DURATION AND BOND RETURNS: EMPIRICAL TESTS

by

RUTH JANINE FREEDMAN

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The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

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ABSTRACT

The purpose of this thesis is to empirically investigate the role of duration in explaining bond price volatility caused by interest rate movements. Specifically, Canadian and American market data are used to test whether Macaulay/Fisher and Weil duration is an adequate measure of basis risk for default-free government bonds during the 20-year sample period January 1961 to December 1980.

The most important result of the study is that in either a Canadian or an American context there is no significant evidence to suggest that, on average, higher duration bonds earn higher returns. Specifically, there appears to be a negative, although insignificant, relationship on average between bond returns and duration. A possible explanation for this result is that interest rates have trended upwards over most of the sample period. American results suggest that when changes in the level of interest rates have been filtered out there is a positive, although insignificant, relationship between bond returns and duration.

Other results of the study are that coupon rates are positively related to bond returns (perhaps due to a tax effect), and that bond pricing errors are often related to subsequent returns.

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Chapter 1

INTRODUCTION

Dramatic increases in both the volatility and the level of interest rates since the late 1960s have resulted in a burgeoning interest in bond analysis, and have motivated researchers and practitioners alike to search for tools to aid in the measurement and management of interest rate risk. The quest for an appropriate measure of bond risk has led to the rediscovery of a concept originally developed in 1938 - Frederick Macaulay's measure of bond term known as "duration." The concept of duration has attracted widespread attention within the investment community, and, as a single factor in explaining bond returns, has shown signs of becoming as widely used as beta is for equities.

In recent years, it has been widely recognized that, just as the price behavior of stocks contains a strong market component, the price of bonds tends to be markedly affected by changes in the level of interest rates. The idea of risk adjustment has been widely accepted with respect to stock portfolios: high risk (high beta) portfolios generate the highest returns in rising markets and generate the lowest returns in declining markets. The interesting question has been whether or not an analogous concept would work for the measurement of bond sensitivity.

The customary way of computing beta coefficients for equity issues is to regress security returns against index returns. However, this approach does not work well for bonds because bonds change their sensitivity characteristics as time passes and as the market itself changes. For example, a bond about to mature is far less sensitive to interest-rate changes than a bond with a long time to maturity. The result is that the regression technique used for beta computations does not apply very well to the derivation of bond risk measures.

Frederick Macaulay's bond sensitivity measure, called "duration," is far simpler to compute than the beta coefficients for stocks. Macaulay and others have pointed out that the most important single source of risk for default-free bonds is basis risk; that is, price fluctuations caused by shifts in interest rates. For a given shift in the yield curve, and holding other factors unchanged, longer maturity bonds generally suffer greater price changes than shorter maturity bonds. This characterization is not exact because high coupon bonds are less volatile than low coupon bonds. "Intuition says that this is to be expected because, other things being equal, high coupon bonds have a greater percentage of their value due to the interim coupons and, hence, have a shorter 'effective' maturity." [16; p.627]. Duration may be thought of as an attempt to quantify this qualitative statement through the use of a single, numerical measure.

Duration, as developed by Macaulay, is a weighted average of the time to each bond payment, coupon as well as principal, where the weights are the present value of each of the payments as a percent of the total present value of all the flows. There are different definitions of duration; each corresponds to a different model of the yield curve and shifts in it. The measure of duration used in this study is the Macaulay/Fisher and Weil measure, which computes present value weights using discount bond yields from the term structure.

This study empirically investigates the role of duration in explaining average rates of return and price volatility in the Canadian and American bond markets over the 20-year period January 1961 to December 1980. Chapter 2 contains a review of some of the important literature on the subject of duration, and Chapter 3 outlines testable implications of the duration model. Chapter 4 describes the methodology used in the study, and Chapter 5 presents the major results. A summary with conclusions is presented in Chapter 6.

Chapter 2

LITERATURE REVIEW

2.1 MACAULAY'S DURATION

Frederick Macaulay [20] developed the basic concept of *duration* in 1938 and presented it in his monumental study of interest rates and railroad bond prices. Duration was originally conceived by Macaulay as a better way to summarize the timing of bond flows than maturity. Maturity provides information only about the date of final payment. However, coupon bonds make regular payments before the repayment of principal at maturity. Therefore, maturity is an incomplete measure of the life (or length) of a coupon bond. Macaulay concluded that "...it would seem highly desirable to have some adequate measure of 'longness.' Let us use the word 'duration' to signify the essence of the time element in a loan. ...It is clear that 'number of years to maturity' is a most inadequate measure of 'duration'"[20; p.74].

Macaulay proposed that duration be computed as a weighted average of the time periods in which payments are made. In Macaulay's definition each period is weighted by the present value of the corresponding payment as a percent of the total present value of all the flows. Macaulay originally defined duration as:

$$D_{t_0} = \frac{\sum_{i=1}^{\infty} (t_i - t_0)\, c(t_i)\, q(t_i, t_0)}{\sum_{i=1}^{\infty} c(t_i)\, q(t_i, t_0)} \tag{1}$$

Where:

t_0 is the current date

$c(t_i)$ is the income stream at time t_i (coupon payments and principal)

$q(t_i, t_0)$ is the present value at time t_0 of $1 payable at time t_i, the 'discount function'

Macaulay, however, proceeded to simplify his original definition by using yield to maturity in the present value calculations, rather than the individual period discount rates. His resulting alternative definition is:

$$D2_{t_0} = \frac{\sum_{i=1}^{\infty} (t_i - t_0)\, c(t_i)\, e^{-y(t_i - t_0)}}{\sum_{i=1}^{\infty} c(t_i)\, e^{-y(t_i - t_0)}} \tag{2}$$

Where:

y is the continuously compounded yield-to-maturity on the bond

Definitions (1) and (2) are equivalent if and only if the interest rate is known to be constant over the life of the bond. "When the two measures differ, it is only the original (1) which is appropriate for risk measurement" [16; p.631]. Nevertheless, as Cox, Ingersoll, and Ross [8] point out, either definition has its advantages. D2 duration, which is referred to as "simple duration," can be computed knowing only the price of the bond and the contract payments, and therefore is most commonly used in practice. On the other hand, Fisher and Weil [12] have championed the reemergence of the original definition (1) for duration. Since it is a more valid risk measure, definition (1) will be used in this study, and it will henceforth be referred to as Macaulay/Fisher and Weil duration.

Duration, as defined in equations (1) or (2), is measured in units of time - e.g., in months or years. For zero coupon bonds, duration is equal to maturity. For all other bonds, duration is shorter than maturity. However, the relationship between duration and maturity is nonlinear and complex.

2.2 EARLY DEVELOPMENTS

Macaulay's duration measure attracted very little attention in 1938 and was not widely acclaimed. In fact, the concept of duration remained to be discovered, apparently independently, several times. As Bierwag, Kaufman, and Toevs mention, "Duration, and possibly the entire area of bond pricing, apparently had little appeal at the time" [2; p.16].

In 1939, J.R. Hicks independently developed equation (2), and discussed it in his major work, Value and Capital [14]. Hicks calculated the elasticity of the value of a stream of payments with respect to a discount factor, and referred to the resulting measure as "elasticity of capital value" and sometimes as "average period." He noted that his elasticity measure was not a pure number (like other elasticities), but was instead denominated in units of time. It is interesting that Hicks, who was searching for an elasticity, ended up deriving the same measure as Macaulay, who was searching for a measure of time. Hicks was the first to demonstrate the risk-proxying property of duration. He showed that for a given (infinitesimal) change in yield, the percentage change in an asset's value is proportional to its duration.[1]

In 1945, Paul Samuelson [24], who was apparently unaware of Hicks' or Macaulay's work, effectively derived equation (2). Samuelson analyzed the effect of interest rate changes on the capital values of institutions such as banks, insurance companies, and universities. He computed the first derivatives of the values of the inflows and outflows with respect to the yield to maturity and obtained a measure, essentially equivalent to duration, which he termed the "weighted average time period of payments." He proved that if the duration of an institution's assets is larger than that of its liabilities, then the institution will lose when interest rates rise and profit when interest rates fall [24; p.19].

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[1]This relationship is implicit in Macaulay's analysis of short and long-term interest rates, but is not rigorously developed [20; pp.50-51,61-62]. In fact, a close examination of Macaulay's book reveals that even he was primarily concerned with the risk-proxying properties of his measure despite the assigned name "duration."

In 1952, a British actuary by the name of F.M. Redington set out to determine what allocation of assets and liabilities would mitigate the effects of interest rate changes on life insurance companies' net worth. In the process, Redington [22] computed the first derivatives of the values of the inflows and outflows with respect to interest rates. These statistics were exactly equal to Macaulay's duration measure. Like Samuelson, Redington appeared to be completely unaware of the work of his predecessors. He called his statistic the "mean term" and introduced the term "immunization" to signify the investment of assets in such a way that a firm would be immune to a general change in interest rates. The essence of Redington's theory was to set the duration (or mean term) of the assets equal to that of the liabilities.

Redington's article was published in an actuarial journal, and as a result was not successful in attracting the attention of researchers and practitioners in the field of financial economics. In fact, the entire concept of duration was largely ignored until 1971, when Fisher and Weil [12] published their extensive empirical study entitled "Coping with the Risk of Interest-Rate Fluctuations." Fisher and Weil showed that duration could be used to immunize bond portfolios against interest rate risk. Fisher and Weil's article was widely read and, as a result, duration began to be recognized as an important tool for both managing and measuring interest rate risk.

2.3 DURATION AS A RISK PROXY

Fisher [11] and, shortly thereafter, Hopewell and Kaufman [15] discovered a direct relation between the duration of a bond and its price sensitivity to changes in market intest rates, and proposed that duration could therefore serve as a measure of price risk for bonds. Specifically, Hopewell and Kaufman showed that the derivative of a bond's price with respect to the yield to maturity is proportional to Macaulay/Fisher and Weil duration. For small yield changes and continuous compounding, bond price volatility could be related to changes in the yield to maturity by:

$$\frac{dP_{jt}}{P_{jt}} = -D_{jt}dr_{jt} \qquad (3)$$

Where:

P_{jt} is the initial price of bond j

dP_{jt} is the price change of bond j

D_{jt} is the duration of bond j at time t

dr_{jt} is the change in yield to maturity

Equation (3) states that a bond's price changes in inverse proportion to a change in the market yield to maturity, where the proportionality factor is duration. For a given change in the yield, the change in bond price will be relatively greater the longer the duration of the bond.

As Bierwag, Kaufman, and Khang [1] point out, equations (1) and (3) permit three widely used rules about bond pricing to be collapsed into one. These rules stipulate that for a given change in interest rates, the proportional change in the price of a bond will be greater:

1. the lower the coupon rate
2. the lower the market yield and
3. the longer the maturity.

From equation (1) it follows that the lower the coupon rate or the market yield and (for most bonds) the longer the maturity, the longer the duration. From equation (3), "The one complete and accurate rule of thumb is that the longer the duration, the greater the proportional price volatility for a given change in yield" [2; p.18]. To the extent that price volatility is viewed as price risk, duration may be viewed as an index of risk for a given change in yields.

However, Ingersoll, Skelton, and Weil [16] prove that Macaulay/Fisher and Weil duration is only a valid risk measure for shape-preserving shifts in the entire yield curve. That is, this comparative static property of duration is useful only to the extent that yield curve shifts are parallel over time. Cooper [7] notes that parallel shifts are somewhat unrealistic and are also inconsistent with equilibrium conditions in financial markets since they imply arbitrage profits.

Due to the highly restrictive assumptions needed to validate the notion of duration, the usefulness of duration has been increasingly questioned in recent years. In fact, duration has fallen from academic favor somewhat as more sophisticated equilibrium models of the term structure have been put forth by Brennan and Schwartz [4], Cox, Ingersoll, and Ross [8], and others. However, in a recent paper, Brennan and Schwartz [6] demonstrate quite clearly that in practical portfolio applications, the simple concept of duration performs as well as a highly technical equilibrium bond pricing model. Brennan and Schwartz conclude that "The duration model performs remarkably well by comparison with the more sophisticated equilibrium model, and in view of the computational cost of the latter, the duration model appears to be more useful for practical bond portfolio management" [6; p.6].

In recent years, many authors have expanded on the Macaulay/Fisher and Weil concept of duration as a surrogate for risk measurement and have developed a variety of new measures of duration. Khang [18], Cooper [7], and Bierwag, Kaufman, and Khang [1] have all proposed alternative measures of duration which they claim are theoretically superior to the traditional measure. Gultekin and Rogalski [13] empirically investigate seven different duration measures and their role in explaining price volatility caused by interest rate movements. Gultekin and Rogalski's results "...do not support the important testable implications of any of the various durations as measures of basis risk" [13; p.263]. In fact, despite the flood of articles claiming superiority for particular measures of duration, the measures examined were virtually indistinguishable empirically. Surprisingly, none of them did much better than maturity in explaining bond returns, and all duration measures proved to be inferior to simple factor models.

Chapter 3

TESTABLE IMPLICATIONS

3.1 HYPOTHESES ABOUT EXPECTED RETURNS

As Gultekin and Rogalski mention, a close examination of the duration literature yields several explicit hypotheses about duration's ability to serve as a proxy for price volatility, "...that is, as an index for cross-sectional comparisons of bond risk" [13; p.246].

The duration literature indicates that, under certain restrictive assumptions about yield curve shifts over time, Macaulay/Fisher and Weil duration is theoretically a good measure of the variability of bond returns. To the extent that price volatility is considered to be price risk, duration may be viewed as a measure of basis risk for bonds.[2] In a market of risk-averse investors this risk should be priced; that is, higher risk should be associated with higher expected return.

The duration literature also suggests strongly that duration is a complete measure of risk because it incorporates the effects of both couon and maturity differences on price volatility. Duration permits the three rules of bond pricing noted earlier to be collapsed into one

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[2]Langstein and Sharpe [19] suggest that measures of duration calculated ex-ante predict the expected reactions of bond prices to unexpected changes in interest rates. This, in effect, implies that duration is a risk measure similar to beta for common stocks.

and to stand without exception.

These important implications obtained from the duration literature give rise to the following three hypotheses about expected bond returns:

1. On average, there is a positive relationship between bond returns and duration. One would expect that, on average, default-free bonds with higher duration would have higher returns.
2. Duration is a complete measure of the risk of bond j. Duration incorporates the effect of coupon and maturity differences on price volatility.
3. The capital market for bonds is efficient; that is, bond pricing errors are unrelated to subsequent returns.

3.2 A STOCHASTIC MODEL FOR RETURNS

The three hypotheses mentioned earlier can be tested with actual data on government bonds (and portfolios of government bonds) for many time periods. It is important to choose a model of period-by-period returns that uses observed average returns to test the three expected-return hypotheses, but one that is nevertheless as general as possible. The following stochastic model for returns is therefore suggested:

$$\tilde{R}_{jt} - \tilde{rf}_t = \tilde{\gamma 0}_t + \tilde{\gamma 1}_t D_{jt} + \tilde{\gamma 2}_t C_{jt} + \tilde{\gamma 3}_t M_{jt} + \tilde{\gamma 4}_t E_{jt} + \tilde{e}_{jt} \tag{4}$$

Where:

$\tilde{R}_{jt}$ is the realized holding period return for bond j

$\tilde{rf}_t$ is the riskfree rate of return over the holding period

D_{jt} is the duration of bond j at time t

C_{jt} is the coupon on bond j

M_{jt} is the time to maturity of bond j, measured at time t

E_{jt} is the pricing error for bond j at time t (actual price of bond minus predicted price)

$\tilde{e}_{jt}$ is the disturbance term

Equation (4) allows $\tilde{\gamma}0_t$, $\tilde{\gamma}1_t$, $\tilde{\gamma}2_t$, $\tilde{\gamma}3_t$, and $\tilde{\gamma}4_t$ to vary stochastically from period to period. The disturbances ($\tilde{e}_{jt}$) are assumed to have mean zero and to be independent of all other variables in equation (4). The variables $\tilde{e}_{jt}$, $\tilde{\gamma}0_t$, $\tilde{\gamma}1_t$, $\tilde{\gamma}2_t$, $\tilde{\gamma}3_t$, and $\tilde{\gamma}4_t$ are assumed to follow approximately a multivariate normal distribution.

The first hypothesis, which concerns duration's ability to serve as a risk-proxy, implies that the expected value of the risk premium $\tilde{\gamma}1_t$ is positive.

The second hypothesis concerns the expected values of $\tilde{\gamma}2_t$ and $\tilde{\gamma}3_t$, the coefficients for the coupon rate and maturity. The coupon rate (C_{jt}) and maturity (M_{jt}) are included in the model as independent variables in order to test whether duration is a complete measure of risk. Completeness implies that $E(\tilde{\gamma}2_t)=0$ and $E(\tilde{\gamma}3_t)=0$.

The third hypothesis (market efficiency) implies that bond pricing errors are unrelated to subsequent returns, so that $E(\tilde{\gamma}4_t)$ equals zero. However, to the extent that pricing errors are attributable to temporary market disequilibria or defficiencies in the data, one should expect the coefficient to be negative.

As yet, no hypothesis has been presented about the intercept term, $\tilde{\gamma}0_t$, in (5). A reasonable hypothesis is that $E(\tilde{\gamma}0_t)$ would equal zero, particularly because excess return (over the risk-free rate) is the dependent variable in the model.

Chapter 4

METHODOLOGY

4.1 DATA

4.1.1 AMERICAN DATA

Tests are carried out using monthly price, coupon, and maturity data on United States Treasury bonds and notes for the twenty-year sample period December 1960 to December 1980. The data are obtained from the Center for Research in Security Prices (CRSP) Government Bond File, which contains month-end data on virtually all direct negotiable obligations of the U.S. Treasury. For each security, the CRSP Government Bond File contains:

1. Bid price at month end
2. Ask price at month end
3. Coupon rate (per cent per annum)
4. Accrued interest as of month end
5. Interest payable during month
6. Maturity date at time of issue

The sample is limited to U.S. Treasury bonds and notes with maturities between one and ten years. "Flower bonds," callable and deep discount bonds are excluded from the study.

The data for the risk-free rate are the yields on United States 91-day Treasury bills (taken from the Bank of Canada Review, Cansim Series 2545.1).

4.1.2 CANADIAN DATA

The data on Canadian Government securities for the twenty-year sample period December 1960 to December 1980 are obtained from the Wood Gundy Monthly Bond Price Database, which contains monthly (generally the last Thursday or Friday) price, maturity, and coupon information for a series of bonds that have been listed on published quote sheets.

The sample includes all Government of Canada bonds with maturities between one and ten years for which prices are available in the Wood Gundy Bond Database and which are neither callable nor exchangeable.

The data for the risk-free rate of interest are the yields on Government of Canada 91-day Treasury bills. The interest rate series are average yields at Thursday tender following the last Wednesday of each month from December 1960 to December 1980 (taken from the Bank of Canada Review, Cansim Series 2560.1).

4.2 REGRESSION MODEL

As mentioned earlier, the linearity, completeness, and efficiency hypotheses can be tested with actual Canadian and American market data for many time periods with the use of government bonds and portfolios of government bonds. More specifically, a regression model that uses observed price changes to test the three hypotheses is estimated monthly over the twenty-year period December 1960 to December 1980.

For each month of this period, the following cross-sectional regression--the empirical analog of equation (4)--is run:

$$R_{jt} - rf_t = \hat{\gamma}0_t + \hat{\gamma}1_t\hat{D}_{jt} + \hat{\gamma}2_tC_{jt} + \hat{\gamma}3_tM_{jt} + \hat{\gamma}4_t\hat{E}_{jt} + \hat{e}_{jt} \tag{5}$$

Where:

R_{jt} is the realized holding period return for bond j (the return on bond j from time t to time t+1)

rf_t is the risk-free rate of return over the holding period

$\hat{D}_{jt}$ is the Macaulay/Fisher and Weil duration of bond j at time t (calculated with error because the discount function must be estimated)

C_{jt} is the coupon on bond j

M_{jt} is the time to maturity of bond j, measured at time t

$\hat{E}_{jt}$ is the pricing error for bond j at time t (calculated as the actual price of the bond minus the predicted price)

The results from (5)--the time series of month-by-month values of the regression coefficients $\hat{\gamma}0_t$, $\hat{\gamma}1_t$, $\hat{\gamma}2_t$, $\hat{\gamma}3_t$, and $\hat{\gamma}4_t$ for the twenty-year period 1961-1980--are the inputs for the tests of hypotheses.

As yet, the length of the holding period has not been specified. It is useful to examine different holding periods, partly because the duration literature does not generally specify any particular holding period for the return-duration relation, and also because "...it is important to determine the robustness of duration over different holding periods." [13; p.252]. For these two reasons, analysis is carried out for holding periods of one month and three months. Thus, results are obtained when bond returns are calculated on a monthly basis and also when returns are calculated on a quarterly basis. In either case, the independent variables, including the price prediction error, are calculated as of the beginning of the observation interval.

Equation (5), which is estimated month-by-month using individual bond data, can also be estimated using portfolios of government bonds. To do this, securities are combined into portfolios each month according to maturity. Returns, durations, and maturities for the portfolios are obtained by equally weighting the assigned securities. The (Macaulay/ Fisher and Weil) duration of a portfolio is, quite conveniently, the weighted average of the durations of the bonds in the portfolio.

Two different equally weighted portfolio schemes are considered:

1. The first scheme places all bonds with maturities between 0 and 1 year in portfolio 1, between 1 and 2 years in portfolio 2, and so on until the tenth portfolio which contains bonds with maturities between 9 and 10 years. This scheme is used by Brennan and Schwartz [4] and [6].
2. The second scheme, which takes into account the scarcity of data in the 7 to 10 year maturity range, combines these three previous portfolios into a single one.

Both methods are included in the study in order to examine the sensitivity of the estimates of the coefficients to the particular portfolio scheme used.

4.3 COMPUTATIONAL DETAILS

4.3.1 RETURN COMPUTATIONS

The realized one-month holding period return for bond j is defined to be the price change in the bond plus interest divided by last month's price and is calculated as:

$$R_{jt} = \frac{(P_{j,t+1} + AI_{j,t+1}) - (P_{jt} + AI_{jt}) + I_{j,t+1}}{(P_{jt} + AI_{jt})} \quad (6)$$

Where:

P_{jt} is the average of bid-ask prices for bond j at the end of month t

AI_{jt} is the accrued interest on security j at the end of month t

$I_{j,t+1}$ is the interest paid on security j between the end of month t and the end of month t+1

Likewise, the realized three-month holding period return for bond j is defined to be the price change in the bond over the quarter plus interest divided by the price of the bond at the beginning of the quarter.

The risk-free rate of return is calculated as the return on government treasury bills over the relevant holding period.

4.3.2 DURATION COMPUTATIONS

The duration of each bond is calculated every month of the sample period according to the Macaulay/Fisher and Weil formula, equation (1), which computes present value weights using discount bond yields from the term structure. Application of the Macaulay/Fisher and Weil duration measure presupposes that the discount function is known. In reality, since there exist no discount government bonds and the set of coupon bonds is not complete,[3] the discount function for each period must be estimated. The discount function is therefore estimated each month of the sample period using data on all taxable government bonds (excluding "flower bonds," callable bonds, and deep discount bonds). The function is approximated by a continuously differentiable piecewise quadratic function as suggested by McCulloch [21].

4.3.3 ERROR COMPUTATIONS

The pricing error for bond j at time t, E_{jt}, is defined to be the actual price of the bond minus the predicted price of the bond. The predicted price of the bond at time t is calculated by discounting the bond's cash flows (interest and principal) using the discount

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[3] A complete set of coupon bonds would be said to exist if one bond matured each period in the future up to the horizon.

function estimated at time t.

Table 1 conveys information about the average magnitude of these pricing errors. The first line of the table shows the results of the predictions for each month of the sample period for all U.S. government bonds with maturities up to 10 years. The root mean square error (RMSE) is a statistic that provides a measure of the average absolute error (unlike the mean error, which allows positive and negative errors to be offsetting). The root mean square price prediction error for the whole period is $2.25 per $100 of par value. A predicted yield to maturity is also calculated each period based on the predicted bond price, and the root mean square error of this predicted yield is also reported. The root mean square yield prediction error for the whole period is 1.02 per cent. The balance of the table shows the results obtained for predictions in December of each year.

Table 2 contains the price and yield prediction errors for Canadian government bonds. The root mean square price prediction error for the 20-year sample period is $2.12 per $100 of par value. The corresponding root mean square yield prediction error is 0.80 per cent.

TABLE 1: U.S. Bond Price and Yield Predictions

	Number of Observations	Prices[a] Mean Error[b] ($)	Prices[a] RMSE ($)	Yields Mean Error (%)	Yields RMSE (%)
Full Period[c]	9168	-0.08	2.25	0.06	1.02
1961[d]	29	-0.04	1.54	0.02	0.58
1962	32	-0.03	1.20	0.02	0.48
1963	32	-0.02	1.07	0.01	0.37
1964	31	-0.03	1.10	0.02	0.42
1965	26	-0.03	1.04	0.02	0.43
1966	25	-0.07	1.72	0.05	0.66
1967	26	-0.05	1.63	0.02	0.69
1968	26	-0.04	1.16	0.02	0.51
1969	28	-0.14	2.80	0.10	1.27
1970	31	-0.21	3.53	0.16	1.43
1971	31	-0.26	4.15	0.17	1.56
1972	33	-0.20	3.43	0.11	1.32
1973	35	-0.19	3.53	0.14	1.51
1974	40	-0.22	3.65	0.16	1.55
1975	44	-0.02	1.43	0.01	0.70
1976	55	-0.03	1.50	0.02	0.68
1977	62	-0.02	1.39	0.02	0.76
1978	63	-0.03	1.62	0.02	0.83
1979	66	-0.04	1.88	0.03	1.15
1980	70	-0.17	3.03	0.13	1.69

a. Per $100 par value.
b. Actual - Predicted.
c. January/61-December/80.
d. December.

TABLE 2: Canadian Bond Price and Yield Predictions

	Number of Observations	Prices[a] Mean Error[b] ($)	Prices[a] RMSE ($)	Yields Mean Error (%)	Yields RMSE (%)
Full Period[c]	4109	-0.05	2.12	0.02	0.80
1961[d]	15	-0.08	1.79	0.07	0.43
1962	14	-0.05	1.31	0.03	0.34
1963	13	-0.04	1.20	0.05	0.45
1964	14	-0.01	0.92	0.01	0.50
1965	15	0.01	0.77	-0.03	0.39
1966	13	-0.02	0.77	0.02	0.40
1967	14	-0.02	0.86	0.02	0.36
1968	21	-0.03	1.41	0.02	0.67
1969	17	-0.04	1.71	0.04	0.80
1970	22	-0.04	1.83	0.03	0.68
1971	27	-0.03	1.73	-0.01	0.79
1972	22	-0.02	1.53	0.00	0.62
1973	19	0.05	1.72	0.05	0.70
1974	15	0.01	1.95	-0.09	1.26
1975	15	-0.03	1.32	0.05	0.66
1976	15	-0.08	2.28	0.04	0.90
1977	19	-0.02	2.19	-0.08	1.11
1978	22	-0.09	2.39	0.05	1.00
1979	24	-0.15	2.97	0.08	1.25
1980	20	-0.11	2.76	0.06	1.47

a. Per $100 par value.
b. Actual - Predicted.
c. January/61-December/80.
d. December.

4.4 TESTS OF HYPOTHESES

The three hypotheses are tested by constructing the means of the time series of the coefficients of equation (5) and calculating the t-statistics in the manner first suggested by Fama and MacBeth [10] in a related context. This method has also been used by Brennan and Schwartz [4] and [5], and by Gultekin and Rogalski [13]. Essentially, a time series for $\hat{\gamma}$ is created "...in order to correct for any statistical biases in obtaining the period-by-period estimates of $\tilde{\gamma}i$." [13; p.249]. A series of cross-sectional regressions are run, thereby generating five series of estimates $\hat{\gamma}i_t$ $i=0,\ldots,4$. The sample means are taken as the final estimates of $\gamma 0$, $\gamma 1$, $\gamma 2$, $\gamma 3$, and $\gamma 4$, and standard errors and t-statistics are computed as if the series are random samples (independently and identically distributed over time).

The t-statistics for testing the hypothesis that $E(\gamma i)=0$ are computed by taking the ratio of the average $\bar{\hat{\gamma}}i$ times the square root of the number of months in the sample period considered over the standard deviation of the monthly estimate. These t-statistics are:

$$t(\bar{\hat{\gamma}}i) = \frac{\bar{\hat{\gamma}}i\ \sqrt{n}}{s(\hat{\gamma}i)} \tag{7}$$

Where:

n is the number of months in the period, which is also the number of estimates $\hat{\gamma}i_t$ used to compute $\bar{\hat{\gamma}}i$ and $s(\hat{\gamma}i)$

$\bar{\hat{\gamma}}i$ is the average of the month-by-month regression coefficients

$s(\hat{\gamma}i)$ is the standard deviation of the monthly estimates

If successive values of $\hat{\gamma}i_t$ are independently and identically distributed normal random variables, the t-statistic of (7) is a drawing from the student distribution with n-1 degrees of freedom. Since the shortest subperiod in this study is five years, n-1 is always greater than 59 and the student distribution is well approximated by the unit normal distribution.

4.5 LIMITATIONS OF METHODOLOGY

The regression procedure in this study is essentially the method proposed and used by Fama and MacBeth [10] in a related context. The Fama-MacBeth procedure, which involves running a series of cross-sectional regressions, is equivalent to joint generalized least squares (GLS) on the whole system if in each cross-sectional regression:

1. the return processes are independent; that is, there is no cross-correlation of returns, and if
2. the independent variables are not measured with error.

The first assumption, like any assumption, is not a completely accurate description of the world. In fact, it is rather unrealistic to assume that the return processes are independent, given that changes in the yield curve tend to have a somewhat similar effect on all bonds. To the extent that the assumption is inaccurate, the Fama-MacBeth approach will produce estimates of the regression parameters which are not efficient, but which are nevertheless unbiased.

If the second assumption is violated (that is, if any independent variables are measured with error), an "errors-in-variables problem" arises. In intuitive terms, the problem centers on the fact that if a proxy explanatory variable is used in a least squares regression, the computed coefficients do not have the same properties as if the true explanatory variable were used. Unfortunately, two of the independent variables in the linear regression model specified in equation (5) are measured with error. Since the discount function must be estimated each month, both the duration calculations and the pricing error calculations involve measurement error. As is well known, the OLS estimator of the regression parameters is biased under these circumstances. The two possible solutions to this problem involve using correction factors or iterative procedures, but these methods will not be employed in this study. At any rate, the discount function will in all likelihood be estimated with minimal error because, as McCulloch [21] points out, the discount function is never extrapolated beyond the maturities of the securities observed. However, any estimation error that does exist will tend to influence all the coefficients in the model because the duration measure is derived from the coupon and maturity values.

Chapter 5

RESULTS

Results are presented for seven periods: the overall period January 1961 to December 1980; two 10-year subperiods, January 1961 to December 1970 and January 1971 to December 1980; and four subperiods starting in January 1961 and covering five years each. Concise summary tables are reported, with other results highlighted in the discussion.

Results are presented for three different versions of the return-duration regression equation (5): the first version suppresses three of the variables in (5); the second version suppresses two of the variables in (5); and the third version is based on equation (5) exactly.

5.1 AMERICAN RESULTS

The cross-sectional regression results for U.S. government bonds are presented in Table 3. For each period and model Table 3 shows the average of the (1-month holding period) regression coefficient estimates, $\bar{\bar{\gamma}}i$. The table also shows t-statistics for testing the hypothesis that $E(\gamma i)=0$.

The results in Table 3 reveal that over the 20-year period, or any subperiod, and for any model, the value of $t(\bar{\bar{\gamma}}1)$ is not large. Thus, there is no evidence of a significant relationship between bond returns (measured on a monthly basis) and Macaulay/Fisher and Weil duration. It is surprising that although $\bar{\bar{\gamma}}1$ is not significantly different from zero it is negative on average.

From Table 3 it can also be observed that the test statistic for the maturity coefficient, $t(\bar{\bar{\gamma}}3)$, is generally very small. Therefore, it appears that duration incorporates the effect of maturity differences on price volatility.

For most time periods, however, the t-value for the coupon coefficient, $t(\bar{\bar{\gamma}}2)$, is large and positive. There appears to be a statistically observable positive relationship between coupon rate and bond returns. The positive sign of $t(\bar{\bar{\gamma}}2)$ is surprising because it is not consistent with the premise that coupon is a measure of risk.[4] A possible explanation for the positive sign is that the coupon rate as an independent variable is essentially capturing a tax effect. Low coupon bonds have a large amount of their return coming in the form of a capital gain, which is taxed at a lower rate. One would therefore expect that the return on high coupon bonds would be higher to compensate for the fact that taxes would be higher.

The results reported in Table 3 also reveal that for the whole period and for the first 10-year subperiod a significant relation exists between the pricing error and the rate of return over the next interval. These results, which tend to suggest that market inefficiencies exist, are, however, sensitive to the inclusion of the coupon

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[4]Other things being equal, price volatility is lower for higher coupon bonds, indicating that if coupon were a measure of risk the coefficient would be negative

TABLE 3: Cross-Sectional Regression Results for U.S. Bonds

(taxable bonds with maturities less than 10 years; t-ratios in parentheses)

Holding Period = One Month

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1980	0.313 (0.80)	-0.208 (-1.14)			
	0.340 (0.88)	-0.213 (-1.16)			0.001 (0.02)
	-1.958 (-1.78)	0.027 (0.18)	0.797 (2.93)	-0.216 (-1.68)	-0.237 (-2.24)
January 1961 to December 1970	0.507 (2.06)	-0.170 (-0.90)			
	0.511 (2.06)	-0.169 (-0.89)			0.079 (0.87)
	-0.645 (-1.44)	0.109 (0.84)	0.629 (3.25)	-0.240 (-1.48)	-0.168 (-2.29)
January 1971 to December 1980	0.116 (0.15)	-0.247 (-0.79)			
	0.168 (0.23)	-0.258 (-0.81)			-0.077 (-0.55)
	-3.282 (-1.52)	-0.055 (-0.20)	0.967 (1.89)	-0.180 (-0.93)	-0.306 (-1.54)

Note: $\bar{\hat{\gamma}}0$, $\bar{\hat{\gamma}}1$, $\bar{\hat{\gamma}}2$, $\bar{\hat{\gamma}}3$, $\bar{\hat{\gamma}}4$ are the arithmetic means of the monthly cross-sectional estimates. The $\hat{\gamma}i$ numbers have been multiplied by 1000. The t-values (shown in parentheses) test the null hypothesis that the mean values are zero.

(cont. over)

TABLE 3 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1965	0.155 (0.44)	-0.026 (-0.21)			
	0.113 (0.43)	-0.026 (-0.20)			0.050 (0.56)
	-0.478 (-1.16)	0.046 (0.35)	0.401 (2.42)	-0.084 (-0.90)	-0.105 (-1.31)
January 1966 to December 1970	0.900 (2.17)	-0.314 (-0.88)			
	0.908 (2.18)	-0.313 (-0.87)			0.108 (0.67)
	-0.812 (-1.01)	0.171 (0.77)	0.857 (2.45)	-0.408 (-1.29)	-0.232 (-1.88)
January 1971 to December 1975	0.511 (0.66)	-0.103 (-0.31)			
	0.515 (0.68)	-0.094 (-0.28)			-0.022 (-0.13)
	-1.765 (-0.96)	0.069 (0.20)	0.771 (1.42)	-0.156 (-0.57)	-0.266 (-1.73)
January 1976 to December 1980	-0.900 (-0.22)	-0.393 (-0.73)			
	-0.186 (-0.15)	-0.425 (-0.79)			-0.133 (-0.58)
	-4.826 (-1.22)	-0.182 (-0.41)	1.166 (1.33)	-0.204 (-0.74)	-0.346 (-0.93)

Note: the $\bar{\hat{\gamma}}i$ values have been multiplied by 1000.

and maturity variables.

Table 4 shows the first order autocorrelations, $\hat{\rho}(\gamma i)$, of the various monthly γi_t.[5] From Table 4 it can be seen that the serial correlations $\hat{\rho}(\gamma 0)$, $\hat{\rho}(\gamma 1)$, $\hat{\rho}(\gamma 2)$, $\hat{\rho}(\gamma 3)$, and $\hat{\rho}(\gamma 4)$ are often greater than zero. The autocorrelations appear most significant for $\hat{\gamma}4$ and least significant for $\hat{\gamma}1$.

As mentioned earlier, $\bar{\hat{\gamma}}1$ does not appear to be significantly positive. One possible explanation for $\hat{\gamma}1$ being negative on average is that interest rates have trended upwards over most of the sample period. In any given period $\hat{\gamma}1$ will be influenced by changes in interest rates in the economy, so that in periods of rising interest rates one would expect $\hat{\gamma}1$ to be negative on average. Hypothesis 1 should hold on average (that is, over repeated experiments) if interest rates both rise and fall but average to zero.

In order to determine what portion of $\hat{\gamma}1$ is not explained by changes in interest rates, the month-by-month values of $\hat{\gamma}1$ are regressed on changes in the interest rate. The intercept term in such a regression can be interpreted as that portion of $\hat{\gamma}1$ which is not determined by changes in the level of interest rates. One would anticipate that the intercept would have a positive sign and that the coefficient for the change in the level of interest rates would be equal to -1.0.

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[5]Autocorrelations for lags greater than one have been computed but are not reported here.

TABLE 4: Estimated First Order Autocorrelations of the Monthly Cross-Sectional Parameter Estimates

(U.S. bonds with maturities less than 10 years)

Holding Period = One Month

	Statistic				
Period	$\hat{\rho}(\gamma 0)$	$\hat{\rho}(\gamma 1)$	$\hat{\rho}(\gamma 2)$	$\hat{\rho}(\gamma 3)$	$\hat{\rho}(\gamma 4)$
1/61-12/80	-.097	.042			
	-.091	.033			-.394*
	-.451*	-.132*	-.400*	-.145*	-.419*
1/61-12/70	.070	.071			
	.061	.055			-.395*
	-.104	-.093	-.258*	-.191*	-.210*
1/71-12/80	-.122	.031			
	-.144	.025			-.401*
	-.477*	-.141	-.423*	-.095	-.451*
1/61-12/65	-.140	.230*			
	-.137	.232*			-.111
	-.018	-.098	-.085	-.079	.079
1/66-12/70	.122	.038			
	.107	.021			-.490*
	-.136	-.091	-.310*	-.217	-.355*
1/71-12/75	-.183	-.120			
	-.187	-.135			-.296*
	-.409*	-.364*	-.246*	-.097	-.321*
1/76-12/80	-.085	.092			
	-.072	.093			-.463*
	-.498*	.009	-.498*	-.095	-.478*

*Significant at the 0.05 level.

The following regression models are estimated:

$$\hat{\gamma}1_t = a + b_1 \Delta YL_t \tag{8}$$

and

$$\hat{\gamma}1_t = a + b_1 \Delta YL_t + b_2 \Delta YS_t \tag{9}$$

Where:

$\hat{\gamma}1_t$ is the estimated value of $\gamma 1$ for month t (obtained from the cross-sectional regression for month t which contains only an intercept term and duration)

ΔYL_t is the corresponding change in the yield on long-term U.S. Government bonds

ΔYS_t is the corresponding change in the U.S. Treasury bill rate

The regression coefficients and t-statistics are reported below in Table 5.

TABLE 5: Estimated Coefficients and T-statistics from Regressing $\hat{\gamma}1$ on the Corresponding Changes in the Long and Short Rates of Interest

	a	b_1	b_2
Estimated Coefficient	0.00597	-0.80974	
t-statistic	(0.41)	(-12.05)	
	R^2=0.3798	F=145.13	N=239
Estimated Coefficient	0.00600	-0.99096	0.10668
t-statistic	(0.42)	(-11.68)	(3.38)
	R^2=0.4085	F=81.488	N=239

From the results in Table 5 it can be seen that the intercept term is positive, as predicted, although it is not significant, and that the coefficient for the change in the long rate is close to one and highly significant. These results suggest that, when changes in the level of interest rates have been accounted for, there is possibly a positive relationship between bond returns and duration.

The analysis summarized in Table 3 has also been carried out for a holding period of three months. Surprisingly, the length of the holding period turns out not to affect the basic conclusions. Table 6 contains the 3-month holding period regression coefficient estimates, and Table 7 shows the estimated first order autocorrelations of the various monthly $\hat{\gamma}i_t$ when the holding period is three months. It should be noted at the outset that returns for overlapping 3-month holding periods have been used in the regressions. This implies that the $\hat{\gamma}i$ will not be independent over time (as can be seen from the first-order autocorrelations in Table 7), and hence that the t-statistics in Table 6 cannot be interpreted as being correct. To a certain extent, this problem is corrected for later when the means and standard errors of the 3-month holding period regression coefficients, $\hat{\gamma}i$, are recalculated under the assumption that each of the $\hat{\gamma}i$ follows an ARIMA process (see Tables 14, 15, and 16).

The results contained in Table 6 reveal that over most time periods, and for most models, the value of $\bar{\hat{\gamma}}1$ is quite small. Hence, there is little evidence of a significant (positive) relationship between duration and bond returns. During some time periods $\bar{\hat{\gamma}}1$ is, in fact, negative (although insignificantly so). As mentioned earlier, a possible explanation for the negative sign is the upward movement in interest rates over most of the sample period.

A comparison of the results in Tables 3 and 6 also reveals that there are great similarities between the one-month holding period and three-month holding period regression coefficients for coupon, maturity, and pricing error. Table 6 reveals that for the overall period 1961-1980, and for all subperiods, the value of $\bar{\hat{\gamma}}2$, the coupon coefficient, is large and systematically positive for all models. As in the one-month holding period case, the coupon coefficient is perhaps capturing a tax-effect. Table 6 also shows that, regardless of the model or the time period examined, the values of $\bar{\hat{\gamma}}3$ are consistently negative. Thus, it is possible that duration does not incorporate the effect of maturity differences on price volatility. Finally, evidence in Table 6 suggests that pricing errors are related in a systematic fashion to subsequent bond returns. For the third model and for all time periods the coefficient for the pricing error is large and negative.

From Table 7, it can be seen that the serial correlations $\hat{\rho}(\gamma 0)$, $\hat{\rho}(\gamma 1)$, $\hat{\rho}(\gamma 2)$, $\hat{\rho}(\gamma 3)$, and $\hat{\rho}(\gamma 4)$ are in the majority of cases significantly greater than zero. This is undoubtedly due to the use of overlapping data.

TABLE 6: Cross-Sectional Regression Results for U.S. Bonds

(taxable bonds with maturities less than 10 years; t-ratios in parentheses)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$	$\bar{\hat{\gamma}}_3$	$\bar{\hat{\gamma}}_4$
January 1961 to December 1980	1.487 (2.66)	-0.764 (-2.42)			
	1.480 (2.64)	-0.758 (-2.40)			0.015 (0.16)
	-4.947 (-4.92)	0.132 (0.60)	2.171 (8.10)	-0.732 (-4.21)	-0.606 (-6.46)
January 1961 to December 1970	1.349 (3.28)	-0.344 (-0.97)			
	1.335 (3.20)	-0.337 (-0.96)			0.270 (2.16)
	-1.301 (-2.02)	0.248 (1.09)	1.535 (6.21)	-0.540 (-2.42)	-0.329 (-3.42)
January 1971 to December 1980	1.629 (1.55)	-1.195 (-2.28)			
	1.629 (1.55)	-1.190 (-2.26)			-0.248 (-1.83)
	-8.687 (-4.65)	0.013 (0.04)	2.823 (5.97)	-0.936 (-3.46)	-0.891 (-5.61)

Note: the $\bar{\hat{\gamma}}_i$ values have been multiplied by 1000.

(cont. over)

TABLE 6 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.429	-0.142			
	(1.14)	(-0.55)			
	0.406	-0.136			0.130
	(1.07)	(-0.53)			(1.09)
	-1.601	0.195	1.330	-0.324	-0.371
	(-2.64)	(0.99)	(5.76)	(-2.23)	(-2.82)
January 1966 to December 1970	2.269	-0.546			
	(3.17)	(-0.82)			
	2.263	-0.539			0.411
	(3.11)	(-0.82)			(1.87)
	-1.000	0.301	1.739	-0.756	-0.288
	(-0.87)	(0.73)	(3.97)	(-1.78)	(-2.03)
January 1971 to December 1975	1.943	-0.581			
	(1.86)	(-1.17)			
	1.927	-0.556			0.011
	(1.85)	(-1.10)			(0.05)
	-3.674	0.208	1.914	-0.660	-0.588
	(-2.06)	(0.56)	(3.15)	(-1.75)	(-3.71)
January 1976 to December 1980	1.300	-1.843			
	(0.69)	(-1.96)			
	1.316	-1.857			-0.521
	(0.70)	(-1.98)			(-3.03)
	-13.964	-0.191	3.781	-1.212	-1.209
	(-4.34)	(-0.28)	(5.28)	(-3.18)	(-4.39)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 7: Estimated First Order Autocorrelations of the Monthly Cross-Sectional Parameter Estimates

(U.S. bonds with maturities less than 10 years)

Holding Period = Three Months

	Statistic				
Period	$\hat{\rho}(\gamma 0)$	$\hat{\rho}(\gamma 1)$	$\hat{\rho}(\gamma 2)$	$\hat{\rho}(\gamma 3)$	$\hat{\rho}(\gamma 4)$
1/61-12/80	.558*	.635*			
	.560*	.631*			.298*
	.253*	.384*	.159*	.509*	.159*
1/61-12/70	.611*	.685*			
	.610*	.679*			.329*
	.453*	.605*	.217*	.535*	.409*
1/71-12/80	.551*	.605*			
	.554*	.601*			.219*
	.176*	.306*	.111	.480*	.015
1/61-12/65	.513*	.629*			
	.511*	.627*			.421*
	.534*	.596*	.432*	.568*	.478*
1/66-12/70	.617*	.692*			
	.617*	.685*			.287*
	.425*	.608*	.149	.526*	.299*
1/71-12/75	.612*	.597*			
	.623*	.579*			.292*
	.528*	.080	.467*	.481*	.345*
1/76-12/80	.532*	.595*			
	.532*	.595*			.036
	-.004	.371*	-.214	.470*	-.146

*Significant at the 0.05 level.

The analysis in Table 3 (one-month holding period) and Table 6 (three-month holding period) has been repeated for portfolios of Treasury securities. As mentioned earlier, two alternative equally-weighted portfolio schemes are considered. Table 8 reports the results obtained when portfolio selection method 1 is used, and Table 9 reports the results obtained using portfolio selection method 2. In both Table 8 and Table 9 bond returns are calculated on a monthly basis. In Tables 10 and 11 bond returns are calculated on a quarterly basis.

The portfolio selection method appears to be unimportant when the holding period is one month, since the results in Table 8 are very similar to those in Table 9. It seems that over the 20-year sample period there is no evidence of a significant relation between one-month portfolio returns and Macaulay/Fisher and Weil duration. On the positive side, there is no evidence to suggest that a significant relation exists between any of the other independent variables and portfolio returns.

The results are somewhat different when the holding period is chosen to be three months, in that the portfolio selection method has an apparent impact (see Tables 10 and 11). When portfolio selection method 1 is used, the average value of the coefficient for duration, $\bar{\gamma}1$, is insignificant, but the coefficients for the other independent variables are, on average, quite large (particularly in the subperiod 1971-1980). However, when the second selection method is

TABLE 8: Cross-Sectional Regression Results for Portfolios of U.S. Bonds

(portfolios selection method 1 used; t-ratios in parentheses)

Holding Period = One Month

Period	Intercept $\bar{\bar{\gamma}}0$	Duration $\bar{\bar{\gamma}}1$	Coupon $\bar{\bar{\gamma}}2$	Maturity $\bar{\bar{\gamma}}3$	Pr.Error $\bar{\bar{\gamma}}4$
January 1961 to December 1980	0.265	-0.218			
	(0.58)	(-1.30)			
	0.197	-0.185			0.343
	(0.44)	(-1.08)			(1.33)
	-3.774	0.038	1.104	-0.157	-0.183
	(-1.50)	(0.18)	(1.47)	(-0.91)	(-0.59)
January 1961 to December 1970	0.476	-0.177			
	(1.58)	(-0.97)			
	0.502	-0.153			1.138
	(1.75)	(-0.79)			(2.60)
	-0.030	0.061	0.319	-0.157	0.267
	(-0.03)	(0.23)	(0.53)	(-0.65)	(0.60)
January 1971 to December 1980	0.053	-0.258			
	(0.06)	(-0.92)			
	-0.111	-0.218			-0.459
	(-0.13)	(-0.76)			(-1.83)
	-7.518	0.015	1.890	-0.156	-0.633
	(-1.54)	(0.05)	(1.37)	(-0.64)	(-1.44)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 8 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.067 (0.22)	-0.023 (-0.20)			
	0.129 (0.43)	-0.040 (-0.31)			0.131 (0.47)
	-0.725 (-0.72)	0.035 (0.27)	0.591 (1.03)	-0.089 (-0.89)	-0.396 (-1.43)
January 1966 to December 1970	0.885 (1.70)	-0.332 (-0.96)			
	0.875 (1.79)	-0.267 (-0.73)			2.145 (2.65)
	0.676 (0.31)	0.088 (0.17)	0.042 (0.04)	-0.227 (-0.47)	0.288 (1.12)
January 1971 to December 1975	0.429 (0.52)	-0.080 (-0.22)			
	0.070 (0.08)	0.009 (0.03)			-0.799 (-2.07)
	1.842 (0.48)	0.246 (0.60)	-0.464 (-0.36)	-0.181 (-0.53)	-0.259 (-0.57)
January 1976 to December 1980	-0.329 (-0.22)	-0.439 (-1.02)			
	-0.296 (-0.20)	-0.448 (-1.00)			-0.114 (-0.36)
	-17.035 (-1.91)	-0.220 (-0.43)	4.284 (1.77)	-0.131 (-0.37)	-1.014 (-1.34)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 9: Cross-Sectional Regression Results for Portfolios of U.S. Bonds

(portfolio selection method 2 used; t-ratios in parentheses)

Holding Period = One Month

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1980	0.302 (0.70)	-0.225 (-1.23)			
	0.203 (0.47)	-0.181 (-0.98)			0.372 (1.45)
	-1.267 (-0.51)	-0.131 (-0.41)	0.431 (0.57)	-0.029 (-0.14)	-0.006 (-0.02)
January 1961 to December 1970	0.476 (1.55)	-0.179 (-0.98)			
	0.495 (1.72)	-0.154 (-0.80)			1.117 (2.58)
	0.131 (0.11)	0.024 (0.09)	0.212 (0.34)	-0.122 (-0.47)	0.274 (0.61)
January 1971 to December 1980	0.126 (0.16)	-0.272 (-0.85)			
	-0.092 (-0.11)	-0.209 (-0.66)			-0.380 (-1.48)
	-2.665 (-0.55)	-0.285 (-0.50)	0.651 (0.47)	0.064 (0.20)	-0.287 (-0.62)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 9 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.068 (0.21)	-0.026 (-0.22)			
	0.116 (0.38)	-0.040 (-0.32)			0.088 (0.35)
	-0.404 (-0.36)	-0.038 (-0.17)	0.378 (0.58)	-0.018 (-0.09)	-0.382 (-1.24)
January 1966 to December 1970	0.885 (1.70)	-0.332 (-0.96)			
	0.875 (1.79)	-0.267 (-0.73)			2.145 (2.65)
	0.676 (0.31)	0.088 (0.17)	0.042 (0.04)	-0.019 (-0.47)	0.941 (1.12)
January 1971 to December 1975	0.553 (0.66)	-0.126 (-0.34)			
	0.179 (0.21)	-0.026 (-0.07)			-0.950 (-2.20)
	3.573 (0.93)	-0.141 (-0.33)	-0.951 (-0.73)	0.098 (0.27)	-0.286 (-0.67)
January 1976 to December 1980	-0.308 (-0.22)	-0.420 (-0.80)			
	-0.369 (-0.26)	-0.394 (-0.76)			0.200 (0.79)
	-9.009 (-1.01)	-0.432 (-0.40)	2.281 (0.92)	0.029 (0.06)	-0.287 (-0.34)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 10: Cross-Sectional Regression Results for Portfolios of U.S. Bonds

(portfolio selection method 1 used; t-ratios in parentheses)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1980	1.047	-0.690			
	(1.50)	(-2.38)			
	1.246	-0.743			0.355
	(1.82)	(-2.46)			(1.15)
	-5.313	0.304	2.286	-0.785	-0.814
	(-2.17)	(1.37)	(3.08)	(-3.75)	(-2.84)
January 1961 to December 1970	1.194	-0.339			
	(2.38)	(-1.06)			
	1.061	-0.291			0.797
	(2.21)	(-0.84)			(1.79)
	-1.504	0.237	1.662	-0.457	-0.626
	(-1.20)	(0.98)	(2.56)	(-1.65)	(-1.69)
January 1971 to December 1980	0.896	-1.051			
	(0.68)	(-2.16)			
	1.436	-1.207			-0.099
	(1.10)	(-2.43)			(-0.24)
	-9.254	0.372	2.931	-1.123	-1.009
	(-1.93)	(1.00)	(2.16)	(-3.60)	(-2.29)

Note: the the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 10 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1965	0.327 (0.81)	-0.159 (-0.76)			
	0.232 (0.56)	-0.129 (-0.53)			0.078 (0.18)
	-3.517 (-2.54)	0.268 (1.76)	2.689 (3.23)	-0.458 (-2.51)	-1.215 (-3.61)
January 1966 to December 1970	2.061 (2.27)	-0.519 (-0.85)			
	1.891 (2.20)	-0.452 (-0.69)			1.517 (1.96)
	0.508 (0.25)	0.207 (0.45)	0.635 (0.65)	-0.457 (-0.87)	-0.036 (-0.05)
January 1971 to December 1975	1.410 (1.18)	-0.419 (-0.75)			
	2.266 (1.82)	-0.667 (-1.21)			0.294 (0.42)
	-2.531 (-0.54)	0.766 (1.55)	1.613 (1.04)	-0.997 (-2.27)	-0.882 (-1.45)
January 1976 to December 1980	0.355 (0.15)	-1.716 (-2.14)			
	0.562 (-0.24)	-1.776 (-2.12)			-0.513 (-1.10)
	-16.456 (-1.93)	-0.049 (-0.09)	4.344 (1.92)	-1.259 (-2.81)	-1.143 (-1.78)

Note: the $\bar{\gamma}i$ values have been multiplied by 1000.

TABLE 11: Cross-Sectional Regression Results for Portfolios of U.S. Bonds

(portfolio selection method 2 used; t-ratios in parentheses)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1980	1.401	-0.757			
	(2.29)	(-2.41)			
	1.524	-0.788			0.437
	(2.48)	(-2.47)			(1.38)
	0.077	-0.207	0.518	-0.361	-0.237
	(0.03)	(-0.46)	(0.61)	(-1.37)	(-0.72)
January 1961 to December 1970	1.232	-0.351			
	(2.43)	(-1.09)			
	1.088	-0.302			0.741
	(2.25)	(-0.87)			(1.59)
	-0.207	-0.129	0.910	-0.156	-0.170
	(-0.15)	(-0.43)	(1.27)	(-0.50)	(-0.43)
January 1971 to December 1980	1.574	-1.173			
	(1.40)	(-2.16)			
	1.971	-1.287			0.125
	(1.73)	(-2.40)			(0.29)
	0.366	-0.287	0.119	-0.569	-0.304
	(0.07)	(-0.33)	(0.08)	(-1.34)	(-0.58)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 11 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1965	0.403 (0.94)	-0.183 (-0.84)			
	0.285 (0.66)	-0.152 (-0.62)			-0.035 (-0.70)
	-0.744 (-0.46)	-0.507 (-1.36)	1.056 (1.02)	0.212 (0.63)	-0.290 (-0.66)
January 1966 to December 1970	2.061 (2.27)	-0.519 (-0.85)			
	1.891 (2.20)	-0.452 (-0.69)			1.517 (1.96)
	0.370 (0.18)	0.255 (0.55)	0.761 (0.77)	-0.531 (-1.00)	-0.048 (-0.07)
January 1971 to December 1975	1.902 (1.60)	-0.535 (-0.93)			
	2.771 (2.24)	-0.794 (-1.43)			0.075 (0.10)
	6.649 (1.32)	-0.206 (-0.35)	-1.408 (-0.82)	-0.218 (-0.42)	-0.131 (-0.20)
January 1976 to December 1980	1.229 (0.63)	-1.845 (-1.97)			
	1.129 (0.58)	-1.805 (-1.93)			0.177 (0.42)
	-6.247 (-0.66)	-0.372 (-0.22)	1.726 (0.66)	-0.938 (-1.37)	-0.486 (-0.59)

Note: the $\bar{\hat{\gamma}}i$ values have been multiplied by 1000.

used, the values of $t(\bar{\gamma}1)$, $t(\bar{\gamma}2)$, $t(\bar{\gamma}3)$, and $t(\bar{\gamma}4)$ are all fairly small.

5.2 CANADIAN RESULTS

The cross-sectional regression results for Canadian government bonds are presented in Tables 12 and 13. Table 12 reports the results obtained when bond returns are calculated on a monthly basis; Table 13 reports the results obtained using quarterly returns. It is interesting to note that there is no apparent difference between the Canadian and American results over the sample period.

When returns are measured on a monthly basis the value of $t(\bar{\gamma}1)$ is not large during any subperiod for any model. Therefore, there is no significant evidence to suggest that, on average, higher duration bonds earn higher returns. In addition, for most time periods the value of $\bar{\gamma}2$ is significant and positive in sign. Once again, this could be attributed to a tax effect. Finally, the results reported in Table 12 reveal that for many time periods a significant relation exists between the pricing error and the rate of return over the next interval. This result is not sensitive to the inclusion of the coupon or maturity variables.

When returns are measured on a quarterly basis, the basic results are the same. The coefficient for maturity is on average insignificant, and the average value of the coupon coefficient, $\bar{\gamma}2$, is once again positive (and significantly so in some subperiods). As far as Hypothesis 1

TABLE 12: Cross-Sectional Regression Results for Canadian Bonds

(taxable bonds with maturities less than 10 years; t-ratios in parentheses)

Holding Period = One Month

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1980	-0.169	0.087			
	(-0.20)	(0.25)			
	0.184	-0.034			-0.333
	(0.32)	(-0.13)			(-2.57)
	-4.214	-0.123	1.723	0.004	-0.555
	(-2.23)	(-0.50)	(3.07)	(0.27)	(-3.77)
January 1961 to December 1970	0.425	0.090			
	(1.11)	(0.43)			
	0.415	0.087			-0.178
	(1.08)	(0.41)			(-2.34)
	-4.648	0.188	2.302	-0.013	-0.640
	(-3.91)	(0.97)	(4.76)	(-1.02)	(-5.26)
January 1971 to December 1980	-0.767	0.084			
	(-0.46)	(0.13)			
	-0.049	-0.156			-0.490
	(-0.04)	(-0.31)			(-1.97)
	-3.775	-0.436	1.140	0.021	-0.468
	(-1.05)	(-0.96)	(1.12)	(0.79)	(-1.74)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 12 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.329 (0.66)	0.129 (0.64)			
	0.321 (0.63)	0.124 (0.60)			-0.170 (-1.64)
	-3.737 (-2.74)	0.150 (0.80)	2.381 (3.36)	-0.013 (-0.96)	-0.668 (-3.94)
January 1966 to December 1970	0.521 (0.89)	0.050 (0.14)			
	-0.509 (-0.87)	0.050 (0.13)			-0.186 (-1.65)
	-5.560 (-2.85)	0.226 (0.66)	2.223 (3.35)	-0.012 (-0.58)	-0.613 (-3.48)
January 1971 to December 1975	-1.091 (-0.34)	0.664 (0.55)			
	0.281 (0.14)	0.206 (0.24)			-0.911 (-1.88)
	-4.477 (-0.64)	-0.641 (-0.80)	1.715 (0.86)	0.052 (1.16)	-0.708 (-1.35)
January 1976 to December 1980	-0.438 (-0.59)	-0.507 (-1.04)			
	-0.384 (-0.52)	-0.525 (-1.08)			-0.061 (-0.98)
	-3.062 (-2.15)	-0.228 (-0.53)	0.555 (1.46)	-0.012 (-0.48)	-0.225 (-1.98)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 13: Cross-Sectional Regression Results for Canadian Bonds

(taxable bonds with maturities less than 10 years; t-ratios in parentheses)

Holding Period = Three Months

Period	Intercept $\bar{\bar{\gamma}}0$	Duration $\bar{\bar{\gamma}}1$	Coupon $\bar{\bar{\gamma}}2$	Maturity $\bar{\bar{\gamma}}3$	Pr.Error $\bar{\bar{\gamma}}4$
January 1961 to December 1980	0.365	-0.164			
	(0.33)	(-0.33)			
	0.867	-0.324			-0.325
	(0.95)	(-0.71)			(-2.50)
	-3.651	-0.077	1.911	-0.019	-0.628
	(-1.44)	(-0.26)	(2.28)	(-0.96)	(-3.46)
January 1961 to December 1970	0.799	0.353			
	(1.00)	(0.82)			
	0.814	0.342			-0.115
	(1.02)	(0.79)			(-1.15)
	-2.467	0.787	2.111	-0.429	-0.639
	(-0.68)	(2.44)	(1.66)	(-2.16)	(-3.55)
January 1971 to December 1980	-0.080	-0.693			
	(-0.04)	(-0.75)			
	0.921	-1.007			-0.542
	(0.56)	(-1.25)			(-2.24)
	-4.866	-0.963	1.706	0.006	-0.617
	(-1.36)	(-1.93)	(1.56)	(0.16)	(-2.34)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 13 CONTD.

Period	Intercept $\bar{\gamma}_0$	Duration $\bar{\gamma}_1$	Coupon $\bar{\gamma}_2$	Maturity $\bar{\gamma}_3$	Pr.Error $\bar{\gamma}_4$
January 1961 to December 1965	0.926 (0.93)	0.313 (0.91)			
	0.958 (0.96)	0.291 (0.83)			-0.164 (-1.16)
	-6.139 (-3.27)	0.388 (1.50)	3.988 (4.45)	-0.023 (-1.12)	-0.982 (-5.07)
January 1966 to December 1970	0.672 (0.54)	0.393 (0.50)			
	0.670 (0.53)	0.393 (0.50)			-0.065 (-0.46)
	1.205 (0.17)	1.185 (2.01)	0.235 (0.10)	-0.063 (-1.85)	-0.296 (-0.64)
January 1971 to December 1975	0.604 (0.15)	0.251 (0.17)			
	2.257 (0.83)	-0.348 (-0.28)			-0.942 (-2.05)
	-5.029 (-0.74)	-1.056 (-1.31)	2.597 (1.25)	0.044 (0.85)	-0.869 (-1.74)
January 1976 to December 1980	-0.800 (-0.72)	-1.687 (-1.67)			
	-0.769 (-0.70)	-1.700 (-1.69)			-0.120 (-1.31)
	-4.695 (-2.48)	-0.864 (-1.48)	0.767 (1.63)	-0.035 (-0.80)	-0.351 (-2.70)

Note: the $\bar{\gamma}_i$ values have been multiplied by 1000.

is concerned, Table 13 indicates that for the overall period 1961-1980 there is no statistically observable relationship between duration and bond returns. However, for the 10-year subperiod 1961-1970 (model 3 only) the value of $\bar{\hat{\gamma}}1$ is significant and positive, and for the following 10-year subperiod 1971-1980 $\bar{\hat{\gamma}}1$ is significant and negative.

5.3 LIMITATIONS OF ANALYSIS

Before drawing any strong conclusions from the results presented in this chapter, it is important to establish whether or not the important assumptions underlying the regression methodology hold.

To begin with, several assumptions have been made about the disturbances, e_{jt}; in particular, that they are normally distributed. A close examination of the residuals of some of the cross-sectional regressions (10 years selected at random) reveals that there are no serious deviations from normality. However, plots of studentized residuals reveal that in each cross-sectional regression there are one or two outliers (typically long maturity bonds), which could perhaps be exerting an influence on the results.

Next, the time series behavior of $\hat{\gamma}0$, $\hat{\gamma}1$, $\hat{\gamma}2$, $\hat{\gamma}3$, and $\hat{\gamma}4$ is examined. It is necessary to determine whether the $\hat{\gamma}i_t$ are uncorrelated through time because the t-statistics for testing the hypothesis that $E(\gamma i)=0$ have been computed on the assumption that successive values of $\hat{\gamma}i_t$ are independently and identically distributed normal random variables. An examination of the sample autocorrelation functions for the various $\hat{\gamma}i_t$ reveals that this assumption does not hold.

When returns are calculated on a monthly basis some of the autocorrelations of the various γi are significantly different from zero, and in the case of the coupon coefficient, $\hat{\gamma}3$), a strong seasonal pattern emerges. When returns are calculated on a quarterly basis the seasonality problem disappears, but almost all of the autocorrelations are significantly different from zero for several lags and they decline in an exponential fashion. This result is undoubtedly due to the fact that returns for overlapping 3-month holding periods are used in the regressions. An autoregressive model of order one appears to describe the time series of each of the $\hat{\gamma}i$ very well.

This evidence indicates that when the holding period is three months the $\hat{\gamma}i$ are serially correlated and, hence, that the means of these coefficients should be recalculated under the assumption that each of the $\hat{\gamma}i$ follows an AR(1) process.

The analysis for U.S. bonds is therefore repeated, taking into account this recommendation. A holding period of three months is chosen, AR(1) models are fitted to the time series of the γ coefficients, and the means of the coefficients are reestimated. AR(1) models are estimated using three different methods: General Nonlinear Least Squares; Conditional Least Squares; and Backcasting. The results are presented in Tables 14, 15, and 16. Although the estimates of the means vary from one method to the next, the basic conclusions are all the same: there is no evidence to suggest that, on average, higher duration bonds earn higher returns; there is some evidence of a significant positive relationship between bond returns and the coupon rate; and for some time periods a significantly negative relationship apparently exists between the pricing error and the rate of return on the bond over the next interval.

TABLE 14: Cross-Sectional Regression Results for U.S. Bonds (Mean Values of Regression Parameters Calculated under Assumption of AR(1) Model)

(taxable bonds with maturities less than 10 years; AR(1) models estimated using General Nonlinear Least Squares method)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\gamma}0$	$\bar{\gamma}1$	$\bar{\gamma}2$	$\bar{\gamma}3$	$\bar{\gamma}4$
January 1961 to December 1980	1.444	-0.702			
	(1.38)	(-1.06)			
	1.435	-0.698			0.013
	(1.37)	(-1.06)			(0.10)
	-4.948	0.126	2.170	-0.712	-0.607
	(-3.80)	(0.38)	(6.89)	(-2.33)	(-5.50)
January 1961 to December 1970	1.283	0.180			
	(1.55)	(0.20)			
	1.278	0.148			0.260
	(1.52)	(0.17)			(1.47)
	-1.287	0.514	1.530	-0.507	-0.341
	(-1.22)	(1.00)	(4.95)	(-1.26)	(-2.29)
January 1971 to December 1980	1.499	-1.177			
	(0.77)	(-1.13)			
	1.470	-1.165			-0.250
	(0.76)	(-1.12)			(-1.47)
	-8.665	-0.022	2.817	-0.864	-0.891
	(-3.88)	(-0.04)	(5.31)	(-1.90)	(-5.51)

Note: the $\bar{\gamma}i$ values have been multiplied by 1000.

(cont. over)

TABLE 14 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.401 (0.60)	0.096 (0.18)			
	0.375 (0.56)	0.101 (0.19)			0.143 (0.73)
	-1.671 (-1.53)	0.374 (0.96)	1.341 (3.69)	-0.321 (-1.17)	-0.361 (-1.58)
January 1966 to December 1970	2.278 (1.60)	-0.064 (-0.04)			
	2.300 (1.58)	-0.093 (-0.06)			0.400 (1.35)
	-0.764 (-0.42)	0.544 (0.58)	1.717 (3.35)	-0.695 (-0.93)	-0.275 (-1.42)
January 1971 to December 1975	1.609 (0.78)	-0.481 (-0.50)			
	1.489 (0.71)	-0.455 (-0.48)			0.001 (0.00)
	-3.511 (-1.12)	0.207 (0.52)	1.803 (1.81)	-0.586 (-0.93)	-0.585 (-2.58)
January 1976 to December 1980	0.774 (0.23)	-1.453 (-0.80)			
	0.786 (0.24)	-1.466 (-0.81)			-0.520 (-2.89)
	-13.963 (-4.32)	-0.140 (-0.14)	3.733 (6.48)	-1.159 (-1.81)	-1.208 (-5.03)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 15: Cross-Sectional Regression Results for U.S. Bonds (Mean Values of Regression Parameters Calculated under Assumption of AR(1) Model)

(taxable bonds with maturities less than 10 years; AR(1) models estimated using Conditional Least Squares method)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1980	1.524 (1.44)	-0.837 (-1.24)			
	1.516 (1.43)	-0.830 (-1.24)			0.017 (0.13)
	-4.947 (-3.78)	0.090 (0.27)	2.172 (6.87)	-0.726 (-2.35)	-0.604 (-5.45)
January 1961 to December 1970	1.479 (1.74)	-0.221 (-0.24)			
	1.481 (1.71)	-0.229 (-0.25)			0.273 (1.53)
	-1.174 (-1.09)	0.297 (0.57)	1.526 (4.88)	-0.530 (-1.28)	-0.324 (-2.15)
January 1971 to December 1980	1.737 (0.88)	-1.266 (-1.18)			
	1.760 (0.88)	-1.264 (-1.18)			-0.239 (-1.39)
	-8.755 (-3.88)	0.015 (0.03)	2.865 (5.36)	-0.947 (-2.06)	-0.893 (-5.47)

Note: the $\bar{\hat{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 15 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1965	0.601 (0.86)	-0.345 (-0.64)			
	0.576 (0.83)	-0.338 (-0.63)			0.177 (0.87)
	-1.393 (-1.22)	0.045 (0.12)	1.314 (3.50)	-0.357 (-1.23)	-0.313 (-1.31)
January 1966 to December 1970	2.400 (1.58)	-0.054 (-0.00)			
	2.419 (1.56)	-0.062 (-0.03)			0.398 (1.31)
	-1.062 (-0.57)	0.573 (0.57)	1.818 (3.53)	-0.720 (-0.92)	-0.337 (-1.74)
January 1971 to December 1975	2.251 (1.03)	-0.577 (-0.57)			
	2.297 (1.05)	-0.562 (-0.56)			0.036 (0.13)
	-3.656 (-1.11)	0.233 (0.57)	2.008 (1.95)	-0.731 (-1.12)	-0.582 (-2.50)
January 1976 to December 1980	1.807 (0.52)	-2.219 (-1.17)			
	1.823 (0.53)	-2.235 (-1.18)			-0.519 (-2.83)
	-13.390 (-4.13)	-0.488 (-0.47)	3.708 (6.33)	-1.148 (-1.72)	-1.195 (-4.90)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

TABLE 16: Cross-Sectional Regression Results for U.S. Bonds (Mean Values of Regression Parameters Calculated under Assumption of AR(1) Model)

(taxable bonds with maturities less than 10 years; AR(1) models estimated using Backcasting method)

Holding Period = Three Months

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\bar{\gamma}}0$	$\bar{\bar{\gamma}}1$	$\bar{\bar{\gamma}}2$	$\bar{\bar{\gamma}}3$	$\bar{\bar{\gamma}}4$
January 1961 to December 1980	1.468	-0.757			
	(1.40)	(-1.13)			
	1.461	-0.750			0.014
	(1.38)	(-1.13)			(0.11)
	-4.947	0.121	2.170	-0.716	-0.607
	(-3.79)	(0.36)	(6.88)	(-2.33)	(-5.49)
January 1961 to December 1970	1.355	-0.011			
	(1.61)	(-0.01)			
	1.352	-0.033			0.261
	(1.58)	(-0.04)			(1.47)
	-1.263	0.429	1.530	-0.513	-0.338
	(-1.19)	(0.82)	(4.92)	(-1.26)	(-2.26)
January 1971 to December 1980	1.569	-1.210			
	(0.80)	(-1.14)			
	1.557	-1.201			-0.249
	(0.79)	(-1.14)			(-1.46)
	-8.669	-0.023	2.817	-0.883	-0.891
	(-3.86)	(-0.04)	(5.30)	(-1.93)	(-5.48)

Note: the $\bar{\bar{\gamma}}i$ values have been multiplied by 1000.

(cont. over)

TABLE 16 CONTD.

	Intercept	Duration	Coupon	Maturity	Pr.Error
Period	$\bar{\hat{\gamma}}0$	$\bar{\hat{\gamma}}1$	$\bar{\hat{\gamma}}2$	$\bar{\hat{\gamma}}3$	$\bar{\hat{\gamma}}4$
January 1961 to December 1965	0.455 (0.66)	-0.066 (-0.12)			
	0.429 (0.63)	-0.061 (-0.11)			0.149 (0.75)
	-1.594 (-1.43)	0.271 (0.65)	1.336 (3.63)	-0.332 (-1.18)	-0.349 (-1.49)
January 1966 to December 1970	2.322 (1.58)	-0.033 (-0.02)			
	2.343 (1.56)	-0.078 (-0.05)			0.399 (1.33)
	-0.814 (-0.44)	0.556 (0.57)	1.719 (3.36)	-0.701 (-0.92)	-0.281 (-1.45)
January 1971 to December 1975	1.835 (0.86)	-0.513 (-0.52)			
	1.782 (0.83)	-0.488 (-0.50)			0.071 (0.03)
	-3.552 (-1.11)	0.208 (0.51)	1.845 (1.82)	-0.617 (-0.96)	-0.584 (-2.55)
January 1976 to December 1980	1.042 (0.31)	-1.708 (-0.91)			
	1.059 (0.31)	-1.721 (-0.92)			-0.520 (-2.86)
	-13.960 (-4.36)	-0.190 (-0.18)	3.770 (6.49)	-1.156 (-1.78)	-1.208 (-5.00)

Note: the $\bar{\hat{\gamma}}i$ values have been multiplied by 1000.

Chapter 6

CONCLUSIONS

In this study Canadian and American market data have been used to test whether Macaulay/Fisher and Weil duration is an adequate measure of basis risk for default-free government bonds during the period January 1961 to December 1980.

The most important result of this study is that in either an American or a Canadian context there is no significant evidence to suggest that, on average, higher duration bonds earn higher returns. Specifically, there appears to be on average a negative (although insignificant) relationship between bond returns and Macaulay/Fisher and Weil duration. This result, which applies equally to government bonds and to portfolios of government securities, is not sensitive to the length of the holding period or to the portfolio selection method used.

One possible explanation for the duration coefficient being negative on average is that interest rates have trended upwards over most of the sample period. In any given period the coefficient for duration, $\hat{\gamma}1$, will be influenced by changes in the level of interest rates in the economy, so that when interest rates are rising $\hat{\gamma}1$ is likely to be negative on average.

To determine what portion of $\hat{\gamma}1$ is not explained by changes in the level of interest rates the month-by-month values of $\hat{\gamma}1$ (American, monthly holding period values only) are regressed on changes in the level of interest rates. The results suggest that when changes in the level of interest rates have been filtered out there is a positive, although insignificant, relationship between bond returns and duration. This indicates that there may be considerable value in a more detailed analysis of the relationship between changes in interest rates and the duration coefficient, $\hat{\gamma}1$.

Another result of the study is that duration seems to incorporate the effect of maturity differences on price volatility, in that the coefficient for maturity is on average insignificantly different from zero. The coefficient for the coupon rate is, however, significantly positive on average. A possible explanation for the positive relationship between coupon rate and bond returns is that the coupon coefficient is capturing a tax effect.

Finally, the results indicate that in some periods a significant relation exists between the pricing error and the rate of return over the next interval.

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