A LIFE CYCLE MODEL OF LABOR SUPPLY

by

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ABSTRACT

This thesis focusses on three areas in the theory of intertemporal utility maximization. First, I integrate the theory of labor supply and human capital accumulation. I formulate a model of intertemporal utility maximization in which time is allocated between leisure, schooling and work. It is assumed that the wage rate is a function of years of schooling and experience which, in turn, is a function of the total number of hours that the individual has worked so far. Second, I develop a new technique which allows us to estimate functional relationships derived from optimal control problems for which no analytic solution exists. Third, I estimate the proposed model for two different data sets. Flexible functional forms are employed for estimation purposes and every effort is made so that the empirical model approximates as closely as possible the theoretical one.

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CHAPTER I

INTRODUCTION

This research project presents a model of consumption, labor supply and human capital accumulation based on the postulates of inter-temporal utility maximizing behavior. It generalizes most of the models in human capital accumulation which have been derived in the tradition of Arrow's [1961], Becker's [1967], and Ben-Porath's [1967] models. Furthermore, it contains as a special case the standard life cycle models of consumption.[1] The model is tested using U.S. data.

Over the past decade there have been quite a few attempts to estimate labor supply and/or human capital accumulation functions. Most of those studies have been hampered by the following shortcomings:

i) The estimated functions have not been derived explicitly from a utility maximization problem.

ii) Labor supply and human capital accumulation functions have been estimated independently of each other.

iii) The opportunity cost of time and the price of human capital have been mis-specified by ignoring the shadow value of experience.

iv) Some of the individuals who have been included in the sample might not have been in equilibrium because of restrictions on the number of hours of work and/or because they might have had a second job.

In this research project, every effort has been made to rectify those shortcomings and take into account the peculiarities of the labor market.

i) The empirical model is an isomorphic mapping of the theoretical one.

ii) Flexible functional forms[2] have been used for the utility function and the wage function. Thus, the structure of the empirical model is almost as rich as the structure of the theoretical one.

iii) The employed data set satisfies most of the restrictions imposed by the theoretical model.

This study is in seven chapters. Chapters II, III and IV consider critically some of the issues related to the modelling and estimation of labor supply and human capital accumulation functions. Chapter V presents a life cycle model of human capital accumulation, consumption and labor supply. The properties of the trajectories of the control and state variables are explored as well as the impact on them of certain exogenous variables. Furthermore, a new technique is proposed which allows estimation of functions derived from optimal control problems for which an analytic solution does not exist. In the last two chapters, the estimates of the model are presented, as well as the conclusions and directions for related research.

Footnotes to Chapter I

1. See, e.g, Modigliani and Brumberg [1954].

2. Flexible functional forms are capable of providing local second-order approximation to any function.

Chapter II

ON MODELLING HUMAN CAPITAL

A. Introduction

The major problem faced by any student of human capital is the definition and measurement of human capital. From an economist's point of view human capital can be defined as any ability, capacity or skill possessed by an individual which commands a nonnegative price.

I will define by the term "human capital" all marketable skills, abilities or capacities of an individual. Assuming that a composite index of human capital can be constructed, human capital might be modelled as follows:[1]

(II.1) $H_j = g(s_j, L_j, OJT_j, MA_j, P_j, u_j)$

where for the jth individual

H_j: human capital,

s_j: years of schooling,

L_j: experience,

OJT_j: on-the-job training,

MA_j: innate mental abilities,

P_j: personality characteristics,

u_j: all other variables which might affect the accumulation of human capital.

Appealing to the marginal productivity theory and assuming that markets are both static and perfectly competitive, we derive the following relationship:

(II.2) $w_j = MP_j = f(H_j; CI)$

where w_j: the wage rate,

MP_j: the marginal product for an "hour" of input,

CI: a composite index of all other inputs used in the production process.

A few caveats are in order here:

i) Except for some primitive technologies the wage rate depends on the relative prices of all other inputs and the technology. Equation (II.2) is, basically, the inverse demand function for labor. Hence, most attempts to estimate such a function will result in biased estimates because of the specification error.

ii) By definition a significant number of the variables that determine human capital are controllable by the individual. Furthermore, investment in some of those factors depends on the real wage, ceteris paribus. For example, investment in formal education and/or on-the-job training depend on the expected wage rate in general. Ignoring those relationships will result in biased estimates of the parameters of the wage equation because of the simultaneity bias.

iii) Equation (II.2) cannot capture certain non-pecuniary benefits associated with specific occupations, e.g., prestige. Furthermore, equation (II.2) cannot capture possible payments in kind, e.g., paid and/or extended vacations, pension plans, etc. If it is assumed that firms offer fringe benefits, observed wages will under-estimate the marginal product of the worker, *ceteris paribus*.

From an empirical point of view the most severe problem will be simultaneity bias, mentioned in (ii). The reason is that, if the markets are "reasonably" perfectly competitive, they will be characterized by perfect information and free mobility of resources. Then the rate of return on human capital, i.e., the wage rate, should be the same for all industries, thus the specification bias will be minimal. It should be emphasized that the aforementioned result is not valid for intertemporal comparisons. Abundant evidence exists that the structure of the economy is changing over time which results in changes in relative prices. If this is the case, i.e., if relative prices are changing over time, the specification bias will be minimized by using cross section rather than time series data. Finally, the impact of the errors in variables and/or specification error discussed in (iii) can be reduced if:

i) The observed wage rate is adjusted to include payments in kind, and

ii) Equation (II.2) is embedded in a utility maximization framework which can accommodate the non-pecuniary rewards of human capital and employment.

Such a model is presented in Chapter V.

In the rest of this chapter I will present a selective survey/ taxonomy of previous attempts to measure human capital.

This exercise will allow us to put into the proper context the model that I will present and discuss later on. All studies that will be reviewed are based on the previously described model, that is, equations (II.1) and (II.2). However, they differ significantly in the information that is required to identify the determinants of human capital. Moreover, the focus as well as the possible policy implications and applications of those studies vary considerably.

B. The Investment Model

Becker [1964] suggested a surprisingly simple technique which allows us to estimate simultaneously the internal rate of return on human capital, the costs of investing in human capital in each period and the total cost of investing in human capital. The only information required is net earnings.

The suggested methodology might be described as follows. Consider two activities; the first one, say $\tilde{E}$, has no investment and provides the net earnings stream $\tilde{E}_0$, $\tilde{E}_1$,... $\tilde{E}_n$. The second activity, say E, requires investment for at least one period and provides the net earnings stream E_0, E_1....E_n. The amount invested, cost, during the j^{th} period is defined as:

$$\text{(II.3)} \qquad C_j = \tilde{E}_j - E_j + r\sum_{k=0}^{j-1} C_k$$

where r is the rate of return, $C_0 = \tilde{E}_0 - E_0$, $C_1 = \tilde{E}_1 - E_1 + rC_0$ etc.

Total undiscounted costs can be estimated as:

$$\text{(II.4)} \quad C = \sum_{j=0}^{\infty} C_j$$

But the present value of net earnings should be the same for both activities, say,

$$\text{(II.5)} \quad \sum_{j=0}^{\infty} \frac{\tilde{E}_j}{(1+r)^{j+1}} \equiv \sum_{j=0}^{\infty} \frac{E_j}{(1+r)^{j+1}}$$

The internal rate of return can be determined utilizing equation (II.5). Investment costs and the investment period can be calculated recursively using equation (II.3), Finally, total costs can be estimated using equation (II.4),

The model can be, somehow, generalized by modelling explicitly the maximization process. Ben-Porath [1967], Becker [1967], Rosen [1976] have specified and/or estimated models where it is assumed that the individual maximizes the present value of earnings subject to an earnings function which is assumed to be a function of human capital. It turns out that the underlying methodology as well as the informational requirements are identical to the technique suggested by Becker [1964], so this "sophisticated" version of the investment model will not be discussed here.

C. The Experience-Schooling Model

Even a cursory study of age-earnings profiles indicates that earnings increase with years of schooling and age/experience. The natural vehicle for studying earnings as a function of schooling and age/experience is Becker's [1964] model. Suppose that $\tilde{E}_j = 0$ for every j. Define gross earnings as:

(II.6) $$GE_j = E_j + C_j$$

Given the definition of gross earnings and the assumption that $\tilde{E}_j = 0$[2], (II.3) can be written as:

(II.7) $$GE_{j-1} + rC_{j-1} = r\sum_{k=1}^{j-1} C_k = GE_j.$$

Solving (II.7) recursively and noting that during the years when the individual is a full-time student investment costs are equal, by definition, to gross earnings, we can rewrite equation (II.7) as:

(II.8) $$\ell n GE_j = \ell n GE_0 + r_s s + r_p \sum_{t=0}^{j-1} \frac{C}{GE_t}$$

where r_s: rate of return to school investment

r_p: rate of return to post-school investment,

(II.8) is not empirically tractable since data about the cost of the investment are not readily available. Assuming that investment in human capital follows a specific path over the lifetime of the individual $\frac{C}{GE_t}$ can be implicitly defined as a function of time, e.g.,

(II.9) $$k_t = k_0 - \frac{k_0 t}{T} \quad \text{or} \quad k_t = k_0 e^{-pt}$$

where T is total period of net investment and $k_t \equiv \frac{C}{GE_t}$

Transforming (II.8) into a continuous function and substituting (II.9) we derive two different specifications for the gross earnings function. Quite a few authors have estimated different "approximations" to the gross earnings function or they have tried to extend it either by introducing more explanatory variables or by considering other functional forms.[3] Unfortunately most of those analyses are rather *ad hoc*. Whatever the limitations of the estimated models we can still derive some lessons from them.

i) The hypotheses that the rates of return to schooling and experience are not constant cannot be rejected by the data.

ii) Returns to education vary with the type of education.

iii) Besides education and experience other factors may influence the level of earnings.

D. The Direct Approach

A few authors have attempted to measure directly the marginal productivity of workers of different educational backgrounds, thus, they have attempted to measure directly the marginal productivity of human capital. The two studies that are of some interest for our purposes, are the ones by Berg [1969] and Layard *et al* [1971]. Berg examined the performance of blue and white collar workers with different educational backgrounds. The interpretation of his results requires extreme caution since his analysis does not go beyond a simple presentation of the data and some descriptive statistics. The study by Layard *et al* is a much more ambitious one.

However, the methodology that they employed, as well as the validity of their conclusions, are quite questionable, as pointed out by Blaug [1972].

E. Concluding Remarks

The picture that emerges from the review of the literature on human capital indicates that the theory, almost twenty years old, has not yet matured. Contrary to Blaug [1976], who asked: "what refutations have been encountered in the 'protective belt' of the program" I believe that the question should be whether the protective belt of the research program allows us to assess its degree of corroboration. The research program on human capital is so poorly or ingeniously designed - depending upon one's point of view - that one can hardly refute its predictions.

i) The model proposed by Becker [1964] is a perfect example of an untestable model for at least two reasons. First, there is a serious definitional/measurement problem. According to Becker [1964; 38] "'real' earnings are the sum of monetary earnings and monetary equivalent of psychic earnings." It is is obvious that there is no possible way of measuring real earnings in the Beckerian framework, unless we embed the problem in a utility maximization framework and include in the utility function such variables as human capital. Second, the fundamental assumption of "earnings maximization" is a maintained but not testable hypothesis.

ii) The "second generation" of the investment in human capital models - what I called "The Sophisticated Investment Model" - present, mutatis mutandis, the same difficulties and inflexibilities. The testable prediction of the model that investment in human capital is declining throughout the life-cycle is fully compatible with other maintained hypotheses, e.g., utility maximization.

iii) The apparent success of the extended earnings function model illustrates, quite forcefully, the naivete of the schooling and/or schooling and experience models, There is no doubt today that genetic pre-disposition and the family environment do affect earnings. However, this should not be interpreted as an unqualified acceptance of the extended earnings function. To the best of my knowledge none of the estimated earnings functions is based on a well defined theory of learning and/or education. Thus, the researchers cannot claim that by estimating those earnings functions they are assessing the degree of corroboration of a well defined theory. All they can claim is that the conjecture that certain variables affect the earnings of an individual cannot be refuted by the data.

iv) Suppose that the estimated earnings functions were models of a theory rather than ad hoc constructions. The assumption that a number of socio-economic variables determine the earnings of an individual suggests that the proper way to model such a situation is via a simultaneous equation model (see Griliches [1976]). Hence, the earnings function is a reduced form equation. But we know from elementary econometrics that estimating only one of the equations of a set of simultaneous equations will result in biased coefficients (see Morgenstern [1973]).

The previous analysis suggests that the economics of education is in bad shape. Hopefully, research in the future will attempt to investigate in a direct way the following two theoretical issues:

i) The construction of operationally useful theories which can describe and analyze the accumulation of knowledge - human capital - as well as the relationship between knowledge and performance.

ii) The behavior of the family unit towards the accumulation of human capital.

Footnotes to Chapter II

1. See Taubman and Wales [1974].

2. See Becker and Chiswick [1966], Mincer [1974].

3. A number of other variables have been introduced in the earning function, e,g., coqnitive abilities/intelligence/ personality traits (Griliches and Mason [1973]), Griliches [1976], Duncan et al [1972], Taubman and Wales [1974], House [1976], Wise [1975]; quality of schools (Wales [1973], Taubman and Wales [1974]; occupational status (Stolzenberg [1975b], Farley [1977], Goodman [1979]); family background/race/social class (Harrison [1972a], [1972b], Weiss [1970], Bowles [1973], Bowles and Gintis [1975], Flanagan [1974], Featherman and Hauser [1976], Stolzenberg [1975a], Farley [1977], Duncan et al [1977]); religious preference (Gockel [1969], Greeley [1976]); migration (Chiswick [1977] [1978]); completion of levels of schooling (Layard and Psacharopoulos [1974], Taubman and Wales [1974], Albrecht [1974], Wise [1975], Goodman [1979]); sex (Mincer and Polachek [1974]).

Chapter III

ON MODELLING THE SUPPLY OF LABOR: THE STATIC MODEL

A. Introductory Remarks

This chapter considers critically some of the empirical and theoretical studies concerned with the supply of labor. The subsequent discussion is limited, in the sense that I will consider only quantitative aspects of labor supply. More specifically, I will focus on issues related to the construction and empirical implementation of operationally useful models of the supply of labor. Until recently, the majority of economists working within the neoclassical paradigm have failed to recognize the distinctive features of the demand for leisure - i.e., the supply of labor - vis â vis the demand for other commodities, Consequently, most of the techniques that I will present and discuss below are fairly recent and deserve some attention.

B. The Conventional Neoclassical Model of Labor Supply

According to the neoclassical model it is assumed that the consumer-worker maximizes a nondecreasing, quasiconcave continuous utility function subject to an income constraint. The model is summarized below:

(III.3) $$\max u = U(x_1, x_2, \ldots, x_N, \ell)$$

w. r. t.

$$x_1 \geq 0, \ldots x_N \geq 0$$

$$h \geq \ell \geq 0$$

subject to

$$I + w(h-\ell) \geq \sum_{i=1}^{N} p_i x_i$$

where u: the utility function,

x_i: the quantity of the i^{th} commodity (rental),

p_i: the price (rental price) of the i^{th} commodity (rental),

ℓ: hours of leisure,

h: total number of hours available,

I: non-labor income.

The comparative statics of the model are readily available and will not be considered here. It suffices to remind ourselves that a change of the wage rate may result in an increase or decrease of the individual's hours of work. (It seems that Robbins [1930] was the first to derive that result.) Hence, the only route that is available in order to evaluate the impact of a change of the wage rate on the supply of labor is by computing the elasticity of labor supply. The importance of reliable estimates of the preference ordering of the consumer-worker for the policy maker cannot be overstated.

The major advantage of the conventional neoclassical model lies in its simplicity. It can be easily estimated and it allows us to investigate a number of theoretical and empirical issues, some of which are important for purposes of economic policy. However, the major advantage of the model - its simplicity - may be considered its major disadvantage. Because of its simplicity one might be tempted to estimate the model without paying attention to some "technicalities" or"minor points". As will be shown in sections (D) and (E) of this chapter, the price that we have to pay in ignoring those "minor points" can be quite high.

C. Becker's Model

The finest partition of the variable "time" that the conventional neoclassical model of labor supply can handle is "leisure" and "hours of work". Such a partition is unrealistic and inadequate for analytic purposes. Activities, such as, preparing a meal, cleaning the house, etc., cannot be classified as leisure. On the other hand one cannot "consume" a book, the only way to derive utility from a book is by reading it, or even by looking at it, or by glancing at it! Contrary to the traditional neo-classical models which cannot handle those situations, Becker's [1965] model can. In his pathbreaking paper he suggested that the utility function of the consumer, or consumer-worker, be defined over activities. The activities are produced by both commodities and time. Hence the demand for activities depends on the price (rental price) of commodities (rentals) and the opportunity cost of time.

Becker's model can be described as follows:

(III.2) $$\max u = U(Z_1, Z_2, \ldots Z_N)$$

w.r.t.

$$x_1 \geq 0, \qquad x_M \geq 0$$

$$T_1 \geq 0, \qquad T_N \geq 0$$

$$Z_1, Z_2 \ldots Z_N$$

$$h \geq \ell \geq 0$$

subject to

(III.3) $$Z_i = f_i\ (X_i,\ T_i)$$

(III.4) $$I + w(h-\ell) \geq \sum_{i=1}^{M} p_i x_i$$

(III.5) $$\sum_{i=1}^{N} T_i + (h-\ell) = H$$

where: u: the utility function,

Z_i: the i^{th} activity

p_i: the price (rental price) of the i^{th} commodity (rental),

x_i: the i^{th} commodity (rental),

f_i: linear homogeneous production function,

I: non-labor income,

$(h-\ell)$: the supply of labor,

w: the wage rate,

H: total number of "hours" available per time period,

T_i: the input of the individual's own time.

Although Becker's work provides us with an important generalization of the neoclassical model, it should not be considered as the ultimate model of labor supply. As Diewert [1971] shows, the model can be extended to include occupational choice of the individual as well as the labor force participation decision.

Although the model can include a significant number of labour supply issues its attractiveness is diminished when one con- templates estimating it. The informational requirements for estimation purposes are excessive. Wales and Woodland [1977] have estimated a simple version of Becker's model. An important conclusion of this study was that, for the data set employed a variant of Becker's model performed better than the traditional model.

D. Estimating the Supply of Labor I

Analytic/Econometric Problems

In this paragraph I will consider a number of analytic and econometric problems which complicate the specification and estimation of the labor supply model. It should be emphasized that most of the topics that will be presented in this paragraph are mutually interdependent. Moreover, the taxonomy is by no means an exhaustive one.

1/. Functional form. Except for a few authors (e.g., Wales and Woodland [1976], [1977], Lau *et al* [1978]) who have employed flexible functional forms for estimation purposes, most authors have used quite restrictive and/or *ad hoc* functional forms. A maintained thesis of this study is that the use of flexible functional forms is imperative for labor supply models. The reason is that flexible functional forms may be interpreted as second order approximations to an arbitrary utility function at a point.

2/. Taxes. The introduction of taxes in the consumer-worker maximization model - either the traditional neoclassical model or Becker's - does not create any difficulties, *per se*. What does create problems is the structure of the tax function. Given the progressivity of the income tax, the concavity of the budget constraint is not violated. However, for most tax systems the after tax budget will be a segmented linear function. Traditional econometric techniques cannot handle segmented linear functions, hence an approximation of the budget constraint is necessary. Fortunately, as Diewert [1971] has shown, the tax function can be linearized around each observation, therefore traditional econometric techniques can be employed. This technique suffers from the following shortcomings:

i) It introduces an approximation error in the estimation process.

ii) It ignores the endogeneity of the net wage rate.

An algorithm - due to Wales and Woodland [1979] - will be presented below which eliminates the approximation error and takes into consideration the endogeneity of the net wage rate.

3/. Non-labor income. Non-labor income might be disaggregated into two components.

i) Capital/Property income which includes dividends, interest paid on bonds and savings accounts, rents, income from life insurance policies, etc.

ii) Transfers which include family and child benefits, negative income tax schemes and wage subsidies paid directly to the worker.

Although from a purely technical point of view (i) and (ii) can be easily introduced into the budget constraint of the consumer, from a theoretical point of view their very existence illustrates the inability of the ordinary static model to handle such situations. Clearly, the presence of capital income violates the assumption of static optimization behavior, while transfer income results in an endogenously determined budget constraint.

4/. Costs of working. There are two kinds of expenses associated with working:

i) Money costs, which include expenses for tools, uniforms, travel to work, etc. and

ii) Time costs, which include commuting time, etc.

One can safely ignore (i) since for most workers such expenses are small. Time costs cannot be accommodated by the neoclassical model. One has to use Becker's model in order to study and/or estimate such costs.[2]

5/. Endogeneity of wages. We have already discussed a number of reasons for treating the wage rate as an endogenous variable, e.g., taxes, transfers, costs of working. Even if we had been living in a frictionless world without taxes and transfers, the wage rate would have been endogenous because of the influence of other variables. Consider the following factors which determine, up to a certain point, the wage rate, the supply of labor and the occupational choice of the individual.

i) Education,

ii) Experience,

iii) The "agreeableness of the job",

iv) Probability of unemployment,

v) "Probability of success",

vi) "The trust to be reposed in the worker",

vii) "The cost of learning the trade",

viii) The relationship between productivity - hence, the wage rate - and the number of hours of work per day, and

ix) Pecuniary and non-pecuniary rewards of the job.

The impact of the first two variables on the supply of labor will be discussed in the next two chapters. The next five, i.e., (iii)-(vii) have been taken from Adam Smith's The Wealth of Nations [1961; 112-113]. Practically none of those can be included in the traditional static model of labor supply which ignores uncertainty and possible non-pecuniary rewards of the occupation chosen by the individual. Now let us be more specific about the non-pecuniary rewards of the job and the relationship between the wage rate and hours of work.

5A. Non-pecuniary rewards. For pedagogical purposes we may distinguish two different kinds of non-pecuniary rewards of a job:

i) Positive (negative) characteristics associated with a specific occupation which increase (decrease) the utility of the consumer-worker. Some of those characteristics might be available through certain commodities, but the consumer-worker would have to pay (be paid) for them.

ii) The consumption of certain goods provided at the place of work.

As Lancaster [1971] has shown the neoclassical model is a special case of the characteristics model. Hence, the first category of the non-pecuniary rewards of the job encompasses the second category. From an empirical point of view, the important question is how those attributes or characteristics of an occupation can be introduced into a model of labor supply. Three possible answers to the previous question have been suggested in the literature.

i) Tinbergen [1956] assumes that the utility function of the consumer-workers is defined over income and the characteristics/attributes of the occupation.

ii) Lancaster [1971] assumes that the utility function is defined over characteristics. The consumer maximizes his/her utility subject to the transformation function which describes the relationship between characteristics and commodities - including time - and his/her budget constraint.

iii) Becker's "Theory of the Allocation of Time" provides a less ambitious but, possibly, empirically tractable model, compared to the Tinbergen or Lancaster models. Define the utility function over activities and hours of work at different occupations (see Diewert [1971]). The budget constraint is, of course, defined conformably with the utility function. Given the Wales and Woodland [1977] results, the specification of the commodities -time/ activities technology should not pose any estimation problem. Some complications may arise due to the existence of corner solutions and the endogeneity of the wage rate

5B. Wages and hours of work. Wages might depend on the hours of work for the following two reasons:

i) The productivity of the worker is a concave function of the hours of work. Initially the productivity is low because it takes some time for the worker to "warm up", while by the end of the working day the productivity might decline because the worker is tired (see, e.g., Barzel [1973]). Note that if the firms are not able to offer variable wage schedules they may impose constraints on the number of hours of work per day.

ii) Certain fixed costs of employment paid by the firm might result in a variable wage schedule (see Rosen [1976]).

Both (i) and (ii) can be easily handled by the neoclassical model. The researcher has to verify whether the supply of labor is a free variable, i,e., whether the individual faces any restrictions on the number of work hours.

6/. Measurement of the supply of labor. We decided to refer to this issue last because of its important effect on the previously mentioned theoretical and empirical considerations. Both the neoclassical model and Becker's model are defined over time. Usually, time is measured in hours, while the model refers to calendar years. These measurements are employed for practical purposes - i.e., availability of data - rather than for theoretical ones. Such an approach is correct if and only if hours, weeks and months are perfect substitutes. However, as Hanoch [1980] points out, this is not necessarily the case. For example, fifty hours of leisure spread over the year are not equivalent - for most individuals - to one non-working week. Hence, the utility function perhaps should be defined over hours, days, weeks, and months.

Assuming that the opportunity cost of time is fixed, one can always aggregate all types of leisure into one. The aggregated model can be used for forecasting purposes provided that there are no costs of working, the hourly wage rate is fixed, and there are no restrictions on the hours of work. If any of those conditions is violated, then the opportunity costs of time cannot be treated as a constant, since Hick's aggregation theorem is not applicable.

E. Estimating the Supply of Labor II

Econometric/Statistical Problems

1/. Sample selectivity. Because of lack of data, or theoretical considerations, economists estimate labor supply functions using a sample of families or individuals that satisfy certain criteria. For example suppose that the researcher imposes the sample selection rule that the individual's income is less than a specific upper limit. This sample selection rule can be expressed as:

$$Y_i = I_i + w_i \ (h-\ell_i) \leq \overline{Y} \quad i = 1,\ldots.N$$

where Y_i: total income of the i^{th} individual,

I_i: non-labor income,

w_i: the wage rate,

$(h-\ell_i)$: hours of work,

$\overline{Y}$: The upper limit of income for the selected sample.

It can be shown that if the sample is censored (see Amemiya [1973], and Wales and Woodland [1981]), estimation of the labor supply function by ordinary least squares will result, in general, in biased and inconsistent estimates. A number of estimation techniques have been proposed which yield unbiased and/or consistent estimates. The only drawback of these algorithms is that they are highly non-linear even in the case of linear supply functions.

2/. Endogeneity of wages. As we have already seen, the introduction of taxes in the budget creates two problems:

i) the observed net wage rate and the income of the consumer-worker are endogenous variables.

ii) The utility maximizing point may be located on a segment of the budget constraint other than the observed one.

Wales and Woodland [1979] proposed an ingenious alqorithm for obtaining unbiased and consistent estimates of the parameters of the utility function solving at the same time for the utility maximizing point. The basic steps of the algorithm are the following ones:

i) For each individual and for every set of parameters of the utility function solve the optimization problem of the individual, hence calculate $\ell_j = \hat{\ell}(W_j, p, M_j)$ for every segment of the budget, where ℓ is leisure, W the wage rate, p the price of the composite good, and M is the full income.

ii) Check if ℓ_j is located on segment j.

iii) If it is, then ℓ_j is the optimal one, say $\ell_j = \ell^*_j$.

Define $e^*_\ell = (S_\ell - W\ell^*)M^*$ where S is the share.

iv) If it is not, check all corner solutions. Pick up the one corresponding to the highest indifference curve.

Define $e^*_\ell = (S_\ell - W^\ell{}_c)M_c$ where $M_c =$

$\rho Y_c + \omega \ell_c$

v) Evaluate $\Sigma(e^*_\ell)^2$ for any set of parameters and minimize it

The experimental evidence presented by Wales and Woodland proves, beyond any doubt, the superiority of their algorithm. Unfortunately, the attractiveness of the technique is diminished because of the computational burden involved in the implementation of the alqorithm. (A similar algorithm has been proposed by Burtless and Hausman [1978]).

Footnotes to Chapter III

1. See Diewert [1974], Lau [1974]

2. The neoclassical model can accommodate time costs if they are treated as exogenous variables (see Cogan [1981]).

Chapter IV

ON MODELLING THE SUPPLY OF LABOR: THE DYNAMIC MODEL

A. Introductory Remarks

The models of human capital accumulation and labor supply presented in the previous two chapters are undoubtedly limited in scope. Neither is human capital accumulated in a vacuum nor is the supply of labor unaffected by the stock of human capital. Moreover the simplifying assumption of instantaneous utility maximization is unnecessary as optimal control theory can be applied. In this chapter I will present a number of models that study the accumulation of human capital within the framework of inter-temporal utility maximization. It will be shown that theoretically oriented models can be tested with approximations; it will also be shown that empirically oriented ones can be as general, consistent and rigorous as the theoretically oriented ones. I will demonstrate that one important aspect of the construction of a model is the specification of it. Even a minor modification or rearrangement of the variables might be the crucial point which allows us to transform a not empirically tractable model into an estimable one.

B. The Basic Model

According to the basic model it is assumed that the consumer worker tries to solve the following maximization problem:

$$\text{(IV.1)} \qquad \max \int_0^T e^{-\rho t}\, U(x(t) \quad \ell(t))dt$$

w.r.t.

$x(t)$, $\ell(t)$

subject to:

$$\text{(IV.2)} \qquad \dot{I} = rI + w(t)(h - \ell(t)) - p(t)x(t)$$

$$\text{(IV.3)} \qquad I(0) = \bar{I}$$

where $p(t)$: the price of the consumption good,

$x(t)$: the composite good,

$\ell(t)$: hours of leisure per period,

$h(t)$: total number of hours available per period

ρ: the rate of time preference,

r: the interest rate,

$I(0)$: initial wealth,

T: terminal time.

Assuming that an interior maximum exists, the first order conditions for the maximization problem are the following:

$$\text{(IV.4)} \qquad \frac{\partial U(t)}{\partial x(t)} = \lambda(t)e^{(\rho - r)t}p(t)$$

$$\frac{\partial U(t)}{\partial \ell(t)} = \lambda(t)e^{(\rho - r)t}w(t)$$

where $\lambda(t)$ is the costate variable associated with the state variable $I(t)$; it has the usual interpretation as the marginal utility of wealth in the sense that it measures the contribution of an additional unit of $I(t)$ to the utility sum from t onwards.

In order to facilitate the subsequent analysis it is convenient to derive the dynamic demand functions for leisure and consumption by inverting equation (IV.4)

The dynamic demand functions may be written as:

(IV.5) $x(t) = x(\lambda(t)e^{(\rho-r)t}p(t),\ \lambda(t)e^{(\rho-r)t}w(t))$

(IV.6) $\ell(t) = \ell(\lambda(t)e^{(\rho-r)t}p(t),\ \lambda(t)e^{(\rho-r)t}w(t))$

A few caveats are in order here:

i) The demand for leisure and for the composite good, during any period, depends on the prevailing prices in this period - the wage rate, the price of the composite good, and $\lambda(t)$. Thus $\lambda(t)$ is the only link between the past and the future.

ii) $\lambda(t)$ depends on life-cycle wages, initial assets, the interest rate and the rate of time preference. While $\lambda(0)$ is the marginal utility of wealth at the start of life-cycle, such marginal utility does vary over the life-cycle, because p(t) and w(t) most certainly will. Contrary to the spirit of the permanent income theory, our focus is on the pattern of variations in consumption and leisure of the individual over his life-cycle.

iii) Provided that an estimate of the trajectory of $\lambda(t)$ can be obtained for every period including 0 and T one can estimate the system of dynamic demand functions, equations (IV.5) and (IV.6).

C. Accumulation of Human Capital and the Supply of Labor

A major drawback of the basic model is the assumption of an exogenous wage rate. There is sufficient evidence supporting the hypothesis that the wage rate depends on the human capital of the worker.

Heckman [1976] presented an empirically tractable model of human capital accumulation. He formulated the maximization problem of the consumer as follows:

$$\max \int_0^T e^{-\delta t}\, u(x(t), \ell(t)H(t))dt \qquad \text{(IV.7)}$$

w.r.t. $\ell(t)$, $x(t)$, $s(t)$

subject to:

$$\dot{H}(t) = f(bs(t)H(t), D(t)) - \sigma H(t) \qquad \text{(IV.8)}$$

$$\dot{I}(t) = arI(t) + aw(t)(h-\ell(t)-s(t)) - p_D(t)D(t) - p(t)x(t) \qquad \text{(IV.9)}$$

$$H(0) = H_0 \qquad I(0) = I_0$$

where $x(t)$: the composite good,

$\ell(t)$: hours of leisure per period,

$H(t)$: human capital,

$s(t)$: time devoted to human capital accumulation,

$D(t)$: the input of market goods,

$p_D(t)$: the price of direct educational expenses,

σ: an exponential rate of depreciation of human capital,

$p(t)$: the price of the composite good,

$I(t)$: the wealth (debt) of the individual,

r: the interest rate,

b: a productivity parameter

δ: the rate of time preference,

h(t): total number of hours available per period,

w(t): the wage rate per unit of human capital, say, w(t) = RH(t) where R can be normalized to unity for every t, and

1-a: the proportional income tax rate

Heckman examined certain properties of the model and specified the earnings function of the individual as

(IV.10) $E(t) = [w(t)(h-s(t)] - p_D(t)D(t) - w(t)\ell(t)$

For estimation purposes it was assumed that the human capital production function and the utility function were Cobb-Douglas. The values of the estimated parameters, using U.S. census data, seem to be quite plausible. However, the asymptotic t statistics were, except for two parameters, less than 0.5. Furthermore, as Heckman shows, a simple quadratic model performs almost as well as (IV.10).

Ghez and Becker [1975] proposed an intertemporal model of labor supply/accumulation of human capital which extends and generalizes Becker's [1965] "Theory of the Allocation of Time". Their model is much more general than any model which has been presented so far. Unfortunately, the estimation of the model is next to impossible because of the excessive information requirements.

D. Occupational Choice and Human Capital Accumulation

It is widely recognized that the two major components of human capital are formal education and experience. In principle, the specification of accumulated work experience as "total number of hours that the individual has worked so far" is erroneous. There is no information available about on-the-job training and the characteristics of different occupations so that we cannot introduce them in an empirically tractable model of labor supply. Nevertheless it is interesting to see how the conclusions of the theoretical model are affected by these variables. Blinder and Weiss [1976] were the first to introduce on-the-job training explicitly in the model. Furthermore they paid attention to periods of specialization, such as full-time schooling and retirement. They were able to derive a number of concrete conclusions by imposing rather severe restrictions on the preferences of the consumer worker and the human capital generating function. I believe that their model is worth studying because it illustrates the limits of theoretical models.

Even a cursory study of their model indicates that a tractable version of it is not possible. Three of the variables of the model are unobservable: human capital, its shadow price and the occupational index. The ambiguity about the first two can be resolved by normalizing one of them and getting a measure of the second one. Estimation of the third variable is not feasible since detailed information is not available.

Now let us consider the model proposed by Blinder and Weiss. The maximization problem of the consumer is specified as follows:

$$\max \quad \int_0^T e^{-\rho t} U(c, \ell)dt + B[A(T)]$$

$$\text{w.r.t.} \quad x(t),\ h(t),\ c(t)$$

subject to:

$$\dot{A} = rA + g(x)hK - c$$

$$\dot{K} = (axh - \delta)K$$

$$h(t) + \ell(t) = 1$$

$$h(t) > 0$$

$$0 < x(t) < 1$$

where c: the composite good,

$\ell(t)$: the fraction of time devoted to leisure,

A(t): the wealth of the individual,

h(t): the fraction of time devoted to market activity (including work and education),

x(t): the occupational index, and

g(x): the wage rate.

The main lesson that we can derive from the model is that a theoretically oriented model, whatever its degree of originality and sophistication, might not be empirically tractable. However, the last statement does not imply that we should sacrifice the consistency and concreteness of the model simply to make it tractable. It implies rather that the researcher has to find a compromise between the two objectives of generality and tractability.

E. An Empirically Oriented Model

Heckman and MaCurdy [1980] presented an empirically tractable model of labor supply.[1]

The theoretical model proposed by Heckman and MaCurdy is a very simple, almost naive, one, but all variables and parameters of it can be easily computed and estimated. The econometric technique implemented by Heckman and MaCurdy was to treat $\lambda(0)$ - the shadow price of wealth during the first period - as a fixed effect variable, that is, they estimated simultaneously the utility function, the wage equation and $\lambda(0)$.

Heckman and MaCurdy assumed that the individual tries to solve the following optimal control problem:

$$\max \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} [A(t)(\ell(t))^{\alpha} + B(t)(c(t))^{\gamma}]$$

w.r.t. to $\ell(t)$, $c(t)$

subject to

$$I(0) = \sum_{t=0}^{T} \frac{1}{(1+r)^t} [c(t) - w(t)(h-\ell(t)]$$

$$A(t) = \exp \{Z(t)\Phi\}$$

$$\ell n w(t) = X(t)\beta$$

where $A(t)$, $B(t)$: "age specific modifiers of "tastes" or "household production"",

$Z(t)$: "a vector of measured determinants of leisure choices",

$X(t)$: a vector of exogenous variables,

$\ell(t)$: hours of leisure,

$c(t)$: consumption,

$w(t)$: the wage rate.

It can be shown that the demand function for leisure is:

$$\ell^*(t) = \left[\frac{1}{A(t)\alpha} \left\{ \frac{1+\rho}{1+r} \right\} \lambda(0)w(t) \right]^{\frac{1}{\alpha-1}} \quad \text{if the individual works}$$

$\ell^*(t) = h$ otherwise

Heckman and MaCurdy estimated the demand function for leisure and the wage equation recognizing that the distribution of the error term is truncated because of sample selectivity. They estimated in two stages. In the first stage they maximized the likelihood with respect to the parameters of the model. In the second stage they regressed the "fixed effect" variable--the shadow price of wealth - on some exogenous variables.

The specification and estimation of this model raises a number of issues.

i) Given their estimation procedure - i.e., fixed effect $\lambda(0)$ - the estimated parameters are inconsistent.

ii) The specification of the likelihood function and the estimation procedure are hampered by the omission of the demand function for the composite good. Therefore, estimates of B(t) - the tastes modifiers for consumption - have not been obtained.

iii) It is assumed that human capital is an exogenous variable. Therefore, the model is mispecified.

iv) It is not clear if this technique can be employed for optimal control problems with more than one equations of motion. (If human capital is endogenously determined at least two equations of motion are needed.)

F. Concluding Remarks

What general conclusions and directions for related research can we draw from the body of work discussed in this chapter? The only unambiguous conclusion is that no "definite" or "generally acceptable" model has been presented so far. This section is designed to justify the last statement and highlight some of the difficulties faced by the researcher attempting to model a life-cycle model of labor supply. It will also be shown that specialized models, ones that focus on specific facets of the labor market can be sufficiently general and rigorous.

The most important difficulties that we face in life-cycle models of labor-supply - in addition to the ones discussed in Chapter III - can be summarized as follows:

1/. <u>Longitudinal data versus synthetic cohorts.</u> As a rule of thumb we can classify the available models into two classes. 1) ones for which data sets are available and the researchers have implementation problems; 2) ones for which no data sets are available and have implementation problems. During the last decade economists have been able to estimate labor supply functions using either longitudinal data or synthetic cohorts. The simulation of the life-cycle profile of an individual using synthetic cohorts suffers from the following drawbacks (see Smith [1977], Heckman and MaCurdy [1980]):

i) If expectations about the future are biased then the variation between the age cells will not correspond to the variation over the life-cycle.

ii) Because of increases in real wage rates, during the recent years, the expected wealth of younger cohorts is greater than that of older cohorts.

iii) It is assumed that all individuals belonging to a specific age cell have the same level of utility and the same amount of wealth.

It is obvious that studies based on longitudinal data will not be affected by the formation of expectations - in the sense of (i) - but they will be affected by the change of the real wage rate, unless complete profiles are available. So the only comparative disadvantages of synthetic cohorts are related to the formation of expectations and the level of utility and wealth. I find the previous treatment of expectations slightly schizophrenic. Except for one model all other models assume either implicitly or explicitly that the consumer is able to foresee the future. Given this assumption there is no place in the model for formation of expectations. The discussion would have been relevant if uncertainty and/or inability to get a perfect forecast of the future had been included, explicitly, in the model. The question that should be asked is not which approach allows for variations in expectations but rather if the model is equipped to handle expectations at all. Mutatis mutandis, the same is true about real wages. Therefore, from a technical point of view, the question is whether $\lambda(0)$, which is a function of "expectations", is correctly specified. I will return to this point in (2) below.

An advantage of longitudinal data is that the resulting estimates are more efficient because the estimation procedure utilizes a ceteris paribus, larger and richer sample. That advantage of longitudinal data should be carefully contrasted with a serious disadvantage, that certain variables, such as experience, are subject to measurement error.

The only unambiguous shortcoming of synthetic cohorts is the assumption that all individuals belonging to a particular age cell are at the same level of utility and possess the same amount of wealth. This can be viewed as some sort of specification error. The impact of the aforementioned shortcoming can be substantially reduced by stratifying the sample according to more than one characteristic. For example, a reasonable stratification is by age and years of education, or age and initial wealth or age, education and initial wealth. Furthermore, the utility function can be defined over consumption, leisure and years of education. On a priori grounds, one expects that as the partition becomes finer the differences in the level of utility and the amount of wealth of the individuals within each cell will decrease. This proposed technique is especially attractive in view of the fact that measurement of the wealth of the individual with longitudinal data sets is not possible yet.

We can draw the following lesson from the previous discussion: Neither of the two techniques has a definite advantage over the other. Errors in variables are more probable with longitudinal data sets, while panel data result in some sort of misspecification. The choice depends on the preferences of the researcher, the subjective evaluation of the advantages and disadvantages of each technique.

2/. Estimating the shadow value of wealth. The demand functions for leisure and the composite good which correspond to the simplest model - which I called the Basic Model - can be written as:

$$x(t) = f[\lambda e^{(\rho-r)t}w(t),\ \lambda e^{(\rho-r)t}p(t)]$$

$$\ell(t) = g[\lambda e^{(\rho-r)t}w(t),\ \lambda e^{(\rho-r)t}p(t)]$$

The main difficulty in estimating those equations is the lack of observations for $\lambda(0)$. In principle, data for $\lambda(0)$, or any $\lambda(t)$ are not required; it suffices to solve the optimal control problem and express $\lambda(t)$ as a function of the other variables and parameters of the model. It is quite surprising that no researcher has pursued this straightforward approach. Although the resulting differential equations may be so "messy" that an analytic solution is impossible, it is up to the researcher to specify the model in such a way that an analytic solution of the two boundary problems can be obtained. Without any loss of generality we can classify all techniques for estimating the foregoing intertemporal equations as follows:

i) The first approach implemented by Ghez and Becker [1974] manages to get around the problem by estimating approximations of the two demand equations, so that $\lambda(t)$ is subsumed into the parameters of the estimated functions. This technique can be employed either with cross section or longitudinal data. With some luck, this approach allows us to recover, at most, Hicks- Slutsky income and substitution effects.

ii) The second approach suggested by Mincer [1962] is to decompose the current income and the wage rate into a permanent and a transitory component. The underlying assumption is that the response of the demand for leisure with respect to those two components differs. As Heckman and MaCurdy (1980] point out, neither the theoretical rationale for this decomposition nor the empirically oriented definitions of these two components are well defined.

iii) The third approach due to MaCurdy [1978] is to proceed by treating $\lambda(0)$ as a fixed effects variable. As we have seen, this approach results in inconsistent estimates.

iv) The fourth approach implemented in this research project is to solve explicitly for the $\lambda(0)$.

Now let us focus on the proposed technique. Notice that for most life-cycle models the equation of motion of $\lambda(t)$ is very simple:

$$\dot{\lambda}(t) = -r \ \lambda(t)$$

This differential equation can be easily solved if either $\lambda(0)$ or $\lambda(T)$ are known. Define the bequest function as:

$$B[I(T);\zeta]$$

where $B[\ ]$: the bequest function,

$I(T)$: wealth at time T, i.e., terminal time,

ζ: a vector of characteristics of the individual and/or variables evaluated at time T.

It is well known that $\lambda(T)$ is given by:

$$\lambda(T) = \frac{\partial B[I(T);\zeta]}{\partial I(T)}$$

Hence, the equation of motion of $\lambda(t)$ can be written as:

$$\lambda(t) = \lambda(T)e^{-r(t-T)}$$

For estimation purposes it suffices to specify a functional form for the bequest function,[2] substitute the last equation into the system of demand equations and estimate simultaneously the parameters of the utility function and the bequest function.

The suggested technique is quite useful because it allows us to estimate intertemporal demand functions without imposing restrictive approximations of the utility function or employing techniques which result in inconsistent estimates. Furthermore, it is useful for simulating the impact of pension plans, taxes, and the like, on the trajectories of the costate variables of the individual.

The same technique can be employed, mutatis mutandis, for any costate variable, provided that the associated state variable is an argument of the bequest function. If it is not, one may still approximate the trajectory of the costate variable by utilizing the transversality conditions; such a case is presented in the next chapter.

3/. Modelling human capital. I have already demonstrated that human capital and its shadow price are not observable. The only observable variable is the wage rate, which is assumed to be a function of the stock of human capital of the individual. In order to simplify the subsequent analysis let us suppose that human capital is a function of years of schooling and experience. The human capital accumulation/earnings part of the model can be written as follows:

$$\dot{H}(t) = f(H(t), \dot{s}, \dot{L})$$

$$w(t) = h(H(t))$$

where H(t): human capital,

s: years of schooling,

L: experience, and

w: the wage rate

Footnotes to Chapter IV

1. MaCurdy [1978] [1981] presented a similar model.

2. Alternatively, T can be defined as date of retirement and B[I(T)] as the utility yield of net wealth at retirement.

Chapter V

THE PROPOSED MODEL

A. Description/Properties of the Model

This paragraph presents a life-cycle model of labor supply that is almost empirically tractable. The model generalizes most of the models which have been proposed so far, save for the one due to Becker and Ghez. The maximization problem of the consumer is written as follows:

(V.1) $$\max \int_{t_0}^{T} e^{-\delta t} U[x,(h-t(s)-t(w)),s]\,dt + B[I(T)]$$

w.r.t.

x, $t(s)$, $t(w)$

subject to

(V.2) $$\dot{s} = t(s)$$

(V.3) $$\dot{L} = t(w) - \gamma L$$

(V.4) $$\dot{I} = rI + f[w(L,s)t(w)] - px - C[t(s)]$$

(V.5) $$s(0) = \bar{s} \gg 0$$
$$L(0) = \bar{L} \gg 0$$
$$I(0) = \bar{I}$$

where U: the utility function,

δ: the rate of time preference,

B: the utility yield of net wealth at retirement (UYONWAR),

t_0: initial time,

T: date of retirement,

x: the composite good,

t(w): hours of work per period,

t(s): hours of schooling per period,

h−t(s)−t(w): hours of leisure per period,

s: "years" of schooling,

L: experience,

I: the wealth of the individual,

r: the interest rate,

w: the wage rate

p: the price of the composite good,

C: cost of schooling,

γ: the depreciation rate of experience

f[ ]: after tax earnings

The following assumptions are made with regard to the optimization problem.

i) The utility function and the UYONWAR function are defined over the positive orthant and are continuous, increasing, strongly concave functions, are finite for all finite values of their arguments and satisfy the usual Inada conditions, e.g.:

$$\lim_{x \to o} U_x = \infty,\ V[h-t(s)-t(w)] \text{ e.t.c.}$$

ii) The after tax earnings function is a positive, continuous, strongly concave function.

iii) Capital income is non-taxable, or r is the after tax return on capital income.

iv) The cost of schooling function is a strictly positive, continuous, strongly convex function.

Before proceeding to study the properties and the predictions of the model, it will be necessary to examine the maintained hypotheses.

i) The reqularity conditions imposed on the utility function are slightly more restrictive than required in order to generate convex indifference curves. Let us consider each condition:

- The continuity assumption is imposted in order to simplify the calculus and, in any case, is quite harmless.
- The Inada conditions exclude the possibility of corner solutions; it is assumed that the subsistence level of consumption is strictly greater than zero and the individual has to take some rest during each period.
- The strong concavity of the utility function is imposed in order to ensure the existence of a unique intertemporal plan.

  Furthermore, if the Hamiltonian is strongly concave, then the necessary conditions are also sufficient for an optimum, a result which considerably simplifies manipulation of the model.

ii) All except one assumption for the after-tax-earnings function are imposed in order to facilitate the calculus and, in any case, are quite harmless. The only assumption that must be justified is strong concavity. Abundant evidence supports the hypothesis of a concave earnings function, (see, e.g., [Mincer, 1974]). This well-established property has been strengthened in order to ensure the strong concavity of the Hamiltonian.

iii) The assumption that capital income is non-taxable is imposed in order to simplify the notation as well as the handling of the model. The assumption also facilitates the modelling of the tax function, albeit at some loss of generality. Three things should be noted. First, for most working individuals, capital income is pretty low. Second, a significant part of capital income is non-taxable. Third, in the empirical implementation of the model, every effort has been made to approximate as accurately as possible the actual amount of taxes paid by the individual.

iv) It was decided to present the model in continuous form so that the notation could be kept at manageable levels. A discrete version of the model will be examined in the second part of this chapter.

(v) The utility function is defined over consumption, leisure and education. There are two reasons for doing so. First, abundant evidence supports the hypothesis that education level affects the utility function.[1] Second, there is enough evidence suggesting that non-pecuniary factors affect the demand for schooling and occupational choice.[2]

The Hamiltonian of the optimal control problem can be written as

$$e^{-\delta t}U(\ ) + \sigma t(s) + \lambda[t(w)-\gamma L] + \mu[rI + f(\ ) - px - C(\ )]$$

The first order conditions for a maximum are:

(V.6) $e^{-\delta t}U_x = \mu p$

(V.7) $-e^{-\delta t}U_\ell = e^{-\delta t}U_{t(s)} = \sigma + \mu C_{t(s)}$ $\quad t^s > 0$

(V.8) $-e^{-\delta t}U_\ell = e^{-\delta t}U_{t(w)} = -\lambda - \mu f'w$ $\quad t^w > 0$

(V.9) $[e^{-\delta t}U_{t(s)} + \sigma - \mu C_{t(s)}] \leq 0$ $\quad t^s = 0$

(V.10) $[e^{-\delta t}U_{t(w)} + \lambda + \mu f'w] \leq 0$ $\quad t^w = 0$

(V.11) $\dot{\sigma} = -e^{-\delta t}U_s - \mu f'w_s t(w)$

$\sigma(T) = 0$

(V.12) $\dot{\lambda} = -\mu f'w_L t(w) + \lambda\gamma$

$\lambda(T) = 0$

(V.13) $\dot{\mu} = -\mu r$

$\mu(T) = B'[I(T)]$

Very few individuals are part time students and part time workers for the entire length of their working lives. For most individuals, the first phase of their life-cycle is the one of full time schooling, succeeded possibly by part time schooling and part time working, while during the third phase they are working full time.

A most interesting prediction of the model is that the aforementioned succession of phases is, in general, the optimal one for an individual who does not face any constraints, save for the already presented ones. I will return to this point below. For the time being, let us suppose that this is the optimal path.

Before studying the three phases of the life-cycle, it is necessary to develop some notation; the three phases will be denoted by T_1, T_2, T_3.

1/. Full time schooling. During the first phase of the life-cycle, the first order conditions can be written as:

(V.14) $$e^{-\delta t}U_{t(s)} + \sigma - \mu C_{t(s)} = 0$$

$$\dot{\sigma} = -e^{-\delta t}U_s < 0$$

(V.15) $$e^{-\delta t}U_{t(w)} + \lambda + \mu f'w \leq 0$$

$$\dot{\lambda}(t) = \lambda\gamma \qquad t \varepsilon T_1$$

The first order conditions can also be written as:

(V.16) $$e^{-\delta t}U_{\ell} = \sigma - \mu C_{t(s)}$$

(V.17) $$e^{-\delta t}U_{\ell} \geq \lambda + \mu f'w$$

Hence,

(V.18) $$\sigma - \mu C_{t(s)} \geq \lambda + \mu f'w$$

That is, the net discounted benefits of education are greater than the net benefits of working during the first phase.

Rearrange equations (V.16) and (V.17) as follows:

$$\dot{\lambda}(t) = \lambda\gamma \qquad t \varepsilon T_1$$

The first order conditions can also be written as:

(V.16) $e^{-\delta t}U_{\ell} = \sigma - \mu C_{t(s)}$

(V.17) $e^{-\delta t}U_{\ell} > \lambda + \mu f'w$

Hence,

(V.18) $\sigma - \mu C_{t(s)} > \lambda + \mu f'w$

That is, the net discounted benefits of education are greater than the net benefits of working during the first phase. Rearrange equations (V.16) and (V.17) as follows:

(V.19) $\sigma > \lambda + \mu f'w + \mu C_{t(s)}$

Equation (V.19) implies that during the first phase, the shadow price of schooling is greater than the sum of foregone earnings, shadow price of experience, and marginal cost of schooling.

A few caveats are in order here:

i) Equations (V.18) and (V.19) should not be interpreted as a "proof" of the succession of phases. Full-time schooling implies (V.18) and (V.19) and vice-versa. (Mutatis mutandis the previus discussion applies to (V.20) and (V.21)).

ii) A true proof must be that a particular sequence and only this sequence guarantees the continuity of shadow prices over-time. As it will be shown below such a proof appears to be impossible. However, it will be shown that for "most" individuals the aforementioned succession of phases guarantees "almost everywhere" the continuity of shadow prices over time. Obviously, this is not a general proof. Therefore, the results that are presented below should be interpreted with caution.

2/. Full time working. In order to facilitate the subsequent analysis, I will consider the third phase first and then discuss the second one. The first order conditions for the third phase can be written as:

(V.20) $$e^{-\delta t}U_{t(s)} + \sigma - \mu C_{t(s)} \leq 0$$

$$\dot{\sigma} = - e^{-\delta t}U_s - \mu f'w_s t(w)$$

(V.21) $$e^{-\delta t}U_{t(w)} + \lambda + \mu f'w = 0$$

$$\dot{\lambda} = - \mu f'w_L t(w) + \lambda\gamma$$

Equations (V.20) and (V.21) can be written as:

(V.22) $$e^{-\delta t}U_{\ell} \geq \sigma - \mu C_{t(s)}$$

(V.23) $$e^{-\delta t}U_{\ell} = \lambda + \mu f'w$$

(V.24) $$\sigma - \mu C_{t(s)} \leq \lambda + \mu f'w \qquad t\varepsilon T_3$$

Equation (V.23) shows that the marginal utility of leisure is not equal to the marginal after tax wage rate but it is equal to the marginal after tax wage rate plus the shadow value of experience. Therefore, all static models underestimate the price of leisure which, of course, results in inconsistent estimates. Equation (V.24) summarizes nicely the characteristics of that phase; it is the period during which the benefits, of schooling pecuniary and non-pecuniary, are less than the benefits of working.

During the third phase, hours of work and earnings reach a peak and then decline. The wage rate by assumption is a non-decreasing function of experience and will reach a peak at a point around the age of retirement. Fuchs [1967] shows that this conjecture is supported by U.S., cross-section data for all levels of education, except for those with post-graduate degrees. Given the shape of the wage rate function, one would expect hours of work to reach a peak before earnings reach their maximum point. Assuming that the utility function is strongly separable in consumption and leisure, the foregoing statement is supported by the model.

Let us consider the case of maximum hours of work: Suppose that the utility function is strongly separable. Define the after tax wage rate as:

(V.25) cw

where c: a constant.

Notice that the last transformation is absolutely harmless since we are going to investigate the properties of the trajectory of the wage rate around a point. Taking the derivative, with respect to time, of equation (V.23) we obtain:

$$(V.26) \quad -\delta e^{-\delta t} U_{\ell} - e^{-\delta t} U_{\ell\ell}\, \dot{t}(w) = \dot{\lambda} - \mu r c w + \mu c [w_L t(w) + w_s t(s)]$$

Noticing that $\dot{t}(w)$ and $t(s)$ are by assumption equal to zero and utilizing the equation of motion for $\dot{\lambda}$ we get

$$(V.27)\quad -\delta e^{-\delta t}U_{\ell} = -\mu r c w + \lambda\gamma$$

Using equation (V.23), equation (V.27) can be written as

$$(V.28)\quad -\delta[\lambda + \mu c w] = -\mu r c w + \lambda\gamma$$

or,

$$(V.29)\quad \frac{\lambda[\delta + \lambda]}{\mu[r-\delta]} = cw \qquad r \neq \delta$$

λ/μ is the <u>relative</u> shadow price of experience in terms of non-human capital. Equation (V.29) shows that hours of work will reach a peak at the point where the relative price of experience is equal to the marginal after tax wage rate, if δ=r and γ=0. If r>δ and γ = 0 hours of work will reach a peak at a point where the relative price of experience is lower than the marginal after-tax wage rate. Finally, if the rate of time preference is greater than the interest rate, then, obviously, the previous analysis does not go through and we have to conclude that hours of work reach a peak during the second phase.

Now let us consider the other important point of the third phase the point where earnings reach a peak. Imposing strong separability on the utility function, multiplying equation (V.23) by t(w) and taking the derivative of the product with respect to time, we obtain:

$$(V.30)\quad -\delta e^{-\delta t}U_{\ell}t(w) - e^{-\delta t}U_{\ell\ell}t(w)\dot{t}(w) + e^{-\delta t}U_{\ell}\dot{t}(w) =$$

$$= \dot{\lambda}t(w) + \lambda\dot{t}(w) - \mu rcwt(w) + \mu c[w\dot{t}(w)]$$

But,

$$\mu c[w\dot{t}(w)] = 0$$

$$e^{-\delta t}U_{\ell}\dot{t}(w) = \lambda\dot{t}(w) + \mu cw\dot{t}(w)$$

$$\dot{\lambda}t(w) = -\mu cw_L t(w)\ t(w) + \lambda\gamma t(w)$$

$$= \mu c[w\dot{t}(w)]\ t(w) + \lambda\gamma t(w)$$

$$= \lambda\gamma t(w)$$

Substituting the previous equations into (V.30) we obtain

(V.31) $(\delta+\lambda)e^{-\delta t}U_{\ell}\ t(w) +$

$$e^{-\delta t}U_{\ell\ell}\dot{t}(w)t(w) - \mu rcwt(w) - \lambda\mu cwt(w)$$

Hence,

(V.32) $(\delta+\gamma)U_{\ell} + U_{\ell\ell}\dot{t}(w) >> 0$ if $r > \gamma$

A sufficient but not necessary condition for (V.32) to hold is $\dot{t}(w)$ to be negative, which implies that hours of work are decreasing when earnings reach a peak, As δ becomes smaller, the probability that hours of work will reach a peak increases. Another way to look at the problem is by writing (V.32) as:

(V.33) $(\delta+\gamma) + \dfrac{U_{\ell\ell}\dot{t}(w)}{U_{\ell}} >> 0$

which illustrates that the point where earnings reach a peak depends on the curvature of the utility function, or the elasticity of marginal utility.

3/. <u>Part time working - Part time schooling</u>. Very few new things can be said about the second phase of the life-cycle. The only interesting exercise is to determine the duration of this phase. This exercise has already been performed when we determined the terminal point of the first period and the initial point of the third period.

This is a good point to consider the optimality of the particular succession of phases that we have studied. That is, it has to be shown that an individual working on a full time basis will not, in general, return to school. As we have seen, during the phase of full time working, the optimality conditions can be written as:

(V.24) $\sigma - \mu C_{t(s)} < \lambda + \mu f'w$

Suppose that the individual returns to school on a part time basis. Then the optimality conditions can be described by the following equation:

$$\sigma - \mu C_{t(s)} = \lambda + \mu f'w$$

Finally, if the individual switches to full time schooling inequality (V.24) is reversed, i.e.,

$$\sigma - \mu C_{t(s)} > \lambda + \mu f'w$$

Equivalently, a necessary but not sufficient condition for an individual to return to school is that the net shadow price of schooling increases at a faster rate than the opportunity cost of working, or equivalently, the opportunity cost of working decreases at a faster rate than the net shadow price of schooling. The change of the net shadow price of schooling during full time working is given by

$$\dot{\sigma} - (\dot{\mu C}_{t(s)}) = \dot{\sigma} + r\mu C_{t(s)}$$

since $C_{t(s)}$ is independent of time. Given (V.20) $\dot{\sigma}$ is negative, hence the net shadow price of schooling might be increasing over time because of the decrease of the shadow value of wealth.

Assuming that the marginal tax is constant, the chanqe of the opportunity cost of working is given by

$$\dot{\lambda} + (\dot{\mu cw}) = -\mu cw_L t(w) + \lambda\gamma - r\mu cw + \mu cw_L t(w)$$
$$= \lambda\gamma - r\mu cw$$

The last equation implies that the opportunity cost of working might be a decreasinq function of time because of the decrease of the shadow value of wealth.

Now let us collect the previous results. It has been shown that the net shadow price of schooling might be increasing and the opportunity cost of working might be decreasing. Hence, one cannot exclude the case that the net shadow price of schooling will overtake, i.e., will become larger, the opportunity cost of working. The question that arises is if such an event should be viewed as an integral part of the optimal trajectory of a "typical" individual. The answer to the last question is probably negative. The previous analysis indicates that only individuals who have accumulated enough assets will consider the possibility of returning to school, either on a part time or full time basis. Moreover, the prime motivation for doing so is not an increase of their wage rate, which may or may not increase when they will return to full time working. (If experience depreciates over time, the wage rate will definitely decrease if they become full time students. On the other hand, the wage rate will increase because their level of education will improve).

The ramifications of the statement that "the prime motivation for returning to school is not an increase of the wage rate" are quite important. The aforementioned statement implies that the behavior of individuals who return to school in order to increase their wage rate is, in general, suboptimal. Intuitively speaking, if investment in education is profitable, then the individual is better off by investing in it as early as possible. There are at least two reasons for a consumer to return to school in order to improve his/her earnings capacity. The first one is the possibility of unforeseeable events which might alter drastically the intertemporal plans of an otherwise rational individual. The second one is the existence of various constraints which might hamper the implementation of an optimal plan during the early stages of the life cycle. Imperfect capital markets, time constraints, lack of part time jobs and a host of other variables may force the consumer/worker to enter the labor force although such a strategy can be suboptimal. We will return to this point in the next chapter.

We now turn to the comparative dynamics of the model. Even in this simplified model, the influence of certain exogenous variables, like taxes or the interest rate, on the trajectories of the state and control variables is rather moot. First, an increase of the interest rate reduces the discounted wealth of the individual who, as a consequence of the increased shadow price of wealth, will decrease his/her demand for leisure. Contrariwise, an increase of the interest rate reduces the price of future consumption, hence the age consumption profile will be pushed upwards. A similar problem arises in static models of labor supply where nothing can be said about the relative importance of the income and substitution effects. In a dynamic model, the wealth effect should also be taken into consideration; this makes things more complicated. The inability of the model to provide us with unambiguous qualitative results underscores the necessity of empirical work.

Second, the life-cycle response to a variation of the marginal tax is ambiguous. Ignoring, temporarily, wealth, we can see that an increase of the marginal tax rate is equivalent to a decrease of the wage rate. By introducing wealth, the picture becomes foggier since the wealth may decrease while its shadow price will increase. An increase of the shadow price of wealth represents an increase of all prices, including leisure; thus the final outcome cannot be determined. The impact of taxes is quite profound during the first phase. Here an increase of the marginal tax reduces the potential wage rate and hence encourages heavier investment in formal education, provided that the increase of the shadow value of wealth does not outweigh the decrease of the potential wage rate.

The purpose of this paragraph was to present a potentially empirically tractable life-cycle model of labor supply. Therefore my task is to show how the model can be estimated given the available information. Before doing so, it is necessary to indicate possible generalizations of the model.

i) It appears as a straightforward exercise to define the model over a family rather than an individual. It suffices to include the hours of leisure of the second member (or members) of the family into the utility function and modify appropriately the equations of motion. Unfortunately, such a model will be limited in scope and realism. Nowadays storks do not bring children, which implies that family formation is an endogenous variable. The introduction of family formation in the model will complicate it and it is not clear if the final product will be a testable one.

ii) By disaggregating the composite good, we can generate a system of intertemporal demand functions. As a matter of fact, this is the only technique which will generate a system of intertemporal demand functions consistent with neoclassical economics.

iii) The wealth can be disaggregated into its components, such as pension plans, savings, bonds, etc. The estimation of such a model should be quite useful for policy purposes since it provides us with a system of demand functions for assets.

iv) Finally, other possible extensions may include introduction of the costs of vocational training, cost of equipment necessary for the job, time required for commuting, etc.

B: A Tractable Model

Given the available data sets, information for the first two phases of the life-cycle is not available. The best that we can do is to estimate the parameters of the model using information from the third phase only. The estimates will be efficient given the available information. The estimate of only a part of the life-cycle can be justified by appealing to the well celebrated principle of optimality. Bellman [1957] expresses the principle as

> An optimal policy has the property that, whatever the initial state and decision, etc., the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision

The principle of optimality can be described as follows. Consider a consumer/worker who has solved the optimal control problem presented at the beginning of the chapter. Therefore, he/she has derived the optimal trajectories for the control and state variables for the entire time horizon, i.e., (t_o,T). Suppose that the optimal policy is such that the individual switches to full time schooling at time t_1, $t_1 > t_o$. Denote the accumulated stocks of wealth, experience and education at time t_1 as I^* (t_1), L^* (t_1) and s^* (t_1) respectively. The principle of optimality states that the remaining part of the trajectories of the said optimal control problem - the trajectories for the time interval (t_1,T) - must be optimal with respect to the initial conditions I^* (t_1), L^* (t_1), and $s^*(t_1)$. Equivalently, the trajectories for the time interval (t_1,T) may be viewed as if they had been derived from the aforementioned optimal control problem defined over the period (t_1,T) and with initial conditions $I^*(t_1)$, $L^*(t_1)$ and $s^*(t_1)$. Therefore, the optimal trajectories for an individual who optimizes over the time interval (t_1,T) and starts with initial stocks of $I^*(t_1)$, $L^*(t_1)$ and $s^*(t_1)$ will be identical to the trajectories of an individual who has accumulated $I^*(t_1)$, $L^*(t_1)$ and $s^*(t_1)$ at t_1. (It is, of course, assumed that both individuals are of the same age and the two optimal control problems are, save for initial time,identical). In the context of the model, the followinq two points should be treated with some caution:

i) The utility function during the third phase is variable with respect to education. Hence, it should be written as $U(x,\ell;s)$ rather than as $U(x,\ell,s)$. In any case, since the demand for schooling is by assumption equal to zero during the third phase, it is not possible to recover all the parameters of an ordinary utility function using data from the third phase.

ii) Initial stocks should be evaluated at the initial time of the third period.

The maximization problem of the consumer can be written as follows:

$$\text{(V.34)} \qquad \text{maximize} \int_{t_1}^{T} e^{-\delta t} U(x, \ell; s)\,dt + e^{-rT} I(T)$$

w.r.t. x, ℓ

subject to

$$\text{(V.35)} \qquad \dot{L} = (h-\ell) - \gamma L$$

$$\text{(V.36)} \qquad \dot{I} = rI + f[w(L,s)(h-\ell)] - px$$

$$\text{(V.37)} \qquad s(t) = \bar{s}(t_1) \gg 0 \qquad t \varepsilon [t_1, T]$$

$$\text{(V.38)} \qquad I(t_1) = \bar{I}_0$$

$$\text{(V.39)} \qquad L(t_1) = \bar{L}_0 \gg 0$$

The first order conditions for the optimal control problem are the following:

$$\text{(V.40)} \qquad e^{-\delta t} U_x = \mu p$$

$$\text{(V.41)} \qquad e^{-\delta t} U_\ell = \mu f' w + \lambda$$

$$\text{(V.42)} \qquad \dot{\mu} = -\mu r \qquad \mu(t) = e^{-rT}$$

$$\text{(V.43)} \qquad \dot{\lambda} = -\mu f' w_L (h-\ell) + \gamma L, \quad \lambda(T) = 0$$

Quite clearly, the previous set of equations cannot be estimated directly because they are in continuous form. There are two ways to get around this difficulty. The first approach would be some sort of discrete time approximation, e.g.,

$\dot{x} = x_t - x_{t-1}$. The second approach would be to set up the model in discrete time. The former technique creates some problems of its own, such as introducing an approximation error in the estimation process. The latter technique not only is free of the aforementioned shortcoming but it also has the extra advantage of presenting the model in a more "realistic" form. The optimal control of an individual who has completed his academic endeavours can be written, in discrete time form, as follows:

(V.44) $$\max \sum_{t=t_1}^{T-1} \frac{1}{(1+\delta)^t} U(x(t), \ell(t); s(t_1)) + \frac{1}{(1+r)^T} I(T)$$

w.r.t. x, ℓ

subject to

(V.45) $$L(t+1) = [h-\ell(t)] + (1-\gamma)L(t)$$

(V.46) $$I(t+1) = (1+r)I(t) + f[w(L,s)(h-\ell)] - px$$

(V.47) $$I(t_1) = \bar{I}_o$$

(V.48) $$L(t_1) = \bar{L}_o \qquad 0$$

The first order conditions are the following:

(V.49) $$(1+\delta)^{-t} U_x(t) = \mu(t+1)p$$

(V.50) $$(1+\delta)^{-t} U_\ell(t) = \mu(t+1)f'w(t) + \lambda(t+1)$$

(V.51) $$\mu(t) = \mu(t+1)(1+r) \qquad \mu(T) = \frac{1}{(1+r)^T}$$

(V.52) $$\lambda(t) = (1-\gamma)\lambda(t+1) + \mu(t+1)f'w_L(t)[h-\ell(t)], \quad \lambda(T)=0$$

The model is not empirically tractable yet because $\lambda(t)$ is an unobservable quantity. There are two ways to solve this particular problem. The first one is to solve the optimal control problem, hence express $\lambda(t)$ as a function of the parameters of the problem and time. The second one is to approximate the path of $\lambda(t)$. The former approach can be implemented if and only if one is willing to impose very restrictive assumptions on the utility function and the wage function so that an analytic solution to the optimal control problem can be derived. Therefore, this technique will not be pursued in this research project. Before considering the second technique, it is necessary to dispose of some technicalities. First, notice that an analytic solution for $\mu(t)$ is available, i.e,:

(V.53) $$\mu(t) = \frac{1}{(1+r)^t}$$

Second, after some experimentation, it was clear that the results were sensitive to the value of the interest rate. Moreover, information about the exact interest rate that every consumer faced is not available. So it was decided to estimate the difference between the interest rate and the rate of time preference. To that effect, the following approximation is sufficient:

(V.54) $$\frac{(1+r)^{t+1}}{(1+\delta)^t} \simeq (1+r-\delta)^t$$

Using equations (V.53) and (V.54) equations (V.49) (V.50) and (V.52) can be written as

(V.55) $$(1+r-\delta)^t U_x(t) = p$$

(V.56) $$(1+r-\delta)^t U_\ell(t) = f'w(t) + \tilde{\lambda}(t+1)$$

(V.57) $$\tilde{\lambda}(t) = (1-\gamma)\tilde{\lambda}(t+1) + f'w_L[h-\ell(t)]$$

where $$\tilde{\lambda}(t) = \lambda(t)(1+r)^{t+1}$$

that is, $\tilde{\lambda}(t)$ is the relative price of experience.

Consider the following approximation for $\tilde{\lambda}(t)$:

$$\tilde{\lambda}(t) = (1-\gamma)\tilde{\lambda}(t+1) + f'w_L[h-\ell(t)]$$

$$\simeq (1-\gamma)\tilde{\lambda}(t+1) + c_t \frac{\Delta w_t}{\Delta L_t}[h-\ell(t)]$$

(V.58) $$\simeq (1-\gamma)\tilde{\lambda}(t+1) + \frac{[c_{t+1}w_{t+1} - c_t w_t][h-\ell(t)]}{h-\ell(t) - \gamma L_t}$$

$$\tilde{\lambda}(T) = 0$$

where c_t is equal to one minus the marginal tax

Inspection of the last set of equations reveals that the path of $\tilde{\lambda}(t)$ can be computed recursively for every individual in the sample.

For estimation, the following flexible functional form for the variable utility function will be employed.

(V.59)

$$U = 4(a_{01}x^{1/4}s+b_{01}x^{1/4}s^{1/2}+a_{02}\ell^{1/4}s+b_{02}\ell^{1/4}s$$

$+ 1/2a_{11}x^{1/2}s + 1/2b_{11}x^{1/2}s^{1/2} + a_{22}\ell^{1/2}s +$
$1/2b_{22}\ell^{1/2}s^{1/2} + a_{12}x^{1/4}\ell^{1/4}s + b_{12}x^{1/4}\ell^{1/4}s^{1/2})$

One can easily show that if all the parameters of the utility function are nonnegative and at least one strictly positive then for fixed s, U is nonnegative, nondecreasing and quasiconcave in x and ℓ. For fixed x and ℓ, U is nonneqative, nondecreasing and quasiconcave in s. The inverse demand functions (V.55) and V.56) corresponding to (V.59) are:

$$\text{(V.60)} \quad (1+r-\delta)^t [a_{01}x^{-3/4}s + b_{01}x^{-3/4}s^{1/2} + a_{11}x^{-1/2}s + b_{11}x^{-1/2}s^{1/2} + a_{12}x^{-3/4}\ell^{1/4}s + b_{12}x^{-3/4}\ell^{1/4}s^{1/2}] = p$$

$$\text{(V.61)} \quad (1+r-\delta)^t [a_{02}\ell^{-3/4}s + b_{02}\ell^{-3/4}s^{1/2} + + a_{22}\ell^{-1/2}s + b_{22}\ell^{-1/2}s^{1/2} + a_{12}x^{1/4}\ell^{-3/4}s + b_{12}x^{1/4}\ell^{-3/4}s^{1/2}] = f'w(t) + \tilde{\lambda}(t+1)$$

Now let us consider the models which will be estimated. Model 1 is defined by equations (V.60) and (V.61) and $\tilde{\lambda}(t)$ is computed via (V.58). The stochastic structure of the model is specified as follows:

Define by e_x, e_ℓ the disturbance term corresponding to equations (V.60) and (V.61) respectively. Denote by e_{it} the 2 X 1 vector of disturbances, i.e., (e_x, e_ℓ), for individual i of time t. It is assumed that e_{it} are independently and normally distributed and $E(e_{it}) = 0$, $E(e_{it}e_{it}) = \Sigma$, $E(e_{it}e'_{it+s}) = 0$ for $s \neq 0$, $E(e_{it}e'_{jt+s}) = 0$. The previous set of assumptions implies that the vector of disturbances is both, serially and contemporaneously independently distributed, while for individual i at time t the disturbances in the two equations are jointly distributed. Maximum likelihood estimates of the parameters of the utility function can be derived by maximizing the logarithm of the concentrated likelihood function

$$\text{(V.62)} \qquad L = \frac{-NK(\ell n 2\pi + 1)}{2} - \frac{K\,\ell n}{2}\left|S\right| + \sum_{k=1}^{K} \ell n(\text{abs}\left|J_k\right|)$$

where N is the number of equations, K the number of observations, S is the 2 x 2 sample covariance matrix of disturbances, and abs $|J_k|$ is the absolute value of the determinant of the 2 x 2 matrix of the derivatives of the error term with respect to the endogenous variables of the model, that is x, ℓ. It is assumed that the disturbances are due to unexpected (unforseeable) events which forced the individual away from the optimal path.

Two points should be noted:

i) The system of demand equations should be, in general, exactly identified. Notice that without any loss of generality we have dropped the demand for "savings" or "wealth" function. The dynamic model should be carefully contrasted to the static one. In the static model one demand equation is redundant, because given the leisure/labor supply choice of the individual and the demand for n-1 commodities, the demand for the last one can be computed as a residual. In the dynamic model, the savings (or debt payment) decision can be treated as the residual.

ii) For the model that I have presented, normalization of the parameters of the utility function is not required. Such a normalization has already been imposed throuqh the UYONWAR function which has been defined as

$$aI(T) + b$$

and it has been normalized to:

$$I(T)$$

that is, a was set to be equal to unity and b equal to zero.

Model II A maintained hypothesis of the proposed model is that the wage rate is an endogenous variable; as a result estimates of the parameters derived from the previous model may be biased because the wage rate was assumed to be exogenous. The model is extended by estimating simultaneously the inverted demand functions and the wage equation:

$$\text{(V.60)} \quad (1+r-\delta)^t U_x = p$$

$$\text{(V.61)} \quad (1+r-\delta)^t U_{\ell} = f'w(t) + \tilde{\lambda}(t+1)$$

$$\text{(V.63)} \quad w = a_0 + a_1 L + a_2 s + a_3 s^{1/2} L^{1/2}$$

where $\tilde{\lambda}(t)$ is estimated via equation (V.61)

The stochastic structure of the model is specified as follows: Denote by e_x, e_ℓ, e_W the disturbances corresponding to the three equations. I assume that the (3x1) vector of disturbances $e = (e_x, e_\ell, e_W)$ is normally and both contemporaneously and serially independently distributed. Maximum likelihood estimates of the parameters of the utility function and the wage function can be obtained by maximizing the logarithm of the concentrated likelihood function with respect to those parameters

$$L = \frac{-NK}{2}(\ell n 2\pi + 1) - \frac{K}{2}\ell n\left|S\right| + \sum_{k=1}^{K} \ell n(\text{abs}\left|J_k\right|)$$

where N is the number of equations, K the number of observations, S is the 3 x 3 sample covariance matrix of disturbances, and abs $\left|J_k\right|$ is the absolute value of the determinant of the 3 x 3 matrix of the derivatives of the error term with respect to the endogenous variables of the model, that is, x_t, ℓ_t, w_t.

Footnotes to Chapter V

1. See Wales and Woodland [1976]. A number of authors have used education level as an explanatory variable in labor supply models [see Masters and Garfinkel [1974], Ham [1982]). Hence, they have shown that education level affects directly the consumption/leisure choice of the individual.
2. See Freeman [1971], Lucas [1977].

Chapter VI

ESTIMATES

A. Data

The model is estimated for 1967 and 1975. The two samples are drawn from the University of Michigan Research Center's "A Panel Study of Income Dynamics". Following the methodology of Ghez and Becker [1975] I use synthetic cohorts to approximate the profile of a typical individual. That is, the data are cross-tabulated according to age and years of schooling; the representative individual for every cohort is simply the "average" individual. Thus the number of observations is equal to (years of schooling)x(time periods). The two samples are constructed as follows:

i) I include in the samples only male, married, not self-employed individuals, 28-50 years old, whose education ranges from grade six up to college graduation. This restriction was imposed in order to make the sample as homogeneous as possible.

ii) I exclude individuals who receive any welfare payments, or any bonuses, commissions or extra income from overtime work, since such payments may affect the labor/leisure choice of the individual.

iii) I exclude individuals whose wives were working, as well as those who hold a second job or were attending school. Quite clearly the behavior of such individuals cannot be captured by the model which was presented in Chapter V.

iv) I also exclude households for which the taxable income of all dependents - other than the wife - is qreater than $1000 (1967) or $1612 for 1975[1], since the labor supply response of the head of the household may be affected if the contribution of the dependents to the family budget is substantial.

v) Finally, I exclude individuals who were not satisfied with the number of hours spent on the job. This restriction was imposed in order to ensure that the maximization process of all individuals included in the sample corresponds as closely as possible to the theoretical model. If the individual is not satisfied with the number of hours spent on the job, that implies that institutional constraints are binding. Hence the proposed model is not directly applicable.

Before discussing the construction of the variables, it is important to understand the limitations of the model.

i) Sample selectivity is a problem in my samples. Unfortunately, given the large number of restrictions, it is next to impossible to incorporate those constraints in the likelihood function. Hence,some caution is required in the interpretation of the results.

ii) Individuals belonging to minority groups, e.g., blacks, are not excluded from the samples for two reasons. First, there is no reason to believe that their preferences are different. Second, there is evidence supporting the hypothesis that differences of the wage rate between whites and blacks are minimal, if any, provided that one controls for education.[2]

The variables of the model are constructed as follows:

i) Consumption. Given the restrictions on the samples, total income is equal to labor income plus interest payments, dividends and rents. An index of annual consumption is constructed as total income plus the rental value of the house - if the household owned one - plus the rental value of the car(s), minus income taxes, minus property taxes, minus annual mortgage payments, minus annual payments on car debts. Information about the rental value of the car(s) and annual payments on car debts was not available for the 1975 sample.

ii) Experience. The index of experience is defined as follows:

$L(t+1) = (1-\gamma)L(t)+[h-\ell(t)]$, $L(0)$ is given. Thus the index is constructed simultaneously with the estimation of the model. Notice that there is a one-to-one mapping between the definition of experience in the theoretical model and the index employed in my empirical work. The only problem in constructing the index is that a measure of experience up to age 28 must be provided. Hours of experience up to age 28 can be approximated by (hours of work per year)x(years of work). Years of work can be estimated as 21 minus years of schooling, Hours of work per year can be approximated as 45x40. Notice that I assumed that the individual was working only 45 weeks per year, hence this ad hoc measure might underestimate the actual experience of individuals with few years of schooling. However, one might claim that most employed young adults and/or adolescents are casual workers, so that their experience should be weighted by a lower weight.

iii) The rest of the variables, such as years of schooling, gross wage rate and hours of work are taken directly from the data set.[3]

A few caveats are in order here:

i) Since there is no information on local and state taxes, I use as a proxy for the marginal tax rate the federal marginal income tax rate. The taxable income is calculated as total income minus the standard deduction minus total exemptions.[4] The after tax marginal wage rate is the gross wage rate multiplied by one minus the marginal tax rate.

ii) Exact information about savings is not available. The households were asked if their savings "were greater than......., but less than,..." or "if savings amount to as much as .....months' income'". For most households reported savings were less than two months' income, a substantial part of which should be transactions balances. Moreover, there is no information about contractual savings. One might be tempted to approximate total savings as capital income divided by the interest rate. But this technique suffers from two drawbacks. First, it ignores contractual savings and assets with accruing forms of income. Second, the actual rate of return on the savings is unknown. As a result, I had to assume that savings were equal to zero. The lack of information about savings is the most serious deficiency of the employed data set.

iii) The rental value of automobiles for 1967 was computed as the price of automobiles times thirty per cent. According to the U.S.A. Tax Guide, the allowable depreciation rate for automobiles is between thirty and forty per cent dependinq on the nature of the business. Since those depreciation rates are too large for privately owned automobiles, a depreciation rate of twenty-five per cent was employed. The rental price was calculated as twenty-five per cent plus five per cent, the prevailing interest rate during the period. This approach leaves much to be desired because the depreciation rate of automobiles is, a choice variable for the household.

iv) The rental value of houses is calculated at the value of the house times the interest rate, less capital appreciation, plus depreciation, plus property taxes.[5] It should be kept in mind that the depreciation rate of houses is also an endogenous variable of the household.

v) An index of the shadow value of experience is constructed using equation (V.58), i.e.,

$$\tilde{\lambda}(t) \approx (1-\gamma)\tilde{\lambda}(t+1) + \frac{[C_{t+1}w_{t+1} - C_t w_t][h-\ell(t)]}{h-\ell(t) - \gamma L_t}$$

$$\lambda(T) = 0$$

The following assumptions are employed in the construction of this index.

(a) It is assumed that the shadow value of experience is zero by the age of 60 $[\tilde{\lambda}(60)=0]$ (Note that the index of the shadow value of experience is constructed using a larger data set - 28-60 years old individuals - than the one used for estimation purposes - 28-50 years old.)

(b) Whenever cohorts are missing the wage rate and hours of work are approximated by interpolation so that the index of the shadow value of experience and the index of experience can be computed using the approximations. For example, if the ith cohort is missing, the wage rate is approximated as the arithmetic mean of the wage rates of the i-1 and i+1 cohorts.

vi) The index of consumption and the after tax wage rate for the 1975 sample are expressed in 1967 prices. They are deflated by the consumer price index - so that the two samples would be as comparable as possible.

The numbers of observations per cohort for the two years are presented in Tables VI.1 and Table VI.2.

B. Results

Before discussing the empirical results of the model, it is necessary to consider a few technical points.

i) As Heckman and MaCurdy [1980] point out, the estimates of labor supply models are not invariant with respect to the choice "total number of hours available per year" Most authors define "total number of hours per year" as equal to 8760, i.e., 24 x 365. Given the definition of leisure as the number of hours in a year minus hours of work, we overestimate hours of leisure by including such activities as sleep as part of leisure. I assume that approximately ten hours per day are required for sleep, commuting, preparation of meals and other home productions so that leisure is defined as 5000 minus hours of work.

ii) The index of education is constructed as follows:

(a) 1967 sample

7. 6-8 grades

10. 9-11 grades

12. 12 grades (completed high school)

13. 12 grades plus non-academic training

14. college, no degree

16. college, bachelor degree (A.B., B.S., etc.,)

(b) 1975 sample

7. 6-8 grades

10. 9-11 grades

12. 12 grades

14. 13-15 grades (including individuals who have completed high school and have some non-academic training)

16. 16 grades

Notice that the 1975 sample has been aggregated into 5 groups rather than 6. This is due to the fact that the 1975 sample is much smaller than the 1967 sample (see Tables VI.1 and VI.2).

iii) For estimation purposes, the index of consumption and hours of leisure are divided by 10,000. The index of education is divided by 2. Time-t-is measured in increments of .5.[6]

TABLE VI.1

NUMBER OF OBSERVATIONS PER COHORT - 1967 SAMPLE

AGE	EDUCATION					
	6-8 Grades	9-11 Grades	12 Grades	13 Grades	14-15 Grades	16 Grades
28	2	-	-	1	1	1
29	1	1	1	1	-	-
30	-	1	2	1	2	1
31	1	-	1	2	1	1
32	2	1	1	-	1	-
33	3	1	1	-	1	-
34	2	3	3	1	-	2
35	3	2	2	1	1	1
36	1	1	-	-	-	1
37	3	-	1	-	2	2
38	1	-	3	1	1	-
39	1	4	2	-	1	1
40	1	1	4	-	-	3
41	3	3	2	2	-	1
42	3	2	1	-	1	1
43	2	1	-	1	-	2
44	2	2	-	-	2	-
45	1	1	1	-	2	2
46	1	-	-	-	-	-
47	-	-	2	1	2	1
48	1	2	-	1	-	-
49	1	2	1	-	1	-

TABLE VI.1 (cont'd.)

AGE	6-8 Grades	9-11 Grades	12 Grades	13 Grades	14-15 Grades	16 Grades
50	2	3	-	-	1	1
51	2	2	2	1	-	-
52	1	3	-	-	-	1
53	-	1	-	-	1	-
54	1	1	-	-	1	-
55	2	1	-	-	1	1
56	3	1	1	-	1	-
57	1	-	-	-	-	-
58	1	3	1	-	-	-
59	1	1	3	-	2	-
60	1	2	-	-	1	-
	50	46	35	14	27	23

TABLE VI.2

NUMBER OF OBSERVATIONS PER COHORT - 1975 SAMPLE

AGE	EDUCATION				
	6-8 Grades	9-11 Grades	12 Grades	13-15 Grades	16 Grades
28	1	-	5	2	3
29	1	1	2	1	1
30	-	-	3	1	-
31	1	2	3	2	-
32	-	3	3	1	-
33	-	2	2	1	-
34	-	2	1	2	2
35	-	1	4	-	1
36	2	2	1	-	1
37	-	-	3	1	1
38	1	3	-	-	-
39	-	1	1	-	-
40	-	2	1	1	-
41	1	-	-	-	-
42	-	-	2	-	-
43	1	1	1	1	3
44	1	-	-	-	1
45	1	-	2	2	1
46	-	-	-	-	-
47	1	2	-	1	2
48	-	2	-	1	-
49	-	2	-	-	-

TABLE VI.2 (cont'd.)

AGE	6-8 Grades	9-11 Grades	12 Grades	13-15 Grades	16 Grades
50	2	2	1	-	-
51	-	-	-	-	1
52	-	1	2	-	1
53	3	2	2	2	-
54	2	-	2	-	1
55	-	-	1	-	-
56	1	1	1	-	-
57	-	-	2	-	-
58	-	-	-	1	2
59	1	1	1	-	-
60	1	-	1	-	-
	21	33	47	20	21

v) The likelihood functions were maximized using a quasi-Newton algorithm due to Fletcher [1972] in order to locate the approximate maximum point and the Berndt et al [1974] algorithm to obtain the point exactly. All first partials of the likelihood functions were computed analytically, save for the one corresponding to γ which was computed numerically. The convergence of the algorithm was quite time consuming for two reasons. First, the Jacobian had to be evaluated for every observation and function evaluation. Second, the indices for $\tilde{\lambda}$ and L had to be reevaluated at every iteration.

Now let us consider the results.

1. Utility function. Inspection of the parameters of the estimated utility functions (see Table V1.3) indicates that the functions cannot satisfy globally the properties of monotonicity and quasiconcavity. However, monotonicity was satisfied everywhere, while quasiconcavity was satisfied for most sample points, (see Table VI.4). This result indicates that the assumption of utility maximization cannot be rejected by the data. Quasiconcavity was tested by checking the following Hessian

$$\begin{matrix} U_{xx} & U_{xl} \\ U_{xl} & U_{ll} \end{matrix}$$

TABLE VI.3

UTILITY FUNCTION PARAMETER ESTIMATES*

	1967 SAMPLE		1975 SAMPLE	
	Model I	Model II	Model I	Model II
a_{01}	0.127 (1.4)	-0.007 (1.6)	-0.014 (2.1)	0.026 (1.9)
b_{01}	-0.102 (0.9)	0.145 (1.1)	0.136 (0.9)	-0.298 (1.3)
a_{11}	0.061 (2.3)	-0.171 (2.4)	0.278 (2.8)	0.008 (2.9)
a_{12}	-0.210 (3.9)	-0.198 (3.9)	0.035 (3.6)	-0.149 (4.9)
b_{12}	0.214 (1.8)	0.561 (2.2)	0.124 (2.2)	0.233 (1.5)
a_{02}	0.640 (1.8)	0.349 (1.5)	0.417 (1.3)	0.562 (1.6)
b_{02}	-1.103 (3.1)	-1.924 (2.7)	-1.633 (2.7)	-1.326 (4.1)
a_{11}	-0.275 (0.9)	0.413 (0.9)	-0.182 (1.3)	-0.491 (1.2)
b_{11}	0.597 (2.4)	0.442 (1.9)	0.469 (1.9)	0.762 (1.7)
b_{22}	1.369 (3.7)	2.051 (4.6)	1.691 (5.8)	1.776 (4.2)

*Asymptotic "t" statistics in parentheses.

TABLE VI.4

TEST OF THE CURVATURE PROPERTIES OF UTILITY FUNCTIONS*

1967 SAMPLE		1975 SAMPLE	
Model I	Model II	Model I	Model II
87/95	88/95	56/64	57/64

*Fraction of sample points for which the utility function is concave.

The function satisfied quasiconcavity locally if U_{xx} and U_{ll} were negative and the determinant of the matrix positive.

2. Rate of time preference (see Table VI.5) Although the rate of time preference was not estimated, it was shown that the difference (interest rate minues rate of time preference) is positive. This supports some of the theoretical predictions of the model which required the rate of time preference to be smaller than the interest rate. Furthermore, it is reasonable to expect the interest rate to be at least equal or larger than the rate of time preference because of market forces and transaction costs. The estimated rates of time preference are consistent with results obtained by Rosen [1976] and Zabalza [1979]. Their estimates were between 0.064 and 0.0875. Given the prevailing interest rates during 1967 and 1975 - 5% and 8.5% respectively - my estimates are, practically, of the same magnitude.

3. Elasticity of labor supply (see Table VI.6).

In evaluating these elasticities, caution is in order. These elasticities should not be compared with elasticities derived from static Marshallian functions. The ones which are presented here can be interpreted as short-run or impact effect elasticities. They are calculated as follows:

$$\frac{d(h-l)}{dw} \cdot \frac{cw}{(h-l)} = -\frac{(1+r-\delta)^{-t}U_{xx}}{U_{xx}U_{ll} - U_{xl}^2} \cdot \frac{cw}{(h-l)}$$

TABLE VI.5

ESTIMATES OF THE DIFFERENCE

(RATE OF INTEREST MINUS RATE OF TIME PREFERENCE)*

1967 SAMPLE		1975 SAMPLE	
Model I	Model II	Model I	Model II
0.0159 (2.4)	0.0172 (1.9)	0.0128 (1.9)	0.0156 (1.6)

*Asymptotic "t" statistics in parentheses.

TABLE VI.6

ELASTICITIES OF LABOR SUPPLY WITH RESPECT TO THE WAGE RATE*

1967 Sample

Education	Model I	Model II
6-8 grades	0.16	0.06
9-11 grades	0.16	0.09
12 grades	0.10	0.05
12 grades plus non-academic training	0.11	0.08
College no degree	0.12	0.07
College bachelor's degree	0.09	0.04

1975 Sample

Education	Model I	Model II
6-8 grades	0.31	0.25
9-11 grades	0.25	0.18
12 grades	0.23	0.20
13-15 grades	0.18	0.14
16 grades	0.21	0.18

*Aggregate elasticities were calculated by weighting cohort elasticities by hours of labor supply and number of individuals per cohort.

where an estimate of the derivative is computed by employing the implicit function theorem. The computed elasticities measure the response of an individual given a temporary change of the net wage rate. As Nagatani (1978) has shown, this response should not be confused with steady-state comparative dynamics.

It can be easily shown that the "wealth effect constant" elasticity of leisure is always nonpositive if the individual is a utility maximizer. Moreover, this result can be justified in terms of traditional economic theory (see Layard and Walters [1978, p. 138]). The sign of substitution effects is always negative. Notice that if we allow for wealth effects - technically by allowing $\lambda(t)$ to vary - the sign of the elasticity will be ambiguous.

All elasticities presented in Table VI.6 have the correct sign and are reasonably low. It should be noted that the elasticities reported by Smith [1977] and Ghez and Becker [1975] - who employed a completely different life-cycle model - are of the same magnitude. Moreover, elasticities of labor supply computed via static models (see Wales and Woodland [1976] [1977]) are quite consistent with the results of this paper.

4. Depreciation rate of experience (see Table VI.7).

The estimated depreciation rates of experience are quite consistent with the observed decline of wages around the

TABLE VI.7

ESTIMATES OF THE DEPRECIATION RATE OF EXPERIENCE*

1967 Sample		1975 Sample	
Model I	Model II	Model I	Model II
0.045	0.037	0.039	0.047
(0.9)	(1.3)	(0.9)	(0.8)

*Asymptotic "t" statistics in parentheses.

late forties or early fifties. Moreover, my estimates are quite similar to the ones reported by Heckman [1976] and Rosen [1976]. However, the interpretation of the estimates requires some caution. First, the standard errors of the estimates are quite large, as shown by the small tabulated asymptotic "t" statistics. Second, the estimates are very sensitive with respect to the specification of the model and the employed data set.

5. <u>Wage function</u> (see Table VI.8)

The estimated wage functions are consistent with the assumptions of the theoretical model in the sense that the wage rate is an increasing function of education and experience. However, the results should be interpreted with extreme caution for the following reasons:

(a) Given the large number of restrictions on my samples, sample selectivity is a problem. Hence, the estimates may be inconsistent.

(b) In view of the complexity of the likelihood function, it was not possible to introduce in the wage functions certain variables which may affect the wage rate, such as quality of schooling.

(c) It is well known that the real wage rate is an increasing function of time because of economic growth. The last statement suggests that time should be introduced as an explanatory variable in the wage function. Unfortunately, "time" and experience are

TABLE VI.8

WAGE FUNCTION PARAMETER ESTIMATES[a,b]

	1967 Sample	1975 Sample
a_1	-0.002 (1.1)	-0.0009 (0.6)
a_2	0.442 (6.1)	0.507 (6.4)
a_3	0.013 (1.6)	0.021 (2.3)
a_0	1.051 (2.6)	1.376 (1.8)

a The wage function is defined as

$$w = a_0 + a_1 L + a_2 s + a_3 s^{\frac{1}{2}} L^{\frac{1}{2}}$$

b Asymptotic "t" statistics in parentheses highly correlated, so it was not possible to introduce time in the model.

Footnotes to Chapter Vl

1. The consumer price index for 1975 was 161.2 (1967 = 100) see, Economic Report of the President, U.S.G.P.O. Washington, D.C., 1979, p. 240.

2. See Ham [1977] [1982] and Masters [1975]. There is a lot of contrary evidence (see Stolzenberg [1975a] and Lazear [1979]). However, this evidence should be interpreted with caution because of possible misspecificationsof the employed models. Ham's analysis suggests that race acts as a proxy for underemployment. Therefore, by excluding individuals who were not satisfied with the number of hours spent on the job we eliminate the spurious correlation between race and labor supply.

3. The following variables have been taken from the 1967 (1975) data sets:

   5(18): House value, 8(22): Annual Mortgage Payments, 6(25): Property taxes, missing (24): Whether Mortgage Payments include property taxes, 22(missing): Annual payments on car debts, 47(32): Head's annual hours working for money, 53(44): Wife's annual hours working for money, 74(22): Head's income from wages, 76(86): Total taxable income of head and wife, 79(106): Taxable income of others in family unit, 114(15): Family composition, 252(75): Bonus, overtime, Commissions, 117(136): Age of Head, 119(137): Sex of Head, 196(158): Employment status, 197(159): Occupation Code, 198(161): Whether work for self, someone else or both, 227(218): Whether extra job, 228(219): Occupation extra job, 313(384):

Education, 230(224): Whether could have worked more, 231(227): Whether wanted more work, 232(228): Whether could have worked less, 233(229): Whether wanted less work, 256(736): Bkt. ADC, AFDC - Head and wife, 257(737): Bkt. other welfare - Head and wife, 258(738): Bkt. Social Securlty - Head and wife, 259(739): Bkt. Other retirement, 260(740): Unemployment, 261(741): Alimony, 145(missing): value of all cars owned.

4. Due to lack of data, I was not able to take into account the exclusion of half of long-term gains.

5. The rental value of houses for 1967 is constructed as follows:

   Long-term interest rate: 5% (See Review, October 1981, Federal Reserve Bank of St. Louis), Capital appreciation: 3.3% (See 1970 HUD Statistical Yearbook, U.S. Department of Housing and Urban Development, U.S.G.P.O. Washington, D.C., 1971, p. 313), Property taxes and depreciation: 4.8% (See Shelton [1968]).

   During the early and mid-seventies house prices exhibited substantial short-term fluctuations. As a result, the estimated rental value is sensitive to alternative assumptions about the appreciation rate. Gillingham [1980] suggested to apply a 15-year weighted average of the appreciation rate to house values. His analysis indicates that the rental values of houses increased by 30 to 60 per cent between 1966/1967 and 1974/1975. So, a rental value of 9.5% is employed for the 1975 data set.

6. The results are not sensitive to these rescalings.

CHAPTER VII

CONCLUSIONS

A. Introductory Remarks

What general conclusions and directions for related research can we draw from this research? What is the usefulness and significance of the proposed model as a piece of empirical research? In order to be able to answer the latter question the former one should be answered first. In the next section I present the main results of this study, those which may be viewed either as original ones or as improvements of previously established results. In the last section, I present possible extensions of the model and directions for related research.

B. Results

1. Estimation of systems of intertemporal functions. The well known aphorism, "in the long run we are all dead" is hardly an acceptable excuse for ignoring time as a parameter of the utility maximization problem. After all, the human being is distinguished by his ability to plan over a long time horizon. Assuming that inter-temporal utility maximization is the appropriate framework for modelling the behavior of the consumer and assuming that all prices are given exogenously, the specification of the model is a straightforward exercise. However, certain issues have to be confronted by estimating the derived demand functions. As we have seen, the demand functions depend on current prices and the shadow value of wealth. This can be expressed, for every instant of time, as the appropriately discounted

shadow value of wealth evaluated at either the initial time or at the terminal time.

The main drawback to using the initial time is that it restricts drastically the number of procedures which can be employed for estimation purposes. Indeed, the only workable estimation procedure is to treat the shadow value of wealth as a fixed effects variable, a procedure which in general yields inconsistent estimates. Note that we are forced to use this technique because an estimate of the shadow value of wealth evaluated at the initial time cannot, in general, be derived.[1]

The technique which was suggested and implemented in this research provides a clear solution to the problem of estimating intertemporal demand functions. It was shown that the shadow value of wealth can be expressed as a function of the bequest function of the individual. Given a functional form for the bequest function and information about terminal wealth,[2] one can estimate simultaneously the parameters of the instantaneous utility function and the bequest function. Furthermore, if one is willing to assume a linear bequest function that accompanies risk neutral behavior, then the shadow value of wealth depends on only the parameter of the bequest function.

In general, there is no reason to believe that all prices are exogenously given to the consumer. Consequently, empirical work based on the assumption that all prices are exogenous will result in biased estimates as a consequence of the misspecification of the model. Although the specification of a model treating the wage rate as an endogenous variable does not create any difficulties, per se, the estimation of the corresponding costate variables does raise some difficulties. It was shown that it is possible to approximate the time path of the costate variables using information from the sample.

Even if the empirical results of this paper were not satisfactory, the usefulness of the proposed techniques would not be affected. Their usefulness lies in the fact that they transform a practically untractable model into one that can be readily estimated.

2. Specification of the model.

The proposed model generalizes most of the models which have been suggested so far, except for those developed in the tradition of the "Theory of the Allocation of Time". The model integrates the two approaches of modelling human capital, the one due to Ben-Porath and the one due to Arrow. The human capital accumulation part of the model is quite similar to the one suggested by Mincer [1974], yet, the underlying methodoloqy of Mincer's approach is substantially different. Mincer's work is based on the assumption of income maximization while in the present paper human capital accumulation was embedded into the far more general framework of utility maximization. The utility maximization framework not only includes income maximization as a special case but also allows us to integrate the model within the paradigm of mainstream neoclassical economics.

The major advantage of the proposed model lies in its specification. All variables and equations are directly observable and measurable , which departs from the tradition of the literature to present such models in terms of unobservable quantities, such as human capital. Moreover, the relationship between the theoretical model and its stochastic structure is clear and the latter can be justified and rationalized in terms of the former. Finally, if the theoretical model is identical to the estimated one, the theoretical one provides us with the required restrictions for the empirical one. In our case, the close similarity between the two models provided us with at least two useful "techniques" for estimation purposes - the use of a conditional utility function and the algorithm for approximating the shadow value of experience.

C. Possible Extensions of the Model/Directions for Related Research.

Possible extensions of the model can be classified into two categories. "Pedestrian" ones simply generalize the model by appealing to "brute force", and other extensions may require reconsideration of the maintained hypotheses. We have already discussed possible extensions which belong to the first class and we will only summarize them here.

i) Household utility function.

ii) Disaggregation of the composite qood.

iii) Disaggregation of the wealth of the individual (family).

iv) Correction for sample selectivity.

v) Introduction of other characteristics, besides education, in the utility function.

vi) Endogeneity of taxes.

Now let us consider some extensions of the model which appear to be promising and interesting. It seems strange that such an "expensive toy" as a life-cycle model cannot produce a "trivial" statistic such as the elasticity of the supply of labor. The only way to get estimates of the elasticity of the supply of labor is by simulating the full model. Note that simulation does not mean to substitute in the model a couple of numbers and solve for the resultinq estimate of the elasticity of the supply of labor. We have to solve the complete optimal control problem using the same time series for wages, except for one observation, and compare the estimated responses to the observed ones. Using this technique yields an estimate of the instantaneous elasticity, that is, the short run impact effect, as well as a measure of the impact effect over the life cycle. Although this exercise is not beyond the capacities of a modern high speed computer, the computational cost will be significant. Be that as it may, I think it would be unwise to invest in this kind of study for the time being, since we do not know yet what is the best approach to model the supply of labor over the life cycle. The results that Heckman and MaCurdy derived as well as the results of this model are reasonable, quite conventional and promising for further research. Nevertheless, more research is required in order to get a more complete picture. Given these results, the extra effort, and money required for estimating more complicated models can be justified.

It is well known that quite a few consumers face a number of time and credit constraints which may affect their optimal paths. For example, as I have shown in the previous chapter, a rational individual who returns to school most probably does so because his/her choice set was constrained when he/she was younger. Therefore, it would be interesting to introduce explicitly those constraints in the optimization problem. Technically speaking, the said generalization of the model will not, in general, create any insurmountable difficulties. Unfortunately, such an extension of the model seems to be quite difficult if not impossible. The reason is the lack of data. One can only hope that statistical agencies will understand the importance and usefulness of such studies and will generate appropriate data sets.[3]

The economics of uncertainty and replanning seem to be other fruitful areas of research. The maintained hypothesis of perfect knowledge of the future is limited in scope and realism. A most reasonable conjecture is the one suggested by Nagatani [1972] that the consumer adjusts every "year" his/her plan according to his/her expected wealth. There are two possible ways to model such a situation. Note that the demand for leisure and the composite good depends on current prices and the shadow value of wealth, which, in turn, depends on the parameter of the bequest function and the terminal wealth. Therefore, it suffices to substitute the value of expected wealth into the system of equations. Since information about expected wealth is not available, it must be estimated as a fixed effects variable. Furthermore, if the prices that will prevail during a period are not precisely known at the beginning of the period, we may assume that the consumer maximizes his/her utility subject to a vector of planning prices which will be functions of the prices that prevailed in the previous period. The equations for the planning prices can be estimated simultaneously with the demand functions. My only qualm about these models is that they are *ad hoc*. The formulation of expectations about wealth and prices is treated as a black box. Even if the estimates are acceptable from a statistical point of view, we cannot assess whether that is due to the robustness of the model or, simply, the time trend of the employed data set. If the planning equations are generated within the model - for example, the wage rate - one cannot raise any objections to their usefulness as analytic and empirical tools. The model that I proposed can easily handle the above mentioned technique for estimating the value of expected wealth. It suffices to define the bequest function as, say, a quadratic one, hence the shadow value of wealth will be a linear function of wealth. Expected wealth can be estimated as a fixed effects variable.

The second approach to uncertainty is to attempt to model explicitly the formation of expectations. The advantage of this approach is that it subsumes a number of other problems that may arise because of uncertainty, such as unemployment and the unknown returns to investment. Unfortunately, this area of research is still in its infancy.

We conclude by examining two crucial question; (i) are there any limitations to the model, and (ii) if so, will the empirical model identify such situations? The answers to both questions are affirmative. (i) If any of the assumptions of the model is violated, then the model is not directly applicable.[4] (ii) By construction, the empirical model is, practically, an isomorphic mapping of the theoretical one. Consequently, if any of the assumptions of the theoretical model are violated, the empirical model will detect them.

In other words, the possible limitations of the theoretical moddel is not a serious issue for the simple reason that all its assumptions are refutable. Thus, if the theoretical model can not pass the test that will provide supportive empirical evidence, this should prompt the researcher to consider alternative structures that might not otherwise have been considered. A caveat is in order here. It is, of course, assumed that the researcher will not employ defensive strategies in testing for evading refutations of hypotheses.

Footnotes to Chapter VII

1. Such an estimate can be derived if an analytic solution of the optimal control problem is available.

2. As we have already seen, terminal time can be interpreted as the date of retirement and the bequest function as the utility yield of net wealth at retirement.

3. Such data sets will include information about savings, contractual savings, assets with accruing forms of income and possible credit constraints.

4. As with any other model, the present model has some possible limitations, such as: uncertainty about future prices, wages and employment opportunities, myopic expectations (Strotz [1956]), unanticipated changes of the utility function and so on. However, as we have already discussed, the model can in principle be extended to handle such situations.

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