STUDENTS' PRECONCEPTIONS OF THREE VECTOR QUANTITIES

by

JOSE M. AGUIRRE
Teacher of Physics, University of Chile, Chile, 1968
M.A., The University of British Columbia, 1978

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF EDUCATION in THE FACULTY OF GRADUATE STUDIES
DEPARTMENT OF MATHEMATICS AND SCIENCE EDUCATION

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
August, 1981

© Jose M. Aguirre, 1981
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Mar. 43. T. SCIENCE LTDUCATION

The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date SEPTEMBER 15, 1981
ABSTRACT

This study was directed towards the identification and analysis of students' intuitive ideas or preconceptions about three kinematic vector quantities -- vector position, displacement, and velocity. A list of ten implicit vector characteristics derived from a task analysis of these three vector quantities, served as the framework for the development of the interview protocols used in the study and the subsequent analysis of the data.

The study proceeded in two phases; in the first, clinical-type interviews were conducted with twenty grade ten students using two tasks which involved describing the movement of a boat on a lake and a river; in the second phase, a group interview protocol (G.I.P.) requiring a written response was developed (based upon the tasks and the results of the first phase) and administered to 8 classes of grade ten students, producing a total sample of 176 subjects.

Analysis of the results from the first phase yielded a small but variable number (2 to 5) number of inferred rules for each of the ten implicit vector characteristics
dealt with in the individual interviews. These 'inferred rules' are hypothesized sets of beliefs (preconceptions) or cognitive structures used by the subjects to explain a problem situation posed by the interviewer. Further categorization yielded three broad types of preconceptions -- scalar, transitional, and vectorial -- which were based in part upon criteria derived from the current physics perspective. The overall results from the first phase indicated that most students possessed fairly consistent preconceptions regarding the directional and distance components involved in the description of the movement of a boat on a lake. For example, all students recognized the necessity of using a reference point and a frame of reference, however, they employed different 'inferred rules' in the selection of these references. Overall, for four vector characteristics, most of the subjects used vectorial type 'inferred rules' (where they considered the quantitative aspects of the variables involved, e.g., direction and distance); for three of the vector characteristics, a high percentage of subjects made use of scalar-type 'inferred rules' which corresponded to fairly primitive notions (where they considered only one qualitative or quantitative variable); for the remaining three vector characteristics, most of the subjects demonstrated transitional-type 'inferred rules' (which consisted in some combination of the two prior extreme categories).
The results from the second phase of the study, in general, corroborated the findings from the first phase. While some new 'inferred rules' were identified, about 80 percent of the inferred rules used were common to those discovered in the interview sample. Hence, it would seem that the results were neither an artifact of the interview procedure, nor limited to a small group of grade ten students.

In summary, the study illustrated the application of a particular analytical technique to generate sets of 'inferred rules' which were hypothesized to account for the way that grade ten subjects intuitively approached a problem situation involving a directional as well as other components. By developing a group questionnaire, the general validity of the approach and results were enhanced by finding similar response patterns in a larger more heterogeneous group of subjects. The educational implications of both the rational task analysis, which yielded the list of implicit vector characteristics, and the substantive results, expressed in terms of the various sets of 'inferred rules', were discussed in terms of their applicability to classroom teachers and curriculum developers.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiv</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>xv</td>
</tr>
<tr>
<td><strong>CHAPTER ONE</strong></td>
<td></td>
</tr>
<tr>
<td>1.0 Background of the Study</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Definition of Terms</td>
<td>4</td>
</tr>
<tr>
<td>1.2 Implicit Vector Characteristics of Three Physical Vector Quantities</td>
<td>7</td>
</tr>
<tr>
<td>1.3 General Statement of the Problem</td>
<td>10</td>
</tr>
<tr>
<td>1.4 Specific Statements of the Problem</td>
<td>13</td>
</tr>
<tr>
<td>1.5 Research Questions</td>
<td>14</td>
</tr>
<tr>
<td>1.6 Overview of Methods of Study</td>
<td>15</td>
</tr>
<tr>
<td>1.7 Delimitation of the Study</td>
<td>17</td>
</tr>
<tr>
<td>1.8 Justification of the Study</td>
<td>19</td>
</tr>
<tr>
<td><strong>CHAPTER TWO</strong></td>
<td></td>
</tr>
<tr>
<td>2.0 Introduction</td>
<td>21</td>
</tr>
<tr>
<td>2.1 Literature Review on Students' Understanding Vector Quantities</td>
<td>22</td>
</tr>
<tr>
<td>2.1.1 Students' Difficulties in Understanding Vector Quantities</td>
<td>22</td>
</tr>
</tbody>
</table>
### 2.1.2 Viewing Students Difficulties as a Critical Barrier Phenomena

Page 28

### 2.1.3 Previous Research About Students' Understanding on Vector Quantities

A. Piagetian-Type Studies

Page 31

B. Special Experiments to Teach Vector Quantities

Page 36

C. Vector Quantities in Mathematics Education

Page 36

### 2.2 Psychological Context of the Study

Page 37

#### 2.2.1 Introduction

Page 37

#### 2.2.2 Two Aspects of Developmental Psychology: Formal and Functional

Page 38

#### 2.2.3 The Neo-Piagetian Approach of Development

Page 42

Siegler's Approach

Page 43

Case's Approach

Page 46

#### 2.2.4 Task Analysis

Page 47

### 2.3 Educational Context of the Study

Page 51

#### 2.3.1 Teaching-Learning Problems in Science Education

Page 52

#### 2.3.2 Curriculum Problems in Science Education

Page 55

### 3.0 Introduction

Page 58

### 3.1 The Pilot Study

Page 58

### 3.2 Rational Task Analysis of Physical Phenomena

Page 60
3.2.1 Networks of the Three Vector Quantities .... ..... ..... ..... ..... 63
3.2.2 The Physics Approach .... ..... ..... ..... ..... 68
3.3 The Two Tasks of the Study .... ..... ..... ..... ..... 69
3.3.1 Task One .... ..... ..... ..... ..... ..... 70
3.3.2 Protocol of Task One .... ..... ..... ..... ..... ..... 73
3.3.3 Task Two .... ..... ..... ..... ..... ..... 79
3.3.4 Protocol of Task Two .... ..... ..... ..... ..... ..... 81
3.3.5 Further Discussion of Interview Protocol .... ..... ..... ..... ..... ..... 86
3.4 Methods of Data Collection: Clinical Interview .... ..... ..... ..... ..... ..... 89
3.4.1 Reliability and Validity in Clinical Interview .... ..... ..... ..... ..... ..... 92
3.4.2 Interview Format .... ..... ..... ..... ..... ..... 94
3.4.3 Sample of Subjects .... ..... ..... ..... ..... ..... 95

CHAPTER FOUR
4.0 Introduction .... ..... ..... ..... ..... ..... 98
4.1 Data Analysis .... ..... ..... ..... ..... ..... 99
4.1.1 Inferred Rules .... ..... ..... ..... ..... ..... 99
4.1.2 An Example of the Derivation of Inferred Rules .... ..... ..... ..... ..... ..... 101
4.1.3 Categorization of the Inferred Rules and Construction of the Rule-Model for RPS .... ..... ..... ..... ..... ..... 105
4.2 Format for Presenting Results of Interview Data Analysis .... ..... ..... ..... ..... ..... 110
4.3 Results of Analysis of Interview Data .... ..... ..... 113
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.1</td>
<td>Inferred Rules for Reference Point for Stationary Bodies (RPS)</td>
<td>113</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Inferred Rules for Frame of Reference for a Stationary Body (FR)</td>
<td>119</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Inferred Rules About Displacement or Change of Location (D)</td>
<td>130</td>
</tr>
<tr>
<td>4.3.4</td>
<td>Inferred Rules for Addition of Displacements (AD)</td>
<td>139</td>
</tr>
<tr>
<td>4.3.5</td>
<td>Inferred Rules for Subtraction for Vector Positions (SV)</td>
<td>146</td>
</tr>
<tr>
<td>4.3.6</td>
<td>Inferred Rules for Reference Bodies for Moving Objects (RPM)</td>
<td>154</td>
</tr>
<tr>
<td>4.3.7</td>
<td>Inferred Rules for Analysis of Components (AC)</td>
<td>163</td>
</tr>
<tr>
<td>4.3.8</td>
<td>Inferred Rules for Components of Velocities (CV)</td>
<td>170</td>
</tr>
<tr>
<td>4.3.9</td>
<td>Inferred Rules for Independence of Magnitudes of Components (IMC)</td>
<td>184</td>
</tr>
<tr>
<td>4.3.10</td>
<td>Inferred Rules for Simultaneity of Component Velocities (SC)</td>
<td>191</td>
</tr>
<tr>
<td>4.3.11</td>
<td>Subjects' Preconceptions About Perspective</td>
<td>196</td>
</tr>
<tr>
<td>4.4</td>
<td>Two New Vector Characteristics</td>
<td>202</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Independence of Location from Path (ILP)</td>
<td>203</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Independence of Directions of Components (IDC)</td>
<td>204</td>
</tr>
<tr>
<td>4.5</td>
<td>Summary of Inferred Rules for Each Subject in the Interview Sample</td>
<td>205</td>
</tr>
<tr>
<td>4.6</td>
<td>Three Types of Preconceptions</td>
<td>207</td>
</tr>
<tr>
<td>4.6.1</td>
<td>Example of the Categorization of Inferred Rules</td>
<td>210</td>
</tr>
</tbody>
</table>
4.6.2 Categorization of Inferred Rules into the Three Types .... .... 211
4.6.3 Categorization of Inferred Rules of each Subject into the Three Types .... .... 211
4.6.4 Discussion of Table 4.25 .... .... 215
4.7 Conclusion of Phase One .... .... 222

CHAPTER FIVE

5.0 Introduction .... .... .... .... 225
5.1 Meaning of Generalizability in this Study .... .... .... .... 226
5.2 The Group Interview Technique .... .... 226
5.3 Construction of the Group Interview Protocol .... .... .... .... 228
5.3.1 Tasks of the Group Interview .... .... 231
5.3.2 The Group Interview Protocol (GIP) .... .... .... .... 232
5.3.3 The Pilot Study ... .... .... .... 233
5.4 Administration of the GIP .... .... .... 233
5.4.1 Sample of Subjects .... .... .... 233
5.4.2 Description of Administration Procedures .... .... .... .... 235
5.5 Methodology of Data Analysis ... .... .... 238

CHAPTER SIX

6.0 Introduction .... .... .... .... 240
6.1 Results for each Vector Characteristic .... 240
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1.1</td>
<td>Results for Reference Point for Stationary Bodies (RPS)</td>
<td>242</td>
</tr>
<tr>
<td>6.1.2</td>
<td>Results for Frame of Reference (FR)</td>
<td>244</td>
</tr>
<tr>
<td>6.1.3</td>
<td>Results for Displacement or Change of Location (D)</td>
<td>246</td>
</tr>
<tr>
<td>6.1.4</td>
<td>Results for Addition of Displacements (AD)</td>
<td>249</td>
</tr>
<tr>
<td>6.1.5</td>
<td>Results for Subtraction of Vector Positions (SVP)</td>
<td>251</td>
</tr>
<tr>
<td>6.1.6</td>
<td>Results for Reference Bodies for Moving Objects (RPM)</td>
<td>253</td>
</tr>
<tr>
<td>6.1.7</td>
<td>Results for Analysis of Components (AC)</td>
<td>255</td>
</tr>
<tr>
<td>6.1.8</td>
<td>Results for Composition of Velocities (CV)</td>
<td>258</td>
</tr>
<tr>
<td>6.1.9</td>
<td>Results for Independence of Magnitudes of Components (IMC)</td>
<td>266</td>
</tr>
<tr>
<td>6.1.10</td>
<td>Results for Simultaneity of Components (SC)</td>
<td>269</td>
</tr>
<tr>
<td>6.1.11</td>
<td>Results for the Two New Vector Characteristics</td>
<td>270</td>
</tr>
<tr>
<td>6.1.12</td>
<td>Inferred Rules for Independence of Locations from Path (ILP)</td>
<td>270</td>
</tr>
<tr>
<td>6.1.13</td>
<td>Inferred Rules for Independence of Directions of Components (IDC)</td>
<td>274</td>
</tr>
<tr>
<td>6.2</td>
<td>Summary of the Inferred Rules for All Subjects in the Sample</td>
<td>279</td>
</tr>
<tr>
<td>6.3</td>
<td>General Discussion of Group Interview Technique and Results</td>
<td>279</td>
</tr>
</tbody>
</table>
## CHAPTER SEVEN

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0 Overview of the Study</td>
<td>282</td>
</tr>
<tr>
<td>7.1 Conclusions of the Study</td>
<td>283</td>
</tr>
<tr>
<td>7.2 Educational Implications</td>
<td>287</td>
</tr>
<tr>
<td>7.3 Recommendations for Further Research</td>
<td>290</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>294</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>301</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>303</td>
</tr>
<tr>
<td>APPENDIX C</td>
<td>322</td>
</tr>
<tr>
<td>APPENDIX D</td>
<td>337</td>
</tr>
<tr>
<td>APPENDIX E</td>
<td>345</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.1</td>
<td>List of Implicit Vector Characteristics of Three Vector Quantities (LIVC)</td>
</tr>
<tr>
<td>3.1</td>
<td>List of Interview Protocol Questions for each Vector Characteristic</td>
</tr>
<tr>
<td>4.1</td>
<td>List of Inferred Rules About Reference Point for Stationary Bodies (RPS)</td>
</tr>
<tr>
<td>4.2</td>
<td>Rules of Rule-Model for RPS</td>
</tr>
<tr>
<td>4.3</td>
<td>List of Inferred Rules for Frame of Reference (FR)</td>
</tr>
<tr>
<td>4.4</td>
<td>Rules of Rule-Model for FR</td>
</tr>
<tr>
<td>4.5</td>
<td>List of Inferred Rules for Displacement (D)</td>
</tr>
<tr>
<td>4.6</td>
<td>Rules of Rule-Model for D</td>
</tr>
<tr>
<td>4.7</td>
<td>List of Inferred Rules for Addition of Displacements (AD)</td>
</tr>
<tr>
<td>4.8</td>
<td>Rules of Rule-Model for AD</td>
</tr>
<tr>
<td>4.9</td>
<td>List of Inferred Rules for Subtraction of Vector Positions (SVP)</td>
</tr>
<tr>
<td>4.10</td>
<td>Rules of Rule-Model for SVP</td>
</tr>
<tr>
<td>4.11</td>
<td>List of Inferred Rules for Reference Bodies for Moving Objects (RPM)</td>
</tr>
<tr>
<td>4.12</td>
<td>Rules of Rule-Model for RPM</td>
</tr>
<tr>
<td>4.13</td>
<td>List of Inferred Rules for Analysis of Components (AC)</td>
</tr>
<tr>
<td>4.14</td>
<td>Rules of Rule-Model for AC</td>
</tr>
<tr>
<td>4.15</td>
<td>List of Inferred Rules for Composition of Velocities</td>
</tr>
<tr>
<td>4.16</td>
<td>Table of Paths Drawn by Subjects</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.17</td>
<td>Table of Paths drawn by Subjects</td>
</tr>
<tr>
<td>4.18</td>
<td>Rules of Rule-Model for CV</td>
</tr>
<tr>
<td>4.19</td>
<td>List of Inferred Rules for Independence of Magnitudes of Components (IMC)</td>
</tr>
<tr>
<td>4.20</td>
<td>Rules of Rule-Model for IMC</td>
</tr>
<tr>
<td>4.21</td>
<td>List of Inferred Rules for Simultaneity of Components (SC)</td>
</tr>
<tr>
<td>4.22</td>
<td>Rules of Rule-Model for SC</td>
</tr>
<tr>
<td>4.23</td>
<td>Table of Inferred Rules for Each Subject in the Sample About Each Vector Characteristic</td>
</tr>
<tr>
<td>4.24</td>
<td>Categorization of the Inferred Rules of Each Vector Characteristic into one of the Type of Preconceptions (Scalar, Transitional, or Vectorial)</td>
</tr>
<tr>
<td>4.25</td>
<td>Categorization of Inferred Rules of Each Subject into the Three Groups</td>
</tr>
<tr>
<td>5.1</td>
<td>Vector Characteristics and the Quantity in the G.I.P. for Each Vector Characteristic</td>
</tr>
<tr>
<td>6.1</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for RPS</td>
</tr>
<tr>
<td>6.2</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for FR</td>
</tr>
<tr>
<td>6.3</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for D</td>
</tr>
<tr>
<td>6.4</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for AD</td>
</tr>
<tr>
<td>6.5</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for SVP</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>6.6</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for RPM</td>
</tr>
<tr>
<td>6.7</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for AC</td>
</tr>
<tr>
<td>6.8</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for CV</td>
</tr>
<tr>
<td>6.9</td>
<td>Paths Drawn for Question 21 of the GIP</td>
</tr>
<tr>
<td>6.10</td>
<td>Paths Drawn for Question 31 of the GIP</td>
</tr>
<tr>
<td>6.11</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for IMP</td>
</tr>
<tr>
<td>6.12</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for SC</td>
</tr>
<tr>
<td>6.13</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for ILP</td>
</tr>
<tr>
<td>6.14</td>
<td>Contingency Table of Female and Male Subjects Holding the Inferred Rules for IDC</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>2.1</td>
<td>A Particle Moves in a Semicircular Path</td>
</tr>
<tr>
<td>2.2</td>
<td>Reflection in a Plane Mirror</td>
</tr>
<tr>
<td>3.1</td>
<td>Network of a Kinematic Vector Quantity</td>
</tr>
<tr>
<td>3.2</td>
<td>Network of Vector Position</td>
</tr>
<tr>
<td>3.3</td>
<td>Network of Vector Displacement</td>
</tr>
<tr>
<td>3.4</td>
<td>Network of Vector Average Velocity</td>
</tr>
<tr>
<td>3.5</td>
<td>Diagram of the Lake for Task One</td>
</tr>
<tr>
<td>3.6</td>
<td>Diagram of the River for Task Two</td>
</tr>
<tr>
<td>6.1</td>
<td>Common Students' Drawings for Illustrating the Direction in which the Motor-Boat was Heading</td>
</tr>
<tr>
<td>6.2</td>
<td>Students' Drawings Illustrating the Direction in which the Motor-Boat was Heading</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

A number of people have been instrumental in assisting me to complete this research. I would like to thank them for their assistance.

The most influential person in my graduate program has been Gaalen Erickson, my thesis advisor; I am deeply indebted to him; he has always been a great source of intellectual stimulation, practical help and emotional support.

I would like to express my gratitude to the members of my committee -- Patricia Arlin, Walter Boldt, Michael Chandler, Peter Matthews, and especially to the late Professor Harry Cannon, who not only served in my committee but was a continuous source of academic and personal support. Also, I would like to thank Lillian McDermott, Betty Howard, and David Whittaker for agreeing to serve as external members of the examination committee.

Special gratitude to Doug Black for his great help in obtaining the subject for the study. Numerous graduate students provided me constant support and constructive criticism. Among them I wish to especially thank Jophus Anamuah-Mensah, David Bateson, and Sharon Haggerty.

Lastly, I must thank my wife, Betty, for the much needed support; and my daughters Carmen and Alejandra, for their patience during my studies. Carmen de Silva, my typist, must receive my gratitude for her wonderful work.
CHAPTER ONE

PROBLEM AND RESEARCH QUESTIONS OF THE STUDY

1.0 Background of the Study

It is generally acknowledged by both teachers and educational researchers that students encounter difficulty in understanding physical vector quantities. The problem seems to be related to the fact that students do not distinguish between physical quantities with only magnitude (scalar quantities) and physical quantities with magnitude and direction (vector quantities); furthermore the factor of direction brings about operations (i.e., addition and subtraction of vectors) different from the operations that scalar quantities deal with (i.e., arithmetic operations: addition, subtraction, multiplication, and so on). Research findings show that problems with these quantities exist at both high school and college levels (Fisher, 1979; Leboutet-Barrel, 1976; Kass, 1976; Warren, 1971). The literature and research regarding these problems are thoroughly reviewed in Chapter 2. It is apparent that researchers have not been looking for explanations with
respect to this area of students' difficulties. Instead, their purpose appears to have been one of identifying the problem rather than analyzing it.

Why do most high school students seem to have difficulty in grasping physical vector quantities? Is it a developmental problem? Must students possess certain mental structures (e.g., the Piagetian formal operations for controlling variables and proportionality) to understand vector quantities and their vector characteristics? Is it that physical vector quantities are too abstract or complex to be grasped by high school students? Or are the difficulties related to instructional procedures? All the factors included in these questions could be part of the problem.

Some educators and psychologists (Ausubel, 1968; Novak, 1977) believe that the most important factor in teaching a content-area is the knowledge that subjects bring to the learning situation. Ausubel felt so strongly about it that he introduced a book (1968) with the following:

If I had to reduce all the educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.

It seems clear, then, that in relation to the topic of this study, the questions to ask are: Do students develop conceptions about vector quantities and their vectorial characteristics prior to formal instruction? Do these
conceptions follow any pattern? Are these conceptions rule-governed? It is the author's contention that students have some intuition regarding the characteristics of vector quantities based on their previous experiences; and it may be that one of the difficulties students encounter in formal instruction is that these intuitive notions are interfering with more formal presentations of solutions to these problems. These intuitions are termed the students' preconceptions about vector quantities and their characteristics.

Little research has been done to uncover students' preconceptions about vector quantities. Piaget (1973) studied the development of conceptions in children regarding several aspects of forces (force is a typical vector quantity), particularly their directional aspect and composition. An extensive analysis of Piaget's results will be presented in Chapter 2.

Aguirre (1978) studied children's beliefs about forces in static equilibrium, and found that students at grade 8 (13 to 14 years of age) have already acquired some intuitive notions of vector characteristics associated with the concept of force. These notions were identified through students' responses to several tasks on forces in an interview setting. An example of those intuitions is that some students in the sample were able to predict the direction of the resultant of two force components. It would be interesting to find out if 14 year old subjects can also associate vector characteristics with other physical vector
quantities such as displacement and velocity.

While little work has been done specifically to study students' preconceptions of vector quantities, more attention has been paid to the problems of representing students' knowledge prior to formal instruction. Some of these attempts have been summarized in Driver and Easley (1978) and Archenhold et al. (1979). However, it was judged that the most practical model, for the present study, is that described by Siegler (1976, 1979). He attempted to identify the rules used by subjects to solve problems in specific content-areas. He further proposed a methodology to assess students' use of those hypothetical rules. A description of his rule-assessment methodology is presented in Chapter 2.

In summary, it is the author's contention that students develop some intuitive notions about physical reality that are related to what physicists define as vector properties. This is illustrated when students are confronted with concrete problem situations in which vector properties are clearly present. This belief is based on the fact that Piaget (1973) and Aguirre (1978) both found evidence in 12 to 14 year old subjects of intuitive ideas which are clearly vectorial.

1.1 Definition of Terms

Some of the terms being used in this study are presented below. It is assumed that the reader is familiar with all the physical quantities included in the study, such as
vector position, displacement, and velocity. If this is not the case, the definitions of those vector quantities are presented in Appendix A.

**Definition of vector quantity** (Jordan, 1969, p.1)

It is a quantity that possesses magnitude and direction and obeys the vector addition operation and the scalar multiplication operation. For each pair of vectors \( \mathbf{u} \) and \( \mathbf{v} \), the vector addition operation requires that:

i) \( \mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u} \) (Commutative property)

ii) \( \mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w} \) (Associative property)

iii) a unique vector 0 (zero) has the property that \( \mathbf{v} + 0 = \mathbf{v} \)

iv) a unique vector \(-\mathbf{v}\) has the property that \( \mathbf{v} + (-\mathbf{v}) = 0 \).

**Definition of preconception**

It is a type of intuitive notion used by a subject to explain a particular phenomenon and which is a product of the subject's experience. This intuitive notion lacks the scope and organization of the formal conception commonly accepted by the discipline.
Definition of inferred rule

A rule is inferred to exist if a subject appears to use a consistent preconception in a given situation to deal with a particular question or problem posed by the interviewer.

Operational definition of inferred rule

The presence of a rule is inferred if a subject uses the same basic preconception to deal with more than half of the questions regarding the same vector characteristic (e.g., choice of reference points to locate a boat on a lake).

Definition of vector characteristic

A vector characteristic describes an aspect of physical phenomena in which the direction plays an important role.

Note: In this study, physical phenomena in which the variable direction is a decisive factor are called vectorial physical phenomena.

Definition of implicit vector characteristic

An implicit vector characteristic represents an aspect of vectorial physical phenomena that is not explicitly elicited in the definition of a vector quantity, or in the definitions of the three vector quantities with which this study deals.
1.2 Implicit Characteristics of Three Physical Vector Quantities

The formal mathematical definition of a vector quantity and its properties was presented in the previous section. This formal definition is generally used by mathematicians and theoretical physicists who do not need to concretize those properties in describing some particular aspect of physical reality. Typical school science text-books (P.S.S.C; 1980, Fundamentals of Physics by Heath, Macnaughton, & Martindale, 1979; Physics by D.C. Giancoli, 1980) also usually introduce vector quantities through such formal definition. However, the author believes that the formal definition does not make explicit some aspects or characteristics of physical phenomena that these vector quantities represent. Furthermore these characteristics are thought to be important in terms of actually understanding these physical quantities. It may be that when teachers use the formal definition, they take for granted that the definitions cover all the important aspects of the physical phenomena that these vector quantities represent. This may be one of the causes of the difficulties encountered by students in understanding these quantities.

The definitions of the three kinematic vector quantities: vector position, displacement, and velocity
(see Appendix A) that this study deals with includes vector characteristics that are not explicitly considered in the definition of a vector quantity. For example, the three quantities require the selection of appropriate reference points for a complete description of them. However, the definition of a vector quantity does not include such a requirement.

Those vector characteristics that are not obviously considered in the usual formal definitions, according to the author, were uncovered by employing a rational task analysis of the three quantities of interest to this study. This is fully explained in Chapter Three. The vector characteristics of interest are listed in Table 1.1.

The last two implicit vector characteristics listed in Table 1.1 (ILP and IDC) were not actually anticipated by the Rational Task Analysis performed upon the three vector quantities. These vector characteristics were uncovered by empirically analyzing the subjects' responses in the first phase of the study. (There were two phases in the study which are explained later). It was decided to consider the two new vector characteristics in the second phase of the study.
Table 1.1  List of Implicit Vector Characteristics of Three Vector Quantities: Vector Position, Displacement, and Velocity (LIVC)

1. Reference Point as a means to describe the location of a stationary body or point (RPS).

2. Frame of Reference associated with a reference point as a means to provide quantitative location of a stationary body (FR).

3. Displacement, a quantity with magnitude and direction, which is independent from all possible paths to join two locations (D). In this study, change of location is a synonym of displacement.

4. Addition of displacements as an operation that relates two consecutive changes of locations with the total change of location (AD).

5. Subtraction of vector position as an operation that relates the vector position associated to two locations of a moving body and the displacement between the locations (SVP).

6. Reference bodies (stationary or moving bodies) as a means to describe the motion of an object (RPM).

7. Analysis of the number of component velocities that combined form an apparent single motion (AC).

8. Composition of two simultaneous velocities that are affecting an object (CV).

9. Constancy of each one of the magnitude of two velocities even when they are interacting to form an apparent single motion (IMC).

10. Simultaneity of effect of two component velocities affecting a body to create an apparent single motion (SC).

11. Independence of a location of a body from the path (ILP). It refers to the fact that it is not necessary to know the path followed by a body when describing an instantaneous position of it.

12. Constancy of each one of the directions of two velocities even when they are interacting to form an apparent single motion (IDC).
The items in this list are not necessarily presented in hierarchical order. The relationships among all these vector characteristics are presented through networks in Chapter Three.

A detailed description of each one of these characteristics and the reasons why they were considered in the present study are being presented in Chapter Four. In that chapter, sections describing each vector characteristic together with the analysis of data collected are developed. It was decided to do so to facilitate the understanding of the relationships between the identified preconceptions with each one of the vector characteristics.

Furthermore, the list was not meant to be exhaustive. As was mentioned earlier, the major selection criterion of the items of Table 1.1 was the condition of not being explicitly included in the formal definition of a vector quantity.

The major aim of this study was to identify and analyze the kinds of preconceptions students possess regarding these implicit vector characteristics. Hereafter, reference is made to the list of implicit vector characteristics, this has been shortened to just vector characteristics.

1.3 General Statement of the Problem

Knowledge of what the learner brings to the learning situation is an important component in planning an educational
program. Although there has been a widespread agreement among educators and educational researchers with respect to the above statement, it is indeed surprising how little research has been directed towards the problem of ascertaining what the learner already knows. Shulman and Tamir suggest that:

There should also be an increase in a genre of basic research that is science-specific rather than experiments to test aspects of psychological theories where science content happens to be a convenient subject-matter to use for research material. Such science-relevant basic research would be on a topic like the cognitive development of science-relevant concepts in young children, e.g., cause and effect, space, time, mass, momentum. This sort of research is clearly in the tradition of Piaget. The purpose would be to identify some general normal expectancies for the evolution of particular concepts around which curriculum developers and program writers could plan their creative endeavours. (Emphasis added) (Shulman and Tamir, 1973, p. 1139).

The shortage of this type of research in the specific area of vector quantities -- as a set of distinctive physical quantities -- may in part be responsible for some of the difficulties encountered by students when confronted with vector-type problems in an instructional setting. This is, there is a lack of knowledge on the part of curriculum makers and teachers about the preconceptions held by students on vector characteristics and consequently these preconceptions are not reflected in the instructional procedures. The difficulties, then, may not be due to the incapacity of students to grasp the vectorial concepts but may be due to the failure to consider students' preconceptions. This lack of consideration may
cause an interference or mismatching between what the teacher wants to teach and what students already know. It was conjectured that this is the case in the instruction of vector quantities.

The broad aim of this study was to develop appropriate methods of data collection and analysis which would permit a detailed examination of the students' preconceptions about vector quantities. The first task required, then, was to carry out an in-depth analysis of the vectorial nature of the three kinematic concepts dealt with in the study. This task analysis yielded the List of Implicit Vector Characteristics, which were described in Section 1.2. Two different methods of data collection were used hence, the results of the study are reported in two distinct phases.

The first phase utilized individual, clinical-type, interviews with students based upon concrete problem situations which illustrated these vector characteristics. In the second phase a group interview instrument was developed which could be administered to intact classes of students. This latter technique allowed data to be gathered on a much larger and more heterogeneous group of students, thus providing a type of validity cross-check of the interview data. In addition it has the potential to be refined for use as a diagnostic instrument by classroom teachers. The analytical techniques developed for revealing the students' preconceptions in Phase One were also used in Phase Two so that comparisons between the two data sets could be made.

In summary, this study makes a significant contribution to the field of science education in two areas: the design and
application of the various methodological techniques used in the study and the substantive results of students' preconceptions about vector quantities.

The broad problem of this study was broken down into a number of more specific problems and further described in terms of a number of research questions in the following two sections.

1.4 Specific Statements of the Problem

The general problem was further subdivided into the following specific statements:

1) To uncover, analyze, and look for the patterns in students' preconceptions about the 12 vector characteristics.

2) To determine whether there was any significant difference between female and male subjects of the study regarding the kind of preconceptions uncovered about the vector characteristics.

3) To categorize the preconceptions for each vector characteristic using a complexity criterion. That is, by analyzing the number and the nature of the variables (qualitative or quantitative) associated with each of the identified preconceptions.

4) To determine the degree of correspondence between the students' preconceptions and the accepted version of vector characteristics as held by the current scientific community.
1.5 Research Questions

The research questions which the present study attempted to answer were as follows:

(1) What preconceptions do students hold about each of the following implicit vector characteristics?
   1.1 Reference point for stationary bodies.
   1.2 Frame of reference.
   1.3 Displacement or change of location.
   1.4 Addition of displacements.
   1.5 Subtraction of vector position.
   1.6 Reference bodies for objects in motion.
   1.7 Analysis of component velocities.
   1.8 Composition of simultaneous velocities.
   1.9 Independence of magnitudes of interacting velocities.
   1.10 Simultaneity of component velocities.
   1.11 Independence of locations from path.
   1.12 Independence of directions of interacting velocities.

(2) How consistent are the preconceptions employed by students when attempting to solve problems involving the listed vector characteristics?

(3) Are the identified preconceptions able to be categorized in some fashion?

(4) Is there a significant difference between female and male students regarding the identified preconceptions?

(5) How generalizable are the identified preconceptions to other groups of students?
Which identified preconceptions are predominant in the study sample?

1.6 Overview of Methods of Study

Phase one consisted in a series of semi-structured individual interviews with students at the grade 10 level and whose ages ranged between 15 and 17 years. The interviews were video-taped and audio-taped for further transcription. Preconceptions with respect to each vector characteristic in the LIVC were identified; consistent preconceptions were expressed as inferred rules, which were categorized according to their content; finally, the inferred rules were presented in a more general and less contextual form, and were called simply rules. These rules should be considered as hypothetical in nature since they were only identified in one type of task situation. The set of rules for each vector characteristic was called a rule-model. In summary, research questions 1 to 3 were addressed in this first phase.

Since the main objective of Phase Two was to uncover intuitive notions about the vector characteristics listed in LIVC using a larger sample of subjects, it was decided to use a group interview methodology. Because one of the objectives was to compare the results from both phases the same context (tasks) had to be used, and also a similar protocol. Therefore, the protocol used in the individual interviews was adapted for use with written response format to facilitate the administration of the
group interview. When administering the group interviews, the interviewer explained the experimental situations depicted in the tasks in sufficient detail, read the questions very carefully and after being satisfied that all students understood the nature of the questions, asked them to think carefully about their response and to write down their ideas in the space provided underneath the questions.

The written instrument used in this second phase was called a Group Interview Protocol (GIP). This instrument cannot be considered as a kind of standardized questionnaire or as a Psychometric test because it was not constructed to serve the purpose of such normative tests. While it was prepared specifically for use in the present study, it may be considered as a type of teachers' diagnostic test. In summary, research questions 5 and 6 were addressed in this second phase. Research question 4 was addressed in both phases.

In both phases, the subjects had not yet been taught any kinematic vector quantities; their preconceptions corresponded to their intuitive notions for dealing with actual problem situations. Here precisely lies the importance of the study since this intuitive knowledge can either help the understanding of concepts delivered in formal instruction or interfere with them. This author
believes that some of the subjects' consistent preconceptions ("inferred rules") could be considered as "stumbling blocks" (Hobbs, 1977) or "critical barriers" (Hawkins, 1980) and could well interfere with the formal instruction of vector quantities.

1.7 Delimitation of the Study

The topic chosen for the study, vector quantities, is a broad one which is also associated with a number of other physics concepts. In addition to the three physical quantities selected for the present study, there are others such as: acceleration, force, and linear and angular momentum. Likewise the study was limited to the analysis of several vector properties; those described in the list of implicit characteristics of physical vector quantities. Others, such as the inner or scalar product of vectors and cross-product of vectors, were omitted because their meaning is remote from students' informal experience. This study, then, was limited to only three vector quantities, (vector position, displacement, velocity) and only to the basic characteristics of those quantities. The reasons for this restriction were related to the purpose of this study, which was focused on uncovering the intuitive notions or preconceptions of students with respect to physical quantities. These three vectorial quantities, which are usually considered to be more simple, are the ones introduced first in a formal instruction context. These three
quantities describe the most relevant directional aspect of motion, which is the attribute that defines them as vector quantities. Hence this seemed like a reasonable starting point for the study.

Furthermore it was felt that other vector quantities, such as angular momentum, torque, or angular acceleration, are very abstract and remote from the experiences of most students, and are only taught in advanced physics courses in the high school curriculum.

Another important limitation was the restricted range of ages (15 to 17) which was used in the study. This narrow range was chosen on the basis of Piaget's (1973) and Aguirre's (1978) studies, both of which reported that vectorial preconceptions were not found until students are over 14 years of age.

The author is aware that an interview technique has some inherent problems with respect to making inferences from small samples. It may also introduce problems of bias with respect to the interview itself and the analysis of data. To overcome these problems in part, the findings of phase one were cross validated in phase two. That is, the generalizability of the inferred rules that were generated in phase one were further checked out in the second phase. The problem of subjectivity in an interview setting was minimized by using a semi-structural protocol for all subjects in the sample.
1.8 Justification of the Study

Physical vector quantities were chosen as the topic of the present research for the following reasons: (1) these quantities are included in most physics programs in grade 11 and 12 in North America; (2) most students appear to have difficulty understanding vector quantities, and it is very important to find out some of the reasons for these difficulties; (3) students' lack of understanding of vector quantities has restricted them from further understanding of a wide range of physics concepts (e.g., see Hawkins (1978) discussion of critical barrier phenomena); (4) most students have gone through informal experiences in which a directional factor, an underlying aspect of vector quantities, has been present and therefore they have had the chance to formulate an intuitive set of ideas about the nature of vector characteristics; (5) it is an area of study that is conspicuously absent from the research literature.

The results of research into students' intuitive notions about physics vector quantities and their specific characteristics can be applied to problems of instruction in two broad ways: Firstly, in the development of instructional packages based upon knowledge obtained from the study. The main purpose of these packages would be
to make the "right connection" between what students already know and what the school curriculum is trying to teach. Secondly, the study would provide science teachers with a well-documented and rich description of the intuitive notions and consistent preconceptions that students use in coping with problematic situations involving vector quantities.
CHAPTER TWO

LITERATURE REVIEW, AND PSYCHOLOGICAL AND EDUCATIONAL CONTEXTS OF THE STUDY

2.0 Introduction

This chapter deals with three main topics: (1) the literature related to students' understanding of vectorial concepts; (2) the general psychological context for the study; (3) the educational perspective in which the findings of the research could be made useful. With regard to the first topic the following aspects are reviewed: (i) an extensive literature review is presented to show the general lack of understanding of vector quantities by students; (ii) the research carried out so far on students' preconceptions about vectorial concepts; and (iii) a review of experiments and instructional units about vectors, which have been created with the specific purpose of alleviating the difficulties usually encountered by students. In relation to the psychological context, a discussion is
presented of the so-called neo-Piagetian psychological approach, and the methodological tools recommended in this new view of development. The psychological components of a new theory of instruction is also introduced in this section, which is based on this neo-Piagetian view of development.

The final section of this chapter deals with the topic of the Educational Context of the study: development of the general implications of this type of research for the curriculum and the corresponding teaching strategies are discussed.

2.1 Literature Review on Students' Understanding of Vector Quantities

Several studies related to the difficulties encountered by students in understanding vectorial concepts are discussed in this section. A connection is made between students' difficulties and what Hawkins (1978) has called critical barrier phenomena; this is done in a special subsection. Previous research of students' preconceptions about vector quantities is also reviewed in this section.

2.1.1 Students' Difficulties in Understanding Vector Quantities

Research findings have shown that difficulties in understanding vector quantities and their characteristics
exist at both high school and college levels. For instance, Fisher (1979) asked teachers to rate a set of physics concepts in terms of perceived difficulty for their students. She found that among the five most difficult concepts (5 of 22) in grade 11, two were related to vector quantities; projectile motion and three dimensional vectors. In grade 12, two vector quantities: oblique momentum interactions and centre of mass reference frame were among the most difficult concepts for students. One source of difficulty, identified by teachers, was the vector nature of the concepts.

Johnstone and Mughol (1976) tried to establish which of the concepts taught in school physics were proving troublesome; they found that five physical quantities among the 20 considered most difficult were strictly vectorial. These concepts were presented to school and college students who were asked to rate them on a scale of difficulty after the concepts had been taught. A high percentage of grade 11 and 12 (44 percent) felt that vector concepts were among the most difficult.

Kass (1976) asked 353 grade 12 students to rate the difference in difficulty between 20 concepts of mechanics matched in pairs. The summary of her results shows that the principal clusters of difficulty are circular motion, centrifugal and centripetal forces, velocity, projectile motion, moments, and composition and
resolution of forces. All of these physical phenomena are strictly vectorial.

In another article Warren (1971) reported results from the following problem which was given to 148 university entrants in engineering and science:

A motor vehicle travels with uniform speed on level ground, turning to the right in a path of uniform curvature. There is no wind. Sketch a plan showing (a) an arrow marked \( R \) representing the resultant of all forces acting on the vehicle in the horizontal plane. (b) An arrow marked \( F \) representing the resultant force of friction exerted by the ground on the vehicle. (c) Suitable labelled arrow(s) representing any other force or forces acting on the vehicle in this plane.

This is a simple vectorial problem. While nearly all the students attempted the problem, their figures were unintelligible projective drawings; only 14 answered question a correctly, 3 answered b, and 13 responded to c. The next year the same problem was given to 193 new students; the number of correct answers for each question were the following: 23, 11, and none for a, b, c respectively. Warren adds that the most popular answer gives the resultant force in or near the direction of motion.

This concept of circular motion is surrounded by mystery and confusion on the part of the students. One reason may be that mechanics is so often taught in only one dimension (scalar perspective) before extension to two dimensions (vectorial perspective). In one dimension the motion considered is in the direction of the applied force, hence the one-dimensional solution reinforces the students'
initial intuitive notion on force and motion. To understand circular motion it is necessary to have some appreciation of simple vector displacement, velocity, and acceleration.

Usually, one of the first vector quantities introduced in school is displacement; it is the difference of position of two points in terms of magnitude and direction. In everyday speech, velocity is often used as a synonym of speed; consequently students have much difficulty in distinguishing clearly the stipulated scientific meaning of these two terms. This confusion between velocity and speed was also examined in the test given by Warren. The question given was:

"A particle moves in the path shown (Figure 2.1), the speed increasing uniformly with time in the semicircular section, from 10 ms\(^{-1}\) to 12 ms\(^{-1}\). For this section of the path calculate the averages of (a) the velocity (b) the acceleration. (Assume \(\pi=22/7\))."
The semicircular path is taken at an average speed of 11 ms\(^{-1}\), hence it takes 2/7 sec. in going along that path. The change of displacement is 2 metres downward in the Figure, hence the average velocity is 7 ms\(^{-1}\) downwards. This answer was given by only one student among the 148 responding. Although nine tenths of the students had distinguished velocity from speed when asked to classify them as scalars or vectors, with but one exception they gave the average speed when asked for the average velocity. The change of velocity is 22 ms\(^{-1}\) to the left in Figure 2.1, hence the average acceleration is 77 ms\(^{-2}\) to the left. This magnitude is obtained by the ratio of the magnitude of the change of velocity, 22 ms\(^{-1}\), and the time taken between the two points, 2/7 sec. The correct answer was given by only two students.

Warren concluded the report by saying that the students' difficulties can be seen to be related to certain common aspects of the teaching of physics in pre-university courses, but he did not go beyond this point to identify the real roots of the problem.

Leboutet-Barrel (1976) reported that more extensive studies about vector understanding have been conducted by a team of university professors in Paris (Malgrange, Saltiel, and Viennot 1973, 1975) using several hundred students. One of their first published papers concerned the distinction between vectors, scalars, and physical measurements.

Two other studies by this team were concerned with students' understanding of the relations between force and velocity, and between force and motion. It appears that
two intuitive types of forces exist for students: real interaction forces which act upon masses (static field of forces, gravity, tension of a spring) and a kind of force associated with motion which has the same direction as the velocity and which becomes zero at the same time as the velocity; this type of force is thought of as a property of the mass, like impulse or force of inertia.

The second study reported (which involved about 300 students) by the above team of researchers examined students' ideas about the composition of the movement of translation in two dimensions. Results showed that students perceived the speed of a moving body as the speed communicated by its motor (it was assumed that the body had a motor) without any relation to the frame of reference. Motion and, therefore, speed are intrinsic properties of the moving object as well as its mass and its dimensions. The results of these studies indicate the difficulty experienced by students in coordinating motion within two frames of reference. This difficulty was also demonstrated by Piaget et al (1973).

Leboutet-Barrel finishes his article saying that much more research is needed concerning the growth of such concepts as force, momentum, velocity, speed, energy in students at each age level. He adds that:

We need to know how students integrate networks of new concepts and structure them. It would be indispensable to explore the area of preconceptions and it would be of extreme importance to make an inventory of the entire range of erroneous concepts elaborated subsequently by students in the course of learning and which distort the construction of coherent system of knowledge. (Leboutet-Barrel, 1976, P.465)
2.1.2 Viewing Students' Difficulties as a Critical Barrier Phenomena

Hawkins (1980) discussed a specific class of learning difficulties in school science and mathematics which he called critical barrier phenomena. It is very likely that the difficulties that most students experience in understanding vectors fit in his definition.

Hawkins' describes critical barriers as follows:

(1) Are conceptual obstacles which confine and inhibit scientific understanding, (2) are 'critical' and so differ from other conceptual difficulties in that they: (a) involve preconceptions which the learner retrieves from past experiences that are incompatible with scientific understanding, (b) are widespread among adults as well as children, among the academically able but scientifically naive as well as those less well educated, (c) involve not simply difficulty in acquiring scientific facts but in assimilating conceptual frames for ordering and retrieving important facts, (d) are not narrow in their application but when once surmounted provide a key to the comprehension of a wide range of phenomena. To surmount a critical barrier is not merely to overcome one obstacle but to open up stimulating new pathways to scientific understanding. (Hawkins, 1980, p.3)

Hawkins has used the following example to investigate people's ideas about mirror vision:

I have sometimes asked subjects to imagine that one wall of the room we are sitting in is a large mirror and then to draw, on a map of the room, the direction in which they would look to see a given object 'in the mirror'.
(Hawkins, 1978, p.1)

He has found a wide range of directions as answers but most of them cluster around the two extreme labelled as
Subjects consisted of upper elementary school children, elementary school teachers, and a graduate class in the philosophy of science. About 15 percent cluster at A (the correct response), 50 percent at B (the incorrect response), and the rest scatter in between. The idea of selecting direction B is a clear example of critical barrier phenomena. This concept and others related to other physical phenomena (Hawkins names size and scale, heat and temperature, and action and reaction) are easily observed in many contexts, and represent barriers to learning for at least a clear majority of high school and college students. These conceptions are generally present when the related subject-matter is introduced, but even when students may see the conflict, they do not seem to be able to discuss it. In regard to this specific problem Hawkins has said that:
a teacher concerned to 'cover the subject' — meaning, of course, to get through a textbook or promised outline — will become exasperated with students' disabilities or with his own inability to make such elementary things clear. The fact that patient explanation is no immediate cure is a hallmark of the class of critical barrier phenomena. (Hawkins, 1978, p.5)

The field of mechanics potentially may contain many examples of critical barrier phenomena. Particularly, in phenomena in which direction is a decisive factor in their comprehension.

It seems reasonable, at this point, to speculate that students' pre-instruction conception about physical vector quantities may be considered as a critical barrier phenomenon. For instance, Warren (1971) found that students tend to ignore the directional aspect in physical situations in which this factor is decisive. Findings such as these would suggest that most of students tend to treat physical vector quantities as though they were scalar. This belief may be a conceptual obstacle and may inhibit further scientific understanding. These beliefs about mechanics phenomena could become "critical barriers" for the instruction of students on current views of mechanics phenomena. It is with this kind of problem that the present study is concerned. The questions addressed in this research are aimed at uncovering students' beliefs that may well qualify as critical barriers. The search for these kinds of beliefs seems imperative. Hawkins has said that:

Though the existence of individual critical barriers appears to have been recognized by many thoughtful and experienced teachers, they appear not to have been considered collectively
as providing important clues to the improvement of the teaching art, to curriculum making or to cognitive science. (Hawkins, 1980, p.1)

This author believes that students' preconceptions can be used as a starting point for instructional procedures which can help to surmount the critical barriers or obstacles that they produce.

2.1.3 Previous Research About Students' Understanding on Vector Quantities

As it was mentioned in Chapter One, there has been little research on students' preconceptions about vector characteristics of physical quantities. In fact, only two studies were found in the literature (Piaget, 1973; Aguirre, 1978) which were mainly concerned with students conceptions of vectors. Both of these studies focused on the students' concept of force; and both used a Piagetian approach for data collection. Two other studies related to vector quantities have been reported but these quantities were considered as scalar (Raven, 1968, 1972). All these studies are reviewed below. Other investigations, not directly related to students' conceptions, but aimed at helping subjects to understand vector quantities are also reviewed in this section.

A. Piagetian-Type Studies

Piaget (1973) created nine tasks to find out how children (4 to 12 years of age) develop notions about several
aspects of forces -- force being a vector quantity. The aspects considered by him are precisely the vector characteristics of force. For each task Piaget categorized the children's beliefs in terms of three developmental stages, with two levels (A and B) at each stage. Reviewing his results, it is evident that children do not understand the vector characteristics of force until stage III. This corresponds to subjects over 12 years of age. Some general results of Piaget's study are reviewed below.

In one of the tasks, which uses a circular table, with three non-parallel forces acting upon a small object (Piaget, 1973, Chapter 6), children in stage II (7 to 10 years of age) cannot solve problems where forces are either at an angle or are different in magnitude. At the start of the task, when forces are parallel, they use strict arithmetic additivity of forces. However, when the forces act at an angle and the observed results seem to contradict their predictions, they return to non-additivity. In stage III some subjects appear to consider the effect of the angle on the resultant force. In another task (Piaget, 1973, Chapter 7), subjects in stage III seem to perceive the role of the direction of the forces as well. They said that weights (an example of a gravitational force) pull less when they pull in different directions and more when they come together. Piaget summarizes these experiments on forces and other studies about children's beliefs in another book called Understanding Causality (1974). In this
book, Piaget claims that:

When we come to directions, we find new problems, since we are no longer concerned with so many general coordinations and relationships between the operational forms and the experimental contents, but rather concerned with spatial operations, the peculiar characteristic of which is the construction of forms of which some correspond, in other respects, to similar forms existing in the objects themselves and subject to being reached perceptively or through physical experience. (Piaget, 1974, p.35)

This statement indicates the importance of the students' preconceptions on spatial operations. It is not expected that these preconceptions include ideas of the way that physicists formally define vector position, displacement, and their vector operations; but it is possible that they may contain some features of vector characteristics, particularly the directional factor of spatial locations of objects.

From the data collected Piaget concludes that:

In all cases studied, the geometric structuring of directions is linked to dynamics. That is, what is causing the direction of motion. For instance, the spatial operation of displacement, direction, etc., become explanatory only when consonant with dynamics and attributed to objects, since these determine, in their causal connections, the geometric relationships isomorphic with those appropriate to the operations. Needless to say, the causal interpretation is possible only if the shapes and initial position of the objects are given, that is, by granting them spatial as well as kinetic and dynamic properties. (Piaget, 1974, p.41)

Efforts will be made in the present research to concentrate only on the kinematic aspects of the concrete problem situations depicted in the tasks, that is, the cause of the physical phenomena will not be considered.
If the subjects refer to dynamic factors in their responses the researcher will cautiously try to move the discussion to the kinematic aspects of the task.

In summary, Piaget was interested only in some vector characteristics about one vector quantity -- force. It is the purpose of the present study to uncover how students develop conceptions about other vector quantities before formal instruction. To this end, it may be that students hold vectorial conceptions about vector position, displacement, and velocity that are similar to the conceptions that Piaget found for forces. However, it is important to point out that the aim of this study was not to categorize the subjects' preconceptions in developmental stages as Piaget (1973) did with his study on forces, but to uncover the specific preconceptions that students develop for the vector characteristics that this study deals with.

Aguirre (1978) carried out a study about children's beliefs about forces in static equilibrium. The main emphasis of this research was not directed toward looking for vectorial characteristic of forces in children's minds, but their beliefs about different aspects of the concept of force. The research questions in that study were mainly related to the children's beliefs on action of forces, static equilibrium in systems with two and three component forces, action and reaction, and composition of forces. Among the findings of the research, the author found that children about 12 years of age have already grasped physical conceptions that clearly have vectorial features.
These were related to composition of forces. Children aged 14 were able to solve problems containing components with different magnitude and various angles. Subjects in the sample had not yet been instructed about composition of forces. This shows that students develop some physical intuitions with vectorial features; it may be however, that these intuitive vectorial conceptions are only related to forces. One of the intents of this study is to determine whether students may also develop some vectorial intuitions in relation to other physical entities.

Raven (1968, 1972) has carried out studies on children's beliefs about two vector quantities; acceleration and linear momentum. But he did not emphasize the vectorial properties of these concepts at all, since he used primary school children in both studies. He was able to avoid the vectorial properties of linear momentum by taking an unidimensional physical situation; that is, he considered momentum as a scalar quantity. The purpose of the momentum study was to determine the developmental sequence within a set of items postulated as being necessary for understanding momentum. A similar purpose was behind his study of the concept of acceleration. Again, by using an unidimensional situation, acceleration was treated as a scalar concept. This is a common approach taken by researchers; they set up experimental tasks in which the directional factor is being controlled. This produces a very simplified situation, transforming complex phenomena into simpler ones,
in which subjects have to work with fewer variables. This is a totally acceptable approach, but it does not allow one to uncover all the potentially rich conceptions that students may have developed through formal and informal experiences with their environment.

B. Special Experiments to Teach Vector Quantities

Physicists and physics teachers, being aware of the students' difficulties in understanding vector quantities, have created special experiments in which the vectorial properties of physical quantities are dramatically shown. These experimental tasks are prepared with the aim of helping students to grasp the vector characteristics of certain quantities. Unfortunately there is no data available to determine if these tasks have been successful. A list of some of these vectorial experiments follows:

Does pressure have direction? (Dempster, 1951); a dramatic distinction between pressure and force (Miller, 1952); a model to illustrate some properties of vectors in three dimensions (Baez, 1957); vector navigation: a game for physics (Perry and Campbell, 1971); versatility of the screw nail approach to teaching the concept of cross product (Greig, 1972).

C. Vector Quantities in Mathematics Education

Researchers in mathematics education have also been interested in students' understanding of vectors. But they usually treat vectors as abstract mathematical entities, and do not make connections with physical reality. Their main interest has been to use vectors as a tool to improve the
teaching of other areas in mathematics. To accomplish this task, mathematics educators have created instructional units for use in school situations. The titles of some of these units are listed below: Complex Numbers and Vectors in High School Math (Pedley, 1966); Vectors: An Aid to Mathematical Understanding (Smith, 1959); Approach to Vector Geometry (Troyer, 1963); Using Vectors to Solve Simultaneous Equations (Salzarulo, 1963); Vectors in Algebra and Geometry (Glicksman, 1965); Plane Geometry by Vector Methods (Scott, 1970). Some of the studies (Lamon, 1971, Rosembloom, 1969) have used concrete objects to improve the understanding of vector space, but none of them have used a physical quantity like velocity, acceleration, or force. This may be an acceptable approach for the study of vectors from a mathematical point of view; however, physics teachers should look for concrete experimental situations obtained directly from physical reality.

2.2 Psychological Context of the Study

2.2.1 Introduction

This section deals with the broad psychological context in which this study is embedded. First, a theoretical issue involving the dichotomous approach to the study of development as proposed by some psychologists is discussed — the formal - functional (or competence and performance models of development) dichotomy. Reasons are advanced as
to why the performance model was chosen as the preferred approach for the present study. This model is explained through the work of Siegler, and Case. Although, this study is not directly related to instruction, a separate subsection is devoted to the psychological component of Case's Theory of Instruction. Case proposes (the second step in a three-step theory of instruction) to assess students' current level of functioning in specific content-areas. This proposition is in sympathy with the basic aim of this research. Finally, the methodology approach currently used by some researchers in the 'Neo-Piagetian tradition' -- rational task analysis -- is presented.

2.2.2 Two Aspects of Developmental Psychology: Formal and Functional

Some psychologists (Flavel and Wohlwill, 1969; Wollman, 1978; Case, 1978) have made the distinction between two interdependent aspects of development: formal and functional. Flavell et al. (1969) have explained the meaning of the two aspects in the following way:

The formal aspect has to do with the morphology of the process: the sorts of cognitive entities that make up the successive outputs of development and how these entities are causally, temporally, and otherwise interrelated ... The other aspect ... has to do with function and mechanism, somewhat specified in relation to environmental inputs, by which it in fact makes the cognitive progress that has been formally characterized. (Flavell and Wohlwill, 1969, p.67).
This dichotomy of development has been shown to be a useful way to organize or to structure research questions; however, it should be applied cautiously and reflectively. One problem is that it implies that the hypothesized mental structures are transsituational, atemporal, and context-free — a position that is difficult to defend. Even Piaget (1972) has stated that his formal operations were not totally context-free.

Another theoretical model that is closely related to the above dichotomy is the competence versus performance or automaton model of development. The former is related to the formal aspect and the latter to the functional aspect. Flavel and Wohlwill have explained this position as follows:

A competence model, which is a formal, logical, representation of the structure of some domain ... an automaton model (an elaborated version of what Chomsky apparently meant by "performance"), which represents the psychological process by which the information embodied in competence actually gets accessed and utilized in real situations. The competence model gives an abstract, purely logical representation of what the organism knows or could do in a timeless, ideal environment, whereas the automaton model has the job of describing a real device that could plausibly instance that knowledge or skill, and instance it within the constraints (memory limitation, rapid performance, etc.) under which human beings actually operate. (Flavell and Wohlwill, 1969, p.71).

Using these descriptive categories, Piaget's theory of development is normally considered as an example of a formal theory or competence model of development. This is because Piaget's main interest was to look for general mental structures that were transsituational and context-free.
Considering Piaget to be predominantly a structuralist or formalist, rather than a functionalist means, according to Wollman (1978), that he is less concerned with the content of human knowledge than with its organization or structure. For instance, Wollman claims that:

Piaget's concern is with the fundamental underlying organization of performance on tasks such as the seriation task. He feels that such tasks reveal an identical organization or 'structure', that is, successful performance on all such tasks involves one basic structure or scheme, the seriation scheme. (Wollman, 1978, p.8)

Flavell and Wohlwill (1969) have also pointed out that Piaget's primary research program has been to provide a logical description of the systems of intellectual operations that children possess at different points in their development, not to provide a psychological description of the processes by which these operations are acquired and utilized.

Other psychologists such as Pascual-Leone (1966), Wollman (1978), Siegler (1976, 1978), and Case (1978) have been more concerned with what has been defined as a functional aspect of development. According to this viewpoint the result or performance of a cognitive encounter with the physical environment is partly a function of what the subject already knows about a specific content-area and how that specific knowledge is represented and organized. In this account, the functional aspect of development (or the performance model) is more closely related to the growth of understanding of specific content-areas and the possible
factors that could influence such growth. This is specifically shown by the work of Pascual-Leone and Siegler.

It is worth stressing again, however, that this dichotomy (formal and functional aspects, or competence and performance models) should not be accepted unreflectively. Even if one finds it useful to distinguish these two aspects of development, their manner of interaction must be considered. It would be difficult, in the view of this author, to defend a position that treated them as two totally independent aspects of development.

If the competence-performance dichotomy is cautiously considered, however, it does have some utility and heuristic value, and on these grounds was used to characterize the nature of the research undertaken in this study. The intention, then, is not to solve the problems of the dichotomy or to settle the issue but to draw upon some of the theoretical and methodological knowledge for use in the present research. Since the broad goals of this study were to find out what a subject already knows about a specific content-area (i.e., vector quantities), and to determine how that specific knowledge is organized, it was clear that the performance model of development provides the more appropriate framework.

In summary, it was thought that the analysis of students' preconceptions, obtained through their performance when solving specific tasks, might shed light on the process by which students develop their beliefs with regard to particular content-areas. This means that the research was very
contextual oriented in nature. The aim was not to see if students' preconceptions about vector characteristics fit, for instance, into the logical description of the systems of intellectual operations of concrete and formal stages of Piaget's theory. Nor was it intended to determine what concrete or formal operations are necessary to learn to understand vector quantities, rather, the aim was to uncover students' preconceptions about a specific content-area and to find out if such preconceptions possess any degree of consistency or organization.

The approach taken in this study, then, fits well with the so-called "Neo-Piagetian" Approach to Development, and is primarily concerned with the functional or performance account of the subject's behaviour.

2.2.3 The Neo-Piagetian Approach of Development

As the name implies, this approach has been recently advanced by several theorists. (c.f. Wollman, 1978). The approach is so named because it takes Piaget's account of development as its starting point and attempts to supplement it with an account of the factors associated with the process of acquiring new conceptions. In other words, the Neo-Piagetian approach considers the process of development and the factors affecting that development, so that the students' current level of functioning in a given content-area might more easily be assessed. The reason for this is that the factors that constrain performance would be better understood
and the type of assessment of interest would be more closely related to observed performance (Wollman, 1978).

The groundwork for the movement was done mainly by Pascual-Leone (1969) in his doctoral dissertation and subsequent work. Besides Pascual-Leone, two other Neo-Piagetian researchers have recognized the importance of emphasizing the functional aspect of development; they are Siegler (1976, 1978) and Case (1978). Their work is closely related to the present study and it is discussed in more detail below.

Siegler's Approach: Siegler's work (1976, 1978, 1979) is predominantly concerned with describing the subjects' current level of functioning in given content-areas. His aim is to characterize and explain developmental differences in thinking. In one of his earlier studies (1976) the focus was upon three aspects of development: specific knowledge governing task performance, responsiveness to experience, and basic processes that underlie differences in these two areas. For the present study, only the first aspect is relevant. Siegler's main concern is to determine children's knowledge about specific content-areas (i.e., balance, time speed, and distance) in an accurate and unambiguous way. Specifically, he characterizes students' conceptions in terms of rules. This entails the assumption that children organize knowledge in terms of rules or schemes which they then apply to solve a series of problems which appear to be similar in some way. In other words, Siegler (1976, p.111) assumes, that students' problem-solving strategies are rule-governed with the rules progressing from
less sophisticated to more sophisticated with age. For the classic, Piagetian balance task, he proposed 4 rules: the first rule typically being used by the younger children (5 years) and the fourth rule by the oldest (over 12 years).

But what does it mean to say someone is using a rule? Siegler answers by saying:

In one sense, rules can be thought of as a means of summarizing data, as when a set of responses is symbolized by a single verbal sentence, a mathematical equation, a production system, or some other formalism. Within this usage it makes little sense to inquire if rule-governedness develops, because the rules are merely representations (they are in the experimenter's head); any failure of the rules to fit the particular responses tells us only that the representation is inadequate. An alternative approach, however, is that the rule statements not only summarize data but also have some correspondence to the way in which the data were generated (they are in the subject's head as well as the experimenter's) (Siegler, 1978, p.19).

For the present study, the second approach was considered. That is, rules were inferred as a means of summarizing data, but it was also assumed that they have some correspondence to the way how subjects generate them.

Siegler (1977, p.394) says that a number of other tasks have been studied using rules as a representation approach. He cites Inhelder and Piaget's (1958) projection of shadows task, Bruner and Kenney's (1966) water jar task, and a probability concept task described by Chapman (1975). Siegler reports that in each case:
The proposed rules have been found to characterize the performance of more than 80% of children aged 5 to 17 years at a level of probability extremely unlikely to be the product of a random response process. (Siegler, 1977, p. 394)

Even though the broad, theoretical approach taken in the present study is similar to that of Siegler, the collection and data analysis methodologies proposed by him (Siegler, 1978, 1979) were not used. The reasons for this decision rested on the methodology itself; in this methodology (Rule-Assessment Methodology) the first steps requires the formulation of models of the rules that subjects might use to perform the tasks. So far, Siegler has formulated these models of rules on the basis of previous research findings. For instance, in his studies on the balance (Siegler, 1978) and on the concepts of time, speed, and distance (Siegler, 1979) he has formulated the models of the rules based on Piaget's initial investigations in these conceptual areas. But, as was mentioned earlier, no study related to students' preconceptions about kinematic vector quantities was found in the literature so there was no data upon which to formulate models of the rules for these concepts. Therefore, the Siegler's Rule-Assessment methodology could not be used in the present study. One of the aims of this study was in fact to provide data such that it would be possible to formulate the Siegler-type rule-model. These can be subsequently tested using the cited methodology.
Case's Approach: Case, a developmental psychologist, believes that research based on the Neo-Piagetian approach can be used to address instructional problems (Case, 1978a). He argues that a functional approach may show the process by which students acquire and utilize operational structures and other more contextual types of conceptions. One of the important questions Case wants to answer "is how to adopt the content of instruction to the structures that children already have available". He proposed in his theory a process of three steps:

The first step is to analyze the structural underpinnings of the academic discipline that is to form the content of instruction. The second step is to assess children's current level of functioning in that discipline. The third step is to tailor the instructional activities that are presented to the level of functioning that is assessed, either with a view to promoting a transition from this level of functioning to a higher one (if that is the goal), or with a view to adopting the material presented to the current level of functioning (if that is the goal). (Case, 1978a, p.204)

Although this instructional procedure was developed to optimize the development of students' operational structures, it can also be used to study and make use of the process by which conceptions related to specific content-areas are acquired and utilized by students.

From the three steps suggested by Case, only the last one is strictly instructional. The second step is actually related to the performance aspect of development; therefore, it is not strictly related to instruction. However, it is very useful for instructional purposes. The first step is
neither directly related to the functional aspect of development nor to instruction. But the result of this analysis, which should identify the major concepts (in a given disciplinary teaching unit) and the relationships among them, may help to focus the search for students' conceptions about specific aspects of that content area.

For the present study, only the first two steps are relevant; the third step, which relates to instruction, can be considered only in terms of the possible applications of this work.

Case's first step, a structural analysis of the academic content, has been partially done in Chapter 1 -- the development of the list of implicit vector characteristics. This analysis is further expanded in Chapter 3 where the relationships among these vector characteristics is displayed in terms of a network of conceptual relationships. The methodology used to perform this type of structural analysis is presented below in the next section. The second step of Case's approach actually corresponds to the main objective of the study; that is, to study the current level of functioning of grade 10 students about the vector characteristics listed in Table 1.1.

2.2.4 Task Analysis

One of the more promising lines of current psychological research in cognitive development utilizes
some form of Task Analysis (Siegler, 1978, Wollman, 1978; Case, 1978). The value of this type of analysis for education is that such research appears to be directly applicable to instruction. Hence this kind of approach was taken in the present study.

The type of analysis alluded to in the previous section (Step 1 in Case's sequence) is generally called a rational task analysis. (Thereafter abbreviated as R.T.A.). One common feature of R.T.A. is its hierarchical nature. Task hierarchies of a certain kind have been described and used for instructional purposes by Gagné (1970) and others. But according to Wollman (1978), Gagné's analysis is not really psychological and certainly not developmental, in that it refers to a kind of expert's performance. Resnick (1976) argues that Gagné-type hierarchies do not arise as psychological descriptions of performances on a variety of tasks; that is, psychological models are "left entirely implicit in Gagné's work". She adds that this is the reason that Gagné's hierarchies need validation. Resnick goes on to describe R.T.A. in terms of:

Typically a rational task analysis is derived from the structure of the subject-matter and makes few explicit assumptions about the limitation of human memory capacity or perceptual encoding processes (Resnick, 1976, p.65).

In contrast to the rather formal, logical analysis of conceptual relationships from a disciplinary perspective, which characterizes R.T.A., another type of analysis has
been described as an empirical task analysis (hereafter abbreviated as E.T.A.). This type of analysis attempts to consider the question of how that particular task is seen by the student. What schemes or preconceptions do the students possess which allow them to understand the task? How are these schemes or preconceptions coordinated? What irrelevant cues are significant for them? Are they likely to be easily influenced by the cues? The answer to these questions obviously requires the investigator to become familiar with the "knowledge" (preconceptions) possessed by the student and how this knowledge is coordinated.

Resnick describes an empirical task analysis in the following way:

It is based on interpretation of the data (errors, latencies, self-reports, eye or hand movements, etc) from human performance on a task; the aim of such analysis is to develop a description (model) of processes that would account for those data". (Resnick, 1976, p.65).

Case (1978) and Siegler (1976, 1979) have used both types of task analysis (Rational and Empirical) in their research. In their work they attempt to uncover the difficulties that students show when encountering specific tasks. They suggest that the tasks or content-areas to be taught (e.g., tasks related to vector quantities) must be carefully analyzed, that the learner's initial knowledge must be carefully assessed, and that the learning activities must be rationally and psychologically (in the developmental sense) sequenced to bring the learner from his initial state to the desired state. This means that Case and Siegler's
approaches differ markedly from that of most Piagetian educational researchers, in that there is no attempt or need to classify learners as concrete or formal. Instead, this neo-Piagetian approach advocates assessment of the learner's initial state in terms of the strategies which he or she spontaneously applies to the criterion task.

Wollman (1978) thinks that some content-areas are more suitable than others to be treated from a functional point of view, and for the use of rational and empirical task analyses. Wollman says that:

Task analysis should be applied to some several key science and science-related mathematics topics. To qualify as a key topic, two conditions should be met: (1) the topic should play a central role in the structure of the subject matter and (2) the topic should be particularly recalcitrant to current teaching attempts. (Wollman, 1978, p.107)

This author believes that physical vector quantities and their special characteristics fit these two conditions stated by Wollman, as was shown in Chapter 1. Therefore, Task Analysis, in its two forms: Rational and Empirical, was used in the present study.

In summary, in this study two kinds of task analyses were used: a Rational Task Analysis that consisted of an analysis from the discipline expert's point of view, and an Empirical Task Analysis that consisted of analyzing the students' preconceptions about the content-area included in the research tasks.
2.3 Educational Context of the Study

In the last two decades the underlying curriculum approach in science teaching has focussed upon the process of inquiry, which emphasizes the practice of scientific method. (Shulman & Tamir, 1973; Hawkins, 1978; Wittrock, 1966; Bruner, 1961; Schwab, 1962). But, according to Hawkins (1978), method is the use of knowledge to gain more knowledge; he claims that if not much knowledge is organized and available for use not much will be learned (p.20). Others in turn have proposed a "Concept Approach" (Shulman & Tamir, 1973; Schwab, 1962, 1964; Hawkins, 1978). They were reacting against that limited curriculum approach, which emphasizes only the method of acquiring knowledge instead of considering the way the disciplines themselves acquire that knowledge. According to this concept approach a curriculum is organized in successive phases, each explaining and illustrating some basic scientific concept.

In this context, it is expected that when the course is finished students will have risen to a higher level of scientific literacy, and will possess a foundation for understanding scientific statements and explanations. Hawkins states that:

This view, like the method view, represents conditions necessary but not sufficient. It suffers from the ambiguity or confusion of two very different levels of 'learn' and 'understand'. Verbal transmission and reception focused on the use and illustration of scientific
concepts is not the same as teaching and learning which succeeds in reconstructing older thought-habits ... What has been wrong with traditional didactic instruction has not been its emphasis upon the transmission of factual knowledge but its neglect of the developmental preconditions for genuine and productive assimilation of this knowledge. There is a time for didactic teaching but it is a time which presupposes that thought-habits which can accept, store, and retrieve for new uses are already developed or developing ... 'Optimal teaching must, therefore, be both diagnostic and nurtural. (Hawkins, 1978, p. 20-21)

This author has a view of science education very similar to that of Hawkins, particularly with regard to the diagnostic aspect of teaching. It is argued that by considering the knowledge, the learning strategies, and the skills that students bring to the instructional situation, the teaching-learning process has much to gain. But to practice this approach, research about these three aspects is necessary. The present study tried, in particular, to examine the first aspect by examining the knowledge or preconceptions that students possess prior to instruction. This was done by uncovering and analyzing students' pre-instructional conceptions in a specific content-area.

2.3.1 Teaching-Learning Problems in Science Education

The general problem of the present research is a teaching-learning problem. Research has shown that students do not easily grasp vectorial quantities that are usually
included in physics Grade 11 and 12. (Fisher, 1979; Kass, 1976). Where do the problems lie? Is it the instruction methodology? Is it that students are not "ready" to grasp these concepts? Is it that the content-area is very remote from students' experience and, therefore, is too complex to be understood?

This author believes that some of the problems lie in the instruction methodology. Present methodologies usually do not consider students' current knowledge. Obviously this methodology is part of an overall curriculum view. In this view, curriculum makers, among other considerations, plan a content-area program mainly taking into account the structure of that content-area or discipline. The program is usually arranged in hierarchical order; that order being suggested by the expert in the field. As we see, in this approach there is no consideration of students' views.

To change to an approach that considers students' conceptions is not an easy task. First of all, research is needed to uncover students' beliefs in many topics included in science programs. For instance, not much is known about students' preteaching conceptions about vectorial conceptions.

Another important reason to find out the students' preconceptions is that some of these (also called by some alternate frameworks or misconceptions, see Driver and
Easley, 1978, p.62) are very resistant to extinction, and interfere with formal instruction. This is another important reason to uncover and analyze these alternate frameworks, and also to investigate how these intuitive conceptions can be used in instruction.

Knowledge of students' alternate frameworks has been used in constructing learning tasks in science. Cole and Raven (1969) report a study with 12-15 year old students which indicates the possible benefit of giving students the opportunity to reject irrelevant factors in understanding the principle of flotation. Common "misconceptions" related to floating and sinking bodies were also considered in the design and implementation of an instructional sequence by Rowell and Dawson (1977). Their report indicates that despite efforts to refute students' misconceptions, some persisted after instruction. Driver (1973) explored some of the difficulties science students (11-12 years of age) have in accommodating their thinking to new experiences. Prior to teaching, students were interviewed and a number of alternate frameworks about the topics being studied (balancing systems, centers of gravity, the law of moments, action and reaction, and relation between force and motion) were identified. After instruction of the units related to these topics, it was apparent that the counter-examples and conflicting evidence did not, of itself, encourage a change in students' thinking,
and at times produced only confusion. This study also illustrated the communication difficulties between teacher and students when each enter into dialogue with different interpretive frames.

The above results show that if we persist in seeing "a student as a filing system" (Hawkins, 1978) and not as an active organism, students' difficulties in understanding certain topics will also persist.

2.3.2 Curriculum Problem in Science Education

In the introductory section of the Educational Perspective (see Section 2.3) it was mentioned that the inquiry approach has been the underlying approach used in science teaching in the last two decades. This approach looks for a method of instruction that could match the techniques of making scientific claims; this method is the inquiry approach. The rationale behind this methodology is that it is thought that by emphasizing student inquiry within a process-oriented program, the subtle regularities of nature, which are apparent to the scientist, can be "discovered" by an inquiring student. But research and practice (Shulman & Tamir, 1973; Hawkins, 1978; Deadman and Kelly, 1978) have shown that students see the world from very different perspectives than that of adults in general, and scientists in particular. Erickson (1975) has said that:
Thus perhaps the real utility of the 'child as a scientist' metaphor is not that it suggests one can simplify or reduce the complex body of scientific knowledge and methods into pedagogically digestible programs, as has been done to date. Rather, it is to view both child and scientist as engaged in the same basic activity of attempting to perceive some sense of order in their world -- only at different levels of abstraction. (Erickson, 1975, p.16).

The implications of this position for the development of science curricula seems clear to Erickson and to this author. The approach is that curriculum development ought to proceed from the simple and more concrete phenomena associated with the student's world toward the more powerful and sophisticated conceptions of the expert's world.

Curriculum development in science subjects has typically followed a procedure whereby the content, materials, and strategies for teaching a topic are established first, and then tried out on students. According to their success, or otherwise, the curriculum packages are then accepted or modified, or alternative ones are sought. This could be referred to as the curriculum-to-students mode in contrast to the students-to-curriculum mode (Deadman & Kelly, 1978). This author prefers the latter approach, in which students' understanding is investigated first and then, through a gradual building-up process in which development, research, and teaching are combined, ways are explored directly with
the students by which their understanding can be increased. Deadman and Kelly (1978) think that: "The curriculum-to-students mode is suitable for developing the broad aspects of courses but it is less suitable for the in-depth development of individual courses". (p. 8)

This may be the reason, or at least one of the reasons, why it has not been possible so far to determine wholly suitable ways for dealing with the topic of vector quantities and their special characteristics. The development methods employed have not provided sufficient details of the teaching and learning problems involved. This is also the reason why this author is favouring the students-to-curriculum mode to attempt to overcome some of the problems in learning vectorial concepts.
3.0 Introduction

This chapter deals with the instruments and the procedures pertinent to the data collection of the first phase. First, a short report of the pilot study is presented, which showed that the tasks were suitable to carry out the first part of the research. The use of rational task analysis is explained next, which shows the method of presenting the vector characteristics in terms of a network of concepts. These networks show broad relationships among the vector characteristics listed in Table 1.1. Following these is a description of the two tasks used in the study, with their respective protocols. The chapter ends by describing the data collection methodology, the format of the interview, and the characteristics of the subjects who were interviewed.

3.1 The Pilot Study

According to Pines et al:

every research program which is to involve the administration of clinical interviews should have a pilot study in which the interview can be developed. The pilot interview should be carried out in a population similar in all respects to the subjects of the research study. (Pines et al, 1978, p.8).

The main purposes of the pilot study were: (i) to determine if the interview tasks to be used in the study are suitable to attain the goals of the study; (ii) to
determine if the protocol questions can be understood by the target population (if not, they should be paraphrased and improved until they are sufficiently clear); (iii) to become familiar with the relevant characteristics of the target population such as language ability and manipulation dexterity.

During the preliminary work for the pilot study, eight children of various ages (ranging from 13 to 17) were interviewed which resulted in numerous changes in both the tasks that were eventually used in the study and the questions about the task situations. The two tasks which eventually emerged for the study seemed to be sufficiently appealing and clear for the subjects; particularly when an 8 mm film-loop and large (100 cm x 100 cm) cardboard model were used. Previously, when smaller paper models were used, the subjects responded to the tasks in terms of a geometry problem because the models permitted the students to use their hands to point to the various objects and drew imaginary or real lines on the models.

Initially the subjects experienced some difficulty in understanding a few of the questions. This was likely due to the fact that the original questions were just the vector characteristics (see Table 1.1) presented in question form. These questions were revised several times until it was evident that the students understood the intent of the question. Likewise the number of questions on each vector characteristic was reduced, after pilot testing, from five to two or three. Even with this reduction in questions,
however, it still seemed feasible in the pilot-study to determine if the subject held consistent preconceptions for a particular vector characteristic by examining their responses to the two or three related questions. Furthermore, the fewer questions made it possible to conduct the interview in 45 to 50 minutes.

Three of the 8 Ss of the final phase of the pilot study were younger than 15 years (2Ss were in grade 8 and 1S in grade 9). They gave little thought to the vector characteristics included in task two, demonstrated great difficulty both in understanding the question of task two and in expressing their ideas. Based on these observations, it was decided to only choose subjects in grade 10 for the actual study. Furthermore, the grade ten students have not been formally taught vectors and so it enabled the study to focus on the kinds of preconceptions students possess about vector quantities before they receive any instruction on the topic.

In summary, the pilot study was useful to improve the tasks and the interview protocol. It also showed that the tasks and the use of individual interviews would be suitable to address the major questions posed in the study.

3.2 Rational Task Analysis of Physical Phenomena

The vector characteristics that were presented in Table 1.1 in Chapter One were obtained by analyzing various physical situations, part of which can be represented by
the three vector quantities dealt with in the present study. After finding the most relevant kinematic aspects of the situations, these aspects were then compared to the properties that the formal definitions generally included for that particular vector quantity. It was determined that some of the uncovered characteristics were not explicitly introduced in the definitions. This was the reason that they were called implicit vector characteristics. It was hoped that by clearly stipulating these characteristics it would be easier to find out if students had any intuitive understanding of any of these characteristics; which characteristics seemed to be most troublesome to students; and if there were any other notions students had about vector quantities.

Many school physics textbooks (e.g., PSSC, 1980; Fundamentals of Physics: Health, Macnaughton, and Martin-dale, 1979; Physics: Giancoli, 1980) define formally the main vector characteristics of several vector quantities, but these books do not present clearly the essential characteristics presented in Chapter One. This author did an extensive review of physics textbooks currently used in B.C. schools. In summary, it can be said that all of them present some vector quantities in isolation, without showing relationships among them, and without strongly emphasizing that vector quantities represent specific properties of physical phenomena.
The task of analyzing in detail the vectorial characteristics of the three physical quantities was carried out with the use of Rational Task Analysis. Resnick says that:

Rational task analysis can be defined as an attempt to specify processes or procedures that would be used in highly efficient performance of some tasks. The result is a detailed description of an "idealized" performance - one that solves the problem in minimal moves, does little 'back-tracking', makes few or no errors. Typically a Rational Task Analysis is derived from the structure of the subject matter and makes few explicit assumptions about the limitations of human memory capacity or perceptual encoding process. (Resnick, 1976, p.65).

Usually, the highly efficient performance and the derivation of properties from the structure of the subject matter are carried out by an expert in the field. This was the procedure used in the present study.

After selecting three of the earliest vector quantities generally taught in high school physics (vector position, displacement, and velocity), an analysis was performed to determine the physical characteristics that they represent. That is, what aspects of physical reality do these three quantities deal with. Both common and different elements in these quantities were examined along with their mathematical abstractions. At the end of this analysis, a set of essential vector characteristics of the three physical quantities was produced. This set was shown to three university physics professors for comments and criticism. Private discussions were held with each one of the professors, and notes were taken of suggested changes
and improvements. A revised set was produced, and discussed again with the same professors. At the end an agreement was reached and a final set of vector characteristics was produced.

The whole process constitutes a procedure of Rational Task Analysis. This derived list of implicit characteristics (see Table 1.1) directly relates to the first research question of the study, which is the search for students' preconceptions of these implicit characteristics.

3.2.1 Networks of the Three Vector Quantities

However, it was felt that the list of implicit vector characteristics does not show clearly the relationships among the three different concepts and their properties. So, it was decided to present the structural relationships among these physical quantities and their vector characteristics as four networks of concepts, which diagramatically depict the various relations among the concepts and their properties. The first network shows the general characteristics of any vector quantity. This network provides the overall framework for the other three. The other three networks depict the vector characteristics associated with each one of three physical quantities selected for this study.

The rationale for preparing these networks is based primarily on the semantic networks of Lindsay and Norman (1977) and the active structural networks of Norman and
Figure 3.1

Network of a Kinematic Vector Quantity

Physical Entity

kinematic scalar quantity

kinematic vector quantity

composition of vectors

an instantaneous location is independent of others loc.

one or more simultaneous components

independent components

parallelogram method

vector addition

vector subtraction

triangle method

parallel method

magnitude

direction

real number

unit

frame of reference

real number

units

reference point

Key of Relational Terms:

sub = has a (non-intersecting) subclass

charac. = has as a characteristic of obeying that operation

prop. = has as a property

cond. = needs certain conditions to make quantitative
Figure 3.2

Network of Vector Position (V.P.)

Physical Entity

\[ \text{kinematic scalar quantity} \]

\[ \text{kinematic vector quantity} \]

\[ \text{triangle method} \]

\[ \text{vector subtraction} \]

\[ \text{composition of two V.P.} \]

\[ \text{an instantaneous location is independent of other locations} \]

\[ \text{vector position} \]

\[ \text{magnitude} \]

\[ \text{direction} \]

\[ \text{real number} \]

\[ \text{units} \]

\[ \text{frame of reference} \]

\[ \text{real number} \]

\[ \text{units} \]

\[ \text{reference point} \]

Note: Meaning of relational terms same as in Figure 3.1
Figure 3.3

Network of Vector Displacement (or Change of Location)

Physical Entity

\[ \text{kinematic scalar quantity} \]
\[ \text{kinematic vector quantity} \]
\[ \text{displacement of change of location} \]

vector addition

composition of displacements

independent from path

magnitude

direction

real number

units

frame of reference

real number

units

reference point

Note: Meaning of relational terms same as in Figure 3.1
Network of Vector Average Velocity

Physical Entity

kinematic scalar quantity

kinematic vector quantity

velocity

average velocity

instantaneous velocity

composition velocities

parallelogram vector addition

one or more instantaneous components

independence of magnitudes

independence of directions

magnitude

direction

real number units

frame of reference

real number units

reference body

stationary body

moving reference body

Note: Meaning of relational terms same as in Figure 3.1
Rumelhart (1975).

The tasks proposed for use in the study are based on all the vector quantities and their characteristics depicted in the four networks of concepts. Efforts were made to show the precise correspondence between the information contained in the networks and the protocols of the tasks.

The diagrams and discussions of the four networks are provided in this section. Figure 3.1 depicts the network for any kinematic vector quantity. The relationships among all vector characteristics listed in Table 1.1 are shown in this network. The lines connecting the different boxes, which contain the vector characteristics, do not define explicit mathematical relationships between the two vector characteristics. The lines only indicate that the two vector characteristics connected by it are in some way related. Some more specific relationships are explained at the bottom of the network (i.e., sub, charac, prop, and cond.). Figures 3.2 to 3.4 show respectively the networks for the vector position, the vector displacement, and vector average velocity.

3.2.2 The Physics Approach

The physics approach represents the physicists' current knowledge (i.e., theories, concepts, definitions, applied mathematics, research methodologies, problem solving methodologies) used to deal with experimental situations
similar to the ones depicted as the tasks of the study.

The physicists' current knowledge—the physics approach—is partially represented in the networks of concepts (Figures 3.1 to 3.4). Then, they represent the physics approach in the present study. It is important to remind the reader that those networks do not represent the formal definitions of the three vector quantities; definitions are usually succinct and leave important aspects out of the concept being defined. The networks actually represent a structural form of the list of implicit vector characteristics (see Table 1.1).

The reason for defining the physics approach was for the specific purpose of using it in the categorization of the inferred rules identified for each vector characteristic. This approach was used by simply comparing the content of an inferred rule of a vector characteristic with the physicist's current view, shown in the networks of concepts, of treating that vector characteristic. Further explanations of this categorization is presented in Section 4.1.3 (Chapter Four).

3.3 The Two Tasks of the Study

Two tasks were created for the present study. Task One included the vector characteristics outlined in the network of vector position (see Figure 3.2) and the network of displacement (see Figure 3.3). These characteristics
correspond to items one through five in Table 1.1. Task Two included the vector characteristics outlined in network of average velocity (See Figure 3.4), which also corresponded to items six through ten in Table 1.1. (Items 11 and 12 were only treated in the second phase).

It was thought that the tasks should be as related as possible to some common experiences of the subjects since a major purpose in the study was the search for intuitive notions which may be possessed by the students. Since boating on lakes and rivers is very prevalent in British Columbia one task referred to a lake situation and the other one to a river situation.

In summary the criteria used for task selection were:

1) that the task setting should be familiar to most of the subjects,
2) that the tasks must illustrate dramatically the vector characteristics, and finally
3) that the tasks should be of some interest or appeal to the student.

3.3.1 Task One

Briefly, the situation simulated in task one was as follows: a lake in which there are a couple of youngsters fishing in a row-boat. The subjects being interviewed is asked to imagine herself/himself standing on a dock for the
purpose of keeping track of the good fishing spots found by the couple. This person is waiting for some friends who are coming to fish later and he/she has to tell them where to locate the good fishing spots.

The following materials were used in the interviews:
- a 8 mm film loop (approximately 4 min. long) showing the round trip of a row-boat on a lake. The film was produced especially for the study; the location was Deer lake, Burnaby, British Columbia.
- a cardboard model of the lake (1m by 1m),
- a toy boat and three toy human figures,
- sheets of paper (27 cm by 21 cm) depicting a schematic diagram of the lake.

The concrete problem situation was presented to the interviewee using both the film loop and the cardboard model to accompany an oral description. A schematic diagram of the lake is shown in Figure 3.5. This diagram shows the starting location of the row-boat (the wharf), the permanent location of the interviewee (represented by S on the dock), and several good fishing spots \( P_1, P_2, P_3, \) and \( P_4 \).

The task consisted in questioning the subject on how he/she would describe the trip of the row-boat and the locations of the fishing spots.

This was judged to be a suitable task to find out subjects' preconceptions about vector position, whose characteristics are included in the network of Figure 3.2.
Preconceptions about the following aspects of this network were looked for: (i) the need for a reference point to locate an object; (ii) the need for a frame of reference to provide a precise location of an object, (this is shown through the kind of coordinates used by the subject which can be qualitative or quantitative); (iii) the need to provide a direction of the location of an object, this can be considered part of aspect (ii).

Task one was also used to find out preconceptions about some aspects of vector displacement, which were included in the network of Figure 3.3. Preconceptions about
the following aspects of this network were looked for:

(i) the need for a way to describe changes of location of a moving object (which can be given by the length of a straight line joining the two consecutive locations — its magnitude — and the angle that the straight line forms with a selected reference line — its direction); (ii) the need for a frame of reference and reference point; (iii) the need for an operation to relate consecutive changes of location (addition of vector displacements); (iv) the need to realize that a change of location is independent of the path followed by the object between two locations; and (v) the need for an operation to relate two different locations of an object relative to a chosen reference point with the resultant change of location (subtraction of vector position to produce the resultant displacement). This aspect is also part of the network of vector position, see Figure 3.2.

Task one was also suitable to find out the understanding of adolescent subjects about perspective taken. This perspective aspect was not considered as another vector characteristic in the present study. It was included to find out if subjects at these ages still have an egocentric conception of perspective.

3.3.2 Protocol of Task One

There were an average of two questions for each vector characteristic. Sentences in parenthesis referred
to what the interviewer (I) was doing during the interview. The film-loop and the cardboard model were used in the appropriate times in the introduction; furthermore the subjects were encouraged to make use of the sheets with the schematic diagram of the lake to explain their views. The introduction of the protocol, given below, was repeated almost verbatim to each of the subjects interviewed.

**Introduction of Protocol:** "Imagine that you are standing on a dock of a calm lake and you suddenly see a couple of youngsters getting ready to go fishing in a row-boat. Since you are waiting there for a couple of friends who are coming later to fish, you decide to carefully observe the trip of the row-boat. You are particularly interested in remembering the places where they seem to catch more fish. You will see a speeded up version of this fishing trip in the film; the camera shooting the film was put on the same dock where you are supposed to be standing so that what you will see in the film corresponds to what you would see if you actually were on the dock. Here is a cardboard model of the lake which represents what you will see in the film. Is everything clear? ... Do you have any questions? If not we can now see the film".

The entire film was shown, which contains several locations of the row-boat in its round trip. The trip started and ended at the wharf, the locations of the fishing spots depicted in Figure 3.5 were shown in the film. While the film was being shown, the interviewer was describing the trip and was pointing out the locations of
the fishing spots. Immediately after the film ended, the interviewer made use of the cardboard model to begin the interview.

The questions related to each one of the vector characteristics discussed in task one are given below.

Questions About Reference Point for Stationary Bodies (RPS)
Perspective Taken, and Frame of Reference (FR)

Question 1: (Items 1 and 2 of LIVC)

You are here on the dock, and as you saw in the film the first fishing spot found by the youngsters was more or less here (I located the toy boat in the corresponding spot on the cardboard model; \( P_1 \) on Figure 3.5).

How would you describe the location of the first fishing spot to your friends?

Question 2: (Perspective taken)

a) (If S uses a particular body or place as a reference point)

Could you describe the location of that fishing spot from other places as well? What places?

b) (If S describes the location from other places)

Is the description of this location the same or different when it is done from other places?

Question 3: (Item 2 of LIVC)

(If S has used no quantity or only one, either distance or direction, when answering question 1)

You have just described the location of the first fishing spot. How could you make the description
of that location (first fishing spot) more precise for your friends?

**Question 4:** (Items 1 and 2 of LIVC)

The boat continued moving, it went to this second fishing spot (I located the toy boat in that corresponding spot; $P_2$ on Figure 3.5). How would you describe the location of this second fishing spot to your friends?

**Question 5:** (Items 1 and 2 of LIVC)

Another fishing spot was about here (I located the toy boat at a point, $P_4$, about the same distance from S as $P_1$ was). (a) How would you describe the location of this last fishing spot to your friends? (b) (If S uses only distance in her/his description) The distance from the last spot to you and from the first spot to you are almost the same. How could you distinguish these two locations?

**Questions About Perspectives**

**Question 6:** (Perspective taken and Item 2 of LIVC).

a) From where you are (on the dock), could you approximately describe the path followed by the boat?

b) Please draw that path in this map of the lake (After S has drawn the path) Do you think this is the actual path?

c) What do you need to know in order to have a better approximation of the path taken by the boat?
d) A friend of yours is by the tree and draws the path he sees for the boat, you compare both drawings, yours and his. Do you expect to find them the same or different?

Questions About Displacements

Question 7: (Item 3 of LIVC)

The initial location or starting point of the boat was here beside the wharf \( (P_0) \). We can say that the boat has had a change of location when it moved from the starting point \( (P_0) \) to the first fishing spot \( (P_1) \). (a) How could you describe the location of the first fishing spot with respect to the starting spot? (If S uses distance in her/his description). Is that the distance along the path followed by the boat between the two spots?

(b) Then, the boat moved from the first fishing spot to the second spot (I moves the boat from \( P_1 \) to \( P_2 \) ). How could you describe the location of the second fishing spot with respect to the first one?

Question 8: (Item 3 of LIVC)

(If S uses the length of the straight line between two locations). Does your description for a change of location say anything about the path followed by the boat between the two locations?

Questions About Addition of Displacements

Question 9: (Item 4 of LIVC)

Suppose you have estimated the distances from the
Starting point (P₀) to the first fishing spot (P₁) to be 100 m, and from the first spot to the second one to be 150 m. If your friends were at the wharf, how could you tell them to go directly to the second spot?

**Question 10:** (Item 4 of LIVC)

Suppose you also estimated the distance from the second spot to the third one to be 100 m. How could you tell your friends to go directly from the starting point to the third spot?

**Questions About Subtraction of Vector Position**

**Question 11:** (Item 5 of LIVC)

Suppose you have estimated the distance from where you are standing on the first fishing spot to be 200 m, and also from the dock to the second spot to be 300 m. If your friends were at the first spot. How could you tell them to go directly to the second spot?

**Question 12:** (Item 5 of LIVC)

Suppose you also estimated the distance from where you are standing on the dock to the third fishing spot to be 300 m. If your friends were at the first spot. How could you tell them to go directly to the third spot?

Immediately after finishing with task one the interview continued with task two. The interviewer tried to make a smooth transition between the two tasks by saying that the same two friends were now using a motor-boat on the river.
3.3.3 Task Two

The situation simulated in task two was the following: a river on which there is a motor-boat with two people, a person on a bridge, (again the subject is asked to imagine that he/she is the person who is observing the movement of the motor-boat, which moves against and with the current, and across the river. During the interview the subject was asked to describe and explain the motor-boat's movements.

The concrete problem situation was presented to the subjects using the following materials:

- a 8 mm film loop (approximately 4 min. long) showing the different movements of the motor-boat on the river. This is a film produced for the Harvard Project Physics course,
- a cardboard model of the river and the bridge (1 by 1 m), which also includes one dock on each side of the river,
- a toy boat and three toy human figures,
- sheets of paper (27 cm by 21 cm) depicting a schematic diagram of the river (see Figure 3.6).

An introduction explaining the task was presented to the subject, after which a semi-structured protocol was used. As before, the subjects were questioned on how he/she would describe the movements of the boat. This was judged to be a suitable task, to determine the students' preconceptions about
Figure 3.6

Diagram of the river, including the bridge (where the subject stands), the two docks, and the motor-boat

the vector characteristics six to ten listed in LIVC, all of which are included in the network of average velocity in Figures 3.4. Preconceptions about the following aspects of this network were examined: (i) the need for describing the motion of an object with respect to any reference body, (be it stationary or in motion); and the need to distinguish that a single motion can be described differently if viewed
from two different reference bodies, (particularly if one of reference body is in relative motion with respect to the other); (ii) the need for realizing that an apparent single motion can be formed by two or more component movements; (iii) the need for realizing that a moving object may be the result of the interaction of two simultaneous movements (i.e., the composition of vector quantities); (iv) the need for realizing that even when two movements interact to produce a single resultant movement, each one of them maintains its own physical characteristics (e.g., magnitude and direction) during the interaction; and finally (v) the need for realizing that two or more movements could affect a body at the same time.

3.3.4 Protocol of Task Two

The general format of this interview protocol is similar to that of task one.

Introduction of Protocol: "Imagine now that you are standing on a bridge over a wide and calm river. You are here on the bridge (I show the subject the point S on the cardboard model, diagrammed in Figure 3.6) observing your friends, who are riding in a motor-boat. The speed of the motor cannot be changed so the speed of the boat due to the motor remains the same. Your friends on their ride will go against and with the stream, and also across the river. As before, you decide it would be interesting to observe and describe all
the moves that the boat will make. This film will show you how the boat might move; the camera to take this film was placed on the bridge where you are to imagine yourself to be standing. So what you will see in the film corresponds to what you would see if you actually were on the bridge. We will see part of the film now and then we can discuss what we have seen using this cardboard model. Again, I am interested in your ideas and explanations of what you will see in the film. Is everything clear? ... Do you have any questions? ... If not we can see the first part of the film".

In this task the film was shown in two parts; discussion and questions were asked after each part. The cardboard model was used as a concrete referent when discussing various questions in the interview.

The questions related to each one of the vector characteristics corresponding to task two are presented in the following sections.

Questions About Reference Body for Moving Objects (RPM)

Opening Question: Have you had any experience with motor-boats on lakes or rivers? (This question was asked to only find out if the subject had any previous experience with motor-boats).

The first part of the film was viewed showing the water moving, the boat going against the stream and two stationary buoys beside the bridge.
Question 13*: (Item 6 of LIVC)

In the film you saw the motor-boat moving.
(a) With respect to what was the boat moving? (If S does not understand this question, it was paraphrased as follows: How could you tell that the boat was moving? (If S mentions that the boat was moving relative to the bridge or shore). (b) Was the boat moving with respect to the water? Why do you think so? (If S says that the boat was also moving relative to the moving water) (c) Was the movement of the boat with respect to the bridge (or whatever stationary body the subject mentions) and the movement of it with respect to the water the same or different? Why do you think so?

Questions About Analysis of Components (AC) and Composition of Velocities (CV)

Question 14: (Items 7 and 8 of LIVC)

a) Imagine that the motor-boat is moving upstream against the current. (I moves the toy boat in that direction in the cardboard model). How would you describe its motion from the bridge?

b) What different movements are affecting the boat when it goes upstream? What are they?

*Numbering of questions for task two continued on from task one.
c) Imagine now that the motor-boat is moving downstream with the current (I moves the toy boat in the appropriate direction). How would you describe its motion from the bridge?

d) What different movements are affecting the boat when it goes downstream? What are they?

**Question 15:** (Item 7 and 8 of LIVC)

Imagine this time that your friends in the motor-boat want to cross the river. They leave from this dock (dock 1, see Figure 3.6) heading that way (I puts the boat beside the dock 1 and explains the way how the boat will start heading).

a) How would you describe its motion from the bridge? Please, take the toy boat and show me the kind of movement you will see using the cardboard model.*

b) Now could you draw that path on this diagram of the river? (I hands S a sheet of paper with the schematic diagram on it). Why do you think it follows the path you have drawn?

---

*Almost at the end of the data collection in phase one, a question that seemed appropriate to find out students' conceptions about the independence of direction of components was suggested through the response of one subject. The question was phrased as follows: For the case of the boat going across the river, will the boat reach the other side heading this way ⬅️ or this way ➡️? The interviewer used the toy-boat and the cardboard model to show the two heading ways. This question was included immediately, after question 15a, and was asked of only 5 subjects.
c) What different movements are affecting the motor-boat when it goes across the river? What are they?

The part of the film with the motor-boat crossing the river is shown.

Question 16:

a) You just saw the motor-boat moving across the river in the film. Was the movement of the boat due to the motor or due to the current?

b) Suppose the speed of the current is 5 km/hr and the speed produced by the motor on the boat is 10 km/hr. Could you guess or estimate the speed of the boat when it was going across the river as you saw it in the film?

Questions About Simultaneity of Components

Question 17: (Item 10 of LIVC)

(If S mentions that the boat is being affected by the speed of the current and the speed due to the motor; otherwise the I suggests the existence of the two components). Are these movements, the movement of the stream and the one due to the motor, affecting the motion of the boat together or one after the other? How could you explain your responses?

Questions About Independence of Magnitudes of Components

Question 18: (Item 9 of LIVC)

(If S mentions that the boat is being affected
simultaneously by the speed of the current and the speed
due to the motor). When the boat was just crossing the river,
you said that both the speed of the current and the speed
due to the motor were affecting the boat. Do you think that
the speed of the boat due to the motor is affected or
changed because of the speed of the current? Why do you
think so?

Questions About Composition of Velocities

Question 19: (Item 8 of LIVC)

Finally, suppose your friends want to cross the
river with the motor-boat and reach dock 2. That is, leave
from dock 1 and end up at dock 2. Is there any way they
could do it? How?

Could you please draw the path on this sheet of
paper that you might see if they followed your suggestion.

3.3.5 Further Discussion of Interview Protocol

Three relevant issues related to the interview
protocol are discussed below. These issues are: (1) the
use of different number of questions for some of the
vector characteristics, (2) occasionally, use of one
question to identify inferred rules for more than one
vector characteristic, and (3) a summary table of the
interview protocol questions associated with each of the
vector characteristics.
1. Each vector characteristic in the interview protocol was addressed with a varying number of questions (see Table 3.1). The main reason for this is due to the nature of the vector characteristic itself and on the tasks chosen for the study. For example, the nature of addition of displacements in task one allowed for at least two questions for this vector characteristic — each one trying to uncover the subjects' ideas with respect to a situation that requires adding two given displacements. To determine whether a subject possessed a consistent view about addition of displacements several other questions, each time using a distinct pair of displacements, could have been used. However, due to the time limitations associated with an interview only two questions were asked. On the other hand, considering the nature of the simultaneity characteristic of two velocity components, either the subject sees the two velocities affecting the boat (in task two) at the same time or one after the other. There are only these two possibilities, and both are covered in the same question. In the tasks used in the study there was no other way to check for this vector characteristic. The nature of other vector characteristics and the tasks chosen for the study in some instances allowed for a larger number of questions to be asked about a specific vector characteristic.

2. At least four questions were used to uncover the subject's preconception of two vector characteristics.
These questions were broad in scope (i.e., Qs.1, 4, 5, 13d) such that the responses to them permitted inferred rules to be assigned for two vector characteristics. This was the case for the introductory questions for each one of task, question 1, 4, and 5 in task one; and question 13d in task two. This is a legitimate approach but it is recommended that the preconceptions obtained through these broad questions be checked by using proving questions, which should be more specific in scope. For instance, preconceptions about frame of reference were obtained from questions 1, 4, and 5; these questions were also suitable for ascertaining possible preconceptions about reference point. These preconceptions were then cross-checked by analyzing the subject's response to question 3.

3. The following table presents a list of questions from the interview protocol which was used to obtain the inferred rules for each of the vector characteristics.

Table 3.1

<table>
<thead>
<tr>
<th>Vector Characteristic</th>
<th>Number of Questions in Protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Task One:</strong></td>
<td></td>
</tr>
<tr>
<td>1. Reference Point for Stationary Bodies (RPS)</td>
<td>1, 4, 5</td>
</tr>
<tr>
<td>2. Frame of Reference (FR)</td>
<td>1, 3, 4, 5a, 5b, 6c</td>
</tr>
<tr>
<td>3. Displacement (D)</td>
<td>7a, 7b, 8</td>
</tr>
<tr>
<td>4. Addition of Displacements (AD)</td>
<td>9, 10</td>
</tr>
</tbody>
</table>
Table 3.1 - continued

<table>
<thead>
<tr>
<th>Vector Characteristic</th>
<th>Number of Questions in Protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Subtraction of Vectors Position (SVP)</td>
<td>11, 12</td>
</tr>
<tr>
<td>(Perspective Taken)</td>
<td>2a, 2b, 6d</td>
</tr>
<tr>
<td>Task Two:</td>
<td></td>
</tr>
<tr>
<td>6. Reference Point for Moving Objects (RPM)</td>
<td>13a, 13b, 13c</td>
</tr>
<tr>
<td>7. Analysis of Components (AC)</td>
<td>14b, 14d, 15c</td>
</tr>
<tr>
<td>8. Composition of Velocities (CV)</td>
<td>14a, 14c, 15a, 15b, 16a, 16b, 19</td>
</tr>
<tr>
<td>9. Independence of Magnitudes of Components (IMC)</td>
<td>17</td>
</tr>
<tr>
<td>10. Simultaneity of Components (SC)</td>
<td></td>
</tr>
</tbody>
</table>

3.4 Methods of Data Collection Methodology: Clinical Interview

Due to the fact that one of the main goals of this study was to uncover students' preconceptions of selected physical quantities, it was thought that a method that will "dig" deeply in subjects' thinking would be more adequate. A kind of Piagetian clinical interview would accomplish that aim. The inadequacy of traditional testing procedures (i.e., written questionnaires or tests) has been noted by several researchers interested in conceptual development. For instance, Easley reminds us that:
Piaget pointed out in 1929 that tests cannot provide enough information to decide what structures are involved in a child's thinking, and Piaget and Inhelder later characterized tests as giving only the 'results of efficiency of mental activity without grasping the psychological operations in themselves' (p.281)...

The overriding quest for reliability which appears to be the dominant concern in the test paradigm, is doomed to generate many errors in the identification of cognitive structures...

Analysis of protocol, in the structuralist paradigm, is necessarily a slow and nonmechanical procedure. It begins in subjective, but hopefully educated, judgements and moves toward objectivity as it attains completeness in accounting for the total protocol. (Easley, 1974, p.289)

Pines (1980) comments further that there is still some dissatisfaction with sophisticated testing and associationalistic methods for evaluating and describing cognitive structures. He cites as an example Stewart (1979), and says that there is sufficient evidence of a move towards capitalizing upon personal, flexible interview techniques (e.g., Pines, 1979; Pines et al, 1978). Pines goes on to say that "The transition, although slow and difficult, will, I believe, ultimately result in a distinct advancement for the field of science education". (Pines, 1981, p.361)

One of the problems of the clinical method mentioned by Pines (1980) is that of non-response by the subject. He claims this problem is generally more acute with younger children, but no difficulties of this nature were encountered with the adolescent subjects in this study. Since no non-responders were present in the sample, the internal
and external validity of the study is increased (Pines, 1980, p. 362).

A clinical interview is literally a face-to-face meeting. It has been defined as "a conversation directed to a definite purpose other than satisfaction in the conversation itself", (Bingham, et al., 1959, p.3) or what could be called a "professional conversation" (Garret, 1972, p.5). Its chief goal is to ascertain the nature and extent of an individual's knowledge about a particular domain by identifying the relevant conceptions the subject holds and the perceived relationships among those conceptions.

The clinical interview has two important features. One is using the interview to elicit and collect specific kinds of information. The second is allowing the subject to take the lead, inducing her/him to talk more and more freely.

The objections to the clinical method as being unstructured, subjective, and prone to experimental bias can be diminished by planning a protocol interview in advance. The analysis of the vector characteristics of the three physical quantities selected for this study, using Rational Task Analysis, provided enough information to prepare relevant and in depth questions with respect to the selected content-area. It was also expected that this researcher's years of experience using the individual interview as a data collection methodology would result in an objective and insightful interpretation of the data generated.
by the interviews. Finally, the use of videotape for
recording the interviews allowed for accurate transcription
and added a degree of objectivity to the data.

3.4.1 Reliability and Validity in Clinical Interview

Any research project must take into consideration
the problems concerning the external and internal validity
of the data collection methodology. Internal and external
validity, are obviously related since if internal validity
is lacking, external validity is no longer an important
consideration. The reliability of a measurement pro­
cedure is also concerned with accuracy and so it is also
related to the issue of internal validity. In other words,
if a measurement is not accurate or cannot be reproduced,
it does not make much sense to determine how well it is
measuring what it is supposed to measure. However, an in­
strument can measure something very reliably, but this does
not imply that it is measuring what it is supposed to
measure — i.e., it may still have low validity. One can
find tests with high reliability but the construct being
measured is either unknown or very trivial.

Referring to validity and reliability when using
clinical interview methodology, Pines et al (1978) have
stated: "that the clinical interview does not produce
identical results from one interview to the next with the
same subject, even with the same interviewer". However, it
is their contention that: "the clinical interview is reliable in that uniformity of knowledge assessment is possible and trained interviewers will show [interrater reliability] correlations in the assessment of $r = .9$ or better"

An important aspect of reliability assessment with clinical interview data, then actually relates to the interviewer judgments about the data; that is, what information or segment of the subject's activity is considered as evidence for the understanding of a certain content-area. The problem is to obtain the exact wording of the subject's responses; from which the parts that elicit the subject's understanding (or his/her conception) about the problematic situation under discussion can be selected. This issue can be solved, in part, by videotaping or audio-taping the interview, so the analyst can identify at what point the subject seemed to have used the language or exhibited the behaviour that represents a given conception. However, the selection and interpretation of interview data still remains very much a matter of good judgment on the part of the analyst. Reliability on these judgments of data can be estimated, according to Pines et al. (1978), by asking other experienced interviewers to go through the same interview data -- in written or taped form -- making the same types of judgments. Agreement or disagreement of judgments provides estimates of reliability. In the present study, an independent analyst went through the interview data
of 6 randomly selected subjects (6 of 20) for 3 randomly selected vector characteristics (3 of 10). The investigator explained the analytic procedure to the analyst. The transcribed interview protocols for one subject was used to demonstrate the analytic procedure. Results of the independent analyst showed an agreement of 94.4 percent regarding the classification of the consistent preconceptions.

Validity judgments in clinical interview data attempt to ensure that the analyst's claims are a reasonable interpretation of the subject's cognitive structure. Again, one way of checking the interpretation of interview data is through the use of external analysts. Another route, employed in this study, was to ask the subject several questions about the same aspect of the content-area (i.e., same vector characteristic). In this way the analyst is able to carry out a type of consistency check for particular conceptions held by a subject.

3.4.2 Interview Format

Before starting the interview itself, the experimenter tried to have an informal chat with the student, asking her/his name, age, and interests. After the subject seemed to feel comfortable with the situation, the author explained in general terms the purpose of the present research and general nature of the interview. (For example, he/she was assured that the interview was not a test but that the interviewer was very interested in their answers
and the way that they thought about problems like this so that we could develop more interesting and helpful science materials for use in the schools).

After this chat, the cardboard model of task one was shown and the introduction of this task was presented. When it was clear that the subject understood the problem situation, the use of the semi-structured protocol together with the film projection began.

The transition to task two was made smoothly, showing first the cardboard model and then presenting the task introduction. Again, introductory questions and the film projection were initiated when the subject seemed to understand the problem situation.

The interviews were carried out in a designated room of the school attended by the subjects. The time required to complete the entire interview ranged from 45 to 50 minutes. Videotape equipment was set up prior to the subject entering the room. The subject was asked if he/she minded having the interview taped. At the end of the interview, the subject was thanked for her/his cooperation.

3.4.3 Sample of Subjects

Piaget (1973), in his studies of students' understanding of forces, found that his subjects do not develop conceptions that could be defined as vectorial until after age 12. Aguirre (1978) did not find clear vectorial conceptions until age 14 or later. These results suggested that it may be nonproductive to select subjects younger
than 13 years of age for this study. Also, since the physical concepts included in this study are usually taught in grade 11, the target population selected for this study was grade 10 (ages 15 to 17 years).

For Phase One of the study the sample consisted of 20 subjects — eleven males and nine females. Two important aspects were considered in the selection of subjects: (1) that they were all relatively healthy and well-adjusted, and (2) they had not been formally taught the concepts included in the study. These two criteria were checked with the students' teachers.

The subjects for the sample were selected at a voluntary basis and after they had obtained consent from their parents. Science teachers in two junior high schools of Coquitlam School District (Sir Frederick Banting Junior Secondary School and Mary Hill Junior Secondary School) explained the need of volunteers for a research study to their classes, the subjects who demonstrated interest were given the parent consent letters. Among the subjects who obtained permission, considering the constraint of sampling 50 percent boys and 50 percent girls, 20 were randomly selected by the teachers.

Actually, for this phase of a research program on concept development the use of a typical sample is sufficient since the search is for individuals' conceptions about a particular content-area. In other words, the aim of this phase of the study was not to statistically infer that all students in some defined population of grade 10
have similar preconceptions about vector characteristics as those possessed by the subjects in the sample. Rather, the aim was to obtain data which would permit the formulation of a set of tentative hypotheses regarding the preconceptions about vector characteristics held by students at this age level. These hypotheses were then submitted to further cross-checking in Phase Two of the study.
CHAPTER FOUR

RESULTS OF PHASE ONE OF THE STUDY

4.0 Introduction

The chapter is divided into the following seven major sections. The sections refers to:

1) the description of the methods of data analysis and examples of those methods;

2) the description of the format of results presentation;

3) the findings of phase one which are presented in a list of inferred rules and rule-models for all vector characteristics;

4) the inferred rules for each subject in the sample;

5) the description of an analytical step to further categorize the interview data;

6) the description of results obtained using this type of categorization;

7) the conclusions of phase one are presented according to the research questions being addressed by this phase.
4.1 Data Analysis

The data analysis of phase one was done in four steps. The first one consisted of looking for patterns or consistent preconceptions in the responses of each subject to all questions for each vector characteristic. The second and third steps corresponded to the process of expressing the consistent preconceptions in terms of inferred rules and categorizing these inferred rules according to a contemporary Physics perspective. The fourth step entailed the task of expressing the inferred rules in less contextual and more general terms to produce a 'rule-model' for each vector characteristic.

4.1.1 Inferred Rules: The search for consistent preconceptions in the subjects' responses to the questions on a particular vector characteristic was done by means of empirical task analysis. This was basically an analysis and interpretation of a student's performance on a particular aspect of the task; the goal was to uncover the subject's preconception of dealing with that aspect. If a subject appeared to be using a common "strategy" to cope with more than half of the questions related to a particular vector characteristic, it was assumed that the subject held a consistent preconception about that particular aspect. The presence of a consistent pattern of responses could mean that subjects hold particular
kinds of ideas which become explicit when they are faced with certain types of cognitive tasks. If this were the case, it could be hypothesized that a subject possesses some type of rule to deal with these situations. This is actually an inferential step because there is no straightforward way to know if a subject is really using these rules; hence they are referred to as 'inferred rules' in this study. In summary, a consistent preconception used by a subject to deal with a particular aspect (i.e., one of the vector characteristics) in one of the tasks was expressed as an inferred rule.

It cannot be claimed that a subject will use an inferred rule for a specific vector characteristic (e.g., reference point) for an experimental situation different from the one depicted in the task, even when the questions be related to the same vector characteristic (e.g., reference point). To make this claim it would be necessary to create several tasks suitable for that vector characteristic, and determine whether the subject uses the same "inferred rule" to cope with the tasks. In this sense, the present research is very contextual; the inferred rules are only applicable to the context of the tasks (i.e., boats on lakes and rivers).

The definition and the operational definition of inferred rules are repeated here (see section of definition of terms in Chapter One) to facilitate the comprehension of the developed examples in the following section.
Definition of Inferred Rule: An inferred rule is a consistent preconception that a subject appears to be using in a given situation to deal with a particular question or problem posed by the interviewer. (An example of the use of this definition: in task one, the experimental situation was a row-boat moving on a lake; a given situation was the location of several fishing spots; and a consistent preconception held by some subjects was the use of several bodies as reference to locate the spots).

Operational Definition of Inferred Rule: A rule is inferred if a subject uses the same basic preconception to deal with more than half of the questions regarding the same vector characteristic.

4.1.2 An example of the Derivation of Inferred Rules for the Implicit Vector Characteristic: Reference Point for Stationary Bodies

Three consistent preconceptions and therefore three rules were inferred from the interview data for Reference Point for Stationary Bodies (RPS). Examples of how each one of the inferred rules was identified are now presented.

The questions about Reference Point used in the interview protocol were as follows:

Question 1: How would you describe the location of this first fishing spot to your friends?

Question 4: How would you describe the location of this second fishing spot to your friends?
Question 5: How could you describe the location of this last fishing spot to your friends?

Case One: A subject using more than one reference point to locate each one of the fishing spots.

The following responses are from Tracy (15 years):

(Q.1) "I will probably point them and say so many feet that way ... I can say 40 feet from the dock ... it will be about 20 feet from the wharf to my right ..."

Researcher's Comments: Subject was using two bodies as reference: the dock and the wharf.

(Q.4) "... I could say 20 feet pass the red one (buoy or red thumb tack placed at the first fishing spot on the model) ... from here (the dock) about 100 feet or something like that".

Researcher's Comments: Subject again was using two bodies as reference: the buoy and the dock.

(Q.5) "I will say it's near that bay there ... very near the shore line there ..."

Researcher's Comments: Subject kept using two bodies or places as reference: bay and shore line.

Derivation of Inferred Rule: The responses of Tracy to the three questions regarding selection of reference point show a consistent pattern when asked to locate objects ... that of using more than one body or place as reference. Based on this consistent preconception and using the operational definition the following rule can be inferred for Reference Point for Stationary Bodies:

"Each one of the locations of the fishing spots is described using several reference bodies or places".
This inferred rule was called RPS-1.

Case Two: A subject using one body or place as a reference but a distinct one when locating each one of the fishing spots.

The following responses are from Hans (17 years):

(Q.1) "...how far it's from the shore ... (I: what shore?) ... that one by the forest (shore in front of the wharf)."

Researcher's Comments: Subject was using one place as a reference: a shore in front of the wharf.

(Q.4) "... how far it's from the other fishing spot (the first one) ...

Researcher's Comments: Subject was again using one place as reference but a different one than when locating the first fishing spot, and it was precisely that spot.

(Q.5) "... from the shore ... how far out it's from the shore" (the shore closer to the last fishing spot).

Researcher's Comment: Subject kept using only one place as a reference but a different reference point was used on each occasion.

Derivation of Inferred Rule: The consistent view of Hans to locate the different fishing spots is expressed in his responses to the questions. Each response contains only one reference body or place but a distinct one for each description. Based on this consistent preconception, the following rule can be inferred for Reference Point for Stationary Bodies:
"Each of the locations of the fishing spots is described using only one reference body but a distinct one for each location".

This inferred rule was called RPS-2

Case Three: A subject using one body or place as a reference and the same one when locating each one of the fishing spots.

The following responses are from Preston (16 years):

(Q.1) "... from here (the dock) I will probably use degrees ... maybe 40 degrees north from where I am standing (the dock)"

Researcher's Comments: Subject was using one body as a reference: the dock.

(Q.4) "... from here (the dock) ... it will be about 15 degrees to the north ... or using the wharf it is straight across".

Researcher's Comments: Subject was again using one body as a reference, the dock. He also mentioned the possibility of using the wharf, but he expressed in the sense that he will use either one and not both.

(Q.5) "... it is in the indentation of land (the closer shore to the last fishing spot) ... it will be in the south ... or it would be about 180 degrees toward the south from here (the dock)".

Researcher's Comments: Subject kept using only one body as a reference, even though he mentioned two places he will use either one, one of which is the dock.

Derivation of Inferred Rule: From Preston's responses, it is clear that he was consistently using the same body as a reference to locate each of the fishing spots. Based on this consistent preconception the following inferred rule can be produced.
"Each one of the locations of the fishing spots is described using only one reference body and the same one for each location".

This inferred rule was called RPS-3.

In the above example, the inferred rules have been presented in a particular order. This process of ordering the inferred rules was done by analyzing the level of differentiation of each inferred rule. That is, by analyzing how the content of the inferred rule could be differentiated from that of a physicist's perspective. This view is contained in the so-called Physics Approach in Section 3.1.2.

A detailed explanation of how this categorization was done is presented in the next section, which also describes the construction of a rule-model.

The methodology described above was used to produce the inferred rules for each subject in the sample and for each implicit vector characteristic. At the end of this process, each subject was assigned a maximum of 10 inferred rules, one for each vector characteristic. If a subject did not possess a consistent preconception for a vector characteristic, then, no rule was assigned to her/him for that vector characteristic.

4.1.3 **Categorization of the Inferred Rules and Construction of the Rule-Model for RPS**

Two aspects were considered in the categorization of the inferred rules presented in Section 4.1.2. First, the
content of each inferred rule was compared with the corresponding vector characteristic - reference point - in the network of vector position (see Figure 3.2) to determine how differentiated it was from the one in the network. Content, in this context, includes both the number of variables present and the nature of these variables (that is, qualitative or quantitative). Second, the inferred rules were ordered according to the Physics Approach; the inferred rules for RPS whose content most differed from the physicist's perspective was considered as the least differentiated among the inferred rules, and it was categorized at the lowest level, (the first inferred rule in the example developed in Section 4.1.2). The inferred rule for RPS whose content was most similar to the corresponding one in the network -- one unique reference point -- was considered as the most differentiated preconception and it was classified at the highest level (the third inferred rule in Section 4.1.2). The inferred rule whose content was considered in between the two extreme levels of differentiation was classified in the intermediate level (the second inferred rule in Section 4.1.2). Other vector characteristics may have more than one intermediate inferred rules.

After categorizing the inferred rules, they were expressed in more general and less contextual terms. The results of this last procedure yielded the rule-model for the first vector characteristic, reference point for stationary bodies. The general rules in this rule-model
should still be considered as being very tentative since
they were identified in a very specific context and still
lack empirical support in other related contexts.

The general rules in the hypothetical rule-model
are closer in form to those used by Siegler (1978, 1979)
with his rule-assessment methodology.

Reference Point in the Physics Approach: A short description
of the approach used by physicists to choose reference points
is presented below.

The network of vector position (see Figure 3.2 and
definition in Appendix A), which actually represents the
Physics Approach, show that this quantity (vector position)
needs magnitude and direction to be completely defined.
Both magnitude and direction need to be measured with respect
to a specific frame of reference which is related to a
specific reference point (the so-called origin of the frame
of reference or coordinates system). As is shown in Figure
3.2, physicists usually use the same frame of reference and
for extension the same reference point to describe (magnitude
and direction) the locations of different bodies or the instan-
taneous locations of a moving body. This is the current
approach used by physicists: and provides the norm to which
the inferred could be compared.

Categorization of the First Inferred Rule: Based on the
approach described above, the content of the consistent pre-
conception that most differed from it was included in the
following inferred rule:
"Each of the locations of the fishing spots are described using several reference bodies or places" (Inferred rule RPS-1)

This inferred rule shows the use of more than one reference to locate bodies. Subjects holding this conception did not perceive the importance of selecting only one body as a reference to locate all spots. Instead, they used several different ones to describe the location of each fishing spot.

The inferred rule RPS-1 was rephrased in the following general form and it corresponded to the first rule in the rule-model for reference point:

"Use of multiple reference points (bodies or places) to locate a stationary body" (Rule RPS-1).

Categorization of the Second Inferred Rule: The consistent preconception that was judged to be next closest to the physicists perspective was:

"Each one of the locations of the fishing spots is described using only one reference body but a distinct one for each location" (Inferred rule RPS-2).

The inferred rule shows the use of only one reference body as a physicist would do it but that reference body is not always the same. This aspect makes this conception still different from a physicist's perspective, even though it contains an important feature of it.

The inferred rule RPS-2 was rephrased in the following general form and it corresponded to the second rule of the rule-model:
"Use of one unique reference point (body or place) to locate a stationary body but different unique reference points are used to locate other bodies". (Rule RPS-II)

Categorization of the Third Inferred Rule: The consistent preconception whose content was closest to the physicist's perspective was:

"Each one of the locations of the fishing spots is described using the same reference body". (Inferred rule RPS-3)

This inferred rule shows the use of only one reference body for each one of the locations. This actually corresponds to the procedure used by physicists.

The inferred rule RPS-3 was rephrased in the following general form and it corresponded to the third rule in the rule-model:

"Use of one unique reference point (body or place) to locate all required stationary bodies". (Rule RPS-III)

This set of three rules, summarized below, thus constitutes the rule-model for the vector characteristic, reference point for stationary bodies.

Rule Model for RPS:

**Rule RPS-I:** Use of multiple reference points (bodies or places) to locate a stationary body.

**Rule RPS-II:** Use of one unique reference point (body or place) to locate a stationary body but different unique reference points are used to locate other bodies.

**Rule RPS-III:** Use of one unique reference (body or place) to locate all required stationary bodies.
This methodology was used to produce the rule-models for each one of the implicit vector characteristics. At the end of this process there were 10 rule-models, one for each vector characteristic.

4.2 Format for Presenting Results of Interview Data Analysis

The results of the analysis of the interview data for each vector characteristic in terms of inferred rules will be presented in a format which contain five sub-sections. Each of these sub-sections are fully explained below before reporting the actual results.

1. Introduction of Vector Characteristic: This sub-section consists of a short description of the physics background related to the vector characteristic, reasons to consider it as an implicit characteristic, and reasons to include it in the present study.

2. Questions in the Interview Protocol Related to the Vector Characteristic: This sub-section consists of a list of all questions from the interview protocol of the tasks related to that specific vector characteristic.

3. List of Inferred Rules: This sub-section contains all the inferred rules identified from the interview data for the specific vector characteristic. The inferred rules were obtained
by following the same procedure illustrated in Section 4.1.2, and are presented in the order obtained from the level of differentiation analysis — which was explained in Section 4.1.3. The inferred rules are identified by using the abbreviation for the vector characteristic plus a digit. These digits are used to differentiate among inferred rules of a particular vector characteristic (i.e., the first inferred rule for reference point for a stationary object would be RPS-1).

The number of female and male subjects assigned to each inferred rule in the list is provided in parenthesis (e.g., 6F, 7M for RPS-1) at the end of the inferred rule statement.

At the bottom of the list of inferred rules the calculated value of the Chi-square test statistic is provided. The two variables used in the statistical test was inferred rule classification and sex.

It was decided to study if there was significant difference, regarding preconceptions about vector characteristics, between female and male subjects because it is usually found in the literature that boys do better than girls in science (Hobbs, et al., 1979; National Assessment of Educational Progress, (1978); Erickson, et al,
1980). Research findings reported usually relate to science achievement, this means after instruction. It was thought it would be interesting to find out if there was significant sex difference regarding pre-instruction knowledge about vector characteristics. Even though, a small sample was used (11 boys and 9 girls), the results of the Chi-square test statistic is reported.

The calculated Chi-square values have indicated that there was no significant difference between the inferred rules of each vector characteristic according to sex.

4. Discussion of Results: Under this sub-section each of the inferred rules presented in subsection 3 is discussed and analyzed in detail. The analysis here is supported by providing selective interview excerpts from a few subjects. Presentation of excerpts will have the following format: name of subject, age, number of question in parenthesis (i.e., Q. 4). Within an excerpt, any explanation of terms or further interviewer's questions (I) are denoted in parenthesis.

5. Rule Model: In the final sub-section the inferred rules were rephrased in a more general and less contextual form thus yielding the
general rules. These more general rules will be identified using the abbreviation for the given vector characteristic and a Roman numeral. The Roman numeral rule corresponds to the Arabic numeral inferred rule (e.g., RPS-1 corresponds to RPS-I).

4.3 Results of Analysis of Interview Data

This section contains the results of using the interview analysis methodology, which was delineated in Sections 4.1.2 and 4.1.3, for each of the first ten vector characteristics listed in Section 1.2.

Some transcriptions of interview data are presented in Appendix B. This corresponds to responses to the questions on two vector characteristics of all subjects in the sample.

Results are presented separately for each vector characteristic following the format described in the previous section.

4.3.1. Inferred Rules for Reference Point for Stationary Bodies (RPS)

At the beginning of the study it was conjectured that a reference point was one unique vector characteristic
which was used when describing the locations of bodies. The data collected in this study showed that the descriptions used by students to locate a stationary body or place (e.g., a fishing spot in task 1) and those used to locate a moving body (e.g., a moving boat on the river in task 2) were two different things. Thus, it became evident that it was not possible to identify common criteria for both cases. Because of these findings it was decided to treat reference point as two distinct vector characteristics. The first, which is treated in this section, refers to the choice of reference bodies or points when describing stationary objects or places. The second type refers to the choice of stationary reference bodies and/or moving reference bodies (e.g., water of the river in task 2) when describing objects in motion. This became vector characteristic number 5 in LIVC (see Chapter One).

This section deals only with the vector characteristic reference point for stationary bodies (RPS) and the aim was to find out how students dealt with the problem of describing the location of stationary bodies or instantaneous locations of a moving object. Did they see the need for a reference point to describe a location? Did they describe these locations with respect to a specific reference point? What did they choose as reference points?

**Questions of Interview Protocol for RPS**

**Questions:** 1, 4, and 5 in task one.

**Question 1:** How would you describe the location of the first fishing spot to your friends?
Question 4: How would you describe the location of the second fishing spot to your friends?

Question 5: How would you describe the location of this last fishing spot to your friends?

Table 4.1

List of Inferred Rules About Reference Point for Stationary Bodies (RPS)

Three inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"Each one of the locations of the fishing spots is described using ..."

Inferred Rule RPS-1: Several reference bodies or places". (6F, 7M).

Inferred Rule RPS-2: Only one reference body but a distinct one for each location" (2F,2M).

Inferred Rule RPS-3: One unique reference body for all locations" (1F,2M).

\( \chi^2 = .21, \, df = 2, \, p>.05 \)

Discussion of Results

Inferred Rule RPS-1: This can be referred to as the "multiple reference point conception" because several bodies or places are used to locate a stationary object. From the sample of 20, 13 used this conception. These subjects believed that the location of an object can best be described
by using several different reference bodies. Some excerpts from interview data will illustrate more clearly this conception.

Suzzane, 17: (Q.4) "It (task 1, second fishing spot) is sort of in the middle but it's closer to the wharf ... there are two buoys on the water ... tell them (her friends who are coming to fish) where the first one (first fishing spot) is and tell them that the second one is farther but not much and it's closer to the tree".

Dale, 15 (Q.4) "It (task 1, second fishing spot) is just past the first fishing spot, in the same angle, a couple of hundred of yards from me and it is right in front of this corner".

These two excerpts show that both subjects were using several reference points. Suzzane made use of five: the middle of the lake, the wharf; the buoys, the first fishing spot, and a tree; and Dale made use of three: the first fishing spot, himself, and a corner. It appears to be a "try to include everything you can" type strategy so as to try and maximize their chance of success.

Inferred Rule RPS-2: This rule can be referred to as the "changing reference point conception", because a distinct reference body is used to locate each fishing spot.

Three places were more often used as a reference point, they were: the observer's location (task 1, the dock), the starting point for the trip (the wharf), and the last stop (fishing spot) of the boat. These points were also found among the reference points selected by subjects holding
RPS-I. A common practice was to select the closest object or place to the location being described. Among the four Ss holding this rule, three followed this procedure. To describe the location of the first fishing spot they used the wharf (the closest object to the spot), for the second fishing spot they used the tree (the closest object to that spot), and for the last fishing spot they used the shore (the closest place to the last fishing spot). Excerpts from the interview data clearly illustrated this practice.

Julie, 15: (Q.1) "I'd tell them (her friends) to go from this point (she showed the wharf) out straight in angle ... a little bit in angle".

(Q.4) "... turn the boat in angle and head for the trees or whatever landmark from the shore (closest shore to the second fishing spot).

(Q.5) "... go to where the curve is (closest shore to the last fishing spot) and go about 15 feet and there it is (she meant to carry the boat to that shore and start rowing from there).

These excerpts show that the subject changed the reference point to describe each one of the locations and selected the closest object to the location as a reference point.

Inferred Rule RPS-3: This rule can be called the "constant reference point conception" because each subject used the same object as a reference point to locate the different
fishing spots. The most common places selected are the observer's location (the dock in task 1) or the starting point for the trip (the wharf). For the 3Ss holding this conception, two consistently used their position (the dock) as a reference point to describe the locations of three fishing spots; the other subject made consistent use of the starting point (the wharf) as a reference point. Excerpts from interview data show this consistent practice:

Preston, 16: (Q.1) from here (the dock) I will probably use degrees ... maybe 40° north from where I am standing here ... "
(Q.4) "... from here (the dock) it will be about 15° to the north ...
(Q.5) "... it is in the indentation of the land ... or it'd be about 180° toward the south from here (the dock)".

These excerpts clearly show that this subject consistently used the same place to describe the location of different fishing spots.

Table 4.2
Rule-Model for Reference Point for Stationary Bodies

Rule RPS-I: "Use of multiple reference points (bodies or places) to locate a stationary body".
Rule RPS-II: "Use of one reference point (body or place) to locate a stationary body but distinct ones to locate other stationary bodies".
Rule RPS-III: "Use of one unique reference point (body or place) to locate a number of stationary bodies".
Rule RPS-III actually corresponds to the physicists' view of selecting a reference point. This is the reason that it occupies the highest position in the rule-model. It was not an easy task to decide which inferred rule between RPS-1 and RPS-2 should follow in complexity. RPS-2 was taken because this also considers only one reference point, even though it was not the same reference point for the different spots.

4.3.2 Inferred Rules for Frame of Reference for a Stationary Body (F.R.)

The reason for including the frame of reference characteristic was to uncover what kind of conceptions students have about making more precise quantitative descriptions of locations of stationary bodies or places. This characteristic is closely related to the previous one, reference point -- it essentially is an extension of the notion of a reference point. Reference point refers only to the need for the choice of a body or place from which to relate the location of another body but no quantitative description is required. After a reference body (or point) has been chosen, the distance to and direction of (i.e., as expressed by an angle) a specific location with respect to that reference point can be measured. When these quantities are expressed, the reference point has then been extended to become a frame of reference.
Specifically, then, the reason for including this vector characteristic was to determine how subjects deal with the problem of describing quantitatively the locations of different points (i.e., fishing spots in task 1) on a flat surface. Did they see the need of using a quantitative description (i.e., distance expressed in units and/or angle expressed in degrees) in order to locate the various fishing spots? Did they use more than one variable (distance and direction) in their descriptions and which reference points did they use to relate these variables to the various locations?

Originally it was intended to include another vector characteristic — that of vector position — but after analyzing the first interviews it was realized that the preconceptions uncovered through the questions related to vector position were similar to those obtained for frame of reference. This was the case because both vector characteristics, frame of reference and vector position, focussed on the quantitative aspect of describing locations. Thus, it was decided to consider only the frame of reference characteristic, and the responses for the questions related to vector position were used as data for the identification of preconceptions and inferred rules for frame of reference.

Frame of reference was selected rather than vector position because it was thought that the former is more intuitive in nature than the latter, and basic notions
of it would more likely be evident in subjects prior to formal instruction.

Questions of Interview for FR

Questions 1, 3, 4, 5a, and 6c in Task One.

Question 1: How would you describe the location of the first fishing spot to your friends?

Question 3: (If S has used no quantity or only one, either distance or direction, when answering question 1). How could you make the description of that location (first fishing spot) more precise for your friends?

Question 4: How would you describe the location of this second fishing spot to your friends?

Question 5: (a): How would you describe the location of this last fishing spot to your friends? (This fishing spot is at the same distance from the dock than the first fishing spot, but in a different direction).

(b): These two fishing spots are at the same distance from you (S on dock). How could you distinguish them?

Question 6: (c): (After S has drawn the path followed by the boat) What do you need to know in order to have a better approximation of the path taken by the boat?
These inferred rules were expressed in both quantitative and qualitative terms. The use of units (e.g., meters, degrees) in the description was interpreted as being quantitative in nature. Where units were not used and other descriptions such as north, south, closer, farther, to the right were used, the descriptions were interpreted as qualitative in nature.

Five inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"Precise description of each one of the locations of the fishing spots is done by using ...

Inferred Rule FR-1: Only a quantitative distance (between a given reference point and the location) as a coordinate" (1F, 2M).

Inferred Rule FR-2: Only a quantitative direction (expressed as an angle in degrees) as a coordinate." (0F, 3M).

Inferred Rule FR-3: Both qualitative distance and qualitative direction as coordinates but two reference points are used -- one for the distance and another one for the direction" (1F, 1M).

Inferred Rule FR-4: Both quantitative distance and qualitative direction as coordinates but two reference points are used -- one for the distance and another one for
the direction" (5F,3M)

**Inferred Rule FR-5:** Both quantitative distance and qualitative direction as coordinates with the same reference point for both distance and direction" (1F,1M)

\( \chi^2 = 3.67, \ df = S p > 0.05 ; 2 \ Ss \ (1F,1M) \) were not assigned inferred rule).

Discussion of Results:

**Inferred Rule FR-1:** This rule could be interpreted as the "only one quantitative distance as coordinate conception". Even after asking question 5b, the only coordinate subjects used between a reference point and the location of a fishing spot was distance. Question 5b put subjects in a position where the distances to the first and last fishing spots were identical, thus distance alone was not sufficient in distinguishing between the two locations of the spots. However, the subjects using this rule did not appear to recognize this difficulty and maintained their adherance to the distance criterion. Only 3 out of 20 subjects made use of this inferred rule; same excerpts from their interview data follow.

Hans, 17: (Q.1) "... how far away [they] are from the shore..."

(Q.3) "... how far it is away from me..."

(Q.5a) "... from the shore ... how far out is from the shore"

(Q.5b) "What do you mean? (Question is rephrased) ... I don't know ... maybe how many metres across are (S referred to the distance between the two fishing spots)".
These two excerpts show that these Ss made use of only distance as a coordinate. There is some indication of direction appearing here but I judged it to be insufficient to qualify as a (FR-3 Rule).

Inferred Rule FR-2: This rule could be summarized as "only quantitative direction as coordinate". Subjects holding this preconception differentiated locations in a two dimensional situation by use of quantitative direction only, which was expressed by angles in degrees. It is interesting to observe that they did not use distance as a coordinate, which seems to be more primary than direction. There were 3 of 20 subjects holding this conception; excerpts from

Kelley, 15: (Q.1) "... find a point in that shore that is in line with the spot ... then, guess the distance (I: What distance?)...

(Q.3) "I don't know any other way".

(Q.4) "... from the tree so far ... it is about 60 yards out ... and then so far out from where I am ...

(Q.5a) "How far away it is from where I am"

(Q.5b) "... I don't know ... from the angle that they are (S referred to the angle formed by the two straight lines joining the two spots with the dock) ... you could say 200 m in one direction (S referred to the distance between the two spots) ... or you make an isosceles triangle and make these sides 200 metres, then they will be equal"

(Q.6c) "... the distances, from the points through they went and the angles of how much they turned (S referred to the angles formed by consecutive displacements)"
their interview data follows.

Preston, 16: (Q.1) "... from where ... I'll probably use degrees ... maybe it's 40° north from where I am standing".

(Q.3) (Not asked to this subject).

(Q.4) "from here ... it will be about 15° to the north"

(Q.5a) "it is in the indentation of the land ... or at 9 o'clock using that direction as 12 o'clock, it will be in the south ..." about 180° toward the south from here".

(Q.5b) "it'd be 200 metres to the north west and 200 m to the south-east (S used distance expressed in metres because I introduced them in the question 5b. What did you mean before by degrees?) ... oh, by degrees I mean ... this is my zero line (a line passing through the dock and wharf) and my fraction is going up like this (S showed several angles; 20°, 30°, 90°, using his location as a reference point)

(Q.6c) "... probably the distances across the lake (I: What distances?) ... from the wharf to here (first fishing spot), from here (first fishing spot) to here (2nd fishing spot) ...".

Steve, 15: (Q.1) "... I would say: start from the wharf there and go out about 70 degrees angle to almost be parallel to the tree (I: anything else that you could tell to your friends?) ... straight in front of this curve (a bay on the lake) is"

(Q.3) "... (long pause) ...

(Q.4) "when you are in the first fishing spot turn toward the shore ... from where I am standing, it will be right behind the first fishing spot".

(Q.5a) "first end up in the bank and parallel to the first one (first fishing spot) I guess ... in the first curve in that bank (I: What else could be important to mention to your friends?) ... (pause) ...
(Q.5b) "that fishing spot is 200 metres (the variable distance was introduced by interviewer in question 5b) to the right about 70 degrees angle ... and the other one is 200 m to the left about 70 degrees angle from this point of view (the dock)"

(Q.6c) "... distances ... from point to point ... from the wharf to this (first fishing spot) and so on".

Both subjects did not mention the distance until the interviewer introduced it as part of question 5b. But, even after the distance was introduced, the Ss did not use both, angle and distance, as a pair of coordinates measured with respect to the same reference point.

Inferred Rule FR-3: This rule is made up of several aspects. First of all it is a type of a qualitative rule since the two variables (distance and direction) mentioned are not associated with units. Secondly, subjects holding this conception saw the need to use two qualitative coordinates in a two dimensional situation. And thirdly, although these subjects used two variables they related them to different reference points. This rule could be summarized as: "qualitative coordinates with different reference points conception". Only two of the 20 subjects held this inferred rule. Excerpts from their interview data are presented below.

Suzzane, 17 (Q.1) "... straight in front of the wharf ... you probably have to say the degrees ... or a map (response to question 3) "... just to tell them to go straight or go with them ..."."
(Q.3) "... just to tell them to go straight or go with them".

(Q.4) "it's sort of on the middle but it's closer to the wharf ... there are two buoys on the water, tell them where the first one is and tell them that the second one is farther but not much and it's closer to the tree".

(Q.5a) "It's right off the side, to my left ... it is past the green buoy".

(Q.5b) "One is at angle (I: What do you mean?) ... well one is to your right and the other to the left ... on the right side is at an angle ... the other one is a bit in an angle"

(Q.6c) (Question not asked of this subject).

Inferred Rule FR-4: The only difference between this inferred rule and the previous one is that in this case the subjects used quantitative distance, along with a qualitative direction coordinate. Like the previous inferred rule these subjects used different reference points for distance and direction. This preconception was the most common one found in the sample of subjects, since 8 of 20 held it.

Excerpts from their interview data follows:

Becky, 15: (Q.1) "... just about half way to the middle, and half way to the right side ... it's about 50 feet from me"

(Q.3) (not asked to this subject)

(Q.4) "... you explain to them again by distance ... about 100 feet ... maybe you could say along the shore just pass the tree ..."

(Q.5a) "it is near the shore on the left side of the lake ... it's not far out, just about 5 feet from the edge".
(Q.5b) "... you could say in an angle and 50 feet (distance of P4 respect to a spot on right side of subject) ... you could say about 120° angle if you start from here ... (I: What about the other spot?) It's about 75° angle..."

(Q.6c) "... the waves it produces when it moves in order to see it ...

Larry, 16: (Q.1) "I could give them the distance from this (the wharf) in front of it, and that there is a tree to the right".

(Q.3) "... It is exactly in the middle of the lake. If I am in this dock and you look that tree the boat is in the middle of them".

(Q.4) "I will tell them: find the tree and this dock (the wharf), the straight line coming from that tree to here (second fishing spot) and back to the dock, and there is the spot about 30 feet from the tree".

(Q.5a) "From the small dock up to the left side ...

(Q.5b) "... they are both 200 yards and one is on the left side of the coast and the other in the center of the lake ..."

(Q.6c) "... I need a compass (I: How could you use it?) ... I don't know ... we need markers like the tree, corners ...

Inferred Rule FR-5: The only difference between this rule and rule FR-4 is that these subjects used the same reference point to express quantitative distance and qualitative direction. This rule is most similar to what physicists define as a polar frame of reference, the only ingredient missing is the quantitative direction, which can be expressed as the angle in degrees that the vector position forms with one reference line passing through the reference
This conception was held by only 2 of 20 subjects which indicates that it is not very common among grade 10 students. Excerpts from their interview data follow:

Ian, 15: (Q.1) "You can tell how far off from the dock it is, in what direction ... you can see from here in the direction of the tree ..."

(Q.3) (Not asked to this subject).

(Q.4) "It'd be directly to the north-east about 5° to the N.E. straight out from the other one (first fishing spot).

(Q.5a) "... you can do it of how far is from here (the dock) ... from here (the dock) is 150 yards off ...

(Q.5b) "... this one is really close to the shore, the other one is out just in the middle of the lake ... or one is on the left (of observer's location) and the other on the right ..."

(Q.6c) "to have a rough idea where it went, how many times it stopped, what direction went in and came back ..."

Although, Ian made use of quantitative direction in some point, it was not a consistent criterion. He was more inclined to mention qualitative direction combined with quantitative distance.

Table 4.4

Rule-Model for Frame of Reference

<table>
<thead>
<tr>
<th>Rule FR-I:</th>
<th>&quot;Use of only one quantitative distance as a coordinate to locate a stationary body in a two dimensional situation&quot;.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule FR-II:</td>
<td>&quot;Use of only one quantitative direction (as an angle expressed in degrees) as a coordinate to locate a stationary body in a two dimensional situation&quot;.</td>
</tr>
</tbody>
</table>
Rule FR-III: "Use of both qualitative direction and qualitative distance as coordinates to locate a stationary body in a two dimensional situation; but two reference points are used, one for each coordinate".

Rule FR-IV: "Use of both quantitative distance and qualitative direction as coordinates to locate a stationary body in a two dimensional situation; but two reference points are used, one for each coordinate".

Rule FR-V: "Use of both quantitative distance and qualitative direction in a two dimensional situation; with the same reference point for both coordinates".

4.3.3 Inferred Rules about Displacement or Change of Location (D)

Although this is a well defined kinematic vector quantity, it was considered important to include it as an implicit vector characteristic for the following reasons: (1) It can be considered a more intuitive concept in comparison to the vector position concept, which is more "artificial" or remote from experience, (2) It is a suitable concept to clearly differentiate between a scalar quantity (e.g., the length of the path) and a vectorial quantity (e.g., displacement). The first is scalar because it only needs a magnitude, the length of the path between two locations, to be completely described; and the second is vectorial because in addition to magnitude it also needs direction to be completely described, and finally, (3) It corresponds, in part, to a very common and intuitive notion
that the closest distance between two locations is the length of the straight line that joins them.

From the pilot study a trend was observed for the subjects to choose the last stopping point of a moving object as a reference point (e.g., use of the first fishing spot to describe location of the second fishing spot). If this is an intuitive conception of selecting reference point, it may help students to understand the concept of displacement. Particularly, it should facilitate the understanding of partial consecutive displacements, since partial displacements associate two consecutive locations of a moving body.

Questions of Interview Protocol

Questions: 6, 7a, 8a in task one.

Question 6: From where you are (on the dock) could you approximately describe the path followed by the boat? Please, draw that path in this map of the lake.

Question 7a: We can say that the boat has had a change of location when it moved from the starting point (S.P.) to the first fishing spot (P₁). How could you describe the location of the first fishing spot with respect to the starting point? (If S includes the distance in her/his description) Is that distance along the path followed by the boat?

Question 7b: Then, the boat moved from the first fishing spot to the second one at here (P₂). Then how could you describe the location of this fishing spot with respect to the first one?
Question 8a: (If S includes the distance in her/his description). You mentioned the distance in your description. Does a change of location tell anything about the path followed by the boat between the two locations?

Table 4.5

List of Inferred Rules for Displacement (D) or Change of Location

In these instances the physical phenomena being described was the boat moving from one fishing spot to another; this means, a change of location, which was used as a synonym of displacement.

Five inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"A change of location of the boat moving from one fishing spot to another indicates ..."

Inferred Rule D-1: The starting and final locations". (1F,1M)
Inferred Rule D-2: The length of the straight line joining the two locations and that length is independent from the path". (0F,3M)
Inferred Rule D-3: Both the distance along the path and the quantitative direction measured from the observer's location
Inferred Rule D-4: Both the length of the straight line joining the two locations which is independent from path and qualitative direction". (7F,3M)
Inferred Rule D-5: Both the length of the straight line joining the two locations which is independent from path and quantitative direction". (OF,1M)

\[ \chi^2 = 6.80 \text{ df} = 5 \text{ p} > .05; \text{ IS(F) was not assigned inferred rule} \]

Discussion of Results

Inferred Rule D-1 This rule states that a change of location indicates the aimed location of a trip and starting location. There was no mention of the possible ways or path in reaching that final location. No qualitative or quantitative variables such as distance to the final location or direction of motion was provided. In summary, this conception relates only the starting and the ending points of a moving object but not what happens in between. This was not a common conception among adolescents (15 to 17 years of age) since only 2 among 20 held this conception.

Excerpts from interview data best illustrate this conception.

Suzzane, 17: (Q.7a) "Just it goes straight ... (I: What could you tell to your friends about this change of location?) ... I don't know ..."

(Q.8a) "Yeh, it is ... it tells where the boat went ..."

Dale, 15: (Q.7a) "Well it started here (wharf) and went to that point (first fishing spot) ... they paddled straight off from the wharf".

(Q.7b) "... yeh ... from here (wharf) to here (first fishing spot) ... it was straight off.

Both subjects solely emphasized the starting and ending points in their responses.
Inferred Rule D-2: Subjects holding this conception thought that the distance between the two locations was an important factor to consider when describing a change of location. Notice that they were not considering the distance along the path; these subjects were differentiating between the two fishing spots. This conception did not include the direction associated with a change of location. Among the 20 subjects, only 3 held this conception, which is shown through the following excerpts.

Hans, 17: (Q.7a) "How far out it is ... (I: Is that distance along the path?) ... from there (wharf) to there (first fishing spot. (He meant the straight distance).

(Q.7b) "Same thing as before ... how far it is"

(Q.8a) "I don't understand what you mean (I: question is repeated) ... no (I: Why?) You can't tell the actual path, you could go around in different ways".

This subject as the other two included only the straight distance in his responses; there was no mention of direction at all.

Inferred Rule D-3: The only subject holding this conception considered the distance along the path. This was calculated by the product of the speed (assumed to be constant) times the interval of time taken in moving from the wharf to the first fishing spot. The direction was expressed by the
angle formed by the two straight lines passing through the starting and ending points from the observer's location. Actually, this was not a direction associated with the change of location but rather with the direction of the vector position associated with the first fishing spot. Only one subject among 20 held this conception which is shown in the excerpts from interview data.

Wayne, 15: (Q.7a) "You could look at the first fishing spot and make an imaginary line (from observer's location on the dock); then look at the wharf (starting point), you notice it will be an angle between the lines. You can note how far over it is (size of the angle). You can also tell how far apart they are (the two locations). If they travel at the same speed all the time, you could time how long it takes to go from one spot to the other, this will give you the distance. (I: What is important to consider for a change of location?) ... the speed, the direction ..."

(Q.8a) "No ... (I: Why not?) ... because he went from the starting point to the fishing spot ... he went zigzagging ..."

Inferred Rule D-4: This conception of change of location was very close to the formal definition of displacements. The only ingredient missing was the quantitative direction; subjects holding this conception included a qualitative direction. These students could readily grasp the formal concept of displacement if they were instructed how to express the qualitative direction in a quantitative form. This was a common conception among grade 10 students since 10 of 20 subjects held it. It is important to mention here that these students were explicitly aware that the distance considered was not along the path;
some of them said that there were many ways (or paths) of moving from one location to the other. Considering the conceptions of all vector characteristics analyzed so far, it seems that most of the students seemed to concentrate on only one variable in a quantitative way, and when they considered a second variable it was in a qualitative way. This was the case for the vector displacement, and for the frame of reference characteristic. The following are excerpts from interview data:

Lisa, 15: (Q.7a) "The starting point is right on the end of the wharf and the first spot is a bit to the right in a slight angle ... and estimates the amount of miles between the two ... along the way followed by the boat".

(Q.7b) "From this (first fishing spot) to this (second fishing spot) is almost the same distance (as the distance from wharf to first fishing spot) ... maybe 50 or 75 metres more ... and they have to turn some degrees".

(Q.8a) "... no ... because the boat can go around in different ways ..."

Although in the response to the first question Lisa said: "along the way followed by the boat", this meant the distance along the path; the responses for the other two questions show that she was referring to the straight distance between the locations.

Lori-Ann, 15: (Q.9a) "How far it is, using feet or miles (I: Is that distance along the path or the straight distance?) ... along the path, how far the boat will go to reach that point. (The first fishing spot. I: Suppose the path is this one. I drew a curve path) ... oh,
I see ... I am talking about the straight distance (I: anything else to consider for this change of location?) ... if the boat were going back and forth it will be a much greater distance ... also if you want to describe the exact path you could use south, north, east, or west, and go so much this way and then turn around the other way".

(Q.8a) "it tells the end of the path but you can't tell the path because it could go straight or like that ... or like that (she sketched two different curve paths)"

**Inferred Rule D-5:** This was the conception closest to the formal definition of displacement. Although, the subjects holding this conception made use of quantitative direction, they were not referring to the direction of the change of location but to the direction of the vector position associated with the fishing apots and using the observer's location as a reference point. Since these subjects realized about the need of using these two quantitative variables (distance and direction), the instructional task would be to show them the importance of knowing the direction of moving from one location to another; it would be expected that these students would easily transfer their knowledge of measuring angles to a new but similar situation.

The conception, included in this inferred rule also considers the distance between the two locations. (The length of the straight line joining them) thus these subjects were differentiating between this distance and the distance along the path. Three students among
20 held this conception, which is shown in the following excerpts from the interview data.

Becky, 15: (Q.7a) "... the distance and in what side of the lake?

(Q.7b) "go about 110 degrees (line joining dock as a reference point) angle by using the protractor toward that corner on the right side of the lake and about 50 feet (I: Is this distance along the path or the straight distance between the two location?) ... the straight distance ... (I: What do you think is important when you refer to a particular change of location?) ... where it starts and where it goes, the distance and the angle".

(Q.8a) "... it does not tell you the path ..."

Steve, 15: (Q.7a) (Q not understood when first asked. I: What do you think is important to consider for a change of location? ... direction and distance ... (I: distance along the path or straight?) ... the straight distance..."

(Q.7b) "... in the same way ... the degrees and distance ..."

(Q.8a) "No ... they can go anywhere around the whole lake to arrive to that point, they can go straight too".

Table 4.6

Rule-Model for Displacement

Rule D-I: "A displacement or change of location indicates the starting and final locations".

Rule D-II: "A displacement or change of location indicates the length of the straight line joining two locations".

Rule D-III: "A displacement or change of location indicates both the length of the path and a quantitative direction measured from the observer's location".
Rule D-IV: "A displacement or change of location indicates both the length of the straight line joining two locations and a qualitative direction of the final location with respect to the initial one".

Rule D-V: "A displacement or change of location indicates both the length of the straight line joining two locations and a quantitative direction given by the angle of the straight line joining the two locations with another reference line".

4.3.4 Inferred Rules for Addition of Displacement (AD)

The addition of displacements is a defined vectorial operation, which is applied to the combination or composition of any vector quantity. The reason for including this operation as an implicit vector characteristic is based on this author's belief that for a formal concept or mathematic operation to be more easily understood by students, the concept must be based on (or related to) some intuitive notion of the concept. Usually, this operation is first taught in mathematics and it is generally done without connecting the vectorial entities with physical quantities. These vectorial entities are represented by "arrows" whose length (of the arrow) corresponds to the magnitude and the arrow itself shows the direction. Physics teachers know very well that this sort of abstract knowledge is not easily transferred to solve kinematic vectorial problems. This author also believes that students develop some preconceptions to cope with
situations in which they have to combine displacements (or distances) with different directions. These preconceptions could be part of the implicit aspects that would enhance the understanding of the formal concept.

This section, then, considers the combination of two consecutive displacements, which possess magnitude and direction, as an implicit characteristic of vector quantities. The aim of this part of the study is to uncover what kinds of conceptions students have developed about combining partial and consecutive displacements.

Questions of Interview Protocol for AD

Questions: 9 and 10 in task one.

Question 9: Suppose you have established the distance from the starting point (S.P.) to the first fishing spot (P₁) as about 100 m, and you have also estimated it from this spot (P₁) to the second fishing spot (P₂) as about 150 m. If your friends are at the starting point how are you going to tell them to go directly to the second fishing spot?

Note: This question was used only with the purpose of introducing the addition of displacement. It was expected that all or most of the subjects will use the arithmetic addition as a solution, and 100 percent of subjects actually did use it as a strategy to find the correct response. Because the major purpose of these questions was to familiarize
the subjects with this kind of situation the responses for question 9 were not considered in the analysis of the inferred rules.

**Question 10:** Suppose you could also estimate the distance from this spot (P2) to this third fishing spot (P3) and it is about 100 m. How could you tell your friends to go directly from the starting point to this third fishing spot?

**Note:** The purpose of this question was to find out how the subjects go about solving this kind of situation. It was not expected that students would discover by themselves the operation of vector addition, but it is important for further instructional use to uncover the students' intuitive ideas about vector addition. It was thought that one question would be sufficient to illustrate the subjects' conceptions about this vector characteristic.

**Table 4.7**

**List of Inferred Rules for Addition of Displacements (AD)**

These inferred rules contain two aspects. First, the criterion to obtain the magnitude of the resultant displacement; and second, the direction (qualitative or quantitative). Both aspects are separated by the conjunction **and**.

**Note:** Despite the fact that there was no search for a
consistency of responses, because there was only one question for this vector characteristic, the different subjects' conceptions were classified as inferred rules to maintain parallelism in the data presentation.

Three inferred rules were obtained for this vector characteristic, which have the following common opening sentence "The resultant of two non-parallel consecutive displacements of the boat has ...

Inferred Rule AD-1: A magnitude equal to the arithmetic addition of the magnitudes and no direction is cited". (1F, 1M)

Inferred Rule AD-2: A magnitude smaller than the arithmetic addition of the magnitudes and no direction is cited" (5F, 7M).

Inferred Rule AD-3: A magnitude smaller than the arithmetic addition of the magnitudes and direction is cited" (2F, 2M)

\( \chi^2 = .14 \text{ df } = 3 \text{ p > .05; 2Ss (1F, 1M) were not assigned inferred rule).} \)

Discussion of Results:

Inferred Rule AD-1: This rule could also be called "the arithmetic addition conception" of composing vector displacements. Subjects holding this conception made use of arithmetic addition to obtain the magnitude of the resultant of two non-parallel displacements. The angle formed by the two consecutive displacements was about 120° and hence was much too far from being parallel to justify the use of arithmetic addition. The two subjects under this rule did not mention direction at all; it may be due to the fact that they concentrated only on figuring out how to find the distance. Excerpts from interview data will illustrate
this conception.

Note: The estimated distances S.P. to $P_1$, $P_1P_2$, and $P_2P_3$ were different for each subject depending upon whether they had earlier offered an estimation.

Suzzane, 17 (Q.10) (SP.$P_1$ = 50 m, $P_1P_2$ = 60m, $P_2P_3$ = 100)

... If I need a map ... about 200m (I: Why do you think so?) ... about 220m or something ..."

Larry, 16: (Q.10) (SP.$P_1$ = 200 yards, $P_1P_2$ = 200, $P_2P_3$=200)

"... 600 yards ... (I: But you asked your friends to go directly from the wharf to this spot, $P_3$?) ... well, it is 600 yards ...")

Both subjects appear to be using strict arithmetic addition, particularly Larry: he just added the three distances. Suzzane hesitated between 200 and 220: it seems she roughly estimated the distance by adding the partial distances.

**Inferred Rule AD-2:** Subjects holding this conception realized that they could not use arithmetic addition because the distances were not in the same direction, and also they observed that the result should be smaller than the arithmetic addition of the component distances. They tried different kind of strategies to obtain a numerical result. It was not expected that they would solve the problem the way a physicist would do it but it is worth examining the students' methods of tackling this sort of problem. The most common strategies were what could be called the
'distance comparison strategy' and the 'Pythagoras' strategy'. The first one consisted in estimating the length of the line SP.P₃ by comparing by eye with a known distance, for instance with the distance SP to P₁. This strategy very much resembles the 'scale approach' to find the magnitude of resultant. This consists in choosing an adequate and arbitrary unit (draw at scale) and seeing how many times that unit fits into the desired distance to be estimated. This requires the understanding of direct proportionality. The 'Pythagoras' strategy' consisted in assuming the triangles to be right angle triangle (even though they were not) and using the Pythagoras' Theorem to find the resultant. Of course, it would be a valid strategy if the triangle were a right angle triangle.

Subjects' classified under this rule did not mention direction at all. More than half of the subjects in the sample, 12 of 20, used these strategies, which will be shown in the following excerpts from interview data.

Kevin, 15: (Q.10) (SP.P₁P₂ = 200, P₂P₃ = 300)
"... I have to figure out the distance from here (SP) to here (P₃) ... 300 squared plus 300 squared ... 900 plus 900 ... 1800 ... and the squared root of that will be the distance".

Russ, 15: (Q.10) (SP.P₁ = 400m, P₁P₂ = 300m P₂P₃ = 300)
"... take from the math ... about 800m ... we have to estimate because it's not going through the curve line (he meant passing through all spots) ... (I: could it be 1000m?) ... no because it goes in straight line".

Tracy, 15: (Q.10) (SP.P₁ = 200 feet, P₁P₂ = 150, P₂P₃ = 200)
"... going in this path (passing through all
spots) or just straight? ... (I: no, directly from here, SP, to here, P3) ... it will be about 475 feet because they will be losing some feet ... if you go this way (passing through all spots, zigzagging path) is longer if you go straight ... this way is shorter".

Sandy, 15: (Q.10) (SP. P1 = 100 yards, P1P2 = 120 P2P3 = 200)

(Question not clear for her. I: How far will you tell them to go?) ... 300 yards ... (I: Why do you think so?) ... say this spot (P2) is here (she assumed translating P2 to be in line with SP. and P3, it sorts of geometrical projection of P2 over the line SP.P3) ... it is about 200 yards ... because from this (new location of P2) to this (P3) is 200 ... then, it may be 400 ...

Inferred Rule AD-3: This rule is similar to the previous one with regard to the strategies used to find the magnitude of resultant displacement; but in addition the subjects under this rule mentioned the direction as an important ingredient. Excerpts of interview data of some of the 4 subjects holding this conception follow.

Preston, 16: (Q.10) (SP. P1 = 200m, P1P2 = 100 P2P3 = 200)

"... go to a westerly direction with a few degrees off ... like about 400m from the starting point to this (P3) ... (I: Why do you think is 400?) ... because the angle will be cut off unless you go in a zigzagging motion, or from here (P2) to here (P3), but if you go straight the angle will be shorter so it will be a straight line shorter in approximately 100m".

Becky, 15: (Q.1) (SP. P1 = 60 feet, P1P2 = 40, P2P3 = 40).

"... you could estimate about 120 feet (I: Why is 120?) ... because from here (SP) to here (P2) is 100, then you add about 20. From this (P2) to here (P3) is 40 but you are going that way, you are going straight path. So it's a 120 feet and about 85 degrees angle and about 50 feet away from the left side".
Both subjects explicitly mentioned a quantitative direction in addition to the belief that the magnitude of the resultant has to be smaller than the arithmetic addition of the components.

Table 4.8

Rule-Model for Addition of Displacements

Rule AD-I: "The resultant of the addition of two non-parallel displacements has a magnitude equal to the arithmetic addition of the magnitudes and direction is not considered".

Rule AD-II: "The resultant of the addition of two non-parallel displacements has a magnitude smaller than the arithmetic addition of the magnitudes and direction is not considered".

Rule AD-III: "The resultant of the addition of two non-parallel displacements has a magnitude smaller than the arithmetic addition of the magnitudes and direction is considered".

4.3.5 Inferred Rules for Subtraction for Vector Position (S.V.P.)

Similar to the case of the vectorial addition of displacements, the subtraction of vector quantities is an explicit and well defined vectorial operation. The reasons for including this operation as an implicit vector characteristic are similar to the ones given for including the addition of displacements, but the inclusion of this vector characteristic needs other kinds of arguments as well.
First it can be argued that the operation of subtraction of vectors is more abstract than the addition operation because it is more remote from students' experience. Hence it can be considered as a more "artificial" conception when compared to addition of displacement which is more "natural". Also there are several previous concepts to be understood before formally grasping subtraction of vector. These concepts are: use of one reference point to describe locations of different points, frame of reference, association of coordinates (magnitude and direction) for points, and the association of vector position to particular stationary locations or instantaneous locations of a moving body. Obviously, it is not expected that subjects will be able to handle all these concepts before formal instruction, but the purpose of including this operation as an implicit vector characteristic was to uncover how subjects cope when confronted with this sort of situation. Did they have any intuitive notion of how to deal with the problem? What kind of strategies did they use to relate the two given distances? Did they consider the fact that the two distances were not parallel?

It may be that the operation of subtraction of vectors contains more implicit aspects that were not detected in the rational tasks analysis. If that is the case, these unknown implicit aspects could be revealed through the students' preconceptions, which may become manifest when dealing with the problematic situation.
Questions of Interview Protocol

Questions: 11 and 12 in task one.

Question 11: Suppose you have estimated the distance from yourself (on the dock) to this spot (thumb tack is placed at \( P_1 \)) and it is about 200 metres (different distances were used for different subjects) and you have also estimated the distance from yourself to this other spot \( P_2 \) (the dock, \( P_1 \), and \( P_2 \) are on a straight line). Your friends are at this spot (\( P_1 \)). How are you going to tell them to go directly to this spot (\( P_2 \))? 

Note: This question was used only with the purpose of familiarizing the subjects with this sort of situation. It was expected that most of the subjects would make use of strict arithmetic subtraction to find a solution. Results agreed with expectation: 18 among 20 subjects did use arithmetic subtraction. As with the addition operation, because this question served as a familiarization procedure their responses are not being considered in the analysis of the inferred rules.

Question 12: Suppose you have also estimated the distance from yourself to this spot (\( P_3 \), see Figure 3.5 for best understanding of situation) and it is about 300 metres. Your friends are again in this spot (\( P_1 \)). How could you tell them to go directly to this other spot (\( P_3 \))?
Note: It was thought that one question on this vector characteristic would show the subjects' conceptions about subtraction of vector quantities.

Table 4.9

List of Inferred Rules for Subtraction of Vector Position (SVP)

These inferred rules contain two aspects. The first refers to the magnitude of the resultant of subtracting two vector positions (magnitude of resultant displacement); and the second, the direction (qualitative or quantitative). Both aspects are separated by the conjunction and.

Again, as in the case of addition of displacements, although there was one question for this vector characteristic the different subjects' conceptions were classified as inferred rules to maintain parallelism in the data presentation.

Three inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"The resultant of the subtraction of two non-parallel vector positions for two fishing spots has ...

Inferred Rule SVP-1: A magnitude equal to the arithmetic subtraction of the magnitudes and direction is not cited." (3F,4M)

Inferred Rule SVP-2: A magnitude different from the arithmetic subtraction of the magnitudes and direction is not cited." (1F,3M)
Inferred Rule SVP-3: A magnitude different from the arithmetic subtraction of the magnitudes and direction is cited. (5F, 3M)

\[ \chi^2 = 2.46 \text{ df } = 3 \quad p > .05; \text{ IS (1M) was not assigned an inferred rule).} \]

Discussion of Results

Inferred Rule SVP-1: This rule could also be referred to as the 'arithmetic subtraction conception' of subtracting the magnitudes of two non-parallel vector positions to obtain the magnitude of the resultant displacement. In the sample, 7 of 20 used this strategy, 35 percent of the sample. This was a high percentage for a notion which is very different from the accepted one. This result confirmed what was expected. Earlier it was suggested that it would likely be rare to find that students develop the abstract operation of vectorial subtraction by themselves. The angle formed by the two vector position was sufficiently different from 0° to 180° to say that the straight lines were in line or parallel (the angle was actually about 40 degrees).

Excerpts from interview data will show the subject's conceptions.

(S: stands for subject's location, on the dock)

Dale, 15: (Q.12) (S.P1 = 100 metres, S.P3 = 250m) "... it will be 150 metres ... you just add them up to have 250".

Hans, 17: (Q.17) (S.P1 = 100m, S.P3 = 400m) "... about 300m ... (I: Why do you think it is 300?) ... because this distance (S.P1) is 100 and the other one is 400 ..."
Kelly, 15: (Q.12) \( S.P_1 = 200\text{m}, S.P_3 = 600\text{m} \)

"... tell the angle between the first spot and that spot (the angle formed by the lines joinging each spot with the subject's location) and tell them to go 400m ... (I: Why 400m?) ... well there are 200 here and 600 there ... then it's about 400m"

These three subjects were strictly arithmetically subtracting the magnitudes of the two non-parallel vector positions.

Inferred Rule SVP-2: Subjects holding this preconception were aware of the fact that the two distances were not in line which would not allow them to use arithmetic subtraction. Another important aspect that subjects observed was that the length of the distance between \( P_1 \) and \( P_3 \) must be greater than the result of applying arithmetic subtraction to the two distances. Subjects in their endeavour to find a numerical answer tried several strategies. Among the more common were again what can be called the 'length comparison strategy', which consists in estimating a distance by comparing by eye with a known length, and the 'Pythagoras strategy', which was also found for addition of displacements. Excerpts from interview data will better show the use of these and other strategies by four of the 20 subjects.

Wayne, 15: (Q.12) \( S.P_1 = 200\text{m}, S.P_3 = 600\text{m} \)

"... you could estimate about 400m ... (I: Why 400m?)... it's not perfect continuous straight line ... (I: Will it be more or less
than 400?) ... a little bit less ... (I: How could you get the exact distance between the two spots?) ... measure the angle between the lines going from me to the two spots, using trigonometry or the angle to calculate the distance between the two spots"

Lisa, 15: (Q.12) (S.P₁ = 200m, S.P₃ = 450m)
"I subtract the distance from me to the first spot from the distance from me to the second spot ... and subtract also the distance from the first spot to here (the projection point of P₁ on the straight line connecting the subject with P₃) ... I have to subtract that because I am going in straight line, the other way is farther out".

Although both subjects used two different strategies, it is clear that they did so because they realized that the two distances were not parallel and conventional arithmetic operations could not be used.

Inferred Rule SVP-3: This rule is similar to the previous one with the difference that subjects holding this preconception cited the direction (qualitative or quantitative) as an important aspect to consider. A high percentage of subjects, 8 of 20, appeared to hold this conception. This could mean that these subjects would be in a better position to understand the operation of vector subtraction when it is formally presented in an instructional setting. Excerpts from interview data follow.

Becky, 15: (Q.12) (S.P₁ = 50m S.P₃ = 200m)
"... it's about 150 away (I: Why?) ... well, that is 200 and this 50, and it is not very much in an angle ... then it is 150 ... maybe a little bit more (I: Why a little bit more?) ... a bit more or less and it is 85 degrees angle".
Tracy, 15: (Q.12) (S.P₁ = 200m, S.P₃ = 600m)

"I tell them to go ... (pause) ... about 400 or 500 ... because they have to go this way ... they have to go farther ... I tell to go to the left about 450m ... or 500m ... (I: Why 500?) ... because is quicker for me to go ... like from me to there (a point which corresponds to the projection of P₁ on the straight line joining subject's location and P₃) is about 200m and from me to her (P₃) is 600 ... then it will be 400 but he is here (P₁) then he has to take a longer way ... about 450".

Both subjects provided direction in addition to ways of obtaining the magnitude of the resultant.

Table 4.10

Rule-Model for SVP

Rule SVP-I: The resultant of the subtraction of two non-parallel vector positions has a magnitude equal to the arithmetic subtraction of the magnitudes and direction is not considered".

Rule SVP-II: "The resultant of the subtraction of two non-parallel vector positions has a magnitude different from the arithmetic subtraction of the magnitudes and direction is not considered".

Rule SVP-III: "The resultant of the subtraction of two non-parallel vector positions has a magnitude different from the arithmetic subtraction of the magnitudes and direction is considered".
4.3.6 Inferred Rules for Reference Bodies for Moving Objects (RPM)

The aim of this second part about reference point was to uncover how students dealt with the problem of describing bodies affected by different kinds of motions. The specific situation (task 2) was to describe the motion of a motor-boat which was affected both by the motion created by its motor, and also by the current of the river.

This section attempted to address the following questions. How did students tell that the boat is in motion? Did they see the need for a reference point to describe a motion? Did they realize that the boat had different kinds of motions with respect to different reference points? What did they choose as reference points?

For this particular vector characteristic, the phrase "reference body" was used instead of "reference point ". This was done because the moving body of water could be used as a second reference frame to which the motion of the boat might refer.

Questions of Interview Protocol for RPM

Questions: 13b, 13c, and 13d in task two.

Question 13b: (After subject has seen part of the film with the boat moving on the river). With respect to what is the boat moving?

Question 13c: Was the boat moving with respect to the water?
**Question 13d:** (If subject has a "yes" answer for question 13c): Is the speed of the boat with respect to you (or bridge or shore) and the speed of the boat with respect to the water the same or different?

**Note:** It is very important to mention here the difficulties that some subjects encountered in understanding the meaning of these questions. In particular the subjects had difficulties in understanding the phrase: *with respect to* (this is part of questions 13b and 13c). The following equivalent question was used to facilitate the understanding of the situation: "How could you tell that the boat is moving?"

To understand the seriousness of the problem, we have to consider that only 10 out of 20 subjects understood question 13b when first asked, and only 6 understood question 13c. Although this is closer to a semantic problem, it is an important factor to consider when preparing curriculum material. Textbook writers and science teachers often seem to take for granted that students understand the phrase "with respect to", but the findings of the present study show that this appears not to be the case.
Each inferred rule covers two aspects. The first relates to stationary reference bodies (i.e., observer on the bridge, the bridge, or the shore), and the second aspect refers to the selection of the moving water (the river) as a reference body. In the description of the inferred rules, both types of reference bodies (stationary and moving) are separated by the conjunction and.

Four inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"Description of the motion of the motor-boat on the river by an observer on the bridge..."

**Inferred Rule RPM-1**: Does not distinguish movements relative to a stationary body (e.g., bridge) and relative to a moving body (e.g., the water)" (3F,0M).
Inferred Rule RPM-2: Distinguishes movement relative to an implicit reference point (i.e., the observer) and relative to a moving body (e.g., the water). (1F, 2M)

Inferred Rule RPM-3: Distinguishes movements relative to several stationary bodies (e.g., the bridge, shore) and relative to a moving body (e.g., the water). (4F, 6M)

Inferred Rule RPM-4: Distinguishes movements relative to one stationary body (e.g., the bridge) and relative to a moving body (e.g., the water). (0F, 3M).

\( \chi^2 = 2.90 \text{ df} = 4 \quad p > .05; \text{ 1S(1M) was not assigned an inferred rule).} \)

Discussion of Results

Inferred Rule RPM-1: This rule can be interpreted to mean that only "stationary bodies" can be used as reference bodies. Subjects holding this conception did not relate the motion of the boat "with respect to" the moving water. They said that it was moving "with" the water. This could be interpreted as meaning that they did not see the independent motion of the boat with respect to the water, compared to the motion with respect to the bridge. This indicates that the moving water itself was not used as a reference body to describe the motion of the boat. For example, if the motor of the boat is not running, then the velocity of the boat with respect to the water would
be "zero". Since only 3 out of 19 subjects hold this conception this would indicate that it is not a common conception among the grade 10 subjects. Excerpts from interview data best illustrates this conception.

Julie, 15 (Q.13b) "... it was moving slowly because it was against the current ... (I: How could you tell that the boat was moving?) ... when it moves because the engine and the current (I: Was it moving with respect to you?) ... it could it be ... (I: was it moving with respect to something else?) ... not really.

(Q.13c) "... a little bit ... it wasn't moving fast but a bit

(Q.13d) "... it is the same ... (I: Why?) ... well I am not moving so if the boat is moving and the river is moving carrying it so it'll be the same speed".

Becky, 15: (Q.13b) "... (after several attempts to make the question understandable, I: How could you tell when an object is moving?) ... well, it goes past you and I guess you could see the scenery ... that is, moving away from it ... it's not at the same stationary scenery".

(Q.13c) "yeh, but the water was also moving".

(Q.13d) "I think it will be the same (I: Why do you think so?) ... because it can't speed up".

The responses of both subjects show that they were not differentiating the independent motion of the boat with respect to the water. They realized that the water had its own speed, and that it carried the boat along with it but in their answers it was not clear if they differentiated the speed of the boat with respect to themselves, or with respect to the water.
Inferred Rule RPM-2: Students holding this rule did not explicitly mention any stationary reference body to describe the motion, but it can be inferred that they were describing it with respect to themselves. It cannot be said that these subjects are consciously aware of the need for a reference body to describe a motion. On the other hand, these students could differentiate the motion of the boat with respect to themselves and the motion with respect to the water. That is, they saw that the boat moved with respect to the water independent of the fact that the water carried the boat. Three subjects among twenty held this conception of describing the motion of the boat, which is shown through the following excerpts:

Suzanne, 17: (Q,13b) "... it looked like the boat was moving ... I saw a little wave (I: Was it moving with respect to what?) ... it was moving slowly ... (was it moving with respect to you?) ... yeh ... (pause) ...

(Q,13d) "... it is different ... the water rushes it more"

Lori, 15: (Q.13b) "By looking at it you can tell it's moving and faster than the water (I: Is the boat moving with respect to the shore?) ... yeh ... (pause) ... (Is it moving with respect to you?) ... yeh, it's getting closer".

(Q.13c) "Yeh, ... the water is coming towards me and so is the boat"

(Q.13d) "I don't know ... I don't understand ...

(I rephrased the question) ... it will be different because the water and the boat they are both moving".
Both subjects did not explicitly mention any reference point, but it was clear that they were implicitly using themselves as a reference body. Suzzane said: "it looked like the boat was moving" and Lori said "by looking at it you can tell it's moving". Both excerpts show the implicit reference point. They also differentiated the speeds with respect to themselves and the speed with respect to the water.

Inferred Rule RPM-3: The subjects holding the preconception expressed in this inferred rule used more than one stationary reference body and a moving body to describe the motion of the motor-boat. Ten of 20 subjects held this conception, which is shown through the following excerpts:

Dale, 15:(Q.13b) (I: How could you tell that the boat was moving?) ... because the engine ... it was going with the current ... it passed this (the docks) and it will be going under the bridge (I: Was it moving with respect to you?) ... yeh, it was coming towards me".

(Q.13c) "probably it was going faster than the water ... the water was helping it to move" ...

(Q.13d) "the engine was running, then, it will be different ... because ... there is different speed between the water and the boat. If the engine is off maybe it will go the same speed as the water and if the engine is on it will probably be faster than the water. If the water goes 10 miles/hr, the boat will go maybe 12 miles/hr."
Hans, 17: (Q.13b) (How could you tell that the boat was moving?) ... "by those fixed buoys ... (pause) ... (I: Was it moving with respect to you?) ... yeh ... (I: Was it moving with respect to anything else?) ... yeh, the shore ...

(Q.13c) "Was the motor on? ... (I: yes) ... it could go faster

(Q.13d) "Different speeds ... because with the motor it will go faster"

Steve, 15: (Q.13b) "... respect to the water (I: Was it moving with respect to anytning else?) ... with respect to you and where I am and both sides of the river".

(Q.13d) "It will be different (I: Why do you think so?) ... on the water it'll go only 20 miles/hr (He assumed before that speed of boat due to the engine is 20) ... by looking from my point of view it'll seem to move faster because the current is pushing it ..."

All subjects chose more than one stationary body (bridge, dock, buoy, or shore) as a reference point, and they differentiated the speed of the boat with respect to different reference bodies.

Inferred Rule RPM-4: Subjects holding this conception have explicitly chosen a body (the bridge) as a reference. These students also differentiated the motion of the boat with respect to the water, and were aware that the water was a moving body. Thus they believed that the boat was moving with respect to the bridge and also with respect to the water. This does not necessarily mean that they conceived a relationship between these two motions of the boat.
Excerpts from interview data best illustrated this conception:

Kevin, 15: (Q.13b) "... to the water ... (I: Was it moving with respect to anything else?) ... the bridge ... (pause) ...

(Q.13d) "I think it will be different because the water is moving and I am just standing there, the boat will be going faster compared to me but less fast compared to the water"

Larry, 16: (Q.13b) "it was moving with respect to the river (I: Was it moving with respect to something else?) ... to the bridge".

(Q.13c) "... it will be different because I am in a fixed position and the river is moving so it will be different.

These two subjects have explicitly chosen one specific stationary body (the bridge) as a reference, and at the same time they have differentiated the speeds with respect to one stationary body and with respect to a moving body.

Table 4.12
Rule-Model for RPM

Rule RPM-I: "Descriptions of the motion of an object relative to a stationary reference body and relative to a moving reference body are equivalent".

Rule RPM-II: "Description of the motion of an object relative to an implicit stationary reference point and relative to a moving reference body are unequivalent".
Rule RPM-III: "Descriptions of the motion of an object relative to several stationary reference bodies and relative to a moving reference body are unequivalent".

Rule RPM-IV: "Descriptions of the motion of an object relative to one stationary reference body and relative to a moving reference body are unequivalent".

4.3.7 Inferred Rules for Analysis of Components (A.C.)

The notion of differentiating the number of the component motions that simultaneously affect a moving body is a decisive aspect to consider if an understanding of the composition of vectors is to be accomplished. Usually, this aspect is treated very superficially in teaching vector quantities, or even worse it is not considered as a separate aspect by itself. This aspect represents a clear example of what has been called an implicit characteristic in the present study, in that, it is an important aspect to consider in comprehending vector quantities yet it is not explicitly treated in most physics textbooks. The problem is that some students do not easily recognize that an apparent single motion can be the result of combining several motions. A common textbook example is the case of projectile motion in the gravitational field. A few students realize that motion is the result of at least two motions, a horizontal component with constant velocity and a vertical component with constant acceleration. Some students, however, have
the belief that if the path of a moving body is a straight line, the body is only being affected by one motion; if it is a curved path, the body is being affected by more than one motion.

An important aspect in working with any vector quantity is to see how many components are present in a particular situation, how these are combined (i.e., composition of vector quantities), and also how they can be analyzed when only the resultant of them is apparent. For the last two steps, the finding of the number of components is a crucial aspect. For this reason, the main purpose of this part of the study was to uncover what kind of conceptions students have about analysis of apparent single motion. Do students realize that a motion can be the result of combining a number of motions? Do students differentiate all or some of the component motions affecting a body?

Questions of Interview Protocol

Questions: 14b, 14d, 15c in task two.

Question 14b: (Boat moving against the current) What different motions (or speeds) are affecting the boat? What are they?

Question 14d: (Boat moving with the current) What different motions (or speeds) are affecting the boat? What are they?
Question 15c: (Boat moving across the river)

What different motions are affecting the boat? What are they?

Table 4.13

List of Inferred Rules for Analysis of Components (A.C.)

Two inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"Analysis of components for the motion of the motor-boat on the river produces ..."

Inferred Rule AC-1: One component affecting it, this being either the motion due to the current or the motion due to the motor". (0F, 1M)

Inferred Rule AC-2: Two components: the one due to the current and one due to the engine". (9F, 9M)

\[ \chi^2 = 1.80 \text{ df } = 2 \text{ } p > .05; \text{ 1S (1M) was not assigned an inferred rule}. \]

Discussion of Results

Inferred Rule AC-1: The preconception expressed in this inferred rule refers to the fact that only one component affects the motion of the boat, it was not necessarily the same component in each situation. The responses of the only subject holding this conception best illustrate this inferred rule.
Hans, 17: (Q.14b) (Question not clear for him, which is repeated) still I don't understand ... (I: when the boat is going against the current, what is affecting its motion?) ... the current ... (I: anything else?) ... (long pause) ... (I: Recall that the engine is on) ... then, you have the current and the engine ..."

(Q.14d) "... just one motion ... (I: Which one?) ... just the motor".

(Q.15c) "... just two, the current and the motor"

Considering the responses to 14b (excluding the prompted response) and 14d, it is clear that this subject saw only one component in action upon the boat, although it seems to shift from the current to the motor. It is difficult to determine what effect occurred when the interviewer introduced the component due to the motor in 14b, but even after that hint the subject mentioned only one component affecting the boat in Q. 14d. On the other hand, the subject seemed pretty sure in his response to Q. 15c, he mentioned two components. It seems that this subject can clearly differentiate the two components only when they are not acting in a parallel way upon the boat, as soon as the components become parallel, the subject became confused and he did not see how the components interact. For these reasons the subject was classified as holding the inferred rule related to one component.

Inferred Rule AC-2: Subjects holding this conception differentiated the two components acting on the boat. That is, the speed of the boat due to the current, and the speed of the boat due to the motor. A high percent
of the sample, 90 percent or 18 of 20, held this conception. This result might be expected in that the two motions affecting the boat in the experimental situation are quite obvious. This task was chosen for that specific reason. Other tasks such as projectile motion are much more complex and might not be suitable for uncovering students' intuitive notions. Some subjects holding this preconception considered other components affecting the boat such as the wind and the "internal current of the river". To make the experimental situation easier for the students the interviewer told them that there was no wind and the river was flowing smoothly with no whirlpools or eddys. Under these circumstances there were only two components affecting the boat.

It was also taken for granted that subjects would assume that the boat acquires the speed of the current independently of the speed produced by the engine. There was no discussion or questions with regard to this issue.

Excerpts from interview data will show more clearly what has been expressed above.

Kevin, 15: (Q.14b) "... the motor is pushing that way (against the current) and the river this way ... There are two motions affecting the boat..."

(Q.14d) (Q. not asked of this subject)

(Q.15c) "Two, forward and sideway ... the motor pushing forward and the river this way" (along the direction of the current).
Wayne, 15: (Q.14b) "... I don't understand ... you mean the forces ... (I: Well, you could say that) ... there are two the motor and the stream".

(Q.14d) "... two ... the motor and the stream pushing and speeding up the boat".

(A.15c) "two ... the engine and the river".

Randy, 15: (Q.14b) "... the engine, the current going against it ... (I: anything else, suppose that there is no wind). ... not, that's all".

(Q.15c) "... the current and the motor".

For all of these subjects, the action of at least two components were affecting the boat. It was also observed that for some subjects the experimental situation depicted in the task is a complex one, this was shown when they introduced the internal currents, the waves, the wind, and the different currents present underneath the bridge. Obviously, if all the natural conditions of a river were considered in the task; i.e., wind, whirl pools, stronger current in the middle than in the shore; it would become a much more complex and difficult task.

An interesting finding regarding this vector characteristic was that a few subjects considered the resultant velocity of the two component velocities (velocity due to the motor and velocity due to the stream) as another independent component affecting the motor-boat when this moved across the river; this brought to three the number of components affecting the boat. These
same subjects responded that there were two components when the boat moved along the stream (upstream or downstream). This is quite understandable for subjects who have not yet been formally introduced to the composition of vector quantities. It is actually quite surprising that some subjects in the sample differentiated these three components; unfortunately these subjects were not asked if they saw any relationship for these three components.

The 'three component conception' and a conception discussed under inferred rule AC-1 seem to point to a similar factor, that the directions or the angle formed by the components make a difference. There is the possibility that subjects differentiate components more easily when they are not parallel. Excerpts from interview data of the three subjects holding the 'three component conception' will best illustrate it. These subjects identified the two components in Q. 14b and Q.14d.

Dale, 15: (Q.15c) "Three ... because the water goes this way, the boat goes that way (S showed the direction in which the boat was pointing - the motion due to the motor), and then it goes like that (S showed the motion along the actual path followed by the boat along the diagonal)...."

Ian, 15: (Q.15c) "... still three ... the propeller, the current and the boat being sideways makes it a lot harder".
Becky, 15 (Q.15c) "two or three ... it's going sideways because it's against the current, the current is pushing down and the motor is trying to push it across. (I: before you mentioned only two motions, the current and the engine, how this third one is being produced?) ... I think it is because the boat is going sideways of the current instead of going down".

The response of the three subjects to Q.15c clearly show the "third component".

Table 4.14
Rule-Model for Analysis of Components

Rule AC-I: "The analysis of component velocities for a moving body, which is actually being affected by two simultaneous and independent velocities, produces only one component".

Rule AC-II: "The analysis of component velocities for a moving body, which is actually being affected by two simultaneous and independent velocities, produces the two components".

4.3.8 Inferred Rules for Composition of Velocities (C.V.)

This vector characteristic can be considered as the reciprocal aspect of the previous one, the analysis of components. Since if a composition or combination of simultaneous velocities needs to be performed, the components or partial velocities must be known. With respect to composition of vector quantities two aspects
must be considered: how the magnitudes of the components will be combined (this will be called the interaction of magnitudes) and how the direction of the components will be combined (this will be the interaction of directions). The interaction of both magnitudes and directions will produce an apparent single motion.

It was expected that students would have definite conceptions about combination of component velocities, even when they could not differentiate the exact number of components affecting a moving body. However, it was not expected that students would have developed the formal method of composing vector quantities, including precise calculations of magnitude and direction. The purpose of this part of the study was to uncover the students' preconceptions about composition of velocities. What kind of intuitive notions about the interaction of motions do they have? How do they combine the magnitudes of velocities? How do they combine the directions? What kind of resultant path do they anticipate for given component motions?

It was realized that this is not an easy task. Particularly, if it is considered that one possible reference point or reference object is a moving body -- the water on the river. The experimental situation is facilitated if the discussion about composition of velocities is done from a fixed reference body, -- i.e., the bridge. If the subjects already possess the notion of the composition of velocities, they may attempt to use it when
trying to explain the interaction of velocities observed from the bridge.

Questions of Interview Protocol for CV
Questions: 14a, 14c, 15a, 15b, 16a, 16b, and 19.

Question 14a: (Motor-boat moving upstream against the current) How would you describe the motion from the bridge?

Question 14c: (Motor-boat moving downstream with the current). How would you describe the motion from the bridge?

Note: These two questions were used as an introduction to familiarize the subjects with the task. It was expected that all or most of the subjects would predict the actual resultant motion as was shown later in the film loop. Hence, the response to these questions were not considered in the analysis of the inferred rules for this characteristic.

Question 15a: (Motor-boat moving across the river). How would you describe the motion that you will see from the bridge? Please take the boat and show me the path you will see.

Question 15b: In this diagram of the river, could you draw that path? Could you explain that prediction?

Question 16a: (After seeing the boat crossing the river in the film) Was the speed of the boat that you saw in the film due to the water or due to the motor?
Question 16b: Suppose the speed of the current is 5 km/hr and the speed of boat due to the motor is 10 km/hr, what do you think is the speed of the boat with respect to you?

Question 19: Finally, suppose your friends want to cross the river from this dock on this side to the other dock just in front on the other side. Is there any way they could do it? Could you explain and draw the path that the boat will follow in this diagram?

The format of these rules is different from the previous ones. They have two aspects. The first aspect refers to the interaction of directions of the two component velocities. This was taken by considering the resultant path drawn by subjects; if they drew a path which shows a direction of motion different from the direction of each one of the components, it was assumed that these subjects were considering the interaction of directions. The second aspect refers to the interaction of magnitudes of the two component velocities. This was done by comparing the magnitude of the resultant velocity with the magnitudes of the components; if the magnitude of the resultant velocity was different from the magnitudes
of the components; it was assumed to be an interaction of magnitudes. The two aspects are separated by the conjunction and.

Three inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"There are two non-parallel velocities affecting simultaneously the motor-boat, the predicted resultant velocity shows that ..."

**Inferred Rule CV-1:** Its direction corresponds to the direction of the component with greater magnitude, showing with this no interaction of directions; and the magnitude corresponds to the magnitude of the component with greater magnitude, showing with this no interaction of magnitude". (1F,0M).

**Inferred Rule CV-2:** Its direction corresponds to one in between of those of the components, showing with this an interaction of directions; and its magnitude corresponds to the magnitude of the component with greater magnitude, showing with this no interaction of magnitudes". (3F,5M)

**Inferred Rule CV-3:** Its direction corresponds to one in between those of the components, showing with this an interaction of directions; and its magnitude
corresponds to one* different from those of the components showing with this an interaction of magnitudes". (5F,6M) 

\[
\chi^2 = 1.40, \text{ df } = 2 \quad p > .05
\]

Discussion of Rules:

Inferred Rule CV-1: Only one subject among the 20 held this conception. She did not see any kind of interactions between directions or magnitudes. Her responses to the questions related to the vector characteristic will best demonstrate her conception.

Suzzanne, 17: (Q.14a) "... not very fast (I: Why?) because it's going against it and the water it is pulling back".

(Q.14c) "... faster because the water pushes it a bit too"

(Q.15a) "... that the water is coming toward me, to go across it will be a little harder because the water pushes it from aside but it could make it across (she meant: toward the other dock).

(Q.15b) stream \[\downarrow\] yeh they will reach the other side"

Note: Questions 16 and 19 were not asked because of her response to question 15.

* Several kinds of values were given by the magnitude of the resultant: values in between the two magnitudes (e.g., 7.5 km/hr), values in between the larger magnitude and the arithmetic addition of the magnitudes (e.g., 12.5 km/hr), and values equal to the arithmetic addition of the magnitudes.
Her responses showed that there was not much interaction between the directions of the two motions affecting the boat. It seems that she believed that the speed of the boat due to the motor was large enough to totally cancel the speed of the boat due to the current. This does not necessarily mean that the resultant magnitude is equal to the speed of the boat due to the engine; that is not clear from her responses. But because it seems that she was so sure about her view of the interaction, she was classified under inferred rule CV-1.

Inferred Rule CV-2: The subjects holding this conception made clear through their responses that they were considering only the interaction of directions. Their answers did not show a clear differentiation between the magnitude of the resultant velocity and the magnitudes of the components. Eight of the 20 subjects possessed this view, which shows that it is a common view among grade 10 students. These subjects might have seen the interference between the speed due to the motor and the speed due to the current, but they did not consider that the resultant of the two can be a "mixed value". They saw this interference as a "sort of fighting" between the two speeds with one of them as a winner, the greater speed being the winner. Excerpts from interview data follow.
Russ, 15: (Q.14a) "it will move slower ... (I: Why do you think so?) ... because it is fighting the current ..."

(Q.14c) "... it will be going faster ..."

(Q.15a) "it will start going sideways because the current goes this way"

(Q.15b) (First, he drew this path)

I: Remember that they only want to cross the river?) ... oh, then it is a straight line:

(Q.16a) (Not asked to this subject)

(Q.16b) "... it will be 10 km/hr because that's how fast can go"

(Q.19) "... yeh, just cross over ... it will go down first, then they turn up to the dock. ( )

Hans, 17: (Q.14a) "... slower ... (I: Why do you think so?) ... because the current is pushing it back"

(Q.14b) "... faster ... because the boat is going with the stream ..."

(Q.15a, 15b) "... the boat moving ... (I: What will be the path?) ... straight across ... the current will push it down a bit ... (He drew this path.) I: remember that they only want to cross the river?) ... oh they will go to this point (a point certain distance down the dock) ... because the current against the side, it'll probably come down (He drew this path: I: Is it a straight line?) ... it's probably a curve a little bit".

(Q.16a) "... due to the motor"
(Q.16b) "... 10 km/hr because the speed of the boat due to the engine is not being affected much"

(Q.19) "... they go up a bit so the current push them down, \( \downarrow \) (I: How much do they have to go up?) ... not too far..."

Kevin, 15: (Q.14a) "... slower ..."

(Q.14b) (not asked to this subject)

(Q.15a) "... the river will push them down the way

(Q.15b) "\[\]

(Q.16a) "... that speed (observed from the bridge) is due to the engine"

(Q.16b) (not asked to this subject)

(Q.19) "they have to sort of turn in this way so the current pushing them keeps them in straight line. It looks like they are going that way \( \downarrow \) but the current wouldn't let them to go that way so they will be finishing going in straight line. They aim the boat that way (\( \nearrow \)) but the current keeps the boat going straight across.

For these three subjects, the interaction of directions was clearly observed in their responses, particularly through the answers for Q.15b and Q.19. On the other hand, the responses for Q.16 show no interaction of magnitudes.

Inferred Rule CV-3: Subjects classified under this inferred rule expressed both kinds of interactions: directions and magnitudes. The resultant velocity has
direction and magnitude different from those of the components. More than 50 percent of the sample, 11 of 20, held this preconception. Even when none of these subjects made explicit use of precise methods to find the actual direction and the actual magnitude of the resultant velocity, the basic understanding seems to be present in a large percentage of students.

In the case of the boat just crossing the river, an interesting view observed was that some subjects believed that the result of the interaction of magnitudes will produce a magnitude whose numerical value will be in between the magnitudes of both components (i.e., 5 km/hr and 10 km/hr are magnitudes of components, 7 km/hr magnitude of resultant). Other subjects believed that the magnitude of the resultant will be greater than the larger magnitude of the components.

Excerpts from interview data follows.

Dale, 15: (Q.14a) "... if the engine is running and it has a speed of 5 miles/hr and the river also has 5 miles/hr, the boat won't move it will stay ... if the boat could go 10 miles/hr, the speed (of the boat with respect to bridge) will be 5 miles/hr."

(Q.14c) "you see the boat goes a lot faster ... the water will be moving and it will be moving through the water ... (I: Why will it move faster?) ... because it's moving with the water, without the engine it is already going ... it has the speed of the water plus the speed of the boat".

(Q.15a) "it'll go in straight line and the water will be pushing it like that (He took
the toy boat and showed the drifting due to the current) ...

(Q.15b) 

Note: Question 16b was asked before Q. 16a.

(Q.16b) "... it goes 7 or 8 miles/hr... (I: Why don't you add them?) ... because the water goes this way (↑) and the boat that way (→) ... I think it'll go 15 but in that angle (↗) ... I am not too sure about that"

(Q.19) "... yeh, they can head the boat up this way a bit and the current will push them down a bit. The path is a slight arc: ...
... it could be a straight line but I doubt it ... the boat will be heading that way (↗) and the river will be pushing that way (↑) so it becomes even".

Tracy,15:(Q.14a) "... it's moving slower because it's moving against the current"

(Q.14b) "... same speed as the current or more because it depends of the power of the motor..."

(Q.15a) "... it'll go straight but down a bit ... it'll go this way (↗) with the current pushing down and here (P in drawing: ) they back up and go very slow to the dock".

(Q.15b) (I: please, draw the path if they only want to cross the river)

(Q.16a) "It's both, it's a kind of combination of the current and the engine ... the engine push to go forward and the current push to go down ... actually the speed (of boat) will be slower than if it were going with the current ... the speed is combined pushing it in a general direction will be with the motor and the current".

(Q.16b) "... it'll go about 7 miles/hr., it'll slow down a bit".

(Q.19) (She proposed two methods) 
"... the slower and logical way and the quicker way
These subjects felt pretty sure that the magnitude and direction of the resultant velocity were different from the correspondent values of the components. Both of these subjects believed that the magnitude of the resultant was smaller than the speed of the boat due to the motor. They said that that speed of the boat slows down for the boat has to "fight" the speed of the current.

In summary, it can be said that the paths drawn by the subjects reveal their conceptions about the results of interaction of motions. At least, it is possible to detect the direction of the resultant motion. It does not explicitly show the magnitude of the resultant or ways by which it can be estimated; it seems necessary to ask further questions to find out if students have preconceived ideas to more precisely estimate the magnitude of the resultant. However, the paths drawn still give sufficient information to provide some interpretative conjectures about the students' preconceptions regarding the interaction of motions.

It was decided to report the different paths drawn by subjects in the sample. Because most subjects tackled the problems of just crossing the river (question 15) and reaching the other dock (question 19) in different ways; both kinds of strategies are presented separately.

The table consists of drawing of the type of path and the number of subjects who used that path.
Table 4.16
(Question 15b: Motor-Boat Moving Across the River)

<table>
<thead>
<tr>
<th>Path</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Path" /></td>
<td>1</td>
</tr>
<tr>
<td><img src="image2" alt="Path" /></td>
<td>15</td>
</tr>
<tr>
<td><img src="image3" alt="Path" /></td>
<td>1</td>
</tr>
<tr>
<td><img src="image4" alt="Path" /></td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
</tr>
</tbody>
</table>

Note: When question 15b first asked 3 Ss drew this path ![Path](image5). After mentioning to them that people on the boat only wanted to cross the river, they drew this ![Path](image6).
Table 4.17

(Question 19: Motor-Boat Trying to Reach Dock 2)
... Could you ... Draw the Path that the Boat Will follow in this Diagram?

<table>
<thead>
<tr>
<th>Path</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram 1" /></td>
<td>4</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram 2" /></td>
<td>1</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram 3" /></td>
<td>9</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram 4" /></td>
<td>3</td>
</tr>
<tr>
<td><img src="image5" alt="Diagram 5" /></td>
<td>2</td>
</tr>
<tr>
<td>No method</td>
<td><img src="image6" alt="Fraction" /></td>
</tr>
</tbody>
</table>

Table 4.18

Rule-Model for Composition of Velocities

**Rule CV-I:** "The composition of two non-parallel velocities, which are simultaneously affecting a moving body, produces a resultant velocity whose magnitude and direction are those of the component velocity with greater magnitude".

**Rule CV-II:** "The composition of two non-parallel velocities, which are simultaneously affecting a moving body, produces a resultant velocity whose magnitude
is that of the component velocity with greater magnitude and whose direction is in between the directions of the two component velocities".

**Rule CV-III:** "The composition of two non-parallel velocities, which are simultaneously affecting a moving body, produces a resultant whose magnitude* is different from those of the components and whose direction is in between the directions of the component velocities".

4.3.9 **Inferred Rules for Independence of Magnitudes of Components (I.M.C.)**

This vector characteristic is one of the characteristics that best fits with the condition of being implicit. In fact, curriculum developers, textbooks, and science teachers very rarely mention the independence condition when introducing composition of vector quantities.

This section refers to the independence of the magnitudes of the components. In the present case, one magnitude is the one corresponding to the velocity of the boat due to the motor and measured with respect to the water; and the other magnitude is the one corresponding to the velocity of the boat due to the current, (which actually corresponds to the magnitude of the velocity of the current) and measured with respect to the shore or bridge. These two velocities are simultaneously affecting

*See note after inferred rule CV-3.
the motor-boat, and their combination produces a velocity of the boat which is different from the component velocities when observed, for instance, from the bridge. The important aspect to consider in this interaction of velocities is that the magnitudes of the component velocities expressed with respect to their reference bodies maintain their values. That is, despite the fact that a "new" magnitude is produced as a result of the interaction of the magnitudes of components, each component measured with respect to a specific reference point conserves its magnitude. In other words, this means that the magnitudes of components are independent among themselves and from the resultant magnitude.

With regard to this vector characteristic, independence of magnitudes, it was thought that it would be sufficient to ask only for the constancy or independence of magnitude of one of the components. Recall that in task two there are two independent components affecting the boat (velocity of boat due to the motor and velocity of boat due to the current). The velocity due to the motor was the one considered. It was assumed that if subjects realized the independence or dependence of one of the components from the other component; that would show their view about this vector characteristic. Furthermore, a question about the effect of the speed of the boat due to the motor upon the speed of the boat due to the current was included in the pilot study; all eight
subjects held the view that the speed due to the current was always the same because the boat was being carried by the river and the speed of the river was always the same.

In summary, the purpose of including this vector characteristic was to find out if subjects in the sample whether or not would realize the independence of magnitude of one component (i.e., velocity due to the motor) from the other component.

Questions of Interview Protocol

Questions: 18

Question 18: In the case of the boat just crossing the river: Is the speed of the boat due to the motor (or with respect to water) affected or changed by the speed of the boat due to the current? How?

Table 4.19

List of Inferred Rules for Independence of Magnitudes of Components (I.M.C.)

Four inferred rules were obtained for this vector characteristic, which have the following common opening sentence:

"There are two component velocities affecting simultaneously the motor-boat, the magnitude of the component due to the motor is ..."

Inferred Rule IMC-1: changed because of the interaction with the component due to the current". (5F, 4M)
Inferred Rule IMC-2: decreased because of the interaction with the component due to the current". (1F, 2M)

Inferred Rule IMC-3: increased because of the interaction with the component due to the current". (2F, 1M)

Inferred Rule IMC-4: not changed because of the interaction with the component due to the current". (1F, 3M)

\( \chi^2 = 1.73 \) \( df = 3 \) \( p > .05; 1S(1M), \) was not asked the question)

Discussion of Results

Inferred Rule IMC-1: Subjects holding this conception mentioned that the speed (speed is used here as a synonym of magnitude of velocity) of the boat due to the motor was affected, yet they did not elaborate whether the speed was decreased or increased. A high proportion of subjects held this view, 9 of 19 (one subject was not asked Q.18), which shows that it was a common view. Furthermore, if subjects were divided according to dependence among the magnitudes (considering preconceptions in inferred rules IMC-1, IMC-2, and IMC-3 as one package) or independence among the magnitudes, almost 80 percent (15 of 19) would be in the dependence category.

Excerpts from the interview data follow

Sandy, 15: (Q.18 ) "... yeh, because the river wants to push the boat with it and go in the same way ... and if the boat is fighting against it ... then it has to ..."
Wayne, 15: (Q.18) "it is changed because the river is pushing against it. If it were calm water, there wasn't anything to stop, but the current is pushing it, it will slow down or speed up".

Inferred Rule IMC-2: These subjects believed that as a result of the interaction of velocities, the speed of the boat due to the motor was decreased or was slowed down. Only 3 of 19 held this view; excerpts of interview data for two of them follow.

Tracy, 15: (Q.18) "yes, it will slow down because the current is a kind of pulling down this way (^_^) and the motor is fighting with it to go to the shore, then, it is slowing down".

Hans, 17: (Q.18) "yes ... it's slowing down because the current is pushing aside".

Inferred Rule IMC-3: These subjects believed that as a result of the interaction of velocities, the speed of the boat due to the motor was increased or speeded up. Only 3 of 19 held this view; excerpts of interview data for two of them follow.

Dale, 15: (Q.18) "yes, affected ... it makes the boat goes faster ... it (the current) will affect the boat but not the river (he meant the speed of the boat due to the motor does not affect the speed of the river)."
Julie, 15: (Q.18) "yeh, a little bit (I: Why?) ... if it is heading this way the current is going to push it down then it's going to help the speed".

Inferred Rule IMC-4: Subjects classified under this inferred rule held the view that the speed of the boat due to the current does not affect the speed due to the motor. These subjects have grasped the independence of magnitudes of velocities in the situation where these velocities interact among themselves. A low percentage of subjects in the sample (4 of 19) held or understood this view; even though it was a low proportion, this result shows that some students seem to have an intuitive grasp of a sophisticated physics concept.

Excerpts from interview data follow.

Lisa, 15: (Q.18) "... you mean the total speed? (I: No, the speed of the boat due to the engine) ... oh, it stays the same ... just the water pushing it ... (I: What do you mean by the total speed?) ... the speed of the water and the speed of the boat
Kevin, 15: (Q.18) "No, I don't think so if it's going this way (crossing the river) ... only the direction of the boat (he meant direction of motion of boat) is being affected.

Table 4.20
Rule-Model for IMC

**Rule IMC-I**: "There are two simultaneous component velocities affecting a body; the magnitude of the component with greater magnitude changes due to the interaction with the other one".

**Rule IMC-II**: "There are two simultaneous component velocities affecting a body; the magnitude of the component with greater magnitude decreases due to the interaction with the other one".

**Rule IMC-III**: "There are two simultaneous component velocities affecting a body; the magnitude of the component with greater magnitude increases due to the interaction with the other one".

**Rule IMC-IV**: "There are two simultaneous component velocities affecting a body; the magnitude of the component with greater magnitude remains constant during the interaction".

Rules II and III are actually very contextual; it cannot be said that these will be found in other contexts. Rules I and IV really represent the two general cases: either subjects observe the independence of magnitudes for interacting velocities or they do not observe it.
4.3.10 Inferred Rules for Simultaneity of Component Velocities (S.C.)

The notion of simultaneity of components is an important one in understanding physical phenomena. During the pilot data collection it was found that some subjects had the belief that one of the components affects the boat before the other, and it was the one with greater magnitude. In task two, the velocity with greater magnitude corresponds to the one due to the motor; then, this one would affect the boat before the velocity due to the current. Due to this finding, it was decided to maintain the simultaneity aspect as one of the implicit vector characteristics.

The composition of components (i.e., vectorial composition of velocities, or forces) is taught in grade 11, but generally without mentioning the simultaneity aspect; and the same operation of vectorial addition is used to obtain the resultant of consecutive displacements. Usually, the addition of consecutive displacements is taught first; later the same operation is used to compare simultaneous displacements*. It seems to this author that, the difference between consecutive and simultaneous cases should be made explicitly. However, previous to

*They are obtained through the use of the equation \( v \Delta t \), (velocity times interval of time) which is used according to the number of components.
doing that it may be best to find out if students, before formal instruction, can discern the difference between the two cases.

Thus the purpose of this part of the study was to uncover the students' intuitive notions about simultaneity of interacting velocities upon a body. Did students realize that the two velocities affecting the motor-boat (in task two) occur at the same time?

Questions of Interview Protocol

After trying several techniques for indirect and alternative ways of finding out students' beliefs about simultaneity (i.e., do the two velocities affect the boat at the same time?); it was decided that the most effective approach was to ask the students the question directly. Question 17 (Task Two): How many motions are affecting the motor-boat when it was crossing the river? (If S mentions the two velocities: velocity of boat due to the engine and the velocity due to the current) Were these motions affecting the boat together or one after the other? Why do you think so?

Table 4.21

Inferred Rules for Simultaneity of Components

The author did not find a large variation of views about simultaneity of components in the sample of subjects.
In fact, it seemed, from the data collected, that students at grade 10 (ages from 15 to 17) have already grasped the conception of simultaneity of components since practically all subjects, 15 of 17 (three subjects were not asked this last question because it was evident that they were exhaustive) held quite firmly that both velocities are affecting the motor-boat together or simultaneously. This finding was sort of surprising in view of the pilot data which showed the existence of some ideas about sequential action of motions. One possible explanation is that the sample of the pilot data included two students from each of grades 8 and 9 (ages from 13 to 15) as well as two from each of grades 10 and 11. It might be that students in grades 8 and 9 have not yet grasped the simultaneity characteristic of certain physical phenomena; however, this cannot be generalized because the sample used in the pilot study was small.

Whatever may be the explanation of the finding of not much variance in the results, the fact is that most of the subjects, after differentiating the two variables, held the view that the two velocities affect it at the same time. Only two subjects of 17 thought that one velocity affects the boat before the other.

Two inferred rules were then obtained for this vector characteristic, which have the following common opening sentence:
"The single apparent movement of the motor-boat is the result of two velocities that are affecting it ...

Inferred Rule SC-1: one after the other". (0F,2M)

Inferred Rule SC-2: At the same time" (9F,6M)

\[\chi^2 = 2.4 \text{ df } = 1 \quad p > .05; \quad 3 \text{ Ss (3M) were not asked the question).}\]

Discussion of Results:

Inferred Rule SC-1: Only two subjects of 17 seemed to hold the view that the two velocities do not act simultaneously. However, their views were expressed in tentative language and when one was asked the question a second time (because the interviewer acted as he did not hear him very well the first time) he changed his mind as is shown in the second excerpt below.

Ian, 15: (Q.17) "... probably the river, the river is trying to push it down at the same time than the propeller is pushing forward ... the river affects it first.

Hans, 17:(Q.17): "... one after the other ... they are not going with each other. If they will be going with each other they will go this way (in the direction of the current). They are two different motions ...

Because his response was not clearly heard by the interviewer the question was repeated. His "second thought" answer was:

"... probably at the same time (I: What do you mean?) ... the motor is pushing the boat and the current is pushing at the side ... they happen at the same time ... (I: Before you were saying something different, what do you actually think?) ... at the same time".
Notice that these two subjects made use of the term probably. Both were not sure or did not have a definite view about the problematic situation.

**Inferred Rule SC-2:** The following are some excerpts from interview data of students holding the simultaneity conception.

Lisa, 15: (Q.17) "... when it's going down the stream (boat moving downstream) it's together because the boat and river go in the same way, then, you add them. When it's going up the river ... you're taking it off (he meant subtracts speed of river from speed of boat due to engine) (I: and when it is crossing?) ... the river is affecting it ... the river is pushing it down ... together ... I think".

Preston, 16: (Q.17) "more likely together ... the engine pushing forward at the same time than the current pushing down so it'll be a very slight angle, otherwise it'll go in steps ( ) if it were one on a time.

Kevin, 15: (Q.17) "About the same time ... because the motor keeps going and the river keeps going so they have to affect the boat at the same time"

Becky, 15: (Q.17) "... together I think (I: Why do you think it's together?) ... because one (the engine?) is pulling it and one (the current?) is pushing it the other way ... it's together".

For all these subjects the two velocities were affecting the boat together. This claim can be made even when some subjects made use of terms such as: more likely, about the same time, I think, which could make the subjects' views not very stable. However, their explanations show
a steady conviction.

### Table 4.22

**Rule-Model of Simultaneity of Component Velocities**

**Rule SC-I:** "The single apparent movement of a body is the result of two component velocities that are affecting it sequentially".

**Rule SC-II:** "The single apparent movement of a body is the result of two component velocities that are affecting it simultaneously".

### 4.3.11 Subjects' Preconceptions About Perspective

In this study the importance of understanding the need of referring the locations of objects to points and frames of reference has been emphasized. Findings of the present study have shown that individual subjects use a coordinate system oriented around themselves when choosing reference points. It was decided to find out how these subjects would describe a particular location from several different reference points. Do they have a different perspective of something that is described from different positions or angles?

With regard to the above question, Piaget (1956) has found that young children (until 7 years of age) have an egocentric view of seeing things. They believe that if they are observing an object from a particular location, other subjects located in other places must see it in the
same way that they see it. According to Piaget, it is not until children are in the range of 10-12 years of age or more that they develop a more "progressive discrimination and coordination of perspectives". That is, they accept that perspectives seen from other points of view are different from theirs. Children at this stage are also able to coordinate different viewpoints. Piaget has said that a child will master simple perspective relations when he/she is able to coordinate a number of possible points of view.

Although the subjects of this study are well over 12 years of age, it was thought important to check their conceptions of perspective using a situation different from that used by Piaget ('The three mountain task' in The Child's Conception of Space, 1956). It was reasoned that if subjects still have difficulty in understanding perspective relations and points of view coordinations; then this could be one of the sources of difficulty in understanding kinematic vector quantities.

It should be emphasized that this aspect of perspective was not considered as an implicit vector characteristic; its inclusion was rather to check Piaget's findings in subjects older than his sample.

Questions of Interview Protocol for Perspectives
Questions: 2a, 2b, 6d in task one.
Question 2a; (If S mentions one or two places as reference points as response to Q.1) Could you describe that location (first fishing spot) from other places as well? What places?
Question 2b: (If S mentions several places or bodies as reference points). Will the descriptions of the locations of this fishing spot done from other places be the same or different?

Question 6d: (After S has drawn the path followed by the boat). If a friend of yours is by the tree and draws the path he sees for the boat can you compare both drawings, yours and his, later. Do you expect to find them the same or different?

Discussion of Results: The findings regarding this part of the study were not analyzed in terms of inferred rules because the notion of perspective was not considered here as an implicit vector characteristic.

Results were reported in terms of number of subjects holding different views and excerpts from interview data showing those views.

Among the 20 subjects, 18 did not have much problem in describing the location of the first fishing spot from other places; this was shown in their responses to question 2a. Some of them even said that a description can be done from anywhere. This was not a surprising result considering the subjects' ages, but it was a surprise to find 5 of 14 subjects (question 2b and 6d was asked these 14 subjects) did not appear to understand question 2b. The other nine subjects did mention that the descriptions were different because the spot was being observed from different points
of view. This would suggest that subjects at these ages, 15 to 17, do not appear to have an egocentric viewpoint, and are able to discriminate perspectives of an object observed from different reference points. Excerpts from interview data will illustrate the above points.

Larry, 16: (2b) "... they will different ... if you are standing in different spots you will be seeing the same point but the descriptions will be different"

Kevin, 15: (2b) "it will be different, you are looking at it from a different angle

Lori-Ann, 15: (2b) "it will be different because it is from different places, it is the same point (fishing spot) but it would have to be different because from this place (dock) will be farther than from there (wharf)"

Randy, 15: (2b) "probably they will be different (I: Why do you think so?) ... you probably have to go either farther from the wharf, or from the tree, or you might go in different angle from the shore than it would from the wharf"

From these excerpts it was clear the these subjects did not possess an egocentric view of perspectives of a body seen from different reference points.

While analyzing the transcript data the author realized that a possible reason why 5 of the 14 subjects did not understand the intention of Q.2b, was that these subjects seemed to interpret the question as a request to describe a method for locating the object. That is, the method should consider the fact of using distances or angles,
they believed that if they have to describe a location from different places, they must use distance, angle or both. Again, excerpts from data will explain this point.

Suzzane, 17: (2b) "they (descriptions) are similar in a way, they (distances) are all straight in front of the wharf ... like the buoy (buoy located it the first fishing spot) is in front of the wharf.

Wayne, 15: (2b) "... it will be the same (I: Why do you think so?) ... it doesn't really matter where you are, you could be anywhere it wouldn't matter much difference (he showed the distances from different places to the spot)."

When asked about the path observed from different reference points (responses for question 6d), 8 of 14 subjects said that the path seen from different places should be the same. Considering the range of ages of subjects it was expected that a larger percentage of subjects would respond with the correct answer — that is, the path is the same no matter where the observer is. The unexpected result was to find that a few subjects, 4 of 14, expressed that the path would be different if observed from different places; and finally two subjects did not understand the question.

Excerpts from the interview data will show the views of the group of subjects who understood that the path was independent of the observer.
Steve, 15: (6d) "it's about the same ... because he (the observer by the tree) will be able to judge the distances of the spots near him better than me (on the dock) ... then, it will be only a little bit different".

Kevin, 15: (6d) "Generally the same because they (friends in the boat) are going to the same spot ... she (the observer by the tree) will be able to see more exactly the spots closer to her, near me I'll be able to see it more accurate".

Tracy, 15: (6d) "it will not be exactly but they will be similar ... first of all, he (the observer by the tree) is looking at from a different point of view and it will be difficult just from memory to remember and obtain exactly the same ... but essentially they should be the same".

Essentially these subjects possessed the conception that the path must be the same when observed from different places; the drawings of the paths (made by the students) might be slightly different due to the difficulties of observers to locate instantaneous locations. The following excerpts are from subjects who believed that the paths look different from different viewing positions.

Lori-Ann, 15: (6d) "They probably will be different because she (the observer by the tree) is looking at from different angle ... when the boat is leaving here, the boat is going away from me but it'll be coming to her ... she will get this angle which I don't get ... paths will probably be different because two different people see different things and also different paths"

Becky, 15: (6d) "they (the paths) will be different because ... it's not going to be exactly the same because everybody sees it different ... because they are standing in different position ..."
It may be that these subjects were not sure about their responses for several of them used the term "probably" in their explanations. While some elements of uncertainty were also present in the previous group of subjects, the important difference was the former group of subjects stated clearly that paths must really be the same, while the latter subjects basically stated that the paths would be different.

4.4. Two New Vector Characteristics (Items 11 and 12 of LIVC)

While analyzing the interview data it appeared to this author that two other implicit vector characteristics could be considered. One of those was not anticipated at all through the rational task analysis performed on the three vector quantities. As was mentioned earlier in Section 1.2 (Chapter One), the list of implicit vector characteristics is not exhaustive, which means that other characteristics that are not explicitly present in the definitions can be identified. The two extra vector characteristics were actually uncovered by empirically analyzing the subjects' responses to questions related to the following vector characteristics: frame of reference, displacement, composition of velocities, and independence of magnitudes of components.
The two new vector characteristics were called: Independence of Locations from Path (ILP) and Independence of Directions of Components (IDC). How each of these was identified is discussed in the next two subsections.

4.4.1 Independence of Locations from Path (ILP)

Actually, this characteristic was originally thought to be an implicit vector characteristic. Since, the pilot data showed that it was obvious to the subjects it was not considered in the first phase of the study. Later, however, while analyzing the data in this first phase, it was evident that some subjects seemed to imply through their responses that a particular location was dependent on the path. Based on this finding, it was thought to reconsider this vector characteristic in the second phase.

The importance of understanding this independent aspect to later comprehend certain vector quantities is explained below.

For physicists a path is the sequence of an infinite number of points or locations through which a particle (or body) passes. The location of each one of these points can be described as a set of coordinates with respect to a determined frame of reference (i.e., a coordinate system). One important aspect is that the location of each point can be determined independently, of the path that it belongs to. This means that the quantitative description of a
location (set of coordinates or magnitude and direction) is independent of the path followed by the particle. The fact that instantaneous locations or stop points (e.g., rowboats stop at fishing spots) of a moving body are independent of the path followed (by it) is important to understand kinematic vector quantities such as vector position, displacement, and velocity. Usually, physics textbooks and science teachers do not emphasize this independence aspect. For all these reasons, it was decided to include this vector characteristic in the second phase of the study.

4.4.2 Independence of Directions of Components (IDC)

The main factor in identifying this vector characteristic was given by a number of subjects answering question 18. Subjects classified under inferred rule IMC-4 said that the speed of the boat due to the motor does not change when interacting with the speed of the current, but its direction is affected. The interviewer pursued this view with further questions such as: Will the boat reach the other side of the river heading this way: \( \rightarrow \) or this way: \( \rightarrow \) (Interviewer showed the two cases with the boat toy on the cardboard model). The responses to this question illustrated whether a subject had or had not grasped the independence of directions of components. There was variance between the responses of subjects who were asked this question. It was then decided to
include the independence of directions as another implicit vector characteristic in the second phase of the study.

The importance of considering this characteristic (IDC) relates to the complete understanding of composition of vector quantities as explained below. This vector characteristic in a sense complements the one about independence of magnitudes of components. The two characteristics completely describe the independence of components. This means that to actually understand the independence property of vector quantities, subjects must understand both the independence of magnitudes and directions of interacting components. This and the subjects' suggestion given through responses to question 18 were the main reasons in deciding to include this vector characteristic (IDC) in the second phase of the study.

4.5 **Summary of Inferred Rules for Each Subject in the Interview Sample**

Table 4.23 is a summary table of data collected in phase one. It shows the inferred rules for each subject about each one of the vector characteristics. Each row corresponds to one subject and shows the particular inferred rule for each vector characteristic. In addition to the name of the subject, the first column also includes the following information: sex (F or M), age in years, and
Table 4.23
Table of Inferred Rules for Each Subject in the Sample About Each Vector Characteristic

<table>
<thead>
<tr>
<th>Subjects</th>
<th>RPS 1 2 3 NR</th>
<th>PR 1 2 3 4 5 NR</th>
<th>D 1 2 3 4 5 NR</th>
<th>AD 1 2 3 NR</th>
<th>SVP 1 2 3 4 5 NR</th>
<th>RPM 1 2 3 4 NR</th>
<th>AC 1 2 3 NR</th>
<th>CV 1 2 3 4 NR</th>
<th>T MC 1 2 3 4 NR</th>
<th>SC 1 2 NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dale</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Lisa</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Suzanne</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 17, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Dwayne</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 16, --</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Larry</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 16, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Russ</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Sandy</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Tracy</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Ian</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Julie</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Preston</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 16, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Hans</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 17, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Becky</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Steve</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Kevin</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, --</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Wayne</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, YN</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Lori</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YN</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Lori Ann</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, YN</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Randy</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>M, 15, YY</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>Kelley</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td>F, 15, NN</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 3 4 3 0</td>
<td>3 3 1 8 2 2</td>
<td>2 3 1 10 3 1</td>
<td>2 1 2 4 2</td>
<td>7 4 8 1</td>
<td>3 3 10 3 1</td>
<td>1 1 8 1</td>
<td>1 8 1 6 0</td>
<td>9 3 3 4 1</td>
<td>2 1 5 6</td>
</tr>
</tbody>
</table>
experience of the subject with boats. Regarding this latter aspect, the first letter (Y—yes or N—no) refers to experience with boats on lakes; and the second (Y or N) to experience with boats on rivers.

There are 10 columns in the table, each one corresponds to each one of the vector characteristic; and each column is subdivided in other columns corresponding to the number of inferred rules found for a vector characteristic in the sample of subjects. NR. means no inferred rule found for a vector characteristic. If a cell in the table does not show a dot (•), it means that no question regarding that vector characteristic were asked to that subject.

4.6 Three Types of Preconceptions

The list of inferred rules for each vector characteristic contains a summary of all the conceptions held by the subjects in the sample regarding that characteristic (see Section 4.3). As it is shown in each list of inferred rules the level of differentiation of these conceptions range from descriptive or direct reporting of the subject's observations to more elaborate and abstract ones; in between these two extremes are conceptions that contain both descriptive and abstract features.

On the other hand, the subject matter that this study deals with can also be categorized from simple to
complex. Vector quantities can be considered more complex than scalar quantities because the former entails relationships between at least two variables; with these relationships being described by relatively complex mathematical operations.

Considering both the level of differentiation of the inferred rules and that vector quantities are more complex than scalar quantities, a way of comparing the complexity of content of the inferred rules for each vector characteristic was created. This consisted in categorizing the content of the inferred rules in one of the following three types: scalar, transitional, or vectorial.

Two criteria were used to categorize the inferred rules. The first criterion considered the number of variables contained in the inferred rule: to be categorized as a 'scalar type' of inferred rule, it must contain preconceptions with at least one variable, which can be qualitative or quantitative in nature; to be categorized as a 'vectorial type of inferred rule, it must contain two or more variables, at least one of these variables must be quantitative in nature. Inferred rules with preconceptions that fall in between the two previously discussed types are considered 'transitional type' inferred rules. They contain two or more variables but are qualitative in nature. This approach can be easily applied to the following vector characteristics (see Table 1.1 in Chapter One): 2, 4, 5, 6, 8 and 9.
A second criterion was used to categorize the inferred rules of vector characteristics that could not easily be analyzed in terms of the number of variables. The approach essentially consisted of comparing the content of each inferred rule to a physicist's perspective of dealing with that specific vector characteristic. If the content of the inferred rule conveyed or was very close to the physicist's perspective, the inferred rule was categorized as 'vectorial'. Otherwise it was categorized as 'scalar' or 'transitional', depending on how closely it resembled the physicist's view. For instance, to describe locations of several bodies or instantaneous locations of a moving physicists use a unique body, or point as a reference; inferred rule RPS-3 conveys this view and it was categorized as 'vectorial', whereas RPS-1 and RPS-2 were categorized as scalar and transitional respectively. This criterion then was used to categorize the inferred rules corresponding to the following vector characteristics: 1, 3, 7, and 10. (See Table 1.1 in Chapter One).

To describe the categories of the inferred rules, the following abbreviations were used:

'Scalar type' of preconception = S
'Transitional type' of preconception = T
'Vectorial type' of preconception = V

For most vector characteristics, the first inferred rule was clearly categorized as scalar; for some vector characteristics, the first two inferred rules were classified
in the scalar category, leaving the rest minus the last inferred rule to be categorized as transitional. For most vector characteristics, the only exception being Reference Point for Moving Objects, the last inferred rule was categorized as vectorial... even though in some cases it was not a complete vectorial conception from a physicist's perspective. An example will better explain the task of categorization of inferred rules into the three type of conceptions.

4.6.1 Example of the Categorization of Inferred Rules

The vector characteristic chosen to develop this example was Frame of Reference for Stationary Bodies. Five inferred rules were identified for this characteristic. (See Section 4.3.2). The first two inferred rules (FR-1 and FR-2) refer only to one variable as a coordinate; each inferred rule having a different variable. Since both inferred rules refer to only one coordinate they were considered as 'scalar type' of conception and coded as S. The next two inferred rules (FR-3 and FR-4) included two variables as coordinates. For one inferred rule (FR-3) only the direction was quantitative, and for the other (FR-4) only the distance was quantitative. Considering these aspects, in addition to the selection of different reference points for each coordinate in both inferred rules, they were considered as 'transitional type' of conception and coded as T. The last inferred
rule (FR-5) includes two variables as coordinates. Even though one of them is qualitative in nature, the fact that one unique reference point was selected for both coordinates resulted in this inferred rule being categorized as a 'vectorial type' of conception. Similar reasoning, as illustrated in this example, was used to categorize the inferred rules into the three type of conceptions for all other vector characteristics.

4.6.2 Categorization of Inferred Rules into the Three Types

The categorization of the inferred rules of each vector characteristic into the three types is shown in Table 4.2.4. This table has three columns. The first containing the 10 vector characteristic considered in phase one. The second column contains all the inferred rules found for each vector characteristic. And the third column shows the results of collapsing the inferred rules of the second column into the three types (scalar, $S$; transitional, $T$; and vectorial, $V$).

4.6.3 Categorisation of Inferred Rules of each Subject into the Three Types

After categorizing the inferred rules of each vector characteristic into the three types of preconceptions (see Table 4.24), the next step was to categorize the
Table 4.24

Categorization of the Inferred Rules of each Vector Characteristic into One of the Type of Preconception (Scalar, Transitional, or Vectorial)

<table>
<thead>
<tr>
<th>Vector Characteristic</th>
<th>Inferred Rules for each Vector Characteristic</th>
<th>Type of Conception in the Inferred Rules (Scalar; S, Transitional; T, Vectorial; V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Reference Point for Stationary Objects (RPS)</td>
<td>RPS-1, RPS-2, RPS-3</td>
<td>S</td>
</tr>
<tr>
<td>2. Frame of Reference for Stationary Objects (FR)</td>
<td>FR-1, FR-2, FR-3</td>
<td>T</td>
</tr>
<tr>
<td>3. Independence of locations from Path (ILP) (only analyzed in second phase)</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>4. Displacement (D)</td>
<td>D-1, D-2, D-3, D-4</td>
<td>S</td>
</tr>
<tr>
<td>5. Addition of Displacements (AD)</td>
<td>AD-1, AD-2, AD-3</td>
<td>T</td>
</tr>
<tr>
<td>6. Subtraction of Vector Position (SVP)</td>
<td>SV-1, SV-2, SV-3</td>
<td>V</td>
</tr>
<tr>
<td>7. Reference Point for Moving Objects (RPM)</td>
<td>RPM-4</td>
<td>S</td>
</tr>
<tr>
<td>8. Analysis of Components (AC)</td>
<td>AC-1, AC-2</td>
<td>V</td>
</tr>
<tr>
<td>9. Composition of Velocities (CV)</td>
<td>CV-1, CV-2, CV-3</td>
<td>S</td>
</tr>
<tr>
<td>10. Independence of Magnitudes of Components (IMC)</td>
<td>IMC-1, IMC-2, IMC-3</td>
<td>T</td>
</tr>
<tr>
<td>11. Independence of Directions of Components (IDC) (only analyzed in second phase)</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>12. Simultaneity of Components (SC)</td>
<td>SC-1, SC-2</td>
<td>S</td>
</tr>
</tbody>
</table>
inferred rules for each subject in the sample into the three types. Table 4.25 shows the result of that categorization; Tables 4.23 and 4.24 provided the basic data to construct Table 4.25. This last table contains 11 columns and 20 rows. The first column shows the name of the subject, and there is one column for each vector characteristic which is subdivided in three columns, each one corresponding to one of the three types of preconceptions (S, T, and V); with two exceptions: Analysis of Components (A.C.) and Simultaneity of Components (S.C.) which have only two types, S. or V. There is one row for each subject, which is also further subdivided in three rows. Each "sub-row" shows the type of preconception (S, T, or V) held by each subject for each vector characteristic.

There were two reasons for organizing Table 4.25 in this fashion. The first relates to the possibility of analyzing the types of preconceptions corresponding to each one of the inferred rules for each subject. This information is provided in each row. At the end of each row, each of the three types of preconceptions are summated for each subject. By analyzing these numbers -- the number of each of the three types of preconceptions (e.g., Preston: 1S, 1T, 8V, see Table 4.25) -- it is possible to see if a subject can be easily diagnosed in terms of scalar or 'vectorial' preconceptions. This information may be very relevant for the teacher or the curriculum developer.
Table 4.25
Categorization of Inferred Rules of Each Subject into the Three Types

<table>
<thead>
<tr>
<th></th>
<th>RPS</th>
<th>F R</th>
<th>D</th>
<th>A D</th>
<th>SVP</th>
<th>RPM</th>
<th>AC</th>
<th>CV</th>
<th>IMC</th>
<th>S C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dale</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lisa</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Suzanne</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dwayne</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Larry</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Russ</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandy</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tracy</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ian</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Julie</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preston</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hans</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Becky</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steve</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kevin</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wayne</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lori</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lori-Ann</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Randy</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kelley</td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td>STV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total Ss: 13 4 3 6 10 2 2 14 3 2 12 4 7 4 8 3 3 13 1 18 1 8 11 9 6 4 2

Percentage: 65 20 15 33 55 12 10 74 16 11 67 22 37 21 42 16 16 68 5 95 5 40 55 47 32 21 12 88

*Percentages were calculated using only the number of subjects holding inferred rules.
The second reason relates to the possibility of finding out what types of preconceptions (S, T, or V) were more common among the subjects in the sample. This can be shown in the percentage of subjects holding each type of preconception for each vector characteristic. This information may also be important for curriculum makers and teachers. At the bottom of each column, the number and percentage of subjects holding each type of preconceptions is provided. This information could also be related to the degree of difficulty that students might encounter when learning certain aspects of vector quantities based on learning materials oriented around the implicit vector characteristics. If for a particular vector characteristic, most of the subjects tend to hold a 'scalar type' of preconception, it could mean that aspects of vector quantities based on that vector characteristic might prove to present considerable difficulties for students (e.g., the vector characteristic RPS, 65 percent of the Ss are scalar typed). On the other hand, if for other vector characteristic most of the subjects held 'vectorial types' of preconceptions, it could mean that these vector characteristics may be easier to comprehend.

4.6.4 Discussion of Table 4.25

This discussion is developed in three parts. Starting with those vector characteristics with high
percentage (of subjects) on the 'scalar type' of preconception; followed by those with high percentage on the 'transitional type'; and ending with the 'vectorial type' of preconceptions.

It has to be mentioned at the outset that this discussion is based on only the two tasks of the study; therefore, it is very contextual in nature and it cannot be generalized to other contexts without carrying out proper research. But, it is argued, that the results of the discussion can be applied to other similar contexts (i.e., boats moving on lakes and rivers) when teaching the vector quantities that the present study deals with; that is, vector position, displacement, and velocity.

Three of the ten vector characteristics may prove to be troublesome for the students, since a high percentage of the subjects held only the 'scalar type' of preconception about them. These are: Reference Point for Stationary Bodies (R.P.S.), subtraction of vector position (S.V.P.), and Independence of Magnitudes of Component Velocities (I.M.C.). The respective percentages were the following: 65, 37, and 47. Regarding Reference Point, the findings showed that the use of several bodies as references (the scalar type of preconception) appealed to many of the subjects. These grade 10 students seemed to have difficulty accepting the use of only one body as a reference to describe the location of other bodies, even
when it could be shown to them that the choice of one body as a reference point would facilitate the task of locating other bodies. It is interesting to note that for the subtraction of vector position a similar percentage of subjects held the 'scalar' and 'vectorial type' of preconceptions -- 37 and 42 respectively. It seems to this author that 37% is a high figure if one considers that by grade 10 subjects should intuitively realize that the use of arithmetic subtraction to find the resultant of two non-parallel vector position is not possible. It was not anticipated that these subjects would actually solve the problem by using the formal operation of subtraction of vectors, but it was expected that most subjects would see the impossibility of using the arithmetic operation. Finally, the third vector characteristic, which is independence of magnitudes of component velocities, 45 percent of the subjects affirmed that the speed (speed is considered as the magnitude of the velocity in the context of the study) of the boat produced by the motor changes because of the effect of the speed of the stream. This shows that the concept of independence of components may be difficult to grasp and efforts should be made to create experimental situations in which this concept is clearly demonstrated.

Three vector characteristics were approached by the majority of students with 'transitional type' of preconceptions. They were Frame of Reference (F.R.), Displacement or Change of Location (D.), and Addition of Displacement (A.D.); their respective percents were: 55, 74, and 67. One possible interpretation of these data is that students may not have
as much difficulty in comprehending the vectorial concepts related to these vector characteristics. Regarding frame of reference, most subjects in the sample saw the need of using two variables to locate a body. It is true that these were qualitative in nature and often referred to different reference points, but one would expect these subjects could readily grasp the need of using quantitative coordinates and refer them to a unique reference point. It was surprising to find that with the displacement vector characteristic most of the subjects (74 percent) held the conception that it was more important to consider the length of the straight line connecting two locations than the length of a possible curved path joining them. This conception actually corresponds to the magnitude of the displacement, a vector quantity. This could be interpreted to mean that most of the students should not have major difficulties in understanding this concept. In this case, the emphasis should be placed on pointing out the directional aspect of the concept. With respect to the addition of the displacements vector characteristic, the majority of subjects in the sample (67 percent) held the conception that it is not possible to use arithmetic addition to obtain the magnitude of the resultant of two consecutive displacements; instead they suggested several possible strategies to obtain the resultant. Again, it was not expected that they would handle the formal operation of vector addition, but at least the results show that students
possess intuitive conceptions upon which to base the comprehension of the vectorial operation. Since, these subjects apparently were mainly concerned with producing a result for the magnitude, they did not mention the direction of the resultant displacement.

Finally, four of the ten vector characteristics were mainly described in terms of the 'vectorial type' of preconceptions. They were: reference point (or body) for moving objects R.P.M.), analysis of components (A.C.), composition of velocities (C.V.), and simultaneity of components (S.C.). Their respective percentages were: 68, 95, 55, and 88. Since most of the subjects in the sample appeared to possess a vectorial type of preconception in all these vector characteristics, it could mean that, by Grade 11, students should grasp vectorial aspects based on their characteristics without too much difficulty. However, it is wise to be cautious in claiming that this will always be the case, since these results are obtained from a specific and familiar context.
The main purpose of including in the study the characteristic, Reference Body for Moving Objects, was to find out if subjects realized that the movement of the motor-boat relative to a stationary body (i.e., bridge, shore, docks) and the movement relative to a moving body (i.e., water of the river) were different. The intention was not to check the understanding of the relative motion of one frame of reference with respect to the other or relationship of the movements of the motor-boat relative to both frames of reference. The fact that most subjects in the sample (68 percent) saw the difference between the two kinds of movements indicates that the students holding this conception should not have much difficulty in grasping the concept of relative motion.

It is suspected that a reason for such a high percentage of subjects holding a 'vectorial type' of preconception for the Analysis of Components vector characteristic could be due exclusively to the task used in the study. The component velocities (velocity due to the motor and velocity due to the stream) affecting the motor-boat were fairly concrete and obvious. The author decided to use this fairly straightforward task to check this vector characteristic because it was thought that subjects may not realize the possibility of the existence of more than one component affecting a moving body for other more subtle tasks. At least, it can be said that most of the subjects in the sample did
observe the characteristic of two movements affecting a moving object. But it is not necessarily the case that the same subjects would recognize the two component velocities affecting a moving object following a parabolic path (e.g., projectile motion).

Turning now to the composition of velocities characteristic, more than half of the Ss in the sample saw an interaction of directions and an interaction of magnitudes of component velocities. That is, these subjects held the conception that the resultant movement of the motor-boat had direction and magnitude different from those of the components. It is worth mentioning that the same subjects holding this 'vectorial type' of preconception also held 'vectorial types' in the previous vector characteristic, analysis of components, as well (see Table 4.23). The fact that most subjects held this vectorial type of preconception shows that they should be able to readily comprehend the formal vectorial approach of combining or composing simultaneous velocities affecting an object.

Finally, as was expected, most of the Ss (88 percent) saw the simultaneity characteristic of the velocities affecting the motor-boat. It is possible that the rest of the subjects (12 percent) experienced some difficulty in understanding the question related to this characteristic even after it was rephrased. However, on balance, this characteristic does not appear to be too troublesome for most of the subjects in this study.
4.7 Conclusion of Phase One

There were a total of 6 research questions posed for this study (see Chapter 1). The first three questions and part of the fourth were addressed by the interview phase of the study. This section presented the findings as they pertain to these four research questions. A summary of the results are presented for each of the four questions below. The overall conclusions of the study are discussed in Chapter 7.

1) The first research question referred to uncovering the preconceptions that students may possess about the first ten vector characteristics of LIVC (Table 1.1). Findings showed that the subjects in the sample held preconceptions about each one of the vector characteristics. These preconceptions were identified and expressed as inferred rules. Results also showed that the individual interview methodology and the tasks were suitable for the purpose of the study.

2) The second research question referred to the consistency of students' preconceptions. Again, findings show that most of the subjects in the sample made use of a similar conception when dealing with the different situations included
in the tasks. The use of similar strategies to cope with specific problematic situations showed that subjects held definite ideas or consistent preconceptions about each one of vector characteristics treated in phase one of the study.

3) The third research question referred to the possibility of categorizing the students' consistent preconceptions in some fashion. Findings showed that the identified consistent preconceptions ranged from primitive to more abstract notions. Based on this finding, a nomothetic approach - use of the Physics Approach - was used to categorize the inferred rules for each vector characteristic. Furthermore, a second categorization of the inferred rules was done using a three types of categories: scalar, transitional, and vectorial.

4) The fourth research question referred to determining if there was a significant difference between female and male subjects regarding the identified preconceptions. Chi-square test statistics were used to test for sex differences in the inferred rules obtained for each vector characteristic. The calculated chi-square values showed that there was no significant
difference between female and male subjects in the sample regarding the inferred-rules classification.
CHAPTER FIVE

CONSTRUCTION AND ADMINISTRATION OF
THE G.I.P.

5.00 Introduction

The second phase of the study is reported in Chapters five and six. The present chapter deals with the construction and the administration of the Group Interview Protocol (G.I.P.), which was the written instrument used in phase two of the study. However, before describing these procedures this chapter deals with two other related issues. The first concerns the problem of generalizability in the study. The second discusses the use of the group interview as a data collection methodology. After the construction and administration of this instrument are outlined, the chapter concludes with a section on the methods used to analyze the data. The results of these analytical procedures are presented in Chapter Six.
5.1 Meaning of Generalizability in this Study

The fifth research question outlined in Section 1.5 addressed the issue of generalizability. Generalizability normally refers to the ability to make inferences about a defined population, based on findings from a sample using some type of appropriate statistical procedure. It should be made clear at the outset of this section that the objective of proceeding with Phase Two, using a larger sample of subjects, was not an attempt to meet in a rigid way the technical definition of generalizability discussed above. Rather, the main aim of Phase Two was to determine the feasibility of using a group interview technique for the purpose of gathering data on student preconceptions about vector characteristics using larger groups of students. Once this methodology is judged to be sound, then the task of making broad generalizations to a more general population can commence. While the substantive results from Phase Two, reported in Chapter Six, are considered important in the context of the present study, the reader is cautioned from making unwarranted inferences to the general population of grade ten students.

5.2. The Group Interview Technique

In developing a written instrument that could be used as the basis for the administration of a group interview,
three objectives were considered: (1) to create a situation that would be as close as possible to an individual interview situation and through which it would be possible to "interview" a group of subjects*; (2) to determine whether a different and larger group of students hold a similar set of inferred rules to those identified in the individual interviews; (3) to determine the patterns of inferred rules used by classroom groups of students responding in a written format for the purpose of examining the potential educational implications of both the results and the technique.

Two methodological characteristics were present in the second phase as compared to the first phase. The sample was formed of intact grade 10 classes as opposed to individual students; and second, a written instrument was used for data collection rather than an oral discussion. Intact classes were used for two reasons: it is the only practical way to obtain group data from a school without causing major disruption in regular school procedures; but perhaps more importantly the class interview procedure may be useful as a diagnostic technique for teachers. However, the present instrument would have to be altered to a more

*It is obvious that a group interview does not have some of the advantages of an individual interview which allows for the use of probing or paraphrased questions, but it does have the important advantage of reaching a larger number of subjects.
standardized format before it could be used in this diagnostic capacity.

The group interview technique, then, consists of a series of demonstrations and large scale cardboard models, the same as those used in the individual interview, plus a Group Interview Protocol. The latter is a written instrument on which the subjects write their responses instead of verbally stating them to the interviewer. This author is aware of the differences between an individual (interview) and group interviews, with the former being, in general, a more valid means of gathering data about student beliefs. Since the format of the data collection was perfected in an interview setting and the format of the data analysis, rule inferring, was based on the results of Phase One, it is argued that the validity issue is not as much of a problem as in studies which rely solely upon group data.

5.3 Construction of the Group Interview Protocol

The Group Interview Protocol (hereafter abbreviated as G.I.P.) basically consisted of the same protocol used in phase one. But considering that there are substantial differences between individual and group interviews, most serious being the lack of flexibility to paraphrase questions in a group setting, some questions of the original protocol were rephrased in order to facilitate understanding.
All questions used on the G.I.P. were thoroughly discussed with a group of five science educators and then four individual high school students until they were judged to be clear to the students. After this, the questions were put into a questionnaire format and piloted with two grade ten classes.

While the original format for the written instrument was designed to use the rule models, identified in Phase One, as a set of fixed choice responses, it was later decided to use an open-ended response format with the students writing in their own responses. This latter format was felt to be more flexible and provided a better check on the responses obtained from Phase One.

In this second phase, the 12 vector characteristics listed in Table 1.1 were considered*. Although most of the vector characteristics are assessed with three or more independent questions, some vector characteristics had fewer than three questions. The number of questions depended on the nature of the vector characteristics. For some, the type of tasks used readily lead to the creation of several relevant questions; for others, it was more difficult and so only one or two questions were asked. For instance, for the vector characteristic simultaneity of components, only

*The eleventh and twelveth vector characteristic, independence of locations from path and independence of directions of components, were actually not considered in Phase One, but were included in the G.I.P.
one question was used. A list of the numbers of the questions for each vector characteristic is provided in Table 5.1.

Table 5.1

Vector Characteristics and the Questions in the G.I.P. for each Vector Characteristic

<table>
<thead>
<tr>
<th>Vector Characteristics</th>
<th>Questions in the G.I.P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Point for Stationary Bodies (RPS)</td>
<td>1, 3, 4</td>
</tr>
<tr>
<td>Frame of Reference (FR) (Path)</td>
<td>1, 2, 3, 4, 5, 6</td>
</tr>
<tr>
<td>Independence of Locations from Path (ILP)</td>
<td>7, 9, 11</td>
</tr>
<tr>
<td>Displacement (D) or Change of Location</td>
<td>8, 9, 10, 12</td>
</tr>
<tr>
<td>Addition of Displacements (AD)</td>
<td>13, 14</td>
</tr>
<tr>
<td>Subtraction of Vector Position (SVP)</td>
<td>15, 16</td>
</tr>
<tr>
<td>Reference Bodies for Moving Objects (RPM)</td>
<td>17, 18, 24</td>
</tr>
<tr>
<td>Analysis of Components (AC)</td>
<td>19, 20, 23</td>
</tr>
<tr>
<td>Composition of Velocities (CV)</td>
<td>21, 25, 26, 30, 31</td>
</tr>
<tr>
<td>Independence of Magnitudes of Components (IMC)</td>
<td>28, 29</td>
</tr>
<tr>
<td>Independence of Direction of Components (IDC)</td>
<td>22, 31</td>
</tr>
<tr>
<td>Simultaneity of Components (SC)</td>
<td>32</td>
</tr>
</tbody>
</table>

The decision rule for judging the consistency of the subject's responses was the same as that used in the first phase; that is, the presence of the same preconception in the responses to
more than half of the questions on a vector characteristic. The G.I.P. cannot be considered as a standardized questionnaire or as a psychometric test because it was not constructed to serve the purpose of such normative tests. This written instrument was prepared specifically to guide the administration of the group interview and to ensure that the subjects, in addition to listening to the questions from the interviewer, could read the questions themselves, allowing a better understanding of the tasks. The possibility of putting the G.I.P. in a more standardized form is discussed later in Chapter Seven.

5.3.1 Tasks of the Group Interview

Since one of the purposes of the Second Phase was to find out if a larger group of grade 10 students possessed preconceptions similar to the ones used by the subjects in the First Phase, the same context and tasks were used. Furthermore, it is quite possible that the preconceptions about vector characteristics are context-dependent and so it was important to control this variable. The author is aware of the broad issue of limited generalizability of the findings that this control of context brings. However, it gives more stability to the results for this particular context. The problem of the generalizibility of the rule-models to other contexts remains a problem for further research studies.
5.3.2 The Group Interview Protocol (G.I.P.)

The title of the instrument was "Describing Locations and Motion of Boats on Lakes and Rivers". A copy of this instrument is presented in Appendix C. The format of the G.I.P. can be described in terms of the following six steps. First, an introduction which explained the purpose of the group interview was included. Second, three questions were asked to determine if subjects had any previous experience with boats; these questions also seemed to familiarize the subjects with the tasks. Third, task one (situation one in the G.I.P.) was introduced and explained in detail, after which an example question was used to explain how the subjects should respond to the questions. Fourth, the first 16 questions were individually addressed to the class (leaving sufficient time for the subjects to respond to each question). These 16 questions dealt with the six vector characteristics considered in task one (items 1 to 5 and 11 in Table 1.1). Fifth, task two (situation two in the G.I.P.) was introduced and explained in detail. Finally, the remaining 16 questions which dealt with the other six characteristics (items 6 to 10 and 12 in Table 1.1), were individually presented to the class.

The G.I.P. had a total of 32 questions. Table 5.1 shows the questions for each vector characteristic and identifies the G.I.P. questions corresponding to each. Note that some questions permitted identification of preconceptions for two vector characteristics (questions 1, 3, 4, 9, and 31).
In addition to the G.I.P., two sheets showing a schematic diagram of the lake and the river were provided to each subject. They were told that these diagrams were similar to the larger cardboard model (which was standing upright in the front of the classroom). The appropriate dimensions of the lake were provided on this sheet.

5.3.3 The Pilot Study

Prior to administering the G.I.P. to the final sample of subjects, it was tried in two grade 10 classes. The author was concerned about three important aspects: (i) if one class period (55 min.) would allow enough time for students to complete the G.I.P., (ii) if the questions were sufficiently clear to avoid misunderstanding of the questions by the students, and (iii) if the presentation procedures were adequate.

The information obtained from the pilot administration and the analysis of pilot data were used to identify required changes which were included in the final version of the G.I.P.

5.4 Administration of the G.I.P.

5.4.1 Sample of Subjects

The final version of the G.I.P. (see Appendix C) was administered in eight grade 10 classes from three junior high schools of a large suburban school district in the lower
Mainland of British Columbia. While no systematic data were gathered on the social-economic status of the families of the subjects participating, the researcher was informed by local school board officials that the three schools represented a wide range of parental occupations and income.

A total of 176 students participated. However, those students who either did not respond to most of the questions (for example, at least one question for each vector characteristics) were eliminated from the analysis. This left a final sample of 163 students: 77 females and 86 males. As long as there was no evidence of random or frivolous responses, those students who partially responded the G.I.P. were included in the analysis. This meant that for those subjects who responded to all the questions for a particular vector characteristic, their responses were considered in the analysis, and those subjects who responded only to some of the questions for a particular vector characteristic were not considered in the analysis for that vector characteristic. This approach resulted in different numbers of subjects in the data analysis for each vector characteristic. The number of subjects for any given characteristic ranged from 123 to 163.

Two possible reasons for incomplete responses are: the time may have been insufficient for some subjects, or secondly, some subjects may not have understood some questions. The time factor was a problem in at least three classes due to interruptions for school announcements.
5.4.2 Description of Administration Procedures

The researcher was present and responsible for introducing the G.I.P., and presenting the group interviews, with accompanying demonstrations, for all eight grade 10 classes. In all cases the "procedures consisted of an introduction by the researcher indicating that he was from the University of British Columbia and that he was there because of his interest in finding out their ideas about motion of boats on lakes and rivers. The researcher told the subjects that he would be asking them some questions and explaining them using films and the cardboard models. The subjects were asked to follow the questions on their papers while the researcher was reading them, to think carefully about their responses, and to write them in short sentences in the space provided below each question. They were told that the time for each question was limited to approximately one minute. After being sure that all subjects understood "the rules of the game", the researcher asked subjects to silently read the introduction while he read it aloud. The complete G.I.P. was then administered.

The following were the steps of the G.I.P. administration. (I: interviewer, S6: subjects, Q: question)

1. I prepared all materials and distributed the G.I.P. on the students' benches before class in order to have everything ready when subjects came.
2. I introduced himself, telling S6 his name and

*The time allowed to answer each question may be a factor in the results, particularly to the slower thinkers.
that he was a graduate student of The University of British Columbia; Ss were asked to write their first name on the front page of the G.I.P.; then, I explained the objectives of the "questionnaire".

3. I explained the format of the presentation of the questions and the procedures to answer the questions.

4. I read the introduction of the G.I.P. aloud and asked Ss to silently read along.

5. I read the questions about Ss experience with boats and asked Ss to respond to them.

6. I read and explained situation one (task one) of the G.I.P. I asked Ss to simultaneously read their paper while he was reading it aloud. The cardboard model, placed in front of the class in an upright position, was used to explain task one.

7. I showed the complete film loop of the row-boat on the lake, while explaining the trip and the locations of the fishing spots.

8. I made use of question 0 to indicate how to respond to the questions; I asked Ss to orally suggest some possible responses.

9. I explained to Ss that some of the questions also required them to try and explain or give reasons for their responses.

10. I read each question individually (from Q.1 to Q.16), using the cardboard model to explain
each of the questions (Ss were reminded that they also had a sheet with the schematic diagram of the lake). I waited until all Ss finished responding to the first question before going on to the next question.

11. After Ss have finished their response to Q.16, I read situation two (task two) and explained it using the second cardboard model of the river with the bridge.

12. I showed the first part of the film with the motor-boat on the river. The motor-boat was seen moving underneath the bridge and going against the stream.

13. The Ss then answered questions 17 to 20 one at a time after they read and explained by I.

14. I showed the second part of the film. In this part, the motor-boat was seen going upstream against the current and downstream with the current.

15. Questions 20 to 23 were given, one at a time.

16. I showed the third part of the film. In this part, the motor-boat was seen leaving dock 1 and travelling perpendicular to the stream; from the bridge, where the camera was located, the motor-boat appeared to be heading perpendicular to the stream and ending at a point farther downstream on the other side of the river.

17. I presented questions 24 to 32 one at a time.

18. I thanked the Ss for their cooperation and the group interview was ended.
5.5 Methodology of Data Analysis

The data collected through the group interview protocol (G.I.P.) were analyzed using essentially the same methodology that was used for the individual interview data. This consisted of the following steps. First, one subject's responses to questions for a particular vector characteristic were read and analyzed by the researcher; the search was for any notion or element that could reveal something about the subject's observation and understanding of the situation being discussed.

Second, these 'substantive elements' were written down for each question and common elements were identified. If the subject's response was judged to be similar for more than half the questions on that vector characteristic, then it was claimed that the subject held a consistent preconception about that vector characteristic. Third, a consistent preconception was later expressed as an inferred rule. These analytical procedures were applied to the responses to all questions for each vector characteristic for each subject in the sample.

Although some vector characteristics had fewer than three questions in the G.I.P. this did not create a problem in deciding if a subject had a consistent preconception about a particular vector characteristic.
These vector characteristics were easily differentiated and the subjects' responses to these questions readily indicated whether they did or did not possess a particular preconception. This was the case for the following vector characteristics (see Table 5.1). A.D., S.V.P., I.M.C., and I.D.C.; all of which had only two questions in the G.I.P. A similar approach was applied to the vector characteristic, simultaneity of components, where only one question was used.

After analyzing the responses of all subjects in the sample, the identified inferred rules were compared to those found in the first phase. Most of the inferred rules were identified in both phases and were designated with the same nomenclature (e.g., RPS-1, RPS-2 etc.). A few new inferred rules were identified and were designated with similar nomenclature but with a different number; the number 0 and 0' were used for them (e.g., RPS-0, RPS-0'). The new inferred rules are presented in Chapter Six.

In addition to identifying the inferred rules held by the subjects in the sample and comparing them to those found in phase one, the frequency of female and male subjects holding each inferred rule for each vector characteristic was determined. All these data are presented and discussed in Chapter Six.
CHAPTER SIX

RESULTS OF PHASE TWO OF THE STUDY

6.0  Introduction

This chapter exclusively deals with the findings of phase two. In the first section, results are presented and discussed separately for each vector characteristics; only the new inferred rules are discussed in detail. The two extra implicit vector characteristics (independence of locations from path and independence of directions of components) are also extensively discussed. The next section consists of a short discussion of results for each subject, presented in tabular form in Appendix D. A final section contains a general, summarizing discussion of all the results and a comparison of the results obtained in both phases is made.

6.1  Results for each Vector Characteristic

Findings for each vector characteristic are presented separately. The format of the sub-sections for each of the vector characteristics consists of a contingency table, a short discussion, and presentation of any
new inferred rules. The contingency tables contain the number and percentages of female and male subjects holding the inferred rules, and a chi-square test of independence to determine if there were any differences between the preconceptions and females and males. Only 'new' inferred rules (those identified in Phase 2 but not Phase 1) are discussed in some detail because of each of the 'old' inferred rules were discussed in detail in Chapter Four. The new inferred rules were designated by 0 (zero) and 0' after the usual abbreviation for the vector characteristic. These new inferred rules were then put in more general terms and placed in sequence corresponding to the general rule model obtained from the interview data. While a new table is not reproduced, for the sake of brevity, a footnote is used to indicate the new numbering sequence.

The format of the subsections for the two extra vector characteristics (ILP and IDC) is similar in format to the one used in Chapter Four when presenting the results of phase one (see section 4.3).
6.1.1 Results for Reference Point for Stationary Bodies (RPS)

Table 6.1
Contingency Table of Female and Male Subjects Holding the Inferred Rules for Reference Point for Stationary Bodies

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>RPS-1</th>
<th>RPS-2</th>
<th>RPS-3</th>
<th>RPS-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>26</td>
<td>42</td>
<td>9</td>
<td></td>
<td></td>
<td>77</td>
</tr>
<tr>
<td>M</td>
<td>23</td>
<td>48</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>86</td>
</tr>
<tr>
<td>Total</td>
<td>49</td>
<td>90</td>
<td>18</td>
<td>2</td>
<td>4</td>
<td>163</td>
</tr>
<tr>
<td>%</td>
<td>30%</td>
<td>55%</td>
<td>11%</td>
<td>1%</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>

\( (\chi^2 = 5.87\) \ df = 4 \ p > .05)

Discussion: More than 95 percent of the subjects possessed consistent preconceptions similar to the ones found in phase one. It is interesting to observe that more than 50 percent of the subjects held the conception of using one body as a reference point to locate an object, even though they used different bodies as reference points to locate different objects (RPS-2). This result is somewhat in contrast to the findings of phase one, where more than 50 percent of the subjects (13 to 20) held the conception of using several bodies as reference points (RPS-1).
New inferred rules: Only one new consistent pre-conception was identified for this vector characteristic (RPS-0).

Inferred rule RPS-0: "Each one of the locations of the fishing spots is described using the observer's location implicitly as reference."

This preconception was used by two subjects who did not explicitly use any one body as a reference. From their responses it was apparent that they were using themselves as a reference. It may be that these subjects do not see the need for using reference bodies to locate objects, but apparently they were using an implicit reference body.

Inferred rule RPS-0 can be categorized between RPS-2 and RPS-3 of Table 4.1 (see section 4.3.1), and can be expressed in the following form to be included in the Rule-Model for RPS:

Rule RPS-III*: "Use of a unique but implicit reference point (e.g., the observer's location) to locate all bodies or different locations of a given body."

* The former rule RPS-III in section 4.3.1 becomes RPS-IV
6.1.2 Results for Frame of Reference (FR)

Table 6.2
Contingency Table of Female and Male Subjects Holding the Inferred Rules for Frame of Reference

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>FR-1</th>
<th>FR-2</th>
<th>FR-3</th>
<th>FR-4</th>
<th>FR-5</th>
<th>FR-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>10</td>
<td>11</td>
<td>24</td>
<td>2</td>
<td>17</td>
<td>12</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>27</td>
<td>2</td>
<td>14</td>
<td>14</td>
<td>2</td>
<td>9</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>2</td>
<td>25</td>
<td>38</td>
<td>4</td>
<td>26</td>
<td>30</td>
<td>162</td>
</tr>
<tr>
<td>%</td>
<td>23%</td>
<td>1.3%</td>
<td>15%</td>
<td>23%</td>
<td>2.5%</td>
<td>16%</td>
<td>19%</td>
<td></td>
</tr>
</tbody>
</table>

( \( \chi^2 = 4.26 \), \( df = 6 \), \( p > .05 \))

Discussion: More than 60 percent of the subjects possessed consistent preconceptions similar to the ones found in phase one. Two aspects of these data are worthy of discussion in greater detail. First, there was a relatively large percentage of subjects (19%) for whom it was not possible to attach a consistent preconception; therefore no rule was inferred for them.* Second, a reasonable percentage of subjects (16%) held a conception not found in the first phase.

* This could be one disadvantage of using a paper and pencil instrument to uncover subjects' conceptions; probing questions could have been used with these subjects in an individual interview situation. No large percentage of subjects with inconsistent conceptions was found in the first phase.
New inferred rules: Only one new consistent preconception, expressed in the following inferred rule, was identified.

Inferred rule FR-0: "Precise description of each one of the locations of the fishing spots is done using only a qualitative direction."

Subjects holding this conception (26 of 162) used only qualitative factors such as "to the left" or "to the west" to locate a fishing spot. This conception is more primitive than the one expressed in inferred rule FR-1 (see Table 4.3 in section 4.3.2), so it can be categorized prior to that.

The inferred rule FR-0 can be expressed in the following general form to be included in the Rule-Model for FR:

Rule FR-I*: "Use of only a qualitative direction as a coordinate to locate stationary bodies in a two dimensional situation."

* The former rule FR-I (see table 4.4) becomes rule FR-II. All other rules are moved one Roman digit up.
6.1.3 Results for Displacement or Change of Location

(D)

Table 6.3

Contingency Table of Female and Male Subjects Holding the Inferred Rules for Displacement or Change of Location

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>D-1</th>
<th>D-2</th>
<th>D-3</th>
<th>D-4</th>
<th>D-5</th>
<th>D-0</th>
<th>D-0'</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>12</td>
<td>8</td>
<td>2</td>
<td>20</td>
<td>2</td>
<td>23</td>
<td>4</td>
<td>5</td>
<td>76</td>
</tr>
<tr>
<td>M</td>
<td>15</td>
<td>7</td>
<td>1</td>
<td>28</td>
<td>3</td>
<td>20</td>
<td>3</td>
<td>7</td>
<td>84</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>15</td>
<td>3</td>
<td>48</td>
<td>5</td>
<td>43</td>
<td>7</td>
<td>12</td>
<td>160</td>
</tr>
<tr>
<td>%</td>
<td>17%</td>
<td>9%</td>
<td>2%</td>
<td>30%</td>
<td>3%</td>
<td>27%</td>
<td>4%</td>
<td>8%</td>
<td></td>
</tr>
</tbody>
</table>

\( \chi^2 = 2.16 \quad df = 7 \quad p > .05 \)

Discussion: More than 60 percent of the subjects held consistent preconceptions similar to the ones found in the first phase. As in the first phase, the conception contained in the inferred rule D-4 was more common among the subjects, 50 and 30 percent respectively. Two new consistent preconceptions were identified -- one of them held by a fairly high percentage of subjects (27%), while only 4% held the other new preconception. A relatively low percentage of subjects (8%) did not show any consistent preconceptions about this vector characteristic.
New inferred rules: The two new consistent preconceptions which were found are expressed in the following inferred rules:

Inferred rule D-0: "A change of location of the boat moving from one fishing spot to another indicates only the direction (qualitative or quantitative) of the second location with respect to the first location or with respect to other bodies."

Inferred rule D-0': "A change of location of the boat moving from one fishing spot to another indicates both the time it takes to go from one location to another and the direction of the second location with respect to the first location or with respect to other bodies."

Both inferred rules contain the directional ingredient of the vector displacement. The variable time is indirectly related to displacement; in the first phase, some subjects considered the time and a constant speed to find a distance. However, that distance is more closely related to the length of the path followed by the boat moving between two locations than to the length of the straight line joining the two locations.

Inferred rule D-0 can be categorized between D-1 and D-2 of table 4.5 (see section 4.3.3), but D-0 is at the same level of complexity as D-1 because both contain
only one content variable. Inferred rule D-0' can be
categorized between D-2 and D-3 (see Table 4.5), placing
D-0' at the same level of complexity as D-3 because both
have two content variables. These inferred rules (D-0
and D-0') can be expressed in the following forms to be
included in the Rule-Model for D.:

Rule D-II*: "A displacement or change of location
indicates the direction of one location with respect to
another one or with respect to another body."

Rule D-V**: "A displacement or change of location
indicates both the time it takes to move from one loca-
tion to another and the direction of the second location
with respect to the first or with respect to other body."

* The former rule D-II (see Table 4.6) becomes D-III.

** The former rule D-IV (see Table 4.6) becomes D-VI,
and the former rule D-V becomes D-VII. In summary,
the general rules for the vector characteristic
Displacement are increased to 7.
6.1.4 Results for Addition of Displacements (AD)

Table 6.4
Contingency Table of Female and Male Subjects Holding the Inferred Rules for Addition of Displacements

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>AD-1</th>
<th>AD-2</th>
<th>AD-3</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>25</td>
<td>13</td>
<td>9</td>
<td>21</td>
<td>64</td>
</tr>
<tr>
<td>M</td>
<td>12</td>
<td>19</td>
<td>5</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>32</td>
<td>14</td>
<td>51</td>
<td>134</td>
</tr>
<tr>
<td>%</td>
<td>28%</td>
<td>24%</td>
<td>10%</td>
<td>38%</td>
<td></td>
</tr>
</tbody>
</table>

\(x^2 = 4.69\quad df = 3\quad p > .05\)

Discussion: More than 60 percent of the subjects demonstrated similar consistent conceptions to the ones found using the individual interview. A high percentage of subjects (28%) used the arithmetic addition conception (AD-1) to deal with two non-parallel partial displacements. This result is very different from the 10 percent obtained in the first phase. This value could mean that Grade 10 students may encounter difficulty in understanding the vectorial addition of displacements or other vector quantities. This assumption can be further supported by the fact that only 10 percent of the subjects held inferred rule AD-3 which contains a more vectorial view.
of addition of displacements. The most remarkable finding for this vector characteristic is that a high percentage of subjects (38%), did not hold any consistent conception. This may suggest a general lack of understanding of the questions regarding this vector characteristic. Many subjects indicated on their questionnaires that they did not understand the questions and that they did not know what to do with the numerical distances provided. Similar problems were found when proceeding with the individual interviews. In that situation the questions were paraphrased until the interviewee understood the questions. Paraphrasing can be used in a group interview to a certain extent if the majority of the subjects seem not to understand, but not to the degree possible in an individual interview setting.

No new preconceptions were found for this vector characteristic.
6.1.5 Results for Subtraction of Vector Position (SVP)

Table 6.5

Contingency Table of Female and Males Subjects Holding the Inferred Rules for Subtraction of Vector Position

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>SVP-1</th>
<th>SVP-2</th>
<th>SVP-3</th>
<th>SVP-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>9</td>
<td>9</td>
<td>14</td>
<td>4</td>
<td>26</td>
<td>62</td>
</tr>
<tr>
<td>M</td>
<td>7</td>
<td>12</td>
<td>8</td>
<td>4</td>
<td>35</td>
<td>66</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>21</td>
<td>22</td>
<td>8</td>
<td>61</td>
<td>128</td>
</tr>
<tr>
<td>%</td>
<td>13%</td>
<td>16%</td>
<td>17%</td>
<td>6%</td>
<td>48%</td>
<td></td>
</tr>
</tbody>
</table>

\( \chi^2 = 3.46 \quad df = 4 \quad p > .05 \)

Discussion: Less than 50 percent of the subjects held conceptions similar to the ones found in the individual interviews. What is more striking is that 48 percent appeared not to possess any consistent view to deal with the problematic situation related to subtraction of vectors. Actually, this author was expecting difficulty with these questions because similar kinds of problems occurred with the individual interviews. However, such a high percentage of subjects lacking intuitive notions about subtraction was not expected. This result also shows one of the disadvantages of a paper and pencil questionnaire, the lack of communication between the
subjects and the researcher. During the questionnaire administration, efforts were made to introduce carefully each question, particularly those which caused more difficulty in the individual interviews. The cardboard model was used to explain and show the points in discussion, but still it was not enough. There is, of course, a high probability that most of the subjects in Grade 10 actually do not possess the intuitive notions required to deal with these kinds of situations without instruction. More research is necessary to test this position.

**New inferred rules:** One new consistent conception was identified for this vector characteristic and it was expressed in the following inferred rule.

**Inferred rule SVP-0:** "The resultant of the subtraction of two non-parallel vector positions for two fishing spots has a magnitude equal to the arithmetic subtraction of the magnitudes and direction (qualitative or quantitative) is cited."

This conception, which was held by a few subjects (8 of 128), is interesting because the directional factor of the resultant was considered, but the direction of the components was not considered when finding the magnitude of the resultant. Subjects holding this conception would
have difficulty understanding the operation of subtraction of vectors because they apply strict arithmetic subtraction to obtain the resultant length from the two non-parallel straight lines.

The inferred rule SVP-0 can be categorized between the inferred rules SVP-2 and SVP-3 in Table 4.9 (see section 4.3.5); and it can be expressed in the following general form to be included in the rule-model for SVP.

 Rule SVP-III*: "The resultant of the subtraction of two non-parallel vector positions has a magnitude equal to the arithmetic subtraction of the magnitudes and direction is considered."

6.1.6 Results for Reference Bodies for Moving Objects (RPM)

Table 6.6

Contingency Table of Female and Male Subjects Holding the Inferred Rules for Reference Bodies for Moving Objects

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>RPM-1</th>
<th>RPM-2</th>
<th>RPM-3</th>
<th>RPM-4</th>
<th>RPM-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>29</td>
<td>4</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>14</td>
<td>73</td>
</tr>
<tr>
<td>M</td>
<td>28</td>
<td>3</td>
<td>4</td>
<td>26</td>
<td>10</td>
<td>8</td>
<td>79</td>
</tr>
<tr>
<td>Total</td>
<td>57</td>
<td>7</td>
<td>11</td>
<td>38</td>
<td>17</td>
<td>22</td>
<td>152</td>
</tr>
<tr>
<td>%</td>
<td>38%</td>
<td>5%</td>
<td>7%</td>
<td>25%</td>
<td>11%</td>
<td>15%</td>
<td></td>
</tr>
</tbody>
</table>

\( \chi^2 = 7.90 \quad df = 5 \quad p > .05 \)

* The former rule SVP-III in Table 4.10 becomes rule SVP-IV
Discussion: About 75 percent of the subjects held conceptions similar to the ones found through the individual interviews for this vector characteristic. However, there were some differences in the distribution of these preconceptions in the group interviews as compared to the individual interviews. For instance, in the latter case 50 percent of the subjects made use of the conception expressed in inferred rule RPM-3 as compared to only 7 percent in the group format. Whereas a larger percentage (38%) in the group interview employed the simplest conception RPM-1.

A relatively small percentage of subjects (15%) did not show any consistent conception about this situation.

New inferred rules: The only new consistent conception identified for this vector characteristic follows:

Inferred rule RPM-1: "Description of the motion of the motorboat on the river by an observer on the bridge distinguishes movement only relative to a moving body (i.e., the water)."

The subjects holding this conception (17 of 152) seemed to be considering the causes of the motion rather than the description of the motion. This was shown when they wrote, "The boat was moving with the water" or
"going with the water". These subjects had difficulty in understanding the phrase, "with reference to". This could be the reason they concentrated more in the causal aspect than in the description aspect.

Inferred rule RPM-0 seems more primitive than RPM-1 (see Table 4.11 in section 4.3.6), as it can be categorized at a lower level than RPM-1. It can be expressed in the following general form to be included in the rule-model for RPM.

Rule RPM-I*: "Description of the motion of an object is done only relative to a moving body."

6.1.7 Results for Analysis of Components (AC)

Table 6.7

Contingency Table of Female and Male Subjects Holding the Inferred Rules for Analysis of Components

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>AC-1</th>
<th>AC-2</th>
<th>AC-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>31</td>
<td>20</td>
<td></td>
<td>13</td>
<td>64</td>
</tr>
<tr>
<td>M</td>
<td>28</td>
<td>34</td>
<td>2</td>
<td>6</td>
<td>70</td>
</tr>
<tr>
<td>Total</td>
<td>59</td>
<td>54</td>
<td>2</td>
<td>19</td>
<td>134</td>
</tr>
<tr>
<td>%</td>
<td>44%</td>
<td>40%</td>
<td>2%</td>
<td>14%</td>
<td></td>
</tr>
</tbody>
</table>

( $\chi^2 = 8.04$, df = 3, $p > .05$)

* The former rule RPM-I (see Table 4.12) becomes RPM-II. All rules in Table 4.12 are then increased by one.
Discussion: More than 80 percent of the subjects showed to possess the two consistent preconceptions identified in the first phase. This percentage is split almost in half, 44 percent of the subjects considered only one component affecting the boat (inferred rule AC-1); and 40 percent considered two component velocities (inferred rule AC-II). Only two subjects held a conception different from the previous two, which is explained below. Finally a small percentage of subjects (14%) appeared not to possess a consistent preconception about this vector characteristic.

New inferred rules: The only new consistent conception identified was expressed as follows:

Inferred rule AC-0: "Analysis of the components for the motion of the motorboat on the river produces one component -- either the motion due to the current or the motion due to the motor -- affecting the boat when it moves with or against the river and two components when the boat moves across the river -- motion due to the current and the motion due to the motor."

This is a very interesting conception and it seems a logical one because when the motorboat moves with or against the current the presence of the two components is not explicitly clear; this may be reinforced because the
interaction of directions is not clear either. For the case of the motorboat going across the river the presence of the two components is clear because the interaction of them is clearly observed. Subjects may have thought that the boat was following the diagonal path because the two components were affecting it simultaneously. Even though only 2 of 134 subjects held this view, it could be an intuitively appealing one.

The inferred rule AC-0 can be categorized between AC-1 and AC-2 of Table 4.13 (see section 4.3.7); and it can be expressed in the following general form to be included in the rule-model for AC:

Rule AC-II*: "The analysis of component velocities for a moving body, which is being affected by two simultaneous and independent velocities, produces one component when the two components affect the body in the same direction or in opposite direction and two components when these affect it in different directions (different from 90° and 180°)."

* The former rule AC-II in Table 4.14 becomes rule AC-III
6.1.8 Results for Composition of Velocities (CV)

Table 6.8
Contingency Table of Female and Male Subjects Holding the Inferred Rules for Composition of Velocities

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>CV-1</th>
<th>CV-2</th>
<th>CV-3</th>
<th>CV-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F</strong></td>
<td>7</td>
<td>26</td>
<td>34</td>
<td>6</td>
<td></td>
<td>73</td>
</tr>
<tr>
<td><strong>M</strong></td>
<td>3</td>
<td>29</td>
<td>46</td>
<td>5</td>
<td>2</td>
<td>85</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>10</td>
<td>55</td>
<td>80</td>
<td>11</td>
<td>2</td>
<td>158</td>
</tr>
</tbody>
</table>

| %              | 6%   | 35%  | 51%  | 7%   | 1.3% |       |

\[ \chi^2 = 4.74 \quad df = 4 \quad p > .05 \]

Discussion: More than 80 percent of the group interview subjects demonstrated conceptions similar to the ones found in the first phase for this vector characteristic. As in the case of the analysis of components, subjects were mainly holding two distinct consistent conceptions expressed in the inferred rules CV-2 and CV-3, respectively. When considering the content of these inferred rules, most of the subjects (86 percent) recognized the interaction of direction of the components, and 51 percent observed the interaction of both direction and magnitude of the components. It cannot be claimed that similar results will occur if other experi-
mental situations or contexts were used, but at least this result shows that a large percentage of subjects are able to observe that different motions affecting an object interact to produce an apparent single motion.

**New inferred rules:** In examining the three inferred rules identified in phase one, it is apparent that one potential inferred rule was missing — that which accounts for interaction of magnitudes only. This conception was identified using the G.I.P. and it was expressed in the following inferred rule.

**Inferred rule CV-0:** "When there are two non-parallel velocities affecting simultaneously the motorboat , the predicted resultant velocity shows that its direction corresponds to the direction of the component with greater magnitude, showing with this no interaction of direction; and the magnitude corresponds to one different from those of the components, showing with this an interaction of magnitudes."

A small percentage of subjects (7%) held this conception, but it should be considered because it seems to represent a legitimate intuitive notion held by subjects about interaction of velocities. Some subjects might argue that this is the case when two parallel velocities are simultaneously affecting an object in the
same direction (e.g., motorboat moving downstream with the motor running), saying that they will observe the boat moving faster with no change of direction.

The inferred rule CV-0 can be categorized between the inferred rules CV-2 and CV-3 in Table 4.15 (see section 4.3.8); and it can be expressed in the following general form to be included in the rule-model for CV:

**Rule CV-III*: "The composition of two non-parallel velocities, which are simultaneously affecting a moving body, produces a resultant whose magnitude is different from the magnitudes of the components and whose direction corresponds to the direction of the component with greater magnitude."**

*The former rule CV-III in Table 4.18 becomes rule CV-IV*
Since the type of path drawn by the subjects can provide additional information about their preconceptions, the distinct paths drawn by all subjects are presented below.

First, question 21 relates to the situation of the motorboat just crossing the river without going to a specific place (Table 6.9). Second, question 31, refers to the path of a motorboat going from dock 1 to dock 2 (Table 6.10). The number and percentage of subjects providing each drawn path are also provided.
Table 6.9

Paths drawn for question 21
(150 subjects answered the question)

<table>
<thead>
<tr>
<th>Paths</th>
<th>Number of Subjects</th>
<th>Percentage of Subjects (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Path 1" /></td>
<td>26</td>
<td>17</td>
</tr>
<tr>
<td><img src="image2" alt="Path 2" /></td>
<td>31</td>
<td>21</td>
</tr>
<tr>
<td><img src="image3" alt="Path 3" /></td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td><img src="image4" alt="Path 4" /></td>
<td>1</td>
<td>6%</td>
</tr>
<tr>
<td><img src="image5" alt="Path 5" /></td>
<td>6</td>
<td>4%</td>
</tr>
<tr>
<td><img src="image6" alt="Path 6" /></td>
<td>2</td>
<td>1.3%</td>
</tr>
</tbody>
</table>
Table 6.10
Paths drawn for question 31
(116 subjects answered this question)

<table>
<thead>
<tr>
<th>Paths</th>
<th>Number of Subjects</th>
<th>Percentage of Subjects (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Path 1]</td>
<td>61</td>
<td>53</td>
</tr>
<tr>
<td>![Path 2]</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>![Path 3]</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>![Path 4]</td>
<td>4</td>
<td>3.4</td>
</tr>
<tr>
<td>![Path 5]</td>
<td>1</td>
<td>.9</td>
</tr>
<tr>
<td>![Path 6]</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>
6.1.9 Results for Independence of Magnitudes of Components (IMC)

Table 6.11

Contingency Table of Female and Male Subjects Holding the Inferred Rules for Independence of Magnitudes of Components

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>IMC-1</th>
<th>IMC-2</th>
<th>IMC-3</th>
<th>IMC-4</th>
<th>IMC-0</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F</strong></td>
<td>51</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>67</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M</strong></td>
<td>56</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>107</td>
<td>5</td>
<td>4</td>
<td>11</td>
<td>11</td>
<td>138</td>
<td></td>
</tr>
<tr>
<td><strong>%</strong></td>
<td>77%</td>
<td>4%</td>
<td>3%</td>
<td>8%</td>
<td>8%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[( \chi^2 = 6.62 \quad df = 4 \quad p > .05)\]

Discussion: As in the first phase, about 80 percent of the subjects possessed conceptions that do not consider the independence of the magnitudes of the component velocities. There is the possibility that the subjects were referring to the velocity of the boat relative to the bridge because they were observing from it. In this case the subjects observed the changes of motion of the boat; but the questions clearly stated that the situations referred to the velocity of the boat due to the motor, which is equivalent to the velocity of the boat relative to the water. No subject held the conception that was
expressed in inferred rule IMC-3; and only a few (4 of 138) held the conception that the magnitudes of the component velocities are maintained even when they are interacting to produce an apparent single motion (IMC-4). These results indicate that this vector characteristic, independence of magnitudes, does not seem to be intuitively grasped by most subjects in Grade 10. It is actually an abstract aspect of vector quantities, and it may be that this characteristic must be pointed out by teachers. A small percentage of subjects (8%) did not show a consistent conception about this vector characteristic; it seemed to this author that most of these latter subjects did not understand the questions because their responses were generally incoherent.

**New inferred rules:** A new consistent conception was identified through the G.I.P., a conception which is closely related to the one expressed in inferred rule AC-0. The new inferred rule follows.

**Inferred rule IMC-0:** "There are two component velocities affecting simultaneously the motorboat; the magnitude of the component due to the motor is affected (changed) when the motorboat moves with or against the stream but the same magnitude is not affected when the motorboat moves across the stream."
This conception shows the difficulty that subjects encounter when passing from a situation in which the boat moves with and against the stream -- components acting along the same line, to a situation in which the boat moves across the stream -- components not acting along the same line. A small number of subjects (11 of 138) held the view contained in this inferred rule.

The inferred rule IMC-0 can be categorized between IMC-3 and IMC-4 in Table 4.19 (see section 4.3.9). It can be expressed in the following general form to be included in the rule-model for IMC:

**Rule IMC-IV**: "There are two simultaneous component velocities affecting a body; the magnitude of one component is affected (changed) when the two components have the same direction or opposite direction, but that magnitude is not affected (changed) when the two components have different directions (e.g., they form an angle of 90°)."

* The former rule IMC-IV in Table 4.20 becomes rule IMC-V
6.1.10 Results for Simultaneity of Components (SC)

Table 6.12

Contingency Table of Female and Male Subjects Holding the Inferred Rules for Simultaneity of Components

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>SC-1</th>
<th>SC-2</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>3</td>
<td>55</td>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>M</td>
<td>2</td>
<td>58</td>
<td>3</td>
<td>63</td>
</tr>
<tr>
<td>Total %</td>
<td>5%</td>
<td>92%</td>
<td>4%</td>
<td>123</td>
</tr>
</tbody>
</table>

\[ \chi^2 = .49 \quad df = 2 \quad p > .05 \]

Discussion: The overwhelming majority of the subjects (92%) who responded to the question regarding this vector characteristic held the conception that the two component velocities were affecting the motorboat at the same time. It is important to know that students at Grade 10 possess this knowledge, because it is a crucial ingredient in understanding composition of velocities; Only by recognizing that two or more movements are simultaneously affecting a body, can they be combined to produce an apparent single motion.

A small percentage of subjects (4%) held the conception that the two component velocities did not
affect the boat at the same time; some of them wrote that the component velocity due to the river affects the boat first. They could have referred to the effect prior to starting the motor, but it is impossible to know without discussing it with them. A small group of subjects (4%) also appeared to have no definite view about this vector characteristic.

6.1.11 Results for the Two New Vector Characteristics

As was mentioned in the introduction of this section 6.1, the format of presentation of results for the two new implicit vector characteristics, which were identified during the First Phase, is similar to the format of results used in Chapter Four (see section 4.3).

6.1.12 Inferred Rules for Independence of Locations From Path (ILP)

All subjects in the sample seem to understand the meaning of the term path. For all of them it meant the route that a body makes when it moves from one point to another. The path's drawings (see question 6 of G.I.P., Appendix C) ranged from joining the stop points (or fishing spots) with straight lines, producing a ziz zag path, to smooth curves. This shows that before format instruction students grasp the concept of path.

The interest in including this vector characteris-
tic was to find out the students' preconceptions about the relationships between the path and the points that belong to it. Do they understand that a location of any point along the path can be described independently of knowing the actual path? How attached are they to the concept of path if a location of a point is required?

Questions of Group Interview Protocol (G.I.P.) Related to Path

Question 6: A diagram of the lake with all fishing spots is provided below. Draw or sketch the path followed by the boat in its round trip?

Question 7: Is it necessary to know the path followed by the boat to describe a particular location of it?

Question 11: Do you need to know the actual path followed by the boat to locate the second stop of the boat?

List of Inferred Rules for Independence of Locations from Path (ILP)

Two inferred rules were obtained for this vector characteristic, which have the following common opening sentence:
"The location of the fishing spots (or the stop points of the rowboat) can be quantitatively described ...

Inferred rule ILP-1: only if the path to reach those spots is known."

Inferred rule ILP-2: even though the path to reach those spots is unknown."

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>ILP-1</th>
<th>ILP-2</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>17</td>
<td>41</td>
<td>18</td>
<td>76</td>
</tr>
<tr>
<td>M</td>
<td>19</td>
<td>49</td>
<td>17</td>
<td>85</td>
</tr>
<tr>
<td>Table %</td>
<td>36%</td>
<td>56%</td>
<td>35%</td>
<td>161%</td>
</tr>
</tbody>
</table>

\( \chi^2 = .48 \quad df = 2 \quad p > .05 \)

Discussion of results: Subjects were separated in two groups regarding consistent preconceptions. One group (comprising 22%) did not see the independence of location from path. Their responses to questions 7 and 11 of the G.I.P. showed that it was necessary to know the path
followed by the rowboat to quantitatively describe a particular location of it. The other group (56 percent) held the conception that there was no need to know the path to quantitatively describe particular stop points of the rowboat. These subjects should not have great difficulty in understanding the concept of vector position if they also held the conception of using one unique body as a reference point.

A relatively high percentage of subjects, 22 percent, appeared not to hold a consistent preconception about this vector characteristic. These subjects responded with a 'yes' answer to question 7 and with a 'no' answer to question 11, or vice versa; thus indicating a non-consistent view about the situation. Based on these responses they were not assigned an inferred rule.

**Rule-Model for ILP:** The two inferred rules were expressed in general terms to produce the rule-model for independence of locations from path.

**Rule ILP-I:** Quantitative description of instantaneous locations of a moving body can be done only if the path followed by it is known.

**Rule ILP-II:** Quantitative description of instantaneous locations of a moving body can be done even though the path followed by it is unknown.
6.1.13 Inferred Rules for Independence of Directions of Components (IDC)

As was explained previously (section 4.4.2) this vector characteristic complements the one about independence of magnitudes (IMC). If students possessed preconceptions about both kinds of independence they would be in a much better position to fully understand the independence of components and composition of vector quantities.

The purpose of including this vector characteristic in the second phase was to find out if subjects were aware of this aspect of vector quantities and to determine the kinds of preconceptions they might have about it.

Questions of Group Interview Protocol (see G.I.P., Appendix C)

Question 22: Copy the path, you drew in the previous question in the diagram below. Choose three well-spaced points in the path and draw the way in which the boat may be heading when crossing the river.

Question 31: (Second part of this question). Choose three well-spaced points in the path sketched above, draw the boat at these points showing clearly how the boat might be heading.
List of Inferred Rules for Independence of Directions of Components (IDC)

The two inferred rules were obtained for this vector characteristic are given below. Both of these rules refer to a common introduction:

"There are two component velocities affecting simultaneously the motorboat, the direction of the component due to the motor ...

Inferred rule IDC-1: is changed because of the interaction with the component due to the current."

Inferred rule IDC-2: is unchanged despite the interaction with the component due to the current."

Table 6.14
Contingency Table of Female and Male Subjects Holding the Inferred Rules for Independence of Directions of Components

<table>
<thead>
<tr>
<th>Inferred Rules</th>
<th>IDC-1</th>
<th>IDC-2</th>
<th>NR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>53</td>
<td>6</td>
<td>6</td>
<td>65</td>
</tr>
<tr>
<td>M</td>
<td>54</td>
<td>11</td>
<td>7</td>
<td>72</td>
</tr>
<tr>
<td>Total</td>
<td>107</td>
<td>17</td>
<td>13</td>
<td>137</td>
</tr>
<tr>
<td>%</td>
<td>78%</td>
<td>12%</td>
<td>10%</td>
<td></td>
</tr>
</tbody>
</table>

( $\chi^2 = 1.23$ \hspace{1cm} df = 2 \hspace{1cm} p > .05)
Discussion: As in the case of independence of magnitudes most subjects did not demonstrate a grasp of the independence of directions. A high percentage of subjects (78%) did not consider this vector characteristic in their drawings. For the motorboat crossing the river (question 22 of G.I.P.), most of the subjects thought that the boat was heading along the direction given by the diagonal (see Figure 6.1a); Another common path drawn was in general given by the tangent to the point of the curve (see Figure 6.1b). This was particularly the case when they drew a kind of semi-circular path to go from dock 1 to dock 2 (question 31 of G.I.P.).

Figure 6.1: Common student drawings (78%) for illustrating the direction in which the motorboat was heading: (a) when crossing the river, (b) when going from dock 1 to dock 2.
The drawings in Figure 6.1 show that subjects did not differentiate the directional aspect of the components from the direction of the resultant velocity. They believed that the front of the boat precisely follows the shape of the path. Some subjects (12 percent of the sample), held the view that even though the overall movement of the boat was along the diagonal, the boat was still heading in the direction given by the velocity due to the motor at the starting point (beside dock 1), as shown in Figure 6.2a.

![Diagram]

**Figure 6.2:** Student drawings (12%) illustrating the direction in which the motorboat was heading: (a) when crossing the river and (b) when going from dock 1 to dock 2.

These subjects realized that the direction of the component velocity due to the motor was independent from the direction of the component velocity due to the stream. Some of the same subjects also suggested that the way to reach dock 2 was to aim the boat into the current (as
depicted in Figure 6.2b). This drawing also clearly shows their understanding of the independence of directions of components. Even though a relatively small percentage of subjects (12%) demonstrated this conception, it does indicate that this vectorial aspect may be present in some students prior to formal instruction. This is a noteworthy finding when it is considered that the independence characteristic is one of the more abstract aspects of vector quantities.

Ten percent of the subjects did not hold any consistent conception about this vector characteristic. Most of these students did not seem to understand the questions, as their written responses were somewhat confusing and difficult to interpret.

**Rule-Model for IDC:** The two inferred rules IDC-1 and IDC-2 can be expressed in the following form in order to obtain a rule-model for independence of directions of components.

**Rule IDC-I:** "There are two simultaneous component velocities affecting a body; the direction of one of the components is changed due to an interaction with the other component."

**Rule IDC-II:** "There are two simultaneous component velocities affecting a body; the direction of one of the
components is unchanged despite an interaction with the other component."

6.2 **Summary of the Inferred Rules for All Subjects in the Sample**

The inferred rules for each vector characteristic held by each subject in the sample are presented in tabular form in Appendix D. This condenses into a summary format all of the data obtained from the use of the G.I.P. This was the data base used to obtain the frequency tables of subjects holding each of the inferred rules for each vector characteristic (as presented in Tables 6.1 to 6.14 in this chapter).

6.3 **General Discussion of Group Interview Technique and Results**

Given the results presented in this chapter, it would seem reasonable to assent that the Group Interview Technique is an effective method for uncovering subjects' preconceptions about a specific content area. This was indicated by two sources of data: first, consistent preconceptions for given subjects were found for vector characteristics which had three or more related questions; and second, most of the preconceptions identified were similar to the ones found in the individual interviews. While a few other consistent preconceptions, different
from the ones identified in the individual interviews, were found for some vector characteristics; it cannot be said that a group interview methodology is a better instrument to uncover subjects' conceptions solely because new conceptions were found. This could simply be a product of the size of the sample. That is, the chance of finding a more diverse number of preconceptions may increase with the size of the sample. However, even though the sample used in the second phase was eight times the size for the first phase only a few new consistent preconceptions were found. Furthermore, since the group interview did not permit the investigator to probe further into the students' responses for clarification, it is not surprising that there were some shifts in the types of inferred rules which were used and also that there were a larger percentage of non-classified responses.

As in the first phase, no significant difference was found between female and male subjects regarding the kinds of consistent preconceptions held. A chi-square test of independence performed on the data for each vector characteristic statistically showed no significant differences (Tables 6.1 to 6.14) in the response patterns between males and females.

In summary, the results on students' preconceptions about vector characteristics, as illustrated in this chapter, should provide further confidence in the inter-
view data presented in Chapter Four. While there was some deviation in the response patterns between the two data collection techniques, as noted above, the high percentage of correspondence of inferred rules between the two techniques certainly outweighs these discrepancies. Perhaps even more important than the level of agreement between the two techniques, however, is the overall finding that for most of the vector characteristics over 85%* of the subjects were found to possess consistent physical intuitions (preconceptions) about two concrete situations which could be analyzed in vectorial terms. Although the types of preconceptions held by these students varied from those using a simple analysis to those which entailed a fairly abstract analysis of the situation, the point to be emphasized is that this range of preconceptions ought to be recognized and acknowledged by both teachers and curriculum developers. These broad educational implications of the study will be discussed in greater detail in the next chapter.

*This percentage was estimated by adding the percentages of subjects holding the various inferred rules (consistent preconceptions) for each vector characteristic (e.g., see Table 6.1, the addition of percentages of subjects holding the various inferred rules makes 97%).
Chapter Seven

CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

7.00 Overview of the Study

The initial analytical procedure used in this study was to carry out a Rational Task Analysis on three vector quantities (i.e., vector position, displacement, and velocity). The result of this analysis (List of Implicit Vector Characteristics) was the basis for developing the six research questions addressed in the study. In answering these questions the research procedures were carried out in two phases. Phase One sought to identify and analyze students' preconceptions about the first ten implicit vector characteristics, listed in LIVC (Table 1.1), held by grade 10 students. The procedures used to investigate this problem consisted of individual interviews with students ranging in age from 15 to 17 years. The interview procedures, the tasks and interview protocol of the study were reported in Chapter Three. The analysis of these interview data was reported in Chapter Four.

Phase Two also attempted to uncover students'
preconceptions about these vector characteristics but employed a group interview technique instead of individual interviews. The group interview was used to obtain a larger sample of students and also to cross-validate the results from Phase One. To this end a paper and pencil instrument was constructed (G.I.P.), which consisted of a similar interview protocol to that used in Phase One. Questions related to the final two vector characteristics were also added. The group interview with the paper and pencil instrument was administered to eight grade 10 classes (176 subjects). The group interview procedures and related issues were reported in Chapter Five; the written instrument is given in Appendix C.

An analysis of the data collected in the group interview resulted in both the identification of preconceptions similar to the ones found in Phase One and a few new preconceptions. These new preconceptions were also expressed as inferred rules, and later as general rules to be included in the corresponding rule-model developed from the individual interview data. The analysis of the results for these group data were reported and discussed in Chapter Six.

7.1 Conclusions of the Study

A number of tentative conclusions can be offered in response to the six research questions that have
provided direction for the study. These conclusions are presented in the same order as the questions were introduced in Section 1.5 of Chapter One.

1) What preconceptions do students hold about each of the following implicit vector characteristics?

From the analysis of the interview data it was clear that the student sample possessed definite intuitive notions about most of the vector characteristics considered in this study. This was one of the broad assumptions made by the author prior to starting the research and actually corresponded to the general statement of the problem.

2) How consistent are the preconceptions employed by students when attempting to solve problems involving the listed vector characteristics?

The intent of this research question was to find out how stable or structured the preconceptions were held by subjects. The use of three or more questions for most of the vector characteristics in the interview protocol helped to shed light on the consistency of subjects' views. The students displayed a fairly high degree of consistency for most groups of questions and it would seem that these preconceptions are fairly stable, at least for the tasks employed in this study.

3) Can the identified students' preconceptions be categorized in some fashion?
Results of interview data analysis showed that there were not many distinct preconceptions or response patterns for each vector characteristic, -- the number ranged between two and five. Further analysis indicated that these preconceptions also varied in complexity, ranging from rather primitive conceptions to more abstract ones. The criterion used to categorize the preconceptions in terms of complexity was obtained from the Rational Task Analysis. The network of conceptual relationships developed from this task analysis provided the normative model for judging the preconceptions. On the basis of these judgements the preconceptions were also classified into three broad groups: scalar, transitional and vector.

4) Is there a significant difference between the preconceptions female and male subjects regarding the identified preconceptions?

A chi-square test statistic was used to test for possible sex differences in the types of preconceptions identified in both phases of the study. The calculated chi-square values showed that there was no significant difference between the preconceptions held by female and male subjects for any of the vector characteristics for either of the two phases.

5) How generalizable are the identified preconceptions (in phase one) to other groups of students?

Given the results from Phase Two of the study it would appear that the inferred rules identified using 20 individual interviews, can be cautiously generalized to a
wider population of grade ten. This is particularly true for some of the vector characteristics (e.g., Reference Simultaneity of Components, etc.) where there was up to 90% agreement in the data between the two phases. The agreement for other vector characteristics was not so high (e.g., Displacement, Addition of Displacements, Subtraction of Vector Positions, etc.) but this can be explained by the problems encountered in explaining the format of the response required for these more abstract vector characteristics. While the percentage of subjects who did not display consistent preconceptions was much higher in Phase Two than in Phase One, this result can also be explained in terms of the lack of communication between the interviewer and the subjects in a group interview setting. Furthermore, there were more time constraints in Phase Two as the group interview took place during a regular 50 minute period and in several classes there were many external interruptions.

6) Which identified preconceptions are predominant in the study sample?

This discussion considers only the sample of Phase Two. The Contingency Tables of Chapter Six showed the preconception that was most predominant for each vector characteristic; as a criterion the preconceptions that were subscribed by more than 50 percent of the sample are taken as the most predominant. Using this criterion, the following were the predominant preconceptions found for some vector
characteristics:

i) For Reference Point for Stationary Bodies:

"Use of one object as a reference to locate a body but distinct reference objects to locate other bodies" (RPS-2).

ii) For Composition of Velocities:

"The composition of two non-parallel velocities, which are simultaneously affecting a moving body, produces a resultant whose magnitude is different from those of the components and whose direction is in between the directions of the component velocities" (CV-3).

iii) For Independence of Magnitudes of Components:

"If there are two simultaneous and non-parallel component velocities affecting a body, the magnitude of the component with greater magnitude changes due to the interaction with the other component" (IMC-1).

iv) For Independence of Directions of Components:

"If there are two simultaneous and non-parallel component velocities affecting a body, the direction of the component with greater magnitude changes due to the interaction with the other component" (IDC-1).

v) For Simultaneity of Components:

"A single apparent movement of a body is the result of two component velocities that are affecting it simultaneously" (SC-2).

vi) For Independence of Locations from Path:

"The locations of instantaneous locations of a moving body can be described even when the path followed by the body is unknown" (ILP-3).

7.2 Educational Implications

The overarching question of this research was:
Why do students encounter difficulties in understanding vector quantities? The intention was not to fully answer the question, but to shed some light on possible causes for difficulties. Two other questions were posed in Chapter One. Are vector quantities too abstract to be grasped by school students? Are the difficulties which students experience with vector quantities, as reported in the literature, related to instructional procedures? While the data in this study cannot answer these questions, directly, the data can be used to speculate or conjecture about possible responses to these questions.

With regard to the first specific question, subjects in both phases of the study possessed consistent intuitive notions of the vector characteristics, -- with some of these notions being close to the formal properties of vector quantities. Given this information, it may be argued that these students would be able to readily understand problem situations involving vectorial analysis. However, some caution must be exercised. As was pointed out earlier these intuitive notions may well be very context-dependent and not directly applicable to other conceptual systems. But the majority of the preconceptions identified were much more simple or 'primitive than those introduced in formal physics classes. The widespread existence of these more commonsense beliefs (also found in many other science areas other than vectors) has lead some authors to speculate about how they may actually interfere with formal instruction. For example, Hobbs (1977) talks about "stumbling blocks"
while Hawkins (1980) discusses this issue in terms of "critical learning barriers".

Both of the above points, students' capacity to grasp some aspects of vector quantities and the interference of intuitive concepts with formal teaching, are related to instructional procedures. That is, the second specific question posed above. It may be that part of the difficulties encountered by students with vectorial concepts is due to the instructional methods employed by textbooks and teachers. It is this author's contention that if students' preconceptions were considered in the instructional process some of these difficulties could be overcome. Findings of the present study can certainly be used in that regard.

Another important educational implication of the study is that the students' preconceptions about vector characteristics identified in this study can be very useful in the preparation of teaching strategies. The actual preparation of strategies was not part of this study but the findings certainly could be used by either curriculum developers or classroom teachers in the preparation of specific teaching strategies for use in the area of vectors. These strategies would be based upon an understanding of the kinds of preconceptions currently held by the students with the intent of getting the students to alter their conceptions to a more encompassing view of the phenomena -- that is, to move toward the type of perspective currently used by physicists.
7.3 Recommendations for Further Research

The overall implications of this study could be greatly enhanced by a number of follow-up studies. This section includes a list of these broad problems to be addressed with brief comments on some of the issues accompanying each of these problem areas.

1) The most obvious follow-up study would be to test the generalizability of the proposed rule-models in other contexts and using more controlled experimental investigations. One type of experimental design for this research would be the Rule-Assessment Methodology proposed by Siegler (1978, 1979).

2) A second useful follow-up study would be to transform the paper and pencil instrument, used in Phase Two of the study, into more of a standardized questionnaire. The purpose would be to provide the curriculum maker and science teacher with a statistically valid and reliable instrument, which can be used to obtain results that can be generalized to well-defined populations. The instrument prepared for the study (the G.I.P.) could very well be used as the basis to carry out the proposed psychometric procedures in the area of vectors, however, if a new context were to be explored, new items would have to be prepared.
In summary, the goal of this type of work would be to produce instruments with high external and internal validity and high reliability.

3) If the type of work carried out in this type of study is to have any educational impact then it is important that a development program be undertaken to prepare specific teaching strategies based upon the preconceptions identified in this study. One approach for such a program has been proposed by Erickson (1979). It essentially consists of four kinds of teaching maneuvers: experimental, clarification, anomaly, and restructuring. The main purpose of these maneuvers is to produce a 'conceptual shift' (or "paradigm shift" using Kuhn's terminology) from the students' more primitive preconceptions to ones that more adequately explain the phenomena being studied.

4) A natural follow-up study to the work just described would involve the design of a research program to assess the effectiveness of these strategies using standard experimental techniques. That is, an experimental group would be taught using the teaching strategies based upon their prior knowledge, whereas a control group would receive the standard textbook treatment. This type of study should illustrate the influence of students' preconceptions on
understanding vector quantities and also shed further light on some of the difficulties encountered by students in comprehending these quantities.

5) Other recommended follow-up studies although not directly related to substantive issues of the present one, would entail the use of rational task analysis for other content-areas, -- specifically content-areas in which students encounter conceptual difficulties. One of the values of using R.T.A. is that of making explicit those aspects that are not usually discussed in school curricula and text-books. In some instances it appears as those both teachers and textbooks assume that the students already understand certain background skills or knowledge. In other cases it is questionable whether the many implicit characteristics of some science concepts are even considered by textbook writers or teachers. A possible program of R.T.A. is the one used in this research which was delineated in Chapter Three. Other programs are discussed by Resnick (1976) and Siegler (1980).

In conclusion, the author feels strongly that it is exactly the kind of research presented in this thesis and the recommended studies for further research which have the greatest potential to make significant contributions to the overall
instructional process in the area of science education. To acquire a real understanding of the students' prior knowledge in particular content areas and, then, through appropriate instructional techniques, to bring about a fuller and more complete appreciation of that area is this author's view of effective instruction.
BIBLIOGRAPHY


Wollman, W. Developmental Implications of Science Teaching. Early Adolescence. ERIC Clearinghouse for Science, Mathematics and Environmental Education. The Ohio State University, Columbus, Ohio, 1978.
APPENDIX A

The definitions of the following vector quantities: Vector Position, Displacement, and Average Velocity.

Definition of Vector Position (in a two dimension situation)

A vector position determines the location of a body with respect to a reference point* by providing a magnitude and a direction. The magnitude is given by the length of the straight line distance between the reference point and the body. The direction is given by the angle formed by the straight line mentioned above and another arbitrary line that passes through the reference point (see Figure A.1)

Definition of Displacement (in a two dimension situation)

A vector displacement determines the locations of a later position of a moving body with respect to an earlier one by providing a magnitude and a direction. The magnitude is given by the length of the straight line distance between the later and earlier positions. The direction is given by

*The reference point, plus the arbitrary but fixed straight line that passes through it, constitutes a frame of reference. More than one arbitrary straight line passing through the reference point may be used as a frame of reference (i.e., The Cartesian system consists in three perpendicular lines passing through the origin or reference point). If the arbitrary but fixed straight lines are labelled with real numbers, the frame of reference constitutes a Coordinate System.
the angle formed by the straight line mentioned above and another arbitrary straight line (see Figure A.2).

Figure A.2

Definition of Average Velocity

A vector average velocity determines the rate of change of position by providing a magnitude and a direction. The magnitude is given by the ratio of the magnitude of the vector displacement associated with the change of position to the time elapsed for that change. The direction corresponds to that of the vector displacement.
These data correspond to the responses to the questions related to the following vector characteristics: Reference point for stationary bodies (RPS), and frame of reference (FR).

Format of Data Presentation

The data is presented in tables with 5 columns.

Each column contains the following information:

**First column:** name of subject, sex, age in years, and experience with boats. This last aspect is represented by two letters: Y and N; Y stands for Yes, meaning that S has had experience with boats, and N stands for not, meaning that S has not had experience with boats. There are 2 letters, the first relates to experience with boats on lakes and the second to experience with boats on rivers. (e.g., Dale, the first subject in the table, has had experience with boats on lakes and rivers; then, it is YY).

**Second column:** Number of the question of the interview protocol.

**Third column:** Responses of subjects to questions presented in the second column. Other description, further explanations, researcher's remarks, and rephrasing of questions are presented in parenthesis.
Fourth column: Researcher's analysis and comments on subjects' responses (empirical task analysis). Results of analysis and comments are presented for each vector characteristic. The vector characteristics are abbreviated by the first letters of its name. The referents are the following:

RPS : Reference point for stationary bodies.

Fifth column: Preconceptions found for each vector characteristic. The search for consistent preconceptions is done by comparing the preconceptions contained in the responses for all questions of a vector characteristic. The consistent preconception is expressed as an inferred rule.

Questions of Interview Protocol

It follows the questions for each vector characteristic presented in this appendix.

Questions for Reference Point for Stationary Bodies (RPS)

Q. 1: How would you describe the location of the first fishing spot to your friends?

Q. 4: How could you describe the location of this second fishing spot to your friends?

Q. 5a: How could you describe the location of this last fishing spot to your friends?

Questions for Frame of Reference (FR)

Q. 1: Same as above.

Q. 3: How could you make the description of this location more precise for your friends?
Q. 4: Same as above.

Q. 5b: These two fishing spots (the first and the last) are at the same distance from you. How could you distinguish them?
<table>
<thead>
<tr>
<th>Name of Subject, Sex, Age, Experience with Boats</th>
<th>Question</th>
<th>Subject’s Responses</th>
<th>Researcher's Analysis</th>
<th>Identified Preconceptions and Inferred Rules</th>
</tr>
</thead>
</table>
| 1) Dale M, 15, YY | 1 | You can use some landmark here and here also it is a couple of hundred yards from the dock (wharf) and it is in front of a little bay on the side here (right side of S) | RPS:  
- Use of several reference points simultaneously (tree, wharf, dock, bay)  
- S do not use his location as one of the R.P.  
- Tendency to use the starting point as a R.P. | RPS:  
- "Simultaneous use of four objects as a reference point". |
| | 3 | If I look from here the boat is right down (showing direction with hand) on the right corner of the lake and in straight angle and straight across the lake. | FR:  
- Distance measured from the starting point (wharf)  
- Direction mentioned quantitatively (using hand, in front of, straight across)  
- Different frame of reference to measure the distance (wharf) and the direction (dock). | FR:  
- "Simultaneous use of distance and qualitative direction with difference reference points criterion". |
| | 4 | It's just past of the last fishing spot, in the same angle, a couple of hundred of yards from me and it is right in front of this corner (What do you mean by the same angle?)... oh, if you look from here ... the last fishing spot is right up on the corner the second spot is just behind the first one farther toward the other end of the lake. | RPS:  
- Use of 3 RPs simultaneously (1st spot, dock, corner) | RPS:  
- "Simultaneous use of three objects as a R.P.". |
| | 5a | It's just next to the shore and it's right on that bay there on the left side of the lake (could you estimate the distance from you to the blue buoy?) If the lake is 1 mile (and the distance from you to the buoy?) ... about the same (How could you distinguish them?) One is closer to the shore and the other closer to the dock and the middle of the lake (any other way to distinguish them?) ... one is at one side and one on the other | FR:  
- Distance: S-P2  
- Qualitative direction  
- Common RP to measure distance and direction | \begin{align*}  
RPS: \quad \text{"Simultaneous use of two objects as a R.P."}  
\quad \text{Inferred Rule for RPS}  
\quad \text{(Derived by comparing the responses to Q. 1,4,5a) "Each one of the locations of the fishing spots is described using several reference bodies or places" (RPS-1)}  
\quad \text{FR:}  
\quad \text{"Simultaneous use of distance and qualitative direction with uncommon RP".}  
\quad \text{Inferred Rule for FR}  
\quad \text{(Derived by comparing responses to Rs: 1,3,4, and 5b) "Precise description of each one of the locations of the fishing spots is done by use of both quantitative direction as coordinates but two reference points are used: one for the distance and another one for the direction" (FR-4)}  
\end{align*} |
Could I describe it from this (wharf) or just from here ... you never know if we could know how many miles (you could estimate : the lake is = 1 mile long and .5 miles wide) ... it was directly in front of the little dock (wharf) and about 1/6 of the length of the lake ... (where from are you describing it?) ... from the little dock (wharf) (... but you are in that dock) ... I would tell them to go to the little dock and go straight the same distance as the length of the dock ... from where I am it is 45° angle and about double the length (why do you think about the 45°?) ah... 90° is right angle ... and it will be half. (where is the right angle?) ... It's in here (she shows:

(... you were saying 45° from you, what else could you tell them?) ... (Long pause ... to the right ... (That's fine))

I'd say to them - go to the little dock and go straight ahead ... about 1/6 on the water and then turn slightly to the right and keep going until you are only half way on the lake ... near the centre ... in front of the tree

I'd say, go to the left and keep going along and when you get the first curve there it is, it is very close to the shore.

(How could you distinguish the 2 locations?) ... Well, one is farther out than the other, almost 1 mile difference, one is at the middle and the other one is close to the shore (Any other way to distinguish them?) ... (Long pause)...
<table>
<thead>
<tr>
<th>3) Suzanne</th>
<th>1</th>
<th>... Straight in front of the wharf ... you probably have to say the degree ... or a map</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>... just to tell or go straight or go with them ... (pause)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>It's sort of on the middle but it's closer to the wharf - there are 2 buoys on the water, tell them where the first one is and tell them that the second one is farther but not much and it's closer to the tree.</td>
</tr>
<tr>
<td></td>
<td>5a</td>
<td>It's right off the side, to my left ... and it is past the green buoy</td>
</tr>
<tr>
<td></td>
<td>5b</td>
<td>It's right off the side, to my left ... and it is past the green buoy</td>
</tr>
<tr>
<td></td>
<td>5b</td>
<td>... one is at angle (what do you mean?) ... well, one is to your right and the other to the left ... on the right side is at an angle ... the other one is a bit at an angle.</td>
</tr>
</tbody>
</table>

| RPS: | - Use of one RP, the SP (wharf) |
| FR:  | - no distance mentioned - direction mentioned qualitatively even when S mentions angle |
| RPS: | - Use of three RP - Use of first spot as a RP |
| FR:  | - Use of qualitative distance - Use of qualitative direction |
| RPS: | - Use of two RPs - Use of observer's location as one of the RP |
| FR:  | - Use of qualitative direction (at an angle. This means that it is not straight in front of) - Use of qualitative distance |

<table>
<thead>
<tr>
<th>Inferred Rule for RPS:</th>
<th>- &quot;Use of one object as a RP: starting point&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inferred Rule for FR</td>
<td>- &quot;Use of only qualitative direction&quot;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4) Dwayne</th>
<th>1</th>
<th>It's close to the dock (wharf) ... it's a couple of hundred yards from the dock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>... (long pause) ... I don't know</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>... (it appears that he refers the second fishing spot to the first one. Not clear)</td>
</tr>
<tr>
<td></td>
<td>5a</td>
<td>... (he describes all the locations of the boat. It seems that he refers this spot with respect to the other locations)</td>
</tr>
<tr>
<td></td>
<td>5b</td>
<td>... (long pause) ... (any idea?) ... not</td>
</tr>
</tbody>
</table>

| RPS: | - Use of only one reference point, the SP |
| FR:  | - distance between RP and the spot - no direction mentioned |
| RPS: | - Use of 1st spot as a RP |
| FR:  | - No use of distance - No use of direction |

<table>
<thead>
<tr>
<th>Inferred Rule for RPS:</th>
<th>- &quot;Use of one object as a RP: starting point&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inferred Rule for FR</td>
<td>- &quot;Use of only distance&quot;</td>
</tr>
</tbody>
</table>

Inferred Rule for RPS: 
"Use of three objects as reference points" 
Inferred Rule for FR: 
"Simultaneous use of qualitative distance and qualitative direction with different RP"
5) Larry
M, 16, YY

1... I could give them the distance from this dock (wharf) in front of it, and that there is a tree to the right

RPS:
- Use of several reference points simultaneously (wharf, tree)
- tendency to use the SP as one of the RP
- no use of observer's location as a RP

FR:
- Use of distance from the SP (wharf)
- direction mentioned qualitatively (in front of, to the right)
- Use of same F of R for distance and direction.

2 It is exactly in the middle of the lake. If I am in this dock and you look at that tree the boat is in the middle of them.

FR:
- "Simultaneous use of distance and qualitative direction with common reference point for both".

3 I will tell them: Find the tree and this dock (wharf), the straight line coming from that tree to here (location of fishing spot) and back to the dock, and there is the spot, about 30 feet from the tree.

RPS:
- Use of 2 RPs

FR:
- Use of distance
- Use of qualitative direction

RPS:
- Use of two RPs.
- Use of SP as one of the RP

FR:
- Use of quantitative distance
- Use of qualitative direction
- Use of different RP to refer to distance and direction

5a... from the small dock go to the left side and on the small... (bay) is a good fishing spot

RPS:
- "Use of two objects as reference points"

5b... tell them they are both 200 yards and one is on the left side of the coast and the other in the centre of the lake... well, you first find the spot near the shore and then tell them that the other is about 200 yards toward the middle

RPS:
- "Use of several reference points simultaneously (wharf, tree)
- use of the SP (wharf) as one of the RP's. 
- no use of observer's location as a RP.

FR:
- Use of distance and measured from the SP

6) Russ
M, 15, YY

1 Look at the dock (wharf) look where the boat is and guess how many yards it is away ... also look at from the tree ... it is just across from the tree ... (you say about yards?) ... yes, guess how many from the dock to where the boat is.

RPS:
- Use of one RP (maybe it is because it is very close to a peculiar place: shore)

FR:
- "Use of one object as reference point"

3 not, what I said before only.

4 I guess it is in straight line from the tree and in straight line from this dock. (wharf) (Could you describe it in another way?)... that position is difficult. I don't know what to tell them.

FR:
- "Use of qualitative direction only"

5a... we need a landmark over there to describe it

FR:
- "Use of qualitative direction"

5b... first I would tell them the distance (but these 2 spots are at the same distance from you?) ... well, tell them there is 4 mile to the red buoy and to the other buoy...

RPS:
- Use of one RP (maybe it is because it is very close to a peculiar place: shore)

FR:
- Use of distance
- Use of qualitative direction

Inferred Rule for RPS:
"RPS-1"

Inferred Rule for FR:
"FR-4"
7) Sandy
F, 15, YY

Tell them to go to the end of the wharf and it is straight out (what else could you tell them?) ... how far it is

... well, how far it is

... a little farther than the first spot from the wharf and it's more to the right

5a ... It's right close to the shore ... (what else?) ... I don't know

5b ... to tell how far the spots are we have to know how long the lake is (it's about 1 mile long) ... (5b)... one is at the edge ... and the other one is at the centre ... (any other way to distinguish them?) ... I don't think so ...

Inferred Rule for RPS:
"Each one of the locations of the fishing spots is described by using only one reference point and the same one for each location" (RPS-3)

Inferred Rule for FR:
"Precise description of each one of the locations of the fishing spots is done by use of both quantitative distance and qualitative direction as coordinates with the same reference point for both distance and direction" (FR-5)
Tracy, F, 15, YY

1. I'll probably point them and say as many feet that way... I can say 40 feet from the dock... to the right or to the left or whichever way will be... In this case it will be about 20 feet from the dock (wharf) to the right.

2. I would say how many degrees but I wouldn't do that (why?)... because it would not occur to me at that time. (What about if you decide to use?) How could you use them... you have to know it and I don't know it... if I knew it I would use it.

3. ... I could say 20 feet pass the red one... from here (the dock) about 100 feet or something like that.

4. ... I would say 40 feet from the dock... to the right or to the left... this one is beside the shore and the other one is out in the water.

5a. I'd say it's near that "bay" then, ... very near the shore line there.

5b. One is on the right side and one on the left... this one is beside the shore and the other one is out in the water.

RPS:
- Use of two RP's: dock and wharf
- Use of the SP as one of the RP
- Use of observer's location as one of RP

RP:
- Use of distance measured from the SP and dock direction mentioned qualitatively (that way, to the right, degrees?)

RPS:
- Use of two RP

RP:
- Use of two RP: the closest one

PB:
- Use of qualitative direction
- Use of qualitative distance

Inferred Rule for RPS:
- "Simultaneous use of the objects as reference points"

Inferred Rule for FR:
- "No Rule" (NR)
You can tell how far off from the dock it is, in what direction, or you could look from here and look across and see if there is any landmark on the other side and you can go straight toward that ... or you can time how long it took to the spot out (where from are you describing it?) From this dock (S) and you can judge how far the dock is or you can time it and in what direction they went from this dock (wharf), you can see from here in the direction of the tree (you say direction: how could you describe that direction?) ... to the right or north of the east.

It'd be directly to the northeast about 50° to the N.E. straight out from the other one (first spot). Usually when I am in the water I use a compass ...

... from the shore line, or you can do it. How far is it from here (dock) ... from here (the dock) is 150 yards off shore ... or you could say it is about 25 yards from there (a point near the shore) ...

... this one is really close to the shore, that one is out just in the middle of the lake ... (Any other way how do you tell them apart?) ... not, only how far is it from the shore and the other in the middle ... or one is on the left and the other on the right.

FR:
- Use of one RP: Dock
- Use of distance
- Use of qualitative direction

RPS:
- Use of one RP: dock or wharf
- Use of observer's location as a RP

FR:
- Use of distance and measured from the dock
- Direction mentioned qualitatively (right, north of the east)
- Use of same F of R (dock) to measure distance and direction.

RPS:
- Use of one RP (1st. spot)

FR:
- Use of quantitative direction (50° north-east)
- No use of distance in Q.4

RPS:
- Use of one RP: Dock

FR:
- Use of distance
- Use of qualitative direction

Inferred Rule for RPS
"RPS-3"

Inferred Rule for FR
"RF-5"
<table>
<thead>
<tr>
<th>1</th>
<th>... I'd tell them to go from this point (wharf) our straight ... in an angle ... a little bit in an angle (What do you mean by angle?) ... don't go perfectly straight ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>... I'd try to line it up with trees and banks ..., about 4 points ... (why 4 points?) ... because it gives more direct center</td>
</tr>
<tr>
<td>4</td>
<td>... Turn the boat in angle and head for trees or whatever landmark and stays off from the shore certain distance</td>
</tr>
<tr>
<td>5a</td>
<td>... go to where the curve is and go about 15 feet and there it is (boat leaving from that shore)</td>
</tr>
<tr>
<td>5b</td>
<td>Pointing them the direction in which they should go (Any other way to explain them?) ... (pause) ... not</td>
</tr>
</tbody>
</table>

**RPS:**
- Use of one RP: wharf
- use of SP as a RP
- no use of observer's location as a RP.

**FR:**
- distance not mentioned
- direction mentioned qualitatively (a bit in an angle ≠ don't go perfectly straight)

**RPS:**
- Use of one RP, the closest to the spot

**FR:**
- Use of qualitative direction
- Use of distance with respect to the closest RP

**RPS:**
- Use of one RP, closest to the spot

**FR:**
- Use of distance measured from the closest RP chosen
- Use of qualitative direction (hand-pointing)

**Inferred Rule for RPS**
- "RPS-2"

**Inferred Rule for FR**
- "FR-4"
<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Location &amp; Description</th>
<th>RPS:</th>
<th>FR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>11) Preston M, 16, YY</td>
<td>... from here (the dock) ... I'll probably use degrees ... maybe 40° north from where I am standing here ... oh, I could use the time clock and it'd be around 2 o'clock from here (the dock) ... (any other way?) ... I can't think of any other...</td>
<td>RPS: - Use of one RP, the dock - Use of observer's location as a RP - Name of SP as a RP</td>
<td>FR: - distance not mentioned - direction mentioned quantitatively (40° north; 2 o'clock)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>From here (the dock)... it'll be about 15° or 5° to the north (Where is the N?) or 1 o'clock toward there or using the wharf it is straight across ... it is in the indentation of the land ... (pause) ... or at 9 o'clock using that direction as 12 o'clock it'll be in the south and it'd be about 180° toward the south from here</td>
<td>RPS: - Use of one RP (dock)</td>
<td>FR: - Use of quantitative direction (15°, north)</td>
<td></td>
</tr>
<tr>
<td>5a</td>
<td>... it'd be 200 m to the north west and 200 m to the S-E (What did you mean before by degrees?) ... oh, by degrees I mean: this is my zero line and my fraction is going up like this... This will be fraction line 10°, 20°, etc. 90°.</td>
<td>RPS: - Use of quantitative direction - No use of distance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>... from the shore ... how far out is from the shore ... what do you mean? (Q. is rephrased) ... I don't know ... maybe how many metres across are (distance between the two fishing spots)...</td>
<td>RPS: - Use of one RP: shore across the lake from the wharf</td>
<td>FR: - Use of distance and measured from shore across the lake - direction not mentioned</td>
<td></td>
</tr>
</tbody>
</table>

Inferred Rule for RPS

RPS-2

Inferred Rule for FR

FR-2

12) Hans M, 17, YY

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Location &amp; Description</th>
<th>RPS:</th>
<th>FR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(Video tape did not record the beginning of interview) How far away are you from the shore (what shore)? that one by the forest (other end of lake) ... how far it is away from me ... how far is it away the other fishing spot... (anything else?) ... look for something around it (there is only water around) ... (long pause) ... from the shore ... how far out is from the shore ... what do you mean? (Q. is rephrased) ... I don't know ... maybe how many metres across are (distance between the two fishing spots)...</td>
<td>RPS: - Use of one RP: shore across the lake from the wharf - Use of one RP: the closest shore</td>
<td>FR: - Use of distance from spot to closest SP - No use of direction</td>
<td></td>
</tr>
</tbody>
</table>

Inferred Rule for RPS

RPS-2

Inferred Rule for FR

FR-1
13) Becky
F, 15, YN

... just about half way to the middle, half way to the right side ... and I have to try to Point to the spot if I can't try to describe it in more de­tail ... it's about 50 feet from me ... (anything else that it could help your friends to find the place?)... (long pause) ... maybe seeing a fish jumping ... I think it will be more in a bit of an angle from here (What do you mean?) ... for a person being here (on the dock) it looks like in an angle than from seeing from the tree

5a ... it is near the shore on the left side of the lake ... it's not that far out, just about 5 feet from the edge

5b ... you could say in an angle and 50 feet (spot on right side of subject) and you could point and the other in the corner of that (bay?) ... (How could you talk about angle?) ... you could say about 120° (degrees) angle if you start from here 

You can use a protractor and you start from this side counting (left side, of S) and it is in front diagonal with the far corner in the other end of the lake (What about the other one?) ... It's about 75° (degrees) angle.

14) Steve
M, 15, YY

I'd say: start from the dock (wharf) there and go out about 70 degrees angle to almost be parallel to the tree ... (anything else you could tell your friends?) ... straight in front of this curve out is (a curve on the shore)

When you are in the first fishing spot turn toward the shore ... from where I am standing it will be right behind it the first fishing spot ... they will be in line from this point of view
... first end up in the bank and parallel to the first one. I guess ... In the first curve in that bank (What else could be important and to mention to your friends?)... (pause) ...

that fishing spot is 200 m to the right... about 70 degrees angle... and the other one is 200 m on the left... about 70 degrees angle from this point of view (dock)... (Both are 70°?)... maybe this one (left side) is a little more... 75° or 80°... (from where you measure these angles?)... from right here... 90° is straight ahead

... I would point something in the other side, like exact on the opposite, tell them to sail toward that and they should see the buoy

I can tell them the way the boat went, in a kind of a circle ... all I can think is to give them the direction in a kind of landmark (What kind of landmark?) ... tree ... it has to be a big one in such a way you could see it.

... from here it will be a little bit farther, farther that way (pointing with finger) it is a little bit closer to the shore in that end of the lake (What do you mean by that way?) ... in a kind of diagonal with respect to the first fishing spot

It is very close to the shore. I can say the distance along the shore (he shows the distance along the edge of the lake, from where he is, to the point on the shore in front of the fishing spot)

I could say go 200 m towards the spot along the shore (he meant the spot closer to shore), go straight toward that spot. (How will you explain the location of the other spot?) ... the spots are in different directions, one is that way and this is this way... if they go to this spot first I would say travel straight out that way and pick something along the shore ... and for this one is right along the shore... pick also something a big rock along the shore
1. If you are here (dock), you could look at the boat, make an imaginary line from yourself to the boat, and then from yourself you look at across of the lake to another object like a tree, something that is distinguishable, then you have that imaginary line which you could show to your friends, you could also walk to here (wharf) when the boat is still there (wharf) and then makes another imaginary line to the other side (of the lake), you make a point at the center (where both imaginary meet), then you have an idea where it would be.

3. If you have a compass you could establish which direction is north, then you could see how many degrees is that line (the imaginary line) away from north ... (anything else important to consider?) ... look for something that is distinguishable that you could make out if you don't have a compass.

4. If you know where the first fishing spot is, you make a triangle and measure the angle you could do the same for the other spot, make a triangle using the two spots and you, then, measure the angle. (Suppose you don't have the first fishing spot!) ... then you do the same thing. You use yourself, the spot and the tree again.

5a. This spot appears to be in line with the tree, you could again, if you have a compass, measure from yourself to there (last spot) and from there to there (dock to tree) that angle (how will you measure that angle?)... with the compass you find your north, then if you have a tape ... with a sight ... to the other point (last spot) measure the angle, then subtract them.

5b. From here you take a sighting and measure the angle, suppose the tree is at north, suppose it is 50°, then, if you look at that one it is about 190°.
| 17) Lori  
F, 15, YN | 1 | ... about halfway across the lake, straight ahead off the wharf |
| | 3 | ... I don’t know ... |
| | 4 | ... start from the dock, facing the other direction (direction toward the other end of the lake) go to the right about 1/3 with respect to the shore line, about 1/3 of the lake across, about halfway across the lake, 1/3 away from that shore |
| | 5a | ... start from the dock again, this time go to your left (her left) and go directly to the shore on the other side about half-way up, the middle of the lake |
| | 5b | ... one is to the right and one to the left, and one is closer to the shore... (Any other way how to distinguish them?)... well, one area is deeper and the other one is shallower... respect to this wharf, that one is closer to the other one |

**RPS:**
- Use of one RP: wharf  
- use of the RP as one of the RP  
- no use of observer’s location as one of the RP  

**FR:**
- Use of a kind of coordinate system: the spot is where two lines meet (one coming from the wharf and the other from the tree)  
- direction mentioned qualitatively (straight ahead off wharf)  
- Use of distance  

**RPS:**
- Use of several RPs  

**FR:**
- Use of qualitative direction  
- Use of distances measured from several RP  

**RPS:**
- Use of several RPs  

**FR:**
- Use of qualitative direction  
- Use of distance and measured from middle of lake  

Inferred Rule for RPS

"RPS-1"  

Inferred Rule for FR

"FR-2"
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th><strong>RPS:</strong></th>
<th><strong>FR:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>It's line up with this dock (wharf) ... it is a 100 feet out and parallel to the side of the dock (wharf) ... it's not quite in the middle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>... the tree could help ... are there more trees... (yeh, there are more)... if it were a special tree, like a big one, that is like 90 degrees there</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>... you mean from the first spot as they have already been there ... (yeh, or other places!)... if they are at the first spot you keep lining up to another tree or look on the bank over there (shores on the right side of her) and say go so far in that direction 20 feet or something toward that tree</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5a</td>
<td>...that spot is so close to the bank. Starting out from the dock (wharf) you say, from the last spot? (It's up to you!) ... either way you could say...maybe there is a land-mark here (shore in front of spot)...and tell how far is or say drive so many minutes at certain speed.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>...one is more on the middle of the lake and the other is closer to the bank, from where I am, one is to my right so far out and the other to my left, but this one is in the opposite it's coming from that side of the lake</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Inferred Rule for RPS**

"RPS-1"
| 1 | ... from the wharf is about 3 or 4 minutes out ... about 90° angle, from the tree to the shore as it came out. (How do you measure the minutes?)... you see your watch and watch and see how it takes to go out. |
| 3 | ... I don't know ... |
| 4 | ...it's closer to the shore than the other one, if you are looking from the opposite shore (shore in front of the wharf) is about 25° angle toward the side there it's closer to the tree and straight out from the tree. |
| 5a | ...it's very close, suppose that is north, to the west. shore, if you just follow the shore along it's about the middle of the shore line. |
| 5b | ...well this is about 200 m but it is in the middle of the lake and this one is very close to the shore (any other way to distinguish them?) ...not, I can't think any other way... |

| RPS: | Use of two RP: wharf and tree. |
|      | Use of SP as one of the RP. |
|      | No use of observer's location as one of the RP. |
|      | Use of other objects as RP (shore near the boat). |
| FR:  | Use of time as a variable (2 or 3 min. from wharf). |
|      | Direction mentioned qualitatively (90° angle). |
|      | Distance not mentioned clearly. |

RPS: Use of several RPs. FR: Use of qualitative distance. Use of qualitative direction. Use of quantitative direction (25°) not clear.

RPS: Use of one RP. FR: Use of closest objects to the spot as a RP.

FR: Use of qualitative distance. Use of different F of R to refer to distance and direction.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Kelley F, 15, NN</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>... find a point in that shore (shore in front of the wharf) that is line with the spot ... then, guess the distance (What distance?) ... from me to the spot</td>
<td>RPS:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of two RP: observer's location and a point on the other end which is in line with spot and observer's location</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- use of observer's location as one of the RP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- no use of SP as a RP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FR:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of distance from observer to spot</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- use of direction not clearly stipulated (2 points in line with the spot)</td>
</tr>
<tr>
<td>3</td>
<td>... I don't know any other way</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>... do they know where the first fishing spot is (you can use that but you can tell only the location of the yellow spot)... in the same way ...if I am on the dock, I guess from the tree so far ... it is about 60 yards out and then so far out from where I am ... (do you want to include both distances or only one?)... I guess both</td>
<td>RPS:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of two RPs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FR:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of two distances, measured from different RPs</td>
</tr>
<tr>
<td>5a</td>
<td>... how far away is it from where I am...</td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>... I don't know... from the angle that they are at... you could say 200 m in one direction... or you make an isosceles triangle and make three sides 200 m, then they will be equal...</td>
<td>RPS:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of one RP: the observer's location</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FR:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of distance, measured from observer's location to the spot</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use of qualitative direction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inferred Rule for RPS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&quot;RPS-1&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inferred Rule for FR</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&quot;FR-1&quot;</td>
</tr>
</tbody>
</table>
This questionnaire is not an examination but we are interested in your ideas about two common situations which many of you may have already experienced. These situations will be demonstrated for you by showing films, and explaining the situations to you verbally. Finally you will be asked to briefly answer and explain some questions about these situations. Enough time will be allowed to explain your answers.

First, we would like to know if you have had the following experiences:

(Answer yes or no)

1) Have you ever taken a ride in a boat (row or motor) on a lake? Ans: ________

2) Have you ever taken a ride in a boat (row or motor) on a river? Ans: ________

3) Have you ever driven a motor boat on a river? Ans: ________

The first situation will be explained next.
Situation One

"Imagine that you are standing on a dock of a calm lake waiting for a couple of friends who are coming later to fish. Suddenly you see another couple getting ready to go fishing in a row boat. You decide to observe them and keep track of the spots where they are catching fish. You will see the trip of this couple in this film; the camera to shoot the film was put on the same dock where your are; so what you will see in the film will correspond to what you would see if you actually were on the dock. There is also here a cardboard model of the lake, in which I can show what you will see in the film. The lake is approximately $\frac{1}{2}$ km long and $\frac{3}{4}$ km wide.

The first question is an example to show you how you should answer the questions

Question 0:

How would you describe to your friends the time it might take them to reach the first fishing spot?

Response: __________________________________________

________________________________________________________________________

Explanation: __________________________________________

________________________________________________________________________

When a question requires an explanation it is explicitly asked, otherwise you provide only the response.
Question 1:

You are here on the dock and suppose the first fishing spot found by the couple is here (P1). How would you describe the location of this spot to your friends?

Response:


Question 2:

How could you make your description more precise in such a way that your friends can be sure to find the same fishing spot?

Response:


Question 3:

How would you describe the location of this second fishing spot (P2) to your friends?

Response:


Question 4: How would you describe the location of this last fishing spot (P4) to your friends?

Response:


Question 5:
You can observe that the distance from the dock to the first fishing spot (P1) and to the last fishing spot (P4) is about the same. How could you differentiate the two locations?

Response: ________________________________

Question 6:
A diagram of the lake with all fishing spots is provided below. Draw or sketch the path followed by the boat in its round trip?
Question 7:

Is it necessary to know the path followed by the boat to describe a particular location of it?

Response:

Explanation:

Question 8:

When the boat has moved from the wharf (P₀) to the first fishing spot (P₁) it has had a change of location. How would you describe the location of the first spot (P₁) with respect to the wharf (P₀)?

Response:

Question 9:

You have just described a change of location, a movement from P₀ to P₁. Does a change of location tell anything about the length of the path followed by the boat between the two locations?

Response:

Explanation:
Question 10:

Then, the boat moved from the first spot (P1) to the second (P2). How would you describe the location of the second spot with respect to the first one?

Response: ________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

Question 11: Do you need to know the actual path followed by the boat to locate the second stop of the boat?

Response: ________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

Explanation: _____________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________

Question 12:

In the round trip of the boat there were several "changes of location", the boat moved from one spot to another. What aspects do you consider important to mention when describing a particular change of location?

Response: ________________________________________________________

______________________________________________________________

______________________________________________________________

______________________________________________________________
Question 13:

Suppose you have estimated the distance from $P_0$ (the wharf) to $P_1$ (first fishing spot) 200 metres, and from $P_1$ to $P_2$ (second fishing spot) in 150 m. Suppose your friends would like to go directly from the wharf ($P_0$) to the second fishing spot ($P_2$). Could you use this information to tell them how to get there?

Response: ____________________________

Explanation: ________________________

Question 14:

The estimated distance from $P_2$ to $P_3$ (third fishing spot) is 200 m, and $P_3$ to $P_4$ (last fishing spot) is 250 m. Suppose your friends are at $P_2$. Could you tell them how to go directly from $P_2$ to $P_4$?

Response: ____________________________

Question 15: Suppose you have estimated the distance from where you are standing on the dock to all fishing spots. The estimated distance from the dock to $P_1$ is 250 m; and from the dock to $P_2$ is 400 m. Assume your friends are at $P_1$. Could you tell them how to go directly to $P_2$?

Response: ____________________________

Explanation: ________________________
Question 16:

If the estimated distance from where you are standing on the dock to P₃ is 450 m. Could you tell your friends how to go directly from P₁ to P₃?

Response:

Situation Two:

"Imagine now that you are standing on a bridge of a wide and calm river. You are at point S on the bridge observing your friends, who are taking a ride in a motor-boat on the river. The speed of the motor cannot be changed so the speed of the boat due to the motor remains constant. Again, you decide it would be interesting to watch your friends in the motor-boat. A film will show you the motor-boat just as if you were actually on the bridge. There is also a cardboard model of the river. As before you will be asked some questions about the boat in the river".

Question 17:

Was the motor-boat you saw in the film moving with reference to anything?

Explain your Response:
Question 18:

Was the motor-boat moving with reference to the water? (Consider that the motor was constantly running)

Response and Explain:

If you think the boat was moving with reference to the water, could you explain if the movement of the motor boat with reference to the bridge and with reference to the water is the same or different?

Question 19:

If the motor-boat was moving upstream against the current. What factors would be influencing the overall movement of the motor-boat? 

Response:

Question 20: If the motor-boat was moving downstream with the current, what factors would be influencing the overall movement of the motor-boat?

Response:
Question 21:

Suppose now that your friends on the motor-boat want to cross the river leaving from dock 1 and heading in the direction shown in the diagram provided below. Draw the path on the diagram that you think the boat might take.

Explain the reasons for choosing the path you just drew:
Question 22:

Copy the path, you drew in the previous question, in the diagram below. Choose three well-spaced points in the path, draw the way in which the boat may be heading when crossing the river (i.e., ▼)  

If possible explain the reasons for your drawings:
Question 23:

For the case of the motor-boat crossing the river, what factors are influencing the overall movement of the motor-boat?

Response:

---

Question 24:

Was the movement of the motor-boat you saw in the film with reference to the water or with reference to the bridge?

Response:

---

Explanation:

---

Question 25:

Was the movement of the motor-boat you saw in the film (motor-boat crossing the river) due to the effect of the motor, due to the effect of the current, or both?

Response:

---

Explanation:

---
Question 26:

Suppose that the boat is able to go 10 km/hour on a still lake. If the speed of the river current is 5 km/hour. What will be the approximate speed of the boat crossing the river as seen from the bridge?

Response:

---

Question 27:

When the motor-boat is going downstream with the current. Is the speed of the boat that is produced by the motor affected or changed by the speed of the current?

Response:

---

Explanation:

---

Question 28:

When the motor boat is going upstream against the current, is the speed of the boat that is produced by the motor affected or changed by the speed of the current?

Response:

---

Explanation:
Question 29:

When the motor-boat is crossing the river, is the speed of the boat that is produced by the motor affected or changed by the speed of the current?

Response:

Explanation:

Question 30:

Now, your friends want to go from dock 1 to dock 2 which is directly across the river. Is there any way they could do it? Explain your response:

Could you estimate the speed of the motor-boat to go from dock 1 to dock 2 if the motor-boat is able to go 10 km/hr in a still lake and the speed of the current is 5 km/hr?
Question 31:

In the diagram provided below draw the path that you think you might see from the bridge if the boat could follow your suggestion in the previous question.

Choose three well-spaced points in the path sketched above, draw the boat in those points showing clearly how the boat might be heading in those points.

Question 32:

In earlier questions some students could have indicated that the motion of the boat was being influenced by two movements: the speed of the boat due to the motor and the speed due to the current. Do you think these movements were influencing the boat one after the other or at the same time?

Response:

Explanation:
Fourteen columns are included in the table of the Appendix D. The first column contains the identification code for each subject in the sample. All 167 subjects were registered in grade ten and their ages ranged between 15 and 17 years. The second column identifies the sex of each subject (F or M) an indication of their experience with boats. Three questions dealt with subject's experiences with boats (see G.I.P. in Appendix C); a single capital "Y" in the second column means responses to all three questions was Yes, a capital "N" means all responses were No. Otherwise, there is one lower-case letter (y or n) for each one of the three responses.

Each subsequent column corresponds to one of the 12 vector characteristics. Each is divided into sub-columns, corresponding to the inferred rules found for each vector characteristic, with the last subcolumn corresponding to no inferred rule (N.R.). Each of the 167 rows corresponds to one subject in the sample; thus, each row shows the inferred rules for each vector characteristic held by each subject in the sample. The final row show the number of subjects holding each of the inferred rules for each vector characteristic.
<table>
<thead>
<tr>
<th>Subject</th>
<th>Sex/Exp</th>
<th>RPS</th>
<th>FR</th>
<th>D</th>
<th>AD</th>
<th>SVP</th>
<th>RPM</th>
<th>AC</th>
<th>CV</th>
<th>IMC</th>
<th>SC</th>
<th>ILP</th>
<th>IDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F/N</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>F/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>F/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>F/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>M/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>F/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>F/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>M/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>M/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>M/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>M/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>F/YYN</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>M/Y</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>Sex/Exp</td>
<td>RPS</td>
<td>FR</td>
<td>D</td>
<td>AD</td>
<td>SVP</td>
<td>RPM</td>
<td>AC</td>
<td>CV</td>
<td>IMC</td>
<td>SC</td>
<td>ILP</td>
<td>IDC</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>-----</td>
<td>----</td>
<td>---</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 2</td>
<td>3</td>
<td>0</td>
<td>1 2</td>
<td>3 4 5 0</td>
<td>0</td>
<td>1 2</td>
<td>3</td>
<td>4 0</td>
<td>1 2</td>
<td>3</td>
<td>4 0</td>
</tr>
<tr>
<td>27</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>Sex/Exp.</td>
<td>RPS</td>
<td>FR</td>
<td>D</td>
<td>AD</td>
<td>SVP</td>
<td>RPM</td>
<td>AC</td>
<td>CV</td>
<td>IMC</td>
<td>SC</td>
<td>ILF</td>
<td>IDC</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>-----</td>
<td>----</td>
<td>---</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>80</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>M/YYY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>86</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>87</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>88</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>89</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>102</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>103</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>104</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>Sex/Exp.</td>
<td>RPS</td>
<td>FR</td>
<td>D</td>
<td>AD</td>
<td>SVP</td>
<td>RPM</td>
<td>AC</td>
<td>CV</td>
<td>IMC</td>
<td>SC</td>
<td>ILP</td>
<td>IDC</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>-----</td>
<td>-----</td>
<td>---</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>106</td>
<td>M/YYN</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>N</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>N</td>
<td>1</td>
</tr>
<tr>
<td>107</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>109</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>111</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>113</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>114</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>115</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>116</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>117</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>118</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>119</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>121</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>122</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>123</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>126</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>128</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>129</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subject</td>
<td>Sex/Exp.</td>
<td>RPS</td>
<td>FR</td>
<td>D</td>
<td>AD</td>
<td>SVP</td>
<td>RPM</td>
<td>AC</td>
<td>CV</td>
<td>IMC</td>
<td>SC</td>
<td>ILP</td>
<td>IDC</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>-----</td>
<td>----</td>
<td>---</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>131</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>132</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>133</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>134</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>136</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>137</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>138</td>
<td>F/NYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>139</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>142</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>143</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>145</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>146</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>147</td>
<td>M/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>148</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>149</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>151</td>
<td>F/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>152</td>
<td>F/YYN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>153</td>
<td>M/YNN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>154</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>155</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>156</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## APPENDIX D

<table>
<thead>
<tr>
<th>Subject</th>
<th>Sex/Exp.</th>
<th>RPS</th>
<th>FR</th>
<th>D</th>
<th>AD</th>
<th>SVP</th>
<th>RPM</th>
<th>AC</th>
<th>CV</th>
<th>IMC</th>
<th>SC</th>
<th>ILP</th>
<th>IDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>157</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>158</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>159</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>M/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>161</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>162</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>163</td>
<td>F/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>164</td>
<td>F/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>165</td>
<td>M/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>166</td>
<td>M/Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>167</td>
<td>M/YN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Percentages

<table>
<thead>
<tr>
<th></th>
<th>4.9</th>
<th>9.0</th>
<th>12</th>
<th>7.4</th>
<th>11</th>
<th>15</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>39</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>37</td>
<td>51</td>
<td>52</td>
<td>41</td>
<td>44</td>
<td>56</td>
<td>61</td>
<td>65</td>
<td>51</td>
<td>47</td>
<td>42</td>
<td>39</td>
<td>40</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>52</td>
<td>51</td>
<td>34</td>
<td>30</td>
<td>28</td>
<td>26</td>
<td>20</td>
<td>17</td>
<td>12</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>37</td>
<td>57</td>
<td>67</td>
<td>75</td>
<td>85</td>
<td>65</td>
<td>44</td>
<td>38</td>
<td>33</td>
<td>29</td>
<td>26</td>
<td>22</td>
</tr>
</tbody>
</table>

### List of Predominant Preconceptions

<table>
<thead>
<tr>
<th>Consistent Preconceptions (Inferred Rules)*</th>
<th>Percentage of Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPS-2</td>
<td>55</td>
</tr>
<tr>
<td>CV-3</td>
<td>51</td>
</tr>
<tr>
<td>IMC-1</td>
<td>77</td>
</tr>
<tr>
<td>SC-2</td>
<td>92</td>
</tr>
<tr>
<td>ILP-2</td>
<td>56</td>
</tr>
<tr>
<td>IDC-1</td>
<td>78</td>
</tr>
</tbody>
</table>

*For the descriptions of the inferred rules see Chapter Four and Chapter Six.

### NOTE:

The last column of this table contains the percentages of the subjects holding the inferred rules for each vector characteristic. These percentages were calculated using the number of subjects answering the questions for each vector characteristic.

As a criterion the preconceptions that were subscribed by more than 50 percent of the subjects (i.e. subjects answering the questions for a given vector characteristic) were taken as the most predominant. The list of these predominant consistent preconceptions for six vector characteristics are presented in the following list.
APPENDIX E

This appendix presents in a hierarchical order the list of the ten vector characteristics studied in Phase One according to the level of difficulty encountered by the subjects. The list starts with the vector characteristic that subjects seemed to have less difficulty and ends with the one that they encounter most difficult. This was mainly done by comparing the percentage of the subjects holding a vectorial type of preconception for each vector characteristic, but the percentages of the subjects holding scalar and transitional types of preconceptions were also considered. The last row of Table 4.25 was used to prepare this list.

1. Analysis of Components (AC)
2. Simultaneity of Components (SC)
3. Reference Bodies for Moving Objects (RPM)
4. Composition of Velocities (CV)
5. Subtraction of Vector Positions (SVP)
6. Independence of Magnitudes of Components (IMC)
7. Addition of Displacements (AD)
8. Frame of Reference (FR)
9. Displacement (D)
10. Reference Point for Stationary Bodies (RPS).