C.1

DIE AND MOULD MAKING USING THE POLYHEDRAL CONCEPT

by

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Bachelor Of Engineering, McMaster University, 1982

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

Department Of Mechanical Engineering

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

January 1984

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Date: January 31, 1984

Abstract

The ultimate goal of many engineering pursuits is the application of science and mathematics to the production of manufactured products. Manufacturing is the transformation of a designers's ideas into three-dimensional objects with practical application in the real world.

Manufacture of products require tools (dies, moulds, punches, etc.) in processes ranging from casting and injection-moulding to forging, punching and coining. These tools, as are all three-dimensional solids, are bounded by surfaces. Different manufacturing processes present different problems to designers; for example, shrinkage and flash in casting and spring-back in forging or deep drawing. The traditional approach in tool and die-making is based on experienced pattern-makers or sculptors making the required object based on engineering blue-prints as well as their own intuition and judgement. With the advent of high speed computers and numerically controlled machines, these traditional procedures can be incorporated into an integrated approach by applying CAD/CAM techniques. The purpose of this research is to develop such general methods for the modelling and making of dies and moulds.

Cavity dies consist of bounding surfaces that are either analytical or non-analytical. Analytical shapes are usually designed surfaces which are combinations of surface-elements represented by well known mathematical equations. Non-analytical shapes are often natural surfaces defined by randomly

measured data. These require sorting and ordering. In addition, shapes such as ducts, shells and bottles lend themselves to special treatments requiring the input of particular parameters for production of similar items over a long production run.

In the work which follows, all of these types of die-cavities have been examined. Examples are given to show how various requirements may be handled by an integrated CAD/CAM approach. Computer routines have been developed in such a way that no special skills in mathematics and programming are required on the part of the user of the programs which can be incorporated into a low cost, fully automated turn-key system.

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Acknowledgement

The author wishes to express his most sincere gratitude to his supervisor, Professor James P. Duncan, for his guidance throughout the preparation of his thesis, particularly for being so considerate, encouraging and enlightening.

The author also wishes to thank Mr. Alan Steeves for his help in developing many of the software routines used for this research, to Mr. Kenneth Law and Mr. Paul Louie for their help in the measurement of data, and to everybody in the Department of Mechanical Engineering for their warmth, cooperation and especially their sense of humour, without which the months spent at the Department would be a lot less enjoyable.

Support for this research was provided by the National Science and Engineering Research Council of Canada.

I. INTRODUCTION

1. THE INCIDENCE OF SURFACES IN ENGINEERING

All objects that exist in physical space are bounded or contained by surfaces. A surface can be considered as the interface between parts of space having different physical attributes. In most engineering applications, a surface is viewed as the interface between a solid object and its atmospheric surroundings.

Engineers have always been concerned with the design of three-dimensional objects, the characteristics of which are their bounding surfaces. An example is shown in Figure 1.1 which shows the model of a punch for forming an automobile rear lamp housing. This item has a surface which is a combination of simple analytical surface elements. Other surfaces, such as human anatomical parts, may not be analytical in nature. Figure 1.2 shows a model of a human face used for biomedical engineering research applications.

2. RELATIONS BETWEEN SURFACE DESIGN AND MANUFACTURING

Nearly all engineering pursuits lead to the design and manufacture of three-dimensional components. The ultimate goal of an engineer is the application of science and mathematics to the production of manufactured products with practical applications in the real world.

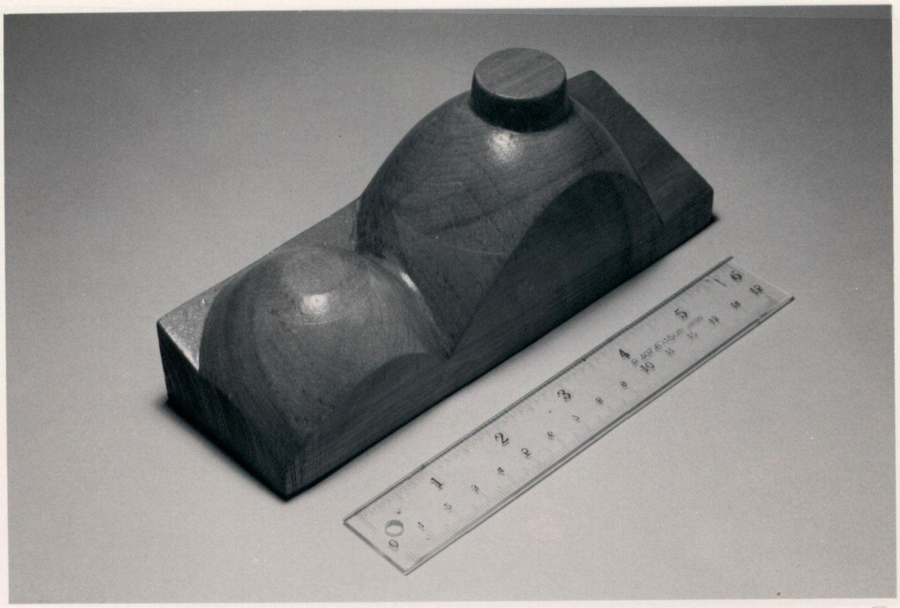


Figure 1.1 Model for the punching die of an automobile rear lamp housing

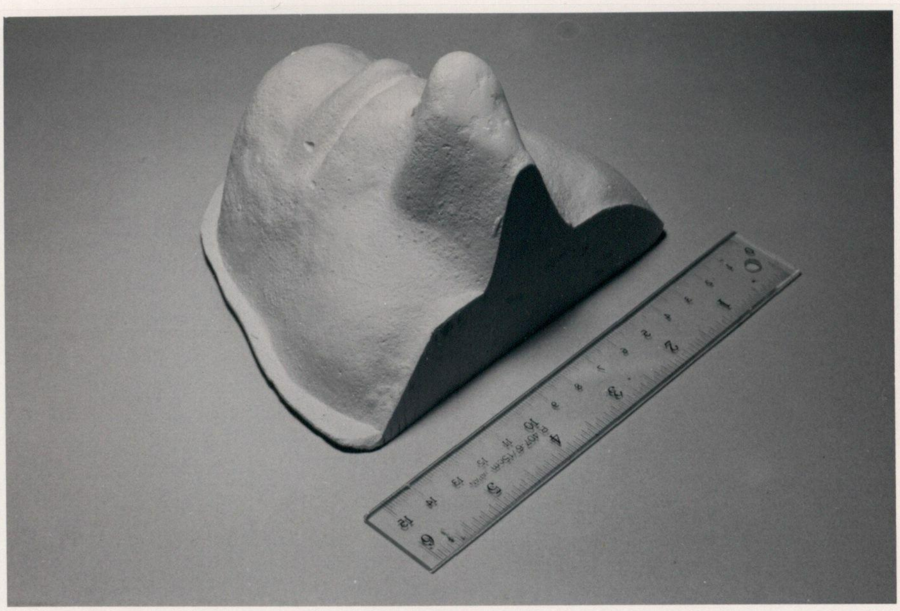


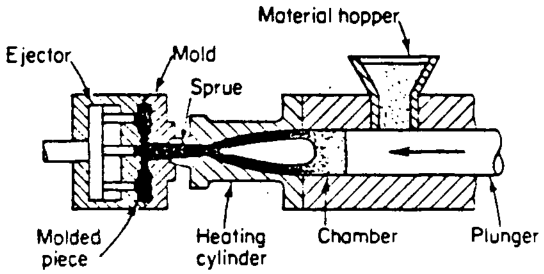
Figure 1.2 Model of a human face

The choice of surface-form for any engineering component is often the result of compromises between low manufacturing cost and functional requirements. For example, planes and cylinders can easily be generated and turned on simple machine tools, and are adopted as the building blocks of most designs. However, requirements in solid mechanics, fluid dynamics, acoustics, optics, etc., may necessitate complex surface-shapes and overrule the considerations of easy manufacture.

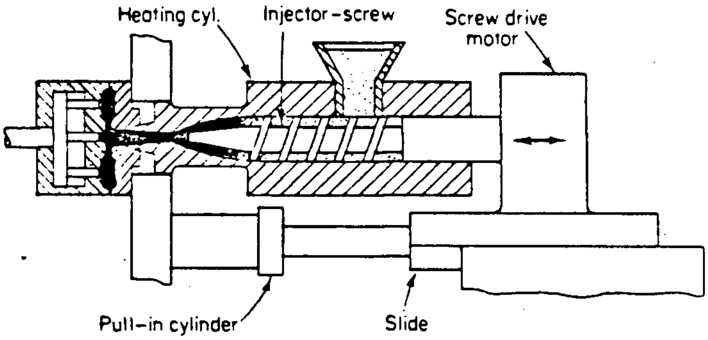
Manufacturing is the transformation of a designer's ideas into three-dimensional objects. More specifically, it can be considered as the forming of the bounding surfaces of a particular component by manipulation of various raw materials. Most manufacturing processes can be categorised into one of the following basic processes :

i <u>Casting / Moulding :</u>

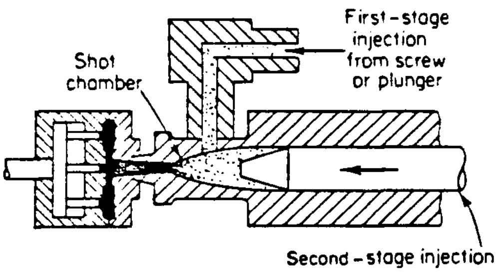
This includes processes such as sand and die casting, injection moulding, etc., and generally involves filling cavity-moulds or dies with liquid or plastic materials. Some examples are shown in Figure 1.3.



(i) Single-stage Plunger type



(ii) Single-stage Reciprocating Scew Type



(iii) Two-stage Plunger or Screw-Plasticsor types

Figure 1.3 Sketches of injection-moulding systems ii Mechanical Working :

Many shapes and forms are produced by mechanical working of metals in processes ranging from sheet metal rolling, forging, drawing to punching, hobbing and coining. Examples are shown in Figure 1.4

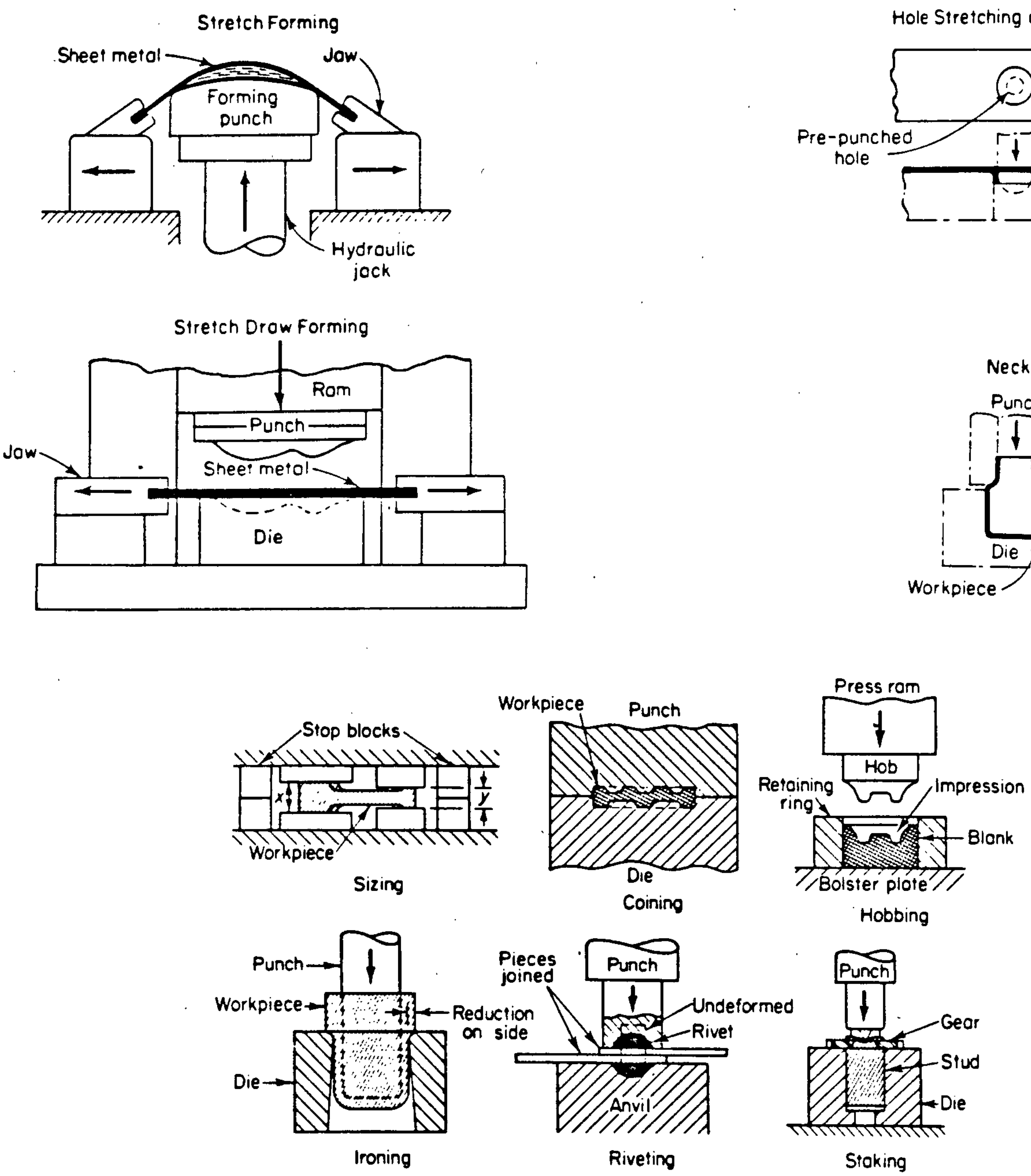


Figure 1.4 Some common metal stretching and squeezing operations

iii Joining

Complex structures are often fabricated by joining simpler elements using processes such as welding, brazing, soldering or adhesive bonding.

iv Cutting / Erosion

Many components are cut into their final shapes by means such as machining or flame cutting. Others are formed by chemical or electrical erosion in processes such as ECM (electrochemical machining) or EDM (electric discharge machining), to name only a few.

3. THE ROLES OF MOULDS AND DIES IN SURFACE-FORMING

In many of the processes described above (and shown in Figures 1.3 and 1.4), notably in casting, moulding, forging, punching, coining and hobbing, tools in the form of dies, moulds or punches are required. The design and making of dies and moulds are therefore very important for manufacturing.

These tools, as are all three-dimensional objects, are bounded by surfaces. In designing their bounding surfaces, a designer is faced with additional problems presented by different manufacturing processes. For instance, shrinkage and flash in casting processes, 'spring-back' in forging and deep-drawing, etc., must be taken into consideration during the design stage.

4. THE MAKING OF DIES AND MOULDS

Although all engineering designs exist in three-dimensional space, the traditional approach has always been for the designer to convey his ideas in two-dimensional drawings. In tool and die-making, the geometrical specifications have been presented in the form of blueprints. An experienced pattern-maker, following the instructions on the drawings as well as his own intuition and judgement, has, in the past, devised procedure of execution and the direction of machine tools to make the required product.

When surface-geometry is complicated, the object must be sculptured. Traditionally, this type of sculptured surface has been hand-made by experienced sculptors. A physical model is first sculptured on soft materials such as wax, plaster, clay or wood. Then the required mould is made using one of many reversal processes. Invaribly, the form of the final mould depends largely on the experience and skill of the sculptor and a certain degree of artistic license is always present. This may not necessarily be desirable in many scientific and technical applications where accuracy and repeatability is of critical importance.

With the development of numerically controlled machines in the past two decades, a more efficient and coherent approach based on the integration of digital computers and automated manufacturing systems can be adopted. Numerous CAD/CAM systems are available for different applications, but many of them are still following the traditional approach based on two-dimensional drawings. Figure 1.5 shows the drawing of a component to be made by an NC machine. The designer specifies the geometry and tool-paths are then deduced from the drawing. More advanced systems can automatically calculate the tool-paths and devise the machining sequence but generally they employ a 'two-and-a-half-D' approach. This is satisfactory for most engineering applications for which only simple analytic surface-elements such as planes and cylinders are present. Difficulty arises, however, when more complicated sculptured surfaces are required. It is the purpose of this research to develop a general and integrated approach to the modelling and making of dies and moulds using CAD/CAM techniques.

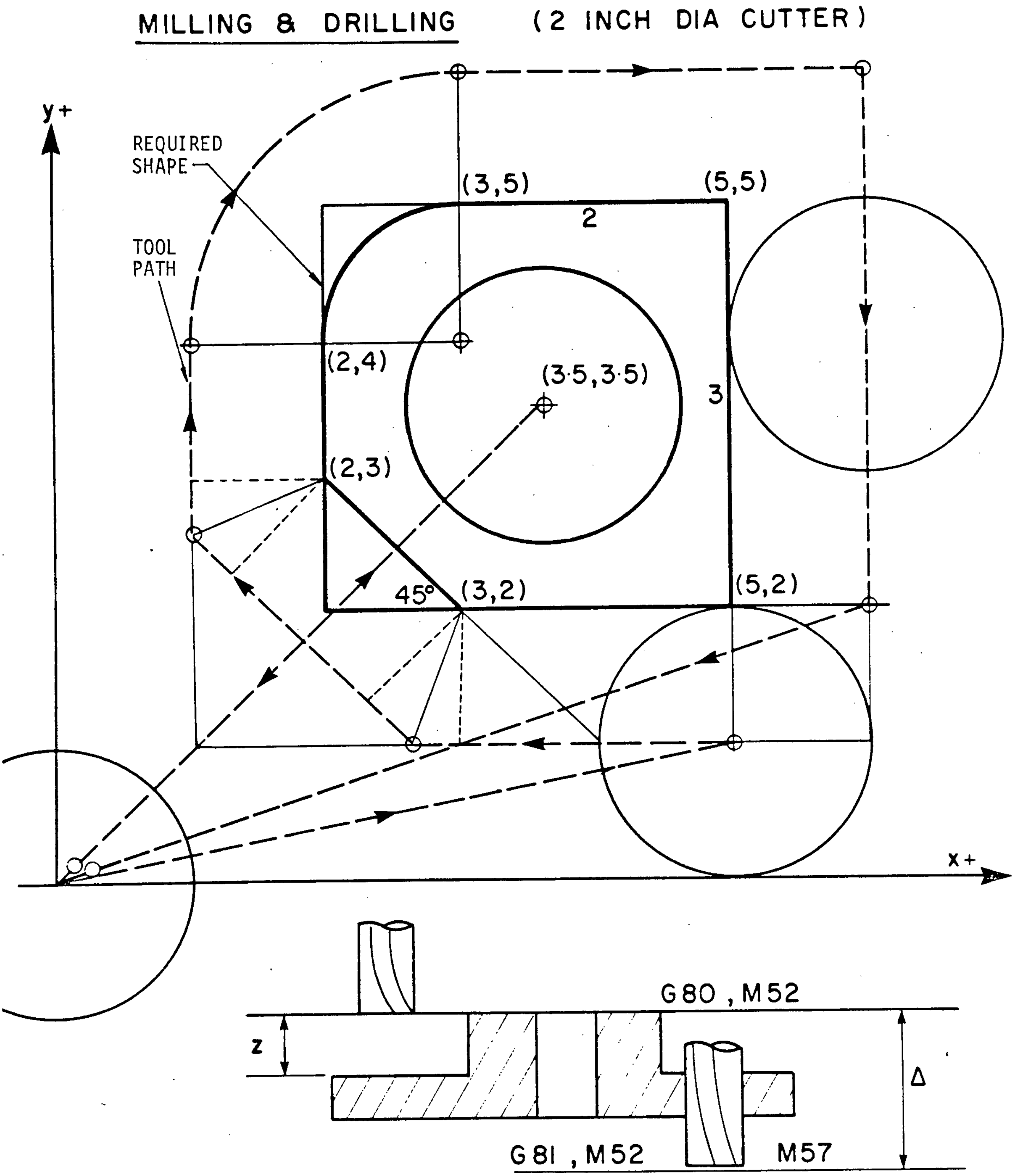


Figure 1.5 Tool-path for a cutter for milling and drilling

5. OBJECTIVES OF RESEARCH

Cavity moulds consist of bounding surfaces that are either analytical or arbitrary. Analytical surfaces are usually designed shapes containing surface-elements represented by mathematical equations. These include most engineering components. Arbitrary surfaces are usually natural surfaces defined by measured data. These ranges from natural landscapes to anatomical parts. Special surfaces, such as ducts, bottles and shells may require special treatments.

Examples of all three classes of surface described above have been examined in the work following. An integrated CAD/CAM approach for modelling and machining of these cavity-surfaces has been developed in this research. The main objectives are :

1. Modelling of Cavity-Surfaces :

   to develop general computer routines to generate analytical surfaces as encountered in many engineering applications ; to develop general procedures for the modelling of non-analytic surfaces .

2. Organization of the Machining Process :

   to generate cutter location data (CLD) to machine the generated surface .

3. Machining and Moulding :

   to machine dies and moulds from the cutter location data using numerically controlled machines, and to test techniques for forming physical components from such dies and moulds.

II. THE TECHNICAL/MATHEMATICAL FEATURES OF SURFACE AS AN ENTITY

1. PHYSICAL SURFACES DEFINED BY ANALYTICAL FUNCTIONS

A general surface can be considered as a continuous manifold of an infinite number of points in space determined by a space function. If this surface is imagined to exist in a Cartesian co-ordinate frame, the points representing the surface can be related by a functional relationship between the coordinates $F(x,y,z) = 0$. Any point P on the surface may be represented by its coordinates (x_p, y_p, z_p) or by the point position vector $\underline{R} = x_p\underline{i} + y_p\underline{j} + z_p\underline{k}$. Thus the manifold of points may be modelled mathematically by some function of two independant variables (x,y) or parameters (u,v). Three fundamental general forms are shown below :

Classical Form : $$F(x,y,z) = 0 \qquad (2.1)$$

Monge's Equation : $$z = F(x,y) \qquad (2.2)$$

Gauss' Form : $$x = F_1(u,v)$$
$$y = F_2(u,v) \qquad (2.3)$$
$$z = F_3(u,v)$$

If the function F(x,y) in z = F(x,y) is given or determined in some way as a mathematical equation, the value z with respect to (x,y) can be determined by analysis. If z exists within specified ranges of (x,y), the resulting surface is considered as an analytical surface.

2. PHYSICAL SURFACES AS A MANIFOLD OF POINTS

Most natural surfaces, such as anatomical surfaces cannot be represented by simple analytical functions. A single point P on one of these surfaces may be measured and its coordinates thought of as a vertical distance Zp above an arbitrary location whose horizontal coordinates are (x_p, y_p). In this case, the measured surface may still be a continuous manifold of points in space, but only a limited number of points on the surface are measured or defined (Figure 2.1). A large number of closely spaced measured points can readily be obtained and give an approximation to the surface; if more points at particular locations not in the given measured set are subsequently required, interpolations must be performed. The closeness of the approximation depends on the number of data points measured, and interpolations are based on the postulation that the surface has continuity of position, slope, and in some cases, curvature.

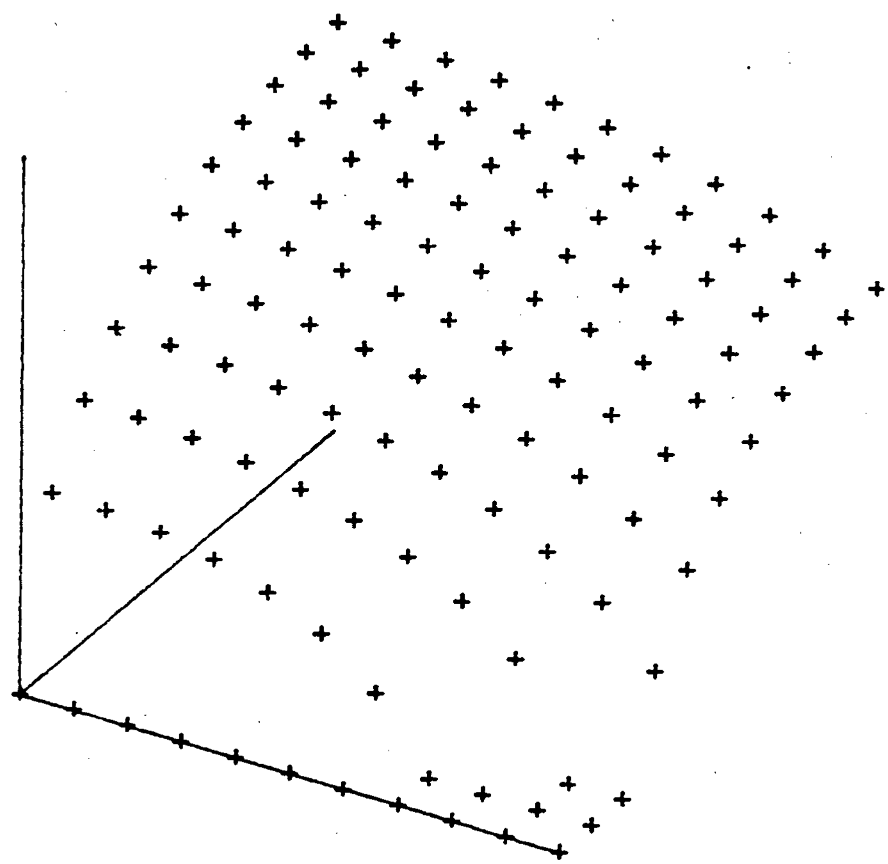


Figure 2.1 Physical surface defined by closely spaced data points

3. METHODS OF SURFACE DEFINITION

In general, surfaces can be defined in the following ways :

a) Analytical Surfaces

Most surfaces involved in engineering design consist of an assembly of simple surface-pieces (eg. cylinders, spheres) whose characteristic equations are well known. Such surfaces can thus be generated from analytical equations.

b) Physical Models

Frequently, surfaces are defined in the form of physical models. These require measurements by various means : mechanical, optical, or acoustical. In this case, the surfaces are represented by closely spaced random surface-points.

c) Surfaces Developed from Spatial Boundaries

In many engineering designs, a surface is determined by drawings of projections of its boundaries. To span a three dimensional surface from these two dimensional projections, various algorithms have been developed employing the ideas of proportional development or vector equations.[Duncan & Forsyth, 1977]

d) Computed Axial Tomography (CAT scan),
Positron Emission Tomography (PET scan), or
Nuclear Magnetic Resonance (NMR scan)

The technique of CAT scanning produces closely spaced slices of sections of bones and internal organs of the human body. By superimposing these slices, the shapes of internal organs can be obtained.[Portugual, 1982]

4. SURFACE INTERPOLATION

Unlike analytic surfaces, sculptured surfaces cannot be described by simple mathematical relationships. Although contour-measuring equipment has been highly developed and NC contouring machines can follow almost any surface, efficient means for mathematical description of arbitrary surfaces are required for development of 'smart' CAD/CAM systems.

Numerous methods to handle free-form sculptured surfaces been suggested, among which are Ferguson's Multivarible Curve Interpolation [Fergusion, 1964], Coon's Bi-Cubic Surface Patch [Coons, 1967] and Bezier's UNSURF system [Bezier, 1972]. These interpolation techniques generally employ high degree vector equations to build sets of elementary surface-patches interconnecting one another over a global field; and the solutions of these equations are found by specifying displacement and slope continuities at the boundary of each surface-elements.

To machine the sculptured surface using NC equipment, the cutter location data must be generated. When a spherically-ended milling cutter is used, the cutter location data is represented by an offset surface which is the locus of tool-centre points. Tool-positions can be determined by calculating the co-ordinates of a point which has an offset distance equal to the tool-radius along the normal vector at a surface-point. The polyhedral concept, an approach taken for this research, approximates the surface as an irregular polyhedron by connecting neighboring data points with facets. No attempt is made to avoid slope discontinuities, since the characteristics of all NC machines are such that they move linearly from one data point to the next; the end result is that all machined surfaces are actually polyhedrons, and no slope continuity is ensured. A more detailed description is provided in the next section.

5. THE POLYHEDRAL CONCEPT

POLYHEDRAL NC is a computer software package developed at the University of British Columbia in the years 1969 to 1976. It consists of a system of programs for the defintion and machining of sculptured surfaces using numerically controlled machines.

The basis of the polyhedral approach is to define the surface by a network of closely spaced discrete points in Cartesian coordinates and then approximate the surface by an irregular polyhedron with vertices being the surface-points. By joining adjacent points in sets of 3, triangular plane facets of the polyhedron are formed. The result resembles a cut gem stone. (Figure 2.2)

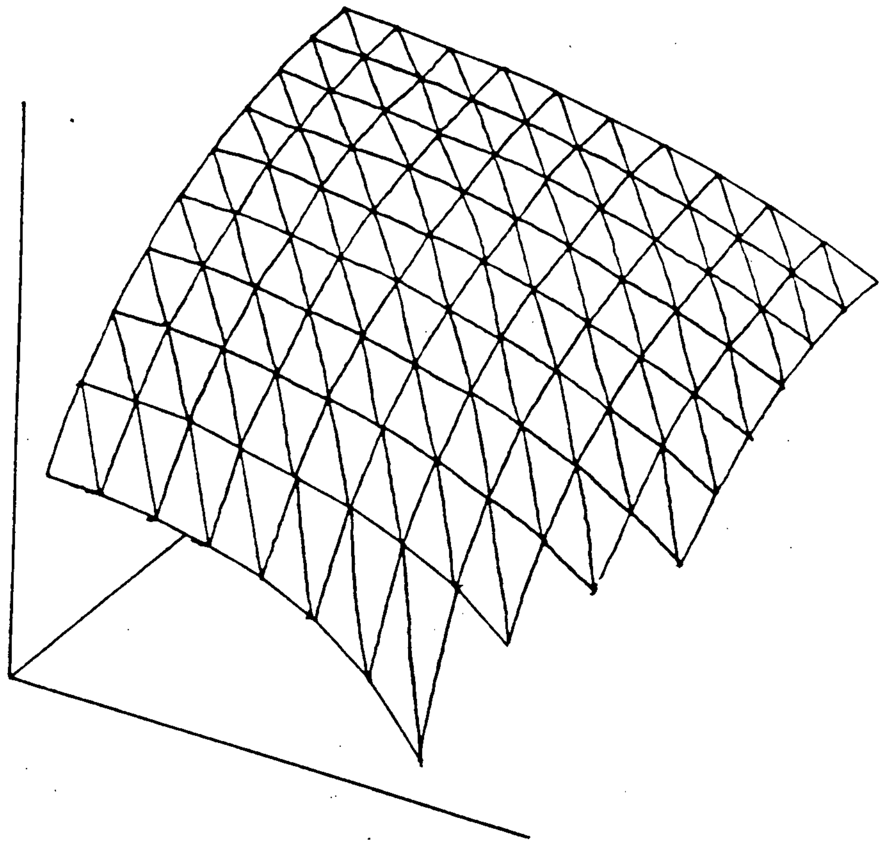


Figure 2.2 Surface approximation as an irregular polyhedron by joining adjacent points in sets of three to form triangular plane facets

Once the coordinates of the vertices are processed and the facets arranged in a logical order, a spherically-ended cutting tool can be directed to touch every facet of the polyhedron, one at a time. The position of the cutting tool, defined by coordinates known as the cutter location data (CLD), is found as follows :

Let $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, $P_3(x_3,y_3)$ represents the vertices of one facet. Since 3 points define one plane, a plane can be represented by the equation :

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \qquad (2.4)$$

or : $$Ax + By + Cz + D = 0 \qquad (2.5)$$

Dividing equation 2.5 by $\sqrt{A^2 + B^2 + C^2}$, the equation becomes :

$$\alpha x + \beta y + \gamma z + p = 0$$

where α, β and γ are the direction cosines of the normal the facet. (Figure 2.3)

Let C be the centroid of the facet whose coordinates (x_c, y_c, z_c) are found from :

$$x_c = (x_1 + x_2 + x_3) / 3$$

$$y_c = (y_1 + y_2 + y_3) / 3 \qquad (2.7)$$

$$z_c = (z_1 + z_2 + z_3) / 3$$

Let T be a point of distance R from C along the normal to the plane through C, then :

$$x_t = x_c + \alpha R$$

$$y_t = y_c + \beta R \qquad (2.8)$$

$$z_t = z_c + \gamma R$$

Now, if R is the radius of the spherically-ended cutting tool, T will be the tool-centre position at which the tool just touches (ie. is tangential) to the facet at its centroid. By repeating the above calculations for each facet, a series of points representing the CLD path can be obtained. (Figure 2.3)

Program SUMAIR in the POLYHEDRAL NC system employs the logic described above to calculate the tool-path for machining. Extensive mathematical analysis is performed to guide the tool in such a way to avoid undercutting of neighbouring facets when 'visiting' each facet.[Duncan & Mair, 1976]

Programs of the POLYHEDRAL NC system have been extensively used and tested in numerous projects throughout the years. It can be claimed that, as long as a single valued surface is represented by a table of points, the system is capable of replicating to a specified accuracy any physical surface. Documentation of system is found in Mair and Duncan, 1978.

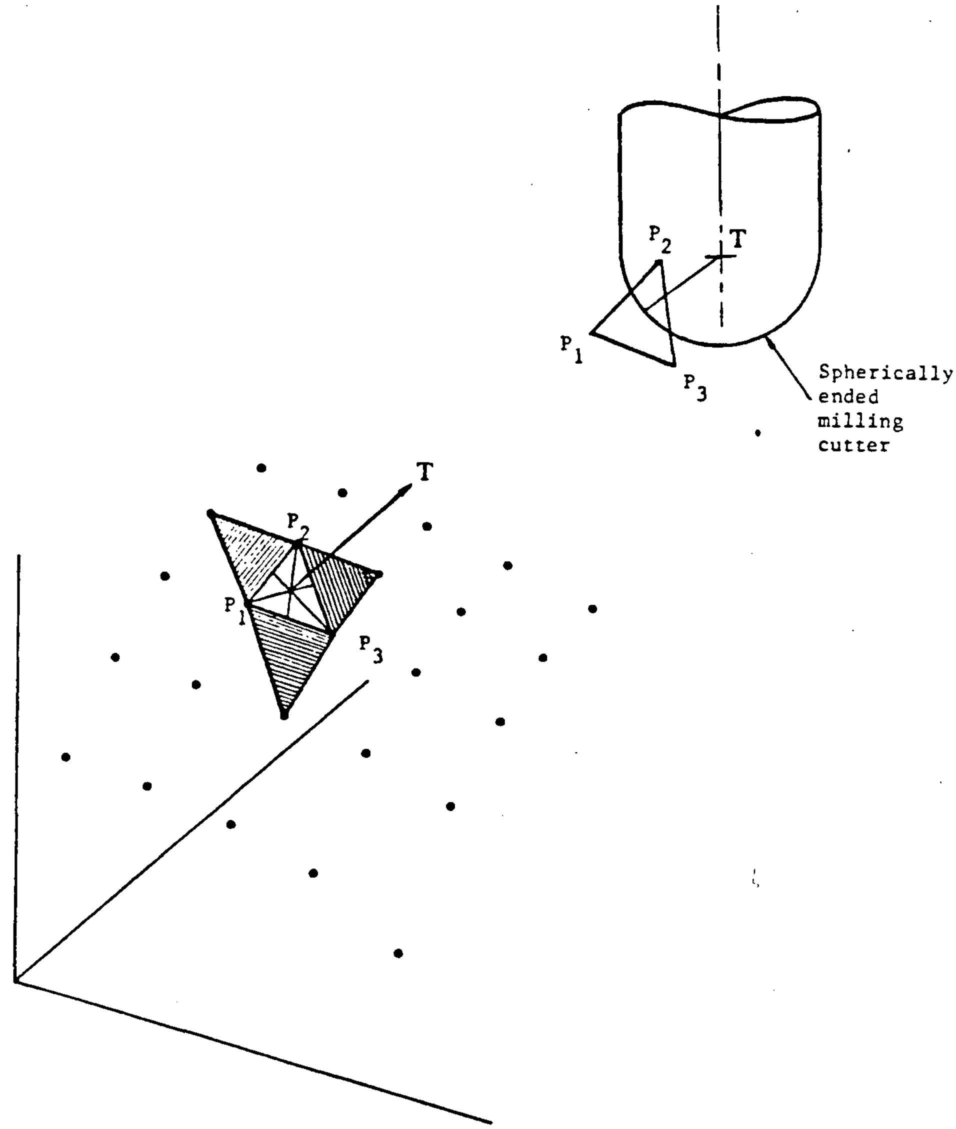


Figure 2.3 Calculation of cutter location data by polyhedral concept

III. SCULPTURED DIE-SURFÂCES

1. GENERAL FEATURES OF DIES

As described in the previous chapters, all engineering designs lead ultimately to some form of products with characteristic bounding surfaces that are either analytical or arbitrary (sculptured). In industries in which metals, plastics, ceramics and other materials are shaped by casting, moulding, mechanical working and other processes, many replications of these products are usually required.

To aid manufacturing either master forms, closely resembling the final product, or cavity-dies shaped to enclose it, are used. The design of such forms and cavities is based on the geometry of the required item as well as the problems imposed by different manufacturing processes. For example, a general dilation of the volume of a cavity-die used in hot casting is needed to account for the contraction of metal upon cooling. In this case, the die must differ in shape from the finished cold item.

1.1 Characteristics Of Die Cavities

Cavity-dies are designed to enclose or limit the flow of liquid or plastic material. When such material has solidified, the moulded product has to be extracted. In most common manufacturing processes, this requires the cavity to be split into two half cavities.

Figure 3.1 shows the typical features of a die cavity. The two half die-blocks are brought together along a common parting surface which is usually, but not necessarily, a plane. The cavity itself is enclosed by the 'ceiling' surface of the upper block, the 'floor' surface of the lower, and the side walls spanning the depth between the ceiling and the floor. The side walls usually slope, or 'draft', towards the parting surface to facilitate the removal of the solidified product. The 'parting line' is the intersection of the parting surface and the cavity-surface. Since materials tend to escape along the parting surface to form a 'flash', this line is also known as the 'flash-line'.

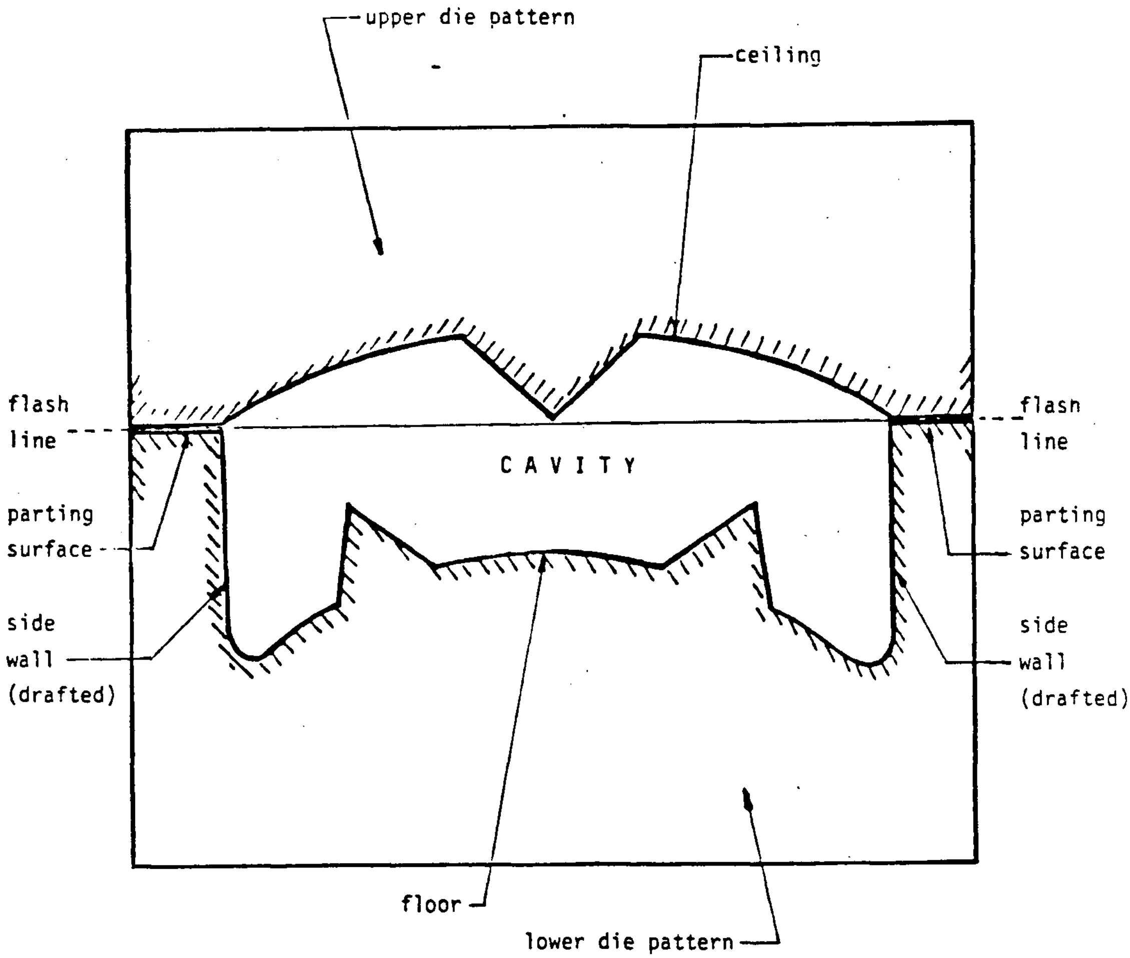


Figure 3.1 Typical features of a cavity-die

2. DESIGN AND MACHINING OF DIES USING THE CAD/CAM APPROACH

The traditional craft-based approach in the design and shaping of dies and forms can now be incoporated into a unified and integrated approach through the use of computers. Geometrical specifications of die-surfaces can be defined by either mathematical equations or measured data and stored in computer memories. Surface-properties can be computed and design adjustments may be applied virtually instantaneously using high speed computers and interactive graphics. Machine instructions are then generated to guide the cutting tool of a numerically controlled machine to create the surface.

2.1 Machining Of Dies By The POLYHEDRAL NC System

Many cavity-dies contain planes and right prisms or cylinders of general cross-section. These are well defined analytically and can be easily machined by a two-and-a-half-D (2-1/2 D) approach using one of the many available CAD/CAM systems. Others, however, incorporate difficult-to-define surfaces, usually compound in nature (ie. an assembly of many individual contiguous pieces), and cannot be generated in a 2-1/2 D manner.

Sculptured surfaces can be machined easily with the POLYHEDRAL NC system. With this approach, it is more satisfactory in many respects to machine the cavity directly. Besides the obvious advantage of saving manufacturing time, direct machining of the female mould generally gives a better surface-finish than machining the male model whenever a spherically-ended milling cutter is used. As can be seen from Figure 3.2, asperities or cusps are left between touches as the tool moves from one facet to the other. The heights of these cusps are dependant on the length of increments between touches as well as the local radii of curvature of the surface at the points which are touched by the tool.

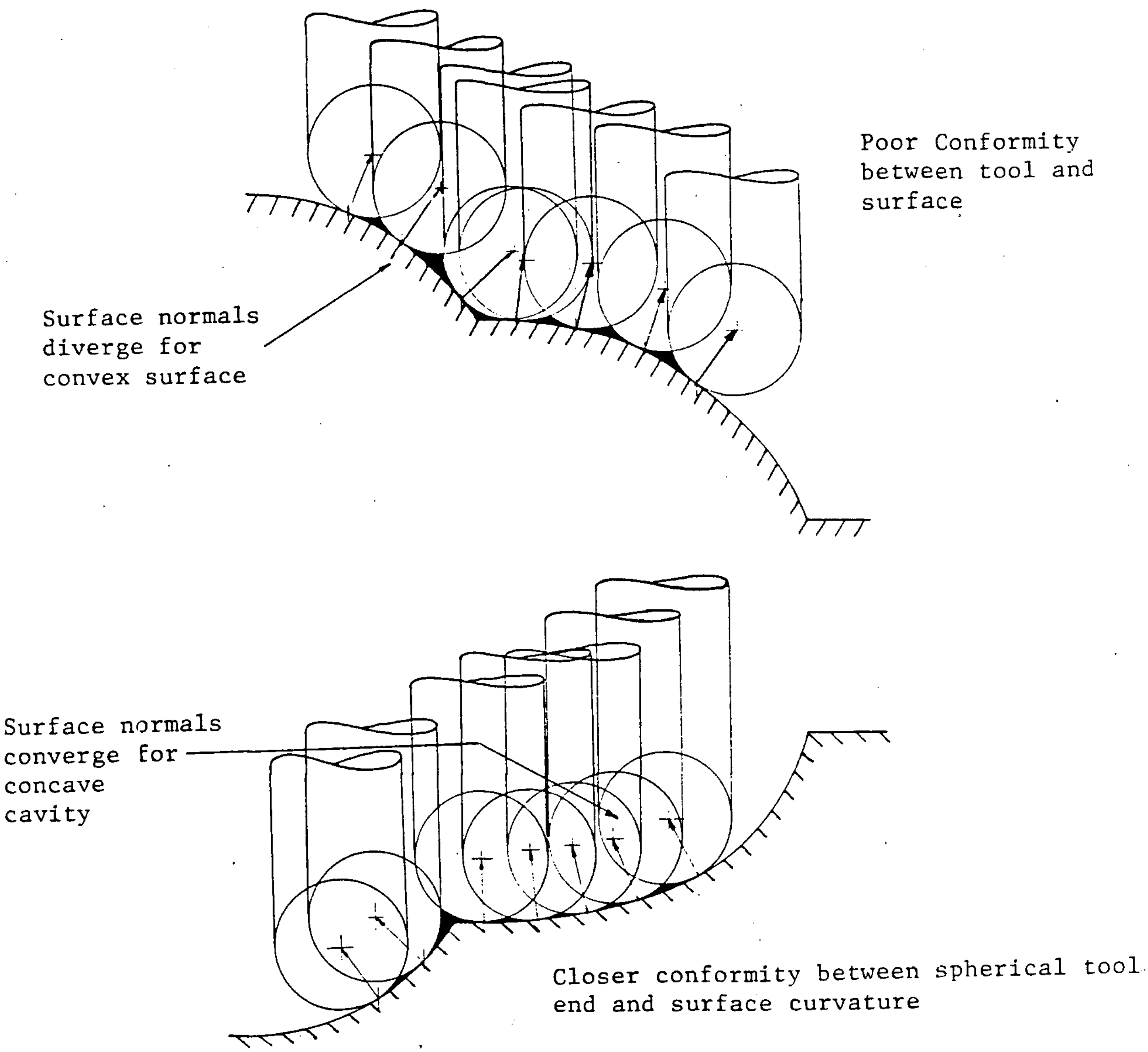


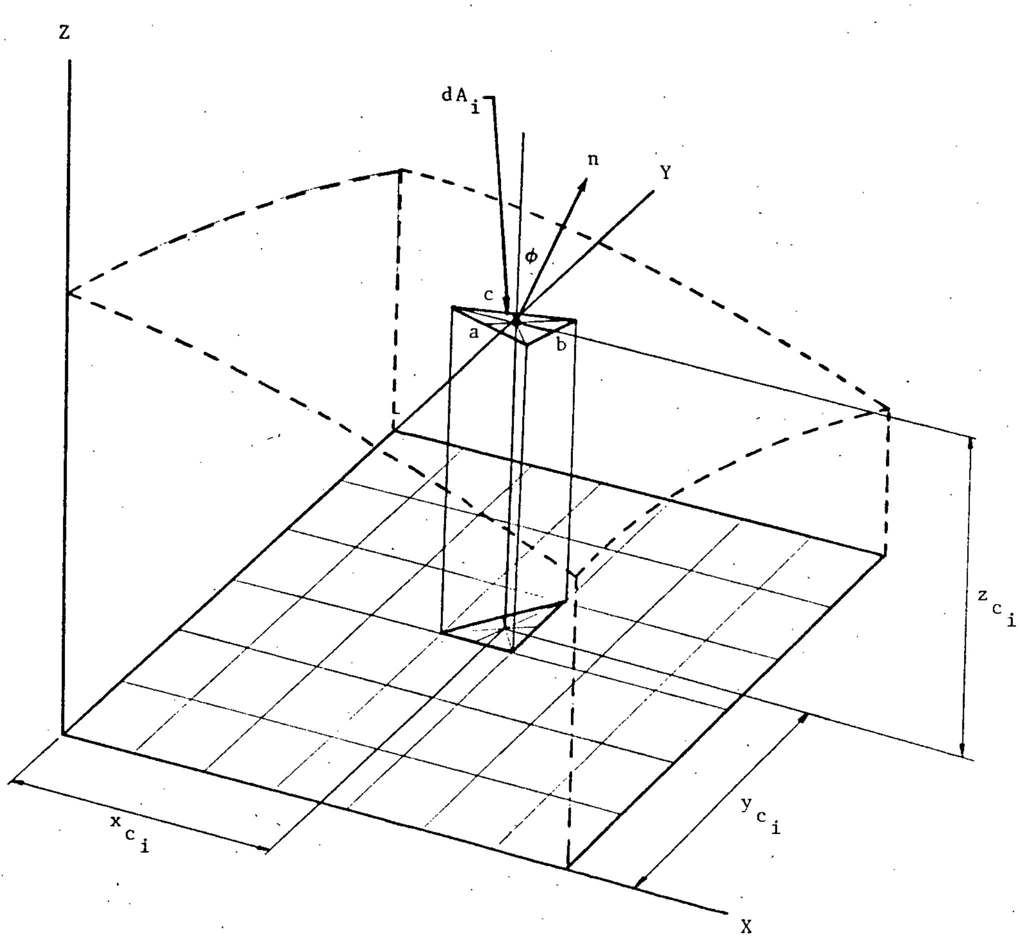
Figure 3.2 Closer conformity of spherically-ended milling cutter to die-surface curvatures gives better surface-finish when machining the female cavity

For a concave upwards surface, the height of the cusp around a surface-point is the function of the difference between the magnitudes of the tool radius and the radius of curvature of the surface at that point. Whereas for a concave downwards (convex) surface, the cusp height is a function of the sum of the two. Obviously, the female mould, which is usually concave upwards, will have better surface-finish when machined by a spherically-ended cutter. Moreover, the surface-normals on a convex surface diverge whereas they converge for a concave surface. The tool positions for different facets are closer together when machining the female mould, which in turn gives a better surface-finish in terms of asperities.

2.2 Computation Of Surface-related Properties Using The Polyhedral Concept

In many instances, it is desirable to have control over such properties as the enclosed volume or the surface-area of a cavity die. This is important, for example, when the volumetric content of a bottle has a prescribed value; or when the heat transfer characteristics of a casting are to be controlled.

By approximating the surface as a multi-faceted polyhedron, the calculations for many surface-properties can be easily achieved. For example, the volume, surface-area and centre of mass of an object can be computed as shown in Figure 3.3



dA_i - surface area of facet i

$(x_{c_i}, y_{c_i}, z_{c_i})$ - centroid coordinates of facet i

<u>Surface Area</u>

$$A = \sum_{i=1}^{n} dA_i$$

$$= \sum_{i=1}^{n} s_i(s_i-a_i)(s_i-b_i)(s_i-b_i)$$

where : $s = \frac{1}{2} (a_i + b_i + c_i)$

<u>Volume</u>

$$V = \sum_{i=1}^{n} \gamma_i dA_i z_{ci}$$

where : $\gamma_i = \cos \phi_i$

<u>Centre of Mass</u>

$$x_{CM} = \frac{\sum_{i=i}^{n} (\gamma_i dA_i z_{ci} * x_{ci})}{V}$$

$$y_{CM} = \frac{\sum_{i=1}^{n} (\gamma_i dA_i z_{ci} * y_{ci})}{V}$$

$$z_{CM} = \frac{\sum_{i=1}^{n} (\gamma_i dA_i z_{ci} * z_{ci})}{V}$$

Other parameters, such as moment of inertia, can also be found.

<u>Figure 3.3</u> Computation of surface-related properties using the polyhedral concept

IV. THE ANALYTICAL DIE

1. PIECEWISE ANALYTICAL AND COMPOUND SURFACES

Many die-cavities and punches are defined geometrically as compound interpenetration of several surface-elements blended together at their junctions. In engineering design, such compound surfaces are usually comprised of elements of various common analytical types intersecting one another at boundaries of discontinuity where the elements interpenetrate. Usually these surface types are second degree quadric surfaces, the most common being spheres and cylinders (Figures 4.1). Since these surfaces are represented by well-known analytical equations, suitable algorithms can be developed to model the required compound surfaces for many engineering applications.

2. MODELLING OF COMPOUND SURFACES USING THE METHOD OF HIGHEST POINT

A compound analytical surface is usually generated by simple surface elements interpenetrating one another. Each individual element is bounded by twisted space-curves of intersection. Although explicit solutions for these curves of interpenetration can be found by solving the equations of the intersecting surface-pieces, the mathematics involved are usually tedious and complicated, and the solutions one can expect may not yield any simple forms.

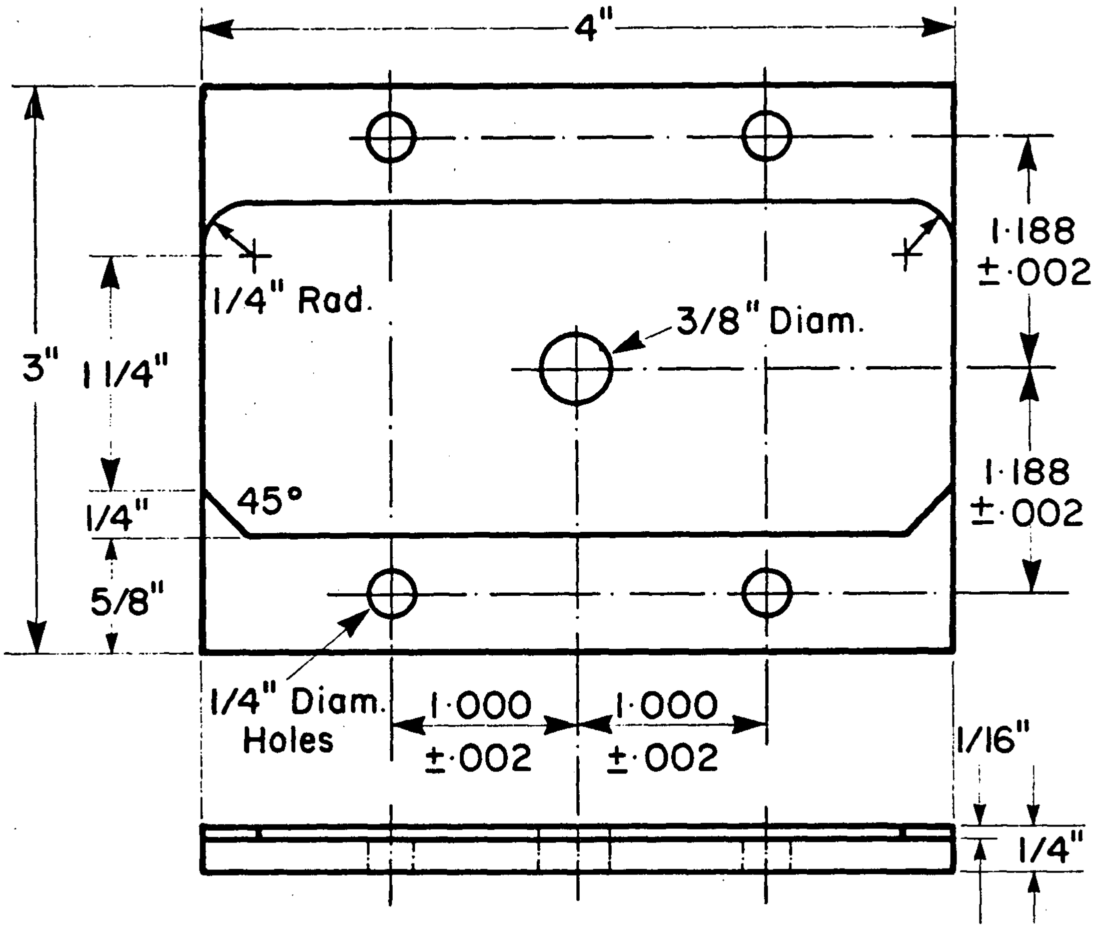


Figure 4.1 Typical engineering component containing simple analytic elements of prisms and cylinders

A simpler approach, known as the Method of Highest Point [Duncan & Mair, 1982], has been developed to define these compound surfaces. The basic approach is to take the highest point calculated from any set of surface-element equations in the domain of interest. If the surface-piece is defined by :

$$z_i = F_i (x,y) \qquad i = 1,2,3,....$$

At each location (x,y) over a fine rectangular grid in the plan view, the height z (when defined) of each piece can then be found. The height of the global surface at (x,y) is taken to be the maximum (ie the highest point) among the z s.

A two dimensional analogy is shown in Figure 4.2. Suppose 3 surface pieces f_1, f_2 and f_3 are defined within the global domain. By scanning along direction X with an increment Δ and calculating z_1, z_2 and z_3 at each grid point, the height of the global surface z at each point is taken to be the maximum of z_1, z_2 and z_3.

It can be seen that this method does not explicitly calculate the exact location of intersection between the surface-pieces, and the actual intersection may lie between neighboring grid points of different surface-pieces. However, if the increment Δ is small enough (ie., the rectangular grid is very dense), the curves of intersection can be closely approximated. When machined by a spherical cutter, as used in the POLYHEDRAL NC system, the sharp discontinuities are automatically filleted and smoothed.

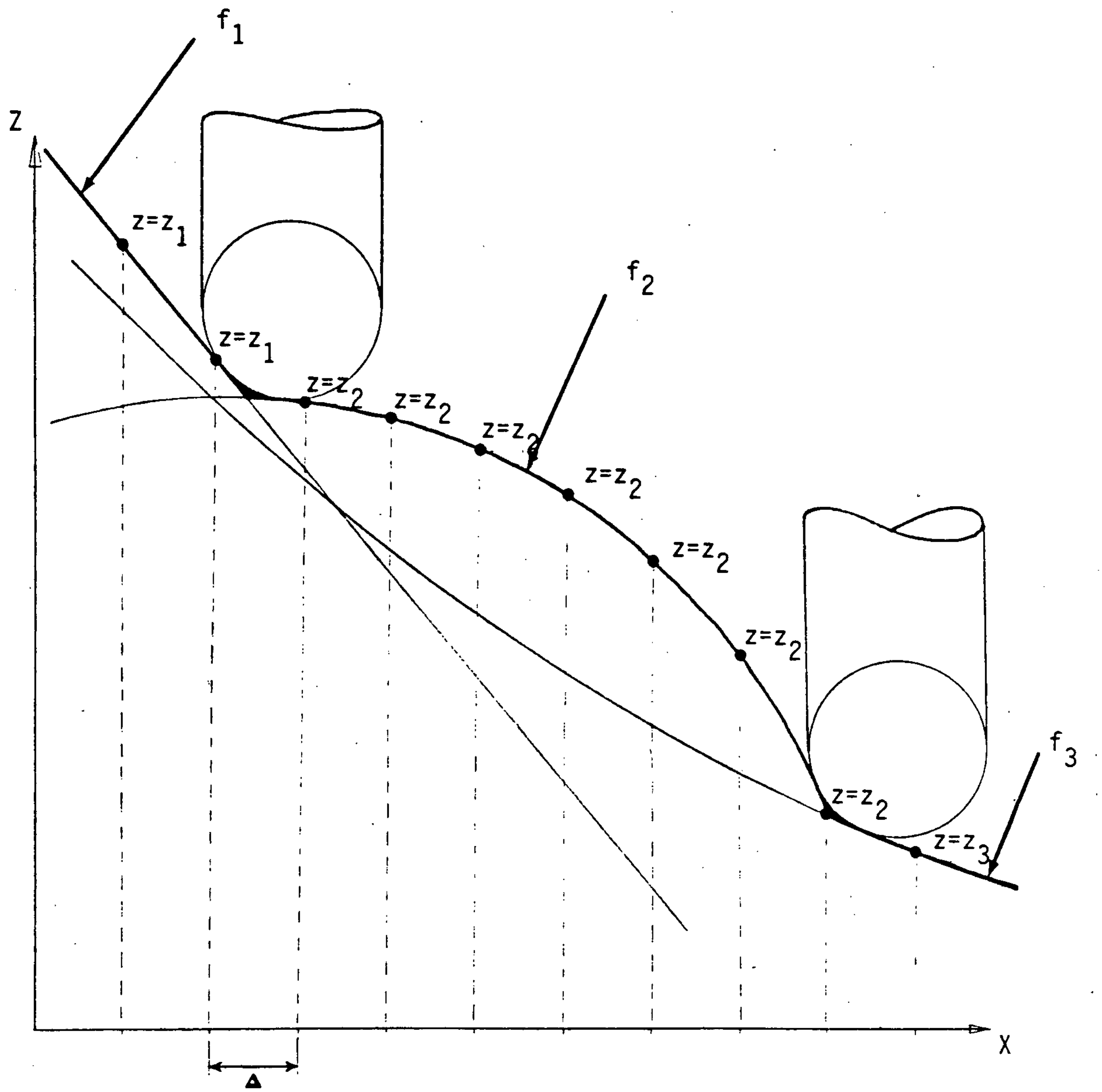


Figure 4.2 Modelling of compound surfaces using the Method of Highest Point.

By performing the above calculations over the entire global field, the tabulated points form the vertices of a multifacted polyhedron subtending the required continuous compound surface. Machining can then be automatically performed by the POLYHEDRAL NC system.

2.1 Subdomains Within The Global Domain

Often designers may wish to impose a 'window' on a specific surface-piece beyond which the piece does not exist. Usually such a window is a rectangular sub-domain within the rectangular global domain, with sides parallel to the global field. (Figure 4.3) In other instances, surface-adjustments may have to be performed at certain regions within the global field. Surface-adjusting functions, such as bi-β functions, can be applied over any sub-domains speicified by the designer. [Duncan & Vickers, 1980] Consequently, any general purpose surface definition program should allow a user to define sub-domains if so required.

2.2 Multivalued Surfaces And Natural Limits

When defining a surface in the form $z = F(x,y)$, it is possible that at any point (x,y), there is more than one z. (eg. spheres and ellipsoids) When using a milling machine for which turning is not possible, a multivalued surface can not be machined. When such cases occur, engineering judgement is required to choose one z value among the possibilities. Figure

4.4 shows some examples of the limits of existance of surface-pieces.

Surfaces such as non-vertical planes and paraboloids exist over the entire X, Y domain; others, such as spheres and cylinders, exist only within certain specific natural limits. (Eg. A sphere does not exist beyond its equator.) These natural limits must be tested to avoid undefined results when computing z.

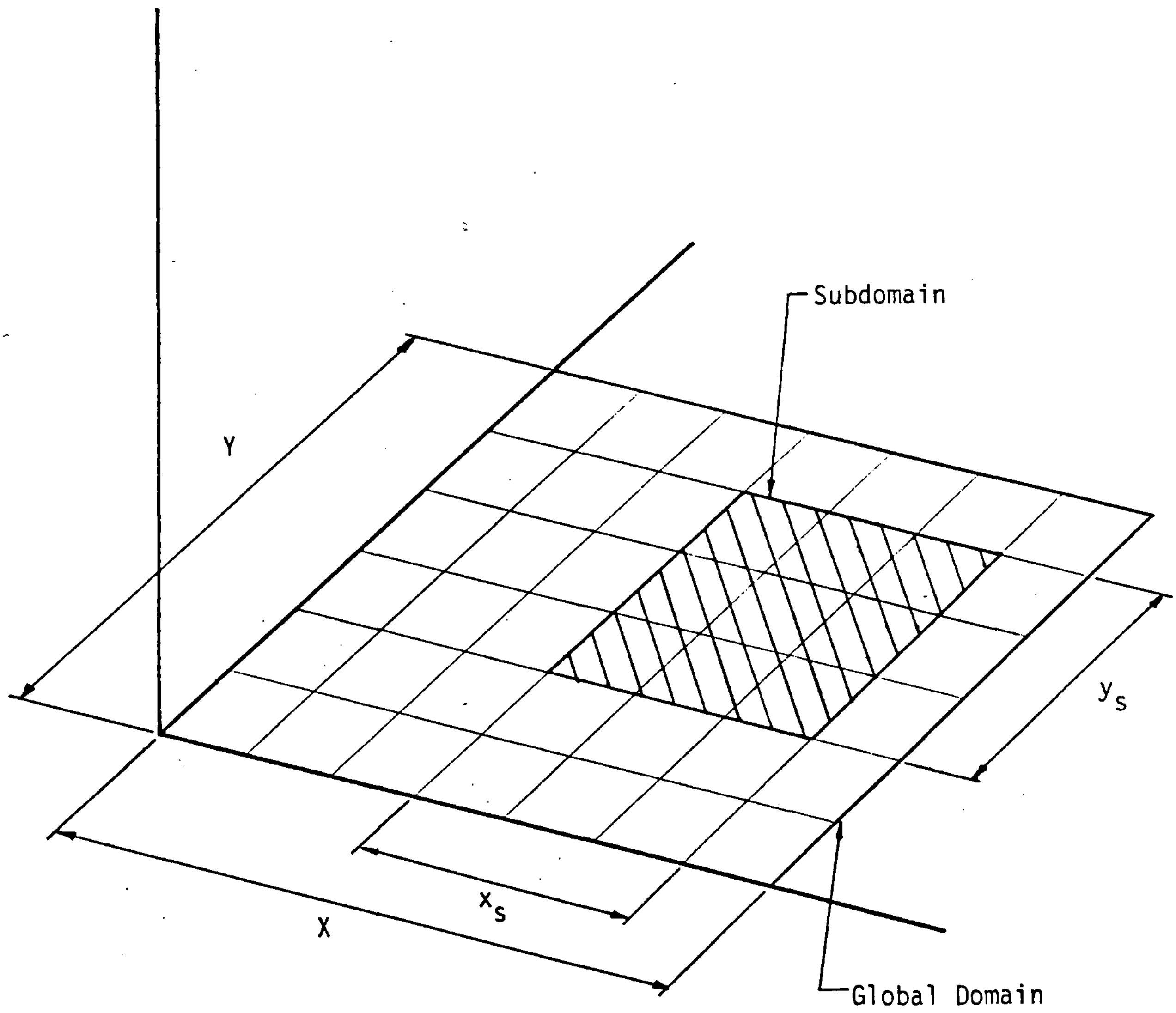


Figure 4.B Subdomain within global domain

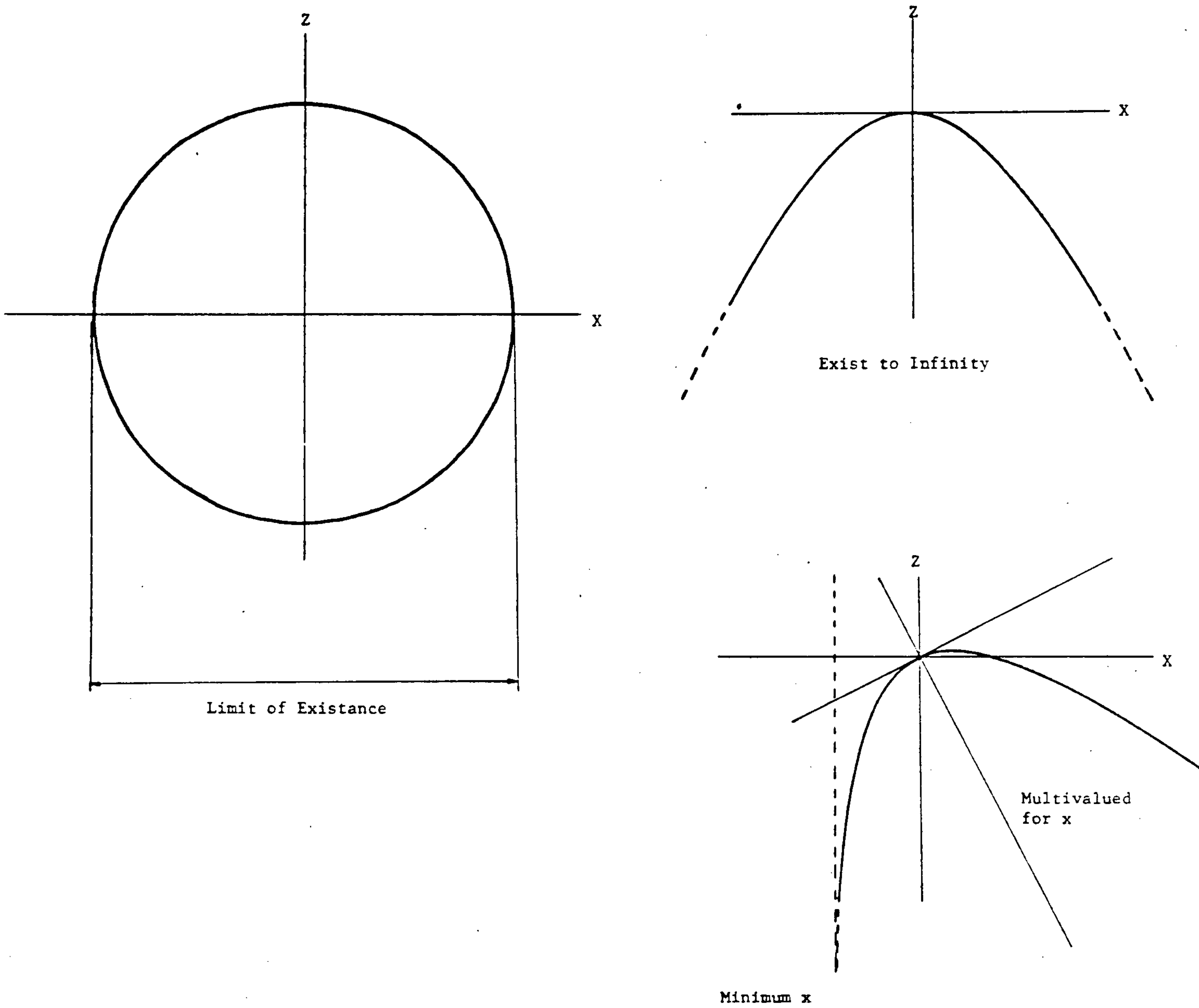


Figure 4.4 Natural limits for surface-elements

2.3 General Procedure For Executing The Method Of Highest Point

A macro algorithm for the execution of the Method of Highest Point is shown below :

1 Define the following parameters:

a) Global field limits: (XMIN,YMIN) & (XMAX, YMAX);

b) Increment for scan: Δ

c) Number of surface-pieces N and their types;

d) Limits ('window') for each individual piece.

2 Start scanning

```
FOR X:= XMIN to XMAX ;
    X:= XMIN + Δ ;
    FOR Y:= YMIN to YMAX ;
        Y:= YMIN + Δ ;
        FOR each surface-piece i:= 1 to N ;
            - check user defined window ;
            - check natural limit ;
            IF out-of-limit skip to next surface-piece ;
                ELSE :- calculate  Z(i) := F ( X,Y ) ;
                      - check highest point ;
                        IF  Z(i)  >  Z(i-1) keep z ;
        REPEAT for next surface-piece i+1 ;
    REPEAT for next Y ;
REPEAT for next X ;
Store ( X,Y,Z ) for each data point in data file ;
Stop.
```

3. GEN7 : A GENERAL PROGRAM FOR EXECUTING THE METHOD OF HIGHEST POINT

A general program, known as GEN7, has been developed to execute the Method of Highest Point for piecewise analytical surfaces. In its present form, the program can handle up to 3 each of the following types of surface-pieces (Figure 4.5) :

Quadric : ellipsoids (which include spheres) ;
elliptic (circular) paraboloids ;
hyperbolic paraboloids ;
quadratic cones ;
elliptic (circular) cylinders ;

Non-quadric : planes ;
tori ;
tubular surfaces of varying sections.

The user is prompted interactively for inputs in the form of convenient identifiable data, such as vertices, semi-axes, etc. In addition, rotations about the X, Y, Z axes for each quadric piece, the 'window' for each surface, and the truncation height can be specified.

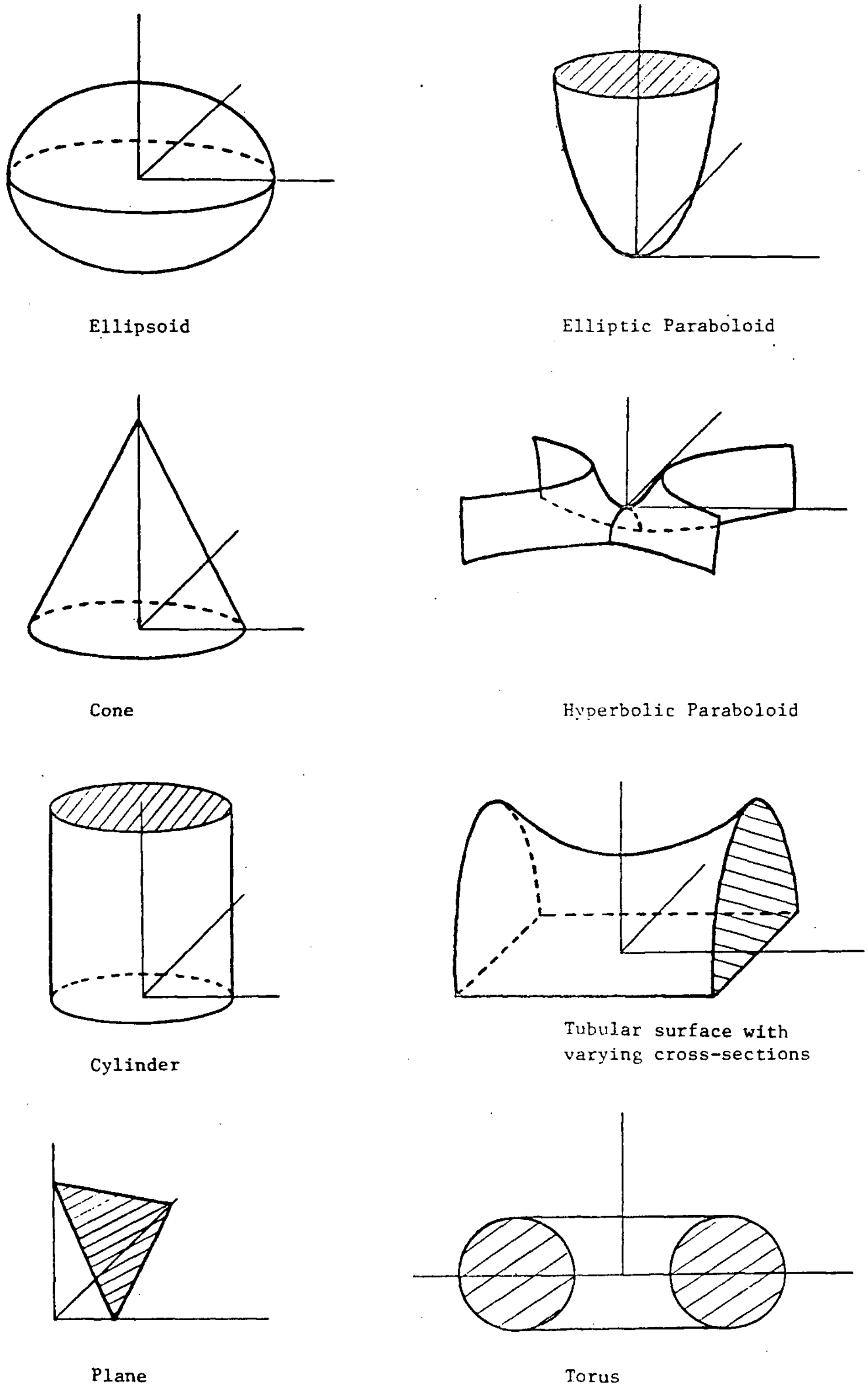


Figure 4.5 Surface elements for program GEN7

3.1 General Equation Of A Quadric Surface

Any quadric surface, in any orientation, can be represented by the following equation :

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Kz + L = 0 \quad (4.1)$$

Hence, for every known x and y, equation 4.1 can be simplified into :

$$A_1 z^2 + B_1 z + C_1 = 0 \quad (4.2)$$

where:

$A_1 = C$

$B_1 = Ey + Fx$

and: $C_1 = Ax^2 + By^2 + Dxy + Gx + Hy + L$

For $A_1 \neq 0$, equation 4.2 is a quadratic equation with variable z. To solve for z :

$$z = (-B_1 \pm \sqrt{B_1^2 - 4A_1C_1}) / 2A_1 \quad (4.3)$$

Two things can be noted from equation 4.3 :

i) For $B_1^2 - 4A_1C_1 > 0$, z has two values for every (x,y). Since POLYHEDRAL NC does not allow multivalued surfaces, GEN7 chooses the maximum (highest) between the two solutions for z. Thus equation (4.3) becomes:

$$z = (-B_1 + \sqrt{B_1^2 - 4A_1C_1})) / 2A_1 \quad (4.4)$$

ii) Equations 4.3 and 4.4 are undefined when:

$B_1^2 - 4A_1C_1 < 0$

These corresponds to the region beyond the natural boundary of the surface.

For $A_1 = 0$, $\quad z = -C_1 / B_1 \quad$ (4.5)

Equation 4.5 gives a single valued surface, natural boundary is exceeded when $B_1 = 0$

3.2 General Transformation Of Axes

Let the co-ordinate axes X, Y and Z in the Cartesian system be rotated by an angle of θ_3 about the x-axis, followed by a rotation of θ_2 about the y-axis, and then by θ_1 about the z-axis, as shown in Figure 4.6; and let the rotated axes be X', Y' and Z' repectively. A point P with coordinates (x,y,z) would have coordinates (x',y',z') in the X'Y'Z' frame. They are related by equation 4.6 .

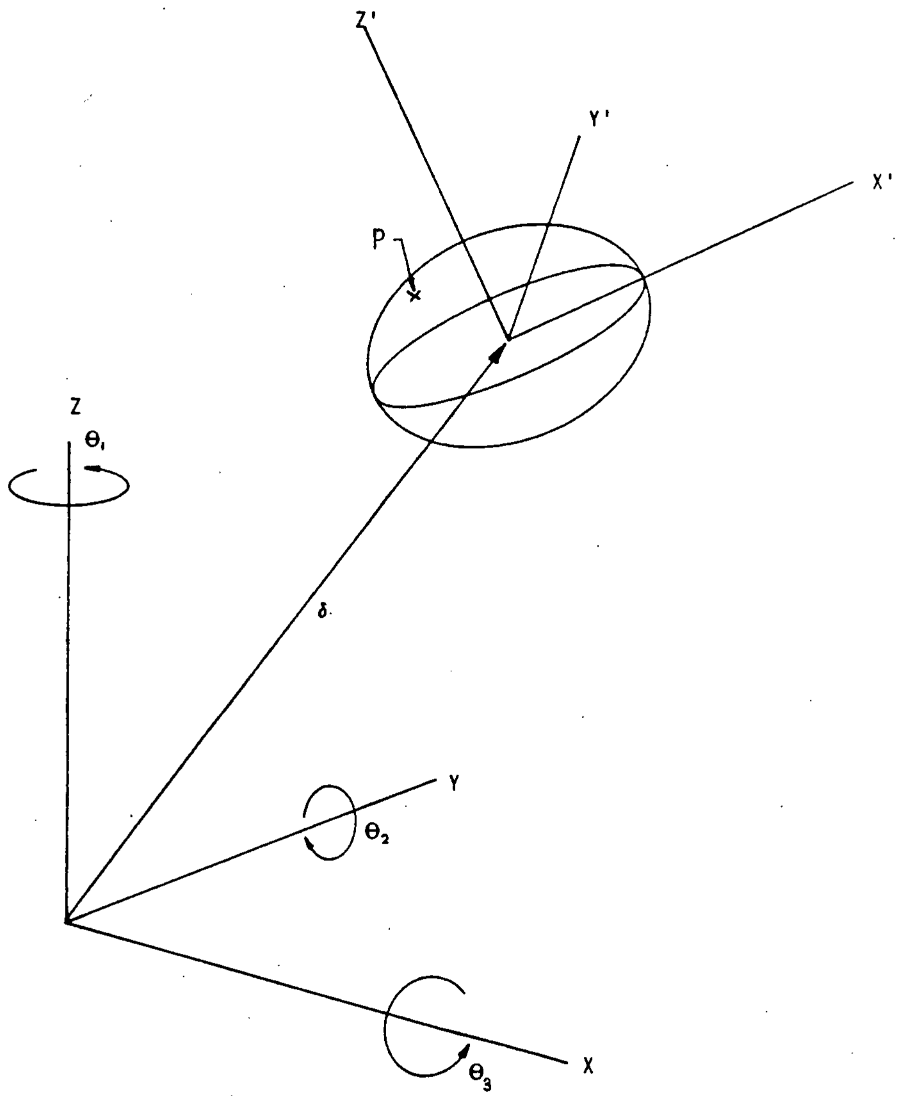


Figure 4.6 Transformation of axes for a quadric surface piece

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \qquad (4.6)$$

The matrix $\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$ is the rotational transformation matrix and is derived from equation 4.7 :

$$\begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ 0 & 1 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 1 \end{bmatrix} \qquad (4.7)$$

or: $l_1 = \cos\theta_1 \cos\theta_2$

$m_1 = \sin\theta_1 \cos\theta_2$

$n_1 = -\sin\theta_2$

$l_2 = \cos\theta_1 \sin\theta_2 \sin\theta_3 - \sin\theta_1 \cos\theta_3$

$m_2 = \sin\theta_1 \sin\theta_2 \sin\theta_3 + \cos\theta_1 \cos\theta_3$

$n_2 = \cos\theta_2 \sin\theta_1$ (4.8)

$l_3 = \cos\theta_1 \sin\theta_2 \cos\theta_3 + \sin\theta_1 \sin\theta_3$

$m_3 = \sin\theta_1 \sin\theta_2 \cos\theta_3 - \cos\theta_1 \cos\theta_3$

$n_3 = \cos\theta_2 \cos\theta_3$

It can be shown that equation 4.6 can be rewritten as :

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (4.9)$$

If the coordinate axes X, Y, Z have also been translated to (x_0, y_0, z_0) in addition to rotation (Figure 4.6), equation 4.9 becomes :

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & m_3 \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \qquad (4.10)$$

An example is given below to illustrate the application of general transformation to a quadric surface. The case of an ellipsoid is shown here.

The characteristic equation of an ellipsoid is :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0 \qquad (4.11)$$

Applying general transformation to 4.11 :

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} - 1 = 0 \qquad (4.12)$$

With respected to the original axes X Y Z, 4.12 becomes :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} \quad \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} \quad \frac{(l_3x_1+m_3y_1+n_3z_1)^2}{c^2} - 1 = 0 \qquad (4.13)$$

where :

$$x_1 = x - x_0$$
$$y_1 = y - y_0$$
$$z_1 = z - z_0$$

Converting into the form similar to equation 4.2 :

$$A_1z_1^2 + B_1z_1 + C_1 = 0 \qquad (4.14)$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2 + \left(\frac{n_3}{c}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)}{a^2} + \frac{2n_2(l_2x_1+m_2y_1)}{b^2} + \frac{2n_3(l_3x_1+m_3y_1)}{c^2} \qquad (4.15)$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 + \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 + \left(\frac{l_3x_1+m_3y_1}{c}\right)^2 - 1$$

Put :

$$D = B_1^{\,2} - 4A_1C_1$$

For $D \neq 0$ and taking the positive square root of D :

$$z_1 = \frac{-B_1 + \sqrt{D}}{2A_1} \qquad (4.16)$$

$$z = z_1 + z_0 \qquad (4.17)$$

Transformations to other quadric surfaces can be performed similarly. Appendix A includes the general transformations to the quadric surface pieces that are handled by the program GEN7.

3.3 Structure Of GEN7

GEN7 has been written in such a way that a user has to specify only the basic parameters of each surface-piece (eg. for an ellipsoid, the semi-axes a, b and c); translations and rotations; as well as user-defined sub-domain and truncation height. The input phase is performed interactively in a step-by-step manner guided by easy-to-understand prompts.

The program first asks for the global field dimensions and the increment between grid points. Next, for each surface type, the user is prompted for the number of pieces (maximum of 3). For each quadric piece, input includes the 3 basic parameters a, b and c defined by the characteristic equation of the surface-type; then translations (x_0, y_0, z_0) and rotations ($\theta_1, \theta_2, \theta_3$); window for the piece (Xmin,Xmax), (Ymin,Ymax); the user-defined height of the piece beyond its natural limit (off-limit height); and the truncation height. Inputs for the non-quadric types of plane and torus are similar, except that no rotations are allowed. A summary of the input parameters for each surface-piece is given in Table I; and Figure 4.7 shows a typical prompting sequence when running the program.

Once the input phase is completed, the program scans along each grid point. At each point, the x and y coordinates are first tested to check if the sub-domain is exceeded; if not, general transformation to the surface-piece is applied and the natural limit is checked. If the surface is within this limit, the value z is calculated using equation 4.4; whereas if it is beyond the natural boundary, z is set to the user-defined 'offlimit height'. The process is repeated for every surface-piece at every grid point, and the maximum z calculated is retained and written onto a data file. The output file can then be processed for graphics or machining purposes.

TABLE I INPUT PARAMETERS FOR GEN7

Surface Type	Characteristic Equation	User Inputs	
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Centroid	x_0, y_0, z_0
	For spheres :	Semi-axes	a, b, c
	$a = b = c = r$ (radius)	Rotations	$\theta_1, \theta_2, \theta_3$
		Subdomain Limits	x_{min}, y_{min} x_{max}, y_{max}
		Offlimit Height	z_{off}
		Truncation Height ...	z_{tr}
Elliptic Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$	Vertex	x_0, y_0, z_0
		Semi-axes	a, b, c
		Rotations	$\theta_1, \theta_2, \theta_3$
		Subdomain Limits	x_{min}, y_{min} x_{max}, y_{max}
		Offlimit Height	z_{off}
		Truncation Height ...	z_{tr}
Hyperbolic Paraboloid	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$	Vertex	x_0, y_o, z_0
		Major & Minor Axes ..	a, b, c
		Rotations	$\theta_1, \theta_2, \theta_3$
		Subdomain Limits	x_{min}, y_{min} x_{max}, y_{max}
		Offlimit Height	z_{off}
		Truncation Height ...	z_{tr}

TABLE I (cont'd)

Surface Type	Characteristic Equation	User Inputs	
Quadratic Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$	Centroid	x_0, y_0, z_0
	For circular Cones :	Semi-axes	a, b, c
	$a = b = \tan \phi$	Rotations	θ_1, θ_2, θ_3
	ϕ = semi-angle	Subdomain Limits	x_{min}, y_{min} x_{max}, y_{max}
	$c = 1$	Offlimit Height	z_{off}
		Truncation Height ...	z_{tr}
Quadratic Cylinder	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Centroid	x_0, y_0, z_0
	Length $= 2r_0$	Semi-axes	a, b
		Half Length	r_0
		Rotations	θ_1, θ_2, θ_3
		Subdomain Limits	x_{min}, y_{min} x_{max}, y_{max}
		Offlimit Height	z_{off}
		Truncation Height ...	z_{tr}
Plane	$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	Intercepts	a, b, c
		Subdomain Limits ...	x_{min}, y_{min} x_{max}, y_{max}
		Truncation Height ..	z_{tr}

TABLE I (cont'd)

Surface Type	Characteristic Equation	User Inputs
Torus	$\left(\sqrt{x^2 + y^2} - a\right)^2 + z^2 = b^2$	Centroid .... x_0, y_0, z_0 Ring Radius .... a Tube Radius .... b Subdomain Limits .... x_{min}, y_{min} x_{max}, y_{max}
Tubular surface with parabolic profile	$z = (cx^2 + b)\sqrt{1 - \frac{y^2}{a^2}}$	Centroid .... x_0, y_0, z_0 Parameters .... a, b, c Rotation (z-axis) . θ_1 Subdomain Limits .... x_{min}, y_{min} x_{max}, y_{max} Offlimit Height ..... z_{off} Truncation Height ... z_{tr}

```
BAS

IAS/RSX BASIC V02-01

READY
RUN GEN7

ENTER FIELD DIMENSION X AND Y ? 600.,230.
ENTER INCREMENT D ? 15.

NUMBER OF ELLIPSOIDS ( max 3 ) ? 0
NUMBER OF ELLIPTIC PARABOLOIDS ( max 3 ) ? 2

ENTER (XO,YO,ZO) FOR ELLIP-PARA(1) ............. ? 141.7,129.4,98.
ENTER A, B, C FOR ELLIP-PARA(1) ................ ? 1.,1.,-192.
FNTER ROTATIONS 1, 2 AND 3 FOR ELLIP-PARA(1) ... ? 0.,-7.3,0.
ENTER LOWLIMX, UPLIMX FOR ELLIP-PARA(1) ........ ? 0.,600.
ENTER LOWLIMY, UPLIMY FOR ELLIP-PARA(1) ........ ? 20.,230.
ENTER OFFLIMIT HEIGHT FOR ELLIP-PARA(1) ........ ? 0.
ENTER TRUNCATION HEIGHT FOR ELLIP-PARA(1) ...... ? 999.

ENTER (XO,YO,ZO) FOR ELLIP-PARA(2) ............. ? 430.4,129.4,183.
ENTER A, B, C FOR ELLIP-PARA(2) ................ ? 1.,1.,-192.
ENTER ROTATIONS 1, 2 AND 3 FOR ELLIP-PARA(2) ... ? 0.,-7.3,0.
ENTER LOWLIMX, UPLIMX FOR ELLIP-PARA(2) ........ ? 0.,600.
ENTER LOWLIMY, UPLIMY FOR ELLIP-PARA(2) ........ ? 20.,230.
ENTER OFFLIMIT HEIGHT FOR ELLIP-PARA(2) ........ ? 0.
ENTER TRUNCATION HEIGHT FOR ELLIP-PARA(2) ...... ? 999.

Pausing, type 1 to alter input, any no. to continue ? 999


NUMBER OF HYPERBOLIC PARABOLOIDS ( max 3 ) ? 0
NUMBER OF QUADRATIC CONES ( max 3 ) ? 0

ENTER (XO,YO,ZO) FOR CONE(1) ................... ? 148.1,129.4,591.6
ENTER A, B, AND C FOR CONE(1) .................. ? 0.0875,0.0875,0.
ENTER ROTATIONS 1, 2 AND 3 FOR CONE(1) ......... ? 0.,0.,0.
ENTER LOWLIMX, UPLIMX FOR CONE(1) .............. ? 0.,600.
ENTER LOWLIMY, UPLIMY FOR CONE(1) .............. ? 20.,230.
ENTER OFFLIMIT HEIGHT FOR CONE(1) .............. ? 0.
ENTER TRUNCATION HEIGHT FOR CONE(1) ............ ? 110.

ENTER (XO,YO,ZO) FOR CONE(2) ................... ? 447.5,129.4,847.
ENTER A, B, AND C FOR CONE(2) .................. ? 0.0875,0.0875,0.
ENTER ROTATIONS 1, 2 AND 3 FOR CONE(2) ......... ? 0.,0.,0.
ENTER LOWLIMX, UPLIMX FOR CONE(2) .............. ? 0.,600.
ENTER LOWLIMY, UPLIMY FOR CONE(2) .............. ? 20.,230.
ENTER OFFLIMIT HEIGHT FOR CONE(2) .............. ? 0.
ENTER TRUNCATION HEIGHT FOR CONE(2) ............ ? 230.

Pausing, type 1 to alter input, any no. to continue ? 999

NUMBER OF ELLIPTIC ( CIRCULAR ) CYLINDER ( max 3 ) ? 0
NUMBER OF PLANES ? 1

ENTER INTERCEPTS X, Y AND Z FOR PLANE(1) ....... ? 1.E99,20.,-65.76
ENTER LOWLIMX, UPLIMX FOR PLANE(1) ............. ? 0.,600.
ENTER LOWLIMY, UPLIMY FOR PLANE(1) ............. ? 0.,230.
ENTER TRUNCATION HEIGHT FOR PLANE(1) ........... ? 999.

Pausing, type 1 to alter input, any no. to continue ? 999

NUMBER OF TORUS ( max 3 ) ? 0
NUMBER OF PARABOLIC ELLIPTICAL CYLINDER ? 0


End of file
```

Figure 4.7 Typical Prompting Sequence of GEN7

3.4 Sample Runs Of GEN7

3.4.1 Pipe-Tee Pattern

Figure 4.8 shows a typical pipe-tee pattern. At the junction, two circular cylinders interpenetrate at right angle to each other. The junction can be modelled by GEN7 from inputs specifying the parameters of the two cylinders and their rotations. The generated surface is shown in Figure 4.9.

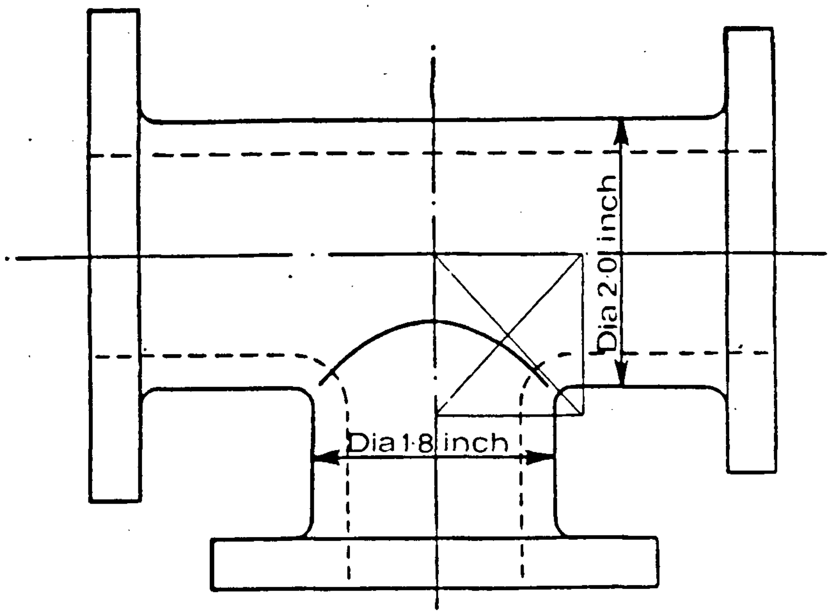


Figure 4.8 Sketch of a pipe-tee pattern

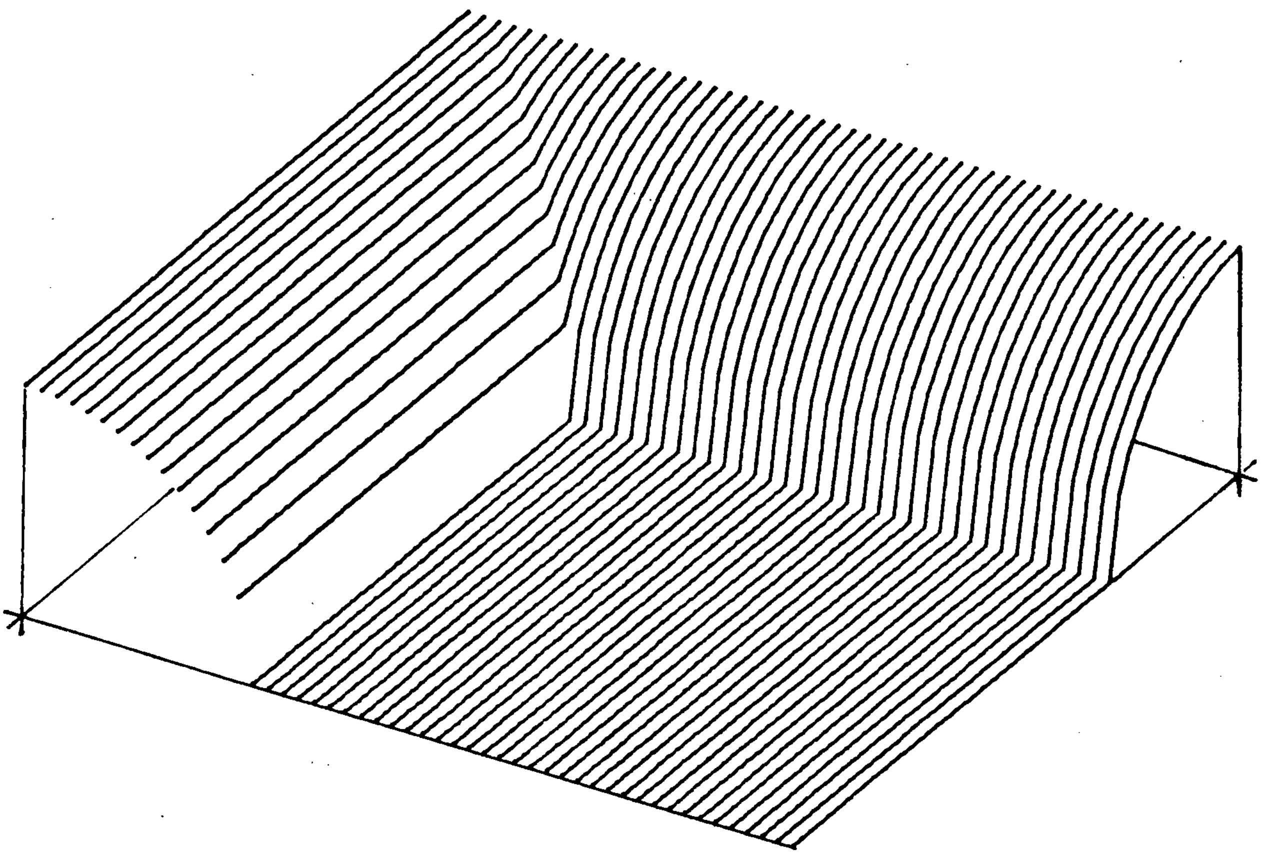


Figure 4.9 Pipe juntion tee modelled by GEN7

3.4.2 Automobile Rear Lamp Punch Model

Figure 4.10 shows a commercial drawing of an automobile tail lamp punch. The surface is an interpenetration of 5 regular analytical pieces : 2 skewed paraboloids (reflectors), 2 truncated cones (lamp sockets), and one inclined plane (to suit the automobile body design). All necessary dimensions are provided from the drawing and converted into inputs for GEN7. Figure 4.11 shows the output generated from GEN7

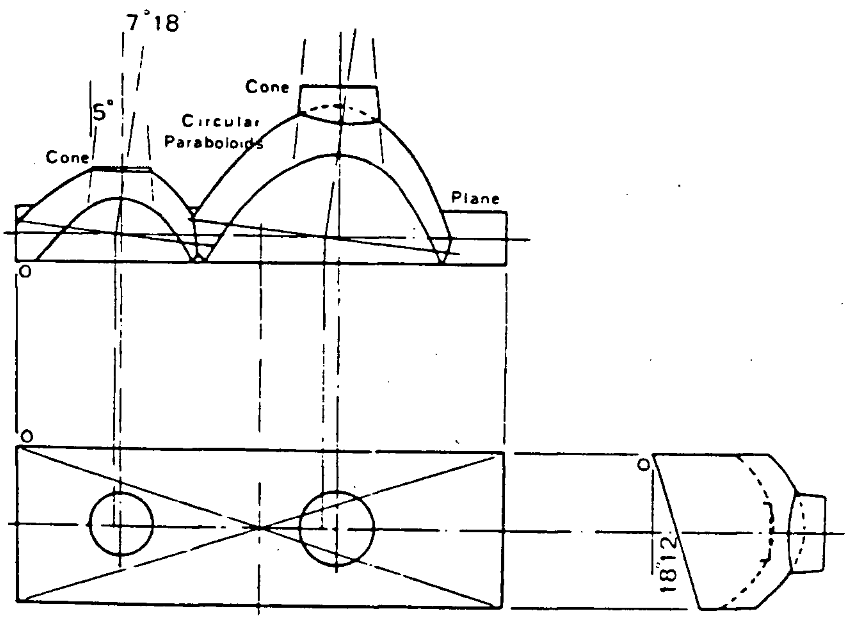


Figure 4.10 Principal sections of an automobile rear lamp punch

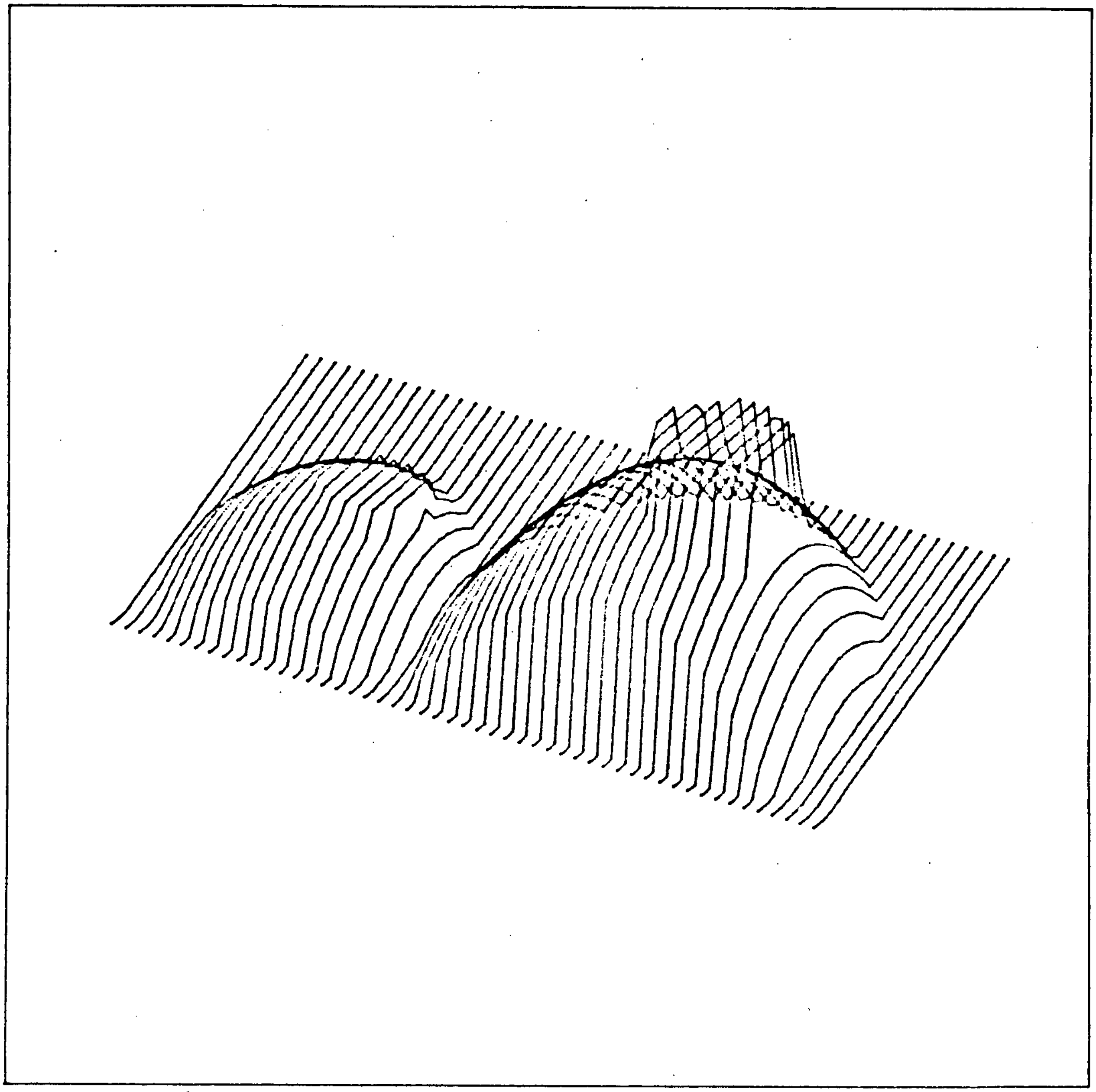


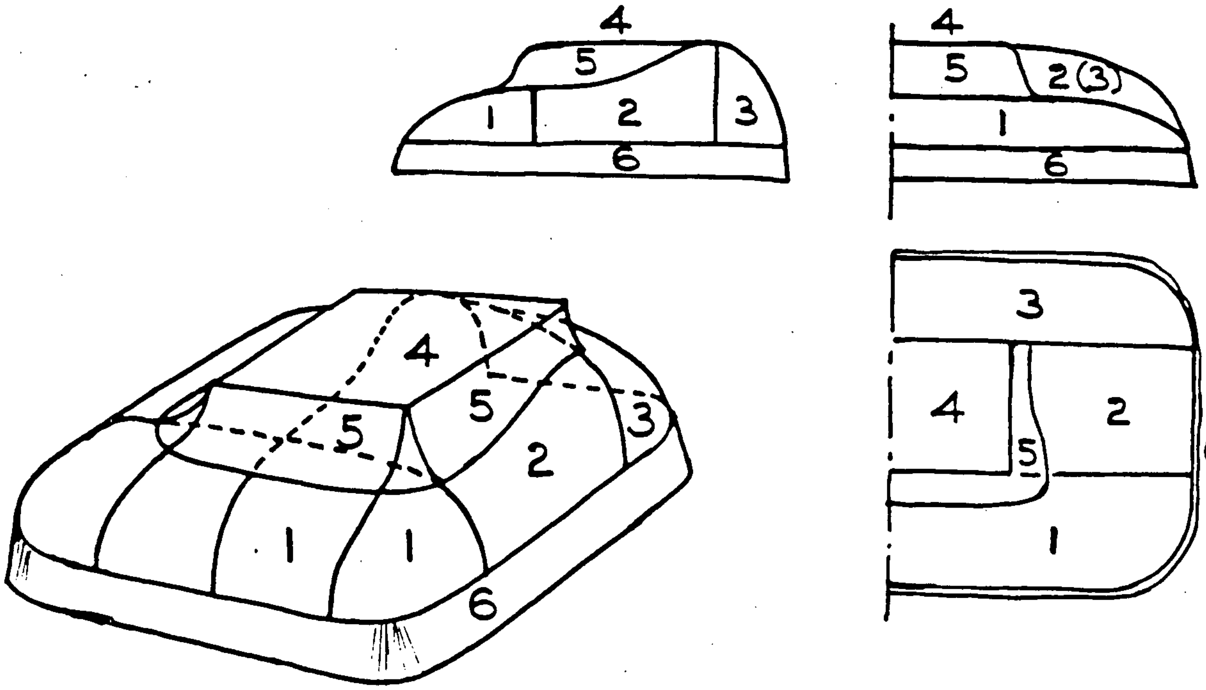
Figure 4.11 Tail lamp punch modelled by GEN7

3.4.3 Vacuum Cleaner Housing Punch Model

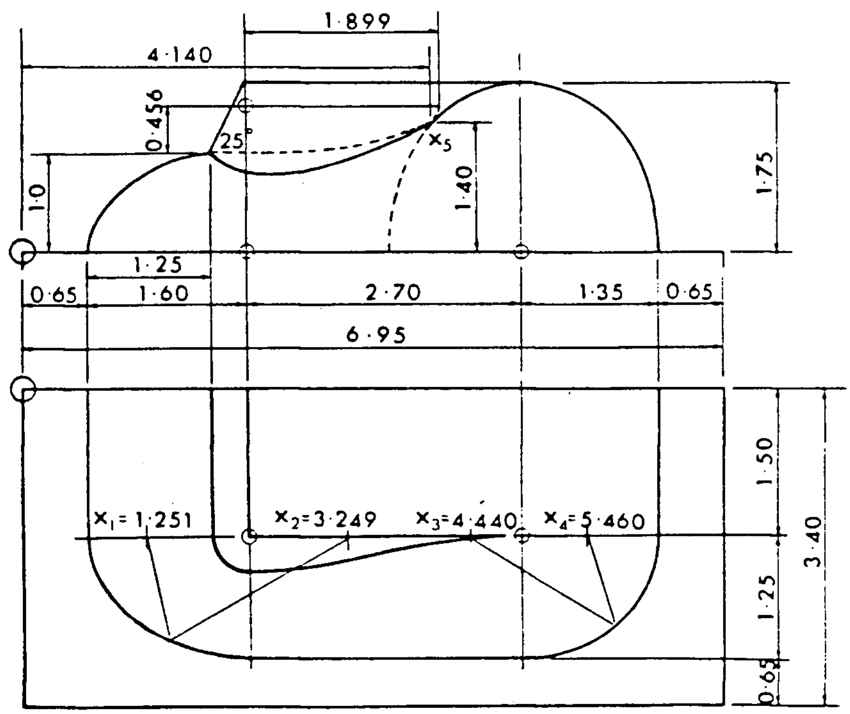
The initial design of a vacuum cleaner housing was done in the form of free-hand sketches, as shown in Figure 4.12. In the next stage, the principal dimensions were chosen and orthogonal projections sketched (Figure 4.13). Suitable conic sections were then adopted as elementary surface-pieces to be blended. For the half-section, these pieces include : 2 ellipsoids, 2 elliptical cylinders, 3 planes, 1 cone, and 1 tubular surface with variable cross-section.

The tubular surface was considered as a combination of 2 duct-type surfaces with vertical sections varying parabolically. A special function was developed to handle this surface type and its characterstic equation is shown in Figure 4.14.

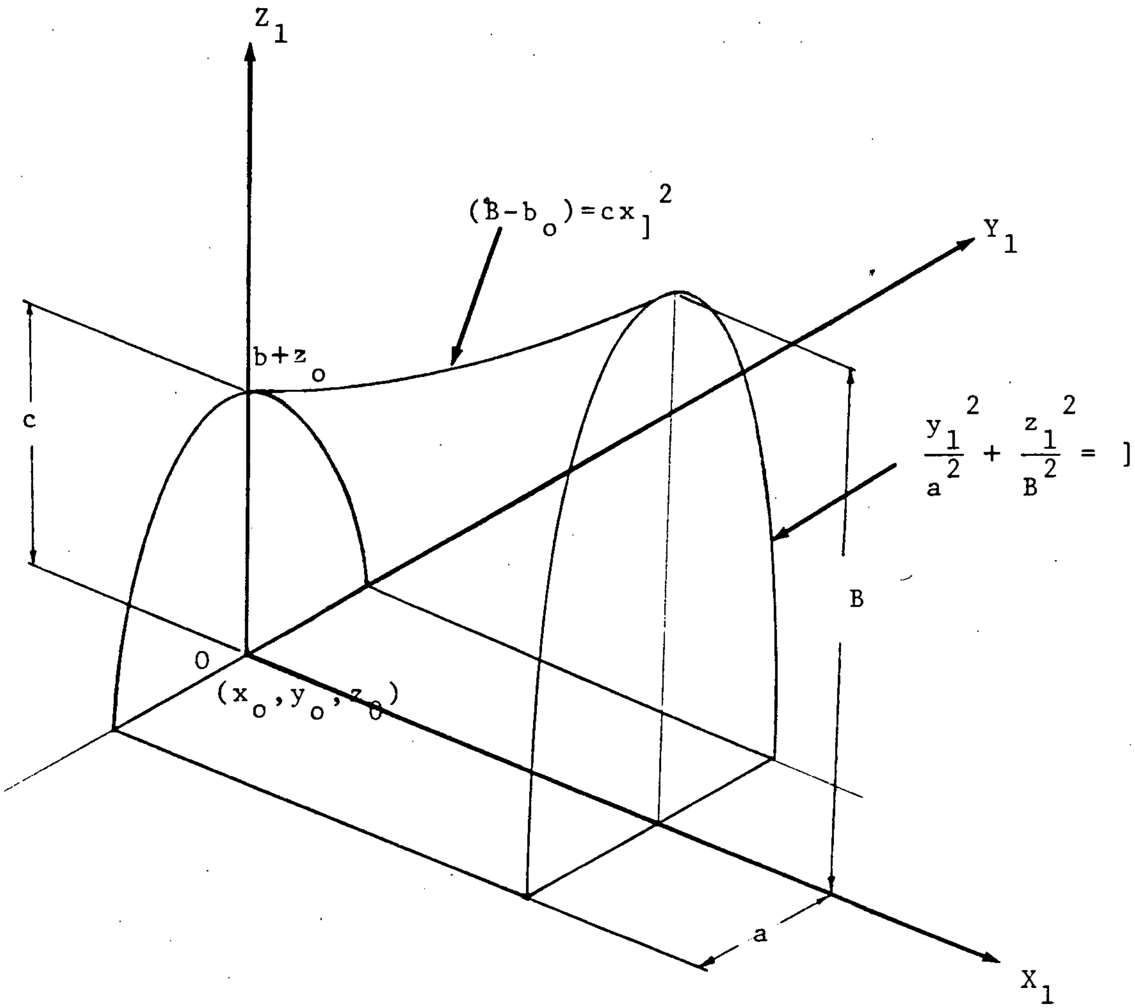
All lines of interpenetrations were generated automatically. The result is shown in Figure 4.15



<u>Figure 4.12</u> Initial proposed sketch of vacuum cleaner housing mould



<u>Figure 4.13</u> Principal sections adopted for vacuum cleaner housing mould



Equation of projection onto Y-Z plane :

$$\frac{y_1^{\,2}}{a^2} + \frac{z_1^{\,2}}{B^2} = 1 \qquad \text{where :} \qquad \begin{array}{l} x_1 = x - x_0 \\ y_1 = y - y_0 \\ z_1 = z - z_0 \end{array}$$

$$z_1 = B \quad] - \frac{y_1^{\,2}}{a^2}$$

But : $B - b_o = cx_1^{\,2}$

or : $B = cx_1^{\,2} + b_0$

Thus : $z_1 = (cx_1^{\,2} + b_0) \quad 1 - \frac{y_1^{\,2}}{a^2}$

When rotated by θ_1 about the z-axis :

$$z_1 = (cx'^2 + b_0) \quad 1 - \frac{y'^2}{a^2}$$

where

$$x' = x_1 \cos\theta_1 + y_1 \sin\theta_1$$
$$y' = y_1 \cos\theta_1 - x_1 \sin\theta_1$$

Figure 4.14 Tubular surface with parabolic profile

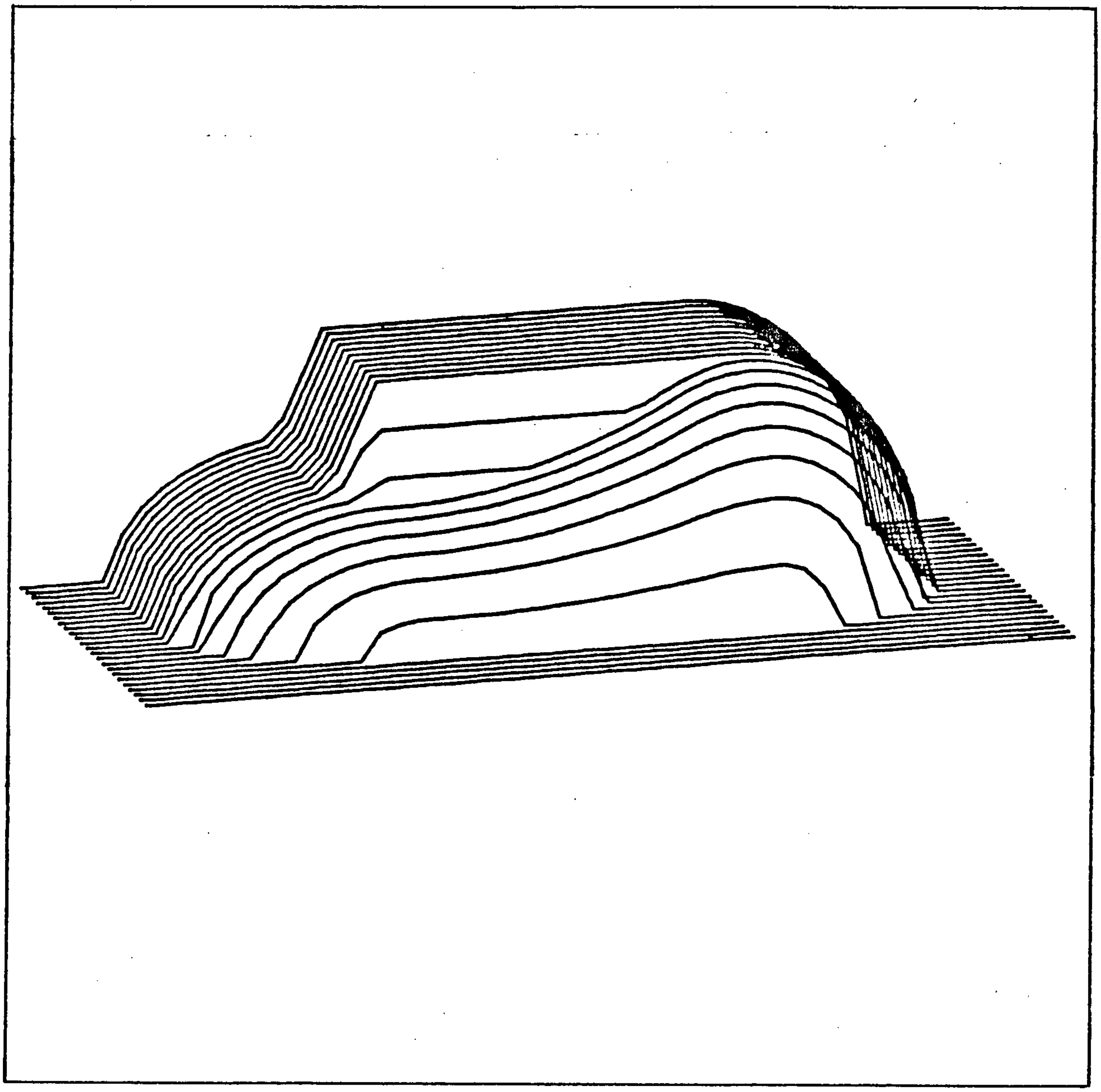


Figure 4.15 Vacuum cleaner housing mould modelled by GEN7

3.4.4 Other Examples

Other sample outputs from GEN7 are shown in the following figures:

Figure 4.16 : inclined cylinder with ellipsoid

Figure 4.17 : sphere, inclined cylinder, elliptic paraboloid and cone.

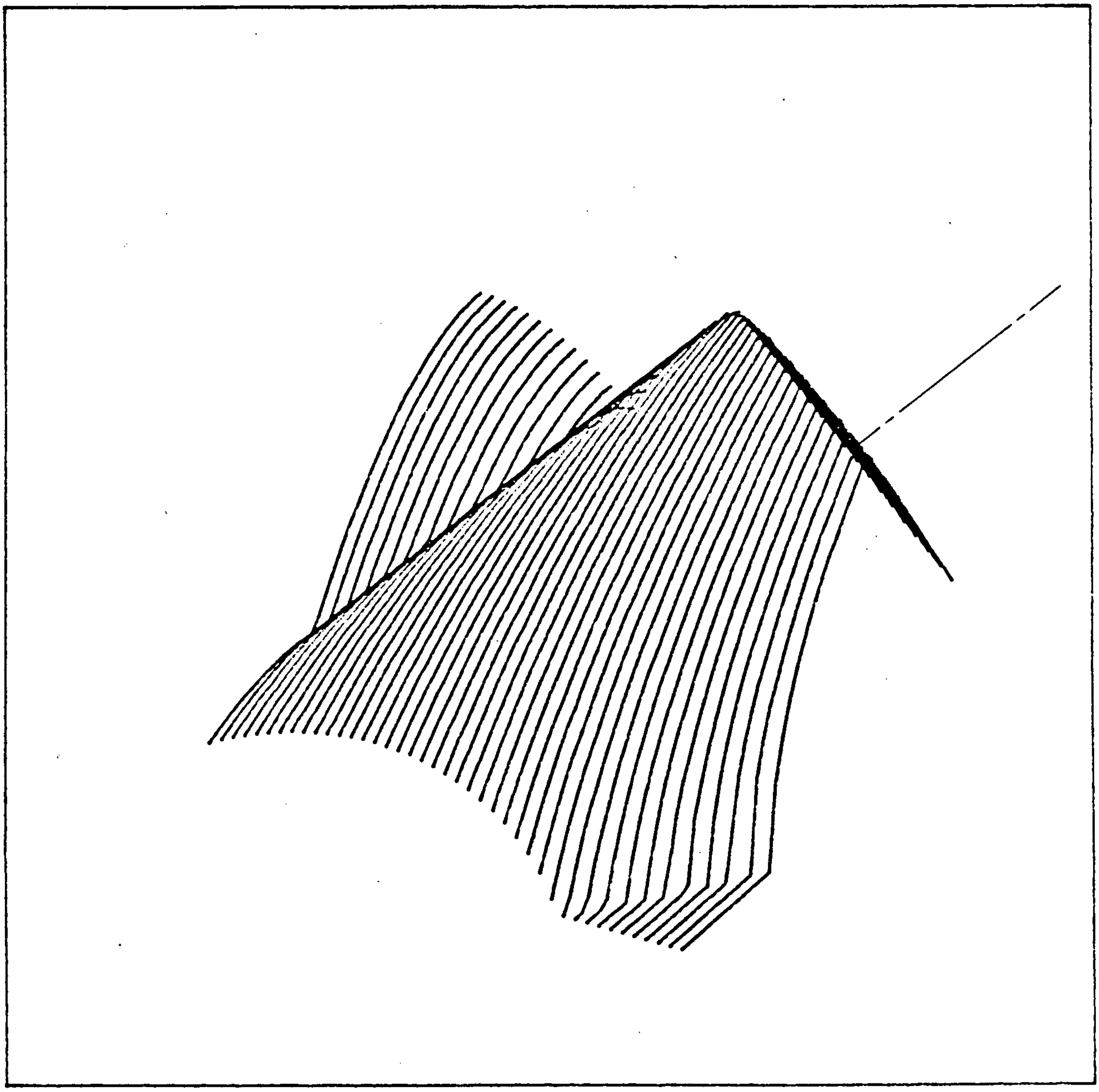


Figure 4.16 Inclined cylinder with ellipsoid

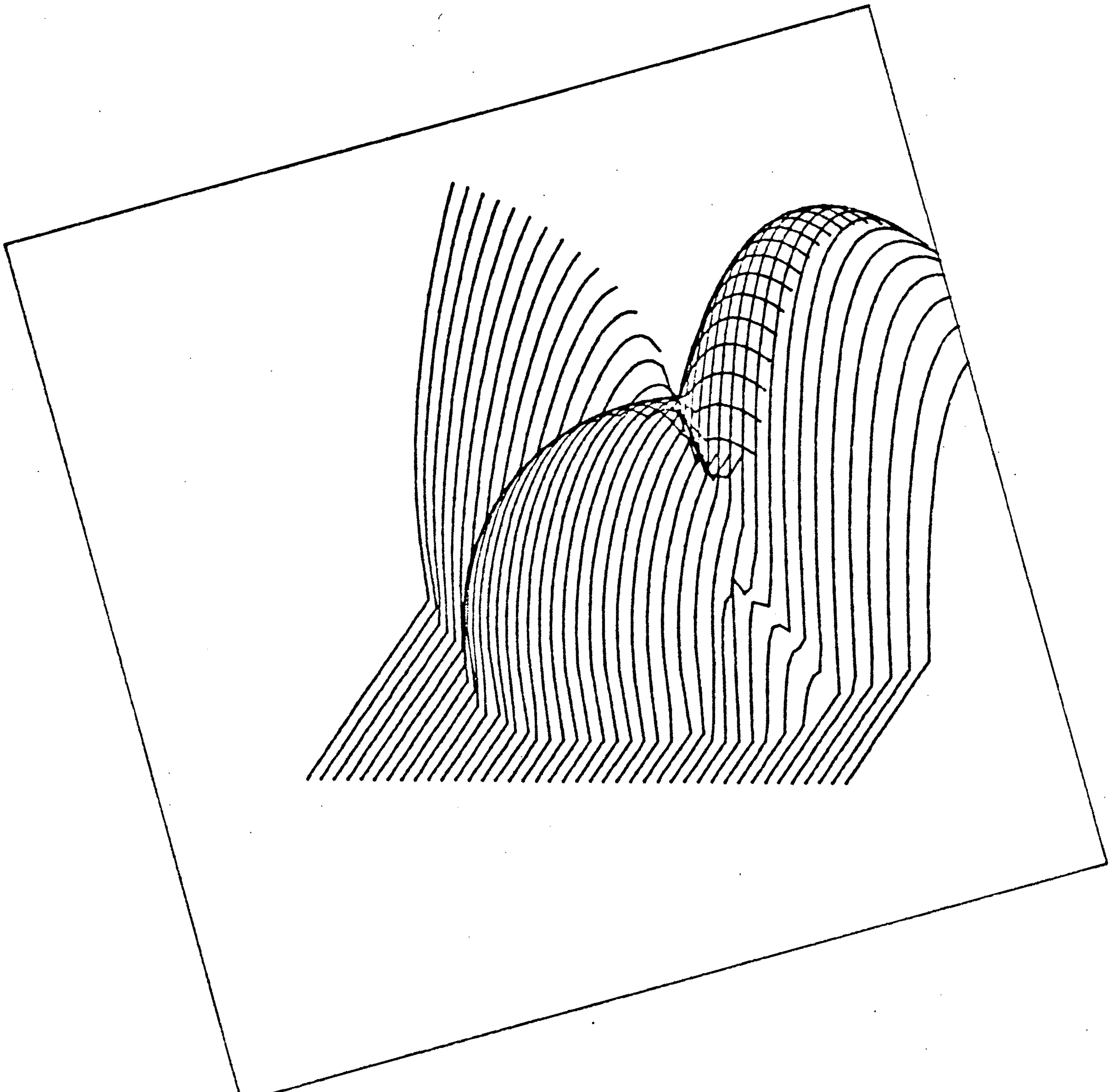


Figure 4.17 Compound surface modelled by GEN7

4. COMPOUND SURFACES WITH NON-ANALYTICAL SURFACE-PIECES

The discussions so far have been dealing with compound analytical surfaces. It does not, however, mean that the Method of Highest Point is limited to such cases. Non-analytical surfaces can also be associated with analytical as long as they are defined by tabulated points arranged in rectangular arrays. Figure 4.18 shows an example of 'marrying' a non-analytical surface with an ellipsoid using the Method of Highest Point.

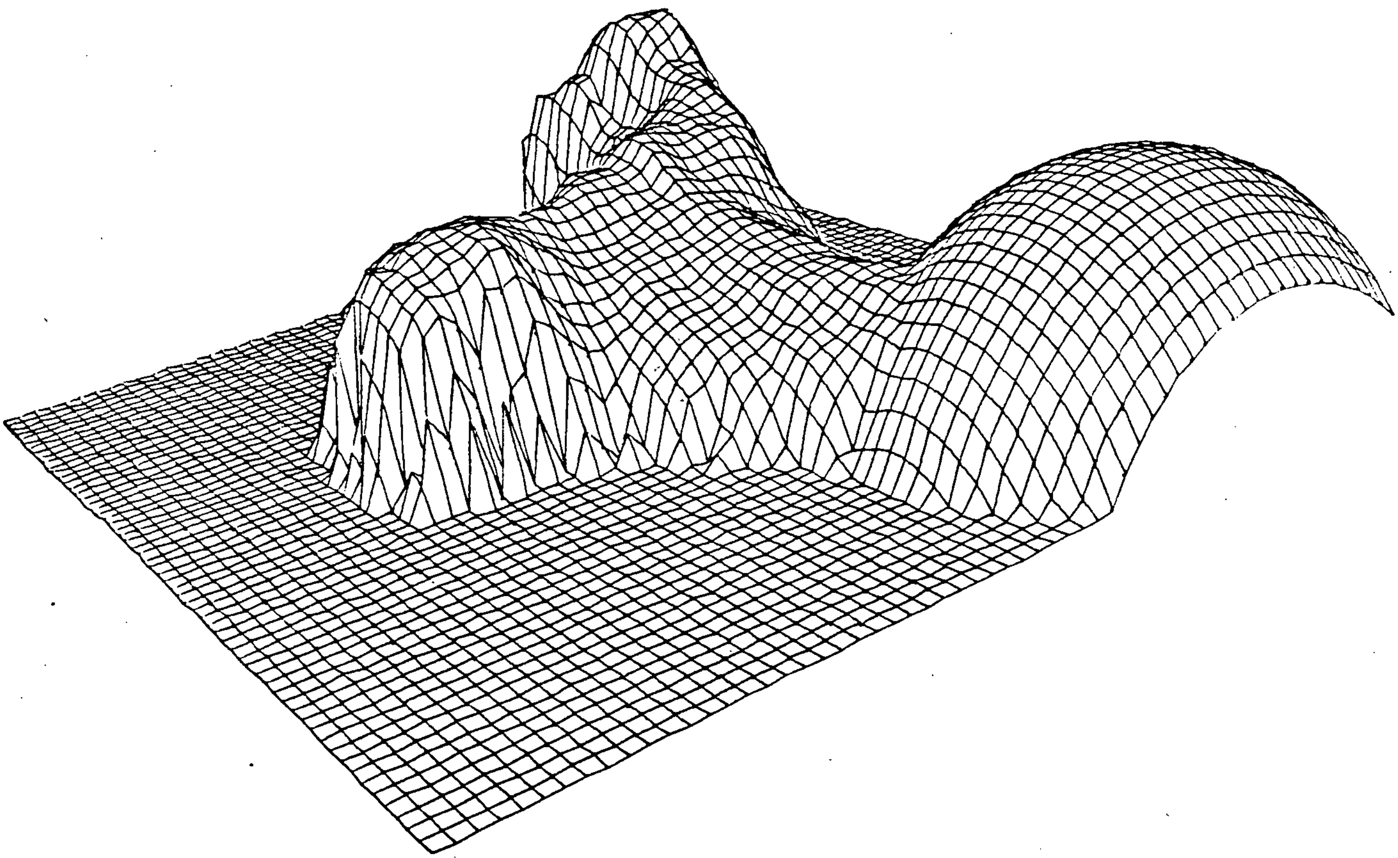


Figure 4.18 'Marriage' of analytical and non-analytical surface-pieces by Method of Highest Point

5. MACHINING OF A DIE

To make the corresponding cavity die to the generated surface, one can machine the model and then make the die using reversal techniques. As discussed earlier, it is more satisfactory to machine the cavity die directly instead. The surfaces of the female mould can be generated by obtaining the mirror image of the male model. This can be done by simply rotating the tabulated points calculated by program GEN7 by 180 degrees.

In cases where the characteristic surface of a die is different from the final product due to constraints imposed by different manufacturing processes, additional manipulations of data are necessary. These may include dilation of volume or surface area, surface-adjustment over a sub-domain, or drafting of cavity walls. The polyhedral concept provides easy means of calculating the geometrical properties of physical surfaces (see Chapter III), and these manipulations may be performed using simple algorithms following pattern-maker's rules

The generated die surface can be viewed over a graphics terminal or computer generated plots, and properties can be calculated and analysed. Once a satisfactory surface has been obtained, the tool path can be generated by program SUMAIR or NEWSU of the POLYHEDRAL NC system.

V. THE NON-ANALYTICAL DIE

1. ARBITRARY (FREE FORM) SURFACES

Most naturally occuring surfaces, such as human anatonmy and geographical landscapes, cannot be represented by simple analytical functions. Others, such as artists' sculptures, are often replications of natural objects that are arbitrary in form. These surfaces must be defined by measured data and subsequently functionalized in order to generate cutter location data for machining purposes.

1.1 Measurement Of Arbitrary Surfaces

Arbitrary surfaces are often defined by measured data obtained from various mechanical, optical, acoustical or electromagnetic techniques. These include mechanical measurements of physical objects or marine soundings of sea-beds, yielding randomly measured data points. Optical measurements, such as shadow moire Technique or photogrammetry, give partially organized data in the form of contour maps.

One of the modern techniques of viewing and measuring concealed surfaces is the method of computed axial tomography (CAT scanning or CT scan). Its chief application is to perform diagnosis of internal organs of human bodies. Figure 5.1 shows a schematic configuration of a typical CAT scanner. The patient is placed on a table which moves through an X-Ray scanning device. An X-Ray source rotates rapidly around the patient, making individual measurements of the densities of thin slices of cross-sections as the table moves. The data are stored in a computer and reassembled to form the image of the patient's interior. Further processings isolate and display a desired internal structure or organ, providing a data base for analysis and surface replications when necessary.

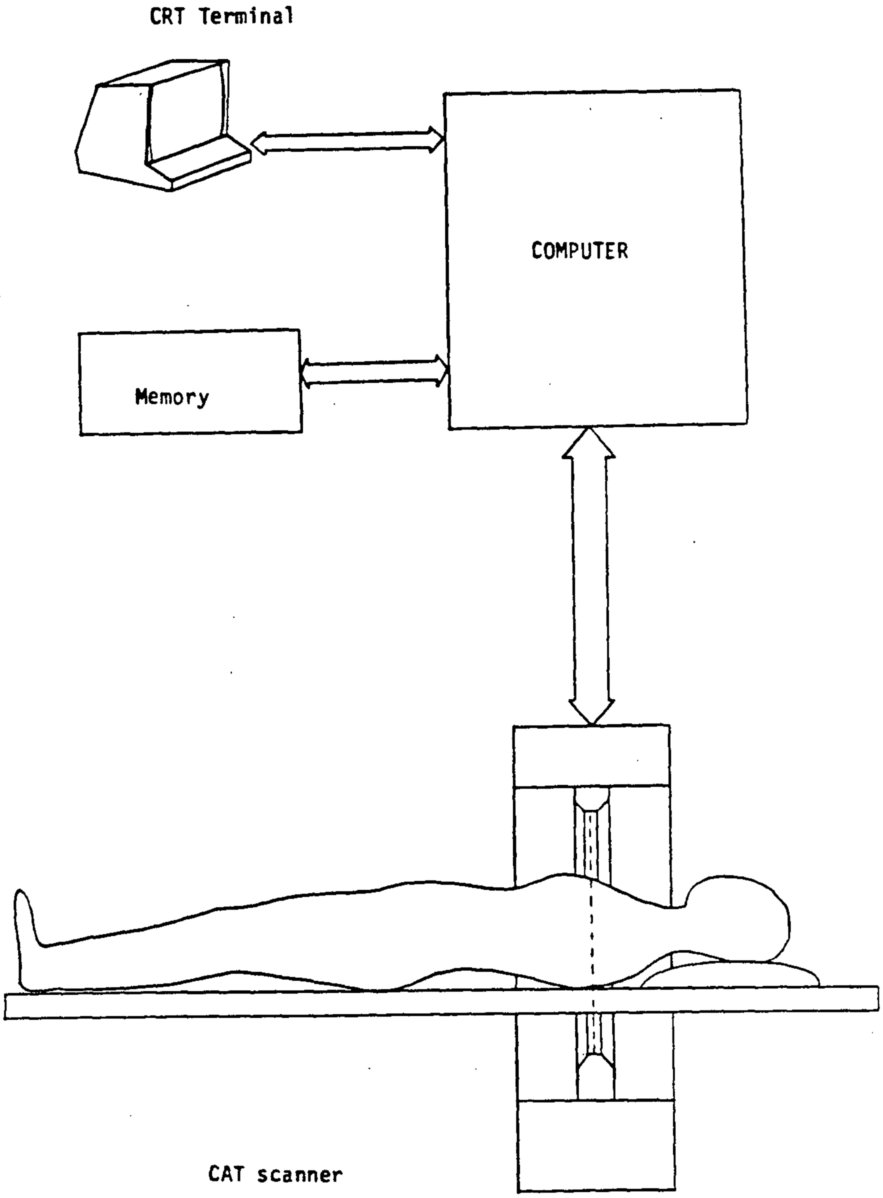


Figure 5.1 Schematic Configuration of a CAT scanner

A surface can also be initially defined by two dimensional profile projections of its spatial boundaries. In this case, algorithms must be developed to 'span' a three-dimensional surface from these boundaries. Figure 5.2 shows the measured boundary-curves of a violin top plate ____ from which a three-dimensional surface is spanned using bi-beta functions.

When the measured data is in analog form, such as contour maps or outlines of cross-sections, digitization is necessary. In order to store data in a digital computer, discrete points must be measured using a digitizer pad or other analog-to-digital converters.

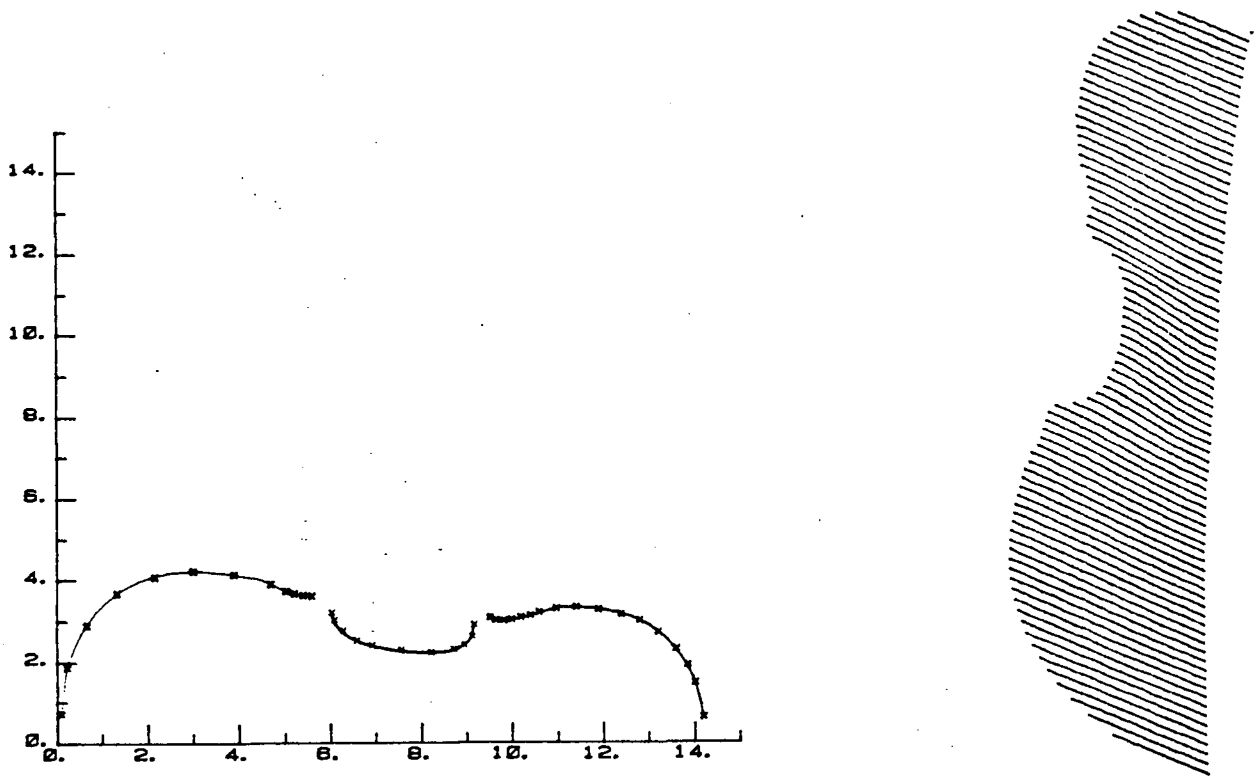


Figure 5.2 Governing boundary curve of a violin top-plate and the spanned surface using bi-beta function

2. MACHINING OF CAVITY MOULDS FOR MEASURED SURFACES

To reproduce a measured surface using NC machining, data must first be sorted and organized before the tool-path can be calculated. Since all of these surfaces appear to be smooth and slope-continuous, it can be assumed that they can be represented, at least locally, by mathematical expressions. By fitting analytical surface-pieces to the measured data, random points can be transformed and tabulated into an orthogonal grid and subsequently approximated as a multifacet polyhedron.

Although the measured data can either be totally random or partially organized, it is advantageous to treat them all as random so that one general purpose surface-fitting routine can be used to handle all cases. A program, known as TRUEPERS (proprietary, by Taylor, Richards and Halstead; Energy, Mines and Resources, Canada, 1971), has been used to transform the data points into an orthogonal grid. It incorporates features that enable a user to specify the degree of smoothness of the fitted surface, and to view it in the form of perspective plots.

Once the data is organized into an orthogonal grid, the tool-path can be generated using program SUMAIR or NEWSU of the POLYHEDRAL NC system. Before machining, additional steps must be taken to check whether the orientation of the surface is suitable for end-milling, whether the parting plane is properly defined, and whether surface-adjustments are necessary.

3. EXAMPLES ON REPLICATING MEASURED SURFACES

3.1 Radius Bone

Part of a human radius bone was to be replicated for research purposes. The surface was measured using Shadow Moire Technique giving a contour map as shown in Figure 5.4. The contour lines within the region of interest were digitized into discrete points using a digitizer pad. The discrete points were replotted and compared with the original contours to check for discrepencies. (Figure 5.4)

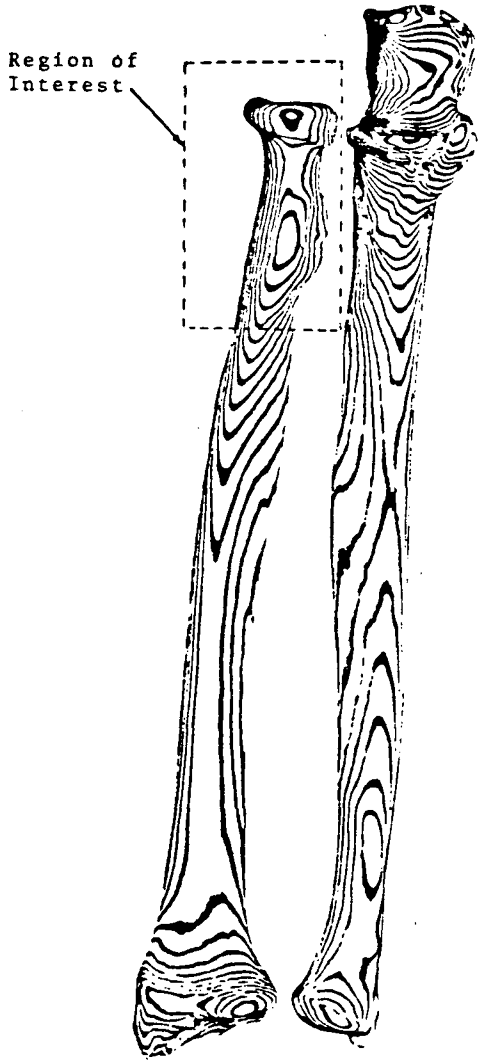


Figure 5.3 Shadow Moire fringes of human radius bones (from Terada, The Skeletal Atlas)[Ref 23]

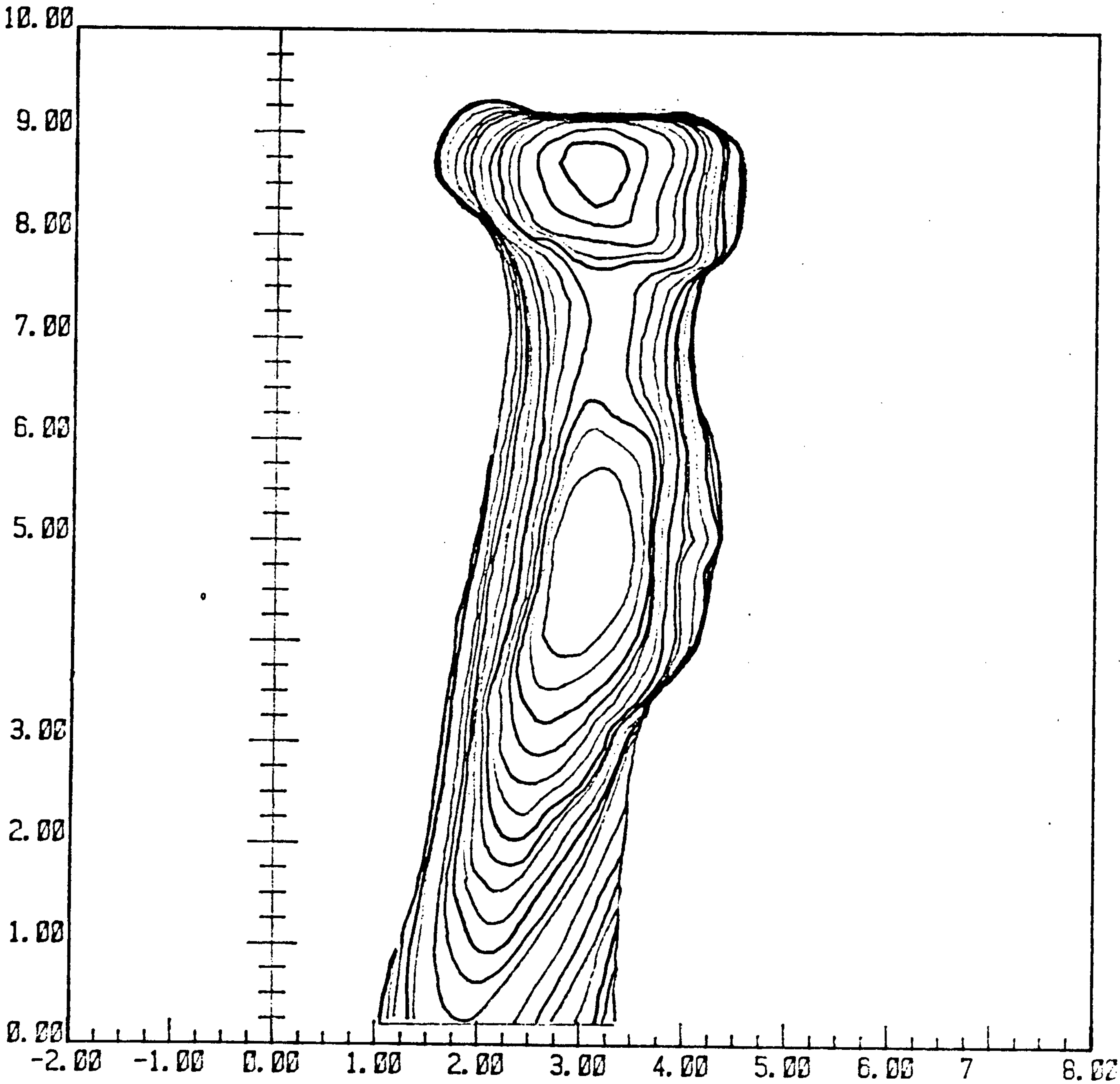


Figure 5.4 Computer replot of the region of interest of the contour-map

The digitized points were then treated as random data for input to the surface-fitting routine TRUEPERS. A perspective plot of the fitted surface is shown in Figure 5.5. Since the orientation of the data presents no difficulty for end-milling operations, no transformation of data was necessary.

To obtain the female mould, the fitted surface was rotated by 180 degrees.(Figure 5.6) This was input to the machining program SUMAIR to generate the cutter location data. For comparision purposes, the male surface was machined using the same procedure.

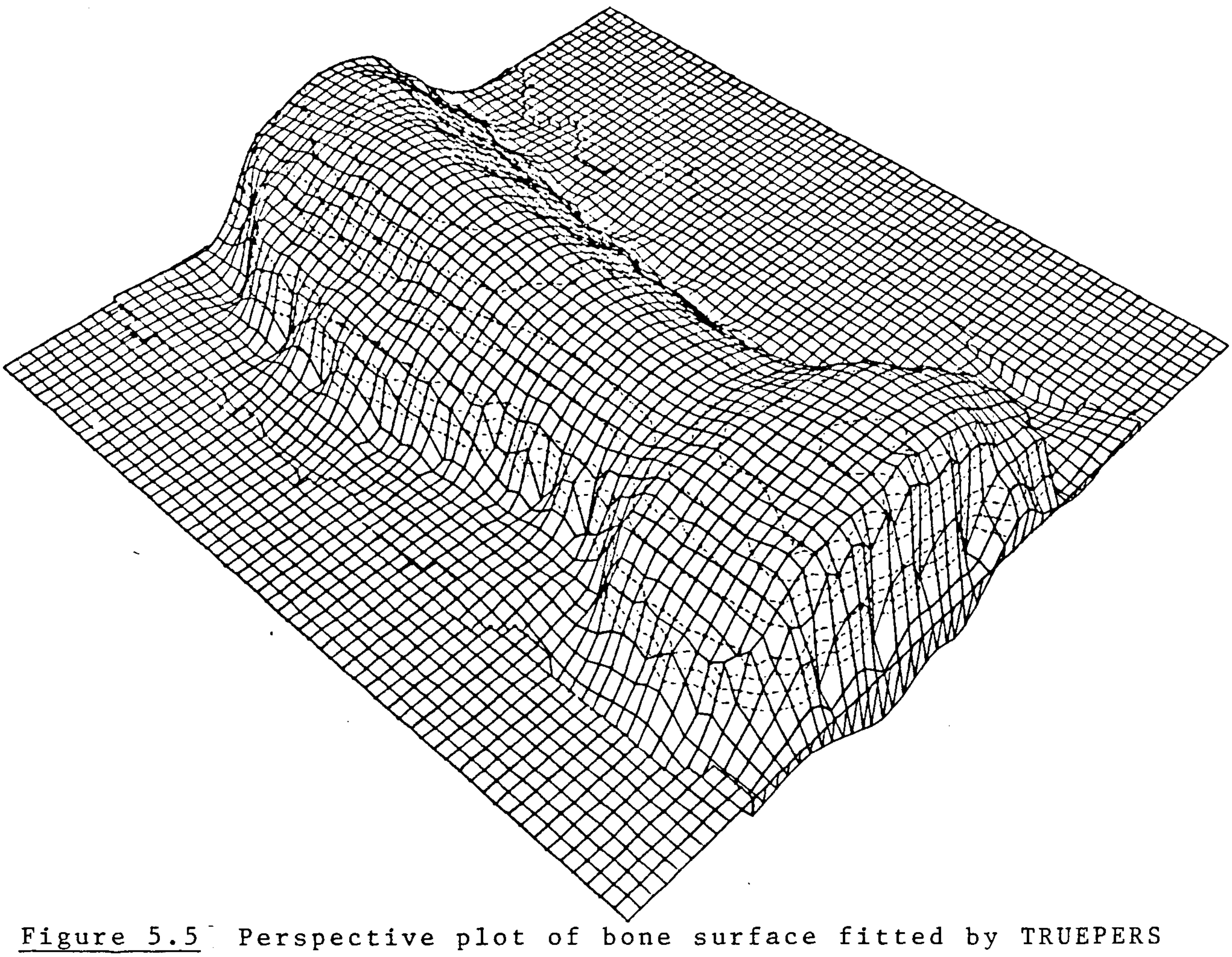


Figure 5.5 Perspective plot of bone surface fitted by TRUEPERS

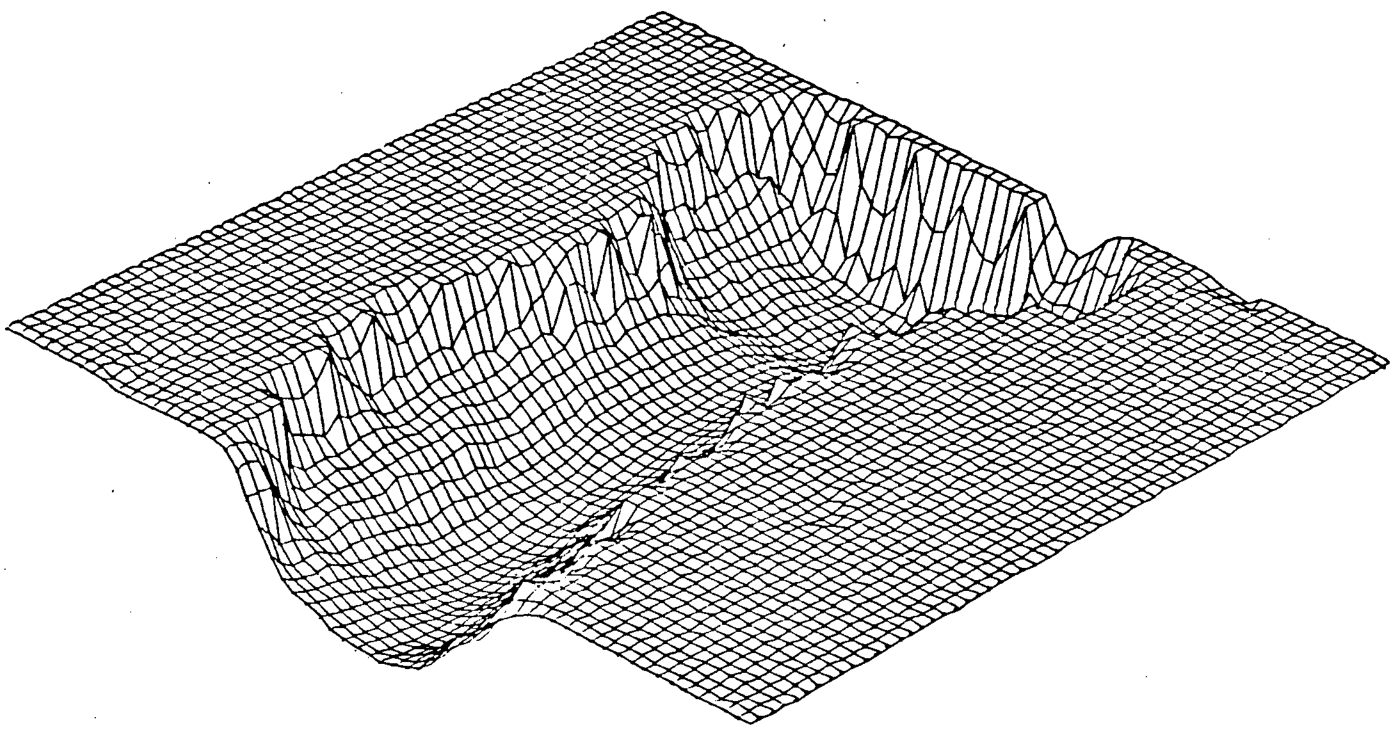


Figure 5.6 Plot of cavity mould (female surface) for radius bone

Machining was performed using a half-inch diameter spherically-ended milling cutter on polyurethane foam and also on a resin-based syntactic plastic called SYNCAST. The moulding materials used were silicone rubber (on the foam mould) and dental plaster (on the plastic mould). The surfaces and moulds are shown in Figure 5.7.

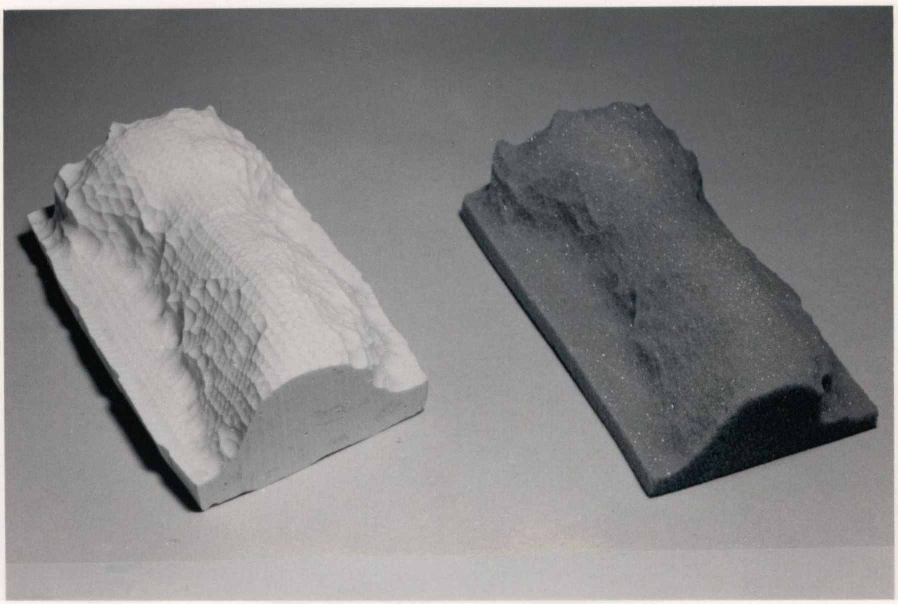


Figure 5.7 Machined models for male and female surfaces

As expected, the moulded surfaces give a better surface-finish than the machined ones when no hand-finishing was done. To achieve a very smooth surface, "hand-finishing to witness" was required. This hand-finishing can be minimized by reducing the step-size and by using a large tool. Experience shows that a step-size corresponding to one-tenth of the tool-diameter gives good results for most applications.

Figure 5.5 indicates that the fitted surface from TRUEPERS does not yield a perfectly flat base-plane at regions just beyond the boundary of the cavity, which implies that the corresponding parting surface of the female mould is not a plane. This is due to the fact that no data was provided to define the base-plane as input to TRUEPERS. Thus an undefined region was created beyond the boundary, the result is that over- and under- shootings, plus oscillations, appear in interpolation. To remedy this, data for the base-plane must also be included, as will now be explained.

3.2 Facial Mould

A model of a human face was required for surgical applications. Data was obtained from stereo photography and a contour map was generated. This was digitized and replotted as shown in Figure 5.8. To prevent oscillations at regions beyond the boundary, additional data defining the base-plane was required. This was obtained by generating artificial 'contour' lines which were actually offset curves at various distances from the boundary-curve. A routine called ATKIN [Law, 1984] was used to automatically generate the data. Thus input data for TRUEPERS appears as shown in Figure 5.9.

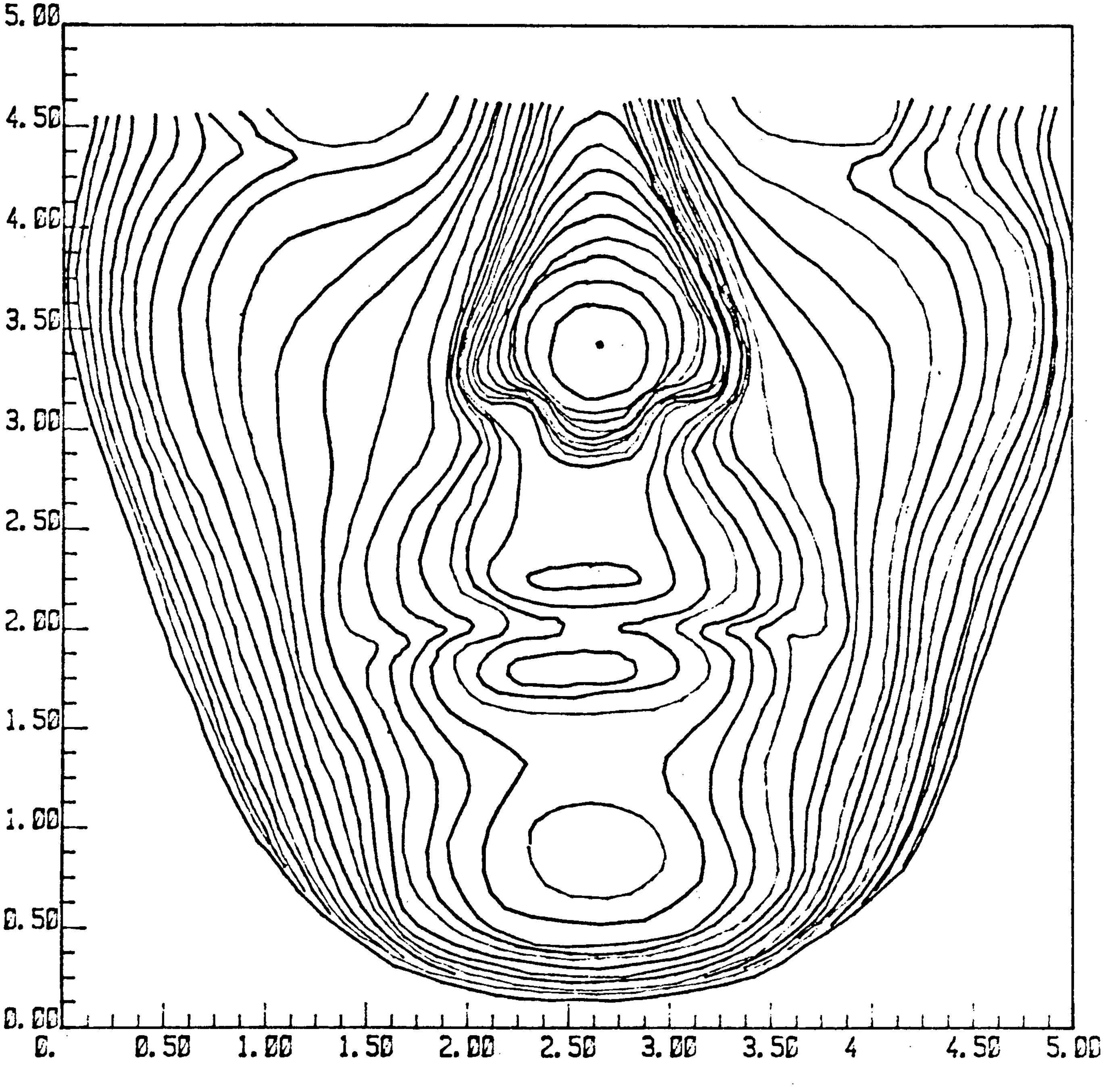


Figure 5.8 Computer replot of contour map defining human face

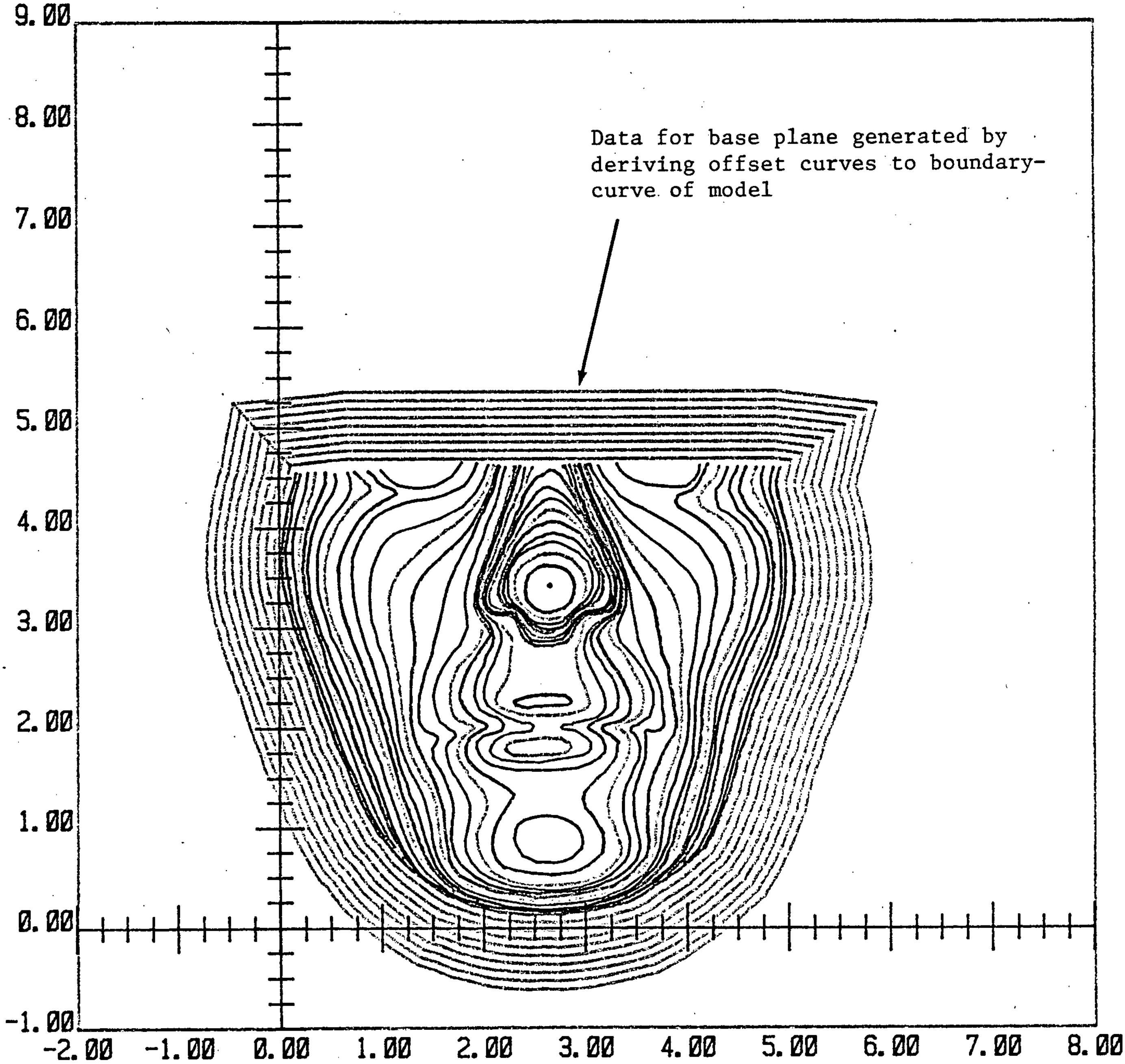


Figure 5.9 Contour-map with added data for base plane

The fitted surface by TRUEPERS is shown in Figure 5.10. Since TRUEPERS assumes position- and slope-continuity over the entire global field, the junction between the base-plane and the model is filleted. When transformed into the female mould, the parting curve, which is the intersection between the cavity-surface and the parting plane and thus represents a junction of discontinuity, is not distinct. This may or may not be desirable, according to different manufacturing processes. One method of generating the discontinuity is to set the data of the base-plane at a level <u>lower</u> than the actual base-plane. After surface-fitting by TRUEPERS, this plane can be raised back to its original level, in effect artificially creating the parting curve.

Figure 5.11 shows the female mould surface. It was machined using CLD generated from SUMAIR on both polyurethane foam and dental plaster. Figure 5.12 shows the cavity-mould and a plaster mould of the model.

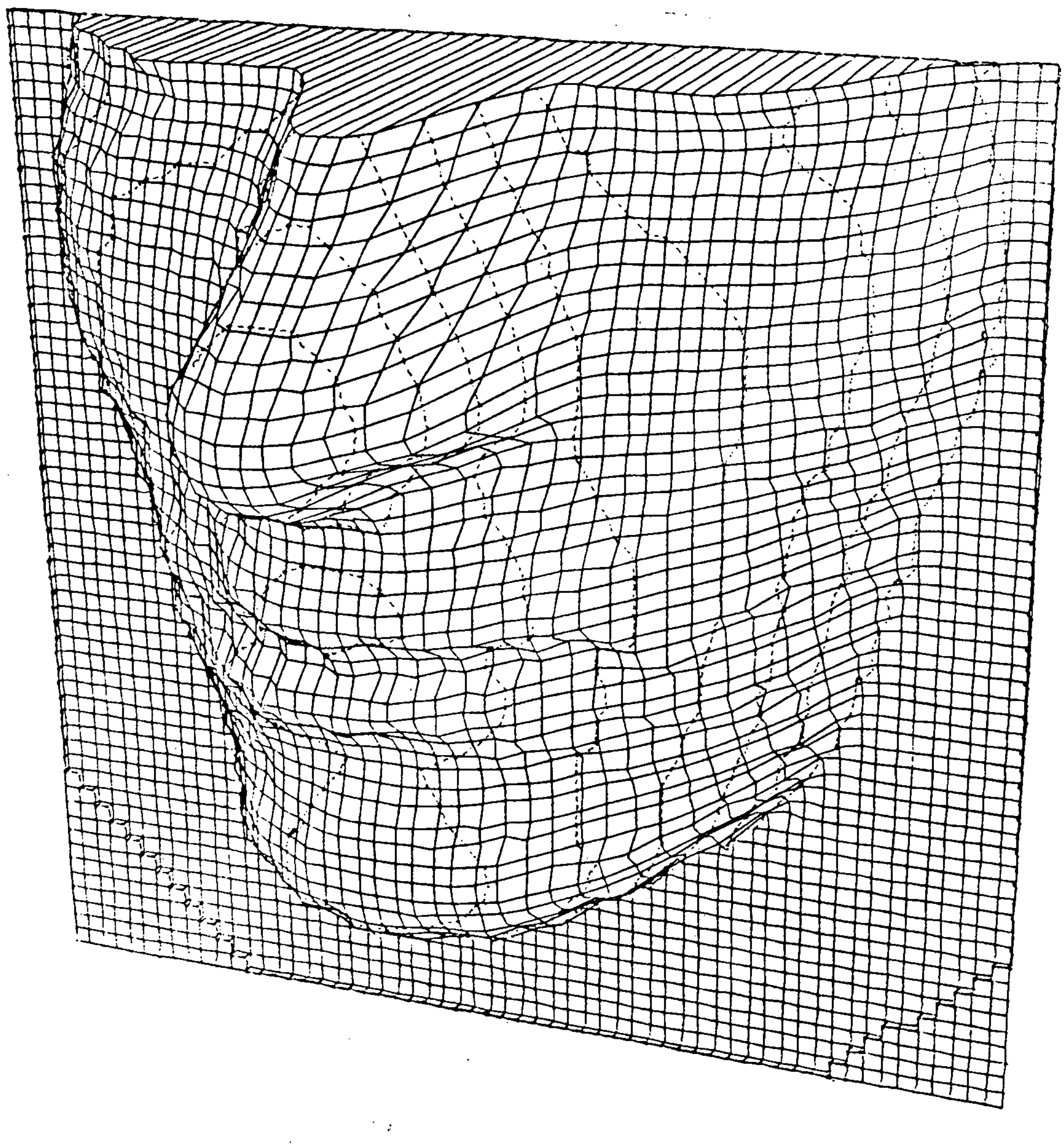


Figure 5.10 Fitted surface of human face by TRUEPERS

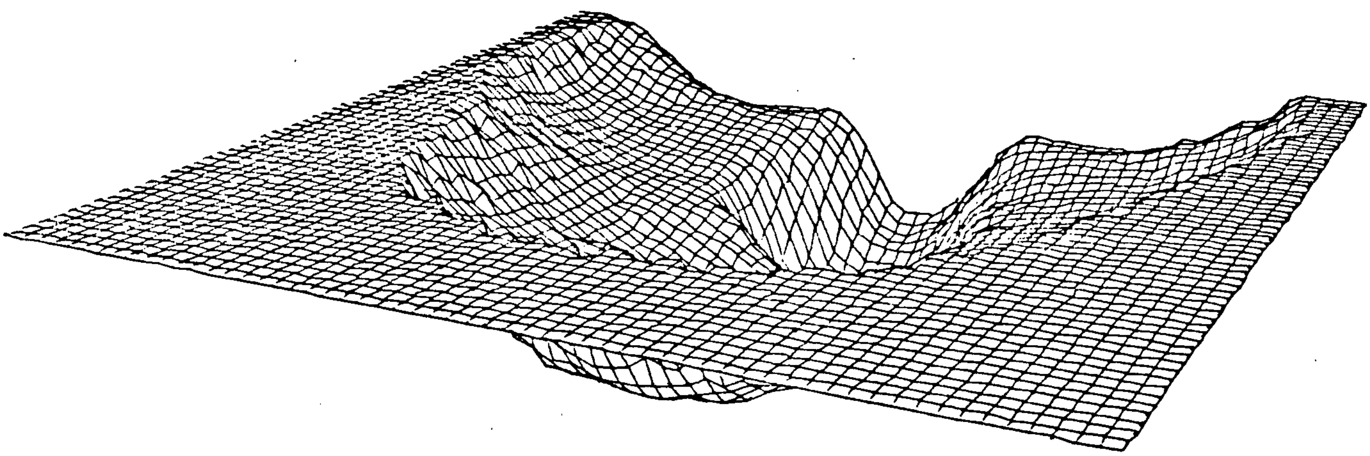


Figure 5.11 Perspective plot of female mould

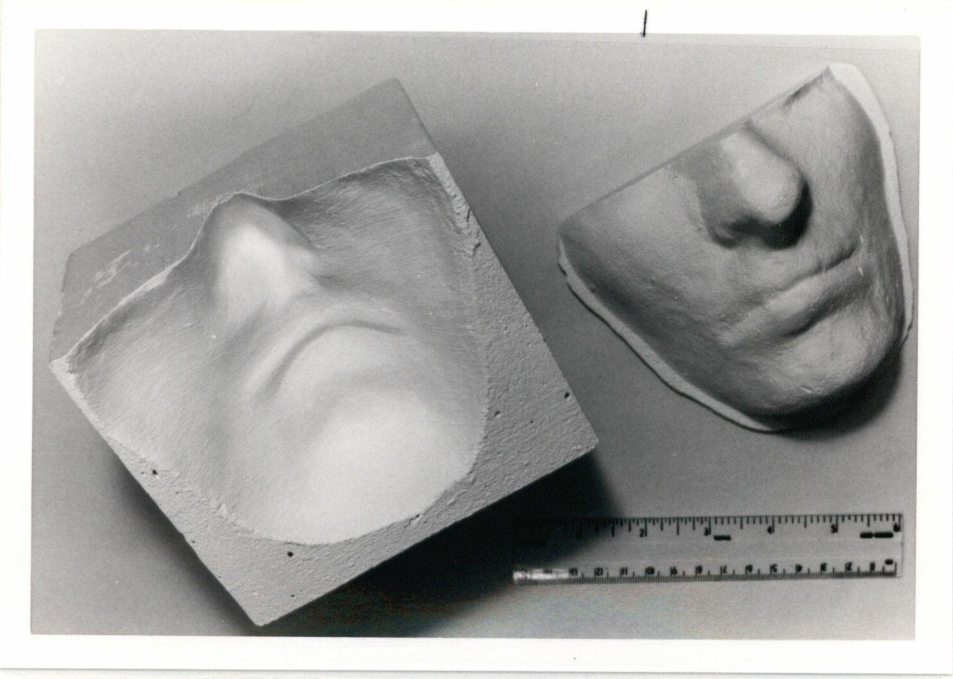


Figure 5.12 Machined cavity-mould and plaster-model of a human face

3.3 Ox Tibia Bone

With the advance of CAT scanning and other similar modern imaging techniques, it is now possible to replicate internal organs or bone-structures that until quite recently have been unable to be measured accurately. Successful efforts were made to machine a human skull from CAT scanning data [Parviti & colleagues, 1983], and similar work can be done on other bone-structures. One major obstacle is that the orientations of a patient's internal structures are constrained by their position and his posture during scanning. It may be inconvenient or even impossible to orient a specific structure of interest in order to make the measured data correspond to the orientation desirable for machining. This problem did not arise when a skull was machined, for the obvious reason that a human skull model can be rotated quite freely. However, this situation is an exception to the general rule.

Since expensive CAT scanning equipment was not available to the author for the purpose of this research, an alternate mean of obtaining data was adopted. To simulate the slicing of cross-sections, an ox tibia bone was place in a box and rigidly embedded with polyurethane foam. The whole assembly; box, foam and bone; was then cut along parallel planes with regular intervals to reveal 35 parallel cuts through the inclined bone. The cuts were then digitized to produce slices of cross-sections. A few examples of replots are shown in Figure 5.13. The replots were then superimposed to reveal the projected shape of the inclined bone. (Figure 5.14) The orientation was arranged in such a way that transformation was necessary to rotate the bone to a position suitable for machining (ie., with no negative draft). General transformation (equations 4.6 - 4.10) was applied to achieve the desired orientation and a parting plane was selected. (Figure 5.15) The points above and below the parting plane were separately stored into two different data files for input to TRUEPERS to generate the two separate half moulds. Asperities and holes which cannot be machined by end-milling were blanked out, and data for the base-plane was added. (Figures 5.16 and 5.17)

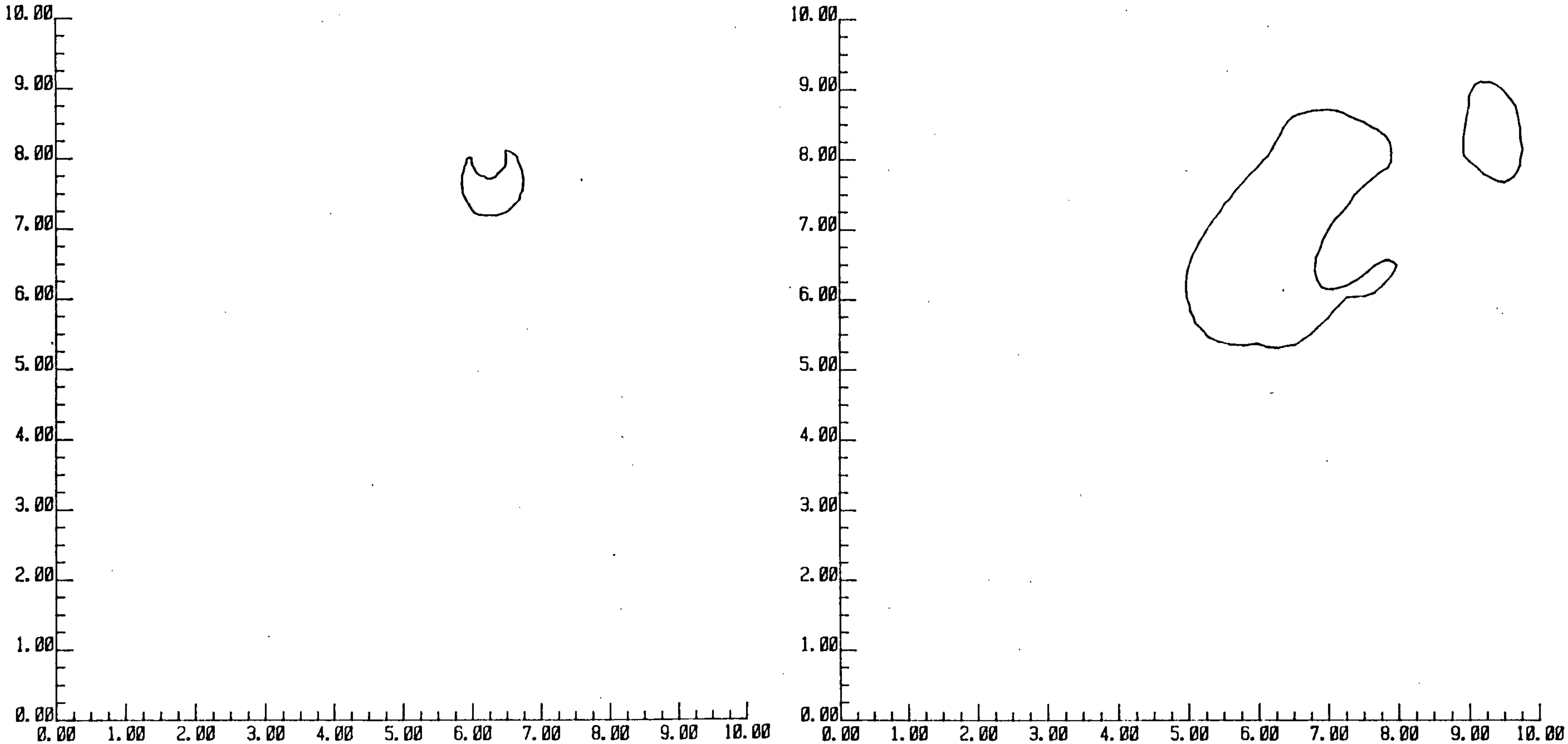
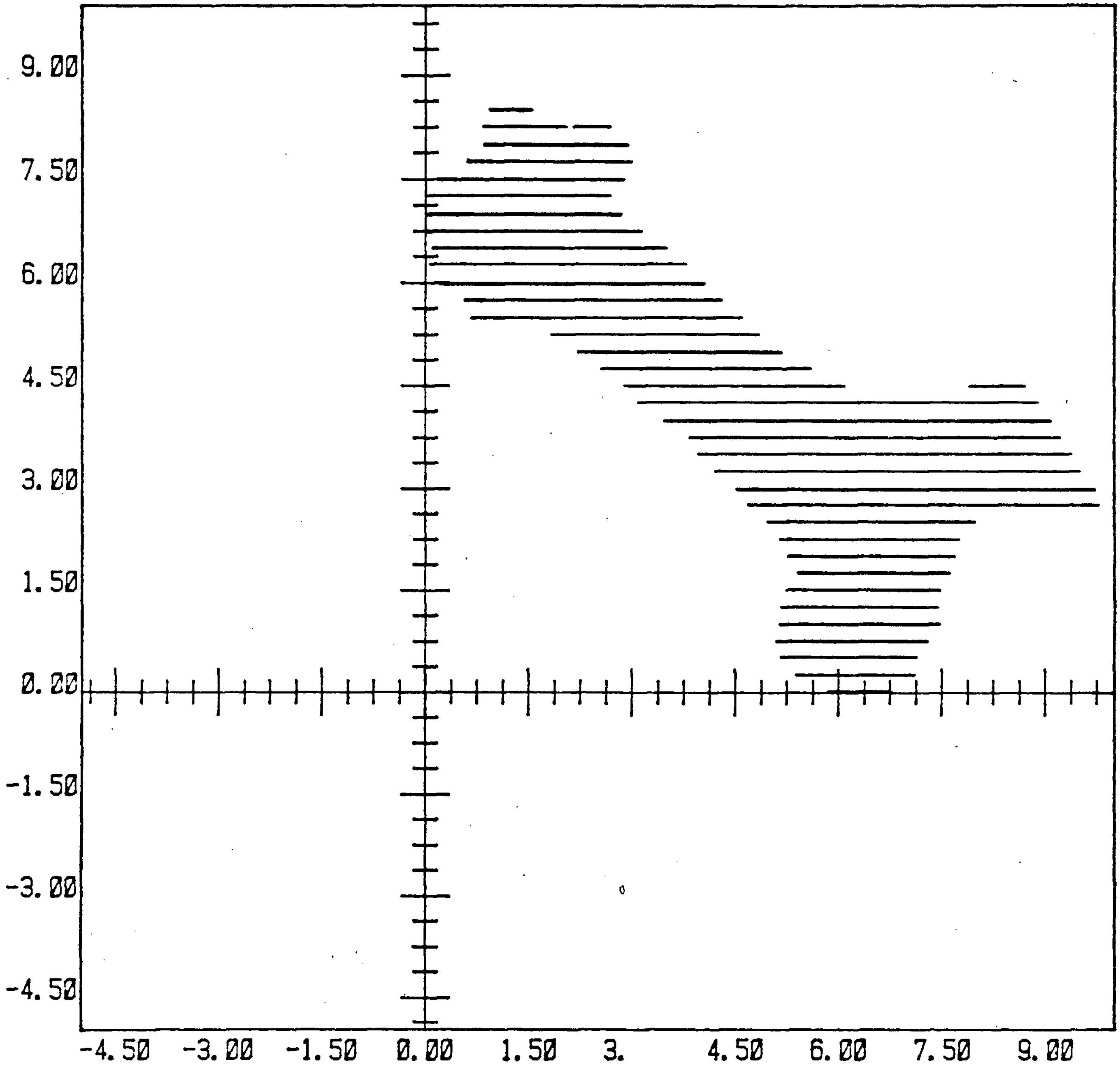
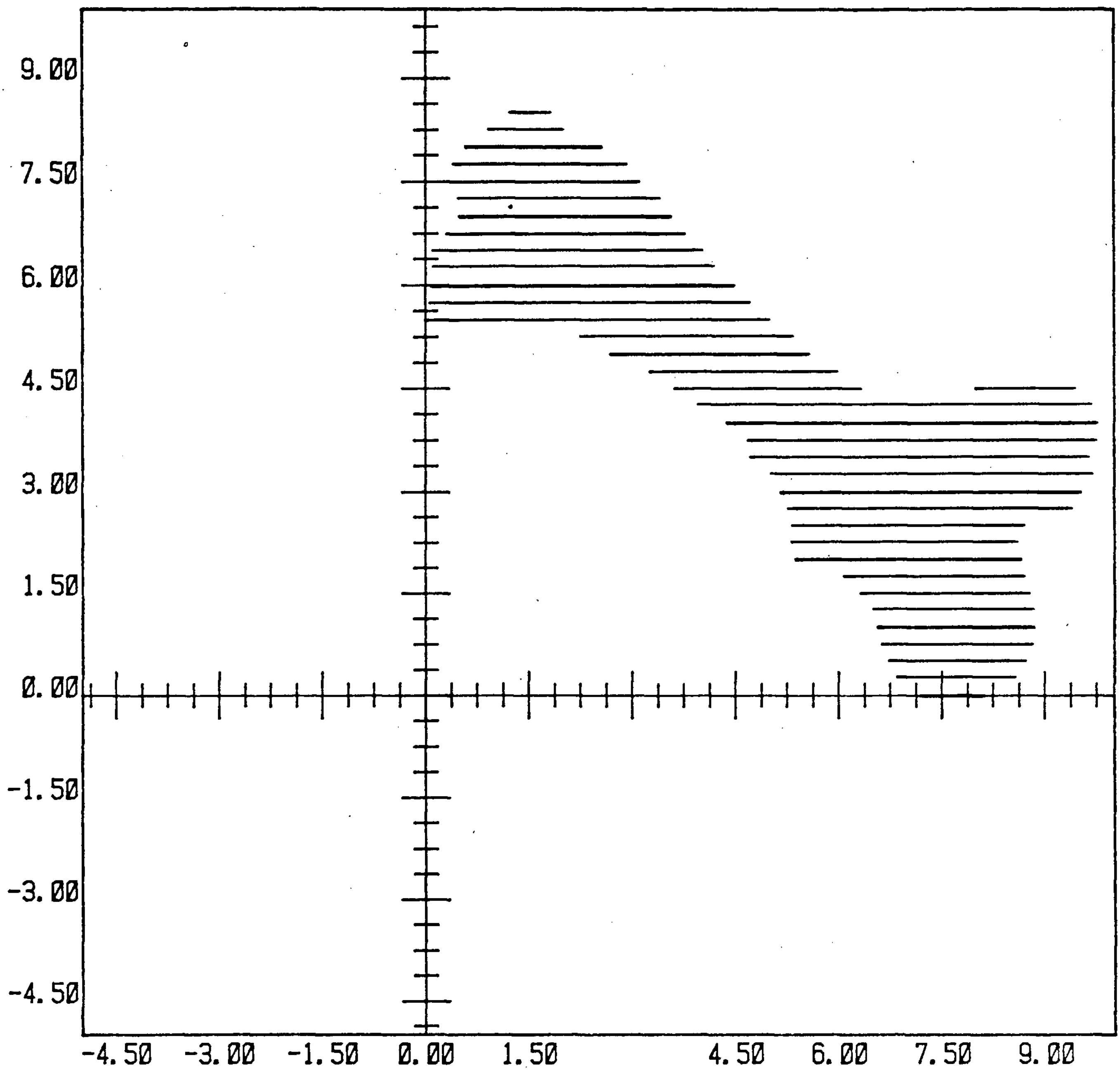


Figure 5.13 Replots of slices of cross-sections of a tibia bone

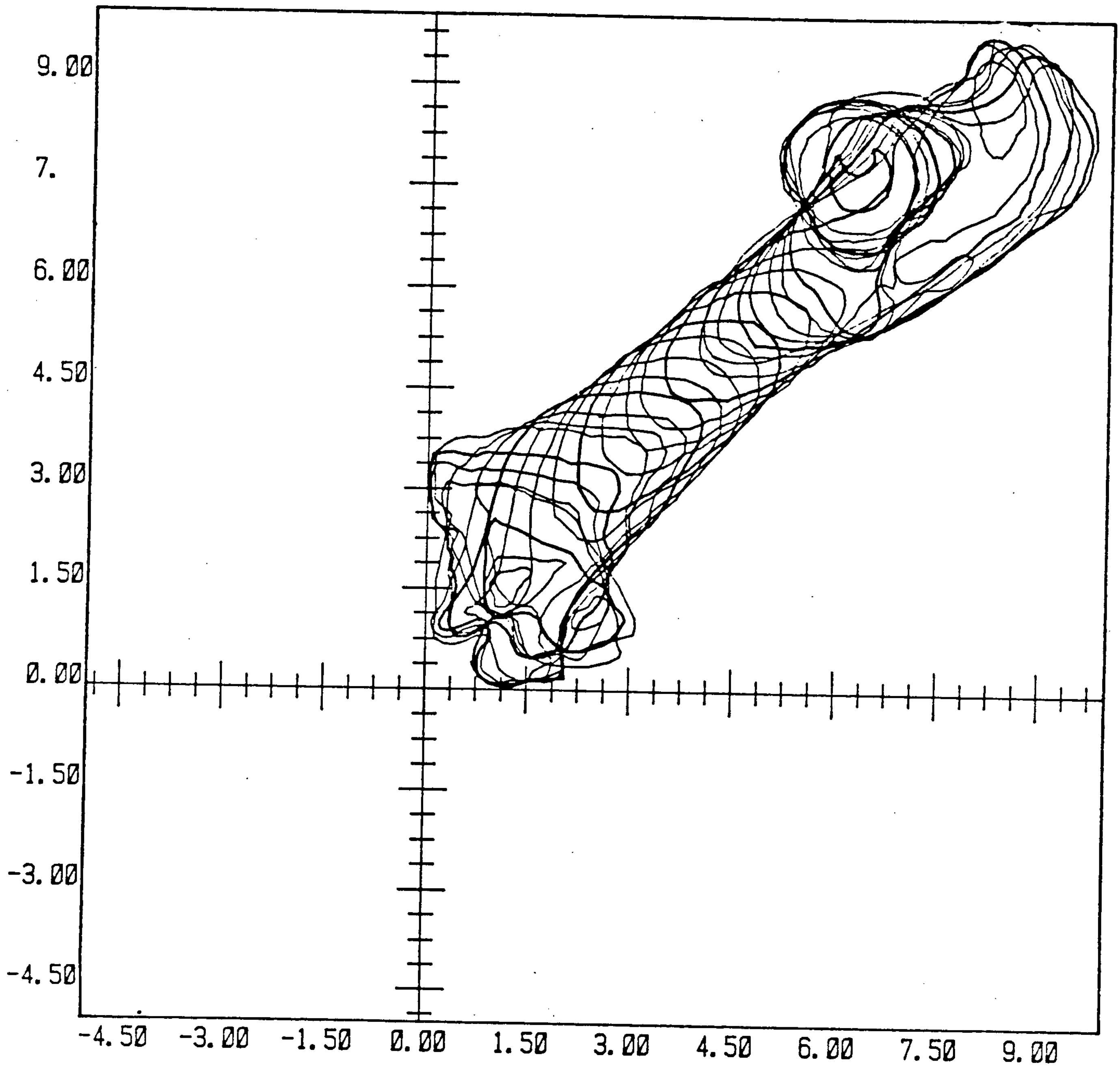


(a)

Figure 5.14 (a) - (c) Three views of the projected shape of tibia bone by superimposition of slices


9.00
7.50
6.00
4.50
3.00
1.50
0.00
-1.50
-3.00
-4.50
-4.50 -3.00 -1.50 0.00 1.50 4.50 6.00 7.50 9.00

(b)


9.00
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-3.00
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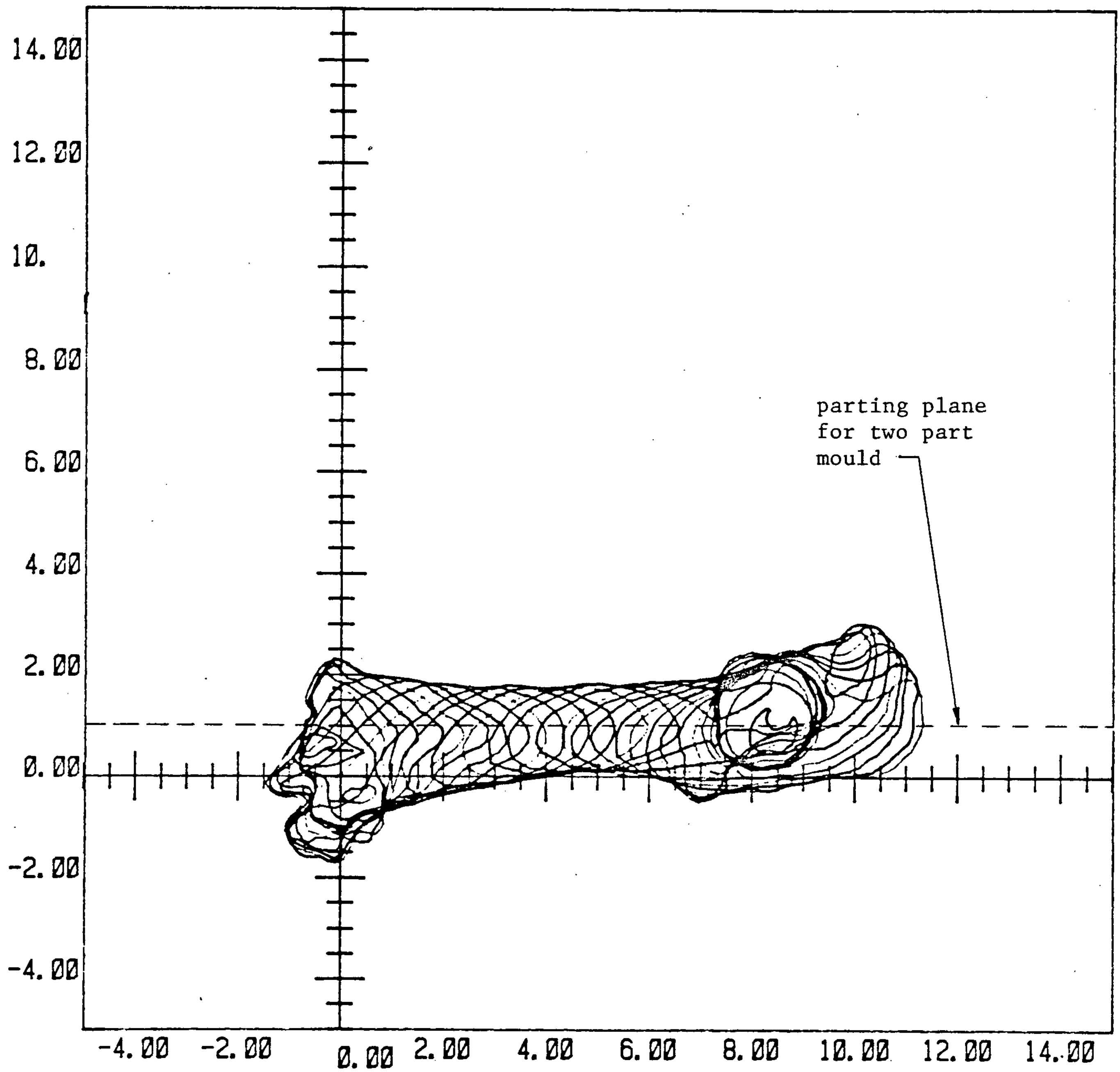
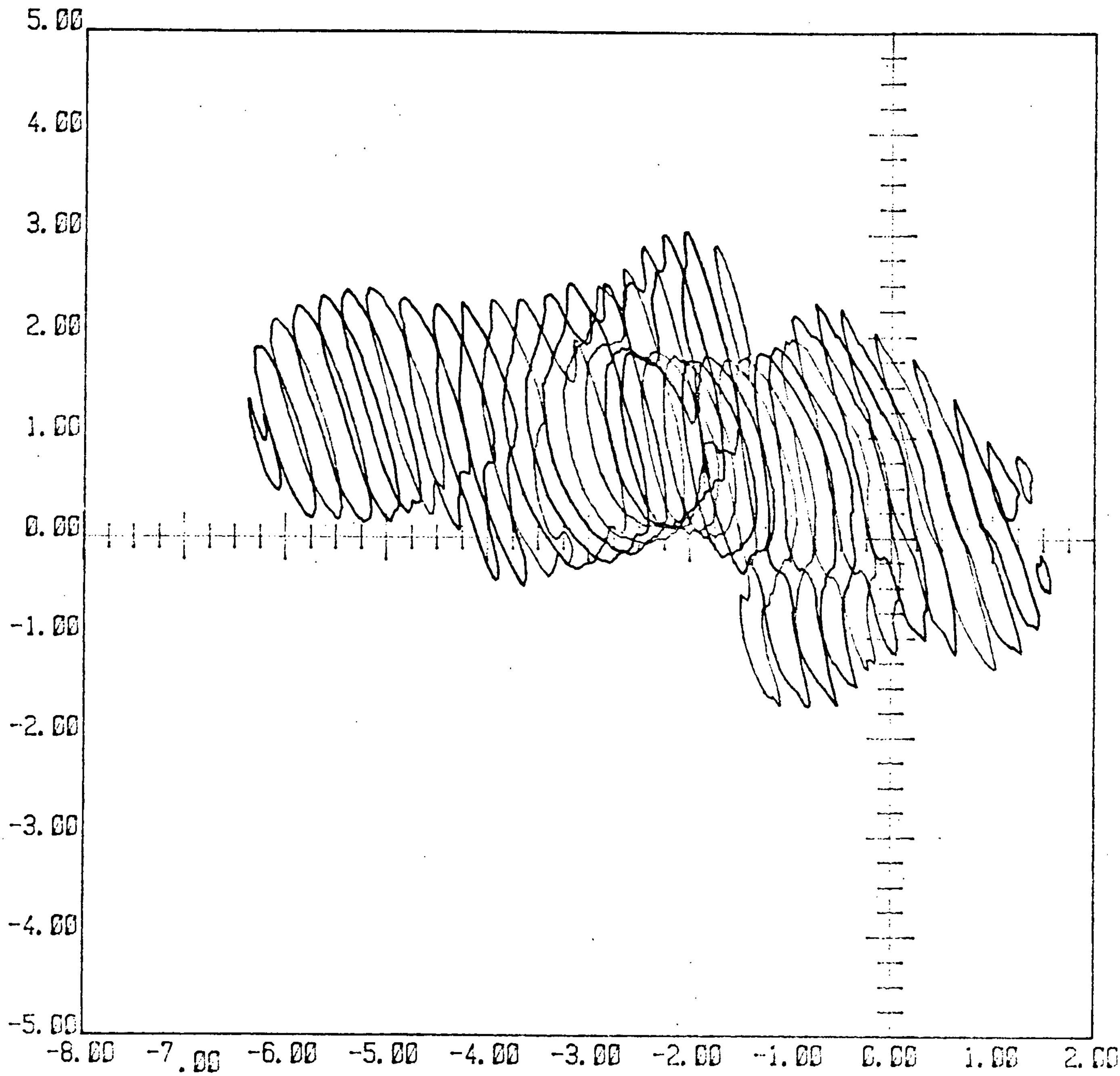
-4.50 -3.00 -1.50 0.00 1.50 3.00 4.50 6.00 7.50 9.00

(c)

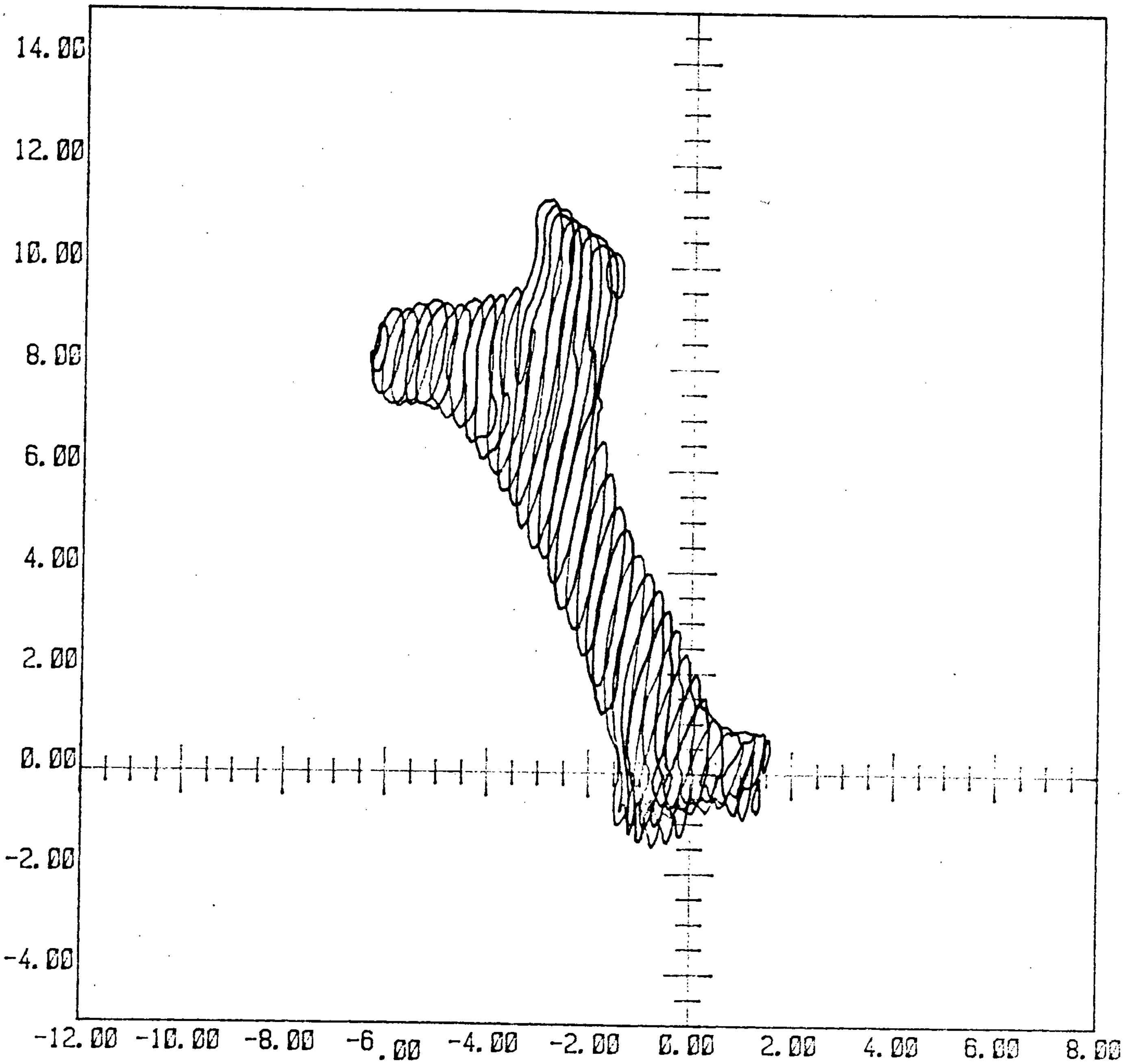


(a)

Figure 5.15 (a) - (c) Three views of superimposed slices after transformation for machining with no negative draft


5.00
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
-5.00
-8.00
-7.00
-6.00
-5.00
-4.00
-3.00
-2.00
-1.00
0.00
1.00
2.00

(b)



14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-12.00
-10.00
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00

(c)

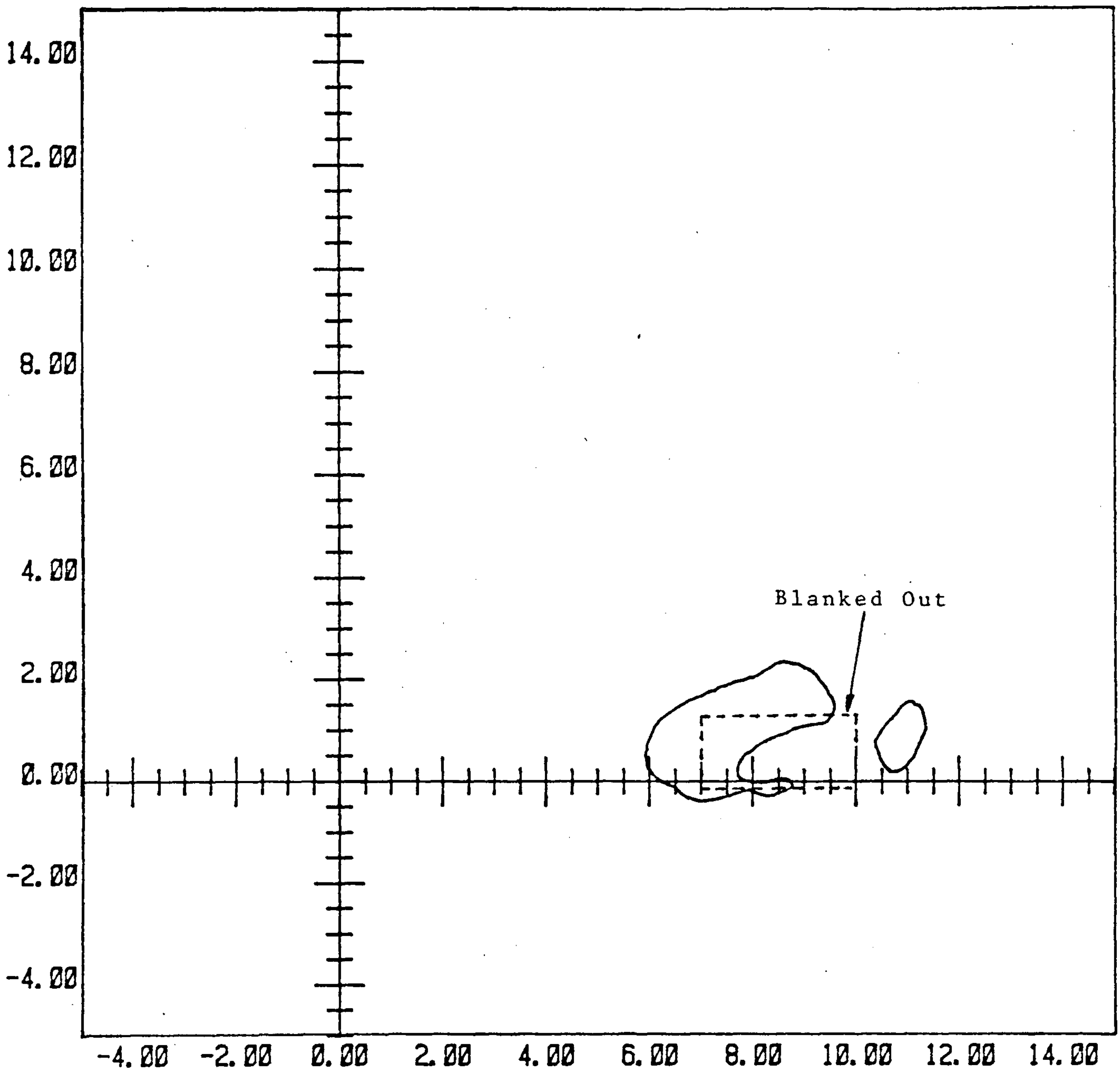


Figure 5.16 Data for 'holes' are blanked out since they cannot be machined

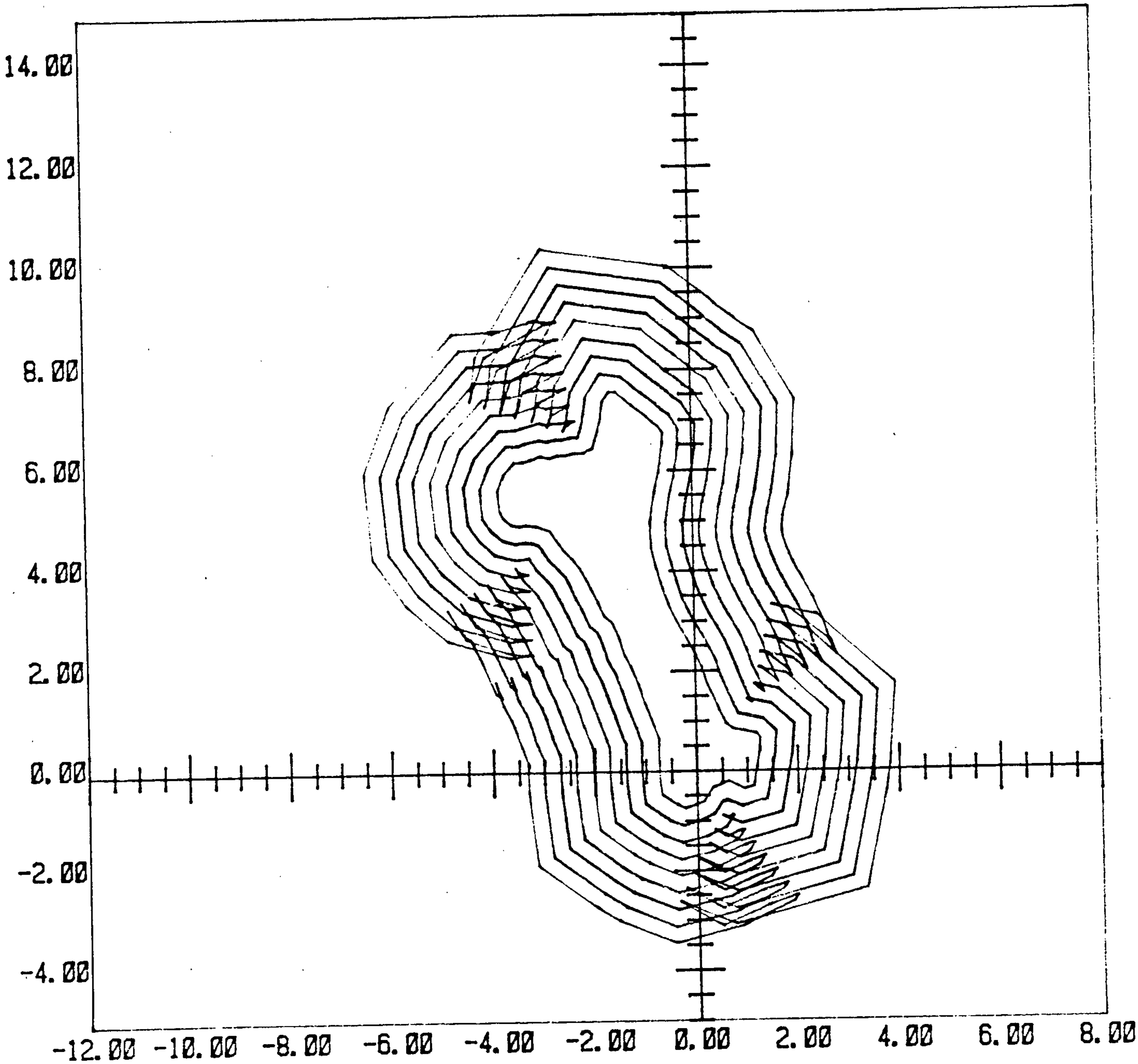


Figure 5.17 Data for base plane

Figure 5.18 shows the fitted surface of a half mould. The mould was machined by the POLYHEDRAL NC System similar to the previous examples.

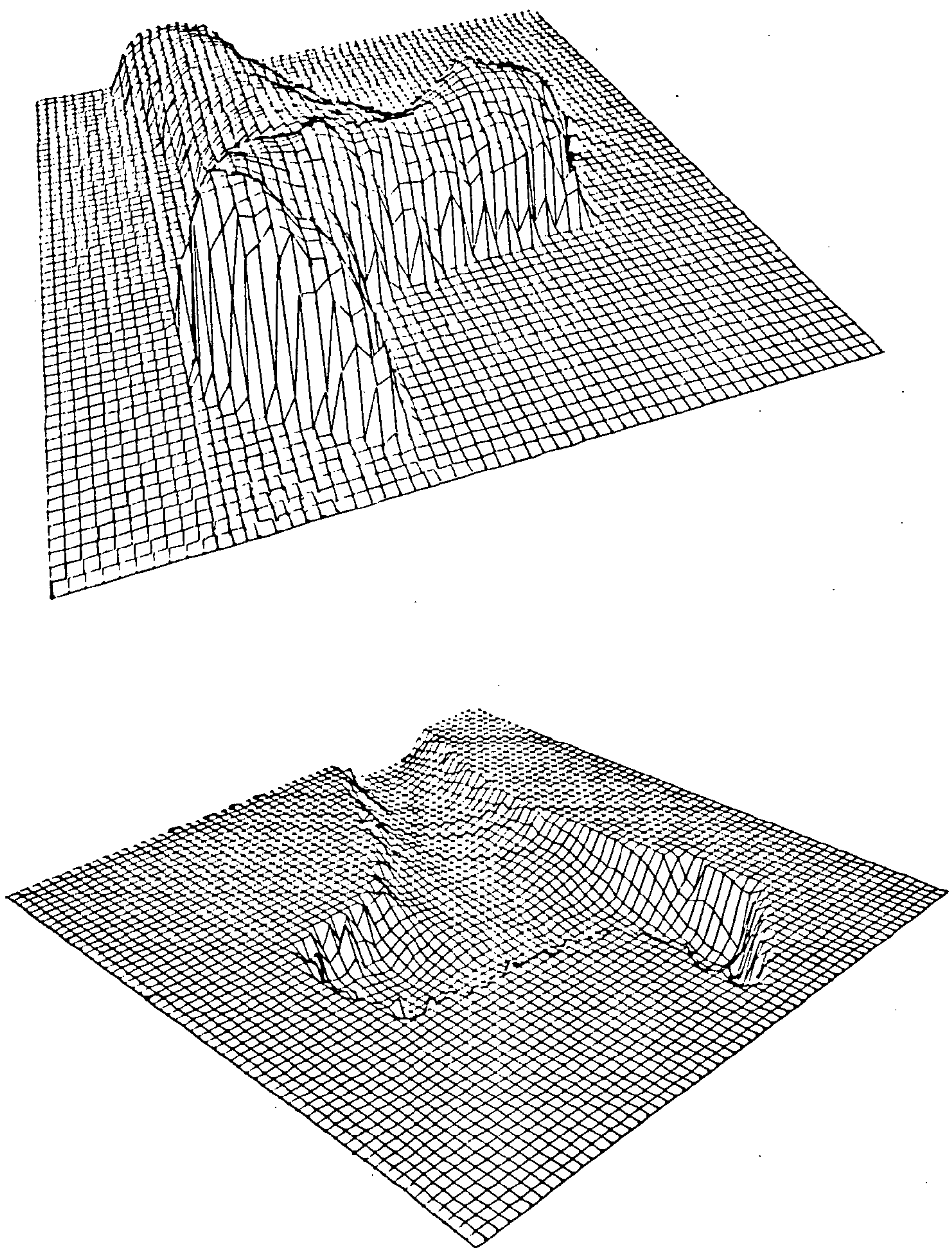


Figure 5.18 Fitted surface for bone and corresponding female mould

4. GENERAL PROCEDURE FOR REPLICATING A MEASURED SURFACE

A general procedure for making a mould of an arbitrary surface is described below:

1) Measure surface geometry by one of many measuring techniques;
2) Digitize measured data;
3) Check if surface orientation is suitable for machining, apply general transformation (rotations and translations) if necessary;
4) Add data for the parting plane;
5) Organize data into a rectangular array by surface-fitting;
6) Apply surface-adjustments if necessary;
7) Machine mould using the POLYHEDRAL NC system.

5. OTHER CONSIDERATIONS

In the above examples, the assumption was made that the cavity-surfaces were identical to the measured ones, ie., no allowances were made for shrinkages. This assumption holds true when such materials as silicone rubber, dental plaster or SYNCAST are used as moulding materials. The degree of shrinkage is largely dependant on the nature of the mould-material as well as the surface-area of dies and moulds. Estimate has to be based on detailed analysis as well as engineering judgement and experience, and is beyond the scope of this work.

VI. SPECIAL DIE-CAVITY SURFACES

1. SPECIALIZED DIES

Many die-cavities have characteristic surfaces which lend themselves to special treatments and require no transformation of surface-points into an orthogonal grid. These surfaces are usually analytical in nature and machining tool-paths can be analytically determined. For example, the tool-path for machining a circular cylinder follows a circular path that is concentric with the cylinder itself.(Figure 6.1) A cavity-surface may have the shape of a duct which follows a guiding curve called a spine, normal to which is a cross-section having a shape that is a function of arc-length along the spine. In this case, surface-normals can be calculated to determine the tool-positions to guide a cutter along the spine.(Figure 6.2)

If an item with these types of surfaces are to be made in large numbers over a long production period, it pays to develop specialized treatments to make dies for a particular manufacturing run. Software may have to be written for modelling of cavity-surfaces and organization of machining operations. For instance, bottles of different sizes and volumes following one standard shape may be required for mass manufacturing. An analytical model of this standard shape may be created for automatic machining of dies. By specifying general parameters of the model, characteristic surfaces of different sizes can be modelled and subsequently machined.

Another example is in the making of shoe-moulds. Figure 6.3 shows the governing boundary-curves of a shoe-moulding cavity-die. From these curves the die surfaces can be developed and tool path generated. (Figure 6.3) [Duncan & Forsyth, 1977]

During the course of this research, a specialized treatment was developed for modelling and machining of a special die for moulding of shell-type components; the approach is described in the following sections.

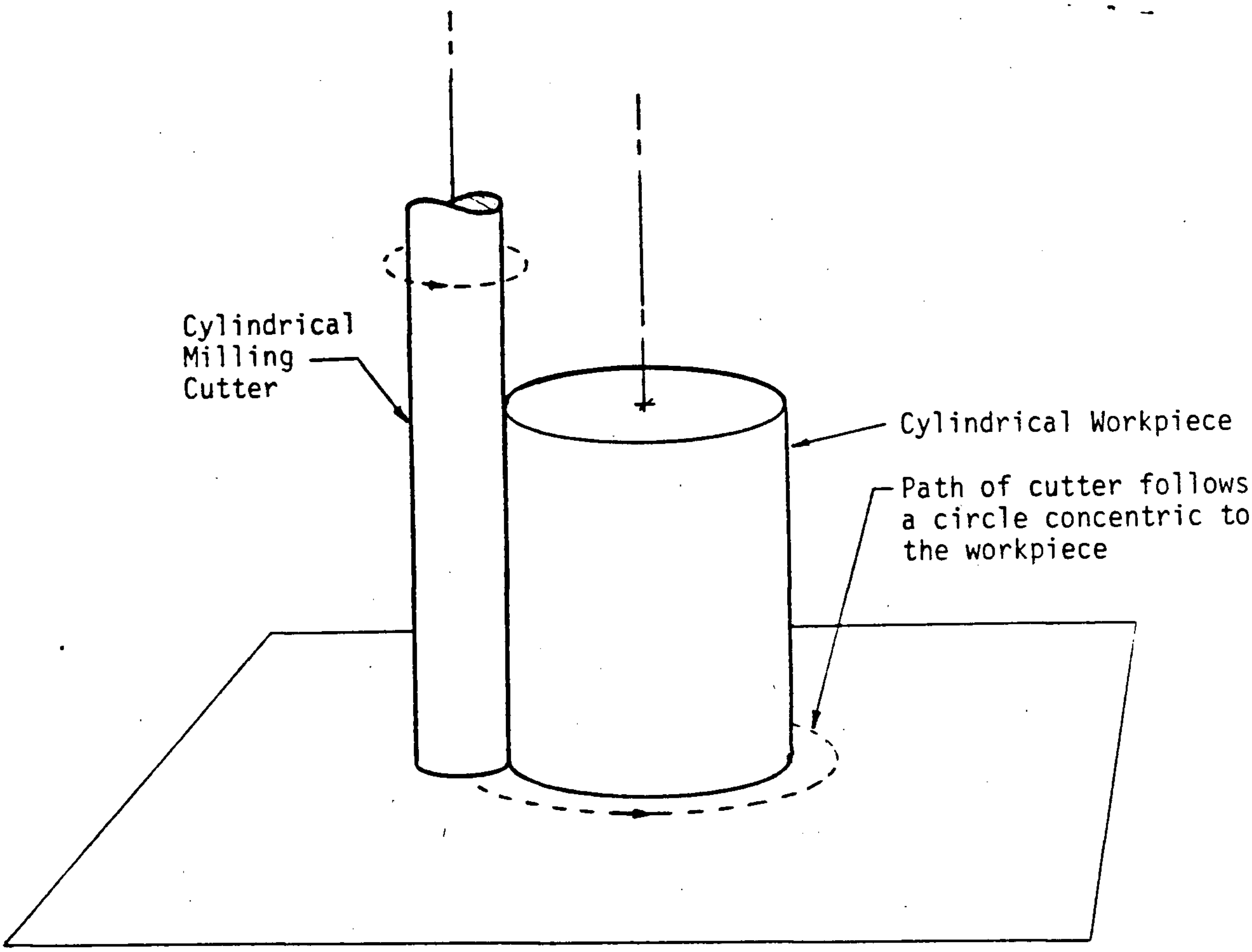


Figure 6.1 Tool path for machining a circular cylinder

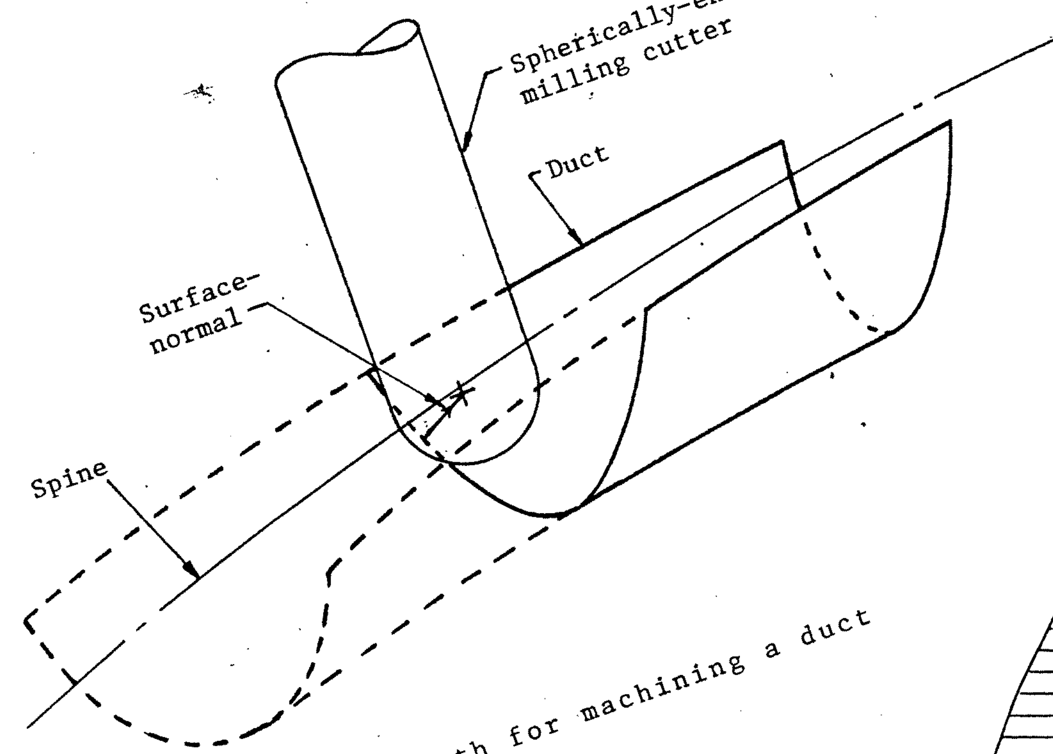


Figure 6.2 Tool-path for machining a duct

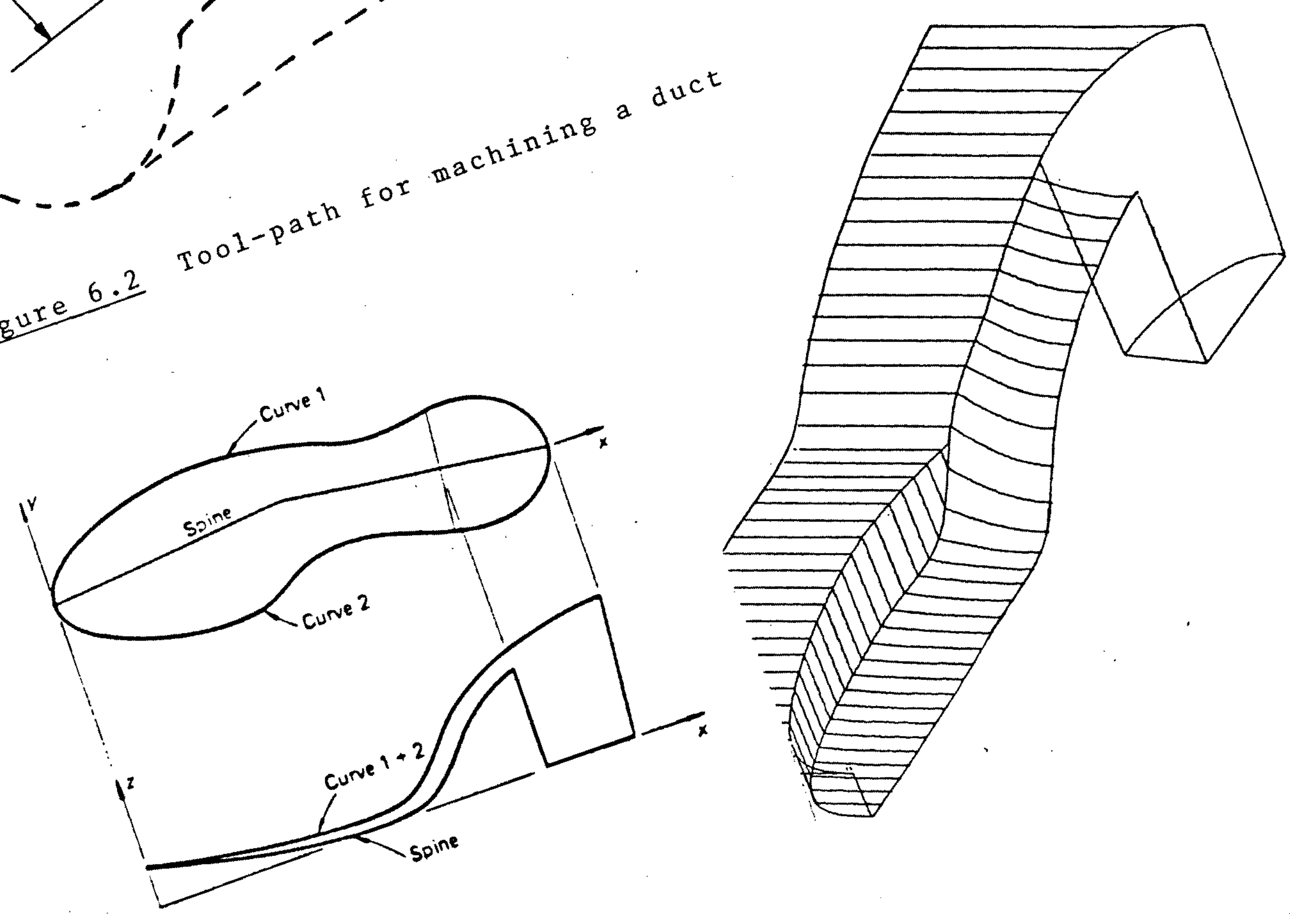


Figure 6.3 Governing boundary-curves of a shoe-mould and corresponding mould-surface modelled by the method of proportional development

2. SPECIALIZED MOULD FOR A SHELL

2.1 Introduction

Figure 6.4 shows two half-dies of a cavity-mould for a shell-like component required for a special application in reconstructive surgery. The lower cavity (floor) has a shape of an elliptic paraboloid and the upper surface (ceiling) is an offset surface at a specified distance away from the floor. The boundary walls are normal to the paraboloid everywhere, and the parting plane is inclined to facilitate the removal of the moulded shell.

It was decided that the lower die-block could be machined directly. To obtain good edge-definition, the top surface can be machined as a concave-upward cavity and the upper die made by casting into the cavity to form the ceiling surface.(Figure 6.5)

2.2 Organization Of Machining Process

The requirements of the machining process can be stated as follows :

a) upper and lower surfaces should be machined as concave-upwards cavities ;

b) boundary-walls normal to the lower surface should be machined to give good edge definition (the rim, see Figure 6.4) and flash-line ;

c) the inclined parting planes and the upper and lower base-planes require machining.

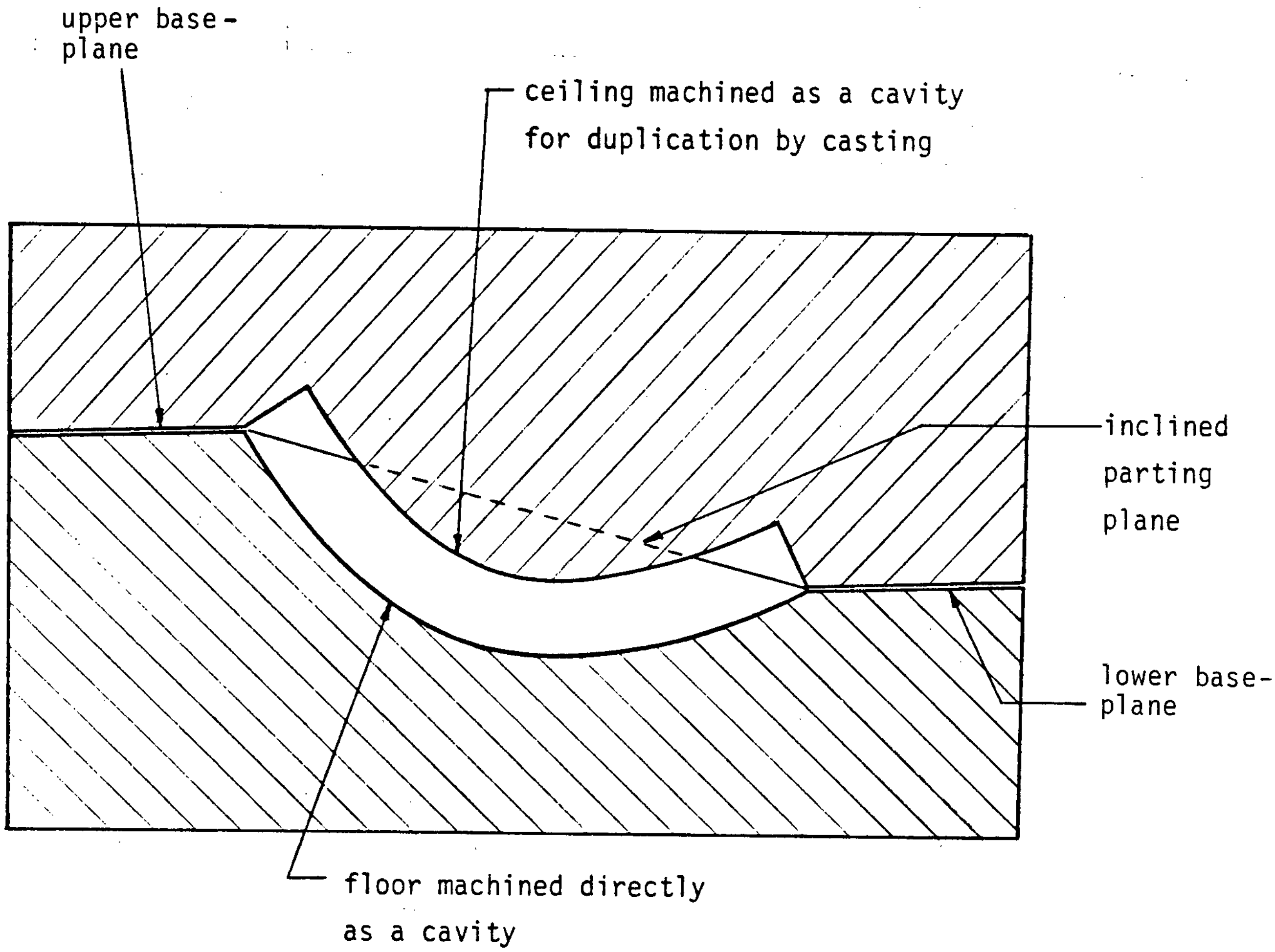
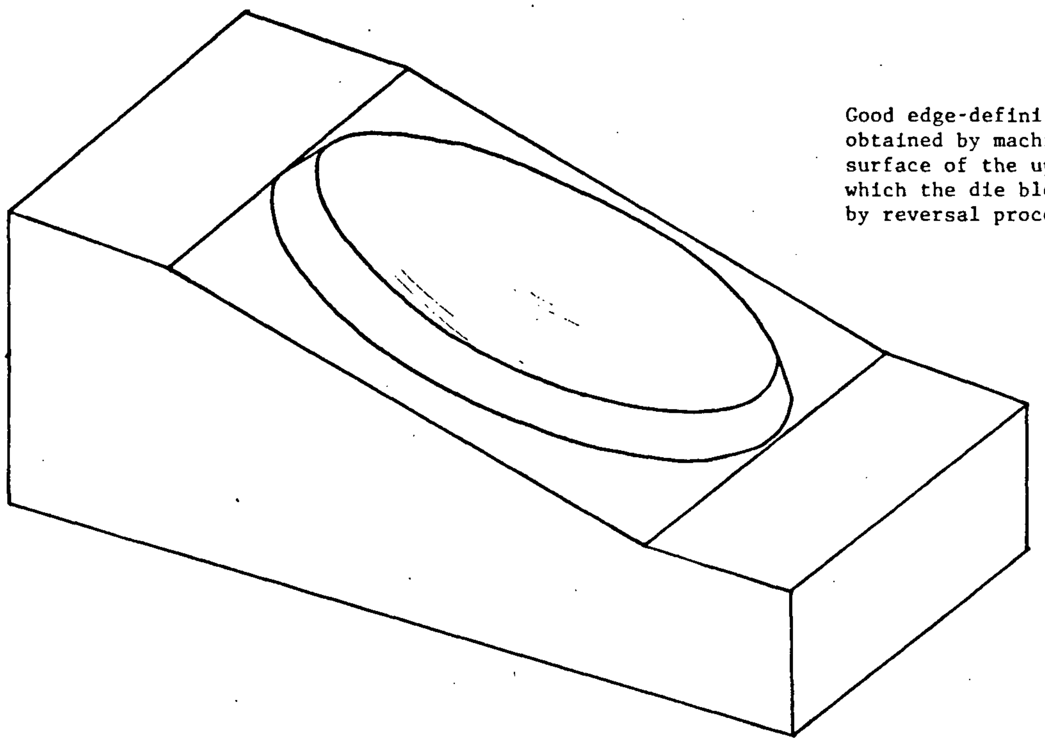


Figure 6.4 Cavity-die of a shell-mould



Good edge-definition can be obtained by machining the mirror surface of the upper die, from which the die block can be obtained by reversal process

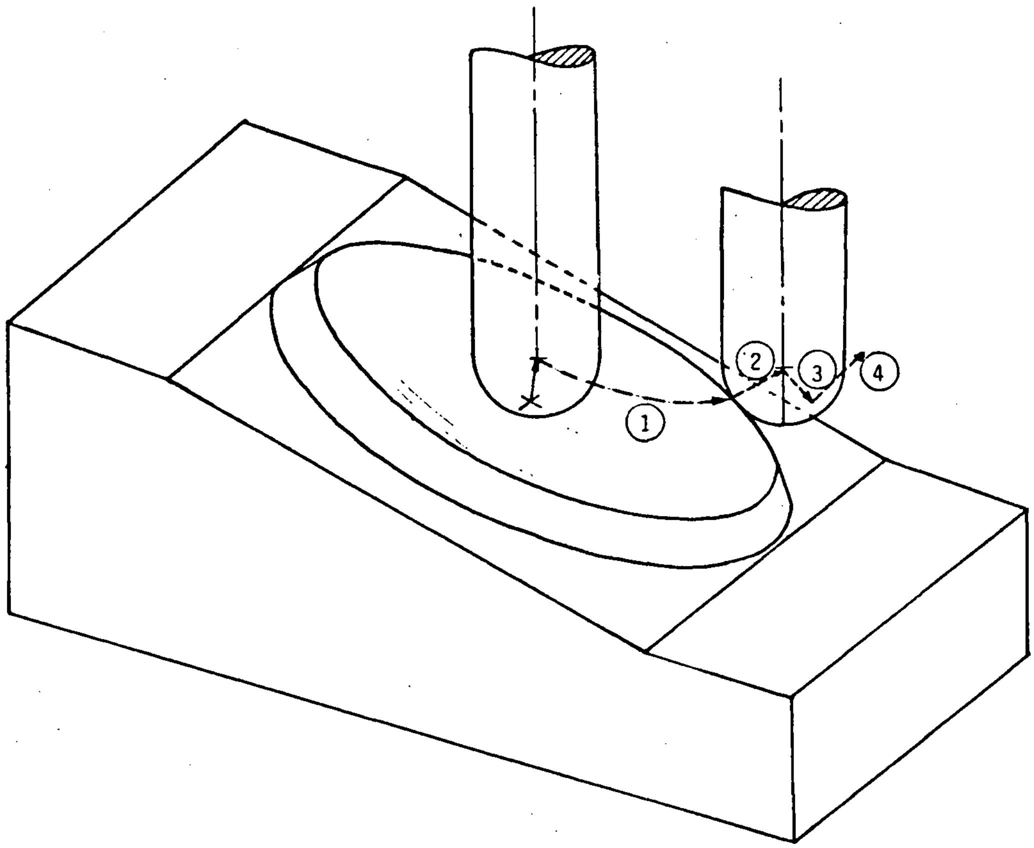
Figure 6.5 Upper die-block is made by casting into machined cavity

Cutter location data can be computed for both the upper and lower die-surfaces using a single program. The upper die-surface is an offset surface to an elliptic paraboloid. This is an analytic surface and tool-center positions can be calculated by employing general offset theories. The lower cavity can be considered as the same type of surface but with zero offset.

Program CAVITY6 (Appendix C) was developed to generate the CLD for both die-blocks. User inputs include the general parameters of the characteristic equation of an elliptic paraboloid, tool-radius, shell-thickness, and step-size for machining. In this way, different sizes and thicknesses of the particular shape can be handled, providing flexibility when different sizes are required for different production runs.

A spherically ended milling cutter was used, the size of which was determined by the minimum radius of curvature of the die-surface, as this would ensure the best surface-finish by using the biggest tool possible without the risk of undercutting.

The starting position for machining was the vertex of the paraboloid, from which the tool moved along at a constant increment over the cavity-surface. Once the limit of the cavity was reached, the tool moved first outwards (to avoid cutting the cavity-surface), and then downwards to cut the edge surface (side-walls), the directions of the cutter being determined by the vector products of the surface-normal and the boundary-curve (flash-line) tangent vector. After the side-wall was cut, the tool was guided across the parting plane to generate the inclined parting surface. This scan repeated at increments of y, until the whole die was machined. Figure 6.6 shows the cutter path for a single scan.



Tool Path Sequence

1) Tool Offset Path for Upper Cavity-Surface
2) Tool Moves Outwards to Avoid Undercut of Edge
3) Tool Moves Downwards to Cut the Edge Wall
4) Tool Moves Outwards to Generate Parting Plane

Figure 6.6 Cutting path for upper cavity

2.2.1 Calculation of Cutter Location Data

The characteristic equation of an elliptic paraboloid is :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - c'z = 0 \qquad (6.1)$$

or :

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z}{c} = 0 = F(x,y,z) \qquad (6.2)$$

The offset tool-centre position (x_t, y_t, z_t) is found by :

$$\begin{aligned} x_t &= x + \alpha_1 R \\ y_t &= y + \beta_1 R \\ z_t &= z + \gamma_1 R \end{aligned} \qquad (6.3)$$

where :

$$\begin{aligned} \alpha_1 &= \frac{1}{S}\frac{\partial F}{\partial x} & \frac{\partial F}{\partial x} &= \frac{-2x}{a^2} \\ \beta_1 &= \frac{1}{S}\frac{\partial F}{\partial y} & \frac{\partial F}{\partial y} &= \frac{-2y}{b^2}, \\ \gamma_1 &= \frac{1}{S}\frac{\partial F}{\partial z} & \frac{\partial F}{\partial z} &= \frac{1}{c} \end{aligned} \qquad (6.4)$$

and :

$$S = \left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2$$

When the tool reaches the edge (ie. when : $x > a\sqrt{\left(1 - \frac{y^2}{b^2}\right)}$) ;

it moves outwards along the binormal vector to avoid cutting the cavity-surface. The binormal is the cross product of the boundary tangent and the surface-normal.

The boundary-curve (flash-line) is an ellipse (Figure 6.7) with the formula :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Taking partial derivatives with respect to x and y, the direction cosines of the tangent vector $\underline{t}$ is :

$$\alpha_2 = \frac{1}{H}$$

$$\beta_2 = \frac{-H_1}{H} \qquad (6.5)$$

$$\gamma_2 = 0$$

where : $H = 1 + H_1^{\,2}$

and : $H_1 = \frac{b^2}{a^2} * \frac{x}{y}$

The binormal vector $\underline{b}$ is the cross product of the surface normal $\underline{n}$ and the edge tangent $\underline{t}$ where :

$$\underline{n} = \alpha_1\underline{i} + \beta_1\underline{j} + \gamma_1\underline{k}$$

$$\underline{t} = \alpha_2\underline{i} + \beta_2\underline{j} + \gamma_2\underline{k}$$

Thus : $\underline{b} = \underline{n} \times \underline{t}$ (6.6)

$$= \alpha_3\underline{i} + \beta_3\underline{j} + \gamma_3\underline{k}$$

$$\alpha_3 = -\frac{\gamma_1\beta_2}{H_2}$$

$$\beta_3 = \frac{\gamma_1\alpha_2}{H_2} \qquad (6.7)$$

$$\gamma_3 = \frac{\alpha_1\beta_2 - \beta_1\alpha_2}{H_2}$$

$$H_2 = \sqrt{(\gamma_1\beta_2)^2 + (\gamma_1\alpha_2)^2 + (\alpha_1\beta_2 - \beta_1\alpha_2)^2}$$

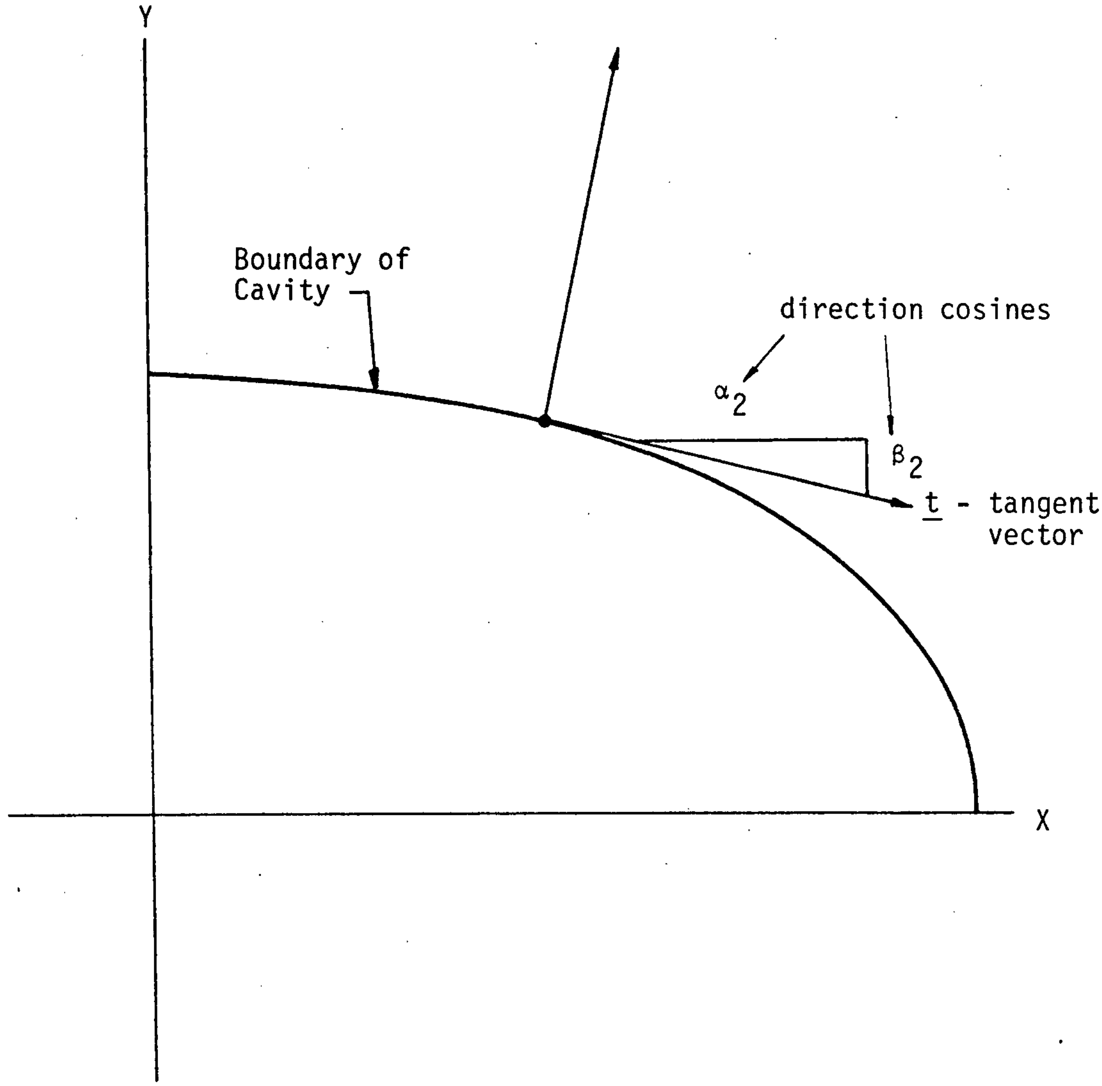
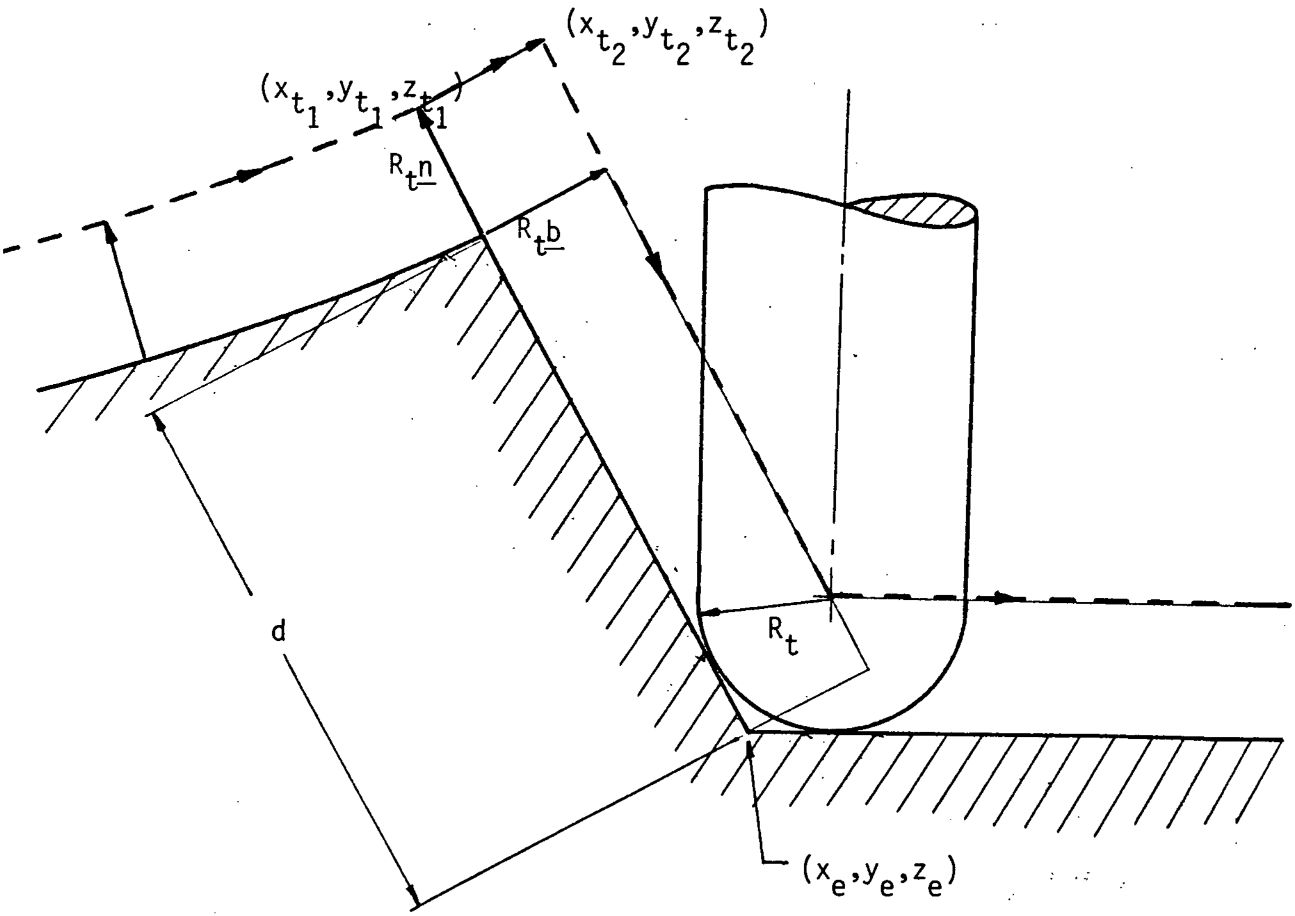


Figure 6.7 Flash Line of Die (Boundary of Lower Cavity) is an Ellipse

$$x_{t_1} = x_e + \alpha_1(R_t+d) \qquad x_{t_2} = x_{t_1} + \alpha_3 R_t$$
$$y_{t_1} = y_e + \beta_1(R_t+d) \qquad y_{t_2} = y_{t_1} + \beta_3 R_t$$
$$z_{t_1} = z_e + \gamma_1(R_t+d) \qquad z_{t_2} = z_{t_1} + \gamma_3 R_t$$



<u>Figure 6.8</u> Tool-motion for cutting the edge-wall

2.2.2 Machining Of Dies

Figures 6.9 and 6.10 show the upper and lower die-surfaces with their corresponding CLD path computed by program CAVITY6, and Figures 6.11 and 6.12 show the finished dies. Machining was performed using different materials ranging from polyurethane foam, SYNCAST, dental plaster, to plexiglass and zinc-aluminium alloy.

Surface asperities were imperceptible when materials with coarse surfaces, such as foam, were used. Cusps were observable on metal and plexiglass, but no hand-finishing was necessary for the particular application in which the dies are to be used. The step-size used was one-tenth the diameter of the milling cutter, and the total machining time for one die was of the order of three hours. Surface-finish would be further improved when machining is done in 'vector' mode as opposed to 2 - 1/2 D mode used for this research, a limitation imposed by the machine with which the author performed all his work. This was subsequently proven when the same machining procedure was performed in another installation. [1]

The advantage of the good surface-finish provided by a large cutting tool was partially offset by the large fillet it created at the flash-line (Figure 6.13). In practice, a 'retouching' operation might be necessary by guiding a smaller cutter around the edge to minimize any excessive 'flash' that

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[1] Vector Mode implies simultaneous motion on 3 axes; whereas an 2 1/2 D machine only allows simultaneous motion of 2 axes at one time.

may occur during the moulding process.

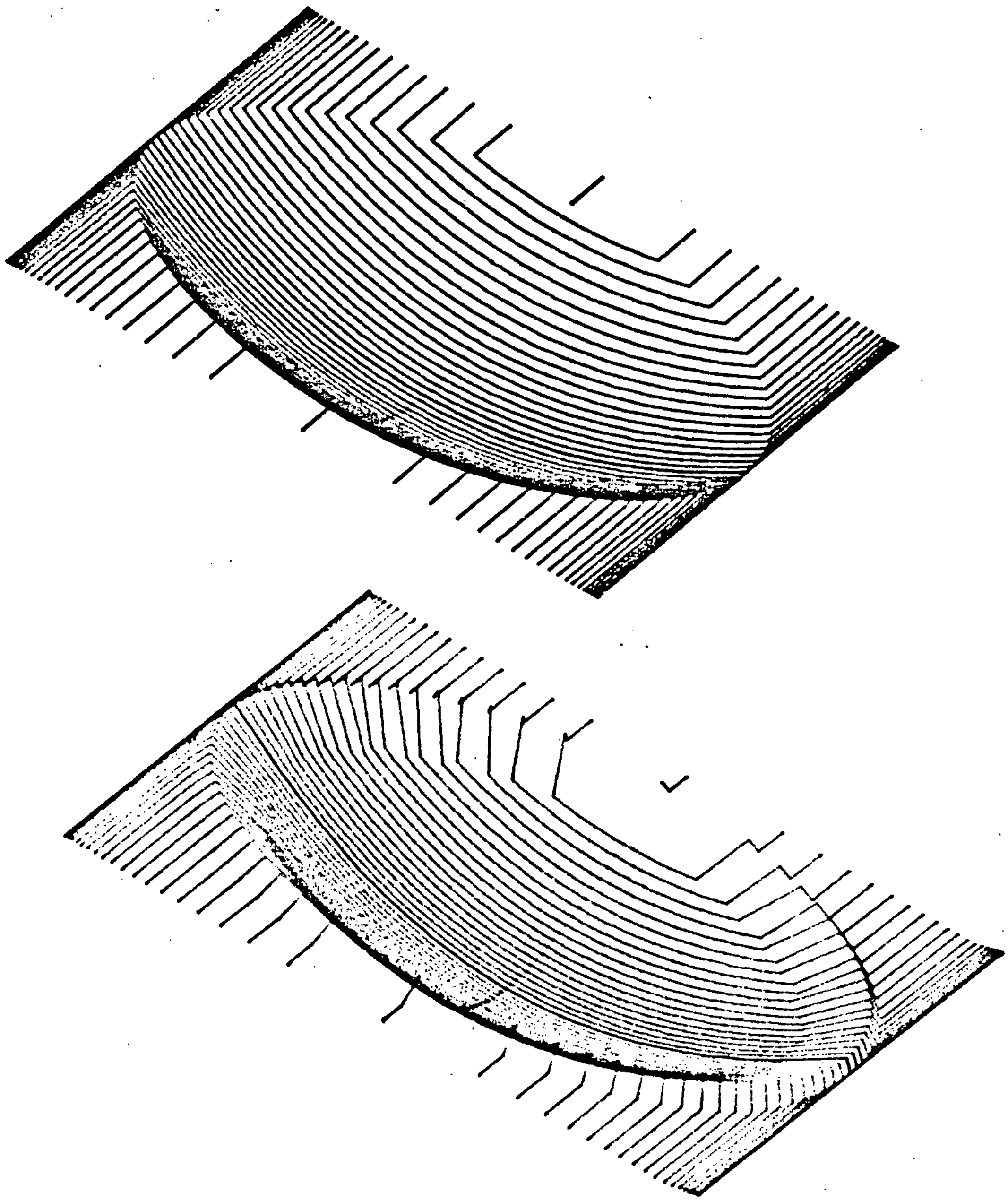
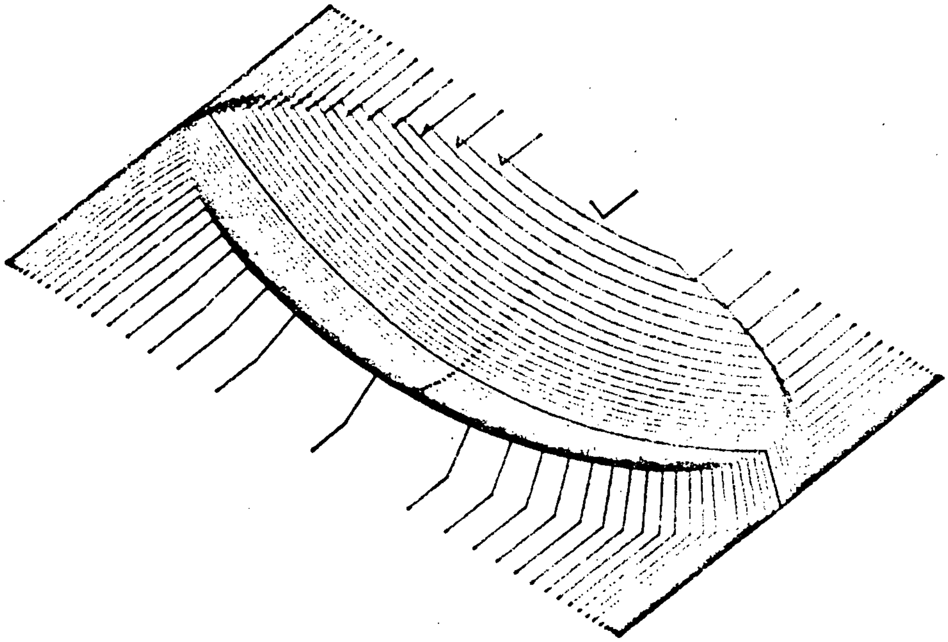


Figure 6.9 Lower cavity-surface and corresponding tool-path



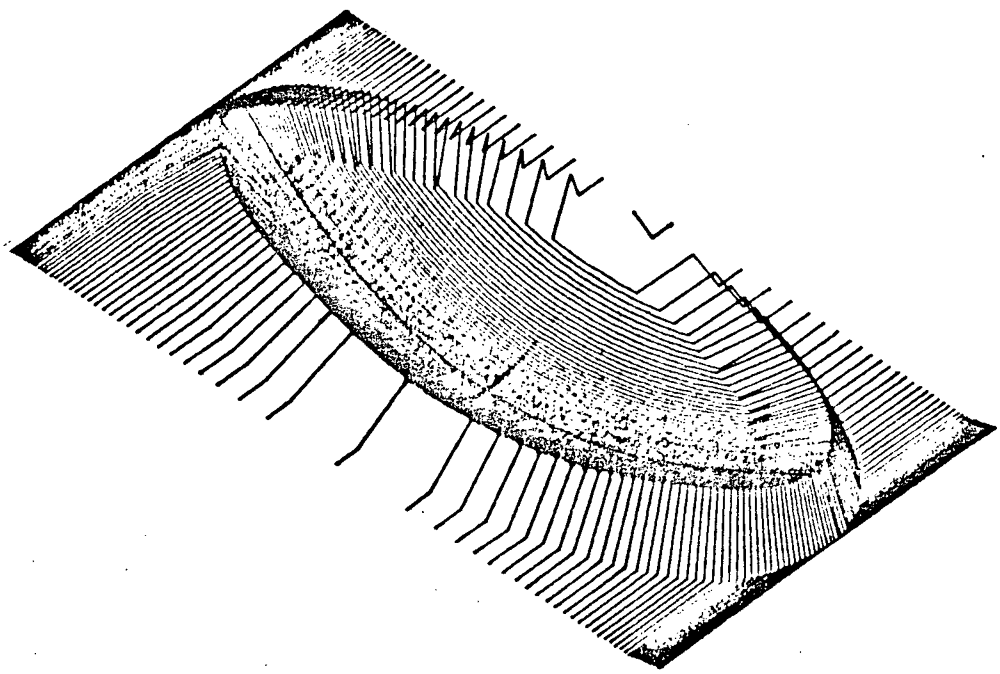


Figure 6.10 Upper cavity-surface and corresponding tool-path

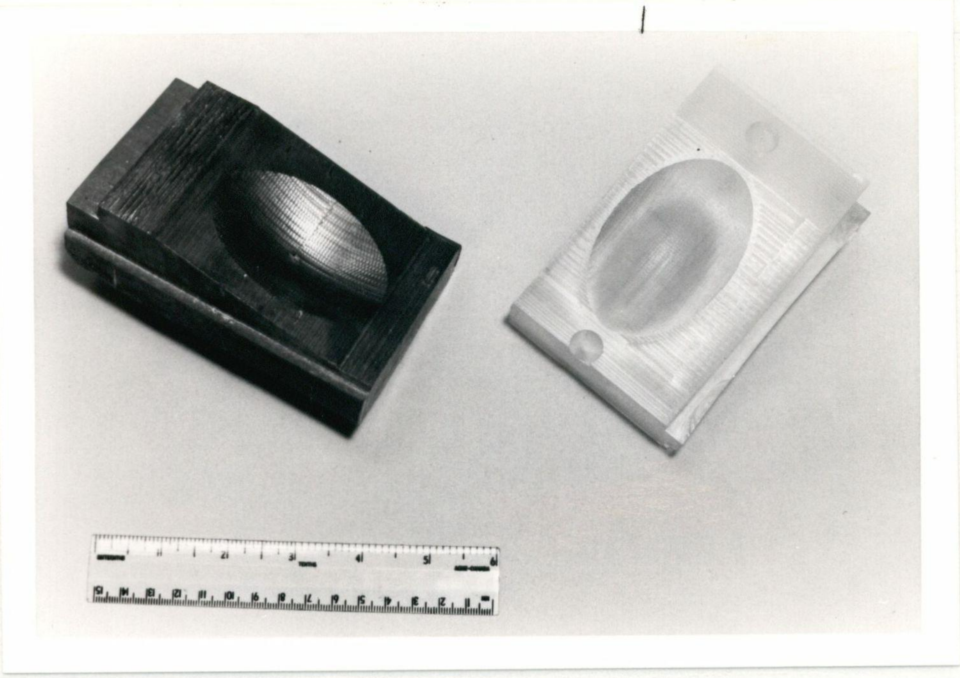


Figure 6.11 Machined die-block for lower cavity

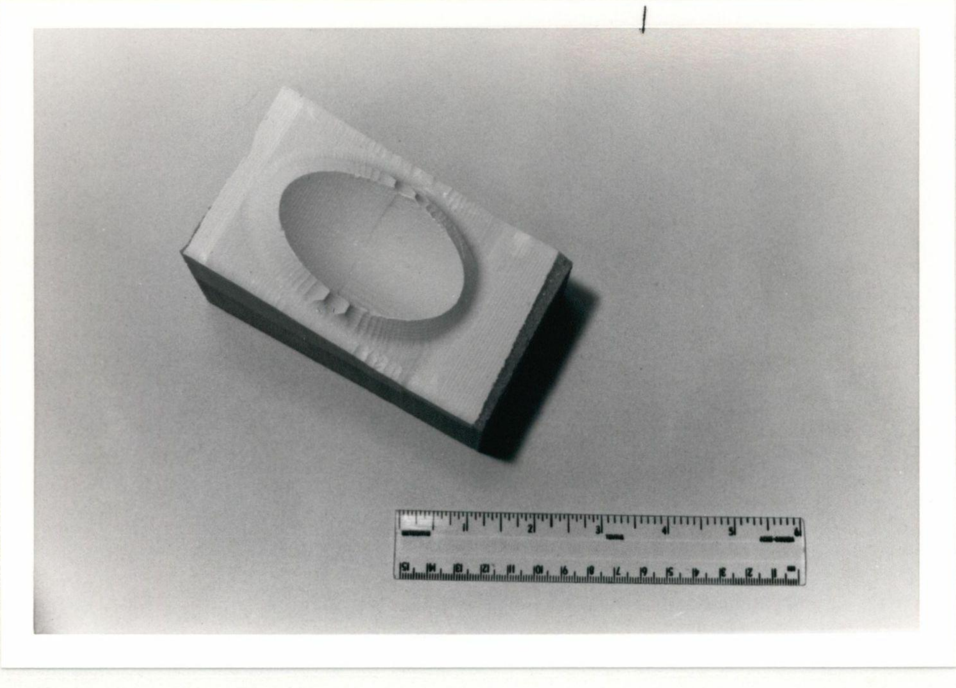


Figure 6.12 Machined surface for upper cavity

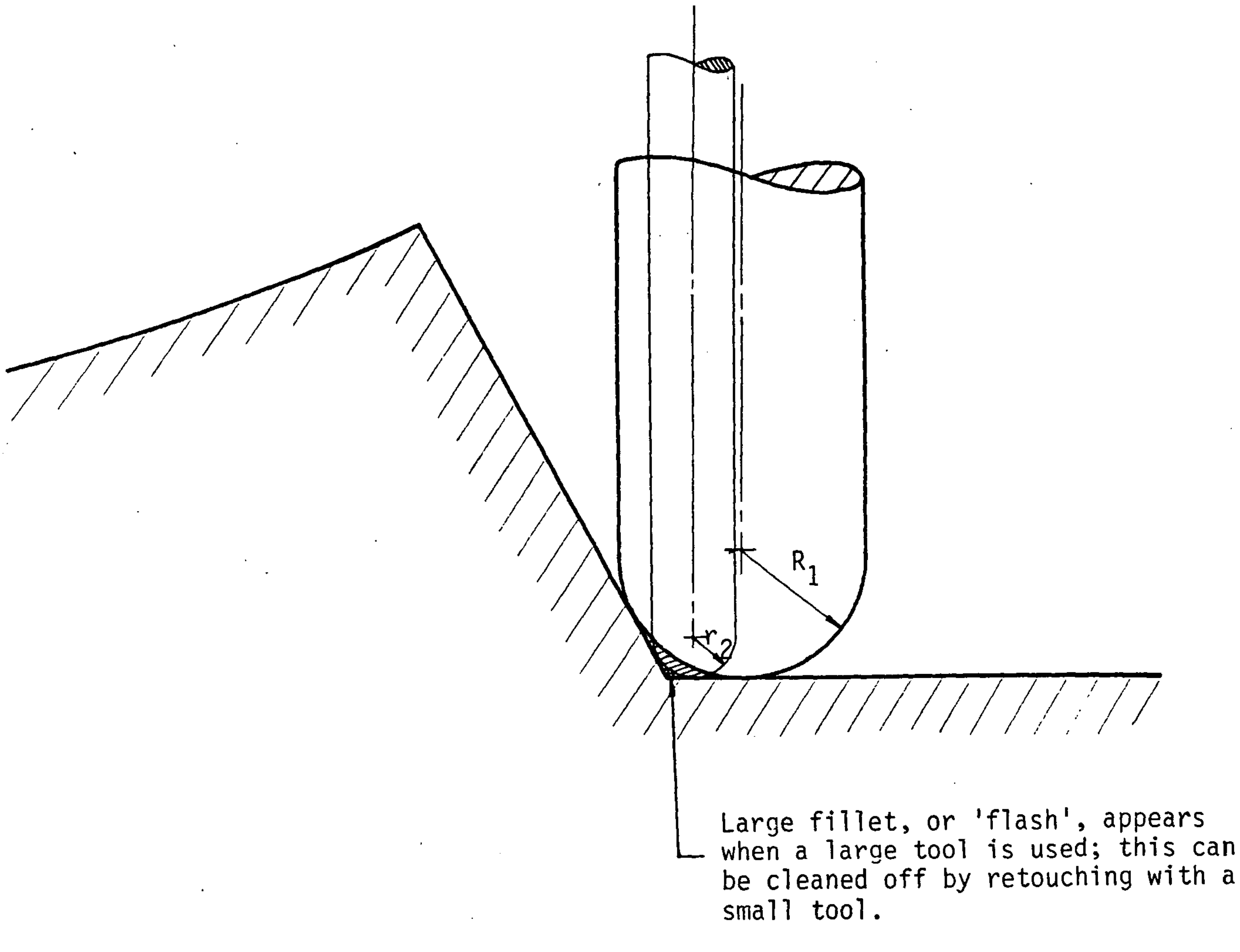


Figure 6.13 Machining upper cavity with a big tool results in large amount of flash at flash-line

2.3 Extension Of Method

The procedure described above deals with cavity-surfaces that are analytical, and with parting surfaces as planes. The same procedure can be applied to non-analytical cavity-surfaces and parting lines.

Figure 6.14 shows a cavity-die for a shell-type component such as a piece of human skull. Both the upper and lower surfaces are non-analytical, and the parting line is a three-dimensional space-curve of an arbitrary shape (Figure 6.15).

To generate the cutter location data, it is necessary to calculate the surface-normal and the tangent of the boundary-curve (ie., flash line). The surface-normal can be computed using the polyhedral concept (see Chapter II, Section 4); whereas the arbitrary space-curve representing the flash-line can be functionalized using one of the many available curve-fitting routines and its tangent calculated by finding the partial derivatives of the fitted curve.

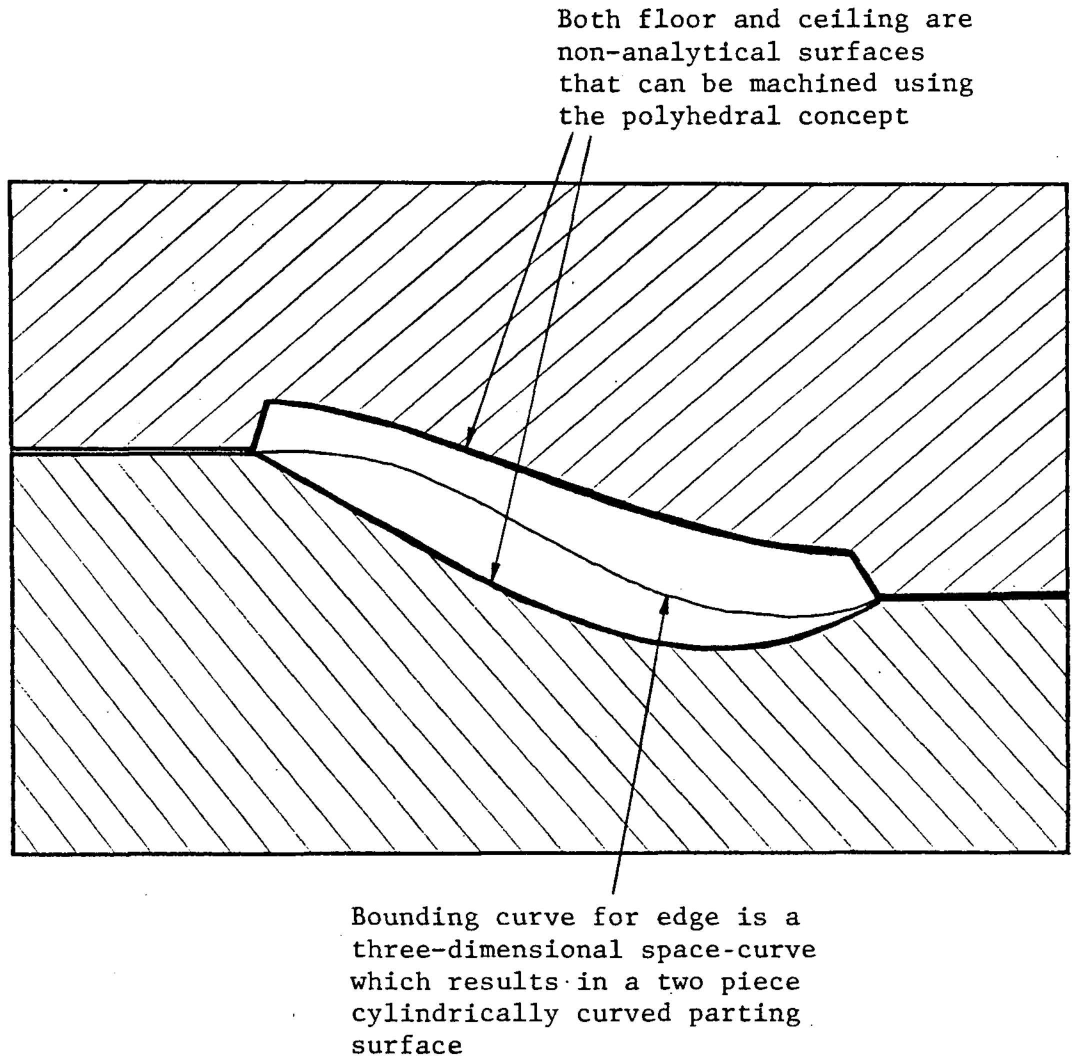


Figure 6.14 Shell-mould with arbitrary bounding surfaces and curved parting surface

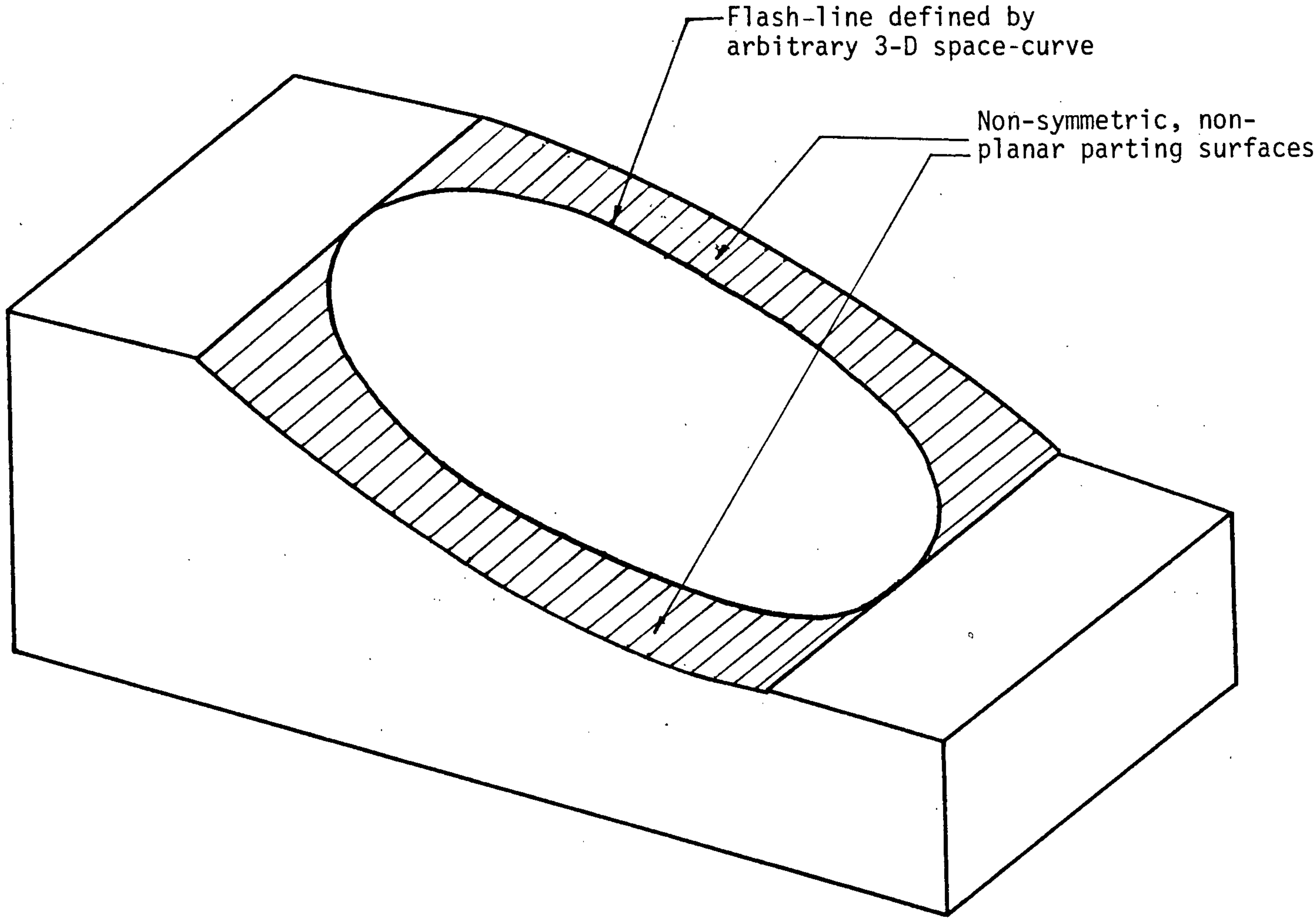


Figure 6.15 Arbitrary flash-line gives non-symmetric, cylindrically curved parting surfaces

VII. DISCUSSIONS AND CONCLUSIONS

1. CONSIDERATIONS IN DIE AND MOULD MAKING

When designing dies and moulds, many factors must be considered. In addition to the problems of shrinkage, spring-back, etc., the methods of die and mould making are also dependant on the manufacturing processes.

Traditionally, a pattern of desired shape is made first and used to mould a cavity of matching shape. In sand-casting, this pattern is usually made in wood, or more recently, machined in polystyrene foam. It is then buried in foundry sand to give the required cavity-mould. In investment-casting (also known as lost-wax process), the pattern is made of wax. This is dipped into slurries to form a shell around it. The wax is removed by baking the shell-mould and burning out the pattern.

Dies made of hard and tough materials, such as those for forging and injection-moulding can be made from pressing a hard male master model into a temporary softened (heated) block of metal. This is known as hobbing, and it has the advantage of being able to make multiple die-cavities from one single master model; but with the advent of modern automatic machine tools, the making of such dies may be more efficiently done by direct machining of the cavities, as discussed in Chapters III, V and VI.

The advantages of direct machining of moulds, such as those discussed in Chapter III, may be offset by the requirement that a large number of moulds have to be made. In this case, it is more efficient to make a male master pattern from which any number of moulds can be derived. This is especially apparent in processes like sand-casting and investment-casting, in which moulds are destroyed during the removal of cast products, and thus can be used only once. Moreover, materials commonly used in casting processes , such as foundry sand, are impossible to machine.

Direct machining of dies is advantageous when many replications are to be cast from one single mould in processes such as injection-moulding, forging or the laying-up of fibre-glass materials.

2. CASTING AND MOULDING OF MODELS

The simplest form of moulding process is what is known as open moulding -- in which liquid material is poured into a single mould - cavity. This method was used for making most of the models for this research. More elaborate models require two or even more seperate die-blocks, such as the shell-mould and the shoe-mould discussed in previous chapters.

To facilitate the removal of material after the moulding process, side-walls of a die are usually 'drafted', ie., they have a slope to the vertical at a small angle. (See Figure 3.1) Negative draft is usually not allowed, since this cannot be machined using simple milling operations. In addition, it locks the component in, although this may not present any problem for flexible and pliable materials. To prevent the moulded piece from sticking to the cavity surfaces, suitable release agent may have to be applied.

The open moulding approach is most suitable in 'surface-reactive' casting. Fluid or fibrous material is applied to the mould by pouring or spraying where it solidifies by some surface-related mechanism. An example of this is the method of 'slip-casting'. A female cavity made of porous material (usually some form of plaster) is filled with a ceramic slurry (usually clay). Capillary action removes water from the slurry, leaving a uniform semi-rigid shell of dewatered slurry (a cake) on the cavity-wall, its thickness depends on time allowed. Surplus slurry, which is not yet dewatered, is then poured out, producing a shell-mould without the use of a male former model (core) within the cavity.

This method was examined using the plaster mould of the facial model (see Chapter V), and the resulting shell-mould is shown in Figure 7.1

Figure 7.1 Shell-mould of facial mask from slip-casting

3. DIE DESIGN AND MACHINING SYSTEM BASED ON POLYHEDRAL NC SYSTEM

The ultimate objective of this research is to develop a general die design and manufacturing system using modern high speed computers and automatic machine tools. Such a system should have the following capabilities.

a) It should allow designers to design and model surfaces from analytical equations and measured physical models, as well as from two-dimensional sketches of characteristic boundary-curves.

b) It should incorporate features for visualization and manipulation of surfaces so that designers can interactively adjust and modify designed surface-shapes and properties.

c) It should be able to support different types of machine tools, from the simplest to the most sophisticated. Moreover, it should incorporate real-time control of machines to permit a fast turn-around time.

d) It should be 'user-friendly', by not requiring experts in computer programming to operate the system. On the other hand, it should allow special programs to be developed for specific types of dies similar to those discussed in Chapter VI.

3.1 Work Achieved In This Research

Elements of a proposed automatic die design and machining system, based on the polyhedral concept, have been developed. They incorporate programs written over the past few years as well as new routines that were developed for the purpose of this research. To summarize, three major goals were achieved :

a) The Method of Highest Point was extended into a general geometric modelling routine for piecewise compound analytic surfaces with the development of program GEN7.

b) A general approach in replicating arbitrary surfaces by casting into machined cavity-moulds were developed by utilizing surface-fitting program TRUEPERS and machining program SUMAIR. Moreover, techiques were developed to handle complicated surfaces (such as those measured from CAT scanning) and to transform them into an orientation most suitable for machining. A new approach to formation of a 'sharp' parting plane and flash line was developed.

c) A specialized approach in the making of dies for shell-type components was proposed and tested. In particular, good edge-definition (ie., sharp edge) was obtained from reversal techniques. Good surface-finish and minimum flash were achieved by cascading large and small tools during the machining process. This approach also permits arbitrary bounding surfaces with non-symmetric, spatially curved flash-lines.

3.2 Scheme For Proposed Die Design And Machining System

Figure 7.2 shows scheme for an integrated general approach for die and mould making. Different classes of surfaces, whether specified by equations, measured data, or projected boundaries, can be modelled by routines GEN7, TRUEPERS and PROPDEV respectively. The point-defined surfaces thus generated can then be processed by program SUMAIR or NEWSU to give the machining path of a spherically ended milling cutter.

Programs such as TRUEPERS and GEN7 also incorporate graphics subroutines so that the modelled surfaces can be viewed on a CRT screen or from hardcopy plots. This facilitates error checking and surface-adjustments (when necessary) by providing easy means for users to visualize any intermediate or final results.

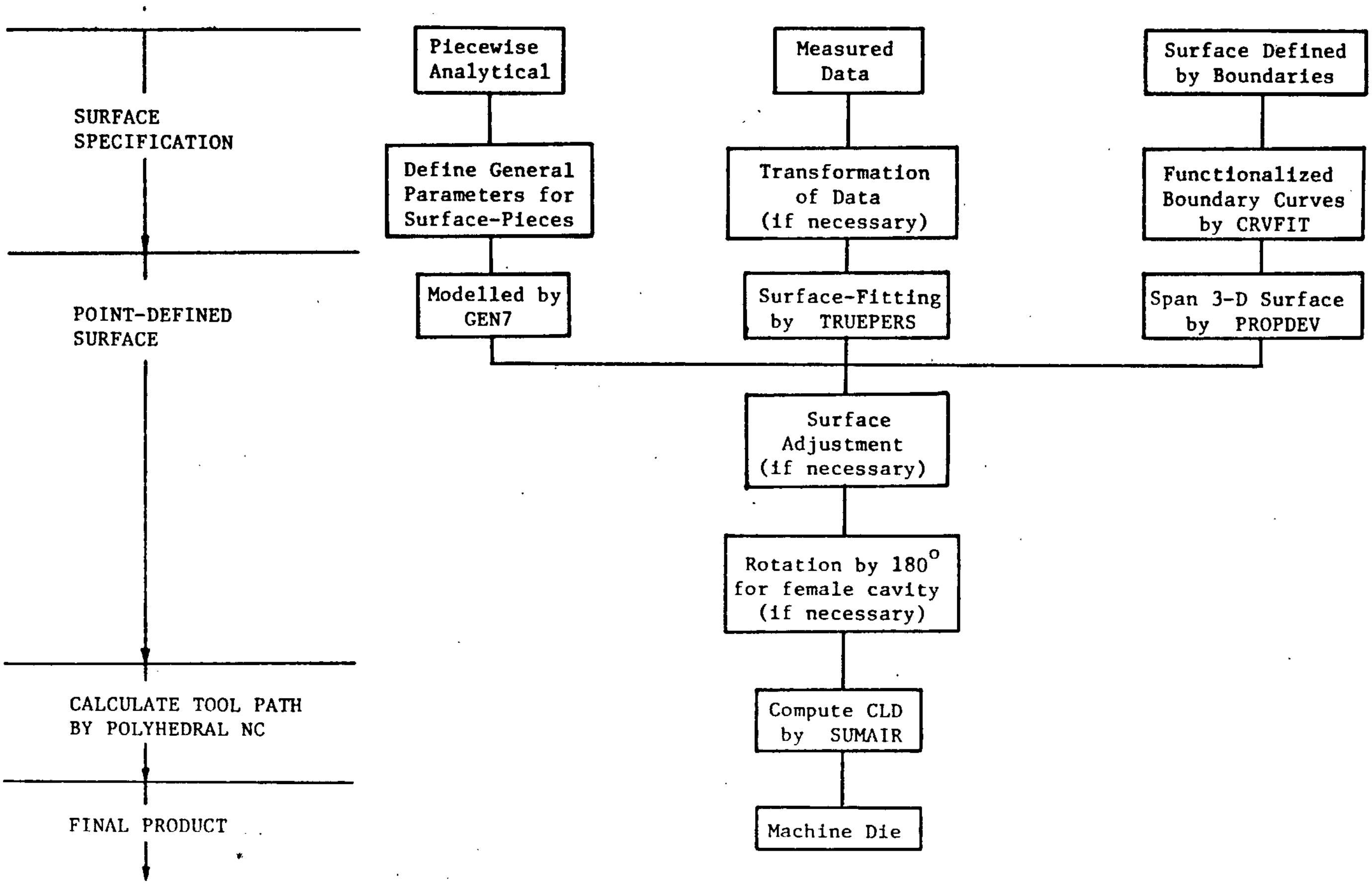


Figure 7.2 Schematic approach of a general die design and machining system based on POLYHEDRAL NC

Machining programs SUMAIR and NEWSU have been modified so that they are now 'machine-independent', ie., the format of their outputs does not limit them to be used by only certain particular CNC machines. Previously SUMAIR and NEWSU wrote the machining commands onto EIA-formatted paper tapes. The tapes had to be physically transferred and mounted onto a SLO-SYN NC machine before machining. This was both unreliable (paper tapes tend to break or jam), inefficient and resulted in long turn-around time. The programs have been modified so that CLDs are now written onto data files as x,y,z coordinates of tool centre positions. This data can then be electronically transmitted via data-link from an Amdahl 470/V8 mainframe computer (where the POLYHEDRAL NC system resides) to a PDP 11/34 minicomputer which controls the milling machine. Automatic routines in the PDP convert the cutter co-ordinates into coded machine commands.

The main advantage of specifying the cutter location data as Cartesian points is that they can be processed for use on different machines using different command codes. Such codes can then be loaded onto a NC machine via electronic links (as the author used), magnetic or paper tapes, or floppy disks etc., depending upon the facilities of a particular installation.

Electronic transmission of data dramatically shortens the time lag between computation and actual machining. This is especially important when the physical distance between the computing and machining site is very long. This would be done in a few hours in an established set-up.

Table II shows a summary of various computer routines that can be incorporated into the proposed system. As mentioned previously, this system should allow routines to be developed for specialized die-surfaces, such as program CAVITY6 for the shell-mould (see Chapter VI). To simplify the task of program development, standard modules such as PLTXYZ (for plotting) and CNCPKG (for generating machine commands, see Table II) have been written so that they can be linked to analysis routines that are required for specialized surfaces.

Table II - Summary of Routines to be used for Proposed Die Design and Machining System

SURFACE MODELLING

GEN7 ...... modelling piecewise compound analytical surfaces

TRUEPERS .. general surface-fitting program (with graphics)

CRVFIT .... general curve-fitting program employing conic-fit

PROPDEV ... spanning 3-D surface from projection of surface boundaries by proportional development

SURFACE VISUALIZATION AND MANIPULATION

TRUEPERS .. general surface-fitting program (with graphics)

PLTXYZ .... trimetric plotting of point-defined surfaces from outputs of GEN7, CAVITY6, TRANSFORM etc. (no hidden line removel)

PBONE3D ... trimetric plotting of digitized contour lines

TRANSFORM . general transformation of data

MACHINING

SUMAIR, NEWSU ... compute CLD path by polyhedral concept, incorporating anti-interference feature

CNCNEWSU .... generate command code for SLO-SYN machine from output of SUMAIR or NEWSU

CNCPKG .... package of FORTRAN callable subroutines for generating command codes for machining by SLO-SYN NC milling machine

4. PROPOSED FURTHER WORK

To incorporate the elements developed for the proposed die design and machining system, further work is necessary to merge them into one single unit so that a 'turn-key' system, which includes both hardware and software, can be made. Recommended items of work are as follows :

a) development of 'master control program' to direct and allocate tasks among various elements of the system;

b) development of interactive 'front-ends' to facilitate communication between user and computer, perhaps in the form of screen menus;

c) development of data accquisition apparatus directly compatible with the system to eliminate the need for digitization;

d) development of analysis routines for evaluating surface properties as well as surface-adjustments.

5. CONCLUSIONS

Cavity-dies consist of bounding surfaces that are either analytical or non-analytical. Analytical surfaces are usually combinations of various individual surface-elements that can be represented by mathematical equations. For simple, developable surfaces, tool-paths can be computed and special algorithms written to organize the machining processes. Others may contain standard analytical surface-pieces intersecting and interpenetrating one another at curves of discontinuity. These have to be sculptured. Most of these types of surfaces can be modelled by routine GEN7 for subsequent machining by the POLYHEDRAL NC system.

Arbitrary shapes defined by measured data can be transformed into a point-defined surface over a regular orthogonal grid using program TRUEPERS. Various shapes measured by different techniques have been so treated and reproduced successfully by moulding into machined cavity-moulds.

Elements of an automatic die design and machining system have been developed and tested. The results, as shown in previous chapters, prove that the polyhedral approach provides a feasible mean for automatic modelling and machining of dies and moulds. This in turn can be developed into an integrated and efficient manufacturing system.

APPENDIX A

General Transformation of Quadric Surfaces

Ellipsoids

Characteristic Equation :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$$

After transformation onto the X'Y'Z' coordinate system :

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} - 1 = 0$$

In terms of the original XYZ coordinate system :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} + \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} + \frac{(l_3x_1+m_3y_1+n_3z_1)^2}{c^2} - 1 = 0$$

Converting into the form :

$$A_1z_1^2 + B_1z_1 + C_1 = 0$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2 + \left(\frac{n_3}{c}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)}{a^2} + \frac{2n_2(l_2x_1+m_2y_1)}{b^2} + \frac{2n_3(l_3x_1+m_3y_1)}{c^2}$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 + \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 + \left(\frac{l_3x_1+m_3y_1}{c}\right)^2 - 1$$

where :

$$x_1 = x - x_0$$
$$y_1 = y - y_0$$
$$z_1 = z - z_0$$

Elliptic Paraboloids

Characteristic Equation (in X'Y'Z' coordinate system) :

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - cz' = 0$$

In terms of the XYZ coordinate system :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} + \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} - c(l_3x_1+m_3y_1-n_3z_1) = 0$$

Converting into the form :

$$A_1z_1^2 + B_1z_1 + C_1 = 0$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)}{a^2} + \frac{2n_2(l_2x_1+m_2y_1)}{b^2} - cn_3$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 + \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 - c(l_3x_1+m_3y_1)$$

Elliptical (Circular) Cylinders

Characteristic Equation (in X'Y'Z' coordinate system) :

For :
$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} \le 1 \qquad z' = c$$

In terms of the XYZ coordinate system :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} + \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} - 1 = 0$$

Converting into the form :

$$A_1z_1^2 + B_1z_1 + C_1 = 0$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)^2}{a^2} + \frac{2n_2(l_2x_1+m_2y_1)^2}{b^2}$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 + \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 - 1$$

The limit of the cylinder is defined by c.
This can be tested by substituting (x_1,y_1,z_1) into :

$$l_3x_1 + m_3y_1 + n_3z_1 \le c$$

Limit of the cylinder is exceeded if the above condition is not satisfied.

Characteristic Equation (in X'Y'Z' coordinate system) :

$$\frac{x'^2}{a^2} - \frac{y'^2}{b^2} - cz' = 0$$

In terms of the XYZ coordinate system :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} - \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} - c(l_3x_1+m_3y_1-n_3z_1) = 0$$

Converting into the form :

$$A_1z_1^2 + B_1z_1 + C_1 = 0$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)}{a^2} - \frac{2n_2(l_2x_1+m_2y_1)}{b^2} - cn_3$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 - \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 - c(l_3x_1+m_3y_1)$$

Quadratic Cones

Characteristic Equation (in X'Y'Z' system) :

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} = 0$$

In terms of the XYZ coordinate system :

$$\frac{(l_1x_1+m_1y_1+n_1z_1)^2}{a^2} + \frac{(l_2x_1+m_2y_1+n_2z_1)^2}{b^2} + \frac{(l_3x_1+m_3y_1+n_3z_1)^2}{c^2} = 0$$

Converting into the form :

$$A_1z_1^2 + B_1z_1 + C_1 = 0$$

$$A_1 = \left(\frac{n_1}{a}\right)^2 + \left(\frac{n_2}{b}\right)^2 + \left(\frac{n_3}{c}\right)^2$$

$$B_1 = \frac{2n_1(l_1x_1+m_1y_1)}{a^2} + \frac{2n_2(l_2x_1+m_2y_1)}{b^2} + \frac{2n_3(l_3x_1+m_3y_1)}{c^2}$$

$$C_1 = \left(\frac{l_1x_1+m_1y_1}{a}\right)^2 + \left(\frac{l_2x_1+m_2y_1}{b}\right)^2 + \left(\frac{l_3x_1+m_3y_1}{c}\right)^2$$

APPENDIX B - USER MANUAL FOR PROGRAM GEN7

Program Name GEN7
Purpose .... Geometrical modelling program for piecewise compound analytical surfaces using the Method of Highest Point
Availability .... Available from the Department of Mechanical Engineering, The University of British Columbia.
Software Required ...... IAS/RSX BASIC V02-01, running under Digital Equipment Corporation RSX-11M operating system.

1. HOW TO RUN

Program GEN7 has been running on a PDP11/34 computer at the Department of Mechanical Engineering, UBC. It is stored in a magnetic tape named LAU (ANSI formatted, 1600 bpi). To retrieve the program from the tape, the following procedure must be followed :

1) Mount magnetic tape onto tape-drive according to instructions provided with the tape-drive ;
2) Log on to PDP using the HELLO command, then type in account name and password ;
3) Mount tape using the mount command :
   MOUNT MT0:LAU
4) Copy tape onto system by typing :
   PIP =MT0:GEN7.BAS
5) Invoke BASIC interpreter by typing BAS
6) To run the program, type the command :
   RUN GEN7

2. USER-INPUT

Program GEN7 accepts user-inputs in an interactive manner. It first prompts for the global field dimension X and Y, followed by the increment of scan. Then, for each of the surface-type, it asks for the number of pieces, and for each of the pieces, user-inputs are translations, characteristic parameters, rotations, subdomain limits, offlimit height as well as truncation height. A typical prompting sequence is shown in Figure 4.7, and the required input for each surface piece is shown in Table I.

3. PROGRAM OUTPUT

Output from GEN7 is contained in the file DATA.DAT. Output formats are as follows :
1st line : NPNTS (format ###) - number of points per X-scan
Next (NPNTS+1) lines : x,y,z (format ###.###,###.###,###.###) - cartesian coordinates of nodal point

Repeat 1st to (NPNTS+1)th line for each X-scan

4. SAMPLE INPUTS

Sample inputs for the vac um cleaner housing mould discussed in Chapter IV are as follows :

Field Dimensions : 3.5 # 7.0
Increment for Scan : 0.1

Number of Ellipsoids : 2

Ellipsoid(1):
$(x_o , y_o , z_o) = (1.50, 1.25, 0.)$
$(a, b, c) = (1.25, 1.60, 1.)$
$(\theta_1 , \theta_2 , \theta_3) = (0^\circ, 0^\circ, 0^\circ)$
(Xmin, Xmax) = (1.5, 2.75)
(Ymin, Ymax) = (0.65, 2.25)
Off-limit Height = 0
Truncation Height = 99

Ellipsoid(2):
$(x_o , y_o , z_o) = (1.50, 4.95, 0.)$
$(a, b, c) = (1.25, 1.35, 1.75)$
$(\theta_1 , \theta_2 , \theta_3) = (0^\circ, 0^\circ, 0^\circ)$
(Xmin, Xmax) = (1.5, 2.75)
(Ymin, Ymax) = (4.95, 6.30)
Off-limit Height = 0
Truncation Height = 99

Number of Cylinders : 2

Cylinder(1):
$(x_o , y_o , z_o) = (0., 2.25, 0.)$
$(a, b, c) = (1.0, 1.6, 1.5)$
$(\theta_1 , \theta_2 , \theta_3) = (0^\circ, -90^\circ, 0^\circ)$
(Xmin, Xmax) = (0., 1.50)
(Ymin, Ymax) = (0.65, 2.25)
Off-limit Height = 0
Truncation Height = 99

Cylinder(2)
$(x_o , y_o , z_o) = (0., 4.95, 0.)$
$(a, b, c) = (1.75, 1.35, 1.50)$
$(\theta_1 , \theta_2 , \theta_3) = (-90^\circ, 0^\circ, 0^\circ)$
(Xmin, Xmax) = (1.5, 2.0)
(Ymin, Ymax) = (4.95, 6.30)
Off-limit Height = 0
Truncation Height = 99

Number of cones : 1

Cone(1) :
$(x_o , y_o , z_o) = (1.5, 2.25, 1.75)$
$(a, b, c) = (0.46631, 0.46631, 1)$
$(\theta_1 , \theta_2 , \theta_3) = (0^\circ, 0^\circ, 0^\circ)$
(Xmin, Xmax) = (1.5, 2.0)
(Ymin, Ymax) = (1.8, 2.25)
Off-limit Height = 0

```
          Truncation Height = 99

Number of Variable Cylinders  :  2

Vari-Cyl(1) :  ( xo , yo , zo ) = ( 1.5, 2.25, 0. )
                  ( a, b, c ) = ( 1.25, 1.0, 0.127463 )
                        θ     = 90°
              ( Xmin, Xmax ) = ( 1.5, 2.75 )
              ( Ymin, Yamx ) = ( 2.25, 4.429 )
           Off-limit Height = 0
          Truncation Height = 1.6054

Vari-Cyl(2) :  ( xo , yo , zo ) = ( 1.5, 4.95, 0. )
                  ( a, b, c ) = ( 1.25, 1.75, -0.5335 )
                        θ     = 90°
              ( Xmin, Xmax ) = ( 1.5, 2.75 )
              ( Ymin, Ymax ) = ( 4.429, 4.90 )
           Off-limit Height = 0
          Truncation Height = 1.75

Number of Planes  :  2

Plane(1) :        ( a, b, c ) = ( 10⁹, 1.4339, -3.0751 )
              ( Xmin, Xmax ) = ( 0., 1.5 )
              ( Ymin, Ymax ) = ( 0., 4.95 )
          Truncation Height = 1.75

Plane(2) :        ( a, b, c ) = ( 2.1361, 10⁹, 4.9668 )
              ( Xmin, Xmax ) = ( 0., 2.75 )
              ( Ymin, Ymax ) = ( 2.25, 4.95 )
           Off-limit Height = 0
          Truncation Height = 1.75
```

5. PROGRAM LISTING FOR GEN7

```
5  REM GENERAL PROGRAM FOR EXECUTING THE METHOD OF HIGHEST POINT
20 OPEN "DATA" FOR OUTPUT AS FILE #1
40 DIM C(44),E(44),G(44),P(44),R(44),T(44),V(44),F(26)
50 REM
51 REM INITIALIZATION
52 REM
61 PRINT "ENTER FIELD DIMENSION  X AND Y";
62 INPUT A1, A2
65 PRINT "ENTER INCREMENT D ";
66 INPUT A3
69 LET M=INT(A1/A3)
70 LET N=INT(A2/A3)
75 PRINT #1,USING '###,';N
96 LET D1=.01745329252
100 PRINT "NUMBER OF ELLIPSOIDS ( max 3 )";
101 INPUT C
102 IF C=0 GO TO 106
105 GOSUB 200
106 PRINT "NUMBER OF EL. PARAB ( max 3 ) ";
107 INPUT E
108 IF E=0 GO TO 112
111 GOSUB 400
112 PRINT "NUMBER OF HYP. PARAB ( max 3 )";
113 INPUT G
114 IF G=0 GO TO 116
115 GOSUB 600
116 PRINT "NUMBER OF QUADRATIC CONE ( max 3 ) ";
117 INPUT P
118 IF P=0 GO TO 120
119 GOSUB 800
120 PRINT "NUMBER OF ELLIPTIC ( CIRCULAR ) CYLINDER ( max 3 ) ";
121 INPUT R
122 IF R=0 GO TO 124
123 GOSUB 1000
124 PRINT "NUMBER OF PLANES ";
125 INPUT T
126 IF T=0 GO TO 128
127 GOSUB 1200
128 PRINT "NUMBER OF TORUS ( max 3 )";
129 INPUT V
130 IF V=0 GO TO 138
131 GOSUB 1400
138 PRINT "NUMBER OF PARABOLIC ELLIPTICAL CYL ( max 3 )";
139 INPUT F
140 IF F=0 GO TO 145
142 GOSUB 1450
145 GOTO 1500
150 REM
151 START OF DATA ENTRY
152 REM
189 REM
190 REM SUB 200 IS FOR AN ELLIPSOID
191 REM
200 FOR I=0 TO C-1
201 PRINT
205 B$=STR$(I+1)
210 A$="ENTER (X0,Y0,Z0) FOR ELLIP("+B$+") ............. "
215 PRINT A$;
220 INPUT C(I*15+3),C(I*15+4),C(I*15+5)
225 A$="ENTER A, B AND C, FOR ELLIP("+B$+") ............. "
```

```
230 PRINT A$;
235 INPUT C(I*15),C(I*15+1),C(I*15+2)
240 A$="ENTER ROTATIONS 1, 2 AND 3, FOR ELLIP("+B$+") ... "
245 PRINT A$;
246 INPUT C(I*15+10),C(I*15+11),C(I*15+12)
300 A$="ENTER LOWLIMX, UPLIMX FOR ELLIP("+B$+") ... "
305 PRINT A$;
310 INPUT C(I*15+6), C(I*15+7)
315 A$="ENTER LOWLIMY, UPLIMY FOR ELLIP("+B$+") ... "
316 PRINT A$;
320 INPUT C(I*15+8), C(I*15+9)
322 A$="ENTER OFFLIMIT HEIGHT FOR ELLIP("+B$+") ... "
324 PRINT A$;
326 INPUT C(I*15+13)
328 A$="ENTER TRUNCATION HEIGHT FOR ELLIP("+B$+") . "
330 PRINT A$;
332 INPUT C(I*15+14)
345 NEXT I
346 A$="Pausing .. Type 1 to alter input, any no. to continue "
347 PRINT A$;
348 INPUT O
349 IF O=1 GOTO 200
350 RETURN
389 REM
390 REM SUB 400 IS FOR EL. PARAB
391 REM
400 FOR I=O TO E-1
401 PRINT
405 B$=STR$(I+1)
410 A$="ENTER VERTEX (XO,YO,ZO) FOR EL PARAB("+B$+") .. "
415 PRINT A$;
420 INPUT E(I*15),E(I*15+1),E(I*15+2)
455 A$="ENTER A, B AND C, FOR EL PARAB("+B$+") ........ "
460 PRINT A$;
465 INPUT E(I*15+3),E(I*15+4),E(I*15+5)
470 A$="ENTER ROT 1, 2 AND 3, FOR EL PARAB("+B$+") .... "
475 PRINT A$;
480 INPUT E(I*15+10),E(I*15+11),E(I*15+12)
500 A$="ENTER LOWLIMX, UPLIMX FOR EL PARAB("+B$+") .. "
505 PRINT A$;
510 INPUT E(I*15+6),E(I*15+7)
520 A$="ENTER LOWLIMY, UPLIMY FOR EL PARAB("+B$+") .. "
525 PRINT A$;
530 INPUT E(I*7015+8),E(I*15+9)
531 A$="ENTER OFFLIMIT HT FOR EL PARAB("+B$+") ...... "
532 PRINT A$;
533 INPUT E(I*15+13)
541 A$="ENTER TRUNCATION HT FOR EL PARAB("+B$+") .... "
542 PRINT A$;
543 INPUT E(I*15+14)
545 NEXT I
546 A$="Pausing .. Type 1 to alter input, any no. to continue "
547 PRINT A$;
548 INPUT O
549 IF O=1 GOTO 400
550 RETURN
589 REM
590 REM SUB 600 IS FOR HYP. PARABOLOID
591 REM
600 FOR I=O TO G-1
```

```
601 PRINT
605 B$=STR$(I+1)
610 A$="ENTER CENTER (XO,YO,ZO) FOR HYP PARAB("+B$+") .. "
615 PRINT A$;
620 INPUT G(I*15),G(I*15+1),G(I*15+2)
655 A$="ENTER A, B AND C, FOR HYP PARAB("+B$+") ........ "
660 PRINT A$;
665 INPUT G(I*15+3),G(I*15+4),G(I*15+5)
670 A$="ENTER ROT 1, 2 AND 3, FOR HYP PARAB("+B$+") .... "
675 PRINT A$;
680 INPUT G(I*15+10),G(I*15+11),G(I*15+12)
700 A$="ENTER LOWLIMX AND UPLIMX, FOR HYP PARAB("+B$+") .... "
705 PRINT A$;
710 INPUT G(I*15+6),G(I*15+7)
720 A$="ENTER LOWLIMY AND UPLIMY, FOR HYP PARAB("+B$+") .... "
725 PRINT A$;
730 INPUT G(I*15+8),G(I*15+9)
732 A$="ENTER OFFLIM HT FOR HYP PARAB("+B$+") .............. "
733 PRINT A$;
735 INPUT G(I*15+13)
737 A$="ENTER TRUNCATION HT FOR HYP PARAB("+B$+") .......... "
739 PRINT A$;
740 INPUT G(I*15+14)
745 NEXT I
746 A$="Pausing .. Type 1 to alter input, any no. to continue "
747 PRINT A$;
748 INPUT O
749 IF O=1 GOTO 600
750 RETURN
789 REM
790 REM SUB 800 IS FOR QUADRATIC CONE
791 REM
800 FOR I=0 TO P-1
801 PRINT
805 B$=STR$(I+1)
810 A$="ENTER VERTEX (XO,YO,ZO), FOR CONE("+B$+") .. "
815 PRINT A$;
820 INPUT P(I*15),P(I*15+1),P(I*15+2)
825 A$="ENTER A, B  AND C, FOR CONE("+B$+") ........ "
830 PRINT A$;
835 INPUT P(I*15+3),P(I*15+4),P(I*15+5)
840 A$="ENTER ROT 1, 2 AND 3, FOR CONE("+B$+") ..... "
842 PRINT A$;
844 INPUT P(I*15+10),P(I*15+11),P(I*15+12)
860 A$="ENTER TRUNCATION HT FOR CONE("+B$+") ....... "
865 PRINT A$;
866 INPUT P(I*15+14)
870 A$="ENTER OFFLIMIT HT FOR CONE("+B$+") ......... "
875 PRINT A$;
880 INPUT P(I*15+13)
900 A$="ENTER LOWLIMX AND UPLIMX FOR CONE("+B$+") .. "
905 PRINT A$;
910 INPUT P(I*15+6),P(I*15+7)
930 A$="ENTER LOWLIMY AND UPLIMY FOR CONE("+B$+") .. "
935 PRINT A$;
940 INPUT P(I*15+8),P(I*15+9)
945 NEXT I
946 A$="Pausing .. Type 1 to alter input, any no. to continue "
947 PRINT A$;
948 INPUT O
```

```
949 IF O=1 GOTO 800
950 RETURN
989 REM
990 REM SUB 1000 IS FOR ELLIPTICAL (CIRCULAR) CYLINDER
991 REM
1000 FOR I=0 TO R-1
1001 PRINT
1005 B$=STR$(I+1)
1010 A$="ENTER CENTRE POINT (XO,YO,ZO) FOR CYL("+B$+") .. "
1015 PRINT A$;
1020 INPUT R(I*15),R(I*15+1),R(I*15+2)
1025 A$="ENTER A, B AND RO, FOR CYL("+B$+") ....... "
1027 PRINT A$;
1030 INPUT R(I*15+3),R(I*15+4),R(I*15+5)
1035 A$="ENTER ROT 1, 2 AND 3 FOR CYL("+B$+") ..... "
1040 PRINT A$;
1045 INPUT R(I*15+10),R(I*15+11),R(I*15+12)
1100 A$="ENTER LOWLIMX AND UPLIMX FOR CYL("+B$+") .... "
1105 PRINT A$;
1110 INPUT R(I*15+6),R(I*15+7)
1115 A$="ENTER LOWLIMY AND UPLIMY FOR CYL("+B$+") .... "
1120 PRINT A$;
1125 INPUT R(I*15+8),R(I*15+9)
1130 A$="ENTER OFFLIM HT FOR CYL("+B$+") ............. "
1135 PRINT A$;
1140 INPUT R(I*15+13)
1141 A$="ENTER TRUNCATION HT FOR CYL("+B$+") ......... "
1142 PRINT A$;
1143 INPUT R(I*15+14)
1145 NEXT I
1146 A$="Pausing .. Type 1 to alter input, any no. to continue "
1147 PRINT A$;
1148 INPUT O
1149 IF O=1 GOTO 1000
1150 RETURN
1189 REM
1190 REM SUB 1200 IS FOR A PLANE
1191 REM
1200 FOR I=0 TO T-1
1201 PRINT
1205 B$=STR$(I+1)
1210 A$="ENTER INTERCEPTS X,Y AND Z FOR PLANE("+B$+") .. "
1215 PRINT A$;
1220 INPUT T(I*8),T(I*8+1),T(I*8+2)
1255 A$="ENTER LOWLIMX AND UPLIMX FOR PLANE("+B$+") .... "
1260 PRINT A$;
1265 INPUT T(I*8+3),T(I*8+4)
1285 A$="ENTER LOWLIMY AND UPLIMY FOR PLANE("+B$+") .... "
1290 PRINT A$;
1295 INPUT T(I*8+5),T(I*8+6)
1297 A$="ENTER TRUNCATION HT FOR PLANE("+B$+") ......... "
1298 PRINT A$;
1299 INPUT T(I*8+7)
1350 NEXT I
1360 A$="Pausing .. Type 1 to alter input, any no. to continue "
1361 PRINT A$;
1362 INPUT O
1363 IF O=1 GOTO 1200
1365 RETURN
1389 REM
```

```
1390 REM SUB 1400 IS FOR A TORUS
1391 REM
1400 FOR I=O TO V-1
1401 B$=STR$(I+1)
1402 A$="ENTER CENTER (XO,YO,ZO) FOR TORUS("+B$+") .."
1403 PRINT : PRINT A$;
1404 INPUT V(I*9),V(I*9+1),V(I*9+2)
1411 A$="CLRADTUBE ("+B$+") = "
1412 PRINT A$;
1413 INPUT V(I*9+3)
1414 A$="SECRADTUBE ("+B$+")= "
1415 PRINT A$;
1416 INPUT V(I*9+4)
1417 A$="ENTER LOWLIMX AND UPLIMX FOR TORUS("+B$+") ... "
1418 PRINT A$;
1419 INPUT V(I*9+5),V(I*9+6)
1423 A$="ENTER LOWLIMY AND UPLIMY FOR TORUS("+B$+") ... "
1424 PRINT A$;
1425 INPUT V(I*9+7),V(I*9+8)
1430 NEXT I
1436 A$="Pausing .. Type 1 to alter input, any no. to continue "
1437 PRINT A$;
1438 INPUT O
1439 IF O=1 GOTO 1400
1440 RETURN
1441 REM
1442 REM SUBROUTINE 1450 IS FOR PARABOLIC ELLIPTICAL CYLINDERS
1445 REM
1450 FOR I=O TO F-1
1451 B$=STR$(I+1)
1453 A$="ENTER (XO,YO,ZO) FOR PARA-EL-CYL("+B$+") ... "
1455 PRINT : PRINT A$;
1457 INPUT F(I*13),F(I*13+1),F(I*13+2)
1459 A$="ENTER a, b, c FOR PARA-EL-CYL("+B$+") ...... "
1460 PRINT A$;
1462 INPUT F(I*13+3),F(I*13+4),F(I*13+5)
1465 A$="ENTER ROTATION FOR PARA-EL-CYL("+B$+") ...... "
1466 PRINT A$;
1468 INPUT F(I*13+6)
1470 A$="ENTER LOWLIMX AND UPLIMX FOR CYL("+B$+") .. "
1471 PRINT A$;
1473 INPUT F(I*13+7),F(I*13+8)
1475 A$="ENTER LOWLIMY AND UPLIMY FOR CYL("+B$+") .. "
1476 PRINT A$;
1477 INPUT F(I*13+9),F(I*13+10)
1480 A$="ENTER OFFLIM HT FOR CYL("+B$+") ......"
1481 PRINT A$;
1482 INPUT F(I*13+11)
1483 A$="ENTER TRUNCATION HT FOR CYL("+B$+") .."
1484 PRINT A$;
1485 INPUT F(I*13+12)
1487 NEXT I
1488 A$="Pausing .. Type 1 to alter input, any no. to continue "
1489 PRINT A$;
1490 INPUT O
1491 IF O=1 GOTO 1450
1492 RETURN
1493 REM
1494 REM END OF INPUT SUBROUTINES
1495 REM
```

```
1497 REM
1498 REM START LOOKING FOR THE HIGHEST POINTS
1499 REM
1500 PRINT ¯ PRINT " Program running ... " ¯ PRINT
1502 FOR K=O TO M
1505 LET X=K*A3
1507 PRINT " Loop ";K
1510 FOR J=O TO N
1515 LET Y=J*A3
1520 LET Z=O
1522 REM
1523 REM FIRST CHECK ELLIPSOIDS
1524 REM
1525 IF C=O GO TO 1600
1530 FOR I=O TO C-1
1535 IF X<C(I*15+6) GO TO 1585
1540 IF X>C(I*15+7) GO TO 1585
1545 IF Y<C(I*15+8) GO TO 1585
1550 IF Y>C(I*15+9) GO TO 1585
1555 LET R1=C(I*15+10)*D1 ¯ LET R2=C(I*15+11)*D1 ¯ LET R3=C(I*15+12)*D1
1556 LET A = C(I*15) ¯ LET B = C(I*15+1) ¯ LET C2 = C(I*15+2)
1557 LET XO = C(I*15+3) ¯ LET YO = C(I*15+4) ¯ LET ZO = C(I*15+5)
1560 GOSUB 3000
1562 LET A1=(N1/A)α2 + (N2/B)α2 + (N3/C2)α2
1563 LET B1=(2*N1*(L1*X1+M1*Y1))/(Aα2)+(2*N2*(L2*X1+M2*Y1))/(Bα2)
1564 LET B1=B1 + (2*N3*(L3*X1+M3*Y1))/(C2α2)
1565 LET C1=((L1*X1+M1*Y1)/A)α2+((L2*X1+M2*Y1)/B)α2+((L3*X1+M3*Y1)/C2)α2-1
1566 LET D = B1α2 - 4*A1*C1
1567 IF D<O THEN LET Z2 = C(I*15+13) ¯ GOTO 1580
1568 LET Z2=((SQR(D)-B1)/(2*A1)) + ZO
1570 IF Z2>C(I*15+14) THEN LET Z2=C(I*15+14)
1580 IF Z2>Z THEN LET Z=Z2
1585 NEXT I
1589 REM
1590 REM CHECK EL. PARAB
1591 REM
1600 IF E=O GO TO 1700
1605 FOR I=O TO E-1
1610 IF X<E(I*15+6) GO TO 1686
1615 IF X>E(I*15+7) GO TO 1686
1620 IF Y<E(I*15+8) GO TO 1686
1625 IF Y>E(I*15+9) GO TO 1686
1630 LET R1=E(I*15+10)*D1 ¯ LET R2=E(I*15+11)*D1 ¯ LET R3=E(I*15+12)*D1
1632 LET A=E(I*15+3) ¯ LET B=E(I*15+4) ¯ LET C2=E(I*15+5)
1634 LET XO=E(I*15) ¯ LET YO=E(I*15+1) ¯ LET ZO=E(I*15+2)
1638 IF (R1+R2+R3)=O THEN LET Z3=(((X-XO)/A)α2+((Y-YO)/B)α2)/C2+ZO ¯ GOTO 1675
1640 GOSUB 3000
1645 LET A1=(N1/A)α2 + (N2/B)α2
1650 LET B1=(2*N1*(L1*X1+M1*Y1))/(Aα2)+(2*N2*(L2*X1+M2*Y1))/(Bα2)-C2*N3
1655 LET C1=((L1*X1+M1*Y1)/A)α2+((L2*X1+M2*Y1)/B)α2-(L3*X1+M3*Y1)*C2
1660 LET D =B1α2 - 4*A1*C1
1665 IF D<O THEN LET Z3=E(I*15+13) ¯ GOTO 1685
1667 IF A1=O THEN LET Z3=-C1/B1+ZO ¯ GOTO 1675
1670 LET Z3=(SQR(D)-B1)/(2*A1) + ZO
1675 IF Z3>E(I*15+14) THEN LET Z3=E(I*15+14)
1685 IF Z3>Z THEN LET Z=Z3
1686 NEXT I
1689 REM
1690 REM CHECK HYP. PARAB
1691 REM
```

```
1700 IF G=O GO TO 1800
1705 FOR I=O TO G-1
1710 IF X<G(I*15+6) GO TO 1785
1715 IF X>G(I*15+7) GO TO 1785
1720 IF Y<G(I*15+8) GO TO 1785
1725 IF Y>G(I*15+9) GO TO 1785
1730 LET R1=G(I*15+10)*D1 ` LET R2=G(I*15+11)*D1   LET R3=G(I*15+12)*D1
1732 LET A=G(I*15+3) ` LET B=G(I*15+4)   LET C2=G(I*15+5)
1735 LET XO=G(I*15) ` LET YO=G(I*15+1)   LET ZO=G(I*15+2)
1738 IF (R1+R2+R3)=O THEN LET Z4=(((X-XO)/A)α2-((Y-YO)/B)α2)/C2+ZO ` GOTO 1775
1740 GOSUB 3000
1742 LET A1=(N1/A)α2-(N2/B)α2
1745 LET B1=(2*N1*(L1*X1+M1*Y1))/(Aα2)-(2*N2*(L2*X1+M2*Y1))/(Bα2)-C2*N3
1747 LET C1=((L1*X1+M1*Y1)/A)α2-((L2*X1+M2*Y1)/B)α2-C2*(L3*X1+M3*Y1)
1750 LET D = B1α2 - 4*A1*C1
1752 IF D<O THEN LET Z4=G(I*15+13) ` GOTO 1780
1753 IF A1=O THEN LET Z4=-C1/B1+ZO ` GOTO 1775
1755 LET Z4=(SQR(D)-B1)/(2*A1) + ZO
1775 IF Z4>G(I*15+14) THEN LET Z4=G(I*15+14)
1780 IF Z4>Z THEN LET Z=Z4
1785 NEXT I
1789 REM
1790 REM CHECK QUADRATIC CONE
1791 REM
1800 IF P=O GO TO 1900
1805 FOR I=O TO P-1
1810 IF X<P(I*15+6) GO TO 1885
1815 IF X>P(I*15+7) GO TO 1885
1820 IF Y<P(I*15+8) GO TO 1885
1825 IF Y>P(I*15+9) GO TO 1885
1830 LET R1=P(I*15+10)*D1 ` LET R2=P(I*15+11)*D1 ` LET R3=P(I*15+12)*D1
1832 LET A=P(I*15+3)   LET B=P(I*15+4) ` LET C2=P(I*15+5)
1835 LET XO=P(I*15) ` LET YO=P(I*15+1) ` LET ZO=P(I*15+2)
1840 GOSUB 3000
1845 LET A1=(N1/A)α2 + (N2/B)α2 - (N3/C2)α2
1847 LET B1=(2*N1*(L1*X1+M1*Y1))/(Aα2)+(2*N2*(L2*X1+M2*Y1))/(Bα2)
1848 LET B1=B1-(2*N3*(L3*X1+M3*Y1))/(C2α2)
1850 LET C1=((L1*X1+M1*Y1)/A)α2 + ((L2*X1+M2*Y1)/B)α2 - ((L3*X1+M3*Y1)/C2)α2
1852 LET D = B1α2 - 4*A1*C1
1855 IF D<O THEN LET Z5=P(I*15+13)   GOTO 1880
1860 LET Z5=(SQR(D)-B1)/(2*A1) + ZO
1875 IF Z5>P(I*15+14) THEN LET Z5=P(I*15+14)
1880 IF Z5>Z THEN LET Z=Z5
1885 NEXT I
1889 REM
1890 REM CHECK ELLIPTICAL (CIRCULAR) CYLINDERS
1891 REM
1900 IF R=O GO TO 2000
1905 FOR I=O TO R-1
1910 IF X<R(I*15+6) GO TO 1985
1915 IF X>R(I*15+7) GO TO 1985
1920 IF Y<R(I*15+8) GO TO 1985
1925 IF Y>R(I*15+9) GO TO 1985
1930 LET R1=R(I*15+10)*D1 ` LET R2=R(I*15+11)*D1   LET R3=R(I*15+12)*D1
1932 LET A=R(I*15+3)   LET B=R(I*15+4)   LET RO=R(I*15+5)
1935 LET XO=R(I*15) ` LET YO=R(I*15+1)   LET ZO=R(I*15+2)
1940 GOSUB 3000
1945 LET A1 = (N1/A)α2 + (N2/B)α2
1950 LET B1 = (2*N1*(L1*X1+M1*Y1))/(Aα2) + (2*N2*(L2*X1+M2*Y1))/(Bα2)
1957 LET C1 = ((L1*X1+M1*Y1)/A)α2 + ((L2*X1+M2*Y1)/B)α2 - 1
```

```
1958 LET D  = B1α2 - 4*A1*C1
1960 IF D<0 THEN LET Z6=R(I*15+13)   GOTO 1980
1962 IF A1=0 THEN LET Z6=R0+Z0   GOTO 1972
1965 LET Z6=(SQR(D)-B1)/(A1*2)+Z0
1966 IF ABS(N3)<1E-05 GOTO 1970
1968 IF (L3*X1+M3*Y1+N3*Z6)>R0 THEN LET Z6=((R0-L3*X1-M3*Y1)/N3)   GOTO 1980
1970 IF ABS(L3*X1+M3*Y1)>R0 THEN LET Z6=R(I*15+13)   GOTO 1980
1971 GOTO 1975
1972 IF ((X1/A)α2+(Y1/B)α2)>1 THEN LET Z6=R(I*15+13)   GOTO 1980
1975 IF Z6>R(I*15+14) THEN LET Z6=R(I*15+14)
1980 IF Z6>Z THEN LET Z=Z6
1985 NEXT I
1989 REM
1990 REM CHECK PLANES
1991 REM
2000 IF T=0 GO TO 2100
2005 FOR I=0 TO T-1
2010 IF X<T(I*8+3) GO TO 2045
2015 IF X>T(I*8+4) GO TO 2045
2020 IF Y<T(I*8+5) GO TO 2045
2025 IF Y>T(I*8+6) GO TO 2045
2030 LET Z7=(1-X/T(I*8)-Y/T(I*8+1))*T(I*8+2)
2035 IF Z7<0 THEN LET Z7=0
2038 IF Z7>T(I*8+7) THEN LET Z7=T(I*8+7)
2040 IF Z7>Z THEN LET Z=Z7
2045 NEXT I
2089 REM
2090 REM CHECK TORUS
2091 REM
2100 IF V=0 GO TO 2200
2105 FOR I=0 TO V-1
2110 IF X<V(I*9+5) GO TO 2180
2115 IF X>V(I*9+6) GO TO 2180
2120 IF Y<V(I*9+7) GO TO 2180
2125 IF Y>V(I*9+8) GO TO 2180
2130 IF X<=V(I*9)-V(I*9+3)-V(I*9+4) GO TO 2180
2135 IF X>=V(I*9)+V(I*9+3)+V(I*9+4) GO TO 2180
2140 IF Y>=V(I*9+1)+V(I*9+3)+V(I*9+4) GO TO 2180
2145 IF Y<=V(I*9+1)-V(I*9+3)-V(I*9+4) GO TO 2180
2150 LET R1=SQR((X-V(I*9))α2+(Y-V(I*9+1))α2)
2155 IF R1<=V(I*9+3)-V(I*9+4) GO TO 2180
2160 IF R1>=V(I*9+3)+V(I*9+4) GO TO 2180
2165 LET R2=R1-V(I*9+3)
2166 LET Z8=SQR(V(I*9+4)α2-R2α2)+V(I*9+2)
2170 IF Z8<0 THEN LET Z8=0
2175 IF Z8>Z THEN LET Z=Z8
2180 NEXT I
2190 REM
2191 REM CHECK PARABOLIC ELLIPTICAL CYLINDERS
2192 REM
2200 IF F=0 GOTO 2600
2205 FOR I=0 TO F-1
2210 IF X<F(I*13+7) GOTO 2500
2212 IF X>F(I*13+8) GOTO 2500
2214 IF Y<F(I*13+9) GOTO 2500
2216 IF Y>F(I*13+10) GOTO 2500
2220 LET R1=F(I*13+6)*D1
2222 LET X0=F(I*13)   LET Y0=F(I*13+1)   LET Z0=F(I*13+2)
2224 LET A1=F(I*13+3)   B1=F(I*13+4)   C1=F(I*13+5)
2226 LET X1 =  (X-X0)*COS(R1) + (Y-Y0)*SIN(R1)
```

```
2228 LET Y1 = -(X-XO)*SIN(R1) + (Y-YO)*COS(R1)
2230 IF ABS(Y1)>A1 THEN LET Z9=F(I*13+11)   GOTO 2250
2232 LET Z1 = (X1α2*C1+B1) * SQR(1-((Y1)/A1)α2)
2235 LET Z9 = Z1 + ZO
2240 IF Z9>F(I*13+12) THEN LET Z9=F(I*13+12)
2250 IF Z9>Z THEN LET Z=Z9
2500 NEXT I
2502 REM
2503 REM HIGHEST POINT FOUND
2504 REM
2600 PRINT #1,USING '###.###,###.###,###.###,';X;Y;Z
2630 NEXT J
2640 NEXT K
2650 STOP
2997 REM
2998 REM  SUBROUTINE 3000 FINDS DIRECTION COSINES OF ROTATED AXES
2999 REM
3000    LET L1 =  COS(R1)*COS(R2)
3040    LET L2 =  COS(R1)*SIN(R2)*SIN(R3) - SIN(R1)*COS(R3)
3050    LET L3 =  COS(R1)*SIN(R2)*COS(R3) - COS(R1)*SIN(R3)
3060 REM
3070    LET M1 =  SIN(R1) * COS(R2)
3080    LET M2 =  SIN(R1)*SIN(R2)*SIN(R3) + COS(R1)*COS(R3)
3090    LET M3 =  SIN(R1)*SIN(R2)*SIN(R3) - COS(R1)*SIN(R3)
3100 REM
3110    LET N1 = -SIN(R2)
3120    LET N2 =  COS(R2)*SIN(R3)
3130    LET N3 =  COS(R2)*COS(R3)
3140 REM
3150    LET X1 =  X - XO
3160    LET Y1 =  Y - YO
3170 RETURN
5000 END
End of file
```

5. PROGRAM LISTING FOR CAVITY6

```
10   REM
20   REM    Program to calculate the CLD path of a spherical end mill
30   REM    to machine a die cavity in the shape of an elliptical
40   REM    paraboloid
50   REM
60          OPEN "CAVITY1.DAT" FOR OUTPUT AS FILE#1
65          OPEN "CAVITY2.DAT" FOR OUTPUT AS FILE#2
70   REM
80          PRINT " Enter a, b and c for paraboloid .. ";
90          INPUT A, B, C
100         PRINT " Enter tilting angle of cavity .... ";
110         INPUT F1
120         PRINT " Enter thickness of die cavity .... ";
130         INPUT T
140         PRINT " Enter tool radius ............... ";
150         INPUT RO
160         PRINT " Enter increment for  X-scan ...... ";
165         INPUT D1
170         PRINT " Enter increment for  Y-scan ...... ";
175         INPUT D2
180  REM
190  REM    Set tool centre to be (RO+T) away from surface of
200  REM    paraboloid
210  REM
220         LET R = RO + T
230  REM
240  REM    Initialize parameters for scan
250  REM
260         LET G  = ABS(INT(B/D2))
270         LET F1 = F1 * 0.01745329252
280  REM
290  REM  Start Y-scan
300  REM
305         PRINT ˉ PRINT " Program running .... " ˉ PRINT
310         FOR N = 0 TO G
312            LET Y = N * ABS(D2)
315            PRINT" Loop - ";N
320  REM
330  REM        Find boundaries for X-scan
340  REM
350            IF D1<0 THEN LET B1=-A*SQR(1-(Y/B)α2)
355            IF D1>0 THEN LET B1= A*SQR(1-(Y/B)α2)
360            LET B2 = Y
370            LET B3 = C
380            LET Q  = ABS(INT(B1/D1))
390            PRINT #1, USING "###";Q+3
392            PRINT #2, USING "###";Q+3
400  REM
410  REM        For each Y, scan along X
420  REM
430            FOR M = 0 TO Q
440  REM
450  REM           Using the equation of paraboloid, calculate Z for
460  REM           each X and Y
470  REM
480               LET X = M * D1
490               LET Z = C * ( (X/A)α2 + (Y/B)α2 )
500  REM
510  REM           Calculate direction cosines and tool centre positions
515  REM           If tool on boundary, check for interference
```

```
520  REM
530            GOSUB 1000
540            IF M=Q THEN GOSUB 1200
550  REM
560  REM         Calculate tool centre positions w.r.t. tilted base plane
565  REM         and write results onto data file
570  REM
580            LET U = Z1*SIN(F1) + X1*COS(F1)
590            LET V = Y1
600            LET W = Z1*COS(F1) - X1*SIN(F1)
605            IF M=Q THEN GOSUB 2000
610            PRINT#1, USING "##.#####,##.#####,##.#####";U,V,W
611            PRINT#2, USING "##.#####,##.#####,##.#####";U,-V,W
612  REM
613  REM         Move tool along inclined parting plane if tool is beyond
614  REM         boundary
615  REM
616            IF M=Q THEN GOSUB 2500
620         NEXT M
630      NEXT N
640      STOP
650      END
1000 REM
1010 REM   Subroutine 1000 finds the direction cosines of the tool offset
1020 REM   path at any point on the paraboloid and computes the tool
1030 REM   centre positions
1040 REM
1050      LET S  = SQR((2*X/Aα2)α2 + (2*Y/Bα2)α2 + 1/Cα2)
1060      LET L1 = -2 * X/(A*A*S)
1070      LET M1 = -2 * Y/(B*B*S)
1080      LET N1 = 1/(C*S)
1090 REM
1100      LET X1 = X + R*L1
1110      LET Y1 = Y + R*M1
1120      LET Z1 = Z + R*N1
1130      RETURN
1200 REM
1210 REM   Subroutine 1200 moves tool along edge of boundary
1230 REM
1240 REM   First find direction cosines of tool offset path
1250 REM
1260      LET X=B1 ¯ LET Y=B2 ¯ LET Z=B3
1270      GOSUB 1000
1280 REM
1290 REM   Then find direction cosines of tangent
1300 REM
1305      IF Y=0 THEN LET L2=0 ¯ GOTO 1345
1310      LET H1 = ( (B/A)α2 * (X/Y) )
1320      LET H2 = 1 + (H1)α2
1330      LET L2 = 1 / SQR(H2)
1340      LET M2 = -H1/SQR(H2) ¯ GOTO 1350
1345      IF X<0 THEN LET M2 = 1
1347      IF X>=0 THEN LET M2 = -1
1350      LET N2 = 0
1360 REM
1370 REM   The outward normal is obtained from the vector product
1380 REM   of the tool offset path with the tangent
1390 REM
1400      LET H5 = (N1*M2)α2 + (N1*L2)α2 + (L1*M2-M1*L2)α2
1410      LET H6 = 1/SQR(H5)
```

```
1420      LET L3 =-N1 * M2/H6
1422      LET M3 = N1 * L2/H6
1424      LET N3 = ( L1*M2 - M1*L2 ) / H6
1430 REM
1432 REM   Move tool along outward normal to avoid the boundary
1436 REM
1438      LET X1 = X1 + RO*L3
1440      LET Y1 = Y1 + RO*M3
1442      LET Z1 = Z1 + RO*N3
1444      LET U1 = Z1*SIN(F1) + X1*COS(F1)
1446      LET V1 = Y1
1448      LET W1 = Z1*COS(F1) - X1*SIN(F1)
1450      PRINT#1, USING "##.#####,##.#####,##.#####";U1,V1,W1
1460      PRINT#2, USING "##.#####,##.#####,##.#####";U1,-V1,W1
1480 REM
1486 REM   Then move tool downward to create the boundary
1487 REM
1488 REM   First find CLD for lower ellipse
1489 REM
1500      LET X5 = B1 + RO*L3
1510      LET Y5 = B2 + RO*M3
1520      LET Z5 = B3 + RO*N3
1525 REM
1530 REM   Avoid interference with inclined parting plane
1535 REM
1540      LET Z7 = C + RO
1545      LET S7 = (Z7-Z5)/(Z1-Z5)
1550      LET X7 = S7*(X1-X5) + X5
1555      LET Y7 = S7*(Y1-Y5) + Y5
1590 REM
1592 REM  Return tool position data to main program
1595 REM
1600      LET X1=X7 ˉ LET Y1=Y7 ˉ LET Z1=Z7
1620      RETURN
1630      END
2000 REM
2010 REM  Subroutine 2000 checks for interference with horizontal base
2020 REM  plane
2030 REM
2032 REM  First save co-ordinates of tool
2033 REM
2034      LET UO=U ˉ LET VO=V ˉ LET WO=W
2035
2038 REM
2040 REM  Calculate Z for base plane
2050 REM
2060      LET Z9 = C*COS(F1) - A*SIN(F1) + RO
2070 REM
2080 REM  Check for possible undercut, if not, return
2090 REM
2100      IF WO => Z9 THEN GOTO 2200
2110 REM
2120 REM  If there is undercut, raise tool to base plane level
2130 REM
2140      LET S9 = (Z9-WO)/(W1-WO)
2150      LET U  = S9*(U1-UO) + UO
2160      LET V  = S9*(V1-VO) + VO
2170      LET W  = Z9
2200      RETURN
2500 REM
```

```
2510 REM
2520 REM  Subroutine 2500 moves tool along the inclined parting plane
2530 REM  if tool is beyond boundary of the paraboloid
2550 REM
2570      LET V = B*1.25
2590      PRINT#1, USING "##.#####,##.#####,##.#####";U,V,W
2592      PRINT#2, USING "##.#####,##.#####,##.#####";U,-V,W
2600      RETURN
End of file
```

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