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AN INVESTIGATION INTO A TWO STAGE TANDEM QUEUE FOR AN OPTIMUM SAWMILL DESIGN

By

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ABSTRACT

The applicability of queueing theory as an operations research tool for modelling sawmills is described. The model selected is a two-stage tandem queue with two stations in the second stage. Each of the stations has general service times and finite buffer storage capacities which create the possibility of blocking pieces coming out of the first stage. A three-variable service time distribution is proposed to model sawmill machinery processing the pieces. This distribution creates the potential to functionally describe the sawmilling process, in contrast to the traditional method of using empirical distributions gathered at an existing sawmill.

The literature in tandem queues reveals the lack of work done and the degree of difficulty in this field of study. Analytical solutions do not exist for the queue system studied. Numerical approximation techniques were not used to model the queue system, but they have good potential for being utilized.

A simulation study was performed on the queue system. A computer program was written with the intention of obtaining results anticipated in a mathematical analysis. Two separate queue disciplines were studied: saturated and unsaturated first-stage queues. The unsaturated queue investigated the dependency of system performance on the arrival rates. It also examined the operation of the queue under different second-stage arrival intensities. The saturated queue analysis focused on the phenomenon of piece blocking in the first stage.

Some conclusions could be made from the simulation study with regards to design procedures for a sawmill. The study showed that in certain cases, improvement to system production by increasing a second-stage machine rate can be comparable to increasing the headrig rate. In addition, two stations in the second stage can complicate the analysis significantly. The simulation study also examined the possibility of utilizing tandem queue analysis to provide solutions for optimum second-stage buffer capacities.

The feasibility of modelling sawmills by tandem queues exists, but the designer must choose the appropriate analytical method to use. Numerical approximation techniques would likely prove to be the most successful method. Machine service times should also be studied in a sawmill to establish the statistical nature of the sawmilling process. This will improve the solutions provided by queueing theory analysis.

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Robert L. Zwick

INTRODUCTION

1.1 Queueing Theory and Sawmill Design

Management decisions in the sawmill industry can often involve a lot of financial resources - particularly in the design stages of the sawmill. A pivotal question that one encounters in a design project for a modern sawmill is, "What is the optimal sawmill design"? Not only is the designer concerned about constructing the best producing sawmill, but his senior company management are devoted to obtaining the best mill for the money available.

Almost all facets of a sawmill operation involve the uninterrupted processing of discrete pieces. The important local design characteristics for a processing station are that an adequate input of pieces are available, and that a smooth, uninterrupted output ensues. Simply restated, a machine center should be neither "starved" for incoming pieces, nor blocked or constrained on the output side. A good discussion of considerations for sawmill design is given by Williston.[33]

Queueing theory lends itself well to a quantitative solution of the previously stated design problems. Designers have applied this operations research (O.R.) technique to provide some of the answers to sawmill design problems. To date, most solutions to these queueing problems have involved discrete time-event simulation. Solutions by simulation have evolved quickly since many good simulation languages are readily available, and apply well to sawmill design. The engineer can first analyze the sawmill layout by computer to determine the adequacy of the design. The advantages of sawmill simulation have been well described by Aune.[2,3]

Simulation however does have several disadvantages. Primarily, it does not provide an optimum solution to the design. The designer must compare the simulation results of a particular design to alternative design simulation runs. He must vary the input parameters, to arrive at a best solution amongst all the trials. He must then make an empirical decision as to whether the design layout is suitable. This design method can be expensive, tedious and even a waste of time.

Sawmills are high-productivity industrial systems. The trend of higher machine processing speeds has not achieved its plateau. However, the problems associated with high speed equipment becomes apparent when one evaluates a sawmill system in its entirety.

Modern small-log sawmills allow little room for mistakes. An operating error of only a few seconds, with a modern machine running at 1.60 meters/sec. (5.33 ft./sec.) will cause logs on a conveyor to travel large distances. Even minor machine or personnel inefficiencies can result in a serious reduction in piece throughput. Bottlenecks and blocking of up-stream processors are the product of these inefficiencies. Therefore it is important to be able to predict potential problems in the mill design before the sawmill is built.

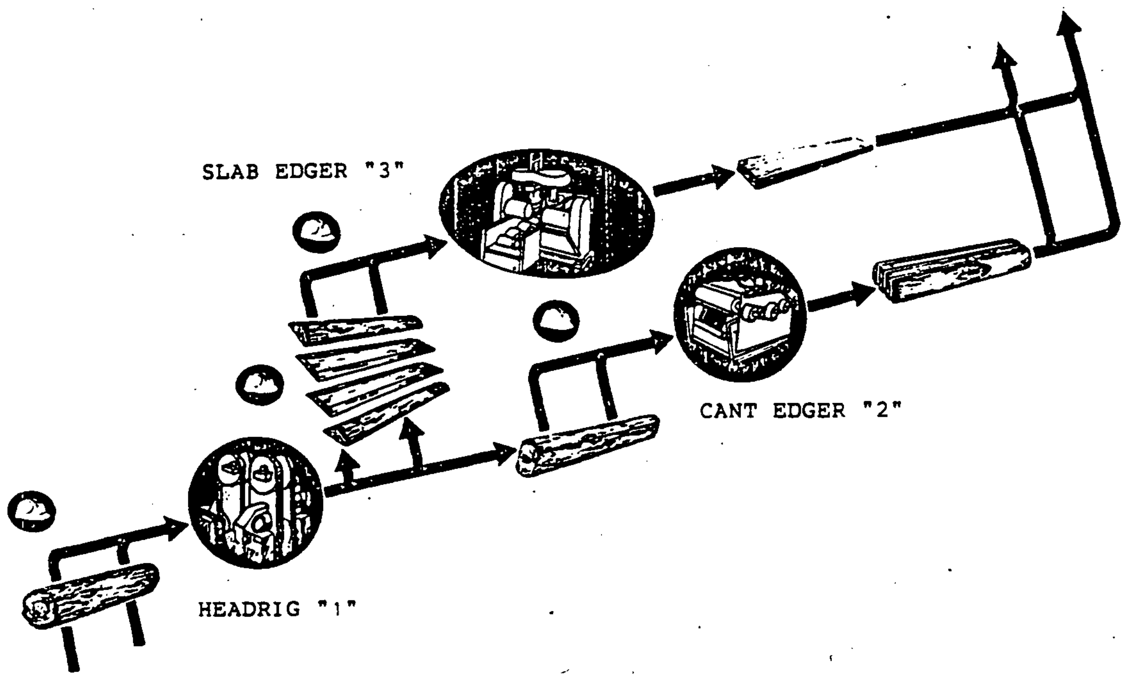
Interest of sawmill designers has consequently focused upon analytical solutions to these queueing problems. For example, a computer program has been written by Carino and Bowyer[10] to take into account many sawmill design layouts, using queueing theory to provide the analytical solutions. They also incorporated the direct search method algorithm (a nonlinear program) to arrive at the optimum solutions for various design parameters to the queue system.

1.2 Tandem Queues and the Model

The sawmill designer is generally concerned with tandem queues. In a tandem queue, the output of the first queueing stage provides input directly into a second stage. Tandem queueing theory is relatively new, with the bulk of the literature starting in the mid-70's. Some of the simpler tandem queue disciplines can be solved mathematically. However, they often are of no use in modelling the system because of the simplistic assumptions used to describe the tandem queue. Unfortunately, increased complexity of the tandem queue configuration often creates limits to the tractable solution. The problem can become mathematically difficult to solve. Sometimes the more general or useful problems have only be solved by numerical approximation.

The purpose of this study is to examine the potential use of queueing theory to solve one aspect of a sawmill design layout, which could prove useful to the sawmilling industry. The tandem queue system considered (refer to Fig. 1) represents one of the small-log sawmill layouts discussed by Aune and Lefebvre.[1] This particular sawmill layout also represents a queue system that had not been analyzed by Carino and Bowyer.[8,9]

Fig. 1. Sawmill System Investigated



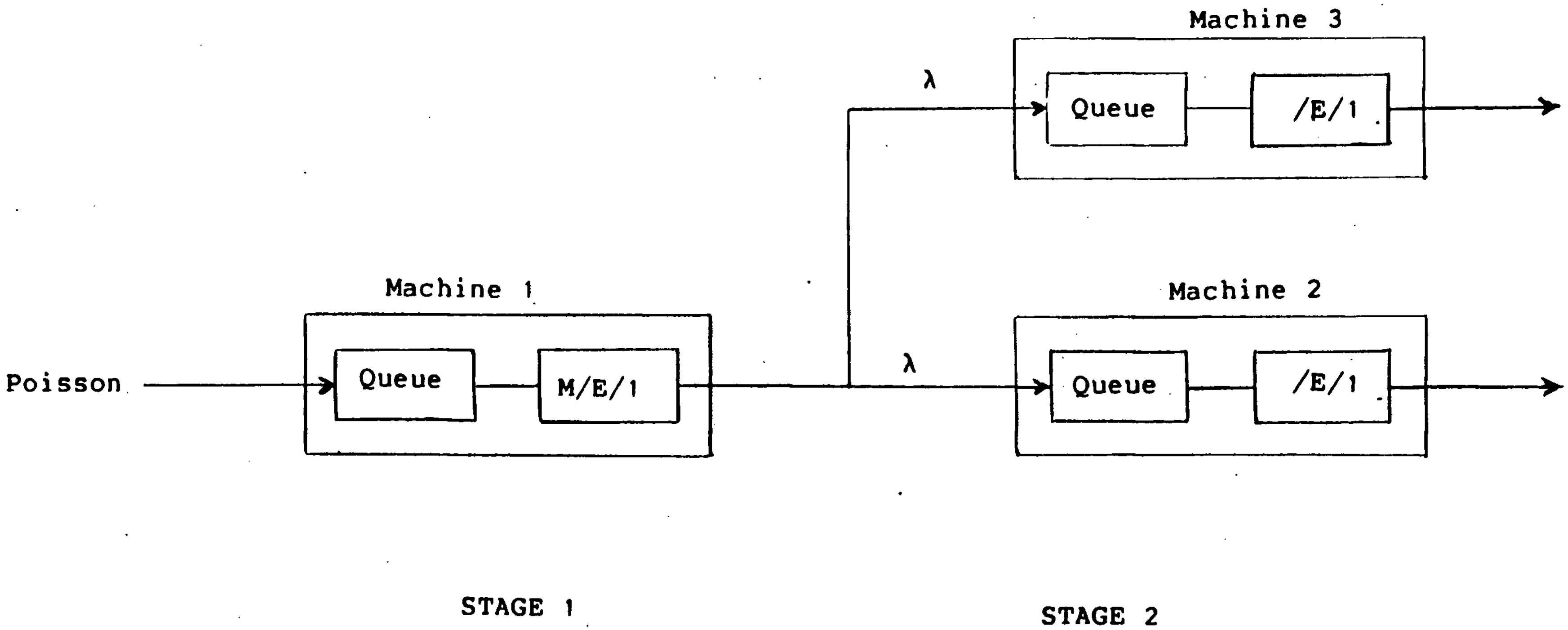
Source: Aune and Lefebvre,[1] p 15.

The sawmilling system in Figure 1 depicts a headrig "1" with a cant edger "2" directly downstream, and an off-stream reman edger "3" handling the slabs. There is a queue (of sawlogs in this case) before the headrig. The logs are processed, with the output (of cants) directly becoming the input into the cant edger. A proportion of the headrig output is in the form of slabs, which are redirected to a reman edger. There are queues before both edgers, and each have the possibility of blocking the headrig output should either of the edger buffer zones become filled. In queueing theory parlance, we have a two-stage tandem system. The first stage is a single station and consists of a single storage buffer and a single server. The second stage consists of two stations, and each station has a single storage buffer and server (refer to Fig. 2). A brief explanation of queueing theory is given in section 1.3 and of queueing theory notation in section 1.5.

The merits of examining this specific queue system are multifold:

1. This queue system is very common in small-log sawmills.
2. The queueing analysis may be solved by using some of the recently introduced methods of obtaining tandem queue solutions.
3. The sawmill layout may continue to be used in future sawmill designs.

Fig. 2. Schematic Representation of Queue System



1.3 A Brief Explanation of Queueing Theory

Queueing theory is the mathematical study of "queues" or waiting lines. There are three aspects that are characteristic of a queue system. They are the arrival process, the waiting line (and its "discipline"), and the service distribution. Each of these points will be briefly touched upon.

The customers arrive to be serviced. The arrival process into the queue is generally a random process and usually exhibits some statistical distribution. The customers can come from either an infinite or a finite population source. An infinite assumption makes the mathematical analysis simpler and is often used for a large population source. The queueing analysis is usually concerned with the distribution of the interarrival time. The time interval between each arrival is a random variable and usually conforms to a specific statistical distribution.

The queue contains customers waiting to be serviced. The queue can have a finite or infinite capacity. A finite capacity can result in customers being refused entry into the line. It also means a more complicated analysis of the queue.

The waiting line is characterized by a discipline that the customer exhibits. Customers can present pecularities such as balking or waiting in line for a certain time and then leaving. All of the queue irregularities have to be written into the analysis if accurate modelling is desired.

The waiting customer is then serviced. A service mechanism consists of one or more service facilities, each of which contains one or more service channels, called servers. Servers can be parallel or in series (tandem) with each other. The time elapsed from the commencement to completion of service is called the service time. The service time is usually a random variable and follows a specific statistical distribution. For queueing analysis, the service-time probability distribution for each server must be specified. To complicate the analysis, the service rate can be dependent upon the amount of customers in the queue or by possibly some other influencing factors.

In a sawmill, the machine centers and their associated conveyor equipment are regarded as servers. The customers are the logs, cants, boards, etc., that are to be processed by the machines. These pieces queue up before the machines in storage buffers or transfer chains. Therefore almost any aspect of a sawmilling environment can be modelled by queueing theory. Figure 3 depicts a simple queue mechanism.

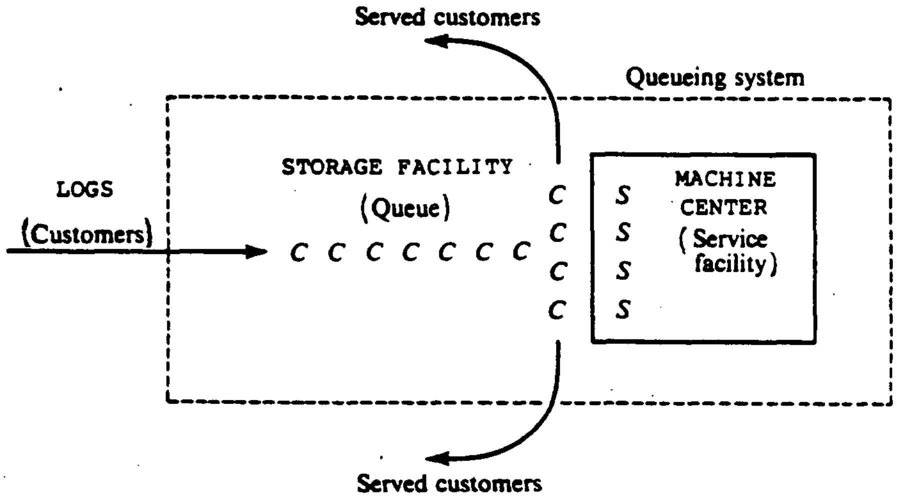


Fig. 3. A Simple Queue Mechanism

Source: Hillier and Lieberman,[19] p 404.

1.4 The Appeal of Analytical Models

Gershwin and Berman's[16] analysis of a tandem queue highlights many of the exciting possibilities for mathematical solutions to tandem queuing problems. Their paper explores in detail the operating characteristics of two unreliable machines in tandem with random processing times and finite storages before each machine (refer to Fig. 4).

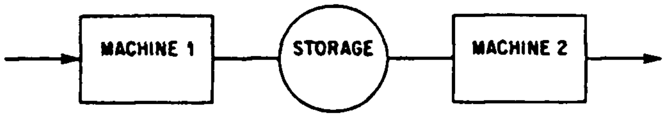


Fig. 4. Tandem Queue Studied by Gershwin and Berman

Source: Gershwin and Berman,[16] p 1.

Gershwin and Berman provide an extensive literature review on the topic of tandem queues. In addition, their results show that if a tandem system can be mathematically modelled, many system features of interest to the designer can be examined. Quantitative decisions can be made on design problems such as buffer sizes, machine production rates and machine reliability. Some of their results are shown in Figures 5 and 6.

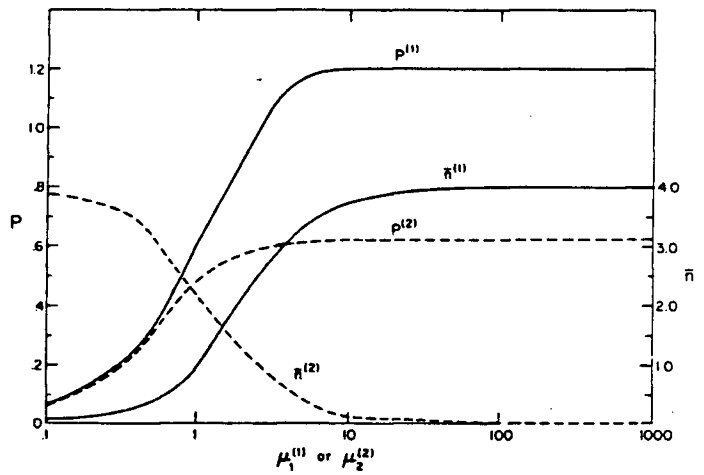


Fig. 5. **Effect of machine speed on production rate and average in-process inventory.**

Source: Gershwin and Berman,[16] p 8.

Figure 5 shows what effect varying machine 1 and 2 operating speeds has on system production and in-process inventory. The abscissa parameters "μ_1" and "μ_2" represent machine 1 and 2 operating speeds respectively. The bracketed number in superscript indicates the graph number. The left ordinate is system production "p" and the right is in-process inventory "$\bar{n}$" for the finite storage buffer between the two machines.

Graphs 1 and 2 display the effect of varying the speed of machine 1 and 2 respectively, on the system characteristics production and in-process inventory. The curves represent non-linear functional relationships between the parameters. Results of a simulation study of this system would not reveal a continuous line. Instead, a simulation study can only generate discrete data points. A regression line can then be drawn through the points to show the functional relationship. This is what was done for the simulation studies in chapters 3 and 4.

Figure 6 shows the effect of varying the storage size "N" on p and $\bar{n}$. Once again, non-linear functions are exhibited. These graphs imply that if a system can be modelled analytically, various system characteristics can be quickly determined for different operating ranges. Optimum or "threshold" solutions can also be predicted beyond which further improvements in the system may not substantially increase the desirable returns from an operating characteristic. This mathematical analysis could assist the sawmill designer immensely.

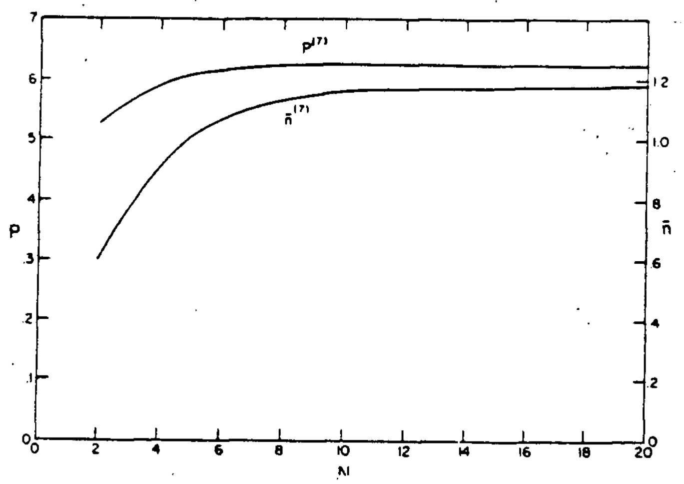


Fig. 6. Effect of storage size on production rate and average in-process process inventory...

Source: Gershwin and Berman,[16] p 9.

In addition, queue systems exhibit a classic operations research dilemma. An increase in the number, or the competency of the server generally increases capital or operating costs. However a service mechanism that is over-utilized can result in dissatisfied customers, excessive waiting lines, system bottlenecks, etc., which also burdens management with operating costs. A conflict exists and an optimimum trade-off can be analytically determined for the system. Arriving at optimum cost solutions is one of the most desirable results from queueing theory.

Figure 7 describes how optimum cost solutions are determined. The expected cost of service E(SC) increases with increasing level of service. The level of service can be defined as the rate, quality or number of servers found at a queueing station. Conversely, the expected cost of waiting E(WC) decreases with an increase in the level of service. If the two graphs are summed, the total expected cost E(TC) is obtained. This curve represents a concave function, with the lowest point (the lowest cost) being the optimum cost solution.

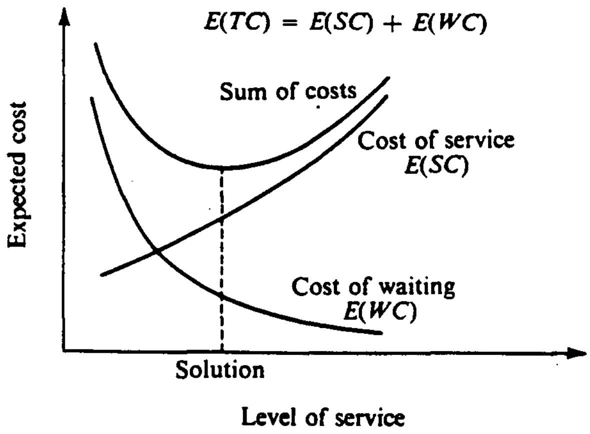


Fig. 7. Queueing Theory and Optimum Cost Decisions

Source: Hillier and Lieberman,[19] p 465.

1.5 Queueing Theory Notation

There is a shorthand notation commonly used to describe a queue system. Listed below is a brief outline of the notation. The symbols and notation used in the thesis are summarized in Appendix A.

M = a "Memoryless" or Exponential distribution

G = a General statistical distribution

GI = General Independent distribution (usually used to describe an arrival process)

E_k = a k-Erlang distribution

D = a "Degenerate" (constant) distribution

N = the finite number of customers allowed in a waiting line (queue capacity)

c = the number of parallel servers in a queue mechanism

These symbols are used to describe the queue system and are written in the form:

interarrival dist./service dist./no. of servers/queue capacity

The following examples illustrate how queue systems are described:

M/M/1 is an exponential interarrival process/exponential service time/with one server

GI/D/c/N is a general independent interarrival process/degenerate service times/c servers/a queue capacity of N customers

M/G/1->/G/2 are two queues in tandem. The first queue has exponential interarrival time/general service time/one server. The output process from the first queue provides the input process to the second-stage tandem queue. The second queue also has a general service distribution with two servers. Figure 2 is a diagram of a tandem queue with characteristics indicated by this same form of notation. The servers have Erlang service time distributions in the diagram.

We can now focus on the description and the approach to the solution of this queue discipline.

THE QUEUE DISCIPLINE

2.1 The Assumptions of the Queue System

The first crucial step in a queueing analysis is to establish certain assumptions about the system. As more intricate descriptions of a system are included in the analysis, the more difficult it becomes to mathematically solve the system. However, if the assumptions are too simplistic, the system cannot be modelled realistically.

This chapter lists the assumptions used in this queueing analysis and the rationale for the selection. The intent is to obtain tractable solutions and still have the potential for modelling a sawmill sytem. The analyst must examine the operation of the system and incorporate the important characteristics into his model. He must also be aware of the limits of analytical methods available in queueing theory.

2.2 The Arrival Process

Two cases are considered for the sawlog input, representing two common situations in a sawmill. These cases represent two important distinctions in the method of analysis for the tandem queue studied.

In the first case, debarked logs are fed directly from a full log pit or transfer chain into the headrig, the stage-one processing machine. In this case, the first queue can be considered as saturated. There is always a customer available for service. Arrivals into this queue consistently keep the buffer zone in the first stage full. Consequently, the departure process from the first station is independent of the arrivals into the first stage queue. The service time distribution of the leaving customer becomes the interarrival time distribution into the second-stage queue system.

In the second case, the arrival into the first queue is a random process, depending for example, on the outfeed from the barker. Here the queue system is unsaturated (with the assumption that steady-state conditions exist). The unsaturated queue in the first stage results in dependent departure times from the first-stage server. The departure times depend upon the arrival process into the first stage, which represents a tandem queue situation that is more useful in modelling sawmills.

In both cases, the first-stage arrival process is represented by a Poisson distribution that is stationary and homogenous. A tandem queue that is saturated in the first stage is insensitive to what arrival distribution is chosen but for simplicity, the same distribution is used.

Poisson arrivals will provide exponential distribution interarrival times into the queue. The Poisson assumption is frequently used in queueing analysis because it allows for convenient mathematics. In addition, general literature on queueing theory support that a Poisson arrival distribution is in many cases, a realistic assumption.

There are three basic assumptions to justify the Poisson distribution as the arrival process under investigation:

(1) The expected time for a Poisson event is constant at μ time units per event.

(2) Pr {one event in the time interval $x, x+\Delta x$} = $\Delta x \cdot \mu$

(3) Pr {more than one event occurs in $x, x+\Delta x$} = $0(\Delta x)$

The variable μ is the scale parameter used in the Poisson and exponential distribution.

The above statements are very general and could be verified for numerous situations in a sawmill environment. Therefore a Poisson arrival process is a good arrival distribution for the model.

2.3 Additional Assumptions

(a) The sawlogs entering the queue represent blocked customers delayed. This assumes that the sawlogs do not leave the queue once they arrive.

(b) The output process from the first stage provides the input process into the second stage.

(c) The two second-stage buffer storages before machines 2 and 3 have finite capacity.

(d) Breakdown and repair rates of the machines are not considered.

(e) The queueing system is considered to be an open network. Upon completion of the service, the customers leave the system immediately.

(f) The condition of blocking exists when either of the two second-stage buffers are full. This results in a shut down of the first-stage machine.

2.4 Customer Types

In sawmills, there often is an increase in the number of pieces on the output side of a queue system, compared to the input side. In the tandem queue studied, individual logs enter the first stage, but there becomes two types of "customers" in the second stage: slabs and cants. Each of these customer types enter separate second-stage queues.

The second-stage arrival process is similar for both customers. However, the respective machine centers can have different service rates for the two customer types. In addition, the two second-stage storage facilities may have different capacities for the two types of customers.

In order to apply queueing theory to the system, operating parameters are selected to reflect a sawmill scenario. Often the edger processing the cants operates at a speed similar to the speed of the headrig, with the buffer capacity being quite small. However the slab-processing edger may operate at speeds faster than the headrig, and generally has a very large buffer capacity. In addition, the slab edger is usually off-stream to the headrig. The slabs which are conveyed on the transfer chains to the slab edger are often slower than for example, the log conveyor speeds for the headrig.

For ease of analysis, the slabs exiting the first stage headrig are considered to be one "customer", even though there may be as many as four slabs, as in the case of a "quad" bandmill. It is assumed that the slabs arrive in one batch to the second-stage reman edger. Therefore the calculation of piece throughput in the total system should reflect the average number of pieces found in a batch.

In the case of the cant-processing edger, only one cant enters the edger at one time, but several boards usually exit. The calculation of piece output must consider the average number of boards created from the cants.

The average number of slabs that come from a specific type of headrig, with a given log diet, can be determined by existing computer programs which determine optimum cutting solutions. This is also applicable for determining the average number of boards created by the cant edger. Total system production can then be calculated, even though each processor handles individual customers.

2.5 Service Time Distributions

2.5.1 The Distribution Assumption

In this study, a difficult challenge was the determination of service time distributions for each of the machines. In existing sawmills, statistics can be obtained, an inference made on the type of distribution and an estimation of the distribution parameters performed. For a theoretical sawmill being designed by analytical methods, this approach is generally not feasible. An assumption for the service time distribution is required.

Simulation studies do not require a distribution assumption. Field data can be gathered at the sawmill to obtain an empirical distribution. The simulation program can then randomly sample from the empirical distribution.

The service time distribution can be subjective to the theoretician in the case of modelling a sawmill. Statistics are often available to the designer on the characteristics of the logs being consumed in the small-log sawmill. Since machine service times are directly related to the length of the log, a frequency distribution of the log lengths should adequately represent the frequency distribution of the processor service times.

It is not the intent of this study to establish or confirm the relationship between machine processing times and the log length frequency distribution. The designer should also be aware that many other factors come into play in an operating environment. These include operator errors, piece-loading machine variability, time delays, etc. They all contribute to the random nature of service (and therefore second-stage arrivals). These contributing factors also affect the service distribution and can reduce the accuracy of the analysis if ignored. With little other information available on machine service times, a log-length frequency distribution is a reasonable estimate of the log-processing time distibution. The total time it takes to serve a customer will consider the other contributing items mentioned above. These factors are often unique to a particular machine center.

2.5.2 The k-Erlang Distribution

The k-Erlang distribution (refer to Fig. 8) has been selected to describe the log length distribution. For a full description of this distribution, see Appendix D. The reasons for its selection are:

-It is an extremely flexible distribution that has a scale parameter "λ" and a shape parameter "k".

-The random variables are always positive.

-Estimation of the parameters uses established statistical methods.

-A k-Erlang random variable realization is the sum of k independent exponential random variables. The result is that it often allows mathematically tractable solutions for queueing problems. This is because a service facility with a k-Erlang service time can be considered as being k exponential servers working in series. Good examples of queueing solutions for

Erlangian service times are found in Jackson[20] or Heyman and Sobel.[18]

-The random variables are reasonably easy to generate for simulation studies; however it may require extra computer time. The generation of a k-Erlang realization as the sum of "k" exponential realizations is not efficient for large values of k.

Fig. 8: k-Erlang Distribution With Various Shape and Scale Parameters

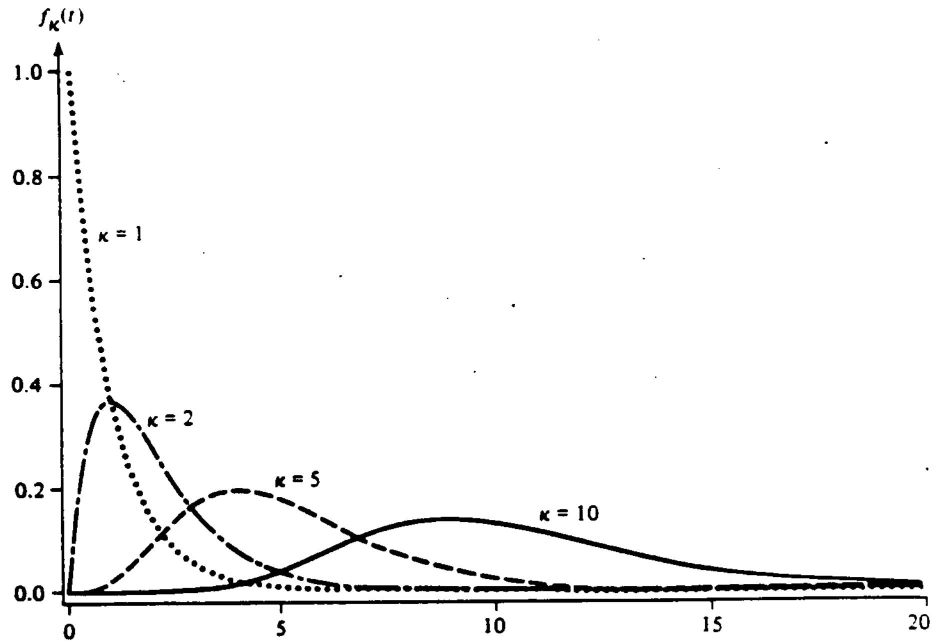


Fig. 8-A. $f_\kappa(t)$ with $\lambda = 1$ and $\kappa = 1, 2, 5, 10$.

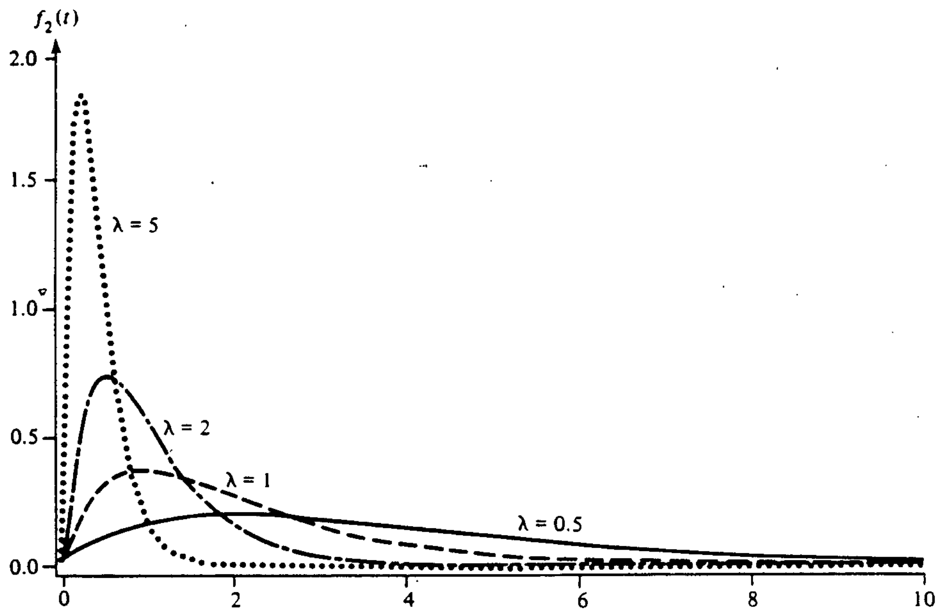


Fig. 8-B. $f_\kappa(t)$ with $\kappa = 2$ and $\lambda = \frac{1}{2}, 1, 2, 5$.

Source: Heyman and Sobel[18], pp 514- 515.

2.5.3 The Total Service Time

The random variable associated with the total service time can be composed of three separate variables. One is the random time to process the log entering the machine, which will be called "L". This random variable is fit with a k-Erlang distribution (refer to App. D). The distribution parameters are estimated and a Kolmogorov-Smirnov goodness-of-fit test is performed (refer to App. E). Data for the log-length frequency distribution came from Dobie.[13] The goodness-of-fit test was accepted in two out of three sets of data obtained from the report.

The second variable "D" is a displacement parameter. There is a minimum processing time associated with the minimum log length and also the time it takes for a log to traverse a given transfer chain. The parameter D simply represents a constant value.

The third random variable "S" is the random nature of "setting up" the log to be processed. This variable can be a result of machine equipment, conveyance of the pieces to the stage-two machines and the operator inefficiencies. For the queueing analysis in this thesis, the description of this variable can be a highly subjective topic, with no available information to make a distributional assumption.

The variable D can be regarded as a displacement parameter with the k-Erlang log distribution mentioned above. The log-length frequency distribution is therefore considered as a 3-parameter k-Erlang model. This statistical model was used for the estimation of the log processing time distribution.

An alternative method of introducing a displaced service time is by adding in series, another k-Erlang server where $k \to \infty$. As k becomes very large, with λ becoming very small (which provides a constant k-Erlang mean), the distribution approaches a vertical line. The displacement parameter can be introduced to wherever the vertical line is required. Jackson[20] outlines a good practical solution to a M/G/1 queue with a time-displaced Erlang server.

The random variable S is unique to the machine and its associated equipment. An exponential distribution is assumed in this study. From my own practical experience, I feel the exponential distribution is a good assumption for this random variable. The values of the distribution scale parameters assumed in the simulation study are "educated estimates". The headrig and slab edgers generally have long set-up times. Conversely, the cant edger has a very short set-up time.

In summation, there are three variables associated with the total service time "T":

$$D + L + S = T \qquad \text{(another random variable)}$$

The above assumptions for a service distribution seem reasonable. They allow a tractable analysis of the queue and also permit reasonable simulation work.

ANALYSIS OF AN UNSATURATED QUEUE

3.1 The Theoretical Analysis

A literature review was made to determine the available methods for analysis of the studied tandem queue. The assumptions outlined in the previous chapter provided a basis for locating the relevent papers. In addition, the listed assumptions were what I felt, necessary to adequately model a sawmill system. A theoretical analysis was not performed because the assumptions that were used would result in analytical work that was beyound the scope of this study. The relevent analytical methods for modelling sawmills, that are available in queueing theory, is given in this section.

3.1.1 The Departure Process of a Queue

The M/G/1->/G/1 queue which is not saturated can be used to model a wide variety of situations. Unfortunately the mathematical solution to this problem is difficult. Judging from the literature review, there has been a great deal of interest in recent years among applied mathematicians to research this important area.

The problem arises from the departure process of the first queue. The departure times for this queue arrangement in most applicable situations, are non-renewal processes (refer to App. C for a discussion on renewal processes). Early work by Burke[6] showed that the departure epochs from an M/M/1 queue form a Poisson process with a departure rate that is identical to the arrival rate. Therefore tandem M/M/1 queues can be analyzed by evaluating each queue individually.

This case is not true for the M/G/1/N queue. The specific conditions when the queue output is recurrent (a renewal process) is detailed by Disney et al.[12] These conditions exist when $N=1$ and $G=D$ or when $N=\infty$ and $G=M$. The output flow is Poisson only in the case $N=\infty$ and $G=M$. The applicability of analyzing the proposed sawmill system is limited if one is constrained by these conditions.

Berman and Wescott[4] have recently outlined certain conditions for the GI/G/c ($1 \leq c < \infty$) queue having a renewal departure process. They show that a necessary condition for the departure process to be a renewal process is if its interval distribution (the time spent in the queue system) is the same as that of the arrival process.

Approaches for evaluating general service time/tandem queue problems have lately been focused on numerical approximations. In addition, simulation studies have recently been performed to define the properties and behaviour of the unsaturated queue. Shimshak and Spichas[32] carried out an extensive study of the M/E/1->M/1 queue, but the dependency of the departure process on the arrival process in the first stage was not considered. The arrivals into the second stage were approximated as a general independent distribution. An equation for the numerical approximation of the tandem queue was derived. The numerical approximation was then compared to the simulation results. Their simulation study took into consideration the dependency of second-stage arrivals on the first-stage arrival process.

Several important conclusions were obtained from Shimshak and Spichas which can directly affect the works done by Carino and Bowyer.[8,9,10] It was shown that the intensity of the arrival rate into the first queue significantly affects the assumption of independent arrivals. Shimshak and Spichas define the first-stage traffic intensity "ρ_1" as:

$$\rho_1 = \eta \cdot \tau$$

where η = mean arrival rate into the first stage

τ = mean service time in the first stage

The intensity of the second-stage queue "ρ_2" is:

$$\rho_2 = \eta \cdot \gamma$$

where γ = mean service time in the second stage

The study results showed that their numerical approximation was accurate only at low traffic intensity for both stages.

Accuracy of the tandem queue approximations were especially sensitive to high intensity in the second stage. Errors of up to 40 percent can be made under the independence assumption, with high-traffic intensity values.

Another influencing factor to the assumption of independent second-stage arrivals is the k value used for the k-Erlang service distribution in the first stage. Shimshak and Spichas' comparison between numerical approximation and simulation shows that a k value greater than five will cause at least a 20 percent difference between the two methods.

Therefore a low-variance k-Erlang distribution or high-intensity second-stage traffic can seriously affect the numerical approximation results. The assumption of independent arrivals into the second stage becomes no longer valid. A dependency exists with the arrival process and the covariance must be considered.

These conclusions are significant in view of the results of Rosenshine and Chandra[28] (and subsequently used in Carino and Bowyer's computer program). Both pairs of authors used a numerical approximation method for evaluating tandem queues with general service times. The departure process from the first stage was assumed to be independent of the first-stage arrival process. Therefore the user of the DSMIN program package must be wary of the limitations of this program, owing to its simplified distribution assumptions.

The covariance structure of the M/E/1 queue departure process has been studied. Jenkins[21] determined the joint distribution between departing customers from a stationary queue system (no transient conditions) with k-Erlang service times. His results gave a description of the correlation between traffic intensity and the dependence of the departure process. Jenkins worked with the first two moments of the k-Erlang distribution and did not provide results for the case of a general distribution. Jenkins also verified Burke's work by showing that exponential service times (a k-Erlang distribution with k=1) result in an independent departure process.

King[23] studied the departure process of an M/G/1 queue. His results gave an expression for the covariance structure of the departure process from any M/G/1 queue. His conclusions on the case of renewal departure processes are in agreement with Disney et al.[12]

3.1.2 Tandem Queues with Correlated Service Times

An important assumption in the queueing analysis assumes independence of the arrival and service distributions. As mentioned previously, this study considers the service distribution to be dependent upon the log-length frequency distribution. The second-stage interarrival distribution (for both tandem queue cases studied), is similar to the first-stage service distribution. Therefore a correlation exists between the second-stage interarrival distribution and the second-stage service distribution.

As a result, the second-stage service and interarrival distributions are not independent. A major assumption used in queueing analysis is violated, where it is assumed the service and arrival distributions are independent of each other. The mathematical analysis of this queue system becomes an increasingly difficult task with the further assumption of correlated service times.

Reports on tandem queues with similar service times at each stage have been recently appearing in technical journals. Boxma[5] began the investigation of tandem queues with identical general service times at each station. He obtained asymptotic and numerical results for the sojourn times (time spent in a queue system) and the actual and virtual waiting times at the second stage. This is a M/G/1->/G/1 queue with highly correlated service times.

Boxma's results are relevant because he encounters a problem that is similar to this sawmill analysis. Because his study was in message-communicating switching networks, Boxma's efforts were directed at finding solutions to some tandem queue characteristics that were different to the interests of a sawmill designer. Nonetheless, his results on sojourn time can be used for determining production of tandem queue systems, as envisioned in this present study. In addition, Boxma's results could be applicable to both the unsaturated and saturated cases discussed in this report. The approach of modelling sawmills by numerical methods may be possible, even with the existence of highly correlated service times.

Boxma concluded that the means and variances of the sojourn time in the second stage are smaller when the first stage is under heavy traffic, compared to an ordinary M/G/1 system with the same traffic intensity and service time distribution. This is an interesting conclusion for computer network applications.

Pinedo and Wolff[27] made a comparison between tandem queues with dependent (correlated) and independent service times. The queue system investigated was a M/M/1->M/1 queue. A simulation study was made for the case of correlated service times where both servers had equal service times. The simulation results were then compared with a queueing analysis that had an independent service time assumption.

Pinedo and Wolff showed that correlated service times affect the determination of waiting times in a queue system. They attribute the waiting time discrepency between dependent and independent service times by a "length biasing effect". This is where it is more likely that the customer with a long stage-one service time will have in addition, a long stage-two service time. Traffic intensity also influences the waiting time of the simulation-studied queue. In addition, the results of Pinedo and Wolff were in agreement to Boxma's method of determining waiting times for correlated-service-time tandem queues.

3.2 Simulation Analysis of the Queue System

The simulation part of this chapter models the queue system with the assumptions outlined in chapter 2. This simulation study provides solutions of the queue system in a similar fashion to what one would expect analytical solutions would provide. This section also investigates some of the important sawmill design characteristics one would look for, to be provided by a queueing analysis.

3.2.1 The Queue Assumptions for the Simulation Study

As was shown in Figure 2, the model is a simple two-stage tandem queue. The first stage has a single server and a finite storage buffer. The second stage has two stations, each with a finite buffer and server. The following queue disciplines for this simulation model are considered:

(a) The piece (sawlog) input rate into the first stage is represented by a Poisson distribution which is stationary and homogenous. This will provide an exponential distribution for the interarrival time into the first queue.

(b) To ensure the queue in the first stage remains unsaturated, the traffic intensity is always less than one. This implies the mean service time is less than the mean interarrival time.

(c) The sawlogs upon entering the first stage do not leave the queue.

(d) The machine processor at each station serves the customer according to the statistical distribution described in section 3.2.2.

(e) If either buffer in the second stage is full, the machine in the first stage shuts down. The first stage becomes blocked.

(f) If the buffer in the first stage is full, the generation of piece arrivals stops until some unused buffer capacity is restored. This ensures that an infinite queue will not exist at the first stage. Thus a "safety" switch is provided for the simulation model to ensure that the allotted memory for the program is not used up.

(g) Two new pieces are created at the first-stage machine. One piece always continues on directly to machine 2 (the cant edger) in the second stage. The other piece is directed to machine 3 (the slab edger) in stage two. There is usually more than one piece created by the headrig and directed toward machine 3 in real life. However, it can be assumed that these pieces arrive at the slab edger as one batch.

(h) The time to convey a piece to the second-stage facility is not considered, except in the "set-up" exponential random variable of the service distribution.

(i) The boards do not leave the second stage without passing through a machine station.

(j) The length of a log is generated from a k-Erlang distribution. The log length (and therefore the service processing time) remains unchanged through the entire process. However, the set-up time of a piece at the three machines are different and therefore are individually generated from an exponential distribution.

(k) Upon exiting the second stage, the pieces leave the system.

3.2.2 The Service Time Random Variable

In accordance with a real-life situation, the processing time of a particular log is used for all three machines. The same k-Erlang random variable is used for the piece in each machine

The set-up time is a random variable independent of the customer. Each machine is provided with its own exponential set-up time distribution and respective scale parameter. The psuedo-random number generator used to sample from the exponential distribution employs a different "seed" than the k-Erlang distribution. In addition, the simulation model uses antithetic variates (refer to App. B) to reduce the variance of the simulation estimates.

In the simulation study, k-Erlang random variables are generated. The random variable is derived from a log-length frequency distribution. Refer to section 2.5.3 for a review of the service time distribution. The generation of k-Erlang random variables for simulation is outlined by Bury.[7]

The exponential random number is generated in the simulation model from the following equation:

$$X_i = \mu \cdot (-\log(1-u_i)) \qquad (1)$$

where u = the i'th random number sampled from the uniform distribution.

μ = the mean of the exponential distribution

The simulation program generates a pseudo-random number sequence of uniform random variables. Schriber[31] details how this is done for the simulation model.

A k-Erlang realization is the sum of "k" exponential random variables:

$$X_E = \sum_{i=1}^{k} X_i$$

Once the k-Erlang random variable is realized, the processing time is then calculated. If the feed speed of the machine is "s", the processing time L is:

$$L = X_E/s \qquad \text{(a random variable)}$$

The variable "m" is the minimum length log allowed in the sawmill. Therefore the displacement time D is:

$$D = m/s$$

Authentic values are assigned to the above variables for realism to the simulation model. For instance, assume the following values:

$$m = 2.5 \text{ meters (8 ft.)}$$

$$X_E = 3 \text{ meters (10 ft.)}$$

$$s = 1.0 \text{ meters/sec. (3.1 ft./sec.)}$$

Then $$D + L = (2.5 + 3)/1.0$$
$$= 5.5 \text{ seconds}$$

The other random variable associated with the total service time is the set-up time. As mentioned previously, the assumption taken is that set-up time is exponentially distributed and independent of the log length. It is a variable dependent upon the type of machine processing the log. Exponential random variables are generated as outlined in equation (1) of this section. Suppose the mean set-up time "μ" is 3 seconds. If an exponential realization of the mean is made, we can now calculate the total service time "T" for this log. It is the sum of the mean set-up time and the processing time:

$$T = D + L + S$$
$$= 2.5 + 3.0 + 3.0$$
$$= 8.8 \text{ seconds}$$

3.3 Discussion of the Simulation Model

The simulation model was written in the General Purpose Simulation System (GPSS) language. This language is extremely flexible and has a short learning time. The computing center at UBC has a GPSS-H compiler, which is the most up to date version. It allows for improved interaction amongst other compilers, the programmer and files or devices over the earlier version, GPSS-V. There is also an improvement in controlling or reiterating runs. Finally, the compiler is about five times faster than the previous version; therefore the simulation costs are markedly reduced. It is a new language that should seriously be considered by the person interested in simulating queue systems.

The selection of values for the operating parameters in the studied queue system is subjective, so the simulation program written allowed considerable flexibility. The approach taken was to first establish what characteristics of the queue system would be of interest for the mathematical analysis. Important characteristics to the sawmill design engineer were also considered. Once these were established, values for the parameters were chosen to best reflect the actual system operation. Realistic operating values found in a sawmill were of primary importance in assigning numbers to the parameters.

The simulation of an unsaturated queue was primarily intended to exhibit the dependency of the second-stage queue process on the arrival rate in the first stage. For a listing of the simulation program and all the parameters selected, one should refer to the appropriate Appendix (F-H) associated with the simulation runs that will be discussed in the following sections.

3.3.1 Production vs. Interarrival Time

The intention of the first simulation run was to observe the dependency of system production on the first-stage arrival rate. System production is the total number of pieces to have been through either second-stage machine in an hour. The program is described in Appendix F and computer-generated graphs were drawn to make more effective use of the simulation results (refer to fig. 9).

The first-stage intensity "ρ_1" is always less than one. This ensures that the first stage queue remains unsaturated. The arriving logs may find no queue, or even machine 1 not busy. Therefore the departure time from the first stage is not only the service time, but can include part of the first-stage interarrival time as well. One can consider the investigation in this section as examining the effect of headrig idle time on system production.

The assignment of values to the multitude of system variables is a subjective task. An example of how this was done is shown below so the reader can familiarize himself with the equations and variables (refer to secton 3.2.2 for a description of the service time distribution. For the simulation runs in subsequent sections, one can simply refer to the proper Appendix for the parameters and the respective values used in the computer program.

(a) Machine 1 Total Service Time

Let machine 1 operate at a feed speed s_1 of 1.0 m/sec.
machine 1 mean set-up time μ_1 be 2 seconds
mean log length be 2.8 meters
minimum log length be 2.5 meters

Then $$T_1 = \frac{2.50 \text{ m} + 2.80 \text{ m}}{1.0 \text{ m/sec.}} + 2.0 \text{ sec.}$$
$$= 7.3 \text{ seconds}$$

To ensure that $\rho_1 \leq 1$, the stage-one arrival rate must be greater than 7.3 seconds. Therefore let the first-stage interarrival time vary from 7.5 to 17.0 seconds in the simulation run and examine the results.

(b) Machine 2 Total Service Time

A high stage-two intensity implies slow second-stage machines

Let machine 2 operate at a feed speed s_2 of 0.8 m/sec.
machine 2 mean set-up time μ_1 be 2 seconds.

Then $T_2 = 5.30/s_2 + \mu_2$ with s_2=0.8 and μ_2=2.0
$= 9.6$ seconds
(which is slightly slower than machine 1)

(c) Machine 3 Total Service Time

Let machine 3 operate at a feed speed s_3 of 0.6 m/sec.
machine 3 mean set-up time μ_3 be 2 seconds.

Then $T_3 = 5.30/s_3 + \mu_3$ with s_3=0.6 and μ_3=2.0
$= 10.8$ seconds

Machine 3 (slab edger) total service time is slower than machine 2 (cant edger), but has a larger storage capacity. This reflects a typical sawmill operating layout. All the system operating parameters of interest now have assigned values. A simulation run can be performed to determine how production is affected. Twenty simulation runs were performed, varying the mean interarrival times from 7.5 to 17.0 seconds, in steps of

0.5 seconds.

The first moments of the service and arrival distributions were used to determine the values for the parameters. The example shown above is for a medium-intensity second-stage arrival rate (ρ_2). High and low second-stage intensities were also studied in this section. The parameter values used for these other two simulation runs are found in Appendix F. All three cases were graphed in Figure 9.

To obtain steady-state conditions, the model is run for an equivalent real-life time of about 8 minutes and then all the statistics are reset. It is assumed that the effect of intitial conditions, where there are no logs in the system at the start, have disappeared by then. The model is then run for another 60 minutes and the statistics are subsequently gathered.

The duration of a simulation run directly affects the accuracy of what one is attempting to estimate. Complicated procedures exist for determining the required duration to provide an estimate at a given confidence level. This study however did not approach this topic. Instead, a subjective approach was made and judging by the results, the goal of obtaining good statistics were met. For each case, a regression curve was drawn through the points. The variation about the regression line was small, implying that the simulation runs were long enough duration. This curve should represent the functional relationship that exists between the parameters graphed.

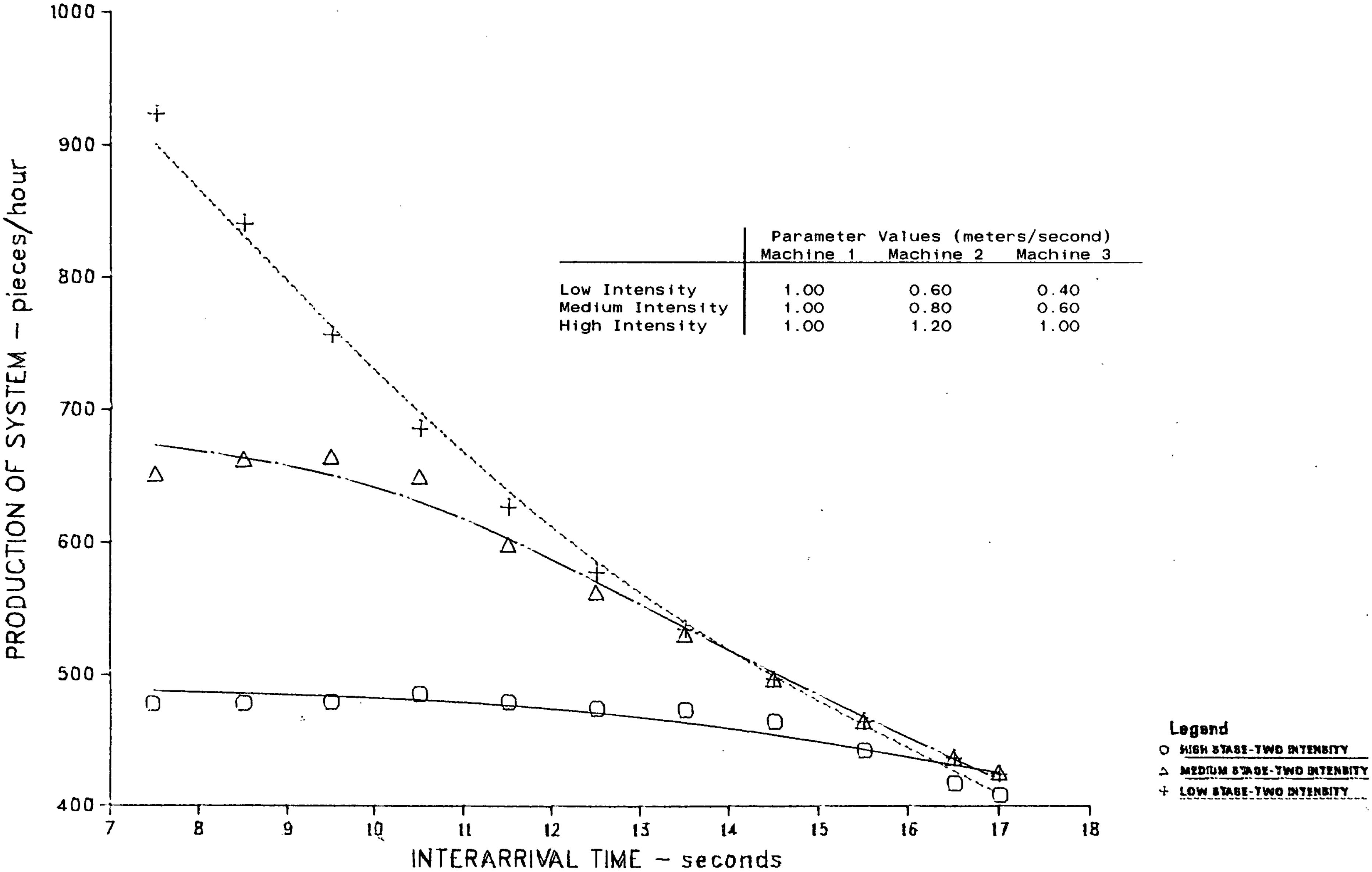
As shown in Figure 9, there appears to be a direct relationship between production and the first-stage interarrival time. Every second data point was omitted on the graph for clarity. As expected, different second-stage intensities affect the total system production. A low intensity (high stage-two machine speeds) provides greater output than the other two higher-intensity cases, especially at short interarrival times. It is interesting to observe that the three curves converge at a common interarrival time of 17 seconds. This is because the queue system production becomes increasingly dependent upon the longer interarrival times rather than the machine rates.

A low stage-two intensity culminates to a point at large interarrival times, where further improvements may not increase production (at least in a linear fashion). In this instance, the second-stage machines would be "starved" for pieces and the storages empty. This implies that if expensive machines of high capacity were installed in the second stage, system production may not be improved.

This section shows that if a mathematical model were made of the system, desirable information for designing a sawmill could be obtained. In addition, different first-stage interarrival times affect production a great deal. The amount of production is also dependent upon the second-stage intensity, as shown by the three different slopes in Figure 9.

The linear relationship for the high-intensity case implies that for the range of interarrival time values studied, the system dependency on the arrival rate is straight forward. The medium and low intensity curves exhibit properties of a non-linear function. A further investigation to the possibilities of why these relationships were found is given in the next section.

FIG. 9 — PRODUCTION VS. INTERARRIVAL TIME



3.3.2 Other System Measures Affected by Interarrival Time

Two other system measures studied here in the unsaturated queue analysis are the average number of pieces in the machine-3 storage facility and the percentage of time stage one is blocked. This simulation run describes the medium stage-two intensity case (refer to App. F). The percentage of time stage one is blocked "B", comes from the equation:

$$B = (\epsilon - \zeta)/\epsilon \times 100$$

where ϵ = average time at machine 1

ζ = average machine 1 service time

The mathematical analysis of this queue system can be difficult if blocking is taken into consideration. The complication exists when both second-stage stations provide the possibility of first-stage blocking. The simulation run performed in this section intentionally allowed for independent blocking to occur. There was only one storage facility that contributed to the blocking.

The machine 3 storage capacity was assigned to 50 pieces. The parameter values chosen for this simulation run ensured that the average piece content before machine 3 was at the most, about half the storage capacity (refer to Fig. 10). Machine 2 was therefore responsible for the blocking. Refer to Appendix F for the parameter values used in the simulation program.

Blocking was an important system characterstic to investigate. For the medium-intensity case, there is no first-stage blocking at an interarrival time of greater than 11.0 seconds (refer to Figure 11). Figure 9 shows that the medium stage-two intensity graph becomes non-linear with no stage-one blocking. The production of the system becomes dependent only upon the interarrival times and in a function of higher order than one.

The high second-stage intensity case (refer to Fig. 9) portrays a linear function in the range of interarrival times investigated. The data in Appendix F shows that blocking occurred throughout this range of interarrival times. The low second-stage intensity case exhibits a slight non-linear graph. The data in Appendix F shows that no blocking occurred through the range of interarrival times. Therefore the non-linear graph is a result of the arrival process only.

Prediction of average queue content before second-stage machinery could be complicated if blocking is considered. Figure 10 shows a curve of average piece content at the buffer before machine 3 as a function of interarrival times (for the medium-intensity case). There was a marked decrease in the average piece content as first-stage blocking (provided by machine 2) decreased. With the cant edger blocking the headrig, determining the proper size for the storage capacity before the slab edger is difficult. A sawmill where blocking occurs (intentionally or not) can result in many complications downstream of the blocked machine.

If both storage facilities contribute to blocking, a covariance must be taken into account. A correlation would exist between the two second-stage buffers, with either of them having the probability of blocking the first stage. The mathematical determination of blocking in stage one is straight-forward if only one second-stage storage facility has to be considered. The percentage of time stage one is blocked is then equal to the percentage of time the storage facility in the second stage is full. This can generally be solved with analytical methods.

FIG. 10 — AVERAGE CONTENTS IN STORAGE 3 VS. INTERARRIVAL TIME

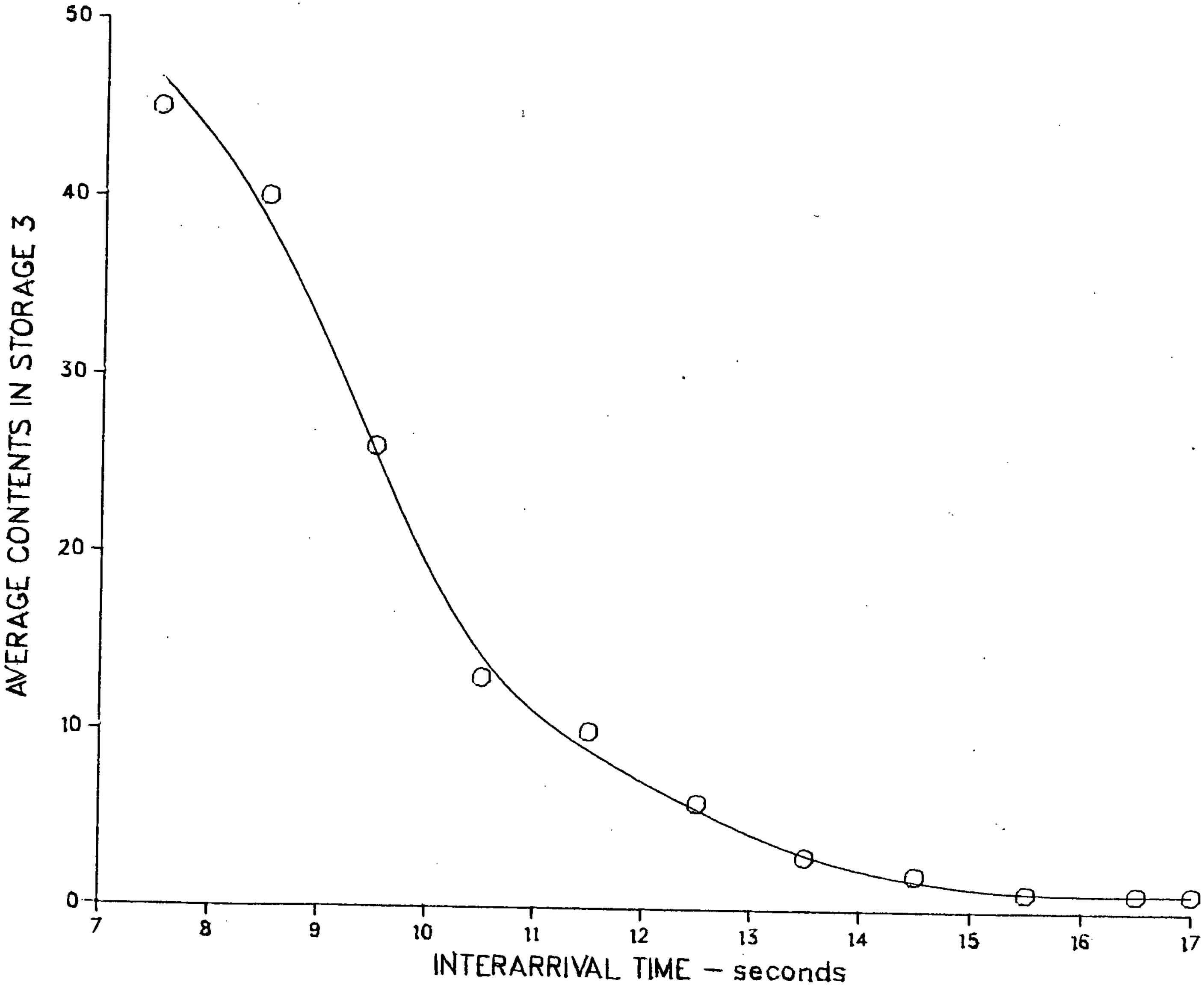
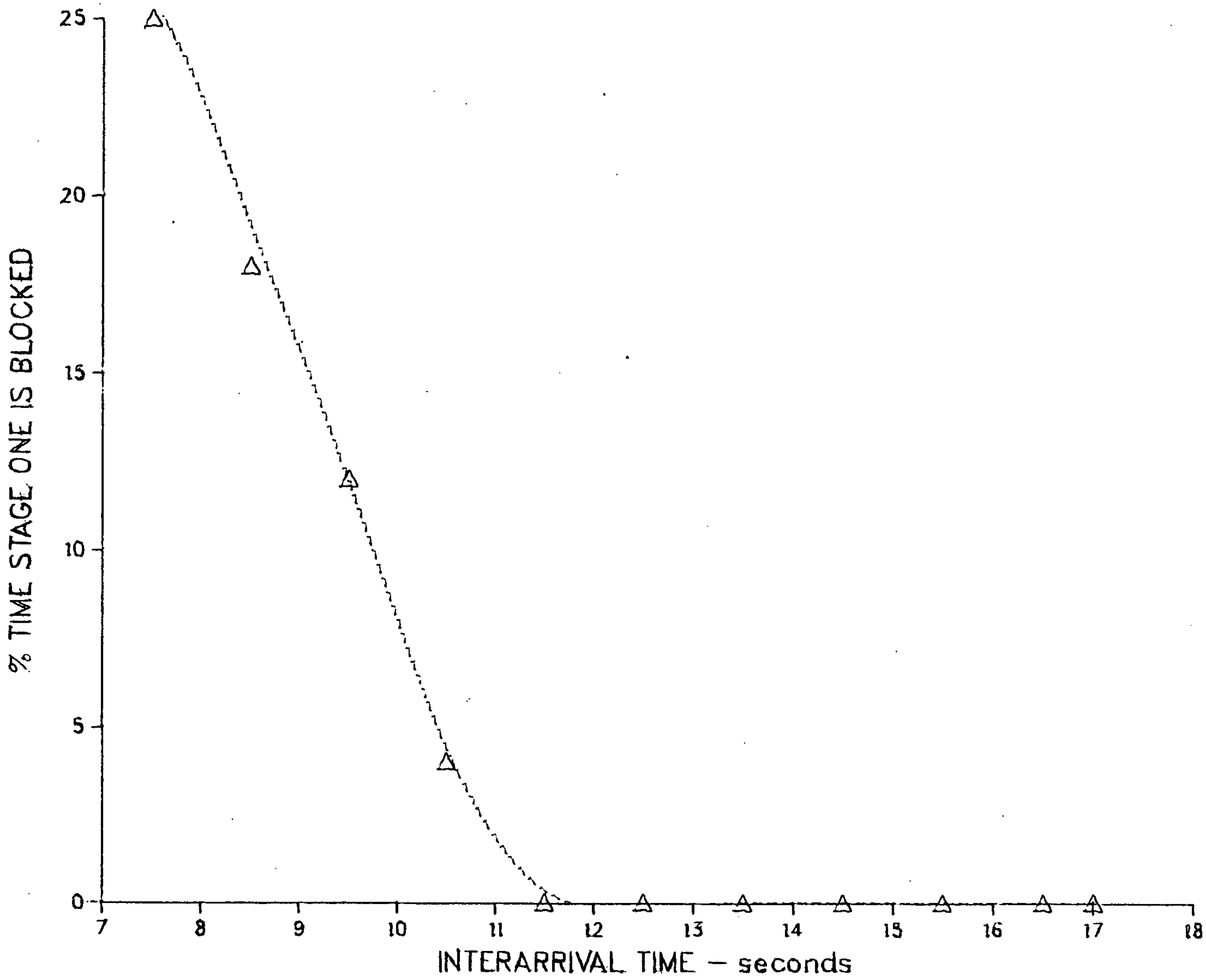


FIG. 11 – % TIME STAGE ONE IS BLOCKED VS. INTERARRIVAL TIME



ANALYSIS OF A SATURATED QUEUE

4.1 The Theoretical Analysis

Analysis of the queue system could be done with present numerical approximation methods if the queue in the first stage were to be considered as saturated. The departure process from the first stage becomes independent of its arrival process. Interest is then focused only on the second stage. If the second-stage arrival process is assumed to be independent of the second-stage service process, the queue system can be described as a GI/G/1/N queue. A good discussion on this queue analysis is found in Neuts.[24]

Neuts discusses the problem of blocking in a tandem queue with a finite buffer in the second stage and he provides a comprehensive list of references on this subject. He also provides solutions to some of the queue systems that exhibit blocking. He explains the numerical method of matrix geometric programming to model a queue system. This method is used for so-called quasi birth and death processes and uses a computer algorithm for their analysis.

With two finite-buffer stations in the second stage, there is the probability that either of them can block the first stage. Blocking is studied in the section of saturated queue analysis because it is a second-stage phenomenon. Since the first stage is saturated, the arrival process can be disregarded and blocking can be studied more effectively.

4.2 Simulation of an Saturated Queue

The simulation of a saturated queue utilizes a computer program similar to the case of a unsaturated queue. All the assumptions outlined in chapter 3 are also used in this analysis. A major difference between the saturated and unsaturated case is that the first-stage traffic intensity ρ_1 is greater than one in the saturated case. As a result, the first-stage buffer is always full of waiting customers. If there was not a limited-capacity first-stage buffer, an infinite queue would result. A first-stage interarrival time of 1.0 second was used in the simulation runs.

With a saturated first-stage queue facility, the arrival process into the second stage is dependent on only the service distribution in the first stage. The purpose of analyzing the saturated queue is to study the second-stage characteristics more effectively, which is possible since the first-stage interarrival times are no longer considered.

The calculation of the parameter values used for the simulation run were similar to the example given in section 3.2.2. Once again, the intention was to examine operating characterstics of the queue system and not to become too involved in assigning values to the parameters. The simulation programs in this section are presented in Appendices G and H.

4.2.1 Production of System as a Function of Machine Rates

Simulation of a saturated queue can more effectively examine the effect of individual machine processing rates. Therefore, the intention of the first simulation run was to observe the effect each machine has on system production. This is important for sawmill design because it may provide information on which machine to give special consideration to, with regards to total production.

Figure 12 displays the effect of machine rates on system production. Each of the three machines was independently varied to produce separate curves. Selection of the base values for the three machines reflected operating rates anticipated in a sawmill. Refer to Appendix G for all the machine operating rates used in the simulation run. Machines 1 and 2 would be expected to have similar rates, with machine 2 (cant edger) being slightly faster in order to "pull away" the pieces from the headrig. Machine 2 also has a small storage capacity. Machine 3 (slab edger) has a longer service time than the other two machines. The long service time reflects the slow transfer of slabs to machine 3. In addition, machine 3 has a large storage capacity. For clarity of reading the three graphs, they were provided with different maximum production values.

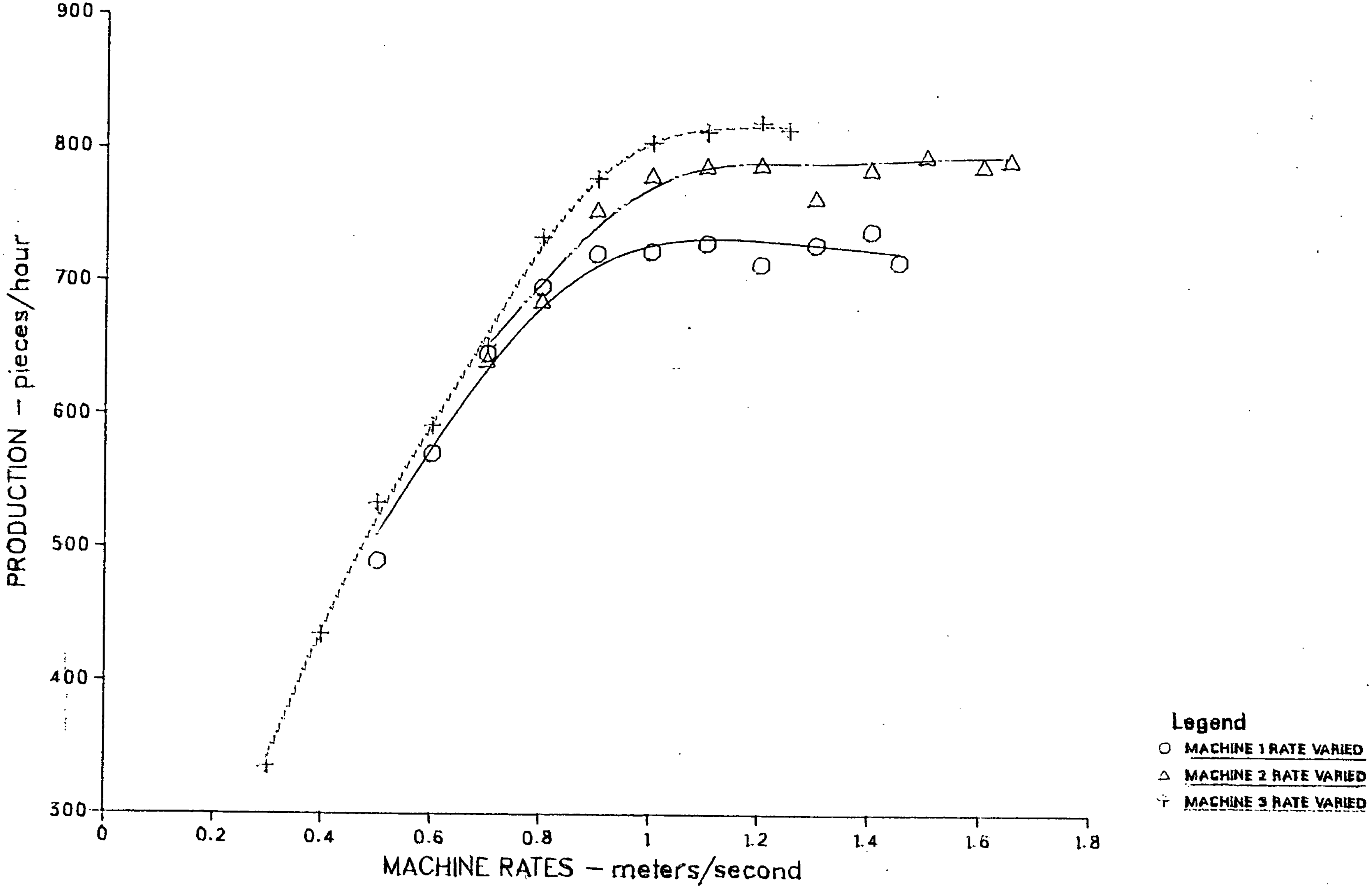
The intention of varying the rate of machine 1 was to investigate the importance of the headrig. The early steep slope indicates the large effect on production the first stage has. The decline in the slope is a result of blocking in the second stage. As the headrig becomes faster, bottlenecks downstream decrease the system production.

The influence of the cant edger on improvement to production is not as great. At slow service rates, production is decreased because of blocking, which is not shown in Figure 12 (refer to App. G for this statistic). As the speed of the cant edger increases, system production increases. The total system production gradually approaches a limit as the cant edger speed is increased. With the cant edger becoming faster than the headrig, the average piece content before machine 2 decreases. The decrease in blocking and the subsequent decrease in average piece content of the small-capacity storage result in the production limit.

Finally, varying the rate of machine 3 can have a large influence on production. The parameter values were selected for this simulation run to encompass a full machine 3 buffer that has a large storage capacity. The shape of the curve (refer to Fig. 12) representing the slab edger shows that system production can be improved as effectively as increasing the service rate of the headrig.

The improvement in system production came from decreasing the time of blocking and then decreasing the average piece content in the large-capacity storage of the slab edger (refer to App. G). The decrease in the curve slope at high machine 3 speeds was a result of the slab edger becoming "starved" for pieces. The similarity between the curves for machine 1 and 3 imply that the influence of the slab edger, under certain conditions, can be as influential in system production as the headrig.

FIG. 12 — PRODUCTION VS MACHINE RATES



4.2.2 The Effect of Blocking on Second-Stage Queues

Another question addressed in the simulation study was what effect blocking provided by one second-stage station has on the other station average piece content? The intention of this simulation run is to examine the effect two second-stage stations have between each other (their correlation). The simulation program is found in Appendix H.

The rate of machine 2 was selected to provide the necessary blocking of the first stage. The parameter values were chosen to initially have the average piece content of machine 3 to be empty. Refer to Appendix H for the parameter values used in the simulation run. Machine 2 speed was increased (to decrease blocking) and the effect on the average piece content behind machine 3 was observed. Figure 13 displays first-stage blocking and Figure 14 displays machine-3 average piece content, as functions of machine 2 operating speed.

As expected, blocking by the cant edger in the second stage affects the average queue contents of the slab edger. As blocking decreases to zero, the average piece content of the slab edger increases, approaching a limit (refer to Fig. 14). The parameter values chosen in this simulation run intentionally display how large an influence blocking provided by one second-stage station can have on the other station.

The occurence of blocking in a sawmill system can make the design of a second-stage storage facility difficult. One can approach this problem (and the results in this section are only an example) by designing the slab-edger storage capacity to accomodate the worst case. This is where it is assumed that there is no blocking done by the cant edger.

The general intention in designing a sawmill is to avoid the event of blocking and attempt to predict operating conditions so that it will not take place. On the other hand, if blocking is to be permitted by the cant edger, it is possible to save space (and money) by designing a smaller storage facility behind the slab edger. Blocking can potentially influence the average piece content in a second-stage buffer a great deal. The designer must be aware of this and if he wishes to allow blocking to occur in the system, he should attempt to understand all the ramifications.

FIG. 13 — % TIME STAGE ONE IS BLOCKED VS. MACHINE 2 RATE

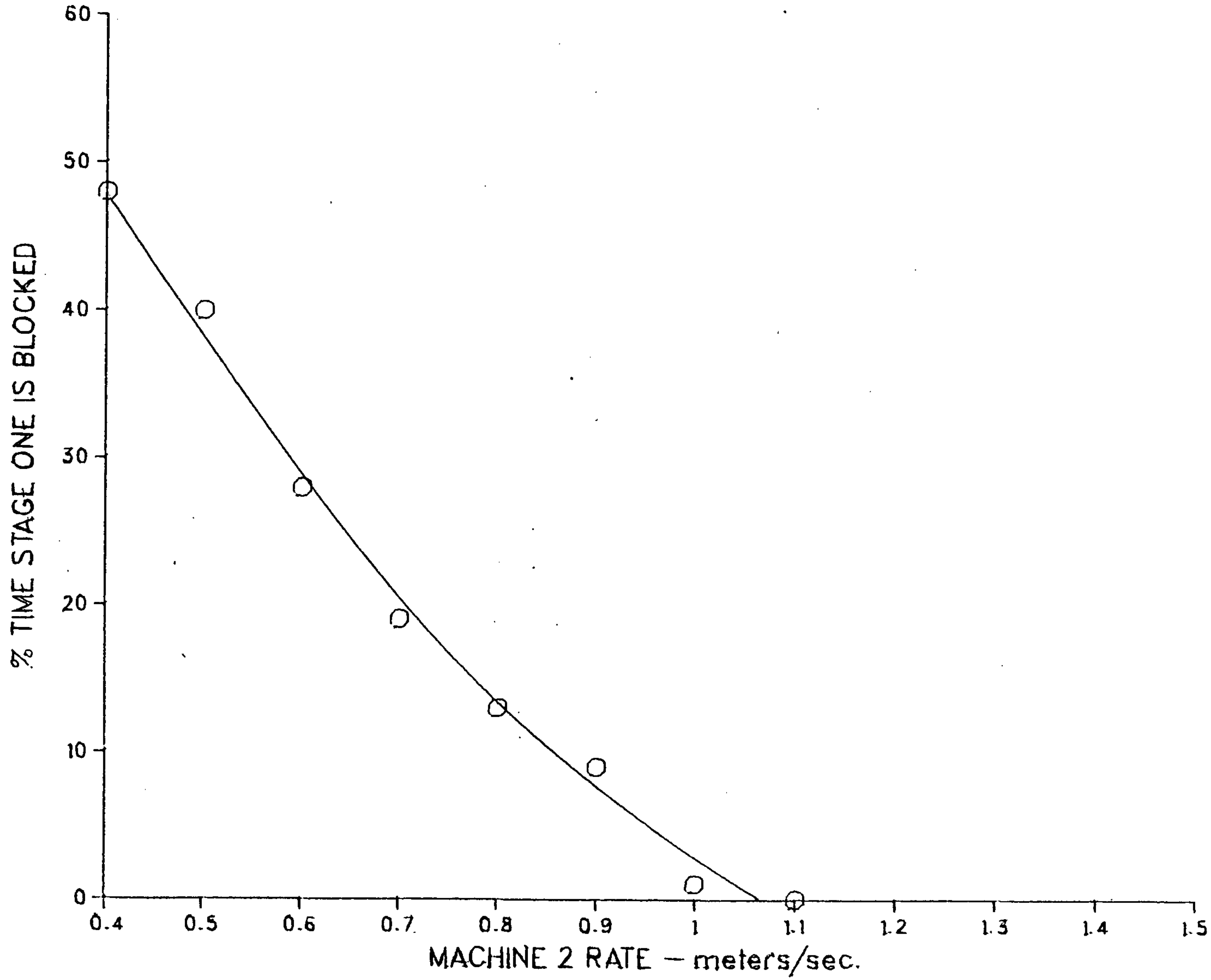
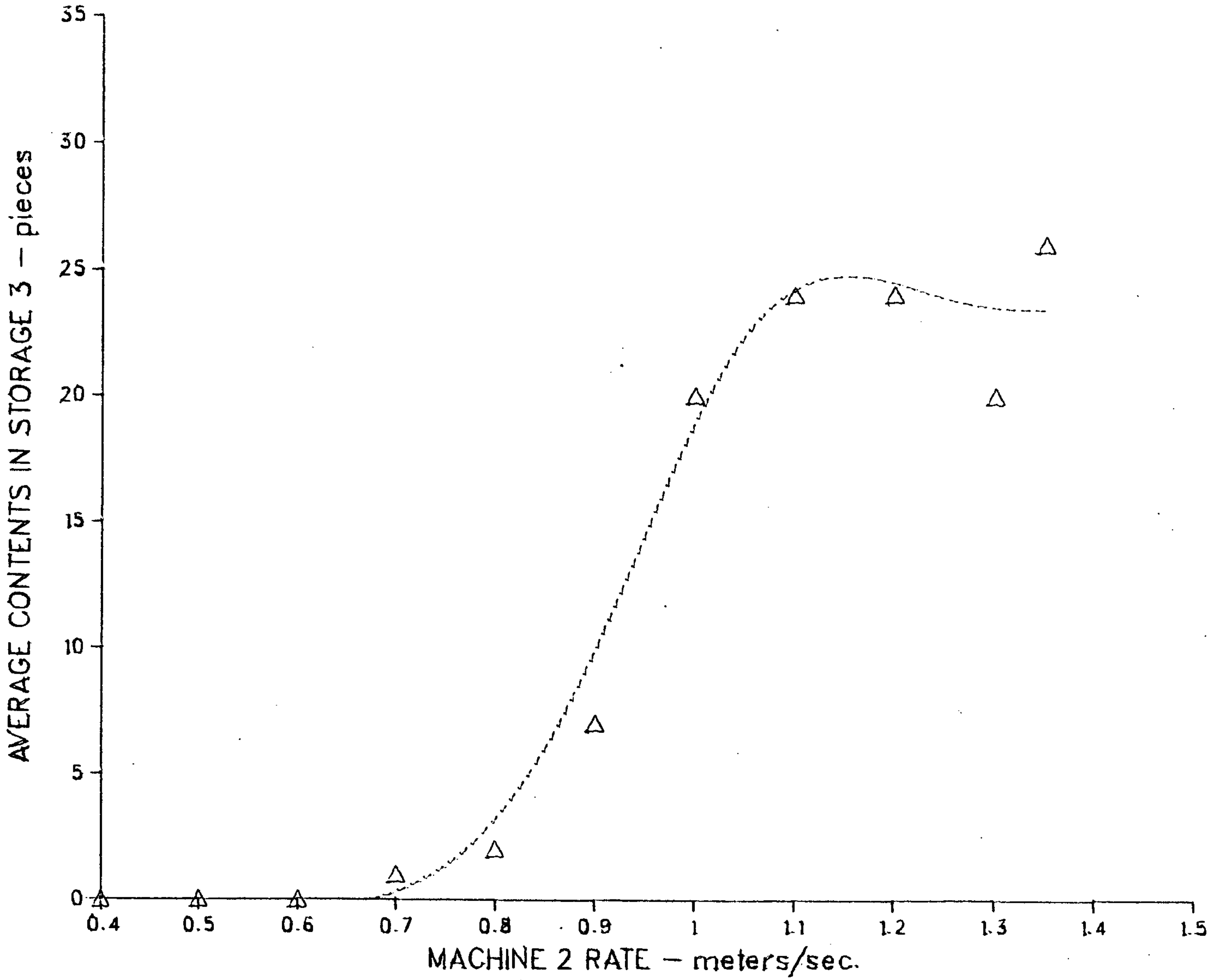


FIG. 14 — AVERAGE CONTENTS IN STORAGE 3 VS. MACHINE 2 RATE



4.2.3 System Production Affected by Second-Stage Storage Capacity

The objective of this simulation run was to determine if an optimum size can exist for a second-stage storage facility. The designer could save capital by installing the smallest second-stage storage facility possible, while at the same time retaining total system production. This simulation run was in part, attempting to duplicate Gershwin and Berman's[16] result (refer to Fig. 6).

In this simulation run, we take the previous results in section 4.2.2 one step further by saying that the designer will intentionally allow blocking to occur. A common situation could be where the cant-edger storage facility is constrained for space. This is often the case because cants are usually stored in facilities longitudinally (ends facing the machines). In addition, the operating speed of the cant edger is generally as fast as the headrig. Therefore, the storage facility can rapidly fill to capacity.

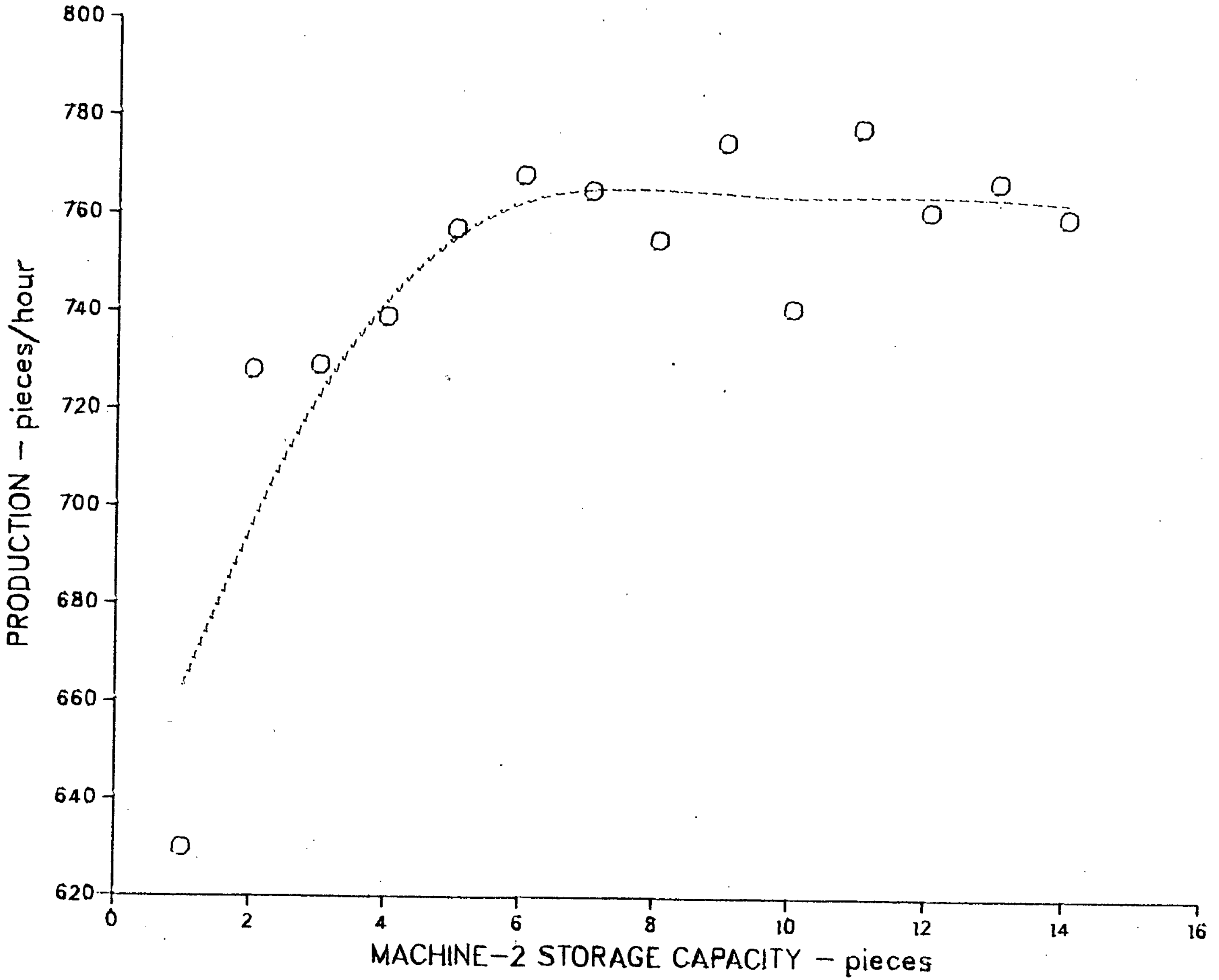
The machine rates are kept at constant values (refer to App. H). This may represent the situation where the machines may already have been selected for the sawmill. The machine 2 storage capacity was varied from 1 to 20 pieces. The selected machine rates provide an average piece content of about 14 pieces in the cant edger storage facility. The average piece content before the slab edger is small enough to ensure no first-stage blocking. However, there are always enough pieces before machine 3 to keep the slab edger busy.

The results of the simulation run are shown in Figure 15. The curve shows that there is only marginal improvement to system production at a storage capacity of larger than 5 pieces. Why would a storage facility of greater than 5 pieces not improve system production, if the average piece content before this cant edger would be 14 pieces if it had an infinite capacity?

Two other simulation runs were performed (but were not graphed) to answer this question. The rate of the cant edger was varied. One simulation run had an average piece content of about 17 pieces, the other 7 pieces. The data from these simulation runs reveal the same characteristic, where system production is not reduced significantly by decreasing the cant-edger storage capacity. Production is reduced only in the range of 6 - 10 percent, depending upon the arrival intensity into the cant edger. In addition, this decrease in production is with respect to the worst situation - a storage capacity of 1 piece. Production however, was significantly changed by the difference in machine-2 operating speed.

Figure 15 duplicates the results found by Gershwin and Berman (as shown in Fig. 6). However, the conclusions made from this simulation run indicate that system production is not highly dependent upon storage capacity. The rates of the second-stage machines can influence total system production at a greater magnitude.

FIG. 15 – PRODUCTION VS. MACHINE–2 STORAGE CAPACITY



SUMMARY AND CONCLUSIONS

The difficulty of modelling a system increases with the amount of detail the designer adopts to describe his model. The assumptions specified in this study for analyzing a queue system describe what is commonly found in a sawmill. Even though these assumptions are descriptive of what occurs in a sawmilling process, they are difficult to employ in an analytical model, thereby precluding a rigorous mathematical analysis.

The advantages of mathematically modelling queue systems make it desirable to pursue the modelling of sawmills by analytical techniques. Research into the field of departure processes from the M/G/1 queue has shown that these processes cannot be mathematically described except for a few restrictive cases. If more general solutions were available, the modelling of a sawmill system would be possible. Research into this area is certainly warranted, but it would be difficult to guarantee that applicable solutions could be obtained.

For the unsaturated case, the investigated sawmill system could not be modelled mathematically because of the assumptions used in this study. A simplifying assumption of exponential service times might improve the potential for mathematical modelling. Exponential service times would result in a recurrent departure process, and there has been extensive work done in this field of queueing theory. The usefulness of modelling sawmills with this service distribution is unknown.

The errors introduced by using this distributional assumption could be investigated. If the errors were tolerable, this would be a reasonable method for approaching the sawmill system analysis.

In cerain cases, numerical approximation methods derived from queueing theory can be applied to model the investigated sawmill layout. Boxma's research (tandem queues with highly correlated service times) has provided solutions that can potentially be useful for sawmill modelling. I believe the assumtion of correlated service times is representative of the sawmilling process. Boxma considered an infinite storage capacity between machine centers, which is a simplifying assumption that can limit the use of the analytical model. In addition, Neuts' solutions can be applied to the saturated queue case, if correlated service times are disregarded.

The service time distribution of machines found in a sawmill should be investigated further. A good distribution should adequately describe how the material is processed. The service time distribution used in this study is flexible and is a good representation of the sawmilling process. It can consider many of the influencing factors that affect sawmill systems. These factors include: the size and quality of logs, the effect of log lengths on processing times, conveyor speeds, machine processing rates and operator inefficiencies.

The service distribution suggested in this study is capable of being used for queueing analysis. It also enables the modeller to use functional equations to describe service times, rather than empirical distributions, which are presently used. This can be advantageous for modelling sawmills by both queueing theory and simulation methods. Field research to confirm the service distribution suggested in this study should be done.

This report discusses some of the possible applications for sawmill designers using queueing theory. The extensive problems which could be encountered by analytical methods were shown by the simulation analysis. Simulation provided a great deal of insight into the sawmill system and enabled the study of important operating characteristics.

Queueing theory is incapable of providing some important solutions of interest to the sawmill designer. Production statistics on the wide variety of board dimensions, species or grades made in a sawmill can only be determined by simulation methods. This simulation study attempted to duplicate results to what one would find from queueing theory. Consequently, the simulation work only considered one customer "type". Valuable information can still be obtained with this simplifying assumption. Sawmill operating characteristics such as buffer capacity, bottlenecks and blocking are generally not dependent on customer types. Simplistic production estimates could also be determined.

This simulation study provided information for sawmill design procedures on its own merit. The study showed that the relationship between production and the arrival rate into this simple queue system can be linear if blocking occurs (in the range of operating values studied). If interarrival times are very large, a slight non-linear relationship exists with respect to production if there is absolutely no blocking. There is only a minor dependency of interarrival times on system production. Therefore, in many applicable situations, system production estimates could be very easily calculated.

Although blocking is difficult to incorporate in an analytical model, it should not be disregarded. With two stations in the second stage of the studied queue system, the determination of first-stage blocking time can be difficult. The analysis could be simplified if independent blocking, contributed by only one second-stage station, is considered.

Blocking provided by one second-stage station can have a large effect on the average queue content of the other station. Therefore, it is desirable to predict the occurrence of blocking, if the designer is attempting to determine the capacity of a second-stage buffer. The assumption of an infinite second-stage storage capacity would allow the modeller to determine average queue contents in the storages without blocking. This would be a desirable procedure for sizing the storage to the worst case.

System production can be affected if any one of the three machines contribute to first-stage blocking. Production of the studied sawmill appears to be most sensitive to the machine speed of the headrig. Production improvement by increasing the headrig speed approaches a limit as blocking by either of the two second-stage stations increases.

The simulation study reveals that another possible method of increasing production is to decrease the average queue content of a second-stage storage; particularly if the storage is large, as in the case of the slab edger. This could be a non-conventional method to get more production from a sawmill, since most sawmill designers focus on the importance of the headrig as the primary production machine. The limit to production improvement by increasing the speed of the slab edger occurs when the average piece storage content is zero.

A designer can intentionally allow some second-stage blocking to occur, by decreasing the storage capacity of a second-stage machine. This might be necessary if saving space in the second stage is an important consideration. The simulation study showed that system production can be quite insensitive to the second-stage storage capacity. System production is not substantially reduced by utilizing very small storages in the second stage, which I consider as a non-intuitive result. Further investigation into optimum second-stage storage capacity should be done. If an analytical approach is undesirable, a simulation study could be done to research this unusual characteristic.

In summation, this thesis outlines some of the problems encountered in attempting to model a sawmill by queueing theory. It also shows some of the disadvantages of simulation for analyzing a sawmill system. It is desirable to have queueing theory as another tool available to the sawmill designer. The most important aspect of queueing theory analysis is to know when, where and how to use it (like using any other O. R.

technique). This is particularly important with user-friendly software packages that are beginning to appear on the market. Unless the designer understands the theory behind these programs, serious errors of interpretation may be made from the computer output.

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APPENDIX A

NOTATION AND SYMBOLS

Notation and Symbols

		Page
μ_1	Exponential distribution scale parameter representing machine 1 operating speed	11
μ_2	Exponential distribution scale parameter representing machine 2 operating speed	11
p	Queue system production	11
$\bar{n}$	Average in-process inventory behind a queue station	11
E(SC)	Expected cost of service	13
E(WC)	Expected cost of waiting	13
E(TC)	Expected total cost	13
M	"Memoryless" or exponential distribution	14
G	General distribution	14
GI	General independent distribution	14
E_k	k-Erlang distribution	14
D	"Degenerate" (constant) distribution	14
N	Finite number of customers allowed in a waiting line (queue capacity)	14
c	Number of parallel servers in a queue mechanism	14
λ	k-Erlang distribution scale parameter	23
k	k-Erlang distribution shape parameter	23
L	Random number representing the time to process a log	25
D	Displacement parameter (a constant)	25
S	Random number representing the set-up time at a machine to process a log	25

		Page
T	Random number representing the total service time to process a log	26
η	Mean arrival rate into the first stage	29
τ	Mean service time in the first stage	30
γ	Mean service time in the second stage	30
ρ_1	Traffic intensity in the first stage	30
ρ_2	Traffic intensity in the second stage	30
X_i	The i'th Exponential distribution random number realization	37
u_i	The i'th Uniform distribution random number realization	37
s	Speed of a machine processor	38
m	Minimum length of a log permitted in the sawmill	38
T_1	Total service time of machine 1	42
T_2	Total service time of machine 2	43
T_3	Total service time of machine 3	43
s_1	Feed speed of machine 1	43
s_2	Feed speed of machine 2	43
s_3	Feed speed of machine 3	43
μ_1	Mean set-up time of machine 1	43
μ_2	Mean set-up time of machine 2	43
μ_3	Mean set-up time of machine 3	43
B	Percentage of time stage 1 is blocked	47
ϵ	Average time at machine one	47
ζ	Average machine 1 service time	47

APPENDIX B

ANTITHETIC VARIATES

Antithetic Variates

Antithetic variables is a variance reducing technique used in simulation. It is used for reducing the variance of two random variable (outputs) X and Y. Antithetic variates are based on on the equation:

$$VAR(X+Y) = VAR\ X + VAR\ Y + 2\ COV\ (X,Y)$$

The purpose of antithetic variables is to induce a negative correlation between X and Y (have the covariance of X and Y become negative). This will result in:

$$VAR\ (X+Y) \leq VAR\ X + VAR\ Y$$

There is no direct control in simulation over the output of the random variable. However, simulation does have the control of the input random variables that are generated. Therefore, the approach for antithetic variables is to induce negatively correlated input random variables and hope that the ensuing output random variables have a negative corellation. Henriksen and Crain[17] states that the resulting negative correlation in output random variables is usually substantially less than the negative correlation induced in input random variables because the statistical models act as "filters".

Antithetic variates are induced by the sampling of the uniform distribution. Numbers are sampled from (0,1) and a negative correlated number is then used. For example:

sampled random number = 0.300

antithetic random number = 1-0.300 = 0.700

Antithetic random variables result in better point estimates of the simulation results, for the same number of samples.

APPENDIX C

RENEWAL PROCESS

Renewal Process

Renewal theory forms the foundation for analysis of queues. The theory is based on counting processes. A counting process for which times between successive events are independent and identically distributed with an arbitrary distribution is defined as a renewal process.

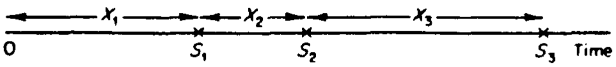

Fig. 16. The Renewal Process

Source: Ross,[29] p 227.

Let $\{N(t),\ t\geq 0\}$ be a counting process and let Xn denote the time between the (n-1)st and the nth event of the process, $n\geq 1$. If the sequence of nonnegative random variables $\{X_1, X_2, \ldots\}$ are independent and identically distributed, then the counting process is a renewal process. For a renewal process having interarrival times $X_1, X_2 \ldots.$

Let $S_0 = 0$ $$S_n = \sum_{i=1}^{N} X_i \qquad \text{for } n\geq 1$$

Then $$N(t) = \max\{\, n:\ S_n \leq t\}$$

Therefore N(t) is the number of renewals that have occurred by time t. For further reference to renewal theory and related processes, see Ross[29,30], Parzen[26], Karlin and Taylor[22] or Heyman and Sobel[18].

APPENDIX D

k-ERLANG DISTRIBUTION

k-Erlang Distribution

A random variable X is said to by a 3-parameter gamma model if the probability density function is of the form:

$$f_E(x;\theta,\lambda,k) = \frac{1}{\lambda\Gamma(k)} \cdot \left(\frac{x-\theta}{\lambda}\right)^{(k-1)} \exp\{-(x-\theta)/\lambda\}$$

where θ= location parameter

λ= scale parameter

$\Gamma(\cdot)$ = gamma function

k = shape parameter

The k-Erlang distribution is a gamma distribution with integer values for the shape parameter. The k-Erlang distribution is an extremely flexible distribution that accomodates a wide variety of shapes. Figure 8-A shows the k-Erlang distribution with a constant scale parameter λ and different values for the shape parameter k. Figure 8-B shows the k-Erlang distribution with a constant value for k and different values for λ.

The 3-parameter k-Erlang distribution possesses the following properties:

$$E(X) = \theta + \lambda \cdot k$$

$$VAR(X) = \lambda^2 \cdot k$$

$$\text{Mode} = \theta + \lambda\cdot(k-1)$$

The maximum likelihood estimators for the gamma distribution are:

$$\hat{\lambda}\cdot\hat{k} = \bar{x}$$

where $\bar{x} = (1/n)\cdot \sum_{n=1}^{n} Y_i$

Y_i = a gamma random variable realization

If the geometric mean is defined as:

$$G = \prod_{i=1}^{n} Y_i^{1/n}$$

and $g = \ln\,(\bar{x}/G)$

Bury[7] states that a highly accurate approximation of $\hat{k}$ can then be made from the following equations:

$$\hat{k} = (0.5001 + 0.1649g - 0.0544g^2)g^{-1}$$

with $0 < g \le 0.577$

or $$\hat{R} = (17.80 + 11.97g + g^2)^{-1} \cdot (8.99 + 9.060g + 0.977g^2)g^{-1}$$

with $0.577 \le g \le 17$

In simulation, a k-Erlang realization is the sum of k exponential distibution realizations:

$$Pr[X_E > t] = \sum_{i=1}^{k} Pr[X_i > t]$$

Where X_E = a k-Erlang realization

X_i = an exponential realization

Thus, a uniform random number u is converted into an exponential random number, and the sum of k such values gives a single realization of a k-Erlang variable:

$$X_E = \mu \cdot \sum_{i=1}^{k} (-\ln (1-u_i))$$

APPENDIX E

ESTIMATION OF k-ERLANG PARAMETERS

Estimation of the k-Erlang Parameters

The data used to estimate a 3-parameter k-Erlang distribution came from Dobie.[13] The page containing the data from Dobie's report is included in this Appendix. One can refer to the data for the numbers that were used in the estimation of the parameters. The equations used to estimate the parameter values are found in Appendix D.

(1) Estimate the k-Erlang parameters for chipper headrig - mill "A"

Minimum log length is 8 ft. Therfore 6 will have to be subtracted from all log-length frequency values. This is because it is desirable to include the 2 ft. between the 8 and 10 ft. frequency class as contributing to the processing time. This will reduce the 3-parameter k-Erlang distribution to a 2-parameter distribtuion. From the data, we obtain the following values:

$$\bar{x} = 9.44 = \lambda \cdot k$$

$$G = \prod_{i=1}^{n} X_i^{1/n}$$

$$= 2^{5/75} \cdot 4^{2/75} \cdot 6^{8/75} \cdot 8^{15/75} \cdot 10^{20/75} \cdot 12^{14/75} \cdot 14^{11/75}$$

$G = 8.629$

$$g(\hat{k}) = \ln(\bar{x}/G) = \ln(9.44/8.629)$$
$$= 0.0898$$

$$\hat{k} = (0.5001 + 0.1649g - 0.0544g^2)g^{-1} \qquad \text{for } 0<g\leq 0.577$$

$$\hat{k} = 5.72 \qquad \hat{\lambda} = 1.65$$

These are estimates for the gamma model. The k-Erlang distribution uses only integer values for k therefore let:

$$\hat{k} = 6 \qquad \hat{\lambda} = 1.6$$

Thus our 3-parameter k-Erlang function takes the form:

$$f_E(x;6,1.6,6) = \frac{1}{1.6\Gamma(6)} \cdot \left(\frac{x-6}{1.6}\right)^{(5)} \exp\{-(x-6)/1.6\}$$

The above equation is used for case (1) in the Kolmogorov-Smirnov test

(2) Estimation of k-Erlang parameters for a scrag headrig - mill "G"

Minimum log length is 10 ft., therefore subtract 8 from all data values to obtain a 2-parameter k-Erlang model.

$\bar{X} = 8.87$

$G = 8.31$

$g(\hat{k}) = 0.0652$

$\hat{k} = 7.82$ $\hat{\lambda} = 1.13$

Integer values of k are required, therfore let:

$\hat{k} = 8$ $\hat{\lambda} = 1.1$

The k-Erlang distribution has the parameters:

$$f_E(x;8,1.1,8) = \frac{1}{1.1\Gamma(8)} \cdot \left(\frac{x-8}{1.1}\right)^{(7)} \exp\{-(x-8)/1.1\}$$

(3) Estimation of k-Erlang parmaters for a log gang headrig - mill "J"

Minimum log length is 10 ft., therefore subtract 8 from all data values to obtain a 2-parameter k-Erlang model.

$\bar{X} = 8.10$

$G = 7.54$

$g(\hat{k}) = 0.0709$

$\hat{k} = 7.21$ $\hat{\lambda} = 1.12$

Integer values of k are required, therfore let:

$\hat{k} = 7$ $\hat{\lambda} = 1.1$

The k-Erlang distribution has the parameters:

$$f_E(x;8,1.1,7) = \frac{1}{1.1\Gamma(7)} \cdot \left(\frac{x-8}{1.1}\right)^{(6)} \exp\{-(x-8)/1.1\}$$

KOLMOGOROFF TEST

Chipper Heading "A"

(1) Class	(2) Observed Value	(3) Observed Frequency	(4) Cumulative Frequency	(5) Sample Distribution	(6) Erlang c.d.f. F_E =	(7) Absolute Deviation $\lvert(5)-(7)\rvert$
i	x_i	n_i	n_i	$\frac{(4)}{n}$	$(x;8,1.6,6)$	
1	8	5	5	0.0067	0.0092	0.0025
2	10	2	7	0.0933	0.1088	0.0155
3	12	8	15	0.2000	0.3225	0.1225
4	14	15	30	0.4000	0.5595	Fails→0.1595
5	16	20	50	0.6667	0.7470	0.0803
6	18	14	64	0.8533	0.8679	0.0146
7	20	11	75	1.0000	0.9340	0.0660

Test fails at $\alpha = 0.10$ $C/\sqrt{n} = 0.14087$

Scrags "G"

					$(x;10,1.1,8)$	
1	10	1	1	0.00099	0.0006	0.0093
2	12	5	6	0.0594	0.0733	0.0139
3	14	25	31	0.3069	0.3062	0.0007
4	16	30	61	0.6040	0.5940	0.0100
5	18	18	79	0.7822	0.8022	0.0200
6	20	11	90	0.8911	0.9162	0.0251
7	22	2	92	0.9109	0.9681	0.0572
8	24	9	101	1.0000	0.9891	0.0109

Test passes at $\alpha = 0.10$ $C/\sqrt{n} = 0.1214$

Log Gangs "J"

					$(x;10,1.1,7)$	
1	10	2	2	0.0238	0.0104	0.0134
2	12	9	11	0.1310	0.1559	0.0249
3	14	17	28	0.3333	0.4469	0.1136
4	16	31	59	0.7024	0.7119	0.0095
5	18	13	72	0.8571	0.8781	0.0210
6	20	7	79	0.9405	0.9536	0.0131
7	22	2	81	0.9643	0.9845	0.0202
8	24	3	84	1.0000	0.9945	0.0059

Test passes at $\alpha = 0.10$ $C/\sqrt{n} = 0.133$

— DISTRIBUTION OF LOG LENGTHS IN STUDY SAMPLES.

Log length (ft.)	Chipper headrigs					Scrags			Log gangs		
	A	B	C	D	E	F	G	H	I	J	K
					(No. of logs)						
8	5					6					
10	2			3	2		1			2	
12	8			24	9	9	5	3	6	9	5
14	15	11	3	21	12	4	25	8	5	17	10
16	20	24	13	68	106	55	30	47	56	31	22
18	14	10	19		14	15	18	2	12	13	18
20	11	12	40		36	13	11		8	7	21
22		4	31		5		2		6	2	6
24							9		11	3	9
26									1		
Number of logs	75	61	106	116	184	102	101	60	105	84	91
Average length (ft.)	16	18	20	15	16	16	16	16	18	16	18

Source: Dobie,[13] p 33.

APPENDIX F

PRODUCTION VS. INTERARRIVAL TIME

AND

OTHER SYSTEM MEASURES VS. INTERARRIVAL TIME

(INCL. SIMULATION PROGRAM AND DATA)

```
      SIMULATE
*
*      THIS PROGRAM VARIES THE INTERARRIVAL TIME
*           OF LOGS ENTERING THE QUEUE SYSTEM
*
*      FUNCTION DEFINITIONS
*
*      THIS IS AN EXPONENTIAL DISTRIBUTION FUNCTION
*      WHICH WILL BE USED FOR THE K-ERLANG DISTRIBTUION
*
 5    FUNCTION   RN1,C24

,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*      THIS IS A SEPARATE EXPONENTIAL DISTRIBUTION FUNCTION
*      FOR THE SET-UP TIMES OF EACH MACHINE.  IT HAS A DIFFERENT
*      "SEED" THAN THE ABOVE EXPONENTIAL DISTRIBUTION
*
 6    FUNCTION   RN2,C24

,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*      THIS STATEMENT IDENTIFIES THE EXTERNAL SUBROUTINE THAT
*      STORES X-Y VALUES OF R1 VS. P INTO A FILE ACCESSIBLE FOR
*      THE TELLAGRAF GRAPHING SOFTWARE PACKAGE
*
      EXTERNAL   &DATA
*
*      STORAGE CAPACITY DEFINITIONS
*
      STORAGE    S$SMACH1,100/S$SMACH2,10/S$SMACH3,50
*
      REALLOCATE COM,100000
*
*
*      DECLARE ALL AMPERVARIABLES USED IN THE PROGRAM
*
      REAL       &RATE1,&RATE2,&RATE3,&SET1,&SET2,&SET3,&SCALE,&M,&K
      REAL       &BLOCK,&P1(20),&AC3(20),&B1(20),&ARR(20)
      INTEGER    &I,&J
*
*      &RATE1  -  RATE OF MACHINE 1
*      &RATE2  -  RATE OF MACHINE 2
*      &RATE3  -  RATE OF MACHINE 3
*      &SET1   -  SETUP MEAN TIME FOR MACHINE 1
*      &SET2   -  SETUP MEAN TIME FOR MACHINE 2
*      &SET3   -  SETUP MEAN TIME FOR MACHINE 3
*      &SCALE  -  MEAN VALUE OF EXPONENTIAL DIST. FOR GENERATION OF
*                 ERLANG RANDOM VARIABLES
*      &K      -  K VALUE FOR ERLANG DISTRIBUTION
*      &M      -  MINIMUM LOG LENGTH
*
*
*      MODEL SEGMENT 1
*
CNTL  GENERATE   750,FN5      GENERATE INTERARRIVALS OF 750 TIME UNITS
```

```
      GATE SNF   SMACH1       IF STORAGE SMACH1 IS FULL, BLOCK ARRIVALS
      ENTER      SMACH1       ENTER STORAGE SMACH1
      SEIZE      MACH1        SEIZE MACH1 FACILITY
      GATE SNF   SMACH2       IF STORAGE MACH2 IS FULL, DO NOT LEAVE STORAGE SMACH1
      GATE SNF   SMACH3       IF STORAGE MACH3 IS FULL, DO NOT LEAVE STORAGE SMACH1
      SEIZE      AVE          STATISTICS FOR NO BLOCK SERVICE TIME
      LEAVE      SMACH1
*
*
*      THIS ROUTINE CREATES ERLANG RANDOM NUMBERS WITH A K VALUE OF 7.
*      THESE RANDOM VARIABLES ARE THEN ASSIGNED TO PARAMETER 1
*      TO GIVE THE MACHINES 1,2 OR 3 THEIR RESPECTIVE PROCESSING TIMES.
*      THE AMPERVARIABLES &R1, &R2 AND &R3 GIVES THE MACHINES THEIR
*      RESPECTIVE PROCESSING RATES, WHICH CAN BE VARIED BY EXTERNAL
*      CONTROL CARDS.
*
*
ONE   ASSIGN     4,&SCALE,5   ASSIGN AN EXPONENTIAL RANDOM NUMBER TO PARAMETER 4
      ASSIGN     1+,P4        ADD THE EXPONENTIAL VALUE TO PARAMETER  1
      ASSIGN     5+,1
      TEST GE    P5,&K,ONE
ERL1  FVARIABLE  (&M+P1)/&RATE1    ASSIGN THE RATE OF MACH1 TO THE EXPONENTIAL VARIABLE
ERL2  FVARIABLE  (&M+P1)/&RATE2    ASSIGN THE RATE OF MACH2 TO THE EXPONENTIAL VARIABLE
ERL3  FVARIABLE  (&M+P1)/&RATE3    ASSIGN THE RATE OF MACH3 TO THE EXPONENTIAL VARIABLE
TOT1  FVARIABLE  &SET1*FN6+V$ERL1
TOT2  FVARIABLE  &SET2*FN6+V$ERL2
TOT3  FVARIABLE  &SET3*FN6+V$ERL3
      ADVANCE    V$TOT1       PROCESS THE PIECE AT TOTAL SERVICE TIMEFOR MACHINE ONE
      RELEASE    MACH1        LEAVE MACH1 (INCLUDES BLOCKING)
      RELEASE    AVE          GATHER STATS ON NO BLOCKING
      SPLIT      1,MACH3      SPLIT THE PIECE INTO TWO: ONE GOES TO MACH2, THE OTHER TO MACH3
      ENTER      SMACH2       ENTER STORAGE FOR MACH2
      SEIZE      MACH2        SEIZE MACH2 FACILITY
      LEAVE      SMACH2       LEAVE THE MACH2 STORAGE
      ADVANCE    V$TOT2       PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE TWO
      RELEASE    MACH2        LEAVE MACH2
      QUEUE      PROD         COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
      DEPART     PROD         GATHER STATISTIC FOR PRODUCTION OF SYSTEM
BYBY  TERMINATE
MACH3 ENTER      SMACH3       ENTER THE STORAGE FOR MACH3
      SEIZE      MACH3        SEIZE FACILITY MACH3
      LEAVE      SMACH3       LEAVE THE STORAGE MACH3
      ADVANCE    V$TOT3       PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE THREE
      RELEASE    MACH3        LEAVE MACH3
      QUEUE      PROD         COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
      DEPART     PROD         GATHER STATISTICS FOR SYSTEM PRODUCTION
      TRANSFER   ,BYBY
*
*      MODEL SEGMENT 2
*
      GENERATE   10000
      TERMINATE  1
*
*      CONTROL CARDS
*
      LET        &M=250       SET MINIMUM LOG LENGTH TO 0.25 METERS
      LET        &K=7         SET K VALUE TO 7
      LET        &SCALE=40    SET ERLANG SCALE PARAMETER TO .40 SECONDS
      LET        &SET1=200    MEAN SET-UP TIME FOR MACHINE 1 IS 2 SECONDS
      LET        &SET2=200    MEAN SET-UP TIME FOR MACHINE 2 IS 2 SECONDS
```

```
         LET         &SET3=200    MEAN SET-UP TIME FOR MACHINE 3 IS 2 SECONDS
         UNLIST      CSECHO
*
*         SET THE RATES OF THE MACHINES FOR THE MEDIUM INTENSITY CASE
*
*         THE MACHINE RATES FOR THE LOW INTENSITY CASE ARE :
*         MACHINE 2 = 1.20 METERS PER SEC. AND MACHINE 3 = 1.00 METERS PER SEC.
*
*         THE MACHINE RATES FOR THE HIGH INTENSITY CASE ARE :
*         MACHINE 2 = 0.60 METERS PER SEC. AND MACHINE 3 = 0.40 METERS PER SEC.
*
*
         LET         &RATE1=1.00  RATE OF MACHINE 1 IS 1.0 METERS PER SEC.
         LET         &RATE2=0.80  RATE OF MACHINE 2 IS 0.80 METERS PER SEC.
         LET         &RATE3=0.60  RATE OF MACHINE 3 IS 0.60 METERS PER SEC.
         LET         &J=750
         DO          &I=1,20
         START       5,NP
         RESET
         RMULT       1,2
         START       18,NP
         RMULT       -1,-2
         START       18,NP
         LET         &P1(&I)=QC$PROD
         LET         &AC3(&I)=SA$SMACH3
         LET         &BLOCK=(FT$MACH1-FT$AVE)/FT$MACH1*100
         LET         &B1(&I)=&BLOCK
         LET         &ARR(&I)=&J
         LET         &J=&J+50
CNTL     GENERATE    &J,5
         CLEAR
         ENDDO
         CALL        &DATA(&P1(1),&AC3(1),&B1(1),&ARR(1))
         END
```

651.0	45.2	25.6	750.0
657.0	43.9	23.7	800.0
662.0	40.0	17.8	850.0
654.0	33.4	15.7	900.0
664.0	26.2	11.5	950.0
663.0	17.1	7.5	1000.0
649.0	13.1	3.6	1050.0
626.0	11.3	1.7	1100.0
598.0	9.8	0.0	1150.0
584.0	7.3	0.0	1200.0
562.0	6.0	0.0	1250.0
549.0	4.2	0.0	1300.0
530.0	3.3	0.0	1350.0
514.0	1.8	0.0	1400.0
496.0	1.9	0.0	1450.0
480.0	1.4	0.0	1500.0
464.0	1.1	0.0	1550.0
452.0	0.9	0.0	1600.0
436.0	0.6	0.0	1650.0
425.0	0.5	0.0	1700.0

THIS DATA IS FOR THE MEDIUM STAGE-TWO INTENSITY CASE
WITH MACHINE 2 = 0.80 METERS PER SECOND
AND MACHINE 3 = 0.60 METERS PER SECOND
COLUMN 1 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 2 IS AVERAGE PIECE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS INTERARRIVAL TIME (IN HUNDREDTH SECONDS)

923.0	3.2	0.0	750.0
874.0	2.8	0.0	800.0
840.0	1.4	0.0	850.0
802.0	1.5	0.0	900.0
756.0	0.4	0.0	950.0
721.0	0.3	0.0	1000.0
685.0	0.2	0.0	1050.0
654.0	0.2	0.0	1100.0
626.0	0.4	0.0	1150.0
601.0	0.1	0.0	1200.0
577.0	0.2	0.0	1250.0
552.0	0.2	0.0	1300.0
534.0	0.1	0.0	1350.0
514.0	0.2	0.0	1400.0
496.0	0.1	0.0	1450.0
480.0	0.2	0.0	1500.0
464.0	0.1	0.0	1550.0
451.0	0.1	0.0	1600.0
436.0	0.1	0.0	1650.0
424.0	0.1	0.0	1700.0

THIS DATA IS FOR THE LOW STAGE-TWO INTENSITY CASE
WITH MACHINE 2 = 1.20 METERS PER SECOND
AND MACHINE 3 = 1.00 METERS PER SECOND
COLUMN 1 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 2 IS AVERAGE PIECE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS INTERARRIVAL TIME (IN HUNDREDTH SECONDS)

478.0	46.5	45.3	750.0
484.0	47.0	44.6	800.0
478.0	46.4	45.5	850.0
482.0	47.3	44.1	900.0
479.0	46.5	44.2	950.0
480.0	45.7	39.5	1000.0
485.0	44.0	35.9	1050.0
482.0	41.7	31.7	1100.0
479.0	38.3	29.5	1150.0
478.0	35.1	26.6	1200.0
474.0	30.1	27.0	1250.0
474.0	26.5	24.8	1300.0
473.0	21.5	16.4	1350.0
474.0	16.7	4.5	1400.0
464.0	12.4	0.0	1450.0
453.0	10.3	0.0	1500.0
442.0	8.7	0.0	1550.0
429.0	8.2	0.0	1600.0
417.0	7.3	0.0	1650.0
408.0	6.0	0.0	1700.0

THIS DATA IS FOR THE HIGH STAGE-TWO INTENSITY CASE
WITH MACHINE 2 = 0.60 METERS PER SECOND
AND MACHINE 3 = 0.40 METERS PER SECOND
COLUMN 1 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 2 IS AVERAGE PIECE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS INTERARRIVAL TIME (IN HUNDREDTH SECONDS)

APPENDIX G

PRODUCTION OF SYSTEM AS FUNCTION OF MACHINE RATES

(INCL. SIMULATION PROGRAM AND DATA)

```
      SIMULATE
*
*
*      THIS PROGRAM VARIES THE THREE MACHINE RATES AND PRODUCTION
*               OF THE SYSTEM IS SUBSEQUENTLY OBSERVED
*
*      FUNCTION DEFINITIONS
*
*      THIS IS AN EXPONENTIAL DISTRIBTUION FUNCTION WHICH WILL BE
*      USED TO OBTAIN A K-ERLANG DISTRIBUTION FUNCTION
*
 5    FUNCTION   RN1,C24

,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*      THIS IS A SEPARATE EXPONENTIAL DISTRIBUTION FUNCTION FOR
*      THE SET-UP TIMES OF THE MACHINES.  IT HAS A DIFFFERENT
*      "SEED" THAN THE ABOVE EXPONENTIAL DISTRIBUTION.
*
 6    FUNCTION   RN2,C24

,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*      THIS STATEMENT IDENTIFIES THE EXTERNAL SUBROUTINE THAT
*      STORES X-Y VALUES OF R1 VS. P INTO A FILE ACCESSIBLE FOR
*      THE TELLAGRAF GRAPHING SOFTWARE PACKAGE
*
      EXTERNAL   &DATA
*
*      STORAGE CAPACITY DEFINITIONS
*
      STORAGE    S$SMACH1,100/S$SMACH2,10/S$SMACH3,50
*
      REALLOCATE COM,100000
*
*
*      DECLARE ALL AMPERVARIABLES USED IN THE PROGRAM
*
      REAL       &RATE1,&RATE2,&RATE3,&SET1,&SET2,&SET3,&SCALE,&M,&K
      REAL       &BLOCK,&R1(20),&R2(20),&R3(20),&P1(20),&P2(20),&P3(20),&B1(20),&B2(20),&B3(20)
      INTEGER    &I
*
*      &RATE1  -  RATE OF MACHINE 1
*      &RATE2  -  RATE OF MACHINE 2
*      &RATE3  -  RATE OF MACHINE 3
*      &SET1   -  SETUP MEAN TIME FOR MACHINE 1
*      &SET2   -  SETUP MEAN TIME FOR MACHINE 2
*      &SET3   -  SETUP MEAN TIME FOR MACHINE 3
*      &SCALE  -  MEAN VALUE OF EXPONENTIAL DIST. FOR GENERATION OF
*                 ERLANG RANDOM VARIABLES
*      &K      -  K VALUE FOR ERLANG DISTRIBUTION
*      &M      -  MINIMUM LOG LENGTH
*
*
*      MODEL SEGMENT 1
*
```

```
       GENERATE    100,FN5       GENERATE INTERARRIVALS OF 100 TIME UNITS
       GATE SNF    SMACH1        IF STORAGE SMACH1 IS FULL, BLOCK ARRIVALS
       ENTER       SMACH1        ENTER STORAGE SMACH1
       SEIZE       MACH1         SEIZE MACH1 FACILITY
       GATE SNF    SMACH2        IF STORAGE MACH2 IS FULL, DO NOT LEAVE STORAGE SMACH1
       GATE SNF    SMACH3        IF STORAGE MACH3 IS FULL, DO NOT LEAVE STORAGE SMACH1
       SEIZE       AVE           STATISTICS FOR NO BLOCK SERVICE TIME
       LEAVE       SMACH1
*
*
*       THIS ROUTINE CREATES ERLANG RANDOM NUMBERS WITH A K VALUE OF 7.
*       THESE RANDOM VARIABLES ARE THEN ASSIGNED TO PARAMETER 1 TO GIVE
*       THE MACHINES 1,2 AND 3 THEIR RESPECTIVE PROCESSING TIMES.
*       THE AMPERVARIABLES &R1, &R2 AND &R3 GIVES THE MACHINES THEIR
*       RESPECTIVE PROCESSING RATES, WHICH CAN BE VARIED BY EXTERNAL
*       CONTROL CARDS.
*
*
ONE    ASSIGN      4,&SCALE,5    ASSIGN AN EXPONENTIAL RANDOM NUMBER TO PARAMETER 4
       ASSIGN      1+,P4         ADD THE EXPONENTIAL VALUE TO PARAMETER 1
       ASSIGN      5+,1
       TEST GE     P5,&K,ONE
ERL1   FVARIABLE   (&M+P1)/&RATE1     ASSIGN THE RATE OF MACH1 TO THE EXPONENTIAL VARIABLE
ERL2   FVARIABLE   (&M+P1)/&RATE2     ASSIGN THE RATE OF MACH2 TO THE EXPONENTIAL VARIABLE
ERL3   FVARIABLE   (&M+P1)/&RATE3     ASSIGN THE RATE OF MACH3 TO THE EXPONENTIAL VARIABLE
TOT1   FVARIABLE   &SET1*FN6+V$ERL1
TOT2   FVARIABLE   &SET2*FN6+V$ERL2
TOT3   FVARIABLE   &SET3*FN6+V$ERL3
       ADVANCE     V$TOT1        PROCESS THE PIECE AT TOTAL SERVICE TIMEFOR MACHINE ONE
       RELEASE     MACH1         LEAVE MACH1 (INCLUDES BLOCKING)
       RELEASE     AVE           GATHER STATS ON NO BLOCKING
       SPLIT       1,MACH3       SPLIT THE PIECE INTO TWO: ONE GOES TO MACH2, THE OTHER TO MACH3
       ENTER       SMACH2        ENTER STORAGE FOR MACH2
       SEIZE       MACH2         SEIZE MACH2 FACILITY
       LEAVE       SMACH2        LEAVE THE MACH2 STORAGE
       ADVANCE     V$TOT2        PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE TWO
       RELEASE     MACH2         LEAVE MACH2
       QUEUE       PROD          COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
       DEPART      PROD          GATHER STATISTIC FOR PRODUCTION OF SYSTEM
BYBY   TERMINATE
MACH3  ENTER       SMACH3        ENTER THE STORAGE FOR MACH3
       SEIZE       MACH3         SEIZE FACILITY MACH3
       LEAVE       SMACH3        LEAVE THE STORAGE MACH3
       ADVANCE     V$TOT3        PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE THREE
       RELEASE     MACH3         LEAVE MACH3
       QUEUE       PROD          COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
       DEPART      PROD          GATHER STATISTICS FOR SYSTEM PRODUCTION
       TRANSFER    ,BYBY
*
*       MODEL SEGMENT 2
*
       GENERATE    10000
       TERMINATE   1
*
*       CONTROL CARDS
*
       LET         &M=250        THE MINIMUM LOG LENGTH IS 0.25 METERS
       LET         &K=7          K-ERLANG K VALUE IS 7
       LET         &SCALE=40     K-ERLANG SCALE PARAMETER IS 0.40 SECONDS
       LET         &SET1=300     SET-UP TIME FOR MACHINE 1 IS 3 SECONDS
```

```
LET        &SET2=300    SET-UP TIME FOR MACHINE 2 IS 3 SECONDS
LET        &SET3=300    SET-UP TIME FOR MACHINE 3 IS 3 SECONDS
UNLIST     CSECHO
*
*      IN THE FIRST SIMULATON RUN, VARY THE RATE OF MACHINE 1.  THE
*      BASE VALUES FOR THE OTHER TWO MACHINES ARE:
*      MACHINE 2 RATE = 1.20 METERS PER SEC.
*      MACHINE 3 RATE = 0.80 METERS PER SEC.
*      THE RATE OF MACHINE 1 IS VARIED FROM 0.50 TO 1.50 METERS PER SECOND
*
LET        &RATE1=0.50
LET        &RATE2=1.20
LET        &RATE3=0.80
DO         &I=1,20
START      5,NP
RESET
RMULT      1,2
START      18,NP
RMULT      -1,-2
START      18,NP
LET        &R1(&I)=&RATE1
LET        &P1(&I)=QC$PROD
LET        &BLOCK=(FT$MACH1-FT$AVE)/FT$MACH1*100
LET        &B1(&I)=&BLOCK
LET        &RATE1=&RATE1+0.05
CLEAR
ENDDO
*
*      IN THE SECOND SIMULATION RUN, VARY THE RATE OF MACHINE 2
*      THE BASE VALUES FOR THE OTHER TWO MACHINES ARE:
*      MACHINE 1 = 1.10 METERS PER SEC.
*      MACHINE 3 = 0.90 METERS PER SEC.
*      VARY THE RATE OF MACHINE 2 FROM 0.70 TO 1.70 METERS PER SECOND
*
LET        &RATE1=1.10
LET        &RATE2=0.70
LET        &RATE3=0.90
DO         &I=1,20
START      5,NP
RESET
RMULT      1,2
START      18,NP
RMULT      -1,-2
START      18,NP
LET        &R2(&I)=&RATE2
LET        &P2(&I)=QC$PROD
LET        &BLOCK=(FT$MACH1-FT$AVE)/FT$MACH1*100
LET        &B2(&I)=&BLOCK
LET        &RATE2=&RATE2+0.05
CLEAR
ENDDO
*
*      IN THIS SIMULATION RUN, VARY THE RATE OF MACHINE 3
*      THE BASE VALUES FOR THE OTHER TWO MACHINES ARE:
*      MACHINE 1 RATE = 1.00 METERS PER SEC.
*      MACHINE 2 RATE = 1.20 METERS PER SEC.
*      VARY THE RATE OF MACHINE 3 FROM 0.30 TO 1.30 METERS PER SECOND
*
LET        &RATE1=1.00
LET        &RATE2=1.20
```

```
LET      &RATE3=0.30
DO       &I=1,20
START    5,NP
RESET
RMULT    1,2
START    18,NP
RMULT    -1,-2
START    18,NP
LET      &R3(&I)=&RATE3
LET      &P3(&I)=QC$PROD
LET      &BLOCK=(FT$MACH1-FT$AVE)/FT$MACH1*100
LET      &B3(&I)=&BLOCK
LET      &RATE3=&RATE3+0.05
CLEAR
ENDDO
CALL     &DATA(&R1(1),&R2(1),&R3(1),&P1(1),&P2(1),&P3(1),&B1(1),&B2(1),&B3(1))
END
```

0.500	0.700	0.300	490.0	641.0	336.0	0.0	23.9	56.0
0.550	0.750	0.350	532.0	662.0	399.0	0.0	22.4	47.1
0.600	0.800	0.400	571.0	686.0	435.0	0.0	21.6	43.6
0.650	0.850	0.450	605.0	732.0	482.0	0.0	15.6	36.3
0.700	0.900	0.500	646.0	754.0	534.0	0.0	11.3	27.6
0.750	0.950	0.550	661.0	769.0	553.0	0.0	8.2	28.6
0.800	1.000	0.600	696.0	780.0	592.0	0.0	5.0	23.3
0.850	1.050	0.650	707.0	795.0	620.0	0.0	3.8	20.7
0.900	1.100	0.700	721.0	787.0	652.0	0.0	2.3	14.4
0.950	1.150	0.750	735.0	788.0	696.0	1.4	4.3	9.6
1.000	1.200	0.800	723.0	788.0	733.0	6.5	4.1	2.5
1.050	1.250	0.850	728.0	783.0	754.0	8.6	5.7	0.0
1.100	1.300	0.900	729.0	763.0	777.0	11.6	7.6	0.0
1.150	1.350	0.950	740.0	792.0	793.0	10.3	3.5	0.0
1.200	1.400	1.000	713.0	785.0	804.0	19.0	3.8	0.0
1.250	1.450	1.050	725.0	796.0	815.0	19.2	4.4	0.0
1.300	1.500	1.100	728.0	795.0	812.0	20.2	2.6	0.0
1.350	1.550	1.150	715.0	803.0	810.0	24.6	0.2	0.0
1.400	1.600	1.200	738.0	788.0	819.0	21.1	4.8	0.0
1.450	1.650	1.250	715.0	792.0	814.0	28.4	3.4	0.0

COLUMN 1 IS RATE OF MACHINE 1 VARIED (IN METERS PER SECOND)
COLUMN 2 IS RATE OF MACHINE 2 VARIED (IN METERS PER SECOND)
COLUMN 3 IS RATE OF MACHINE 3 VARIED (IN METERS PER SECOND)
COLUMN 4 IS SYSTEM PRODUCTION FROM MACHINE 1 VARIED (IN PIECES PER HOUR)
COLUMN 5 IS SYSTEM PRODUCTION FROM MACHINE 2 VARIED (IN PIECES PER HOUR)
COLUMN 6 IS SYSTEM PRODUCTION FROM MACHINE 3 VARIED (IN PIECES PER HOUR)
COLUMN 7 IS % BLOCKED TIME FROM FIRST RUN
COLUMN 8 IS % BLOCKED TIME FROM SECOND RUN
COLUMN 9 IS % BLOCKED TIME FROM THIRD RUN

APPENDIX H

THE EFFECT OF BLOCKING ON SECOND-STAGE QUEUES

AND

SYSTEM MEASURES AFFECTED BY SECOND-STAGE STORAGE CAPACITY

(INCL. SIMULATION PROGRAM AND DATA)

```
       SIMULATE
*
*       THE FIRST SIMULATION RUN OF THIS PROGRAM VARIES THE RATE OF
*       MACHINE 2 TO VARY FIRST-STAGE BLOCKING.  THE SECOND SIMULATION
*       RUN VARIES THE SIZE OF MACHINE 2 STORAGE FACILITY
*
*       FUNCTION DEFINITIONS
*
*       THIS IS AN EXPONENTIAL DISTRIBUTION FUNCTION WHICH WILL BE
*       USED TO OBTAIN A K-ERLANG DISTRIBUTION FUNCTION
*
 5     FUNCTION   RN1,C24
,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*       THIS IS A SEPARATE EXPONENTIAL DISTRIBUTION FUNCTION FOR
*       THE THREE MACHINE SET-UP TIMES.  IT HAS A DIFFERENT "SEED"
*       THAN THE ABOVE EXPONENTIAL DISTRIBUTION
*
 6     FUNCTION   RN2,C24
,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
*
*       THIS STATEMENT IDENTIFIES THE EXTERNAL SUBROUTINE THAT
*       STORES X-Y VALUES OF R1 VS. P INTO A FILE ACCESSIBLE FOR
*       THE TELLAGRAF GRAPHING SOFTWARE PACKAGE
*
       EXTERNAL    &DATA
*
*       STORAGE CAPACITY DEFINITIONS
*
       STORAGE     S$SMACH1,100/S$SMACH3,50/S$SMACH2,10
*
       REALLOCATE COM,100000
*
*
*       DECLARE ALL AMPERVARIABLES USED IN THE PROGRAM
*
       REAL        &RATE1,&RATE2,&RATE3,&SET1,&SET2,&SET3,&SCALE,&M,&K
       REAL        &BLOCK,&R2(20),&P1(20),&P1B(20),&S2(20),&B1(20),&AC3B(20),&AC3(20),&AC2B(20)
       INTEGER     &I,&J
*
*       &RATE1  -  RATE OF MACHINE 1
*       &RATE2  -  RATE OF MACHINE 2
*       &RATE3  -  RATE OF MACHINE 3
*       &SET1   -  SETUP MEAN TIME FOR MACHINE 1
*       &SET2   -  SETUP MEAN TIME FOR MACHINE 2
*       &SET3   -  SETUP MEAN TIME FOR MACHINE 3
*       &SCALE  -  MEAN VALUE OF EXPONENTIAL DIST. FOR GENERATION OF
*                  ERLANG RANDOM VARIABLES
*       &K      -  K VALUE FOR ERLANG DISTRIBUTION
*       &M      -  MINIMUM LOG LENGTH
*
*
*       MODEL SEGMENT 1
*
```

```
      GENERATE    100,FN5       GENERATE INTERARRIVALS OF 100 TIME UNITS
      GATE SNF    SMACH1        IF STORAGE SMACH1 IS FULL, BLOCK ARRIVALS
      ENTER       SMACH1        ENTER STORAGE SMACH1
      SEIZE       MACH1         SEIZE MACH1 FACILITY
      GATE SNF    SMACH2        IF STORAGE MACH2 IS FULL, DO NOT LEAVE STORAGE SMACH1
      GATE SNF    SMACH3        IF STORAGE MACH3 IS FULL, DO NOT LEAVE STORAGE SMACH1
      SEIZE       AVE           STATISTICS FOR NO BLOCK SERVICE TIME
      LEAVE       SMACH1
*
*
*      THIS ROUTINE CREATES ERLANG RANDOM NUMBERS WITH A K VALUE OF 7.
*      THESE RANDOM VARIABLES ARE THEN ASSIGNED TO PARAMETERS 1
*      TO GIVE THE MACHINES 1,2 AND 3 THEIR RESPECTIVE PROCESSING TIMES.
*      THE AMPERVARIABLES &R1, &R2 AND &R3 GIVES THE MACHINES THEIR
*      RESPECTIVE PROCESSING RATES, WHICH CAN BE VARIED BY EXTERNAL
*      CONTROL CARDS.
*
*
ONE   ASSIGN      4,&SCALE,5    ASSIGN AN EXPONENTIAL RANDOM NUMBER TO PARAMETER 4
      ASSIGN      1+,P4         ADD THE EXPONENTIAL RANDOM NUMBER TO PARAMETER 1
      ASSIGN      5+,1
      TEST GE     P5,&K,ONE
ERL1  FVARIABLE   (&M+P1)/&RATE1     ASSIGN THE RATE OF MACH1 TO THE EXPONENTIAL VARIABLE
ERL2  FVARIABLE   (&M+P1)/&RATE2     ASSIGN THE RATE OF MACH2 TO THE EXPONENTIAL VARIABLE
ERL3  FVARIABLE   (&M+P1)/&RATE3     ASSIGN THE RATE OF MACH3 TO THE EXPONENTIAL VARIABLE
TOT1  FVARIABLE   &SET1*FN6+V$ERL1
TOT2  FVARIABLE   &SET2*FN6+V$ERL2
TOT3  FVARIABLE   &SET3*FN6+V$ERL3
      ADVANCE     V$TOT1        PROCESS THE PIECE AT TOTAL SERVICE TIMEFOR MACHINE ONE
      RELEASE     MACH1         LEAVE MACH1 (INCLUDES BLOCKING)
      RELEASE     AVE           GATHER STATS ON NO BLOCKING
      SPLIT       1,MACH3       SPLIT THE PIECE INTO TWO: ONE GOES TO MACH2, THE OTHER TO MACH3
      ENTER       SMACH2        ENTER STORAGE FOR MACH2
      SEIZE       MACH2         SEIZE MACH2 FACILITY
      LEAVE       SMACH2        LEAVE THE MACH2 STORAGE
      ADVANCE     V$TOT2        PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE TWO
      RELEASE     MACH2         LEAVE MACH2
      QUEUE       PROD          COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
      DEPART      PROD          GATHER STATISTIC FOR PRODUCTION OF SYSTEM
BYBY  TERMINATE
MACH3 ENTER       SMACH3        ENTER THE STORAGE FOR MACH3
      SEIZE       MACH3         SEIZE FACILITY MACH3
      LEAVE       SMACH3        LEAVE THE STORAGE MACH3
      ADVANCE     V$TOT3        PROCESS THE PIECE AT TOTAL SERVICE TIME FOR MACHINE THREE
      RELEASE     MACH3         LEAVE MACH3
      QUEUE       PROD          COUNT TOTAL PRODUCTION THROUGHPUT OF SYSTEM
      DEPART      PROD          GATHER STATISTICS FOR SYSTEM PRODUCTION
      TRANSFER    ,BYBY
*
*      MODEL SEGMENT 2
*
      GENERATE    10000
      TERMINATE   1
*
*      CONTROL CARDS
*
      LET         &M=250        MINIMUM LOG LENGTH IS 0.25 METERS
      LET         &K=7          K-ERLANG K VALUE IS 7
      LET         &SCALE=40     K-ERLANG SCALE PARAMETER IS 0.40 SECONDS
      LET         &SET1=300     SET-UP TIME FOR MACHINE 1 IS 3 SECONDS
```

```
       LET        &SET2=300    SET-UP TIME FOR MACHINE 2 IS 3 SECONDS
       LET        &SET3=300    SET-UP TIME FOR MACHINE 3 IS 3 SECONDS
       UNLIST     CSECHO
*
*       THE FIRST SIMULATION RUN INTENTIONALLY CREATES FIRST-STAGE
*       BLOCKING BY MACHINE 2.  THE RATE OF MACHINE 3 ENSURES A
*       QUEUE IN STORAGE 3, BUT WILL NOT PROVIDE FIRST-STAGE BLOCKING.
*       THE RATE OF MACHINE 2 IS VARIED, TO VARY THE FIRST-STAGE BLOCKING.
*
       LET        &RATE1=1.00
       LET        &RATE2=0.40
       LET        &RATE3=0.90
       DO         &I=1,20
       START      5,NP
       RESET
       RMULT      1,2
       START      18,NP
       RMULT      -1,-2
       START      18,NP
       LET        &R2(&I)=&RATE2
       LET        &P1(&I)=QC$PROD
       LET        &BLOCK=(FT$MACH1-FT$AVE)/FT$MACH1*100
       LET        &B1(&I)=&BLOCK
       LET        &AC3(&I)=SA$SMACH3
       LET        &RATE2=&RATE2+0.05
       CLEAR
       ENDDO
*
*       THIS SIMULATION RUN INTENTIONALLY HAS AN AVERAGE STORAGE 2
*       PIECE CONTENT OF 14 PIECES.  THERE IS NO BLOCKING BY MACHINE 3.
*       MACHINE 2 HAD A SPEED OF 0.9 METERS PER SECOND.
*       THE MACHINE 2 STORAGE CAPACITY IS VARIED FROM 1 TO 20 PIECES.
*       TWO MORE SIMULATION RUNS WERE MADE.  ONE RUN HAD A MACHINE 2
*       RATE OF 0.85 METERS PER SECOND.  THIS CREATED AN AVERAGE PIECE
*       CONTENT OF 17 BEFORE MACHINE 2.  THE OTHER RUN HAD A MACHINE 2
*       RATE OF 0.95 METERS PER SECOND.  THIS CREATED AN AVERAGE PIECE
*       CONTENT OF 7 BEFORE MACHINE 2.
*
       LET        &RATE1=1.00      METERS PER SECOND
       LET        &RATE2=0.95        "      "     "
       LET        &RATE3=1.00        "      "     "
       LET        &J=1
       DO         &I=1,20
       STORAGE    S$SMACH1,100/S$SMACH3,50/S$SMACH2,&J
       START      5,NP
       RESET
       RMULT      1,2
       START      18,NP
       RMULT      -1,-2
       START      18,NP
       LET        &P1B(&I)=QC$PROD
       LET        &AC3B(&I)=SA$SMACH3
       LET        &S2(&I)=&J
       LET        &AC2B(&I)=SA$SMACH2
       LET        &J=&J+1
       CLEAR
       ENDDO
       CALL       &DATA(&R2(1),&P1(1),&B1(1),&AC3(1),&P1B(1),&AC3B(1),&S2(1),&AC2B(1))
       END
```

0.400	410.0	48.1	0.1	630.0	0.1	1.0	0.2
0.450	448.0	42.6	0.1	728.0	0.5	2.0	0.7
0.500	483.0	40.0	0.1	729.0	0.8	3.0	1.3
0.550	526.0	33.2	0.3	739.0	0.6	4.0	2.5
0.600	568.0	27.8	0.4	757.0	1.5	5.0	3.3
0.650	591.0	24.7	0.3	768.0	1.7	6.0	3.7
0.700	643.0	19.1	0.7	765.0	1.7	7.0	4.5
0.750	672.0	15.7	0.8	755.0	2.5	8.0	6.1
0.800	699.0	13.0	1.6	775.0	2.4	9.0	5.0
0.850	723.0	9.8	2.7	741.0	0.9	10.0	7.8
0.900	744.0	9.0	7.2	778.0	5.2	11.0	5.4
0.950	759.0	5.2	13.6	761.0	1.5	12.0	9.3
1.000	777.0	1.1	19.8	767.0	2.2	13.0	9.8
1.050	777.0	0.6	30.2	753.0	1.7	14.0	11.2
1.100	790.0	0.0	24.0	755.0	2.5	15.0	12.3
1.150	779.0	0.0	25.7	752.0	1.7	16.0	13.5
1.200	773.0	0.1	24.4	759.0	3.4	17.0	12.9
1.250	780.0	0.0	20.5	761.0	3.0	18.0	15.1
1.300	777.0	0.0	20.1	753.0	2.1	19.0	16.3
1.350	766.0	1.8	36.1	774.0	1.2	20.0	13.7

FOR THE FIRST SIMULATION RUN:
COLUMN 1 IS RATE OF MACHINE 2 (IN METERS PER SECOND)
COLUMN 2 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
FOR THE SECOND SIMULATION RUN:
COLUMN 5 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 6 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 7 IS CAPACITY OF STORAGE 2 (IN PIECES)
COLUMN 8 IS AVERAGE QUEUE CONTENT IN STORAGE 2 (IN PIECES)

0.400	410.0	48.1	0.1	623.0	0.2	1.0	0.2
0.450	448.0	42.6	0.1	703.0	0.4	2.0	0.8
0.500	483.0	40.0	0.1	705.0	0.7	3.0	1.7
0.550	526.0	33.2	0.3	728.0	0.9	4.0	2.4
0.600	568.0	27.8	0.4	724.0	1.1	5.0	3.6
0.650	591.0	24.7	0.3	723.0	0.7	6.0	4.3
0.700	643.0	19.1	0.7	731.0	1.0	7.0	5.2
0.750	672.0	15.7	0.8	729.0	1.0	8.0	6.5
0.800	699.0	13.0	1.6	725.0	0.8	9.0	7.1
0.850	723.0	9.8	2.7	713.0	0.7	10.0	8.7
0.900	744.0	9.0	7.2	728.0	0.8	11.0	8.8
0.950	759.0	5.2	13.6	731.0	1.0	12.0	9.6
1.000	777.0	1.1	19.8	729.0	1.3	13.0	11.1
1.050	777.0	0.6	30.2	730.0	0.9	14.0	12.1
1.100	790.0	0.0	24.0	732.0	1.2	15.0	13.2
1.150	779.0	0.0	25.7	734.0	1.0	16.0	14.0
1.200	773.0	0.1	24.4	729.0	0.8	17.0	14.7
1.250	780.0	0.0	20.5	731.0	0.6	18.0	14.7
1.300	777.0	0.0	20.1	738.0	1.8	19.0	16.6
1.350	766.0	1.8	36.1	741.0	1.5	20.0	16.6

THIS DATA IS FOR THE SIMULATION STUDY WHERE THE SECOND RUN HAS
A MACHINE 2 RATE OF 0.60 METERS PER SECOND.
FOR THE FIRST SIMULATION RUN:
COLUMN 1 IS RATE OF MACHINE 2 (IN METERS PER SECOND)
COLUMN 2 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
FOR THE SECOND SIMULATION RUN:
COLUMN 5 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 6 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 7 IS CAPACITY OF STORAGE 2 (IN PIECES)
COLUMN 8 IS AVERAGE QUEUE CONTENT IN STORAGE 2 (IN PIECES)

0.400	410.0	48.1	0.1	660.0	0.3	1.0	0.2
0.450	448.0	42.6	0.1	747.0	0.9	2.0	0.6
0.500	483.0	40.0	0.1	758.0	1.0	3.0	1.0
0.550	526.0	33.2	0.3	772.0	1.2	4.0	2.0
0.600	568.0	27.8	0.4	775.0	3.0	5.0	2.7
0.650	591.0	24.7	0.3	769.0	0.9	6.0	3.4
0.700	643.0	19.1	0.7	774.0	2.1	7.0	4.6
0.750	672.0	15.7	0.8	774.0	1.8	8.0	5.6
0.800	699.0	13.0	1.6	783.0	4.2	9.0	5.0
0.850	723.0	9.8	2.7	787.0	5.4	10.0	7.2
0.900	744.0	9.0	7.2	785.0	3.9	11.0	6.7
0.950	759.0	5.2	13.6	778.0	4.3	12.0	9.2
1.000	777.0	1.1	19.8	785.0	2.1	13.0	9.5
1.050	777.0	0.6	30.2	784.0	1.8	14.0	10.6
1.100	790.0	0.0	24.0	789.0	5.0	15.0	10.1
1.150	779.0	0.0	25.7	788.0	3.0	16.0	8.7
1.200	773.0	0.1	24.4	778.0	1.4	17.0	7.3
1.250	780.0	0.0	20.5	775.0	1.8	18.0	6.8
1.300	777.0	0.0	20.1	774.0	6.8	19.0	14.9
1.350	766.0	1.8	36.1	786.0	6.4	20.0	13.8

THIS DATA IS FOR THE SIMULATION STUDY WHERE THE SECOND RUN HAS
A MACHINE 2 RATE OF 0.90 METERS PER SECOND.
FOR THE FIRST SIMULATION RUN:
COLUMN 1 IS RATE OF MACHINE 2 (IN METERS PER SECOND)
COLUMN 2 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 3 IS % TIME STAGE ONE IS BLOCKED
COLUMN 4 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
FOR THE SECOND SIMULATION RUN:
COLUMN 5 IS SYSTEM PRODUCTION (IN PIECES PER HOUR)
COLUMN 6 IS AVERAGE QUEUE CONTENT IN STORAGE 3 (IN PIECES)
COLUMN 7 IS CAPACITY OF STORAGE 2 (IN PIECES)
COLUMN 8 IS AVERAGE QUEUE CONTENT IN STORAGE 2 (IN PIECES)

```
      SUBROUTINE DATA(R2,P1,B1,AC3,P1B,AC3B,S2,AC2B)
      REAL R2(20),P1(20),B1(20),AC3(20),P1B(20),AC3B(20),S2(20)
     +,AC2B(20)
      INTEGER I
      CALL FTNCMD('ASSIGN 1=DATAS2;')
      DO 30 I=1,20
      WRITE (1,20) R2(I),P1(I),B1(I),AC3(I),P1B(I),AC3B(I),S2(I)
     +,AC2B(I)
20    FORMAT (' ',F5.3,2X,7(F6.1,2X))
30    CONTINUE
      RETURN
      END
```