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SOME MATHEMATICAL PROGRAMMING MODELS IN THE DESIGN AND MANUFACTURE OF PLYWOOD

by

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ABSTRACT

One factor of wood loss in the manufacture of plywood is implicit in the form of excess thickness in plywood due to the choice of veneer thicknesses and plywood designs used in assembly. The thickness and designs currently in use appear to have come largely from tradition and there is no evidence in the literature to show what constitutes the most economical veneer thicknesses and plywood designs for a mill. The problem of determining them is very complex since many types of plywood are assembled in each mill as some integral multiple combination of a few veneers satisfying the 'balanced design' and other structural specifications. The consumption of logs is dependent on the excess thickness in plywood and the economics of the mill further depend on how efficiently a given set of veneers and designs are used to satisfy the orderfile requirements. In this dissertation, these aspects of the Plywood Design and Manufacturing (PDM) problem are addressed using a mathematical programming approach.

The problem of finding the optimal veneer thicknesses, associated plywood designs and product mix is formulated as a non-linear mixed integer mathematical programming model. Utilizing the structure of the constraints and by selecting appropriate variables to branch on, it is demonstrated that the PDM problem can be solved efficiently through an implicit enumeration algorithm involving a tree search procedure. The subproblem to be solved at each feasible node of the tree is a Linear Multiple Choice Knapsack (LMCK) problem whose solution can be obtained explicitly following its coefficient structure. A computer code is written in FORTRAN for the implicit enumeration algorithm.

Data obtained from a plywood mill in B.C. is analysed using the PDM model and this code. It is demonstrated that the annual net revenue of the mill can be substantially increased through the use of the PDM model.

The PDM model is further extended to mill situations involving more than one species and varying orderfile requirements. The model is reformulated in each case and it is demonstrated that essentially the same tree search procedure can be used to solve all these models. When the orderfile is independent of species, the subproblem to be solved at each node of the tree is a Generalized Network problem. It is shown that this Generalized Network problem can be reduced to a Generalized Transportation problem utilizing the structure of the coefficients and solved as an ordinary Transportation problem. When the orderfile is dependent on species, the subproblem decomposes into several Linear Multiple Choice Knapsack problems. If more than one species of veneer can be mixed within a plywood panel, the subproblem is a linear programming problem.

The PDM model is further shown to be a special case of a disjunctive programming problem. Following the development of the PDM model, methods to determine the efficiency of plywood designs and the optimum number of veneer thicknesses for a plywood mill are developed..

TABLE OF CONTENTS

ABSTRACT			(ii)
ACKNOWLEDGEMENTS			(viii)
CHAPTER 1		INTRODUCTION	1
	1.1	Introduction	1
	1.2	Operations Research in Plywood Industry	6
	1.3	Wood Losses in Plywood Manufacture	11
CHAPTER 2		FORMULATION OF THE PROBLEM	14
	2.1	The Veneer Thickness Problem	14
	2.2	Terminology Related to Plywood Designs	16
	2.3	The Design and Manufacturing Problem	19
	2.4	Formulating the PDM Model	22
	2.5	Discussion and Extensions	34
CHAPTER 3		SOLUTION TO THE PROBLEM	38
	3.1	Solution to the PDM Model	38
	3.2	An Implicit Enumeration Algorithm	40
	3.3	Branching Tests and Bounds	48
	3.4	Computational Results	60
CHAPTER 4		EXTENSIONS TO MORE SPECIES	63
	4.1	Extensions to the PDM Model	63
	4.2	Orderfile Independent of Species	64
	4.3	Orderfile Dependent on Species	81
	4.4	Mix of Species within a Panel	85
CHAPTER 5		DISJUNCTIVE PROGRAM	91
	5.1	Disjunctive Programming	91
	5.2	The PDM as a Disjunctive Program	93

CHAPTER 6	NUMBER OF VENEER THICKNESS	101
6.1	The Optimum Number of Veneer Thicknesses	101
6.2	Alternate Face Veneer Thickness	110
CHAPTER 7	CONCLUSIONS	113
BIBLIOGRAPHY	....	115
APPENDIX I:	Listing of FORTRAN Program	120
APPENDIX II:	Plywood Mill Data	133
APPENDIX III:	Results of PDM Model: Four Veneers	136
APPENDIX IV:	Results of PDM Model: Three Veneers	138
APPENDIX V:	Results of PDM Model: Alternate Face Veneer ...	140

LIST OF TABLES

1. Number of Veneers for Face, Core and Centre 17

2. Example of Plywood Designs with Four Veneers 19

3. Permissible Construction Alternatives for Three Ply Plywood 27

4. Permissible Construction Alternatives for Five Ply Plywood 28

5. Permissible Construction Alternatives for Seven Ply Plywood 28

6. Permissible Construction Alternatives for Nine Ply Plywood 29

7. Symmetry in Core/Centre Veneers 54

8. Bounds on x_1 and $\dot{x}_K$ 56

9. PDM Subproblem Example 76

10. Species Lay-up Alternatives 86

11. Computation of the Upperbound $\overline{Z}_0$......................... 107

12. Optimum Number of Veneer Thickness and Design Efficiency.. 108

LIST OF FIGURES

1. A Flowchart of the Algorithm to Solve the PDM Problem 59

2. The PDM Subproblem as a Network Flow Problem 74

3. Feasible Region of Plywood Design Constraint 92

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CHAPTER 1

1.1 Introduction

In the conversion of logs to veneer and plywood, wood losses occur at several stages due to factors such as the size and shape of logs and processing limitations. The yield of plywood is generally 45-55 per cent of the log input by volume, the remaining being conversion losses in the form of residuals and losses due to shrinkage and compression. Though much of the residuals is converted into byproducts such as wood chips and hog fuel and used elsewhere, their economic value is considerably reduced. In the context of dwindling forest resources, increasing manufacturing costs and fluctuating and competitive market structure, the importance of efficient utilization of timber is all the more greater now than it was ever before.

Though many of the factors of wood loss are either biological or technological, there are some areas in plywood manufacture where decision-making or policy factors contribute to the reduction in the yield of plywood. One such factor is implicit in the form of excess thickness in plywood due to the choice of veneer thicknesses and plywood designs used in assembly. While the extent of wood loss in this form may appear to be small, the savings any improvement in this area may bring about can prove to be substantial. The importance stems from the fact that processed wood with an added value from manufacture is lost indirectly. The veneer peel thickness and plywood designs currently in use in most mills have come largely from age old tradition with intuitive "improvements', if any, over time. There is no evidence in the published literature to show what constitutes the most economical peel thicknesses and plywood designs for a mill and how to find them. In recent years, technological innovations have lead to improved manufacturing processes in the plywood industry. Adoption of sophisticated quantitative techniques and computers as aids in decision-making and process control have brought considerable savings to the industry. Yet, this one aspect of the wood loss and plywood design problem has not been given the attention it deserves.

The problem of determining the optimum veneer thicknesses and associated plywood designs is very complex due to the following reasons. Many types of plywood are assembled by each plywood mill using only a few (three or four) veneer thicknesses. Generally, all the plywood so assembled should conform to the 'balanced design', a requirement which regulates thedirection of grain and the order in which veneers of different species and thicknesses can be assembled into plywood. Specifically, the assembly of veneers should be such that they are symmetrical about the central ply(ies). There should be at least one balanced design for each type of plywood which, in addition, should satisfy other structural requirements and specifications. The production of plywood using any design alternative is conditional upon that design alternative being feasible for the balanced design requirement and other specifications. When veneer thicknesses are themselves decision variables, the relation between consumption of logs and the production of veneers is non-linear. The volumetric wood loss in the form of excess thickness in plywood, or equivalently, the consumption of logs, also depends on the distribution of the orderfile requirements. The economics of the mill further depend on how efficiently a set of veneer thicknesses and plywood designs are utilised to form the best product mix subject to constraints on resource availability and product demand.

Associated with the veneer thickness and plywood design problem described above is the problem of determining the optimum number of veneer thicknesses for a mill. Generally, the higher the number of veneer thicknesses used by a mill, the lower is the wood loss and log costs. However, a higher number of veneer thicknesses is associated with higher setup costs in peeling, drying, storage, lay-up, handling and record-keeping. On the other hand, the lower the number of veneer thicknesses, the higher is the wood loss and log costs, but the setup costs are reduced considerably. The problem of determining the optimum number of veneer thicknesses which balances the trade-off between the two is therefore a direct sequel of the veneer thickness, plywood design and product mix problem.

In this dissertation, we address the above aspects of these two problems using a quantitative approach. We formulate the problem of determining the veneer thickness, associated plywood designs and product mix as a mathematical programming model in which the objective function and some of the constraints are non-linear and, in addition, some of the decision variables are restricted to 0-1 values. The model takes on different forms depending on the factors considered in the problem and mill practices. Some of the general problems are considered and, in each case, solution procedures which exploit the structure in the model are developed. A computer code is written and data obtained from a plywood mill is analysed to demonstrate the suitability of the model.

To our knowledge, there are no literature references which directly consider the evaluation of optimum veneer thicknesses, plywood designs and product mix. Whatever little work done in this field relates to the study of strength or structural properties of known thicknesses and designs (Colebeck and Northcott (1958), Norris, Werren and McKinnon (1961)). However, there is considerable work done in the area of optimum product mix for plywood using, mostly, linear programming techniques. We consider this and other Operations Research applications in the plywood industry in section 1.2 of this chapter. In addition, some work is done in the general area of wood losses in plywood production. We briefly describe this work as well, analyse the factors of wood loss and optimization techniques associated with them in section 1.3.

The organisation of the rest of this dissertation is as follows. In chapter two we present the terminology and factors associated with the plywood manufacturing process and the design problem. We define the decision variables and develop the constraints and the objective of the problem as functions of these variables. We further demonstrate how the balanced design requirement can be achieved through the development of a set of design coefficients and associated indicator variables and we formulate the Plywood Design and Manufacturing (PDM) problem as a non-linear mixed integer mathematical programming model.

We present an implicit enumeration algorithm for solving the PDM problem in chapter three. Utilising the constraint structure in the PDM problem, we demonstrate how the size of the search tree, employed by the implicit enumeration algorithm, can be substantially reduced. We further show that the subproblem to be solved at each node of this search tree is a Linear Multiple Choice Knapsack (LMCK) problem whose solution can be obtained explicitly following its coefficient structure. We describe a computer code written in FORTRAN for the implicit enumeration algorithm to solve the PDM problem. Using this code, we analyse data from a plywood mill in British Columbia and show that for the particular configuration of the problem parameters, the solution obtained from our model is far superior than those currently used.

Extension of the PDM model to mill situations involving more than one species and varying orderfile restrictions are considered in chapter four. The PDM model is reformulated in each case and it is demonstrated that essentially the same tree search procedure can be used to solve all these models. When the orderfile is independent of species, the linear subproblem to be solved at each node of the tree is a Generalized Network problem. It is shown that this Generalized Network problem can be reduced to a Generalized Transportation problem following some properties of the PDM model. When the orderfile is dependent on species, the subproblem decomposes into several linear multiple choice knapsack problems whose solution can be given explicitly. If more than one species of veneer can be mixed within a panel, the subproblem is a linear program.

In chapter five, we show how the PDM model can be naturally cast as a special case of a Disjunctive Programming problem. We analyse the characteristics of this Disjunctive Programming problem and show how the PDM problem can be viewed as a large scale linear programming problem.

In chapter six, we consider the problem of determining the optimum number of veneer thicknesses for a plywood mill. Using computer codes of the implicit enumeration algorithm and data from the plywood mill we derive the maximum revenue associated with various number of veneer thicknesses. Together with hypothetical data on setup costs we demonstrate how the optimum number of veneer thicknesses can be determined.

1.2 Operations Research In The Plywood Industry

Extensive information on Operations Research (OR) techniques in forestry and the forest products industry (plywood, lumber, pulp and paper mills and other processing industries using wood as the raw material) are available from the bibliographies of Bare (1971), Martin and Sendak (1973) and Field (1976, 1977). However, much of the work reported in these sources refers to application of OR techniques in forestry operations such as harvesting, logging, forest management and other similar areas. A general appraisal of possible OR applications in the forest products industry mentioning the areas where these can be effectively used is given by Holmes (1976). Diaz (1974) has prepared a 'bibliography' of OR in lumber production and 'other forestry industries'. However, his bibliography suffers from the drawback that it neither exclusively contains literature related to the forest products industry nor is exhaustive of all OR applications in the industry. In order to complement these sources with recent literature and unreported use of OR in the industry, with particular reference to plywood and sawmilling, a survey was undertaken in British Columbia (Raghavendra (1979)). OR work related to the plywood industry as found from published literature and industrial practices in British Columbia are briefly described in this section. The description is by the technique employed for solution such as Linear Programming, Dynamic Programming and Simulation.

1.2.1 Linear Programming

As is the case in many other industries, the most widely used OR methodology in plywood industry is Linear Programming (LP). From simple transportation problems to complex operational planning models involving log allocation, production scheduling and distribution, this technique is extensively used. Numerous introductory articles describing how LP can be effectively used in different contexts of the plywood industry can be found in the literature [refer Bare (1971), Diaz (1974), Martin and Sendak (1973) and Field (1976, 1977)].

Application of Linear Programming to plywood manufacture has been considered by Bethel and Harrel (1957), Koenigsberg (1960), Donnelly (1966), Ramsing (1965, 1968), Everett (1967) and Lee (1968). An extensive account of an actual application of LP in plywood production is described by Kotak (1976). The model, developed for the plywood and hardboard division of Canadian Forest Products, New Westminster, BC and in use since 1969, basically determines an optimum balance between the available wood mix and orderfile requirements so as to maximize the contribution margin of the division. The model serves as a basis for an annual operating plan giving details of production schedules, keeping track of raw-material inventory, orderfile requirements and targets on a biweekly basis. The strategies are further revised on a day to day basis with schedules for log peeling, veneer drying, press production and shipping based on availability of raw materials and cumulative performance up to the day. A financial variance analysis is prepared to analyze the effect of variations due to price and mix of raw-materials on the operating income. The net contribution margin of the division is reported to have increased by an average of one million dollars per year during the period 1969-1975 due to the use of this model.

Wellwood (1971) describes how the orderfile requirements of different types of plywood can be linked to press, drying and lathe schedules using LP. His article is similar to that of Kotak but gives greater detail on scheduling material flow at different processing centres and has the objective of minimizing the penalty associated with unused veneer and downgrading of veneer. Yaptenco and Wylie (1970) consider a hypothetical illustration in which a characteristic production scheduling problem of a plywood mill involving lathes, dryers, edge gluers, patchers and glue spreaders is brought down to linear programming formulation through 'algebraic and difference equations'. Dobson (1971) describes the use of LP for the allocation of logs to plywood, sawmill and open market sales.

There are several other LP models that are actually in use in mills but are not reported in the literature (Raghavendra (1979)). Many of these models are meant for intermediary stages of manufacture and their results may not prove to be optimal when the overall problem is considered. Nevertheless, these serve the purpose of analysing efficiency or productivity factors in a decentralized framework. One such LP is used to give lathe schedules for peeling different species and veneer thicknesses for each one of the lathes with the objective of minimizing peeling time. Another LP is meant for improving efficiency in drying since the drying process is a bottleneck in plywood manufacture. The drying time of veneer depends on the size and type of dryer, the number of decks, species and thickness of veneer and the objective of the LP is to minimize total drying time. A third LP model is designed to give optimum pressing schedules on a weekly basis with constraints on pressing time, glue-spreader press configurations, orderfile requirements and setup time between batches.

There are several other areas in the industry where LP models have been used or recommended for use. Sitter (1969) describes in general how LP can be effectively used in an integrated woodworking company consisting of pulpmill, sawmill, and veneer mills. Klamecki (1978) utilises LP to determine the least cost energy mix for a forest products complex considering alternative sources from mill residues, oil, natural gas and solar energy. Holecek (1975) and McKillop (1974) describe the use of LP as a systems model for an integrated forest products firm in California.

1.2.2. Simulation

Simulation studies in the plywood industry have been made in the area of veneer peeling and drying only. Resch and Scheurman (1977) simulated the softwood veneer drying operation so as to determine the optimum flow of veneer through two jet dryers. Simulated computer runs demonstrated the effect of veneer thickness, drying temperature and pre-sorting of veneer on drying time and costs. Tobin and Bethel (1969) describe an analytical procedure to evaluate the quality and quantity of veneers recoverable in the rotary cutting process using simulation.

1.2.3 Dynamic Programming

Bailey (1970, 1972, 1973) uses Dynamic Programming for log allocation decisions by analysing hypothetical problems related to log supply from several areas with non-linear costs. Pnevmaticos and Mann (1972) and Briggs (1978) demonstrate that Dynamic Programming can be used for bucking of tree-length or long logs into small logs. When the values of small logs are known, the cutting process is viewed as a sequential decision process in which the optimal values from bucking at several 'stages' of the log length are determined through Dynamic Programming recursion.

1.2.4 Non-Linear Programming

Klamecki (1978) has used Non-Linear Programming to determine the optimum lathe settings for producing the best quality of veneer. Three basic variables, namely, knife rake angle, roller bar compression and knife roller lead in the veneer peeling process are restricted by physical considerations as linear constraints. The objective function is based on the formation and severity of lathe checks in the veneer and is expressed as a non-linear function of the three decision variables. For various process parameters such as stress, shear and strength in the cutting process, the optimum lathe settings are derived using a Non-Linear Programming model.

1.2.5 Other OR Techniques

Tyre and Screpetis (1978) propose an inventory system based on weight scaling of roundwood logs and describe a method for control of veneer, sawntimber and pulpwood volumes. Traditional inventory control techniques such as EOQ are reported to be used (Raghavendra (1979)) in the area of log boom control, in-process veneer or lumber inventory and finished products inventory. Ramalingam (1976) is reported (in Briggs (1978)) to have used a Branch and Bound approach to the tree bucking problem. Successive reductions of the tree stems or long logs at the merchandiser is postulated to follow a finite-horizon discrete state Markov Process by Luken (1978) (as reported in Luken et al. (1980)). Economic impact of forest based industries is analysed using Leontief input-output models by Reimer (1969) and Raizada and Nautiyal (1974).

1.3 Wood Losses In Plywood Manufacture

The foregoing analysis of literature sources and actual practice of OR in the plywood industry indicate the wide variety of problems that can be tackled through some facet of the OR methodology. Examined closely, the success of many OR techniques lay in their ability to give efficient operational strategies which, either directly or indirectly, reduces wood loss in the conversion of trees to logs, veneer and plywood. Even when the objective of an OR model is the maximization of value, it implicitly results in efficient utilisation of the principal raw-material namely, wood. This leads us to the basic questions: why wood losses occur in the conversion of logs to veneer and plywood and how can it be prevented or reduced.

Extensive information on the areas of wood loss in plywood manufacture, factors responsible for them and the extent of these losses are available in the literature [Dobie and Hancock (1972), Woodfin (1973), Nagaraju, Raghavendra and Venkataraman (1974), Meriluoto (1965), Heiskanen (1966), Brackley (1968), Baldwin (1975) and Wood (1962)]. Many of the factors of wood loss in plywood are interactive. However, they can be broadly classified into three categories:

i) Biological Factors: Due to the inherent nature and variability of the raw material. Losses due to rounding of logs, defects in wood, shrinkage in drying and compression in pressing can be classified to fall into this category;

ii) Technological Factors: Due to processing limitations or non-availability of better technology. Core losses which occur due to the limitation of the spindle in veneer lathe, rounding losses due to centering errors, loss due to spur trim and handling losses are some of the technological factors responsible for wood losses in plywood manufacture and

iii) Decision-Making/Design Factors: Due to manufacturing designs or decision-making practices within a given technological setup. At least part of the bucking losses in the log yard, trimming losses caused by oversize allowance for veneer and loss in the form of excess thickness in plywood are some of the decision-making factors responsible for wood loss.

Investigation of the biological or technological factors of wood losses in plywood manufacture is beyond the scope of this dissertation. Among the decision-making or design factors responsible for wood losses, evaluation of some of these factors with an economic objective has received some attention in recent years. The problem of log bucking for maximising value, though not specifically in the context of plywood manufacture, has been studied by Conway (1978), Pnevamaticos and Mann (1972), Briggs (1978), Ramalingam (1976), Lefebvre (1978) and Western Forest Products Laboratory (1978). Evaluation of the trim allowance in veneer for plywood has been studied to some extent by Hawkins and Clarke (1970) and Raghavendra and Nagaraju (1975). The concept of evaluating veneer peel thicknesses and plywood designs for economic optimality is, to our knowledge, not considered in the literature. A contribution of this thesis is the development of a quantitative technique to derive the optimum veneer thicknesses, plywood designs and product mix. The characteristics of this problem are described in the next chapter.

CHAPTER 2

2.1 The Veneer Thickness Problem

Most plywood mills manufacture a variety of plywood types, each varying, apart from species and surface quality characteristics, in the number, thickness and order of veneers or plies and the total thickness of plywood. In any one mill logs are peeled to one of three, four or five basic veneer thicknesses and all the plywood types are assembled as some integral multiple combination of these veneers. The veneers as well as the plywood assembled from them should meet some specifications with regard to thickness, strength, stiffness, surface quality and other factors. Most of the plywood so produced should also conform to the "Balanced Design", a requirement which regulates the order in which veneers of different species and thicknesses can be assembled into plywood.

The use of a large number of veneer peel thicknesses tends to increase the cost of plywood because of the added costs of peeling, drying, storage, handling, lay-up and record-keeping involved (Colebeck and Northcott (1958)). Alternatively, inability to manufacture certain types within plywood thickness specification, the possibility of most plywood types ending up in higher than required thickness and requirement of specific customer orders dictate the use of two or more peel thicknesses. For example, the two plywood types 5 ply 20.5 mm having 5 veneers adding up to 20.5 mm thickness and 7 ply 20.5 mm having 7 veneers adding up to the same thickness cannot both be assembled using a single veneer thickness within a tolerance of $\pm$ 0.5 mm.

Similarly, a 7 ply 20.5 mm plywood and a 7 ply 23.5 mm plywood cannot both be assembled to within $\pm$ 0.5 mm using a single veneer thickness.

The economics of plywood manufacture depend not only on the costs of labor, raw material, equipment and services but also on the designs used in the assembly of plywood. Traditional linear programming models which have been hitherto used and are being extensively used [Dobson (1971), Kotak (1976), Lee (1968) and Ramsing (1965, 1968)] mainly centre around optimal product mix of log grades, species and veneer lay-up alternatives. However, one basic information which goes into these LP models as input is the existing set of veneer peel thicknesses, associated yield factors and design or construction alternatives. There is no evidence in published literature to claim that the veneer thicknesses and the associated designs so used are truly the best for any particular mill. Most of the plywood designs currently used appear to have been developed "partly from theoretical considerations, partly from tradition and partly from manufacturing requirements" (Colebeck and Northcott (1958)).

The foregoing analysis naturally raises the following two questions: (1) what is the optimum number of veneer thicknesses for a mill, and (2) given the number of veneer thicknesses, what should be these thicknesses and what plywood designs or construction alternatives are the best for a mill. The answer to the first question depends on how best the "best" veneer thicknesses are for each number in the second problem and how responsive the changes in costs are to the addition of each veneer thickness. Both these, in turn, are dependent on the distribution of the mill's orderfile requirements, log availability and other resource restrictions.

It is demonstrated in this chapter that the problem of finding the optimum veneer thicknesses can be formulated as an optimization model which is a mixed 0-1 non-linear programming problem. Some of the terminology related to plywood designs and the manufacturing process are described in section 2.2. The veneer thickness, plywood design and product mix problem is described in terms of this terminology in section 2.3. The model formulation with details of the decision variables, the constraints and the objective function are presented in section 2.4. In section 2.5 a discussion of the characteristics of the optimization model, its variations and extensions are presented.

2.2 Some Terminology Related to Plywood Design and Manufacture

Before presenting the mathematical formulation of the plywood design problem some of the terminology associated with the manufacture of plywood are considered. Generally, most plywood sheets consist of an odd number of layers or plies of veneer bonded together by an adhesive in such a way that the grain direction of adjacent plies is at right angles to each other. In recent years, however, plywood assembled from an even number of plies is also being made (Parasin (1976), COFI (1978)).

In a plywood sheet with odd number of plies, counting from the top or bottom veneer,

a) The first and the last veneers are called face veneers,

b) All even numbered veneers with grain direction perpendicular to that of the face are called core veneers or cross-band and

c) All other odd numbered veneers, if any, with grain direction parallel to that of the face are called <u>centre</u> veneers.

The number of veneers for face, core and centre in plywood sheets having odd number of plies and their respective positions would therefore be as listed in Table 1. In an even-ply plywood, the face, core and centre veneers are similarly defined relative to their position and alignment of grain direction.

# of plies in plywood	# of veneers for		
	Face	Core	Centre
3	2(1,3)	1(2)	-
5	2(1,5)	2(2,4)	1(3)
7	2(1,7)	3(2,4,6)	2(3,5)
9	2(1,9)	4(2,4,6,8)	3(3,5,7)

<u>Table 1</u>: Number of Veneers for Face, Core and Centre

A specification which describes the number, thickness, species of veneer and the order in which they are assembled into a plywood sheet is called the <u>design</u> or <u>construction</u> of plywood. Since the number of veneers for face, core and centre plies can be determined once the number of plies in plywood is known (Table 1), a design can be specified by describing the thickness and species for each one of face, core and centre veneers.

The <u>balanced design/construction</u> requires that within a plywood panel, the species, thickness and direction of grain of veneers should be symmetrical about the central ply(ies). This is stipulated from considerations related to the strength properties and warping of the panel. Together with general mill practices, a balanced design implies that within a plywood panel

a) all face veneers should be of the same thickness and species,

b) all core veneers should be of the same thickness and species,

c) all centre veneers should be of the same thickness and species and

d) the species and/or thickness in any one group a), b) or c) above might be the same as those of the other(s).

An unbalanced design or modified construction refers to plywood panels which vary from the requirements for a balanced design in that the grain direction, species and/or thickness of inner plies may be unbalanced about the central ply(ies).

Briefly, the process of plywood manufacture involves the conversion of logs to veneer, drying the veneer to remove excessive moisture and gluing, assembling and pressing the veneers to form a plywood panel. The thickness of veneer before the drying process is called the green thickness while that after drying is called the dry thickness. Plywood thicknesses generally refer to the thickness of the finished panel ready for market, after accounting for sanding losses, if necessary. Throughout our analysis veneer thicknesses refer to the green thickness and plywood thicknesses refer to the thickness of unsanded panels.

Finally, since most mills manufacture a variety of plywood, a plywood type is normally designated by the number of plies it contains and the thickness of plywood. Thus,a 7 ply 20.5 mm plywood implies that this plywood has seven veneers in it and that its thickness is

20.5 millimetres.

Using the notations and terminology introduced above, we can now describe the plywood design and manufacturing problem.

2.3 Description of the Plywood Design and Manufacturing Problem

Consider an example of four veneer peel thicknesses and a sample of plywood types and associated designs as presented in Table 2. These are taken from the actual practices of a plywood mill in British Columbia

Plywood Type	Plies - Thickness(mm)	Design Alternative	Veneers for Face	Core (inches)	Centre	Plywood Thickness(mm) Green	Dry	Excess(+) or Shortage(-)
1	3 ply 7.5	(i)	1/10	1/10	-	8.07	7.60	+1.33%
2	3 ply 9.5	(i)	1/10	3/16	-	10.36	9.74	+2.53%
		(ii)	1/8	1/8	-	10.06	9.46	-0.42%
3	5 ply 12.5	(i)	1/10	1/10	1/10	13.46	12.65	+1.20%
4	5 ply 15.5	(i)	1/10	1/8	3/16	17.06	16.04	+3.48%
		(ii)	1/10	1/7	1/7	17.27	16.24	+4.77%
		(iii)	1/8	1/8	1/8	16.76	15.76	+1.68%
		(iv)	1/8	1/7	1/10	17.32	16.28	+5.03%
5	7 ply 18.5	(i)	1/10	1/10	1/8	20.16	18.96	+2.49%
		(ii)	1/10	1/8	1/10	20.83	19.58	+5.84%
		(iii)	1/8	1/10	1/10	20.16	18.96	+2.49%

Table 2: Plywood Designs with Four Veneer Thicknesses; Veneer Thickness in mm (inches) are: 2.69 (1/10), 3.35 (1/8), 3.96 (1/7) and 4.98 (3/16).

whose design and manufacturing problem will be fully considered in chapter 3. The veneer thicknesses used were 2.69, 3.35, 3.96 and 4.98 mm corresponding approximately to 1/10, 1/8, 1/7 and 3/16 of an inch respectively. For some plywood types there is more than one design alternative, which is described in the form of veneers for face, core and centre plies. The 'green' thickness of plywood represents the sum total of the thickness of veneer in them while the 'dry' thickness refers to the actual final thickness of plywood after accounting for losses due to shrinkage in drying, compression in pressing and glue-line additions. The last column, excess or shortage, represents the percentage deviation from the intended thickness of the plywood type induced by the choice of design. Within permissible tolerances, shortages preceeded by a negative sign indicate savings in wood while excesses preceeded by a positive sign indicate loss of wood. The actual extent of wood loss or gain can be computed by multiplying the absolute deviations with the respective quantities of plywood produced.

We emphasise that the volume of wood loss or gain depends not only on the veneer thickness and plywood design but also on how efficiently the designs are used to meet the orderfile under constraints of log availability, machine capacities and other mill restrictions. This leads to two aspects of the problem, namely, (1) the veneer thickness and design problem and (2) the product mix and manufacturing problem.

In the veneer thickness and design problem, the list of plywood types, their specifications and the number of veneer peel thicknesses are known. The objective is:

(i) To determine the thickness and

(ii) To specify how these veneers should be assembled so as to produce all types of plywood within specification.

On the other hand, in the product mix and manufacturing problem the veneer thicknesses, the plywood designs, the availability of logs, the demand for end products and other mill restrictions are known. The objective is:

(i) To find the optimum quantity of veneers of each species and thickness to be produced and

(ii) To find the optimum quantity of plywood to be assembled under each design alternative.

The two aspects of the problem are interrelated as veneer thickness and plywood designs are input to the product mix part of the problem. The objective for both problems combined would be (i) to minimize implicit wood loss in the form of excess thickness in plywood which, as a result, would minimize total log consumption, or (ii) to maximize net revenue. When more than one species with varying log costs are used, the two objective functions need not necessarily give the same results. This can happen, for example, when an expensive species has a lower yield compared to an inexpensive species. Recognizing that the ultimate trade-off can be measured in terms of value, we use maximizing of net revenue as the objective.

The product mix part of the problem can be solved using linear programming (Kotak (1976), Dobson (1971), Lee (1968) and Ramsing (1965, 1968)) which is now an accepted mill practice. However, the veneer thickness and plywood design problem as presented above has not been considered to date. Whatever little research has been undertaken in this regard relates to strength or structural properties of known thicknesses and designs (Colebeck and Northcott (1958), Norris, Werren and McKinnon (1961) and Biblis, Hsu and Chiu (1972)).

In this thesis, we consider simultaneously both aspects of this problem, referred to as the Plywood Design and Manufacturing (PDM) problem. In the PDM problem we seek the veneer thicknesses, associated plywood designs and quantities of veneers and plywoods to be produced which will maximize the net revenue for a mill.

2.4 Formulating the Model

To simplify the presentation, we do not consider explicitly factors such as species, log grades, surface quality of veneers, plywood grades and machine capacities in the formulation of the PDM model in this chapter. These factors can be easily incorporated in the model as demonstrated by the existing plywood L.P. models (Lee (1968), Ramsing (1965, 1968), Kotak (1976)). Further, though we consider in our formulation plywood designs with an odd number of veneers having balanced designs, our model can be extended to even-ply construction and/or unbalanced designs. We discuss the implications of some of these in section 2.5. Constraint coefficients are illustrated for plywood types having up to nine plies and three veneer thicknesses but can be extended to any number of plies and any number of veneer thicknesses. Sizes of veneer and plywood sheets are expressed in equivalents of the standard size of plywood (8' x 4' or 2.44 m x 1.22 m).

Decision Variables

Let K be the number of veneer thicknesses in general and N be the number of plywood types. Define the decision variables as follows:

x_k = k^{th} veneer thickness (in mm); x_1 is conventionally treated as the thickness of the face veneers,

L_k = Quantity of logs peeled into veneer thickness x_k (in cubic metres),

V_k = Quantity of veneer sheets produced of thickness x_k (in number of sheets of size equivalent to 2.44 m x 1.22m size of plywood),

P_{ij}= Quantity of plywood of type i produced using construction alternative j (in number of sheets of standard size 2.44 m x 1.22 m or equivalent),

δ_{ij}= An indicator (0-1) variable for plywood type i made using construction alternative j

k = 1,2...K; j = 1,2...n_i and i = 1,2...N.

Generally, the number of veneer thicknesses, K, used in most mills is three or four and seldom more than four peel thicknesses are used. The number of design alternatives, n_i, depends on the number of plies in plywood.

The Constraints

In this section we will describe the various constraints arising in the PDM problem.

a) Veneer Thickness Tolerance and Constraints

Tolerances for veneer thickness are normally laid down in company standards or specifications related to the product. Council of Forest Industries of British Columbia (COFI (1978)) standards for exterior plywoods, for example, specify tolerances for face veneers separately from those for core or centre veneers. To establish tolerances for veneer thicknesses, we should first consider the capabilities of the veneer peeling lathe of a mill. Most peeling lathes can produce only a discretely finite set of veneer thicknesses. However, some peeling lathes might be capable of producing veneer thicknesses in a continuous range of values. But, veneer thicknesses beyond a certain degree of accuracy may not be possible from practical considerations. Thus, we let the veneer thickness take only discrete values, if necessary, by transforming the range of peel thicknesses into a discrete set in steps of, say, $1/10^{th}$ of a millimetre.

Let $T = \{T_1, T_2, \ldots T_M\}$ be the set containing all possible veneer thicknesses the peeling lathe(s) of a mill can produce. Of this, let T_f and T_c be the subset representing all thicknesses within tolerances specified for face and core/centre veneers respectively. Then

$$x_k \in T_k = \{T_k^1, T_k^2, \ldots\ldots T_k^{m(k)}\}, \quad k = 1,2,\ldots K \qquad \text{—— (2.1)}$$

where $m(k)$ is the number of elements in T_f if $k = 1$ and that in T_c otherwise. Since x_k can take on only one value in T_k, it can be expressed as follows:

$$x_k = \sum_{m=1}^{m(k)} \eta_{km} T_k^m \qquad \text{— (2.1A)}$$

$$\sum_{m=1}^{m(k)} \eta_{km} = 1 \qquad \text{— (2.1B)}$$

and $$\eta_{km} = \begin{cases} 1 & \text{if } x_k = T_k^m \\ 0 & \text{otherwise} \end{cases} \qquad \text{— (2.1C)}$$

Clearly, in all the constraints of the PDM model in which x_k appears it can be replaced by the right hand side of (2.1A) and in that case the other two constraints (2.1B) and (2.1C) are to be included explicitly. However, for simplicity in presentation, we retain x_k as a variable that can assume one value from the set T_k as expressed by (2.1).

b) Plywood Thickness Constraints

These are perhaps the most complicated constraints in the PDM problem due to the following reasons:

i) All the N plywood types are to be assembled using some permissible combination of one or more of the x_k's, with the number of plies adding to 3, 5, 7 or 9 (or higher, if the case dictates);

ii) There should be at least one construction alternative satisfying the balancing requirement and thickness tolerance for each plywood type;

iii) There might be more than one plywood type having the same number of veneers in it but differing in thickness, and

iv) There might be more than one plywood type having the same thickness but differing in the number of plies.

All of these problems were overcome in our formulation by a careful evaluation of the balanced construction requirement. Since all the plywood types will have face veneers, the convention that x_1 is the face veneer leads to the fact that all construction alternatives will have at least two veneers of x_1. Analysis of the balancing requirement with the number of veneers required for face, core and centre veneers indicate that there are only a few permissible combinations of veneers in which a plywood of a given number of plies can be assembled. If K=3, a 3 ply plywood can be assembled in one of 3 alternate ways and there are 9 possible ways for each one of higher ply construction having odd number of plies (5, 7, 9 or higher odd). These permissible construction alternatives specifying the veneers for face, core and centre, using some or all of the three veneer thicknesses are listed in Tables 3, 4, 5 and 6 for plywood made of 3, 5, 7 and 9 plies respectively. Similar permissible construction alternatives can be listed for any number of veneer thicknesses and/or plywood made with any number of plies. Using Tables 3-6, the balanced construction and plywood thickness tolerance can be specified by the following set of constraints:

$$b_i^L - M(1-\delta_{ij}) \leqslant \sum_{k=1}^{K} a_{ijk}x_k \leqslant b_i^U + M(1-\delta_{ij}) \qquad \text{—— (2.2)}$$

$$\sum_{j=1}^{n_i} \delta_{ij} \geqslant 1 \qquad \text{—— (2.3)}$$

$$\delta_{ij} = \begin{cases} 1 & \text{If plywood type } i \text{ is assembled using construction alternative } j \\ 0 & \text{Otherwise} \end{cases} \qquad \text{—— (2.4)}$$

for all $j = 1, 2, \ldots n_i$ and $i = 1, 2, \ldots N$, where,

a_{ijk} = Number of veneers of thickness x_k used in construction alternative j for plywood type i; The a_{ijk} are taken from Tables 3 - 6,

$b_i^L(b_i^U)$ = Lower (Upper) tolerance for thickness of plywood type i,

M = A large positive number,

n_i = Number of permissible construction alternatives for the i^{th} type of plywood.

Construction Alternative (j)	(a_{ijk}): Number of Veneers of x_1	x_2	x_3	Veneers for Face	Core	Centre
1	3	0	0	x_1	x_1	-
2	2	1	0	x_1	x_2	-
3	2	0	1	x_1	x_3	-

Table 3: Permissible Construction Alternatives for Three Ply Plywood

Construction Alternative (j)	(a_{ijk}): Number of Veneers of x_1	x_2	x_3	Veneers for Face	Core	Centre
1	5	0	0	x_1	x_1	x_1
2	4	1	0	x_1	x_1	x_2
3	4	0	1	x_1	x_1	x_3
4	3	2	0	x_1	x_2	x_1
5	3	0	2	x_1	x_3	x_1
6	2	3	0	x_1	x_2	x_2
7	2	0	3	x_1	x_3	x_3
8	2	2	1	x_1	x_2	x_3
9	2	1	2	x_1	x_3	x_2

Table 4: Permissible Construction Alternatives for Five Ply Plywood.

Construction Alternative(j)	(a_{ijk}): Number of Veneers of x_1	x_2	x_3	Veneers for Face	Core	Centre
1	7	0	0	x_1	x_1	x_1
2	5	2	0	x_1	x_1	x_2
3	5	0	2	x_1	x_1	x_3
4	4	3	0	x_1	x_2	x_1
5	4	0	3	x_1	x_3	x_1
6	2	5	0	x_1	x_2	x_2
7	2	0	5	x_1	x_3	x_3
8	2	3	2	x_1	x_2	x_3
9	2	2	3	x_1	x_3	x_2

Table 5: Permissible Construction Alternatives for Seven Ply Plywood

Construction Alternative (j)	(a_{ijk}): Number of Veneers of x_1	x_2	x_3	Veneers for Face	Core	Centre
1	9	0	0	x_1	x_1	x_1
2	6	3	0	x_1	x_1	x_2
3	6	0	3	x_1	x_1	x_3
4	5	4	0	x_1	x_2	x_1
5	5	0	4	x_1	x_3	x_1
6	2	7	0	x_1	x_2	x_2
7	2	0	7	x_1	x_3	x_3
8	2	4	3	x_1	x_2	x_3
9	2	3	4	x_1	x_3	x_2

Table 6: Permissible Construction Alternatives for Nine Ply Plywood

Observe that in constraint (2.2), $\sum_{k=1}^{K} a_{ijk}x_k$ represents the total thickness of veneer which is treated as the thickness of plywood. In actual practice, this would not be the same as shrinkage in drying, compression in pressing and spreading of glue between veneer layers affect the final thickness of plywood. Appropriate correction factors can be used in the actual application of the model.

From Tables 3 - 6 it can be observed that when there are three veneer thicknesses, $n_i = 3$ for three ply plywood and $n_i = 9$ for 5, 7 or 9 ply plywood. In general, since x_1 is treated as face veneer by convention, the number of possible combinations in which K veneer thicknesses can be used for core and centre veneers would be $K \times K = K^2$. As three ply plywood won't have centre plies in it, the corresponding number of combinations for them would only be K. Thus, in general, $n_i = K$ for 3 ply plywood and $n_i = K^2$ for any higher ply (odd) plywood. It should be noted again here that most plywood mills use 3, 4 or at most 5 veneer thicknesses and we need not therefore be concerned about large number of construction alternatives associated with higher values of K.

c) Constraint Linking Log Consumption to Veneer Production

To relate the quantity of veneer produced to the consumption of logs, the following assumption is made (we discuss the implications of this assumption in section 2.5). The volume of veneer obtainable from a log remains the same irrespective of the thickness of veneer into which it is converted. This is equivalent to saying that the number of veneer sheets of a fixed size obtainable from a log is inversely proportional to the veneer thickness into which it is peeled. With this assumption, these variables satisfy the equation $yx_kv_k = L_k$, or

$$yx_kv_k - L_k = 0 \qquad \text{for all } k \qquad \text{—— (2.5)}$$

where y is a correction factor for yield of veneer from logs and for units and dimensions of the three variables x_k, v_k and L_k. For example, if x_k is in millimetres, v_k is the number of veneers of size 2.6 m x 1.4 m, L_k is in cubic metres and the yield of veneers is 60% of log volume, then y is given by

$$y = (0.001 \times 2.6 \times 1.4)/0.6 = 0.006067$$

d) Log Availability Constraint

These are typical constraints of resource availability. The quantity of logs peeled into different veneer thicknesses should be less than or equal to the quantity of logs available. This is given by

$$\sum_{k=1}^{K} L_k \leq W \qquad \text{—— (2.6)}$$

where W is the quantity of logs available. Using (2.5) as a definition for L_k, (2.6) can be replaced by

$$y \sum_{k=1}^{K} x_k v_k \leq W \qquad \text{—— (2.7)}$$

e) Feasibility of Plywood Construction Alternative

A plywood construction alternative can be used for production of plywood only if that alternative is feasible for plywood assembly and thickness tolerance. This is equivalent to saying that if $\delta_{ij} = 0$ for any particular (i,j), then the corresponding P_{ij} must be zero. With non-negativity constraints on P_{ij} this can be expressed by $P_{ij} \leq M\delta_{ij}$, or,

$$P_{ij} - M\delta_{ij} \leq 0 \qquad \text{for all } i, j \qquad \text{—— (2.8)}$$

where M is a large positive number as in (2.2).

f) Constraints Linking Veneer Consumption to Plywood Production

The quantity of veneers of each thickness used by various construction alternatives for different types of plywood should be within the total quantity of veneers produced of that thickness. This is expressed by the constraint $\sum_i \sum_j a_{ijk} P_{ij} \leq v_k$, or

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} a_{ijk} P_{ij} - v_k \leq 0 \qquad \text{for all } k \qquad \text{—— (2.9)}$$

The slack in this constraint represents the excess quantity of veneer produced, but not used, in any of the construction alternatives.

g) Demand/Orderfile Constraints

The quantity of each type of plywood produced should meet the demand or orderfile requirements. These are specified by typical constraints of the form

$$\sum_{j=1}^{n_i} P_{ij} \geqslant d_i \qquad \text{for all } i \qquad \text{—— (2.10)}$$

where d_i is the demand for product i.

Finally, all the decision variables used in the formulation are required to be non-negative, i.e.,

$$v_k, L_k, P_{ij} \geqslant 0 \qquad \text{—— (2.11)}$$

for all k = 1,2...K; j = 1,2...n_i and i = 1,2...N.

The Objective Function

The objective is to maximize net revenue for the mill. If we assume that revenues and costs are linear functions, the objective function can be expressed as

$$\text{Max} \sum_{i=1}^{N} \sum_{j=1}^{n_i} r_i P_{ij} - C \sum_{k=1}^{K} L_k$$

where

r_i = Revenue per plywood sheet of type i

C = Cost of log per unit

Again, utilising (2.5) as a definition for L_k, this can be written as

$$\text{Max} \quad \sum_{i=1}^{N} \sum_{j=1}^{n_i} r_i P_{ij} - Cy \sum_{k=1}^{K} x_k v_k \qquad \text{—— (2.12)}$$

The Overall Formulation of the PDM Problem

Using the notation, decision variables and constraints introduced above, the PDM problem can be written as:

$$\text{Max} \quad \sum_{i=1}^{N} \sum_{j=1}^{n_i} r_i P_{ij} - Cy \sum_{k=1}^{K} x_k v_k$$

Subject to

$$x_k \in T_k = \{T_k^{1}, T_k^{2}, \ldots T_k^{m(k)}\}$$

$$b_i^{L} - M(1-\delta_{ij}) \leq \sum_{k=1}^{K} a_{ijk} x_k \leq b_i^{U} + M(1-\delta_{ij})$$

$$\delta_{ij} = \{0, 1\}$$

$$\sum_{j=1}^{n_i} \delta_{ij} \geq 1$$

$$y \sum_{k=1}^{K} x_k v_k \leq W$$

$$P_{ij} - M\delta_{ij} \leq 0$$

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} a_{ijk} P_{ij} - v_k \leqslant 0$$

$$\sum_{j=1}^{n_i} P_{ij} \geqslant d_i$$

$$P_{ij}, v_k \geqslant 0$$

for all $i = 1,2...N$; $j = 1,2,...n_i$ and $k = 1,2...K$.

The above is a formulation of the PDM problem as a mathematical programming problem in which the objective function and some of the constraints are non-linear and, in addition, some of the decision variables are restricted to 0-1 values. The size of the problem depends on factors such as the number of veneer thicknesses, the number of plywood types and the number of plies in each of them.

2.5 Discussion and Extensions

In the formulation of the PDM model in areas related to the product mix part, we have deviated from the traditional L.P. models (Kotak (1976), Ramsing (1965), Lee (1968)). In particular, the quantity of veneer and plywood are defined in terms of the number of sheets rather than volume, and yield factors are used as direct percentages rather than inverse multipliers or recovery ratios. This offers several advantages in the design problem since the construction alternatives can be specified in terms of number of veneers, the feasible designs can be identified through δ_{ij} and veneer consumption can be directly linked to production of plywood through δ_{ij} and a_{ijk}.

The model assumes that the volume of veneer from a log remains the same irrespective of the thickness of veneer into which it is peeled. This assumption is made in the absence of relevant information and is valid theoretically, since, the volume of wood peeled from a log remains the same. However, in practice, thicker veneers may result in lower volumetric yield due to factors such as splits or lathe checks in veneer. In such cases, if information is available on the relation between veneer thickness and yield, it can be included in the model in the form of y_k, a factor dependent on k, replacing y in constraint (2.5). The rest of the model formulation or solution procedure will not be affected by this change. The plywood thickness tolerance b_i^L and b_i^U in constraint (2.2) refer to design tolerances and not the thickness tolerance of an individual plywood sheet at a random point. If only individual panel tolerances are available they can be adjusted in design for chance variations through statistical concepts such as 3 - sigma limits. The orderfile requirements specified in constraint (2.10) can be of the 'less than or equal to' form or, may combine both type of inequalities.

In the formulation of the PDM model we considered only those plywood types with odd number of plies having balanced designs as they constitute the majority of the panels currently manufactured. The model can be easily extended to even ply and/or unbalanced designs of plywood. For even-ply plywood with balanced designs, design coefficients similar to the a_{ijk} of tables 3 - 6 can be constructed for any number of plies. For example, with K = 3, for a four-ply plywood, there would be three design alternatives represented by the vectors (4, 0, 0), (2, 2, 0) and (2, 0, 2) for (a_{ij1}, a_{ij2}, a_{ij3}). If plywood can be made with unbalanced designs, the number of design alternatives n_i will increase considerably, especially for those plywood types having large number of plies. However, if there are restrictions on the face veneers to be of the same thickness, the increase in the number of design alternatives will not be substantial and they can be handled within the framework of our PDM model.

We have treated x_1 as the face veneer by convention since there are more restrictions on the thickness and surface quality characteristics of face veneer than any other veneer in a plywood sheet. Additionally, in the manufacturing process, face veneers must be peeled to the full length of plywood sheets while core veneers which go across the face can be peeled in lengths relative to the width of plywood. Having more than one face veneer thickness would therefore result in more scheduling, handling, sorting and surface preparation costs. However, having a second face veneer thickness might result in better designs for some plywood types. Such a situation, i.e. having more than one face veneer thickness, can also be included within the framework of our PDM model. We illustrate this briefly through our plywood mill example in chapter six.

Existing algorithms for large scale problems can solve either integer linear programs or non-linear programs in continuous variables. The unique features of the PDM model is that it contains both discrete and continuous variables as well as non-linearities in the constraints and objective function. Relaxing the integer variables of the PDM problem to continuous values will produce a non-linear non-convex optimization problem in which a local optimum is not necessarily a global one. Thus, to our knowledge, no algorithm or solution procedure that can produce a global solution to the PDM model is available.

One of the important contributions of this thesis is the development of an efficient algorithm for solving the PDM problem. By utilising the constraint structure in the model and by selecting appropriate variables to branch on, we develop an efficient implicit enumeration algorithm to derive a global solution to the PDM problem. This algorithm is described in the next chapter.

When factors such as species, log grades, veneer types and plywood grades are included, the size of the problem increases manyfold. In each situation, however, the essence of the problem formulation remains the same since the design aspect of the problem (constraints (2.1) through (2.4)) is unaffected. Only the product mix part of the problem changes. A set of optimal veneer thicknesses and optimal plywood designs for a PDM problem with one species need not remain the optimal solution when more than one species are included in the PDM model. We consider some of these extensions to the PDM problem, analyse the corresponding structures and solution procedures in chapter four.

CHAPTER 3

3.1 Solution to the PDM Model

From chapter 2 we recall that the plywood design and manufacturing (PDM) problem can be formulated as the following non-linear mixed 0-1 programming problem.

$$Z_0 = \text{Max} \sum_{i=1}^{N} \sum_{j=1}^{n_i} r_i P_{ij} - C\,y \sum_{k=1}^{K} x_k v_k$$

s.t.

$$b_i^L - M(1-\delta_{ij}) \leq \sum_{k=1}^{K} a_{ijk} x_k \leq b_i^U + M(1-\delta_{ij})$$

$$\sum_{j=1}^{n_i} \delta_{ij} \geq 1$$

$$y \sum_{k=1}^{K} x_k v_k \leq W$$

$$P_{ij} - M\delta_{ij} \leq 0$$

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} a_{ijk} P_{ij} - v_k \leq 0$$

$$\sum_{j=1}^{n_i} P_{ij} \geq d_i$$

$$\delta_{ij} \in \{0,1\}$$

$$x_k \in T_k = \{T_k^1,\ T_k^2,\ \ldots\ T_k^{m(k)}\}$$

$$p_{ij} , v_k \geqslant 0$$

for all $k = 1,2,...K$; $j = 1,2,...n_i$ and $i = 1,2,...N$.

T_k is the set of veneer thicknesses for x_k, $k = 1,2,...K$. Further, recall that by convention x_1 is the face veneer thickness and in most mills K, the number of veneer thicknesses is generally three or four.

We develop an implicit enumeration algorithm for solving the PDM problem. The efficiency of our implicit enumeration algorithm stems from the fact that for a given set of veneer thicknesses the PDM problem reduces to a special structure Linear Multiple Choice Knapsack (LMCK) problem [Zemel (1980), Glover and Klingman (1979)]. Moreover, each one of these LMCK problems can be solved explicitly. Further, we show that due to the special structure of the PDM problem, we can reduce substantially, from the outset, the number of veneer thicknesses that need to be considered in the implicit enumeration algorithm.

Our implicit enumeration algorithm was coded in FORTRAN and was used to solve some real world problems. Data obtained from a plywood mill in British Columbia was used to evaluate the PDM model and the algorithm. For the particular configuration of the problem parameters used in the study our model gave a set of veneer thicknesses and plywood designs which can increase the annual net revenue by more than 6.8 per cent.

The plan of this chapter is as follows. In section 3.2 we describe our implicit enumeration algorithm and analyze the LMCK problems. In section 3.3 we show how we can use the structure of the PDM problem to accelerate the performance of the implicit enumeration algorithm, while in section 3.4 we present our computational results.

Though much of the contents of this chapter is mathematical, the implications of the end result are straightforward. What is shown here is that a seemingly complicated non-linear mixed 0-1 mathematical programming problem can be solved efficiently for a global solution by exploiting the structures in the model. In fact, the main job of the computer code written for our algorithm to solve the PDM problem is more of a house-keeping nature than one in which complicated optimization routines are involved.

3.2 An Implicit Enumeration Algorithm to Solve the PDM Problem

Our implicit enumeration algorithm can be, in a very rudimentary manner, described as follows:

Algorithm A: (Rudimentary algorithm for solving the PDM problem)

Step 1: Let $x_t = x_t^*$, $x_t^* \in T_t$, $t = 1, 2, \ldots, K-1$ denote a previously unselected set of values for the thicknesses $x_1, x_2, \ldots, x_{K-1}$ in the PDM problem. If none exists, terminate. Otherwise, denote by PDM $(x_1^*, \ldots, x_{K-1}^*)$ the PDM problem in which $x_t = x_t^*$, $t = 1, \ldots, K-1$, and go to step 2.

Step 2: Attempt to fathom PDM $(x_1^*, \ldots, x_{K-1}^*)$. If successful, go to step 1. Otherwise,

Step 3: Solve PDM $(x_1^*, \ldots, x_{K-1}^*)$. Store the optimal solution if better than the incumbent, and go to step 1.

Clearly, algorithm A will terminate after a finite number of iterations with an optimal solution to the PDM problem. In the discussion which follows we show how to execute efficiently step 3 of algorithm A. In section 3.3.we develop tests which assist in fathoming the PDM $(x_1^*, \ldots, x_{K-1}^*)$ problem in step 2 of algorithm A. Further, we show in that section how we can eliminate, from the outset, values of $x_t = x_t^*$, $t = 1, 2, \ldots, K-1$ which are not consistent with an optimal solution of our PDM problem.

3.2.1 The Linear Multiple Choice Knapsack Problem

We will consider now the PDM problem in which all thicknesses have been determined, i.e. $x_k = x_k^*$, $x_k^* \in T_k$ for $k = 1, 2, \ldots K$. For all $j = 1, 2, \ldots, n_i$ and $i = 1, 2, \ldots, N$, let

$$\delta_{ij}^* = \begin{cases} 1 & \text{if } b_i^L \leqslant \sum_{k=1}^{K} a_{ijk} x_k^* \leqslant b_i^U \\ 0 & \text{Otherwise} \end{cases} \qquad \text{— (3.1)}$$

Note that if $\sum_{j=1}^{n_i} \delta_{ij}^* = 0$ for any i, then plywood type i cannot be assembled with the set of thicknesses x_k^*, $k = 1, 2, \ldots, K$, and therefore the remaining PDM problem is infeasible. If, on the other hand, $\sum_{j=1}^{n_i} \delta_{ij}^* \geqslant 1$ for all $i = 1, 2, \ldots N$, let

$$I_i = \{j \mid \delta_{ij}^* = 1\} \qquad \text{— (3.2)}$$

I_i represents the index set of feasible design alternatives for plywood type i with $x_k = x_k^*$, $k = 1, 2, \ldots, K$. Now, the constraints $P_{ij} - M\delta_{ij} \leqslant 0$ and $P_{ij} \geqslant 0 \; \forall \, i,j$ in the PDM problem can equivalently be replaced by

$$P_{ij} \geqslant 0 \quad \forall \; j \in I_i; \; i = 1, 2, \ldots, N \qquad \text{— (3.3)}$$

Thus, when the thicknesses x_k are assigned the values x_k^*, $k = 1, 2, \ldots, K$, the PDM problem reduces to

$$\text{Max} \sum_{i=1}^{N} \sum_{j \in I_i} r_i P_{ij} - Cy \sum_{k=1}^{K} x_k^* v_k \qquad (3.4)$$

s.t.

$$y \sum_{k=1}^{K} x_k^* v_k \leq W \qquad (3.5)$$

$$\sum_{i=1}^{N} \sum_{j \in I_i} a_{ijk} P_{ij} - v_k \leq 0 \qquad (3.6)$$

$$\sum_{j \in I_i} P_{ij} \geq d_i \qquad (3.7)$$

$$P_{ij}, v_k \geq 0 \qquad (3.8)$$

for all $k = 1, 2, \ldots K$; $j \in I_i$ and $i = 1, 2, \ldots N$. Let us denote by PDM $(x_1^*, x_2^*, \ldots x_K^*)$, problem (3.4) - (3.8). Clearly, PDM $(x_1^*, x_2^*, \ldots x_K^*)$ is a linear program. Notice further that at an optimal solution to this problem,(3.6) would be satisfied as an equality. Substituting v_k, given by

$$v_k = \sum_{i=1}^{N} \sum_{j \in I_i} a_{ijk} P_{ij} \quad \forall \; k \qquad (3.9)$$

in (3.5) produces

$$\sum_{i=1}^{N} \sum_{j \in I_i} b_{ij} P_{ij} \leq W \qquad (3.10)$$

where

$$b_{ij} = \sum_{k=1}^{K} y a_{ijk} x_k^* \qquad \text{— (3.11)}$$

Note that $b_{ij} > 0 \; \forall \; i, \; j \in I_i$ since $y > 0$, $x_k^* > 0$, $a_{ijk} \geq 0$, and there exists at least one k for each j such that $a_{ijk} > 0$. Substituting (3.9) in the objective function (3.4) we get that

$$\sum_{i=1}^{N} \sum_{j \in I_i} r_i P_{ij} - Cy \sum_{k=1}^{K} x_k^* v_k = \sum_{i=1}^{N} \sum_{j \in I_i} \hat{r}_{ij} P_{ij} \qquad \text{— (3.12)}$$

where

$$r_{ij} = r_i - Cy \sum_{k=1}^{K} a_{ijk} x_k^*$$

$$= r_i - Cb_{ij} \qquad \text{— (3.13)}$$

Thus, the PDM $(x_1^*, x_2^*, \ldots, x_K^*)$ has been reduced to an optimization problem of the form:

$$Z = \text{Max} \sum_{i=1}^{N} \sum_{j \in I_i} r_{ij} P_{ij}$$

s.t.

$$\sum_{i=1}^{N} \sum_{j \in I_i} b_{ij} P_{ij} \leq W$$

$$\sum_{j \in I_i} P_{ij} \geq d_i \qquad \text{— (3.14)}$$

$$P_{ij} \geq 0 \quad \forall \quad i = 1, 2, \ldots N \text{ and } j \in I_i$$

The PDM (x_1^*, x_2^*, ... x_K^*) problem given by (3.14) is easily recognized as a linear multiple choice knapsack (LMCK) problem. (It differs slightly from the traditional LMCK [Zemel (1980), Glover and Klingman (1979)], in that, the latter problem has (i) equality sign in the multiple choice constraints, (ii) $d_i = 1$ for all i, and (iii) minimization as the objective). Using proposition 3.1 which follows, we show that due to the special structure of the coefficients in the PDM (x_1^*, x_2^*, ..., x_K^*) problem, (3.14) can be solved explicitly, without having to go through any of the LMCK algorithms [such as Zemel (1980)].

3.2.2 An Explicit Solution for the PDM (x_1^*, ..., x_K^*) Problem

Let

$$b_{iJ_i} = \underset{j \in I_i}{\text{Min}} \{b_{ij}\} \quad \forall\, i \qquad \text{—— (3.15)}$$

and observe from (3.13) it follows that

$$r_{iJ_i} = r_i - Cb_{iJ_i}$$

$$= \underset{j \in I_i}{\text{Max}} \{r_{ij}\} \quad \forall\, i \qquad \text{—— (3.16)}$$

Proposition 3.1: There exists an optimum solution to (3.14) such that

$$P_{ij} = 0 \quad \forall\, j \neq J_i \text{ and } \forall\, i = 1,2,...N \qquad \text{—— (3.17)}$$

Proof: Suppose (3.17) does not hold. That is, there exists a solution (P'_{ij}) which is optimum for (3.14) with Z = Z' but $P'_{pt} > 0$ for some $i = p$, $t \in I_p \setminus \{J_p\}$. Consider the new solution given by

$$P^*_{ij} = \begin{cases} P'_{pt} + P'_{pJ_p} & i = p,\ j=J_p \\ 0 & i = p,\ j=t \\ P'_{ij} & \text{Otherwise} \end{cases} \qquad \text{—— (3.18)}$$

Observe first that (P^*_{ij}) is feasible for (3.14). Indeed, (P'_{ij}) feasible for (3.14) implies:

(i) $P^*_{ij} \geqslant 0 \qquad \forall\ i, j \in I_i$

(ii) $\sum_{j \in I_i} P^*_{ij} = \sum_{j \in I_i} P'_{ij} \geqslant d_i \quad \forall\ i$

and (iii) $\sum_{i=1}^{N} \sum_{j \in I_i} b_{ij} P^*_{ij} = \sum_{i=1}^{N} \sum_{j \in I_i} b_{ij} P'_{ij} + b_{pJ_p} P'_{pt} - b_{pt} P'_{pt}$

$$\leqslant \sum_{i=1}^{N} \sum_{j \in I_i} b_{ij} P'_{ij} \text{ since, } b_{pJ_p} \leqslant b_{pj}\ \forall\ j \in I_p,$$

from (3.15)

$$\leqslant W$$

Next, suppose b_{pt} is such that $b_{pt} = b_{pJ_p}$; i.e. there exists more than one $j \in I_p$ minimizing b_{ij} for $i = p$ in (3.15). Then it follows from (3.16) that $r_{pJ_p} = r_{pt}$ and therefore $\sum_i \sum_{j \in I_i} r_{ij} P^*_{ij} = \sum_i \sum_{j \in I_i} r_{ij} P'_{ij} = Z'$. On the other hand, if J_p is unique for $i = p$; i.e. $b_{pJ_p} < b_{pj}$, $j \in I_p$, then it follows from (3.16) that $r_{pJ_p} > r_{pt}$ and observe that

$$r_{pt} P^*_{pt} + r_{pJ_p} P^*_{pJ_p} = r_{pJ_p} [P'_{pJ_p} + P'_{pt}]$$

$$> r_{pJ_p} P'_{pJ_p} + r_{pt} P'_{pt}$$

Therefore, $\sum_i \sum_{j \in I_i} r_{ij} P^*_{ij} \geqslant \sum_i \sum_{j \in I_i} r_{ij} P'_{ij}$, contradicting the optimality of (P'_{ij}) and the proof of proposition 3.1 follows.

Following proposition 3.1, the LMCK reduces to solving

$$\text{Max} \quad \sum_{i=1}^{N} r_{i.} P_{i.}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} b_{i.} P_{i.} \leqslant W \qquad (3.19)$$

$$P_{i.} \geqslant d_i \quad \forall\ i = 1, 2, \ldots N.$$

Where $b_{i.} \equiv b_{iJ_i}$ and $r_{i.} \equiv r_{iJ_i} = r_i - Cb_{i.}$

This is a Linear Knapsack problem in bounded variables. Observe that (3.19) is feasible if and only if $\sum_{i=1}^{N} b_{i.} d_i \leqslant W$. Let

$$\frac{r_{p.}}{b_{p.}} = \underset{i}{\text{Max}} \left\{ \frac{r_{i.}}{b_{i.}} \right\} \qquad (3.20)$$

If $r_{p.} > 0$, that is, if there exists at least one plywood type with positive net profit, the solution of (3.19) is given by

$$P^*_{i.} = \begin{cases} d_i & i \neq p \\ \dfrac{W - \sum_{i \neq p} b_{i.} d_i}{b_{p.}} & i = p \end{cases} \qquad (3.21)$$

If $r_{p.} \leq 0$, that is, if the net profit of all the plywood types is non-positive, the solution of (3.19) is given by

$$P^*_{i.} = d_i \qquad \forall\ i = 1, 2, \ldots N \tag{3.22}$$

Transforming these results to our LMCK problem, we have that the solution to the PDM $(x^*_1, \ldots, x^*_k)$ is given by

$$P^*_{ij} = \begin{cases} d_i & i \neq p,\ j = J_i \\ \begin{cases} d_p & i = p,\ j = J_p \text{ if } r_{pJ_p} \leq 0 \\ \dfrac{W - \sum_{i \neq p} b_{iJ_i} d_i}{b_{pJ_p}} & i = p,\ j = J_p \text{ if } r_{pJ_p} > 0 \end{cases} \\ 0 & \text{Otherwise} \end{cases} \tag{3.23}$$

From (2.9) it follows that the quantity of veneers to be produced is given by

$$v^*_k = \sum_{i=1}^{N} a_{iJ_ik} P^*_{iJ_i} \qquad \forall\ k \tag{3.24}$$

Thus, when the x_k's are assigned values x^*_k, $k = 1, 2, \ldots K$, we have shown that the PDM problem can be explicitly solved. The solution to δ_{ij}, P_{ij} and v_k would be as given by (3.1), (3.23) and (3.24) respectively. In order to produce (P^*_{ij}) as given by (3.23), we need to find the indicies J_i, $i = 1, 2, \ldots N$ and the index p. The computational difficulty in generating (P^*_{ij}) is therefore $O(|I_1| + |I_2| + \ldots + |I_N| + N)$.

3.3 Branching Tests and Bounds

In the first part of this section we develop bounds on the value of x_k when values of x_t are fixed for $t = 1, 2, \ldots K-1$. Using these bounds, and making use of the results from the previous section, we demonstrate how an upper bound on the objective function value of the linear subproblem can be established for all branches emanating from the node associated with the PDM $(x_1^*, \ldots x_{K-1}^*)$ problem. In the second part, we use the structure of the PDM problem to develop branching tests which, from the outset, can substantially reduce the number of sets of thicknesses $x_1^*, \ldots, x_{K-1}^*$ that need to be considered in step 1 of our algorithm A.

3.3.1 Bounds on x_K and an Upperbound on the Objective Function Value of the PDM $(x_1^*, \ldots x_{K-1}^*)$ Problem.

Assume that $x_t = x_t^*$, $x_t^* \in T_t$, $t = 1, 2, \ldots K-1$ and let

$$T_{ij} = \sum_{t=1}^{K-1} a_{ijt} x_t^* \qquad \forall\, i, j \qquad \text{— (3.25)}$$

Further, let T_K^{min} and T_K^{max} denote the minimum and maximum possible thickness for x_K, i.e., $T_K^{min} = \underset{m}{\text{Min}} \{T_K^m\}$ and $T_K^{max} = \underset{m}{\text{Max}} \{T_K^m\}$. For all i, j for which $a_{ijK} > 0$, let

$$D_i^+ = \left\{ j \,\middle|\, \frac{b_i^L - T_{ij}}{a_{ijK}} \leq T_K^{max},\ \frac{b_i^U - T_{ij}}{a_{ijK}} \geq T_K^{min} \text{ and } a_{ijK} > 0 \right\} \qquad \text{— (3.26)}$$

D_i^+ represents the set of all design alternatives with $a_{ijK} > 0$ which might be feasible for plywood type i with $x_t = x_t^*$, t = 1, 2, ... K-1 and $x_K \in T_K$. Define

$$X_{iK}^{max} = \text{Min} \left[\underset{j \in D_i^+}{\text{Max}} \left\{ \frac{b_i^U - T_{ij}}{a_{ijK}} \right\}, \quad T_K^{max} \right] \quad \text{— (3.27)}$$

and

$$X_{iK}^{min} = \text{Max} \left[\underset{j \in D_i^+}{\text{Min}} \left\{ \frac{b_i^L - T_{ij}}{a_{ijK}} \right\}, \quad T_K^{min} \right] \quad \text{— (3.28)}$$

If for some i and j, $a_{ijK} = 0$ but $b_i^L \leqslant T_{ij} \leqslant b_i^U$, then plywood type i can be assembled without x_K in which case set $X_{iK}^{min} = T_K^{min}$ and $X_{iK}^{max} = T_K^{max}$. Let

$$\overset{\circ}{D}_i = \{j \mid b_i^L \leqslant T_{ij} \leqslant b_i^U, \; a_{ijK} = 0\} \quad \text{— (3.29)}$$

and

$$D_i = D_i^+ \cup \overset{\circ}{D}_i \quad \text{— (3.30)}$$

Now, define

$$X_K^{min} = \underset{i}{\text{Max}} \{X_{iK}^{min}\} \quad \text{— (3.31)}$$

and

$$X_K^{max} = \underset{i}{\text{Min}} \{X_{iK}^{max}\} \quad \text{— (3.32)}$$

Observe that if $X_K^{min} > X_K^{max}$, the PDM problem is infeasible for this set of x_t^* , t = 1, 2, ... K-1, and the node associated with the PDM $(x_1^* , \ldots x_{K-1}^*)$ is fathomed. Further, for values of x_K such that $x_K > X_K^{max}$

or $x_K < X_K^{min}$, PDM $(x_1^*, \ldots x_{K-1}^*)$ is infeasible.

An upperbound on the value of the objective function of the PDM problem when $x_t = x_t^*$, $t = 1, 2, \ldots K-1$ can now be computed by a relaxation of the PDM $(x_1^*, \ldots x_{K-1}^*)$ problem. Recall from proposition 3.1 that if the LMCK is feasible, then in an optimal solution to (3.14), for each i, $P_{ij} > 0$ only for that index j for which $b_{ij} = \sum_{k=1}^{K} y \cdot a_{ijk} x_k$ is minimum. Thus, since y is a constant and when x_K is known, P_{ij} would be positive only for that j for which $(T_{ij} + a_{ijK} x_K)$ is a minimum, $j \in D_i$. Using these results, a relaxation of the constraints related to plywood thickness can be specified as follows.

Determine, for each i, if there exists x_K^i such that

$$x_K^i \in (X_K^{min}, X_K^{max}) \cap T_K \qquad \text{— (3.33)}$$

and

$$b_i^L \leq T_{ij} + a_{ijK} x_K^i \leq b_i^U \text{ for at least one } j \in D_i \qquad \text{— (3.34)}$$

The relaxation is that the value(s) of x_K satisfying (3.33) and (3.34) may differ for each i. If such an x_K^i does not exist for some i, the PDM $(x_1^*, \ldots x_{K-1}^*)$ problem is infeasible and the node associated with $x_t = x_t^*$, $t = 1, 2, \ldots K-1$ is fathomed. Assuming that an x_K^i satisfying (3.33) and (3.34) exists for each i define

$$b_i' = \underset{j \in D_i}{\text{Min}} \{T_{ij} + a_{ijK} x_K^i \mid (3.33) \text{ and } (3.34) \text{ are feasible}\} \qquad \text{— (3.35)}$$

b_i' represents the minimum possible thickness for plywood type i, given that $x_t = x_t^*$, $t = 1, 2, \ldots K-1$ and x_K is an element of $(X_K^{min}, X_K^{max}) \cap T_K$. An upperbound on the objective function value of the PDM $(x_1^*, \ldots, x_{K-1}^*)$ problem is given then by the optimal value, $\bar{Z}$, of the optimization problem given below

$$\bar{Z} = \text{Max} \sum_{i=1}^{N} r_i' p_i$$
$$\text{s.t.} \quad \sum_{i=1}^{N} b_i' P_i \leqslant W/y \qquad (3.36)$$
$$P_i \geqslant d_i$$

where $r_i' = r_i - Cyb_i'$

(3.36) is a Linear Knapsack Problem (LKP) in bounded variables. If $\sum_{i=1}^{N} b_i' d_i > W/y$, (3.36) is infeasible. Otherwise, if $(r_p'/b_p') = \underset{i}{\text{Max}}\{r_i'/b_i'\}$, the solution of (3.36) is given by

$$P_i^* = \begin{cases} d_i & i \neq p \\ \begin{cases} d_p & i = p \text{ if } r_p' < 0 \\ \dfrac{W/y - \sum_{i \neq p} b_i' d_i}{b_p'} & i = p \text{ if } r_p' \geqslant 0 \end{cases} \end{cases} \qquad (3.37)$$

and

$$\bar{Z} = \sum_{i=1}^{N} r_i' P_i^* \qquad (3.38)$$

If $\bar{Z}$, as given by (3.38), is less than the corresponding value of the incumbent solution in step 3 of algorithm A, the node associated with

$x_t = x_t^*$, $t = 1, 2, \ldots K-1$ can be fathomed. If $\bar{Z}$ is higher than the incumbent solution then branching on the value of $x_K \in (X_K^{min}, X_K^{max}) \cap T_K$ should be made. If the number of elements in $T_K \cap (X_K^{min}, X_K^{max})$ is large, this interval can further be subdivided and bounds on Z can be computed in the same way as above for each such subdivision. However, since the number of elements in T_K is generally small and since other branching tests that follow eliminate a substantial number of these elements, further divisions of the interval $T_K \cap (X_K^{min}, X_K^{max})$ may not be necessary.

3.3.2 Branching Tests from the Structure of the PDM Model

We use the special structure of the PDM problem to substantially reduce, at the outset, the size of the tree that has to be searched by algorithm A.

Reduction 1: (Distinct Veneer Thicknesses)

Since the a_{ijk}'s are constructed in such a way that all feasible values of $\sum_{k=1}^{K} a_{ijk}x_k$ with $x_s = x_t$, for some $s \neq t$, can also be obtained with $x_s \neq x_t$, we can assume that in the PDM problem

$$x_s \neq x_t \qquad 1 \leqslant s, t \leqslant K \qquad \text{—— (3.39)}$$

For example, consider $x_1 = 2.5$, $x_2 = 2.5$, $x_3 = 4.0$ and $K = 3$. Suppose the design alternative nine, $(a_{i91}, a_{i92}, a_{i93}) = (2, 1, 2)$, is feasible for a 5-ply plywood. That is, $\sum_{k=1}^{3} a_{i9k}x_k = 2x_1 + x_2 + 2x_3 = 15.5$ mm is within the permissible lower and upper tolerances. Then, precisely the same ordering of the plies for face, core and centre veneers can be produced by using the design alternative five in which $(a_{i51}, a_{i52}, a_{i53}) = (3, 0, 2)$. Indeed, $\sum_{k=1}^{3} a_{i5k}x_k = 3x_1 + 2x_3 = 15.5$ mm.

<u>Reduction 2</u>: <u>(Symmetry in Core/Centre Veneers)</u>

Due to the symmetry in the construction of the design coefficients a_{ijk} for the balancing requirement, identical designs can be produced using different but symmetric set of thicknesses. Explicitly, suppose $x_{s_1} = T_{s_1}$, $x_{s_2} = T_{s_2}$, $2 \leqslant s_1, s_2 \leqslant K$. Then the plywood produced by the set of thicknesses $x_1, \ldots x_{s_1}, \ldots x_{s_2}, \ldots x_K$ can be produced as well by using the set of thicknesses $x_1, \ldots x_{s_2}, \ldots x_{s_1}, \ldots x_K$.

For example, let $x_1 = 2.4$, $x_2 = 2.7$ and $x_3 = 3.1$ be one set of veneer thicknesses designated as set I in Table 7. Let the second set of thicknesses be given by $x_1 = 2.4$, $x_2 = 3.1$, $x_3 = 2.7$, designated set II. Table 7 shows that for all possible values of alternative designs j having three or five plies these two sets produce plywoods of the same thicknesses.

Number of Plies	Design Alternative, j	Plywood Thickness Set I	Plywood Thickness Set II
3	1	7.2	7.2
	2	7.5	7.9
	3	7.9	7.5
5	1	12.0	12.0
	2	12.3	12.7
	3	12.7	12.3
	4	12.6	13.4
	5	13.4	12.6
	6	12.9	14.1
	7	14.1	12.9
	8	13.3	13.7
	9	13.7	13.3

Table 7: Symmetry in Core/Centre Veneers.

The above observation is valid for any number of plies, for any $K \geqslant 3$ and for all x_{s_1}, x_{s_2} such that $2 \leqslant s_1, s_2 \leqslant K$. Therefore, we can assume that our set of thicknesses $x_1, x_2, \ldots x_K$ in the PDM problem is such that

$$x_{s_2} > x_{s_1} \qquad \text{for all } 2 \leqslant s_1 < s_2 \leqslant K \qquad \text{— (3.40)}$$

Reduction 3: (Upper Bound on Face Veneer Thickness)

Let T_1 be the minimum possible veneer thickness, i.e. $T_1 \leqslant T_k^m \;\forall\, m, k$.

In any plywood type, at least two veneer's thickness should be x_1, the face veneer thickness. If L_i is the number of plies in plywood type i, the minimum total thickness the remaining $(L_i - 2)$ veneers can have is $(L_i - 2)T_1$. If b_i^U is the upperbound on plywood thickness of type i, the maximum thickness the two face veneers can have is $[b_i^U - (L_i-2)T_1]$. Thus, an upper bound $\hat{X}_1$ on the face veneer thickness X_1 for that plywood type is $[b_i^U - (L_i-2)T_1]/2$. When all plywood types are considered this upper bound is given by $\underset{i}{\text{Min}}[(b_i^U - (L_i-2)T_1)/2]$. Suppose $T_1^{max} = \underset{m}{\text{Max}}\{T_1^m\}$ is the maximum possible thickness for face veneer. Then in our PDM problem we must have that

$$x_1 \leqslant \hat{X}_1 = \text{Min}\left[\min_i \left\{\frac{b_i^U - (L_i-2)T_1}{2}\right\},\ T_1^{max}\right] \qquad \text{— (3.41)}$$

Consider, for example, the 12 plywood types listed in Table 8 below. These are taken from the actual list of plywood types produced by a mill with $T_1 = 2.4$ mm, $T_1^{max} = 3.2$ mm and b_i^U is + 0.5 mm of the specified plywood thickness for all i. From Column 3 of the table it follows that

$$x_1 \leqslant \hat{X}_1 = \text{Min}\left[\min_i \left\{\frac{b_i^U - (L_i-2)T_1}{2}\right\},\ 3.2\right] = 2.80$$

Notice that a 3 ply 7.5 mm plywood cannot be produced within specification if X_1 exceeds 2.8 mm. Values of x_1 higher than $\hat{X}_1$ would only make the problem infeasible.

Plywood Type i	Ply - Thickness L_i mm	$\frac{b_i^U - (L_i-2)T_1}{2}$	$\frac{b_i^L - 2\hat{X}_1}{L_i-2}$
1	3 - 7.5	2.80	1.40
2	3 - 9.5	3.80	3.40
3	5 - 12.5	2.90	2.13
4	5 - 15.5	4.40	3.13
5	7 - 18.5	3.50	2.48
6	7 - 20.5	4.50	2.88
7	7 - 22.5	5.50	3.28
8	9 - 23.5	3.60	2.49
9	9 - 25.5	4.60	2.77
10	9 - 27.5	5.60	3.20
11	9 - 28.5	6.10	3.34
12	9 - 30.5	7.10	3.49

Table 8: Bounds on x_1 and x_K.

Reduction 4: (Lower Bound on K^{th} Thickness)

From reduction 3, $\hat{X}_1$ as given by (3.41) is the maximum possible face veneer thickness and from (3.40) it follows that $x_K > x_{K-t}$ for all t = 1, 2, ... K-2. Since $2\hat{X}_1$ is the maximum possible thickness of the two face veneers, a lower bound $\check{X}_K$, on the veneer thickness X_K, for a plywood with L_i plies and lower tolerance for thickness b_i^L, is given by $(b_i^L - 2\hat{X}_1)/(L_i-2)$. Since this is true for all plywood types the lower bound on x_K is obtained by $\underset{i}{\text{Max}}[(b_i^L - 2\hat{X}_1)/(L_i-2)]$. If $T_K^{min} = \{\underset{m}{\min}\{T_K^m\}$ is the minimum possible thickness of x_K, we have

$$x_K \geq \check{X}_K = \text{Max}\left[\underset{i}{\text{Max}} \left\{\frac{b_i^L - 2\hat{X}_1}{L_i - 2}\right\}, T_K^{min}\right] \quad \text{—— (3.42)}$$

The reduction implied by (3.42) is illustrated using the previous example in the last column of Table 8 with $T_K^{min} = 2.40$, $\hat{X}_1 = 2.80$ and b_i^L is -0.5 mm of the specified thickness for all i. In that example

$$\check{X}_K = \text{Max}\left[\underset{i}{\text{Max}}\left\{\frac{b_i^L - 5.60}{L_i - 2}\right\}, 2.40\right] = 3.49$$

Observe that a 9 ply 30.5 mm plywood cannot be assembled within specification if x_K is less than 3.49 mm. Values of x_K lower than $\check{X}_K$ would only make the PDM problem infeasible.

To appreciate the importance of the reductions (3.39) - (3.42), consider a typical problem with 12 types of plywood as listed in table 8 with K = 4, m(1) = 9, and m(k) = 27 for k = 2, 3, 4. Then the number of sets of veneer thicknesses that need to be considered in the PDM problem reduces from a maximum of 177,147 to 12,400 by the reductions specified by (3.39) - (3.42).

If plywood can be made with unbalanced designs, some of the branching tests given above may have to be modified. In particular, the reduction specified by (3.40) would depend on the design alternatives considered and the bounds given by (3.41) and (3.42) would not be valid if the two face veneers in a plywood sheet can be of different thicknesses.

Using the analysis developed in section 3.3.1 and the reductions specified by (3.39) - (3.42), we can now present, in greater detail, an efficient algorithm for solving the PDM problem as follows.

Algorithm B: (An algorithm for solving the PDM problem)

Step 0: Initialize $(x_1, x_2, \ldots x_K)$ and Z_0

Step 1: Let $x_t = x_t^*$, $x_t^* \in T_t$, $t = 1, 2, \ldots K-1$ be a previously unselected set of values for the thicknesses $x_1, x_2, \ldots x_{K-1}$ of the PDM problem which satisfy (3.39) - (3.41). If none exists, terminate. Otherwise, go to step 2.

Step 2: Evaluate X_K^{min} and X_K^{max} from (3.31) and (3.32). If $X_K^{min} > X_K^{max}$, PDM $(x_1^*, \ldots x_{K-1}^*)$ is infeasible and go to step 1. Otherwise, compute $\overline{Z}$ from (3.38). If $\overline{Z}$ is less than Z_0 go to step 1. Otherwise,

Step 3: Let $x_K \in T_K \cap (X_K^{min}, X_K^{max})$ along with $(x_1^*, \ldots x_{K-1}^*)$ be a previously unselected set of thicknesses satisfying (3.39) - (3.42). If none exists, go to step 1. Otherwise,

Step 4: Solve the LMCK problem associated with the thicknesses $x_1^*, x_2^*, \ldots x_K^*$ using (3.23). Store the solution and update Z_0 if better than the incumbent. Go to step 3.

In the implementation of this algorithm for practical problems, the existing values of x_k, P_{ij} and the corresponding value of Z_0 for the mill can be used at the initial node. Otherwise, we can start with $Z_0 = -\infty$ as the initial value and update it whenever better solutions are generated. Clearly, algorithm B will terminate in a finite number of iterations with an optimal solution to the PDM problem. As is true with any other algorithm, the computational time required to solve the problem would mainly depend on the size in terms of the number of variables and constraints in the problem. A flowchart of Algorithm B is presented in Fig. 1.

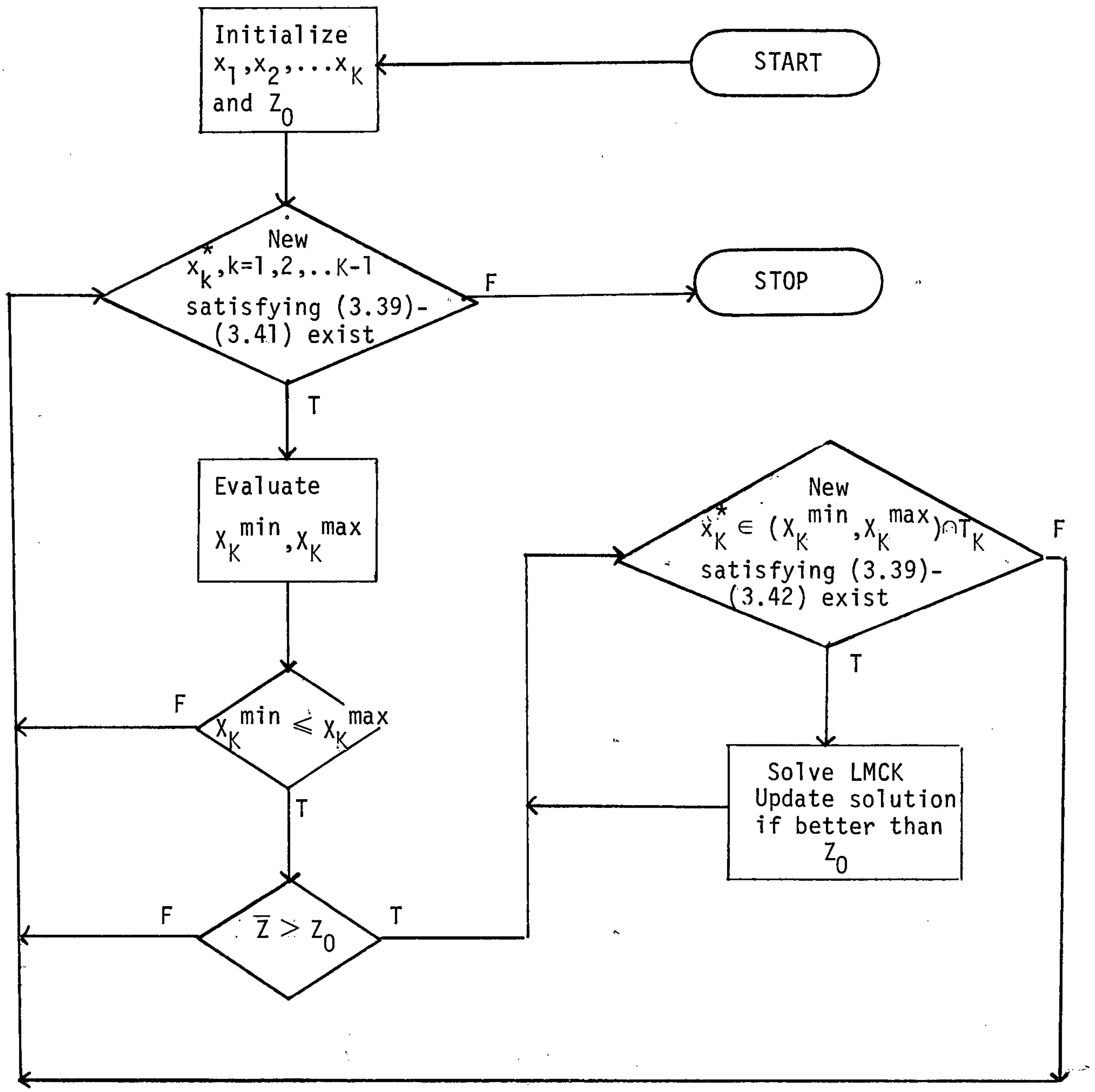

Figure 1: A Flowchart of the Algorithm to Solve the PDM Problem.

3.4 Computational Results

Our implicit enumeration algorithm to solve the PDM problem was coded in FORTRAN. As the design coefficients and the number of design alternatives for each product depended on the number of veneer thicknesses, separate codes were written for different values of K. Input data for the code were the availability of logs, log cost, the yield factor, correction factor for compression and shrinkage, set of all possible veneer thicknesses of the peeling lathe(s), the upper tolerance for face veneer thickness, list of plywood types, their revenues, order file requirements and the respective thickness tolerances. The codes are capable of giving as output, the optimal veneer thicknesses, quantity of logs to be converted to each thickness, maximum net revenue at the optimal solution, marginal value of wood, all the feasible design alternatives for each type of plywood and the quantity of plywood to be produced under each design alternative. Appendix I gives a listing of the FORTRAN program for the PDM problem with four veneer thicknesses (K = 4). The purpose of the code is only to demonstrate that the PDM model can be solved efficiently using our implicit enumeration algorithm and no expertise is claimed on the efficiency in coding.

For testing the suitability of our PDM model and the implicit enumeration algorithm for real-world situations, representative data obtained from a plywood mill in British Columbia was used. The mill was mostly manufacturing exterior plywood adhering to the specifications laid by the Council of Forest Industries of British Columbia (COFI (1978)). Currently the mill was using four veneer thicknesses (K = 4) as listed in Table 2. The number of possible veneer thicknesses that could be considered within the framework of veneer thickness tolerance amounted to $m(1) = 9$ and $m(k) = 27$ for $k = 2$, 3 and 4. The mill's order file consisted of twelve types of plywood. Details of the plywood types, their existing designs, the availability of logs, log cost, yield factor, revenue and order file requirements of plywood, their thickness specifications are given in appendix II.

When the mill's data with the existing set of veneer thicknesses and designs were used in a linear program, the maximum possible annual net revenue for the mill was $13,416,694. Starting with this as an initial solution, our implicit enumeration algorithm codes for the PDM problem were used on the University of British Columbia's AMDAHL 470 V8 computer with a WATFIV compiler. When the number of veneer thicknesses, K, was four, the code took 29.3 seconds of CPU time and gave an optimal solution to the PDM problem with a maximum net revenue of $14,337,370. Details of the corresponding optimum veneer thicknesses, the feasible design alternatives, quantities of plywood to be produced under each design alternative and such other information are given in appendix III. For the particular configuration of the problem parameters used in the study, the annual net revenue obtained from our model was 6.86% higher than that for the existing set of veneer thicknesses and plywood designs used by the mill. The increase resulted from the fact that the wood loss in the form of excess thickness in plywood reduced from the current 7944.7 cubic metres to 1647.4 cubic metres.

In addition to the above, possibilities of manufacturing all the mill's products from only three veneer thicknesses instead of four was tested using our PDM model with K = 3. It was found that all the products of the mill could be assembled within plywood thickness specifications using only three veneer thicknesses, a fact which was not known earlier. Details of the corresponding optimum veneer thicknesses, plywood designs and such other data are given in appendix IV. The maximum possible net revenue associated with three veneer thicknesses was $13,930,670 and the corresponding wood loss in the form of excess thickness in plywood was 3612.1 cubic metres. This indicates that in addition to an increase of 3.83% in annual net revenue, there was substantial savings possible in the form of reduced set up costs associated with the fourth peel thickness. A more detailed analysis of the results of the PDM model for different number of thicknesses and their comparison is given in chapter six.

CHAPTER 4

4.1 Extensions to the PDM Problem

In the formulation of the Plywood Design and Manufacturing (PDM) problem, we had earlier considered the case where only one species is used in the assembly of the end products. However, most plywood mills might use more than one species with varying log costs and yield factors. In addition to this, the orderfile requirements may be independent of species or specified specieswise. As we will show in this chapter, the plywood design and manufacturing problem can be reformulated to incorporate all these factors and solved efficiently.

The inclusion of more than one species in the model does not affect the design part of the PDM problem. Changes in green veneer thickness due to variations in the density or specific gravity of a species are normally nullified by the corresponding shrinkage and compression during drying and hot pressing. However, if significant deviations occur in plywood thickness for any particular species, it can be adjusted by making appropriate corrections in the veneer. In such a case, the veneer thickness can be expressed relative to the thickness of a standard species of veneer such as Douglas fir in the Pacific Northwest region.

The inclusion of more than one species in the model, however, affects the product mix part of the problem. The linear subproblem to be solved at each feasible node of the implicit enumeration's search tree assumes different forms depending on the factors included in the model. However, essentially the same implicit enumeration algorithm described in the previous chapter can be used to solve these various extensions of the PDM problem.

We now consider extensions to the PDM model involving some of these situations. In the first case, in section 4.2, we show that when the orderfile is independent of species, the linear subproblem is a Generalized Network problem. We show that, due to some special structure in the PDM model, this Generalized Network problem reduces to a Generalized Transportation problem which, in turn, can be reduced to a standard Transportation problem. We solve an example of this subproblem and present modifications required in the implicit enumeration algorithm. In section 4.3, we consider the situation when the orderfile is dependent on species. We demonstrate that, in this case, a linear subproblem associated with a node of the search tree decomposes into separable Linear Multiple Choice Knapsack problems. In section 4.4, we consider the situation when veneers of different species can be mixed within a plywood panel.

4.2 Orderfile Independent of Species

4.2.1 Formulation

Define new decision variables and coefficients of the model as follows:

V_{ks} = Quantity of veneer sheets of thickness k from species s (in number of sheets of standard size or equivalent).

P_{ijs} = Quantity of Plywood of type i, species s, made using construction alternative j (in number of sheets of standard size or equivalent).

r_{is} = Revenue for Plywood type i, species s. (\$)

W_s = Quantity of logs available, species s (cu. mtrs.)

C_s = Cost per unit of log, species s (\$/cu.mtr.)

$\ddot{y}_s$ = Yield factor for veneer from species s.

$s = 1, 2, \ldots S$; $i = 1, 2, \ldots N$; $j = 1, 2, \ldots n_i$ and $k = 1, 2, \ldots K$.

All other variables and parameters of the earlier model remain unchanged. Formulating the problem in the same way as in Chapter 2, the PDM model in this case would be

$$Z_0 = \text{Max} \sum_{i=1}^{N} \sum_{s=1}^{S} \sum_{j=1}^{n_i} r_{is} P_{ijs} - \sum_{s=1}^{S} C_s \ddot{y}_s \sum_{k=1}^{K} x_k \dot{v}_{ks} \qquad \text{— (4.1)}$$

s.t.

$$b_i^L - M(1-\delta_{ij}) \leqslant \sum_{k=1}^{K} a_{ijk} x_k \leqslant b_i^U + M(1-\delta_{ij}) \qquad \text{— (4.2)}$$

$$\sum_{j=1}^{n_i} \delta_{ij} \geqslant 1 \qquad \text{— (4.3)}$$

$$P_{ijs} - M\delta_{ij} \leqslant 0 \qquad \text{— (4.4)}$$

$$y_s \sum_{k=1}^{K} x_k \dot{v}_{ks} \leqslant W_s \qquad \text{— (4.5)}$$

$$\sum_{i=1}^{N} \sum\sum_{j=1}^{n_i} a_{ijk} P_{ijs} - \dot{v}_{ks} \leqslant 0 \qquad \text{— (4.6)}$$

$$\sum_{s=1}^{S} \sum_{j=1}^{n_i} P_{ijs} \geqslant d_i \qquad \text{— (4.7)}$$

$$\delta_{ij} = \{0,1\} \qquad \text{— (4.8)}$$

$$x_k \in T_k = \{T_k^{1},\ T_k^{2},\ \ldots\ T_k^{m(k)}\} \qquad \text{— (4.9)}$$

$$P_{ijs}\ ,\ v_{ks} \geqslant 0 \qquad \text{— (4.10)}$$

for all $s = 1, 2, \ldots S$; $i = 1, 2, \ldots N$; $j = 1, 2, \ldots n_i$ and $k = 1, 2, \ldots K$.

Again, the above is a non-linear mixed integer programming problem. Note that in the above model $y_s x_k v_{ks}$ represents the quantity of logs peeled to thickness x_k from species s and $\sum_k y_s x_k v_{ks}$ represents the total quantity of logs of species s. Also, $\sum_{j=1}^{n_i} P_{ijs}$ is the quantity of plywood type i made with species s and $\sum_{s=1}^{S} \sum_{j=1}^{n_i} P_{ijs}$ is the total quantity of plywood type i.

4.2.2 The Generalized Network Subproblem

The feasibility of a set of thicknesses, x_k^* , $x_k^* \in T_k$, $k = 1, 2, \ldots K$, to the extended PDM problem given by (4.1)-(4.10) can be verified by evaluating the indicator variables δ_{ij} using (3.1). Let δ_{ij}^* and I_i be as defined in (3.1) and (3.2) respectively, of the previous chapter. Then, following arguments similar to those used in that chapter, we can show that (4.5) reduces to

$$\sum_i \sum_{j \in I_i} (\sum_k y_s a_{ijk} x_k^*)\ P_{ijs} \leqslant W_s$$

or

$$\sum_i \sum_{j \in I_i} b_{ijs}\ P_{ijs} \leqslant W_s$$

where

$$b_{ijs} = y_s \sum_{k=1}^{K} a_{ijk} x_k^* \qquad \text{— (4.11)}$$

The objective function reduces to

$$\sum_i \sum_{j \in I_i} \sum_s r_{is} P_{ijs} - \sum_i \sum_{j \in I_i} \sum_s C_s \left(y_s \sum_{k=1}^{K} a_{ijk} x_k^* \right) P_{ijs}$$

$$= \sum_i \sum_{j \in I_i} \sum_s r_{ijs} P_{ijs}$$

where

$$r_{ijs} = r_{is} - C_s y_s \sum_{k=1}^{K} a_{ijk} x_k^*$$

$$= r_{is} - C_s b_{ijs} \qquad \text{—— (4.12)}$$

The linear subproblem associated with a set of thicknesses x_1^*, x_2^*, ... x_k^*, would then be

$$Z = \text{Max} \sum_i \sum_{j \in I_i} \sum_s r_{ijs} P_{ijs}$$

s.t.

$$\sum_i \sum_{j \in I_i} b_{ijs} P_{ijs} \leqslant W_s \quad \forall \; s$$

$$\sum_s \sum_{j \in I_i} P_{ijs} \geqslant d_i \quad \forall \; i \qquad \text{—— (4.13)}$$

$$P_{ijs} \geqslant 0 \quad \forall \; i, s \text{ and } j \in I_i$$

(4.13) is known as a Generalized Network (GN) problem and when $|I_i| = 1$ for all i, (4.13) is specialized to a Generalized Transportation (GT) problem. b_{ijs} can be interpreted as the amount of wood required to produce one unit of plywood type i, species s, using design alternative j and r_{ijs} is the corresponding unit net revenue. The unique feature of (4.13) is that each variable in it appears at most twice in the constraints. This special structure is further reflected by the fact that we can associate a graph with a GN problem in which the nodes represent the constraints and the undirected arcs, the variables. Since each variable has at most two non-zero coefficients in the constraints, the basis of a GN problem has some special structure which facilitates its solution in much faster time than a linear program (Kennington and Helgason (1980), Elam, Glover and Klingman (1979), Phillips and Garcia-Diaz (1981)). However, our GN problem as given by (4.13) reduces to a GT problem as shown in the following section.

4.2.3 The Generalized Transportation Subproblem

Recall that the coefficients in our GN problem (4.13) are related by $r_{ijs} = r_{is} - C_s b_{ijs}$. Suppose

$$b_{iJ_is} = \underset{j \in I_i}{\text{Min}} \{b_{ijs}\} \qquad \text{—— (4.14)}$$

Then, it follows that

$$r_{iJ_is} = r_{is} - C_s b_{iJ_is}$$

$$= \underset{j \in I_i}{\text{Max}} \{r_{ijs}\} \qquad \text{—— (4.15)}$$

Proposition 4.1: For any i = 1, 2, ...N, the index J_i (or indicies, if more than one exists) minimizing b_{ijs} in (4.14) is (are) the same for all s = 1, 2, ...S.

Proof: The proof follows from the definition of b_{ijs} since

$$b_{iJ_is} = \min_{j \in I_i} \{\sum_k y_s a_{ijk} x_k^* \}$$

$$= y_s \min_{j \in I_i} \{\sum_k a_{ijk} x_k^* \}$$

and $\sum_k a_{ijk} x_k^*$ is independent of s.

Proposition 4.2: There exists an optimal solution to (4.13), in which

$$P_{ijs} = 0 \quad \forall \quad j \neq J_i \quad \text{and} \quad \forall\, i,\, s \tag{4.16}$$

Proof: Suppose (4.16) does not hold for some i and s. That is, there exists a solution (P'_{ijs}) which is optimum for (4.13) with $Z = Z'$ but $P'_{ptf} > 0$ for some i = p, s = f and $t \in I_p \setminus \{J_p\}$. Assume $r_{pJ_pf} > 0$ and consider the new solution given by

$$P^*_{ijs} = \begin{cases} P'_{pJ_pf} + \dfrac{b_{ptf}}{b_{pJ_pf}} P'_{ptf} & i = p,\ j = J_p,\ s = f \\ 0 & i = p,\ j = t,\ s = f \\ P'_{ijs} & \text{Otherwise} \end{cases} \tag{4.17}$$

Observe that (4.17) is feasible for our GN subproblem (4.13). Since (P'_{ijs}) is a feasible solution, it follows that

(i) $P^*_{ijs} \geqslant 0 \quad \forall\ i, j, s \qquad$ since $b_{ptf} \geqslant b_{pJ_pf} > 0$,

(ii) $$\sum_s \sum_{j \in I_i} P^*_{ijs} = \sum_s \sum_{j \in I_i} P'_{ijs} + \left(\frac{b_{ptf}}{b_{pJ_pf}} - 1\right) P'_{ptf}$$

$$\geqslant \sum_s \sum_{j \in I_i} P'_{ijs} \quad \text{since } b_{ptf} \geqslant b_{pJ_pf}$$

$$\geqslant d_i$$

and (iii) $$b_{ptf}P^*_{ptf} + b_{pJ_pf}P^*_{pJ_pf} = 0 + b_{pJ_pf}\left[P'_{pJ_pf} + \frac{b_{ptf}}{b_{pJ_pf}} P'_{ptf}\right]$$

$$= b_{pJ_pf}P'_{pJ_pf} + b_{ptf}P'_{ptf}$$

so that

$$\sum_i \sum_{j \in I_i} b_{ijs}P^*_{ijs} = \sum_i \sum_{j \in I_i} b_{ijs}P'_{ijs}$$

$$\leqslant W_s \quad \forall\ s.$$

Further, observe that if b_{ptf} is such that $b_{ptf} = b_{pJ_pf}$, i.e. There exists more than one $j \in I_p$ minimizing b_{pjs} in (4.14). Then, it follows from (4.15) that $r_{ptf} = r_{pJ_pf}$ and therefore

$$\sum_i \sum_{j \in I_i} \sum_s r_{ijs}P^*_{ijs} = \sum_i \sum_{j \in I_i} \sum_s r_{ijs}P'_{ijs}.$$

On the other hand, if (i) J_p is unique for $i = p$, i.e. $b_{pJ_pf} < b_{pjf}$, $j \neq J_p$, $j \in I_i$, or (ii) the index t is such that $b_{pJ_pf} < b_{ptf}$, then it follows from (4.15) that $r_{pJ_pf} > r_{ptf}$. Now, observe that

$$r_{ptf}P^*_{ptf} + r_{pJ_pf}P^*_{pJ_pf} = r_{pJ_pf}\left(P'_{pJ_pf} + \frac{b_{ptf}}{b_{pJ_pf}} P'_{ptf}\right)$$

$$> r_{pJ_pf}P'_{pJ_pf} + r_{pJ_pf}P'_{ptf} \qquad \text{from (4.14)}$$

$$> r_{pJ_pf}P'_{pJ_pf} + r_{ptf}P'_{ptf} \qquad \text{from (4.15)}$$

Therefore $\sum\limits_i \sum\limits_{j \in I_i} \sum\limits_s r_{ijs}P^*_{ijs} > \sum\limits_i \sum\limits_{j \in I_i} \sum\limits_s r_{ijs}P'_{ijs}$, contradicting the optimality of (P'_{ijs}). The fact that this result holds even when $r_{pJ_pf} \leqslant 0$ can be shown by dropping the factor b_{ptf}/b_{pJ_pf} in (4.17) and using similar arguments as above. The proof of Proposition 4.2 then follows.

Propositions 4.1 and 4.2 would together imply that in an optimal solution to the generalized network problem (4.13), if a plywood type is made with more than one species, the design alternative for them would be the same.

From propositions 4.1 and 4.2 it follows that the GN problem reduces to solving

$$Z = \text{Max} \sum_i \sum_s r_{i\cdot s}P_{i\cdot s}$$

s.t.

$$\sum_i b_{i\cdot s}P_{i\cdot s} \leqslant W_s \quad \forall\, s$$

$$\sum_{s} P_{i \cdot s} \geq d_i \quad \forall \ i \qquad \text{—— (4.18)}$$

$$P_{i \cdot s} \geq 0 \quad \forall \ i, s$$

where, $b_{i \cdot s} \equiv b_{iJ_is}$ and $r_{i \cdot s} \equiv r_{iJ_is}$. This is a Generalized Transportation problem. Again, $b_{i \cdot s}$ is the amount of wood required to produce one unit of plywood type i, species s, using the design alternative J_i, and $r_{i \cdot s}$ is the corresponding unit revenue. Observe that (4.18) can be represented in the following tabular form of a GT problem with the species as 'sources' and plywood types as 'destinations'.

Plywood Type (i)

Species (s)	1		2		....	N		Supply
1	$b_{1 \cdot 1}$	$r_{1 \cdot 1}$	$b_{2 \cdot 1}$	$r_{2 \cdot 1}$	......	$b_{N \cdot 1}$	$r_{N \cdot 1}$	$\leq W_1$
	$P_{1 \cdot 1}$		$P_{2 \cdot 1}$			$P_{N \cdot 1}$		
. . . .	....		....		....	....		. . . .
S	$b_{1 \cdot S}$	$r_{1 \cdot S}$	$b_{2 \cdot S}$	$r_{2 \cdot S}$	....	$b_{N \cdot S}$	$r_{N \cdot S}$	$\leq W_S$
	$P_{1 \cdot S}$		$P_{2 \cdot S}$			$P_{N \cdot S}$		
Demand	$\geq d_1$		$\geq d_2$		....	$\geq d_N$		-

The solution procedure of a GT problem deviates from that of a standard Transportation problem in that the basis graph of a GT can have more than one maximal connected subgraph or component. The dual variables associated with a GT are unique since any non-degenerate feasible solution will have as many basic variables as the number of constraints. Algorithms to solve a GT problem have been described in the literature [Balas and Ivanescu (1964), Eisemann (1964), Lourie (1964), Balas (1966), Taha (1971), Glover and Klingman (1973)].

The problem as represented by (4.18) deviates from standard GT problems (for example, Balas and Ivanescu (1964), Taha (1971), Glover and Klingman (1973)) in that the standard GT problems will have equality sign in the demand constraints. (4.18) can be put in the standard form of a GT problem by the addition of a dummy row and N + 1 columns, one for each product and a slack. However, we can solve (4.18) in the present form noting that the dual variable(s) would be zero (i) for all columns having allocation ($\sum_s P_{i \cdot s}$) more than demand (d_i) and (ii) for all rows having weighed allocation ($\sum_i b_{i \cdot s} P_{i \cdot s}$) less than the supply ($W_s$).

4.2.4 Scaling the GT problem to a Transportation Problem

Alternate to solving (4.18) as a GT problem, it is possible to solve it as a Transportation problem following a scaling procedure for network problems with gains (Truemper (1976)). Observe that the GT problem can be represented as a network flow problem with the arcs having gain/loss factors. Specifically, in such networks the amount of flow entering an arc need not be equal to the amount of flow leaving the arc. For example, the GT problem (4.18) with two species (S = 2) and two products (N = 2) can be transformed into a network as shown in Fig. 2.

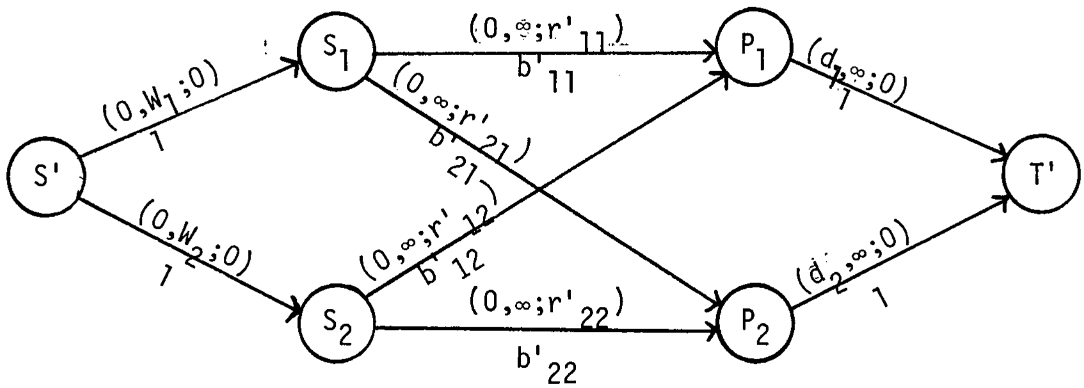

Fig.2: The PDM Subproblem as a Network Flow Problem

In Figure 2, S_1 and S_2 are nodes associated with the two species and P_1 and P_2 are nodes associated with the two products. S' is a 'super source', a consolidation of all the supply (or species) nodes and T' is a 'super sink', a consolidation of all the demand (or product) nodes. The three numbers in bracket on top of the arc represent the lower arc capacity, upper arc capacity and revenue per unit 'flow' respectively. The number below an arc represents the loss/gain factor associated with a unit of 'flow'. In a pure network flow problem these loss/gain factors would all be equal to unity. Our problem (4.18) can be viewed as one of allocating species to products in which $b'_{is} = (1/b_{i \cdot s})$, the quantity of plywood of type i produced per one unit of species s, is the gain factor. $r'_{is} = (r_{i \cdot s}/b_{i \cdot s})$ is the revenue generated by the allocation of one unit of species s to plywood type i. The supply (W_s) and demand (d_i) restrictions of the GT problem can be included as arc capacities as shown in the figure. If f_{is} is the 'flow' from species node S_s to product node P_i in the network, it is related to the variable $P_{i \cdot s}$ of (4.18) by $f_{is} = P_{i \cdot s} b_{i \cdot s}$. Our GT problem (4.18) would then be equivalent to the following minimal cost (or maximal revenue) network flow problem with gains.

$$\text{Max} \sum_i \sum_s r'_{is} f_{is}$$

s.t.

$$\sum_j f_{S'j} - \sum_j b'_{jS'} f_{jS'} \leqslant \overline{F}$$

$$\sum_j f_{tj} - \sum_j b'_{jt} f_{jt} = 0 \qquad t \neq S', T' \qquad \text{—— (4.19)}$$

$$\sum_j f_{T'j} - \sum_j b'_{jT'} f_{jT'} \geqslant -F$$

where, t is the index of all intermediary nodes (i.e., excluding S' and T') in the network, $\overline{F}$ is the flow available at the super source S' (= $\sum_s W_s$) and F is the minimum flow required at the super sink T' ($\doteq \sum_i d_i$). Truemper (1976) gives a scaling procedure by which a network problem with gains can be reduced to a pure network problem. Since (4.18) is a Generalized Transportation problem, the pure network problem obtained by Truemper's scaling procedure yields a standard Transportation problem. This is illustrated through the following example of a PDM subproblem.

Example 4.1: Consider the following example (hypothetical) of a PDM subproblem with two species, Fir (F) and Hemlock (H), four products (N=4) and four veneer thicknesses (K=4) which are set to x_k = 2.69, 3.35, 3.96 and 4.98 mm for k = 1, 2, 3 and 4 respectively; C_f = 35.00,

$C_H = 30.00$, $Y_F = 0.006067$, $Y_H = 0.006276$, $W_F = 70{,}000$ and $W_H = 80{,}000$. Other data for the PDM problem are given in Table 9.

Plywood Type, i	Ply - Thick L_i (mm)	Revenue ($) r_{iF}	r_{iH}	Demand d_i	Design j	a_{ijk}
1	3 - 7.5	4.3	4.1	171107	1	3 0 0 0
2	5 - 12.5	6.1	5.8	502289	1	5 0 0 0
					2	4 1 0 0
3	5 - 15.5	7.6	7.4	350192	1	2 2 0 1
					2	2 1 2 0
4	7 - 18.5	8.9	8.6	423394	1	5 2 0 0
					2	4 3 0 0

Table 9: Input data for Example 4.1; A PDM subproblem with S=2, Orderfile Independent of Species.

We will proceed now with solving this PDM subproblem. Computation of the b_{ijs} $\forall$ i, j, s using (4.11) and evaluation of the minimum from (4.14) gives J_i = 1, 1, 2 and 1 for i = 1, 2, 3 and 4 respectively. For these design alternatives, $b_{i \cdot s}$ and $r_{i \cdot s}$ are presented in the tabular form of a GT problem below.

Products \ Species	1	2	3	4	Supply
F	.04896 2.5864	.08160 3.2440	.10101 4.0645	.12225 4.6213	$\leqslant$ 70,000
H	.05065 2.5806	.08441 3.2677	.10449 4.2651	.12646 4.8062	$\leqslant$ 80,000
Demand	$\geqslant$ 171107	$\geqslant$ 502289	$\geqslant$ 350192	$\geqslant$ 423394	-

The above problem can be solved as a GT problem using known algorithms (Balas and Ivanescu (1964), Taha (1971), Glover and Klingman (1973)) or transformed into a standard Transportation problem using the scaling procedure of Truemper and then solved. We use the latter method for illustration. Observe that the constraints of the GT problem given above can be written as

$$.04896\ P_{1\cdot F} + .08160\ P_{2\cdot F} + .10101\ P_{3\cdot F} + .12225\ P_{4\cdot F} \leqslant 70{,}000 \qquad \text{— (4.20)}$$

$$.05065\ P_{1\cdot H} + .08441\ P_{2\cdot H} + .10449\ P_{3\cdot H} + .12646\ P_{4\cdot H} \leqslant 80{,}000 \qquad \text{— (4.21)}$$

$$P_{1\cdot F} + P_{1\cdot H} \geqslant 171107 \qquad \text{— (4.22)}$$

$$P_{2\cdot F} + P_{2\cdot H} \geqslant 502289 \qquad \text{— (4.23)}$$

$$P_{3\cdot F} + P_{3\cdot H} \geqslant 350192 \qquad \text{— (4.24)}$$

$$P_{4\cdot F} + P_{4\cdot H} \geqslant 423394 \qquad \text{— (4.25)}$$

There exists a set of multipliers, one each for each of constraints (4.20) - (4.25), such that the gain factors (coefficients on the LHS of (4.20) and (4.21)) can be transformed to unity (Truemper (1976), Phillips and Garcia-Diaz (1981)). In particular, starting with (4.20) as the initial constraint, if we use the multipliers 1, 0.9667, 0.04896, .08160, .10101 and .12225 for the constraints (4.20) to (4.25) respectively, we get

$$.04896\ P_{1\cdot F} + .08160\ P_{2\cdot F} + .10101\ P_{3\cdot F} + .12225\ P_{4\cdot F} \leqslant 70{,}000 \quad \text{—— (4.26)}$$

$$.04896\ P_{1\cdot H} + .08160\ P_{2\cdot H} + .10101\ P_{3\cdot H} \pm .12225\ P_{4\cdot H} \leqslant 77336.0 \quad \text{—— (4.27)}$$

$$.04896\ P_{1\cdot F} + .04896\ P_{1\cdot H} \geqslant 8377.40 \quad \text{—— (4.28)}$$

$$.08160\ P_{2\cdot F} + .08160\ P_{2\cdot H} \geqslant 40986.78 \quad \text{—— (4.29)}$$

$$.10101\ P_{3\cdot F} + .10101\ P_{3\cdot H} \geqslant 35372.89 \quad \text{—— (4.30)}$$

$$.12225\ P_{4\cdot F} + .12225\ P_{4\cdot H} \geqslant 51759.92 \quad \text{—— (4.31)}$$

Now, setting $P'_{1s} = .04896\ P_{1\cdot s}$, $P'_{2s} = .08160\ P_{2\cdot s}$, $P'_{3s} = .10101\ P_{3\cdot s}$ and $P'_{4s} = .12225\ P_{4\cdot s}$ for s = F, H, all the coefficients on the LHS of (4.26) to (4.31) are transformed to unity. The resulting transportation problem, in a tabular form, would be as follows

Products / Species	1	2	3	4	Supply
F	52.8268	39.7549	40.2386	37.8020	$\leqslant$ 70,000
H	52.7083	40.0453	42.2245	39.3145	$\leqslant$ 77336.0
Demand	$\geqslant$ 8377.4	$\geqslant$ 40986.8	$\geqslant$ 35372.9	$\geqslant$ 51759.9	-

The numbers in each cell of this transportation problem are the revenues associated with the transformed variables P'_{is}, $s' = F, H$ and $i = 1, \ldots 4$. The above problem is not in the standard form of a Transportation problem since (i) the constraints have inequality signs and (ii) total supply and total demand are not balanced. Converting this problem to the standard form and solving yields the solution: $P'_{1F} = 19216.4$, $P'_{2F} = 40986.8$, $P'_{3H} = 35372.9$, $P'_{4F} = 9796.8$, $P'_{4H} = 41963.1$ and $P'_{1H} = P'_{2H} = P'_{3F} = 0$. Transforming these to the original variables we get $P_{1 \cdot F} = 392492.0$, $P_{2 \cdot F} = 502289.0$, $P_{3 \cdot H} = 350192.0$, $P_{4 \cdot F} = 80137.5$, $P_{4 \cdot H} = 343256.5$ and $P_{1 \cdot H} = P_{2 \cdot H} = P_{3 \cdot F} = 0$ as the solution to the Generalized Transportation problem. The corresponding objective function value is Z = \$6,158,270.

4.2.5 Implicit Enumeration for the Overall Problem

To solve the overall PDM problem as given by (4.1) - (4.10), essentially the same implicit enumeration algorithm described in chapter 3 can be used. Branching is initially done on the veneer thicknesses x_t, $t = 1, 2, \ldots K-1$. All the reductions in the number of nodes of the search tree specified by (3.39) - (3.42) and the bounds on x_K given by (3.31) and (3.32) are equally applicable here. The upper bound on Z given by (3.38), however, changes for this problem. Following the relaxation specified by (3.33) and (3.34), let b'_i be as defined in (3.35). Then, since the linear subproblem in this case reduces to a Generalized Transportation problem, an upper bound on Z for all branches from this node can be obtained as a solution of (4.32) below:

$$\overline{Z} = \text{Max} \sum_i \sum_s r'_{i \cdot s} P_{i \cdot s}$$
s.t.

$$\sum_i b'_i P_{i \cdot s} \leqslant W_s / y_s$$

$$\sum_s P_{i \cdot s} \geqslant d_i \qquad \text{—— (4.32)}$$

$$P_{i \cdot s} \geqslant 0 \quad \forall \ i, s$$

where

$$r'_{i \cdot s} = r_{is} - C_s y_s b'_i$$

Again, (4.32) is a GT problem and can be solved using methods outlined earlier. The node associated with the subproblem in which $x_t = x_t^*$, $t = 1, 2, \ldots K\text{-}1$ can be fathomed if $\overline{Z}$ obtained in (4.32) is less than the incumbent Z_0. If, however, branching on the last veneer thickness x_K in the set $(X_K^{min}, X_K^{max}) \cap T_K$ is found necessary, additional bounds can be calculated for the GT problem using the structure in our PDM problem. Consider the dual of (4.18) given by

$$Z_0 = \text{Min} \sum_s W_s \Pi_s + \sum_i d_i t_i$$
s.t.

$$b_{i \cdot s} \Pi_s + t_i \geqslant r_{i \cdot s} \qquad \text{—— (4.33)}$$

$$\Pi_s \geqslant 0, \quad t_i \leqslant 0 \qquad \forall \ i, s$$

Recall from duality theory that an upper bound on (4.18) can be obtained using any feasible solution to (4.33). An efficient upperbound can be obtained using the structure of the constraints in (4.33).

Observe that in (4.33), the constraints lead to

$$\Pi_s \geq r_{i \cdot s}/b_{i \cdot s} \quad \forall \; s$$

and

$$t_i \geq r_{i \cdot s} - b_{i \cdot s}\Pi_s \quad \forall \; i, s$$

Let

$$\hat{\hat{\Pi}}_s = \underset{i}{\text{Max}} \{r_{i \cdot s}/b_{i \cdot s}\}$$

and

$$\hat{t}_i = \underset{s}{\text{Max}} \{r_{i \cdot s} - b_{i \cdot s}\hat{\Pi}_s\}$$

Then, $\overline{Z}' = \sum_s W_s \hat{\Pi}_s + \sum_i d_i \hat{t}_i$ will give an upper bound on the value of Z in (4.18). Intuitively, this bound is sharp since when S = 1, $\overline{Z}' = Z$, the objective function value at an optimal solution of the corresponding LMCK problem. For the example considered earlier, $\hat{\Pi}_F = 52.8286$, $\hat{\Pi}_H = 50.9497$, $\hat{t}_1 = 0$, $\hat{t}_2 = -1.0330$, $\hat{t}_3 = -1.0586$ and $\hat{t}_4 = -1.6369$. This gives $\overline{Z}' = \$6,191,220$, which deviates from the actual Z by half a percent only.

4.3 Orderfile Dependent on Species

In this section we formulate the PDM problem when the orderfile is dependent on species and analyze its structure. We demonstrate that the subproblem obtained in this case decomposes into Linear Multiple Choice Knapsack problems which can be solved explicitly using results of chapter 3. For solving the overall PDM problem, the same implicit enumeration algorithm described in the previous chapter can be used, with few changes in the computation of the bound on the objective function.

When the orderfile is dependent on species, constraint (4.7) changes to

$$\sum_{j=1}^{n_i} P_{ijs} \geq d_{is} \qquad \forall\ i, s \qquad \text{—— (4.34)}$$

where d_{is} is the quantity of plywood required of type i, species s. All other constraints and the objective function of section 4.2 remain unchanged.

4.3.1 The Separable LMCK problem

When veneer thicknesses are assigned values x_k^* , $x_k^* \in T_k$ for $k = 1, 2, \ldots K$, δ_{ij}^* and I_i are as given by (3.1) and (3.2) respectively, we can show that the linear subproblem reduces to

$$\text{Max} \sum_s \sum_i \sum_{j \in I_i} r_{ijs} P_{ijs}$$

s.t.

$$\sum_i \sum_{j \in I_i} b_{ijs} P_{ijs} \leq W_s \qquad \text{—— (4.35)}$$

$$\sum_{j \in I_i} P_{ijs} \geq d_{is}$$

$$P_{ijs} \geq 0 \quad \forall\ i, j \in I_i \text{ and } s.$$

Where b_{ijs} and r_{ijs} are as defined by (4.11) and (4.12) respectively.

(4.35) is separable into S distinct linear programming problems. For each s, the problem is of the form:

$$\text{Max} \quad \sum_i \sum_{j \in I_i} r_{ijs} P_{ijs}$$

s.t.

$$\sum_i \sum_{j \in I_i} b_{ijs} P_{ijs} \leqslant W_s$$

$$\sum\sum_{j \in I_i} P_{ijs} \geqslant d_{is} \qquad \text{—— (4.36)}$$

$$P_{ijs} \geqslant 0 \quad \forall \; i, j \in I_i$$

Observe that, for each s, (4.36) is a Linear Multiple Choice Knapsack problem. Also, defining b_{iJ_is} and r_{iJ_is} as in (4.14) and (4.15) respectively we can show that propositions 4.1 and 4.2 are equally applicable here. From chapter 3, we know that the solution of (4.36) is given by

$$P^*_{ijs} = \begin{cases} d_{is} & i \neq p,\ j = J_i \\ \begin{cases} d_{ps} \\ \dfrac{W_s - \sum\limits_{i \neq p} d_{is}\, b_{iJ_is}}{b_{pJ_ps}} \end{cases} & \begin{array}{l} i = p,\ j = J_p \text{ if } r_{pJ_ps} < 0 \\ i = p,\ j = J_p \text{ if } r_{pJ_ps} \geqslant 0 \end{array} \\ 0 & \text{Otherwise} \end{cases} \qquad \text{—— (4.37)}$$

where $r_{pJ_ps}/b_{pJ_ps} = \underset{i}{\text{Max}}\, \{r_{iJ_is}/b_{iJ_is}\} = \underset{i,j \in I_i}{\text{Max}}\, \{r_{ijs}/b_{ijs}\}$.

The index p maximising $\{r_{iJ_is}/b_{iJ_is}\}$ may differ from species to species:

4.3.2 The Implicit Enumeration Algorithm

To solve the overall PDM problem in this case, the same implicit enumeration algorithm of chapter 3 can be used. Branching is done on veneer thicknesses x_t, t = 1, 2, ... K-1. All the branching tests and bounds specified by (3.31), (3.32) and (3.39)-(3.42) are applicable here also. The upper bound on Z, given by $\overline{Z}$ in (3.38) is to be modified slightly following the inclusion of more species in the PDM model. In this case, $\overline{Z}$ is obtained as a solution of

$$\overline{Z} = \text{Max} \sum_i \sum_s r'_{i\cdot s} \, P_{i\cdot s}$$
$$\text{s.t.}$$
$$\sum_i b'_i \, P_{i\cdot s} \leq W_s/y_s \qquad \text{— (4.38)}$$
$$P_{i\cdot s} \geq d_{is} \quad \forall \; i, s$$

where b'_i and $r'_{i\cdot s}$ are as given in (4.32). Following the properties of LMCK in our PDM problem, the solution of (4.38) is given, for each s, by

$$P^*_{i\cdot s} = \begin{cases} d_{is} & i \neq p \\ \begin{cases} d_{ps} & i = p \text{ if } r'_{p\cdot s} \leq 0 \\ \dfrac{W_s/y_s - \sum_{i \neq p} b'_i d_{is}}{b'_p} & i = p \text{ if } r'_{p\cdot s} \geq 0 \end{cases} \\ 0 & \text{Otherwise} \end{cases} \qquad \text{— (4.39)}$$

and

$$\overline{Z} = \sum_i \sum_s r'_{i\cdot s} \, P^*_{i\cdot s} \qquad \text{— (4.40)}$$

The node with $x_t = x_t^*$, t = 1, 2, ... K-1 is fathomed if $\overline{Z}$ obtained from (4.40) is less than the incumbent value of Z_0 in the implicit enumeration algorithm.

4.4 Mix of Species Within a Plywood Sheet

In some plywood mills, more than one species might be used within a plywood sheet. This is normally done when there are tradeoff benefits associated with the cost and yield factors of different species of veneer. However, these mixed species panel are assembled in such a way that the balanced design requirement is not affected by the lay-up of species. The veneers for any one of face, core or centre plies would be of the same species or belong to a group of species having similar physical properties. In such cases, there might be added restrictions on lay-up of veneers such as 'fir only for face veneers' or other restrictions imposed by specific customer orders. In this section, we illustrate the formulation of the PDM problem in such a situation and analyze its structure.

We define a lay-up alternative to be a plan which specifies the species or group of species for face, core and centre veneers in a plywood panel. For example, if fir (F), hemlock (H) and other species (O) form three groups of species and there are restrictions on the face veneer to be of species fir only, the possible lay-up alternatives (l) for a balanced design would be as specified in Table 10. As earlier, we let the design alternatives (j) specify the number of veneers of each thickness to be used in the panel.

Lay-up Alternative, l	Species for Face	Core	Centre
1	F	F	F
2	F	F	H
3	F	F	0
4	F	H	F
5	F	H	H
6	F	H	0
7	F	0	F
8	F	0	H
9	F	0	0

Table 10: Species Lay-up alternatives.

4.4.1 Formulation of the PDM Model

Define new decision variables and coefficients of the PDM problem as follows.

P_{ijl} = Quantity of plywood of type i, construction alternative j and lay-up alternative l (in number of sheets of standard size or equivalent).

r_{il} = Revenue for product i, lay-up alternative l (\$); If revenue is independent of l, $r_{il} = r_i$ for all l.

d_{il} = Demand for product i, lay-up alternative l (in number of sheets of standard size or equivalent); If demand is independent of l, $d_{il} = d_i$ for all l.

α_{ijkls} = Number of veneers of species s, thickness k, used in lay-up alternative l and construction alternative j for product i.

l = 1, 2 ... L; L is the number of lay-up alternatives.

All other variables and parameters of the problem remain unchanged. The α_{ijkls} are known numbers similar to a_{ijk} and are related to them by

$$\sum_s \alpha_{ijkls} = a_{ijk} \qquad \forall\, l = 1, 2, \ldots L \quad \text{and} \quad \forall\, i, j, k$$

The model would be

$$\text{Max} \quad \sum_i \sum_j \sum_l r_{il} P_{ijl} - \sum_s C_s y_s \sum_k x_k v_{ks}$$

s.t.

$$b_i^L - M(1-\delta_{ij}) \leqslant \sum_k a_{ijk} x_k \leqslant b_i^U + M(1-\delta_{ij})$$

$$\sum_j \delta_{ij} \geqslant 1$$

$$\sum_k y_s x_k v_{ks} \leqslant W_s$$

$$P_{ijl} - M\delta_{ij} \leqslant 0 \qquad \text{—— (4.41)}$$

$$\sum_i \sum_j \sum_l \alpha_{ijkls} P_{ijl} - v_{ks} \leqslant 0$$

$$\sum_j P_{ijl} \geqslant d_{il}$$

$$\delta_{ij} = \{0, 1\}$$

$$x_k \in T_k = \{T_k^1, T_k^2, \ldots T_k^{m(k)}\}$$

$$P_{ijl}, v_{ks} \geqslant 0 \quad \forall \ i, j, k, l \text{ and } s$$

This is again a non-linear mixed integer programming problem.

4.4.2 The Linear Subproblem

When the veneer thicknesses are assigned values $x_k = x_k^*$, $x_k^* \in T_k$, $k = 1, 2, \ldots K$ the resulting problem reduces to solving

$$\text{Max} \ \sum_i \sum_{j \in I_i} \sum_l r_{ijl} P_{ijl}$$

s.t.

$$\begin{aligned} &\sum_i \sum_{j \in I_i} \sum_l b_{ijsl} P_{ijl} \leqslant W_s \quad \forall \ s \\ &\sum_{j \in I_i} P_{ijl} \geqslant d_{il} \quad \forall \ i, l \\ &P_{ijl} \geqslant 0 \end{aligned} \qquad \text{—— (4.42)}$$

where, $b_{ijsl} = \sum_k y_s \alpha_{ijkls} x_k^*$, $r_{ijl} = r_{il} - \sum_s C_s b_{ijsl}$ and I_i is as defined by (3.2). In the above model we have assumed the orderfile to be dependent on the lay-up alternative. If it is independent of the lay-up alternative, the demand constraint in (4.42) is replaced by

$\sum_{l} \sum_{j \in I_i} P_{ijl} \geq d_i$, where, d_i is the demand for product i.

Irrespective of whether the orderfile is dependent on the lay-up alternative or not, the resulting subproblem is a linear program. b_{ijsl} is the quantity of wood of species s required to produce one unit of plywood type i using design alternative j and lay-up alternative l when the veneer thicknesses are $(x_1^*, \ldots x_K^*)$. Since b_{ijsl} and r_{ijl} are dependent on both species and the lay-up alternative, the subproblems do not reduce to any other simpler structure. Consequently, in an optimal solution to the linear subproblem, a plywood type may have more than one lay-up alternative and/or more than one design alternative. The optimal design and lay-up alternative for any particular plywood type would depend on the trade-off associated with the amount of wood required of each species, their relative cost and yield factors, the availability of logs of each species and the orderfile. This is illustrated by the following example.

<u>Example 4.2</u>: Consider a PDM subproblem with one plywood type, 7 Ply 22mm, with two lay-up alternatives (F, F, H) and (F, H, F) for face, core and centre respectively. The veneer thicknesses are 2.50, 3.10, 3.90 and 4.81 mm and the design alternatives are (5, 0, 0, 2) and (2, 3, 2, 0). Other coefficients of the model are: $C_F = 35.00$, $C_H = 30.00$, $Y_F = 0.006067$, $Y_H = 0.0072$, $W_F = 87.729$, $W_H = 55.080$ and $r_i = 8.0$.

Suppose the orderfile is independent of lay-up alternative and $d_i = 1000$. Then the solution of this subproblem is $P_{111} = P_{122} = 0$, $P_{112} = 500.0$ and $P_{121} = 500.0$. Suppose the orderfile is dependent on the lay-up with $d_{i1} = 500$, $d_{i2} = 500$ and W_H is changed from 55.080 to 60.000, all other coefficients remaining unchanged. Then the solution of the resulting subproblem would be $P_{111} = 0$, $P_{112} = 255.32$, $P_{121} = 531.14$ and $P_{122} = 244.68$ indicating that more than one design and more than one lay-up alternative can be in the final solution.

CHAPTER 5

5.1 Disjunctive Programming

The PDM model formulated in chapter two is a non-linear mixed integer (0-1) mathematical programming problem. The feasible region formed by the constraints of this model is non-convex due to the presence of integer variables δ_{ij} and x_k (through (2.1A) - (2.1C)) and non-linearities in the constraints (2.5). The efficiency of the implicit enumeration algorithm in seeking a global solution to such a problem resulted from the fact that by branching on x_k, the integer variables δ_{ij} were explicitly evaluated and the non-linearities in the constraints and objective function were reduced to linearities.

In recent years,much attention has been focussed on treating integer programming and a host of other non-convex programming problems as linear programs with logical conditions. An outgrowth of this approach is disjunctive programming, in which an integer programming problem can be transformed into an equivalent linear program with disjunctive constraints (Balas (1979)). For example, the 0-1 variables δ_{ij} introduced in the formulation of the PDM model for plywood thickness tolerance and design feasibility can be overcome by replacing constraints (2.2) - (2.4) by the logical condition

$$b_i^L \leqslant \sum_k a_{ijk} x_k \leqslant b_i^U \quad \text{for at least one } j, \text{ for all } i. \qquad \text{—— (5.1)}$$

As a set of constraints, this can be expressed by

$$b_i^L \leqslant \underset{j}{V} \sum_k a_{ijk} x_k \leqslant b_i^U \qquad \forall\ i \qquad \text{—— (5.2)}$$

Where the symbol V stands for disjunction implying that the constraint should be satisfied for at least one j. The non-convexity implied by these constraints can be illustrated by a simple example of one plywood type with two veneer thicknesses. Suppose the plywood type has 3 plies and the design alternatives are (3,0) and (2,1) for the two veneer thicknesses x_1 and x_2. Then, assuming that x_1 and x_2 are continuous variables, the feasible region in the x_1 - x_2 plane satisfying constraint (5.1) or (5.2) would be star-shaped as illustrated in figure 3. When more than one plywood type is included, the feasible region would be the intersection of such star-shaped region of each plywood type.

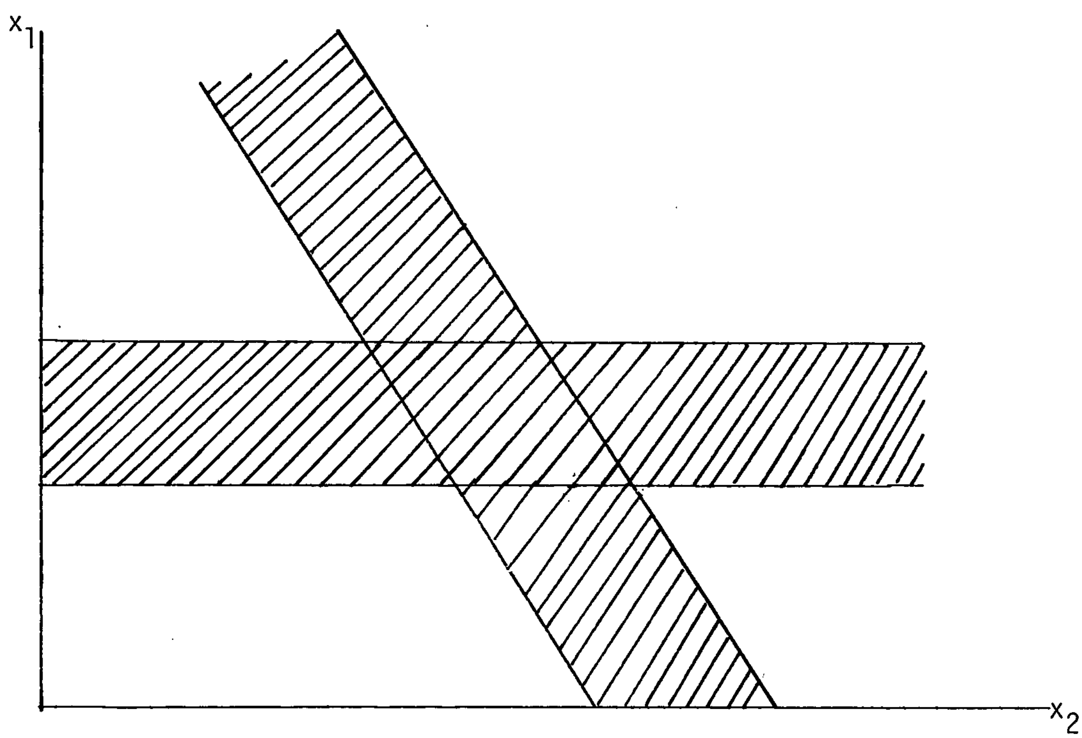

Fig. 3: Feasible Region (Shaded Area) of Plywood Design Constraint

The alternate approach of eliminating the integer variables does not overcome problems associated with non-convexity as it is implied by disjunction. However, the treatment of some integer programs and other non-convex programming problems as disjunctive programming or linear programming problems with logical conditions have lead to some interesting properties (Balas (1979)). In this chapter, we explore the possibilities of treating the PDM problem as a disjunctive programming problem. We study the implications of some of its properties on the PDM model.

5.2 The PDM as a Disjunctive Programming Problem

Expressing the plywood thickness tolerance and design feasibility constraints by (5.2) instead of (2.2) - (2.4) does not itself lead the PDM problem to a disjunctive programming problem since identifying the feasible design constraint (2.8) without δ_{ij} becomes complicated and the non-linearities in (2.5) continue to exist. However, from an alternative approach, the overall PDM problem can be cast as a disjunctive programming problem.

Recall from chapter 3 that when veneer thicknesses are assigned values the resulting subproblem is a linear program and, in particular, an LMCK given by (3.14). Since δ^*_{ij} and hence I_i are dependent on the set of veneer thicknesses through (3.1) and (3.2), the constraints of the LMCK are dependent on the set of veneer thicknesses. Suppose $X^* = (x^*_1, x^*_2, \ldots x^*_K)$ and $P^* = (P^*_{ij})$ is an optimal solution to the PDM problem obtained from the implicit enumeration algorithm. Observe then that for any other set of veneer thicknesses $X' = (x'_1, x'_2, \ldots x'_K)$,

$X' \neq X^*$, and associated δ'_{ij} and I'_i, $P^* = (P^*_{ij})$ need not be feasible for the corresponding LMCK given by

$$\text{Max} \sum_i \sum_{j \in I'_i} r'_{ij} P_{ij}$$

s.t.

$$\sum_i \sum_{j \in I'_i} b'_{ij} P_{ij} \leqslant W$$

$$\sum_{j \in I'_i} P_{ij} \geqslant d_i$$

$$P_{ij} \geqslant 0 \qquad \forall \; i, j \in I'_i$$

where, $b'_{ij} = y \sum_k a_{ijk} x'_k$ and $r'_{ij} = r_i - Cb'_{ij}$. Thus, the (P_{ij}) are also dependent on the set of veneer thicknesses and in an optimum solution to the PDM problem, what matters is that (P^*_{ij}) must be feasible for the LMCK associated with $X^* = (x^*_1, \ldots x^*_K)$. From these observations, the PDM problem can be formulated as a disjunctive programming problem as follows.

Suppose $X^h = (x^h_1, \ldots x^h_K)$, for various values of h, represent distinct sets of veneer thicknesses feasible for the design constraints (2.1) - (2.4). Let Q be the index set of all such h. The PDM model can equivalently be specified by the following disjunctive programming problem.

$$Z_0 = \text{Max } Z$$

s.t.

$$\underset{h \in Q}{\vee} \left\{ \begin{array}{l} Z - \sum_i \sum_{j \in I_i^h} r_{ij}^h P_{ij}^h \leqslant 0 \\ \sum_i \sum_{j \in I_i^h} b_{ij}^h P_{ij}^h \leqslant W \\ \sum_{j \in I_i^h} P_{ij}^h \qquad \geqslant d_i \\ \qquad P_{ij}^h \geqslant 0 \quad \forall\, i,\, j \in I_i^h \end{array} \right\} \tag{5.3}$$

where,

$$I_i^h = \{j \mid b_i^L \leqslant \sum_k a_{ijk} x_k^h \leqslant b_i^U \text{ and } X^h \text{ feasible}\} \tag{5.4}$$

$$b_{ij}^h = y \sum_k a_{ijk} x_k^h \tag{5.5}$$

$$r_{ij}^h = r_i - Cb_{ij}^h \tag{5.6}$$

The disjunction V indicates that the constraints in brackets should hold for at least one h. The number of elements in Q is the number of distinct feasible sets of veneer thicknesses, feasible for the design constraints (2.1) - (2.4) and is bound from above by $\prod_{k=1}^{K} m(k)$. The disjunction results in the constraint set of (5.3) being non-convex.

Our disjunctive program (5.3) deviates from standard problems (see, for example, Balas (1979)) in that in (5.3) the variables (P_{ij}^h) are dependent on h. However, (5.3) can be brought into the standard format by defining a variable set consisting of Z and (P_{ij}^h) for all h and adjusting the coefficients appropriately. A unique feature of

(5.3) is that the RHS of (5.3) is the same for all h while in standard disjunctive programming problems they would be dependent on h.

5.2.1 The Dual of the Disjunctive Program

Balas (1979) defines a 'dual' for the disjunctive programming problem and studies some of the relations between the original problem and the dual so defined. (This is not to be confused with the classical dual of LP). Interestingly, though the original problem is non-convex, the 'dual' of it is a convex linear programming problem. Following Balas, the 'dual' of (5.3) is given by

$$U_0 = \text{Min} \quad U$$

$$\text{s.t.} \quad \bigwedge_{h \in Q} \left\{ \begin{array}{ll} U - W\pi^h + \sum_i d_i t_i^h \geqslant 0 & \\ -r_{ij}^h \theta^h + b_{ij}^h \pi^h - t_i^h \geqslant 0 & \forall\, i,\ j \in I_i^h \\ \theta^h \geqslant 1 & \\ \pi^h,\ t_i^h,\ \theta^h \geqslant 0 & \forall\, i \end{array} \right\} \qquad \text{—— (5.7)}$$

where the symbol $\wedge$ stands for conjunction implying that the constraints in brackets should be satisfied for all $h \in Q$. (5.7) is a convex linear programming problem. Observe that a solution to (5.7) can be obtained by solving, for each h, the problem

$$U^h = \text{Min} \quad W\Pi^h - \sum_i d_i t_i^h$$

s.t.

$$b_{ij}^h \Pi^h - t_i^h \geqslant r_{ij}^h \qquad \forall \; i, j \in I_i^h \qquad (5.8)$$

$$\Pi^h, \; t_i^h \geqslant 0 \qquad \forall \; i$$

and it follows that

$$U_0 = \underset{h}{\text{Max}} \; [U^h] \qquad (5.9)$$

(5.8) is precisely the dual of an LMCK subproblem of the PDM problem with $x_k = x_k^h$, $k = 1, 2, \ldots K$. From the solution of the LMCK derived in chapter 3 and using complementary slackness conditions, the solution of (5.8) is given by

$$\Pi^h = \frac{r_{p.}^h}{b_{p.}^h} = \underset{i}{\text{Max}} \left\{ \frac{r_{i.}^h}{b_{i.}^h} \right\} \qquad (5.10)$$

$$t_i^h = b_{i.}^h \, \Pi^h - r_{i.}^h \qquad (5.11)$$

where,

$$b_{i.}^h \equiv \underset{j \in I_i^h}{\text{Min}} \{b_{ij}^h\} \qquad (5.12)$$

and

$$r_{i.}^h \equiv \underset{j \in I_i^h}{\text{Max}} \{r_{ij}^h\} = r_i - Cb_{i.}^h \qquad (5.13)$$

5.2.2 Relation between the Disjunctive Program and its Dual

Suppose

$$P_h = \{(P_{ij}^h) \mid (P_{ij}^h) \text{ satisfies constraints of } (5.3)\}$$

and

$D_h = \{(\Pi^h, t_i^h, \theta^h) \mid$ constraints of (5.7) are satisfied for $h\}$.

Assume that the following regularity condition holds.

Regularity Condition (Balas): If (5.3) is feasible and (5.7) is infeasible, then there exists $h \in Q$ such that $P_h \neq \phi$ and $D_h = \phi$.

Then, the relation between the disjunctive program and its dual can be characterized by the following theorem which we state without proof (for proof, see Balas (1979)).

Theorem 5.1 (Balas): Assume that the disjunctive program (5.3) and its dual (5.7) satisfy the regularity condition. Then exactly one of the following two situations hold:

a) Both problems are feasible; each has an optimal solution and $Z_0 = U_0$.

b) One of the problems is infeasible; the other one is either infeasible or has no finite optimum.

We assume in the sequel that both problems are feasible, that an optimum solution exists and that $Z_0 = U_0$. Now, since (5.7) is a linear program its traditional dual is given by

$$\eta_0 = \text{Max} \sum_h \eta^h$$

s.t.

$$\bigwedge_{h \in Q} \left\{ \begin{array}{l} \eta^h - \sum_i \sum_{j \in I_i^h} r_{ij}^h \bar{\mu}_{ij}^h \leqslant 0 \\ \sum_i \sum_{j \in I_i^h} b_{ij}^h \mu_{ij}^h - W\lambda^h \leqslant 0 \\ - \sum_{j \in I_i^h} \mu_{ij}^h + d_i \lambda^h \leqslant 0 \\ \mu_{ij}^h, \eta^h, \lambda^h \geqslant 0 \qquad \forall \quad i, j \in I_i^h \end{array} \right\} \quad \text{—— (5.14)}$$

and $\sum_{h \in Q} \lambda^h = 1.$

The dual of the 'dual' as given by (5.14) is a linear program with a block-angular structure, with one block of constraints for each h, linked through the common constraint $\sum_{h \in Q} \lambda^h = 1$. It follows from duality theory of linear programs that when (5.7) and (5.14) are feasible, $U_0 = \eta_0$. Observe that if P_{ij}^H, for some $H \in Q$ is feasible for (5.3) then the corresponding solution with $\mu_{ij}^H = P_{ij}^H$ and $\lambda^H = 1$ is feasible for (5.14).

In the block-angular structure of the constraints of (5.14), W and d_i occur in all the blocks once each for each h. λ^h in (5.14) acts as a scaling factor for different sets of veneer thicknesses X^h. Since the factors they scale are W and d_i, both independent of h, it follows that (5.14) will have an optimal solution in which $\lambda^{h'} = 1$ for some $h' \in Q$ and $\lambda^h = 0$ for all $h \in Q \setminus \{h'\}$. Exception to this solution is the possibility of multiple solution to (5.14) in which case more than one set of veneer thicknesses can yield the same objective function value. Barring situations of multiple solutions to (5.14), it follows that an optimal solution of (5.14) will also be an optimal solution to our original disjunctive programming problem (5.13). (Incidentally, this situation need not hold good for all disjunctive programs since the RHS of many disjunctive programs are dependent on h. The dual of the 'dual' for such disjunctive programs may only be a relaxation of the original problem).

Though the PDM model can be formulated as a disjunctive program and transformed to an equivalent linear program as in (5.14), it does not offer easier solutions for practical situations due to several reasons. First, the number of possible elements in Q is generally very large. Secondly, for each such feasible set of thickness, b_{ij}^h and r_{ij}^h should be computed explicitly and P_{ij}^h for $j \in I_i^h$ should be identified. Third, the number of rows and columns of the LP given by (5.14) would be substantially large. For example, for the plywood mill data of chapter 3, with K = 4, there can be a maximum of 177147 blocks of h, each block having anywhere between 14 to 170 variables and 14 constraints. The disjunctive program approach does not therefore offer any computational advantages over the implicit enumeration algorithm of chapter 3. Nevertheless, it shows that the PDM problem can be cast as a special case of a disjunctive programming problem and gives an insight into its properties.

CHAPTER 6

6.1 The Optimum Number of Veneer Thicknesses

The PDM model presented in the earlier chapters determines the veneer thicknesses, associated plywood designs and the corresponding product mix for a given number of veneer thicknesses. The maximum net revenue so obtained is dependent on the number of peel thicknesses, K, and does not include the setup or overhead costs associated with it. Generally, the lower the number of veneer thicknesses, the lower is the operational costs associated with peeling, drying, storage, assembly and record-keeping but they also result in higher wood losses in the form of excess thickness in plywood and lower revenue. The higher the number of veneer thicknesses, the higher is the operational costs but they also result in higher revenue. For any particular mill therefore, the overall benefits are determined by the trade-off between the two.

6.1.1 Setup Costs Associated with More Peel Thicknesses

a) Peeling Logs to Veneer

Everytime a veneer thickness is changed at the peeling lathe, there would be a setting time required to change the lathe settings such as the horizontal gap, vertical gap or exit gap and pressure bar compression to the appropriate thickness. This is followed by changes in sorting at the clipper where the veneers are clipped to different sizes and sorted by the thickness and size of the veneer. The higher the number of peel thicknesses, the higher would be the setup costs in peeling and clipping of veneers.

b) Drying of Veneers

When veneers are dried to reduce the moisture content, the driers are to be adjusted to proper settings of the temperature and drying speed/ time in conventional dryers. These settings depend on species, thickness and the initial moisture content of veneers. There would be a time loss associated with the changes in dryer setup, each time a change in veneer thickness is made. Again, more veneer thicknesses imply higher setup costs in drying.

c) Handling and Storage of Veneers

This problem is multiplied several times since veneers are classified on the basis of species, sizes (full, half, strips and fishtails) and surface quality characteristics (based on knots, surface smoothness, lathe checks and other factors). The storage and handling costs double since veneers are stored both in the green end (after peeling but before drying) as well as the dry end (after drying but before assembly).

d) Assembling Veneers

In the gluing and assembly of veneers, higher K implies more space and material handling facilities. More veneer thicknesses lead to more design alternatives. Changes in design alternatives result in higher scheduling costs and higher setup time between batches.

e) Record-Keeping

Increased number of peel thicknesses result in increased costs of record-keeping. Routine scheduling problems related to conversion of different species and types of logs to different veneer thicknesses, assigning veneers of different species, size, grade and thickness to different dryers and assembly of a combination of these into several types of plywood become more complicated with the addition of each peel thickness.

All the above factors indicate that the setup costs increase, perhaps exponentially, with increase in the number of peel thicknesses.

6.1.2 Benefits Associated with More Peel Thicknesses

Generally, more peel thicknesses result in more design alternatives per plywood type. Consequently, they result in lower wood loss in the form of excess thickness in plywood and higher revenues. Suppose Z_K is the maximum possible net revenue obtainable from the PDM model when the number of veneer thicknesses is K. That is,

$$Z_K = \text{Max} \sum_i \sum_j r_i P_{ij} - Cy \sum_{k=1}^{K} x_k v_k \qquad \text{—— (6.1)}$$

subject to the usual constraints as described in chapter two. [(6.1) applies to single species model; If more than one species is used, the appropriate objective function value should be used]. Let $Z_K = -\infty$, if the optimization model is infeasible for some K. When the problem is feasible and Z_K is finite, it satisfies the relation

$$Z_{K+1} \geqslant Z_K \qquad \text{for all } K \qquad \text{—— (6.2)}$$

(6.2) follows from the fact that with K + 1 thicknesses, one will have at least as many design options as with K plus the added benefit of the extra peel thickness.

6.1.3 Upper Bound on Z_K

Though the maximum net revenue increases with increase in the number of peel thicknesses, beyond a certain value of K, there would not be any improvement in the value of the objective function. In fact, it can be postulated that the marginal rate of increase in Z_K decreases as K increases, ultimately vanishing at some value of K. Though it is difficult to determine such a value of K, the exact value of the upperbound, $\bar{Z}_0$, for the objective function value Z_K, for all K, can be calculated for the single species model of the PDM problem.

Recall from chapter 3 that when veneer thicknesses are assigned values x_1^*, x_2^*, ... x_K^*, the resulting subproblem is a LMCK problem. Further, in an optimal solution to this LMCK problem, for each i, P_{ij} would be non-negative only for that j for which b_{ij} is minimum. Observe that when veneer thicknesses are assigned values $x_k = x_k^*$, $x_k^* \in T_k$, $k = 1, 2, \ldots K$, it must satisfy

$$b_i^L \leqslant \sum_{k=1}^{K} a_{ijk} x_k^* \leqslant b_i^U \qquad \text{for at least one j and for all i}$$

Thus, for each i, $\sum_{k=1}^{K} a_{ijk} x_k^*$ is bound from below by b_i^L, which is independent of K. Then, it follows that, for each i, $b_{ij} = y \sum_k a_{ijk} x_k^* \geqslant y\, b_i^L$ and $r_{ij} = r_i - Cb_{ij} \leqslant r_i - Cyb_i^L$. Consequently, for any i and j, $\{r_{ij}/b_{ij}\}$ is bound from above by

$$\frac{r_p - Cyb_p^L}{yb_p^L} = \underset{i}{\text{Max}} \left\{ \frac{r_i - Cyb_i^L}{yb_i^L} \right\} \qquad \text{—— (6.3)}$$

From (6.3) and the solution to the LMCK problem as derived in chapter 3, it follows that the upperbound on Z_K, for any K, is given by

$$\overline{Z}_0 = \sum_{i \neq p} \overline{r}_i d_i + \left(\frac{W - \sum_{i \neq p} \underline{b}_i d_i}{\underline{b}_p} \right) \overline{r}_p \qquad \text{—— (6.4)}$$

where, $\overline{r_i} = (r_i - Cyb_i^L)$, $\underline{b}_i = yb_i^L$ and $(\overline{r}_p / \underline{b}_p) = \underset{i}{\text{Max}} \{\overline{r}_i / \underline{b}_i\}$. If a correction factor, C_f, is used for shrinkage and compression, then the plywood thickness is given by $C_f \sum_{k=1}^{K} a_{ijk} x_k^*$ and in that case b_i^L in (6.3) and (6.4) is replaced by (b_i^L / C_f). Intuitively, the upperbound being obtained when $C_f \sum_k a_{ijk} x_k^* = b_i^L$ for all i is meaningful since it implies that all plywood types are assembled to the minimum permissible thickness and there is no loss in the form of excess thickness in plywood.

6.1.4 Design Efficiency

Suppose Z_D represents the maximum net revenue for a set of veneer thicknesses and plywood designs and $\overline{Z}_0$ is the upperbound on the value of Z_K for all K. Then the ratio

$$E_D = \frac{Z_D}{\overline{Z}_0} \times 100 \qquad \text{—— (6.5)}$$

can be termed the design efficiency, a percentage rating for this set of peel thicknesses and plywood designs. Values of E_D closer to 100% indicate better peel thicknesses and designs. Since $\overline{Z}_0$ is independent of K but Z_D is dependent on it, E_D measures the efficiency ignoring the setup costs associated with K. Nevertheless, it gives an indication as to how good a set of veneer thicknesses and plywood designs are and the extent of further improvements, if any, from higher number of peel thicknesses.

6.1.5 Determining the Optimum Number of Thicknesses

For any particular mill, let O_K be the total set up cost associated with all the factors described in section 6.1.1, when the number of veneer thicknesses is K. Let Z_K be the corresponding maximum net revenue obtainable from the PDM model. Then $R_K = Z_K - O_K$ gives the net benefit associated with K. The optimum number of thicknesses for the mill is given by that K for which the net benefit R_K is maximum. Observe that Z_K is bound from above by $\overline{Z}_0$ while O_K is unrestricted.

The derivation of the upper bound $\overline{Z}_0$, the design efficiency and the optimum number of veneer thicknesses is illustrated through the following example.

Example 6.1: The plywood mill data used in chapter 3 is again considered in this example. The plywood types (N = 12), the lower tolerance on plywood thicknesses (b_i^L), the revenue (r_i) and the orderfile (d_i) are as listed in appendix II. For these plywood types, $\underline{b_i}$, $\overline{r_i}$ and $(\overline{r_i}/\underline{b_i})$

required to determine $\overline{Z}_0$ were computed using (6.4) and are presented in Table 11.

Plywood Type plies - Thick	$\underline{b_i}$	$\overline{r_i}$	$(\overline{r_i}/\underline{b_i})$
3 - 7.5	.04518	2.7188	60.1791
3 - 9.5	.05809	2.7670	47.6361
5 - 12.5	.07745	3.3893	43.7625
5 - 15.5	.09681	4.2117	43.5043
7 - 18.5	.11617	4.8340	41.6105
7 - 20.5	.12908	5.2822	40.9219
7 - 22.5	.14199	5.5304	38.9499
9 - 23.5	.14844	6.1045	41.1240
9 - 25.5	.16135	6.4528	39.9923
9 - 27.5	.17426	7.0010	40.1759
9 - 28.5	.18071	7.2751	40.2579
9 - 30.5	.19362	7.8233	40.4054

Table 11: Computations for the Upperbound $\overline{Z}_0$

From table 11, we have that

$$(\overline{r_1}/\underline{b_1}) = \underset{i}{\text{Max}} \{\overline{r_i}/\underline{b_i}\} = 60.1791$$

so that p = 1. Then, from (6.4) we have

$$\overline{Z}_0 = \sum_{i \neq 1} \overline{r_i} d_i + \left(\frac{300000 - \sum_{i \neq 1} \underline{b_i} d_i}{.04518} \right) 2.7188$$

$$= \$14{,}679{,}760$$

Thus, for the plywood mill data of chapter 3 (appendix II), the maximum net revenue can never exceed \$14,679,760, irrespective of the number of veneer thicknesses used.

Computer codes of the implicit enumeration algorithm of the PDM model were used with the mill's data to determine Z_K for values of K from 1 to 5. The maximum net revenue Z_K, and the quantity of wood loss in the form of excess thickness derived from the PDM model are given in Table 12. For comparison purposes, the corresponding values for the existing set of veneer thicknesses and associated designs are also presented. The design efficiencies for each one of these sets were computed using the value of $\overline{Z}_0$ = 14,679,760 obtained above and are presented in column 4 of table 12.

K	Maximum Revenue ($), Z_K	Wood Loss in Excess Thickness (cu. mtr.)	Design Efficiency (%), E_D	Estimated Setup Cost* ($), $\hat{O}_K$	Net Benefit ($) $\hat{R}_K$
Existing (4)	13,416,694	7,944.7	91.40	3,714,770	9,701,924
1	Infeasible	-	-	1,118,870	-
2	Infeasible	-	-	1,669,160	-
3	13,930,670	3,612.1	94.90	2,490,090	11,440,580
4	14,337,370	1,647.4	97.67	3,714,770	10,622,600
5	14,562,530	558.3	99.20	5,541,790	9,020,740

* Estimated from $\hat{O}_K = 750{,}000\exp^{0.4K}$

Table 12: Optimum Number of Veneer Thicknesses and Design Efficiency

For K = 1 and 2, the problem was infeasible implying that the products of the mill cannot be assembled within specifications using one or two peel thicknesses. For K = 3, the optimal veneer thicknesses were 2.6, 3.2 and 4.4mm and the maximum net revenue was $13,930,670 (appendix IV). This is higher than the corresponding value for the existing set of four thicknesses by 3.83%. This indicates that not only better designs existed for the mill but substantial further savings in the setup costs associated with the fourth peel thickness were also possible. When K = 4, the optimum veneer thicknesses were 2.5, 3.1, 3.9 and 4.8 mm and the corresponding net revenue was $14,337,370 (appendix III). Thus, if the mill intends keeping four veneer peel thicknesses for any reason, a better set of thicknesses and plywood designs resulting in additional revenue of $920,676 (equal to 6.86% higher than that for the current set) exists.

When K = 5, the optimal veneer thicknesses were 2.4, 2.7, 3.2, 3.7 and 4.8 mm and the associated net revenue, Z_K, was $14,562,530. Since there were 34 feasible design alternatives for the 12 plywood types of the mill, detailed results of the model are not presented. The design efficiency attained was 99.20% indicating that further benefits from any higher number of peel thicknesses would not be significant. The wood loss in the form of excess thickness in plywood was 558.3 cubic metres. This set of five veneer thicknesses results in additional revenue of $225,160 over the one for four veneer thicknesses obtained from the PDM model. However, this increase in revenue should be compared with the additional setup costs associated with the fifth peel thickness.

Data on setup costs associated with different number of peel thicknesses were not available for the mill. For comparison purposes, some hypothetical data on setup costs were used along with the existing data on revenues. It was postulated that the setup costs were exponentially related with the number of peel thicknesses by $\hat{O}_K = 750{,}000\exp^{0.4K}$. The estimated setup costs obtained under this assumption are given in column 5 of Table 12. The last column of this table gives $\hat{R}_K = Z_K - \hat{O}_K$, the estimated net benefits. For this set of data, the maximum net benefit occurs at K = 3 implying that the optimum number of veneer thicknesses is three.

6.2 Alternate Face Veneer Thickness

In the formulation of the PDM model in chapter two, we had treated x_1 to be the face veneer thickness by convention. This convention follows from the fact that there are more restrictions on the thickness and surface quality characteristics of a face veneer than those on the inner plies in a plywood sheet. The face veneer should normally be a full size sheet with blemish-free surface whereas small sized veneers from strips and fishtails can be used as core or centre plies only. From manufacturing considerations, having thin veneers as face veneer is advantageous since more full size veneer sheets can be obtained from a given log for a thin veneer rather than that for a thick veneer. Further, repairs or rework caused by factors such as knot holes, pitch pockets or splits are easier with thin veneers than with thick veneers. In addition to this, for standard construction (COFI (1978)), the face veneers must have the grain direction along the length of the panel implying that for face veneers the logs must be peeled to the full length of the panel while that for core veneers they can be in lengths relative to the width of the panel.

The convention that x_1 is the face veneer thickness is not a serious limitation of the capabilities of the PDM model or the implicit enumeration algorithm to solve it. If more than one thickness can be used for face veneer, subject to all of them meeting the relevant specifications, such a situation can also be incorporated within the framework of the PDM model. The implicit enumeration algorithm requires a few modifications in that case but its effectiveness would virtually be the same. We illustrate these in the following paragraphs.

Recall that most plywood mills use three or four veneer thicknesses. Suppose there can be two face veneers, x_1 and x_2, then it leads to few more design alternatives which can be obtained by interchanging the columns associated with a_{ij1} and a_{ij2} in tables 3 - 6. However, this can result in some of the design alternatives already in tables 3 - 6 being duplicated and such alternatives may be excluded. For example, for a five-ply plywood with K = 3, when additional design alternatives are generated by interchanging a_{ij1} and a_{ij2} in table 4, the designs represented by the vectors (2, 3, 0), (3, 2, 0) and (2, 2, 1) for (a_{ij1}, a_{ij2}, a_{ij3}) would be repeated. Eliminating such duplications, it can be verified that for K = 3 the resultant number of design alternatives (n_i) would be 6, 15, 15 and 18 for plywood made with 3, 5, 7 and 9 plies respectively.

In solving the PDM model using the implicit enumeration algorithm of chapter 3, some of the branching tests need modification. Treating x_f to be the face veneer thickness for f = 1, 2, the bounds derived in (3.41) and (3.42) would change, respectively to

$$x_f \leqslant \hat{X}_f = \text{Min}\left[\underset{i}{\text{Min}}\left\{\frac{b_i^U - (L_i-2)T_1}{2}\right\},\ T_f^{\max}\right] \qquad \text{— (6.6)}$$

$$x_K \geqslant \check{X}_K = \text{Max}\left[\underset{i}{\text{Max}}\left\{\frac{b_i^L - 2\hat{X}_f}{L_i - 2}\right\},T_K^{\min}\right] \qquad \text{— (6.7)}$$

Where, T_f^{max} is the maximum permissible face veneer thickness. Following arguments similar to that in chapter 3, we can show that the branching test (3.40) would be replaced by the following two tests:

$$x_1 \leq x_2 \qquad (6.8)$$

and

$$x_{s_1} \leq x_{s_2} \qquad \text{for } 3 \leqslant s_1 < s_2 \leqslant K \qquad (6.9)$$

With the modifications described above, the plywood mill data of chapter 3 (appendix II) was used to demonstrate the effectiveness of the PDM model with alternate face veneer thicknesses for K = 3. The results obtained from this modified model are presented in appendix V. The optimum veneer thicknesses in this case were 2.5, 3.1 and 3.8 mm with both 2.5 and 3.1 mm veneers being used as face veneers. It can be observed from these results that the plywood types 3 ply 9.5 mm, 9 ply 28.5 mm and 9 ply 30.5 mm must be assembled using 3.1 mm only as the face veneer and that the plywood types 3 ply 7.5 mm and 5 ply 12.5 mm must be assembled using 2.5 mm as the face veneers. For all other types of plywood, alternate designs having either2.5 mm or 3.1 mm as face veneers existed. However, all these designs are variables for the LMCK problem and as shown in chapter 3, the solution of the LMCK would be such that only one design alternative is used for each plywood type. The maximum net revenue obtained for this model was $14,387,280. However, it should be noted that this figure does not include the costs associated with keeping two veneer thicknesses as face veneers. These results are presented here only to demonstrate that the PDM model and the implicit enumeration algorithm to solve it can be used under varying circumstances.

CHAPTER 7

7.1 Conclusions

In this dissertation, a real-world problem of practical significance which was hitherto not considered due to its complexity has been formulated and solved using a quantitative approach. The importance of the problem stems from the fact that a processed material with value added from manufacture is lost implicitly due to the non-availability of better methods of evaluation. In the wake of dwindling supply of timber resources and fluctuating and competitive market structure the need for efficient utilization of the raw material is all the more greater now than it was ever before.

The problem of determining a set of veneer thicknesses, associated plywood designs and product mix which maximizes the net revenue for a plywood mill has been formulated as a mathematical programming model. A method of evaluating all feasible plywood designs for a set of veneer thicknesses is developed. The non-linear mixed integer (0-1) programming problem so formulated is solved for a global solution using an implicit enumeration algorithm. The efficiency of this algorithm arises from its ability to exploit the structures in the model. A computer code is written and data from a plywood mill is analyzed to demonstrate the practicality of the model.

Variations and extensions of the model under different circumstances have been considered and their solution procedures have been analysed. It is shown that the PDM problem is a non-convex programming problem which can be cast as a special case of a disjunctive program. Following the development of the mathematical programming model and its solution procedure, ethods to determine the efficiency of plywood designs and the optimum number of veneer thicknesses for a mill are developed.

The implication of this dissertation are straight-forward. A plywood mill can use its data on log availability, cost of logs, yield factor list of plywood types, orderfile and such other factors in the PDM model to derive the optimal veneer thicknesses, associated plywood designs and the produce mix. If inclusion of factors such as log grades, veneer sizes and plywood grades prove to be computationally expensive, an abridged version of the PDM model may be used. The veneer thicknesses and plywood designs obtained as a solution of this abridged PDM model can be used as input in a detailed linear programming model to verify if they are indeed better than the existing ones. In this way, the model provides alternatives which never existed before. In the absence of mill LP models, the PDM model can be used as a basis for decisions on the choice of veneer thicknesses, plywood designs and the product mix. When all relevant factors are considered, the PDM model can, at worst, end up with the set of veneers and designs currently used by a mill. The computational time and money involved in generating a solution to the PDM model is relatively insignificant when compared with the potential benefits it can bring about.

BIBLIOGRAPHY

Bailey, G.R. 1970. Log Allocation by Dynamic Programming. Ph.D Thesis, University of British Columbia, Vancouver, British Columbia.

Bailey, G.R. 1972. A New Approach to an old Problem - Log Allocation by Dynamic Programming. Pulp and Paper Magazine of Canada, 73 (9) T247 - T249.

Bailey, G.R. 1973. Wood Allocation by Dynamic Programming. Canadian Forestry Service, Publication #1321, Ottawa, Ontario.

Balas, E. and P.L. Ivanescu (Hammer). 1964. On the Generalized Transportation Problem. Management Science, 11 (1), 188-202.

Balas, E. 1966. The Dual Method for the Generalized Transportation Problem. Management Science, 12 (7), 555-568.

Balas, E. 1979. Disjunctive Programming. Annals of Discrete Mathematics, 5, 3 - 51.

Baldwin, R.F. 1975. Plywood Manufacturing Practices. Miller-Freeman Publications, San Francisco, California.

Bare, B.B. 1971. Applications of Operations Research in Forest Management; A Survey. Quantitative Science Paper #26, Centre for Quantitative Science in Forestry, Fisheries and Wildlife, University of Washington, Seattle, Washington.

Bethel, J.S. and C. Harrel. 1957. The Application of Linear Programming to Plywood Production and Distribution. Forest Products Journal, 7 (7), 221-227.

Biblis, E.J., S-T. Hsu and Y-M. Chiu. 1972. Comparison of Certain Structural Properties among 3-Ply, 4-Ply and 5-Ply ½ inch Southern Plywood Sheathing. Wood and Fiber, 4 (1), 13-19.

Brackely, A.M. 1968. An Investigation of the Yield of Rotary Cut Veneer and Plywood from Eastern Spruce under Selected Conditions of Manufacture. M.S. Thesis, University of Maine, Orono, Maine.

Briggs, D.G. 1978. A Dynamic Programming Model for Bucking Tree Stems into Logs. Tropical Forest Utilization System,Contribution #30, Institute of Forest Products, University of Washington, Seattle, Washington.

COFI. 1978. Exterior Plywood Metric Manufacturing Standards and Mill Guide. Council of Forest Industries of British Columbia, Vancouver, British Columbia.

Colebeck, H.G.M. and P.L. Northcott. 1958. Some Factors Influencing the Design of Douglas-fir Plywood Panels. Forest Products Journal, 8, 226-229.

Conway, S. 1978. Timber Cutting Practices; (3rd Edition). Miller-Freeman Publications, San Francisco, California.

Dantzig, G.B. 1963. Linear Programming and Extensions. Princeton University Press, Princeton, New Jersey.

Diaz, R.E. 1974. Bibliography of Operations Research in Forestry Industries. College of Forest Resources, University of Washington, Seattle, Washington.

Dobie, J. and W.V. Hancock. 1972. Veneer Yields from B.C. Interior Douglas-fir and White Spruce. Canadian Forest Industries, 92(7), 32-33.

Dobson, P.O. 1971. How MB Allocates Logs by Linear Programming. Canadian Forest Industries, 91 (2), 44-45.

Donnelley, R.H. 1966. Linear Programming in Plywood Manufacturing. Seminar on OR in Forest Products Industry, IBM Corporation, Los Angeles, 22-39.

Eisemann, K. 1964. The Generalized Stepping Stone Method for the Machine Loading Model. Management Science, 11 (1), 154-176.

Elam, J., F. Glover and D. Klingman. 1979. A Strongly Convergent Primal Simplex Algorithm for Generalized Networks. Mathematics of Operations Research, 4 (1), 39-59.

Everett, J.P. 1967. An Experimental Application of Linear Programming to Plywood Production. M.F. Thesis, University of Washington, Seattle, Washington.

Field, D.B. 1976. Applications of Operations Research to Quantitative Decision Problems in Forestry and the Forest Products Industries - A Bibliography (Fourth Revision). School of Forestry and Environmental Studies, Yale University, New Haven, Connecticut.

Field, D.B. 1977. Linear Programming; Out of the Classroom and into the Woods. Journal of Forestry, 75 (6), 330-334.

Garfinkel, R.S. and G.L. Nemhauser. 1972. Integer Programming. John Wiley and Sons, New York.

Glover, F. and D. Klingman. 1973. A Note on Computational Simplifications in Solving Generalized Transportation Problems. Transportation Science, 7, 351-361.

Glover, F. and D. Klingman. 1979. A O(n log n) Algorithm for LP Knapsacks with GUB Constraints. Mathematical Programming, 17 (3), 345-361.

Hawkins, B.T. and L.N. Clarke. 1970. Allowance for Trim When Making Plywood. The Australian Timber Journal, 36 (9), 39.

Heiskanen, V. 1966. Studies on the Defects of Birch, Their Influence on the Quality and Quantity of Rotary cut Veneer and Their Considerations in Veneer Birch Grading. Acta Forestalia Fennica, 80 (3), 1-128.

Holecek, D.F. 1975. A Systems Model of an Integrated Forest Products Firm with Decision Applications. Ph.D Dissertation, University of California, Berkeley, California.

Holmes, S. 1976. Introduction to Operations Research Applied to Forest Products Industries. Forest Products Journal, 26 (1), 17-22.

Kennington, J.L. and R.V. Helgason. 1980. Algorithms for Network Programming. John Wiley and Sons, New York.

Klamecki, B.E. 1978. Utilising Solar Energy in the Forest Products Industry. Forest Products Journal, 28 (1), 14-20.

Klamecki, B.E. 1978. Optimization of Veneer Lathe Settings - a Non-linear Programming Approach. Wood Science, 10 (4), 220-224.

Koenigsberg, E. 1960. Applying Linear Programming to the Plywood Industry. Forest Products Journal, 10 (9), 481-486.

Kotak, D.B. 1976. Application of Linear Programming to Plywood Manufacture. Interfaces, 7 (1), 56-68.

Lasdon, L.S. 1970. Optimization Theory for Large Systems. McMillan Publishing Company, New York.

Lee, M.H. 1968. Linear Programming Analysis Applied to a Selected Plywood Manufacturing Firm. M.B.A. Thesis, University of British Columbia, Vancouver, British Columbia.

Lefebvre, E. 1978. How Improper Log Bucking Reduces Lumber Revenues. Canadian Forest Industries, 98 (9), 21-25.

Lourie, J.R. 1964. Topology and Computation of the Generalized Transportation Problem. Management Science, 11 (1), 177-187.

Luken, T.P. 1978. Markov Chains as Planning Tools for Long Log Merchandising and Sawmilling. Paper Presented at ORSA/TIMS meeting, Los Angeles, California (In Luken et al. (1980), Reference Cited).

Luken, T.P., A. Otten and P.S. Haugen. 1980. Quick and Clean; An Aspect of IE/OR Interface. Interfaces, 10 (5), 1-6.

Martin, A.J. and P.E. Sendak. 1973. Operations Research in Forestry - A Bibliography. USDA Forest Service General Technical Report NE-8, Northeast Forest and Range Experiment Station, Upper Darby, Pennsylvania.

McKillop, W. 1974. An Operations Research Study of an Integrated Wood Processing Firm in California. Paper Presented at FPRS Meeting, Northern California Section, # NC-74-S26, Redding, California.

Meriluoto, J. 1965. The Influence of Raw Material Factors on the Quantity and Quality of Rotary cut Birch Veneer. Acta Forestalia Fennica, 80 (1), 1-155.

Nagaraju, S., B.G. Raghavendra and J. Venkataraman. 1974. Green Veneer Yield in Rotary Cutting. IPIRI Journal, Journal of the Indian Plywood Industries Research Institute, 4 (4), 141-151.

Norris, C.B., F. Werren and F.R. McKinnon. 1961. The Effect of Veneer Thickness and Grain Direction on the Shear Strength of Plywood. U.S. Forest Products Laboratory, Report # FPL-1801, Madison, Wisconsin.

Parasin, A.V. 1976. Experimental and Theoretical Comparison of 4 vs. 5 Ply and 6 vs. 7 Ply Plywood Construction by Stiffness and Moment Carrying Capacity. Report #108, Council of Forest Industries of British Columbia, North Vancouver, British Columbia.

Phillips, D.T. and A. Garcia-Diaz. 1981. Fundamentals of Network Analysis. Prentice-Hall, Englewood Cliffs, New Jersey.

Pnevmaticos , S.M. and S.H. Mann. 1972. Dynamic Programming in Tree Bucking. Forest Products Journal, 22 (2), 26-30.

Raghavendra, B.G. and S. Nagaraju. 1975. Optimising the Trim Allowance in Plywood Manufacture. Journal of the Indian Association for Quality and Reliability, 3, 119-124.

Raghavendra, B.G. 1979. A Survey of Operations Research Applications in Wood Products Industries. Term Paper Submitted to Forestry 559: OR in Forestry; Faculty of Forestry, University of British Columbia, Vancouver (unpublished).

Ramalingam, P. 1976. A Branch and Bound Approach to the Tree Bucking Problem. Paper Presented at the ORSA/TIMS Meeting, Anaheim, California (In Briggs (1978), Reference Cited).

Ramsing, K.D. 1965. A Linear Programming Model for the Allocation of Logs for the Manufacture of an Optimum Plywood Mix. D.B.A. Thesis, University of Oregon, Eugene, Oregon.

Ramsing, K.D. 1968. Linear Programming for Plywood Mix Problem. Forest Products Journal, 18 (4), 98-101.

Raizada, H.C. and J.C. Nautiyal. 1974. An Input-Output Model of Ontario Forest Based Industries. Canadian Journal of Forest Research 4 (3), 372-380.

Reimer , D.R. 1969. Development of a Leontief Input-Output Model and its Application in the Analysis of the Economic Impact of a New Forest Industry. Ph.D Dissertation, Purdue University, Lafayette, Indiana.

Resch, H. and H.L. Scheurman. 1977. Computer Simulation of the Softwood Veneer Drying Operation. Forest Products Journal, 27 (2), 32-35.

Sitter, R.M. 1969. LP for Better Profit Levels. BC Lumberman, 53 (5), 31-35.

Taha, H. 1971. Operations Research. MacMillan Publishing Company, New York.

Tobin, L.R. and J.S. Bethel. 1969. Veneer Recovery Prediction and Analysis Through Computer Simulation. Wood and Fiber, 1 (2), 97-107.

Truemper, K. 1976. An Efficient Scaling Procedure for Gains Networks. Networks, 6, 151-160.

Tyre, G.L. and G.D. Screpetis. 1978. System for Roundwood Inventory and Control of Veneer, Sawn timber and Pulpwood Volumes. Forest Products Journal, 28 (1), 40-41.

Wagner, H.M. 1975. Principles of Operations Research; (2nd Edition). Prentice-Hall, Englewood Cliffs, New Jersey.

Wellwood, E.W. 1971. Scheduling and Control of a Douglas Fir Plywood Plant Aided by Linear Programming. Forest Products Journal, 21 (11), 42-50.

Western Forests Products Laboratory. 1978. Program Review 1977-78. Western Forest Products Laboratory (Now, Forintek Canada Corporation), Vancouver, British Columbia.

Wood, A.D. 1963. Plywoods of the World; Their Development, Manufacture and Application. Johnston and Bacon Limited, London, England.

Woodfin, R.O. Jr. 1973. Wood Losses in Plywood Production - Four Species. Forest Products Journal, 23 (9), 98-106.

Yaptenco, R.C. and A.E. Wylie. 1970. A Quantitative Approach to Plywood Production Scheduling. Forest Products Journal, 20 (3), 54-59.

Zemel, E. 1980. The Linear Multiple Choice Knapsack Problem. Operations Research, 28 (6), 1412-1423.

APPENDIX I

Listing of the FORTRAN Program for the Implicit Enumeration Algorithm; Four Veneer Thickness

```
C
C
C  IMPLICIT ENUMERATION ALGORITHM FOR THE PLYWOOD
C  DESIGN AND MANUFACTURING (PDM) MODEL
C
C      B.G.RAGHAVENDRA
C      FACULTY OF COMMERCE AND BUS. ADMN.
C      UNIVERSITY OF BRITISH COLUMBIA
C      VANCOUVER, BC
C
C  NUMBER OF VENEER THICKNESSES (K) = 4
C  THIS CODE CAN CURRENTLY HANDLE UPTO 25 TYPES
C  OF PLYWOOD MADE WITH 3,5,7 OR 9 PLIES. IT CAN
C  BE EXTENDED TO ANY NUMBER OF PLIES AND PLYWOOD
C  TYPES.
C
$COMPILE
      REAL X(4),XSTAR(4),BL(25),BU(25),T(50),TPLY(25),THICK(25,16)
      REAL TW(25,16),TP(25),R(25),B(4),DEM(25),PLY3(4,4),P(25,16)
      REAL PLY5(16,4),PLY7(16,4),PLY9(16,4),BSUM(25,16),RSUM(25,16)
      REAL WOOD(25),REV(25),PSTAR(25,16),V(4)
      REAL TSTAR(25,16),RSTAR(25,16),TWSTAR(25,16),BSTAR(4),LSTAR(4)
      REAL XMIN(25),XMAX(25),TH(25,16),BD(25),PD(25),WD(25),TS(25)
      INTEGER ID(25,16),IDENT(25)
      INTEGER IDELTA(25,16),IPLY(25),ISTAR(25,16),ITEST(25),JBEST(25)
C
C  PLY3(J,K),...PLY9(J,K) ARE THE DESIGN COEFFICIENTS
C
      DO 10 J=1,4
   10 READ,(PLY3(J,K),K=1,4)
      DO 11 J=1,16
   11 READ,(PLY5(J,K),K=1,4)
      DO 12 J=1,16
   12 READ,(PLY7(J,K),K=1,4)
      DO 13 J=1,16
   13 READ,(PLY9(J,K),K=1,4)
C
C  INPUT DATA FOR THE PROBLEM; 'NUMBER' IS THE NUMBER
C  OF PLYWOOD TYPES FOLLOWED BY DETAILS OF ITS NUMBER
C  OF PLIES,THICKNESS,LOWER AND UPPER TOLERANCES,ORDERFILE
C  AND REVENUE
C
      READ,NUMBER
      DO 14 I=1,NUMBER
   14 READ,IPLY(I),TPLY(I),BL(I),BU(I),DEM(I),R(I)
C
C  'N2' IS THE NUMBER OF VENEER THICKNESSES AVAILABLE
C  AND 'TFU' IS THE UPPER TOLERANCE ON FACE VENEER
C  THICKNESS. 'XSTAR(K)' AND 'ZSTAR' ARE THE EXISTING
C  THICKNESSES AND THE CORRESPONDING NET REVENUE.
C
      READ, N2
```

```
      READ,(T(I),I=1,N2)
      READ,W,C,Y,CF,TFU
      READ,(XSTAR(K),K=1,4),ZSTAR
C
C  THE FOLLOWING SECTION PRINTS THE INPUT DATA AS READ
C  BY THE COMPILER FOR VERIFICATION PURPOSES.
C
      PRINT220
  220 FORMAT('1',5X,'INPUT DATA FOR THE PLYWOOD DESIGN PROBLEM')
      PRINT221
  221 FORMAT(' ',5X,'-----------------------------------------')
      PRINT222,NUMBER
  222 FORMAT('-',10X,'NUMBER OF PLYWOOD TYPES=',17X,I3)
      PRINT223,W
  223 FORMAT('-',10X,'WOOD AVAILABILITY (CU.MTRS.) =',11X,F10.2)
      PRINT224,C
  224 FORMAT('-',10X,'LOG COST ($/CU.MTRS.)=',19X,F6.2)
      PRINT225,Y
  225 FORMAT('-',10X,'CORRECTION FACTOR FOR YIELD=',12X,F10.6)
      PRINT226,CF
  226 FORMAT('-',10X,'CORRECTION FACTOR FOR SHRINKAGE=',8X,F6.2)
      PRINT227,TFU
  227 FORMAT('-',10X,'UPPER LIMIT FOR FACE VENEER (MM)=',7X,F6.2)
      PRINT228
  228 FORMAT('-',10X,'OTHER DETAILS OF PLYWOOD TYPES:')
      PRINT229
  229 FORMAT('-',2X,'SL.NO.',2X,'PLY',2X,'THICKNESS',2X,'UPPER LIMIT',2X
     C,'LOWER LIMIT',2X,'REVENUE',2X,'ORDER FILE')
      PRINT230
  230 FORMAT(' ',18X,'(MM)',8X,'(MM)',9X,'(MM)',7X,'($)')
      DO 231 I=1,NUMBER
  231 PRINT232, I,IPLY(I),TPLY(I),BL(I),BU(I),R(I),DEM(I)
  232 FORMAT('-',4X,I2,5X,I2,5X,F4.1,8X,F4.1,9X,F4.1,7X,F4.1,5X,F8.0)
      KSET=0
      TKMAX=TFU
      NODE=0
      IMPR=0
      IBND=0
C
C  COMPUTATION OF THE BOUNDS IMPLIED BY THE BRANCHING
C  TESTS (3.41)-(3.42)
C
      DO 180 M=1,N2
      IF(T(M).LT.TFU)GO TO 180
      MAXIM1=M
      GO TO 181
  180 CONTINUE
  181 MXNODE=MAXIM1*N2**3
      PRINT182
  182 FORMAT('1',5X,'RESULTS OF THE PLYWOOD DESIGN PROBLEM WITH 4 VENEER
     CS')
      PRINT185
  185 FORMAT(' ',5X,'---------------------------------------------------
     C-')
      PRINT183
  183 FORMAT('-',10X,'RESULTS OF THE BRANCHING TESTS')
      PRINT184
  184 FORMAT(' ',10X,'------------------------------')
      DO 20 I=1,NUMBER
      IF(IPLY(I).EQ.5)GO TO 21
```

```
      IF(IPLY(I).EQ.7)GO TO 22
      IF(IPLY(I).EQ.9)GO TO 23
      TMAX=(BU(I)-T(1))/2.
      GO TO 20
   21 TMAX=(BU(I)-3.*T(1))/2.
      GO TO 20
   22 TMAX=(BU(I)-5.*T(1))/2.
      GO TO 20
   23 TMAX=(BU(I)-7.*T(1))/2.
   20 IF(TMAX.LT.TKMAX)TKMAX=TMAX
      PRINT24,TKMAX
   24 FORMAT('-',5X,'MAXIMUM PERMISSIBLE FACE VENEER THICKNESS FROM (3.4
     C1) =',F8.2,' MM')
      TKMIN=0.
      DO 25 I=1,NUMBER
      IF(IPLY(I).EQ.5)GO TO 26
      IF(IPLY(I).EQ.7)GO TO 27
      IF(IPLY(I).EQ.9)GO TO 28
      TMIN=(BL(I)-2.*TKMAX)
      GO TO 25
   26 TMIN=(BL(I)-2.*TKMAX)/3.
      GO TO 25
   27 TMIN=(BL(I)-2.*TKMAX)/5.
      GO TO 25
   28 TMIN=(BL(I)-2.*TKMAX)/7.
   25 IF(TMIN.GT.TKMIN)TKMIN=TMIN
      PRINT29,TKMIN
   29 FORMAT('-',5X,'MINIMUM PERMISSIBLE THICKNESS FOR X4 FROM (3.42)=',
     C6X,F8.2,' MM')
      DO 17 M=1,N2
      IF(T(M).LT.TKMAX)GO TO 17
      MAX1=M
      GO TO 18
   17 CONTINUE
   18 DO 19 M=1,N2
      IF(T(M).LT.TKMIN)GO TO 19
      MIN4=M
      GO TO 199
   19 CONTINUE
C
C  THIS SECTION IS INTENDED TO PRINT AN ITERATIVE SUMMARY
C  AS AND WHEN IMPROVED VENEER THICKNESSES ARE FOUND.
C
  199 PRINT195
  195 FORMAT('-',5X,'RUN STATISTICS:')
      PRINT194
  194 FORMAT(' ',5X,'---------------')
      PRINT196
  196 FORMAT('-',5X,'IMPROVED VENEER SETS FOUND SO FAR AND CORRESPONDING
     C THICKNESS AND OBJECTIVE')
      PRINT1960
 1960 FORMAT(' ',5X,'FUNCTION VALUE ARE AS FOLLOWS:')
      PRINT197
  197 FORMAT('-',5X,'NUMBER',10X,'VENEER THICKNESS',5X,'OBJ. FN. VALUE (
     C$)')
      PRINT74,(XSTAR(K),K=1,4),ZSTAR
   74 FORMAT('-',2X,'EXISTING SET',3X,4F6.2,2X,F15.2)
      DO 30 L1=1,MAX1
      X(1)=T(L1)
      DO 31 L2=1,N2
```

```
      IF(L2.GT.N2-2)GO TO 31
      IF(L2.EQ.L1)GO TO 31
      X(2)=T(L2)
      DO 311 L3=1,N2
      IF(L3.GT.N2-1)GO TO 311
      IF(L3.LE.L2)GO TO 311
      IF(L3.EQ.L1)GO TO 311
      X(3)=T(L3)
C
C  COMPUTATION OF THE BOUNDS ON X4 FROM (3.31)-(3.32)
C  AND THE INFEASIBILITY TESTS ASSOCIATED WITH THEM.
C
      DO 400 I=1,NUMBER
      IDENT(I)=0
      XMIN(I)=T(N2)
      XMAX(I)=T(MIN4)
      IF(IPLY(I).EQ.5)GO TO 410
      IF(IPLY(I).EQ.7)GO TO 420
      IF(IPLY(I).EQ.9)GO TO 430
      DO 401 J=1,4
      ID(I,J)=0
      SUM=0.
      DO 402 K=1,3
  402 SUM=SUM+X(K)*PLY3(J,K)
      TH(I,J)=SUM*CF
      IF(PLY3(J,4).EQ.0)GO TO 403
      AMIN=(BL(I)-TH(I,J))/(PLY3(J,4)*CF)
      AMAX=(BU(I)-TH(I,J))/(PLY3(J,4)*CF)
      IF(AMIN.GT.T(N2).OR.AMAX.LT.T(MIN4))GO TO 401
      IF(AMIN.LT.T(MIN4))AMIN=T(MIN4)
      IF(AMAX.GT.T(N2))AMAX=T(N2)
      GO TO 404
  403 IF(TH(I,J).LT.BL(I).OR.TH(I,J).GT.BU(I))GO TO 401
      AMIN=T(MIN4)
      AMAX=T(N2)
  404 ID(I,J)=1
      IDENT(I)=IDENT(I)+1
      IF(AMIN.LT.XMIN(I))XMIN(I)=AMIN
      IF(AMAX.GT.XMAX(I))XMAX(I)=AMAX
  401 CONTINUE
      IF(IDENT(I).EQ.0)GO TO 311
      GO TO 400
  410 DO 411 J=1,16
      ID(I,J)=0
      SUM=0.
      DO 412 K=1,3
  412 SUM=SUM+PLY5(J,K)*X(K)
      TH(I,J)=SUM*CF
      IF(PLY5(J,4).EQ.0)GO TO 413
      AMIN=(BL(I)-TH(I,J))/(CF*PLY5(J,4))
      AMAX=(BU(I)-TH(I,J))/(CF*PLY5(J,4))
      IF(AMIN.GT.T(N2).OR.AMAX.LT.T(MIN4))GO TO 411
      IF(AMIN.LT.T(MIN4))AMIN=T(MIN4)
      IF(AMAX.GT.T(N2))AMAX=T(N2)
      GO TO 414
  413 IF(TH(I,J).LT.BL(I).OR.TH(I,J).GT.BU(I))GO TO 411
      AMIN=T(MIN4)
      AMAX=T(N2)
  414 ID(I,J)=1
      IDENT(I)=IDENT(I)+1
```

```
      IF(AMIN.LT.XMIN(I))XMIN(I)=AMIN
      IF(AMAX.GT.XMAX(I))XMAX(I)=AMAX
  411 CONTINUE
      IF(IDENT(I).EQ.O)GO TO 311
      GO TO 400
  420 DO 421 J=1,16
      ID(I,J)=O
      SUM=O.
      DO 422 K=1,3
  422 SUM=SUM+X(K)*PLY7(J,K)
      TH(I,J)=SUM*CF
      IF(PLY7(J,4).EQ.O)GO TO 423
      AMIN=(BL(I)-TH(I,J))/(CF*PLY7(J,4))
      AMAX=(BU(I)-TH(I,J))/(CF*PLY7(J,4))
      IF(AMIN.GT.T(N2).OR.AMAX.LT.T(MIN4))GO TO 421
      IF(AMIN.LT.T(MIN4))AMIN=T(MIN4)
      IF(AMAX.GT.T(N2))AMAX=T(N2)
      GO TO 424
  423 IF(TH(I,J).LT.BL(I).OR.TH(I,J).GT.BU(I))GO TO 421
      AMIN=T(MIN4)
      AMAX=T(N2)
  424 ID(I,J)=1
      IDENT(I)=IDENT(I)+1
      IF(AMIN.LT.XMIN(I))XMIN(I)=AMIN
      IF(AMAX.GT.XMAX(I))XMAX(I)=AMAX
  421 CONTINUE
      IF(IDENT(I).EQ.O)GO TO 311
      GO TO 400
  430 DO 431 J=1,16
      ID(I,J)=O
      SUM=O.
      DO 432 K=1,3
  432 SUM=SUM+PLY9(J,K)*X(K)
      TH(I,J)=SUM*CF
      IF(PLY9(J,4).EQ.O)GO TO 433
      AMIN=(BL(I)-TH(I,J))/(CF*PLY9(J,4))
      AMAX=(BU(I)-TH(I,J))/(CF*PLY9(J,4))
      IF(AMIN.GT.T(N2).OR.AMAX.LT.T(MIN4))GO TO 431
      IF(AMIN.LT.T(MIN4))AMIN=T(MIN4)
      IF(AMAX.GT.T(N2))AMAX=T(N2)
      GO TO 434
  433 IF(TH(I,J).LT.BL(I).OR.TH(I,J).GT.BU(I))GO TO 431
      AMIN=T(MIN4)
      AMAX=T(N2)
  434 ID(I,J)=1
      IDENT(I)=IDENT(I)+1
      IF(AMIN.LT.XMIN(I))XMIN(I)=AMIN
      IF(AMAX.GT.XMAX(I))XMAX(I)=AMAX
  431 CONTINUE
      IF(IDENT(I).EQ.O)GO TO 311
  400 CONTINUE
      X4MIN=TKMIN
      X4MAX=T(N2)
      DO 440 I=1,NUMBER
      IF(XMIN(I).GT.X4MIN)X4MIN=XMIN(I)
  440 IF(XMAX(I).LT.X4MAX)X4MAX=XMAX(I)
      IF(X4MIN.GT.X4MAX)GO TO 311
      DO 441 M=MIN4,N2
      IF(T(M).LT.X4MIN)GO TO 441
      MINX4=M
```

```
      GO TO 442
 441  CONTINUE
 442  DO 443 M=MIN4,N2
      IF(T(M).LT.X4MAX)GO TO 443
      MAXX4=M
      GO TO 444
 443  CONTINUE
 444  IF(MINX4.GT.MAXX4)GO TO 311
C
C   COMPUTATION OF THE UPPER BOUND ON Z FROM (3.38):
C   'ZBOUND' IS THE UPPER BOUND.
      DO 450 I=1,NUMBER
      BD(I)=BU(I)
      IDENT(I)=0
      IF(IPLY(I).EQ.5)GO TO 451
      IF(IPLY(I).EQ.7)GO TO 452
      IF(IPLY(I).EQ.9)GO TO 453
      DO 454 J=1,4
      IF(ID(I,J).EQ.0)GO TO 454
      DO 4540 M=MINX4,MAXX4
      SUMA=TH(I,J)+T(M)*CF*PLY3(J,4)
      IF(SUMA.LT.BL(I).OR.SUMA.GT.BU(I))GO TO 454
      IDENT(I)=IDENT(I)+1
      IF(SUMA.LT.BD(I))BD(I)=SUMA
4540  CONTINUE
 454  CONTINUE
      IF(IDENT(I).EQ.0)GO TO 311
      GO TO 450
 451  DO 455 J=1,16
      IF(ID(I,J).EQ.0)GO TO 455
      DO 4550 M=MINX4,MAXX4
      SUMA=TH(I,J)+T(M)*CF*PLY5(J,4)
      IF(SUMA.LT.BL(I).OR.SUMA.GT.BU(I))GO TO 455
      IDENT(I)=IDENT(I)+1
      IF(SUMA.LT.BD(I))BD(I)=SUMA
4550  CONTINUE
 455  CONTINUE
      IF(IDENT(I).EQ.0)GO TO 311
      GO TO 450
 452  DO 456 J=1,16
      IF(ID(I,J).EQ.0)GO TO 456
      DO 4560 M=MINX4,MAXX4
      SUMA=TH(I,J)+T(M)*CF*PLY7(J,4)
      IF(SUMA.LT.BL(I).OR.SUMA.GT.BU(I))GO TO 456
      IDENT(I)=IDENT(I)+1
      IF(SUMA.LT.BD(I))BD(I)=SUMA
4560  CONTINUE
 456  CONTINUE
      IF(IDENT(I).EQ.0)GO TO 311
      GO TO 450
 453  DO 457 J=1,16
      IF(ID(I,J).EQ.0)GO TO 457
      DO 4570 M=MINX4,MAXX4
      SUMA=TH(I,J)+T(M)*CF*PLY9(J,4)
      IF(SUMA.LT.BL(I).OR.SUMA.GT.BU(I))GO TO 457
      IDENT(I)=IDENT(I)+1
      IF(SUMA.LT.BD(I))BD(I)=SUMA
4570  CONTINUE
 457  CONTINUE
      IF(IDENT(I).EQ.0)GO TO 311
```

```
  450 CONTINUE
      TSMAX=0.
      DO 460 I=1,NUMBER
      TS(I)=(R(I)-(C*Y*BD(I))/CF)/((BD(I)*Y)/CF)
      IF(TS(I).LE.TSMAX)GO TO 460
      TSMAX=TS(I)
      IBEST=I
  460 CONTINUE
      SWOOD=0.
      DO 461 I=1,NUMBER
      IF(I.EQ.IBEST)GO TO 461
      PD(I)=DEM(I)
      WD(I)=(BD(I)*PD(I)*Y)/CF
      SWOOD=SWOOD+WD(I)
  461 CONTINUE
      PD(IBEST)=(W-SWOOD)/((BD(IBEST)*Y)/CF)
      IF(PD(IBEST).LT.DEM(IBEST))GO TO 311
      ZBOUND=0.
      DO 462 I=1,NUMBER
  462 ZBOUND=ZBOUND+(R(I)-(C*Y*BD(I))/CF)*PD(I)
      IF(ZBOUND.GT.ZSTAR)GO TO 198
      IBND=IBND+1
      GO TO 311
C
C   SOLVING THE LMCK PROBLEM AFTER BRANCHING TESTS
C   AND BOUNDS: 'ZNODE' IS THE OBJECTIVE FUNCTION
C   VALUE FROM (3.14).
C
  198 DO 32 L4=MINX4,MAXX4
      X(4)=T(L4)
      IF(L4.LE.L3)GO TO 32
      IF(X(4).EQ.X(2))GO TO 32
      IF(X(4).EQ.X(1))GO TO 32
      NODE=NODE+1
      DO 40 I=1,NUMBER
      ITEST(I)=0
      IF(IPLY(I).EQ.5)GO TO 41
      IF(IPLY(I).EQ.7)GO TO 42
      IF(IPLY(I).EQ.9)GO TO 43
      DO 44 J=1,4
      IDELTA(I,J)=0
      THICK(I,J)=TH(I,J)+X(4)*PLY3(J,4)*CF
      IF(THICK(I,J).LT.BL(I).OR.THICK(I,J).GT.BU(I))GO TO 44
      IDELTA(I,J)=1
      ITEST(I)=ITEST(I)+IDELTA(I,J)
   44 CONTINUE
      IF(ITEST(I).GE.1)GO TO 40
      GO TO 32
   41 DO 45 J=1,16
      IDELTA(I,J)=0
      THICK(I,J)=TH(I,J)+X(4)*PLY5(J,4)*CF
      IF(THICK(I,J).LT.BL(I).OR.THICK(I,J).GT.BU(I))GO TO 45
      IDELTA(I,J)=1
      ITEST(I)=ITEST(I)+IDELTA(I,J)
   45 CONTINUE
      IF(ITEST(I).GE.1)GO TO 40
      GO TO 32
   42 DO 46 J=1,16
      IDELTA(I,J)=0
      THICK(I,J)=TH(I,J)+X(4)*PLY7(J,4)*CF
```

```
      IF(THICK(I,J).LT.BL(I).OR.THICK(I,J).GT.BU(I))GO TO 46
      IDELTA(I,J)=1
      ITEST(I)=ITEST(I)+IDELTA(I,J)
   46 CONTINUE
      IF(ITEST(I).GE.1)GO TO 40
      GO TO 32
   43 DO 47 J=1,16
      IDELTA(I,J)=0
      THICK(I,J)=TH(I,J)
      IF(THICK(I,J).LT.BL(I).OR.THICK(I,J).GT.BU(I))GO TO 47
      IDELTA(I,J)=1
      ITEST(I)=ITEST(I)+IDELTA(I,J)
   47 CONTINUE
      IF(ITEST(I).GE.1)GO TO 40
      GO TO 32
   40 CONTINUE
C
C  SETTING LIMITS ON COMPUTATIONS; 'KSET' IS THE
C  MAXIMUM NUMBER OF LMCK PROBLEMS TO BE SOLVED.
C
      KSET=KSET+1
      IF(KSET.GT.5000)GO TO 200
      DO 50 K=1,4
   50 B(K)=Y*X(K)
      TWMAX=0.
      DO 51 I=1,NUMBER
      TP(I)=0.
      IF(IPLY(I).EQ.5)GO TO 52
      IF(IPLY(I).EQ.7)GO TO 53
      IF(IPLY(I).EQ.9)GO TO 54
      DO 55 J=1,4
      TW(I,J)=0.
      RSUM(I,J)=0.
      BSUM(I,J)=0.
      IF(IDELTA(I,J).EQ.0)GO TO 55
      DO 611 K=1,4
  611 BSUM(I,J)=BSUM(I,J)+B(K)*PLY3(J,K)
      RSUM(I,J)=R(I)-BSUM(I,J)*C
      TW(I,J)=RSUM(I,J)/BSUM(I,J)
      IF(TW(I,J).LT.TP(I))GO TO 55
      TP(I)=TW(I,J)
      JBEST(I)=J
      IF(TP(I).LT.TWMAX)GO TO 55
      TWMAX=TP(I)
      IBEST=I
   55 CONTINUE
      GO TO 51
   52 DO 56 J=1,16
      TW(I,J)=0.
      RSUM(I,J)=0.
      BSUM(I,J)=0.
      IF(IDELTA(I,J).EQ.0)GO TO 56
      DO 612 K=1,4
  612 BSUM(I,J)=BSUM(I,J)+B(K)*PLY5(J,K)
      RSUM(I,J)=R(I)-C*BSUM(I,J)
      TW(I,J)=RSUM(I,J)/BSUM(I,J)
      IF(TW(I,J).LT.TP(I))GO TO 56
      TP(I)=TW(I,J)
      JBEST(I)=J
      IF(TP(I).LT.TWMAX)GO TO 56
```

```
      TWMAX=TP(I)
      IBEST=I
   56 CONTINUE
      GO TO 51
   53 DO 57 J=1,16
      TW(I,J)=0.
      RSUM(I,J)=0.
      BSUM(I,J)=0.
      IF(IDELTA(I,J).EQ.0)GO TO 57
      DO 613 K=1,4
  613 BSUM(I,J)=BSUM(I,J)+B(K)*PLY7(J,K)
      RSUM(I,J)=R(I)-C*BSUM(I,J)
      TW(I,J)=RSUM(I,J)/BSUM(I,J)
      IF(TW(I,J).LT.TP(I))GO TO 57
      TP(I)=TW(I,J)
      JBEST(I)=J
      IF(TP(I).LT.TWMAX)GO TO 57
      TWMAX=TP(I)
      IBEST=I
   57 CONTINUE
      GO TO 51
   54 DO 58 J=1,16
      TW(I,J)=0.
      RSUM(I,J)=0.
      BSUM(I,J)=0.
      IF(IDELTA(I,J).EQ.0)GO TO 58
      DO 614 K=1,4
  614 BSUM(I,J)=BSUM(I,J)+B(K)*PLY9(J,K)
      RSUM(I,J)=R(I)-C*BSUM(I,J)
      TW(I,J)=RSUM(I,J)/BSUM(I,J)
      IF(TW(I,J).LT.TP(I))GO TO 58
      TP(I)=TW(I,J)
      JBEST(I)=J
      IF(TP(I).LT.TWMAX)GO TO 58
      TWMAX=TP(I)
      IBEST=I
   58 CONTINUE
   51 CONTINUE
      SUMB=0.
      DO 60 I=1,NUMBER
      IF(I.EQ.IBEST)GO TO 69
      NI=4
      IF(IPLY(I).GT.3)NI=16
      DO 61 J=1,NI
      P(I,J)=0.
      IF(J.NE.JBEST(I))GO TO 61
      P(I,J)=DEM(I)
      WOOD(I)=BSUM(I,J)*P(I,J)
      REV(I)=RSUM(I,J)*P(I,J)
   61 CONTINUE
      SUMB=SUMB+WOOD(I)
      GO TO 60
   69 IP=I
   60 CONTINUE
      NI=4
      IF(IPLY(IP).GT.3)NI=16
      DO 70 J=1,NI
      P(IP,J)=0.
      IF(J.NE.JBEST(IP))GO TO 70
      P(IP,J)=(W-SUMB)/BSUM(IP,J)
```

```
      IF(P(IP,J).LT.DEM(IP))GO TO 32
      REV(IP)=RSUM(IP,J)*P(IP,J)
   70 CONTINUE
      ZNODE=0.
      DO 71 I=1,NUMBER
   71 ZNODE=ZNODE+REV(I)
C
C  UPDATING THE INCUMBENT SOLUTION AND PRINTING
C  IT FOR ITERATIVE SUMMARY.
C
      IF(ZNODE.LE.ZSTAR)GO TO 32
      IMPR=IMPR+1
      ZSTAR=ZNODE
      WOODVL=TWMAX
      DO 72 K=1,4
   72 XSTAR(K)=X(K)
      PRINT75,IMPR,(XSTAR(K),K=1,4),ZSTAR
   75 FORMAT('0',5X,I3,9X,4F6.2,2X,F15.2)
      DO 73 I=1,NUMBER
      NI=4
      IF(IPLY(I).GT.3)NI=16
      DO 73 J=1,NI
      PSTAR(I,J)=P(I,J)
      RSTAR(I,J)=RSUM(I,J)
      TSTAR(I,J)=THICK(I,J)
      TWSTAR(I,J)=TW(I,J)
   73 ISTAR(I,J)=IDELTA(I,J)
   32 CONTINUE
  311 CONTINUE
   31 CONTINUE
   30 CONTINUE
      IF(KSET.GT.0)GO TO 100
C
C  MESSAGE IF THE PROBLEM IS INFEASIBLE.
C
      PRINT101
  101 FORMAT('1',5X,'THE PROBLEM IS INFEASIBLE: VENEER THICKNESS SET SAT
     CISFYING CONSTRAINTS (2.2)-(2.4) DOES NOT EXIST')
      GO TO 999
C
C  MESSAGE IF THE NUMBER OF LMCK PROBLEMS TO BE
C  SOLVED EXCEEDS A PRESET LIMIT.
C
  200 PRINT201
  201 FORMAT('1',5X,'NUMBER OF FEASIBLE VENEER THICKNESS SETS EXCEEDS 50
     COO; PROGRAM TERMINATED PREMATURELY. CURRENT RESULTS ARE PRESENTED'
     C)
C
C  THIS SECTION GIVES A SUMMARY OF THE PERFORMANCE
C  OF THE IMPLICIT ENUMERATION ALGORITHM.
C
  100 PRINT193,MXNODE
  193 FORMAT('-',5X,'MAXIMUM POSSIBLE SETS OF VENEER THICKNESS,ORIGINAL
     C PROBLEM =',5X,I6)
      PRINT470,IBND
  470 FORMAT('-',5X,'NUMBER OF TIMES UPPER BOUND ON Z (3.38) WAS EFFECTI
     CVE =',8X,I6)
      PRINT192,NODE
  192 FORMAT('-',5X,'NUMBER OF VENEER SETS EVALUATED AFTER BRANCHING TES
     CTS=',9X,I6)
```

```
      PRINT191,KSET
  191 FORMAT('-',5X,'NUMBER OF VENEER SETS EVALUATED FOR LMCK PROBLEM=',
     C12X,I6)
      DO 77 K=1,4
      V(K)=0.
      DO 78 I=1,NUMBER
      IF(IPLY(I).EQ.5)GO TO 170
      IF(IPLY(I).EQ.7)GO TO 171
      IF(IPLY(I).EQ.9)GO TO 172
      DO 173 J=1,4
      IF(ISTAR(I,J).EQ.0)GO TO 173
      V(K)=V(K)+PLY3(J,K)*PSTAR(I,J)
  173 CONTINUE
      GO TO 78
  170 DO 174 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 174
      V(K)=V(K)+PLY5(J,K)*PSTAR(I,J)
  174 CONTINUE
      GO TO 78
  171 DO 175 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 175
      V(K)=V(K)+PLY7(J,K)*PSTAR(I,J)
  175 CONTINUE
      GO TO 78
  172 DO 176 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 176
      V(K)=V(K)+PLY9(J,K)*PSTAR(I,J)
  176 CONTINUE
   78 CONTINUE
      BSTAR(K)=Y*XSTAR(K)
      LSTAR(K)=V(K)*BSTAR(K)
   77 CONTINUE
C
C  THIS SECTION GIVES THE SOLUTION TO THE PDM PROBLEM.
C
      PRINT102
  102 FORMAT('1',5X,'RESULTS:')
      PRINT103
  103 FORMAT(' ',5X,'--------')
      PRINT104,(XSTAR(K),K=1,4)
  104 FORMAT('0',5X,'OPTIMAL VENEER THICKNESSES ARE (MM):',4F8.2)
      PRINT177
  177 FORMAT('0',5X,'QUANTITY OF LOGS FOR CORRESPONDING ')
      PRINT1770,(LSTAR(K),K=1,4)
 1770 FORMAT(' ',5X,'THICKNESS (CU. MTRS.):',15X,4F8.0)
      PRINT105,ZSTAR
  105 FORMAT('0',5X,'OBJ.FN. VALUE AT OPTIMAL SOLUTION ($):',F20.2)
      PRINT190,WOODVL
  190 FORMAT('0',5X,'MARGINAL VALUE OF WOOD ($/CU.MTR.):',F19.4)
      PRINT106
  106 FORMAT('0',5X,'CORRESPONDING DESIGN ALTERNATIVES AND PRODUCT MIX
     CARE AS FOLLOWS:')
      PRINT107
  107 FORMAT('-',5X,'PLYWOOD TYPE',8X,'DESIGN ALTERNATIVE',5X,'THICKNES
     CS',5X,'QUANTITY',10X,'NET REVENUE')
      PRINT108
  108 FORMAT(' ',49X,'(DRY-MM)',8X,'(#)',13X,'$/SHEET')
      DO 110 I=1,NUMBER
      PRINT111,IPLY(I),TPLY(I)
  111 FORMAT('-',5X,I2,'PLY',2X,F4.1,'MM')
```

```
      IF(IPLY(I).EQ.5)GO TO 112
      IF(IPLY(I).EQ.7)GO TO 113
      IF(IPLY(I).EQ.9)GO TO 114
      DO 115 J=1,4
      IF(ISTAR(I,J).EQ.0)GO TO 115
      PRINT116,(PLY3(J,K),K=1,4),TSTAR(I,J),PSTAR(I,J),RSTAR(I,J)
  116 FORMAT(' ',22X,4F5.0,7X,F5.2,5X,F12.1,8X,F8.4)
  115 CONTINUE
      GO TO 110
  112 DO 117 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 117
      PRINT118,(PLY5(J,K),K=1,4),TSTAR(I,J),PSTAR(I,J),RSTAR(I,J)
  118 FORMAT(' ',22X,4F5.0,7X,F5.2,5X,F12.1,8X,F8.4)
  117 CONTINUE
      GO TO 110
  113 DO 119 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 119
      PRINT120,(PLY7(J,K),K=1,4),TSTAR(I,J),PSTAR(I,J),RSTAR(I,J)
  120 FORMAT(' ',22X,4F5.0,7X,F5.2,5X,F12.1,8X,F8.4)
  119 CONTINUE
      GO TO 110
  114 DO 122 J=1,16
      IF(ISTAR(I,J).EQ.0)GO TO 122
      PRINT121,(PLY9(J,K),K=1,4),TSTAR(I,J),PSTAR(I,J),RSTAR(I,J)
  121 FORMAT(' ',22X,4F5.0,7X,F5.2,5X,F12.1,8X,F8.4)
  122 CONTINUE
  110 CONTINUE
  999 STOP
      END
C
C   THIS LAST SECTION IS THE INPUT DATA. THE FIRST
C   52 STATEMENTS ARE THE DESIGN COEFFICIENTS. THESE
C   ARE FOLLOWED BY SPECIFIC MILL DATA USED FOR
C   ILLUSTRATION IN CHAPTER THREE.
C
$DATA
3 0 0 0
2 1 0 0
2 0 1 0
2 0 0 1
5 0 0 0
4 1 0 0
4 0 1 0
4 0 0 1
3 2 0 0
3 0 2 0
3 0 0 2
2 3 0 0
2 0 3 0
2 0 0 3
2 2 1 0
2 2 0 1
2 1 2 0
2 1 0 2
2 0 2 1
2 0 1 2
7 0 0 0
5 2 0 0
5 0 2 0
5 0 0 2
```

```
4 3 0 0
4 0 3 0
4 0 0 3
2 5 0 0
2 0 5 0
2 0 0 5
2 3 2 0
2 3 0 2
2 2 3 0
2 2 0 3
2 0 3 2
2 0 2 3
9 0 0 0
6 3 0 0
6 0 3 0
6 0 0 3
5 4 0 0
5 0 4 0
5 0 0 4
2 7 0 0
2 0 7 0
2 0 0 7
2 4 3 0
2 4 0 3
2 3 4 0
2 3 0 4
2 0 4 3
2 0 3 4
12
3 7.5 7.0 8.0 171107. 4.3
3 9.5 9.0 10.0 106378.0 4.8
5 12.5 12.0 13.0 502289.0 6.1
5 15.5 15.0 16.0 350192.0 7.6
7 18.5 18.0 19.0 423394.0 8.9
7 20.5 20.0 21.0 443442.0 9.8
7 22.5 22.0 23.0 2952.0 10.5
9 23.5 23.0 24.0 6135.0 11.3
9 25.5 25.0 26.0 11172.0 12.1
9 27.5 27.0 28.0 2738.0 13.1
9 28.5 28.0 29.0 1278.0 13.6
9 30.5 30.0 31.0 38.0 14.6
27
2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5
3.6 3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5 4.6 4.7
4.8 4.9 5.0
300000.0 35.0 0.006067 0.94 3.2
2.64 3.35 3.96 4.98 13416690.
```

APPENDIX II

Data from a Plywood Mill in B.C.

1. Number of veneer thicknesses (K): 4
2. Veneer thicknesses (x_k): 2.69, 3.35, 3.96 and 4.98 mm
3. Log Cost (C): 35.00 $/cu.mtr.
4. Log availability (W): 300000 cu.mtr.
5. Veneer yield factor (y): 0.006067 (corresponds to a 60% yield by volume)
6. Shrinkage and compression factor (C_f): 0.94
7. Upper limit on face veneer thickness (T_f^U): 3.20 mm
8. Number of plywood types (N): 12

9. Plywood types and designs:

Plywood Type Plies-Thickness (mm)	Design Alternative	Number of veneers of			
		2.69	3.35	3.96	4.98
3 - 7.5	i)	3	0	0	0
3 - 9.5	i)	2	0	0	1
5 - 12.5	i)	5	0	0	0
	ii)	4	1	0	0
5 - 15.5	i)	2	2	0	1
	ii)	2	1	2	0
7 - 18.5	i)	5	2	0	0
	ii)	4	3	0	0
7 - 20.5	i)	2	5	0	0
	ii)	4	0	3	0
7 - 22.5	i)	2	0	5	0
	ii)	2	2	3	0
9 - 23.5	i)	6	3	0	0
9 - 25.5	i)	6	0	3	0
	ii)	5	4	0	0
9 - 27.5	i)	5	0	4	0
	ii)	2	7	0	0
9 - 28.5	i)	6	0	0	3
	ii)	2	4	3	0
9 - 30.5	i)	2	0	7	0
	ii)	5	0	0	4

10. Plywood thickness tolerance, revenue and orderfile:

Number of Plies	Thickness (mm)			Revenue ($/Panel)	Orderfile (# of Panels 2.44m x 1.22m)
	Specified	Lower Limit	Upper Limit		
3	7.5	7.0	8.0	4.3	171107
3	9.5	9.0	10.0	4.8	106378
5	12.5	12.0	13.0	6.1	502289
5	15.5	15.0	16.0	7.6	350192
7	18.5	18.0	19.0	8.9	423394
7	20.5	20.0	21.0	9.8	443442
7	22.5	22.0	23.0	10.5	2952
9	23.5	23.0	24.0	11.3	6135
9	25.5	25.0	26.0	12.1	11172
9	27.5	27.0	28.0	13.1	2738
9	28.5	28.0	29.0	13.6	1278
9	30.5	30.0	31.0	14.6	38

11. Maximum possible Net Revenue ($): 13416694

12. Wood loss in the form of excess thickness in plywood (cu. mtr.): 7944.7

APPENDIX III

Results from the PDM Model with Four Veneer Thicknesses

1.	Optimal veneer thicknesses (mm):	2.5	3.1	3.9	4.8
2.	Quantity of logs for corresponding thicknesses (cubic metres):	205848	48886	31799	13468
3.	Objective function value at optimal solution ($):	14337370.00			
4.	Marginal value of wood ($/cubic metre):	59.5003			
5.	Wood loss in the form of excess thickness in plywood (cubic metres):	1647.4			

6. Plywood design alternatives and product mix:

Plywood Type ply-Thickness (mm)	Design Alternative	Number of Veneers of 2.5	3.1	3.9	4.8	Quantity of Plywood, # of Panels, 2.44m x 1.22 m
3 - 7.5	i)	3	0	0	0	2371653
	ii)	2	1	0	0	-
3 - 9.5	i)	2	0	0	1	106378
5 - 12.5	i)	4	1	0	0	502289
	ii)	3	2	0	0	-
5 - 15.5	i)	2	0	3	0	-
	ii)	2	2	0	1	350192
7 - 18.5	i)	4	3	0	0	423394
7 - 20.5	i)	5	0	0	2	-
	ii)	4	0	3	0	443442
	iii)	2	3	2	0	-
7 - 22.5	i)	4	0	0	3	-
	ii)	2	3	0	2	2952
9 - 23.5	i)	5	4	0	0	6135
9 - 25.5	i)	6	0	3	0	-
	ii)	2	7	0	0	111172
9 - 27.5	i)	2	4	3	0	2738
	ii)	6	0	0	3	-
9 - 28.5	i)	2	3	4	0	1278
9 - 30.5	i)	2	0	7	0	38

APPENDIX IV

Results from the PDM Model with Three Veneer Thicknesses

1.	Optimal veneer thicknesses (mm):	2.6	3.2	4.4
2.	Quantity of logs for corresponding thickness (cubic metres):	231533	32087	36380
3.	Objective function value at optimal solution ($):	13930670		
4.	Marginal value of wood ($/cubic metre):	55.8657		
5.	Wood loss in the form of excess thickness in plywood (cubic metres):	3612.1		

6. Plywood design alternatives and product mix:

Plywood Type Ply - Thickness (mm)	Design Alternative	Number of veneers of 2.6	3.2	4.4	Quantity of Plywood, # of panels, 2.44m x 1.22m
3 - 7.5	i)	3	0	0	2277072
	ii)	2	1	0	-
3 - 9.5	i)	2	0	1	106378
5 - 12.5	i)	5	0	0	502289
	ii)	4	1	0	-
5 - 15.5	i)	3	0	2	-
	ii)	2	2	1	350192
7 - 18.5	i)	5	2	0	423394
	ii)	4	3	0	-
7 - 20.5	i)	5	0	2	443442
7 - 22.5	i)	4	0	3	-
	ii)	2	3	2	2952
9 - 23.5	i)	6	3	0	6135
9 - 25.5	i)	2	7	0	11172
9 - 27.5	i)	6	0	3	2738
9 - 28.5	i)	5	0	4	1278
9 - 30.5	i)	2	3	4	38

APPENDIX V

Results from the PDM Model; Alternate Face Veneers

1.	Optimal veneer thicknesses (mm):	2.5	3.1	3.8
2.	Quantity of logs for corresponding thickness (cubic metres):	185670	82869	31461
3.	Face veneers (mm):	2.5	3.1	
4.	Objective function value at optimal solution ($):	14,387,280		
5.	Marginal value of wood ($):	59.5003		
6.	Wood loss in the form of excess thickness in plywood (cubic metres):	1403.0		

7. Plywood design alternatives and product mix:

Plywood Type Plies - Thick (mm)	Design Alternative*	Number of Veneers of 2.5	3.1	3.8	Quantity of Plywood, # of Panels
3 - 7.5	i)	3	0	0	2383259
	ii)	2	1	0	-
3 - 9.5	i) A	0	2	1	106378
5 - 12.5	i)	4	1	0	502289
	ii)	4	0	1	-
	iii)	3	2	0	-
5 - 15.5	i)	2	0	3	-
	ii) A	0	4	1	350192
	iii) A	0	3	2	-
	iv) A	1	2	2	-
7 - 18.5	i)	5	0	2	-
	ii)	4	3	0	423394
	iii) A	3	4	0	-
7 - 20.5	i)	4	0	3	-
	ii)	2	3	2	-
	iii) A	0	7	0	-
	iv) A	3	2	2	443442
7 - 22.5	i)	2	0	5	-
	ii) A	0	4	3	2952
9 - 23.5	i)	5	4	0	6135
	ii) A	4	5	0	-
9 - 25.5	i)	2	7	0	11172
	ii) A	4	2	3	-
9 - 27.5	i)	2	4	3	2738
	ii)	2	3	4	-
	iii) A	3	2	4	-
9 - 28.5	i) A	0	6	3	1278
	ii) A	0	5	4	-
9 - 30.5	i) A	0	2	7	38

* 'A' indicates alternative face veneer thickness of 3.1 mm