THE DYNAMICS OF CAPITAL STRUCTURE CHOICE

By

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ABSTRACT

This thesis employs two-period state-contingent model based upon the "tax shield plus bankruptcy costs" approach to examine the dynamic capital structure decision. By allowing recapitalization at the end of period one, we can analyse the dynamics of the firm's capital structure choice. Also, the effect of a call provision on bonds can be examined.

Simulated results show that the firm will recapitalize at the end of period one only if the gain in firm value, with- or ex-dividend, resulting from recapitalization exceeds the after-tax flotation costs. There exists a tolerable recapitalization boundary within which the firm will not recapitalize. This implies that the empirically observed capital structure is not necessarily at the acme of the firm value function, as most empirical studies assume.

Another important result is that a call provision on bonds may be wealth reducing; the call provision may reduce the wealth of shareholders by inducing recapitalization in states which is suboptimal if there is no call provision, and incurs flotation costs which could have been avoided. The gain in firm value resulting from recapitalization may be too small to justify the extra flotation costs and thus reduces the overall firm value.
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1. INTRODUCTION

The firm capital structure decision has long been a controversial issue in the academic world of corporate finance. Starting from the pioneering work of Miller and Modigliani [1958], there has been much work in this area. The MM theory proposes capital structure irrelevance in complete and perfect capital markets. Later studies by Kraus and Litzenberger [1973], Baxter [1967] and Kim [1978] suggest a tradeoff between the tax shield from debt financing and the expected bankruptcy costs. This tradeoff gives rise to a concave levered firm value function and thus an optimal capital structure.

There have been numerous empirical studies of the capital structure decision; however, recapitalization costs are usually ignored. One common methodology measures some typical financial characteristics of a sample of companies and attempts to explain the relationship among capital structure, firm value and other variables. Another approach is to examine capital structure and other financial characteristics through time. Heinkel [1984] points out that these cross-sectional tests assume that observed capital structures are optimal; however this assumption ignores the fact that, with flotation costs, there is always a tolerable bound for suboptimal capital structure. Especially in light of the conflicting evidence on the role of capital structure behavior, the incorporation of
recapitalization costs into the capital structure decision may lead to a better understanding of the relationship between observed capital structure, firm characteristics, and firm value.

The call provision in bond indentures has traditionally been treated as a protection for the shareholders. However, this is not necessarily the case since low call price may serve as an incentive for the firm to recapitalize more frequently than without a call provision, thus incurring unnecessary flotation costs; this implies a reduced firm value. The analysis of a call provision in the presence of recapitalization will provide a clear example of this.

This thesis follows a "tax and bankruptcy costs" approach and analyzes the dynamics of capital structure choice in a two-period state-contingent model. The model is subdivided in four cases of progressive complexity. Starting out from the base case in Case I, the firm issues one two-period debt as its only security other than stock. Then we analyse the situation when two one-period bonds are issued. In Case III, optional recapitalization at the end of period one is allowed with fixed flotation costs. Now, the capital structure choice is a recursive decision because the recapitalization decision is made at the end of period one while the capital structure decision is made at the beginning of the same period. Finally, a call provision is introduced in Case IV and this shows the effect of a call provision on the capital structure choice.

With this two-period model, we can analyse the effects of
recapitalization on the firm's capital structure choice and thus hypothesize that the observed capital structure of a firm is not necessarily at the acme of the firm value function and that there is a tolerable bound of debt levels within which the firm will not recapitalize because of flotation costs. The effects of bankruptcy costs, corporate tax rate, flotation costs, discount rate and call price on the capital structure decision will be analysed through sensitivity analysis.

Section II provides an overview of the recent theories and development of the issue of capital structure choice. Section III provides a description of the background, approach, functions and assumptions of the two-period model. Then Section IV describes the layout of the model and the underlying reasons for these mechanisms. All four cases of the two-period model are simulated in the WATFIV computer programs in Section V. With the simulation programs, the firm's capital structure choice is simulated with an example and the results are interpreted in Section VI. Then sensitivity analysis is performed with respect to the changes in bankruptcy costs, flotation costs, corporate tax rates, discount rates and call prices. Finally, the results and findings will be summarized and concluded in Section VIII.
2. RECENT THEORIES AND DEVELOPMENTS

The pioneering work of Miller and Modigliani [1958] suggests that in complete and perfect capital markets, the capital structure decision has no effect on the value of the firm. This point of view is later supported by Stiglitz [1969] and Hirshleifer [1966]. Since then, there has been research on the capital structure controversy modifying the models by adding various market imperfections to the analysis. Hypothesizing tax advantages as a positive incentive and bankruptcy costs as a negative incentive of debt financing has been a popular approach.

In a market corporate taxes, the interest payments to the bondholders are tax deductible to the firm and this increases the value of equity. When the firm cannot meet the payment to bondholders, it is in default and incurs trustee fees and other legal fees. The expected value of these costs reduces the present value of the future cashflows of the firm and hence the market value of the firm decreases. Kraus and Litzenberger [1973], Baxter [1967] and Kim [1978] suggest that there is a tradeoff between tax advantages and expected bankruptcy costs and an optimal capital structure exists as a result of this tradeoff.

Other than the above approach, Jensen and Meckling [1976] suggest that optimal capital structure exists even without tax shields and bankruptcy costs because of the existence of agency costs. Management of the firm will choose investment projects which maximize shareholders' wealth. In cases where there are conflicts of interest between bondholders
and shareholders, projects which favour shareholders will be chosen and this reduces the wealth of the bondholders. As a result, bondholders will insist on bond covenants to protect their interests. Costs of writing these covenants and costs incurred by the firm to keep these covenants represents a certain amount of expected agency costs. Also, there are opportunity costs incurred because the firm may have given up positive net present value projects so as to transfer wealth from bondholders to shareholders. These costs depend on the amount of debt financing issued by the firm. Jensen and Meckling point out that there exists an optimal capital structure even without tax shields and bankruptcy costs and this optimal debt-equity mix will minimize the expected agency costs. There are also other circumstances which give rise to agency problems. Barnea, Haugen and Senbet [1981] suggest numerous agency problems. These include agency problems due to information asymmetry, limited liability, and due to partial ownership with controlling interests.

There have been numerous empirical studies on the capital structure controversy. However, Copeland and Weston [1983, Ch. 13] suggest that there are several difficulties with empirical tests on this topic. First, the anticipated future growth of a firm is hard to estimate. Second, flotation costs for smaller firms are usually higher than large firms. A third difficulty is that all companies chosen in an empirical test do not have the same business risk.
Miller and Modigliani [1966] use a model which assumes that the firm grows more rapidly than the economy for a fixed period of time and they run cross-section multiple regressions on 63 electric utility companies. Their results show that the tax shield advantages of debt have significant positive impact on the overall firm value and this implies that the weighted average cost of capital decreases as leverage increases.

Other studies have focused on the relationship between some firm specific characteristics with the capital structure choice. Ferri and Jones [1979] examine the relationships between a firm's capital structure choice and its industry class, size, business risk and operating leverage. They found a weak and indirect relationship between the debt level of a firm and its industry class. Size of a firm has some effects on its debt level; however, the relationship is not positive and linear. Also, business risk of the firm has insignificant impact on the capital structure choice and they found a negative relationship between debt level and operating leverage.

Bradley, Jarrell and Kim [1984] present a single-period model to simulate the capital structure choice of a firm. They include costs of financial distress, corporate taxes, personal taxes, non-debt tax shields, agency costs and variability of firm value in the model. Their simulated results show that firm leverage is inversely related to expected costs of financial distress and non-debt tax shields. Empirical test on the model shows that there
exists strong industry influences on firm financial leverage. These influences include the volatility of earnings, the amount of research and development, and advertising expenses. The firm financial leverage is inversely related to the above factors. Also, the debt level is directly related to the amount of non-debt tax shield. However, Mikkelson [1984] criticizes the model in the sense that the empirical test does not relate directly to the simulated results of the model and the model does not account for the effect of the variability of firm value on agency costs. Also, measurement of leverage ratio does not incorporate the fact that there are multiple classes of debt and equity claims in the firms they are measuring.

Martin and Scott [1975] demonstrate in an empirical test that industry class is a determinant of capital structure choice. They choose data covering the period 1967-1972 and from firms in 12 industries for the analysis. Common equity ratio is used rather than debt ratio because the latter has omitted the existence of preferred shares. They found that industry class is a key determinant of capital structure choice.
3. A MODEL FOR FIRM RECAPITALIZATION

3.1 BACKGROUND OF THE MODEL

3.1.1 Nature of Recapitalization Behaviour

The main objective of a corporate financial manager is to maximize the equity value of the firm. The same rule also applies to firm capital structure decisions. The manager should arrange the equity mix so as to maximize the total wealth of shareholders. The capital structure decision is a function of, among other things, the corporate tax rate, required rate of return of bondholders and of shareholders, the unlevered value of the firm as the present value of the expected future earnings, the probability of bankruptcy, bankruptcy costs, expected agency costs and numerous other possible firm specific variables. As these variables change, the previous optimal debt-equity mix may not be the one which now maximizes shareholders' wealth. In cases like this, the financial manager might want to shift to another debt-equity mix so that shareholders' wealth, subject to bond covenants, is a maximum among all the different debt-equity mixes. This process is called recapitalization.

During recapitalization, a firm can increase its debt-equity proportion by issuing bonds or by buying back equity shares from the market. On the other hand, decreasing the debt-equity proportion would involve issuing new common
shares, buying back/retiring corporate bonds from the capital market or calling their bonds if there is a call provision and the bond price is higher than the call price. Of course, these transactions will not be costless and recapitalization costs are incurred whenever the firm changes the debt-equity mix. Flotation costs include underwriting fees, brokerage fees, and other transaction costs. Most of these costs are fixed costs with only a small portion variable, so it seems reasonable to assume they are fixed.

3.1.2 Reasons for Recapitalization

Numerous circumstances lead to recapitalization. Changes in the variables determining the original debt-equity mix cause a recapitalization that implies a shift in debt-equity mix and a higher overall value of bonds and stocks.

Traditional capital structure theories propose a concave function of levered firm value against its debt-equity proportion (Baxter [1967], Stiglitz [1969], Kraus & Litzengerger [1973], Kim [1978]). This concavity gives rise to a unique maximum firm value and a corresponding optimal debt-equity mix. The firm should stay with this debt level as long as it maximizes shareholders' wealth and there is no reason for the firm to recapitalize unless, firstly, the debt-equity mix of the firm has changed due to some stochastic events, or secondly, the original debt-equity mix is no longer optimal.
In the first case, stochastic events may require extra funds or supply an extra source of funds. These stochastic cashflows may be dealt with by a temporary change in the debt-equity mix. This change is not under the control of the management because the demand for funds is instantaneous and the firm can only settle this with their short term line of credit from the bank. After the change, the firm will recapitalize to the optimal position with a long term debt or stock issue.

In the second case, changes in the financial characteristics of the firm may change the optimal level of the debt-equity mix. For example, an increase in tax rate would increase the present value of the future tax benefits of debt and thus an optimal capital structure requires a higher debt-equity ratio. Other factors such as probability of bankruptcy, bankruptcy costs and expected future earnings may give rise to a change in the optimal capital structure. Recapitalization is necessary to adjust the suboptimal structure to the optimal level.

3.1.3 Criteria for recapitalization

In a market where there are no transaction costs, it would be in the best interest of the firm to recapitalize whenever there is any deviation, even very slight, from the optimal debt-equity mix. In this theoretical world, the observed debt-equity mix would always be at the optimum.

However, when there are flotation costs associated with
recapitalization, the situation is different. For every time the firm recapitalizes, costs are incurred; these costs serve as negative incentives for the firm to adjust its debt-equity mix continuously. As in evaluating a capital budgeting project, recapitalization takes place only when its net present value is greater than zero. In other words, recapitalization is optimal only if the increment in firm value resulting from recapitalization exceeds the corresponding flotation costs. Given a concave function of firm value against debt ratio, this recapitalization criterion provides a tolerable bound on suboptimal debt, as shown in Figure 1. Inside this bound, the increment of firm value from recapitalization is less than the flotation costs $F$, so recapitalization is not desirable. The firm will allow a suboptimal debt-equity mix only within these bounds; a debt-equity mix outside these bounds will encourage a takeover. The bidding party in a takeover will, if successful, recapitalize the firm and realize a profit from the difference between the increment in firm value from recapitalization and the corresponding costs (Heinkel [1984]).
Figure 1

Criterion for Recapitalization

VL

MAX VL

F

RECAPITALIZE  TOLERABLE BOUND  RECAPITALIZE

DEBT-EQUITY RATIO

3.2 FUNCTIONS AND OBJECTIVES OF THE MODEL

A two period model will be presented to simulate recapitalization behaviour under different sets of assumptions. The model presents a way of analysing the relationship between the value of equity, value of debt, overall firm value, the recapitalization decision under the objective of equity value maximization, and the optimal debt level. Also, we analyze the influence of changes in flotation costs, bankruptcy costs, the corporate tax rate, and the interest rate on the recapitalization decision and firm value.

The model assumes a firm with access to a project with a sequence of two stochastic, serially dependent cashflows and the ability to finance this project with some debt financing. The debt may have a one or two period maturity. The model focuses on the decision to recapitalize the firm after one period given the existence of flotation costs. The recapitalization decision is made to maximize the value of equity at that time.

The objective of this thesis is to understand the impact of the option of costly recapitalization on firm value. Also, we want to determine how the capital structure characteristics, such as optimal debt level, firm value, and the value of debt and equity, change after the recapitalization decision. Finally, sensitivity analysis allows us to examine the impact of changes in flotation costs, bankruptcy costs, the corporate tax rate and the discount rate on the firm.
We hope this study provides insights in the theory of optimal capital structure and possibly a strategy for the capital structure decision for a firm.
3.3 MODEL STRUCTURE

3.3.1 Tax and Bankruptcy Cost Approach

The model employs a traditional corporate tax and bankruptcy cost point of view. Debt financing is favourable because of the resulting interest tax shield; however, there is the offsetting factor of higher expected bankruptcy costs as a disincentive to debt financing.

The most obvious advantage of leverage is the tax shield associated with it. Part of the payments to debtholders is tax deductible and so reduces the cost of borrowing and hence the overall cost of capital. A reduced tax payment means that the value left for shareholders has increased. Thus, considering only the tax advantage of borrowing, leverage increases the market value of the firm by an amount equal to the present value of the future tax deductions resulting from the borrowing. Various models employing the tax deductibility issue of capital structure (state-preference approach by Kraus and Litzenberger [1973] and Hirshleifer [1966], the mean-variance approach by Rubinstein [1973]) reach similar conclusions.

Later theoretical research by Stiglitz [1974] shows that another advantage of debt financing is the fact that the firm can open up new portfolio opportunities for investors from issuing debt and this allows investors to achieve more desirable portfolio positions than they could before. However, since the valuation of this advantage is not the
objective of the model, it will be ignored.

A firm entering any debt financing agreement will enter into an obligation to make some fixed payments over a certain period and a risk of insolvency is involved in the sense that future cashflows are uncertain and there is a chance that the firm would breach the agreement and default. In this case, bondholders have the right to take over the firm. Thereafter, they can liquidate the firm if the market value of the firm as a going concern falls below its dismantled value, or if this is not the case, they can reorganize the firm. In either case, legal fees, trustee fees and other costs of bankruptcy are incurred. The value of the firm in bankruptcy will be reduced because these costs of bankruptcy must be paid to parties other than the bondholders and stockholders. We assume there is always a chance that the firm cannot meet its payments. Investors will perceive this chance of bankruptcy and they will incorporate the expected value of the perceived bankruptcy costs into the valuation of the firm's securities. In a theoretical world of perfect markets where there are no bankruptcy costs, the transfer of ownership from stockholders to bondholders under default is costless and the the possibility of bankruptcy will have no impact at all on the firm's capital structure decision (Stiglitz [1974]).

3.3.2 The State-Preference Approach (Arrow [1964], Debreu [1959], Hirshleifer [1966])
In evaluating the value of a firm and its recapitalization behaviour in the two period model, we assume uncertain streams of future cashflows over the next two periods. An investment in the firm would lead to different payoffs in different states. In the state-preference model presented later, the uncertainty of cashflows takes the form of not knowing what state of nature will obtain at the end of the first and second periods. There are numerous potential states of nature due to the different future economic and business conditions. Each state corresponds to a particular level of earnings and the states are exhaustive and mutually exclusive. The state of nature captures the fundamental causes of economic uncertainty in the economy; for example, the various states can represent prosperity, normalcy, recession, depression, etc. Once the uncertain state of the world is revealed, the earnings of the firm are determined exactly.

The probability of an earnings level is just the probability of the corresponding state of nature and the sum of the state probabilities equals one. The sum of the product of earnings and the corresponding probabilities is equal to the expected earnings. All the evaluations of the variables in the model are based on this state-preference principle.

There are several key assumptions associated with this approach and they are discussed below.

First, we assume the firm and each investor can relate an outcome from the firm's probability distribution of its end
of period earnings with each state of nature that can possibly occur. Second, we assume that the objective of an investor is to maximize the present value of his wealth and the firm will maximize the present value of the shareholders' wealth. Investors are only concerned about end of period payoffs and shareholders' utility functions are state-independent. Third, as pointed out by Myers [1968], both shareholders and bondholders will choose their portfolios such that their expected utility of future return is maximized. Also, the total expected utility associated with their portfolios is just a linear function of the utility function defined for each state.

Let s represent a state and t represent the time that the state may occur. If P(s,t) is an investor's judgement of the probability of occurrence of contingent (s,t) and U(s,t) is the utility of returns to be received in (s,t), then we can formulate the overall utility of the contingent returns of the firm as:

\[ f = \sum_s \sum_t P(s,t) \cdot U(s,t) \]

Fourth and finally, we assume that shareholders and bondholders are risk neutral, i.e. they have a linear utility function.

The model of recapitalization assumes uncertain firm cashflows over the two periods. The recapitalization decision involves a series of recursive decisions and the model will be solved by a dynamic programming approach. The state-preference approach provides a simple analytic model within which to apply a dynamic programming solution. This
approach allows us to observe clearly the capital structure decision of the firm under different economic and business conditions.

Despite the advantages of the state-preference approach, there are disadvantages associated with it. In a real world situation, it is always difficult to define a set of exhaustive and mutually exclusive states of nature. Incompleteness of the states and their overlapping nature are some obvious problems. However, this disadvantage should not be a concern in our analysis since we are interested in analysing the recapitalization behaviour of the firm and the states of nature are just arbitrarily defined to clarify the recapitalization process.
3.4 Environment and Assumptions of the Model

3.4.1 The Basic Framework

In the two period model, we consider a firm having future earnings at the end of each of two periods. There are uncertainties in the earnings and this takes the form of a number of states of nature, at the end of each of the periods, each having their particular earnings. Each state of nature at the end of period one leads to a number of states of nature at the end of period two.

Figure 2 shows the various states of the two period model represented by a tree diagram. The firm starts out at the point So, the current time and state. All the cashflows levels in the figure are arranged with the smallest at the top. Xi represents the ith level of cashflow at the end of period one. Yij represents the jth level of cashflow at the end of period two given that Xi has already occurred.

States of cashflows in a period are mutually exclusive and exhaustive and we have the following properties:

\[ \sum_{i} P(X_i) = 1 \]
\[ \sum_{j} P(Y_{ij}|X_i) = 1 \]  \hspace{1cm} (2)

Also,
\[ P(Y_{ij}) = P(Y_{ij}|X_i).P(X_i) \]  \hspace{1cm} (3)

Where \( P(X_i) \) is the probability of \( X_i \).
Figure 2
Structure of the Two Period Model

$t=0$
Debt Level = D

$t=1$

Expected bankruptcy costs decrease, tax shield decreases

$t=2$

Expected bankruptcy costs = B, lower tax shield
Expected bankruptcy costs = B, higher tax shield
Expected bankruptcy costs increase, tax shield increases
3.4.2 Capital Market Environment

3.4.2.1 Efficiency of the Capital Market

We have assumed that investors are risk neutral and that their utility functions are state-independent. Their objective is to maximize the present value of expected future payoffs. In attaining this objective, we assume an efficient capital market in the sense that it achieves information-arbitrage efficiency, fundamental efficiency (Tobin [1982]) and allocational efficiency.

In terms of information-arbitrage efficiency, it is assumed that prices follow a random walk pattern and no one can gain on the basis of publicly known information. For example, given that we are at So, no individual investor has superior information concerning the earnings of the firm at the end of the first period. We will assume homogeneous beliefs across investors and the firm manager so that no informational inefficiencies can exist. Also, we assume that the true earnings state is observed by all when that state obtains. Once investors can estimate the state-contingent future earnings of the firm, they can readily determine the optimal recapitalization decision and the corresponding firm values with the assumption that the objective of the firm is to maximize shareholders' wealth.

In terms of fundamental efficiency, we assume that the market prices of the unlevered firm, levered firm, stocks and bonds are correct signals for investors, i.e., they fully
and instantaneously reflect all available relevant information which is publicly known. In other words, the market price of each security is equal to the net present value of all expected future streams of payoffs. Since all investors are assumed to have linear state-independent utility functions, maximizing this net present value is at the same time maximizing their utilities.

In terms of allocational efficiency, we assume that investors and the firm will allocate their funds so as to maximize their net present values. If there is a gain from recapitalization, the firm will proceed with it. In the case where there is a call provision on the bonds of the firm, the firm should call the bonds instead of buying them back in the market if the call price is lower than the economic value.

3.4.2.2 Market Imperfections

One of the functions of the model is to analyse the effect of recapitalization under imperfect market conditions; so taxes, flotation costs and bankruptcy costs are included in the model. One additional assumption will be made. We assume that all participants in the capital markets are price takers. None of the investors have sufficient market power to change the price of the securities.
3.4.3 **Objective of the Firm**

The objective of the firm is to maximize the present value of the wealth of its shareholders. At the beginning of the current period, So in Figure 2, investors pay for the securities of the firm to share the cashflows that will accrue to the firm in the later periods. All the proceeds from debt financing accrue to the existing shareholders who in turn, sacrifice part of their future profit to the bondholders. Since the proceeds of debt financing undertaken by the firm at So accrue to existing shareholders, the objective of the decision-maker at time zero is to maximize the debt proceeds (market value of debt) plus the residual equity value (market value of equity). Thus, the objective of the firm at So is to maximize total firm value, equal to debt plus equity value. At the end of period one, recapitalization is possible and the market price of the debt does not directly impact on shareholders' wealth. Thus, maximizing overall firm value is no longer the objective of the firm; instead, the objective is to maximize the present value of the net proceeds to the shareholders, i.e., the market value of equity.
3.4.4 Characteristics of the Firm

Since our aim is to focus on the recapitalization decision and the related financial position of the firm, the production decision is ignored. The firm is considered as an entity which, by means of its nature, possesses an opportunity to realize some earnings patterns at the end of the next two periods. These earnings patterns are described by the state-contingent approach previously outlined and are independent of the firm's capital structure choices. Thus, the value of the firm is affected only by the value of the stocks and bonds since these represent the expected future payoffs to the investors. The assets are merely providing means to generate the earnings in the next two periods. This implies that we consider only the right hand side of the balance sheet.

The earnings of the firm over the next two periods depend on exogenous economic and business conditions. We will assume that earnings in period two are dependent on the earnings of the previous period. Lower earnings in period one would mean lower expected earnings in the next period.

The outstanding securities of the firm are assumed to consist of only debt and equity. This allows us to concentrate on the effects of the changes in debt and equity in the firm's financial position without making adjustments to other claims. In addition, bonds of the firm consist only of pure discount bonds, that is, the firm promises to pay a fixed amount at a certain date and the market value of the discount bond is just the present value of the future
amount that bondholders expect the firm can pay discounted by the required rate of return of the debtholders. So the market value of the discount bond depends only on the prevailing required rate of return on similar bonds and the ability of the firm to pay off the debt, i.e., the future earnings of the firm. An increase in the market interest rate would reduce the market value of the bonds and a decrease in interest rate would increase the market value of the debt. On the other hand, if the earnings of the firm are high in period one, there is a better chance that the firm will have good earnings next period. In this case, the probability of bankruptcy is reduced, expected bankruptcy costs are lower, and the market value of the bonds is higher. In fact, both the value of the bonds and the value of the stock will increase, or at least remain the same, due to the reduction in expected bankruptcy cost.

We assume that the firm follows a pure residual dividend policy. Earnings in each of the two periods are all earnings before interest and taxes (EBIT). The net amount after bond payments, flotation costs and taxes is distributed to the shareholders as dividends. Bankruptcy costs and flotation costs are assumed to be extraordinary expenses and thus are not included in the earnings before interest and taxes.

Because EBIT is positively serially correlated, the market values of the securities of the firm at the end of period one will incorporate the conditional expected earnings in period two, given the state of nature in period
one, and from the relevant financial decisions made. Residual earnings in period one will be paid out as dividends. Thus the effect of the earnings in period one on the market values of the securities in that same period is not due to the amount of earnings in that period, but to the information content in the earnings in period one.
3.4.5 Other Assumptions

Besides the assumptions on the capital market environment and the characteristics of the firm, several conditions are still required.

It is assumed that the discount rate for bonds is constant. This assumption will avoid confusion of the effect of a change in interest rate on the value of debt with that of a change in capital structure. Similarly, the required rate of return to shareholders is assumed to be constant within the two periods. Again, this assumption avoids confusion in the analysis and simplifies the situation. This assumption along with the assumption of risk neutrality of investors implies that both bondholders and stockholders apply the same discount rates to expected cash flows in both periods one and two. Also, instead of having only the interest expense tax deductible, all the proceeds to the bondholders are assumed to be tax deductible. There are two reasons for making this assumption. First, this avoids the complication arising from allocating the debt repayment between principal and interest payments. Secondly and more importantly, making the whole payment tax deductible amplifies the effect of taxes on financial decisions. Changes in the capital structure will now have a more profound effect on the value of the securities. This allows us to observe the effect of recapitalization more dramatically.

We further need the assumption that the firm can buy or sell its securities in the capital market whenever the
manager desires. In addition, flotation costs and bankruptcy costs are both fixed lump sum amounts. Finally, we have ignored the existence of any agency problem. The model only considers the financial decisions of the firm and not the investment and production problems. We are aiming at the effect of recapitalization on the firm, so it does add to the analysis if we include potential agency problems arising in other areas such as giving up positive net present value projects to exploit the bondholders, agency problems caused by co-ownership, etc.
3.5 Four Cases

There are four cases in the model corresponding to different scenarios and assumptions. However, there are some conditions common to all the four cases. The tax rate is assumed to be constant for all levels of earnings. Also, if the firm cannot meet its obligations at the maturity of the debt, a fixed bankruptcy cost is incurred. Flotation costs arising from the recapitalization are a fixed amount. For simplicity, the probabilities of occurrence of the states in a period are assumed to be equal, i.e., all the states in a period have an equal chance of occurrence. Finally, it is assumed that the personal tax rates of the shareholders are irrelevant to the financial policy of the firm.

In case one, if the firm decides to issue any debt at time period zero, they will issue only bonds with a length of maturity of two periods. Once the firm has determined its optimal capital structure in period zero, it will keep this debt-equity mix until the end of the second period. No change in the debt-equity mix is allowed within the two periods. In this case, the firm will make its capital structure decision based only upon the second period earnings. Earnings in period one are all distributed to the shareholders under the pure residual dividend policy so that those earnings will not contribute to debt repayment at the end of period two. Thus, valuation of debt and the determination of the optimal debt level will only be determined by the second period earnings.
In case two, we consider a firm which issues only pure discount bonds with a length of maturity of one period. The firm has the option to issue one-period discount bonds twice, once at the beginning of period one and again at the beginning of period two. In this case, the size of the first debt issue is affected only by the earnings in period one since the bonds have to be paid off before the second period. Similarly, the amount of debt in the second issue is affected only by second period earnings.

Comparing case one and two, it seems that having a single issue of two-period bonds is inferior to having two issues of one-period bonds over the two periods. If two-period bonds are used, we can only determine the optimal capital structure of the firm by looking at all the earnings states at the end of the second period. However, when the earnings level in period one is revealed the distribution of period two earnings changes (due to positive serial correlation). So investors revise the expected earnings level in period two using the conditional distribution of second period earnings. The original debt-equity mix is possibly no longer optimal. Since the two-period bonds have not matured at period one and recapitalization is not allowed in this case, the optimal mix possibly becomes suboptimal and overall firm value will be below the maximum. There are two harmful effects to the shareholders. First, if the debt-equity ratio becomes too high in period one, there is a higher than optimal probability that the firm will go bankrupt. The higher expected bankruptcy costs will reduce
both the market values of the bonds and stocks. Given the objective of maximizing shareholders' wealth, this result is undesirable. Second, if the debt-equity ratio in period one becomes too low, the firm will have a lower than optimal probability of bankruptcy. Also, it will lose some of the advantage of tax benefits from debt financing. Thus the value of the bonds will increase because of lower expected bankruptcy costs but the value of the stock will decrease since the firm must pay more taxes out of earnings and thus reduces the amount of dividends distributed.

Issuing one-period bonds has the advantage of flexibility. Once the earnings level in period one is revealed, the firm can issue new one-period bonds up to the amount desired. The capital structure of the firm can be adjusted so as to attain the maximum equity value.

Despite the advantages, one-period bonds have the disadvantage of higher flotation costs. Each issue of the bonds requires a fixed amount of flotation costs, so that issuing one-period bonds twice will incur twice the amount of flotation costs as in the case of issuing two-period bonds once. One-period bonds are preferred to two-period bonds only if the value of the added flexibility exceeds the extra flotation costs incurred.

In case three, we consider the case in which the firm first issues two-period debt, but the firm has the option to recapitalize at the end of the first period. Once the earnings level of period one is revealed, the firm can adjust its debt-equity mix so as to attain the maximum
equity value.

Cases two and three are similar in the sense that both allow recapitalization at the end of period one. However, in case two, a new debt issue at t=2 is always necessary with the attendant flotation costs. The firm will start from a zero debt at the end of period one and it is very likely that the gain in equity value from issuing new debt will exceed the flotation costs incurred. As a result, part of the wealth of the shareholders will be lost due to the extra expense of flotation costs. In case three, two-period debt is issued and the firm does not start from a zero debt level at the end period one. Thus, in some cases, the increment in equity value resulting from recapitalization may not be able to justify the extra flotation costs and recapitalization is not optimal. Thus two-period debt with a recapitalization option allows more flexibility and avoids mis-allocation of the resources of the firm to perform unnecessary recapitalization every period.

In the final case of the model, we examine a case similar to case three, however, a call provision is added to the bonds. The firm is allowed to call the bonds outstanding at the end of period one and repurchase the bonds at a predetermined call price. Of course, the firm will call the bonds only if the call price is below the market price of the bonds. We add a callable clause to the

1Except in the case of an unlevered capital structure; however, this is suboptimal since the tax advantage is very large.
analysis to allow us to observe the effect of this provision on the recapitalization decision. Actually, having the callable clause is analogous to having a call option and it can be evaluated as such.

All four cases show a pattern of progressive complexity. Case one represents the simplest case with two-period debt outstanding. The one-period debt feature in case two includes an added feature of almost compulsory recapitalization at the end of period one. As we proceed to case three, more flexibility is allowed in the sense that recapitalization now involves an option feature in the equity value maximizing decision. Finally, a call provision is added in case four. This progressive complexity across the four cases allows us to isolate each factor separately so that we can observe the effect of each of the factors on the financial decision of the firm and the valuation of the firm's securities.
4. MODEL LAYOUT

4.1 Debt Level Determination

Theoretically, there are infinite number of debt levels which the firm can take on. However, assuming a finite number of possible earnings levels in any period indicates that only some of these levels need to be studied in the capital structure decision of the firm.

All the states of cashflows in Figure 2 are arranged such that $X_a < X_b$ for $a < b$ and $Y_{ia} < Y_{ib}$ for $a < b$. Let $D_0$ represent the debt level that the firm pursues at $t = 0$ such that $Y_{ia} < D_0 < Y_{ib}$. If $Y_{ia}$ obtains at the end of period $t$, the firm cannot meet the payments to bondholders and goes bankrupt. Bankruptcy costs of $B$ are incurred and this is paid out of the cashflows $Y_{ia}$ before the bondholders and stockholders receive any payments. As long as $Y_{ia} < D_0 < Y_{ib}$ bankruptcy costs of $B$ will be incurred and this is true for any the cashflow levels between the two bounds. When $D$ is less than $Y_{ib}$, both the probability of bankruptcy and the expected bankruptcy costs will not change. On the other hand, the larger $D$ is, the larger the tax shield\(^1\) will be. By keeping $Y_{ia} < D_0 < Y_{ib}$, we can achieve a larger tax

\(^1\)Assuming a constant tax rate of $t_c$ for the firm, since the whole issue of $D$ is tax deductible, tax paid to the government will be reduced by $t_c D_0$. 

shield with a higher D while on the other hand keeping the expected bankruptcy costs constant. So the only way to realize the largest tax shield while keeping bankruptcy costs constant is to have $D = Y_{ib}$, i.e. the debt level at $Y_{ib}$ is superior to all the other debt levels between the range $Y_{ia}$ and $Y_{ib}$.

Now suppose that the cashflow level $Y_{ib}$ obtains instead of $Y_{ia}$. $D = Y_{ib}$ is still the best debt level within the range $Y_{ia}$ and $Y_{ib}$ since it gives the largest tax shield without increasing expected bankruptcy costs.

From the above arguments, we can see that the relevant debt levels for the firm are those which equal the possible cashflow outcomes. This implies that the capital structure decision of the firm involves examining a finite number of debt levels. The amount of debt remains constant within a range of cashflow levels and then jumps to another level after a certain cashflows level is attained. As a result, we consider only those debt levels which equal the cashflow levels."

4.2 Definition of Variables

Before the different cases are discussed, the notation of the model needs to be defined. $X_i$ and $Y_{ij}$ were defined earlier. In the following, the subscript $i$ is used when $D_1 = X_i$ and the subscripts $i,j$ are used together when $X_i$ has obtained and $D_1 = Y_{ij}$.

$D_0, D_1 =$ the face value of debt outstanding, due at $t = 1$ and $t = 2$.

$P_{Di}, P_{Di} =$ the payoff to the bondholders at the end of periods 1 and 2 respectively.

$P_{Ei}, P_{Eij} =$ similar as above, but to shareholders.

$V_{Do}, V_{Di} =$ the market value of debt at the end of periods 0 and 1 respectively.

$V_{EO}, V_{Ei} =$ as above, but for value of equity.

$V_{Lo}, V_{Li} =$ as above, but the sum of debt and equity values.

$V_U =$ unlevered firm value at $S$.

$DIV_1 =$ the amount of dividends distributed at the end of period one.

$W_0 =$ the total wealth of shareholders at $S$. Our objective is to maximize $W_0$.

$W_1 =$ the wealth of shareholders at the end of period 1.

$RECAPCF =$ the extra cash inflow or outflow to equityholders as a result of recapitalization.

$CP =$ call price for the bonds in case IV.

$m =$ number of states at the end of period one.
n = number of states in period two serially dependent of each state in period one.

\( P(X_i), P(Y_{ij}) \) = the probability of occurrence of \( X_i, Y_{ij} \).

F = floatation costs applying to any buying or selling of debts.

B = bankruptcy costs.

R = \( 1/(1+\text{discount rate}) \).

T = corporate tax rate.
4.3 Case I

In this case, the firm issues two-period pure discount bonds which mature at the end of the second period and no recapitalization at t = 1 is allowed. In order to determine the optimal capital structure, each of the relevant debt levels' is evaluated and the one that maximizes shareholders' wealth is the optimum.

In Figure 3, Do is the debt level at t = 0. If the cashflow level at the end of period 2 is less than D, the firm goes bankrupt and bankruptcy costs of B are incurred. Bondholders receive the residual after bankruptcy costs if cashflows are larger than B and receive nothing if cashflows are less than or equal to B. On the other hand, if cashflows are larger than Do, the bondholders receive their full payment. For the shareholders, if cashflows are less than D, the firm goes bankrupt and they receive nothing. If the firm does not go bankrupt, shareholders receive the residual amount after the debt and tax payments, i.e.

\[
P_{Dij} = \begin{cases} 
  Do & \text{if } Y_{ij} > Do \\
  \max(Y_{ij} - B, 0) & \text{otherwise} 
\end{cases} 
\]  

\[
P_{Eij} = \begin{cases} 
  (Y_{ij} - Do)(1-T) & \text{if } Y_{ij} > Do \\
  0 & \text{otherwise} 
\end{cases} 
\]  

Equations (4) and (5) give the payoffs to bondholders and shareholders for each state in period two. In order to find

'As discussed above, the relevant debt levels are those which equal the cashflow levels.
Figure 3
Case 1

$t=0$

Issue 2-period bonds, issuing costs $F$. Debt level = $D_0$

$t=1$

Capital structure not allowed to change

$t=2$

$P(Y_{ij}) = 1/(m \cdot n)$

All cashflows here accrued to shareholders, none to bondholders.
the values of the bonds and stock at time zero, we need to
determine the expected values of payoffs and the
corresponding present values at \( t = 0 \), but with some
adjustments on the value of equity. Flotation costs are
incurred at time zero when the two-period bonds are issued.
The costs are paid out of the shareholders' wealth and this
reduces the value of equity. These costs are assumed to be
written off as the bonds are issued and the firm does not
receive any tax shield from these since there are no
cashflows at \( t = 0 \). Also, the cashflow at the end of the
first period is paid out as a dividend and thus the present
value of equity value increases accordingly. So,

\[
V_{Do} = \sum_{i=1}^{m} \sum_{j=1}^{n} PD_{ij} \cdot P(Y_{ij}) \cdot R^2
\]

(6)

\[
V_{Eo} = \sum_{i=1}^{m} \sum_{j=1}^{n} PE_{ij} \cdot P(Y_{ij}) \cdot R^2
\]

(7)

\[
- F + \sum_{i=1}^{m} X_i (1-T) \cdot P(X_i) \cdot R
\]

We can calculate the value of the bonds and the value of
the stock for a given debt level. At time zero, the firm
possesses an opportunity for investors to share the future
cashflows of the firm. The production and assets of the firm are assumed fixed, so no new investment in real assets will be undertaken. Bondholders share the future cashflows of the firm when they purchase the bonds of the firm. All the proceeds from selling bonds, i.e. the market value of the bonds at time zero, belong to the shareholders because they have sacrificed part of their future cashflows. For this reason, shareholders' wealth at time zero should be equal to the sum of the equity value and debt value, i.e.

\[ W = VL = VD + VE \]

In order to maximize the wealth of the shareholders, we need to determine VL for each of the appropriate debt levels. The debt level D which gives the maximum VL is the optimal debt level.

The unlevered value of the firm is its market value if all equity financed. Theoretically, this reflects the present value of the after-tax expected cashflows within the two periods. This value is independent of the amount of debt financing pursued by the firm and thus \( VU \) should be constant as long as the expected cashflows are unaltered.

In a perfect market where there are no bankruptcy costs, flotation costs and taxes, the value of the levered firm and the unlevered firm should be the same, i.e. capital structure is irrelevant. In an imperfect market like the one we have, the unlevered firm value and the levered firm

\[ \text{the appropriate debt levels are possible cashflow levels at } t=2, \text{ i.e. } Y_{ij}, i=1,m \text{ and } j=1,n. \]
value should be different. The unlevered firm value is reduced by the present value of the after-tax expected bankruptcy costs and increased by the present value of expected tax shields realized by the firm.
4.4 Case II

In case two, the firm issues one-period bonds at the beginning of periods one and two. In order to determine the optimal debt level and the corresponding debt and equity values at $t=0$, we need to determine the optimal debt levels pursued at $t=1$ under different cashflow levels of period one. This is because the equity value at $t=0$, $V_{E_0}$, should include the present value of expected dividends and the present value of the firm value at $t=1$. In Figure 4, we can see that the amount of debt issued at $t=1$ will vary according to the level of cashflow $X_i$, $i=1$ to $m$. If the cashflow level $X_a$ obtains at $t=1$, the only possible cashflow levels at $t=2$ are $Y_{aj}$, $j=1,n$. Let $D_1$ be the face value of debt that the firm issues at $t=1$. If $Y_{aj} > D_1$, bondholders receive their full payment and shareholders receive the residual after bond payments and taxes. On the other hand, the firm goes bankrupt if $Y_{aj} < D_1$. Shareholders are left with nothing and bondholders receive the residual amount, if there is any, after bankruptcy costs.

Each of the relevant debt levels are evaluated and $D_1$ becomes the optimal debt level given $X_i$ obtains if it gives rise to the maximum shareholders' wealth. In this case, expressions for $PD$ and $PE$ are the same as in equations (4)
Case II

Figure 4

$t=0$
Issue 1-period bonds, issuing costs $F$.
Debt level = $D_0$

$t=1$
Issue 1-period bonds, issuing costs $F$.
Debt level = $D_1$

$t=2$

$P(X_1) = 1/m$
$P(Y_{ij}) = 1/(m.n)$
$P(Y_{ij}/X_1) = 1/n$

All cashflows here accrued to shareholders, none to bondholders.
and (5). And for a debt level \( D_1 = Y_{aj} \),

\[
V_{Da} = \sum_{j=1}^{n} PD_{aj}P(Y_{aj}|X_a)R
\]  

(8)

\[
V_{Ea} = \sum_{j=1}^{n} PE_{aj}P(Y_{aj}|X_a)R - F
\]  

(9)

Now we move backward in time to \( t = 0 \) in Figure 4. The firm issues one-period bonds \( D_0 \) at \( t = 0 \) and the market value of this issue depends only on the cashflows levels at \( t = 1 \). If the cashflow obtained is \( X_i > D_0 \), the bondholders are paid with the full amount \( D_0 \) and shareholders receive the residual amount as dividends after the payments to bondholders and taxes. In addition, shareholders are entitled to the present value of the expected market value of the firm at \( t = 1 \) since there are more cashflows coming in at the end of the second period. So the market value of equity at \( t = 0 \), \( V_{E0} \), includes the present value of the expected dividends and the present value of the firm value at \( t = 1 \) less the amount of issuing costs \( F \).

On the other hand, \( X_i < D_0 \) would lead to bankruptcy of the firm. Bankruptcy costs of \( B \) are incurred and

\[ \text{The present value of the expected t = 1 firm value as of t = 0 is given by:} \]

\[
\varphi = \sum_{i=1}^{m} (\text{optimal VLi}).P(X_i).R
\]

(10)

where \( V_{Li} = V_{Di} + V_{Ei} \).
bondholders receive the residual amount, if there is any, after bankruptcy costs. Shareholders receive nothing since Xi has been paid out as bankruptcy costs and bond payments. When the firm goes bankrupt, shareholders can no longer receive the cashflows at t = 2. Bondholders take over the firm and inherit the second period cashflow. Again, the criterion of maximizing shareholders' wealth is equivalent to maximizing the overall market value of the firm. Each of the relevant debt levels are evaluated with the above analysis and the largest VL is the optimum. So for a debt level Do = Xi,

\[ VD = \sum_{i=1}^{m} PD_i P(X_i) R + \sum_{i=1}^{m} I_i \sum_{j=1}^{n} P(Y_{ij}) R \]  

(11)

where \( I_i = 0 \) if no bankrupt in state \( i \)

1 if bankrupt in state \( i \)

\[ VE = [\sum_{i=1}^{m} P E_i P(X_i) R + J_i (optimal VLi) P(X_i) R] - F \]  

(12)

where \( J_i = 1 \) if no bankrupt in state \( i \)

0 if bankrupt in state \( i \)
4.5 Case III

In this case, the firm issues two-period bonds at \( t = 0 \) and it has the option to buy back the bonds issued and/or issue new one-period bonds at \( t = 1 \). The firm has more flexibility in its financing choice and this allows it to alter its capital structure as needed. A large cashflow obtained at \( t = 1 \) would mean high probability of large cashflow at \( t = 2 \). On the other hand, low cashflow this period would imply higher probability of low cashflow next period. Thus at \( t = 1 \), the firm has a revised probability of future cashflows and recapitalization may be necessary to maximize shareholders' wealth.

In evaluating debt and equity at \( t = 0 \), we face the uncertainty of not knowing the recapitalization decision at \( t = 1 \) because cashflow levels in period one have not yet been observed. One of the best ways to approach this problem is to anticipate the expected recapitalization decision based on the objective of maximizing the present value of expected total wealth of shareholders. Each relevant debt level is evaluated with this expectation and the one which gives rise to the largest shareholders' wealth is the optimum.

Let \( D_0 \) be the face value of debt the firm issues at \( t = 0 \) and \( X_a \) be the cashflow level obtained at \( t = 1 \). Given this cashflow level, we know that only \( Y_{aj}, j = 1,2,\ldots,n \), can possibly occur at \( t = 2 \). In Figure 5, we can see that the firm faces two choices at \( t = 1 \), recapitalization or no recapitalization, and we need to determine the present value...
Figure 5
Case III

Recapitalization allowed.
Option to buy back bonds issued at $t=0$
and issues new 1-period bonds at a cost $F$.

Cashflow net of tax and floatation costs is
accrued to shareholders.
of the total wealth of shareholders under each choice.

If there is no recapitalization, the debt level would remain at $D_0$. At $t = 1$, shareholders' wealth includes the expected after-tax dividends at $t = 1$ and the value of equity at $t = 1$. The market value of debt is no longer part of the wealth of shareholders at $t = 1$ because, unlike the previous cases, bonds outstanding at $t = 1$ were issued at $t = 0$ and the proceeds accrued to shareholders at $t = 0$. The expected after-tax dividends are just the after-tax period one cashflows. The value of equity at $t = 1$ is affected by the cashflow levels at $t = 2$ and is evaluated the same way as described in case II. So, given that $X_a$ occurs, the wealth of the shareholders is:

$$W_1 = (1-T)X_a + VE_a(D_0)$$ (13)

The above wealth of shareholders is compared with the case where there is recapitalization. Again, we are assuming that $X_a$ has occurred. If the firm recapitalizes, $D_0$ is changed to $D_1$ and recapitalization costs of $F$ are incurred. We can go back to Case II to determine $D_1$. It is determined exactly the same way as the optimal debt levels at $t = 1$ in case II because they both have the condition that cashflows level $X_a$ has obtained. This gives the values of equity and debt corresponding to the new optimal $D_1$.

$D_1$ is considered as a new issue of bonds which mature at the end of the second period. The firm buys back the old debt issue $D_0$ in the capital market and issues $D_1$ of one-period bonds. Since there is a difference between the market values of the old issue and the new issue, the firm
will have a positive or negative recapitalization cashflow and this accrues to shareholders. So the total wealth of the shareholders at \( t = 1 \) includes the after-tax and after flotation-cost dividends, the value of equity, given the new optimal debt and recapitalization cashflow, i.e.

\[
W_1 = (1-T) \cdot (X_a - F) + \text{RECAPCF} + \text{VEa(D1)}
\]  

(14)

where \( \text{RECAPCF} = \text{VDa(D1)} - \text{VDa(Do)} \)

The wealth of shareholders under the above two decisions is compared and the firm will recapitalize only when that decision generates a larger shareholders' wealth. We can see from Figure 5 that this decision and the corresponding wealth are conditional on \( X_a \), thus we need to similarly determine shareholders' wealth for all the possible cashflows at \( t = 1 \), i.e. for \( X_i, i=1,2,...,m \).

The above analysis will give the optimal decisions and the maximum shareholders' wealth for each cashflows levels obtained at \( t = 1 \). The next step is to determine the wealth of shareholders at \( t = 0 \).

If the firm recapitalizes at \( t = 1 \), the bonds will be repurchased from the bondholders; if there is no recapitalization, the bond value to the bondholders is again the market value of the bonds. Thus, the value of the bonds at \( t = 1 \), and hence also at \( t = 0 \), is independent of the firm's recapitalization at \( t = 1 \). So in either case, bondholders are entitled, at \( t = 1 \), to the market value of their bonds. The market value of the old debt issue at \( t = 1 \) given that \( X_a \) has obtained is just the present value of
expected payoffs to bondholders. This can be evaluated using the methodology of Case II. At \( t = 0 \), the market value of debt given debt level \( D_0 \) is just the discounted expected value of the market value of old debt at \( t = 1 \).

\[
V_{D0} = \sum_{i=1}^{m} V_{Di} P(X_i) R
\]

The value of equity at \( t = 0 \) depends only on the total payoff that is accrued to shareholders at the end of the first and second period. This is equivalent to the wealth of shareholders at \( t = 1 \) under the optimal recapitalization decision. So the value of equity at \( t = 0 \) is the discounted expected value of shareholders' wealth at \( t = 1 \) under the optimal recapitalization decision.

\[
V_{E0} = \sum_{i=1}^{m} W^* P(X_i) R
\]

\( W^* = \) shareholders' wealth at \( t = 1 \) under the optimal recapitalization decision.

Now we have the values of debt and equity at \( t = 0 \). The total shareholders' wealth at \( t = 0 \) is just the sum of these values, i.e. the levered firm value because the market value of bonds at \( t = 0 \) accrues to shareholders.

\[1\] As in Case II,

\[
V_{Di} = \sum_{j=1}^{n} P_{Dij} P(Y_{ij}) R
\]
So far the analysis is done for a given debt level pursued at \( t = 0 \). In order to find out the optimal debt level, each of the relevant debt levels with value equal to the second period cashflows should be substituted into the analysis. The one which maximizes total shareholders' wealth at \( t = 0 \) is the optimum.
4.6 Case IV

Case IV is similar to case III except that a call provision is introduced in the bonds. We can use the same approach as above but with a minor alteration. Figure 6 shows the situation of Case IV. When the firm recapitalizes in case III, it buys back the old debt in the capital market. With a call provision now, the firm will, as it recapitalizes, call the old debt if the call price is below the market price. The firm will exercise the call provision only when market price is higher than call price and the resulted increment in shareholders' wealth is higher than the after-tax flotation costs. If the firm recapitalizes, there is a positive effect on recapitalization cashflow:

\[
\text{RECAPCF} = VD(D1) - \text{MIN}[CP, VD(Do)]
\]  

(15)

With this revised recapitalization cashflow, the process of determining the optimal recapitalization decision is the same as in Case III. However, the value of debt is different from that in Case III. In Case III, the value of debt is just the present value of the market value of debt at \( t = 1 \) and this is independent of the recapitalization decision. Now the value of debt is dependent on the recapitalization decision because when the firm decides to recapitalize and the market value of bonds at that time is higher than the call price, bondholders of the old issue will receive an amount less than the market value.

- Bondholders should anticipate this at \( t = 0 \) and the market price of bonds at \( t = 0 \) should reflect this. Thus, the
Figure 6

Case IV

$t=0$

$X_1$

$X_2$

$t=1$

recapitalize

$X_1$

$X_m$

$t=2$

call price < market price,
call the bonds

$Y_{i1}$

$Y_{i2}$

$Y_{ij}$

$Y_{in}$

call price > market price,
buy bonds in the market

$Y_{i1}$

$Y_{i2}$

$Y_{ij}$

$Y_{in}$

no recapitalize

$Y_{i1}$

$Y_{i2}$

$Y_{ij}$

$Y_{in}$
value of debt at \( t = 0 \) is given by:

\[
V_{Do} = \sum_{i=1}^{m} V_{Di} \cdot P(X_i) \cdot R
\]  

(16)

where \( V_{Di} \) = market value of debt at \( t = 1 \)

if decision = no recapitalize

= market value of debt at \( t = 1 \)

if decision = recapitalize and

market value of debt < call price

= call price if decision = recapitalize

and market value of debt > call price
5. **COMPUTER SIMULATION OF MODEL**

In order to interpret the results of the model, we need to simulate the model with a set of numbers which represents the different cashflow states in periods one and two. For this purpose, the four cases described above are simulated with WATFIV programs. With these programs, we can determine with ease the optimal debt levels and the corresponding debt and equity values. Also, we can see how the value of the firm changes in the four cases as the debt level changes. Larger numbers of cashflow states can be simulated without extensive manual calculations. Also, sensitivity analysis can be performed readily without performing repetitive manual work.

The four cases are shown in Appendices 1a to 1d. A numerical example with cashflow levels shown in Figure 7 is run through the programs and the results are shown in Appendix 1f.
Figure 7
Numerical Example for Simulation Programs

tc=0  K=10%  B=3500  F=1800

D = 7368
VL=2227
VD=1440
VE=787

D = 2376
VL=2227
VD=1440
VE=787

D = 8537
VL=6274
VD=5338
VE=936

D = 14375
VL=11651
VD=10453
VE=1197

0
2376
6704
4091

7368
4042
8537
3684

12670
9246
14375
20961
6. INTERPRETATION OF RESULTS

As mentioned earlier in the section on 'Debt Level Determination', debt levels equal to cashflow levels are superior to all other possible debt levels. For two adjacent states of cashflow, if the debt level is equal to a value inbetween the two states, an increase in debt level will increase the present value of the expected tax shield while keeping the expected bankruptcy costs constant. This trend continues until the debt level reaches the next state of cashflow. Thus our analysis is based on discrete debt levels and the graph plotting firm values against debt levels should show discrete points. Anything in between the points is suboptimal for the firm's capital structure decision. However, for the purpose of clarification, the graphs hereafter will have all the points connected. The following sub-section will analyse the tradeoff of bankruptcy costs and tax shield of debt under the discrete nature of the model and thus resulting a condition for an optimal capital structure. Then the later sub-sections will analyse the simulated results in detail. Finally, this section ends with the discussion of two important implications of the model.
6.1 Optimal Capital Structure in Discrete Cashflow States

Without bankruptcy states, an increase in debt level from \( Db = Yab \) to \( D(b+1) = Ya(b+1) \) would lead to an increment in the present value of expected tax shield equal to \( tc.[D(b+1) - Db].R^2 \). However, the situation is different when there are bankruptcy states. When the face value of debt increases from \( Db = Yab \) to \( D(b+1) = Ya(b+1) \) there is one more state of cashflow, \( Yab \) in which the firm will go bankrupt. If \( Yab \) actually obtains, the firm goes bankrupt and incurs bankruptcy costs \( B \) which could have been avoided if the debt level was \( Db \). In this case, the total present value of the tax shield realized by the firm in the state \( Yab \) is just \( tc.Yab.R^2 \) instead of \( tc.Db.R^2 \). Part of the tax shield is lost because the firm does not have large enough cashflow to realize all the tax benefits. So the amount of tax shield contributed to the increment in firm value is just the portion of it which the firm can actually realize.

For each state of cashflow smaller than \( Ya(b+1) \), only \( tc.Yij.R^2 \) of tax shield can be realized instead of \( tc.D(b+1).R^2 \). Thus an increase in face value of debt cannot increase the tax shields in those states. Only those states equal to or larger than \( Ya(b+1) \) can realize an increase in tax shields resulted from increasing \( Db \) to \( D(b+1) \). In general, the increment in the present value of expected tax shields resulting from increasing the debt level from \( Db = Yab \) to \( D(b+1) = Ya(b+1) \) is not \( tc.D(b+1).R^2 - tc.Da.R^2 \) (or \( tc.Ya(b+1).R^2 - tc.Yab.R^2 \)), but as follows:

\[
m \quad n
\]
\[ \Delta PV[E(TS)] = tc[D(b+1)-Db] \sum_{i=a}^{m} \sum_{j=b+1}^{n} P(Y_{ij})R^2 \]  
where \( P(Y_{ij}) = \frac{1}{m,n} \)  

Using the same example as above, increasing the debt level to \( D(b+1) = Ya(b+1) \) would mean one more state of bankruptcy, \( Yab \), and bankruptcy costs \( B \) are incurred. Thus in general, the increment in the present value of expected after-tax bankruptcy costs resulting from an increase in debt level from \( Db = Yab \) to \( D(b+1) = Ya(b+1) \) is  
\[ \Delta PV[E(B)] = (1 - tc)B \cdot P(Yab) \cdot R^2 \]  
where \( P(Y_{ia}) = \frac{1}{m,n} \)  

Equations (17) and (18) above show the effect of tax benefit from debt financing and bankruptcy costs on the levered firm value as we increase the debt level to the next higher relevant level. Thus the net effect of the two equations will show whether an increase in debt level would increase or decrease the levered firm value.  

Figure 8 shows an example of the arguments described above. The solid lines represent the value of the firm at various debt levels including those irrelevant debt levels with values in between the states of cashflow. The values of the firm at \( D = Yij \) are connected with the dashed line. The slopes of the dashed line show the rate of change of the firm value as the debt level changes its value from one cashflow level to the next. This dashed line only shows whether the firm value has increased or decreased as debt changes from one state of cashflow level to the other. If
Figure 8
Capital Structure in Discrete Cashflow States

- All vertical distances between the highest point of a solid segment to the lowest point of an adjacent segment are the same.

The diagram illustrates the concept of capital structure in discrete cashflow states, showing the relationship between variables VL or W and Y1, Y2, Yij, and Ymn. The optimal point is marked as OPTIMAL, and the floor value is indicated as FLOOR VALUE. The term NON-CONCAVITY highlights areas where the cashflow structure is non-convex.
we increase the debt level from \( D_b = Y_{ab} \) to \( D(b+1) = Y_{a(b+1)} \), the levered firm value will increase or decrease depending on the net of equations (17) and (18):

\[
\Delta V_L = \frac{\{\Delta PV[E(TS)] - \Delta PV[E(B)]\}}{[Y_{a(b+1)}] - Y_{ab}} =
\]

\[
\{tc.(Y_{ab+1}-Y_{ab})\sum_{i=a}^{m} \sum_{j=b+1}^{n} Y_{ij}.P(Y_{ij}).R^2\}
- (1-tc).B.P(Y_{ab}).R^2} / [Y_{a(b+1)}] - Y_{ab}] (19)
\]

The effect of the above value on the capital structure decision is summarized as follows:

- **slope of dashed line** optimal decision
  - \( \Delta V_L > 0 \) positive increase debt
  - \( \Delta V_L = 0 \) zero stay, optimal point
  - \( \Delta V_L < 0 \) negative decrease debt

The solid segments in Figure 8 show the value of the firm when for all debt levels, including those suboptimal debt levels. Only the highest point of each segment will be observed in the firm's capital structure decision because these points are optimal (debt levels equal to cashflow levels). Suppose the debt level starts out at \( D_b = 1 + Y_{ab} \) and begins to increase as shown in Figure 8. Expected bankruptcy costs remain the same while the tax shield is increasing. This situation exists until it reaches \( Y_{a(b+1)} \). Once the debt level exceeds \( Y_{a(b+1)} \), the present value of expected bankruptcy costs increase by an amount equal to
that of equation (18). So there is a sudden drop in the levered firm value at $Y_{a(b+1)}$. As shown in equation (18), this drop in firm value depends on the bankruptcy costs and the probability of occurrence of that state. The larger they are, the larger will be the drop in firm value. Since in our model we have constant bankruptcy costs and equal probability of occurrence in all states, the drop is constant between all the segments in Figure 8.

The slopes of the segments are just the marginal increment of tax shield in the segments and these should be constant for each segment. By modifying equation (17), the slopes are given by:

$$
tc. \sum_{i=a}^{m} \sum_{j=b+1}^{n} P(Y_{ij})R^2
$$

(20)

We can see from the above equation that as debt level increases, the term inside the summation sign decreases and the slope of the solid lines decreases. However, these lines can never have a negative slope. The minimum is zero when the firm increases its debt level into an amount which exceeds the highest possible state of cashflow. In this case the firm will go bankrupt in all the states and value of equity becomes zero. On the other hand, the value of debt cannot be increased with further increases in debt because bondholders are now entitled to the residual
cashflow after paying the certain bankruptcy costs\(^1\) and this residual amount cannot be increased with higher debt levels. Now, for an increase in debt level, the minimum increase in tax shield is zero and there is a point where the expected bankruptcy costs cannot be increased further. So even though the debt level may increase indefinitely, there is always a floor limit to the levered firm value, and thus shareholders' wealth, as long as the firm cannot obtain an infinite amount of cashflow. This minimum amount is shown in Figure 8 as a horizontal line at the end of the VL line (or shareholders' wealth in cases I, III and IV) and both the dashed line and the solid segments coincide here. This floor value is just the expected residual amount, after paying the bankruptcy costs given in equation (21), when the firm goes bankrupt in all the states, i.e.

\[
\text{Minimum VL(or W)} = \sum_{i=1}^{m} \sum_{j=1}^{n} \text{MAX}[0, Y_{ij} - B] \cdot P(Y_{ij}) \cdot R^2
\]  

(22)

\(^1\) Now the firm goes bankrupt in all states and the present value of the after-tax expected bankruptcy costs is given by:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} (1 - tc) \cdot P(Y_{ij}) \cdot R^2 \cdot \text{MIN}[B, Y_{ij}]
\]  

(21)

This amount is the maximum that the firm is expected to pay.
6.2 Basic Properties of the Simulated Results (Case I)

The two period model used here provides the traditional result of a concave firm value function. This can best be shown using the computer simulation on Case I. Figure 9 shows the simulated result of Case I using the states of cashflow in Figure 6. We can see that the levered firm value (which is the same as shareholders' wealth in this case) produces a roughly, though not strictly, concave function as it is plotted against the face value of debt. This result is consistent with the traditional tax and bankruptcy cost approach. From equation (17), we can see that at low debt levels, a and b in the equation are small and the increment of tax shield resulting from an increase in debt level is large. As the debt level increases, the quantity given by equation (17) becomes smaller. On the other hand, equation (17) shows that the increment of bankruptcy costs remains constant for all debt levels. Thus at a low level of debt, an increase in debt level leads to a large increment in tax shield but a constant increment in expected bankruptcy costs and thus an increase in the levered firm value as debt level increases. However, as the debt level becomes larger and larger, the increment in firm value becomes smaller because the quantity in equation (17) has decreased while the quantity in equation (18) remains the same.

Although the line of VL in Figure 9 shows a roughly concave pattern, there are some segments, as marked by asterisks, that are is not strictly concave. However, this
Figure 9

CASE I RESULT

Legend

△ VL₀
× VD₀
□ VE₀
is not a violation of the result of the traditional tax plus bankruptcy costs approach. These points are just the result of the discreteness in the cashflow levels. In equation (17), suppose we are considering to increase the debt level from $D_b = Yab$ to $D(b+1) = Ya(b+1)$. If $D(b+1) - D_b$ is large, the quantity in equation (17) will be large. On the other hand, the increment in expected bankruptcy costs given by equation (18) is independent of $D(b+1) - D_b$. Thus we have a situation which a sudden large increase in expected tax shield resulting from a large increase in debt level has more than offset the constant increment in bankruptcy costs. Non-concavity will follow when we further increase the debt level to $D(b+2)$ such that $D(b+2) - D(b+1)$ is much smaller than $D(b+1) - D_b$. The resulting incremental tax shield cannot offset the constant increment in expected bankruptcy costs. So from $D_b$ to $D(b+2)$ we have first an increase and then a decrease in firm value and this gives rise to the non-concavity on the VL line.

So in general, a violation of concavity occurs whenever there exist $D_b$, $D(b+1)$, $D(b+2)$ such that the two gaps between these three cashflow levels have a large difference and $D(b+1)$ is not the optimal debt level. The non-concavity could be eliminated by assuming constant intervals of cashflow states. However, the two period model is still valid in the sense that the non-concavity will disappear with an infinite number of states of cashflow such that the states become continuous. With the assumption of equal probability of occurrence in the model, a continuous
function is analogous to regular intervals between cashflow states with the states very close one another.

From Figure 9, we can see that the value of debt increases as the face value of debt increases and the value of equity decreases as the face value of debt increases. Again, these are just the general trend of the VD and VE lines. These two lines are not monotonically increasing or decreasing. The peaks and troughs on the lines are due to the irregularity of ranges between debt levels as discussed above. VD has a upward sloping trend because the addition of debt will increase the future claim of bondholders on the firm's cashflow. As D increases, the increment in VD for each debt level increment decreases. Equation (17) has already provided a quantitative treatment of this. We can see in Figure 9 that VD levels off at the largest two debt levels. In fact, the line of VD will start to have a zero slope if D is increased beyond the highest cashflow $20961. In this case, the firm will go bankrupt in all the states and any increase in D cannot increase the payoff to bondholders.

The same arguments can be applied to the VE line which decreases as debt level increases, but levels off at very high debt levels and finally becomes 2587 when Do = 20961. At this debt level, the firm goes bankrupt in all the states and the full amount of period two cashflow is distributed as either bankruptcy costs or bond payment. Thus the value of equity consists only of the expected value of period one dividends which is $2587 in our example.
6.3 Comparison of Cases I and II

Case I is a typical base case with properties that are present in the other three cases. Instead of considering each case individually, we should analyse the effects of the added features in the latter three as compared to the base case. Comparing the levered firm values of Case I and Case II, we can see that Case II has higher values than Case I in general. Case II has the advantage of flexibility over Case I: debt level will adjust to revised beliefs about $t = 2$ cashflows. However, Case II has the disadvantage of incurring two flotation costs rather than one in Case I. In this example, the former effect outweighs the latter, and Case II values are higher. In comparing Case II with other cases, we have to bear in mind that debt levels in Case II are equal to the period one cashflow levels while debt levels in other cases are equal to the period two cashflow levels. Since any debt levels not equal to the end of period cashflow levels are irrelevant in the firm's capital structure decision, comparing Case II values with that of the other cases may produce misleading interpretations. This is the reason why some firm values of Case II are lower than the corresponding values in Case I. At these points, the debt level may be optimal in Case I but suboptimal in Case II.
6.4 Interpretation of the Effects of Recapitalization

6.4.1 Recapitalization as a Call Option

From Figure 10, we can see that the levered firm values for Case III are higher than those of Case I for all the debt levels. This result can be attributed to the fact that in Case III the firm can recapitalize, as necessary, at the end of period one. The firm recapitalizes when the resulting increment in shareholders' wealth is larger than the flotation costs outlay. The option to recapitalize is analogous to a call option which the firm can exercise this option when the gain in wealth is larger than the call price. The call price is analogous to the flotation costs of recapitalization and the gain in wealth in just the resulting increment in shareholders' wealth. Thus the recapitalization feature provides the firm with a tool to realize upside gain potentials. The increment in shareholders' wealth in Case III represents the values of the analogous call option at different debt levels. The value of this option is:

\[ \text{RECAPCF} + \text{VL1(D1)} - \text{VL1(Do)} - (1 - tc) \cdot F \cdot R \]  

(23)

This added value of recapitalization benefits only the shareholders and not the bondholders. As explained earlier, at \( t = 0 \), bondholders are entitled to the present value of the period one market value of bonds and this is independent to whether the firm will recapitalize or not at \( t = 1 \). On
Figure 10

COMPARISON OF VLo

Legend
△ VLo I
× VLo III
□ VLo IV
Figure 11
COMPARISON OF VD

Legend
△ VD_{aI}
× VD_{aIII}
□ VD_{aIV}

VD_{aI} AND III COINCIDE
Figure 12
COMPARISON OF VE

Legend
△ VE I
× VE III
□ VE IV
the other hand, shareholders' wealth depends on the recapitalization cashflow, the new optimal firm value and the dividends after tax and flotation costs. If recapitalization is an optimal decision, these values will increase shareholders' wealth. If there is no recapitalization, shareholders' wealth is the same as that in Case I. Also, the value of equity at $t = 0$ depends on the recapitalization cashflow, the value of equity at $t = 1$ and the dividends after tax and flotation costs. Again, this value cannot be less than those in Case I. So when there is an option to recapitalize, values of equity and shareholders' wealth will increase or at least remain the same under different debt levels. Bondholders do not benefit by the option to recapitalize and values of debt under different debt levels remain the same. These are exactly the simulated results as shown in Figures 11 and 12.

6.4.2 Changes in Financial Position after Recapitalization

In order to determine how the capital structure of the firm changes after recapitalization, we need to compare the levered firm value, at $t = 1$, at various debt levels chosen at $t = 0$, given that there is no recapitalization, with the levered firm value at various debt levels if there is recapitalization. We should evaluate the capital structure of the firm at $t = 1$ because this is the time when the recapitalization decision is made. At $t = 1$, one of the
cashflow states has obtained, so the levered firm values are conditional on the observed cashflow level. The capital structure of the firm using the with- and ex-dividend values are analysed and compared. Figure 13a shows the comparison of the firm's capital structure, ex-dividend and after flotation costs are paid, before and after recapitalization. For a given cashflow level at $t = 1$, the line "VL before RECAP" represents the levered firm values ex-dividend, under different debt levels chosen at $t = 0$, between the observation of $X_i$ and the recapitalization decision. The line "VL after RECAP" represents the levered firm values ex-dividends, under different debt levels chosen at $t = 1$ for recapitalization, just one moment after $t = 1$. At this time, the recapitalization decision has been made.

In Figure 13a, "Y" means that the firm decides to recapitalize given the face value of debt chosen at $t = 0$ and "N" means that the firm does not recapitalize at that face value of debt chosen at $t = 0$. We can see that recapitalization does not occur at the optimal debt level 8537 and neither at debt levels close to 8537. The firm

---

1In evaluating firm values when there is no recapitalization, debt levels relevant at $t = 0$ are used because these are the debt level of the firm until $t = 2$ if it does not recapitalize.

2Flotation costs have also been paid because dividends are the residual of period one cashflow after paying the flotation costs.

3When the firm recapitalizes, it buys back the old debt issue and issues new one-period bonds. At $t = 1$, the firm faces the cashflow levels at $t = 2$ as their relevant debt levels.
recapitalizes only on the extremes of the VL curve. As discussed before in Figure 1, there is a bound of suboptimal capital structure which the firm can tolerate because of the presents of flotation costs. Theoretically, the firm will recapitalize only if the increment in shareholders' wealth at $t = 1$ resulted from recapitalization is larger than the after-tax flotation costs. Figure 13a resembles very closely to Figure 1 because it also has a bound which the firm will not recapitalize. In order to determine this bound, we need to examine the criterion of recapitalization as discussed in the model layout of Case III. Equations (13) and (14) shows the shareholders' wealth at $t = 1$ under the decisions of recapitalization and no recapitalization. The firm will recapitalize only if shareholders after recapitalization is larger than that before recapitalization. Putting equations (13) and (14) together, for a given period one cashflow level $X_a$, the firm will recapitalize only when:

$$(1-tc).X_a + VE_1(Do) < (1-tc).X_a - F + VE_1(D_1) + \text{RECAPCF}$$

Rearranging,

$$(1-tc).F < \text{RECAPCF} + VE_1(D_1) - VE_1(Do)$$

or $$(1-tc).F < \text{VD}_1(D_1) - \text{VD}_1(Do) + VE_1(D_1) - VE_1(Do)$$

or $$(1-tc).F < \text{VL}_1(D_1) - \text{VL}_1(Do)$$

Equation (27) represents a criterion for the firm to recapitalize. It shows that the firm will recapitalize only if the increase in ex-dividend levered firm value resulting from recapitalization exceeds the after-tax flotation costs.
Equations (13) and (14) are based on maximizing shareholders' wealth at \( t = 1 \) and now the result in equation (27) represents a criterion to maximize the levered firm value. So in our two period model, the criterion of recapitalization is the same no matter which objective is used. The reason is that in equation (14), the term RECAPCF has already captured the VD component of the levered firm value, thus the criterion of recapitalization under both objectives are the same.

In Figure 13a, \( VL_1(Do) \) lies on the line "VL before RECAP". This line shows the value of the firm at \( t = 1 \) under different debt levels chosen at \( t = 0 \). \( VL_1(D1) \) on the "VL after RECAP" line is the ex-dividend firm value if it recapitalizes. This is also the value after flotation costs are paid. Both \( VL_{old} \) and \( VL_{new} \) can be determined the same way as the optimal \( VL_i \) in Case II because they are both the levered firm value at \( t = 1 \) given that a particular period one cashflow \( X_a \) has occurred. This explains why the two lines in Figure 13a coincide with each other at some points. Firm values at some debt levels are different because in determining \( VL_1(D1) \), only those second period cashflow levels conditional on the period one cashflow level obtained are relevant debt levels. But for \( VL_1(Do) \), all the period two cashflow levels are relevant debt levels. This discrepancy can be eliminated if the cashflow levels are not discrete as shown in Figure 1. Equation (27) is basically saying that the firm will recapitalize if the difference between the firm value at \( t = 1 \) and the maximum firm value
attainable through recapitalization exceeds the after-tax flotation costs. So we can draw a horizontal line in Figure 13a to create a boundary within which the firm will not recapitalize.

However, in order to analyse the change in capital structure of the firm after recapitalization, we need to look at the with-dividend VL of the firm. The reason is that this is the value which actually affects the wealth of shareholders. Figure 13b shows the changes in the capital structure curve after recapitalization. All the values in Figure 13b are with-dividend values. In this case, the criterion for recapitalization can be determined by modifying Equation (27):

From Equation (27),

\[(1-tc).F < \text{ex-div VL}_1(D1) - \text{ex-div VL}_1(Do)\]

or, \[(1-tc).F < (\text{ex-div VL}_1(D1)+\text{div})-(\text{ex-div VL}_1(Do)+\text{div})\]

or, \[(1-tc).F < \text{with-div VL}_1(D1) - \text{with-div VL}_1(Do)\] (28)

Note: before dividends \(\text{VL}_1(D1)\) is also before flotation costs

So we can draw a boundary on the "VL before RECAP" line within which the firm will not recapitalize. Once the value of the firm drops low enough to justify the after-tax flotation costs, it recapitalizes. The maximum point on the "VL before RECAP" is the with-dividend value of the firm after recapitalization when there are no flotation costs. When there are flotation costs, the value of the firm after recapitalization will be lower than that maximum point by an amount equal to the after-tax flotation costs. From the "VL after RECAP" line in Figure 13b, we can see that the optimal
Figure 13b
CAPITAL STRUCTURE AFTER RECAP

Legend
△ VL before RECAP
× VL of RECAP of F
□
debt level after recapitalization is unaffected by the presents of flotation costs. The reason is that the new debt level that the firm chooses to recapitalize is based on the second period cashflow levels and the optimal level is determined using the methodology of Case II and this is independent of the level of flotation costs. So essentially, the whole VL line shifts downward by an amount of \((1-tc).F\) after recapitalization. From the geometry of the diagram, we can see that the firm can never lose value after recapitalization. This implication is plotted again in Figure 13c which assumes a continuous function of cashflow.

6.4.3 Effect of F on the Recapitalization Boundary

Case III is run through the simulation program in Appendix 1c with various levels of flotation costs and the results are shown in Figure 13d. We can see that the recapitalization bound increases as the level of flotation costs increases. This implies that the probability of recapitalization is lower since now the existing debt level of the firm has to lead to a very low firm value in order for recapitalization to be optimal.

Also, Figure 13d shows that as the level of flotation costs increases, the firm value after recapitalization decreases. The amount of decrease is exactly equal to the after-tax amount of increase in flotation costs.
Figure 13c

Capital Structure after Recap.: Continuous Cashflow

VL1

before recap

(1 - tc).F

D1

D

after recap

D0
Figure 13d
EFFECT OF F ON VL after RECAP

Legend
△ VL before RECAP
× F=500
□ F=1800
⊗ F=2800
6.4.4 **Implications of the Recapitalization Boundary**

With the above findings, the two period model suggests a tolerable bound of debt levels within which the firm will not recapitalize because the increment in firm value cannot justify the after-tax flotation costs. The firm will recapitalize once the present debt level hits this bound; i.e., once the potential increase in firm value (before flotation costs) exceeds the after-tax flotation costs.

The recapitalization boundary also implies the change in the capital structure curve as in Figure 13e. At t = 0, the firm's capital structure decision is at the optimal point A on the VLo curve. At t = 1, the cashflow level of period one is obtained and the firm value function changes because of new information obtained. If the cashflow level obtained is low, the firm value function will shift to VLo(low) and the new optimal debt level is at D1*. This is smaller than Do* because the periods one and two cashflows are serially correlated and the firm will choose a lower optimal debt level to reduce the amount of expected bankruptcy costs. Without recapitalization, the firm is on point B which is now suboptimal. If the firm recapitalizes, the firm value is reduced by the amount of after-tax flotation costs. This causes the firm value function to shift down to VLo'(low). Recapitalization is optimal only if Point C is higher than Point B. The final position of the firm if it recapitalizes is at Point C. Thus we can only observe Point A at t = 0 and either Point B or Point C at t = 1. The same arguments apply if the cashflow level turns out to be high at t = 1.
Figure 13e

Implications of Case IV Results
In Figure 13e, Point E is lower than Point D and recapitalization is suboptimal.

The above findings suggest that the debt level and the levered firm value we observe empirically do not necessarily represent the optimal values. Instead, we may observe those values such that recapitalization is not a positive net present value project. Empirical studies conducted on the capital structure decision usually ignore the presence of flotation costs of recapitalization and the resulting recapitalization bound. This observed capital structure is usually treated as the optimal one and thus may lead to misleading conclusions.
6.5 Effects of a Call Provision on Firm Value

6.5.1 The Value of the Call Provision

Figure 10 shows that shareholders' wealth in Case IV is always lower than or equal to the shareholders' wealth in Case III. The addition of a call provision to the bonds allows the firm to call the bonds when the market price of the bonds at $t = 1$ is higher than the call price. The firm will exercise the call provision only under the conditions that the call price is lower than the market price and that recapitalization is an optimal decision. The value of debt will remain the same when the market price of the bonds is lower than the call price and decrease when the firm calls the bonds. So the value of debt cannot be increased by the call provision. This is shown in Figure 11.

On the other hand, value of equity at $t = 0$ will increase or at least remain constant as a result of the added call provision. This is shown in Figure 12. During recapitalization, the firm can call the bonds if the call price is lower, so recapitalization cashflow will increase and so will the value of equity. It will remain constant only when the firm does not exercise the call provision.

In those states in which recapitalization is an optimal decision in Case III, the call provision cannot change this decision because it will only increase the recapitalization cashflow and thus the value of equity and total shareholders' wealth. The increase in payoff to
shareholders is (market price of debt - call price) and this is also the decrease in payoff to bondholders in that state.

In those states which recapitalization is suboptimal in Case III, it is possible that the call provision will change this decision because recapitalization cashflow can be increased and, as a result, recapitalization may become optimal. Since recapitalization benefits only the shareholders, the value of equity increases. The decrease in the value of debt in those states is still the difference between the market price of debt and the call price because bondholders cannot share the benefit of recapitalization.

In general, value of debt at \( t = 0 \) will decrease and value of equity will increase at \( t = 0 \). The increase in value of debt at \( t = 0 \) as a result of the call provision is:

\[
\sum_{i=1}^{n} (V_D - CP) \cdot \text{PROB}(X_i) \cdot R \cdot I
\]

where CP is the call price

- \( I = 0 \) when recapitalization is suboptimal
- \( I = 1 \) when recapitalization is optimal and call price is below the market price of debt.

However, the increase in the value of equity at \( t = 0 \) is more complicated. In those states where recapitalization is suboptimal in both Cases III and IV, the increase in value of equity is zero. In those states where recapitalization is optimal in Case III, the increase in the value of equity
at \( t = 0 \) is also the amount in equation (24) above. In the states where recapitalization is suboptimal in Case III but optimal in Case IV, the increase in value of equity at \( t = 0 \) is equal to equation (24) minus the extra flotation costs incurred in that state plus the increase in value of equity at \( t = 1 \) due to the optimal new debt level. Thus the present value of the increase in equity value at \( t = 0 \) is given by:

\[
\sum_{i=1}^{n} \left[ (VD_i - CP) \cdot I - F(1-tc) \cdot I \cdot J + (VE_{new} - VE_{old}) \cdot I \cdot J \right] \cdot P(X_i) \cdot R \quad (25)
\]

where \( I = 0 \) when recapitalization is suboptimal

\( I = 1 \) when recapitalization is optimal and call price is below market price of debt.

\( J = 0 \) when recapitalization in that state is optimal in Case III.

\( J = 1 \) when recapitalization in that state is suboptimal in Case III.

The above equation is the increase in the value of equity at \( t = 0 \) and this is also the present value of the increase in expected shareholders' wealth at \( t = 1 \). So the above equation is also the value of the call provision added to the value of the shares at \( t = 0 \). However, since shareholders' wealth at \( t = 0 \) is the sum of debt and equity, the change in total shareholders' wealth at \( t = 0 \) is given
by the difference between equations (24) and (25):

\[ \sum_{i=1}^{n} [V_{\text{new}} - V_{\text{old}} - F(1-tc)].R.P(X_i).I.J \]  

(26)

The levered firm value, and thus total shareholders' wealth, at \( t = 0 \) is a tradeoff between the increase in expected flotation costs and the expected increase in equity value at \( t = 1 \) due to recapitalization. In our simulation example, flotation costs are set at a relatively high level, so the value of the firm decreases as the call provision is introduced. In conclusion, a call provision increases the share price and decreases the bond price. However, total shareholders' wealth, as defined in our model, decreases.

6.5.2 Implications of Case IV Results

The above findings imply that a call provision does not necessarily benefit the shareholders. Bondholders will anticipate when the firm will call the bonds and this anticipation will incorporate the expected loss in bondholders' wealth into the bond price at \( t = 0 \). In fact, it is possible that shareholders' wealth will be reduced because there are some cases where recapitalization becomes optimal as a result of the call provision and the firm incurs flotation costs which could have been avoided if there is no call provision. The increase in firm value due to recapitalization may not be able to offset the flotation costs and thus firm value is reduced and so does
shareholders' wealth.
7. **SENSITIVITY ANALYSIS**

Sensitivity analysis is performed on Case III using the WATFIV program in Appendix 1d and the data set in Appendix 1e. Various levels of bankruptcy costs, discount rate, tax rate and flotation are used so as to analyze the change in capital structure decision to these variables.

7.1 **Bankruptcy costs**

Figure 14a shows the values of the firm under different bankruptcy costs. We can see that for a given debt level, the higher the bankruptcy costs, the lower the levered firm value. Higher bankruptcy costs reduces the amount accrued to the bondholders during bankruptcy and hence reduces the market value of debt at $t = 0$. So the levered firm value, which is also shareholders' wealth, is decreasing as the level of bankruptcy costs is increasing.

Also, Figure 14a shows that the optimal debt level is lower for higher bankruptcy costs. This is because for an increase in debt level the increment in expected bankruptcy costs, as in equation (18), is higher than the increment in expected tax shield as shown by equation (17). Thus lower debt level would reduce the high expected bankruptcy costs and this reduction can more than offset the loss in expected tax shield.

Figure 14b shows that value of debt is decreasing as the
Figure 14a

EFFECT OF BANKRUPTCY COST ON VL/W

Legend

\( \Delta \) B=1000
\( \times \) B=3500
\( \square \) B=6000
Figure 14 b

EFFECT OF BANKRUPTCY COST ON VD

Legend
Δ B=1000
× B=3500
□ B=6000
Figure 14c

EFFECT OF BANKRUPTCY COST ON VE

Legend

△ B=1000
× B=3500
□ B=6000
level of bankruptcy costs is increasing. This is due to the fact that once the firm goes bankrupt, cashflow of the firm belongs to the bondholders and the higher the bankruptcy costs, the less is left for the bondholders. In fact, there are some cases which the market value of debt drops so much that recapitalization now becomes an optimal decision. The reason is that value of the old debt issue drops considerably due to its suboptimality but the value of the new issue is an optimal debt level and thus the level of expected bankruptcy costs is a minimum. So recapitalization cashflow increases as the level of bankruptcy costs increases and the optimal decision is to recapitalize. Since the value of equity is the present value of the expected shareholders' wealth at $t = 1$ under the optimal recapitalization decision, it will increase or at least remain the same when bankruptcy costs increase. Figure 14c shows exactly this result.

7.2 Corporate tax rate

Figure 15 shows the result of sensitivity analysis on the corporate tax rate. One obvious effect of an increase in tax rate is that the optimal debt level increases. Higher tax rate implies that the increment in expected tax shield resulted from an increase in debt level has increased. On the other hand, the increment in expected bankruptcy costs remains constant. Thus the optimal debt level will be at higher debt levels so as to utilize the higher tax shield.
Figure 15

EFFECT OF TAX RATE ON VL/W

Legend

△ tc=0.20
× tc=0.40
□ tc=0.70
With higher tax rate, larger amount of taxes are payable to the tax authority and thus the levered firm decreases.

7.3 **Discount Rate**

The effect of discount rate on the levered firm value is shown in Figure 16. Higher discount rate will reduce the present value of all the values. However, the capital structure decision is not affected by the level of discount rate. We can see in Figure 16 that the whole line shifts downward without any change in the optimal debt level.

7.4 **Call Price**

Sensitivity analysis is done on Case IV and Figure 17a shows the value of the firm in Case IV when call price increases. Call prices are set at 1/3, 1/2 and 2/3 of the face value of debt that the firm issues at t = 0. We can see that the levered firm value increases with increasing call price. Before we can explain why this is so, we need to examine the effect of a change in call price on the value of debt and equity. Figure 17b shows that for a given face value of debt, the value of debt increases as the call price increases. Higher call price would mean that at t = 1, there is less chance for the market value of debt to be higher than the call price. Thus the probability of calling the bonds is reduced. As discussed earlier, bondholders
Figure 16
EFFECT OF DISCOUNT RATE ON VL/W

Legend
△ R=1+0.05
× R=1+0.10
□ R=1+0.20
Figure 17a

EFFECT OF CALL PRICE ON VLo

Legend
△ CP=D/3
× CP=D/2
□ CP=2D/3
Figure 17b

EFFECT OF CALL PRICE ON Vd

Legend

△ CP=D/3
× CP=D/2
□ CP=2D/3
Figure 17c

EFFECT OF CALL PRICE ON VE

Legend

△ CP=D/3
× CP=D/2
□ CP=2D/3
will lose the difference between the market price of the bonds and the call price. So a reduction in the probability of calling the bonds will increase the market value of debt at $t = 0$. The market value of equity increases as call price decreases and this is shown in Figure 17c. As the call price decreases, value of equity increases because the firm has larger probability to call the bonds. This will increase the recapitalization cashflow. Also, value of equity at $t = 1$ will increase as there are higher chance of recapitalizing. In our example, the net effect is a decrease in firm value as call price decreases.

7.5 Flotation Costs

With higher flotation costs, the frequency of recapitalization is fewer than before because the increment in shareholders' wealth resulted from recapitalization has to be higher so as to offset the high flotation costs. These costs are paid out of the cashflow of the firm and higher flotation will reduce the value of equity and thus the levered firm value. The simulated results in Figures 18a and 18c conform to these arguments.

The value of debt will not change when the level of flotation costs increases because, as discussed before, the value of debt is not affected by the recapitalization decision in Case III. This result is shown in Figure 18b.
Figure 18a

SENSITIVITY ON F

Legend

△ F=500
× F=1800
□ F=2800
Figure 18b
SENSITIVITY ON F

Legend
- F=500
- F=1800
- F=2800

$V_{D_0}$ does not change with $F$
Figure 18c
SENSITIVITY ON F

Legend
△ F=500
× F=1800
□ F=2800
8. SUMMARY AND CONCLUSIONS

8.1 Summary

This thesis develops a two period state-contingent model which employs the "tax shield of debt and expected bankruptcy costs" approach with the objective of maximizing shareholders' wealth at the beginning of period one. With this two-period model, we can analyse the dynamics of firm capital structure choice under various conditions. First the firm issues one two-period bond, then it issues two one-period bonds. In Case III, the firm issues one two-period bond and optional recapitalization is allowed at the end of period one with fixed flotation costs. Then a call provision on the bonds is added in Case IV. With these four cases, we can examine the effect of recapitalization and the call provision on the values of debt, equity shareholders' wealth and the levered firm value.

The simulated results are consistent with the concavity of the firm value function. Issuing two one-period bonds has the advantage of flexibility but extra issuing costs are incurred. On the other hand, issuing one two-period bond lacks the flexibility but incurs less issuing costs. Recapitalization is analogous to a call option and it always adds value to the firm. However, only the shareholders benefit and not the bondholders. The value of debt at the beginning of period one is not affected by recapitalization because the new firm value resulting from recapitalization
consists of new debt and equity and the original bondholders receive the market value of the old debt when the firm recapitalizes. The firm will recapitalize only when the gain in shareholders' wealth (which is equal in amount to the gain in with- or ex-dividend firm value) is larger than the after-tax flotation costs. The levered firm value, with- or ex-dividend, will be reduced by an amount equal to the after-tax flotation costs after recapitalization. As a result of recapitalization, the optimal debt level will decrease when period one cashflow level is low and increase when it is high. Because of the presence of flotation costs, there exists a recapitalization boundary within which the firm will not recapitalize. Outside this boundary, the firm will recapitalize.

Simulated results show that the addition of a call provision always decreases the value of debt and increases the value of equity; this follows because bondholders will anticipate the possible loss in value when the firm calls the bonds and thus bond price at the beginning of period one is lowered to reflect this. The overall effect on the levered firm value, and hence shareholders' wealth, at t = 0 is more complicated and is a tradeoff between the increase in expected flotation costs and the expected increase in value of equity as a result of a higher chance of recapitalizing.

Sensitivity analysis shows that both the firm value and the optimal debt level decreases with higher bankruptcy costs. A higher corporate tax rate implies a lower firm
value and a higher optimal debt level. Also, a higher discount rate will lower the firm value but will not affect the optimal debt level. A higher call price will increase the firm value because, in our example, flotation costs are high and a high call price reduces the number of recapitalization states and hence the flotation costs. Finally, higher flotation costs imply fewer recapitalizations and thus a lower firm value.

8.2 Two Important Implications of the Model

There are two important issues arising out of the two-period model. First, empirical studies often assume that the observed capital structure of a firm indicates the acme of the firm value function. This may be incorrect since there exists a tolerable recapitalization boundary within which the firm will not recapitalize and the observed debt level is not necessarily at the acme of the firm value function.

Second, as opposed to the traditional view, the addition of a call provision on bonds does not necessarily enhance shareholders' wealth. Firm value, and hence shareholders' wealth, may be reduced by the call provision because the lower call price may cause the firm to recapitalize in states which would otherwise not involve recapitalization. Thus, extra flotation costs are incurred which may exceed the gain in calling the bonds and thus reduce the firm value.
BIBLIOGRAPHY


APPENDICES
APPENDIX 1A
CASE I SIMULATION PROGRAM

INTEGER X, Y, Z, FLAG
REAL B(10), T(5), R(5), RES, F(10)
REAL OVL, OVD, OVE, OVU, OD
REAL PD(64,64), PE(64,64), BA(64,64)
REAL D(64), SUMD(64), SUME(64), SUMBA(64)
REAL PROB
REAL VD(64), VE(64), VL(64), VU(64)
REAL E1(10), E2(64)
REAL DIVPV

C INPUT
READ(2,*) M,N
READ(2,*) (B(I),I=1,3)
READ(2,*) (F(I),I=1,3)
READ(2,*) (T(I),I=1,3)
READ(2,*) (R(I),I=1,3)
READ(2,*) (E1(I),I=1,M)
MN=M*N
READ(2,*) (E2(I),I=1,MN)
X=2
Y=2
Z=2
G=2
PROB=1.0/(M*N)

C initializing variables
DO 1 I=1,MN
   SUMD(I)=0
   SUME(I)=0
   SUMBA(I)=0
   OVL=0
   OVD=0
   OVE=0
   OVU=0
1 CONTINUE

C calculate expected dividend from period one
DIVPV=0
DO 23 I=1,M
   DIVPV=DIVPV+E1(I)*(1.0/M)*(1-T(Y))*(1/(1+R(Z)))
23 CONTINUE

DO 2 I=1,MN
   D(I)=E2(I)
   DO 3 J=1,MN
      IF (D(I) .LE. E2(J)) THEN DO
         PD(I,J)=D(I)
         PE(I,J)=(E2(J)-D(I))*(1-T(Y))
         BA(I,J)=0
      ELSE DO
         RES=E2(J)-B(X)
         PD(I,J)=AMAX1(0.0,RES)
         PE(I,J)=0
         IF (RES .GE. 0) THEN DO
            BA(I,J)=E2(J)
      END IF
   END DO
2 CONTINUE
ELSE DO
   BA(I,J)=B(X)
END IF
END IF
SUMBA(I)=SUMBA(I)+BA(I,J)
SUMD(I)=SUMD(I)+PD(I,J)
SUME(I)=SUME(I)+PE(I,J)
3 CONTINUE
VD(I)=SUMD(I)*PROB/((1+R(Z))**2)
VE(I)=SUME(I)*PROB/((1+R(Z))**2)+DIVPV-F(G)
VL(I)=VD(I)+VE(I)
VU(I)=VL(I)-T(Y)*D(I)+(1-T(Y))*SUMBA(I)*PROB/
      (((1+R(Z))**2)
C
IF (OVL .GE. VL(I)) GO TO 2
OVL=VL(I)
OVE=VE(I)
OVD=VD(I)
OVU=VU(I)
OD=D(I)
2 CONTINUE
C ARRANGE DEBT LEVELS IN ASCENDING ORDERS
DO 741 J=1,MN
   AMIN=1.0+D(MN)
   DO 143 I=1,MN
   IF (D(I) .LE. AMIN) THEN DO
      AMIN=D(I)
      K=I
   END IF
   143 CONTINUE
   DD(J)=D(K)
   VDD(J)=VD(K)
   VEE(J)=VE(K)
   VLL(J)=VL(K)
   D(K)=2.0+D(MN)
741 CONTINUE
C PRINT
DO 99 I=1,MN
   WRITE(9,234) DD(I),VDD(I),VEE(I),VLL(I)
234 FORMAT (4(F12.4,1X))
99 CONTINUE
WRITE(9,154) OD,OVD,OVE,OVL
154 FORMAT (4(F12.4,1X))
STOP
END
$EXECUTE
$COMPILE

INTEGER X,Y,Z,FLAG,P,INDEX,Q,W
REAL B(3),T(3),R(3),RES,F(3)
REAL E1(8),E2(8,8)
REAL EARN(2,8,8,8)
REAL OV(L(8)),OVE(8),OVD(8),OVU(8)
REAL OD(8)
REAL OV0,OVE0,OVD0,OVU0,OD0
REAL P(2,8,8,8),PE(2,8,8,8),BA(2,8,8,8)
REAL D(2,8,8)
REAL SUMD(2,8,8),SUME(2,8,8),SUMBA(2,8,8)
REAL PROB(2)
REAL VD(2,8,8),VE(2,8,8),VL(2,8,8),VU(2,8,8)
READ(2,*) M,N
READ(2,*) (B(I),I=1,3)
READ(2,*) (F(I),I=1,3)
READ(2,*) (T(I),I=1,3)
READ(2,*) (R(I),I=1,3)
READ(2,*) (E1(I),I=1,M)
MN=M*N
READ(2,*) ((E2(I,J),J=1,N),I=1,M)
X=2
Y=2
Z=2
G=3
PVOVL=0
PVF=0
PROB(1)=1.0/M
PROB(2)=1.0/N
C INITIALIZING VARIABLES
DO 12 P=1,2
   DO 4 I=1,M
      DO 5 J=1,N
         SUMD(P,I,J)=0
         SUME(P,I,J)=0
         SUMBA(P,I,J)=0
         VL(P,I,J)=0
         OVL(I)=0
         OVL0=0
      5 CONTINUE
   4 CONTINUE
   12 CONTINUE
C
   DO 13 I=1,M
      DO 14 J=1,N
         EARN(2,I,J,K)=E2(I,K)
         D(2,I,J)=E2(I,K)
      14 CONTINUE
   13 CONTINUE
C
   DO 16 I=1,M
      DO 17 K=1,M
EARN(1, I, 1, K) = E1(K)
D(1, K, 1) = E1(K)

CONTINUE
CONTINUE

W = N
P = 2

DO 1 I = 1, M
    IF (P .EQ. 1) THEN DO
        U = 1
        Q = 1
        L = N
        W = 1
    ELSE DO
        U = 0
        Q = 2
        L = N
    END IF

DO 2 J = 1, W
    DO 3 K = 1, L
        IF (D(P, I, J) .LE. EARN(P, I, J, K)) THEN DO
            PD(P, I, J, K) = D(P, I, J)
            PE(P, I, J, K) = (EARN(P, I, J, K) - D(P, I, J)) *(1 - T(Y)) + U*PVOVL + U*PVF
            BA(P, I, J, K) = 0
        ELSE DO
            RES = EARN(P, I, J, K) - B(X)
            PD(P, I, J, K) = AMAX1(0.0, RES) + U*PVOVL
            PE(P, I, J, K) = 0
            IF (RES .LT. 0) THEN DO
                BA(P, I, J, K) = EARN(P, I, J, K)
            ELSE DO
                BA(P, I, J, K) = B(X)
            END IF
        END IF
    SUMBA(P, I, J) = SUMBA(P, I, J) + BA(P, I, J, K)
    SUMD(P, I, J) = SUMD(P, I, J) + PD(P, I, J, K)
    SUME(P, I, J) = SUME(P, I, J) + PE(P, I, J, K)

CONTINUE
VD(P, I, J) = SUMD(P, I, J)*PROB(Q)/(1 + R(Z))
VE(P, I, J) = SUME(P, I, J)*PROB(Q)/(1 + R(Z))
    -U*F(G)

PRINT, PVF, PVOVL, VE(P, I, J)
VL(P, I, J) = VD(P, I, J) + VE(P, I, J)
VU(P, I, J) = VL(P, I, J) - T(Y)*D(P, I, J) + (1 - T(Y)) *
    SUMBA(P, I, J)*PROB(Q)/(1 + R(Z))
IF (OVL(I) .LT. VL(P, I, J) .AND. P .EQ. 2) THEN DO

OVL(I) = VL(P, I, J)
OVE(I) = VE(P, I, J)
OVD(I) = VD(P, I, J)
OVU(I) = VU(P, I, J)
OD(I) = D(P, I, J)
END IF
IF (OVL0 .LT. VL(P,I,J) .AND. P .EQ. 1) THEN
  DO
    OVL0=VL(P,I,J)
    OVE0=VE(P,I,J)
    OVD0=VD(P,I,J)
    OVU0=VU(P,I,J)
    OD0=D(P,I,J)
  END IF
2 CONTINUE
1 CONTINUE
IF (P .EQ. 2) THEN DO
  DO 471 I=1,M
    PVOVL=PVOVL+OVL(I)
  CONTINUE
  PVOVL=PVOVL*PROB(1)/(1+R(Z))
  PVF=F(G)/(1+R(Z))
  P=1
  GO TO 11
END IF
C OUTPUT
DO 98 P=1,2
L=N
DO 99 I=1,M
  IF (P .EQ. 1) L=1
  DO 999 J=1,L
    IF (P .EQ. 2) THEN DO
      PRINT, D(P,I,J),VL(P,I,J)
    END IF
    WRITE (9,543) D(P,I,J),VD(P,I,J),VE(P,I,J),VL(P,I,J)
  END IF
  543 FORMAT (4(F12.4,1X))
999 CONTINUE
99 CONTINUE
98 CONTINUE
C
DO 96 I=1,M
  WRITE (9,246) OVD(I),OVE(I),OVL(I),OD(I)
  246 FORMAT (4(F12.4,1X))
  WRITE (7,*) OVD(I),OVE(I),OVL(I),OVU(I),OD(I)
  96 CONTINUE
WRITE (9,85) OD0,OVD0,OVE0,OVL0
  85 FORMAT (4(F12.4,1X))
STOP
END
$EXECUTE
APPENDIX 1C
CASE III SIMULATION PROGRAM

$COMPILE
INTEGER X,Y,Z,FLAG,G
REAL D(8,8),E1(8),E2(8,8)
REAL PD(8,8),PE(8,8)
REAL T(3),R(3),B(3),RES,F(3)
REAL SUMD(8,8,8),SUME(8,8,8)
REAL VD(8,8,8),VE(8,8,8),VL(8,8,8)
REAL DIV(8,8,8),W(8,8,8)
REAL REDIV(8,8,8),CF(8,8,8)
REAL REW(8,8,8)
REAL OVD(8),OVE(8),OVL(8),OVU(8),OD(8)
REAL DECVD(8,8,8),DECVE(8,8,8)
REAL DSUMD(8,8),DSUME(8,8)
REAL VD0(8,8),VE0(8,8),VL0(8,8)
REAL VDD0(8,8),VEE0(8,8),VLL0(8,8),DD(8,8)
REAL OVL0,OVD0,OVE0,OD0
REAL EXDVL(8,8,8),REEXDVL(8,8,8)
REAL PROB(2)
CHARACTER DECIDE(8,8,8)

C
READ(2,*) M,N
READ(2,*) (B(I),I=1,3)
READ(2,*) (F(I),I=1,3)
READ(2,*) (T(I),I=1,3)
READ(2,*) (R(I),I=1,3)
READ(2,*) (E1(I),I=1,M)
READ(2,*) ((E2(I,J),J=1,N),I=1,M)
DO 17 I=1,M
     READ(7,*) OVD(I),OVE(I),OVL(I),OVU(I),OD(I)
17 CONTINUE
X=2
Y=2
Z=2
G=2
PROB(1)=1.0/M
PROB(2)=1.0/N
C INITIALIZING VARIABLES
DO 11 I=1,M
     DO 22 J=1,N
         DO 33 K=1,M
             SUMD(I,J,K)=0
             SUME(I,J,K)=0
             DSUMD(I,J)=0
             DSUME(I,J)=0
             OVL0=0
33 CONTINUE
22 CONTINUE
11 CONTINUE
C MAIN PROGRAM
10 DO 1 I=1,M
     DO 2 J=1,N
         D(I,J)=E2(I,J)
     DO 3 K=1,M
DO 4 L=1,N
  IF (D(I,J) .LE. E2(K,L)) THEN DO
    PD(K,L)=D(I,J)
    PE(K,L)=(E2(K,L)-D(I,J))*(1-T(Y))
  ELSE DO
    RES=E2(K,L)-B(X)
    PD(K,L)=AMAX1(0.0,RES)
    PE(K,L)=0
  END IF
  SUMD(I,J,K)=SUMD(I,J,K)+PD(K,L)
  SUME(I,J,K)=SUME(I,J,K)+PE(K,L)
4 CONTINUE

VD(I,J,K)=SUMD(I,J,K)*PROB(2)/(1+R(Z))
VE(I,J,K)=SUME(I,J,K)*PROB(2)/(1+R(Z))
VL(I,J,K)=VD(I,J,K)+VE(I,J,K)
DIV(I,J,K)=E1(K)*(1-T(Y))
W(I,J,K)=DIV(I,J,K)+VE(I,J,K)
EXDVL(I,J,K)=VL(I,J,K)+DIV(I,J,K)

C
REDIV(I,J,K)=(E1(K)-F(G))*(1-T(Y))
CF(I,J,K)=OVD(K)-VD(l,J,K)
REW(I,J,K)=REDIV(I,J,K)+OVE(K)+CF(I,J,K)
REEXDVL(I,J,K)=VL(I,J,K)+REDIV(I,J,K)

C
IF (W(I,J,K) .GE. REW(I,J,K)) THEN DO
  DECIDE(I,J,K)='N'
  DECVD(I,J,K)=VD(I,J,K)
  DECVE(I,J,K)=W(I,J,K)
ELSE DO
  DECIDE(I,J,K)='Y'
  DECVD(I,J,K)=VD(I,J,K)
  DECVE(I,J,K)=REW(I,J,K)
END IF
DSUMD(I,J)=DSUMD(I,J)+DECVD(I,J,K)
DSUME(I,J)=DSUME(I,J)+DECVE(I,J,K)
3 CONTINUE

VD0(I,J)=DSUMD(I,J)*PROB(1)/(1+R(Z))
VE0(I,J)=DSUME(I,J)*PROB(1)/(1+R(Z))-F(G)
VL0(I,J)=VD0(I,J)+VE0(I,J)

C
IF (OVL0 .LE. VL0(I,J)) THEN DO
  OVL0=VL0(I,J)
  OVD0=VD0(I,J)
  OVE0=VE0(I,J)
  ODO=D(I,J)
END IF
2 CONTINUE
1 CONTINUE
DO 104 K=1,M
  DO 101 I=1,M
    DO 102 J=1,N
      PRINT, D(I,J),VL(I,J,K)
C      PRINT, D(I,J),DECIDE(I,J,K),EXDVL(I,J,K)
C
102 CONTINUE
101 CONTINUE
CONTINUE
C ARRANGE DEBT LEVELS IN ASCENDING ORDERS
    DO 741 J=1,M
    DO 489 JJ=1,N
        AMIN=1.0+D(M,N)
        DO 143 I=1,M
            DO 214 II=1,N
                IF (D(I,II) .LE. AMIN) THEN DO
                    AMIN=D(I,II)
                    K=I
                    KK=II
                    END IF
            214 CONTINUE
        143 CONTINUE
        DD(J, JJ)=D(K, KK)
        VDD0(J, JJ)=VD0(K, KK)
        VEE0(J, JJ)=VE0(K, KK)
        VLL0(J, JJ)=VL0(K, KK)
        D(K, KK)=2.0+D(M,N)
    489 CONTINUE
    741 CONTINUE
C OUTPUT
    DO 99 I=1,M
    DO 999 J=1,N
        WRITE(9,154) DD(I, J), VDD0(I, J), VEE0(I, J), VLL0(I, J)
    154 FORMAT (4(F12.4,1X))
999 CONTINUE
99 CONTINUE
C WRITE(9,742) OD0, OVD0, OVE0, OVL0
C742 FORMAT (4(F12.4,1X))
STOP
END
$EXECUTE
APPENDIX 1D
CASE IV SIMULATION PROGRAM

$COMPILE

INTEGER X,Y,Z,FLAG,G
REAL D(8,8),E1(8),E2(8,8)
REAL PD(8,8),PE(8,8)
REAL T(3),R(3),B(3),RES,F(3)
REAL SUMD(8,8,8),SUME(8,8,8)
REAL VD(8,8,8),VE(8,8,8),VL(8,8,8)
REAL DIV(8,8,8),W(8,8,8)
REAL REDIV(8,8,8),CF(8,8,8)
REAL REW(8,8,8)
REAL OVD(8),OVE(8),OVL(8),OVU(8),OD(8)
REAL DECVD(8,8,8),DECVE(8,8,8)
REAL DSUMD(8,8),DSUME(8,8)
REAL VD0(8,8,8),VE0(8,8,8),VL0(8,8,8)
REAL VDD0(8,8),VEE0(8,8,8),VLL0(8,8,8),DD(8,8)
REAL OVL0,OVD0,OVE0,OD0
REAL CP(8,8)
REAL PROB(2)
CHARACTER DECIDE(8,8,8)

C
READ(2,*),M,N
READ(2,*),B(I),I=1,3
READ(2,*),F(I),I=1,3
READ(2,*),T(I),I=1,3
READ(2,*),R(I),I=1,3
READ(2,*),E1(I),I=1,M
READ(2,*),((E2(I,J),J=1,N),I=1,M)
READ(2,*),((E2(I,J),J=1,N),I=1,M)
DO 17 I=1,M

17 CONTINUE
X=2
Y=2
Z=2
G=2
PROB(1)=1.0/M
PROB(2)=1.0/N

C INITIALIZING VARIABLES
DO 11 I=1,M

11 CONTINUE
C MAIN PROGRAM
DO 10 I=1,M

10 CONTINUE
DO 3 K=1,M
  DO 4 L=1,N
    IF (D(I,J) .LE. E2(K,L)) THEN DO
      PD(K,L)=D(I,J)
      PE(K,L)=(E2(K,L)-D(I,J))*(1-T(Y))
    ELSE DO
      RES=E2(K,L)-B(X)
      PD(K,L)=AMAX1(0.0,RES)
      PE(K,L)=0
    END IF
    SUMD(I,J,K)=SUMD(I,J,K)+PD(K,L)
    SUME(I,J,K)=SUME(I,J,K)+PE(K,L)
  CONTINUE
  VD(I,J,K)=SUMD(I,J,K)*PROB(2)/(1+R(Z))
  VE(I,J,K)=SUME(I,J,K)*PROB(2)/(1+R(Z))
  VL(I,J,K)=VD(I,J,K)+VE(I,J,K)
  DIV(I,J,K)=E1(K)*(1-T(Y))
  W(I,J,K)=DIV(I,J,K)+VE(I,J,K)
  REDIV(I,J,K)=(E1(K)-F(G))*(1-T(Y))
  IF (VD(I,J,K) .LT. CP(I,J)) THEN DO
    CF(I,J,K)=OVD(K)-VD(I,J,K)
  ELSE DO
    CF(I,J,K)=OVD(K)-CP(I,J)
  END IF
  REW(I,J,K)=REDIV(I,J,K)+OVE(K)+CF(I,J,K)
  IF (W(I,J,K) .GE. REW(I,J,K)) THEN DO
    DECIDE(I,J,K)='N'
    DECVD(I,J,K)=VD(I,J,K)
    DECVE(I,J,K)=W(I,J,K)
  ELSE DO
    DECIDE(I,J,K)='Y'
    IF (VD(I,J,K) .GE. CP(I,J)) THEN DO
      DECVD(I,J,K)=CP(I,J)
      DECVE(I,J,K)=REW(I,J,K)
    ELSE DO
      DECVD(I,J,K)=VD(I,J,K)
      DECVE(I,J,K)=REW(I,J,K)
    END IF
  END IF
  END IF
  DSUMD(I,J)=DSUMD(I,J)+DECVD(I,J,K)
  DSUME(I,J)=DSUME(I,J)+DECVE(I,J,K)
  CONTINUE
  VD0(I,J)=DSUMD(I,J)*PROB(1)/(1+R(Z))
  VE0(I,J)=DSUME(I,J)*PROB(1)/(1+R(Z))-F(G)
  VL0(I,J)=VD0(I,J)+VE0(I,J)
  IF (OVL0 .LE. VL0(I,J)) THEN DO
    OVL0=VL0(I,J)
    OVD0=VD0(I,J)
    OVE0=VE0(I,J)
    ODO=D(I,J)
  END IF
CONTINUE

C ARRANGE DEBT LEVELS IN ASCENDING ORDERS
DO 741 J=1,M
   DO 489 JJ=1,N
      AMIN=1.0+D(M,N)
      DO 143 I=1,M
         DO 214 II=1,N
            IF (D(I,II) .LE. AMIN) THEN DO
               AMIN=D(I,II)
               K=I
               KK=II
            END IF
         214 CONTINUE
      143 CONTINUE
      DD(J,JJ)=D(K,KK)
      VDD0(J,JJ)=VD0(K,KK)
      VEE0(J,JJ)=VE0(K,KK)
      VLL0(J,JJ)=VL0(K,KK)
      D(K,KK)=2.0+D(M,N)
   489 CONTINUE
741 CONTINUE

C OUTPUT
DO 99 I=1,M
   DO 999 J=1,N
      WRITE(9,851) DD(I,J),VDD0(I,J),VEE0(I,J),
* VLL0(I,J)
851 FORMAT (4(F12.4,1X))
999 CONTINUE
99 CONTINUE

C WRITE (9,156) OCD0,OVDO,OVE0,OVL0
C156 FORMAT (4(F12.4,1X))
STOP
END
$EXECUTE
APPENDIX 1E

INPUT DATA AND FORMAT

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