AN ANALYSIS OF TWO APPROACHES TO TEACHING
USED BY GRADE 8 MATHEMATICS TEACHERS

By

ROSITA TSENG TAM
B.Sc., The University of British Columbia, 1973

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS

in

THE FACULTY OF GRADUATE STUDIES

Department of Mathematics and Science Education
Department of Educational Psychology and Special Education
Faculty of Education

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
August 1983
© Rosita Tseng Tam, 1983
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Mathematics and Science Education

Department of Educational Psychology
The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date August, 1983
ABSTRACT

The purpose of the study was to perform an analysis of two approaches to teaching used by Grade 8 mathematics teachers in British Columbia. A comparison was made in the orientation to concrete and abstract approaches. The major question addressed in the study was: Were teacher practices in accordance with the theory of instruction proposed by several authors, in which it is recommended that concepts be presented using manipulative materials, pictures, or diagrams, and then through abstract presentations with symbols?

To address this question, an analysis was made of the data collected from approximately 100 mathematics teachers who took part in the Second International Mathematics Study (SIMS) during the 1980-81 school year. The instruments used to collect the data were a set of five topic-specific questionnaires on classroom processes. The items in these questionnaires related to many aspects of classroom practice. This study was concerned with those items that were constructed to collect information on methods used in presenting certain concepts and skills in the Grade 8 mathematics curriculum. Since the nature of the data was not known prior to analysis, the study was designed to explore the nature of the data so that valid comparisons
could be made on the concrete-abstract variable. Several data processing procedures were used. These procedures included the categorization of items by a panel of experts; the scoring of items; the preliminary analysis to investigate the justifiability of aggregation of item scores; and the calculation of concrete, abstract, and difference scores for all the teachers. Exploratory data analysis techniques were then used to compare the teachers' choice between concrete and abstract approaches.

The results of the analysis seemed to indicate that teachers were abstractly-oriented in teaching most of the skills and concepts included in the study. In their teaching of three new concepts and skills in the Grade 8 curriculum, mathematics teachers were concretely-oriented in their teaching of one concept only. This finding led to the conclusion that teacher practices were not in accordance with the theory of instruction which states that new concepts and skills in mathematics should be initially taught using concretely-oriented approaches.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ix</td>
</tr>
<tr>
<td>1 THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>Framework for the SIMS</td>
<td>6</td>
</tr>
<tr>
<td>Definition of Terms</td>
<td>8</td>
</tr>
<tr>
<td>Purpose of the Study</td>
<td>11</td>
</tr>
<tr>
<td>2 REVIEW OF THE LITERATURE</td>
<td>13</td>
</tr>
<tr>
<td>Theories of Instruction and</td>
<td></td>
</tr>
<tr>
<td>Related Research</td>
<td>13</td>
</tr>
<tr>
<td>Classroom Practices and</td>
<td></td>
</tr>
<tr>
<td>Related Research</td>
<td>18</td>
</tr>
<tr>
<td>Reliability and Validity</td>
<td></td>
</tr>
<tr>
<td>of Questionnaires</td>
<td>24</td>
</tr>
<tr>
<td>3 DESIGN AND PROCEDURE</td>
<td>33</td>
</tr>
<tr>
<td>Population and Sample Selection</td>
<td>33</td>
</tr>
<tr>
<td>The Instruments</td>
<td>36</td>
</tr>
<tr>
<td>Data Collection</td>
<td>39</td>
</tr>
<tr>
<td>Data Processing Procedure</td>
<td>39</td>
</tr>
<tr>
<td>Categorization of Items</td>
<td>40</td>
</tr>
<tr>
<td>Scoring of Items</td>
<td>44</td>
</tr>
<tr>
<td>Preliminary Analysis</td>
<td>44</td>
</tr>
<tr>
<td>Scoring of Subtopics</td>
<td>47</td>
</tr>
<tr>
<td>Difference Score for</td>
<td></td>
</tr>
<tr>
<td>Each Subtopic</td>
<td>48</td>
</tr>
<tr>
<td>Method of Analysis</td>
<td>49</td>
</tr>
<tr>
<td>4 RESULTS OF THE STUDY</td>
<td>52</td>
</tr>
<tr>
<td>Concrete and Abstract Scores</td>
<td>52</td>
</tr>
<tr>
<td>Common and Decimal Fractions</td>
<td>53</td>
</tr>
<tr>
<td>Algebra</td>
<td>57</td>
</tr>
<tr>
<td>Geometry</td>
<td>61</td>
</tr>
<tr>
<td>Measurement</td>
<td>61</td>
</tr>
<tr>
<td>Difference Scores</td>
<td>65</td>
</tr>
<tr>
<td>Summary</td>
<td>68</td>
</tr>
</tbody>
</table>
Table of Contents - continued

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONCLUSIONS AND IMPLICATIONS</td>
<td>69</td>
</tr>
<tr>
<td>Answer to Research Question</td>
<td>69</td>
</tr>
<tr>
<td>Limitation of the Study</td>
<td>71</td>
</tr>
<tr>
<td>Implications</td>
<td>72</td>
</tr>
<tr>
<td>Suggestions for Further Research</td>
<td>75</td>
</tr>
<tr>
<td>Summary</td>
<td>78</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>80</td>
</tr>
</tbody>
</table>

APPENDICES

<p>| A | Common and Decimal Fractions Questionnaire | 86 |
| B | Ratio, Proportion and Percent Questionnaire | 104 |
| C | Algebra Questionnaire | 120 |
| D | Geometry Questionnaire | 144 |
| E | Measurement Questionnaire | 168 |
| F | The Initial List of Subtopics and Items as Categorized by the Panel of Experts | 187 |
| G | Profiles of Responses to the Concrete and Abstract Items for Each Subtopic for the Sample of Ten Teachers | 189 |</p>
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correlations of Students and Teacher with Observer</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>Description of Population A Teachers</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>The List of Subtopics and Corresponding Items for the Concrete and Abstract Categories as Validated by the Panel of Experts</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>The List of Subtopics and Corresponding Items Included for Analysis in This Study</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>The Mean Variances of Item Scores from Corresponding Subtopic Scores for the Sample of Ten Teachers</td>
<td>46</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Framework for the international study</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>An abstract treatment of the concept of negative integers</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>An abstract treatment of the concept of fractions</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>A concrete treatment of the concept of negative integers</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>A concrete treatment of the concept of fractions</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Items dealing with methods of instruction for the concept of integers</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>Distribution of abstract scores and concrete scores for the concept of fractions</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>Distribution of abstract scores and concrete scores for the concept of decimal fractions</td>
<td>54</td>
</tr>
<tr>
<td>9</td>
<td>Distribution of abstract scores and concrete scores for the addition of fractions</td>
<td>56</td>
</tr>
<tr>
<td>10</td>
<td>Distribution of abstract scores and concrete scores for the concept of integers</td>
<td>58</td>
</tr>
<tr>
<td>11</td>
<td>Distribution of abstract scores and concrete scores for the subtraction of integers</td>
<td>60</td>
</tr>
<tr>
<td>12</td>
<td>Distribution of abstract scores and concrete scores for the concept Pythagorean Theorem</td>
<td>62</td>
</tr>
</tbody>
</table>
List of Figures - continued

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Distribution of abstract scores and concrete scores for the concept number ( \pi )</td>
<td>64</td>
</tr>
<tr>
<td>14</td>
<td>Distribution of abstract scores and concrete scores for the relationship among various metric units</td>
<td>64</td>
</tr>
<tr>
<td>15</td>
<td>Distribution of difference scores for the eight subtopics</td>
<td>66</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The author would like to express her gratitude to members of her thesis committee -- Dr. David Robitaille, Dr. Todd Rogers, and Dr. Doug Edge -- for their advice and encouragement; to members of her panel of experts -- Dr. G. Bluman, Dr. A. Adler, Dr. W. Szetela, and Mr. M. Dirks -- for consultation services regarding data processing; and to members of her family, Mr. & Mrs. Jorge Tseng, Mr. Theodore Tam, and Miss Genevieve Tam for their support and cooperation.
Chapter 1

THE PROBLEM

Background

It is apparent from the literature that in the last 20 to 30 years many changes in the mathematics curricula have been proposed in Canada and in countries throughout the world (House, 1979; Howosn, 1970; Robitaille, 1980; and Van der Blij, Hilding and Weinzweig, 1980). Nichols (1968) described this period as "a period of revolution" (p. 16). New topics such as probability, algebra and number theory were added to the elementary school curriculum. Topics such as statistics, logic and computer science became a part of the secondary school curriculum (Van der Blij, Hilding and Weinzweig, 1980). New mathematics programs produced by groups such as the University of Illinois Committee on School Mathematics (UICSM) (Howson, Keitel and Kilpatrick, 1981) and the School Mathematics Study Group (SMSG) (Howson, Keitel and Kilpatrick, 1981) were introduced into the schools. New theories in developmental psychology (Piaget, 1963) and in instruction (Bruner, 1966; Dienes, 1971) were developed.

Bruner (1966) presented his theory of instruction when he extended his developmental work into prescriptions for classroom instruction. He was influenced by Piaget's work
and defined three "modes of representation" (p. 46) in concept development. An enactive representation is a mode of representing a concept through appropriate motor response or physical manipulation of relevant concrete objects. An iconic representation involves the use of pictures, maps or other forms of mental imagery. A symbolic representation is a word or mark that stands for some concept or event but in no way resembles that concept or event. Bruner stated that new concepts should be taught in a specific sequence:

The object was to begin with enactive representation... --something that could literally be done or built-- and to move from there to an iconic representation.... Along the way, notation was developed...into a properly symbolic system (p. 64, 65).

Dienes (1971), in his principle of multiple embodiments, emphasized the use of concrete representations in the introduction of new concepts. His argument was that acquaintance with a concept through a variety of concrete materials helps the learner with the abstraction of the concept.

Prominent mathematics educators such as Ashlock (1967), Fennema (1972), Suydam and Dessart (1976), Rathmell (1978), and Resnick and Ford (1981) also urged that new concepts be taught through the use of concrete materials. Suydam (1976) summarized the opinions of numerous mathematics educators to the effect that "mathematical ideas and skills be developed from a concrete, physical base" (p. 6). This summary is consistent with recommendations from current research. After completing a literature review of studies on the use of concrete methods in mathematics instruction, Resnick and
Ford (1981) found that most researchers suggested that new concepts be introduced using concrete material, followed by semi-concrete material such as pictures and finally through abstract presentation with symbols.

There is evidence that certain proposed changes in the curriculum were in accordance with Piaget's developmental theory and Bruner's theory of instruction. For example, programs produced by SMSG reflect various features that are in harmony with the results of Piaget's work. Kilpatrick (1970) wrote:

Piaget's evidence that the child of nine or ten can handle many of the basic concepts of Euclidean spatial representation and measurement is mirrored in SMSG's placement of such topics in the elementary curriculum (p. 249).

Regarding teaching strategies, he also wrote that "Piaget's emphasis on the participation of the learner as he performs real actions on the learning materials...is shown in SMSG's use of discovery exercises in all of its text materials" (p. 249). In the SMSG curriculum, other advanced topics such as probability, statistics and number theory were also introduced at lower grade levels. Van der Blij, Hilding and Weinzweig (1980) contended that this shift of topics was inspired by Bruner who stated that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (Bruner, 1963; p. 33).

Unfortunately, although "teachers found the demonstrations [of new programs] persuasive, they did not adopt the new programs on returning to their schools" (House, 1979; p. 2).
Howson (1979) attributed the failure of diffusion of these new programs into regular classrooms in part to the composition of writing teams of these new products:

In the United Kingdom such teams usually contained a preponderance of teachers, whereas in the United States the balance was more even or was weighted towards the university academic. The dangers of placing responsibility for writing in the hands of university mathematicians are exemplified in the work of many projects whose materials have long since been cast aside as impracticable. Yet teams which lack professional mathematical support produce material lacking in coherence, purpose and balance. Indeed the achievement of 'balance' is notoriously difficult (p. 144).

While the literature (House, 1979; Howson, 1979; Howson, Keitel and Kilpatrick, 1981) is in agreement that dissemination of most of these new programs has failed and that their content was not adopted in mathematics classrooms, it is silent on the subject of whether or not theories proposed by developmental psychologists, mathematicians and mathematics educators have influenced teacher behaviour and methods of instruction in the classroom. In particular, it is not known if teacher practices are in accordance with the theory that concrete approaches be used in the instruction of mathematical concepts.

An assumption held by some educators has been that once a theory of instruction is developed, it would subsequently be implemented in educational practice (Common, 1980). Unfortunately, there has been little research to verify this assumption. The few studies (Goodlad and Klein, 1970; Price, Kelley and Kelley, 1977; Fey, 1979) that have investigated
classroom practices have concluded that many of the changes believed by educators to be taking place have not been accepted and implemented by classroom teachers. However, such a small sample of studies cannot provide valid, generalizable information. Several mathematics educators (Westbury, 1972; Lanier, 1978; Robitaille, 1980; McKnight, 1980; and Fullan and Park, 1981) have written that descriptive analyses of the methods of instruction used by teachers in classrooms are absent in mathematics education literature. The authors of the NACOME Report (1975) stated that "apallingly little is known about teaching in any large fraction of U.S. classrooms. The vacuum of data on classroom practices should give pause to those who present simplified cause-and-effect explanations" p. 68). Thus, the problem is that there is a lack of information on classroom practices. Little is known about the reality of implementation of innovative ideas in mathematics classrooms.

The Second International Mathematics Study (SIMS) (Robitaille, O'Shea and Dirks, 1982) was designed in part to provide highly specific information on classroom practices. For example, information on methods used by teachers when teaching specific topics in the curriculum was collected. The SIMS, then, provided a data base for researchers to examine the methods adopted by teachers. In particular, the data included information on teachers' orientation to the concrete or abstract approaches in teaching selected concepts and skills. A descriptive analysis of this portion of the
data would provide the much needed information regarding the translation of theory into practice.

This present study was designed to perform such an analysis. The data used for the analysis were collected from the SIMS and related to the concrete-abstract variable. The framework for the international study is presented in the next section to provide the context in which the present study was conducted.

**Framework for the SIMS**

According to Robitaille, O'Shea and Dirks (1982), the international study is "a broadly-based, comparative investigation of the mathematics curriculum as prescribed, as taught, and as learned" (p. 5). In this study, the mathematics curriculum is viewed from three perspectives: the intended curriculum, the implemented curriculum, and the attained curriculum. The intended curriculum is the mathematics curriculum as prescribed by the Ministry of Education in each of the participating countries. The implemented curriculum is the curriculum as taught by teachers in their classrooms. The attained curriculum is the curriculum as learned by the students as evidenced by their knowledge of and attitudes toward mathematics. Figure 1 gives a view of the framework for the study.
As shown in Figure 1, the three main components of the SIMS correspond to the three perspectives of the mathematics curriculum: a curriculum analysis; an analysis of classroom processes; and an analysis of student outcomes, both cognitive and affective, in the light of the nature of the curriculum and the kind of instruction received. For the analysis of the intended curriculum, curriculum guides and related documents were examined. For the study of the implemented curriculum, various questionnaires on classroom processes, each dealing with a specific topic, were developed. These instruments were designed to collect highly specific information from teachers regarding the methods they used in teaching these topics in the curriculum. The five specific topics selected for one population of the study were: common
and decimal fractions; ratio, proportion and percent; algebra; geometry; and measurement. The attained curriculum was assessed by test items, questionnaires and attitude scales.

**Definition of Terms**

The following definitions of terms were adopted for use in this study:

An abstract representation is a representation which uses a word or mark that stands for some concept or event but in no way resembles that concept or event. An abstract treatment of content relies primarily on an explanation which is symbolic in nature and derives its meanings from other mathematical content. Examples of abstract treatment of content are given in Figures 2 and 3. Figure 2 is an abstract treatment of the concept of negative integers found in the algebra questionnaire (see Appendix C, item 21).

The need to extend the number system to include the negative integers is discussed in order to find a solution to equations like:

\[
\underline{} + 7 = 5
\]

**Figure 2.** An abstract treatment of the concept of negative integers.
Figure 3 is an abstract treatment of the concept of fractions found in the common and decimal fractions questionnaire (see Appendix A, item 24).

Fractions are defined as quotients:

\[
\frac{3}{4} \text{ means "3 divided by 4"}
\]

Figure 3. An abstract treatment of the concept of fractions.

A **concrete** representation is a representation that involves the use of manipulative materials, diagrams or pictures. A **concrete** treatment of content relies primarily on manipulative materials, diagrams or experiences from the environment. It should be noted that concrete representations in this study included representations which were, in Bruner's definitions, either enactive or iconic in nature. Examples of concrete treatment of content are given in Figures 4 and 5. Figure 4 is an example of a concrete treatment of the concept of negative integers found in the algebra questionnaire (see Appendix C, item 20).
Negative integers are introduced by extending the number ray (0 and positive numbers) to the left. In this way, direction as well as magnitude are shown in the diagram.

-3 means 3 units to the left of 0.

Figure 4. A concrete treatment of the concept of negative integers.

Figure 5 is an example of a concrete treatment of the concept of fractions found in the common and decimal fractions questionnaire (see Appendix A, item 21).

Fractions are defined as parts of regions:

3/4 means

Figure 5. A concrete treatment of the concept of fractions.

A topic is defined by the content covered by each classroom process questionnaire.
A subtopic is a concept or skill within a selected topic. For example, four subtopics within the topic algebra were: concept of integers, addition of integers, subtraction of integers, multiplication of integers and formula.

A new subtopic is a subtopic that is introduced for the first time in Grade 8 according to the curriculum guide used in British Columbia. The new subtopics in this study were: concept of integers, addition of integers, subtraction of integers, multiplication of integers, formulae, and Pythagorean Theorem.

A review subtopic is a subtopic that has been introduced in the mathematics curriculum before Grade 8 according to the curriculum guide used in British Columbia. The review subtopics in this study were: concept of fractions, addition of fractions, concept of decimal fractions, operations with decimal fractions, ratio, proportion, number, and relationship among various metric units.

Purpose of the Study

The present study was focused on the practice of teaching mathematics through the use of concrete materials. The purpose of the study was to describe and compare the teachers' choice of concrete or abstract approaches in
teaching certain concepts and skills within selected topics in Grade 8. The major question addressed in the study was:

Were teacher practices in accordance with the theory of instruction proposed by several authors, in which it is recommended that concepts be presented using manipulative materials, pictures, or diagrams, and then through abstract presentations with symbols?

To address this question, the following aspects of instructional behaviour were examined:

1) The consistency within individual teachers in their orientation to the concrete and abstract approaches in teaching each subtopic.

2) Teachers' orientation to the abstract or concrete approaches in teaching various new and review subtopics.
Chapter 2

REVIEW OF THE LITERATURE

The literature related to three major topics relevant to the purpose of the study is reviewed in this chapter. First, the viewpoints of various psychologists, mathematicians and mathematics educators regarding the use of concrete approaches are reviewed. This is followed by a review of the research on classroom practices. Since classroom process research often is based upon data and information collected using questionnaires, the reliability and validity of such instruments is examined in the last section.

Theories of Instruction and Related Research

Bruner (1966) specified that the most effective sequence in which to introduce new concepts to students is to progress in the order enactive, iconic to symbolic. In his classroom studies, Bruner worked closely with individual students on experiments that were mainly concerned with mathematics learning. For example, in one study (Bruner and Kenney, 1965), quadratic equations were introduced to Grade three students using base ten blocks. Another procedure involving the manipulation of weights on a balance beam was used to reinforce the same concept. During the course of instruction, Bruner (1966) observed the sequence in which concept development occurred:
The children always began by constructing an embodiment of some concept, building a concrete model....The fruit of the construction was an image...that stood for the concept. From there on, the task was to provide [symbolic] representations that were free of particular manipulations and specific images (p. 65).

Bruner emphasized the importance of teaching in this sequence as he wrote:

But what struck us about the children as we observed them is that they not only understood the abstractions they had learned but also had a store of concrete images that served to exemplify the abstractions. When they searched for a way to deal with new problems, the task was usually carried out not simply by abstract means but also by matching up images (p. 65).

On the basis of his observation, Bruner argued that even though some students might be quite ready for a purely symbolic representation, it seemed reasonable and wise to present at least the iconic representation first. He described this theory as conservative, and stated:

When the learner has a well-developed symbolic system, it may be possible to by-pass the first two stages. But one does so with the risk that the learner may not possess the imagery to fall back on when his symbolic transformations fail to achieve a goal in problem solving (p. 49).

Kilpatrick (1970) pointed out that "much of Bruner's interpretation of new concepts and methods in education is made in light of what Piaget has said about intellectual development" (p. 249). For that reason, Piaget's views on cognitive development and the implication of his views for mathematics education are summarized in the following paragraphs in order that the foundation of Bruner's theory can be better understood.
Piaget (1963) summarized the four stages of cognitive development as follows: sensori-motor intelligence, from birth to two years; preoperational thought, from two to seven years; concrete operations, from seven to 11 years; and formal operations, from 11 to 15 years. Each stage of development is characterized by specific behaviour. The period of sensori-motor intelligence is characterized primarily by motor behaviour, whereas the period of preoperational thought is characterized predominantly by language development. During the period of concrete operations, the child develops the ability to apply logical thought to problems that can be presented in a concrete manner. The child then becomes able to apply logic to problems that are presented in an abstract manner when he reaches the period of formal operations.

Regarding the implications of Piaget's views for mathematics education, Easley (1979) wrote:

Piaget's theory emphasized reflective abstraction from one's own physical action in the formation of logico-mathematical mental structures....What he calls concrete operations is the application of internalized logico-mathematical transformations or correspondences, one-at-a-time, to objects or their internal representations...and may involve the manipulation of physical objects or not, as circumstances permit (p. 10).

Thus, physical action and manipulation of objects are important in the process of acquiring mathematical knowledge. The manipulation of objects or their internal representations leads to understanding and application of logical thought to mathematical problems. Herein lies the foundation of
Bruner's theory that enactive or iconic presentations be used before symbolic presentation in the teaching of any mathematical concept. This sequence of instruction is recommended so that the learner will have "not only a firm sense of the abstraction underlying what he was working on, but also a good stock of visual images for embodying them" (Bruner, 1966; p. 66).

Dienes (1973) also advocated the use of concrete materials in mathematics instruction. In his principle of multiple embodiments, Dienes emphasized the use of concrete materials in the introduction of mathematical concepts. He stated that acquaintance with a concept through a variety of concrete representations helps the learner with the abstraction of the concept. In one of his earlier studies with young children, Dienes (1963) used multibase arithmetic blocks (a set of mathematics materials which he designed) to present the factoring principle underlying quadratic equations. In a later study, Dienes and Golding (1971) used attribute blocks to display principles of classification, set theory and logic. In both cases a concrete approach was used when introducing new concepts.

Suydam (1976) has summarized the views of theorists such as Piaget, Bruner and Dienes. She stated:

Generally, researchers have concluded that understanding is best facilitated by the use of concrete materials, followed by semi-concrete materials (such as pictures), and finally by the abstract presentation with symbols....The child should have
many experiences in which real objects are manipulated. Only after an idea has been developed with real materials should pictures, charts, and other less concrete materials be used -- and the use of symbols alone should be delayed until the child has a basic understanding.... These steps should not be limited to the primary level, they are also important at later grade levels when new mathematical ideas are introduced (p. 6, 7).

This same endorsement was also suggested by Rathmell (1978) who contended that

concrete materials should probably be used during the initial instruction for an operation. Concrete materials play an important role in concept development. The materials become a referent for work involving the operation. They provide a link to connect the operation to real-world problem-solving situations (p. 16).

And, more recently, Resnick and Ford (1981) advocated the teaching of new concepts using the concrete approach:

The structures of mathematics may be taught in an intellectually honest way at an early age by presenting them in concrete form, especially in the form of math materials that physically embody those structures (p. 126).

In summary, developmental psychologists, mathematicians and mathematics educators strongly support initially teaching mathematical concepts through concretely-oriented approaches, with the gradual introduction of symbolic representations.
Classroom Practices and Related Research

It is difficult to obtain an accurate picture of what is happening in ordinary classrooms (NACOME Report, 1975). The authors of the NACOME Report (1975) pointed out that when what is "current" is discussed in conferences, it usually means that "current trends at the cutting edge of innovation" (p. 67) are being discussed. However, the impression is left that the present time "is a time of great change and ferment" (p. 67), when in reality, little is known about what is happening in ordinary classrooms. What seems to be known is that classrooms which are part of some well-publicized, well-funded project appear to be implementing the innovative ideas suggested by theorists. But one of the conclusions of the report is that little is known about methods of instruction in the remaining larger number of classrooms. Lanier (1978) summarized the essence of this deficiency with the statement: "Descriptive analyses of teachers planning for and instructing groups of learners in classrooms are obviously absent in mathematics literature" (p. 7).

Very few research studies have been conducted to investigate classroom practices. The few studies in the literature (Goodlad and Klein, 1970; Price, Kelley and Kelley, 1977; Robitaille and Sherrill, 1977; Robitaille, 1981) which were designed to examine classroom practices provide general information such as the content that was taught, resources
used, classroom organization and methods of evaluation. These studies do not provide specific and detailed information on instructional methods. In particular, questions related to the teachers' orientation to concrete or abstract approaches were not addressed. 

Goodlad and Klein (1970) conducted a study to find out whether the innovative educational practices recommended by educators were actually finding their way into the schools. They selected three types of schools for classroom observation: schools enrolling a high proportion of culturally disadvantaged children, schools considered to be typical, and schools considered to be innovative. Their findings were based on observations and interviews conducted in 150 classrooms ranging from kindergarten to third grade. They concluded from their findings that many of the most noted and recommended curriculum innovations were "dimly conceived and, at best, partially implemented" (p. 72) in schools claiming their use. It was their opinion that

novel features seemed to be blunted in the effort to twist the innovation into familiar conceptual frames or established patterns of schooling....Many of the changes believed by educators to be taking place in schooling have not been getting into classrooms (p. 72).

Price, Kelley and Kelley (1977) conducted a study to determine what actual classroom practices occur in second and fifth grade mathematics classes in the United States. A questionnaire designed to gather information
about practices in mathematics classrooms was sent to a sample of second and fifth grade teachers across the United States. This sample of 1200 teachers was selected using the following procedure:

During the spring of 1975, 300 supervisors from a list of more than 800 provided by the National Council of Teachers of Mathematics (NCTM) were randomly selected and asked to distribute 10 questionnaires each. The questionnaires were to be distributed randomly, 5 of them to second-grade teachers and 5 to fifth-grade teachers, in the area served by each supervisor. The supervisors were provided with detailed procedures for the random selection of the teachers (p. 323).

In the study, the researchers investigated the implementation of innovative practices in these classrooms. The innovative practices examined were: individualized and small group instruction, the use of multiple sources of mathematics information for instruction, and the use of concrete materials. The findings indicated that 40% of the teachers used whole-class instruction most of the time; 82% of the teachers used one or predominantly one text; and 72% of the teachers used concretely-oriented approaches less than 10% of their instructional time. Regarding the generalizability of these findings, the researchers wrote:

It is conceivable that the method of selecting respondents through supervisors introduced a bias in the results, since not all school districts have mathematics supervisors. It might be assumed that the teachers were more professional, better prepared, and more likely to try innovative methods in mathematics teaching than the average teacher (p. 324).

The authors concluded that mathematics teachers and classrooms have changed far less in the past fifteen
years than had been supposed: "Teachers are essentially teaching the same way that they were taught in school. Almost none of the concepts, methods, or big ideas of modern mathematics programs have appeared in this median classroom" (p. 330).

The Ministry of Education in the province of British Columbia (B.C.) conducted an assessment of mathematics classrooms in Grades 4, 8 and 12 in 1976-77. In this study (Robitaille and Sherrill, 1977), 3500 mathematics teachers completed a comprehensive questionnaire dealing with numerous aspects of the methods and materials used in the teaching of mathematics in the province. Each questionnaire consisted of five parts: background and general information, learning outcomes, classroom organization, classroom instruction, and use of textbooks. The part on classroom instruction was designed to collect information on the resources, aids and methods teachers used when teaching mathematics. The specific instructional methods included for investigation were: use of concretely-oriented approaches such as learning centers and laboratories, individualized instruction, total class instruction, team teaching, and computer-aided instruction. The results of the assessment indicated that less than 25% of the secondary mathematics teachers used concretely-oriented approaches in their instruction. In the report, the recommendation was made to teachers at all levels that they should "vary their teaching approaches to include
concretely-oriented methods]...such as the use of learning centers and mathematics laboratory activities" (p. 91). A recommendation was also made to teacher educators that they should "encourage their student teachers to develop the skills required to use such techniques" (p. 91).

The second B.C. Mathematics Assessment (Robitaille, 1981) was carried out during the 1980-81 school year. Approximately 1600 teachers of mathematics completed questionnaires which dealt with a number of important aspects of the teaching and learning of mathematics. Much information was collected on various classroom activities including the use of oral, individual and group work; the use of observation, teacher-prepared tests and commercially-made tests for evaluation; the nature of the curriculum; and the use of calculators. However, questions related to the orientation to concrete or abstract approaches in instruction were not included in the second assessment.

Although the few studies reported in the preceding paragraphs involved investigation into classroom practices, such a small sample cannot provide valid, generalizable information on classroom processes. Once again it becomes apparent that there is a lack of description of classroom practices. This opinion is supported by several mathematics educators. For example, Robitaille (1980), in addressing the issue of the reality of curriculum reform in school mathematics since 1960, raised the question of how much
anyone really knows about the actual implementation processes in mathematics classrooms. He described the situation as one in which "the extent to which the widely publicized changes of the past two decades in methods of teaching mathematics have resulted in observable changes in the classroom behaviours of teachers" (p. 90, 91) is not known. In Bulletin No. 4 of the SIMS the same message is echoed:

Very little detailed information is available on what instructional strategies are employed by teachers as they go about teaching....It is essential that we have more information about what students encounter as they study in the mathematics classroom (p. 13).

This same opinion was also expressed by McKnight (1980) who contended that there are "no adequate portraits of what goes on in schools and classrooms as mathematics curricula are implemented" (p. 241). He pointed out that "a descriptive mosaic[ that] covers a variety of aspects related to a variety of specific instructional situations and instructional decision-making" (p. 257) is much needed. However, he suggested that the items of the five topic-specific questionnaires used in the SIMS to describe classroom processes do provide specific, detailed pieces of information that make the construction of such a descriptive mosaic possible.

Another group of studies that involved the investigation of teaching practices is experiments on teaching methods. These studies were designed to compare the effectiveness of certain instructional methods. Medley (1979) completed a literature review of these methods experiments and concluded:
Almost every methods experiment that I have found in the literature was designed to use the pupil (rather than the teacher) as the unit of analysis. As a result, no valid generalization to teachers other than those who actually took part in the experiment could be made. In order to make such generalization possible, many teachers (the more the better) would have to teach by each method, so that an accurate estimate could be made of the consistency of the results obtained by different teachers using the same method. This has rarely been done (p. 14).

In summary, it is evident that there is a lack of description of classroom practices. This is particularly evident when focusing on the concrete-abstract variable. In spite of the strong endorsement by theorists and mathematics educators for concretely-oriented approaches, little is known about the reality of the translation of this theory of instruction into practice.

Reliability and Validity of Questionnaires

In this section, research on the reliability and validity of questionnaires is reviewed. The studies considered seem to provide contradictory results on the validity of teacher self-report data. A possible reason for the contradiction might be that they differ in design, purpose, variable in question and instrument used.

In their study on classroom processes, Goodlad and Klein (1970) interviewed 150 teachers to obtain the teachers' opinion of classroom activities such as individualization of instruction, use of a wide range of
instructional materials, group processes, and inductive or discovery methods in their classrooms. Trained observers also made anecdotal records of the same activities in these classrooms. In comparing the teacher data with the observer records, the researchers found a discrepancy between the reports of observers of classroom activities and the reports of the teachers who were interviewed. When interpreting this finding, Goodlad and Klein pointed out that a major limitation of the study was that the observation records "varied in their comprehensiveness and in the degree to which observational data were separated from evaluations without data" (p. 34). Therefore, no recommendation was made regarding the use of teacher self-report data in research.

Ehman (1970) reported on the varying degree of consistency among three sources of data with respect to the students' freedom to express themselves. The three sources of data were teacher, student and observer. Observations of discussions on controversial issues were made by trained observers in 14 social studies classrooms. At the end of the instructional unit, the teachers completed a paper and pencil self-administered questionnaire about students' freedom to express their opinion during these discussions. A sample of their students also responded to a similar questionnaire. When the data from the three sources were compared, it was found that the teacher data
disagreed with the other two sources. Although Ehman concluded that teacher self-report is an unsatisfactory source of data for the "freedom of students to express opinions" (p. 4) variable, he also pointed out two major limitations to the generalization of the results:

This study is based on a limited number of teachers from a single school, and represents only one subject area -- secondary school social studies instruction. Generalizations from the findings, therefore, cannot be broad, although they can suggest cautions which should be heeded by researchers interested in studying classroom phenomena in general (p. 4).

House and Steele (1971) investigated the validity of two sources of information on classroom characteristics in a state-wide evaluation program in Illinois. These two sources were student observation and teacher self-report information. A study was conducted in 32 classes comparing the student and teacher data to records obtained by trained observers. These observation records were used as external criteria for the "percentage of instructional time the teacher spent in talking" variable. The finding indicated that there was a high discrepancy between teacher self-report data and data collected from students or observers. For example, in a case where the observed teacher talk was 73%, the teacher estimate was 25% and the median student estimate was 75%. Thus, the decision was taken to process the student data for the evaluation of the gifted program.

In comparing teachers and trained observers as alternative sources of instructional time data, Marliave, Fisher and
Filby (1977) compared the records kept by teachers and data collected by trained observers when measuring allocated and engaged instructional time in six grade two classes. Analyses showed that teacher records of allocated instructional time in each class were positively correlated with both allocated and engaged time obtained by direct observation. The researchers concluded that teacher and observer are comparable sources in instructional time data.

Hook and Rosenshine (1979), in their review of the literature on the accuracy of teacher reports of classroom behaviour, concluded that teacher reports of their own specific behaviour might be inaccurate. They recommended that if possible, data other than self-report data should be used in research.

In light of these contradictory findings and recommendations regarding the use of questionnaires, Flexer (1980) conducted a study to assess the validity of the topic-specific questionnaires used in the SIMS. The purpose of the Flexer study was to investigate the validity of these teacher self-report instruments for gathering data about classroom practices.

The two topic-specific questionnaires used were integers; and ratio, proportion and percent. Data were analyzed only on the sections of each questionnaire that dealt with instructional methods and allocation of instructional time. The questions in these two sections
were described as "very specific, low inference questions" (Bulletin No. 4, p. 25), meaning that the questions were unambiguous and clearly worded, and that they required minimal subjective interpretation on the part of the respondents. Flexer did not include the sections on reasons for inclusion and exclusion of topics, on sources of materials, and on providing for individual differences. She reasoned that on these variables, "many teachers will shade the truth, wanting to appear as good teachers to the research staff" (p. 9).

The sample consisted of three eighth grade mathematics classes shared by two teachers. These teachers were observed by trained observers during a period of an instructional unit. At the end of the instructional unit, the teachers, the students of the classes and the observers were all given the appropriate topic-specific questionnaires to fill out. Responses on the questionnaires filled out by students were compiled and means for each item calculated. Correlation coefficients were calculated among student means, teachers' responses, and observers' responses. These results are shown in Table 1. From the findings, Flexer concluded that the "strong-positive correlation of teachers' and observer's response on items relating to topics presented in the classrooms seem to legitimize the use of these questionnaires for gathering such data on classroom practice" (p. 6).
Table 1
Correlations of Students and Teacher with Observer

<table>
<thead>
<tr>
<th>Subgroups</th>
<th>n of items</th>
<th>Students</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Algebra</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introduction to integers and ordering</td>
<td>7</td>
<td>.92**</td>
<td>.73</td>
</tr>
<tr>
<td>Operating with integers (+, -, X)</td>
<td>16</td>
<td>.60*</td>
<td>.65**</td>
</tr>
<tr>
<td>Introduction to integers, ordering, operating with integers @</td>
<td>23</td>
<td>.67**</td>
<td>.69**</td>
</tr>
<tr>
<td>Time spent on each activity</td>
<td>7</td>
<td>.96**</td>
<td>.94**</td>
</tr>
<tr>
<td>Importance of topics to teacher</td>
<td>6</td>
<td>.39</td>
<td>.53</td>
</tr>
<tr>
<td><strong>Ratio, Proportion and Percent</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio and proportion</td>
<td>17</td>
<td>.47</td>
<td>.89**</td>
</tr>
<tr>
<td>Percent</td>
<td>25</td>
<td>.61**</td>
<td>.93**</td>
</tr>
<tr>
<td>Ratio, proportion and percent @</td>
<td>42</td>
<td>.38*</td>
<td>.91**</td>
</tr>
<tr>
<td>Time spent on each activity</td>
<td>5</td>
<td>-.33</td>
<td>.86</td>
</tr>
<tr>
<td>Importance of topics to teacher</td>
<td>8</td>
<td>-.12</td>
<td>.48</td>
</tr>
</tbody>
</table>

Note. Flexer, 1980

*p .05

**p .01

@ Combination of above two
The research on validity concerning the use of questionnaires seems to provide contradictory results. Some studies (Ehman, 1970; House and Steele, 1971) recommended the use of student data or observer data over the use of teacher self-report data. Other studies (Marliave, Fisher and Filby, 1977; Flexer, 1980) concluded that teacher self-report data were reliable when specific, low-inference questions were used. Berdie and Anderson (1974) attempted to explain this phenomenon by attributing the contradiction to the fact that these results usually have been based on experimental designs that were not chosen exclusively to test questionnaire validity. Rather, the results were often by-products of surveys designed for other purposes. Therefore, they concluded that, "...the contradictory reports concerning questionnaire methods are not surprising, as they are based on results from different questionnaires used for different reasons with different people at different times" (p. 12).

The explanation offered by Berdie and Anderson seems to be a logical and suitable explanation for the contradictory results found in the studies reviewed in this section. These studies investigated different variables such as individualization of instruction, use of materials, the students' freedom to express themselves, allocated and engaged instructional time and instructional methods. Different questionnaires were administered to different
grade levels regarding these variables. The result of the House and Steele (1971) study was a by-product of an evaluation study for a gifted program. Therefore, the conclusions of these studies can only be applicable to the specific instruments used, and to the situations in which they were used.

Thus, the results of studies reviewed in this section cannot give valid, generalizable information on the validity of the questionnaires used in the SIMS because of their differences in purpose, design, variable in question and instrument used. Since the small-scale Flexer study is the only source of information on the validity of these topic-specific questionnaires, an investigation into the development of these questionnaires was conducted in order that conclusions regarding their validity could be drawn.

Bulletin No. 5 of the SIMS reported on how validity concerns have been taken into account during various stages of development of the questionnaires. The initial stage consisted of gathering input from mathematics specialists from Canada, West Germany and the U.S.A. to map out the general framework for the questionnaires. Drafts of the questionnaires were then reviewed by the International Mathematics Committee which consisted of prominent members of the mathematics education community. Detailed ratings, at the item level, as to appropriateness, feasibility and suitability were performed by the Committee in the second
stage. On the basis of this information, revisions of the questionnaires were made. Care was taken at each stage to ensure that the questions were unambiguous, clear and low-inference in nature. For example, questions that required subjective interpretation by the teachers, such as whether or not teachers are open to student opinions, were not included in the questionnaires. The final stage consisted of pilot-testing of the instruments. Experienced researchers in mathematics education conducted in-depth interviews with classroom teachers concerning the clarity and intention of the questions, the coverage of the questionnaires with respect to content and method, and the time demands of the instruments. Suggestions collected from teachers during the pilot-testing were then incorporated in subsequent revisions.

Thus, the questionnaires were developed by mathematics specialists, reviewed by a committee consisting of prominent members of the mathematics education community, and pilot-tested by experienced researchers. This lengthy process substantiates the claim that "concerns for the validity of data on classroom processes...have been central to the development of the instruments" (Bulletin No. 5, p. 30). This fact, supported by the findings from the Flexer study lead one to conclude that the sections of the questionnaires in which low-inference questions were used are reliable and valid.
Chapter 3

DESIGN AND PROCEDURE

The present study was designed to compare teachers' use of concrete and abstract approaches in teaching certain subtopics in Grade 8. The comparison was made on the data collected from approximately 100 mathematics teachers who took part in the SIMS. The instruments used to collect this data were a set of five topic-specific questionnaires on classroom processes. Since the nature of the data was not known prior to analysis, certain data processing procedures and analysis techniques were used in order that valid comparisons could be made on the concrete-abstract variable. This chapter outlines the data processing procedures used, and presents the rationale for choosing exploratory data analysis techniques to compare the teachers' choice between concrete and abstract approaches.

Population and Sample Selection

Two populations of students were identified for investigation in the international study. The first, Population A, was defined as consisting of all those students enrolled in the grade where the majority of
students have reached the age of 13 by the middle of the school year. In B.C., this population was defined to include all students enrolled in regular Grade 8 classes in the public schools as of September, 1980.

The second, Population B, was intended to encompass all students in the last year of secondary school who were studying mathematics as a significant part of an academic program. In B.C., this population was defined to include all students enrolled in Algebra 12 classes in the public schools as of September, 1980. The teachers who taught the students belonging to populations A and B formed the population of teachers in the study. A sample size of approximately 100 classes and their teachers from each population was established in B.C. This present study is concerned with Population A only.

Robitaille, O'Shea and Dirks (1982) gave a description of the sample selection for the SIMS in B.C.:

In order to achieve a sample size of approximately 100 Grade 8 and 100 Algebra 12 classes stratified according to geographic zone of the province and by school size, initial samples of 125 classes at each level were drawn. In most cases this resulted in the selection of no more than one class per school. Of the 125 classes, 105 were selected for initial contact and the remainder reserved to be used as needed. In the spring of 1980, letters were sent from the Ministry of Education to all of the principals of the schools selected, soliciting their cooperation in the study and asking them to select a Mathematics 8 or Algebra 12 teacher or teachers at random from among the teachers available. In cases where it was not possible to make a random selection, the principals were asked to exercise their best judgment about which teacher or teachers to select (p. 9).
Regarding the representativeness of the achieved samples, they noted:

On the whole, it appears that, at both levels, the geographic distribution of classes participating in the international study is sufficiently close to that of the design sample to be representative of the province as a whole. Moreover, the gender and age statistics for the students in the two samples compare favorably with those for the populations. In the case of the teachers selected for participation in the study, there are some significant differences between them and the population of teachers of mathematics. In particular, the [SIMS] teachers are more experienced and more likely to have specialized in the teaching of mathematics than their colleagues (p. 13).

Table 2 lists descriptive information that was collected from teachers in the Population A sample.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age in years</td>
<td>38.6</td>
</tr>
<tr>
<td>Years of teaching experience</td>
<td>13.8</td>
</tr>
<tr>
<td>Years spent in teaching mathematics to Grade 8 students</td>
<td>8.5</td>
</tr>
<tr>
<td>Percentage of time spent in teaching mathematics during the 1980-81 school year</td>
<td>80.0</td>
</tr>
<tr>
<td>Number of mathematics courses included in teachers' post-secondary education</td>
<td>9.1</td>
</tr>
</tbody>
</table>
As shown in Table 2, few inexperienced teachers or teachers of subjects other than mathematics who teach one or more mathematics classes in order to complete their teaching schedules were selected for participation in the international study. The fact that experienced and specialized teachers were selected for participation might create a bias in the results of this study and might limit the generalization of the results to the population of mathematics teachers in British Columbia.

The Instruments

Each Population A teacher was asked to complete five questionnaires on classroom processes for the following topics:

- Common and Decimal Fractions
- Ratio, Proportion and Percent
- Algebra (Integers, Formulae and Equations)
- Geometry
- Measurement

The classroom process questionnaires for these five topics are in Appendices A to E.

Each topic-specific questionnaire was designed to collect information on the following aspects of classroom practice:
1) Resources used in teaching the topic

2) Specific subtopics taught

3) Instructional methods used to present specific concepts such as the concept of negative integers and certain operations such as multiplication of integers.

4) Factors teachers perceived as influencing their choice of approach or procedure

5) Time allocated to the topic and subtopics within the same subject

6) Types of application problems utilized by the teacher

7) Teachers' opinions regarding issues such as the use of calculators

The present study was concerned with the items that were constructed to collect information on instructional methods used in presenting specific concepts and skills (category 3). These items were described by Bulletin No. 4 of the SIMS as being "very specific, [and] low-inference" (p. 25) in nature. Examples of these items are given in Figure 6. A list of the items (20-24) dealing with the teaching of the concept of integers is shown in this figure.
The interpretations of integers given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

20. Extending the number ray to the number line:
   I extended the number ray (0 and positive numbers) to the left by introducing direction as well as magnitude.
   Ex: -4 -3 -2 -1 0 1 2 3 4
   -3 means 3 units to the left of 0.

21. Extending the number system to find solutions to equations:
   I discussed the need to extend the positive integers in order to find a solution to equations like \( x + 7 = 5 \).

22. Using vectors or directed segments on the number line:
   I defined an Integer as a set of vectors (directed line segments) on the number line.
   Ex: -2 can be represented by any of:
   \(-10 \quad -5 \quad 0 \quad 5 \quad 10\)
   Ex: +2 can be represented by any of:
   \(-10 \quad -5 \quad 0 \quad 5 \quad 10\)

23. Defining integers as equivalence classes of whole numbers:
   I developed the integers as equivalence classes of ordered pairs of whole numbers.
   Ex: \((0,2),(1,3),(2,4),\ldots\) = \(-2\)
   or \((a,b) \in \mathbb{Z}: b = a + 2\) = \(-2\)

24. Using examples of physical situations:
   I developed integers by referring to different physical situations which can be described with integers.
   Ex: thermometer, elevation, money (credit/debit), sports (scoring), time (before/after), etc.

Figure 6. Items dealing with methods of instruction for the concept of integers.
Data Collection

Data collection took place during the 1980-81 school year. Classroom process questionnaires were sent out to teachers in sealed envelopes at the beginning of that school year. Teachers were instructed to open the envelope for a given questionnaire only after they had finished teaching that particular topic. This procedure was used to minimize the influence of the questionnaires on teaching practices (Robitaille, O'Shea and Dirks, 1982). The teachers were also asked to fill out the topic-specific questionnaires as soon as the unit of instruction on a given topic was finished. In this way, errors on the information gathered by these questionnaires were kept to a minimum.

Data Processing Procedure

The data processing procedure consisted of various steps which were performed in sequence. The initial step involved the categorization of items into subtopics within each topic. Then these items were categorized into the concrete and abstract categories by a panel of experts. Only those items which were agreed upon by four out of five experts as belonging to a category were included in the analysis. Preliminary analysis
was performed on the responses to items to determine the justifiability of aggregation of item scores to form subtopic scores. Finally, subtopic scores were obtained by aggregating the item scores.

**Categorization of Items**

In each questionnaire, the items that were identified by the researcher as pertaining to various subtopics were categorized into the concrete and abstract approaches by a panel of experts. The panel consisted of two mathematicians from the Faculty of Science at the University of British Columbia (U.B.C.), two mathematics educators from the Faculty of Education at U.B.C. and a doctoral student who was an experienced teacher of Grade 8 mathematics. The panel was not given the terms "concrete" and "abstract" to assist in their categorization of items. Instead they were given definitions as outlined below and were asked to classify the items as either A, B or C.

**A** The items in this category involve the use of manipulative materials, diagrams or pictures. A major characteristic of the items in this category is that treatment of the mathematical content relies primarily on manipulative materials, diagrams or experiences from the environment.

**B** The items in this category involve the use of symbols to stand for some concept or event but the symbols may not resemble that concept or event. A major characteristic of the items in this category is that the treatment of the content relies primarily on explanation which derives its meaning from other mathematical content.

**C** The items in this category do not belong to A or B.
Table 3 shows the list of subtopics and classification of items as validated by the panel of experts according to the definitions outlined above. Only the categorizations that were agreed upon by four out of the five experts were included in this list. According to the panel, some subtopics were defined by items in one category only, and these subtopics were excluded from subsequent analyses. For example, the subtopic angles of triangle was excluded from the Geometry section because the items in the questionnaire corresponding to this subtopic do not consist of both the concrete and abstract approaches. (See Appendix F for the original list of subtopics and items.)

Six more subtopics were then excluded from further analyses because they were defined by only one item in either or both categories. These subtopics were: operations with decimal fractions, ratio, proportion, addition of integers, multiplication of integers, and formula. The rationale for excluding these subtopics is that "rarely is one item sufficiently reliable and valid to make it worthwhile to report its score alone" (Allen and Yen, 1979; p. 130). The eight subtopics that remained were all defined by two or more items in both categories. These subtopics with their corresponding items are listed in Table 4.
Table 3

The List of Subtopics and Corresponding Items for the Concrete and Abstract Categories as Validated by the Panel of Experts

<table>
<thead>
<tr>
<th>Topic / Subtopic</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete Items</td>
</tr>
<tr>
<td>Common and Decimal Fractions</td>
<td></td>
</tr>
<tr>
<td>Concept of Fractions</td>
<td>21, 22, 23, 27,</td>
</tr>
<tr>
<td></td>
<td>28, 30</td>
</tr>
<tr>
<td>Addition of Fractions</td>
<td>31, 32, 33, 37,</td>
</tr>
<tr>
<td></td>
<td>38</td>
</tr>
<tr>
<td>Concept of Decimal Fractions</td>
<td>51, 53, 56</td>
</tr>
<tr>
<td>Operations with Decimal Fractions</td>
<td>59</td>
</tr>
<tr>
<td>Algebra</td>
<td></td>
</tr>
<tr>
<td>Concept of Integers</td>
<td>20, 22, 24</td>
</tr>
<tr>
<td>Addition of Integers</td>
<td>25, 27</td>
</tr>
<tr>
<td>Subtraction of Integers</td>
<td>28, 32</td>
</tr>
<tr>
<td>Multiplication of Integers</td>
<td>36</td>
</tr>
<tr>
<td>Formula</td>
<td>45, 47, 48</td>
</tr>
<tr>
<td>Ratio, Proportion and Percent</td>
<td></td>
</tr>
<tr>
<td>Ratio</td>
<td>21</td>
</tr>
<tr>
<td>Proportion</td>
<td>27</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>67, 68, 71</td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
</tr>
<tr>
<td>Number $\bar{N}$</td>
<td>48, 53, 54</td>
</tr>
<tr>
<td>Relationship Among Various Metric Units</td>
<td>69, 70</td>
</tr>
<tr>
<td>Topic / Subtopic</td>
<td>Category</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>---------------------------------------</td>
</tr>
<tr>
<td>Common and Decimal Fractions</td>
<td></td>
</tr>
<tr>
<td>Concept of Fractions</td>
<td></td>
</tr>
<tr>
<td>Concept of Decimal Fractions</td>
<td></td>
</tr>
<tr>
<td>Addition of Fractions</td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td></td>
</tr>
<tr>
<td>Concept of Integers</td>
<td></td>
</tr>
<tr>
<td>Subtraction of Integers</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
</tr>
<tr>
<td>Number $\bar{N}$</td>
<td></td>
</tr>
<tr>
<td>Relationship Among Various Metric Units</td>
<td></td>
</tr>
</tbody>
</table>
Scoring of Items

As shown in Figure 6 (p. 38), three response options were given for each item in the questionnaire. The options were: used as a primary method of explanation, used but not as a primary means of explanation, and not used. These options for this particular section of the questionnaire remained uniform across all five topics.

For the purposes of this study, the responses to the items were assigned values 0, 1 or 2 according to the option chosen. A response that was "used as a primary method of explanation" was assigned a value of 2. A response that was "used but not as a primary means of explanation" was assigned a value of 1. A response that was "not used" was assigned 0.

Preliminary Analysis

The purpose of the preliminary analysis was to investigate the justifiability of aggregation of item scores to obtain a corresponding subtopic score. An analysis of response patterns of a randomly selected sample of teachers was performed. This analysis was used to determine the differences within each teacher in his or her response to the items in the concrete or abstract category for each subtopic considered.

Teacher responses to both the concrete items and the abstract items for each subtopic were graphed. Since the
items in each category were few in number, and there were only three response options to each item, the number of different response patterns was limited. A decision was taken that the sample size, n, should be determined by the patterns displayed by the graphs themselves. If definite patterns appeared and these patterns started to repeat, then the random selection of more teachers for the analysis would stop. The result of this procedure showed that definite patterns were found and that these patterns started to repeat when the responses of the 6th or 7th teacher were graphed. Therefore, it was reasonable to conclude that an increase in sample size beyond ten would not yield more information regarding the teachers' pattern of responses. The graphs showing the response patterns of 10 randomly selected teachers are displayed in Appendix G. The mean and variance for each teacher were calculated for each category for all subtopics. The mean variances were also calculated for the 10 teachers for each category for all subtopics. These means are listed in Table 5.

Table 5 shows that the mean variances obtained for the subtopics range from .15 to .55 for the concrete items and from .13 to .55 for the abstract items. With the exception of two values, all the means are less than half a score point on a scale that ranges from zero to two, thus suggesting that the item scores for each
Table 5

The Mean Variances of Item Scores from Corresponding Subtopic Scores for the Sample of Ten Teachers

<table>
<thead>
<tr>
<th>Subtopics</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete Items</td>
</tr>
<tr>
<td>Concept of Fractions</td>
<td>.37</td>
</tr>
<tr>
<td>Concept of Decimal Fractions</td>
<td>.35</td>
</tr>
<tr>
<td>Addition of Fractions</td>
<td>.25</td>
</tr>
<tr>
<td>Concept of Integers</td>
<td>.55</td>
</tr>
<tr>
<td>Subtraction of Integers</td>
<td>.22</td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>.44</td>
</tr>
<tr>
<td>Number ( \pi )</td>
<td>.15</td>
</tr>
<tr>
<td>Relationship Among Various Metric Units</td>
<td>.30</td>
</tr>
</tbody>
</table>

subtopic do not differ significantly from each other within each category. Hence, it is possible to aggregate item scores to obtain subtopic scores for the 10 teachers without much loss of information.

The profiles of these ten teachers can be considered to be typical of the response patterns of all teachers involved in the study for the following reasons: the
ten teachers were randomly selected from the total sample of teachers, and a sample size of 10 was sufficient because of the fact that definite patterns were found, and these patterns had started to repeat within the 10 teachers that were chosen. Therefore, since the item scores for the 10 teachers can be aggregated, the item scores for all teachers can also be justifiably aggregated to arrive at a single concrete score as well as a single abstract score for each subtopic.

Scoring of Subtopics

Figure 6 (p. 38) shows a list of the items (20 - 24) dealing with the teaching of the concept of integers. Items 21 and 23 represent abstract approaches to teaching this subtopic whereas items 20, 22 and 24 represent concrete approaches to teaching the same concept. Obtaining the sum of assigned values to the responses to these items made it possible to create a score for each teacher for the concrete and abstract categories in teaching the concept of integers. The sum for each category was divided by the number of items involved in that particular category to provide equal weighting and then multiplied by ten to give whole number values. Thus, each teacher had two scores for each subtopic: an abstract score and a concrete score. Both scores range from 0 to 20. A score of 0 means that the teacher chose the option "not used" for all the items
in that category. A score of 20 means that the teacher chose the option "used as a primary explanation" for all items in that category.

**Difference Score for Each Subtopic**

The concrete and abstract scores, when considered for any teacher, give information as to whether or not most concrete or abstract items were emphasized or used in the instruction of a certain subtopic. These scores, however, do not give information on the teacher's preference for either approach. This information can be obtained by combining the concrete and abstract scores to obtain a single score that reflects the preference for or orientation to either approach for each teacher. This opinion was also expressed by Cooney (1980) who suggested that teachers be reassigned discrete scores on a scale of 0 to 5 according to the differences between their concrete and abstract scores. However, an examination of the data showed that the differences between the concrete and abstract scores for the teachers were continuous in nature and, therefore, could not be classified into discrete categories. On the other hand, difference scores obtained by subtracting the abstract scores from the concrete scores do reflect accurately the teacher's orientation to either approach. Hence, the final step in the data processing procedure was to calculate the difference scores for all teachers.
The difference scores have unique properties that are different from the concrete and abstract scores. The difference scores range from -20 to 20. A positive difference score shows that the teacher is concretely-oriented in teaching that subtopic and a negative difference score shows that the teacher is abstractly-oriented in teaching that subtopic. The absolute value of the difference score indicates the degree of orientation for that particular approach. For example, a teacher who scored -15 was more strongly-oriented to the abstract approaches than a teacher who scored -5. Similarly, a teacher who scored 15 was more strongly-oriented to the concrete approaches than a teacher who scored 5. It is this unique property of the difference score that permits the comparison among teachers in their choice or emphasis of approach.

Method of Analysis

Exploratory data analysis techniques were used in this study. These techniques were more appropriate to use for the purposes of this study than conventional statistical procedures for two reasons:

1) Exploratory data analysis techniques concentrate on "simple arithmetic and easy-to-draw pictures" to provide a description of "what [the data] seem to say" (Tukey, 1977; p. v). Since the purpose of this study was to
compare the teachers' choice of instructional methods, it follows that the method of analysis should first of all provide an accurate picture of the data before valid comparisons can be made. For this purpose, exploratory techniques seem to be most appropriate.

2) The use of conventional statistical procedures requires that certain assumptions be satisfied by the data so that results from the analysis can be meaningfully interpreted. Since the nature of the data in this study was not known before the analysis, it is inappropriate to use these procedures.

Two specific techniques for displaying the data were used. These were stem-and-leaf displays and box-and-whisker plots. Tukey emphasized that these techniques are "forgiving" techniques (p. viii), meaning that they do not produce "digit-perfect" results (p. viii), rather they allow us "to look at the general pattern of the [data]" (p. 27).

For each subtopic, the concrete, abstract, and difference score distributions were graphed in side-by-side and back-to-back stem-and-leaf displays. The choice of display depended on the number of distributions involved in a particular comparison (Tukey, 1977; p. 65). The back-to-back display was used to compare concrete and abstract score distributions for each subtopic because it is more suitable for comparisons involving two
distributions. The side-by-side display was used to compare difference score distributions for all the sub-topics because it is more suitable for comparisons involving several distributions.
Chapter 4

RESULTS OF THE STUDY

Stem-and-leaf displays and box-and-whisker plots used to examine the distributions of concrete, abstract, and difference scores are presented and discussed in this chapter. Both stem-and-leaf displays and box-and-whisker plots were employed in order to maximize the information provided by the data. Each x on the stem-and-leaf displays represents the score of one teacher. In the box-and-whisker plots, the dots show the extreme values and the horizontal lines show the rounded whole number values of the median, the 25th and the 75th percentiles of the distribution (Tukey, 1977; p. 33-39).

Concrete and Abstract Scores

The concrete and abstract score distributions for the eight subtopics are displayed in Figures 7 to 14. Each figure consists of two parts: Part a) is the stem-and-leaf display and Part b) is the box-and-whisker plot. Since the subtopics represented by these figures can be grouped into corresponding topics, discussion and interpretation of these figures are made under each topic heading.
Common and Decimal Fractions

The three subtopics under this topic heading are: fractions, decimal fractions and addition of fractions. All three subtopics are considered to be review material for grade 8 students. Two patterns of teacher behaviour can be found in the instruction of these subtopics. The first pattern is a strong orientation to the abstract approaches when teaching fractions and decimal fractions. The second pattern does not give any conclusive evidence as to the preference of approach in the instruction of addition of fractions.

The first pattern is represented by Figures 7 and 8. These figures are characterized by skewed abstract score distributions, with medians that are above 15. This means that more than half of the teachers used the abstract items when teaching these concepts. For example, in Figure 8, an extreme value of 10 in the abstract score distribution means that all the teachers taught decimal fractions as another way of writing fractions (item 52), as a series (item 55) or as an extension of place value (item 54). Furthermore, a median of 17 and a 25th percentile of 13 indicate that more than 75% of the teachers emphasized these items in teaching the concept. Another characteristic of these figures is the skewed concrete score distributions, with medians that are
Figure 7. Distribution of abstract scores and concrete scores for the concept of fractions.

Figure 8. Distribution of abstract scores and concrete scores for the concept of decimal fractions.
below 10. This means that the concrete items were not emphasized by teachers when teaching these concepts. For example, in Figure 8, a 75th percentile for the concrete scores is 10 and the median is 7. This means that less than 25% of the teachers emphasized the use of the number line (item 51), diagrams (item 53) or rods (item 56) when teaching decimal fractions. The third characteristic of Figures 7 and 8 is that the 25th percentile of the abstract scores is higher than the 75th percentile of the concrete scores, showing that teachers scored much higher with the abstract items than the concrete items. All these characteristics constitute a pattern which suggests that teachers showed a definite and strong preference for the abstract approaches in teaching the review concepts fractions and decimal fractions.

The second pattern is represented by Figure 9. The concrete and abstract distributions for this skill are very similar in shape and range. The concrete scores range from 0 to 16 whereas the abstract scores range from 0 to 17. The box-and-whisker plots are similar in length and have the same median. This indicates that the variability of scores is similar for both distributions. While the graphs indicate that teachers used both concrete and abstract approaches in
Figure 9. Distribution of abstract scores and concrete scores for the addition of fractions.
teaching addition of fractions, the similarity gives no conclusive evidence as to teachers' preference of approach.

Algebra

The two subtopics under this topic heading are: integers and subtraction of integers. Both subtopics are considered to be new material for students in Grade 8. Two patterns of teacher behaviour can be found in the instruction of these subtopics. The first pattern is a strong orientation to the concrete approaches when teaching integers. The second pattern is an orientation to the abstract approaches when teaching subtraction of integers.

The first pattern is represented by Figure 10. This figure is almost the mirror image of Figure 8, and is characterized by skewed distributions for both the concrete and abstract scores. A median at 5 and a 75th percentile at 10 for the abstract scores indicate that less than 25% of the teachers emphasized algebraic approaches (item 21) or equivalence classes (item 23) when teaching integers. A median at 13 and a 25th percentile at 10 for the concrete scores indicate that over 75% of the teachers used or emphasized the number line (items 20 and 22) and physical situations (item 24)
Figure 10. Distribution of abstract scores and concrete scores for the concept of integers.
when teaching this concept. In this figure, the 75th percentile of the abstract score distribution is the 25th percentile of the concrete score distributions, showing that teachers scored much higher with the concrete items than the abstract items. This difference in the distributions suggests that teachers showed a strong preference for the concrete approaches in teaching the new concept integers.

The second pattern is represented by Figure 11. The concrete and abstract score distributions in this figure showed that teachers used or emphasized the abstract approaches more than the concrete approaches. A 25th percentile at 10 for the abstract scores showed that more than 75% of the teachers emphasized or used rules (items 29 and 30) in teaching subtraction of integers. The concrete scores are more evenly distributed on the scale. A median at 10 showed that 50% of the teachers used or emphasized the number line (items 28 and 32) when teaching this skill. Comparing the two distributions, a preference for the abstract approaches is indicated in the instruction of this new skill.
Figure II. Distribution of abstract scores and concrete scores for the subtraction of integers.
Geometry

Pythagorean Theorem is the only subtopic under this heading. This subtopic is considered to be new material for students in Grade 8. Figure 12 is the concrete and abstract score distributions for this concept. As shown in this figure, the concrete and abstract distributions, although not mirror images, are quite similar in shape and range. The similar lengths of the box-and-whisker plots indicates that the variability of the two scores are similar. The graph indicates that teachers used concrete approaches such as measuring devices (item 67), geoboards (item 71) and diagrams (item 68) and abstract approaches such as formula (item 69) and algebraic deductions (item 72) in teaching this concept. The similarity in the two distributions gives no conclusive evidence as to the teachers' preference of approach.

Measurement

The two subtopics under this heading are: number Ñ and relationship among various metric units. Both subtopics are considered to be review material for students in Grade 8. A consistent pattern is found in the instruction of these two subtopics: teachers were abstractly-oriented when teaching these concepts.
Figure 12. Distribution of abstract scores and concrete scores for the concept Pythagorean Theorem.
The two subtopics are represented by Figures 13 and 14. In both figures, the concrete scores showed skewed distributions. This is an indication that most teachers scored low on the concrete items. In Figure 13, the median is 3 and the 75th percentile is 7 for the concrete scores, thus suggesting that few teachers emphasized the use of measuring devices (item 48), polygons (item 53) or grids (item 54) when teaching the concept \( \mathbb{F} \). Furthermore, 43% of the teachers scored 0 on these concrete items, indicating that these approaches were not used. In Figure 14, the median is 5 and the 75th percentile is 10 for the concrete scores. This suggests that few teachers emphasized the use of the metre stick (item 69) or centimetre and decimetre cubes (item 70) to establish relationships among various metric units.

Another characteristic of these figures is that the abstract score distributions are quite evenly spread out on the scale, suggesting that most teachers used or emphasized the abstract approaches when teaching these concepts. For example, in Figure 13, the median is 10 and the 25th and the 75th percentiles are 5 and 15 respectively. This indicates that although the abstract approaches such as formula (item 49) and charts (item 51) were used, teachers differ in their degree of emphasis in using these approaches. In both figures, the differences
Figure 13. Distribution of abstract scores and concrete scores for the concept number π.

Figure 14. Distribution of abstract scores and concrete scores for the relationship among various metric units.
between the concrete and abstract score distributions suggests that teachers showed a preference for the abstract approaches in teaching these two review concepts.

Difference Scores

The comparison of difference score distributions confirms the conclusions made about teacher preferences as shown in the stem-and-leaf concrete and abstract score distributions. It was confirmed by this particular comparison that most teachers were concretely-oriented in their instruction of integers; and abstractly-oriented in their instruction of fractions, decimal fractions, subtraction of integers, number \( \pi \) and relationship among various metric units. For the remaining two subtopics, addition of fractions and Pythagorean Theorem, this particular comparison indicated that perhaps most teachers showed no significant preference for either approach in their instruction.

The distributions of the difference scores for the eight subtopics were graphed in side-by-side stem-and-leaf displays in Figure 15. Each display is accompanied by the corresponding box-and-whisker plot. Several patterns can be found in this figure. The first pattern is characterized by the fact that the box in the box-and-whisker plot is completely below the zero line.
Figure 15. Distribution of difference scores for the eight subtopics: fractions, addition of fractions, decimal fractions, integers, subtraction of integers, Pythagorean Theorem, number π, relationship among various metric units.
The second pattern by the box in the box-and-whisker plot extending from the zero line to below. The third by the box in the box-and-whisker plot being completely above the zero line, and the fourth by the box in the box-and-whisker plot straddling the zero line.

In the first pattern, the boxes are completely below the zero line. This indicates that more than 75% of the teachers are abstractly-oriented in their instruction of the following subtopics: fractions, decimal fractions, and relationship among various metric units.

In the second pattern, the boxes extend from the zero line to below, meaning that 75% of the teachers scored zero or negatively in their difference scores. This pattern is found in two distributions: subtraction of integers and number $\pi$. The fact that the medians are below zero indicates that more than 50% of the teachers were abstractly-oriented in their instruction of subtraction of integers and number $\pi$.

In the third pattern, the box is above the zero line, indicating that more than 75% of the teachers are concretely-oriented in their instruction. This pattern can be found in the instruction of integers only.

In the fourth pattern, the boxes straddle the zero line. This pattern is found in two distributions: addition of fractions and Pythagorean Theorem. The fact
that the boxes straddle the zero line might be an indication that the teachers used both concrete and abstract approaches in their instruction without showing any significant preference for either approach.

Summary

Three patterns of teacher behaviour seemed to emerge from the distributions of concrete, abstract and difference scores. The patterns were: abstract-orientation, concrete-orientation and no preference for either approach. Teachers tended to be abstractly-oriented in their instruction of five of the eight subtopics included for investigation in this study. These five subtopics were: fractions, decimal fractions, subtraction of integers, number π and relationship among various metric units. Integers was the only subtopic that was taught with a concrete-orientation. For the remaining two subtopics, addition of fractions and Pythagorean Theorem, most teachers might have showed no significant preference for either approach in their instruction.
Chapter 5

CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to provide an analysis of the approaches used by mathematics teachers in teaching certain concepts and skills in Grade 8. In particular, a comparison was made in the orientation to two approaches: concrete and abstract. The results of this analysis reveal findings which have certain implications for further research in mathematics education.

Answer to Research Question

The major question addressed in the study was: Were teacher practices in accordance with the theory of instruction proposed by several authors, in which it is recommended that concepts be presented using manipulative materials, pictures, or diagrams, and then through abstract presentations with symbols?

To address this question, the following aspects of instructional behaviour were examined:

1) The consistency within individual teachers in their orientation to the concrete and abstract approaches in teaching each subtopic.

2) Teachers' orientation to the abstract or concrete approaches in teaching various new and review subtopics.
When teachers' choice of methods for the instruction of individual subtopics were examined, it was found that teachers were consistent in their emphasis or choice among concrete and abstract approaches for most concepts or skills. Preliminary analysis procedures showed that teachers' responses to the concrete and abstract items do not differ significantly within each category. This means that teachers tended to use with equal emphasis either none or most of the items within each category for the instruction of each subtopic.

When instructional behaviours for the three new subtopics were examined, it was found that teachers were concretely-oriented in their teaching of one subtopic only -- integers. For the two remaining new subtopics, teachers were abstractly-oriented in their instruction of subtraction of integers, and showed no significant preference of approach in their instruction of Pythagorean Theorem.

When instructional behaviours for the review subtopics were examined, it was found that teachers were abstractly-oriented in their instruction of four of the five subtopics. These subtopics were fractions, decimal fractions, number π and relationship among various metric units. For the remaining skill, addition of fractions, teachers used both concrete and abstract
approaches and showed no significant preference toward either approach.

The findings from the investigation of the different aspects of instructional behaviour lead to the conclusion that teacher practices related to the use of concrete and abstract approaches were not in accordance with the theory of instruction proposed by Bruner, Dienes and many mathematics educators. According to this theory, new concepts should be introduced through concretely-oriented approaches, followed by the introduction of symbolic representations. This instructional behaviour has not been found among the Grade 8 mathematics teachers in this study. In their teaching of three new concepts and skills in the Grade 8 curriculum, mathematics teachers were concretely-oriented in their teaching of one subtopic only. If teacher practices were in accordance with this theory of instruction, the finding of the study should indicate that teachers were concretely-oriented in teaching all the new concepts and skills.

Limitation of the Study

A limitation of the study is that while the questionnaires were constructed in order to provide information on classroom practices, they were not constructed for investigation into the concrete-
abstract variable alone. Many subtopics were eliminated from analysis in this study because of insufficient numbers of items in the questionnaires that related to this variable. The questionnaires also did not provide other information that would be valuable such as the teachers' reasons for preferring abstract approaches to concrete approaches in instructing the concepts and skills considered for this study.

Another limitation is that the results of this study might be biased due to the fact that the teachers in the sample were more experienced and specialized in the teaching of mathematics than their colleagues. The findings might not provide accurate, generalizable information on the population of mathematics teachers in Grade 8.

**Implications**

According to Bruner's "conservative doctrine" (Bruner, 1966; p. 49), all concepts and skills, especially new concepts and skills, should be taught with concretely-oriented approaches first, with gradual introduction of abstract representations. This practice has not been found among the teachers in the study. The fact that their behaviours were not in accordance with the suggestions made by theorists leads one to question the reality of the
implementation of the theory in all mathematics classrooms.

There can be several reasons for the failure of the implementation process.

One obstacle to the implementation process might be that teachers were not convinced by the theorists that a concrete-orientation to instruction is a better approach and leads to better understanding of mathematical concepts. While the theorists have based their conclusions on classroom studies, the numbers of students involved in these studies were few, and achievement results between students that were taught with a concrete-orientation and those that were taught with an abstract-orientation were not compared. A significant difference in achievement results in favour of the concretely-oriented approaches will suggest to teachers that students taught with a concrete-orientation do reflect better understanding of the mathematical concepts involved.

The practical aspect of implementation might be an obstacle. Teachers might have found that the relevant materials needed for teaching certain subtopics were not easily accessible. Time might also have been an important factor in affecting teachers' decisions regarding instructional methods. When under the pressure of teaching a set number of prescribed topics in a year according to the curriculum, teachers might have found concrete approaches to be too time-consuming. Therefore,
they might have chosen to use their time economically and taught most concepts and skills abstractly.

A third obstacle to implementation might be the inadequate pre-service training of teachers in using concrete approaches in mathematics instruction. This obstacle might be overcome by changes in teacher-training programs. One possible change might be an increase in emphasis in the instruction of concretely-oriented approaches in specific topics in mathematics. Another change might be a significant increase in time allocated to the instruction of methods in teaching specific concepts and topics in mathematics.

The failure in the implementation process of the theory can also imply that the theory is not a valid one. The fact that experienced teachers in mathematics have not applied this theory in their instruction of a wide variety of concepts could cast doubt on the validity of the theory itself. The small-scale studies on which Bruner and Dienes based their conclusions about mathematics learning might be insufficient to prove that concrete approaches are indeed necessary or advantageous for the understanding of mathematical concepts.
Suggestions for Further Research

A major finding of the study is that teachers typically preferred abstract approaches to concrete approaches in teaching mathematics. However, one limitation of this study is that the questionnaires did not provide detailed information on the reasons for teachers' preference for abstract approaches. Therefore, a further study could be done using an instrument designed specifically to investigate the concrete-abstract variable. A representative sample of Grade 8 mathematics teachers should be used in order to produce more generalizable results. The finding of this further study might reveal teachers' reasons for preferring abstract approaches to concrete approaches, thus casting light on the obstacles to the implementation of the theory.

The validity of the theory can also be examined in further research. Achievement results can be compared between students that are taught with a concrete-orientation and those that are taught with an abstract-orientation. In order to make valid generalizations, a representative sample of classes should be used, and teachers should be trained specifically for the concrete or abstract approaches they use. A wide variety of topics should also be used in order that patterns can be detected across topics.
Another finding of this study is that Grade 8 mathematics teachers were not consistent in their orientation to either the concrete or abstract approaches across the eight subtopics. For example, teachers were more abstractly-oriented in dealing with review subtopics than they were in dealing with new subtopics. Information collected from another portion of the questionnaires reveals that teachers assumed that students had previous knowledge of the review subtopics. Therefore, the implication is that the teachers made the assumption that this knowledge was both correct and sufficient for the students to handle further work in these subtopics in abstract representations. It would be interesting to investigate the basis on which this assumption was made, whether teachers used valid instruments to assess students' knowledge of the review subtopics, or whether the decision to assume this knowledge was made subjectively.

An important finding of this study concerns the nature of the SIMS data regarding the concrete-abstract variable. The exploratory data analysis reveals that the distribution of scores are uni-modal and approximate the normal distribution. This finding suggests that the basic assumption to conventional data analysis procedures is satisfied by this data. Therefore, these statistical procedures can be applied to the data to find out
relationships between teacher practices and other variables dealt with in the international study. Since the ultimate aim of the SIMS was to "relate student achievement to teaching practice" (Robitaille, O'Shea and Dirks, 1982; p. 93), relationships between teacher differences on the concrete-abstract variable and student outcomes can be examined using conventional statistical procedures. A problem that deserves further study is the investigation of the effect of teachers' choice of concrete or abstract approaches on students' achievement in and attitude toward mathematics. Another problem is the investigation of the impact of teachers' choice of approach on the variability of student results. This information can be found in the relationship between the teachers' orientation to concrete or abstract approaches and the variability of student achievement and attitude within each class.

Relationships between teacher practices and other variables concerning teacher data can also be examined. For example, teacher practices might be related to teachers' experience in teaching mathematics, amount of university education, the attitude toward mathematics or how mathematics is perceived as a discipline by the teacher. Information on these variables was collected by other instruments in the SIMS. Hence, it is possible to
investigate and compare the relationships among these variables. Findings from these investigations might create a profile of a concretely-oriented teacher and provide valuable information to the mathematics education community.

Summary

The theory of initially teaching mathematical concepts through concretely-oriented approaches, followed by the introduction of abstract representations, has been proposed by theorists such as Bruner and Dienes, and advocated by many mathematics educators. However, in spite of the strong endorsement for this theory, it was found in this study that this theory has not been implemented to the extent that educators had intended. Several reasons have been suggested for the failure of implementation. These suggestions have yet to be verified by further research. The fact is that the implementation of the theory has not occurred. This finding is in agreement with findings from some other studies on classroom processes. These findings "provide reason to question the extent to which any of these proposals for innovative pedagogy have influenced predominant instructional patterns" (Fey, 1979; p. 493). It appears that the many changes proposed in the mathematics
curricula in the last twenty to thirty years have not resulted in significant changes in teacher behaviour in mathematics classrooms.
REFERENCES


Easley, J. A. Mathematical foundation of forty years of research on conservation in Geneva. Focus on Learning Problems in Mathematics, October, 1979, 7-26.


Fey, J. T. Mathematics teaching today: perspectives from three national surveys. The Mathematics Teacher, October, 1979, 72, 490-504.


APPENDIX A

Common and Decimal Fractions Questionnaire
INTERNATIONAL ASSOCIATION for the
EVALUATION of EDUCATIONAL ACHIEVEMENT

SECOND
Study of
MATHEMATICS

GRADE 8
TOPIC SPECIFIC QUESTIONNAIRE
COMMON AND DECIMAL FRACTIONS

(Booklet 10 L)

FOR NATIONAL CENTRE USE ONLY

PROVINCE OF BRITISH COLUMBIA
MINISTRY OF EDUCATION
DIVISION OF PUBLIC INSTRUCTION
LEARNING ASSESSMENT BRANCH
Check here if neither common fractions nor decimal fractions are included in your program. In that case, disregard the rest of the questionnaire and return it to B.C. Research in the envelope provided.

CHECK the response which best describes the use you made of each of the following materials in your instruction on common and/or decimal fractions.

RESPONSE CODES:

1. primary source, used frequently
2. secondary source, used occasionally
3. not used or rarely used

1. School Mathematics II (Addison-Wesley) 1 2 3
2. Mathematics II (Ginn) 1 2 3
3. Essentials of Mathematics II (Ginn) 1 2 3
4. Other published text materials (e.g., textbooks, workbooks, and worksheets) 1 2 3
5. Locally produced text materials (e.g., textbooks, workbooks, or worksheets) 1 2 3
6. Commercially or locally produced individualized materials (e.g., programmed instruction or computer assisted instruction) 1 2 3
7. Commercially or locally produced films, film-strips, or teacher demonstration models. 1 2 3
8. Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives) 1 2 3
TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. taught as new content
2. reviewed and then extended
3. reviewed only
4. assumed as prerequisite knowledge and neither taught nor reviewed
5. not taught and not assumed as prerequisite knowledge

Fractions

9. Developing the concept
10. Finding equivalent fractions - including reducing fractions
11. Adding and subtracting - including finding common denominators
12. Multiplying
13. Dividing
14. Ordering

Decimals

15. Developing the concept
16. Converting decimals to fractions or vice versa
17. Adding and subtracting
18. Multiplying
19. Dividing
20. Ordering

IF YOU DID NOT TEACH COMMON FRACTIONS, PROCEED DIRECTLY TO ITEM 51.
### Fractions

The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

**RESPONSE CODES:**

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

**Was this interpretation in the students' text?**

<table>
<thead>
<tr>
<th>#</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>21. Fractions as part of regions:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 means ( \frac{3}{4} )</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>22. Fractions as part of a collection:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 means ( \frac{3}{4} )</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>23. Fractions as the coordinates of points on the number line:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>24. Fractions as quotients:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 means &quot;3 divided by 4&quot;</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>
The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in the students' text?

25. Fractions as decimals:
\[ \frac{3}{4} = 0.75 \]

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

26. Fractions as repeated addition of a unit fraction.
\[ \frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \]

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

27. Fractions as ratios:
\[ \frac{3}{4} \]

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

If you EMPHASIZED the given interpretation (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

If you DID NOT USE the given interpretation (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>65-70</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>71-76</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>19 C. 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>20-25</td>
<td></td>
</tr>
</tbody>
</table>
The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in the students' text?

If you EMPHASIZED the given interpretation (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given interpretation (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

28. Fractions as measurements:
this container holds

\[
\frac{3}{4}
\]

this stick is \( \frac{3}{4} \) cm

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

29. Fractions as operators:

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

30. Fractions as comparisons:

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]

\[\begin{array}{c}
1 \\
2 \\
3 \\
\end{array}\]
Addition of Fractions

The interpretations of the addition of fractions given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used by not emphasized.
3. Not used.

31. The sum of two fractions as the union of two regions

Ex. \( \frac{2}{3} + \frac{1}{4} \) as

![Region Diagram]

32. The sum of two fractions as combination of fractional parts of a collection

Ex. \( \frac{2}{3} + \frac{1}{4} \) as

(Note: the collection consists of 20 dots)

33. The sum of two fractions on the number line

Ex. \( \frac{2}{3} + \frac{3}{4} \) as:

![Number Line Diagram]

34. The sum of fractions as the sum of two quotients

Ex. \( \frac{2}{3} + \frac{3}{4} \) as \((2 + 3) + (3 + 4)\)

Since \(2 + 3 = 8 + 12\)
And \(3 + 4 = 9 + 12\)
\((8 + 12) + (9 + 12) = (8 + 9) + 12\)
\(= 17 + 12\)

35. The sum of two fractions as the sum of two decimals.

Ex. \( \frac{3}{4} + \frac{2}{5} = 0.75 + 0.40 \)
\(= 1.15\)

36. The sum of two fractions using fractions as repeated addition of the unit fractions

Ex. \( \frac{2}{5} + \frac{4}{5} \)
\(= \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \)
\(= \frac{6}{5}\)

37. The sum of two fractions as a combination of two measurements

Ex. \( \frac{2}{5} + \frac{1}{3} \) as

38. The sum of two fractions as joining two segments

Ex. \( \frac{2}{3} + \frac{3}{4} \) as
Procedures for Adding Fractions

The procedures for adding fractions given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary procedure, referred to extensively or frequently)
2. Used by not emphasized
3. Not used

39. Using the least common denominator in a horizontal format

\[
\frac{4}{9} + \frac{1}{6} = \frac{4 \times 2 + 1 \times 3}{9 \times 2 + 6} = \frac{8 + 3}{18} = \frac{11}{18}
\]

1 2 3

42. Using any common denominator in a horizontal format

\[
\frac{4}{9} + \frac{1}{6} = \frac{4 \times 6 + 1 \times 9}{9 \times 6} = \frac{24 + 9}{54} = \frac{33}{54} = \frac{11}{18}
\]

1 2 3

43. Using any common denominator in a vertical format

\[
\frac{4}{9} + \frac{1}{6} = \frac{24}{54} + \frac{9}{54} = \frac{33}{54} = \frac{11}{18}
\]

1 2 3

44. Using decimals

\[
1 + \frac{5}{8} = 0.2 + 0.625 = 0.825
\]

1 2 3
Techniques for Adding Fractions

45. Which one of the following best describes the technique you used in teaching the addition of fractions? CHECK the one most appropriate response.

a) I presented only numerical examples demonstrating the procedure(s).

Ex. \[
\frac{3}{4} + \frac{4}{7} = \frac{3 \times 7 + 4 \times 5}{35} = \frac{21 + 20}{35} = \frac{41}{35}\]

b) I first presented the procedure symbolically and then illustrated it with numerical examples.

Ex. Symbolically: \[
\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}
\]

Numerically: \[
\frac{3}{5} + \frac{4}{7} = \frac{3 \times 7 + 4 \times 5}{35} = \frac{21 + 20}{35} = \frac{41}{35}\]

c) I first used numerical examples and then presented the procedure symbolically.
Finding Common Denominators

The procedures for finding common denominators given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

46. Using the product of the denominators

Ex. To find a common denominator of \( \frac{1}{6} \) and \( \frac{1}{8} \), find the product of 6 and 8.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

47. Using common multiples of the denominators

Ex. To find a common denominator of \( \frac{1}{6} \) and \( \frac{1}{8} \), list the multiples of 6 and the multiples of 8, and find common multiples

Multiples of 6 = \{6, 12, 18, 24, \ldots\}
Multiples of 8 = \{8, 16, 24, 32, \ldots\}

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

48. Using prime factorization

Ex. To find a common denominator of \( \frac{1}{6} \) and \( \frac{1}{8} \), factor each denominator and take the product of the prime factors of each denominator.

\[
6 = 2 \times 3 \quad \text{and} \quad 8 = 2 \times 2 \times 2
\]

Take \( 2 \times 2 \times 2 \times 3 \). This product contains the prime factors of 8 and those of 6.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

49. Listing the multiples of the greater of the two denominators until one that is divisible by the other denominator is found.

Ex. \( \frac{1}{6} \) and \( \frac{1}{8} \)

8, 6 does not divide 8
16, 6 does not divide 16
24, 6 divides 24

Hence, 24 is L.C.D.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

50. Using the trial multiples of the denominators until equal products are obtained.

Ex. \( \frac{1}{6} \) and \( \frac{1}{8} \)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

\[
3 \times 6 = 18 \quad \text{and} \quad 2 \times 8 = 16
\]

\[
4 \times 6 = 24 \quad \text{and} \quad 3 \times 8 = 24
\]
Decimals

The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

If you EMPHASIZED the given interpretation (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given interpretation, (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

51. A decimal as the coordinate of a point on the number line.

<table>
<thead>
<tr>
<th>0.28</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Was this interpretation in the students' text?

1st Primary Reason 2nd Primary Reason

52. A decimal as another way of writing a fraction.

<table>
<thead>
<tr>
<th>0.17 = 17/100</th>
<th>0.8 = 8/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

1st Primary Reason 2nd Primary Reason

53. A decimal as part of a region.

<table>
<thead>
<tr>
<th>0.38</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

1st Primary Reason 2nd Primary Reason

19 C5
20-25
The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

<table>
<thead>
<tr>
<th>RESPONSE CODES:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>RESPONSE CODES:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Well known to me</td>
<td>Yes</td>
<td>2</td>
<td>1. Not well known to me</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>2. In B.C. or local curriculum guide</td>
<td>Yes</td>
<td>1</td>
<td>2. Not in B.C. or local curriculum guide</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>3. Easy for students to understand</td>
<td>Yes</td>
<td>1</td>
<td>3. Hard for students</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>4. Enjoyed by students</td>
<td>Yes</td>
<td>1</td>
<td>4. Disliked by students</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>5. Related to math of prior grades</td>
<td>Yes</td>
<td>1</td>
<td>5. Not related to math of prior grades</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>6. Useful in math of later grades</td>
<td>Yes</td>
<td>1</td>
<td>6. Not useful in math of later grades</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>7. I was taught it was appropriate</td>
<td>Yes</td>
<td>1</td>
<td>7. I was taught it was inappropriate</td>
<td>No</td>
<td>2</td>
</tr>
<tr>
<td>8. Emphasized in student text</td>
<td>Yes</td>
<td>2</td>
<td>8. Not emphasized in student text</td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>

54. A decimal as an extension of place value.

<table>
<thead>
<tr>
<th>Tenths</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17 as</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>1</td>
<td>No</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

55. A decimal as a series

| 0.243 = 0.2 + 0.04 + 0.003 | Yes | 1 | No | 2 |
| 10 + 100 + 1000 | Yes | 1 | No | 2 |
| 1 | 2 | 3 |

56. A decimal as a comparison

| 0.45 | Yes | 1 | No | 2 |
| 0.6 | Yes | 1 | No | 2 |
| Yes | 1 | No | 2 |
| 1 | 2 | 3 |
Operations with Decimals

The techniques for teaching operations with decimals given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently).
2. Used but not emphasized
3. Not used

57. Related operations with decimals to operations with fractions.

Ex. 0.7 x 0.6 =

But \[\frac{7}{10} \text{ and } \frac{6}{10}\]

So \[\frac{7}{10} \times \frac{6}{10}\]

\[= \frac{42}{100}\]

Therefore \(0.7 \times 0.6 = 0.42\)

58. Related operations with decimals to operations with whole numbers, teaching rules for placing the decimal point.

Ex. 1.38 x 5.2 =

Since 138

\[
\begin{align*}
\times 52 \\
\hline
276 \\
690 \\
7176
\end{align*}
\]

1.38 x 5.2 = 7.176
2 places 1 place 3 places

59. Used concrete materials to illustrate operations with decimals.

Ex. 3.47 + 2.13 = ______

Using rods I demonstrated that

3.47 m and 2.13 m makes 5.60 m
TIME ALLOCATIONS

60. What was the average length (in minutes) of each class period? 47-48

Fractions

61. How many total class periods did you spend on teaching fractions? (Combine partial lessons when necessary.) 49-50

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

62. Activities related to developing the concept of fraction 51-52
63. Activities related to finding equivalent fractions including reducing fractions 53-54
64. Activities related to adding and subtracting fractions including finding common denominators 55-56
65. Activities related to multiplying fractions 57-58
66. Activities related to dividing fractions 59-60
67. Activities related to ordering fractions 61-62
68. Problem-solving activities related to fractions (textbook word problems, problems arising from real life situations, recreational problems, challenging problems, etc.) 63-64

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 62 TO 68 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 61.
Decimals

69. How many total class periods did you spend on teaching decimals? (Combine partial lessons when necessary.)

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

70. Activities related to developing the concept of decimals...

71. Activities related to converting decimals to fractions or vice versa...

72. Activities related to adding and subtracting decimals...

73. Activities related to multiplying decimals...

74. Activities related to dividing decimals...

75. Activities related to ordering decimals...

76. Problem solving activities related to decimals (textbook word problems, problems arising from real life situations, recreational problems, challenging problems, etc.).

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 70 - 76 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 69.
### OPINIONS

Indicate the extent to which you agree or disagree with each of the following statements for your class. CIRCLE the choice which best describes your feelings.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Strongly Agree</th>
<th>Strongly Disagree</th>
<th>Agree</th>
<th>Undecided</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>77. Computation with common fractions should be taught.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>78. The degree to which the students are skilled at computing is an indicator of their understanding of fractions and/or decimals.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>79. Computations with common fractions should be delayed until students are at least 12-13 years of age.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80. Computation with decimals and common fractions should be done with hand-held calculators.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81. Only common fractions with small denominators should be taught (e.g., 1/2, 1/3, etc.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82. It is important to drill on computation with common fractions and decimals until students are very good at computing.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>83. Rules for operations with common fractions and decimals should be memorized.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>84. Emphasis should be placed on teaching applications involving common fractions and decimals.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85. Problem solving activities and applications with common fractions and decimals should be emphasized more than computations with fractions and decimals.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
86. In teaching common fractions it is important that structural properties (distributivity, associativity, commutativity, identity, inverse elements) be emphasized.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

87. Estimation, approximation, and checking the reasonableness of an answer are more important than becoming skilled in computing with common fractions and decimals.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

88. Decimals and their operations should be emphasized more than common fractions and their operations.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

89. Mental calculation should be emphasized with common fractions and decimals.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

90. Instruction on common fractions should precede instruction on decimals.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

91. Instruction on addition of common fractions (like and unlike denominators) should precede instruction on multiplication of fractions.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

92. It is important for students to know how to find the least common multiple of two whole numbers.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

93. It is important for students to know how to find the greatest common factor of two whole numbers.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

94. When reducing fractions, students should first find the greatest common factor (GCF) of the numerator and denominator and then divide the numerator and the denominator by the GCF.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree
APPENDIX B

Ratio, Proportion and Percent Questionnaire
SECOND Study of MATHEMATICS

GRADE 8
TOPIC SPECIFIC QUESTIONNAIRE RATIO, PROPORTION AND PERCENT
(Booklet II L)
Check here if none of ratio, proportion, or percent are included in your program. In that case, disregard the rest of the questionnaire and return it to B.C. Research in the envelope provided.

CHECK the response which best describes the use you made of each of the following materials in your instruction on ratio, proportion, or percent.

**RESPONSE CODES:**

1. primary source, used frequently
2. secondary source, used occasionally
3. not used or rarely used

| 1. School Mathematics II (Addison-Wesley) | 1 | 2 | 4 | 21 |
| 2. Mathematics II (Ginn) | 1 | 2 | 3 | 22 |
| 3. Essentials of Mathematics II (Ginn) | 1 | 2 | 3 | 23 |
| 4. Other published text materials (e.g., textbooks, workbooks, and worksheets) | 1 | 2 | 3 | 24 |
| 5. Locally produced text materials (e.g., textbooks, workbooks, or worksheets) | 1 | 2 | 3 | 25 |
| 6. Commercially or locally produced individualized materials (e.g., programmed instruction or computer assisted instruction) | 1 | 2 | 3 | 26 |
| 7. Commercially or locally produced films, film-strips, or teacher demonstration models | 1 | 2 | 3 | 27 |
| 8. Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives) | 1 | 2 | 3 | 28 |
TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. taught as new content
2. reviewed and then extended
3. reviewed only
4. assumed as prerequisite knowledge and neither taught nor reviewed
5. not taught and not assumed as prerequisite knowledge

9. The concept of ratio
   1  2  3  4  5  29
10. The concept of proportion
    1  2  3  4  5  30
11. Solving proportional equations
    1  2  3  4  5  31
12. The concept of percent
    1  2  3  4  5  32
13. Computing percents: Find a percent of a given number or find what percent one number is of another
    1  2  3  4  5  33
14. Changing percents to common fractions
    1  2  3  4  5  34
15. Changing percents to decimal fractions
    1  2  3  4  5  35
16. Changing common fractions to percents
    1  2  3  4  5  36
17. Changing decimal fractions to percents
    1  2  3  4  5  37
18. Percents greater than 100%
    1  2  3  4  5  38
19. Percents less than 1%
The interpretations below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently).

2. Used, but not extensively.

3. Not used.

Ratio

20. Ratio as a rate

Exs: i) 13 km in one hour
    ii) 72 heartbeats per minute

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

21. Ratio as comparison

Exs: i) One part cleaner to 10 parts water
    ii) Three pencils per student

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

22. Ratio as a fraction

Ex: 3:5 means 3/5 (three fifths)

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

23. Ratio as a quotient of two whole numbers

Ex: 3:5 means 3 ÷ 5

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

Percent

24. Percent as a fraction (i.e., a synonym for hundredths)

Ex: 83% means 83/100 or 0.83

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]

25. Percent as a ratio with a second term of 100

Ex: 83% means 83:100

\[
\begin{array}{ccc}
1 & 2 & 3 \\
\end{array}
\]
#### Proportion

The interpretations given below may be included in your instructional program. Check the response code which describes the treatment of each topic in your class.

**RESPONSE CODES:**

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in this student's text?

If you EMHASIZED the given interpretation (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

**RESPONSE CODES:**

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. Was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given interpretation (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

**RESPONSE CODES:**

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Distilled by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. This taught it was inappropriate
8. Not emphasized in student text

---

<table>
<thead>
<tr>
<th>Ex.</th>
<th>Proportion</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ex. 12 heartbeats per 10 seconds is the same as 72 beats per min.</td>
<td>Yes</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>No</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ex.</th>
<th>Proportion</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>27. Ex. 9 red cars to 12 blue ones is the same as 3 to 4.</td>
<td>Yes</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>No</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ex.</th>
<th>Proportion</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>28. Exs. 1) 1/3 = 4/12</td>
<td>Yes</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

2) | | | |

3) | | | |

4) | | | |

---- | | | |

<table>
<thead>
<tr>
<th>Ex.</th>
<th>Proportion</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>29. Ex. 3:4 and 9:12 Since 3 + 4 = 0.75 and 9 + 12 = 0.75. the quotients are equal; so 3:4 and 9:12 are equivalent.</td>
<td>Yes</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>No</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Procedures for Solving Proportions

The procedures for solving proportions given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary procedure, referred to extensively or frequently)
2. Used, but not emphasized.
3. Not used.

30. Finding the cross products and then solving the resulting equation.

Ex: Given $\frac{3}{14} = \frac{x}{3}$

$3 \cdot x = 14 \cdot 6$ or $3 \cdot x = 84$, etc.

1 2 3

31. Using multiplication or division to equate numerators and denominators.

Ex: Given $\frac{3}{14} = \frac{x}{3}$

$\frac{3 \cdot 2}{14} = \frac{6}{x}$ or $\frac{6}{28} = \frac{6}{x}$

Since the numerators are equal and the ratios are equivalent, the denominators must be equal.

Hence $x = 28$.

1 2 3

32. Dividing the terms of one ratio and then solving the resulting equation.

Ex: Given $\frac{x}{9} = \frac{17}{4}$

$x = 4.25$, so $x = 9 \times 4.25$, or $38.25$

$\frac{9}{1} = \frac{2}{2} = \frac{3}{3}$
Methods for Solving Problems Involving Proportions

Several methods of solving problems involving proportions are listed below. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:
1. Emphasized (used as a primary method, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

EXAMPLE PROBLEM:
Three neckties cost $20.00. How much do 12 neckties cost?

33. Solved it using proportional reasoning without an equation.
   For example: 12 neckties are four times as much as three neckties, so they would cost four times as much, or $80.00.

   \[
   \frac{1}{3} = \frac{2}{20} = \frac{3}{x}
   \]

34. Solved it using a proportional equation.
   For example: \(\frac{3}{20} = \frac{12}{x}\) where \(x\) is the cost of 12 neckties.

   Solve for \(x\).

   \[
   \frac{1}{20} = \frac{2}{12} = \frac{3}{x}
   \]

35. Solved it using the unit method without an equation.
   For example: one necktie cost $20/3 or $6.67, therefore, 12 neckties cost 12 \(\cdot\) ($6.67) = $80.00.
Techniques for Teaching Solving Proportional Equations

The following statements describe techniques a teacher might use when teaching a procedure for solving proportional equations. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary procedure, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

36. I presented only numerical examples demonstrating the procedure(s).
   Ex: \( \frac{3}{5} = \frac{6}{n} \)
   
   1 2 3

37. I first used numerical examples and then presented the procedure symbolically (i.e., the general case).
   Ex: Numerically Symbolically
   \( \frac{3}{5} = \frac{6}{n} \) \( \frac{a}{b} = \frac{c}{n} \)
   
   1 2 3

38. I first presented the procedure symbolically (i.e., the general case) and then illustrated it with numerical examples).
   Ex: Symbolically Numerically
   \( \frac{a}{b} = \frac{c}{n} \) \( \frac{3}{5} = \frac{6}{n} \)
   
   1 2 3
Applications and Problems

Several applications of ratio and properties are listed below. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used frequently)
2. Used, but not emphasized
3. Not used

39. Scale Models (airplanes, automobiles)
   __1__ 2 3

40. Finding distances from maps
   __1__ 2 3

41. Scale drawings
   __1__ 2 3

42. Calculating the size of a population from a sample estimate
   __1__ 2 3

43. Problems involving buying decisions based on cost rates
   Ex: Pay $1.00 for 3 items or 35¢ for each?
   __1__ 2 3

44. Mixture or recipe problems
   __1__ 2 3

45. Real world problems using similar triangles
   Ex: A 12 foot tree casts a shadow of 4 feet. A building has a shadow of 25 feet. How tall is the building?

   __1__ 2 3

   12

   4

   ?

   25
Applications and Problems

Several applications of percents are listed below. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used frequently)
2. Used, but not emphasized
3. Not used

46. Commission
   1  2  3

47. Discount
   1  2  3

48. General Word Problems

Ex: John bought 25 toys.
   40% were defective.
   How many were defective?
   1  2  3

49. Simple or compound interest
   1  2  3

50. Percent of increase or decrease
   1  2  3

51. Circle or bar graphs
   1  2  3
Sources of Applications and Problems

Several sources of applications of ratio, proportion, and percent are listed below. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:
1. used frequently
2. used occasionally
3. not used at all

52. Students' textbooks
   _____ 1 _____ 2 _____ 3 31

53. Supplementary textbooks or workbooks
   _____ 1 _____ 2 _____ 3 32

54. Worksheets or exercises designed by myself or local teachers
   _____ 1 _____ 2 _____ 3 33

55. The B.C. curriculum guide or a local guide
   _____ 1 _____ 2 _____ 3 34

56. Articles or papers published by professional associations
   _____ 1 _____ 2 _____ 3 35

57. Applications or problems suggested by my students
   _____ 1 _____ 2 _____ 3 36
Methods of Solving Percent Problems

Four methods of solving percent problems are listed below for each of three types of percent problems. CHECK the response which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as primary procedure for this type of problem)
2. Taught, but not as a primary procedure for this type of problem
3. Not taught

Type I: Given the base and percent find the percentage:

EX: Sara bought a new dress priced at $150.00. The sales tax was 3% of the price. What was the sales tax?

58. The equation method:
   Ex: 0.03 x 150 = x
   Solve for x.
   ___1___ 2___ 3____

59. The proportion method:
   Ex: Let x be the sales tax. Then:
   \[
   \frac{x}{150} = \frac{3}{100}
   \]
   Solve for x.
   ___1___ 2___ 3____

60. The arithmetic method:
   Ex: Multiply the percent (in decimal or fractional form) times the base to get the percentage, using only arithmetic.
   \[
   150 \\
   \times 0.03
   \]
   Solve for x.
   ___1___ 2___ 3____

61. The unit method:
   Ex: 3% of $1 is $\frac{3}{100}$
   3% of $150 is $\frac{3}{100} \times 150$
   ___1___ 2___ 3____

Type II: Given the base and percentage find the percent:

EX: The Mathematics Club has 40 members. Twenty-eight of the members were at a meeting to elect officers. What percent of the members attended the meeting?

62. The equation method:
   Ex: 100 (28 / 40) = x
   Solve for x.
   ___1___ 2___ 3____

63. The proportion method:
   Ex: 28 = x
   \[
   \frac{40}{100}
   \]
   Then solve for x.
   ___1___ 2___ 3____

64. The arithmetic method:
   Ex: Divide the base into the percentage and multiply by 100 to find the percent using only arithmetic.
   \[
   \frac{150}{0.03} \times 100
   \]
   Solve for x.
   ___1___ 2___ 3____

65. The unit method:
   Ex: 1 member is 1 of 100%
   28 members is 28 of 100%
   ___1___ 2___ 3____

Type III: Given percent and percentage find the base:

EX: On a certain school day, there were 30 students absent. That was 5% of the total. How many students were there?

66. The equation method:
   Ex: 0.05 x = 30
   Solve for x.
   ___1___ 2___ 3____

67. The proportion method:
   Ex: \[
   \frac{30}{5} = \frac{x}{100}
   \]
   Solve for x.
   ___1___ 2___ 3____

68. The arithmetic method:
   Ex: Divide the percent (in decimal or fractional form) into the percentage to get the base.
   \[
   \frac{0.05}{30}
   \]
   Solve for x.
   ___1___ 2___ 3____

69. The unit method:
   Ex: 5 students are 5% of 100 students.
   1 student is 5% of 100 or 20 students.
   Therefore, 30 students are 5% of 30 x 20 students.
   ___1___ 2___ 3____
TIME ALLOCATIONS

70. What was the average length (in minutes) of each class period? ___ minutes 49-50

71. How many total class periods did you spend on teaching ratio, proportion, and percent? (Combine partial lessons when necessary.) ___ periods 51-52

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answers to the nearest whole number.

72. Activities related to developing the concept of ratio. ____________________________ 53-54

73. Activities related to developing the concept of proportion. ________________________ 55-56

74. Activities related to solving proportional equations. ____________________________ 57-58

75. Applications/problem solving activities related to ratio and proportion -- (textbook word problems, problems arising from real life situations, recreational problems, challenging problems, etc.). ________________________ 59-60

76. Activities related to developing the concept of percent ____________________________ 61-62

77. Activities related to computing with percents. ____________________________ 63-64

78. Activities related to changing percents to common fractions. _______________________ 65-66

79. Activities related to changing percents to decimal fractions. ________________________ 67-68

80. Activities related to changing common fractions to percents. ________________________ 69-70

81. Activities related to changing decimal fractions to percents. ________________________ 71-72

82. Applications/problem solving activities related to percents -- (textbook word problems, problems arising from real life situations, recreational problems, challenging problems, etc.). ____________________________ 73-74

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 72 TO 82 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 71.
OPINIONS

Indicate the extent to which you agree or disagree with each of the following statements relative to your class. CIRCLE the choice which best describes your feelings.

83. The study of percent should be related to the study of proportion.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

84. The study of percent should precede the study of ratio and proportion.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

85. The study of proportion should be delayed until the students learn how to solve linear equations.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

86. Students should be taught to identify each of the three types of percent problems before solving them.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

87. The study of proportion should be delayed beyond this grade level.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

88. The students should initially learn how to solve proportional problems using arithmetical methods (without setting up proportional equations).

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

89. Students should be given a specific procedure for each type of percent problem.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree

90. The degree to which the students are skilled at computing when solving proportions is an indicator of their understanding of proportions.

Strongly\(^1\) Disagree\(^2\) Undecided\(^3\) Agree\(^4\) Strongly\(^5\) Agree
91. Computation with percent should be done with hand-held calculators.

<table>
<thead>
<tr>
<th>Strongly</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

22

92. Applications of proportion should be emphasized more than solving proportional equations.

<table>
<thead>
<tr>
<th>Strongly</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

23

93. Applications involving consumer arithmetic (discount, interest, etc.) should be emphasized when students study percent.

<table>
<thead>
<tr>
<th>Strongly</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

24

94. Ratio should be taught as fractions or quotients rather than as rates or comparisons of collections.

<table>
<thead>
<tr>
<th>Strongly</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

25
APPENDIX C

Algebra Questionnaire
SECOND
Study of
MATHEMATICS

GRADE 8
TOPIC SPECIFIC QUESTIONNAIRE
ALGEBRA (FORMULAS AND EQUATIONS)
AND INTEGERS

(Booklets 14L and 15L Combined)
Check here if none of integers (positive and negative whole numbers), formulae or equations are included in your program. In that case, disregard the remainder of the questionnaire and return it to B.C. Research in the envelope provided.  

CHECK the response which best describes the use you made of each of the following materials in your instruction on integers, formulae and equations.

**RESPONSE CODES:**
1. primary source, used frequently
2. secondary source, used occasionally
3. not used or rarely used

| 1. School Mathematics II (Addison-Wesley) | 1 | 2 | 3 |
| 2. Mathematics II (Ginn) | 1 | 2 | 3 |
| 3. Essentials of Mathematics II (Ginn) | 1 | 2 | 3 |
| 4. Other published text materials (e.g., textbooks, workbooks, and worksheets) | 1 | 2 | 3 |
| 5. Locally produced text materials (e.g., textbooks, workbooks, or worksheets) | 1 | 2 | 3 |
| 6. Commercially or locally produced individualized materials (e.g., programmed instruction or computer assisted instruction) | 1 | 2 | 3 |
| 7. Commercially or locally produced films, filmstrips, or teacher demonstration models | 1 | 2 | 3 |
| 8. Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives) | 1 | 2 | 3 |
TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK
the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. taught as new content
2. reviewed and then extended
3. reviewed only
4. assumed as prerequisite knowledge and
neither taught nor reviewed
5. not taught and not assumed as pre­
requisite knowledge

Integers
9. The concept of positive and
negative integers. __ 1 __ 2 __ 3 __ 4 __ 5 29
10. Addition of integers (+ and -). __ 1 __ 2 __ 3 __ 4 __ 5 30
11. Subtraction of integers (+ and -). __ 1 __ 2 __ 3 __ 4 __ 5 31
12. Multiplication of integers (+ and -). __ 1 __ 2 __ 3 __ 4 __ 5 32
13. Division of integers (+ and -). __ 1 __ 2 __ 3 __ 4 __ 5 33
14. Structural properties of the set of
integers (e.g., commutativity,
associativity, distributivity, etc.) __ 1 __ 2 __ 3 __ 4 __ 5 34
15. Order relations in the set of integers. __ 1 __ 2 __ 3 __ 4 __ 5 35

Formulas and Equations
16. Evaluation of formulas for given
values of the variables.
EX: Given A = L x W. If L = 4 and
W = 5, substitute for L and W
and find the value of A. __ 1 __ 2 __ 3 __ 4 __ 5 36
17. Deriving formulas or equations.
EX: Each weight stretches a spring
3 cm. What formula gives the
stretch (total) for n weights? __ 1 __ 2 __ 3 __ 4 __ 5 37
EX: Solve Y = 2x + r for r. __ 1 __ 2 __ 3 __ 4 __ 5 38
EX: Solve 4x - 3 = 19 __ 1 __ 2 __ 3 __ 4 __ 5 39
The interpretations of integers given below may be included in your instructional program. Check the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in the students' text?

If you EMHASIZED the given interpretation [i.e., if you checked 1 in the first column] WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student test

If you DID NOT USE the given interpretation [i.e., if you checked 3 in the first column] WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disturbed by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student test

20. Extending the number ray to the number line:
   1. I extended the number ray (0 and positive numbers) to
      the left by introducing
      direction as well as magnitude.
   Yes ______
   No ______

   Ex: __________________________
      -4 -3 -2 -1 0 1 2 3 4
      -3 means 3 units to the left of 0.

   ______ ______ ______
   1    2    3

21. Extending the number system to find solutions to equations:
   1. I discussed the need to extend
      the positive integers in order
      to find a solution to equations
      like __ + 7 = 5.
   Yes ______
   No ______

   ______ ______ ______
   1    2    3

22. Using vectors or directed segments on the number line:
   1. I defined an integer as a set
      of vectors (directed line segments) on the number line.
   Yes ______
   No ______

   Ex: -2 can be represented by any of:
   __________________________
   -2 0 2 4 6 8 10
   Ex: +2 can be represented by any of:
   __________________________
   2 0 2 4 6 8 10

   ______ ______ ______
   1    2    3
The interpretations of integers given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in the students' text?

<table>
<thead>
<tr>
<th>RESPONSE CODES:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Well known to me</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. In B.C. or local curriculum guide</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Easy for students to understand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Enjoyed by students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Related to math of prior grades</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Useful in math of later grades</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. I was taught it was appropriate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Emphasized in student text</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Was this interpretation in the students' text?

23. Defining Integers as equivalence classes of whole numbers:

I developed the integers as equivalence classes of ordered pairs of whole numbers.

Ex: 

\[(0,2),(1,3),(2,4),...\] = \(-2\)  
\[(a,b) \in \mathbb{Z}: b = a + 2\] = \(-2\)

1 2 3 4 5 6 7 8

24. Using examples of physical situations:

I developed integers by referring to different physical situations which can be described with integers.

Ex: thermometer, elevation, money (credit/debit), sports (scoring), time (before/after), etc.

1 2 3 4 5 6 7 8
The procedures given below deal with the topic of addition of integers. CHECK the response code which describes the treatment of each procedure in your class.

**RESPONSE CODES:**
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

---

### 25. Addition on the number line:

<table>
<thead>
<tr>
<th>I used the number line to add integers.</th>
<th>Yes</th>
<th>1</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>No</th>
<th>2</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 26. Addition by rules:

<table>
<thead>
<tr>
<th>I used rules to add integers.</th>
<th>Yes</th>
<th>1</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>No</th>
<th>2</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ex: If both addends have the same sign, the sum is found by adding their numerical (absolute) values and adjoining the common sign.

### 27. Use of physical situations:

<table>
<thead>
<tr>
<th>I used physical situations to add integers.</th>
<th>Yes</th>
<th>1</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>No</th>
<th>2</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ex: In climbing out of the Dead Sea Valley, the car started at an elevation of -643 feet and climbed 432 feet to an elevation of ____ feet.

---

If you **EMPHASIZED** the given procedure (i.e. if you checked 1 in the first column) **WRITE** the numbers of the response codes which show the two primary reasons for its emphasis.

**RESPONSE CODES:**
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you **DID NOT USE** the given procedure (i.e. if you checked 3 in the first column) **WRITE** the numbers of the response codes which show the two primary reasons for not using it.

**RESPONSE CODES:**
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

---

1 2 3 70-75
10 20-25
125 26-31
The procedures given below deal with the topic of subtraction of integers. Check the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

If you EMPHASIZED the given procedure [i.e., if you checked 1 in the first column] WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure [i.e., if you checked 3 in the first column] WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disturbed by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

28. Subtraction as addition of opposites:
I used the number line to subtract integers by starting at the minuend and going the number of units indicated by the subtrahend but in the direction opposite of its sign.

Yes ______
No ______

29. Subtraction as inverse of addition:
I used the inverse relation between addition and subtraction to subtract integers.

Yes ______
No ______

Ex: 4 - (-3) = ______
Solve 4 = ______ + (-3)

30. Subtraction by rules:
I used rules to subtract integers.

Yes ______
No ______

Ex: To subtract an integer, add its opposite.
The procedures given below deal with the topic of subtraction of integers. CHECK the response code which describes the treatment of each procedure in your class.

**RESPONSE CODES:**

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

<table>
<thead>
<tr>
<th>31. Subtraction as a number of units:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>I extended the meaning of subtraction of whole numbers (i.e. ( y - x )) means the number of units from ( x ) to ( y ) to integers. Ex: ( 4 - 3 ) means the number of units from 3 to 4.</td>
<td>Yes 1</td>
<td>No 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>32. Subtraction as distance:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>I used the number line to subtract integers by finding the number of units (or distance) from the subtrahend to the minuend.</td>
<td>Yes 1</td>
<td>No 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>33. Subtraction as &quot;what must be added&quot;:</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>I interpreted subtraction to mean &quot;what must be added&quot; to the subtrahend to get the minuend. Ex: ( 4 - 3 = ) means &quot;what must be added to -3 to get 4&quot;.</td>
<td>Yes 1</td>
<td>No 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The following statements describe methods by which a teacher might develop the concept of the product of integers. CHECK the response code to indicate the extent to which that method of developing the concept was used with your class.

RESPONSE CODES:
1. Emphasized (used as a primary method of development, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used.

34. Development by use of repeated addition:
I developed the concept of multiplication by appealing to repeated addition, e.g.,
\[ 4 \times -3 = -3 + -3 + -3 + -3 = -12 \]

35. Development by the extension of properties of the whole number system:
I developed the concept of multiplication of integers by using the commutative, associative, and distributive properties to justify the products, e.g.,
\[ -4 \times -3 = \]
But \[ 0 = (-4 + 4) \times -3 \]
\[ = (-4 \times -3) + (4 \times -3) \]
\[ = (-4 \times -3) + -12 \]
Hence \[ -4 \times -3 = +12 \]

36. Development by use of physical situations:
I developed the concept of multiplication of integers by appealing to physical situations that might illustrate the product of positive and negative numbers, e.g. A refrigerator is cooling at a rate of 4° per minute. Its thermometer is at 0°. What will be its temperature 4 minutes from now?
37. Development by use of patterns:
I developed the concept of multiplication of integers by appealing to patterns of products, e.g.,

\[
\begin{align*}
+4 \times -3 &= -12 \\
+3 \times -3 &= -9 \\
+2 \times -3 &= -6 \\
+1 \times -3 &= -3 \\
0 \times -3 &= 0 \\
-1 \times -3 &= 3 \\
-2 \times -3 &= 6
\end{align*}
\]

38. No development -- students were given rules:
I did not develop the facts for multiplication of integers by using any of the above methods. I instead gave them rules similar to the following.

If the signs are alike, the answer is positive. If they are different the answer is negative. If one factor is zero, the answer is zero.
The procedures given below deal with methods for solving linear equations. **CHECK** the response code which describes the treatment of each procedure in your class.

**RESPONSE CODES:**
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

**Was this procedure in the students' text?**

| 39. Using properties of equality with operations with numbers: |
| Ex: $7x + 5 = 40$ |
| $7x + 5 - 5 = 40 - 5$ (Subtract 5 from both sides) |
| $7x = 35$ (arithmetic fact) |
| $\frac{7x}{7} = \frac{35}{7}$ (divide both sides by 7) |
| $x = 5$ |
| **Yes** | **No** |
| 1 | 2 |

| 40. Using inverse operations with numbers: |
| Ex: $7x + 5 = 40$ |
| $7x + 5 - 5 = 40 - 5$ (add the inverse of 5 to both sides) |
| $7x = 35$ |
| $\frac{1}{7} \cdot (7x) = \frac{1}{7} \cdot 35$ (multiply both sides by the reciprocal of 7) |
| $x = 5$ |
| **Yes** | **No** |
| 1 | 2 |

### 1st Primary Reason
### 2nd Primary Reason

| If you **EMPHASIZED** the given procedure (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis. |

### RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

| If you **DID NOT USE** the given procedure (i.e., if you checked 3 in the first column), WRITE the numbers of the response codes which show the two primary reasons for not using it. |

### RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

73-70

19 C

20-25
The procedures given below deal with methods for solving linear equations. **CHECK** the response code which describes the treatment of each procedure in your class.

**RESPONSE CODES:**

1. Emphasized (used as a primary explanation, referred to extensively or frequently)  
2. Used but not emphasized  
3. Not used  

**Was this procedure in the students' text?**

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

1. Well known to me  
2. In B.C. or local curriculum guide  
3. Easy for students to understand  
4. Enjoyed by students  
5. Related to math of prior grades  
6. Useful in math of later grades  
7. I was taught it was appropriate  
8. Not emphasized in student text

**Example Rules**

--- 

- collect all constant terms on one side of the equation and all variable terms on the other. 

  \[ 7x = 40 - 5 \]

- combine like terms. 

  \[ 7x = 35 \]

- divide by the coefficient of \( x \). 

  \[ x = 5 \]
TEACHING TECHNIQUES

The following statements describe techniques a teacher might use in teaching formulas. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary technique, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

44. Presenting formulas and explaining the meaning of the terms in the formulas:
Ex: Formula: \( A = \frac{1}{2} \text{bh} \)
A stands for the area of a triangle
\( b \) stands for the base of a triangle
\( h \) stands for the height of a triangle

45. Having the students inspect graphs and find formulas to express the relationships portrayed by the graph:
Ex:

\[
\begin{array}{c|c}
\text{x} & \text{y} \\
0 & 0 \\
1 & 3 \\
2 & 5 \\
3 & 7 \\
4 & 9 \\
5 & 11 \\
\end{array}
\]
Hence \( y = 2x + 1 \)
Teaching Techniques (Con't.)

RESPONSE CODES:

1. Emphasized (used as a primary technique, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

47. Having students collect data on related variables and formulate the relationship between the variables:

Example:

\[
\begin{align*}
\text{Ex.} & \quad \text{one revolution} \\
5 \text{m} & \quad 15.6 \text{ cm} \\
\text{Ratio: } & \quad \frac{15.6}{5} = 3.12
\end{align*}
\]

\[
\begin{align*}
\text{one revolution} & \quad 40.9 \text{ cm} \\
13 \text{ m} & \quad \text{Ratio: } \frac{40.9}{13} = 3.15
\end{align*}
\]

Hence \( \frac{c}{d} = 3.1 \), So \( C = 3.1d \)

\[ 1 \quad 2 \quad 3 \]
Teaching Techniques (Con't)

RESPONSE CODES:
1. Emphasized (used as a primary technique, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

48. Having students create new formulas based on known, simpler formulas:

Ex. Create formula for surface area of a cylinder based on formulas for area of the rectangle and the circle.

\[ A_2 = \pi r^2 \quad A_1 = 2\pi rh \]

\[ w = 2\pi r \]

So, surface area = \(2\pi rh + 2\pi r^2\)
SA = \(2\pi r (h + r)\)
APPLICATIONS AND PROBLEMS

Several types of problems are listed below which may have been included in your instructional program. CHECK the response code to indicate the degree to which a particular type of problem was studied by your class.

RESPONSE CODES:
1. Emphasized (used as a primary type of problem, used extensively or frequently)
2. Used, but not emphasized
3. Not used

49. Age problems
Roberta is now 15 years older than Stan. In 3 more years Roberta will be 3 times as old as Stan was 4 years ago. How old is Roberta now?

50. Digit problems
If 4/5 of a number is added to 3/5 of that number, the result is the same as if 10 is added to the number. What is the number?

51. Mixture problems
A feed dealer plans to mix corn (at $1.12 a bushel) with wheat (at $1.74 a bushel) to get a mixture that sells at $1.43 per bushel. How many bushels of corn are needed to make 200 bushels of the mixture?

52. Percent problems
In 1980 about 4/7 of the telephones in Georgia had direct distance dialing capabilities. What percent was this?

53. Distance-Rate-Time problems
How long does it take a rainstorm to travel 360 km at a rate of 45 km per hour?
Applications and Problems (Con't)

RESPONSE CODES:
1. Emphasized (used as a primary type of problem, used extensively or frequently)
2. Used, but not emphasized
3. Not used

54. Interest problems
Les borrowed $3000 from the bank at 11% interest per year. How much interest would he have to pay at the end of 9 months?

55. Area-Volume problems
The Great Pyramid in Egypt has a square base measuring 240 m on a side. Its altitude is 160 m. What is its volume?

56. Physical-Natural Science problems
(lever problems, Hooke's law, ...)
If Sue has a mass of 56 kg and Sara has a mass of 42 kg, how far will Sue have to sit from the middle of the teeter-totter to balance with Sara, if Sara is 1.2 m from the middle?

57. Energy or Ecological problems
An adult guppy requires 60 cm$^2$ of air surface to live in an aquarium. How many adult guppies can live in a rectangular aquarium that is 45 cm long and 30 cm wide?
SOURCES OF APPLICATIONS AND PROBLEMS

Several sources of applications/problems of integers, formulas, and equations are listed below. CHECK the response code to show how frequently each source was used.

RESPONSE CODES:
1. Used frequently
2. Used occasionally
3. Not used at all

58. Students' textbooks

59. Supplementary textbooks or workbooks

60. Worksheets or exercises designed by myself or local teachers

61. The curriculum guide or syllabus

62. Articles or papers published by professional associations

63. Applications or problems suggested by my students

64. Applications or problems from real world sources such as newspapers or individuals involved in the use of mathematics
TIME ALLOCATIONS

65. What was the **average** length (in minutes) of each class period?.. [ ] 65-66

**INTEGERS**

66. How many total class periods did you spend on the development of the integers and operations with integers? (Combine partial periods when necessary) ........................................... [ ] 67-68

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

67. Activities related to the development of the concept of positive and negative integers.......................................................... [ ] 69-70

68. Activities related to the addition of integers (positive and negative).................................................................................. [ ] 71-72

69. Activities related to the subtraction of integers (positive and negative).................................................................................. [ ] 73-74

70. Activities related to the multiplication of integers (positive and negative).................................................................................. [ ] 75-76

71. Activities related to the division of integers (positive and negative).................................................................................. [ ] 77-78

72. Activities related to the structural properties of the set of integers (commutativity, associativity, distributivity, etc.).................................................................................. [ ] 79-80

73. Activities related to order relations with the set of integers.................................................................................. [ ] 19 C. 4. 20-21

74. Application/problem solving activities related to integers (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc. .......................................................... [ ] 22-23

**NOTE:** The sum of the periods given for items 67 to 74 should not exceed the number given for item 66.
### Time Allocations (Con't.)

**FORMULAS AND EQUATIONS**

75. How many total class periods did you spend on teaching formulas and equations? (Combine partial lessons when necessary.)

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>24-25</th>
</tr>
</thead>
</table>

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

76. Activities related to evaluation of formulas (for given values of the variables)

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>26-27</th>
</tr>
</thead>
</table>

77. Activities related to deriving formulas or equations

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>28-29</th>
</tr>
</thead>
</table>

78. Application/problem solving activities related to use of formulas (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc.)

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>30-31</th>
</tr>
</thead>
</table>

79. Activities related to solving literal equations

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>32-33</th>
</tr>
</thead>
</table>

80. Activities related to solving linear equations

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>34-35</th>
</tr>
</thead>
</table>

81. Application/problem solving activities related to the use of equations (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc.)

<table>
<thead>
<tr>
<th>Time Allocated</th>
<th>36-37</th>
</tr>
</thead>
</table>

**NOTE:** The sum of the periods given for items 76 to 81 should not exceed the number given for item 75.
OPINIONS

Indicate the extent to which you agree or disagree with each of the following statements for your class. CIRCLE the choice which best describes your feelings.

82. The use of the number line adds a lot to the teaching of integers.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>38</td>
</tr>
</tbody>
</table>

83. It is very important to justify the rules for multiplying integers.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>39</td>
</tr>
</tbody>
</table>

84. A great deal of practice is required in order for students to acquire competence in performing operations with directed numbers.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>40</td>
</tr>
</tbody>
</table>

85. It is important for students to understand how integers obey general laws like the distributive law, the associative law, etc.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>41</td>
</tr>
</tbody>
</table>

86. Average students are usually not satisfied with knowing only the rules for performing operations with integers; they want to know why the rules work.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>42</td>
</tr>
</tbody>
</table>

87. Most students find it difficult to appreciate the significance of studying the structural properties (additive inverse, order relation, distributive law, etc.) of the set of integers.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>43</td>
</tr>
</tbody>
</table>

88. Most students cannot be expected to master the use of letters for unknowns quickly; they have to become accustomed to this usage slowly over a long period of time.

<table>
<thead>
<tr>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagree1</td>
<td>Disagree2</td>
<td>Undecided3</td>
<td>Agree4</td>
<td>Agree5</td>
<td>44</td>
</tr>
</tbody>
</table>
Opinions (Con't.)

89. Linear equations whose solution is a fraction (like $5x - 2 = 1$) are
generally more difficult for students to solve than linear equations
whose solution is an integer (like $6x - 3 = 15$).

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

90. In solving equations, it is important that students be able to justify
each step in their solution procedure.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

91. Solving linear equations by trial and error helps students understand
the meaning of a solution.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

92. The notion "solution set" (those values of the unknown which make the
relation true) aids the students' comprehension of linear equations.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

93. Average students have difficulty in solving word problems involving
linear equations.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

94. Average students have difficulty in translating verbal and written
sentences into mathematical sentences, and vice versa.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

95. Average students have difficulty with applications involving linear
equations.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
Opinions (Con't.)

96. When solving problems, it is important for students to first identify the type of problem (age, digit, mixture, etc.) being solved.

   Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
   1                2                3                4                5

97. Solving equations requiring students to justify the steps in the solution procedure has a detrimental effect on learning how to solve equations.

   Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
   1                2                3                4                5

98. The notion of equivalent equations is useful in helping students understand solutions.

   Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
   1                2                3                4                5

99. Formulas taught should be memorized by students.

   Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
   1                2                3                4                5

100. Formulas should be used mainly to aid students in solving classes of story problems.

    Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
    1                2                3                4                5

101. Formulas should be used mainly to find volumes, areas, and perimeters of geometric figures.

    Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
    1                2                3                4                5

102. Formulas should be used mainly in applications to practical situations.

    Strongly Disagree¹  Disagree²  Undecided³  Agree⁴  Strongly Agree⁵
    1                2                3                4                5
APPENDIX D

Geometry Questionnaire
INTERNATIONAL ASSOCIATION for the EVALUATION of EDUCATIONAL ACHIEVEMENT

SECOND Study of MATHEMATICS

GRADE 8 TOPIC SPECIFIC QUESTIONNAIRE GEOMETRY

(Booklet 13L)

FOR NATIONAL CENTRE USE ONLY

PROVINCE OF BRITISH COLUMBIA MINISTRY OF EDUCATION DIVISION OF PUBLIC INSTRUCTION LEARNING ASSESSMENT BRANCH
Check here if no geometry content is included in your program for your class. In that case, disregard the remainder of the questionnaire and return it to B.C. Research in the envelope provided.

CHECK the response which best describes the use you made of each of the following materials in your instruction on geometry.

**RESPONSE CODES:**

1. primary source, used frequently
2. secondary source, used occasionally
3. not used or rarely used

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. School Mathematics II (Addison-Wesley)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Mathematics II (Ginn)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Essentials of Mathematics II (Ginn)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Other published text materials (e.g., textbooks, workbooks, and worksheets)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Locally produced text materials (e.g., textbooks, workbooks, or worksheets)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Commercially or locally produced individualized materials (e.g., programmed instruction or computer assisted instruction)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Commercially or locally produced films, film-strips, or teacher demonstration models</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK the response which describes the treatment of each topic in your class.

**RESPONSE CODES:**

1. taught as new content  
2. reviewed and then extended  
3. reviewed only  
4. assumed as prerequisite knowledge and neither taught nor reviewed  
5. not taught **and** not assumed as prerequisite knowledge

<table>
<thead>
<tr>
<th>Topic</th>
<th>Code 1</th>
<th>Code 2</th>
<th>Code 3</th>
<th>Code 4</th>
<th>Code 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles (acute, right, supplementary, etc.)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Transformations (translations, rotations, reflections)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Vectors</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The Pythagorean Theorem</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Triangles and their properties (excluding congruent triangles)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Polygons and their properties (excluding properties related to congruent or similar polygons)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Circles and their properties</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Congruence of geometric figures (including congruent triangles)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Similarity of geometric figures (including similar triangles)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Parallel lines</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Spatial relations</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Geometric solids and their properties</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Geometric constructions with ruler and compass</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Proofs (formal deductive demonstrations)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Tessellations</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Coordinate geometry</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
INSTRUCTIONAL APPROACHES

Several approaches to teaching geometry are given below. CHECK the response code which describes the teaching approach in your class.

RESPONSE CODES:

1. emphasized (used as a primary means of developing geometric content, used extensively or frequently)
2. used but not emphasized
3. not used

25. An informal Euclidean approach based on inductive reasoning, measurement, or students' intuitions

26. A formal Euclidean approach based on an axiomatic system used to prove theorems

27. An informal transformational approach based on inductive reasoning or students' intuitions

28. A formal transformational approach based on an axiomatic system used to prove theorems

29. A coordinate approach (either informal or formal) using coordinates of points, equations, etc.

30. A vector approach (either informal or formal) using addition of ordered pairs, a scalar times an ordered pair, etc.
INSTRUCTIONAL AIDS

For each of the following aids to the teaching and learning of geometry, CHECK the response code which indicates the degree to which you and your students used the aid.

RESPONSE CODES:
1. used extensively or frequently
2. used occasionally
3. not used

<table>
<thead>
<tr>
<th>Aid</th>
<th>Code 1</th>
<th>Code 2</th>
<th>Code 3</th>
<th>Code 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>31. Ruler and compass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32. Protractor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33. Set squares (draftman's triangles)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34. Geoboards</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35. Paper cutouts or patterns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36. Models of solids (cones, pyramids, cylinders, etc.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37. Paper folding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38. Tracing paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39. Graph paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40. Mirrors or translucent reflectors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41. Filmstrips and films</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42. Computer graphics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43. Kits for constructing plane or solid figures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TEACHING TRANSLATIONS

If you did not teach translations, check here and proceed directly to Item 50.

Several interpretations of translations are given below. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

44. I defined the vector $\vec{AB}$ as the set of equivalent pairs of points:
   $$\vec{AB} = \{(M,N) \mid MeP, NeP, (M,N) \sim (A,B)\}$$
   where $(M,N) \sim (A,B) \iff (A,N)$ and $(B,M)$ have the same midpoint.
   Then the translation along the vector $\vec{V}$ was defined as the map of $P$ onto $P$ which associates to each point $M$ a point $N$ such that $\vec{MN} = \vec{V}$ (or $(M,N) \in \vec{V}$).

45. Given $(A,B)$ a pair of points on the plane $P$, I defined the translation associated with the pair as the map of $P$ onto itself which makes each point $M$ correspond to a point $N$ such that $ABNM$ is a parallelogram.

46. I defined a translation as the composition of two central symmetries.

47. I presented translations by a physical approach involving displacements determined by their direction, orientation and magnitude.
Teaching Translations (Con't.)

48. Using graph paper we studied the mappings \( t(a,b) \) from \( \mathbb{Z}^2 \) onto \( \mathbb{Z}^2 \) such that \( t(a,b)(x,y) = (x',y') \) where
\[
\begin{align*}
x' &= x + a \\
y' &= y + b
\end{align*}
\]
Then a translation of the plane \( P \) was defined as the map \( \tau(a,b):P \to P' \) which associates to each point \( M \) with coordinates \( (x,y) \) a point \( M' \) with coordinates \( (x',y') \) such that
\[
\begin{align*}
x' &= x + a \\
y' &= y + b
\end{align*}
\]

49. I presented the axioms of incidence and defined the translation on the plane \( P \) as a bijection of \( P \) satisfying the following axioms:
1. the identity map \( I \) of \( P \) is a translation.
2. the image of any line \( \ell \) under a translation, is a line \( \ell' \) parallel to \( \ell \).
3. for every translation (other than the identity), there exist one and only one direction \( d \), such that any line \( \ell \) with orientation \( d \) has itself for an image.
4. for every \( A \) and for every \( B \), there exists one and only one translation \( t \) such that \( t(A) = B \).
TEACHING VECTORS

If you did not teach vectors, check here and proceed directly to Item 59.

Several interpretations of vectors are given below. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

50. After choosing the axes, the vector $\vec{v}$ associated with the translation $\tau(a,b)$ is defined as the pair $(a,b)$.

Addition of vectors is then defined in terms of the composition of translations.

51. A vector $\vec{v}$ is defined as the set of pairs $(M, \tau(M))$ where $M$ is a point and $\tau$ is given translation.

52. A vector is defined as an equivalence class of pairs of points. The pairs $\vec{AB}$ and $\vec{MN}$ are equivalent if there exists a translation that transforms $A$ into $B$ and $M$ into $N$.

53. A vector $\vec{AB}$ is defined by:
   1. its orientation (that of the line $\vec{AB}$)
   2. its direction (from $A$ to $B$)
   3. its length (distance from $A$ to $B$)

54. A vector is defined as an equivalence class of pairs of points. The pairs $\vec{AB}$ and $\vec{MN}$ are equivalent if and only if $\overrightarrow{AN}$ and $\overrightarrow{BM}$ have the same midpoint.

55. I do not define vectors since a definition is not necessary. It suffices that students know how to work with them.
TEACHING ADDITION OF VECTORS

Several interpretations of addition of vectors are given below. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

56. Given that a vector is associated with a translation, I presented the addition of two vectors as the vector associated with the composition of their translations.

Ex: \( \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \)

Since \( \tau_{\overrightarrow{BC}} \circ \tau_{\overrightarrow{AB}} = \tau_{\overrightarrow{AC}} \)

57. Given \((A,B)\) and \((A,D)\) as the pairs of points representative of the vectors \( \overrightarrow{U} \) and \( \overrightarrow{V} \), respectively, I defined the sum of \( \overrightarrow{U} \) and \( \overrightarrow{V} \) as the vector \( \overrightarrow{W} \) representative of the pair \((A,C)\) where \(ABCD\) is a parallelogram.

Ex:

```
A
\downarrow
\rightarrow
V
B
\rightarrow
W
\rightarrow
U
D
\rightarrow
\rightarrow
\rightarrow
\rightarrow
C
\rightarrow
\rightarrow
\rightarrow
\rightarrow
W = \overrightarrow{U} + \overrightarrow{V}
```

58. Since the equivalence relation is defined as "having the same midpoint" (see 54 above), I introduced translations before defining the addition of vectors.
Several methods for teaching that the sum of the measures of the angles of a triangle is 180° are given below. CHECK the response code which describes the treatment of each method in your class.

**RESPONSE CODES:**
1. Used as a primary method of explanation
2. Used but not as a primary means of explanation
3. Not used

<table>
<thead>
<tr>
<th></th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
</table>
| 59. | **My students measured the angles of a triangle and added the measures to discover that the sum of the measures is 180°.**  
|   | Yes | No | | | 19 | 20 |
|   | 1 | 2 | 3 | | | |
| 60. | **I drew a line through a vertex parallel to the opposite side and used alternate interior angles to show that the sum of the angles of a triangle is 180°.**  
| Ex: | In the figure ∠1 = ∠4 and ∠3 = ∠5,  
| So ∠1 + ∠2 + ∠3 = ∠4 + ∠2 + ∠5  
| = 180° | | | | | 26 |
| | Yes | No | | | |
|   | 1 | 2 | 3 | | |
| 61. | **My students cut the angles off a triangle and arranged them on a straight line.**  
| | Yes | No | | | 32 |
| | 1 | 2 | 3 | | |
| 62. | **I told my students that the sum of the measures of the angles of a triangle is 180° and had them verify it by measuring the angles and adding the measures.**  
| | Yes | No | | | 38 |
| | 1 | 2 | 3 | | |
Several methods for teaching that the sum of the measures of the angles of a triangle is 180° are given below. Check the response code which describes the treatment of each method in your class.

**RESPONSE CODES:**
1. Used as a primary method of explanation
2. Used but not as a primary means of explanation
3. Not used

Was this method in the students' text?

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes 1</td>
<td></td>
</tr>
<tr>
<td>No 2</td>
<td></td>
</tr>
</tbody>
</table>

63. I had my students verify the relationship by paper folding.

64. I used the fact that (as illustrated in the figure) in traveling AB, BC, CA, a complete revolution (360°) is swept.

65. Using tessellations perhaps from the real world, I identified three angles at a point (C) congruent with three angles in a triangle (ABC) embedded in the tessellation.

66. A ruler and compass construction was used to show the relationship.

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes 1</td>
<td></td>
</tr>
<tr>
<td>No 2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes 1</td>
<td></td>
</tr>
<tr>
<td>No 2</td>
<td></td>
</tr>
</tbody>
</table>

For those methods used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

**RESPONSE CODES:**
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

For those methods NOT USED (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using them.

**RESPONSE CODES:**
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text
Several methods for teaching the Pythagorean Theorem are given below. CHECK the response code which describes the treatment of each method in your class.

RESPONSE CODES:
1. Used as a primary method of explanation
2. Used but not as a primary means of explanation
3. Not used

---

For those methods used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

---

67. I presented my students with a variety of right triangles and had them measure and record the lengths of the legs and hypotenuse. The pattern was discussed and then we stated the property.

Ex: leg leg hypotenuse

<table>
<thead>
<tr>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

3² + 4² = 5²
9² + 12² = 15²
∴ a² + b² = c²

Was this method in the students' text?

Yes ☐  No ☐

---

68. I used diagrams like the following to show that, in a right triangle

\[ a^2 + b^2 = c^2 \]

---

69. I gave my students the formula

\[ a^2 + b^2 = c^2 \]

and had them use it in working examples.

---

70. The theorem was presented in the context of a historical account of Pythagoras and Euclid.
Several methods for teaching the Pythagorean Theorem are given below. CHECK the response code which describes the treatment of each method in your class.

RESPONSE CODES:
1. Used as a primary method of explanation
2. Used but not as a primary means of explanation
3. Not used

Was this method in the students' text?

71. I presented an informal area argument using physical, e.g., geoboards, or pictorial models.

Ex: I showed that the two squares had equal area.

\[
\begin{array}{c}
\text{Ex: I showed that the two squares had equal area.}
\end{array}
\]

\[
\begin{array}{c}
\text{Yes} \quad \text{No}
\end{array}
\]

72. I presented a formal deductive "algebraic" argument.

Ex: Using similar right triangles, proportions can be set up to yield \(a^2 + b^2 = c^2\).

\[
\begin{array}{c}
\text{Ex: Using similar right triangles, proportions can be set up to yield } a^2 + b^2 = c^2.
\end{array}
\]

\[
\begin{array}{c}
\text{Yes} \quad \text{No}
\end{array}
\]

73. I presented a formal deductive argument using area.

Ex: This figure is sometimes used to present a formal proof.

\[
\begin{array}{c}
\text{Ex: This figure is sometimes used to present a formal proof.}
\end{array}
\]

\[
\begin{array}{c}
\text{Yes} \quad \text{No}
\end{array}
\]
TECHNIQUES FOR TEACHING CONGRUENT TRIANGLES

Several techniques for teaching congruent triangles are given below. CHECK the response code which describes the technique used in your class.

RESPONSE CODES:

1. used extensively or frequently
2. used occasionally
3. not used

74. State definition and properties
   Students were given a definition and conditions under which two triangles are congruent, e.g., SSS, SAS, or ASA. [1 2 3]

75. Graph paper or tracing paper
   Congruent triangles were constructed using graph paper or tracing paper. [1 2 3]

76. Measurement
   Measurement activities were used to study properties of congruent triangles, e.g., congruence of corresponding sides and angles. [1 2 3]

77. Constructions with ruler and compass
   Students constructed congruent triangles using a ruler and compass. [1 2 3]

78. Geoboard
   Students used the geoboard to make congruent triangles and study their properties. [1 2 3]

79. Environment
   Examples of congruent triangles from the environment were discussed. Ex. Scaffolding

80. Transformations
   Students formed congruent triangles by finding images of triangles using reflections, rotations or translations. [1 2 3]
## TECHNIQUES FOR TEACHING SIMILAR TRIANGLES

Several techniques for teaching similar triangles are given below. Check the response code which describes the technique used in your class.

**RESPONSE CODES:**

1. used extensively or frequently  
2. used occasionally  
3. not used

<table>
<thead>
<tr>
<th>Technique</th>
<th>Code</th>
<th>Description</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>81. State definition and properties</td>
<td></td>
<td>Students were given a definition and conditions under which two triangles are similar, e.g., AAA, SAS or SSS.</td>
<td>57</td>
</tr>
<tr>
<td>82. Graph paper or tracing paper</td>
<td></td>
<td>Similar triangles were constructed using graph paper or tracing paper.</td>
<td>58</td>
</tr>
<tr>
<td>83. Measurement</td>
<td></td>
<td>Measurement activities were used to study properties of similar triangles, e.g., proportionality of sides.</td>
<td>59</td>
</tr>
<tr>
<td>84. Constructions with ruler and compass</td>
<td></td>
<td>Students constructed similar triangles using a ruler and compass.</td>
<td>60</td>
</tr>
<tr>
<td>85. Geoboard</td>
<td></td>
<td>Students used the geoboard to make similar triangles and study their properties.</td>
<td>61</td>
</tr>
<tr>
<td>86. Environment</td>
<td></td>
<td>Examples of similar triangles from the environment were discussed.</td>
<td>62</td>
</tr>
<tr>
<td>87. Dilations (stretching or shrinking)</td>
<td></td>
<td>Students constructed the image of triangles under an enlargement or dilation (stretching or shrinking).</td>
<td>63</td>
</tr>
</tbody>
</table>
Several techniques for teaching parallel lines are given below. CHECK the response code which describes the technique used in your class.

RESPONSE CODES:
1. used extensively or frequently
2. used occasionally
3. not used

88. Definition and examples
Students were given a definition and examples of parallel and nonparallel lines were illustrated. ___1___2___3 64

89. Paper folding
Paper folding activities were used to present and study parallel lines. ___1___2___3 65

90. Measurement
Measurement activities were used to study such properties as: parallel lines are everywhere equidistant, parallel lines form congruent corresponding angles with a transversal, etc. ___1___2___3 66

91. Constructions with ruler and compass
Ex. Given a line \( l \), and a point \( A \) not on the line, students constructed a line \( l' \) through the point parallel to the given line. ___1___2___3 67

92. Tessellations
Given tessellations of the plane such as floor or ceiling tiles, students inspected the tessellations for parallel lines and their properties. ___1___2___3 68

93. Geoboards
Given a geoboard, students inspected lines on the board to determine parallel lines and study their properties. ___1___2___3 69
Techniques for Teaching Parallel Lines (Con't.)

RESPONSE CODES:

1. used extensively or frequently
2. used occasionally
3. not used

94. Constructions with straightedge and set squares (draftsman's triangles)
   Ex. Students constructed \( \ell' \) parallel to \( \ell \)

95. Environment
   Examples of "parallel lines" from the environment, e.g., railroad tracks or telephone lines, were discussed.

96. Translations
   Parallel lines were studied through the use of translations.
   Ex. Given the translation \( T \), points \( A \) and \( B \) and their image points \( A' \) and \( B' \)
   then \( AA' \parallel BB' \).

97. Reflections
   Parallel lines were studied through the use of reflections.
   Ex. Given two lines, students used translucent materials (e.g. miras) to determine whether the lines are parallel.

98. Rotations
   Parallel lines were studied through the use of rotations.
   Ex. Given a line, students determined its image under a half-turn (180° rotation).
Several techniques for teaching spatial relations are given below. CHECK the response code which describes the technique used in your class.

RESPONSE CODES:
1. used extensively or frequently
2. used occasionally
3. not used

99. Using ready-made two-dimensional patterns (nets) to build three dimensional figures.
Ex.

100. Designing a two-dimensional pattern for a given three-dimensional object.
Ex.

101. Making a two-dimensional drawing for a given three-dimensional object.
Ex.

102. Drawing plans and elevation (orthogonal projections) of geometric solids.
Ex.
Teaching Spatial Relations (Con't.)

RESPONSE CODES:

1. used extensively or frequently
2. used occasionally
3. not used

103. Representing the intersection of a plane and a solid by a two-dimensional drawing.

Ex.

104. Finding numerical or algebraic expressions that describe relationships among the parts of a geometric figure.

Ex.

$\angle B = 60^\circ$ because $\triangle ABC$ is equilateral since its sides are the diagonals of the faces of the cube.

105. Building models of intersecting planes in space.

106. Predicting the shape of the shadows cast by various objects under a fixed source of light.
TIME ALLOCATIONS

107. What was the average length (in minutes) of each mathematics period? Minutes 22-23

108. How many total class periods did you spend on geometry? (Combine partial periods when necessary) Periods 24-25

Indicate the amount of time spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises or proofs, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

109. Activities related to the development of the concept of angles (acute, right, supplementary, etc.) Periods 26-27

110. Activities related to transformations (translations, rotations, reflections) Periods 28-29

111. Activities related to vectors Periods 30-31

112. Activities related to The Pythagorean Theorem Periods 32-33

113. Activities related to triangles and their properties (excluding congruent triangles) Periods 34-35

114. Activities related to polygons and their properties (excluding properties related to congruent or similar polygons) Periods 36-37

115. Activities related to circles and their properties Periods 38-39

116. Activities related to congruence of geometric figures (including congruent triangles) Periods 40-41

117. Activities related to similarity of geometric figures (including similar triangles) Periods 42-43
**Time Allocations (Con't.)**

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>118</td>
<td>Activities related to parallel lines</td>
<td></td>
</tr>
<tr>
<td>119</td>
<td>Activities related to spatial relations</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>Activities related to geometric solids and their properties</td>
<td></td>
</tr>
<tr>
<td>121</td>
<td>Activities related to geometric constructions with ruler and compass</td>
<td></td>
</tr>
<tr>
<td>122</td>
<td>Activities related to proofs (formal deductive demonstrations)</td>
<td></td>
</tr>
<tr>
<td>123</td>
<td>Activities related to tessellations</td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>Activities related to coordinate geometry</td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>Application/problem solving activities related to geometry</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** THE SUM OF THE PERIODS GIVEN FOR ITEMS 109 TO 125 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 108.
OPINIONS

Indicate the extent to which you agree or disagree with each of the following statements for your class. CIRCLE the choice which best describes your feelings.

126. The main objective of teaching geometry at this grade level is that of constructing a mathematical model of real situations.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

127. Mastery of deductive procedures (e.g. proving theorems) is the goal of teaching geometry at this grade level.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

128. The objective of teaching geometry at this grade level is to present the students with situations in which they have to formally demonstrate something about which they have an intuitive notion.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

129. It is desirable that the presentation of geometric concepts follow an order determined by an axiomatic approach.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

130. An intuitive approach to geometry is more meaningful to students at this grade level than a formal approach.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

131. Geometry should be taught mainly through transformations (flips, turns, stretches).

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

132. The use of concrete models and instructional aids is essential in teaching geometry.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

133. Three dimensional geometry should be taught only in the context of measurement (volume, surface area, etc.) for these students.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
Opinions (Con't.)

134. The concept of translation should be part of the knowledge of students at this grade level.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

135. The concept of vector should be part of the knowledge of students at this grade level.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

136. It is preferable to delay the study of vectors to a later time.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

137. Activities to improve students' ability to visualize spatial figures should be included in the instructional program.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

138. The study of polygons and their properties should be limited only to triangles and quadrilaterals.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

139. The students should be skilled in geometric constructions using ruler and compass.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

140. Teachers' demonstration of proofs of theorems should be an essential part of an instructional program in geometry for these students.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

141. Geometric topics should be taught only to those students who will pursue higher education.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree

142. Proofs of theorems should be delayed until these students are at least 15 years of age.

Strongly 1  Disagree 2  Undecided 3  Agree 4  Strongly 5
Disagree
APPENDIX E

Measurement Questionnaire
SECOND
Study of
MATHEMATICS

GRADE 8
TOPIC SPECIFIC QUESTIONNAIRE
MEASUREMENT
(Booklet 12L)

FOR NATIONAL CENTRE USE ONLY

PROVINCE OF BRITISH COLUMBIA
MINISTRY OF EDUCATION
DIVISION OF PUBLIC INSTRUCTION
LEARNING ASSESSMENT BRANCH
Check here if measurement is not included in your program for your class. In that case, disregard the remainder of the questionnaire and return it to B.C. Research in the envelope provided.

CHECK the response which best describes the use you made of each of the following materials in your instruction on measurement.

**RESPONSE CODES:**

1. primary source, used frequently
2. secondary source, used occasionally
3. not used or rarely used

1. School Mathematics II (Addison-Wesley)
   - 1
   - 2
   - 3

2. Mathematics II (Ginn)
   - 1
   - 2
   - 3

3. Essentials of Mathematics II (Ginn)
   - 1
   - 2
   - 3

4. Other published text materials (e.g., textbooks, workbooks, and worksheets)
   - 1
   - 2
   - 3

5. Locally produced text materials (e.g., textbooks, workbooks, or worksheets)
   - 1
   - 2
   - 3

6. Commercially or locally produced individualized materials (e.g. programmed instruction or computer assisted instruction)
   - 1
   - 2
   - 3

7. Commercially or locally produced films, filmstrips, or teacher demonstration models
   - 1
   - 2
   - 3

8. Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives)
   - 1
   - 2
   - 3
TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:
1. taught as new content
2. reviewed and then extended
3. reviewed only
4. assumed as prerequisite knowledge and neither taught nor reviewed
5. not taught and not assumed as prerequisite knowledge

9. Concept of measurement (including the selection of appropriate units) _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

10. Names of units of measure in the metric system (SI) _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

11. Names of units of measure in the English system _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

12. Conversion of units within a system
   Ex. 5 centimetres = 50 millimetres _________ 1 _________ 2 _________ 3 _________ 4 _________ 5
   24 inches = 2 feet _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

13. Conversion of units between systems
   Ex. Convert 5 inches to centimetres
   How many miles in 60 kilometres? _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

14. Estimating measurements
   Ex. Find a stick 15 cm long.
   How many metres high is the ceiling? _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

15. Operations with measurements
   Ex. 4 yards 2 feet 8 inches + 2 yards 1 foot 10 inches
   2.5 m + 67 cm = _________ 1 _________ 2 _________ 3 _________ 4 _________ 5

16. Precision, accuracy, percent error, or relative error _________ 1 _________ 2 _________ 3 _________ 4 _________ 5
<table>
<thead>
<tr>
<th>Teaching Topics (Con't.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17. Concept of $\pi$</td>
</tr>
<tr>
<td>18. Linear measurement</td>
</tr>
<tr>
<td>Ex. Find the length of segment $AB$</td>
</tr>
<tr>
<td>19. Perimeter of a polygon (including triangles, quadrilaterals and other polygons)</td>
</tr>
<tr>
<td>20. Circumference of a circle</td>
</tr>
<tr>
<td>21. Area of a triangle</td>
</tr>
<tr>
<td>22. Area of a rectangle (including squares)</td>
</tr>
<tr>
<td>23. Area of a parallelogram other than a rectangle</td>
</tr>
<tr>
<td>24. Area of a trapezoid</td>
</tr>
<tr>
<td>25. Area of a circle</td>
</tr>
<tr>
<td>26. Surface area of a rectangular solid (including a cube)</td>
</tr>
<tr>
<td>27. Surface area of a cylinder</td>
</tr>
<tr>
<td>28. Surface area of a sphere</td>
</tr>
<tr>
<td>29. Volume of a rectangular solid (including a cube)</td>
</tr>
<tr>
<td>30. Volume of a cylinder and prism</td>
</tr>
<tr>
<td>31. Volume of a sphere</td>
</tr>
<tr>
<td>32. Volume of a cone and pyramid</td>
</tr>
</tbody>
</table>
INSTRUCTIONAL AIDS

For each of the following aids to the teaching and learning of measurement, CHECK the response code which indicates the degree to which you and your students used the aid.

RESPONSE CODES:

1. used extensively or frequently
2. used occasionally
3. not used

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>33. Rulers (metrestick, yardstick, 12&quot; ruler, etc.)</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>34. Measuring tape</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>35. Trundle wheel</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>36. Examples of non-standard units (paper clips, hand span, foot length, popsicle sticks, sugar cubes, matchboxes, etc.)</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>37. Geoboards, graph paper, or grids</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>38. Models for standard units of area (cm squares, cm cubes, cm rods, etc.)</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>39. Graduated cylinders</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>40. Containers (litre, gallon, etc.)</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>41. &quot;Fillable&quot; models of geometric solids</td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
TEACHING METHODS

The methods used to introduce the use of units of measurement given below may have been included in your instructional program. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:

1. emphasized (used as a primary method, referred to extensively or frequently)
2. used, but not emphasized
3. not used

42. I have my students use non-standard units of measurement.
   Ex. Measure the length of a desk using paper clips.
   __1__ __2__ __3__ 62

43. I have my students use standard units in measuring objects.
   Ex. Measure the length of the room in metres.
   __1__ __2__ __3__ 63

44. I have my students estimate the size of real world objects.
   Ex. Estimate how many sugar cubes will fit into a given container.
      Estimate the length of the hallway.
   __1__ __2__ __3__ 64

45. I have my students identify objects whose measurement is as close as possible to a given number of units.
   Ex. Which of these four containers has a capacity closest to two litres?
      Cut a long piece of string about 10 centimetres long without using a ruler.
   __1__ __2__ __3__ 65

46. I have my students measure a given object using different units of measure.
   Ex. Measure the width of the paper in mm and cm.
      Find the height of the table in cm and inches.
   __1__ __2__ __3__ 66

47. I have my students increase the precision of their measurements by using smaller units.
   Ex. The length of the stick is between 5 cm and 6 cm.
      More precisely, it is between 53 mm and 54 mm.
   __1__ __2__ __3__ 67
The interpretations of the number \( w \) given below may be included in your instructional program. CHECK the appropriate response code which describes the treatment of each interpretation in your class.

**RESPONSE CODES:**
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

<table>
<thead>
<tr>
<th>Question</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>48. I had my students measure and find the ratio of the circumference to the diameter of a number of circular objects, and approximate ( c ) for any circle. ( d )</td>
<td>Yes</td>
<td><img src="60-73" alt="Diagram" /></td>
<td>Yes</td>
<td><img src="74-75" alt="Diagram" /></td>
</tr>
<tr>
<td>49. I told my students that ( w = \frac{22}{7} ) or 3.14.</td>
<td>Yes</td>
<td><img src="74-75" alt="Diagram" /></td>
<td>Yes</td>
<td>![Diagram](19 C)</td>
</tr>
<tr>
<td>50. My students estimated the value of ( w ) using Buffon's Needle Problem.</td>
<td>Yes</td>
<td><img src="74-75" alt="Diagram" /></td>
<td>Yes</td>
<td><img src="20-25" alt="Diagram" /></td>
</tr>
<tr>
<td>51. I presented a chart relating the values of the circumference to that of the diameter of various circles like the following:</td>
<td><img src="20-25" alt="Diagram" /></td>
<td><img src="20-25" alt="Diagram" /></td>
<td><img src="20-25" alt="Diagram" /></td>
<td><img src="20-25" alt="Diagram" /></td>
</tr>
<tr>
<td>( c )</td>
<td>44</td>
<td>28</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td>14.0</td>
<td>8.9</td>
<td>11.8</td>
<td></td>
</tr>
<tr>
<td>I asked the students to find the ratio of the circumference to the diameter for each circle and generalized that ( c/d = 3.14 ).</td>
<td>Yes</td>
<td><img src="20-25" alt="Diagram" /></td>
<td>Yes</td>
<td><img src="20-25" alt="Diagram" /></td>
</tr>
<tr>
<td>52. I told my students that ( w ) is an irrational number obtained as the result of dividing the circumference of any circle by its diameter.</td>
<td>Yes</td>
<td><img src="20-25" alt="Diagram" /></td>
<td>Yes</td>
<td><img src="20-25" alt="Diagram" /></td>
</tr>
</tbody>
</table>
For those interpretations used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. It was taught it was appropriate
8. Emphasized in student text

For those interpretations NOT USED (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using them.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

53. I had my students use regular polygons inscribed in a circle to obtain successive approximations of π.

Ex. A

Using square ABCD, π < 2.75
Using the octagon, π < 7
and so on, to show that π approaches 3.14 as the number of sides of the polygons increases.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>

54. I introduced π as the area of a circle of radius 1.

Ex. Using successive approximations to the area of the unit circle, I showed that:

2 < area of circle < 4
OR

2 < π < 4

Using a finer grid, I showed that:

68 < area of circle < 80
OR

2.72 < π < 3.52

Using still a finer grid, I showed that:

208 < area of circle < 208
OR

2.68 < π < 3.44

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>
Several methods for teaching the formula for the area of a parallelogram are given below. CHECK the appropriate response code which describes the treatment of each method in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

Was this method in the students' text?

<table>
<thead>
<tr>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
<th>1st Primary Reason</th>
<th>2nd Primary Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>1</td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>

For those methods used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

For those methods NOT USED, (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using them.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

55. I presented the formula $A = bh$ and demonstrated how to apply it by means of examples. 4 cm

![Diagram of parallelogram](image)

A = 4 cm $\times$ 1.7 cm = 6.8 cm$^2$

<table>
<thead>
<tr>
<th>Ex.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>1</td>
<td>No</td>
<td>2</td>
</tr>
</tbody>
</table>

56. I presented a parallelogram on a grid (or a geoboard) like the one below (parallelogram ABCD), and had the students relate the number of square units inside ABCD to the base and altitude of the parallelogram.

![Diagram of parallelogram](image)

| Yes | 1 | No | 2 |

57. I presented a parallelogram on a grid (or a geoboard) like the one shown above and had the students count the square units inside triangles ABE and CDF. Then I had them relate the area of ABCD to that of rectangle BEFC based on the congruence of $\triangle$ ABE and $\triangle$ BCF.

| Yes | 1 | No | 2 |

---

55-55

56-56

62-67
Several methods for teaching the formula for the area of a parallelogram are given below. CHECK the appropriate response code which describes the treatment of each method in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Not emphasized in student text

For those methods used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

58. I derived the formula $A = bh$ by comparing the area of the parallelogram to that of a related rectangle of equal dimensions.

Yes __1__ 2 3 __ No __1__ 2 3 __

59. I gave the student a parallelogram like the one below, and asked them to cut off triangle FDC and to use this to form a rectangle (AF'D'). The students then related the formula for the area of the rectangle to the area of the parallelogram.

Yes __1__ 2 3 __ No __1__ 2 3 __

60. I partitioned the parallelogram by a diagonal into two congruent triangles.

Yes __1__ 2 3 __ No __1__ 2 3 __

Then the area of $\triangle ABD$ is $\frac{1}{2}bh$ and the area of the parallelogram is then $bh$. 
Several methods for teaching the formula for the area of a parallelogram are given below. CHECK the appropriate response code which describes the treatment of each method in your class.

RESPONSE CODES:
1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

61. I partitioned the parallelogram ABCD into \( \triangle ABE, \triangle CDF \) and rectangle AECF so that the area of the parallelogram is obtained by adding the areas of the two triangles and the rectangle.

![Diagram of parallelogram and triangles](image)

Was this method in the students' text?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

62. I obtained the area of the parallelogram by subtracting the areas of \( \triangle ABG \) and \( \triangle DCH \) from the area of the rectangle GHID.

![Diagram of parallelogram and triangles](image)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

For those methods used as PRIMARY explanations (i.e., if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for using them.

RESPONSE CODES:
1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

For those methods NOT USED (i.e., if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using them.

RESPONSE CODES:
1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disturbed by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text
Several methods for teaching the formula for the volume of a rectangular prism are given below. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:

1. Emphasized [used as a primary explanation, referred to extensively or frequently]
2. Used, but not emphasized
3. Not used

63. I presented the formula \( V = 1 \times W \times h \) or \( V = (\text{area of base}) \times \text{height} \) and demonstrated how to apply it by means of examples.

Ex.

\[
\begin{array}{c}
2 \\
2.5 \\
\hline
4 \\
\hline
\end{array}
\]

\( V = 2 \times 2.5 \times 4 \)

\( V = 20 \text{ cm}^3 \)

64. I presented a physical model of a right rectangular prism (box) with its faces marked off in square units, as illustrated below. I had students generate the formula by relating the number of cubic units contained in the prism to the dimensions of the box, giving hints only if necessary.

Ex.

65. I provided my students with unit cubes and asked them to build rectangular prisms of specified dimensions. I asked them to relate the number of unit cubes used to the given dimensions, giving hints only if necessary.
Several techniques a teacher might use in teaching the relationship among various metric (SI) units are listed below. CHECK the response code which describes the treatment of each technique in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

66. I established the analogy between decimal numeration system and the basic metric units of measurement.
Ex. One kilolitre is 1,000 litres and 121 centimetres is 1.21 metres. □ □ □ □

67. I taught my students rules to change from one metric unit to another.
Ex. To convert from a unit to a smaller unit, multiply.
   To convert from a unit to a larger unit, divide. □ □ □ □

68. I presented a table showing definitions and adjacent relationships.
Ex. 1 km = 1000 m = 10 hm
    1 hm = 100 m = 10 dam
    1 dam = 10 m
    1 m
    1 dm = 0.1 m = 10 cm
    1 cm = 0.01 m = 10 mm
    1 mm = 0.001 m □ □ □ □

69. I used a number line or a metre stick (graduated in centimetres and millimetres) to illustrate interrelationships among units. □ □ □ □

70. I used centimetre cubes and decimetre cubes to establish relationships among units. □ □ □ □

71. I demonstrated the relationship between metric units of length, metric units of capacity and metric units of mass using instruments and units.
Ex. 1,000 cubic centimetres = 1 litre
    1 cubic centimetre of H₂O = 1 gram
    Therefore, 1 litre of H₂O = 1 kilogram □ □ □ □
TIME ALLOCATIONS

72. What was the average length (in minutes) of each class period? Minutes 47-48

73. How many total class periods did you spend on teaching measurement? (Combine partial lessons when necessary.) Periods 49-50

Indicate the amount of time spent on each of the following activities (that is, demonstrations, explanations, students doing computations, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

74. Activities related to the concept of measurement (including selection of units and use of unit to assign a number) Periods 51-52

75. Teaching units in the metric system (SI) Periods 53-54

76. Teaching units in the English system Periods 55-56

77. Activities related to conversion of units within a system
   Ex. 5 cm = 50 mm
   24 inches = 2 feet Periods 57-58

78. Activities related to conversion of units between systems
   Ex. Convert 5 inches to centimetres.
   How many miles in 60 kilometres? Periods 59-60

79. Activities related to estimating measurements
   Ex. Find a stick 15 cm long.
   How many metres high is the ceiling? Periods 61-62

80. Activities related to determining precision, accuracy, percent error or relative error Periods 63-64
Time Allocations (Con't.)

81. Activities related to operations with measurements
   
   Ex. 4 yards 2 feet 8 inches
   + 2 yards 1 foot 12 inches
   
   \[2.5 \text{ m} + 67 \text{ cm} =\]
   
   Periods 65-66

82. Activities related to the concept of \(\pi\)
   
   Periods 67-68

83. Activities related to linear measurement
   
   Ex. Find the length of segment \(AB\).
   
   Periods 69-70

84. Activities related to finding the perimeter of a polygon (including triangles, quadrilaterals, and other polygons)
   
   Periods 71-72

85. Activities related to finding the circumference of a circle
   
   Periods 73-74

86. Activities related to finding the area of a triangle
   
   Periods 75-76

87. Activities related to finding the area of a rectangle (including spheres)
   
   Periods 77-78

88. Activities related to finding the area of a parallelogram other than a rectangle
   
   Periods 79-80

89. Activities related to finding the area of a trapezoid
   
   Periods 20-21

90. Activities related to finding the area of a circle
   
   Periods 22-23

91. Activities related to finding the surface area of a rectangular solid (including cubes)
   
   Periods 24-25

92. Activities related to finding the surface area of a cylinder
   
   Periods 26-27
### Time Allocations (Cont.)

<table>
<thead>
<tr>
<th>Activity Description</th>
<th>Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>93. Activities related to finding the surface area of a sphere</td>
<td>28-29</td>
</tr>
<tr>
<td>94. Activities related to finding the volume of a rectangular solid (including cubes)</td>
<td>30-31</td>
</tr>
<tr>
<td>95. Activities related to finding the volume of a cylinder and prism</td>
<td>32-33</td>
</tr>
<tr>
<td>96. Activities related to finding the volume of a sphere</td>
<td>34-35</td>
</tr>
<tr>
<td>97. Activities related to finding the volume of a cone and pyramid</td>
<td>36-37</td>
</tr>
<tr>
<td>98. Application/problem solving activities related to measurement</td>
<td>38-39</td>
</tr>
</tbody>
</table>

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 74 TO 98 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 73.
Indicate the extent to which you agree or disagree with each of the following statements for your class. CIRCLE the choice which best describes your feelings.

99. Estimation and approximation should be emphasized in the teaching of measurement.
   Strongly disagree  Disagree  Undecided  Agree  Strongly agree

100. Students' use of standard instruments for measuring should be emphasized in the mathematics program.
     Strongly disagree  Disagree  Undecided  Agree  Strongly agree

101. Measurements other than length, area, or volume should be taught as part of the school science program and not as a part of the school mathematics program.
     Strongly disagree  Disagree  Undecided  Agree  Strongly agree

102. Work with non-standard units is essential for increasing students' understanding of the concept of measurement.
     Strongly disagree  Disagree  Undecided  Agree  Strongly agree

103. Measurement of time, temperature, mass, and weight should be taught as part of the mathematics program at this grade level.
     Strongly disagree  Disagree  Undecided  Agree  Strongly agree

104. Work with formulae for finding the perimeter, area, and volume should be emphasized.
     Strongly disagree  Disagree  Undecided  Agree  Strongly agree
Opinions (Con't.)

105. Computations involving measurement should be done with hand-held calculators.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
</table>

106. The best way students learn about measurement is by actually measuring things.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
</table>

107. Students should be expected to know and apply standard area and volume formulas.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Undecided</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
</table>
APPENDIX F

The Initial List of Subtopics and Items as Categorized by the Panel of Experts
The Initial List of Subtopics and Items as Categorized by the Panel of Experts

<table>
<thead>
<tr>
<th>Topic / Subtopic</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td><strong>Common and Decimal Fractions</strong></td>
<td></td>
</tr>
<tr>
<td>Concept of Fractions</td>
<td>21, 22, 23, 27, 28, 30</td>
</tr>
<tr>
<td>Addition of Fractions</td>
<td>31, 32, 33, 37, 38</td>
</tr>
<tr>
<td>Concept of Decimal Fractions</td>
<td>51, 53, 56</td>
</tr>
<tr>
<td>Operations with Decimal Fractions</td>
<td>59</td>
</tr>
<tr>
<td><strong>Algebra</strong></td>
<td></td>
</tr>
<tr>
<td>Concept of Integers</td>
<td>20, 22, 24</td>
</tr>
<tr>
<td>Addition of Integers</td>
<td>25, 27</td>
</tr>
<tr>
<td>Subtraction of Integers</td>
<td>28, 32</td>
</tr>
<tr>
<td>Multiplication of Integers</td>
<td>36</td>
</tr>
<tr>
<td>Formula</td>
<td>45, 47, 48</td>
</tr>
<tr>
<td><strong>Ratio, Proportion, and Percent</strong></td>
<td></td>
</tr>
<tr>
<td>Ratio</td>
<td>21</td>
</tr>
<tr>
<td>Proportion</td>
<td>27</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>Measures of Angles of Triangle</td>
<td>59, 60, 61, 62, 63, 64, 66</td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>67, 68, 71</td>
</tr>
<tr>
<td><strong>Measurement</strong></td>
<td></td>
</tr>
<tr>
<td>Number ( \text{\textbar} )</td>
<td>48, 53, 54</td>
</tr>
<tr>
<td>Area of Parallelogram</td>
<td>56, 57, 58, 59, 60, 61, 62</td>
</tr>
<tr>
<td>Volume of rectangular prism</td>
<td>64, 65</td>
</tr>
</tbody>
</table>
APPENDIX G

Profiles of Responses to the Concrete and Abstract Items for Each Subtopic for the Sample of Ten Teachers
Figure 1. Profiles of responses to the concrete items for the concept of fractions.
Figure 2. Profiles of responses to the abstract items for the concept of fractions.
Figure 3. Profiles of responses to the concrete items for the skill addition of fractions.
Figure 4. Profiles of responses to the abstract items for the skill addition of fractions.
Figure 5. Profiles of responses to the concrete items for the concept of decimal fractions.
Figure 6. Profiles of responses to the abstract items for the concept of decimal fractions.
Figure 7. Profiles of responses to the concrete items for the concept of integers.
Figure 8. Profiles of responses to the abstract items for the concept of integers.
Figure 9. Profiles of responses to the concrete items for the skill subtraction of integers.
Figure 10. Profiles of responses to the abstract items for the skill subtraction of integers.
Figure 11. Profiles of responses to the concrete items for the concept Pythagorean Theorem.
Figure 12. Profiles of responses to the abstract items for the concept Pythagorean Theorem.
Figure 13. Profiles of responses to the concrete items for the concept $\tau$. 
Figure 14. Profiles of responses to the abstract items for the concept $\pi$. 
Figure 15. Profiles of responses to the concrete items for the relationship among various metric units.
Figure 16. Profiles of responses to the abstract items for the relationship among various metric units.