect information due to an overinvestment in the signal.

2. THE ADVERSE SELECTION PROBLEM IN THE CAPITAL MARKET

The economic result of the mechanism discussed in Section 1 is a redistribution of wealth among buyers and sellers of a particular product. While the welfare of buyers and sellers has been improved, nothing can be said about the effect on allocations of wealth in the economy. As an example, consider the failure of Akerlof's used car market. While allocations of wealth among buyers and sellers of used cars may be improved if used car dealers offer Grossman's warranties against future breakdown, it cannot be said that the resulting reallocation is Pareto-optimal in the economy where many individuals and firms are making investment and production decisions. It could be possible that it is Pareto-optimal for the used car market to fail so that used cars are scrapped. In order to move from questions about micro efficiency in a product market to those about macro efficiency in the economy, the impact of the adverse selection problem and its resolution on production and investment decisions of all economic agents must be considered.

The role of security prices in the allocation of resources in an economy was described in Chapter One. The allocational role of security prices will be impaired in the presence of a market imperfection such as asymmetric information between issuers of securities and investors. Therefore the existence of an adverse selection problem in the capital market and the means of resolving the problem will have an impact on the efficiency of allocations within the economy.

An adverse selection problem arises in the capital market when the entrepreneur/manager of a firm is endowed with inside information

about the value of his firm. Investors in the market know the distribution of firm values, but cannot observe the value of a particular firm. There are two types of models of mechanisms which may resolve the adverse selection problem in the securities market. In both models, investors infer value from an observable managerial action. The first type of mechanism is a contingent contract similar to Grossman's guaranty and warranty contracts. The manager selects an action which investors observe and use to infer value. A penalty for misrepresentation will be imposed on the manager if ex post observation of value or some event determined by value reveals that the manager was signaling falsely by his action. Signaling with a contingent contract is generally considered to be costless because appropriate penalties induce truthful actions and the signaler in such models is risk neutral. The second type of model is an extension of Spence's model where education was a signal of quality. The manager selects an action which investors observe and use to infer value. However this type of signal differs from the contingent contract because the manager bears the cost of the action ex ante. The cost of the signal must be appropriate to induce truthful signaling in order to be used by investors in valuing the firm. As a consequence it is not necessary that value or some event determined by value is observable ex post. Examples of financial signaling models to be discussed are Ross (1977), which is a model of contingent contracting, and Leland and Pyle (1977) and Bhattacharya (1979), which have both a contingent contract and a costly signal.

Ross (1977) addresses the question of whether the Modigliani-Miller proposition that the value of the firm is independent of its

capital structure would hold in an imperfect market where manager-insiders possess inside information about firm value. In Ross' model, where firms differ in value, the securities market will fail as Akerlof's used car market failed because each manager knows the value of his firm and investors know only the distribution of values. The risk-neutral manager's only act is to select the amount of debt issued by his firm and he selects a debt issue in order to maximize his compensation. A linear managerial compensation scheme is assumed in which the manager's end-of-period compensation is an increasing function of the market price of the firm at the beginning of the period, and includes a penalty if the firm is bankrupt at the end of the period. The manager signals his firm's value through his debt choice in that a high value firm will issue more debt than a low value firm. Investors infer firm value from the amount of debt issued and the manager's compensation therefore is increasing in the amount of debt. Debt serves as a credible signal because a low value firm which signals high value through a large debt issue will be unable to repay the debt at the end of the period and will go bankrupt, resulting in a penalty imposition on the manager. Since the compensation contract is determined by an unspecified third party which presumably also imposes the penalty, it is essential that the third party is able to observe the event of bankruptcy and that the occurrence of bankruptcy is dependent only on firm value after debt has been issued. However the penalty will never be imposed since the threat of imposition is assumed to be sufficient to deter false signaling. The contingent contract of Ross' model is similar to Grossman's warranty contract in that the warranty is not contingent

on ex post verification of value, but on the observable event of bankruptcy which depends only on value. The motivations for the warranties differ. Grossman's warranty is offered to buyers of a product in order to increase the market price of the product which directly increases the seller's wealth. Ross's warranty is offered to existing owners of the product in order to increase the market price of the product which indirectly increases the manager's wealth and directly increases the product owner's wealth. Ross's cost of mispresentation is dissipative where Grossman's is nondissipative. Signaling through such contingent contracts is costless because the event's probability of occurrence depends only on value which is truthfully communicated. The Modigliani-Miller theorem that the value of a firm is dependent only on the distribution of cash flows generated by its investments therefore is invalid when investors cannot observe that distribution. Firm value will be dependent upon whatever credible signal can be supplied to investors by managers.

In Leland and Pyle's (1977) signaling model an entrepreneur is endowed with information about the value of a productive technology and can acquire the capital necessary for investment in the technology through issuing riskless debt or selling equity in the future cash flows to be generated by the investment. In the absence of any market imperfection, the risk-averse entrepreneur would choose to sell all of the equity in the risky cash flows and invest in a well-diversified market portfolio. However, investors do not know the value of the investment project and will pay some average price for ownership in the project/firm. The signal used by the entrepreneur to communicate his information is ownership in his own firm which is

perceived by the market as credible because of the ex ante cost of the signal to the entrepreneur and an ex post penalty for false signaling. While Leland and Pyle focus on the ex ante cost to the entrepreneur of bearing diversifiable firm-specific risk, it is the ex post penalty which makes the signal credible. The benefits of the signal are increases in wealth and the cost is an increase in risk. The level of the signal selected is the level at which the marginal benefit equals the marginal cost. To consider a simple example, consider two firms with different underlying 'true' values V_H and V_L.

Let V_i = true value of firm i

R_i = firm specific risk of firm i

α_i = percentage ownership of entrepreneur in his firm

$V_i(\alpha_i)$ = market's inference of value based upon the signal.

Assume that (i) the riskfree rate of interest is zero

(ii) $V_H > V_L$ because of differences in the expected value of future cash flows.

(iii) V_i is independent of R_i

(iv) $R_H = R_L = R$

If $\alpha_H = \alpha_L = \alpha$ which signals V_H, then the increase in wealth at the time of signaling and the amount of firm-specific risk borne are the same for each entrepreneur. The entrepreneur who signals falsely will be penalized at the end of the period when $\alpha V_H > \alpha V_L$. The signal will be credible if the cost of false signaling exceeds the benefit, or

$$(1-\alpha)V(\alpha) + \alpha V_H \geq \text{Cost of } R$$

$$\text{and } (1-\alpha)V(\alpha) + \alpha V_L < \text{Cost of } R$$

Investors infer firm value through observation of the entrepreneur's portfolio decision and understanding of the cost to him of holding an undiversified portfolio. It is not necessary that the event upon which the penalty contract is contingent is observable because the penalty for false signaling is imposed endogenously. The market's inference is self confirming in that the reaction of the market to the signal is anticipated by the entrepreneur in his selection of the signal and the inferences are correct in the resulting equilibrium.

The role of dividends as a signal of expected cash flows has been analyzed by Bhattacharya (1979) who assumes that the manager of a firm is maximizing after-tax shareholders' welfare. The motivation for signaling is similar to that of Ross: the manager of a product is trying to maximize the welfare of owners of the product. The cost structure is similar to that of Leland and Pyle: there is an ex ante cost of signaling and an ex post penalty for false signaling. The dissipative cost of dividends is borne by shareholders in the personal tax on dividend income which exceeds the tax on capital gains. The penalty for false signaling is the cost to the firm of 'bail out' financing or forced liquidation of assets if the realized cash flow is less than the promised dividend payment. At the time dividends are declared, the end-of-period cash flow is a random variable and therefore there is some risk of a cash deficit under truthful signaling. The cost of such risk is not relevant to the model because risk neutrality is assumed. As in the Ross model, it is not necessary that the realized cash flow is observable. The action which the manager undertakes given a cash flow deficit must be observable to the market. As with the Ross and Leland and Pyle models, the threat

of a penalty for false signaling induces truthful signaling and the market's inferences are self confirming.

The signaling models described in this section satisfy the two criteria essential for signaling to be a viable means of resolving asymmetric information: the signaler must be motivated to undertake the activity and the signal must have a cost structure which makes it credible. Each of the papers had a different motivation: Ross's manager maximized the expected value of his compensation, Leland and Pyle's entrepreneur maximized his expected utility, and Bhattacharya's manager maximized shareholders' wealth. The Leland and Pyle model addressed the adverse selection problem about valuation of a product offered for sale. Ross and Bhattacharya address the problem of valuation of a product owned. The Leland and Pyle signaler is selling the product where the Ross and Bhattacharya signalers manage the product. Potential shareholders and existing shareholders have different objective functions. Current shareholders are better off if share price is maximized so that they can make optimal consumption and investment decisions. Potential shareholders clearly would like shares to be undervalued, but they cannot distinguish undervalued shares from overvalued shares. Therefore, the adverse selection problem in the capital market is of a different nature than that in a product market because the shares are not consumed as other products are.

The three papers discussed in this section are the seminal financial signaling models and have spawned much interest in signaling in the capital market. They ignore or dismiss any role for communication of inside information through accounting reports.

Bhattacharya states, "We ignore the incorporation of other sources of information (e.g., accountant's reports) on the ground that, taken by themselves, they are fundamentally unreliable 'screening' mechanism because of the moral hazard involved in communicating profitability." Such a statement is inherently false since the financial signaling models use contingent contracts to induce truthful signals. Truthful communication through accounting reports presumably could be induced through appropriate contingent contracts.

3. THE ROLE OF ACCOUNTING UNDER ADVERSE SELECTION

Bhattacharya (1979) dismissed any role for accounting reports under asymmetric information because they are 'vitiated by moral hazard'. However, Bhattacharya (1980) states that accounting reports may be credible indicators of ex post profitability free of moral hazard. He develops a nondissipative signaling model in which dividends (or earnings forecasts) are costless signals of firm value at the beginning of a period when end-of-period cash flows can be costlessly communicated due to the absence of moral hazard in accounting reports. The manager's objective function is to select a dividend to maximize the value of the firm at time zero and at time one. The market infers time zero value as a function of the level of promised dividend, and penalizes time one value if the ex post cash flow indicates false signaling. If a low value firm declares a large dividend payment, it will be valued by the market as a high value firm at time zero. However, the penalty imposed by the market at time one when cash flow is observed will be of sufficient size to deter false signaling. A comparison of Bhattacharya (1979) and

Bhattacharya (1980) will indicate that it is not true that the first model is dissipative and the second nondissipative. If taxes are eliminated in the first model as they are in the second, the role of accounting reports is identical to that of costly bail-out financing: both are observable events which provide a basis for contingent contracting. In Bhattacharya (1979), it is not necessary that ex post cash flows are observable, but that the event of acquisition of bail-out financing is observable. In Bhattacharya (1980) it is assumed that ex post cash flows are observable, or costlessly are truthfully communicated through accounting reports. The firm offers a warranty contract in Bhattacharya (1979) and a guaranty contract in Bhattacharya (1980). In Grossman's warranty model, buyers use the warranty offered as a basis for product valuation because the event upon which the warranty payment is contingent (i.e., product breakdown) is independent of any actions of either the seller or buyer (i.e., no moral hazard). Such an assumption is nontrivial when accounting reports provide the basis for contracting. There clearly is a moral hazard problem associated with accounting reports, which limits their usefulness in contracting in the absence of verification or monitoring. The addition of verification or monitoring to Bhattacharya's model would make it more palatable, but also would add costs which would change the model and its results.

Feltham and Hughes (1983) recognize two potential verification roles for accounting reports in a scenario similar to that of Leland and Pyle; but they assume the roles rather than derive or analyze them. Their analysis of a signaling mechanism is more general than that of Leland and Pyle in that Feltham and Hughes do not assume

linear contracts and do not assume that a fully revealing separating equilibrium will be the optimal solution to the adverse selection problem. The risk-averse agent/manager of a firm issues securities in order to acquire capital for investment and production and to share risks with investors. The firm's cash flows are determined by both general economic events and firm-specific events. In the absence of asymmetric information, optimal risk sharing is attainable where investors bear all firm-specific risks and systematic risks are shared. Both investors and the manager have homogenous beliefs about general economic events (i.e., nondiversifiable market risk). Investors have homogenous beliefs about firm-specific events (i.e., diversifiable risk) and know that the manager has inside information about the firm-specific events which affect cash flows. At the beginning of the period when the manager issues securities, he also issues a prospectus. The prospectus discloses the production plan, dividend policy, the manager's investment portfolio, and a message about his inside information. Two potential roles for accounting reports are ex ante verification and ex post verification: the inclusion of an accounting report in the prospectus which verifies disclosure in the prospectus, or the selection of an accounting system which will produce end-of-period accounting reports verifying disclosures made in the prospectus. In the Feltham and Hughes model, the second role of ex post verification is assumed, and the manager discloses the reporting system in the prospectus. Disclosures in the prospectus, other than the message, are assumed to be truthful due to the existence of penalties which will be imposed on the manager if the ex post verification process indicates that he was lying. The message

about his inside information is not verified and investors know that he selects disclosure in order to maximize his self interest. In addition, positive disclosure is assumed under which the manager does not lie, but does not necessarily reveal the entire truth. If the manager's inside information is good news, the expected value of cash flows exceeds that believed by the market, and investors would pay more for the firm's securities if they knew the inside information. The manager selects a prospectus in order to maximize his expected utility of end-of-period consumption, where his consumption is equal to the realized cash flow plus the return on his investment portfolio less dividends paid to investors. Feltham and Hughes' result is that whether a manager reveals his information in his message depends upon the prior belief of investors:

(i) If investors believe that there is a high probability that the manager has good news, a pooling equilibrium results in which no message is reported by the manager with good news. The firm is undervalued by investors by a small amount. Due to the undervaluation, the manager does not transfer all firm-specific risk, but bears some in order to side bet with investors. In other words, the manager agrees to sell some of his firm at an undervalued price, but will not sell all at that price. The manager with bad news also bears firm-specific risk, not to side bet, but to disguise his bad news and be overvalued by investors.

(ii) If investors believe that there is a high probability that the manager has bad news, a separating equilibrium results in which the manager sends a message and bears firm-

> specific risk to 'guaranty' the message. There is no side betting since investors correctly value the firm. The manager with bad news does not bear firm-specific risk since it is too costly for him to try to disguise his bad news.

While Feltham and Hughes refer to accounting reports, it seems that the contribution of the paper is in showing that a separating signaling equilibrium cannot be assumed to be optimal for the manager and the choice between separating and pooling depends upon investors' prior beliefs.

There have been a number of empirical studies which have alluded to the ability of the manager of a firm to communicate his inside information through accounting numbers. Gonedes (1976) tests the relative abilities of dividends, income, and extraordinary items to 'signal' information about the unobservable distribution of cash flows. Gonedes states that he is not directly testing the managerial motivation to signal or the credibility of the signals, and states in a footnote:

> "Note that a signaling management need not think of itself as actually engaging in signaling. The critical issue is whether it ends up providing observables that are potentially useful in making inferences about unobservable characteristics of its production-investment and financing decisions." (p.29)

In a signaling scenario, as has been discussed in this chapter, the manager does indeed think of himself as engaging in signaling in his anticipation of the reactions of investors in his selection of the signal. It appears that, although Gonedes refers to Spence's work on signaling, he is not testing a signaling hypothesis, but rather is testing for correlation among variables. The question he appears to

be asking is: given that income numbers are correlated with the price of a firm's security and can be used to explain cash flows, does the addition of dividend changes and/or extraordinary items add more explanatory power? In order to test a signaling hypothesis, it would be necessary to derive such an hypothesis from a model in which there was motivation for and credibility of communication of inside information through such actions as disclosure of extraordinary items.

Many empirical papers suggest that a managerial act or choice may be correlated with firm value without reference to a signaling model. These papers provide empirical evidence without attempting to explain managerial motivation for such actions. Patell (1976) examines the effect of voluntary corporate forecasts of earnings per share on security prices, and finds that the market reacted to the voluntary forecasts such that forecasts of earnings greater than the market's naive expectation were preceded by positive price adjustments and those less than the expectation were preceded by negative price adjustments. For both groups, there was a positive price reaction in the week of the forecast. Patell does state that one cannot distinguish between the inference that the forecasted numbers convey information or that the act of forecasting is informative.

Harrison (1977) examines market reactions to discretionary and nondiscretionary changes in accounting methods. He states that possible motivations for a discretionary change (e.g., a switch from accelerated depreciation to straight line) may be to avoid violation of bond covenants, to smooth income, or to communicate inside information about future cash flows. Nondiscretionary changes (e.g., a

switch from the equity method to the cost method of valuation of equity investments) are mandated by a regulatory agent. The statistically significant empirical results are that firms which made discretionary changes which increased income earned a lower return than firms making no changes, and firms which made nondiscretionary changes which increased income earned a higher return than no-change firms. He suggests that the discretionary act provides information to investors about changes in either systematic or residual returns. While no clear motivation for such voluntary actions is presented, and no explanation of why investors would react to such actions, Harrison's study does present evidence that managerial discretion is an intervening variable in the market reaction to accounting changes.

The objective of Penman's (1980) empirical study of firms making voluntary earnings forecasts is to provide evidence useful in evaluating the benefit of a mandatory forecasting regulation. He therefore addresses two questions: do such voluntary forecasts provide information to investors and do all firms voluntarily provide forecasts? His empirical results are:

(i) Earnings of firms making forecasts are, on average, higher than the market's expectation (defined in terms of a martingale with drift).

(ii) Forecasting firms have higher unexpected earnings than the average firm in the economy.

(iii) Excess returns are earned by the forecast firms, on average, on the day prior to the forecast date.

(iv) The sample of firms making forecasts earned, on average, greater returns than the market portfolio during the period studied.

Penman concludes that voluntary forecasts do provide information to investors and that the firms which self-select into the group of firms making forecasts are not representative of all firms. However, he also concludes that his empirical findings cannot be interpreted as evidence that forecasts should be mandatory, because firms that do not forecast may communicate information through some other action, such as dividends. In addition, the act of not forecasting may be inferred as communication of poor earnings.

Ricks (1982) analyzes earnings performance and market performance for firms switching from the FIFO method of inventory valuation to LIFO in 1973. His empirical results are:

(i) Firms that switched were more profitable in the year of switch than a control group of non-switching firms, and would have been even more profitable had they not switched.

(ii) Firms that switched showed lower security returns than the non-switching firm.

Ricks admits that there are several potential explanations for the poor market performance. A change in LIFO during inflation has one real economic impact: a reduction in taxes, which presumably increases firm value through the increase in after-tax cash flows. Therefore the act of switching to LIFO must convey negative information to investors such that they reduce their inferred value of the firm.

The empirical studies summarized above reveal that firms which issue voluntary forecasts of earnings and firms which voluntarily change accounting methods are not representative of all firms in the economy. A frequent criticsm of these types of studies is that the results are biased because firms which undertake such voluntary actions are self-selecting into a particular group of firms and that any market reaction may not be a response to the changed accounting number or to the forecast earnings, but to the act of changing or forecasting. However, it is exactly this reaction to self-selection which is interesting empirical evidence. While the motivation for such voluntary actions are not understood, the empirical studies do provide evidence that a manager of a firm may convey information to the market by a choice of an accounting reporting system.

CHAPTER THREE

THE ROLE OF THE INVESTMENT BANKER IN THE NEW ISSUES MARKET

INTRODUCTION

This chapter develops the verification role of the investment banker. Section 1 describes how the informational asymmetry existing in the new issues market may motivate signaling activities. Section 2 shows how a contingent contract between the issuer of a new security and investors may be used to resolve such an informational asymmetry. A review of the literature on intermediation and asymmetric information appears in Section 3. The role of the investment banker as an intermediary which provides credible verification of disclosure is developed in Section 4. It is shown in Section 5 that the role of the investment banker developed in Section 4 is consistent with the role as defined by existing regulation of the securities market.

1. SIGNALING AND THE NEW ISSUES MARKET

New issues of equity securities can be classified as seasoned or unseasoned. Seasoned issues are new issues of securities which are publicly held and for which a market exists. There is no existing

public market for unseasoned new issues which are securities of firms which have been privately held and now are 'going public'. A public offering of unseasoned common stock can be viewed as a situation in which a seller (the entrepreneur of the firm) offers for sale to buyers (the investors in the securities market) a product (shares of common stock) which has a quality or value which is unobservable. Investors can be assumed to know the average quality, or the distribution of quality, based upon their prior information, such as information about other public firms in the same industry; however, they cannot observe the quality or value of the securities of each individual new issue. When a firm is privately held, investors cannot trade in its securities and there consequently is no demand for or production of information about the firm. The entrepreneur/owner of the privately held firm however can be assumed to know the value of the firm due to his information about future cash flows which he acquires during his managerial activities within the firm. The informational asymmetry existing in this scenario is the same as that described by Akerlof (1970) and Spence (1973), and consequently is a situation in which there exists a demand for a signaling mechanism such as those described in Chapter Two.

The previous chapter indicated the two essential elements in a signaling model: the seller is motivated to undertake signaling activities and the signal is credible to the market of buyers. The single entrepreneur/owner of a firm or technology who is attempting to sell equity shares in his firm/technology, where the value of that

firm exceeds the market's average valuation, clearly is motivated to communicate his inside information because he will gain personally if the market increases its valuation of his firm's security. The motivation to signal in such a scenario is more apparent than those in other scenarios discussed in the previous chapter where the manager of a firm is assumed to be motivated to maximize the value of the firm. The communication will be credible to the market only if it is costly in the sense described by Spence, or if a contingent contract can be written on some ex post realization of value and signaled value. If the motivation and cost conditions are met, then the issuer of securities will be motivated to signal and will invest in a costly signal if the benefits of such credible communcation exceed its costs.

2. DISCLOSURE AND CONTINGENT CONTRACTS

Spence (1976) describes two types of signaling devices: exogeneously costly signals, and contingent contracts. As was described in Chapter Two, the financial signaling models of Bhattacharya (1979) and Leland and Pyle (1977) are combinations of exogeneously costly signals and contingent contracts. In such models the informational asymmetry is resolved; but the signaling equilibrium is inefficient due to the dissipative costs incurred. Ross's (1977) study is an example of a model of contingent contracting in which there is a penalty imposed on the risk-neutral manager of the firm if he signals falsely. Ross's model also is dissipative because any penalty imposed on the manager is not distributed to the shareholders who have overvalued the firm.

However, if the potential penalty is appropriate, the manager will be induced to make truthful disclosures. Ross does not consider any costs of inefficient risk sharing because the manager is risk neutral. As Spence indicates, if the signaler is risk averse and there is residual uncertainty about product quality, then the contingent contracting signaling equilibrium may result in non-optimal risk sharing.

The entrepreneur of a firm which is going public can resolve the informational asymmetry through the following contingent contracting process:

(i) The entrepreneur states the ex ante expected value of his firm.

(ii) Investors in the market price the firm as a function of the entrepreneur's statement.

(iii) If actual ex post value is found to be less than that stated, a monetary penalty is imposed upon the entrepreneur and the penalty is redistributed to shareholders.

The above contract is nondissipative and will result in efficient risk sharing if the entrepreneur is risk neutral or there is no residual uncertainty about actual value. However, actual end-of-period value is a realization of a random variable and may be low due to its being an outcome in the lower tail of the distribution. If a penalty were to be imposed upon the entrepreneur when the realized value was less than the stated expected value, there is a high probability (i.e., 50% when the random variable has a symmetric distribution) of a penalty imposition even when the entrepreneur discloses the true expected value. Such a penalty would induce the entrepreneur to disclose an expected value less than the true expected value, while

never reducing the probability of a penalty imposition to zero under truth-telling. While truth-telling, or not overstating value has been induced, a penalty will be incurred for low realizations in order to induce that truth-telling, and such a penalty will be due only to the randomness of ex post firm value.

The legal and regulatory system does indeed impose such a penalty scheme upon issuers of new securities. The Securities Act of 1933 gives purchasers of an issue the right to recover any damages they may suffer as a result of a price decline if the registration statement or prospectus contained an untrue statement of a material fact or materially misleading statement or omission.[1] Due to the SEC Act, all investors, including those who never read the prospectus, may try to recover damages. Clearly all investors would like to recover damages if the price declines; however the courts must be provided with proof that there was a material omission or misleading statement in the prospectus in order to award damages.[2] If the SEC awards damages to investors, the issuer of the security must buy back the shares from the investors at the offering price. The penalty imposed by the SEC therefore is the difference between the price paid by investors when the securities were issued and the current market price of the securities.

The contingent contract described above is costly to the entrepreneur in terms of inefficient risk sharing if the entrepreneur is risk averse and the penalty is contingent upon the realization of a random variable.

The process by which the market infers value and penalizes the entrepreneur is described below. A more precise formulation of the process is developed in section 4.

The value of the firm is assumed to be a function of the expected value of future cash flows. For a specific firm, there is an underlying distribution of future cash flows which is assumed to be a normal distribution in this discussion.

$$\tilde{x} \sim N(\mu, \sigma^2)$$

where $\tilde{x}$ = end-of-period cash flow

μ = expected value of $\tilde{x}$

σ^2 = variance of $\tilde{x}$

The entrepreneur knows the value of μ and σ^2 for his firm.

The market has a prior normal distribution on μ:

$$\mu \sim N(\bar{x}_0, \sigma_0^2)$$

In the absence of additional credible information, the market will value all firms as a function of $\bar{x}_0$, the average value of μ, which will lead to Akerlof's market failure.

The objective of the entrepreneur is to maximize his expected utility of end-of-period wealth. The disclosure will be of the form "The expected value of future cash flows is y." The market will value the firm based only upon the entrepreneur's disclosure:

$$\text{market value} = V(y)$$

and will not use its prior information in valuation. The market and entrepreneurs have implicit agreement about a penalty function under which the entrepreneur must buy back the shares if the realized value of $\tilde{x}$ is sufficiently low relative to y. The threatened penalty will induce truthful disclosure from the entrepreneur if it is of sufficient size and is perceived by the entrepreneur as credible. To be credible, the penalty must be imposed for low realizations of $\tilde{x}$, even though investors know that the entrepreneur was induced to tell the truth. If it were not imposed for low realizations, it would not provide an incentive for truth-telling. The investors and entrepreneur precommit to the penalty scheme.[3] The entrepreneur's end-of-period wealth is reduced by any penalty imposed upon him. Both firm valuation and the amount of the threatened penalty depend upon y: the higher is y, the greater is V(y) and the proceeds of the stock issue; and the higher is y, the greater is the probability that a penalty will be imposed at the end of the period, and the higher will be the penalty which is equal to V(y). Therefore y will be perceived by the market as a credible signal of value.

At the end of the period, investors observe a single realization x which is the end-of-period cash flow. Investors know that $\tilde{x}$ is a random variable and that to impose a penalty when $x < y$ will induce the entrepreneur to disclose $y << \mu$ rather than $y = \mu$ in order to avoid a penalty. In order to induce the entrepreneur to reveal μ, a penalty is imposed if $x < L < y$. In the determination of L, investors form a posterior estimate of μ from their prior estimate

and the realization x, form a posterior confidence interval around the posterior estimate, and penalize the entrepreneur if y falls above the confidence interval. The posterior distribution of μ is derived following deGroot (1970, p. 166-68).

$$\mu \sim N(\bar{x}_1, \sigma_1^2)$$

$$\text{where } \bar{x}_1 = \frac{\bar{x}_0 \sigma^2 + x\sigma_0^2}{\sigma_0^2 + \sigma^2} \tag{3.2.1}$$

$\bar{x}_1$ is a weighted sum of $\bar{x}_0$, the prior estimate of μ, and the realization x. The larger is the variance of $\tilde{x}$, the greater is the weight placed upon the prior estimate. Investors form confidence intervals around $\bar{x}_1$ such that

$$L = \bar{x}_1 + \text{an increasing function of } \sigma,$$

such as $\bar{x}_1 + 2\sigma$ which is a commonly used confidence interval in statistical hypothesis testing. If $y > \bar{x}_1 + 2\sigma$, the entrepreneur is penalized.

Then

$$\Pi = \text{Probability } \{y > \bar{x}_1 + 2\sigma\} .$$

represents the probability that the entrepreneur will be penalized. Figure (3.1) illustrates the posterior distribution of μ and the penalty probability.

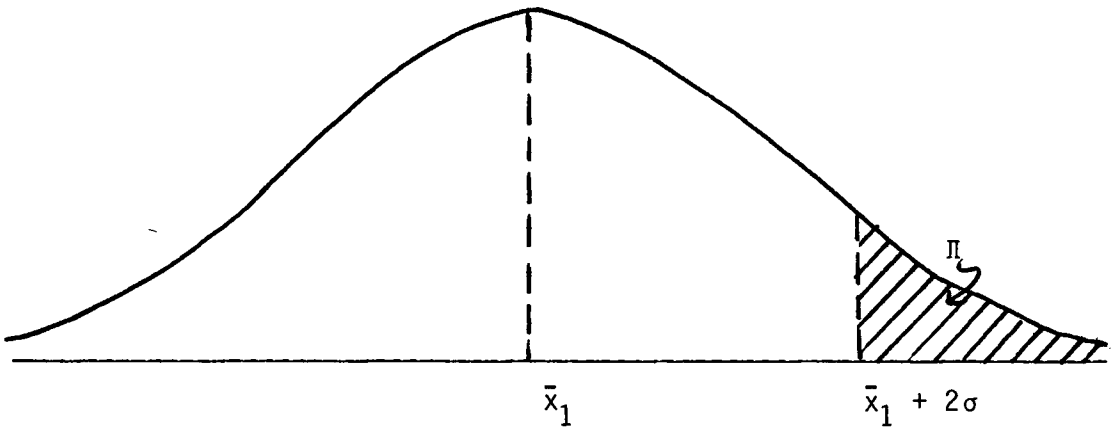

Figure 3.1. Posterior Distribution of μ

The stages in the process therefore are:

1) At the beginning of the period,

   i) the entrepreneur knows μ.

   ii) Investors believe that the expected value of cash flows is $\bar{x}_0$.

   iii) The entrepreneur reveals y, investors infer V(y), and the securities are sold for V(y).

2) At the end of the period,

   i) investors and the entrepreneur observe x.

   ii) Investors form a posterior estimate of the expected value of cash flows, $\bar{x}_1$ from $\bar{x}_0$ and x.

   iii) Investors form a confidence interval such that $L > x_1$ and penalyze the entrepreneur if $y > L$.

iv) If $y > L$, the entrepreneur buys back the securities for $V(y)$ which have a current market value $V_1 < V(y)$ and bears a loss $V(y) - V_1$.

Since the entrepreneur knows the expected value and variance of cash flows, he knows the value of Π at the beginning of the period. The probability that $y > \bar{x}_1 + 2\sigma$ is expressed as

$$\Pi(y;u,\sigma^2) = \Pi(y > \bar{x}_1 + 2\sigma)$$

and the expected value of the penalty conditional on $y > \bar{x}_1 + 2\sigma$, is

$$\Pi \cdot [V(y) - E(V_1)]$$

The end of period loss depends upon the realization of $\tilde{x}$ and therefore is a random variable. The random variation in ex post observed value imposes risk upon the risk-averse entrepreneur. Therefore, even if truth-telling is induced by the contingent contract, the signaling mechanism is costly to the entrepreneur in terms of additional risk which he must bear. In Section 4 it will be shown that the cost of this contingent contract can be reduced through the use of an intermediary.

3. INTERMEDIATION AND ASYMMETRIC INFORMATION

A potential role for an intermediary may be the resolution of an informational asymmetry. Leland and Pyle (1977) suggest at the end of their paper that financial intermediaries may be the most efficient producers of information if there are economies of scale to information production. Information may be produced at a cost, but cannot be sold directly to the market at a price reflecting its true

value due to its public good nature. Therefore the return to costly information production can be realized through returns on a portfolio of assets which are purchased on the basis of inside information. To overcome the moral hazard problem involved in communicating the value of its information, the organizers of the intermediary can signal the value of their information and resulting portfolio through their investment in the equity of the intermediary. Leland and Pyle only suggest that there may be economies of scale in information production in order to justify the existence of an intermediary. However, since the individual entrepreneur can credibly communicate his inside information to the market through his equity position in his own project or firm, a demand for the intermediary will not arise unless the intermediary can resolve the informational asymmetry at a smaller cost to the entrepreneur than his cost in terms of lack of diversification.

Campbell and Kracaw (1980) develop a model of financial intermediation following the suggestion of Leland and Pyle that intermediation resolves the problems of moral hazard and appropriability associated with information production and communication. They assume that firms cannot signal to resolve the asymmetry themselves. Such an assumption is used to justify the existence of the intermediary and demand for its informational production services. However, it may be the case that an intermediary can produce and credibly communicate the entrepreneur's inside information at less cost than the entrepreneur's cost of credible communication; and therefore that such an assumption may not

be necessary. In their model, undervalued firms pay information producers to undertake costly production of information which will reveal the true value of the firms. Such information is credible to the market because the information producer invests in the intermediary. The informational asymmetry is resolved, but the need for the producer to invest in the intermediary provides a wealth constraint on information production which may preclude efficient information producers from entering the information production market. Therefore, the cost to the undervalued firms of credibly communicating their value may be great due to the potential exclusion of efficient information producers.

The Campbell/Kracaw analysis indicates that direct communication of information creates a moral hazard problem which can be resolved, but at a cost. Since all investors are assumed to be risk neutral, the cost is not risk-bearing, but a less efficient production of information due to the exclusion of potential producers with insufficient wealth to satisfy the wealth constraint. Such a solution appears to be somewhat artificial since there arises the motivation for the poor, efficient producers to signal their efficiency in order to borrow to satisfy the wealth constraint. There also is no a priori demand for the information producer if alternative ways of signaling were permitted. The authors do suggest at the end of their paper that a more promising explanation for the existence of intermediaries might be that they produce information and provide other services. It does appear that they have not adequately justified the demand for an intermediary to undertake costly production and costly communication of inside information.

Thakor (1982) models debt insurers as third party information producers when borrowers know their default probabilities and potential lenders do not. The insurers invest in costly information production and offer insurance to borrowers at premia which are a function of insurance coverage and default probability. A borrower purchases insurance coverage which signals its true default probability and compensates insurers for the default risk borne and the cost of information production. If all market participants are risk neutral, the demand for insurance will exist because of its signaling benefits. Under risk aversion, the intermediary provides an insurance service in addition to information production. Therefore, with risk aversion, there is a demand which justifies the existence of the intermediary aside from the demand for a signaling mechanism. Thakor does assume that borrowers cannot signal directly and that no other intermediaries or credit rating agencies exist. Therefore there is no analysis of the efficiency of signaling through debt insurance.

The intermediaries described by Leland and Pyle, Campbell and Kracaw, and Thakor are information producers who invest in costly production of information which is already known by one market participant: the entrepreneur or borrower. There is a potential role for an intermediary as a verifier of information already known by the firm. The cost of verification of information may be less than the production cost of duplicate information. Indeed, it may be impossible to produce information without examination and verification of the information provided by the entrepreneur/firm. The actual role of financial intermediaries and debt insurers may be

that of verification and supplementation of information provided by firms about past and projected cash flows.

Diamond (1984) analyzes the role of intermediation as delegated monitoring rather than information production and develops conditions under which third-party monitoring of an entrepreneur will be less costly than direct monitoring by investors when there is asymmetric information between investors and the entrepreneur. In Diamond's model, the entrepreneur and investors have homogeneous beliefs about the distribution of $\tilde{x}$, the cash flows generated by the firm. The end-of-period realization x is observable only by the entrepreneur, and monitoring costs are sufficiently high so that the free rider problem will prohibit expenditures on monitoring activities by any investor. As a consequence, investors will only agree to purchase a debt security promising a fixed end-of-period return, and will provide the debt capital only if they believe that the expected return exceeds the expected return on an alternative investment. Since x cannot be observed by lenders, the entrepreneur can claim that $x<z$ when $x \geq z$, make a payment less than z to lenders, and pocket the difference. The optimal debt contact between the entrepreneur and lenders involves a non-pecuniary penalty (e.g., bankruptcy) to be imposed if lenders do not receive a minimum expected return. Since $\tilde{x}$ is a random variable bounded below by zero, there is some positive probability that x will be less than the minimum return and that the penalty will be imposed (assuming that the entrepreneur has no other wealth). The deadweight penalty is due to the unobservability of x. The entrepreneur would be better off if x were observable and he could share the risk of x with equity securityholders.

Diamond introduces an intermediary delegated to monitor the entrepreneur's information. The intermediary borrows from lenders at a promised rate, loans to the entrepreneur, spends K to monitor the realization x, and receives a payment from the entrepreneur which is unobservable by its depositors. The intermediary is viable if

(i) its depositors receive an agreed upon minimum expected return,

(ii) the intermediary receives a return of at least zero after incurring monitoring costs and deadweights penalties, and

(iii) the entrepreneur is no worse off than if he contracted directly with the depositors/lenders.

The unobservability of the payment received by the intermediary from the entrepreneur gives rise to the same moral hazard problem which existed between the entrepreneur and lenders. As a consequence, penalties will be imposed if the actual payment made to depositors is less than the promised return, and identical deadweight losses are incurred. Therefore a single intermediary monitoring a single entrepreneur is not viable because the deadweight loss has not been reduced and an additional cost of monitoring has been incurred.

Diamond proves conditions under which delegation is viable under both universal risk neutrality and risk aversion. Under risk neutrality, delegation is viable if the intermediary monitors more than one entrepreneur and projects have less than perfect correlation, and delegation costs reach a minimum when projects are identically and independently distributed. Deadweight losses are reduced because penalties are imposed when the realization x is in

the extreme lower tail, and the probability of the average return across projects being in that tail is decreasing as the number of entrepreneurs monitored increases. If the number is large and returns are independent, the cost per entrepreneur of monitoring approaches K.

In order to focus on the diversification of risk under risk aversion, Diamond eliminates the deadweight penalty cost by assuming that the entrepreneur has no wealth constraint. The entrepreneur cannot share risks with its lenders due to the unobservability of x. He can, however, share risks with the intermediary and will do so if the risk premium charged by the intermediary is less than the entrepreneur's risk premium. One type of diversification is when one individual in the intermediary monitors many entrepreneurs. If all agents in the market have a negative exponential utility function, and project returns (conditional on the market) are independent, then the risk aversion toward the Nth independent gamble is a constant, not decreasing, function of N and therefore the intermediary's risk premium is the same as the entrepreneur's, regardless of the size of N. A second type of diversification which does result in a reduced risk premium is when there are many bankers in the intermediary who share information as well as risks. As the number of agents in the intermediary grows large, the average risk premium grows small and the cost of delegation approaches K.

Therefore, Diamond has identified a set of conditions which lead to diversification within financial intermediaries so that delegated monitoring is preferred by entrepreneurs. They are:

(i) Risk neutrality and the intermediary monitors N entrepreneurs with project returns less than perfectly correlated.

(ii) Risk aversion and N agents within the intermediary share risks.

Under each of these conditions, as N grows very large, the costs of delegated monitoring approach K.

The intermediary of Diamond provides viable monitoring of the unobservable ex post realization of $\tilde{x}$. In the following analysis, where there is asymmetric information about the ex ante expected value of $\tilde{x}$, an investment banker is viewed as an intermediary who verifies (or monitors) the statements made by the entrepreneur about the value of the ex ante mean μ. It will be shown that the investment banker is able to provide such verification at less cost than the entrepreneur due to sub-division of risk within the intermediary.

4. THE ROLE OF THE INVESTMENT BANKER UNDER ASYMMETRIC INFORMATION

It was seen in the preceding section that an efficient resolution of an informational asymmetry may be feasible through the use of an intermediary. The investment banker's role will be shown to be that of such an intermediary. It will be shown that (a) such a role emerges from the provision of other services for which there is a demand (as in Thakor) and, (b) the intermediary is viable (as in Diamond).

In the absence of asymmetric information between an entrepreneur and the market, the investment banker can provide three types of services to the issuer of a new security:

(i) insurance,
(ii) distribution of security, and
(iii) advice about market conditions and pricing of the security.

The first service is provided when the banker underwrites the issue under a firm commitment or a stand-by agreement. In the former, the issuer sells the entire issue to the underwriter at an agreed upon price and the underwriter bears the entire risk of meeting adverse market conditions. In the latter, the underwriter agrees to buy whatever amount of the security the issuer has been unable to sell directly at a specified price. There is no underwriting or insurance provided in a best-efforts agreement where the banker only agrees to use his best efforts to sell the issue and does not buy any of it from the issuer. The distribution service is purchased when the banker sells the issue because he may be better able to generate demand due to his contacts and experience in the market, because he has a clientele, or because he has a "reputation" which the issuer lacks. Baron (1982) alludes to the potential ability of the banker to certify the issue in his discussion of how the banker may generate demand due to his reputation behind it. Theoretically, the banker could provide the underwriting service without actually selling the issue, but such contracts are not observed in North America (they are observed in England). The third service is provided when the banker has better information about market conditions than the issuer.

Therefore, there is a demand for the services of an investment banker in the absence of an informational asymmetry between the entrepreneur and investors. When an asymmetry does exist, the underwriter provides a fourth service: verification of information contained in the selling document. As will be discussed in Section 5, due to the regulatory environment within which new securities are issued, the underwriter is liable for damages if a future

price decline can be proven to be attributable to misrepresentation or fraud in the selling document. The selling document, or prospectus, contains information about the firm or planned investment project which is provided to the market by the entrepreneur. It is a direct disclosure about firm value. The selling document therefore is also a disclosure document and as such is subject to disclosure regulation. It is from these two natures of the prospectus that the benefits and costs of such direct disclosure emerge. As a selling document, its contents are related to selling price in that the higher the value indicated in the prospectus, the higher will be market valuation. As a disclosure document, its contents are related to legal liability in that the higher the value indicated, the greater the probability of misrepresentation and consequent legal liability for price decline. It is this cost of such direct disclosure which makes the disclosure credible.

It is not clear that such regulation by a governmental agency is necessary. As long as the government enforces contracts, the entrepreneur or underwriter and investors could negotiate contingent contracts based upon direct disclosure and ex post realizations for each security issue. However, it is not the purpose of this paper to evaluate the efficiency of governmental specification of contingent contracts, but to take its regulatory existence as given, while recognizing that there may be viable market solutions to the problem of providing credibility to direct disclosure. The model in the subsequent section requires only that contingent contracts are enforced.

It was shown earlier in this chapter that the issuer could assume liability for fraud, which could eliminate the demand for verification of disclosure by the investment banker. In a competitive

market, the issuer will compensate the banker for its assuming the liability. In the absence of regulation, the investment banker will be a viable provider of verification if it is less costly to the issuer to use the banker rather than to provide credible verification himself. It will be shown that since the banker reduces risk through risk dividing, it can provide verification at less cost than the risk-averse entrepreneur. Therefore, it is not necessary to the analysis that the government regulate that the underwriter is liable since it is likely that he would emerge as an efficient provider of verification in the market. In reality, due to regulation, if an issuer purchases the underwriting or distribution services of an investment banker, it must also purchase certification services.

Optimal contracts between the issuer and investment banker for the insurance, distribution, and pricing services have been derived in several published papers. Mandelker and Raviv (1977) look at the insurance function under different assumptions about risk attitudes. Issuers are inferred to be risk averse if they seek insurance against the risk of adverse market conditions. If the issuer is risk neutral, he will bear all risk and will contract with an investment banker only if services other than insurance are provided, in which case a best efforts agreement where the banker bears no risk is optimal. If both issuer and underwriter are risk averse, risks are shared through a stand-by agreement. If the issuer is risk averse and the underwriter risk neutral, a firm commitment contract where the underwriter bears all risk is optimal. The Mandelker/Raviv results

are standard Pareto-optimal risk-sharing results when there are no problems of moral hazard and/or asymmetric information.

Baron (1979) analyzes the moral hazard problem existing when the investment banker recommends an offering price and the selling efforts of the banker are unobservable. A low price promotes good relationships between the banker and his customers and permits a reduction in the amount of effort required to sell the issue. If the issuer is risk averse and the banker risk neutral, the optimal contract is firm commitment in which the issuer determines the offering price. In such a contract, the banker bears all of the cost of his shirking. An incentive problem exists only when the banker is risk averse and in such a case the optimal contract is a stand-by agreement with a bonus paid to the banker when the issue is sold out. Therefore, Pareto-optimal risk sharing between issuer and banker is attainable when the banker is risk neutral.

An issuer of a new security will seek advice about pricing when an informational asymmetry exists between issuer and banker about market conditions. Baron and Holmstrom (1980) investigate contracting under asymmetric information when the banker obtains his superior information after the time of contracting. It may be in the best interests of the banker to suggest a low selling price in order to benefit favored customers and achieve a quick sale. If the banker is risk neutral, a firm commitment contract is optimal, even when the issuer is risk neutral and no insurance is demanded, because the banker then bears the consequences of mispricing the issue. When the banker is risk averse, his compensation is based upon offering price

in order to induce him to set a higher price. Again, Pareto-optimal risk sharing is attainable if the banker is risk neutral.

Baron (1982) analyzes the contracting problem under both asymmetric information and moral hazard when the issuer demands both advisory and distribution services. Both issuer and banker are assumed to be risk neutral in order to eliminate the demand for underwriting. From the earlier results, it is clear that a firm commitment contract will be optimal if the banker obtains his superior information after contracting. However, if the banker has superior information about market conditions prior to contracting, the first-best firm commitment contract is not attainable. Whether the issuer will actually issue the security depends upon capital market conditions and therefore upon the banker's information. The banker must be provided an incentive to truthfully communicate his information to the issuer. Due to the addition of a truth-telling constraint, optimal risk sharing is not attainable and a firm commitment contract will not be optimal, even when the investment banker is risk neutral.

The results of the above papers can be summarized as follows: a firm commitment contract is optimal and results in Pareto-optimal risk sharing between issuer and investment banker when the banker is risk neutral, unless the banker has an informational advantage about capital market conditions prior to contracting. The moral hazard problem due to the banker supplying unobservable distribution effort and the information problem due to the banker's gaining an informational advantage after contacting can be eliminated by imposing all risk on the banker. If the investment banker is risk

averse, optimal contracts involve risk sharing and provision of incentives.

Due to these contracting results, modeling the investment banker as a verifier of information will be simplified considerably if:

(i) the investment banker is risk neutral, and

(ii) there is no informational asymmetry between issuer and banker at the time of contracting.

If (i) and (ii) hold, any moral hazard and/or asymmetric information problems between banker and issuer can be ignored and a firm commitment contract will be optimal.

In any underwriting agreement, the underwriter bears some of the risk of adverse market conditions. If the banker is risk averse, it is costly to him if the entire issue is not sold. The risk to the banker can be reduced by dividing the risk with another banker. In the model to be analyzed in Chapter Four, it will be shown that, in the absence of asymmetric information, it is optimal for the risk-averse entrepreneur to sell 100% of the risky equity in his firm. If the investment banker has the same preferences and attitude toward risk as the entrepreneur, it will be as costly to the banker if he does not sell 100% of the issue as it would be for entrepreneur. Since the entrepreneur must compensate the investment banker for all costs, the entrepreneur is no better off than selling the issue directly to the market. However the banker can reduce risk by dividing the risk with another banker so that each owns 1/2 of the equity in the firm. As the number of risk-averse bankers grows to n,

each buys $\frac{1}{n}$ of the shares sold by the entrepreneur. The risk added to the portfolio of each banker is $\frac{1}{n^2}\sigma^2$. Therefore as n grows large, the risk borne by each member of the underwriting group becomes very small, so that a large underwriting group behaves as if it were risk neutral. Since the notation necessary for a formal proof of this reasoning is not developed until Chapter Four, the proof appears in Appendix 1.

It will be assumed that n is sufficiently large that the underwriter is risk neutral. It also will be assumed that there is no informational asymmetry between issuer and banker at the time of contracting. Therefore a firm commitment contract is optimal.

In addition to risk sharing, the underwriter provides advisory, distribution, and verification services. Since the underwriter is risk neutral, its objective function is to maximize net income, where the issuer compensates the underwriter for all costs which it incurs in supplying the services. The contract between entrepreneur and underwriter will be analyzed because the cost to the entrepreneur of purchasing the services of the underwriter will enter the signaling problem in the next chapter.

The information sets and actions of the entrepreneur, underwriter, and market at various points of time will be described.

(i) Prior to contracting:

(a) The entrepreneur knows μ, σ^2.

(b) The market has prior information about the distribution of μ and knows σ^2.

(c) The underwriter knows about the selling conditions in the capital market.

(d) During the period of time prior to contracting, the underwriter investigates and verifies the information provided by the entrepreneur about firm value.

(ii) Time of contracting between entrepreneur and underwriter:

(a) The entrepreneur and the underwriter have identical information sets about μ, σ^2, and market conditions.

(b) The market has received no additional information with which to revise prior distribution of μ.

(c) At the time of contracting, the entrepreneur and underwriter agree as to the share to be received by the entrepreneur. Since the underwriter is risk neutral and the entrepreneur is risk averse, a fixed payment to the entrepreneur is the Pareto-optimal risk sharing arrangement. The entrepreneur selects y, where y is his direct disclosure about μ.

(d) During the period of time prior to the issue, the underwriter exerts effort in pre-selling activities which may provide him with better information about market conditions.

(e) The market observes y and values the firm based upon the signal.

(iii) Time of issue:

(a) The entrepreneur sells the equity in the project to the underwriter for the agreed upon price.

(b) The underwriter selects the offering price and sells the entire issue.

(c) It will be assumed that the underwriter does not overprice the issue. The entire issue is sold.

The cost of direct disclosure was described earlier in this chapter as an end-of-period penalty if a sufficiently low outcome occurs. The expected penalty, or loss, was shown to be

$$L(y) = \Pi(y;\mu,\sigma^2)[V(y) - E(V_1)], \text{ conditional on } y \gtrsim x_1 + 2\sigma$$

where $\Pi = \text{Prob}\{y > \bar{x}_1 + 2\sigma\}$

Let $P_0 = V(y)$, the issue, or offering price. It will be assumed for ease of analysis that V_1 is sufficiently small if a penalty is incurred that the expected loss will be:

$$L(y) = P_0 \Pi(y;\mu,\sigma^2)$$

In order to obtain an expression for Π, substitute for $\bar{x}_1$ using (3.2.1):

$$\Pi = \text{Prob}\left\{y > \frac{\bar{x}_0\sigma^2 + x\sigma_0^2}{\sigma_0^2 + \sigma^2} + 2\sigma\right\}$$

$$= \text{Prob}\left\{x < \frac{(y-2\sigma)(\sigma_0^2+\sigma^2) - \bar{x}_0\sigma^2}{\sigma_0^2}\right\}$$

which is the correct way to express Π, given y, because x is the realization of the random variable. Π can also be expressed as:

$$\pi(y;\ \mu\ \sigma^2) = \int_{-\infty}^{T} f(x)dx$$

$$\text{where} \qquad T = \frac{(y-2\sigma)(\sigma_0^2+\sigma^2) - \bar{x}_0\sigma^2}{\sigma_0^2}$$

f(x) is the normal density function of $\tilde{x}$.

Consider some characteristics of the loss function.

(i) $\frac{\partial L(y)}{\partial y} > 0$ because the loss occurs when $y > \bar{x}_1 + 2\sigma$. Therefore Π increases with y.

(ii) $\frac{\partial L(y)}{\partial \mu} < 0$ because μ is the mean of the underlying distribution. As μ increases, $\bar{x}_1$ will increase because the distribution of

$\tilde{x}$ shifts to the right and higher realizations x are more likely.

(iii) $\frac{\partial L(y)}{\partial \sigma^2} < 0$ because the confidence interval is

$$CI = \frac{\bar{x}_0 \sigma^2 + x\sigma_0^2}{\sigma_0^2 + \sigma^2} + 2\sigma$$

and $$\frac{\partial CI}{\partial \sigma^2} = \frac{(\sigma_0^2+\sigma^2)\bar{x}_0 - \bar{x}_0\sigma^2 - x\sigma_0^2}{(\sigma_0^2+\sigma^2)^2} + 2$$

$$= \frac{\sigma_0^2(\bar{x}_0 - x)}{(\sigma_0^2+\sigma^2)^2} + 2$$

Then $\frac{\partial CI}{\partial \sigma^2} > 0$ if $x < \bar{x}_0 + 2\sigma_0^2 + 4\sigma^2 + \frac{2\sigma^4}{\sigma_0^2}$

which will be assumed to be so.

If σ^2 increases while μ is constant, there is greater weight in both tails of the distribution and therefore a greater probability of a higher or lower x. With the symmetric distribution, these two probabilities will offset each other in

determining whether a higher or lower x is more likely.

L(y) changes in response to σ^2 due to the effect on the confidence interval. As σ^2 increases, the confidence interval increases as long as

$$x < \bar{x}_0 + 2\sigma_0^2 + 4\sigma^2 + \frac{2\sigma}{\sigma_0^2}.$$

The result of an increased confidence interval is $\frac{\partial L(y)}{\partial \sigma^2} < 0$.

In summary, the loss function is increasing in y and decreasing in μ and σ^2. The underwriter's selection of P_0 <u>after</u> y has been optimally chosen by the entrepreneur is analyzed below. The entrepreneur's choice of y is the subject of Chapter Four.

Let P_0 = Proceeds of issue received by the underwriter.

= Offering price x Number of shares sold

= $P_0(e,\Theta)$

Where e is the underwriter's unobservable distribution effort such that $\partial P_0/\partial e > 0$

Θ represents the state of the capital market about which the underwriter may have superior information.

The P_0 function is a demand price function and represents the maximum price that can be obtained, given a selected level of effort. In Figure 3.2, if θ_1 is the realized state of nature and e* is

the level of selling effort chosen by the underwriter, the P_o^* is the maximum offering price obtainable.

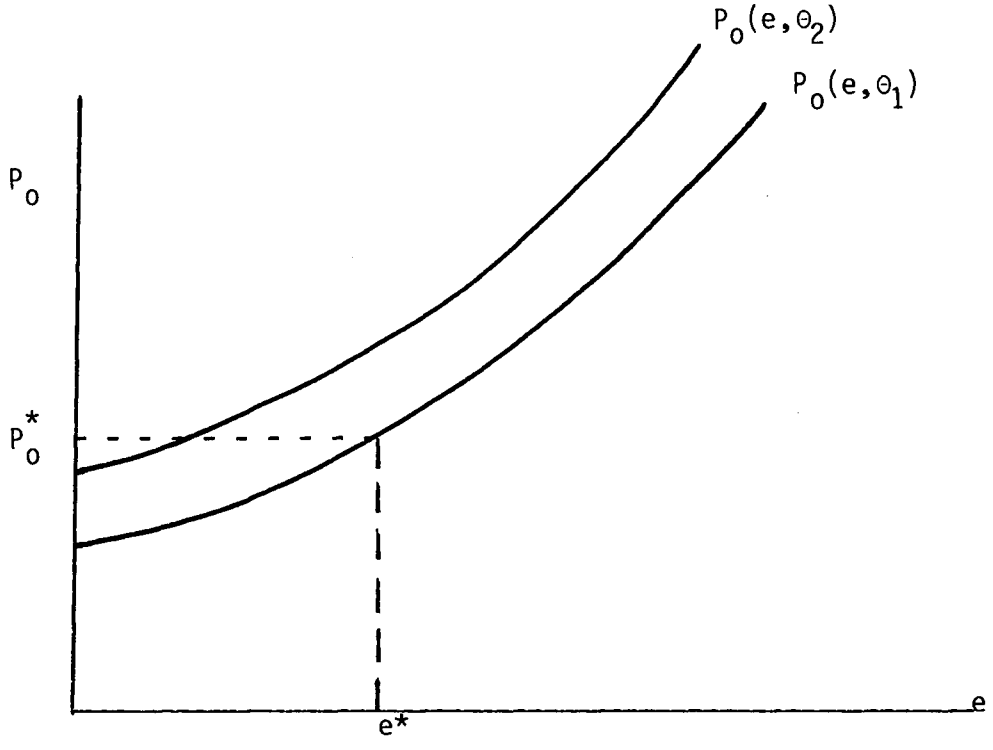

Figure 3.2 Offering Price Function

Let P_N = Net proceeds to entrepreneur

P_E = Equilibrium price based upon μ.

The underwriter and entrepreneur negotiate P_N and the underwriter selects P_o. If $P_o = P_E$, the issue is sold at equilibrium price. If $P_o < P_E$, the issue is underpriced by the underwriter.

Then $P_E - P_N$ = Cost of issuing shares through the underwriter. It is compensation for risk-sharing, advice, selling efforts, and verification.

The value of the firm is P_E, but the entrepreneur receives only P_N for the equity sold.

The problem of the risk-neutral underwriter is to select optimal effort in order to maximize profit, or:

$$\max_{e} \text{Profit} = P_o(e,\Theta) - P_N - K(y) - L(y,P_o) - g(e) \tag{3.4.1}$$

where $K(y)$ = direct out-of-pocket costs of verification

$L(y,P_o)$ = present expected value of end-of-period loss.

$$= \frac{P_o \Pi(y;\mu,\sigma^2)}{1+r} \tag{3.4.2}$$

$g(e)$ = disutility of effort where $g'(e)>0$

The first-order condition to the maximization is:

$$\frac{\partial \text{ Profit}}{\partial e} = \frac{\partial P_o}{\partial e} - \frac{\partial L}{\partial P_o} \cdot \frac{\partial P_o}{\partial e} - g'(e) = 0$$

or

$$\frac{\partial P_o}{\partial e} = g'(e) + \frac{\partial L}{\partial P_o} \cdot \frac{\partial P_o}{\partial e} \tag{3.4.3}$$

It is seen in (3.4.3) that the level of effort is selected so that the marginal benefit of that effort is equal to the marginal cost. The marginal benfit $\frac{\partial P_o}{\partial e}$ is the increase in income arising from the underwriter's share in the offering price. Marginal costs are the marginal disutility of effort $g'(e)$ and the marginal increase in future loss arising from the increased offering price.

There is a large amount of empirical evidence that unseasoned new issues are, on average, underpriced.[4] Equation (3.4.3) suggests that such underpricing would result due to the underwriter's disutility of effort and the dependency of future loss on offering price. Baron (1979) also noted that the disutility of distribution effort would provide incentives to the underwriter to underprice.

The underwriter selects effort after y has been disclosed, and the underwriter assumes legal liability for the truthfulness of the entrepreneur's disclosure. The signal y is credible because the contingent contract between the underwriter and the market prescribes that a penalty will be imposed on the underwriter if the realization of $\tilde{x}$ is sufficiently low relative to y that the courts will believe that y was a misrepresentation of μ. The disclosure is made by the entrepreneur, but the contingent contract is between the underwriter and the market. The entrepreneur will compensate the underwriter for its expected loss. However, the loss to the risk-neutral underwriter is smaller than the effective loss would be to the risk-averse entrepreneur in a contingent contract between the entrepreneur and the market because of the randomness of the outcome reduced by the regulatory system. Therefore the role of the underwriter to verify y and assume liability for y is viable.

In a competitive market for the services of investment bankers, the banker will be compensated so that it earns zero net profit in equilibrium. From (3.4.1), zero profits are earned when

$$P_O - P_N = K(y) + L(y) + g(e). \tag{3.4.6}$$

The underwriter's share of the proceeds of sale is compensation for direct costs, the expected future loss, and the disutility of effort.

In summary, the final optimal contract between the entrepreneur and the underwriter is a firm commitment contract in which:

(i) The entrepreneur and the underwriter agree as to the entrepreneur's share P_N. Such a risk-sharing arrangement is optimal because the entrepreneur is risk averse, the underwriter is risk neutral, and underwriter effort is unobservable.

(ii) The entrepreneur selects y. The selection of y will be analyzed in Chapter Four.

(iii) The underwriter verifies y at a direct cost K(y) and has a contingent contract with the market about the truthfulness of y.

(iv) The entrepreneur exactly compensates the underwriter for all costs and the present value of expected future liability losses so that the underwriter earns zero net profits.

(v) The underwriter selects the offering price and bears the entire risk of its shirking in distribution effort.

The cost to the entrepreneur of issuing securities through the intermediary is

$$C(y) = P_e - P_N \tag{3.4.5}$$

where P_e is the true value of the firm.

It was shown above (in 3.4.4) that the payment to the underwriter is

$$P_o - P_N = K(y) + L(y) + g(e)$$

If $P_o = P_e$, then

$$C(y) = K(y) + L(y) + g(e),$$

or the cost to the entrepreneur is equal to the payment to the underwriter. The combination of (3.4.4) and (3.4.5) results in

$$C(y) = P_e - P_o + K(y) + L(y) + g(e) \ . \qquad (3.4.6)$$

Therefore the cost to the entrepreneur exceeds the payment to the underwriter if $P_o < P_e$. The difference is a gain to the shareholders who purchase the securities at an offering price which is less than equilibrium value. This shareholder gain is a necessary cost to the entrepreneur due to the agency problem arising from the unobservability of the underwriter's effort.

The use of the underwriter by the entrepreneur is a viable means of credible communication of inside information to the market if the three conditions identified by Diamond are satisfied:

1. The entrepreneur receives a return at least as high as he would by contracting directly with the market.
2. The underwriter/intermediary earns a zero expected return.
3. Investors receive the same returns that they would if they contracted directly with the entrepreneur.

Condition one has been shown to be satisfied because the cost to the risk-neutral intermediary of the positive probability of a future penalty is less than the cost to the risk-averse entrepreneur, and the entrepreneur compensates the intermediary for the expected penalty. If there is an agency problem such that the intermediary's distribution effort is unobservable, there is an additional cost to

the entrepreneur. However, as with any principal-agent problem, the principal contracts with an agent only if there is benefit in doing so (e.g., it would cost more for the entrepreneur to directly distribute the securities). Condition two was shown to be satisfied. Condition three is satisfied because the penalty threat induces truthtelling such that investors will never pay more than P_e for the shares. Investors will be better off under the agency problem when $P_o < P_e$.

5. THE REGULATORY ENVIRONMENT

The issuance of new securities is regulated by the Securities and Exchange Commission according to the Securities Act of 1933. The stated purpose of the Act is to protect investors against fraud. The issuing firm is required to provide a minimum amount of information about the firm, its business environment, and its financial condition so that investors can value the firm. The Act specifies conditions under which investors can claim damages if the disclosures made by the issuing firm can be later proven to be misleading or fraudulent or if material information is omitted.

There are three principal parties to a typical stock issue: the issuing firm, investors, and the investment banker.[5] The investment banker provides various services to the issuer which were discussed in Section four. However, in the eyes of the SEC, the chief role of the investment banker/underwriter is to provide additional protection to the investors.

> "The underwriters's obligations and his central role as the intermediary between the issuer and the investing public rightfully cause the public to look to the underwriter for protection against defects in the prospectus and to expect him to verify the accuracy of statements in the registration statement. By associating himself with a proposed offering, an underwriter impliedly represents that he has made an investigation in accordance with professional standards. Investors properly rely on this added protection which has a direct bearing on their appraisal of the reliability of the representations in the prospectus" [SEC 1933 Act Release, No. 5274, July 26, 1972.]

Therefore, the SEC clearly views the investment banker as a third-party intermediary whose role is to verify disclosures made by the entrepreneur. The investment banker is legally liable (along with directors of the company and its auditors) for damages if price declines within three years and material omissions or misstatements are in the prospectus.

A firm can sell unseasoned shares directly to the public market. Since virtually all new issues are sold through underwriters, it is clear that there is a demand for the advisory, insurance, distribution, and verification roles provided by the investment banker.

The issuing firm and the investment banker enter into an informal agreement up to six months prior to the issue date. During the months before issue, the investment banker is required by the SEC to conduct an intensive investigation along with lawyers, auditors, and engineers into the contents of the S-1 registration statement. When the registration statement is filed, a preliminary prospectus (i.e., the red herring) is published which contains most of the information which will appear in the final prospectus if it is approved by the SEC.[6] The offering price and details about the underwriting agreement and the syndicate membership do not appear in the preliminary prospectus. If the SEC does not find the registration deficient, it

becomes effective within a few weeks. At that time, the underwriting agreement is signed, an offering price is chosen, the final prospectus is published, and the issue goes to market. The issuing firm receives the net proceeds of sale from the investment banker about a week after issue.

No preselling or market conditioning activities are permitted prior to the registration filing. After filing, unrestricted oral and prescribed written communications are permitted. The preselling activities of the underwriter include a multi-city road show which is an important part of the marketing function.[7]

There are three types of underwriting agreements between the issuer and investment banker. In a firm commitment agreement, the banker agrees to purchase the entire issue from the firm at an agreed upon price and then bears the entire risk of selling the issue to the market at the stated offering price. In a best efforts agreement, the banker merely agrees to use his best efforts to sell the shares at the offering price. The third and less common type is an all-or-nothing agreement where the banker agrees to sell the entire issue at the offering price within a prespecified period of time, or the agreement is cancelled. In a firm commitment agreement, the principal underwriter forms a large syndicate of investment bankers to sell the issue in order to reduce risk.

The offering price is determined by the banker and issuer and appears on the cover of the final prospectus. The price cannot be increased by the banker. The banker however can prevent price decline before all shares are sold by stabilizing the issue through pegging the market at the offering price and buying back shares at the offering price. Therefore the banker in a firm commitment issue

bears the risk of price decline, but does not benefit directly in a price increase. A partner of the investment banker, Alex Brown and Sons, stated that this after-market support was an important factor in gaining credibility of disclosure.[8]

From this brief description of the regulatory environment for new issues, it is clear that the assumptions of the theoretical role of the investment banker in the preceding section are consistent with the real world. It was assumed that the investment banker and issuer had identical information about the firm at the time the security is sold to the public. Such an assumption is realistic because of the long investigation period prior to issue. A syndicate is formed for firm commitment agreements and therefore the assumption of risk neutrality is appropriate. The preselling activities do require that the underwriter expend effort and may provide the underwriter with information about market conditions which is useful in setting the offering price.

Footnotes to Chapter Three

1. "Sec. 11. (a). In case any part of the registration statement, when such part became effective, contained an untrue statement of a material fact or omitted to state a material fact required to be stated therein or necessary to make the statements therein not misleading, any person acquiring such security (unless it is proved that at the time of such acquisition he knew of such untruth or omission) may, either at law or in equity, in any court of competent jurisdiction, sue --

   (1) every person who signed the registration statement;
   (2) every person who was a director of (or person performing similar functions) or partner in, the issuer at the time of the filing of the part of the registration statement with respect to which his liability is asserted;
   (3) every person who, with his consent, is named in the registration statement as being or about to become a director, person performing similar functions, or partner;
   (4) every accountant, engineer, or appraiser, or any person whose profession gives authority to a statement made by him, who has with his consent been named as having prepared or certified any part of the registration statement, or as having prepared or certified any report or valuation which is used in connection with the registration statement, with respect to the statement in such registration statement, report, or valuation, which purports to have been prepared or certified by him;
   (5) every underwriter with respect to such security.

   .
   .
   .

   (g) In no case shall the amount receivable under this section exceed the price at which the security was offered to the public."

   [U.S. Government Printing Office (1976)]

2. A recent example of such a lawsuit was reported in the September 7, 1983 Wall Street Journal. A shareholder of Computer Devices, Inc. sued the company, its directors, its underwriters, and its accountants for failure to disclose post-palance sheet losses in the July 8, 1983 issue of one million shares at $11.25 per share. The price on the date of the lawsuit was $6.25.

   John Shad of the S.E.C. implied that the motivation for legal action is price deline rather than new information about fraud when he said that once the market breaks and prices decline, lawsuits related to new issue disclosure will start flying (Venture, September, 1983).

3. Such precommitment can be compared to cost variance investigation in a principal/agent setting where the principal knows that the threat of variance investigation induces the desired action from the agent. If investigation is costly, why would it be undertaken when it is known that the agent acted optimally? Unless the principal precommits to costly investigation for certain outcomes, the rational agent will not be induced to take the desired action.

4. Studies by Reilly and Hatfield (1969), McDonald and Fisher (1972), and Logue (1973) found that unseasoned new issues were underpriced. Ibbotson (1975) studied aftermarket performance of unseasoned new issues in the 1960's and concluded that the average return in the first month was 11.4% and was not due to market inefficiency. None of these authors could satisfactorily explain the underpricing. Ritter (1983) examined the "hot issues" market of 1980 and found that the average initial return of 48.4% was attributable to natural resource issues which earned a 110.9% return.

An explanation of the persisting underpricing phenomenon was offered by Rock (1982) in a model in which some investors have inside information about values of new securities. As a result of this informational asymmetry, the informed oversubscribe to undervalued issues and the uninformed subscribe equally to over- and undervalued issues. Therefore, in order for there to exist uninformed demand, the issuer must offer the new shares at a discount. Rock's result appears to depend upon the two questionable assumptions that the issuer is uninformed and informed demand is less than the size of the issue.

5. The auditor of the financial statements contained in the prospectus is a fourth party who shares in the legal liability for disclosures made in the prospectus. The auditor's role is not considered in this dissertation. The intermediary could be viewed as a combined investment banker and auditor.

6. The information contained in the prospectus includes:
   a) Audited financial statements
   b) Proposed use of proceeds
   c) Outline of underwriting agreement
   d) List of underwriters in syndicate and the commitment of each
   e) Five-year summary of income
   f) Detailed description of business
   g. Detailed information about directors and executive officers, including age, remuneration, and shareholdings before and after issue.

7. "William F.X. Grubb, president of Imagic, one of the hottest new-issue prospects, is now spending more time peddling his company's soon-to-be-issued shares than its 'Demon Attack' home video games. Grubb and two of his top executives -- prompted by a cadre of underwriters and armed with impassioned speeches, a slide

show, and game samples -- have swept through seven U.S. and five foreign cities in the past few weeks, pitching their company to potential investors." [Business Week. December 6, 1982,p. 100]

8. Venture. March, 1983. p. 92.

CHAPTER FOUR

SIGNALING BY DIRECT DISCLOSURE: A BIVARIATE SIGNALING MODEL

INTRODUCTION

This chapter develops a signaling model in which there is asymmetric information about two parameters of the distribution of future cash flows. An entrepreneur has inside information about the two paramaters and signals his information with two costly signals. Section 1 derives optimal risk sharing in a perfect market. Section 2 shows how optimal risk sharing is unattainable when there is a market imperfection such as asymmetric information. The bivariate signaling model is formulated and analyzed is section 3. Empirical implications of the analysis are discussed in Section 4.

1. RISK SHARING IN A PERFECT MARKET

Pareto-optimal risk sharing is attainable in a capital market when market participants have identical beliefs about the distribution of the future cash flows to be generated by the firm's investments. The capital asset pricing model describes valuation of a firm or an investment project in a perfect market when investors' preferences are defined over two parameters of the distribution of cash flows. Under the capital asset pricing model, the expected return/risk relationship of a capital asset is:

$$\bar{r}_j = r + \frac{(\bar{r}_m - r) \cdot \mathrm{Cov}\,(\tilde{r}_j, \tilde{r}_M)}{\sigma_M^2} \qquad (4.1.1)$$

where $\tilde{r}_j$ = random return on security of firm j

$\tilde{r}_M$ = random return on market portfolio

r = riskfree rate of interest

$Cov(\tilde{r}_j, \tilde{r}_M)$ = covariance between random returns on firm j and the market portfolio

σ_M^2 = variance of returns of the market portfolio

Expected return is increasing in only the systematic risk of the asset because investors are able to eliminate firm-specific non-sytematic risk through investment in a well-diversified portfolio of securities.

Market value in terms of the risk-adjusted present value of future cash flows is derived from the capital asset pricing model by converting expression (4.1.1) from returns to values.

Let V = current value of the firm

V_m = current market value of the market portfolio

$\tilde{x}$ = random end-of-period value of the firm

$\tilde{M}$ = random end-of-period value of the market portfolio

r = riskfree rate of interest

$$\text{Then } \tilde{r}_j = \frac{[\tilde{X} - V]}{V} = \frac{\tilde{X}}{V} - 1$$

$$\tilde{r}_m = \frac{[\tilde{M} - V_m]}{V_m} = \frac{\tilde{M}}{V_m} - 1$$

$$\sigma_M^2 = \left[\frac{1}{V_m}\right]^2 \sigma_m^2$$

where σ_m^2 is the variance of values of the market portfolio

$$\mathrm{Cov}(\tilde{r}_j, \tilde{r}_M) = \mathrm{Cov}\left[\frac{\tilde{M}}{V_m} - 1, \frac{\tilde{x}}{V} - 1\right]$$

$$= \frac{1}{VV_m} \cdot \mathrm{Cov}(\tilde{x}, \tilde{M})$$

The above four expressions are substituted into (4.1.1) which then becomes:

$$(\mu/V) - 1 = r + \left[\frac{\bar{M}}{V_m} - 1 - r\right] \cdot \frac{\left[\frac{1}{V.V_m}\right] \cdot \mathrm{Cov}\,(\tilde{x}, \tilde{M})}{\left[\frac{1}{V_m}\right]^2 \sigma_m^2} \qquad (4.1.2)$$

where μ = expected value of $\tilde{X}$

$\bar{M}$ = expected value of $\tilde{M}$

After algebraic manipulations, expression (4.1.2) simplifies to:

$$V = \frac{\mu - \lambda^0 \,\mathrm{Cov}(\tilde{x}, \tilde{M})}{1 + r} \qquad (4.1.3)$$

$$\text{where } \lambda^0 = \frac{\bar{M} - (1+r)V_m}{\sigma_m^2}$$

$$\text{or } V = \frac{\mu - \lambda}{1 + r}, \qquad \text{letting } \lambda = \lambda^0 \text{ Cov } (\tilde{x}, \tilde{M}) \tag{4.1.4}$$

In expression (4.1.3), firm value is expressed as the risk-adjusted present value of future cash flows where:

λ^0 Cov$(\tilde{x}, \tilde{M})$ is the risk adjustment in which the covariance term represents risk and λ^0 is the market price of risk

$(1 + r)$ is the present value operator.

Again, the risk adjustment is for systematic covariance risk only. μ is the end-of-period expected value of the future stream of cash flows.

Risk sharing in a perfect market with symmetric information will be analyzed by deriving the risk-sharing contract between a risk-averse entrepreneur seeking to raise funds to finance a capital investment and risk-averse investors who invest in a well-diversified portfolio of securities in other firms or projects.

The entrepreneur's objective is to maximize expected utility of his end-of-period wealth, which is composed of returns from his own investment in his project or firm, the market portfolio, and the riskless asset. His problem can be expressed as:

$$\max_{\alpha,\beta} E\{U(\tilde{W}_1)\}$$

subject to:

$$W_0 + (1 - \alpha)V - I - \beta V_m - Y = 0 \tag{4.1.5}$$

$$\text{where } \tilde{W}_1 = \alpha \tilde{x} + \beta \tilde{M} + (1 + r)Y \tag{4.1.6}$$

Equation (4.1.5) is the budget constraint of the entrepreneur, in which he has an initial endowment of wealth, W_0, and will make a capital investment of I dollars, which is valued by the market according to expression (4.1.3). He sells to the market a proportion $1-\alpha$, where $0 < \alpha < 1$, of ownership in the future cash flows to be generated by the project; the selling price is $1-\alpha$ times the present risk-adjusted value of the future cash flows, V. Any wealth remaining after the capital expenditure is invested in the market portfolio and the riskless asset. Market value of the market portfolio is V_m and β is the proportion of the market portfolio owned by the entrepreneur. Y represents the amount of his investment in the riskless asset.

In order to simplify the analysis of the problem, two assumptions will be made:

1. The entrepreneur has a negative exponential utility function:

$$U(\tilde{W}) = -e^{-b\tilde{W}} \tag{4.1.7}$$

2. Returns on all securities are normally distributed:

$$-b\tilde{W} \sim N(b\bar{W}, b^2\sigma_W^{\ 2})$$

In order to determine the distribution of U(W), take logarithms of equation (4.1.7):

$$\log \{-U(\tilde{W})\} = -b\tilde{W}$$

Using the distribution of $b\tilde{W}$,

$$\log \{-U(\tilde{W})\} \sim N(-b\bar{W}, b^2\sigma_W^{\ 2})$$

Therefore -U(W) has a lognormal distribution such that

$$E\{-U(W)\} = e^{-b\bar{W} + 1/2 b^2 \sigma_W^2}$$

and $$\text{Max } E(U) \equiv \text{Max } G\{\bar{W} - \frac{b}{2} \sigma_W^2\} \tag{4.1.8}$$

Let $$H(\alpha,\beta) = \bar{W} - \frac{b}{2} \sigma_W^2 \tag{4.1.9}$$

Then $G(H) = e^{bH}$ is a monotonically increasing function of H such that:

$$\arg\max_{\alpha,\beta} E\{U(\tilde{W}_1)\} = \arg\max_{\alpha,\beta} H \tag{4.1.10}$$

To solve the entrepreneur's problem, substitute for Y in (4.1.6), using (4.1.5).

$$\tilde{W}_1 = \alpha\tilde{x} + \beta\tilde{M} + (1+r)\,[W_0 + (1-\alpha)\,V - I - \beta V_m] \tag{4.1.11}$$

After substitution for V using (4.1.4) and simplication, (4.1.11) becomes

$$\tilde{W}_1 = \alpha(\tilde{x} - \mu + \lambda) + \beta(\tilde{M} - (1+r)V_m) + \mu - \lambda + (1+r)(W_0 - I) \tag{4.1.12}$$

The expected value and variance of (4.1.12) are

$$E(\tilde{W}_1) = \bar{W}_1 = \alpha\lambda + \beta(\bar{M} - (1+r)V_m) + \mu - \lambda + (1+r)\,(W_0 - I) \tag{4.1.13}$$

$$\sigma_{W_1}^2 = \alpha^2\,\sigma^2 + \beta^2\sigma_m^2 + 2\alpha\beta\,\text{Cov}(\tilde{x},\tilde{M}) \tag{4.1.14}$$

First order conditions are derived, using (4.1.9), (4.1.10), (4.1.13) and (4.1.14). They are:

$$\frac{\partial H}{\partial \alpha} = \lambda - \alpha b\ \sigma^2 - \beta b\ \mathrm{Cov}(\tilde{x},\tilde{M}) = 0 \qquad (4.1.15)$$

$$\frac{\partial H}{\partial \beta} = \bar{M} - (1+r)V_m - \beta b\ \sigma_m^2 - \alpha b\ \mathrm{Cov}(\tilde{x},\tilde{M}) = 0 \qquad (4.1.16)$$

To solve, substitue for βb in (4.1.15) using (4.1.16):

$$\text{from (4.1.16)}\ \beta b = \frac{\bar{M} - (1+r)V_m - \alpha b\ \mathrm{Cov}(\tilde{x},\tilde{M})}{\sigma_m^2}$$

$$= \lambda^o - \frac{\alpha b\ \mathrm{Cov}(\tilde{x},\tilde{M})}{\sigma_m^2}$$

(4.1.15) becomes:

$$\lambda - \alpha b\ \sigma^2 - [\lambda^o - \frac{\alpha b\ \mathrm{Cov}(\tilde{x},\tilde{M})}{\sigma_m^2}]\ \mathrm{Cov}(\tilde{x},\tilde{M}) = 0$$

$$\text{or} \qquad \alpha b\ [\frac{\sigma^2\sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2}] = 0 \qquad (4.1.18)$$

There are three possible conditions under which the necessary equlibrium condition (4.1.18) will be satisfied:

(a) $b = 0$. This solution requires a risk-neutral entrepreneur who is different to the level of α because he is different to the amount of risk imposed on him. The entrepreneur has been described as risk averse and therefore $b > 0$.

(b) $\sigma^2 \sigma_m^2 - Cov(\tilde{x},\tilde{M})^2 = 0$. This expression can be derived from the valuation expression (4.1.13)

$$\mu = (1+r)V + \frac{\left[\bar{M} - (1+r)V_m\right] Cov(\tilde{x},\tilde{M})}{\sigma_m^2}$$

Then:

$$\tilde{x} = (1+r)\ V + \frac{(\tilde{M} - (1+r)V_m)\ Cov(\tilde{x},\tilde{M})}{\sigma_m^2} + \tilde{\varepsilon} \qquad \text{Conditional the Market} \quad (4.1.19)$$

and $$\sigma^2 = \sigma_m^2 \left[\frac{Cov(\tilde{x},\tilde{M})}{\sigma_m^2}\right]^2 + \sigma_\varepsilon^2$$

or $$\sigma_\varepsilon^2 = \frac{\sigma^2\sigma_m^2 - Cov(\tilde{x},\tilde{M})^2}{\sigma_m^2}$$

Therefore this solution requires that the project has zero non-sysematic risk and that $\tilde{x}$ and $\tilde{M}$ are perfectly correlated since

$$\frac{Cov(\tilde{x},\tilde{M})}{\sigma\ \sigma_m} = \rho(\tilde{x},\tilde{M})$$

$$\frac{Cov(\tilde{x},\tilde{M})^2}{\sigma^2\ \sigma_m^2} = \rho(\tilde{x},\tilde{M})^2$$

Then

$$\sigma_{\varepsilon}^{2} = 0 \rightarrow \mathrm{Cov}(\tilde{x},\tilde{M})^{2} = \sigma^{2}\ \sigma_{m}^{2} \rightarrow \rho = \pm 1$$

An investment in the new project is a perfect subsitute for an investment in the market portfolio so that the risk-averse entrepreneur is indifferent to the amount of his investment in his own project. While such a solution is possible, it is not interesting for the purpose at hand.

(c) $\alpha = 0$ means that the entrepreneur will sell 100% of the equity in his project. The risk-averse entrepreneur wants to bear no unsystematic risk, while the well-diversified market can eliminate firm-specific risk.

The optimal solution is

$$\alpha^{*} = 0$$

$$\beta^{*} = \frac{\lambda^{0}}{b} \quad \text{from (4.1.17) when } \alpha = 0$$

The risk-averse entrepreneur will sell the firm and invest in only the market portfolio and riskless asset. The amount he invests in the market depends upon his degree of risk aversion b and the market price of risk λ^{0}. This solution is consistent with risk-sharing results in agency theory and portfolio separation results in portfolio theory.

A standard risk-sharing result first derived by Borch (1962) is that Pareto-optimal risk sharing between a principal and agent requires that:

$$\frac{U'(P)}{U'(A)} = k \text{ in every state.}$$

This expression states that the ratio of marginal utilities of payoffs to the principal and agent must be equal to a constant in every state. Investors in the market are risk neutral as to firm-specific risk because it can be eliminated through portfolio diversification. The market can be viewed as a risk-neutral principal whose marginal utility is a constant. In order to maintain the constant ratio in every state, it is necessary that the risk-averse agent's marginal utility is a constant, which necessitates that he receive a constant payoff. Therefore, it is Pareto-optimal for the market to bear all firm-specific risk.

Portfolio separation looks at the optimal sharing of risks of securities among many investors. The basic two-fund separation result is that all investors will divide their wealth between one risky asset and a riskless asset and that the equilibrium price of the risky asset will be independent of individual preferences or wealth. Portfolio separation obtains under the capital asset pricing model, and more generally when utility functions exhibit linear risk tolerance [Cass and Stiglitz (1970)] (such as with the negative exponential) and when returns are normally distributed [Ross (1976)]. All of the assumptions were used in the present model. The optimal division of wealth between the risky and riskless assets for an individual will depend upon his degree of risk aversion [Mossin (1973)].

The entrepreneur therefore will bear no nonsystematic risk and will bear some proportion of the systematic risk of all risky

assets through his optimal holding of the market portfolio. It should be noted that it has been assumed that the entrepreneur's firm is sufficiently small in relation to the market portfolio so that the parameters:

$$\sigma_m^2$$

$$Cov(\tilde{x}_i, \tilde{M}) \quad i=1,\ldots,N \text{ firms in the market}$$

are unchanged after the entrepreneur sells his firm to the market.

The formulation of the problem in this section has followed that of Leland and Pyle with the exception that a perfect market is assumed. The equilibrium optimal solution to the problem of this section is a "first-best" solution when no constraints are added due to a market imperfection. The solution represents an ideal solution against which to compare results when there exists an informational asymmetry.

2. RISK SHARING UNDER ASYMMETRIC INFORMATION

The model of the preceding section will be modified by assuming that the market knows the distribution of μ such that:

$$\mu \sim N(\bar{x}_o, \sigma_o^2)$$

The entreprenur has perfect information about μ, and both the market and entrepreneur have perfect information about $Cov(\tilde{x},\tilde{M})$, σ^2, and σ_m^2. Market valuation then is:

$$V_o = \frac{x_o - \lambda}{1+r} \gtreqless \frac{\mu - \lambda}{1+r}$$

This scenario is analogous to that of Akerlof's in that the market knows the average value of μ, the entrepreneur knows his μ. In

Akerlof's model, the market unravels until only the lemons are traded.

The formulation of the problem is as in the preceding section, with the exception that market valuation of the present risk-adjusted value of future cash flows is V_0. First order conditions analogous to those of section 1 are:

$$\frac{\partial H}{\partial \alpha} = \mu - \bar{x}_0 + \lambda - \alpha b\ \sigma^2 - \beta b\ \mathrm{Cov}(\tilde{x},\tilde{M}) = 0 \qquad (4.2.1)$$

$$\frac{\partial H}{\partial \beta} = \bar{M} - (1+r)V_m - \beta b\ \sigma_m^{\ 2} - \alpha b\ \mathrm{Cov}(\tilde{x},\tilde{M}) = 0 \qquad (4.2.2)$$

(4.2.1) and (4.2.2) are combined to solve for α^*:

$$\alpha^* = \frac{\mu - \bar{x}_0}{b\left[\dfrac{\sigma^2\ \sigma_m^{\ 2} - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^{\ 2}}\right]} \qquad (4.2.3.)$$

According to (4.2.3):

$\alpha = 0$ if $\bar{x}_0 = \mu$ when market correctly values firm

$\alpha > 0$ if $\bar{x}_0 < \mu$ when market undervalues firm.

$\alpha < 0$ if $x_0 > \mu$ when market overvalues firm.

Clearly, such a story could exist only when the market is irrational. Assuming for the moment that such irrationality does exist, the informational asymmetry does not result in Akerlof's market failure because the product is risky and the risk-averse seller will sell some of his product at an undervalued price because he can diversify risk with the proceeds. Since α^* is

decreasing in b in (4.2.3), a more risk-averse entrepreneur will sell a greater proportion of the firm at an undervalued price.

However, the above result is not a rational equilibrium because investors will know that the firm is undervalued if $\alpha > 0$. Investors seeking more of the undervalued security will bid up the price until $\alpha = 0$ and the equilibrium price is reached. The entrepreneur's action of investing in his own security conveys information to the market. The above scenario also allows for some deception because the entrepreneur might be able to hold a small equity position in order to bid the price above true value.

The above problem must be formulated rationally where the market infers value as a function of α.

Leland and Pyle's model analyzes the above problem with α as a costly signal of μ. The formulation of the problem is the same as above except:

$$V(\alpha) = \frac{\bar{x}(\alpha) - \lambda}{1+r}$$

where $\bar{x}(\alpha)$ is the market's inference about μ.

An additional constraint to the problem is that, in equilibruim:

$$\bar{x}(\alpha^*(\mu)) = \mu$$

This constraint is the market rationality constraint.

The necessary equilibrium condition to their problem is:

$$(1-\alpha)\bar{x}_\alpha = \alpha b \left[\frac{\sigma^2 \sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2} \right] \tag{4.2.4}$$

This expression equates the marginal benefit and marginal cost of signaling with α:

$(1-\alpha)\bar{x}_\alpha$ = marginal increase in proceeds

$\alpha b[\cdot]$ = marginal cost in terms of risk

If $\alpha^* > 0$, Pareto-optimal risk sharing is not achieved because of the informational asymmetry. The asymmetry is completely resolved, but at the cost to the entrepreneur of his bearing project-specific risk which can be costlessly transferred to the market in a perfect market. Leland and Pyle solved the differential equation in (4.2.4):

$$\bar{x}(\alpha) = -b \frac{\sigma^2 \sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2} [\log(1-\alpha) + \alpha] + K \tag{4.2.5}$$

This expression defines a family of inference schedules. The Pareto-optimal schedule is one in which the firm with the minimum value does not signal. The equilibrium schedule is:

$$\bar{x}(\alpha) = -b \frac{\sigma^2 \sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2} [\log(1-\alpha) + \alpha] + (1+r)I + \lambda \tag{4.2.6}$$

$$\text{and } V(\alpha) = \frac{-b\,\dfrac{\sigma^2\sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2}\,[\log(1-\alpha) + \alpha] + I}{1+r} \tag{4.2.7}$$

It is essential to Leland and Pyle's model that the market knows σ^2, σ_m^2, and $\mathrm{Cov}(\tilde{x},\tilde{M})$ (as well as b) in order to interpret the signal α because the cost of α depends upon the magnitude of firm specific risk.

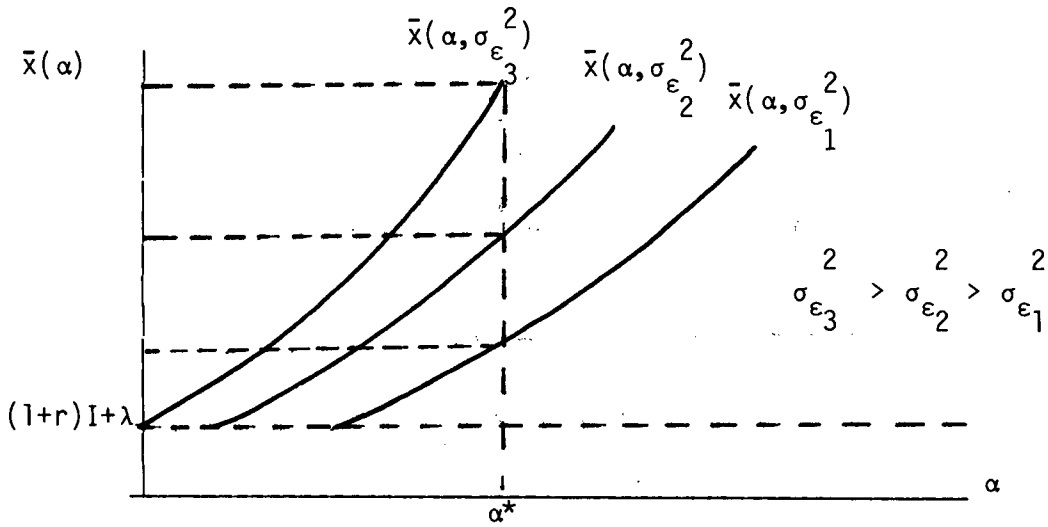

Figure 4.1. Equilibrium Signaling Schedules

If the value of $\sigma_\varepsilon^2 = \dfrac{\sigma^2\,\sigma_m^2 - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^2}$ is not known by the market, α will not be a fully-revealing signal if there is more than one possible value of σ_ε^2, as is shown in Figure 4.1. For a given α, the marginal cost is increasing in σ_ε^2. If the market

observes α^* and knows the distribution of σ_ε^2, it would have to infer σ_ε^2. Otherwise there would be an opportunity for false signaling. Then the entrepreneur whose project has high firm-specific risk $\sigma_{\varepsilon_3}^2$ will be motivated to communicate his information so that the market can correctly value the firm at $V(\alpha^*, \sigma_{\varepsilon_3}^2)$ Firm value appears to be independent of σ^2 when expressed as $V=(\mu - \lambda) / (1+r)$. However, when firm value under asymmetric information is expressed as a function of α as in (4.2.7), it is a function of σ^2 also. Therefore, if an informational asymmetry exists about σ^2 as well as μ, the entrepreneur cannot signal firm value with a single signal when the cost of that signal depends upon σ^2. Market uncertainty about the value of the second parameter will provide a setting where two costly signals will be used to communicate inside information about μ and σ^2. The signaling model will be analyzed in the next section.

3. SIGNALING THROUGH DIRECT DISCLOSURE

The motivation existing for an entrepreneur to invest in a costly signal is the same as in Section 2. The entrepreneur is endowed with initial wealth and a technology. In order to realize returns on the technology, a capital investment in the amount of I must be made. The entrepreneur knows all the parameters of the distribution of future cash flows to be generated by the investment. Investors in the market know only the distribution of μ, and do not know σ^2. In the Leland and Pyle model, investors presumably know σ^2 since they are assumed to know the value of firm specific risk as well as parameters of the distribution of returns on the

market portfolio. In a perfect market, firm value is independent of σ^2 since only systematic risk is priced. However, as was discussed in section 2, if value is inferred from a signal which has a cost function dependent upon σ^2, the market must know σ^2 in order to interpret the signal. Therefore a second signal may be necessary so that investors can interpret the first signal correctly.

The entrepreneur chooses to communicate his inside information about μ by a direct disclosure about the value of his project to the market. The disclosure takes the form of a piece of information which is the entrepreneur's estimate of μ. In other words, he makes a statement such as "the expected end-of-period value of the project in terms of future cash flows is y." The entrepreneur seeks the services of a risk-netural intermediary/investment banker to share risks, help in selling the security, and provide third-party verification of the disclosure. The proof that a large underwriting group behaves as if it were risk neutral was not presented in Chapter Three because the notation is developed in this chapter. The proof appears in Appendix 1, The cost of the intermediary's services was derived in Chapter Three. Investors then infer the value of the project from y.

In order to be able to solve the signaling problem, it is necessary to specify a functional form for the cost function of y. In section four of Chapter Three the cost function was shown in expression (3.4.6.) to be

$$C(y) = K(y) + L(y)$$

where $K(y)$ = direct costs of verification

$$L(y) = \frac{P_0 \, \Pi \, (y;\mu,\sigma^2)}{1+r} \quad \text{from (3.4.2)}$$

$$= \text{Present value of expected future loss.}$$

The probability of loss was shown in section two of Chapter Three as:

$$\Pi \, (y;\mu,\sigma^2) = \int_{-\infty}^{T} f(x) \, dx$$

$$\text{where} \quad T = \frac{(y-2\sigma) \, (\sigma_0^2 + \sigma^2) - \bar{x}_0 \, \sigma^2}{\sigma_0^2}$$

$$f(x) = \text{normal density function of } \tilde{x}.$$

There does not exist a closed form solution to the Π expression and it therefore will be approximated by a function which possesses certain necessary characteristics. Characteristics of L(y) were shown in section two of Chapter Three to be:

1) $L_y > 0$
2) $L_\mu < 0$
3) $L_{\sigma^2} < 0$

The direct costs of disclosure will be assumed to be such that:

$K_y > 0$

$K_\mu = 0$

$K_{\sigma^2} = 0$

which means that out-of-pocket investigation costs are increasing in the level of stated quality, but are unrelated to μ and σ^2. Additional necessary characteristics are:

4) $C_{y\mu} < 0$ from Spence's cost condition to exclude imitation of the signaler's behavior by an entrepreneur with a project with a lower value.

5) $C_{yy} \neq 0$ is frequently necessary to satisfy second-order conditions.

The cost of y will be approximated by:

$$C(y;\ \mu,\sigma^2) = \frac{P_o\ \phi\ y^3}{(1+r)\mu\sigma^2} \tag{4.3.1}$$

which satisfies:

1) $C_y = \dfrac{3\ P_o\ \phi\ y^2}{(1+r)\mu\sigma^2} > 0$

2) $C_\mu = \dfrac{-P_o\ \phi\ y^3}{(1+r)\mu^2\sigma^2} < 0$ for $y > 0$

3) $C_{\sigma^2} = \dfrac{-P_o\ \phi\ y^3}{(1+r)\mu\sigma^4} < 0$

4) $C_{y\mu} = \dfrac{-3P_o\ \phi\ y^2}{(1+r)\mu^2\sigma^2} < 0$

5) $C_{yy} = \dfrac{6\ P_o\ \phi\ y}{(1+r)\mu\sigma^2} \neq 0$

In (4.3.1), the components of the cost of y are combined so that

$$C(y) = K(y) + L(y) = \frac{P_o \ \phi \ y^3}{(1+r) \ \mu\sigma^2}$$

An intuitive explanation of this cost function follows.

(i) $C(y) >$ for $y > 0$ because there are direct costs of certification of y from the K(y) term even when L(y) is very small.

(ii) As y becomes greater, the probability of a penalty becomes a more significant portion of the cost of certification.

(iii) ϕ is a scaling constant to ensure that $0 < \Pi < 1$ and to prevent the penalty from being so high that y is uneconomic.

It is clear that σ^2 must be known in order to interpret the signal y because the cost and marginal cost of y are decreasing in σ^2. Figure 4.2 illustrates C(y) and how C(y) varies with σ^2. Investors will be unable to infer firm value from y when they do not know μ or σ^2 and both appear in the cost function of y. A second signal will be α, entrepreneurial ownership in his own project or firm. The cost of α was seen to be increasing in σ^2 in section II. Therefore α and y have different relationships to σ^2. A high value for α combined with a certain y will indicate a low σ^2, and vice versa. y conveys information about μ, and α conveys information about the cost of y.

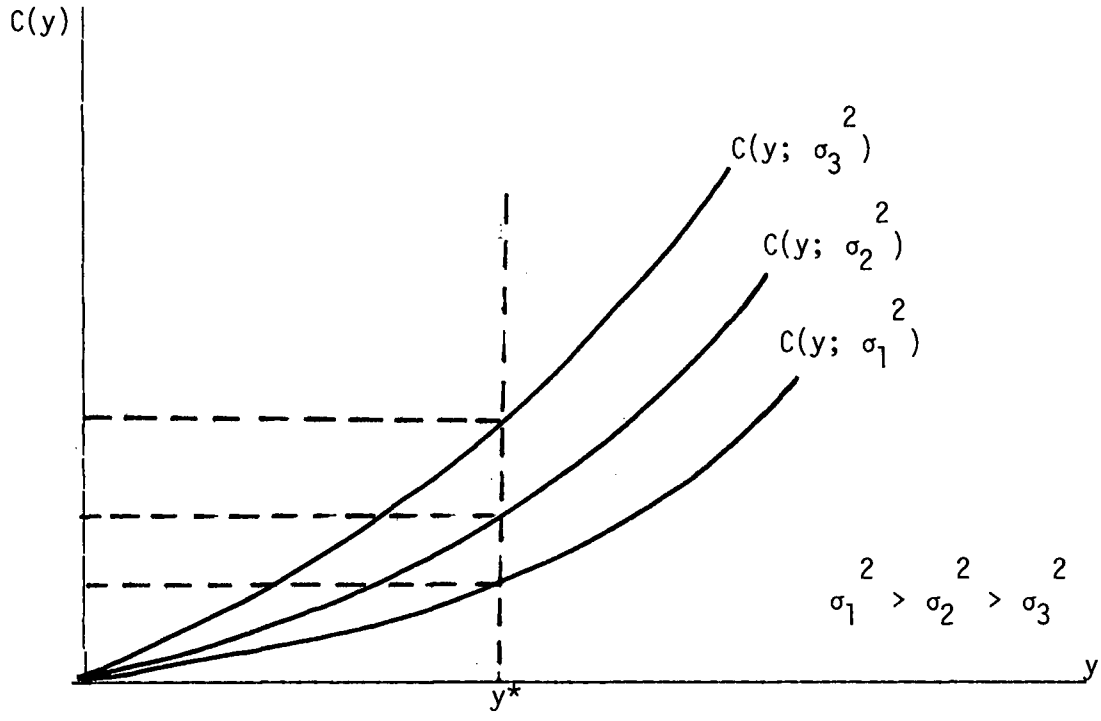

Figure 4.2. Cost of Certification Function for Different Levels of σ^2

Talmor (1981) formulates and attempts to solve a signaling problem in which dividend policy and capital structure are modeled as simultaneous signals of firm value when there is an informational asymmetry between insiders in the firm and the market about the mean and variance of before-tax earnings from operations. The cost of debt is the probability of bankruptcy when earnings are insufficient to meet promised payments. Dividends are costly signals because costly sources of funds must be found if earnings are insufficient to meet promised dividend payments. The cost of signaling with dividends depends upon the debt level because the probability of meeting dividend payments with earnings is negatively related to the level of outstanding debt. Talmor was unable

to derive the market's inference schedule about firm value because of the complexity of his model.

The problem of the entrepreneur in the bivariate signaling model presented here is to maximize expected utility of his end-of-period wealth

$$\tilde{W}_1 = \alpha \tilde{x} + \beta \tilde{M} + (1+r) Y \tag{4.3.2}$$

which is the same objective function as that in section one. In addition to selecting investments in his own firm and the market portfolio, α and β, he chooses a disclosure y, where y is a statement about μ. To simplify analysis, it is assumed that Cov(x,M) = 0: future cash flows generated by the project are uncorrelated with cash flows on the market portfolio. It was shown in section one that systematic market risk will be shared optimally through a market portfolio. The risk-sharing result in sections one and two which is of interest is the sharing of firm-specific risk in an imperfect market. Therefore it is assumed with little loss of generality that the entire risk of the project is project-specific risk so that the value of the firm is

$$V = \frac{\mu}{1+r} \tag{4.3.3}$$

The signals α and y are costly to the entrepreneur, thus serving as credible signals of value to the market. Investors observe the entrepreneur's choices α^* and y^* and value the firm as

$$V(\alpha^*, y^*) = \frac{\bar{x}(\alpha^*, y^*)}{1+r} \tag{4.3.4}$$

where $\bar{x}(\alpha^*, y^*)$ is the value of μ inferred by investors from α^* and y^*.

It is being assumed that a single separating equilibrium exists where entrepreneurs of firms with different values select different levels of α and y such that investors correctly infer value in equilibrium. The existence of such an equilibrium has been questioned [e.g., Riley (1979)] because alternative types of equilibria, such as multiple or pooling, may dominate. Feltham and Hughes (1983) state that multiple equilibria occur when the buyers offer sets of zero-profit contracts to sellers who are constrained to select one contract from the set. When a seller offers the contracts, as in this chapter, he will offer only that contract which maximizes his welfare and a single equilibrium contract will result. Feltham and Hughes prove that it is the prior beliefs of investors which determine whether a pooling or separating equilibrium is optimal. Therefore, a basic assumption to the model in this chapter is that investors do not have prior beliefs which will result in a pooling equilibrium: that is, they do not have a prior belief that the firm has a high value. When there is a continum of values, this condition must hold for every sub-interval.

The problem of the entrepreneur now is:

$$\max_{\alpha,\beta,y} \quad E\{U(\tilde{W}_1)\}$$

$$\text{s.t. } W_0 + (1-\alpha)V(\alpha,y) - C(y) - I - \beta V_m - Y = 0 \tag{4.3.5}$$

$$V(\alpha^*,y^*) = \frac{\bar{x}(\alpha^*,\ y^*)}{1+r} = \frac{\mu}{1+r} = V \tag{4.3.6}$$

$$C(y^*;\mu,\sigma^2) = \frac{Py^3}{(1+r)\ \mu\sigma^2} \quad \text{where } P = P_0\ \phi \text{ from (4.3.1)} \tag{4.3.7}$$

The maximization problem is subject to three constraints which are:

1. The budget constraint (4.3.5) indicates that the entrepreneur sells a proportion $1-\alpha$ of equity in future cash flows for $1-\alpha$ times the market's inferred valuation of the cash flows based upon the two observed signals. The proceeds of the sale of equity are reduced by the cost of verification $C(y)$.

2. Equation (4.3.6) is the market rationality constraint which states that the market's inference is correct in equilibrium.

3. The third constraint (4.3.7) is a competivity constraint which states that, in equilibrium, the underwriter is exactly compensated for its costs so that zero profits are earned.

The budget constraint (4.3.5) can be better understood by re-expressing it in the notation used in Chapter Three. The cost of direct disclosure when the underwriter distributes, prices, and certifies the issue was shown in equation (3.4.7) to be

$$C(y) = P_e - P_N$$

Since the entrepreneur is selling a portion of the equity is his firm, the cost can be re-expressed as

$$C(y) = (1-\alpha)\ (P_e - P_N)$$

or $(1-\alpha)P_N = (1-\alpha)P_e - C(y)$, where P_e and P_N refer to the prices of all shares, not just those sold to investors.

Since P_e is the equilibrium price,

$$(1-\alpha)P_N = (1-\alpha)V - C(y)$$

and after substituting (4.3.6),

$$(1-\alpha)P_N = (1-\alpha)V(\alpha,y) - C(y),$$

Therefore the budget constraint (4.3.5) can be rewritten as

$$W_o + (1-\alpha)P_N - I - \beta V_m - Y = 0$$

which more clearly indicates that the entrepreneur sells $(1-\alpha)$ of the equity to the market for $(1-\alpha)V(\alpha,y)$, receives $(1-\alpha)P_N$ and then invests in the market portfolio and the riskless asset.

The assumption of normally distributed cash flows and constant absolute risk aversion still hold so that the objective function simplifies to:

$$\underset{\alpha,\beta,y}{\text{Max}} \quad H = \bar{W}_1 - \frac{b}{2}\,\sigma_{W_1}^{\ 2} \tag{4.3.8}$$

(4.3.5) and (4.3.6) are used to calculate $\tilde{W}_1$:

$$\tilde{W}_1 = \alpha\tilde{x} + \beta\tilde{M} + (1+r)\left[W_0 + (1-\alpha)\,\frac{\bar{x}(\alpha,y)}{1+r} - C(y) - I - \beta V_m\right] \tag{4.3.9}$$

After simplification, (4.3.9) becomes:

$$\tilde{W}_1 = \alpha[\tilde{x} - \bar{x}(\alpha,y)] + \beta\,[\tilde{M} - (1+r)V_m] + \bar{x}(\alpha,y) + (1+r)\,[W_0 - C(y) - I] \tag{4.3.10}$$

The expected value and variance of (4.3.10) are:

$$\bar{W}_1 = \alpha[\mu - \bar{x}(\alpha,y)] + \beta\,[\bar{M} - (1+r)V_m] + \bar{x}(\alpha,y) + (1+r)\,[W_0 - C(y) - I] \tag{4.3.11}$$

$$\sigma^2_{W_1} = \alpha^2\sigma^2 + \beta^2\sigma^2_m \tag{4.3.12}$$

First-order Conditions to (4.3.8) are obtained, using (4.3.11) and (4.3.12):

$$\frac{\partial H}{\partial \alpha} = \mu - \bar{x}(\alpha,y) + (1-\alpha)\,\bar{x}_\alpha - \alpha b\sigma^2 = 0 \tag{4.3.13}$$

$$\frac{\partial H}{\partial \beta} = \bar{M} - (1+r)V_m - \beta b\sigma_m^2 = 0 \tag{4.3.14}$$

$$\frac{\partial H}{\partial y} = (1-\alpha)\bar{x}_y - (1+r)C_y = 0 \tag{4.3.15}$$

Expressions (4.3.13) and (4.3.14) show that α and y will be selected so that marginal cost equals marginal benefit for each. Any remaining wealth will be invested in the market portfolio and and riskless asset such that $\beta = \frac{\lambda^0}{b}$, where

$$\lambda^0 = \frac{\bar{M} - (1+r)V_m}{\sigma^2_m}$$ is the market price of risk.

Equations (4.3.13), (4.3.14), and (4.3.15) are a system of three equations in the three unknowns α, β, and y. However, a direct solution is not possible because it is not known, a priori, what the specific form of $\bar{x}(\alpha,y)$ is. It is necessary in equilibrium that $\bar{x}(\alpha^*,y^*) = \mu$, but partial derivatives of the inference schedule appear in (4.3.13) and (4.3.15) and these are not known. In order to derive the $\bar{x}(\alpha,y)$ inference schedule, equations (4.3.13) and (4.3.15) will be combined with the market rationality and competivity constraints. These four equations define the necessary equilibrium conditions and together they permit solution of $\bar{x}(\alpha,y)$:

$$\mu - \bar{x}(\alpha,y) + (1-\alpha)\,\bar{x}_\alpha = \alpha\, b\, \sigma^2 \tag{4.3.13}$$

$$(1-\alpha)\bar{x}_y = (1+r)C_y \tag{4.3.15}$$

$$\bar{x}(\alpha^*,y^*) = \mu \tag{4.3.6}$$

$$C(y^*;\ \mu,\sigma^2) = \frac{Py^3}{(1+r)\ \mu\sigma^2} \tag{4.3.7}$$

Differentiate (4.3.7) and substitute into (4.3.15)

$$C_y = \frac{3Py^2}{(1+r)\ \mu\sigma^2}$$

Then (4.3.15) becomes:

$$(1-\alpha)\bar{x}_y = \frac{3Py^2}{\mu\sigma^2} \tag{4.3.16}$$

Since (4.3.13) and (4.3.16) must hold at the optimal choices of α and y, condition (4.3.6) must hold also. Therefore (4.3.6) can be substituted into (4.3.13) and (4.3.16).

(4.3.13) becomes:

$$(1-\alpha)\bar{x}_\alpha = \alpha b \sigma^2 \tag{4.3.17}$$

(4.3.16) becomes:

$$(1-\alpha)\bar{x}_y = \frac{3Py^2}{\bar{x}\ \sigma^2} \tag{4.3.18}$$

Equations (4.3.17) and (4.3.18) are a system of simultaneous partial differential equations to be solved for $\bar{x}(\alpha,y)$ where the inference schedule cannot depend upon σ^2 which is unobservable.

Solve (4.3.17) for σ^2:

$$\sigma^2 = \frac{(1-\alpha)\bar{x}_\alpha}{\alpha b} \tag{4.3.19}$$

and substitute for σ^2 in (4.3.18):

$$(1-\alpha)\bar{x}_y = \frac{3Py^2 \alpha\ b}{\bar{x}\ (1-\alpha)\bar{x}_\alpha} \tag{4.3.20}$$

Rearrange (4.3.20):

$$(1-\alpha)^2\ \bar{x}_\alpha\ \bar{x}_y\ \bar{x} = 3\alpha\ b\ Py^2 \tag{4.3.21}$$

Because (4.3.17) and (4.3.18) both contain σ^2 which must be eliminated, it was possible to convert simultaneous partial differential equations into a single equation.

The method of solution of (4.3.21) is the method of separation of variables [Boyce and Diprima (1969), p. 422-425] whereby a solution will be found of the form

$$\bar{x}(\alpha,y) = g(\alpha)\ h\ (y) \tag{4.3.22}$$

Substitution of (4.3.22) into (4.3.21) yields

$$(1-\alpha)^2(g_\alpha h)(h_y g)(gh) = 3\alpha\ b\ Py^2 \tag{4.3.23}$$

Equation (4.3.23) is equivalent to:

$$\frac{(1-\alpha)^2\ g_\alpha g^2}{\alpha} = \frac{3\ b\ Py^2}{h_y h^2} \tag{4.3.24}$$

In (4.3.24) the left-hand side depends only on α and the right-hand side on y and exogenous parameters. In order for (4.3.24) to hold in equilibrium, it is necessary that both sides be equal to the same constant. The constant of separation will be called δ. Then (4.3.24) becomes:

$$\frac{(1-\alpha)^2 g_\alpha g^2}{\alpha} = \frac{3\ b\ Py^2}{h_y h^2} = \delta \tag{4.3.25}$$

and (4.3.25) results in two ordinary differential equations for $g(\alpha)$ and $h(y)$:

$$g_\alpha g^2 = \delta \frac{\alpha}{(1-\alpha)^2} \tag{4.3.26}$$

$$h_y h^2 = \frac{3\ b\ Py^2}{\delta} \tag{4.3.27}$$

The method of separation of variables has permitted replacing a partial differential equation with two ordinary differential equations and has simplified the derivation of $\bar{x}(\alpha,y)$.

In order to solve for $g(\alpha)$, integrate (4.3.26):

$$\frac{1}{3} g^3 = \delta\left[\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1\right]$$

$$\text{or } g(\alpha) = \{3\delta\left[\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1\right]\}^{1/3} \tag{4.3.28}$$

where k_1 is the constant of integration.

Similarly, the integration of (4.3.27) yields

$$\frac{1}{3} h^3 = \frac{bP}{\delta}\left[y^3 + k_2\right]$$

$$\text{or } h(y) = \{\frac{3\ b\ P}{\delta}[y^3 + k_2]\}^{1/3} \tag{4.3.29}$$

where k_2 is the constant of integration.

Combine (4.3.22), (4.3.28), and (4.3.29) to arrive at

$$\bar{x}(\alpha,y) = \{3\ \delta\left[\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1\right]\}^{1/3}\left[\frac{3\ b\ P}{\delta} y^3 + k_2\right]^{1/3}$$

$$\bar{x}(\alpha,y) = \{9\ b\ P \left[\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1\right]\left[y^3 + k_2\right]\}^{1/3} \qquad (4.3.30)$$

Expression (4.3.30) defines a family of market inference schedules. In order to determine the Pareto-optimal schedule which will be the unique equilibrium schedule, (4.3.30) must be evaluated at the boundaries. There are three boundary conditions for which (4.3.30) must be evaluated:

(a) $\bar{x}(\alpha=0,\ y=0)$

(b) $\bar{x}(\alpha = 0,\ y > 0)$

(c) $\bar{x}(\alpha > 0,\ y = 0)$

The optimal solution for the entrepreneur with the project with the lowest value is no investment in signaling because the signals are costly and there is no benefit derived from false signaling.

Therefore $\bar{x}(\alpha = 0,\ y = 0)$ = the minimum value of μ in the market. It was assumed that investors know the distribution of μ and the minimum value of μ. It will be assumed that the lowest value is $\mu_{min} = 0$. If $\mu_{min} = M \neq 0$, then the solution of the inference schedule includes a function of M which gives rise to a more complex expression without adding economic interest. If $\mu_{min} = 0$,

$$V(\alpha=0,\ y=0) = \frac{\bar{x}(\alpha=0,\ y=0)}{1+r} = 0 \qquad (4.3.31)$$

To determine Pareto-optimal values for k_1 and k_2, combine (4.3.31) and (4.3.30) to obtain:

$$0 = 9bP\ [1 + k_1]\ [k_2] \qquad (4.3.32)$$

The values for k_1 and k_2 must be consistent with condition (4.3.32). Clearly (4.3.32) will hold for the values

$$k_1 = -1 \quad \text{and/or } k_2 = 0$$

Intuition suggests that $\bar{x}(\alpha,y) = 0$ for the three boundaries (a), (b), and (c) since one signal alone cannot be unambiguously interpreted. However, a more rigorous argument will be used.

<u>Proposition 4.1</u>. The boundary conditions are $k_1 = -1$ and $k_2 = 0$ which implies that either $\alpha^*=0$, $y^*=0$ or $\alpha^*>0$, $y^*>0$.

<u>Proof</u>. The first-order conditions (4.3.17) and (4.3.18) must be satisfied for all α^*, y^*. Expressions for $\bar{x}_\alpha(\alpha,y)$ and $\bar{x}_y(\alpha,y)$ will be derived from the $\bar{x}(\alpha,y)$ schedule and substituted into the first-order conditions.

Inference schedule (4.3.30) is differentiated with respect to α:

$$\bar{x}_\alpha(\alpha,y) = \frac{\alpha[9bP(y^3 + k_2)]^{1/3}}{3(1-\alpha)^2 \{\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1\}^{2/3}} \tag{4.3.33}$$

Inference schedule (4.3.30) is differentiated with respect to y:

$$\bar{x}_y(\alpha,y) = \frac{y^2 \{9bP [\frac{1}{1-\alpha} + \ln(1-\alpha) + k_1]\}^{1/3}}{[y^3 + k_2]^{2/3}} \tag{4.3.34}$$

Expression (4.3.33) is substituted into first-order condition (4.3.17):

$$\frac{\alpha\,[9bP\,(y^3 + k_2)]^{1/3}}{3(1-\alpha)\,\{\frac{1}{1-\alpha} + \ln\,(1-\alpha) + k_1\}^{2/3}} = \alpha b\sigma^2 \qquad (4.3.35)$$

Expression (4.3.34) is substituted into first-order condition (4.3.18):

$$\frac{(1-\alpha)\,y^2\,\{\,9bP\,[\frac{1}{1-\alpha} + \ln\,(1-\alpha) + k_1]\}^{1/3}}{[y^3 + k_2]^{2/3}} = \frac{3Py^2}{\mu\sigma^2} \qquad (4.3.36)$$

Expressions (4.3.35) and (4.3.36) must hold for all choices α^*, y^*. Values for k_1 and k_2 will be determined by evaluation of (4.3.35) and (4.3.36) at boundaries (b) and (c).

<u>Case 1</u>. Boundary (b): $\alpha^* = 0$, $y > 0$.

When $\alpha^* = 0$, (4.3.35) is satisfied for all values of k_1, k_2, and y. When $\alpha^* = 0$, (4.3.36) becomes

$$\mu\sigma^2\,\{\,9bP\,[1 + k_1]\}^{1/3} = 3P\,[y^3 + k_2]^{2/3} \qquad (4.3.37)$$

which must be satisfied for all y^*.

(i) Let $k_1 = -1$

Then the left-hand side of (4.3.37) equals zero, and (4.3.37) will hold only if

$$3P[y^{*3} + k_2]^{2/3} = 0$$

One feasible solution is $k_2 = -y^{*3}$.

Such a solution is not permitted because k_2 is a constant of integration over y and cannot be a function of y.

The only permissible solution therefore is $y^*=0$, $k_2 = 0$.

(ii) Let $k_2 = 0$

Then (4.3.37) becomes

$$\mu\sigma^2 \{9bP [1 + k_1]\}^{1/3} = 3Py^2$$

One feasible solution is

$$k_1 = \{\frac{3Py^2}{\mu\sigma^2}\}^3 \frac{1}{9bP} - 1$$

Such a solution is not permitted because k_1 cannot be a function of y since (4.3.26) cannot depend upon y.

The only permissible solution therefore is $y^*=0$, $k_1 = -1$.

Therefore it has been proved that when $\alpha^*=0$, it must be so that $k_1 = -1$ and $k_2 = 0$ and $y^* = 0$.

<u>Case 2</u>. Boundary (c): $\alpha \geqslant 0$, $y^* = 0$.

When $y^* = 0$, (4.3.36) is satisfied for all values of α, k_1, and k_2. When $y^* = 0$, (4.3.35) becomes.

$$[9bPk_2]^{1/3} = 3(1-\alpha)\, b\sigma^2 \{\frac{1}{1-\alpha} + \ln (1-\alpha) + k_1\}^{2/3} \qquad (4.3.38)$$

which must be satisfied for all α^*

(i) Let $k_2 = 0$

Then the left-hand side of (4.3.38) equals zero and (4.3.38) will hold only if

$$3(1-\alpha^*)\, b\sigma^2 \{\frac{1}{1-\alpha^*} + \ln (1-\alpha^*) + k_1\}^{2/3} = 0$$

Since k_1 is a constant of integration over α, k_1 cannot be a function of α. The only permissible solution is $\alpha^* = 0$, $k_1 = -1$

(ii) Let $k_1 = -1$

Then (4.3.38) becomes:

$$[9bPk_2]^{1/3} = 3(1-\alpha)\ b\sigma^2\ \{\frac{1}{1-\alpha} + \ln\ (1-\alpha)\ -1\}^{2/3}$$

Due to the separable property of the problem, k_2 cannot be a function of α [from (4.3.27]. The only permissible solution then is $\alpha^*=0$, $k_2=0$

Q.E.D.

The value of the constants are $k_1 = -1$ and $k_2 = 0$, with the resulting Pareto-optimal inference schedule:

$$\bar{x}\ (\alpha,y) = \left[9bPy^3\ \{\frac{1}{1-\alpha} + \ln\ (1-\alpha)\}\right]^{1/3} \qquad (4.3.39)$$

Proposition 4.1 shows that Pareto-optimal signaling is an investment in both signals or in no signal. Each signal is costly and there is no benefit to be derived from signaling with one signal. It can be seen in (4.4.39) that

$$\bar{x}(\alpha=0,\ y=0) = 0$$

$$\bar{x}(\alpha=0,\ y>0) = 0$$

$$\bar{x}(\alpha>0,\ y=0) = 0$$

It remains to be shown that

(i) the inference schedule used by the market is fully revealing,

(ii) the market rationality condition is satisfied, and

(iii) the signaling mechansim is incentive compatible.

The first condition is verified by examining certain properties of the inference schedule, the second by verifying that $\bar{x}(\alpha^*, y^*) = \mu$ and $\sigma_i^2(\alpha^*, y^*) = \sigma^2$ where σ_i^2 is the inferred variance, and the third by looking at second-order conditions.

<u>Proposition 4.2</u> A fully revealing equilibrium exists when the market's inference schedule is

$$\bar{x}(\alpha,y) = \{9bPy^3 [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{1/3}$$

<u>Proof</u> The signaling equilibrium is fully revealing if $\bar{x}(\alpha,y)$ is strictly monotonic in both α and y. To determine monotonicity with respect to α, given y

$$\frac{\partial\bar{x}(\alpha,y)}{\partial\alpha} = \frac{3\ \alpha\ b\ Py}{(1-\alpha)^2\{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}}\ 2/3 \qquad (4.3.40)$$

In order to sign $\partial\bar{x}/\partial\alpha$ it is necessary to sign $[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]$:

Observe that

$$\frac{\alpha}{1-\alpha} + \ln(1-\alpha) = 0 \quad \text{when } \alpha = 0$$

and

$$\frac{\partial[\cdot]}{\partial\alpha} = \frac{\alpha}{(1-\alpha)^2} > 0 \quad \text{for } 0 < \alpha < 1$$

Therefore $\frac{\partial \bar{x}(\alpha,y)}{\partial \alpha} > 0$

To determine montonicity with respect to y, given α

$$\frac{\partial \bar{x}(\alpha,y)}{\partial y} = \{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{1/3} \tag{4.3.41}$$

$$> 0 \text{ because } \frac{\alpha}{1-\alpha} + \ln(1-\alpha) > 0 \quad \text{for } 0 < \alpha < 1$$

Q.E.D.

<u>Proposition 4.3</u>. The fully-revealing signaling equilibrium satisfies the market rationality conditions.

<u>Proof</u>. The steps in the proof are:

(i) Use the $\bar{x}(\alpha,y)$ schedule in first-order conditions to derive α^*, y^*.

(ii) Use α^*, y^* in $\bar{x}(\alpha,y)$ to determine whether $\bar{x}(\alpha^*, y^*) = \mu$ and $\sigma_{\bar{i}}^2\ (\alpha^*\ ,y^*) = \sigma^2$.

To derive α^* ,y^* , $\bar{x}(\alpha,y)$ is used in the first-order conditions. The first-order conditions are:

$$(1-\alpha)\bar{x}_\alpha = \alpha b\ \sigma^2 \tag{4.3.17}$$

$$(1-\alpha)\bar{x}_y = \frac{3Py^2}{\mu\sigma^2} \tag{4.3.18}$$

It was seen previously that:

$$\bar{x}_\alpha = \frac{3\ \alpha\ b\ P\ y}{(1-\alpha)^2 \{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{2/3}} \tag{4.3.40}$$

$$\bar{x}_y = \{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{1/3} \tag{4.3.41}$$

Combining (4.3.17) and (4.3.40) yields

$$\frac{(1-\alpha)\ 3\alpha\ b\ Py}{(1-\alpha)^2\ \{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{2/3}} = \alpha\ b\ \sigma^2$$

$$\text{or} \quad y = \frac{b\ \sigma^2(1-\alpha)\{9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{2/3}}{3\ b\ P} \tag{4.3.42}$$

Combining (4.3.18) and (4.3.41) yields

$$(1-\alpha)\ \{\ 9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{1/3} = \frac{3\ Py^2}{\mu\ \sigma^2}$$

$$\text{or} \quad y^2 = \frac{(1-\alpha)\ \mu\sigma^2\ \{9\ b\ P\ \left[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)\right]\}^{1/3}}{3P} \tag{4.3.43}$$

Square (4.3.42) and substitute into (4.3.43)

$$\frac{b^2\sigma^4(1-\alpha)^2\{9\ b\ P\left[\frac{\alpha}{1-\alpha}+\ln(1-\alpha)\right]\}^{4/3}}{9\ b^2P^2} = \frac{(1-\alpha)\ \mu\sigma^2\ \{9\ b\ P\left[\frac{\alpha}{1-\alpha}+\ln(1-\alpha)\right]^{1/3}}{3P} \tag{4.3.44}$$

Expression (4.3.44) can be solved for α:

$$\frac{\alpha^*}{1-\alpha^*} + \ln(1-\alpha^*) = \frac{\mu}{3\ b\ \sigma^2(1-\alpha^*)} \tag{4.3.45}$$

which is an implicit function for the choice of α^*.

To solve for y^*, substitute (4.3.45) into (4.3.42)

$$y^* = \{\frac{(1-\alpha^*)\ \mu^2\ \sigma^2}{3P}\}^{1/3} \tag{4.3.46}$$

which describes the choice of y^*.

Substitute (4.3.45) and (4.3.46) into $\bar{x}(\alpha,y)$ to arrive at:

$$\bar{x}(\alpha^*\ ,y^*) = (9bp)^{1/3}\ \mu^{2/3}\left[\frac{(1-\alpha)\sigma^2}{3P}\right]^{1/3}\left[\frac{\mu}{3\ b\ \sigma^2\ (1-\alpha)}\right]^{1/3}$$

$$= \mu \tag{4.3.47}$$

Thereby proving that

$$\bar{x}(\alpha^*(\mu,\sigma^2),y^*(\mu,\sigma^2)) = \mu$$

To determine whether market rationality for σ^2 is satisfied:

from (4.3.19)

$$\sigma^2 = \frac{(1-\alpha)\bar{x}_\alpha}{\alpha\ b}$$

It can be seen that (4.3.19) expresses σ^2 as a function of observables.

Therefore the market will infer that

$$\sigma_i^2(\alpha,y) = \frac{(1-\alpha)\bar{x}_\alpha}{\alpha\ b} \tag{4.3.48}$$

Combining (4.3.48) with (4.3.40) yields

$$\sigma_i^2(\alpha,y) = \frac{(1-\alpha)}{\alpha\ b} \cdot \frac{3\ \alpha\ b\ Py}{(1-\alpha)^2\{9\ b\ P\left[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)\right]\}^{2/3}}$$

$$\text{or} \quad \sigma_i^2(\alpha^*,y^*) = \frac{3\ Py}{(1-\alpha)\{9\ b\ P\left[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)\right]\}^{2/3}} \tag{4.3.49}$$

Substitute (4.3.45) and (4.3.46) into (4.3.49) to arrive at:

$$\sigma_i^2(\alpha,y) = \frac{3P}{(1-\alpha)\ (9\ b\ P)^{2/3}}\ \mu^{2/3}\left[\frac{(1-\alpha)\sigma^2}{3\ P}\right]^{1/3} \cdot \left[\frac{3\ b\ \sigma^2\ (1-\alpha)}{}\right]^{2/3}$$

(4.3.48)

$$\text{or } \sigma_i^2(\alpha^*,y^*) = \sigma^2$$

Thereby proving that

$$\sigma_i^2(\alpha^*(\mu,\sigma^2),y^*(\mu,\sigma^2)) = \sigma^2$$

Q.E.D.

A proof that the second-order conditions are satisfied appears in Appendix 2.

The shape of $\bar{x}(\alpha,y)$ can be determined by examination of several derivatives. $\bar{x}(\alpha,y)$ is increasing and linear in y since $\bar{x}_y > 0$ as shown in Proposition 4.2 and $\bar{x}_{yy} = 0$ from (4.3.41). $\bar{x}(\alpha,y)$ is increasing and convex in α since $\bar{x}_\alpha > 0$ as shown in Proposition 4.2 and $\bar{x}_{\alpha\alpha} > 0$ is shown below. An expression for $\bar{x}_{\alpha\alpha}$ appears in Appendix 2 as equation (A2.13):

$$\bar{x}_{\alpha\alpha} = \frac{b(1+\alpha)\ \mu\sigma^2 - 2\ (\alpha b\sigma^2)^2}{(1-\alpha)^2\mu}$$

which, after simplifcation, becomes

$$\bar{x}_{\alpha\alpha} = \frac{b\sigma^2}{(1-\alpha)^2\mu}\left[(1+\alpha)\mu - 2\alpha^2 b\sigma^2\right]$$

In order to sign the second derivative, consider the values as α approaches zero and one.

$$\lim_{\alpha \to 0} \bar{x}_{\alpha\alpha} = b\sigma^2 > 0$$

$$\lim_{\alpha \to 1} \bar{x}_{\alpha\alpha} = \infty > 0$$

The $\bar{x}(\alpha,y)$ inference schedule is illustrated in Figure 4.3

In univariate signaling models, there is a direct relationship between the observed signal and the inference of the unknown parameter. In the bivariate model, there does not exist a direct relationship between a single signal and a single parameter. Rather, the market observes both signals and simultaneously infers the two

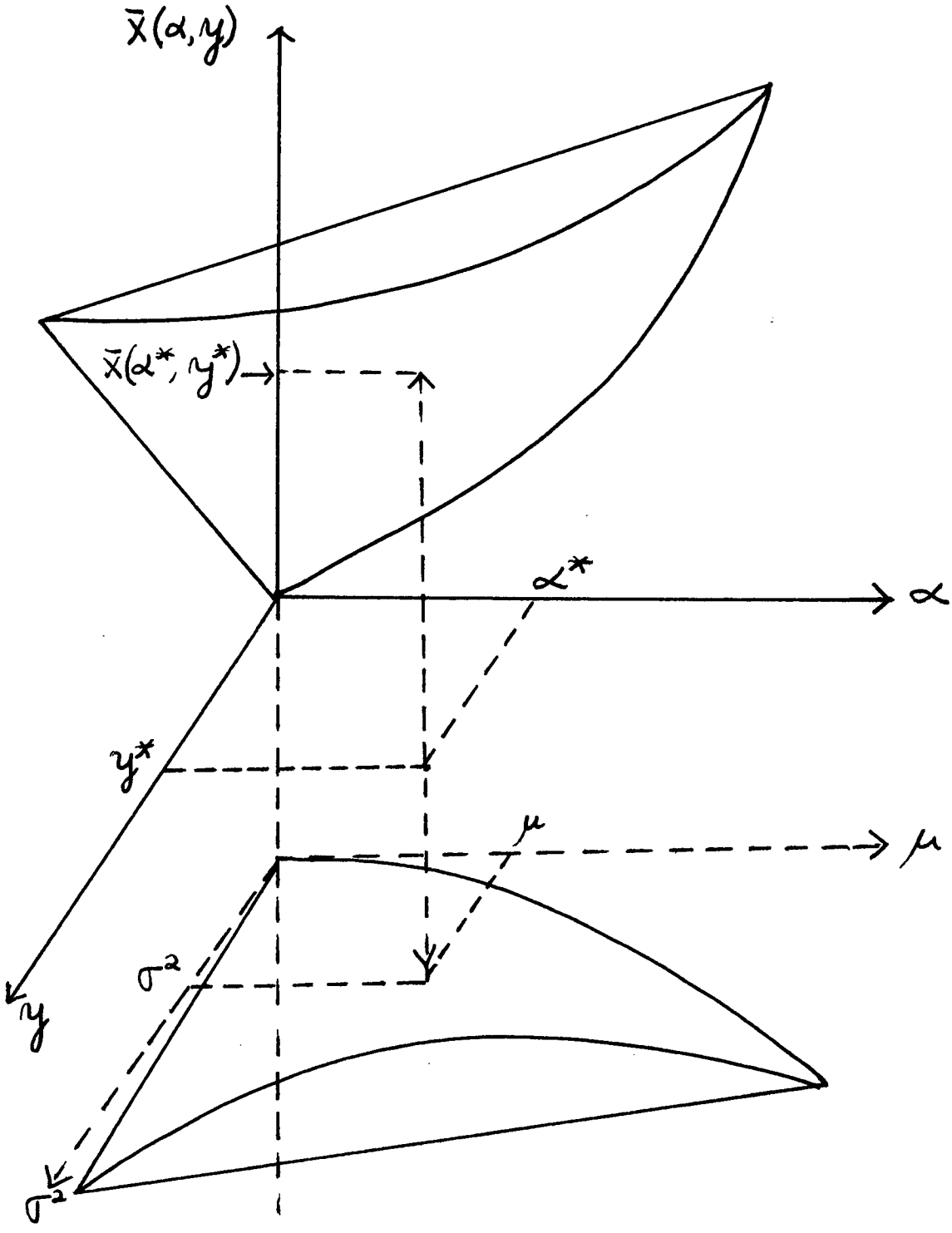

Figure 4.3 $\bar{x}(\alpha,y)$ Inference Schedule.

parameters of future cash flows. In a univariate model, the entrepreneur selects the level of the signal so as to equate marginal costs and marginal benefits. In the bivariate model, there is an added dimension which is the trade-off between the two signals which depends upon relative signaling benefits and costs. The signals are selected simultaneously and the marginal benefit of each depends upon the level of the second signal as well as marginal cost.

The following analysis identifies some of the relationships among the two signals and various parameters. The entrepreneur's choice of signals was shown to be:

$$\frac{\alpha^*}{1-\alpha^*} + \ln(1-\alpha^*) = \frac{\mu}{3\,b\,\sigma^2(1-\alpha^*)} \tag{4.3.45}$$

$$y^* = \left\{\frac{(1-\alpha^*)\,\mu^2\,\sigma^2}{3\,P}\right\}^{1/3} \tag{4.3.46}$$

In (4.3.45), α appears to be independent of y. This seeming independence is a result of the sequence of substitutions used in the algebraic derivation. If y had been derived first, then the optimal choices would appear as $y=y(\mu,\sigma^2,P)$ and $\alpha = \alpha(y,\mu,\sigma^2,P)$.

The choices of α^* and y^* are functions of:

(i) Exogenous unobservable parameters of the distribution of cash flows: μ, σ^2.

(ii) Exogenous observable parameters of the cost functions: b, P.

It will be seen that changes in (i) or (ii) have a cost effect and a substitution effect. The first effect is a change in the level of a signal due to a change in its cost function; and the second is a change in the level of the second signal in response to the first change.

Proposition 4.4. An increase in α^* results in a decrease in y^*.

Proof. Differentiate (4.3.46) with respect to α:

$$\frac{\partial y}{\partial \alpha} = \left\{\frac{\mu^2 \sigma^2}{3P}\right\}^{1/3} \left\{- \frac{1}{3} (1-\alpha)^{-2/3}\right\} < 0$$

Q.E.D.

The direct relationship between α and y is evident in the first-order conditions

$$(1-\alpha)\ \bar{x}_\alpha = \alpha\ b\ \sigma^2$$

$$(1-\alpha)\ \bar{x}_y = (1+r)C_y.$$

Since α and y enter the inference schedule multiplicatively, $\bar{x}_\alpha$ is a function of y and $\bar{x}_y$ is function of α. A change in one signal will be accompanied by a change in the second signal even if the particular parameter which has changed has no affect on the cost function of the second signal. The effect of Proposition 4.4 will be called the Substitution Effect. If one signal becomes more costly and is therefore reduced, it will be substituted for by the second signal.

Proposition 4.5. If σ^2 increases, for a given μ, the effect on the choices of signals will be an increase in y and a decrease in α.

<u>Proof</u>. To see that α decreases, rewrite (4.3.45) as

$$\alpha + (1-\alpha)\ \ln\ (1-\alpha) = \frac{\mu}{3b\sigma^2}$$

and totally differentiate with respect to α and σ^2.

$$\left[1 - \frac{1-\alpha}{1-\alpha} - \ln(1-\alpha)\right] d\alpha = \frac{-\mu}{3\ b\ \sigma^4}\ d\sigma^2$$

$$\text{or} \qquad \frac{d\alpha}{d\sigma^2} = \frac{(-\mu/3)b\ \sigma^4}{-\ \ln\ (1-\alpha)} < 0$$

since $\ln(1-\alpha) < 0$ for $0 < \alpha < 1$

To see that y increases, rewrite (4.3.46) as

$$y = \left[\frac{\mu^2}{3P}\right]^{1/3} (1-\alpha)^{1/3} (\sigma^2)^{1/3}$$

and differentiate with respect to σ^2:

$$\frac{\partial y}{\partial \sigma^2} = \left[\frac{\mu^2}{3P}\right]^{1/3} \left[\frac{1}{3}\ (1-\alpha)^{1/3} (\sigma^2)^{-2/3} - \frac{1}{3}\ (\sigma^2)^{1/3} (1-\alpha)^{-2/3} \frac{\partial \alpha}{\partial \sigma^2}\right]$$

$$> 0 \text{ since } \frac{\partial \alpha}{\partial \sigma^2} < 0$$

Q.E.D.

Figure 4.4 illustrates the trade-off between α and y with respect to σ^2 in μ, σ^2 space.

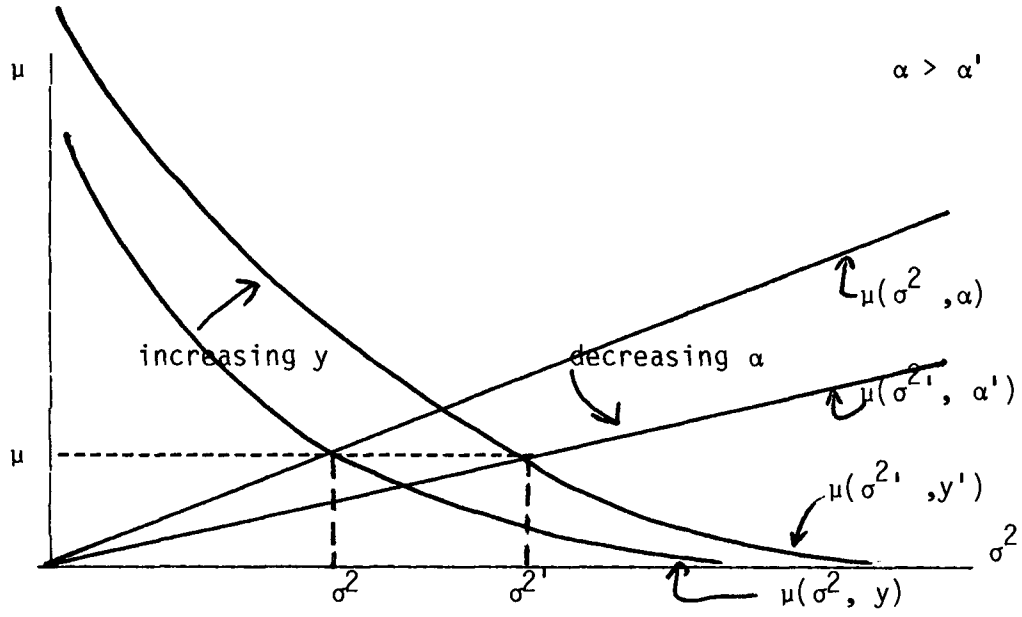

Figure 4.4. Signaling μ, σ^2 by Joint Choices of α, y.

In order to derive the relationships illustrated in Figure 4.4, equations (4.3.45) and (4.3.46) are rewritten to express μ as a function of σ^2 for a given α,y.

From (4.3.45):

$$\text{(a)}\quad \mu = 3b(1-\alpha)\left[\frac{\alpha}{1-\alpha} + \ln\,(1-\alpha)\right]\sigma^2$$

$$\text{Then (b)}\quad \frac{\partial\mu}{\partial\sigma^2} = 3b(1-\alpha)\left[\frac{\alpha}{1-\alpha} + \ln\,(1-\alpha)\right] > 0$$

$$\text{(c)}\quad \frac{\partial^2\mu}{\partial(\sigma^2)^2} = 0$$

$$\text{(d)}\quad \frac{\partial^2\mu}{\partial\sigma^2\,\partial\alpha} = 3b\left[\frac{(1-\alpha)\,\alpha}{(1-\alpha)^2} - \frac{\alpha}{(1-\alpha)} - \ln\,(1-\alpha)\right] > 0$$

From (4.3.46):

$$\text{(a)}\quad \mu = \left[\frac{3Py^3}{1-\alpha}\right]^{1/2} (\sigma^2)^{-1/2}$$

Then

$$\text{(b)}\quad \frac{\partial\mu}{\partial\sigma^2} = \left[\frac{3Py^3}{1-\alpha}\right]^{1/2}\left[-\frac{(\sigma^2)^{-3/2}}{2}\right] < 0$$

$$\text{(c)}\quad \frac{\partial^2\mu}{(\partial\sigma^2)^2} = \left[\frac{3Py^3}{1-\alpha}\right]^{1/2}\left[\frac{3(\sigma^2)^{-5/2}}{4}\right] > 0$$

$$\text{(d)}\quad \frac{\partial^2\mu}{\partial\sigma^2\,\partial y} = \left[\frac{3P}{1-\alpha}\right]^{1/2}\left[-\frac{(\sigma^2)^{-3/2}}{2}\right]\left[\frac{3y^{1/2}}{2}\right] < 0$$

$$\text{(e)}\quad \frac{\partial^2\mu}{\partial\sigma^2\,\partial\alpha} = \left[\frac{3Py^3}{(1-\alpha)^3}\right]^{1/2}\left[\frac{-(\sigma^2)^{-3/2}}{4}\right] < 0$$

The convex curve is the plot of (4.3.46a) and the upward-sloping line is (4.3.45a). Each line shows how y or α alone cannot signal μ because there maybe an infinite number of possible μ,σ^2 combinations consistent with a specific choice of y or α. When both signals are observed simultaneously, a specific μ,σ^2 pair is signaled. If

σ^2 increases while μ is unchanged, firm value has not changed, but the relative costs of the signals have changed. The probability of a penalty imposition decreases and a higher y will be selected, as proved in Proposition 4.5. From (4.3.46d), the curve shifts outward to $\mu(\sigma^{2\prime},y')$. The market cannot unambiguously interpret the increase in y if α is unchanged: either μ has increased to μ' or σ^2 increased to $\sigma^{2\prime}$. The increase in σ^2 has made α more costly, and therefore α will be reduced as proved in Proposition 4.5. From (4.3.45d), the slope of the line increases. Since α decreases, the curve shifts out more according to (4.3.46e). The new $\alpha^{*\prime},y^{*\prime}$ indicate that μ is unchanged and σ^2 has increased to $\sigma^{2\prime}$. A change in σ^2 has a cost effect and a substitution effect: both signals change in response to the effect on cost functions, and there is an additional change due to the signals' interaction. The trade-off between α and y as σ^2 varies illustrates how the two signals are related through their cost structures, how they are chosen simultaneously, and how they are used together to signal μ efficiently.

<u>Proposition 4.6</u>. The effect of an increase in the size of the penalty for false disclosure is a decrease in y.

<u>Proof</u>. Differentiate (4.3.46) with respect to the penalty P:

$$\frac{\partial y}{\partial P} = -\frac{1}{3}\left[\frac{\mu^2(1-\alpha)\sigma^2}{3}\right]^{1/3} P^{-4/3} < 0$$

Q.E.D

If it becomes more costly to signal with y, y* will decrease. The market can observe P and therefore can correctly interpret the change in y alone. However the substitution effect will result in an increase in α^*.

Proposition 4.7. The effect of an increase in the risk aversion of the entrepreneur, b, is a decrease in α^*.

Proof. Totally differentiate (4.3.45) with respect to α and b:

$$-\ln(1-\alpha)\, d\alpha = -\frac{\mu}{3\, b^2 \sigma^2}\, db$$

$$\text{or} \quad \frac{d\alpha}{db} = \frac{(-\mu/3) b^2 \sigma^2}{-\ln(1-\alpha)} < 0 \quad \text{since } \ln(1-\alpha) < 0$$

Q.E.D.

If b increases, the cost of signaling with α increases, and α^* is reduced. The market knows b can therefore interpret unambiguously the reduction in α. There will be an increase in y* due to the substitution effect.

Proposition 4.8. The effect on α^* and y* of an increase in μ depends upon the size of σ^2.

Proof. The cost effect on y of an increase in μ is determined by differentiating (4.3.46) with respect to μ, holding α constant:

$$\frac{\partial y}{\partial \mu} = \left[\frac{(1-\alpha)\ \sigma^2}{3P}\right]^{1/3} 2/3\mu^{-1/3} > 0 \qquad (\text{ignoring } \partial\alpha/\partial\mu)$$

The cost effect on α of an increase in μ is found by totally differentiating (4.3.45) with respect to α and μ.

$$-\ln(1-\alpha)\ d\alpha = \frac{d\mu}{3\ b\ \sigma^2}$$

$$\text{or } \frac{d\alpha}{d\mu} = \frac{1}{3b\sigma^2 \ln\ (1-\alpha)} > 0$$

Therefore the direct cost effects of an increase in μ is an increase in both signals. However, the sign of the substitution effect depends upon the size of σ^2 and therefore on the relative costs of the two signals.

Differentiating the above two derivatives with respect to σ^2 yields:

$$\frac{\partial^2 y}{\partial\mu\ \partial\sigma^2} > 0 \quad \text{and} \quad \frac{d^2\alpha}{d\mu\ \partial\sigma^2} < 0$$

These second-order derivatives indicate that the increase in y is larger for a high σ^2 than a low σ^2, and the increase in α is smaller for a high σ^2 than a low σ^2. Since $\partial y/\partial\alpha < 0$, a sufficiently large or small σ^2 may lead to a sufficiently large negative substitution effect to counteract the positive cost effect on y or α.

To prove the above, differentiate (4.3.46) totally with respect to μ.

$$\frac{\partial y}{\partial\mu} = \left[\frac{\sigma^2}{3P}\right]^{1/3} \left\{\frac{2}{3}\left[\frac{1-\alpha}{\mu}\right]^{1/3} - \frac{1}{3}\left[\frac{\mu}{1-\alpha}\right]^{2/3} \frac{d\alpha}{d\mu}\right\}$$

$$= \left\{\frac{\sigma^2 (1-\alpha)}{9P\mu}\right\}^{1/3} \left\{2 - \frac{\mu}{1-\alpha} \frac{d\alpha}{d\mu}\right\}$$

Therefore sign $\frac{\partial y}{\partial \mu}$ = sign $\left\{2 - \frac{\mu}{1-\alpha} \frac{d\alpha}{d\mu}\right\}$

Substitute $\frac{d\alpha}{d\mu} = - \frac{1}{3\ b\ \sigma^2 \ln\ (1-\alpha)}$

Then $\{\cdot\} = 2 + \frac{\mu}{3\ b\ \sigma^2 (1-\alpha)\ \ln(1-\alpha)}$

from (4.3.45)

$$(1-\alpha)\ \ln\ (1-\alpha) = \frac{\mu}{3\ b\ \sigma^2} - \alpha$$

$$= \frac{\mu - 3\ \alpha\ b\ \sigma^2}{3\ b\ \sigma^2}$$

Substitute this expression into $\{\cdot\}$:

$$\{\cdot\} = 2 + \frac{\mu\ 3\ b\ \sigma^2}{3\ b\ \sigma^2 (\mu - 3\ \alpha\ b\ \sigma^2)}$$

$$= 2 + \frac{\mu}{\mu - 3\alpha\ b\ \sigma^2}$$

Then $\frac{\partial y}{\partial \mu} \gtrless 0$

$$\text{if} \quad 2 + \frac{\mu}{\mu - 3\,\alpha\, b\, \sigma^2} \gtrless 0$$

$$\text{or} \quad 2 \gtrless \frac{-\mu}{\mu - 3\,\alpha\, b\, \sigma^2}$$

$$\frac{1}{2} \lessgtr - \frac{\mu - 3\,\alpha\, b\, \sigma^2}{\mu}$$

$$\mu \lessgtr - 2\,\mu + 6\,\alpha\, b\, \sigma^2$$

$$\mu \lessgtr 2\,\alpha\, b\, \sigma^2$$

From (4.3.45)

$$\mu = 3\, b\, \sigma^2\, [\alpha + (1-\alpha)\, \ln\, (1-\alpha)]$$

Since $[\alpha + (1-\alpha)\, \ln\, (1-\alpha)] = 0$ for $\alpha = 0$

and $\to 1$ for $\alpha \to 1$,

it follows that $\mu < 3\alpha b\sigma^2$ for $0 < \alpha < 1$.

Therefore $\mu < 2\,\alpha b\sigma^2$ if σ^2 is sufficiently large (or equivalently, α sufficiently small).

Then $\frac{\partial y}{\partial \mu} > 0$ if $\mu < 2\,\alpha\, b\, \sigma^2$

$= 0 \qquad \mu = 2\ \alpha\ b\ \sigma^2$

$< 0 \qquad 2\alpha\ b\ \sigma^2 < \mu < 3\ \alpha\ b\ \sigma^2$

Q.E.D.

The cost effect on y of an increase in μ is clearly positive. The substitution effect is negative and may exceed the cost effect if σ^2 is very large. A similar argument should hold for α. The cost effect on α of an increase in μ is positive. The substitution effect is negative and may exceed the cost effect if σ^2 is very low. The total change in α^* and y^* will result in the combined α, y which will efficiently signal that μ has increased.

A summary of the results of comparative statics is

Parameter Changed	Direction of Changed	Effect on y^*	Effect on α^*
b	+	+ (S)	- (C)
	-	- (S)	+ (C)
P	+	- (C)	+ (S)
	-	+ (C)	- (S)
σ^2	+	+ (C,S)	- (C,S)
	-	- (C,S)	+ (C,S)
μ	+ high σ^2	?	+ (C,S)
	low σ^2	+ (C,S)	?
	- high σ^2	?	- (C,S)
	low σ^2	- (C,S)	?

where C is Cost effect

S is Substitution effect

The final section will discuss empirical implications of the analysis in this section.

4. EMPIRICAL IMPLICATIONS OF THE SIGNALING MODEL

The bivariate signaling model formulated in this chapter was proved to have an equilibrium solution which is fully revealing, rational, and incentive compatible. There are empirical implications emerging from the analysis which will be discussed.

The information set available to investors for valuing a new security includes the contents of the prospectus. The value of α is clearly displayed in the prospectus, while the value of y is not so obvious. An examination of prospectuses for new issues in 1981 revealed that the prospectuses contain historical-cost financial statements, a summary of historical earnings for a maximum of five years, and a cautious discussion by management of such matters as a description of the company's business, its competitive environment, and its investment plans. The disclosures made do not include forward-looking information such as management forecasts of future cash flows, but appear to be restricted to historical information. It does appear that the potential imposition of penalties has limited disclosure to "hard" information which can be documented and verified more easily.

The signaling model of Leland and Pyle has been empirically tested by Downes and Heinkel (1982) and Ritter (1982). Both tests used data in prospectuses for a large sample of firms making initial public offerings of common stock, or which were 'going

public'. The data used in both test included historical accounting numbers.

Downes and Heinkel performed a nonlinear estimation of the form:

$$V_j = \sum_{i=0}^{9} (A_i x_i)\ E_j + U_j$$

Where V_j = market value of equity of firm j measured as offering price

E_j = normalized earnings from an exponential smoothing model applied to all available historical earnings per share for a maximum of ten years

x_i = explanatory variable

a_i = regression coefficient on i^{th} independent variable

U_j = error term

The estimation is, in effect, of the form

$$\frac{V_j}{E_j} = \sum_{i=0}^{9} a_i x_i + \frac{U_j}{E_j}$$

where the dependent variable is the price/earnings ratio. The explanatory variables used were:

0)	Constant	* (+)
1)	α	* (+)
2)	Dividend policy dummy	* (-)
3)	Debt/assets ratio after issue	
4)	Industry dummy	* (+)
5)	Age	
6)	Sales of most recent full year	* (-)
7)	Three-year growth rate in sales	* (+)
8)	Underwriter prestige	* (+)
9)	Hot issue market dummy	* (+)

Those variables which had statistically significant regression coefficients (i.e., $t > 1.96$) are noted with asterisks and the sign of significant coefficients is noted parenthetically. The dividend policy dummy variable is included because of Bhattacharya's (1979) and Heinkel's (1978) results on dividend signaling, however the negative coefficient is not explained. The reason presented for inclusion of the remaining variables is to proxy for risk since firm-specific risk appears in the Leland/Pyle valuation equation and is assumed by Leland and Pyle to be known by investors. These surrogate variables are assumed by Downes and Heinkel to be indices which are exogenous and beyond the control of the entrepreneur.

Ritter's test on 559 firms going public in the period 1965-73 also used historical accounting numbers as explanatory variables and explicitly used earnings as an independent variable. His estimation was of the form

$$\frac{V_j}{B_j} = \sum_{i=0}^{8} a_i x_i + U_j$$

where V_j = market value of equity of firm j measured as offering price.

B_j = pre-offering book value, used to control for size.

The explanatory variables used were:

0)	Constant	* (+)
1)	α	* (+)
2)	Age	* (+)
3)	Sales	* (-)
4)	Median P/E on ASE	* (+)
5)	Book Value	* (+)

6) Earnings in most recent fiscal year * (+)
7) Sales growth
8) Investment (I) * (+)

Reasons presented for inclusion of some of the explanatory variables are to control for heterogeneity of firms and heteroskedasticy. There was no justification presented for inclusion of earnings, and no discussion of the highly significant result that the coefficient on earnings/book value was 13.75, with a standard error of 0.98, resulting in a t-statistic of 14. Ritter did not discuss inclusion of a proxy for firm-specific risk.

The results of both empirical tests indicate that historical accounting numbers are correlated with market value of new securities. Both tests include the accounting numbers as exogenous parameters. However, these numbers clearly are chosen, controlled, or at least influenced by the entrepreneur of a private firm going public. If the entrepreneur can increase market value of the security being sold by disclosing high accounting earnings, there is a motivation for him to do so. However, the market is rational and must realize that accounting earnings can be manipulated. Therefore, the inclusion of such numbers in a test of rational, incentive compatible valuation derived from a signaling model without incorporating an incentive for truthful reporting of accounting numbers seems to lack either rationality or incentive compatibility.

The model developed in this chapter has a mechanism through which disclosure of historical accounting earnings is made credible. An observable variable which could be used as a proxy for y

is the past year's accounting earnings or normalized earnings. If the inference schedule is

$$\bar{x}\ (\alpha,y) = \{9\ b\ p_0\ \phi\ y^3 \left[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)\right]\}^{1/3}$$

and $\quad Cov(\tilde{x},\tilde{M})$ is still assumed to be zero,

then $$V(\alpha,y) = \frac{\{9\ b\phi\ P_0\ y^3[\frac{\alpha}{1-\alpha} + \ln(1-\alpha)]\}^{1/3}}{1+r}$$

where b and ϕ are not measurable, but are assumed to be known by investors. No risk term appears in the valuation, and therefore no proxy is required for the market's estimate of firm-specific risk. Therefore, the inferred valuation schedule does have empirical content. Downes and Heinkel and Ritter used offer price P_0 as V_j. They both reported obtaining similar results using price in the after market. The model developed in this chapter specifically uses P_E as V_j, allowing for an underpriced issue. In order to measure P_E, it would be necessary to estimate when equilibrium price is reached. As discussed in Chapter Three, the adjustment process appears to take less than one week on average and frequently is immediate.

The results of comparative statics appear to be empirically testable.

1) The result of Proposition 4.4 that the level of α is decreasing in y is testable once a proxy for y has been specified.

2) Proposition 4.5 showed that y is increasing in σ^2 and α is decreasing. If stationarity of σ^2 is assumed, σ^2 can be estimated from price data on the securities subsequent to issue. The result of the proposition then is testable.

3) Proposition 4.6 showed that y is decreasing in $P_0\phi$. While ϕ cannot be measured, P_0 is published in the prospectus. Therefore the analytical relationship between y and P_0 is testable.

4) Proposition 4.7 does not appear to have empirical content because the entrepreneur's coefficient of risk aversion cannot be measured.

5) Proposition 4.8 showed that the trade-off between α and y as μ increases depends upon σ^2. Assuming market rationality, μ can be estimated from the equilibrium post-offering price. Thus the results of the proposition appear to be testable.

In summary, if historical earnings are used to proxy for y, and if μ and σ^2 are estimated from price data, there are several implications of the analysis which appear to be amenable to cross-sectional empirical testing. Such empirical tests will be the focus of future research.

CHAPTER FIVE

RISK SHARING AND VALUATION UNDER MORAL HAZARD

INTRODUCTION

A second type of asymmetric information between the entrepreneur and investor arises when the entrepreneur manages the firm or technology so that its future cash flows are dependent upon his managerial actions and effort. In this case, if his actions are unobservable by investors, then a moral hazard problem exists. In the preceding chapter, the distribution of cash flows was exogenous and unobservable; in this chapter it is endogenous and unobservable. Section 1 describes the original formulation of the moral hazard problem as a two-person risk-sharing problem. Section 2 reviews recent literature which analyzes the moral hazard problem in a market, where firm valuation depends upon the means of resolution of the problem. In Section 3, the Leland/Pyle and Jensen/Meckling models are integrated in order to look at the risk-averse entrepreneur's problem under moral hazard. A specific example is analyzed in Section 4 in order to derive a closed-form solution. Empirical implications of the analysis are discussed in Section 5.

1. THE PRINCIPAL-AGENT PROBLEM

The principal-agent problem was originally formulated (e.g., Ross (1973) and Holmstrom (1979)) as a two-person risk-sharing problem under moral hazard.

Pareto-optimal risk sharing between two parties under varying assumptions about risk attitudes was described in Chapter Four. When both parties are risk averse, the Pareto-optimal contract specifies that risk is shared so that the ratio of marginal utilities is a constant for any risky outcome, where the ratio is a function of risk tolerances and relative bargaining powers. If one party is risk neutral, he bears all of the risk of the outcome, and the risk-averse party receives a fixed payment. These Pareto-optimal risk-sharing contracts are not attainable when moral hazard, or unobservability of actions, exists. Conditions under which such unobservability prohibits efficient contracting are:

(1) Both principal and agent maximize expected utility, where

(a) the principal's utility function U(W) is defined over wealth and exhibits risk aversion or risk neutrality:

$$U'(W) > 0$$

$$U''(W) \leq 0$$

(b) the agent's utility function V(W,e) is defined over wealth and effort and exhibits strict risk aversion and disutility of effort:

$$V_W(W,e) > 0$$
$$V_e(W,e) < 0$$
$$V_{WW}(W,e) < 0$$

(2) The principal and agent contract to share an observable risky outcome from a distribution which

(a) has a fixed support and about which both parties have homogeneous beliefs, and

(b) depends upon the unobservable actions or effort of the agent, such that a higher level of effort increases the expected value of the distribution.

Since contracts can be written only on inputs, variables, or outcomes which are jointly observable, the principal and agent can contract only on the random outcome. It will be shown that both parties would be better off if effort were observable, and therefore could provide a basis for contracting. Not only does the unobservability of effort preclude its use in contracting, but a basis for shirking is provided because the agent derives disutility from expending effort and the detection of shirking is hindered by the random variation in the outcome. The principal cannot determine, without additional information, whether an extreme value of the outcome is due to the agent's effort or to an extreme realization of the stochastic variable. The agent can trade off the gain in expected utility from his share in the outcome (which is increasing in his efforts) against the disutility from providing such efforts; while the principal wishes only to maximize his share of the outcome. Therefore there is a conflict between the objectives of the principal and agent. The principal must select a contract which will motivate the agent to provide an optimal level of effort as well as share risks.

The agent must be strictly risk averse for the problem to exist. If the agent is risk neutral, the optimal contract involves the principal receiving a fixed share so that the agent bears all of the consequences of his actions.

The usual formulation of the principal's problem is

$$\max_{\Phi(x),e} \; E_e\{U[\tilde{x}-\Phi(\tilde{x})]\}$$

$$\text{s.t.} \quad E_e\{V[\Phi(\tilde{x}),\, e]\} \geq \bar{V}$$

$$e^* \in \text{Argmax } E_e\{V[\Phi(\tilde{x}),\, e]\}$$

$$\text{or} \quad V_W[\Phi(\tilde{x}),e] = V_e$$

In the objective function, the principal selects the agent's share of the random outcome, $\Phi(\tilde{x})$, where the outcome $\tilde{x}$ is conditional on the agent's effort e in that more effort will shift the distribution of $\tilde{x}$ to the right in the sense of first-order stochastic dominance. In the first constraint, $\bar{V}$ is a minimum level of expected utility which must be provided so that the agent will accept the contract. The minimum level of expected utility is determined exogeneously in the market. The second constraint is the Nash constraint which states that the effort selected by the principal must be the same level of effort that the agent will select in his own optimization problem, given $\Phi(x)$ selected by the principal. The principal selects $\Phi(x)$ which then induces the agent to select e* in his problem

$$\max_{e} \; E_e\{V[\Phi(\tilde{x}),\, e]\}$$

The optimal solution to the principal's problem was derived by Holmstrom:

$$\frac{U'[\tilde{x} - \Phi(\tilde{x})]}{V_W[\Phi(\tilde{x}),e]} = k + \ell \frac{f_e(x|e)}{f(x|e)}$$

where $f(x|e)$ is the density function of x conditional on e; and k, ℓ are Lagrangian multipliers.

This solution can be compared to the Pareto-optimal solution derived in Chapter Four:

$$\frac{U'}{V_W} = k$$

It is obvious that Pareto-optimal risk sharing is unattainable if $\ell \neq 0$ since $f_e(x,e) \neq 0$ by assumption. Therefore, due to the unobservability of the agent's actions, the agent will expend less effort and inefficient risk sharing will result.

In the absence of additional information, resolution of the agency problem necessitates that the agent bear some of the risk of the outcome for motivational purposes. Holmstrom shows that if some of the noise can be removed from the measurement of x, or information can be provided about e, then risk sharing can be improved by using the information in contracting.

2. THE PRINCIPAL-AGENT PROBLEM IN A MARKET

The owner of a firm with a risky cash flow can share risk in the capital market. There is no demand for an individual agent with whom to share risks and therefore an agent must provide some expertise

which the principal does not possess. The diffuse ownership structure of a corporation creates the demand for agents to manage operations of the firm. The agency problem arises under separation of ownership and control when the risky return to the shareholders/principal is dependent upon the unobservable managerial actions of the manager/agent who is maximizing his own welfare rather than the welfare of the shareholders. The distribution of cash flows therefore becomes endogenous where it was assumed to be exogenous in the asymmetric information problem of Chapter Four.

The contracting problem between owner and manager of a firm is more difficult to analyze than the two-person problem analyzed by Holmstrom because additional complexities emerge in a market setting. Some of these complexities are:

1) The risk-sharing capabilities of the capital market are available to the principal and agent, and both may be able to use the market to eliminate the risk imposed for incentive purposes. In order to prevent such elimination of risk, various assumptions are made, such as:

    a) The manager has no wealth and cannot borrow or sell short.
    b) The manager's trading and portfolio are observable.
    c) The manager is not permitted to trade in the firm's securities.

2) Opportunities may exist for the manager to reduce the risk he must bear by altering the variance of cash flows generated by the firm's capital investments through his selection of investment projects.

3) The market value of the firm is a function of the distribution of cash flow and therefore is dependent upon managerial actions and the means used to resolve the moral hazard problem.

4) In addition to the existence of a securities market for the risky outcome, there is a market for managerial labor which makes endogenous the agent's reservation level of expected utility which is in fact a market price.

5) Shareholders represent many principals and there may be many agents and a hierarchy of agents which make coordination and contracting costly.

Papers which have addressed these issues will be briefly discussed.

Holmstrom showed that information is valuable if it reduces the noise in estimates of the agent's expenditure of effort so that more efficient risk sharing is attainable. If there is more information about effort or the draw from the distribution of x (i.e., which state of nature occurred), less risk need be imposed upon the risk-averse agent. Diamond and Verrecchia (1982) look at the problem of using valuable information in contracting when there are many principals. Top management is considered to be a single risk-averse agent and the many principals/shareholders act as if they are risk neutral because they can diversify risk across firms. As earlier shown, in the absence of a moral hazard problem, it would be optimal to pay the manager a fixed salary. However, if the manager can improve his welfare by shirking, he can do so with no cost to himself under a fixed salary. There are N identical risk-neutral shareholders, each of whom possesses identical information useful in managerial contracting (by definition, useful information reduces the

noise in estimates of effort). As N grows large, the costs of communication and coordination required to directly incorporate the information in a management compensation contract may grow prohibitively large so that the information is not used, even though all shareholders would be better off if the information was in the contract. A low cost way of using the information is for shareholders to trade in the market on the basis of their information so that market prices reflect the information. The existence of many principals makes direct contracting with the agent infeasible; but due to the existence of a market for the risky security, market price will aggregate the information of the many principals. Diamond and Verrecchia use an example to derive the price and the optimal incentive contract when output is a random variable whose distribution and realization depend upon a systematic risky factor with an ex-post observable realization, a firm-specific risky factor which is non-observable, and effort. The manager's compensation is decreasing in the realization of the systematic factor and in price which has impounded the aggregate information about firm-specific risk. Therefore private information about firm-specific risk and public information about systematic risk are used in contracting because they permit extraction of some risk, which in a sense has become observable, so that managerial shirking becomes more detectable. The risk borne by the manager for incentive purposes has been reduced to a portion of firm-specific risk. Due to the existence of the capital market, systematic risk can be considered to be observable risk and therefore the agency problem can be minimized as firm-specific risk grows small. (Of course, it is necessary that the firm's β is not

influenced by actions and decisions of the manager.) While non-systematic risk is irrelevant to capital budgeting decisions in a perfect market, the magnitude of such risk becomes important in risk sharing under moral hazard and therefore may be quite relevant to capital budgeting decisions when the manager is a risk-averse agent of the shareholders.

The effect of non-optimal risk sharing on managerial investment decisions also is analyzed by Marcus (1982) when the risk-averse manager of a firm is constrained to hold a specified fraction of ownership in the firm. The manager's compensation includes shares in his firm's stock which he is prohibited from trading on the market. In the two-person agency problem, the agent does not have the opportunity to eliminate risk imposed for incentive reasons. In a market setting, if the risk which is borne by the agent is in the form of shares which are tradeable in the market, an additional constraint must be added to the problem: such as that of no trading. Marcus imposes the constraint and analyzes the effects on managerial valuation of the firm's shares and managerial investment decisions. The manager values his shares in the firm at less than market price because he does not hold a well-diversified portfolio and therefore his risk premium is for total risk where the market's risk premium is for systematic risk. The non-optimal risk sharing provides incentives for the manager to undertake less risky investments so as to reduce nonsystematic risk which he must bear. Since the manager is prohibited from eliminating firm-specific risk through portfolio diversifications, he will attempt to eliminate it through the investment decision he is making for the shareholders of the firm and such

a search for low-risk investments may be costly to shareholders if the manager expends resources of the firm as well as diverting his own efforts and expertise to such activities. Marcus' result about consequences of investment behavior can be compared to the results of Diamond and Verrecchia. The latter show that the principal and agent will be better off if firm-specific risk is reduced since improved risk sharing results. In Marcus' model, the search for risk-reducing projects may be sufficiently costly that shareholders are worse off while managers are better off. Marcus' analysis is incomplete for the following reasons:

1) The specified fraction of managerial ownership is not derived, but merely assumed. In a more general analysis, the manager's share of the risky outcome is derived in an optimization problem which is constrained by the need to supply a minimum reservation level of expected utility to the manager.
2) To analyze such a problem in a financial market without consideration of the market for managers is a partial analysis because one determinant of the risk imposed on the manager is the relative bargaining powers.
3) Marcus shows that the manager has an incentive to underinvest in risky projects. He states that the owners of the firm must undertake costly monitoring of investment decisions or suffer the consequences of the suboptimal decisions. Such a statement seems to be allowing for naivety or irrationality on the part of owners of the firm. If investment decisions are observable, the manager can be provided with incentives to undertake the investment projects desired by the owners

through an appropriate forcing contract. If the decisions are not observable, but the available opportunity set of investments is observable, the owners will rationally assume that the manager will select the investment which will maximize his expected utility. Since the owners of the firm own the technology and the hired manager is a price-taker, the owners will adjust compensation so that the manager bears the cost of his suboptimal behavior. It then might be in the best interest of the manager to provide monitoring. When the demand for monitoring arises due to the suboptimal incentives of the agent, and the agent is the price-taker, it would be expected that the agent will bear the monitoring costs in such a scenario.

4) Marcus suggests that initial underpricing of new equity issues may be explained by the lower valuation placed on shares by the selling entrepreneur. Such a statement implies non-economic behavior of the entrepreneur. If the entrepreneur values the shares at a lower price than does the market, he would be very happy to sell them at market valuation and receive greater proceeds with which to diversify his portfolio.

The Marcus analysis does indicate that the agency problem has added complications when opportunities exist for the agent to diversify risk and when the agent can alter specific parameters of the distribution of cash flows by managerial decisions, such as choice of investments. In the two-person problem, the agent must bear risk. The manager of a firm has available various ways of altering the risk

imposed on him and such potential actions must be prohibited, constrained, or monitored in contracting between principal and agent.

A different approach to providing managerial incentives is used by Beck and Zorn (1982) when they analyze the relationship between incentives and share price when the manager must be motivated to purchase shares in his firm. The risky return on each share in the firm is an increasing function of the manager's ownership share because managerial incentives are assumed to be provided through ownership of the firm. While it is not made explicit in the problem, the manager is assumed to be more productive the greater is the proportion of his wealth dependent upon the performance of the firm. The manager is risk averse and therefore derives disutility from ownership in his firm due to lack of diversification rather than through disutility of effort provided to increase return. The approach is unusual because the manager/agent and investors/ principal do not contract to share the risky outcome as in the usual principal-agent problem, but rather the agent must purchase his share in the risky outcome from the principals. It then is the problem of the owner/investor to select a price and the number of shares to be sold to the manager in order to maximize expected wealth (assuming risk neutral shareholders) such that the number of shares purchased at the selected price is the optimal choice of the manager, and the manager receives an exogeneously specified minimum reservation level of expected utility of wealth. Since the agent's disutility does not derive from effort, his utility function is defined over wealth. Therefore Beck and Zorn do not relate firm value to the moral hazard problem, but focus on incentive pricing in which the manager's impact

on productivity and profits are endogenous, and the price charged must be negatively related to the number of shares to be purchased by the manager due to his risk aversion. The derived optimal pricing schedule depends upon the assumption that the manager must receive a reservation level of expected utility: if more risk is to be imposed upon the manager, additional compensation must be provided in order to satisfy the minimum utility constraint, and the additional compensation is provided in the form of a price reduction for the risky shares. The analysis is partial in that the reservation level of expected utility is exogenous. An additional limitation is that the manager derives utility from his increases in productivity through ownership in the firm. There is no disutility to the manager from exerting additional productive effort. The Beck/Zorn model may be consistent with some types of management compenation plans in which managers purchase shares below market price.

The Marcus and Beck/Zorn papers focus on the manager's valuation of shares in the firm which they are constrained to hold. The effect of moral hazard on market valuation has been analyzed by Ramakrishnan and Thakor (1982) where the principal's problem is to select managerial compensation, managerial effort and a costly monitoring system in order to maximize the risk-adjusted present value of net cash flows subject to the two constraints of meeting reservation level of utility and satisfying incentive compatibility. Cash flows can be influenced by the manager through his investment and production decisions and are reduced by managerial compensation and monitoring costs. The manager is precluded from investing in any assets, which eliminates his ability to diversify risk in the market. Since the

manager may provide different levels of effort under different monitoring or information systems, total cash flows from production will vary under different information systems. Therefore firm value is dependent not only upon managerial actions under moral hazard, but also upon the means used to resolve the problem. Ramakrishnan and Thakor suggest that the relationship between accounting information and firm value arises from the agency problem existing in a corporation since accounting reports are a product of an information system which can be viewed as a monitor of managerial actions.

Campbell and Kracaw (1982) integrate the capital and managerial labor markets when there is a moral hazard problem between owners and the manager of a firm which arises because future cash flows depend upon unobservable effort supplied by the manager which may change the expected value of cash flows. Similar to the Ramakrishnan/Thakor analysis, firm value depends upon managerial actions because such actions make endogenous a parameter of the distribution of cash flows. However, the focus of the Campbell/Kracaw paper is asset pricing when managers are also investors who may borrow and therefore risk sharing can be accomplished directly in the capital market. Consequently, there exists an optimal investment for the manager to make in his own firm which is derived in his portfolio formation problem. Due to the moral hazard, the labor market imposes risk upon the manager through his compensation which is composed of a fixed payment plus a share in the equity of his own firm. If the share imposed on the manager by the labor market exceeds his optimal investment determined in his portfolio problem, incentives are provided through suboptimal risk sharing. The portfolio of the manager

is assumed to be observable. Each firm has a single manager whose reservation level of expected utility is exogenous. Investor/managers in the capital market solve their portfolio problem while investors in a particular firm simultaneously solve for optimal managerial compensation. Risk-neutral owners of the firm select compensation in order to maximize their return from the firm subject to the incentive compatibility and individual rationality constraints. Risk-averse investors/managers select investments in all firms and effort to be expended in their own firms in order to maximize expected utility of wealth and effort, taking as given optimal compensation provided by the labor market. Not only is firm value affected by the agency problem, but parameters of the market portfolio also are altered because many investors are managers who must be provided incentives and who therefore hold nondiversified portfolios. While a manager is not permitted to sell his firm-specific capital in the market, he may attempt to take advantage of risk sharing in the capital market to diversify or eliminate the risk by selling short his firm's security or investing in a hedge portfolio with returns negatively correlated with the returns from his firm. The manager must be constrained from selling short the security of his own firm. If he invests in a hedge portfolio he can reduce or eliminate firm-specific risk without reducing or eliminating the incentive effect because variation in his return will then be due only to his effort and not to exogenous sources of uncertainty.

The analyses summarized attempt to address one or more of the five issues indicated at the beginning of this section. One issue which has not been addressed is that of determination of the agent's

reservation level of expected utility, which is a market price determined in the labor market and which has been assumed to be exogenous. An approach which eliminates the need to determine the market price for managerial expertise is to invert the usual principal-agent formulation so that the agent offers an investment contract to potential principals in the market such that he offers that contract which maximizes his expected utility and shareholders accept the contract if it provides a return equal to that of similar investment opportunities available in the market. Such an approach is used by Atkinson and Feltham (1981).

In the Atkinson/Feltham paper, a risk-averse manager is endowed with capital and a productive technology which requires a capital investment in order to generate a return. He offers securities to the market in order to raise external capital, then invests in the capital market and in the productive technology, and both investments earn risky returns. The capital market is complete with respect to systematic risk and therefore such risk can be shared in the market. A moral hazard problem arises because the manager's utility is defined over wealth and effort, and cash flows generated by the productive investment are dependent upon managerial effort. The contract which the manager offers to potential principals specifies a sharing rule for the risky outcome and a reporting system which he will implement. The focus is similar to that of Ramakrishnan and Thakor in that the manager's return is contingent on information provided by a reporting system and therefore managerial actions will depend upon the monitoring system implemented, and the information

system will affect the welfare of managers and investors. The monitoring system is selected by the party who selects the contract: the principal selects the information system in Ramakrishnan/Thakor, while the agent provides it in Atkinson/Feltham. (Atkinson/Feltham assume that the information provided by the agent's reporting system is truthful.) Atkinson/Feltham show how the efficiency of risk sharing depends upon the reporting system. If risk, effort, and outcome are reported, then Pareto-optimal risk sharing is attainable. At the other extreme, if there is no information about risk, effort, or outcome, all firm-specific risk is borne by the agent. Ex-post reporting systems between the two extremes are analyzed as to their effects on risk sharing and incentives in a capital market. A reporting system provides a basis for efficient sharing of nonsystematic risk if the report is correlated with the firm's output, and can be used to provide managerial incentives if it reports agents' actions and thus serves as a basis for performance-contingent compensation. Therefore, Pareto-optimal sharing of systematic market risk can be achieved in the capital market and the degree to which nonsystematic firm-specific risk can be efficiently shared under moral hazard depends upon the informativeness of the reporting system provided by the agent.

An approach similar to that of Atkinson and Feltham is that of Jensen and Meckling (1976). A risk-neutral owner/manager of a firm is endowed with a productive technology and has an insufficient wealth endowment to finance the capital investment necessary to realize a return from the technology. He therefore seeks to raise capital in the market. A moral hazard problem arises because managerial actions are not observable and the manager can divert assets

of the firm for his personal consumption. Jensen and Meckling describe the suboptimal (from the viewpoint of shareholders) behavior as consumption of perquisites. However, such behavior is in effect the same as shirking or reducing effort expenditure in that both reduce the expected value of cash flows and both are unobservable. When the owner/manager owns 100% of the equity in his firm, he receives 100% of the benefits of perquisite consumption and bears 100% of the cost. As his proportion of ownership declines, he bears less than 100% of the cost of his behavior while continuing to receive all the benefit. Non-managing prospective shareholders know the manager's utility function, can rationally infer his behavior as his ownership declines, and will price the equity accordingly. The manager selects his proportion of ownership in his firm in order to maximize his return where the market valuation of the equity sold is increasing in his proportionate ownership and his perquisite consumption is decreasing. Jensen and Meckling suggest that monitoring or bonding activities can be used to control the manager's behavior. Monitoring and bonding through control systems and preparation of audited financial statements represent use of information systems to reduce the noise in effort measurement similar to those activities analyzed more formally by Atkinson/Feltham and Ramakrishnan/Thakor. The costs of monitoring are borne by the manager since market participants price securities rationally. Jensen and Meckling's manager is risk neutral and therefore the real cost of his owning a proportion of his firm is not apparent.

Jensen and Meckling state that their approach differs fundamentally from the agency theory described earlier in this section.

However the two approaches are not substantially different if risk aversion is added to the Jensen/Meckling model and it is compared to the Atkinson/Feltham model. When the agent is risk averse, his ownership in his own firm clearly represents suboptimal risk sharing which he accepts in order to communicate his behavior to the market. Jensen and Meckling are indirectly determining an optimal compensation contract in which a portion of the manager's compensation consists of returns from his own firm which is a nontradeable asset and which imposes risk on him in order to motivate him to act in the best interests of the owners of his firm. The manager chooses to substitute compensation in the form of unobservable nontradeable riskless perquisites for observable nontradeable risky shares in order to maximize his expected utility.

In the next section, the Jensen/Meckling problem of an owner/manager seeking external equity capital will be formulated under risk aversion. It will be seen that the implications of the model for managerial behavior under moral hazard are the same as those in agency models in which the principal offers a contract to an agent in order to share risks and provide incentives.

3. RISK SHARING AND VALUATION WITH MORAL HAZARD

The problem of an entrepreneur seeking to raise funds to finance a capital investment will be analyzed where all market particpants have homogeneous beliefs about the exogenous distribution of future cash flows to be generated by the capital investment. The entrepreneur/owner will manage operations of the firm and can alter the distribution of cash flows through his actions which are unobservable by outside investors in the project or firm. It was shown in

Section 1 of Chapter Four that Pareto-optimal risk sharing results when the risk-averse entrepreneur sells all of the equity in the cash flows generated by the project because systematic market risk can be optimally shared in the capital market and the market is risk neutral as to nonsystematic firm-specific risk. This Pareto-optimal result is attainable in a perfect market, but was shown to be unattainable when there exists a market imperfection such as asymmetric information about the expected value of future cash flows. A similar market imperfection which precludes attainment of efficient risk sharing exists when the manager can reduce the expected value of future cash flows through his unobservable actions in order to maximize his expected utility. Since the manager selects his actions the informational asymmetry is in effect the same as that of Chapter Four: the entrepreneur has inside information about firm value.

The entrepreneur chooses his ownership in his firm, α, and his actions in order to maximize his expected utility of wealth. He can reduce cash flows in every state by:

(a) shirking, or reducing the amount of effort expended in managing the firm, and

(b) taking non-pecuniary benefits, or diverting assets of the firm to his personal use (e.g., supplying himself with a company car, or using the corporate jet for recreational travel).

Shirking will result in reduced cash flows to the extent that they are dependent upon managerial actions. Perquisites reduce cash flows by their cost. The reduction in cash flows in every state due to shirking and perquisite consumption will be denoted F. Perquisites affect only the expected value of cash flows and therefore are

riskless to the entrepreneur. When α is equal to the Pareto-optimal value of zero, the manager will receive 100% of the benefits of F while the firm's shareholders will bear 100% of the cost. When $\alpha > 0$, the manager continues to receive 100% of the benefits of F, but also bears α proportion of the cost. Therefore as α increases, the manager's propensity to consume perquisites should decrease.

In the problem at hand, a linear sharing rule is being assumed to simplify the analysis. A linear sharing rule is not likely to be optimal when a moral hazard problem exists. Therefore a loss in generality is being accepted so that a solution can be derived and analyzed.

The problem of the entrepreneur is to select α, β, F in order to maximize expected utility of end-of-period wealth.

$$\max_{\alpha,\beta,F} \quad E\{U(\tilde{W}_1,F)\} \tag{5.3.1}$$

$$\text{where } \tilde{W}_1 = \alpha(x - F) + \beta\tilde{M} + (1+r)Y \tag{5.3.2}$$

$$\text{s.t.} \quad W_0 + (1-\alpha)V(\alpha) - I - \beta V_m - Y = 0 \tag{5.3.3}$$

$$V(\alpha^*) = \frac{\mu - F(\alpha^*) - \lambda}{1+r} = \frac{\mu - F^* - \lambda}{1+r} = V \tag{5.3.4}$$

End-of-period wealth in expression (5.3.2) is reduced by the proportion of the cost of the perquisites actually consumed by the entrepreneur which he will bear, αF. In the budget constraint (5.3.3), the market pays $(1-\alpha)$ times the value which it infers from observation of α for $(1-\alpha)$ of equity in future cash flows generated by the investment I. Expression (5.3.4) is the market rationality

constraint that the market's inference is correct in equilibrium. Firm value is reduced by F through the reduction in expected value of future cash flows. In the entrepreneur's objective function (5.3.1) perquisites F is shown separate from wealth W_1. Utility is derived from the use of assets consumed as perquisites or from shirking, and both are nonmarketable benefits. There are alternative ways of formulating the objective function. As shown in Section 1, in a two-person agency problem the agent's utility is additively separable in wealth and effort and can be represented with two utility functions (e.g., $U(W) - G(e)$). In the problem at hand, a separate utility function for perquisties could be defined so that the objective function becomes $\max E\{U(W_1) + V(F)\}$. The approach to be taken is to transform the benefits of F into dollars of wealth so that the simplification of the objective function made possible by combining the normal distribution and negative exponential utility function can be used.

The transformation of F into dollars of wealth will be accomplished by defining a transformation function T(F) which must satisfy the following conditions:

(i) $T(0) = 0$

(ii) $T(F) > 0$ for $F > 0$ because benefits are derived from any positive level of perquisites or shirking.

(iii) $0 < T'(F) < 1$ since more perquisites are preferred to less and they are not tradeable.

(iv) $T''(F) < 0$

(v) $T'''(F) > 0$

Conditions (iv) and (v) impose sufficient regularity on T(F) necessary for the following analysis. Thus T(F) is a type of 'utility' function which transforms F to dollars of wealth rather than utiles. Figure 5.1 depicts an acceptable transformation function.

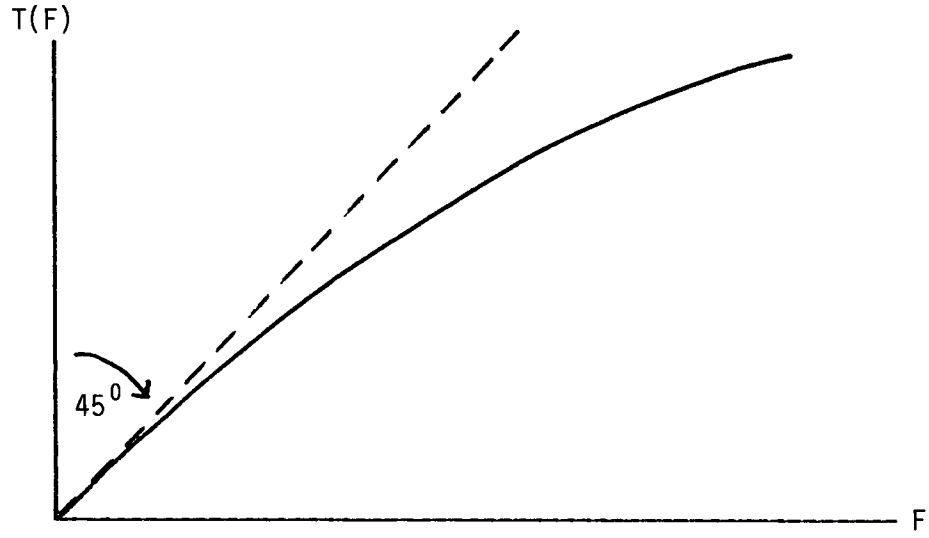

Figure 5.1. Perquisite Transformation Function

T(F) is measured in the same units as wealth and can therefore be entered into the utility function defined over wealth as:

$$U(\tilde{W}_1, F) = -e^{-b(\tilde{W}_1 + T(F))}$$

It was proved in Chapter Four that

$$\arg\text{Max } E\{U(\tilde{W}_1)\} = \arg\text{Max } \{\tilde{W}_1 - \frac{b}{2}\sigma_w^2\}$$

Therefore, because F is riskless, the entrepreneur's objective function (5.3.1) can be rewritten as:

$$\max_{\alpha,\beta,F} \; G\{\bar{W}_1 - \frac{b}{2}\sigma_w^2 + T(F)\}$$

Let $H(\alpha,\beta,F) \equiv \bar{W}_1 - \frac{b}{2}\sigma_w^2 + T(F)$

To solve the entrepreneur's problem, substitute for Y in (5.3.2), using (5.3.3)

$$\tilde{W}_1 = \alpha(\tilde{x} - F) + \beta\tilde{M} + (1+r)\,[W_0 + (1-\alpha)V(\alpha) - I - \beta V_m] \quad (5.3.5)$$

After substitution for $V(\alpha)$ using 5.3.4 and simplification, (5.3.5) becomes:

$$\tilde{W}_1 = \alpha[\tilde{x} - F - \mu + F(\alpha) + \lambda] + \beta[\tilde{M} - (1+r)V_m] \quad (5.3.6)$$

$$+ \mu - F(\alpha) - \lambda + (1+r)(W_0 - I)$$

Recall from Chapter Four that

$$\lambda = \frac{\bar{M} - (1+r)V_m}{\sigma_m^2} \cdot \mathrm{Cov}(\tilde{x},\tilde{M})$$

The expected value and variance of 5.3.6 are:

$$\bar{W}_1 = \alpha[\mu - F - \mu + F(\alpha) + \lambda] + \beta[\bar{M} - (1-r)V_m] \quad (5.3.7)$$

$$+ \mu - F(\alpha) - \lambda + (1+r)(W_0 - I)$$

$$\sigma_{w_1}^2 = \alpha^2\sigma^2 + \beta^2\,\sigma_m^2 + 2\,\alpha\,\beta\,\mathrm{Cov}(\tilde{x},\tilde{M}) \quad (5.3.8)$$

The first order conditions to the problem are:

$$\frac{\partial H}{\partial \alpha} = \mu - F - \mu + F(\alpha) + \lambda + (\alpha-1)F_{\alpha} - \alpha b\sigma^2 - \beta\, b\, Cov(\tilde{x},\tilde{M}) = 0 \tag{5.3.9}$$

$$\frac{\partial H}{\partial \beta} = \bar{M} - (1+r)V_m - \beta b\sigma_m^{\ 2} - \alpha\, b\, Cov(\tilde{x},\tilde{M}) = 0 \tag{5.3.10}$$

$$\frac{\partial H}{\partial F} = -\alpha + T'(F) = 0 \tag{5.3.11}$$

In order to eliminate β from the problem, (5.3.10) is solved for βb and then substituted into (5.3.9):

$$-F + F(\alpha) + (\alpha-1)F_{\alpha} - \alpha b \left[\frac{\sigma^2\sigma_m^2 - Cov(\tilde{x},\tilde{M})^2}{\sigma_m^2}\right] = 0 \tag{5.3.12}$$

The market rationality condition (5.3.4) is combined with (5.3.12) to arrive at:

$$(\alpha-1)F_{\alpha} = \alpha b \left[\frac{\sigma^2\sigma_m^2 - Cov(\tilde{x},\tilde{M})^2}{\sigma_m^{\ 2}}\right] \tag{5.3.13}$$

Equation (5.3.11) is rewritten as:

$$T'(F) = \alpha \tag{5.3.14}$$

Equations (5.3.13) and (5.3.14) are the necessary equilibrium conditions, each equating marginal benefit and marginal cost for a choice variable. In (5.3.13), since $(\alpha-1) < 0$, then $F_{\alpha} < 0$ because the right-hand side is positive. The marginal benefit of α is the increase in proceeds, $(\alpha-1)F_{\alpha}$, and the marginal cost of α is the bearing of firm-specific risk. The marginal benefit of F is T'(F),

or the marginal increase in the wealth-equivalent of perquisites and the marginal cost of F is α, the entrepreneur's share of the cost of F. Figure 5.2 illustrates how the entrepreneur selects his actions on the basis of α. Therefore, the market infers F from observation of α and the entrepreneur does indeed select F based upon α which is selected according to (5.3.13).

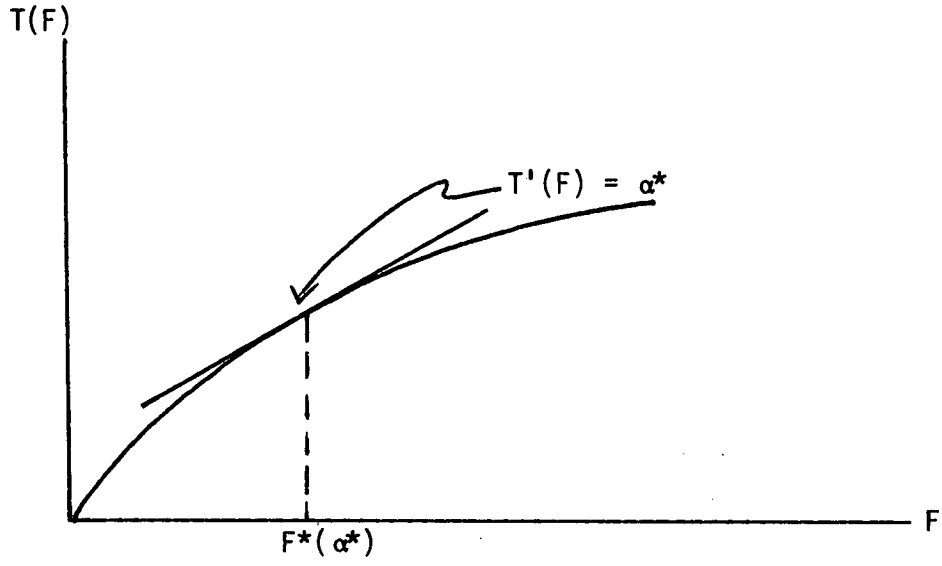

Figure 5.2. Choice of $F(\alpha^*)$

Since $0 < \alpha < 1$, $0 < T'(F) < 1$ which is consistent with Figure 5.1 and the conditions on T(F).

It was suggested earlier in this section that the manager's propensity to consume perquisites or shirk should decrease as α increases. In order to verify this conjecture, totally differentiate (5.3.14) with respect to F and α:

$$T''(F)dF = d\alpha$$

$$\text{or} \quad \frac{dF}{d\alpha} = \frac{1}{T''(F)} < 0 \qquad (5.3.15)$$

from condition (iv) on T(F)

Therefore it is true that F is decreasing in α. The choice of F as a function of α is illustrated in Figure 5.3. Convexity is proved by differentiating (5.3.15) with respect to α:

$$\frac{d^2F}{d\alpha^2} = -\frac{1}{[T''(F)]^2} \; T'''(F)F_\alpha > 0 \qquad (5.3.16)$$

follows from $T''' > 0$

$F_\alpha < 0$

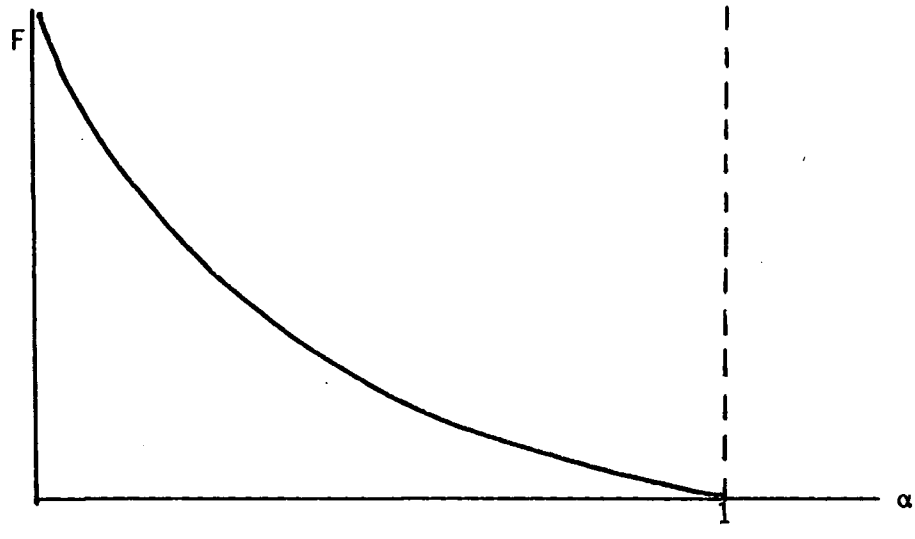

Figure 5.3. The Entrepreneur's F(α) Choice

Since the market's inference is assumed to be rational, the market's inference schedule will also look like Figure 5.3 and (5.3.15) can be substituted into (5.3.13) to arrive at the single necessary equilibrium condition.

$$\frac{(\alpha-1)}{T''(F)} = \alpha b \left[\frac{\sigma^2\sigma_m^2 - Cov(\tilde{x},\tilde{M})^2}{\sigma_m^2}\right] \tag{5.3.17}$$

which has replaced the market's inference F_α with the entrepreneur's choice $F_\alpha = \frac{1}{T''(F)}$. Equation (5.3.17) relates choices of α and F to exogenous parameters. A proof that second-order conditions are satisfied appears in Appendix 5-A.

<u>Proposition 5.1</u>. An increase in firm specific risk will result in a decrease in equilibrium a^* and a corresponding increase in F^*.

<u>Proof</u>. It was shown in Chapter Four that

$$\frac{\sigma^2\sigma_m^2 - Cov(\tilde{x},\tilde{M})^2}{\sigma_m^2} = \sigma_\varepsilon^2 \text{, firm specific risk}$$

Rewrite (5.3.13), replacing [•] with σ_ε^2.

$$(\alpha-1)F_\alpha = \alpha b \sigma_\varepsilon^2 \tag{5.3.18}$$

The total derivative of (5.3.18) with respect to α and σ_ε^2 is:

$$(\alpha-1)F_{\alpha\alpha}\, d\alpha + F_\alpha d\alpha = b\sigma_\varepsilon^2\, d\alpha + \alpha b\, d\sigma_\varepsilon^2$$

$$\text{or} \quad \frac{d\alpha}{d\,\sigma_\varepsilon^2} = \frac{\alpha b}{(\alpha-1)F_{\alpha\alpha} + F_\alpha - b\,\sigma_\varepsilon^2} < 0 \qquad (5.3.19)$$

since $(\alpha-1) < 0$

$F_{\alpha\alpha} > 0$

$F_\alpha < 0$

$b\,\sigma_\varepsilon^2 > 0$

Q.E.D.

If firm-specific risk increases, then 'signaling' with α becomes more costly and therefore the entrepreneur is better off exchanging risky α for riskless F.

Proposition 5.1 can be compared to results of the papers described in Section 2. Systematic risk is 'observable' and therefore can be shared optimally in the capital market. In the absence of moral hazard, non-systematic risk is borne by the risk-neutral principal/investors, as proved in Chapter Four. Due to the need to provide incentives to the risk-averse agent/manager, firm-specific risk is imposed on him. In the model here, the agent selects the firm-specific risk that he will bear in order to signal his actions to the market. If firm specific risk increases, the level of perquisites selected will increase because the manager substitutes a

riskless return from his firm for the risky return. To the manager perquisites serve as insurance against increases in risk.

Proposition 5.2. An increase in the risk aversion parameter b will result in a decrease in equilibrium α^* and a corresponding increase in F^*.

Proof. The total derivative of (5.3.18) with respect to α and b is:

$$(\alpha-1)F_{\alpha\alpha}\, d\alpha + F_\alpha d\alpha = b\ \sigma_\varepsilon^2\ d\alpha + \alpha\sigma_\varepsilon^2\ db$$

or

$$\frac{d\alpha}{db} = \frac{\alpha\sigma_\varepsilon^2}{(\alpha-1)F_{\alpha\alpha} + F_\alpha - b\ \sigma_\varepsilon^2} < 0$$

Q.E.D.

This result is similar to Proposition 5.1. If b increases, communicating through α becomes more costly and therefore risky α is exchanged for riskless F.

Many of the studies described in Section 2 show how the manager may be motivated to reduce the variance of cash flows through his investment policy in order to reduce the risk which he bears. Diamond and Verrecchia describe systematic risk as being 'observable' and conclude that reduction of firm-specific risk should result in improved risk sharing between a risk-averse agent and risk-neutral principal because effort is more easily detectable. It can be seen from (5.3.17) that firm-specific risk can be reduced if σ^2 is decreased or Cov^2 is increased. The following proposition formalizes the Diamond/Verrecchia conjecture.

<u>Proposition 5.3</u>. A reduction in firm-specific risk achieved through a reduction in σ^2 results in a greater expected utility for the entrepreneur and an increase in market value of the firm, <u>for a given covariance</u>.

<u>Proof</u>.

(a) It was earlier shown that

$$\text{Arg Max } E\{U\} = \text{Arg Max } G\{H\} = \text{Arg Max } H$$

since $G' > 0$

Then, $\dfrac{d\ E\ \{U\}}{d(\text{parameter})} = G'(H)\ \dfrac{dH}{d(\text{parameter})}$

and sign $\dfrac{d\ E\{U\}}{d(\text{parameter})} = \dfrac{dH}{d(\text{parameter})}$

as long as the parameter is not b which appears in G.

Let $H^*(\alpha^*, \beta^*\ F^*)$ be the maximal value function.

Then

$$\frac{dH^*}{d\sigma^2} = \frac{\partial H^*}{\partial \sigma^2} + \frac{\partial H^*}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \sigma^2} + \frac{\partial H^*}{\partial F} \cdot \frac{\partial F}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \sigma^2}$$

$$= \frac{\partial H^*}{\partial \sigma^2} \text{ by envelope theorem.}$$

Using (5.3.8),

$$\frac{dH}{d\sigma^2} = -\frac{b}{2}\alpha^2 < 0$$

A decrease in σ^2 reduces $\sigma_{w_1}{}^2$ and therefore improves the welfare of the entrepreneur in that the cost of communicating F by α has decreased.

(b) To show that the market value increases, differentiate $V(\alpha)$ in (5.3.4) with respect to σ^2:

$$\frac{dV(\alpha)}{d\sigma^2} = \frac{\partial V}{\partial F} \cdot \frac{\partial F}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \sigma^2} < 0 \qquad \text{since each term} < 0$$

Q.E.D.

Market valuation increases because $F(\alpha)$ decreases as a result in a decrease in σ^2. In a perfect market, firm value is a function of systematic risk and changes in firm specific risk should have no effect on valuation. However, when moral hazard is present and the manager must bear firm-specific risk as an agency cost, changes in that risk result in changes in his behavior and therefore in firm valuation which depends upon his behavior. Therefore, more efficient risk sharing does result when firm-specific risk is reduced such that both shareholders and manager are better off.

Shareholders are indifferent to the value of and changes in σ^2 ex ante because they correctly infer managerial behavior and value the firm based upon observation of α; while the manager does gain by ex ante reduction in σ^2 as shown in part (a) of the proof. On the other hand, ex post reduction of σ^2 does benefit shareholders as well as the manager because the value of the firm will increase despite no changes in the expected value and systematic risk of cash flows.

As σ_ε^2 becomes very small, it is seen from (5.3.18) that α approaches the value of 1 because the market knows that the cost of α to the entrepreneur is very small. It was shown in Chapter Four that if $\sigma_\varepsilon^2 = 0$, the firm's cash flows are prefectly correlated with those of the market portfolio so that the entrepreneur's investment in his own firm is a perfect substitute for an investment in the market. Therefore no moral hazard problem will exist if $\sigma_\varepsilon^2 = 0$: the entrepreneur will own 100% of the equity in future cash flows, will hold an investment in the market portfolio which complements the systematic risk in his firm, and hence will bear 100% of the cost of his behavior. Therefore it is indeed true that the agency problem is minimized if firm-specific risk is very small.

<u>Proposition 5.4</u>. A decrease in $\mathrm{Cov}(\tilde{x},\tilde{M})$ results in greater expected utility for the entrepreneur <u>even if it results in greater firm-specific risk</u>, given constant σ^2. The effect on market value depends upon the sign of the covariance.

<u>Proof</u>.

(a) By use of the envelope condition as in the proof of Proposition 5.3:

$$\frac{dH^*}{d\ \mathrm{Cov}} = \frac{\partial\ H^*}{\partial\ \mathrm{Cov}}$$

$$= \lambda^0(\alpha-1) - \alpha\ b\ \beta < 0$$

where $\lambda^0 = \dfrac{\bar{M} - (1+r)V_m}{\sigma_m^2}$ is market price of risk.

If $Cov(\tilde{x},\tilde{M})$ decreases, the entrepreneur's expected utility increases because the value of the firm has increased. The first-order condition (5.3.10) simplifies to:

$$\beta^* = \frac{\lambda^0}{b} - \alpha\, Cov(x,M) \qquad (5.3.20)$$

It was shown in Chapter Four that the optimal choice of α and β in a perfect market are:

$$\alpha^0 = 0$$

$$\beta^0 = \frac{\lambda^0}{b}$$

β^0 is the optimal investment in the market portfolio of an investor with a coefficient of risk aversion b in order to bear an optimal amount of systematic market risk. If $Cov(\tilde{x},\tilde{M}) = 0$, then $\beta^* = \beta^0$ regardless of the level of α. Since the firm is uncorrelated with the market, the entrepreneur will bear the optimal amount of systematic risk through his investment in the market portfolio. If $Cov(\tilde{x},\tilde{M}) > 0$, some systematic risk is held through the investment α in his own firm and therefore the entrepreneur reduces his investment in the market by $\alpha Cov(\tilde{x},\tilde{M})$. If $Cov(\tilde{x},\tilde{M}) < 0$, the entrepreneur is 'selling short' systematic risk through his investment α and therefore he increases his investment in the market so as to acquire the desired amount of market risk. Therefore the choices of β^* in the present problem are:

$$\beta^* < \beta^0 \text{ if } Cov(x,M) > 0$$

$$\beta^* > \beta^0 \text{ if } Cov(x,M) < 0$$

$$\beta^* = \beta^0 \text{ if } Cov(x,M) = 0$$

If the covariance changes, while σ^2 is constant, firm-specific risk changes and therefore α will change. As both Cov and α change, the entrepreneur will adjust β^* so that he continues to hold the optimal amount of systematic risk.

If $Cov(\tilde{x},\tilde{M}) > 0$, a decrease in $Cov(\tilde{x},\tilde{M})$ has the following effects:

Increase in $\sigma_\varepsilon^2 \longrightarrow$ decrease in α^*

because the signal α has become more costly.

$$\left.\begin{array}{l}\text{Decrease in Cov}\\ \text{Decrease in } \alpha\end{array}\right\} \longrightarrow \text{ increase in } \beta^*$$

because less market risk is held through α. Less wealth is invested in a non-tradeable asset with systematic risk and therefore a greater investment will be made in the market so that β^0 is reattained.

Increase in $E\{U\}$ because the value of the firm has increased.

If $Cov(\tilde{x},\tilde{M}) < 0$, a decrease in $Cov(\tilde{x},\tilde{M})$ has the following effects:

Decrease in $\sigma_\varepsilon^2 \longrightarrow$ increase in α^*

because the signal α has become more costly.

$$\left.\begin{array}{l}\text{Decrease in Cov}\\ \text{Increase in } \alpha\end{array}\right\} \longrightarrow \text{ increase in } \beta^*$$

Because of the negative correlation, not only does a decrease in covariance mean that less market risk is held, but also the increase in α means that more 'sold short'. Therefore a greater investment is made in the market.

Increase in $E\{U\}$ because the value of the firm has increased.

(b) In order to prove the effect on market value, (5.3.4) is rewritten as:

$$V(\alpha) = \frac{\mu - F(\alpha) - \lambda^0 \, Cov(\tilde{x},\tilde{M})}{1+r} \tag{5.3.21}$$

which is then differentiated with respect to $Cov(\tilde{x},\tilde{M})$.

$$\frac{dV}{d\ Cov} = \frac{\partial V}{\partial\ Cov} + \frac{\partial V}{\partial F} \cdot \frac{\partial F}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial\ Cov} \tag{5.3.22}$$

All of the above partial derivatives are negative, with the exception of $\partial\alpha/\partial$ Cov which was shown in part (a) of this proof to depend upon the sign of the covariance, where:

$$\frac{\partial \alpha}{\partial\ Cov} = \left\{ \begin{matrix} > 0 \\ \\ < 0 \end{matrix} \right. \quad \text{if Cov} \left\{ \begin{matrix} > 0 \\ \\ < 0 \end{matrix} \right.$$

Therefore:

$$\frac{dV}{d\ Cov} < 0 \quad \text{if Cov} < 0$$

$$\lesseqgtr 0 \quad \text{if Cov} > 0$$

The change in value in (5.3.22) has two terms: one representing an exogenous effect which is a change in the risk priced by the market, the second an endogenous effect resulting from managerial behavior. In a perfect market, only the first effect would result from a change in the covariance.

If $Cov(\tilde{x},\tilde{M}) > 0$, a decrease in $Cov(\tilde{x},\tilde{M})$ has the following effects:

Increase in $\sigma_e^2 \rightarrow$ decrease in $\alpha \rightarrow$ increase in F $\rightarrow$ lower value

Decrease in risk adjustment $\rightarrow$ higher value

If $Cov\ (\tilde{x},\tilde{M}) < 0$, a decrease in $Cov(\tilde{x},\tilde{M})$ has the following effects:

Decrease in $\sigma_\varepsilon^2 \rightarrow$ increase in $\alpha \rightarrow$ decrease in F $\rightarrow$ higher value

Decrease in risk adjustment $\rightarrow$ higher value.

Therefore the effect on firm value if $Cov(\tilde{x},\tilde{M}) < 0$ depends upon the relative sizes of the exogenous and endogenous changes.

Q.E.D.

Propositions 5.3 and 5.4 illustrate the complexity of the principal-agent problem in a market in which the agent may make decisions other than how much effort to expend or perquisites to consume. Proposition 5.3 shows that all market participants are better off if the variance of cash flows is reduced. Proposition 5.4 shows that the manager can reduce his risk through adjustment of his personal investment portfolio as well as through the firm's investment policy. Investors in the market are assumed to be rational and can infer that the manager will undertake any feasible action that maximizes his

welfare. Therefore securities will be priced rationally and the entrepreneur will bear the costs of his actions. Since the manager must bear firm-specific risk and will be assumed to select investments to maximize his welfare, he may be motivated to (i) offer an alternative bonding system so that he can reduce his risk and (ii) provide a system which monitors his investment policy.

4. AN EXAMPLE

The solution to the entrepreneur's problem in Section Three was in the form of an implicit function of α. A closed form solution is not attainable without a specification of the form of T(F). Necessary criteria which T(F) must satisfy were described as:

(i) $T(0) = 0$

(ii) $T(F) > 0$ for $F > 0$

(iii) $0 < T'(F) < 1$

(iv) $T''(F) < 0$

(v) $T'''(F) > 0$

A function which satisfies these criteria is the negative exponential function

$$T(F) = 1 - e^{-F} \tag{5.4.1}$$

A parameter in an individual's utility function is the coefficient of risk aversion which describes the attitude of the individual toward risk. A useful parameter in an individual's perquisite transformation function would be a coefficient which would describe the attitudes of the individual toward shirking or consumption of perquisites. Such a parameter will be included in T(F) as:

$$T(F) = \frac{1}{\gamma} - \frac{1}{\gamma} e^{-\gamma F} \tag{5.4.2}$$

where $\gamma > 0$ is the coefficient of the propensity to consume perequisites.

To show that (5.4.2) satisfies the five criteria listed above,

(i) $T(F) = \frac{1}{\gamma} - \frac{1}{\gamma}e^{-\gamma F} = 0$ when $F=0$

(ii) $T(F) > 0$ for $F > 0$

Since $\frac{1}{\gamma}[1-e^{-\gamma F}] > 0$

follows from $e^{-\gamma F} < 1$ for $\gamma F > 0$

(iii) $T'(F) = e^{-\gamma F} = 1$ when $F=0$

$T'(F) > 0$ since $e^{-\gamma F} > 0$

$T'(F) < 1$ since $e^{-\gamma F} < 1$ for $\gamma F > 0$

(iv) $T''(F) = -\gamma e^{-\gamma F} < 0$

(v) $T'''(F) = \gamma^2 e^{-\gamma F} > 0$

The equilibrium conditions which were derived in Section Four are:

$$T'(F) = \alpha \tag{5.3.14}$$

$$(\alpha-1)F_\alpha = \alpha b \sigma_\varepsilon^2 \tag{5.3.18}$$

To solve for F*, substitute for T'F in (5.3.14)

$$e^{-\gamma F} = \alpha \tag{5.4.3}$$

Taking logarithms of (5.4.3):

$$-\gamma F = \ln \alpha$$

or $$F^* = -\frac{\ln \alpha}{\gamma} > 0 \quad \text{Since } 0 < \alpha < 1 \tag{5.4.4}$$

and $$F_\alpha = -\frac{1}{\alpha\gamma} < 0 \tag{5.4.5}$$

and $$F_\gamma = \frac{\ln \alpha}{\gamma^2} > 0 \qquad (5.4.6)$$

Expressions (5.4.4) - (5.4.6) describe the entrepreneur's choice of F as decreasing in α and increasing in γ. From (5.4.4) it is seen that

$$\lim_{\alpha \to 0} F = \infty$$

and $$\lim_{\alpha \to 1} F = 0$$

The manager will consume zero perquisites if he owns the entire firm because he bears all the cost and his benefit is less than cost (from $T(F) < F$). If no equity in the firm is held, he will consume all cash flows, and therefore investors will not purchase 100% of the equity in the project.

The market knows the manager's perquisite transformation function, observes α^*, and can perfectly infer F^*.

$$F(\alpha^*) = F^* \qquad (5.4.7)$$

Substituting for F^* from (5.4.4),

$$F(\alpha^*) = -\frac{\ln \alpha}{\gamma}$$

In order to derive α^*, substitute for F_a in (5.3.18), using (5.4.5)

$$\frac{-(\alpha-1)}{\alpha\gamma} = \alpha b \sigma_\varepsilon^2$$

or $$\alpha^2 \gamma b \sigma_\varepsilon^2 + \alpha - 1 = 0 \qquad (5.4.8)$$

The solution to (5.4.8) is:

$$\alpha^* = \frac{-1 + \sqrt{1 + 4\gamma b\sigma_\varepsilon^2}}{2\gamma b\sigma_\varepsilon^2} \tag{5.4.9}$$

Market valuation of the firm obtains by substituting for $F(\alpha)$ in the valuation function

$$V(\alpha) = \frac{\mu - F(\alpha) - \lambda^0 \operatorname{Cov}(\tilde{x},\tilde{M})}{1+r}$$

so that
$$V(\alpha) = \frac{\mu + \frac{\ln \alpha}{\gamma} - \lambda^0 \operatorname{Cov}(\tilde{x},\tilde{M})}{1+r} \tag{5.4.10}$$

Both α and γ enter into valuation because both are determinants of F.

<u>Proposition 5.5</u>. An increase in the propensity to consume perquisites γ will result in a decrease in α.

<u>Proof</u>. Propositions 5.1 and 5.2 indicated that increases in b or σ_ε^2 would result in a decrease in α. In expression (5.4.9), $\gamma, b, \sigma_\varepsilon^2$ appear multiplicatively.

Totally differentiate (5.4.8) with respect to α and $(\gamma b\sigma_\varepsilon^2)$.

$$\alpha^2 d(\gamma b\sigma_\varepsilon^2) + 2\alpha(\gamma b\sigma_\varepsilon^2)\, d\alpha + d\alpha = 0$$

or
$$\frac{d\alpha}{d(\gamma b\sigma_\varepsilon^2)} = -\frac{\alpha^2}{2\alpha(\gamma b\sigma_\varepsilon^2) + 1} < 0$$

Q.E.D.

α^* decreases if b or σ_ε^2 increase because α becomes a more costly signal of F.

α^* decreases if γ increases because F increases with γ and α and F are inversely related.

<u>Proposition 5.6</u>. The choice of α^* will always be such that $0 < \alpha < 1$.

<u>Proof</u>.

(a) Verification that $\alpha > 0$:

From (5.4.9) it is clear that $\alpha^* > 0$ since $4\gamma b\sigma_\varepsilon^2 > 0$

(b) Verification that $\alpha < 1$:

To be proved:

$$-1 + \sqrt{1 + 4\gamma b\sigma_\varepsilon^2} < 2\gamma b\sigma_\varepsilon^2$$

or $$\sqrt{1 + 4\gamma b\sigma_\varepsilon^2} < 1 + 2\gamma b\sigma_\varepsilon^2$$

Squaring both sides,

$$1 + 4\gamma b\sigma_\varepsilon^2 < 1 + 4\gamma b\sigma_\varepsilon^2 + 4(\gamma b\sigma_\varepsilon^2)^2$$

which is clearly true since $4(\gamma b\sigma_\varepsilon^2)^2 > 0$

Q.E.D.

In the example analyzed in this section, firm valuation is a function of an observable managerial action, α, and an individual managerial behavioral parameter, γ. Investors know that cash flows are reduced by perquisites and that the level of perquisites is determined by α and γ.

5. EMPIRICAL IMPLICATIONS

The information set available to investors for valuing a new security was briefly described in section 4 of Chapter Four. In 1977, the SEC issued Securities Act Release No. 5856 which expanded the mandatory disclosure in registration statements to include information about managerial nonpecuniary compensation. Disclosure of remuneration is mandatory for the three highest paid officers and any officer whose direct remuneration exceeds $40,000. In specifying what is to be considered direct remuneration,

> "The Release divides direct remuneration into two categories, namely 'salaries, fees, bonuses and other payments' (which have traditionally been deemed covered by the existing disclosure requirements) and the 'personal benefits' received by management a euphemism for perquisites."
>
> [Practising Law Institute (1977), p. 504]

There are three categories of personal benefits.

A. Benefits which must be reported as remuneration are those payments made by the company for:

   1. home repairs and improvements;
   2. living expenses;
   3. the personal use of company property such as automobiles, yachts, or vacation houses;
   4. personal travel expenses;
   5. personal entertainment expenses; and
   6. legal and accounting fees unrelated to company business.

B. Benefits which may be reported as remuneration are

   1. the manager's ability to obtain favorable bank loans or similar benefits because the company compensates the third party, and

2. the use of corporate staff for personal purposes.

C. Benefits which are not considered to be remuneration are:

1. directly related to job performance (e.g., use of a parking space), or

2. necessary to the conduct of the company's business (e.g., an expense account).

The total pecuniary and nonpecuniary remuneration received by top management therefore is disclosed to some extent and can be used as a measure of riskless F in empirical tests of the predictions from the analysis in the preceding section.

The principal empirical prediction of both the adverse selection model in Chapter Four and the moral hazard model in Chapter Five is that firm value will be positively related to α. The empirical tests discussed in Section 4 of Chapter Four found such a positive relationship. Since there are two different explanations for the positive relationship, it would be desirable to design an empirical test which could differentiate between the adverse selection and the moral hazard explanations. Such a test may be possible due to the existence of data about perquisites. Both Proposition 4.4 and Proposition 5.1 show that an increase in σ_ε^2 will result in a decrease in α, which is empirically testable. However, Proposition 5.1 also adds that an increase in σ_ε^2 will result in an increase in F, which also is empirically testable since F is measurable. If a negative relationship between σ_ε^2 and α is found empirically, and there is no change in remuneration, then empirical support for the model of Chapter Four is provided. If however, there is also found a positive relationship between σ_ε^2 and F, then

the empirical evidence supports the moral hazard model (perhaps in addition to the adverse selection model). The analytical result that there will be a negative relationship between α and F is testable.

According to the capital asset pricing model, firm value should depend upon systematic risk and be independent of firm-specific risk. Proposition 5.3 shows that under moral hazard, firm value will change if firm-specific risk changes while the covariance is constant. Since σ^2 and $Cov(\tilde{x},\tilde{M})$ can be estimated from price data, then the prediction of Proposition 5.3 is testable.

Firm value is decreasing in $Cov(\tilde{x},\tilde{M})$ according to the capital asset pricing model. Proposition 5.4 indicates that firm value may be increasing in covariance due to the agency problem and the effects of changes in risk on the choice of α. The prediction of this proposition therefore is testable.

In summary, since F and α are disclosed in the prospectus and σ_ε^2 and $Cov(\tilde{x},\tilde{M})$ can be estimated from a time series of price data, there are implications of the analysis in this chapter which can be empirically tested.

CHAPTER SIX

RISK SHARING AND VALUATION UNDER ADVERSE SELECTION AND MORAL HAZARD

INTRODUCTION

This chapter integrates the results of risk sharing and valuation under adverse selection from Chapter Four and under moral hazard from Chapter Five. The entrepreneur employs two signals to communicate his information about both the exogenous parameters of the distribution of cash flows and his endogenous behavior. While a complete solution to the problem cannot be derived due to a mathematical difficulty, the formulation and partial solution indicate how the adverse selection and moral hazard problems are in fact variations of one basic informational problem of unobservability, and how sensitive a solution will be to the nature of the interaction of the signals.

AN INTEGRATION OF THE ADVERSE SELECTION AND MORAL HAZARD PROBLEMS

The informational asymmetry which exists in the integrated model arises from the ex ante unobservability of the expected value of cash flows and the ex post unobservability of managerial effort. As in Chapter Four, it is assumed that the firm's cash flows are uncorrelated with those of the market. In a perfect market, the value of the firm then is a function of the expected value of future cash flows:

$$V = \frac{\mu}{1+r} \tag{6.1}$$

The entrepreneur knows the parameters of the distribution of future cash flows while investors do not. The entrepreneur also expends effort in managing the firm and can reduce the expected value of cash flows through his unobservable consumption of perquisites and shirking. The end-of-period realization of cash flow is a random variable observable by investors and the entrepreneur.

The problem of the entrepreneur is to select ownership in his firm, perquisite consumption, and a disclosure about value so as to maximize his expected utility. In the absence of asymmetric information, investors know firm value, there will be no disclosure and the entrepreneur will sell 100% of the equity in his firm. In the problem of Chapter Four, two costly signals were supplied by the entrepreneur to the market to communicate information about μ because the costs of both signals are functions of σ^2 about which there is asymmetric information. In the moral hazard problem of Chapter Five, the entrepreneur selected his ownership in his firm in order to communicate information about his behavior. In the integrated problem in this chapter, the entrepreneur selects ownership in his firm and disclosure when the market does not know μ or σ^2 and cannot observe F.

There is sufficient separability in the problem as formulated that investors can infer F from α alone. The entrepreneur selects y in order to disclose his information about μ. However, since the choice of α affects the marginal benefits of the signal y, it is also used by investors to infer μ. The market rationality conditions to the integrated problems then are:

$$F(\alpha^*) = F^* \tag{6.2}$$

$$\bar{x}(\alpha^*,y^*) = \mu \tag{6.3}$$

The market observes α^* and correctly infers endogenous managerial behavior; and also observes y and correctly infers the exogenous parameter μ. Since the result of the entrepreneur's behavior is a reduction in cash flows, inferred firm value is

$$V(\alpha^*,y^*) = \frac{\bar{x}(\alpha^*,y^*) - F(\alpha^*)}{1+r} \tag{6.4}$$

The problem of the entrepreneur is

$$\max_{\alpha,\beta,F,y} E\{U(\tilde{W})_1)\} = \max_{\alpha,\beta,F,y} H \tag{6.5}$$

$$\text{s.t.} \quad W_0 + (1-\alpha)V(\alpha,y) - C(y) - I - \beta V_m = Y \tag{6.6}$$

$$V(\alpha^*,y^*) = \frac{\bar{x}(\alpha^*,y^*) - F(\alpha^*)}{1+r} = \frac{\mu-F}{1+r} = V \tag{6.7}$$

$$\text{where } H = \bar{W}_1 - \frac{b}{2}\sigma_W^2 + T(F) \tag{6.8}$$

$$\tilde{W}_1 = \alpha(\tilde{x}-F) + \beta\tilde{M} + (1+r)Y \tag{6.9}$$

The problem is constrained by the budget constraint (6.6) and the market rationality constraint (6.7) which follows from (6.2), (6.3), and (6.4).

In order to derive expression (6.8), substitute (6.6) into (6.9)

$$\tilde{W}_1 = \alpha(\tilde{x}-F) + \beta\tilde{M} + (1+r)\,[W_0 + (1-\alpha)V(\alpha,y) - C(y) - I - \beta V_m] \tag{6.10}$$

After substitution of (6.4) and rearrangement, (6.10) becomes

$$\tilde{W}_1 = \alpha\,[\tilde{x} - F - \bar{x}(\alpha,y) + F(\alpha)] + \beta[\tilde{M} - (1+r)V_m] + \bar{x}(\alpha,y) - F(\alpha) + (1+r)[W_0 - C(y) - I] \tag{6.11}$$

The expected value and variance of (6.11) are

$$\bar{W}_1 = \alpha[\mu - F - \bar{x}(\alpha,y) + F(\alpha)] + \beta[\bar{M} - (1+r)V_m] + \bar{x}(\alpha,y)$$

$$- F(\alpha) + (1+r)\ [W_0 - C(y) - I] \tag{6.12}$$

$$\sigma_w^2 = \alpha^2\sigma^2 + \beta^2\sigma_m^2 \tag{6.13}$$

First-order conditions to (6.5) are:

$$\frac{\partial H}{\partial \alpha} = \mu - F - \bar{x}(\alpha,y) + F(\alpha) + (1-\alpha)\bar{x}_\alpha + (\alpha-1)\ F_\alpha - \alpha b\sigma^2 = 0 \tag{6.14}$$

$$\frac{\partial H}{\partial \beta} = \bar{M} - (1+r)V_m - \beta b\sigma_m^2 = 0 \tag{6.15}$$

$$\frac{\partial H}{\partial F} = -\alpha + T'(F) = 0 \tag{6.16}$$

$$\frac{\partial H}{\partial y} = (1-\alpha)\bar{x}_y - (1+r)C_y = 0 \tag{6.17}$$

Due to the separability in these first-order conditions, the choices of β and F can be isolated from the choice of α and y. The separability results from assuming $Cov(\tilde{x},\tilde{M}) = 0$, a T(F) function independent of β or y, and C(y) function independent of β or F. The choice of β can be derived from (6.15) as:

$$\beta^* = \frac{\bar{M}-(1+r)V_m}{b\sigma_m^2} = \frac{\lambda^0}{b}$$

$$\text{Using } \lambda^{o} = \frac{\bar{M}-(1+r)V_m}{\sigma_m^2}$$

It was shown in Chapter Five that

$$\beta^* = \frac{\lambda^o}{b} - \alpha Cov(\tilde{x},\tilde{M})$$

In the problem at hand when $Cov(\tilde{x},\tilde{M}) = 0$, the entrepreneur holds the optimal amount of market risk through his investment in the market portfolio since ownership in his firm imposes only firm-specific risk on him.

In order to derive the choice of F, equation (6.14) is combined with (6.2) and (6.3)

$$(\alpha-1)F_\alpha = \alpha\ b\ \sigma^2 \tag{6.18}$$

Equations (6.16) and (6.18) are identical to the first-order conditions of the moral hazard problem in Chapter Five. In order to derive an explicit solution for $F(\alpha)$, the perquisite transformation function adopted in section four of Chapter Five will be employed here.

$$\text{Assume } T(F) = \frac{1}{\gamma} - \frac{1}{\gamma} e^{-\gamma F} \tag{6.19}$$

Differentiate (6.19) and use (6.16)

$$T'(F) = e^{-\gamma F} = \alpha$$

or $$-\gamma F = \ln \alpha$$

and $$F^* = -\frac{\ln \alpha}{\gamma} \tag{6.20}$$

In (6.20), both α and γ are observable. The optimal selection of perquisites is unchanged by the additional asymmetric information about μ and σ^2. The market, however, now can infer σ_i^2 from observation of α above since, from (6.18),

$$F_\alpha = -\frac{1}{\alpha\gamma} \tag{6.21}$$

can be substituted into (6.18).

$$-\frac{\alpha-1}{\alpha\gamma} = \alpha\, b\, \sigma^2$$

or

$$\sigma_i^2 \text{ inferred} = \frac{1-\alpha}{\alpha^2 b\gamma}. \tag{6.22}$$

The entrepreneur selects F so that $T'(F) = \alpha$ where the marginal benefit and marginal cost of F are equal. Given the assumed negative exponential form of T(F), he selects F according to (6.20) as a function of α and his propensity to consume perquisites γ. Since they understand the entrepreneur's problem, investors know γ, observe α, and correctly infer F and σ^2. While σ^2 is irrelevant for firm valuation, it is a parameter which is necessary to infer μ from y. The entrepreneur therefore chooses

$$\alpha^* = \frac{-1 + \sqrt{1 + 4b\gamma\sigma^2}}{2b\gamma\sigma^2} \tag{6.23}$$

from (6.22)

and $F^* = \dfrac{-\ln \alpha}{\gamma}$

The market infers

$$F(\alpha^*) = F^* = \frac{-\ln \alpha^*}{\gamma}$$

$$\sigma^2(\alpha^*) = \frac{1 - \alpha^*}{\alpha^{*2} b\gamma} = \sigma^2$$

To show that the market's inference $\sigma_i^2(\alpha^*)$ is equal to the true σ^2, substitute for α^*, using (6.23) in the inference function.

$$\sigma^2(\alpha^*) = \frac{1 - \left[\dfrac{-1 + \sqrt{1 + 4b\gamma\sigma^2}}{2b\gamma\sigma^2}\right]}{b\gamma\left[\left[\dfrac{-1 + \sqrt{1 + 4b\gamma\sigma^2}}{2b\gamma\sigma^2}\right]\right]^2}$$

$$= \sigma^2 \text{ after simplification}$$

In order to derive y^*, it is necessary to derive the market's inference schedule $\bar{x}(\alpha,y)$. The cost function for y which was assumed in Chapter Four will be assumed here:

$$C(y;\mu,\sigma^2) = \frac{Py^3}{(1+r)\mu\sigma^2} \tag{6.24}$$

$$\text{and} \quad C_y = \frac{3Py^2}{(1+r)\mu\sigma^2} \tag{6.25}$$

As in Chapter Four, σ^2 is irrelevant to investors in valuing the firm in a perfect market. However, since the cost of y depends upon σ^2, investors must know or be able to infer σ^2 in order to interpret y. Equation (6.25) is substituted into (6.17):

$$(1-\alpha)\ \bar{x}_y = \frac{3Py^2}{\mu\sigma^2} \tag{6.26}$$

The market rationality condition (6.3) is substitution into (6.26) in equilibrium

$$(1-\alpha)\bar{x}_y\bar{x}(\alpha,y) = \frac{3Py^2}{\sigma^2} \tag{6.27}$$

Condition (6.3) is also substituted into (6.14):

$$(1-\alpha)\bar{x}_\alpha + (\alpha-1)F_\alpha - \alpha b\sigma^2 = 0 \tag{6.28}$$

Expression (6.21) for F_α is substituted into (6.28):

$$(1-\alpha)\bar{x}_\alpha + \frac{1-\alpha}{\alpha\gamma} - \alpha b\sigma^2 = 0 \tag{6.29}$$

The market's inference schedule is the solution to the simultaneous partial differential equations:

$$(1-\alpha)\bar{x}_y\bar{x} - \frac{3Py^2}{\sigma^2} = 0 \tag{6.27}$$

$$(1-\alpha)\bar{x}_\alpha + \frac{1-\alpha}{\alpha\gamma} - \alpha b\sigma^2 = 0 \tag{6.29}$$

The method of separation of variables which was used in Chapter Four cannot be used to solve the system because the solution does not appear to be of the form $\bar{x}(\alpha,y) = g(\alpha)h(y)$ as was conjectured in Chapter Four. The $\frac{1-\alpha}{\alpha\gamma}$ term in (6.29) prohibits separation. In Chapter Four, α provided information about σ^2 so that μ could be inferred. In the problem at hand, α provides information about perquisites and, in a sense, affects how much information is provided

by the disclosure y. As α approaches zero, F becomes very large and investors know that the firm has little value without observing y. As α approaches one, F becomes very small and investors now that the firm value is $\frac{\mu}{1+r}$ and therefore must observe y to infer value. More formally,

As $\alpha \to 0$, $F \to x$

and $E(x-F) \to 0$

and $V \to 0$

As $\alpha \to 1$, $F \to 0$

and $E(x-F) \to \mu$

and $V \to \frac{\mu}{1+r}$

The speculated interaction between α and y is that, for a fixed μ, if α is very small, investors can infer value without y and less y will be provided by the manager because it is costly. If α is very large, investors cannot infer value without y and therefore the marginal benefits of y increase, resulting in a choice of a high y. A second type of interaction is seen in (6.27) in the $(1-\alpha)\bar{x}_y\bar{x}$ term which indicates how the marginal benefit of y depends upon α. The conjectured form of the inference schedule then is

$$\bar{x}(\alpha,y) = b(y) + g(\alpha)h(y)$$

where the multiplicative term captures the interaction between the two signals. The exact solution of the inference schedule will be addressed in future research.

When the problem as formulated in this Chapter is compared to those in Chapters Four and Five, it is clear that the adverse selection and moral hazard problems are variations of one basic informational problem of unobservability of value, or quality, where value is exogenous in one case and endogenous in the second. The mathematical difficulty which prohibits the solution of the integrated problem of this Chapter at this stage is not created by the combination of adverse selection and moral hazard into one problem; rather it is due to the inability to conjecture an appropriate functional form which permits a solution. Based upon the analysis in this dissertation, it appears that the existence of a fully-revealing signaling equilibrium in a multivariate signaling model will depend upon the nature of the interactions of the signals. There also must be a sufficient amount of separability among the choice variables in order for a solution to be tractable.

BIBLIOGRAPHY

Atkinson, A.A. and G.A. Feltham. "Information in Capital Markets: An Agency Theory Perspective." University of British Columbia Working Paper, 1982.

Akerlof, George. "The Market for 'Lemons': Qualitative Uncertainity and the Market Mechanism." Quarterly Journal of Economics 89 (August 1970), 488-500.

Baron, David P. "The Incentive Problem and the Design of Investment Banking Contracts." Journal of Banking and Finance 3 (1979), 157-75.

Baron, David P. "A Model of the Demand for Investment Banking Advising and Distribution Services for New Issues." The Journal of Finance 37 (September 1982), 955-76.

Baron, David P. and Bengt Holmstrom. "The Investment Banking Contract for New Issues Under Asymmetric Information: Delegation and The Incentive Problem." The Journal of Finance 35 (December 1980), 1115-38.

Beaver, William H. Financial Reporting: An Accounting Revolution. Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1981.

Beck, Paul J. and Thomas S. Zorn. "Managerial Incentives in a Stock Market Economy." The Journal of Finance 37 (December 1982), 1151-67.

Bhattacharya, Sudipto. "Imperfect Information, Dividend Policy, and 'the Bird in the Hand' Fallacy." The Bell Journal of Economics 10 (Spring 1979), 259-70.

Bhattacharya, Sudipto. "Nondissipative Signaling Structures and Dividend Policy." The Quarterly Journal of Economics 95 (August 1980), 1-24.

Borch, Karl. "Equilibrium in a Reinsurance Market." Econometrica 30 (July 1962), 424-44.

Boyce, William E. and Richard C. DiPrima. Elementary Differential Equations and Boundary Value Problems. Second edition. John Wiley & Sons, Inc., New York, 1969.

Campbell, Tim S. and William A. Kracaw. "Information Production, Market Signalling, and the Theory of Financial Intermediation." The Journal of Finance 35 (September 1980), 863-82.

Campbell, Tim S. and William A. Kracaw. "The Market for Managerial Labor Services and Capital Market Equilibrium." University of Utah Working Paper, 1982.

Cass, D. and J.E. Stiglitz. "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocation: A Contribution to the Pure Theory of Mutual Funds." Journal of Economic Theory 2 (June, 1970), 122-60.

DeGroot, Morris H. Optimal Statistical Decisions. McGraw-Hill Book Company. New York, 1970.

Diamond, Douglas W. "Financial Intermediation and Delegated Monitoring." Forthcoming, Review of Economic Studies, 1984.

Diamond, Douglas and Robert Verrecchia. "Optimal Managerial Contracts and Equilibrium Security Prices." The Journal of Finance. 37 (May 1982), 275-87.

Downes, David H. and Robert Heinkel. "Signaling and the Valuation of Unseasoned New Issues." The Journal of Finance. 37 (March 1982), 1-10.

Fama, Eugene and Merton Miller. The Theory of Finance. Dryden Press. Hinsdale, Illinois. 1972

Feltham, G.A. and John S. Hughes. "Communication of Private Information in Capital Markets: Contingent Contracts and Verified Public Reports." University of British Columbia working paper, 1983.

Friend, Irwin and Longstreet, Mendelson, Miller, and Hess. Investment Banking and the New Issues Market. The World Publishing Company. New York, 1967.

Gonedes, Nicholas. "Corporate Signaling, External Accounting and Capital Market Equilibrium: Evidence on Dividends, Income, and Extraordinary Items." Journal of Accounting Research 16 (Spring 1978), 26-79.

Grossman, Sanford J. "The Informational Role of Warranties and Private Disclosure About Product Quality." The Journal of Law and Economics 24 (December, 1981), 461-83.

Harrison, Tom. "Different Market Reactions to Discretionary and NonDiscretionary Accounting Changes." Journal of Accounting Research 15 (Spring 1977), 84-107.

Heinkel, Robert. "Dividend Policy as a Signal of Firm Value." from "Essays on Financial Markets with Imperfect Information." Ph.D. dissertation, University of California, Berkeley, 1978.

Heinkel, Robert. "Uncertain Product Quality: The Market for Lemons with An Imperfect Testing Technology." The Bell Journal of Economics 12 (Autumn 1981), 625-36.

Holmstrom, Bengt. "Moral Hazard and Observability." The Bell Journal of Economics 10 (Spring 1979), 74-91.

Ibbotson, Roger G. "Price Performance of Common Stock New Issues." Journal of Financial Economics 2 (1975), 235-72.

Jensen, Michael C. and William H. Meckling "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." Journal of Financial Economics 3 (1976), 305-60.

Leland, Hayne E. "Quacks, Lemons, and Licensing: A Theory of Minimum Quality Standards." Journal of Political Economy 87, No. 6 (1979), 1328-46.

Leland, Hayne E. and David H. Pyle. "Informational Asymmetries, Financial Structure, and Financial Intermediation." The Journal of Finance 32 (May 1977), 371-87.

Logue, Dennis E. "On the Pricing of Unseasoned Equity Issues: 1965-1969." Journal of Financial Quantitative Analysis 8 (January 1973), 91-103.

Mandelker,Gershon and Artur Raviv. "Investment Banking: An Economic Analysis of Optimal Underwriting Contracts." The Journal of Finance 32 (June 1977), 683-94.

Marcus, Alan J. "Risk Sharing and the Theory of the Firm." The Bell Journal of Economics 13 (Autumn 1982), 369-78.

McDonald, J.G. and A.K. Fisher. "New-Issue Stock Price Behavior." The Journal of Finance 27 (March 1972), 97-102.

Mossin, J. Theory of Financial Markets. Prentice-Hall, Englewood Cliffs, N.J., 1973.

Patell, James. "Corporate Forecasts of Earnings Per Share and Stock Price Behavior." Journal of Accounting Research 14 (Autumn 1976), 246-76.

Penman, Stephen H. "An Empirical Investigation of the Voluntary Disclosure of Corporate Earnings Forecasts." Journal of Accounting Research 18 (Spring 1980), 132-60.

Practising Law Institute Corporate Law and Practice Course Handbook Series, Number 262, "Preparation of Annual Documents Disclosure," Practising Law Institute, New York City, 1977.

Ramakrishnan, Ram T.S. and Anjan V. Thakor. "Moral Hazard, Agency Costs, and Asset Prices in a Competitive Equilibrium." Journal of Financial and Quantitative Analysis 17 (November 1982), 503-52.

Reilly, F.K. and K. Hatfield. "Investor Experience with New Stock Issues." Financial Analysis Journal 25 (1969), 73-80.

Ricks, William E. "The Market's Response to the 1974 LIFO Adoptions." Journal of Accounting Reseach 20 (Autumn 1982), 367-87.

Riley, John G. "Informational Equilibrium." Econometrica 47 (March, 1979), 331-59.

Ritter, Jay R. "Inside Holdings and the Pricing of Initial Public Offerings." working Paper, Wharton School, University of Pennsylvania. February, 1982.

Ritter, Jay R. "The 'Hot Issue' Market of 1980." Working Paper, Wharton, School University of Pennsylvania. 1983.

Robinson, Gerald J. and Klaus Eppler. Going Public. Securities Law Series, Volume 1. Clark Boardman Company, Ltd, 1978.

Rock Kevin. "Why are New Issues Underpriced?" Working Paper, University of Chicago, 1982.

Ross, Stephen A. "The Economic Theory of Agency: The Principal's Problem." The American Economic Review 63 (May 1973), 134-39.

Ross, Stephen A. "Mutual Fund Separation and Financial Theory - The Separating Distributions." Journal of Economic Theory. (April, 1978).

Ross, Stephen A. "The Determination of Financial Structure: The Incentive Signalling Approach." The Bell Journal of Economics 8 (Spring 1977), 23-40.

Spence, Michael. "Job Market Signaling." Quarterly Journal of Economics 87 (August 1973), 355-74.

Spence, Michael. "Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution." Journal of Economic Theory 7 (1974), 296-332.

Spence, Michael. "Informational Aspects of Market Structure: An Introduction." Quarterly Journal of Economics 90 (November 1976), 591-97.

Stiglitz, Joseph E. "Information and Capital Markets." Financial Economics: Essays in Honor of Paul Cootner. Edited by William F. Sharpe and Cathryn Cootner. Prentice-Hall. Englewood Cliffs, New Jersey. 1982.

Talmor, Eli. "Asymmetric Information, Signaling, and Optimal Corporate Financial Decisions." Journal of Financial and Quantitative Analysis 41 (November 1981), 413-38.

Thakor, Anjan V. "An Exploration of Competitive Signalling Equilibria with 'Third Party' Information Production: The Case of Debt Insurance." The Journal of Finance 37 (June 1982), 717-39.

U.S. Government Printing Office. Laws Relating to Securities Commission Exchanges and Holding Companies. Washington, D.C., 1976.

APPENDIX

APPENDIX 1

The Portfolio Problem of the Underwriting Syndicate

The risk-averse investment banker can reduce risk-bearing costs by dividing risk with other bankers. Assume that n identical risk-averse bankers underwriter the issue so that each buys $\frac{1}{n}$ of the shares sold by the entrepreneur.

The problem of each banker i becomes:[1]

$$\underset{\alpha_i,\ \beta_i}{\text{Max}}\ E\ \{U(\tilde{W}_{1i})\} \qquad i = 1,\ldots,n \tag{A1.1}$$

$$\text{where } \tilde{W}_{1i} = \alpha_i \frac{\tilde{X}}{n} + \beta_i \tilde{M} + (1+r)Y_i \tag{A1.2}$$

$$\text{s.t. } W_{0i} + (1-\alpha_i)\frac{V}{n} - \beta_i V_m - Y = 0 \tag{A1.3}$$

$$\text{where } V = \frac{\mu-\lambda}{1+r} \tag{A1.4}$$

The portfolio problem of the underwriter is similar to that of the entrepreneur. Each individual banker in the group selects α_i, how much of his equity of $\frac{1}{n}$ to retain in the entrepreneur's security, and β_i and Y_i, his investments in the market portfolio and the riskless asset.

Assuming a negative exponential utility function with risk aversion parameter b and normally distributed cash flows, the maximization of(A1.1), as shown in Chapter Four can be simplified to:

$$\underset{\alpha_i,\ \beta_i}{\text{Max}}\ H_i = \bar{W}_{1i} - \frac{b}{2}\,\sigma_{W_i}^{\ 2}$$

where, using substitutions as in Chapter Four

$$\tilde{W}_{1i} = \alpha_i \frac{\tilde{x}}{n} + \beta_i \tilde{M} + (1+r)\ [W_{oi} + \frac{(1-\alpha_i)\ (\mu-\lambda)}{n(1+r)} - \beta_i\ V_m$$

$$= \alpha_i \frac{\tilde{x} - \mu + \lambda}{n} + \beta_i\ [\tilde{M} - (1+r)V_m] + \frac{\mu-\lambda}{n} + (1+r)\ W_{oi} \qquad \text{(A1.5)}$$

The expected value of (A1.5) is:

$$\bar{W}_{1i} = \alpha_i \frac{\lambda}{n} + \beta_i\ [\bar{M} - (1+r)V_m] + \frac{\mu-\lambda}{n} + (1+r)\ W_{oi} \qquad \text{(A1.6)}$$

The variance of (A1.5) is:

$$\sigma_{Wi}^{2} = \alpha_i^2 \frac{\sigma^2}{n^2} + \beta_i^{\ 2}\ \sigma_m^{\ 2} + \frac{2\alpha_i \beta_i}{n}\ \text{Cov}(\tilde{x},\tilde{M}) \qquad \text{(A1.7)}$$

The first-order conditions to the problem are:

$$\frac{\partial H}{\partial \alpha_i} = \frac{\lambda}{n} - \frac{\alpha_i b \sigma^2}{n^2} - \frac{\beta_i b}{n}\ \text{Cov}\ (\tilde{x},\tilde{M}) = 0 \qquad \text{(A1.8)}$$

$$\frac{\partial H}{\partial \beta_i} = \bar{M} - (1+r)V_m - \beta_i b \sigma_m^{\ 2} - \frac{\alpha_i\ b}{n}\ \text{Cov}(\tilde{x},\tilde{M}) = 0 \qquad \text{(A1.9)}$$

The single necessary condition is derived from the combination of (A1.8) and (A1.9):

$$\frac{\alpha_i \, b}{n^2} \left[\frac{\sigma^2 \, \sigma_m^{\,2} - \mathrm{Cov}(\tilde{x},\tilde{M})^2}{\sigma_m^{\,2}} \right] = 0 \qquad \text{(A1.10)}$$

Feasible solutions to (A1.10) are $\alpha_i = 0$, risk neutrality, and zero firm-specific risk, as discussed in section one of Chapter Four. However, in a group of n risk-averse individuals, a fourth feasible solution is $n \to \infty$. Therefore the cost of $\alpha_i > 0$ is decreasing in n. The larger is the underwriting syndicate, the lower is the cost to the group of the risk of not selling the entire issue. The risk premium charged by a group of more than one investment bankers is less than that of an individual and therefore one of the conditions necessary for viable intermediation described in Chapter Three is satisfied: the entrepreneur earns a return at least as great as he would by contracting directly with the market. Since the risk borne becomes very small with large n, a large underwriting group behaves as if it were risk neutral.

Footnote to Appendix 1:

[1] The notation for this appendix is developed in chapter Four although the reference to the appendix appears in Chapter Three.

APPENDIX 2

Second-Order Conditions To The Bivariate Signaling Model

A sufficient condition for a maximization of the objective function is that the Hessian matrix of second-order partial derivatives is negative definite. Since the choice of β was eliminated from the problem, the Hessian matrix is composed of the second-order partial derivatives of the objective function with respect to α and y.

The Hessian matrix = $\begin{bmatrix} H_{\alpha\alpha} & H_{\alpha y} \\ H_{\alpha y} & H_{yy} \end{bmatrix}$

is negative definite if

(1) $H_{\alpha\alpha} < 0$

(2) $H_{yy} < 0$

(3) $H_{\alpha\alpha} H_{yy} - H_{\alpha y}^{2} > 0$

Values for the second-order partial derivatives are derived from (4.3.13) and (4.3.15):

$$H_{\alpha\alpha} = (1-\alpha)\bar{x}_{\alpha\alpha} - 2\bar{x}_{\alpha} - b\ \sigma^2 \qquad \text{(A2.1)}$$

$$H_{yy} = (1-\alpha)\bar{x}_{yy} - (1+r)C_{yy} \qquad \text{(A2.2)}$$

$$H_{\alpha y} = (1-\alpha)\bar{x}_{\alpha y} - \bar{x}_{y} \qquad \text{(A2.3)}$$

I. Proof that $H_{\alpha\alpha} < 0$

The following equilibrium relationships were derived in Chapter Four.

(i) $C = \dfrac{Py^3}{(1+r)\mu\sigma^2}$ (4.3.7)

Then $(1+r)\ C_{yy} = \dfrac{6Py}{\mu\sigma^2}$ (A2.4)

(ii) $\bar{x} = \{9bPy^3L\}^{1/3}$ (4.3.39)

Letting $L = \dfrac{\alpha}{1-\alpha} + \ln\ (1-\alpha)$ (A2.5)

$$L_\alpha = \frac{\alpha}{(1-\alpha)^2} \qquad \text{(A2.6)}$$

$$L_{\alpha\alpha} = \frac{(1+\alpha)}{(1-\alpha)^3} \qquad \text{(A2.7)}$$

or $\bar{x}^3 = 9bPy^3L$ (A2.8)

(iii) $\alpha + (1-\alpha)\ \ln\ (1-\alpha) = \dfrac{\mu}{3b\sigma^2}$ (4.3.45)

or $(1-\alpha)L = \dfrac{\mu}{3b\sigma^2}$ (A2.9)

(iv) $y = \{\dfrac{(1-\alpha)\ \mu^2\ \sigma^2}{3P}\}^{1/3}$ (4.3.46)

(v) $\bar{x} = \mu$ (4.3.6)

Equations (ii) - (v) must hold in equilibrium and therefore can be used in determining whether the second-order conditions hold at equilibrium.

Totally differentiate (A2.8) with respect to α:

$$3\,\bar{x}^2\,\bar{x}_\alpha = 9bPy^3\,L_\alpha \tag{A2.10}$$

Substitute for L_α using (A2.6):

$$\bar{x}_\alpha = \frac{3\,\alpha\,b\,Py^3}{(1-\alpha)^2\,\bar{x}^2} \tag{A2.11}$$

Expression (A2.11) is simplified by substituting for y from (4.3.46) and for $\bar{x}$ from (4.3.6).

$$\bar{x}_\alpha = \frac{3\,\alpha\,bP\,(1-\alpha)\mu^2\,\sigma^2}{(1-\alpha)^2\,\mu^2\,3P}$$

Then $\bar{x}_\alpha = \dfrac{\alpha\,b\,\sigma^2}{(1-\alpha)}$ which is the first-order condition (4.3.17)

To derive $\bar{x}_{\alpha\alpha}$, totally differentiate (A2.10) with respect to α.

$$3\,\bar{x}^2\,\bar{x}_{\alpha\alpha} + 6\,\bar{x}\,\bar{x}_\alpha^{\,2} = 9\,b\,Py^3\,L_{\alpha\alpha}$$

$$\text{or} \quad \bar{x}_{\alpha\alpha} = \frac{3\ b\ Py^3\ L_{\alpha\alpha} - 2\ \bar{x}\ \bar{x}_{\alpha}^{\ 2}}{\bar{x}^2} \tag{A2.12}$$

Expression (A2.12) is simplified by substituting for $L_{\alpha\alpha}$ from (A2.7) and $\bar{x}$ from (4.3.6).

$$\bar{x}_{\alpha\alpha} = \frac{\dfrac{3bPy^3(1+\alpha)}{(1-\alpha)^3} - 2\ \mu\ \bar{x}_{\alpha}^{\ 2}}{\mu^2}$$

which is further simplified by substituting for $\bar{x}_{\alpha}$ from (4.3.17) and y from (4.3.46).

$$x_{\alpha\alpha} = \frac{\dfrac{3bP(1+\alpha)}{(1-\alpha)^3} \cdot \dfrac{(1-\alpha)\ \mu^2\ \sigma^2}{3P} - 2\mu \left[\dfrac{\alpha\ b\ \sigma^2}{(1-\alpha)}\right]^2}{\mu^2}$$

which is further simplified by substituting for $\bar{x}_{\alpha}$ from (4.3.17) and y from (4.3.46).

$$\bar{x}_{\alpha\alpha} = \frac{\dfrac{3bP(1+\alpha)}{(1-\alpha)^3} \cdot \dfrac{(1-\alpha)\ \mu^2\ \sigma^2}{3P} - 2\mu \left[\dfrac{\alpha\ b\ \sigma^2}{(1-\alpha)}\right]^2}{\mu^2}$$

$$\text{or} \quad \bar{x}_{\alpha\alpha} = \frac{b(1+\alpha)\ \mu\ \sigma^2 - 2\ (\alpha\ b^2 \sigma^2)}{(1-\alpha)\ \mu^2} \tag{A2.13}$$

The expressions for $\bar{x}_{\alpha\alpha}$ and $\bar{x}_{\alpha}$ are substituted into (A2.1):

$$H_{\alpha\alpha} = (1-\alpha)\left[\frac{b(1+\alpha)\ \mu\sigma^2 - 2(\alpha\ b\ \sigma^2)^2}{(1-\alpha)^2\ \mu}\right] - \frac{2\ \alpha\ b\ \sigma^2}{(1-\alpha)} - b\sigma^2$$

$$= \left[\frac{b(1+\alpha)\ \mu\sigma^2 - 2\ (\alpha b\ \sigma^2)^2}{(1-\alpha)\ \mu}\right] - \frac{2\alpha b\ \sigma^2}{(1-\alpha)} - b\sigma^2$$

$$= \frac{b(1+\alpha)\mu\sigma^2 - 2\ (\alpha b\sigma^2)^2 - 2\alpha b\sigma^2\mu - (1-\alpha)\ b\sigma^2\ \mu}{(1-\alpha)\ \mu}$$

$$= \frac{b\mu\sigma^2\ (1+\alpha - 1+\alpha) - 2\ \alpha b\sigma^2\ (\mu + \alpha b\sigma^2)}{(1-\alpha)\ \mu}$$

$$= \frac{2\ \alpha\ b\ \mu\ \sigma^2 - 2\alpha b\sigma^2\ (\mu + \alpha b\ \sigma^2)}{(1-\alpha)\ \mu}$$

$$= -\ \frac{2\ (\alpha\ b\ \sigma^2)^2}{(1-\alpha)\ \mu} < 0 \tag{A2.14}$$

Q.E.D.

II. Proof that $H_{yy} < 0$

It was shown in equation (4.3.41) is Chapter Four that

$$\bar{x}_y = \{ 9\ b\ P\ [\frac{\alpha}{1-\alpha} + \ln\ (1-\alpha)]\}^{1/3}$$

Then $\bar{x}_{yy} = 0.$

The expressions for $\bar{x}_{yy}$ and C_{yy} from (A2.4) are substituted into (A2.2):

$$H_{yy} = (1-\alpha)\ (0) - \frac{6\ Py}{\mu\sigma^2} < 0 \qquad \text{(A2.15)}$$

Q.E.D.

III. Proof that $H_{\alpha\alpha}\ H_{yy} - H_{\alpha y}^2 > 0$

$H_{\alpha\alpha}\ H_{yy} - H_{\alpha y}^2$ can be rewritten:

$H_{\alpha\alpha}\ H_{yy} - \{(1-\alpha)\ \bar{x}_{\alpha y} - \bar{x}_y^2\}$ using (A2.3)

To derive $\bar{x}_y$, totally differentiate (A2.8) with respect to y:

$$3\ \bar{x}^2\ \bar{x}_y = 27bPy^2L$$

or $\bar{x}_y = \dfrac{9\ b\ Py^2\ L}{\bar{x}^2}$ which differs from (4.3.41) because the (A2.16) $\bar{x}(\alpha,y)^2$ term has not been substituted

This expression is simplified by substituting for L from (A2.9) and for $\bar{x}$ from (4.3.6).

$$\bar{x}_y = \frac{9b\ Py^2}{\mu^2} \cdot \frac{\mu}{3b\ \sigma^2\ (1-\alpha)}$$

or $\bar{x}_y = \dfrac{3Py^2}{(1-\alpha)\mu\sigma^2}$ which is the first-order condition (4.3.18)

To derive $\bar{x}_{\alpha y}$, totally differentiate (A2.10) with respect to y.

$$3\,\bar{x}^2\,\bar{x}_{\alpha y} + 6\,\bar{x}\,\bar{x}_\alpha\,\bar{x}_y = 27bPy^2\,L_\alpha$$

$$\text{or}\quad \bar{x}_{\alpha y} = \frac{9bPy^2L_\alpha - 2\,\bar{x}\,\bar{x}_\alpha\,\bar{x}_y}{\bar{x}^2} \qquad \text{(A2.17)}$$

Expression (A2.17) is simplified by substituting for L_α from (A2.6), $\bar{x}$ from (4.3.6), $\bar{x}_\alpha$ from (4.3.17), and $\bar{x}_y$ from (A2.16).

$$\bar{x}_{\alpha y} = \frac{\dfrac{9bPy^2\alpha}{(1-\alpha)^2} - 2\mu\left[\dfrac{\alpha\,b\,\sigma^2}{(1-\alpha)}\right]\left[\dfrac{3\,Py^2}{(1-\alpha)\mu\sigma^2}\right]}{\mu^2}$$

$$= \frac{9\alpha bPy^2 - 6\alpha bPy^2}{(1-\alpha)^2\,\mu^2}$$

$$\text{or}\quad \bar{x}_{\alpha y} = \frac{3\alpha bPy^2}{(1-\alpha)^2\,\mu^2} \qquad \text{(A2.18)}$$

Substitute (4.3.18) and (A2.18) into (A2.3):

$$H_{\alpha y} = \frac{(1-\alpha)3\alpha bPy^2}{(1-\alpha)^2 \mu^2} - \frac{3\ Py^2}{(1-\alpha)\mu\sigma^2}$$

$$= \frac{3\ \alpha\ b\ Py^2\ \sigma^2 - 3\ Py^2\ \mu}{(1-\alpha)^2\ \mu^2\ \sigma^2}$$

$$= 3Py^2 \left\{\frac{\alpha\ b\ \sigma^2 - \mu}{(1-\alpha)^2\ \mu^2\ \sigma^2}\right\} \qquad \text{(A2.19)}$$

After substitution for y using (4.3.46), (A2.19) becomes:

$$H_{\alpha y} = 3P\left\{\frac{(1-\alpha)\ \mu^2\ \sigma^2}{3P}\right\}^{2/3} \left\{\frac{\alpha b\sigma^2 - \mu}{(1-\alpha)\mu^2\ \sigma^2}\right\}$$

and $$H_{\alpha y}^2 = (3P)^2\left\{\frac{(1-\alpha)\mu^2\sigma^2}{3P}\right\}^{4/3} \left\{\frac{\alpha\ b\ \sigma^2 - \mu}{(1-\alpha)^2\ \mu^2\ \sigma^2}\right\}^2$$

or $$H_{\alpha y}^2 = \left\{\frac{3P}{(1-\alpha)\mu^2\ \sigma^2}\right\}^{2/3} (\alpha b\ \sigma^2 - \mu)^2 \qquad \text{(A.2.20)}$$

Using (A2.14) and (A2.15)

$$H_{\alpha\alpha}\ H_{yy} = \frac{-2(\alpha b\sigma^2)^2}{(1-\alpha)\ \mu} \quad \frac{-6Py}{\mu\sigma^2}$$

After substitution for y from (4.3.46),

$$H_{\alpha\alpha} H_{yy} = \left\{\frac{12\ (\alpha b\sigma^2)^2 P}{(1-\alpha)\ \mu^2\ \sigma^2}\right\} \left\{\frac{(1-\alpha)\ \mu^2\ \sigma^2}{3P}\right\}^{1/3}$$

$$= \left\{\frac{3P \cdot 4(\alpha b\sigma)^2}{(1-\alpha)\ \mu^2\ \sigma^2}\right\} \left\{\frac{(1-\alpha)\ \mu^2\ \sigma^2}{3P}\right\}^{1/3}$$

or

$$H_{\alpha\alpha} H_{yy} = 4(\alpha b\sigma^2)^2 \left\{\frac{3P}{(1-\alpha)\ \mu^2\ \sigma^2}\right\}^{2/3} \qquad \text{(A2.21)}$$

$H_{\alpha\alpha} H_{yy} - H_{\alpha y}^2$ is simplified using (A.21) and (A2.20)

$$H_{\alpha\alpha} H_{\alpha y} - H_{\alpha y}^2$$

$$= 4\ (\alpha b\sigma^2)^2 \left\{\frac{3P}{(1-\alpha)\ \mu^2\ \sigma^2}\right\}^{2/3} - \left\{\frac{3P}{(1-\alpha)\ \mu^2\ \sigma^2}\right\}^{2/3} [\alpha b\sigma^2 - \mu]^2$$

$$= \left\{\frac{3P}{(1-\alpha)\ \mu^2\ \sigma^2}\right\}^{2/3} \{4(\alpha b\sigma)^2 - [\alpha b\sigma^2 - \mu]^2\}$$

The first term is > 0. Therefore the third second-order condition is satisfied if

$$4(\alpha b\sigma^2)^2 - [\alpha b\sigma^2 - \mu]^2 > 0$$

Expansion of the expression results in:

$$4(\alpha b\sigma^2)^2 - (\alpha b\sigma^2)^2 + 2\,\alpha b\sigma^2\,\mu - \mu^2$$

$$= 3(\alpha b\sigma^2)^2 + 2\alpha b\sigma^2\,\mu - \mu^2 > 0$$

$$\text{if } 3(\alpha b\sigma^2)^2 + 2\alpha b\sigma^2\,\mu > \mu^2 \qquad \text{(A2.22)}$$

It is known that $\alpha > \frac{\mu}{3b\sigma^2}$ from (4.3.45) since $(1-\alpha)\ln(1-\alpha) < 0$.

$$\text{Then } 3\alpha b\sigma^2 > \mu \qquad \text{(A2.23)}$$

It follows from (A2.23) that

$$9(\alpha b\sigma^2)^2 > \mu^2$$

$$\text{or} \quad 3(\alpha b\sigma^2)^2 > \frac{\mu^2}{3} \qquad \text{(A2.24)}$$

It follows from (A2.23) that

$$\frac{2}{3} \cdot 3\,\alpha b\sigma^2\,\mu > \frac{2}{3}\,\mu^2$$

$$\text{or} \quad 2\,\alpha\, b\sigma^2\,\mu > \frac{2}{3}\,\mu^2 \qquad \text{(A2.25)}$$

Adding together (A2.24) and (A2.25):

$$3(\alpha b\sigma^2)^2 + 2\alpha b\sigma^2\,\mu > \frac{\mu^2}{3} + \frac{2\mu^2}{3} = \mu^2$$

Q.E.D.

APPENDIX 3

Second-Order Conditions to the Moral Hazard Problem

A sufficient condition for a maximization of the objective function is that the Hessian matrix of second-order partial derivatives is negative definite: Since the choice of β was eliminiated from the problem, the Hessian matrix is composed of the second-order partial derivatives of the objective function with respect to α and F.

The Hessian matrix = $\begin{bmatrix} H_{\alpha\alpha} & H_{\alpha F} \\ H_{\alpha F} & H_{FF} \end{bmatrix}$

is negative definite if

(1) $H_{\alpha\alpha} < 0$

(2) $H_{FF} < 0$

(3) $H_{\alpha\alpha}H_{FF} - H_{\alpha F}^{2} > 0$

Values for the second-order partial derivatives are derived from (5.3.12) and (5.3.14):

$$H_{\alpha\alpha} = 2\,F_{\alpha} + (\alpha-1)F_{\alpha\alpha} - b\sigma_{\varepsilon}^{2} \tag{A3.1}$$

$$H_{FF} = T''(F) \tag{A3.2}$$

$$H_{\alpha F} = -1 \tag{A3.3}$$

I. Proof that $H_{\alpha\alpha} < 0$

$H_{\alpha\alpha} < 0$ follows from (A3.1)

$F_{\alpha} < 0$ from (5.3.15)

$(\alpha-1) < 0$

$F_{\alpha\alpha} > 0$ from (5.3.16)

$b\ \sigma_{\varepsilon}^{2} > 0$

II. Proof that $H_{FF} < 0$

$T''(F) < 0$ by assumption

Q.E.D.

III. Proof that $H_{\alpha\alpha} H_{FF} - H_{\alpha F}^{2} > 0$

Expressions (A3.1), (A3.2) and (A3.3) are substituted.

To be shown:

$$[\ 2F_{\alpha} + (\alpha-1)F_{\alpha\alpha} - b\ \sigma_{\varepsilon}^{2}\ ]\ [T''(F)]\ - 1 > 0 \qquad \text{(A3.4)}$$

The following substitutions will be made to simplify (A3.4):

$F_{\alpha} = \frac{1}{T''} < 0$ from (5.3.15)

$F_{\alpha\alpha} = -\frac{T'''}{(T'')^{2}}\ F_{\alpha}$ from (5.3.16)

$$= -\frac{T'''}{(T'')^3} \text{ using (5.13.15)}$$

Then A3.4 becomes:

$$2\frac{T''}{T''} - (\alpha-1)\frac{T'''}{(T'')^2} - b\sigma_\varepsilon^2 T'' - 1$$

$$= 1 - (\alpha-1)\frac{T'''}{(T'')^2} - b\sigma_\varepsilon^2 T'' > 0$$

Since $(\alpha-1) < 0$
$T''' > 0$
$T'' < 0$

Q.E.D.