MARGINAL GAINS IN ACCURACY OF VALUATION
FROM INCREASING THE SPECIFICITY OF PRICE INDEXES:
EMPIRICAL EVIDENCE FOR THE CANADIAN ECONOMY

By
KARIM JAMAL

B.Comm.(Hon.), The University of Manitoba, 1981

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
in Business Administration
in
THE FACULTY OF GRADUATE STUDIES
Faculty of Commerce and Business Administration

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
December 1983

© Karim Jamal, 1983
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of **Commerce and Business Administration**

The University of British Columbia  
1956 Main Mall  
Vancouver, Canada  
V6T 1Y3

Date **December 24/83**
In this paper we present empirical estimates of marginal gains in accuracy of asset valuation from increasing the specificity of price indexes used to adjust Historical Cost Financial Statements. The empirical evidence strongly suggests that the accuracy function for the Canadian economy is highly convex. This implies that the marginal gain in accuracy of valuation declines sharply as the number and specificity of price indexes used for valuation increases. These findings are potentially valuable for Auditors, Academics and Regulatory agencies who are involved in the debate on selection of asset valuation rules. The results are of particular relevance for selection of asset valuation rules when it is costly to use finer measurement methods. CICA handbook (Section 4510) presently allows companies to choose from several alternative methods for adjusting historical prices. In this paper we show the benefits obtained by using finer data, so that companies can made a decision as to the amount of resources they should invest to obtain finer data.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td></td>
<td>ii</td>
</tr>
<tr>
<td>List of Tables</td>
<td></td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>Introduction</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>Description of Valuation Problem</td>
<td>2</td>
</tr>
<tr>
<td>II</td>
<td>Summary of Sunder, Waymire and Ijiri's work</td>
<td>4</td>
</tr>
<tr>
<td>III</td>
<td>Purpose of Empirical Test and Data</td>
<td>9</td>
</tr>
<tr>
<td>IV</td>
<td>Empirical Evidence</td>
<td>12</td>
</tr>
<tr>
<td>V</td>
<td>Discussion</td>
<td>15</td>
</tr>
<tr>
<td>VI</td>
<td>Description of Tables and Figures</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Bibliography</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Footnotes</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Tables</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Figures</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Simplified Calculation of Error for any Index System</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>E matrix</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Computer program for Calculation MSE</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Appendix A</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Appendix B</td>
<td>43</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>Search Algorithm</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>Components of Industry Selling Price Indexes</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>Average Error of 15 most efficient index combinations</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>Error of most efficient index combination</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>Order of combination of goods that results in the most efficient index</td>
<td>25</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

PAGE

Figure 1 - Average accuracy of 15 index combinations 26

Figure 2 - Most accurate index combinations 27

Figure 3 - Average accuracy and most accurate index combinations 28
Historically two competing paradigms, the stewardship and the valuation paradigm have sought to define accounting and explain its purpose. The stewardship paradigm was originally emphasized by Paton in the 1920's, and sought to explain the role of accounting as the provision of information to a principal so that he could assess the performance of an agent. This model requires 'hard' objective data in order to evaluate performance hence objectivity and reliability are the primary criteria for judging the value of an accounting report.

The valuation paradigm was emphasized by Canning in 1930. He tried to relate Fishers concept of economic income to accounting. This model places a premium on relevance to decision making as the primary criterion for judging the value of an accounting report.

The two paradigms draw sharp boundaries in order to define what accounting is and how accountants should function. However, both paradigms have not been successful in explaining what accountants do and why rules requiring historical cost, conservatism, etc., are found in practise.

At present, research is being done to develop an economic theory of information which is holistic in nature, and combines insights obtained in economics, finance and accounting. This theory is called Agency theory and the academic community has great hopes that it will explain the economic forces that affect accounting.
In this paper we shall be dealing with the problem of valuation. A description of the valuation problem is given in section I. A Summary of Sunder, Waymire and Ijiri's work and a discussion of their contribution to our knowledge of the problem is given in section II. A description of the purpose of the empirical test and the data is presented in section III. The empirical evidence is presented in section IV, a discussion of the results follows in section V and a description of the tables and figures used in the study is presented in section VI.

I. Various approaches to valuation have been regarded as empirical proxies for a common unobservable theoretical conception of value. The major aspect of the Ijiri, Sunder approach of research in valuation is to determine how well each method approximates the underlying value. There is no unique measure of how good a proxy is, and several attributes such as relevance, reliability, objectivity, freedom from bias and cost are mentioned in the literature. The major problem of valuation exists in determining the value of used assets for which an organized and efficient market does not exist. These assets can be broken down into two categories:

1) Assets whose prices are unobservable

2) Assets whose prices can be observed after undertaking a costly search for information.
Comparative analysis of alternate valuation rules has traditionally been qualitative in nature. Use of historical cost, general price level adjustments and current cost adjustments have been viewed as distinct valuation rules rather than elements of a continuum. Historical cost represents prices that are unadjusted. General price level adjusts prices using one average economy-wide index. Current cost uses many specific indexes to adjust prices of assets held by a firm. Due to the qualitative nature of the analysis it is impossible to rank these valuation rules. The current legislation (section 4510) allows companies to choose from among many alternative methods in order to adjust historical prices. Major decisions facing managers of these companies are:

a) Should indexes be used or should the company hire an appraiser.

b) If indexes are used, what are the benefits gained by investing resources to obtain finer data.

Since the legislation has been recently introduced, many companies are going through this process for the first time and are looking for guidance in order to comply with the letter and spirit of the legislation. In order to answer these questions we estimated the accuracy function of the Canadian economy by assuming that it consists of 16 assets. We empirically demonstrated Sunder's convexity result and showed that the results obtained by Sunder in the U.S. are also valid for the Canadian economy.
I. Continued

The empirical evidence shows that gain in accuracy drops sharply as finer information is obtained. It also shows that for a large number of industries, indexes provide a very accurate measure of the change in prices of assets owned by a firm.

II. Ijiri (1967,1968) initiated an alternate approach by characterizing valuation rules as aggregation functions. He defined a statistic "the linear aggregation coefficient" which summarizes an important property of valuation rules as a single number. This was extended by Sunder (1978) who showed that most valuation rules, for example historical cost, general price level, current replacement cost, exit values, etc., may be viewed as members of a family of asset valuation rules which he labelled "exchange valuation rules". He developed a unified scheme to algebraically represent each rule in terms of the price index configuration employed to adjust historical cost. Various results pertaining to Bias and Mean Squared Error (MSE) of valuation rules were derived. Sunder and Waymire (1983) extended these findings to examine the amount of accuracy for a given index configuration with respect to any strictly finer index configuration. Hall (1982) provided empirical evidence in support of Sunder's results using data for electrical, gas, pipeline, telephone and water utilities. The most important results of these papers are summarized in Appendix A.
II. Continued

A major problem facing accountants in the past has been to clearly identify the role of accounting in providing information about the value of a firm. Is it the accountants role to value the firm or is that task to be left to the stock market or appraisers? Many prominent accountants like Ijiri were not in favour of providing a firm's value since they did not think that accountants were appraisers. They felt that the valuation function should be carried out by the stock market. However, one should realize that the value reflected by the stock market is a collection of subjective values, and it is the accountants role to provide objective information so that these subjective assessments can be made. It should also be noted that it is more efficient to require each company to provide price level adjusted data, rather than having numerous participants in the stock market duplicating this task. Efficiencies of scale can be gained if there is general acceptance about the manner in which these data should be generated.

If we assume that a demand for this type of information exists we can then focus on the benefits to be gained by investing resources to obtain finer data.

A well known portfolio effect in finance is that the variance of portfolio returns decline in a convex fashion as the number of securities in the portfolio increases. The accuracy of valuation (MSE) declines rapidly as more goods are added to the index since the change in prices of the goods are highly correlated. This results in a convex function, hereafter referred to as the convexity property.
II. Continued

At an intuitive level, this property is well understood by the government for example only a few goods are used to construct the consumer price index. This property is also used extensively in the finance literature where a market index (TSE index), is used as a measure of the market's performance rather than using all the securities traded in the market.

It is important to note that there are three types of assets found in the economy namely:

1) Assets that are heterogenous. These assets possess some unique properties that distinguish them from all other assets in the economy for example the Hotel Vancouver. At the present time Statistics Canada does not publish any indexes for real estate. Since these assets are heterogenous, even if an index were published, we would expect that it would have a very low correlation with the actual change in value of an individual piece of real estate.

2) Assets that are homogenous. The government usually produces indexes for these types of assets. Since they are homogenous the convexity property holds and the index has a very high correlation with the actual change in value of an individual asset.

3) Assets that fall in between the two groups depicted in (1) and (2). If indexes are available for these goods, they have a medium correlation with the actual change in value of an individual asset.
II. Continued

The major contributions of Sunder's paper are as follows:

1) The convexity property obtained from the portfolio theory, which is used extensively in finance and economics can be applied to the valuation problem.

2) If a firm has homogenous assets, the correlation between an appropriate index and the change in value of an individual asset will be high. He also demonstrated the convexity property, and showed clearly that the marginal benefit from using finer indexes decreases rapidly. Thus a coarse index reflects a large portion of the change in the value of underlying assets.

3) He introduced the notion of stochastic dominance and fineness. He showed an objective benefit that can be derived by investing resources to produce finer information.

4) He introduced the concept of a sufficient statistic (MSE) which could be applied to the valuation problem. This measure reflects all desirable attributes such as relevance, reliability, representational faithfullness, etc.
II. Continued

5) He separated the issue of quality of information from the user decision models that utilize this information. This is the concept of fineness of information. He hoped to show that we can order information systems based on fineness (stochastic dominance) i.e. to show that one information system would be preferred over another for all purposes if it could be obtained at the same cost. Unfortunately we can only get a fineness relationship for a subset of all information systems (partial order). A major contribution of Sunder was to show that in this case the convexity property can be applied to a substantial amount of goods in the economy.

6) Since the benefit is measured in terms of lower error (more accuracy) and cost is measured in dollars, Sunder derived a benefit graph on which each individual decision maker can plot his/her own cost curve. It is important to recognize that the cost curve depends on a subjective utility for accuracy i.e. the price that each person is willing to pay to obtain a lower MSE.
II. Continued

An important point to remember is that Sunder used American data to test his results, and that the quality and amount of information available in the U.S. is far superior than that available in Canada. Statistics Canada provides a few highly aggregated price indexes but it is willing to help any group or organization that wishes to obtain finer data. See Appendix B for a chart showing the levels of aggregation of data available in the U.S.

Sunder assumes that the value of the goods can be observed, thus it is important to note that for goods that are heterogenous for example the Hotel Vancouver, it is not appropriate to use this model.

III. CICA Handbook Section 4510: Reporting the Effects of Changing prices requires large publicly held companies to disclose the effect of changing prices on Inventory, Property Plant and Equipment (and Depreciation) as well as the gain/loss accruing to shareholders from holding net monetary assets.

General Price Level data are obtained by applying a single economy-wide price index, to historical cost data. Specific price changes are obtained by applying specific indexes to individual goods. Considerable flexibility is allowed in the choice of sources of information about current costs in order to encourage experimentation and learning.
III. Continued

Section 4510A.52 says: "Detailed rules for measurement of current costs have not been provided in the recommendations. The committee recognizes that selection of appropriate techniques will be a matter for determination by management, taking into account the nature of the assets concerned and the circumstances of the enterprise. Practical experience with the use of various techniques for determining estimates of current cost will increase the present level of knowledge and may permit more detailed recommendations to be made at a later stage".

Those who choose specific price indexes must also decide the specificity of the index system by balancing the gains in accuracy of valuation against the increased costs of using a more specific set of indexes. Those who use other methods (appraisals) may also want to know what they can expect to gain from their efforts.
In this paper we present empirical estimates of marginal gains in accuracy of asset valuation by increasing the specificity of price indexes used to adjust Historical Cost. The results show that the price structure of the Canadian economy is such that marginal gains in accuracy decline sharply as the specificity of price indexes is increased. A large proportion of total potential gain for accurate estimation of current cost is attained by a few broad indexes; additional detail adds relatively little to accuracy. For example increase in accuracy of valuation obtained by using 20 price indexes instead of 10 indexes is smaller than the increase in accuracy from using 10 indexes instead of 5. These results are similar to those in the finance literature where the marginal reduction of diversifiable risk of a portfolio declines as the portfolio size increases.

Assets of a firm as well as price indexes used to estimate their current value can be represented as portfolios of goods in the economy. The accuracy (MSE) of a given set of price indexes in approximating assets of a firm is a function of the mean vector and covariance matrix of price changes in the economy. Previous work by Ijiri (1967,1968) and Sunder (1978) provided analytical results which enable us to estimate the statistical accuracy of valuation rules based on various index systems directly from the data on price structure of the Canadian economy. Relative weights of goods and mean and covariance of price changes from Industry Selling Price Indexes published by Statistics Canada are used in this study.
III. Continued

The Data

The Manufacturing Price Index, published by Statistics Canada, was used rather than the Consumer Price Index since it provides a better valuation of industrial assets. The manufacturing price index consists of two hierarchical levels of price indexes:

1) An overall manufacturing price index (general price level index).

2) Specific Industry price indexes.

I have used fewer indexes than the total available in the data base. All the industries for whom monthly data was available for the period January 1977 - December 1982 were examined. This resulted in the selection of 16 industry specific price indexes out of a total of 20 industries. The weights of each index were adjusted to bring the total to 1. (see Table 2).

IV. Empirical Estimates of the Accuracy Function

We estimated the accuracy function of the Canadian economy by assuming that it consists of 16 assets, each of which is represented by one of 16 industry price indexes which comprise "the manufacturing price index". The weights of these assets indicate the relative proportion of these assets in the economy. Due to the adjustment of these weights it is assumed that these are the only goods in the economy.
IV. Continued

Suppose all the firms in the economy consisted of randomly drawn bundles of these 16 assets in varying proportions which were valued using only, 10 (10=k) price indexes. Which particular combination of assets can be expected to yield the most accurate valuation of individual firms on average? How accurate is this 10 price index system? Table 5 answers the first question, the curve shown in Figure 1 and Figure 2 answers the second question. I used the systematic search procedure (Table 1) to combine the 16 assets into 10 price indexes in fifteen different ways. Point X (Figure 1) shows the average accuracy for these fifteen 10-index values. Point Y (Figure 2) depicts the values of the most accurate of these fifteen 10-index values. Note that in this 16-asset economy, 16 price indexes (k=16), one for each good, automatically yields the most accurate current cost of the hypothetical bundle of 16 assets and therefore the mean squared error for a 16 index system is zero in Figure 1 and Figure 2. At the other end of the scale, valuation based on a single price index (k=1), "the manufacturing price index", is least accurate. Note that the curve in Figure 1 and Figure 2 have the same end points (at k=1 and k=16). There is only one way of combining 16 assets into a single index or 16 individual indexes. Figure 1 and Figure 2 show values of X and Y for all possible values of k (k=1-16).
The X-curve shows the accuracy of the average k-index systems that can be created from these 16 assets. There are two points of interest:

1) The X-curve accuracy function is convex. The marginal gain in accuracy by increasing k (the number of price indexes in this economy) keeps declining. More than half the gain in accuracy can be realized from using only four price indexes and there is hardly any gain beyond nine indexes.

2) X is an upward biased estimate of the accuracy function because further experimentation with this algorithm (and more computer resources), or with more efficient algorithms we can expect to obtain even better k-index system for different values of k.

The Y-curve shows the values of the most accurate k-index system created from the 16 available assets. It is interesting to note that the accuracy function Y, is even more convex than the accuracy function X. Since the estimated accuracy function shows further improvement for a given k as n increases, the accuracy function for, the actual canadian economy is likely to be highly convex.
IV. Continued

The analysis shows that if this 16 goods index (manufacturing) price index) is standardized for current valuation, we can construct finer indexes than those produced by Statistics Canada. This concept is intuitively apparent since Statistics Canada produces data for a variety of users. Therefore modification of this data for valuation of assets only, should yield a more accurate valuation.

V. Discussion:

The empirical evidence strongly suggests that the accuracy function of the Canadian economy is highly convex. This implies that the marginal gain in accuracy of valuation declines sharply as the number and specificity of price indexes used for valuation increases. These findings are valuable for Auditors, Managers, Academics and regulatory agencies who are involved in the debate on selection of asset valuation rules. Especially in a world where it is costly to employ more finer measurement methods.

Prior to practical implementation of the analytical model used in this paper, several research issues need to be resolved. First, it remains to be shown that in general, the accuracy function is convex. If not, under what conditions is it not convex?
V. Continued

This is important because prior to any policy decision on asset valuation rules, it would be useful to know whether the economy under consideration is characterized by a convex accuracy function. If a given economy does not possess this function, then accuracy of valuation will not be improved substantially by using only a few price indexes.

Second, to what extent is the convexity of the accuracy function related to parameters $u$ and $\Sigma$ (the mean and covariance) which describe the underlying process generating relative price changes. By identifying a relationship between the parameters $u$ and $\Sigma$ and the convexity curve of the accuracy function, it would be possible to determine the partitions along the accuracy function without employing costly search procedures for the set of valuation rules.

Any policy process attempting to exploit this framework would have to implement it on an \textit{ex-ante} basis. Thus, additional information is needed to determine whether $u$ and $\Sigma$ are stable over time and can be reliably predicted. If it is not possible to derive the relationship between $u$ and $\Sigma$ and the convexity of the accuracy function, can we identify superior search algorithms for the accuracy function? Note that this is a one period model and we have also assumed that there are no changes in the quantity of goods during the year. The impact of changes in quantities as well as over time would have to be assessed prior to practical implementation of this model.
V. Continued

The government can take a more active role by collecting data and providing finer information. The government, however, has to provide information for a variety of users and the CICA should conduct more research to determine:

1) For which specific industries does the convexity property apply?

2) What specific adjustments need to be made to data provided by the government to improve accuracy of asset valuation?

3) Identify all the alternatives available for valuation of heterogenous assets.

After specifically defining the information required the CICA and Statistics Canada should work together to provide the required information and the CICA can then provide more guidance to companies on how to adjust historical prices.
VI Description of Tables and Figures

Table 1 - describes the search Algorithm used in this study to find the MSE of Index Systems and identify finer partitions of the Index Combinations.
Any individual who wishes to reproduce the results presented in this paper, or who wishes to do a similar analysis using different data, should follow the steps set out in Table 1.

Table 2 - describes the Components of Industry Selling Price Indexes published by Statistics Canada that were used in this paper. All Components of the Index for whom monthly data was available for the period January 1977 - December 1982 were included in the study. The original weights of these goods and their adjusted weights are also shown in the table. The weights are determined by Statistics Canada based on surveys, questionnaires and collection of data by other means at their disposal. Their weights are revised on a periodic basis by Statistics Canada.

Table 3 - shows the Average MSE obtained of the 15 most efficient Combinations we could find for k, where k is the number of Indexes (k=1-16). For example, for k=10, there are a very large number of ways of combining 16 goods into 10 Indexes. Using the Systematic Search Procedure in Table 1, the 15 most efficient Combinations were identified and we calculated an Average for these Combinations hence Average MSE for k=10 is 55. This table shows the decrease in MSE on average as k increases.
Description of Tables and Figures (Continued)

Table 4 - shows the MSE obtained from the most efficient Index Combination we could find for \( k \) (\( k=1-16 \)) using the Systemic Search procedure described in Table 1. The third column shows the marginal gain obtained from increasing the Specificity of Indexes Used.

Table 5 - shows the order in which goods should be combined in order to generate the most efficient Index Systems. For example, we found that using our search procedure, if you wish to combine three goods into an Index than combining goods 6,8, and 12 results in the lowest MSE.

Figure 1 - is a graphical representation of the data presented in Table 3. The point X on the graph shows the average MSE of the 15 most efficient Index Configurations for \( k=10 \). This graph shows the average gain in accuracy as \( k \) increases.

Figure 2 - is a graphical representation of the data presented in Table 4. The point Y on the graph shows the most efficient Index Combination we could identify using the Systematic Search procedure for \( k=10 \). This graph shows the gain in accuracy for the most efficient combination as \( k \) increases.

Figure 3 - is a graphical representation of the data in Table 3 and Table 4. This graph enables us to see the marginal gains for the most efficient Index Configuration and on Average as \( k \) increases. The graph shows that both curves are convex.
Bibliography

1) Canadian Institute of Chartered Accountants
   CICA Handbook Section 4510 - "Reporting the Effects of Changing
   Prices". 1982.

2) Financial Accounting Standards Board
   Statement of Financial Accounting Standards No. 33: Financial
   Reporting and Changing Prices. 1979

3) Hall, T.W. "An Empirical Test of The Effect of Asset Aggregation
   on Valuation Accuracy".
   Journal of Accounting Research (Spring 1982) pages 139-151.

4) Ijiri, Y
   The Foundations of Accounting Measurement : A Mathematical,
   Economic and Behavioural Inquiry. Englewood cliffs, New Jersey:

5) Sunder, S. "Accuracy of Exchange Valuation Rules".

   :Additivity of Accuracy and Estimation Problems:
   Journal of Accounting Research (Forthcoming).

7) Sunder, S. and G. Waymire. "Marginal Gains in Accuracy of
   Valuation From Increasingly Specific Price Indexes: Empirical
Footnotes

1. \[ L_k = \sum_{j=0}^{k-1} \frac{(k-j)^n (-1)^j}{j!(k-j)!} \]


2. Note that mean square error is an inverse measure of accuracy. The valuation system becomes increasingly accurate as the mean squared error decreases.
### Table 1

**Search Algorithm**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Collect Data</td>
</tr>
<tr>
<td>2)</td>
<td>Compute Relative Price Change ( \frac{(P^1 - P^0)}{P^0} )</td>
</tr>
<tr>
<td>3)</td>
<td>Find Mean ((u)) and variance-covariance matrix ((\Sigma)) of relative price changes</td>
</tr>
<tr>
<td>4)</td>
<td>( w ) is given</td>
</tr>
<tr>
<td>5)</td>
<td>( S = \Sigma + UU' )</td>
</tr>
<tr>
<td>6)</td>
<td>( E = w_i w_j (S_{ii} + S_{jj} - 2S_{ij}) )</td>
</tr>
<tr>
<td>7)</td>
<td>Combine assets using ( E ) matrix as a guide to identify goods that have a low mean squared error when combined and compute error for the index.</td>
</tr>
</tbody>
</table>
TABLE 2
Components of Industry Selling Price Indexes:
Monthly Data 1877-1982 (72 observations)

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Original Weights</th>
<th>Revised Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 D500001</td>
<td>Food &amp; Beverage</td>
<td>19.016</td>
<td>23.6</td>
</tr>
<tr>
<td>2 D511200</td>
<td>Tobacco Products</td>
<td>1.084</td>
<td>1.4</td>
</tr>
<tr>
<td>3 D511500</td>
<td>Rubber &amp; Plastics</td>
<td>2.417</td>
<td>3.0</td>
</tr>
<tr>
<td>4 D513400</td>
<td>Leather Industries</td>
<td>.840</td>
<td>1.0</td>
</tr>
<tr>
<td>5 D514500</td>
<td>Textiles</td>
<td>3.369</td>
<td>4.2</td>
</tr>
<tr>
<td>6 D516600</td>
<td>Knitting Mills</td>
<td>.846</td>
<td>1.0</td>
</tr>
<tr>
<td>7 D519100</td>
<td>Wood</td>
<td>4.515</td>
<td>5.6</td>
</tr>
<tr>
<td>8 D523200</td>
<td>Furniture</td>
<td>1.539</td>
<td>1.9</td>
</tr>
<tr>
<td>9 D524200</td>
<td>Paper</td>
<td>7.809</td>
<td>9.7</td>
</tr>
<tr>
<td>10 D527100</td>
<td>Primary Metals</td>
<td>7.970</td>
<td>9.9</td>
</tr>
<tr>
<td>11 D529400</td>
<td>Metal Fabrication</td>
<td>7.169</td>
<td>8.9</td>
</tr>
<tr>
<td>12 D532900</td>
<td>Machinery Industries</td>
<td>4.162</td>
<td>5.2</td>
</tr>
<tr>
<td>13 D537300</td>
<td>Electrical Products</td>
<td>6.470</td>
<td>8.0</td>
</tr>
<tr>
<td>14 D541400</td>
<td>Non-Metallic Industries</td>
<td>3.043</td>
<td>3.8</td>
</tr>
<tr>
<td>15 D544000</td>
<td>Petroleum &amp; Coal</td>
<td>4.044</td>
<td>5.0</td>
</tr>
<tr>
<td>16 D545200</td>
<td>Chemical</td>
<td>6.270</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80.563</td>
<td>100.0</td>
</tr>
<tr>
<td>Number of Indexes (K)</td>
<td>Average Error of 15 Most Efficient Combinations</td>
<td>Marginal Gain from Increase in Specificity of Indexes Used</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------------------------------</td>
<td>------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>K = 1</td>
<td>1,161.</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>K = 2</td>
<td>1,076.</td>
<td>85.</td>
<td></td>
</tr>
<tr>
<td>K = 3</td>
<td>799.</td>
<td>277.</td>
<td></td>
</tr>
<tr>
<td>K = 4</td>
<td>583.</td>
<td>216.</td>
<td></td>
</tr>
<tr>
<td>K = 5</td>
<td>391.</td>
<td>192.</td>
<td></td>
</tr>
<tr>
<td>K = 6</td>
<td>250.</td>
<td>141.</td>
<td></td>
</tr>
<tr>
<td>K = 7</td>
<td>163.</td>
<td>87.</td>
<td></td>
</tr>
<tr>
<td>K = 8</td>
<td>114.</td>
<td>49.</td>
<td></td>
</tr>
<tr>
<td>K = 9</td>
<td>83.</td>
<td>31.</td>
<td></td>
</tr>
<tr>
<td>K = 10</td>
<td>55.</td>
<td>28.</td>
<td></td>
</tr>
<tr>
<td>K = 11</td>
<td>41.</td>
<td>14.</td>
<td></td>
</tr>
<tr>
<td>K = 12</td>
<td>27.</td>
<td>14.</td>
<td></td>
</tr>
<tr>
<td>K = 13</td>
<td>17.</td>
<td>10.</td>
<td></td>
</tr>
<tr>
<td>K = 14</td>
<td>10.</td>
<td>7.</td>
<td></td>
</tr>
<tr>
<td>K = 15</td>
<td>5.</td>
<td>5.</td>
<td></td>
</tr>
<tr>
<td>K = 16</td>
<td>0</td>
<td>5.</td>
<td></td>
</tr>
<tr>
<td>Number of Indexes (K)</td>
<td>Error of Most Efficient Combination</td>
<td>Marginal Gain from Increase in Specificity of Indexes Used</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>------------------------------------</td>
<td>----------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>K = 1</td>
<td>1161.0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>K = 2</td>
<td>894.5</td>
<td>266.5</td>
<td></td>
</tr>
<tr>
<td>K = 3</td>
<td>647.6</td>
<td>247.0</td>
<td></td>
</tr>
<tr>
<td>K = 4</td>
<td>410.0</td>
<td>237.6</td>
<td></td>
</tr>
<tr>
<td>K = 5</td>
<td>255.7</td>
<td>154.3</td>
<td></td>
</tr>
<tr>
<td>K = 6</td>
<td>167.0</td>
<td>88.7</td>
<td></td>
</tr>
<tr>
<td>K = 7</td>
<td>113.0</td>
<td>54.0</td>
<td></td>
</tr>
<tr>
<td>K = 8</td>
<td>87.0</td>
<td>26.0</td>
<td></td>
</tr>
<tr>
<td>K = 9</td>
<td>62.6</td>
<td>24.0</td>
<td></td>
</tr>
<tr>
<td>K = 10</td>
<td>46.0</td>
<td>17.0</td>
<td></td>
</tr>
<tr>
<td>K = 11</td>
<td>32.0</td>
<td>14.0</td>
<td></td>
</tr>
<tr>
<td>K = 12</td>
<td>18.6</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>K = 13</td>
<td>10.5</td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>K = 14</td>
<td>5.5</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>K = 15</td>
<td>2.0</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>K = 16</td>
<td>0</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 5**

Most efficient Index Systems are generated by combining goods in the following order:

6, 8, 12, 3, 5, 13, 4, 11, 2, 16, 14, 9, 10, 15, 7.

Best Index Containing 3 Goods is: (6, 8, 12)

Best Index Containing 5 Goods is: (6, 8, 12, 3, 5)

Best Index Containing 10 Goods is: (6, 8, 12, 3, 5, 13, 4, 11, 2, 16)
FIGURE 1

AVERAGE ACCURACY OF EACH INDEX

MEAN-SQUARE ERROR

NUMBER OF GOODS IN INDEX
FIGURE 2

MOST ACCURATE INDEX COMBINATION

MEAN-SQUARE ERROR

NUMBER OF GOODS IN INDEX

Y(MSE=46)
FIGURE 3

AVERAGE ACCURACY OF EACH INDEX
AND
MOST ACCURATE INDEX COMBINATION

NUMBER OF GOODS IN INDEX

MEAN-SQUARE ERROR

FIGURE 1

FIGURE 2
Simplified Calculation of Error for any Index System

Suppose an index system contains 4 goods 1,2,3,4. Contribution of this index to the mean squared error is:

$$E_{12} + E_{13} + E_{14} + E_{23} + E_{24} + E_{34}$$

where

$$E_{ij} = w_i w_j \left[ \text{Var}(r_i - r_j) + (u_i - u_j)^2 \right]$$

$$E = w_i w_j \left[ \sigma_{ii} + \sigma_{jj} - 2 \sigma_{ij} + u_i^2 + u_j^2 - 2 u_i u_j \right]$$

$$E = w_i w_j \left[ S_{ii} + S_{jj} - 2 s_{ij} \right]$$

where $s_{ij}$ is the $ij^{th}$ element of $S = \sum + uu'$.  

Add up the contribution of each index to obtain the total mean squared error of the index system.

For a detailed Mathematical Analysis, interested readers should consult Sunder (1978) and Sunder and Waymire (1983).
The $E$ Matrix shows all possible combinations of 2 Goods in an Index. The numbers divided by the weights of the respective Goods result in the MSE of combining two goods in an Index. Table E helps us find finer partitions. The reasoning is that a low $E$ value results in a low MSE.
COMPUTER PROGRAM FOR CALCULATING ERROR

Dimension W(16),E(16,16),Index(16,16),Kn(16),Error(16),WK(16)
Dimension Dummy(16)

2 Read (2,020) (W(I),I=1,16)
020 Format (F 11.5)

7 Read (7,030) (E (I,J) J=1,16) (Dummy(J) J=1,16)) I=16)
Write (6,30) (E (I,J) J=1,16)I=1,16)
030 Format (16F 7.3)

50 Write (6,100)

100 Format ('How Many Indexes?')
Read (5,110) K
If (K.EQ.0) Go to 300

110 Format (12)
Write (6,120)

120 Format ('Please enter Number of Variables and 
their Identification in Each Index:')
ERR=0
DO 200 I=1,K
Write (6,130) I

130 Format ('Index No.', I5. ')?')
Read (5,140) KN (I), (Index (I,I1) I1=1,16)
140 Format (I7I2)
WK (I)=0
KNK=KN(I)
DO 150 I1=1, KNK
12=Index (I,I1)

150 WK(I) = WK(I)+W(I2)/100
Error (I)=0
If (KN(I).EQ.1) Go to 185
KKK = KN(I)-1
Do 180 I1=1, KKK
III = I1+1
Do 180
12=Index (I1,I1)

180 Error (I) = ERROR(I)+E(I3,I4)/WK(I)
185 Write (6,190)I, WK(I),ERROR (I)

190 Format ('For Index', I5,Weight ',F 11.5' ERROR ',F 11.5)
ERR = ERR = ERROR (I)

200 Continue
Write (6,210) ERR

210 Format ('Total Error For This Index System = ',F 11.5')
Go To 50

300 Stop
END

This Computer Program enables the User to log-on to the Computer and Use the Computers Computational Speed and memory capacity to combine Various goods in an Index and Compute the MSE of the Index.
COMPUTER OUTPUT

INDEX CONTAINING 2 GOODS

This Printout is an Example of a run on the Computer. Where the reasoning Implicit in the E matrix was tested.

1 Represents the number of goods Combined in the Index.
2 The two specific goods in the Index (i.e.) asset 6 and asset 8.
3 The Weights of goods 6 and 8, Used to divide their E matrix Value of .057.
4 The MSE Obtained by Combining these two goods.

How Many Indexes?
16
Please Enter Number of Variables and Their Identification in Each Index.

Index No. 1?
2  68
For Index 1 WEIGHT 0.2900 ERROR 1.96552

Index No. 2?
2  63
For Index 2 WEIGHT 0.4000 ERROR 2.90000

Index No. 3?
2  65
For Index 3 WEIGHT 0.5200 ERROR 2.17338

Index No. 4?
2  6 12
For Index 4 WEIGHT 0.6200 ERROR 1.96774

Index No. 5?
2  6 13
For Index 5 WEIGHT 0.0900 ERROR 2.67776

Index No. 6?
2  6 11
For Index 6 WEIGHT 0.09900 ERROR 3.22227
### COMPUTER OUTPUT

**INDEX CONTAINING 2 GOODS (Cont.)**

<table>
<thead>
<tr>
<th>Index No.</th>
<th>Weight</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.08800</td>
<td>4.13637</td>
</tr>
<tr>
<td>8</td>
<td>0.0200</td>
<td>7.1500</td>
</tr>
<tr>
<td>9</td>
<td>0.02400</td>
<td>12.20834</td>
</tr>
<tr>
<td>10</td>
<td>0.24600</td>
<td>9.43089</td>
</tr>
<tr>
<td>11</td>
<td>0.04900</td>
<td>4.08163</td>
</tr>
<tr>
<td>12</td>
<td>0.06100</td>
<td>6.22951</td>
</tr>
<tr>
<td>13</td>
<td>0.07100</td>
<td>3.70423</td>
</tr>
<tr>
<td>14</td>
<td>0.9900</td>
<td>7.80808</td>
</tr>
<tr>
<td>15</td>
<td>0.10800</td>
<td>6.79630</td>
</tr>
<tr>
<td>16</td>
<td>0.09700</td>
<td>8.36083</td>
</tr>
</tbody>
</table>
COMPUTER OUTPUT

INDEX CONTAINING 2 GOODS (Cont.)

TOTAL ERROR FOR THIS INDEX SYSTEM = 84.81244

How Many Indexes?
16
Please Enter Number of Variables and Their Identification in Each Index:

Index No. 1?
2 84
For Index 1 WEIGHT 0.02900 ERROR 9.24138

Index No. 2?
2 8 2
For Index 2 WEIGHT 0.03300 ERROR 15.66667

Index No. 3?
2 8 11
For Index 3 WEIGHT 0.25500 ERROR 19.17645

Index No. 4?
2 3 5
For Index 4 WEIGHT 0.07200 ERROR 7.13889

Index No. 5?
2 312
For Index 5 WEIGHT 0.08200 ERROR 4.97561

Index No. 6?
2 313
For Index 6 WEIGHT 0.1100 ERROR 8.79091

Index No. 7?
2 311
For Index 7 WEIGHT 0.11900 ERROR 9.93278
COMPUTER OUTPUT

INDEX CONTAINING 2 GOODS (Cont.)

Index No. 8?
2  316
For Index 8 WEIGHT 0.10800 ERROR 9.49074

Index No. 9?
2  34
For Index 9 WEIGHT 0.04000 ERROR 13.9000

Index No. 10?
2  32
For Index 10 WEIGHT 0.04400 ERROR 15.45455

Index No. 11?
2  31
For Index 11 WEIGHT 0.26600 ERROR 29.04886

Index No. 12?
2  512
For Index 12 WEIGHT 0.09400 ERROR 7.17021

Index No. 13?
2  513
For Index 13 WEIGHT 0.12200 ERROR 11.64754

Index No. 14?
2  511
For Index 14 WEIGHT 0.13100 ERROR 11.54193

Index No. 15?
2  516
For Index 15 WEIGHT 0.12000 ERROR 11.70834

Index No. 16?
2  54
For Index 16 WEIGHT 0.05200 ERROR 13.500

TOTAL ERROR FOR THIS INDEX SYSTEM = 198.38486
Appendix A

In this Appendix we summarize the important analytical results derived by Sunder (1978), and by Sunder and Waymire (1983). Interested readers should consult the original articles for a more thorough analysis.

Consider an economy with $n$ assets. Let $q$ be the vector of quantities of the $n$ assets contained in a given bundle. Suppose that under a given rule, valuation of the bundle is $p_0$ at time 0 and $p_1$ at time 1; the relative price change is $R = (p_1 - p_0)/p_0$.

Let $r$ be the $n$-vector of relative price changes from time 0 to time 1 for the $n$ assets. If valuation of each of the $n$ assets in the bundle is arrived at by applying a specific price index to each asset, the resulting value of $R$ is the relative price change in current value of the bundle ($R_{cv}$) and is defined to be the principal aggregation:

$$R_{cv} = w'r$$  \hspace{1cm} (1)

where

$$W_i = \frac{p^0_i q_i}{\sum_{j=1}^{n} p^0_j q_j}$$ \hspace{1cm} \text{for } i = 1,2,...,n .$$

and

$$p^0_i = \text{Unit Price of asset } i \text{ at time 0.}$$

$W$ represents the vector of relative weights characterizing a given bundle of assets, i.e. the firm.
Appendix A (Continued)

R is used as a generic symbol for valuation rules and two modifiers are added to identify a specific rule. $R_{ki}$ represents a valuation rule which uses $k$ ($k \leq n$) different price indexes to adjust the beginning of the period valuation of all $n$ assets. Since, in most cases, there is more than one way of forming $k$ price indexes from the $n$ assets, $R_{ki}$ represents the valuation obtained by using the $i$th of $L_k$ possible configurations of $k$ indexes. Since there is only one way of forming $n$ indexes from $n$ assets $R_{cv} = R_{nl}$.

Sunder (1978) defined the accuracy of a valuation rule to be the economy-wide average of the mean squared error of valuation for individual firms, $R_{ki}$, with respect to the principal aggregation $R_{nl} (=R_{cv})$. Each individual bundle of assets (representing one firm) is characterized by its vectors of relative weights, $W$, which is assumed to be generated randomly from the economy-wide bundle of relative proportions $w$, using a constant number $(p)$ of multinomial trials. This accuracy measure (denoted $A_{ki}$) is given by:

$$A(R_{ki}) = E (R_{ki} - R_{nl})^2$$
$$= \frac{1}{p} (w'(\mathbf{G} + \mathbf{\hat{u}}) - \mathbf{w}'\mathbf{e}) (\mathbf{w}'\mathbf{\hat{u}} + \mathbf{u}u')\mathbf{w}$$

(2)

where $\mathbf{e} =$ vector of unit elements of appropriate length

$\mathbf{w} = E(\mathbf{w})$; $n$-vector of relative weights of $n$ assets in the economy

$\mathbf{w}'\mathbf{e} = 1$

$\mathbf{u} = E(\mathbf{r})$; $n$-vector of expected relative price changes for $n$ assets,

$\mathbf{\hat{u}} = n$-vector of squared elements of $\mathbf{u}$,

$E = E(\mathbf{r-u}) (\mathbf{r-u})'$, $n \times n$ covariance matrix of relative price changes for $n$ assets,
Appendix A (Continued)

\( \sigma \) = n-vector of diagonal elements of \( \Sigma \)

\( k \) = number of price indexes used in the valuation rule. The set of \( n \) assets is partitioned into \( k \) nonempty subsets and a price index is constructed for each subset. \( w, u \) and \( \Sigma uu \) are the sub-vectors and submatrix respectively corresponding to the \( u \)th of the \( k \) subsets.

\( p \) = number of multinomial trials by which the bundle of assets for individual firms is randomly drawn from the economy-wide bundle defined by \( w \).

Let \( T_{ki} \) denote the \( i \)th of \( L_k \) distinct partitions that can be used to form \( k \) price indexes for \( n \) assets. Similarly, \( T_{k+m,j} \) is the \( j \)th of \( L_{k+m} \), partitions that can be used to form \( (k+m) \) indexes from the \( n \) assets, \( m = 1,2,\ldots,n-k \). Sunder (1978) proved that if \( T_{k+m,j} \) is a strictly finer partition of the \( n \) assets than \( T_{ki} \), then

\[ A(R_{k+m,j}) < A(R_{ki}) \]

In other words, economy-wide average of mean squared error of valuation is a monotonically decreasing function of the fineness of the partitions used to form price indexes for \( n \) goods. \( A_k \), however, is not monotonic in \( k \), the number of price indexes employed.

For each value of \( k \), let \( R_k^* \) denote the most accurate of the \( L_k \) valuation rules (i.e.) \( R_k \) has the smallest economy-wide average of mean squared error.

\[ A(R_k^*) < A(R_{ki}) \quad i = 1,2,\ldots,L_k. \]
Let the corresponding partition of the n assets be denoted by Tk. Thus, for every value of k there exists a partition Tk* which yields the best accuracy (i.e.) smallest mean squared error, through the valuation rule R^k. We define H(k) to be the accuracy function which gives the accuracy for the most accurate k-index valuation rule: 

$$H(k) = A(R^k), k=1,2,...,n.$$  \hspace{1cm} (3)  

The remainder of this paper is devoted strictly to examining the properties of H(k) for the Canadian economy. If one wishes to use k specific price indexes for valuation of n(>k) assets, what is the most accurate partition of n assets for constructing k price indexes from n assets and how accurate is this method. Specifically, we are concerned with whether H(k) is convex in k. We know already from the results in Sunder (1978) that H(k) is strictly decreasing in k. For any partition Tk* (for any $k \leq n$) along the accuracy function, we can generate a strictly finer partition of the n assets into k+1 indices (denoted Tk+1 j). Since T^k and Tk+1 j are comparable with respect to fineness, A(R^k+1 j) must be less than A(R^k*). By applying this argument for every value of k strictly less than n, it is concluded that H(k) is strictly decreasing in k.
Appendix A (Continued)

Convexity of the accuracy function, $H(k)$ implies that the marginal gain in accuracy (i.e.) reduction in mean squared error, declines as $k$ increases. The convexity properties of the accuracy function are important from a practical viewpoint. A highly convex accuracy function implies that use of only a few broad indexes achieves a large proportion of total potential gain towards accurate estimates of current cost and further gains in accuracy may not be worth the additional costs associated with using a more detailed set of price indexes or direct measurement of current cost of individual assets.

II Estimation of Accuracy Function $H(k)$

Two problems must be solved to estimate the accuracy function, $H(k)$, from data. First, accuracy measure $A(R_{ki})$ for index configuration $T_{ki}$, is a function of $\mu$ and $\Sigma$ the mean vector and covariance matrix, respectively, of relative price changes for $n$ assets in the economy. Since these parameters are unknown, they must be estimated from data. Sampling errors bias estimates of $A(R_{ki})$ upward if (2) is applied. We present an estimator which corrects for the bias. Second, even for moderate values of $n$, the set of all possible valuation rules is very large making exhaustive search over the set infeasible. Estimated accuracy function depends on search procedures used to identify the most accurate price index systems for each value of $k$. We present an algorithm which systematically searches the set to identify relatively accurate index systems.
Consider first the issue of sampling errors in $u$ and $\Sigma$. If unbiased parameter estimates (denoted $\hat{u}$ and $\hat{\Sigma}$) are employed in (2), the estimated accuracy for valuation $R_{ki}$ based on partition $T_{ki}$ is:

$$\hat{A}(R_{ki}) = w'(\hat{\sigma} + \hat{u}) - \sum_{u=1}^{k} \frac{w'u}{w'e} \left( \sum_{uu} + \hat{u}\hat{u}' \right) \frac{1}{T} \sum_{k=1}^{K} \hat{R}_{ki}^2$$

(Sunder and Waymire (1983) show that the presence of sampling errors in $u$ biases (4) upward, while sampling errors in estimating the covariance matrix, $\Sigma$, have no effect. The estimator to correct for this bias in (4) is henceforth referred to as the unbiased estimator:

$$\hat{A}(R_{ki}) = w'\left[ \frac{T-1}{T} \hat{\sigma} + \hat{u} \right] - \sum_{u=1}^{k} \frac{w'u}{w'e} \left[ \frac{T-1}{T} \sum_{uu} + \hat{u}\hat{u}' \right] \frac{1}{T} \sum_{k=1}^{K} \hat{R}_{ki}^2$$

where $T$ represents the number of observations of relative price changes used to estimate $u$ and $\Sigma$.

From inspection of Equation (5), it is evident that the unbiased estimator differs from equation (4) only by the term $(T-1)/T$ which is multiplied by the diagonal elements in the covariance matrix and the covariance submatrix for each of $k$ indexes. Estimator (4) converges to the unbiased estimator (5) as $T \to \infty$ and is therefore asymptotically unbiased. The values of $A(R_{ki})$ reported in this paper have been calculated using the unbiased estimator (5).
II  Estimation of Accuracy Function $H(k)$ - Continued

The second estimation problem concerns selection of an appropriate procedure for searching over the set of valuation rules. Identification of the accuracy function $H$ requires that for each value of $k=2,...,n-1$, the index configuration with best accuracy (i.e.) minimum mean squared error, be identified and an unbiased estimate of its accuracy be obtained by using (5). This requires optimization of (5) with respect to alternative index systems defined by $k$ partitions of the set of $n$ assets, a problem that has not been solved analytically. The total number $L_k$ of alternative $k$-partitions of the set of $n$ elements is extremely large, even for moderate values of $n$. Exhaustive search over such a large set is infeasible and efficient search algorithms must be devised to obtain an approximation of the accuracy function $H(k)$ under the constraint of limited computer resources.

In order to estimate $H(k)$ we have used the procedure described in Table 1. This systematic search procedure exploits previous analytical results to identify those index partitions which are likely candidates for inclusion in the accuracy function. Likely candidates for the most accurate $k$-partition are those which are strictly coarser than the most accurate $(k+1)$ partition and strictly finer than the most accurate $(k-1)$ partition. The search procedure focuses on the neighbourhood around the finest partition.
APPENDIX B

Producer Price Index

Rubber and Plastic Products (07)

Rubber Products (071)
- Crude Rubber (0711)
- Tires and Tubes (0712)
- Miscellaneous (0713)

Plastic Products (072)
- Plastic Construction (0721) Products
- Unsupported Plastic Film (0722) and Sheeting
- Laminated Plastic (0723) Sheets

Single Index

Two Digit Index

Three Digit Indexes

Four Digit Indexes

Figure 1

Producer Price Index (PPI) Classification Example