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ABSTRACT

This dissertation proposes a model oriented towards integrating farm households' production and consumption decisions into a unified theoretical and econometric framework. It is argued that some commodities such as household's labour and, in some circumstances, outputs produced by the farm are traded within the household-farm unit. The implication of this is that, in contrast with other forms of economic organization, farm households' utility and profit maximization decisions are not likely to be independent. Thus, the general objectives of the thesis are to develop a model appropriate to estimate farm households' supply and demand responses which explicitly considers the interdependence of utility and profit maximization decisions as well as to formally test the hypothesis of independence using Canadian farm census data.

A model which considers two labour supply equations, i.e., on-farm and off-farm labour supply, and five net output supply equations including one aggregated output and four inputs (land and structures, hired labour, animal inputs, and farm capital) has been jointly estimated using Canadian farm data. The main hypotheses tested are independence of utility and profit maximizing decisions and homotheticity of households' preferences.

This investigation suggests that utility and profit maximizing decisions are not independent and, moreover, that there are significant gains in explanatory power and efficiency by estimating the consumption
(i.e., the labour supply equations) and the production equations jointly. Another finding of the study is that farm households' preferences are not homothetic.

Estimates regarding the quantitative effects of changes in cost of living index, output price, wage rates, and other farm input prices on households' on-farm labour supply, off-farm labour supply, and net output supply are provided. Additionally, the effects of farm operators' educational level on their labour supply, output supply, and input demand decisions are also measured.
# TABLE OF CONTENTS

ABSTRACT ........................................ ii

LIST OF TABLES ....................................... vi

LIST OF FIGURES ....................................... vi

ACKNOWLEDGEMENTS ...................................... vii

Chapter

I INTRODUCTION .................................... 1

II THE MODEL .......................................... 10

2.1 Conditions for Independence of Utility and Profit Maximization Decisions ............ 18
2.2 Seasonality of the Self-Employment Activities ........................................... 25
2.3 Some Comparative Static Results ............................................................. 30
2.4 Review of the Literature ................................................................. 34
2.5 Summary .............................................. 40

III ESTIMATION OF HOUSEHOLD'S SUPPLY RESPONSES WITH FIXED FACTORS OF PRODUCTION ........ 45

3.1 Properties of the Indirect Utility Function .................................................. 49
3.2 Derivation of the Demand and Supply Equations ............................................ 52
3.3 Further Implications of the Utility Maximization Hypothesis ............................ 54
3.4 A Stochastic Specification ........................................................................ 59
3.5 Non-traded Outputs ............................................................................. 63

IV THE ESTIMATING MODEL ........................................ 68

4.1 Functional Forms for the Indirect Utility Function and for the Conditional Profit Function ................................................................. 68
4.2 The Econometric Model ................................................................. 82
4.3 Testing for Independence of Utility and Profit Maximization Decisions ............ 88
LIST OF TABLES

Table
1 Chi-square Statistics for the Various Hypothesis Tests ........................................ 113
2 Parameter Estimates of the Consumption and Production Equations (Equations 55 and 60) ................. 115
3 Labour Supply Elasticities ........................................ 118
4 Compensated Demand Elasticities ................................. 121
5 Labour Supply Elasticities with Respect to Net Output Prices Calculated using Equation (20) .................. 122
6 Conditional Net Output Supply Elasticities ...................... 124
7 Unconditional Net Output Supply Elasticities Calculated using Equation (22) .............................. 124
A.1 Mean, Standard Deviation and Extreme Values of Some Important Variables Considered ................ 165
A.2 Different Expenditures as a Proportion of Family Labour Farm Returns ................................. 167
A.3 Cost Shares of the Different Factors ............................ 167
A.4 Price Indexes of Net Outputs and After Tax Labour Returns, Expenditures and Farm Operator's Schooling Years ................................. 169
A.5 Quantities of Net Outputs by Census Division ............. 171
A.6 Mean Elasticities, Standard Deviations, Minimum and Maximum Values .............................. 174

LIST OF FIGURES

Figure 1 ........................................................................ 20
2 ........................................................................ 29
3 ........................................................................ 30
4 ........................................................................ 73
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CHAPTER I

INTRODUCTION

The focus of attention in this study is the analysis of the economic behaviour of self-employed farmers. Specifically, it is concerned with the supply and demand responses of households which also own and operate a firm (i.e., a farm). Distinctive features of these economic units are: (a) a significant proportion or usually the totality of the labour input used by the household's firm is supplied by its proprietors, i.e., the household's members; (b) the returns from the family farm's operation may constitute an important proportion of the household's income available for consumption and other purposes; (c) in many cases a substantial part of the family farm output is oriented to satisfy directly the household's own consumption necessities. This latter feature may be important for farm households in developing countries. Thus, recognizing these rather close linkages between households viewed as consumers and producers one may conceptualize them as household-firm units.

An important implication of feature (a) is that family labour is traded within individual households. Family labour is "produced" by the household and used by the family firm. Hence, there exists a household's supply schedule and a firm's demand schedule for family labour, and the internal equilibrium shadow price of family labour is given by the
intersection of these two schedules. The supply schedule of family labour is, therefore, dependent on household's preferences, household's income, and on the vector of consumption good prices faced by the household. Thus, a change in any of these variables will affect not only consumption demand responses (including leisure) but also profit maximizing decisions, i.e., output supply and input demand responses.

This is so, because changes on the consumption side will shift the family labour supply schedule faced by the firm and hence the level of family labour used by the firm is altered. Shifts in the supply schedule of family labour (which is usually the most important factor of production in this type of firm) will lead to changes in output supply and input demand responses. Thus, the consumption side exerts an impact on the production side via the supply of family labour. Similarly, the family labour demand curve is dependent on the firm's production technology and output and input prices faced by the household's firm. Hence, changes in these variables will shift the labour demand schedule faced by the household and, hence, the shadow price of leisure will be affected.

This will imply that a change on the production side will exert an influence on utility maximizing decisions not only through the effect of the firm's profit on family income (effect associated with feature [b]) but also by altering the relevant shadow price of leisure. Hence, consumption decisions will also be dependent on the production side. The implication of this is, therefore, that utility maximizing and profit maximizing decisions are not in general independent as occurs in the conventional case.¹

An implication of feature (c), on the other hand, is that if those outputs produced for satisfying the household's own consumption
necessities are not traded outside the household then the prices of these outputs are endogenous to the household-firm unit. The internal trade equilibrium takes place in an identical manner as it occurs with family labour. Hence, non-traded outputs will play the same role as family labour in establishing a linkage between the consumption and the production sides. Thus, interdependence of utility and profit maximizing decisions are in this case related to the endogeneity of the shadow price of non-traded outputs.²

The previous discussion suggests that household-firm unit's utility and profit maximizing decisions are, in general, prone to be interdependent. However, this interdependence may not exist if all goods produced by the household's firm are at least partially traded³ and if the following assumptions are made:⁴

Assumption 1: Households utility depends on total labour supply, not on the allocation of that supply among different working activities. In particular, on-farm work and off-farm work as wage earners are perfect substitutes in consumption.

Assumption 2: Household's members working in the family firm and hired labour used by the family firm are perfect substitutes in production.

It will be shown, however, that although either of these assumptions is necessary for independence of profit and utility maximizing decisions, they are not sufficient.

Assumption 1 implies that households can provide only one type of labour services, i.e., that the salaried working activities performed by households outside the family farm are the same as the self-employment
activities. Indeed, if this were not the case, Assumption 1 could not be justified. It has long been recognized that the disutility associated with diverse working activities is generally different (see, for example, Benewitz & Zucker; Diewert, 1971; Fields & Hosek; and Rottenburg). The one type of labour services assumption is quite restrictive and appears to contradict even casual observation. For example, in the case of the agricultural sector in Canada (where production is organized essentially as household-firm units) more than 75% of farm operators doing off-farm work in 1976 were reported to be in non-agricultural activities (Bollman).

Utility differences associated with different working activities may be even more remarkable when one of the activities is a self-employment one. To the differences attributed to diverse jobs now one may add the greater time flexibility, the "pride" of working for your own business and not being dependent on boss decisions, etc., implied by self-employment activities in contrast with wage earnings occupations. Thus, it appears that the labour choice problem has an even greater importance in the case of self-employed workers.

Assumption 2 is also dubious if one considers differences in supervision costs and differences in educational levels between firm operators and their families and hired labour. Additionally, the absence of perfect substitutability between hired and non-hired labour has been empirically established in studies applied to agriculture (see, for example, Barichello).

The lack of independence between profit and utility maximizing decisions has important implications from the point of view of estimating household's preferences and firm's production technologies. Consider,
for example, two households which face the same prices for consumption goods and leisure and that have equal incomes. If they select different commodity bundles then, using an approach based on independence of the production and consumption sides, one would conclude that the households have different preferences. However, if the hypothesis of independence does not hold then the differences on the commodity bundles selected may be due to differences in the production technology of the household's firm (or differences in prices faced by the firm) rather than to differences in household's preferences or tastes. Similarly, two firms facing same output and input prices which behave differently in the output and input markets would be considered to have different technologies if one uses the conventional dichotomized models based on the independence hypothesis. However, their different production responses may be due to differences in the household's tastes or wealth, for example, rather than to differences in production technologies. Thus, the main problem of the conventional dichotomized models of the household and of the firm is that they cannot discriminate between changes in production technologies and changes in household's preferences and may wrongly identify changes in production technology as changes in consumer's preferences and vice versa.

From the previous discussion the following conclusions emerge:
(a) in contrast with other forms of economic organization, utility maximization and profit maximization are not likely to be independent;
(b) if the problem of no independence prevailed then it will have important implications in modelling farm household's supply and demand responses. Consequently, general objectives of this thesis are to provide an empirical framework for measuring farm household's supply and
demand responses which explicitly considers the interdependence of utility and profit maximizing decisions and to formally test the hypothesis of independence using Canadian farm data.

The specific objectives are the following:

1. To discuss a general model of the economic behaviour of the household-firm unit. It is intended to use this model in analyzing its supply and demand responses as consumers and as producers. The emphasis is placed on studying the interrelations among consumption goods' demand, labour supply, output supply, and input demand of the household-firm unit.

2. To empirically estimate supply and demand responses of Canadian farm households. Specifically, it is intended to estimate the equations of consumption behaviour, labour choice, output supply and input demand for Canadian farm households explicitly derived from the theoretical model. The use of a household-firm model in Canadian agriculture is justified considering that in this sector the basic unit of production is the family farm. Thus, a model which explicitly considers the interdependence of consumption and production activities is used to answer some important agricultural policy related questions such as:

(a) Effects of agricultural commodity price changes on output supply, farmer's labour supply responses and farmer's labour choice between farm and off-farm work.

(b) Effects of input price changes on output supply, input demand, farm families labour supply, and labour choice.

(c) Effects of changes in non-agricultural wages on farm household's supply and demand responses. In particular, it is intended to
quantify the effects of such changes on the supply of agricultural commodities as well as on the derived demand for other inputs.

(d) Effects of changes in farm household's wealth on farmer's labour supply. Changes in farmer's labour supply associated with changes in wealth may have an effect in input demand and supply of agricultural commodities which has not been estimated so far.

3. The model used in analyzing Canadian farm households is based on the assumptions that there are no fixed factors of production and that all outputs produced by the farm are at least partially traded outside the household. Moreover, some restrictions on the production technology were also imposed. Hence, a third objective of the thesis is to provide a framework appropriate to econometrically estimate farm household's supply responses with fixed factors of production and non-traded outputs, based on general assumptions regarding the production technology. This model is expected to have applications particularly for studying supply responses of farm households in under-developed countries.

The remainder of this thesis is organized as follows. In Chapter II a general static model for the household-firm is introduced. Some simplifications of the model using assumptions regarding production technologies and existence of rental markets for durable factors of production are then considered. These simplifications have the advantage of substantially reducing the difficulties involved in the empirical estimation of the model and also facilitate the comparative static analysis. This chapter also includes an analysis regarding the precise conditions under which utility maximization and profit maximization decisions
are independent.

Chapter III considers theoretical and empirical implications of the general model when no restrictions on the production technology are imposed and/or when the assumption of no fixed factors is relaxed. The emphasis is placed on analyzing the existence of duality relations between direct and indirect utility functions and on the derivation of the household's supply and demand schedules using duality, given endogeneity of the family labour's price and/or of outputs produced by the family firm. Furthermore, the implications derived from the utility maximization hypothesis are also discussed. Some comparative static expressions are derived using those restrictions.

In Chapter IV the issues of the empirical implementation of the simplified version of the model developed in Chapter II are discussed. Functional forms for the functions representing the consumption and production sides of the model (the indirect utility function and the profit function, respectively) are specified and used to derive the estimating equations. The next section considers a stochastic framework for the estimating equations and discuss the econometric method used and some econometric problems. In the last section a discussion of the data requirement vis-a-vis the data available is presented. The data available and the transformations performed on the data, stressing the data limitations and hence the necessity of interpreting the results cautiously, are discussed.

Chapter V reports the major empirical findings of the thesis and in Chapter VI a summary and conclusions are provided.
The problem of interdependence of utility maximization and profit maximization decisions is analogous to the household production function analyzed by Lancaster, Becker and Pollack, and Watcher. In this context, utility is a function of commodities which are produced by the household using goods and time. Hence, utility maximizing decisions are dependent not only on goods' prices and household's preferences but also on the household's technology or production function.

The importance of considering that some outputs and family labour are "traded" within households in modelling farm households supply responses in developing countries has been recently emphasized by Nerlove:

In most developing economies the agricultural sector is so large and so central to the whole process of economic growth and demographic change, that supply response cannot be treated as an isolated phenomenon. Moreover, in these economies, markets, at least as we know them in developed economies, may be poorly organized or may not exist at all; it follows that the relevant 'prices' motivating producer behaviour may be difficult or impossible to observe directly. Many of the trade-offs in the allocation of resources may take place within individual households or between those households and relatively isolated labor or product markets. (p. 3)

If all outputs are at least partially traded outside the household-firm unit then the shadow prices will always be identical to the external market prices and hence the relevant prices are the market prices.

Barnum and Squire, Bollman, and Lau, Lin and Yotopolous have implicitly used either of these assumptions.

According to Agriculture Canada, 53% of the total labour force in agriculture were self-employed operators, 19.5% were family workers, and only 27.5% were hired workers. "Canadian farming is characterized by a large number of small family units" (p. 69).

The indirect utility function is defined as the maximum utility attainable given a budget constraint (Roy).

Footnotes

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4. If all outputs are at least partially traded outside the household-firm unit then the shadow prices will always be identical to the external market prices and hence the relevant prices are the market prices.

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6. According to Agriculture Canada, 53% of the total labour force in agriculture were self-employed operators, 19.5% were family workers, and only 27.5% were hired workers. "Canadian farming is characterized by a large number of small family units" (p. 69).

7. The indirect utility function is defined as the maximum utility attainable given a budget constraint (Roy).
CHAPTER II

THE MODEL

In this chapter a general model of the farm household is presented and, using some assumptions regarding the production technology and existence of rental markets for factors of production, a simplified, empirically applicable, version of it is derived. Some comparative static results with emphasis on the interactions between the production and consumption sides of the model are also presented.

The basic idea underlying the model is that households maximize their utility which is a function of the goods consumed, the time spent on household activities, the number of hours of on-farm work and the number of hours of off-farm work supplied by the household members (which are related to the leisure time available). It is assumed that households maximize such a utility function subject to a budget constraint and subject to a time constraint. The budget constraint indicates that the total expenditures on consumer goods cannot be greater than the total income obtained by the household. This income consists of three parts: (a) the net income obtained from the family farm's operation, represented by a dual profit function conditional on the number of hours of work which the household's members supply to their own farm. Thus, the profit function represents the production side of the model whose basic linkage with the consumption side (represented by
the utility function) is the household's labour supplied to the family farm. The net income obtained from the family farm's operation naturally will depend on the production technology, output and input prices faced by the family farm, and on the amount of work which the household's members provide to their farm. (b) A second major component of the household's budget is the income which household's members earn while working for other firms as wage earners. It is assumed that the wage rate obtained while working outside the family farm is exogenous. (c) A third source of income is the non-labour income which includes the returns obtained from financial and real assets owned by the household. These returns are also assumed to be exogenous.

More formally, the utility maximization problem of the household firm is:

$$\max_{L, X, T} f(L_1, L_2; X_1, \ldots, X_N; T_1, \ldots, T_M)$$

subject to:

(i) $$\sum_{n=1}^{N} p_n X_n \leq \pi(q; L_1) + w_2 L_2 + y$$

(ii) $$\sum_{k=1}^{2} L_k + \sum_{m=1}^{M} T_m = H$$

(iii) $$X_n \geq 0, T_m \geq 0, L_k \geq 0$$

where

- $$f$$ = household's utility function
- $$X = (X_1, \ldots, X_N)$$ is the $$N$$-dimensional vector of consumption goods
- $$T = (T_1, \ldots, T_M)$$ is the $$M$$-dimensional vector of time which household members spend on the household activities excluding productive work in the household's firm
- $$L_1$$ = number of hours of work supplied to the family firm by
household's members

$L_2 =$ number of hours of work supplied to other firms by household's members

$p_n =$ rental price of commodity $n$ consumed by household members

$y =$ non-labour income

$q =$ price vector of the net outputs the family firm can produce (using the convention of representing outputs as positive quantities and inputs as negative quantities)

$\pi(q;L_1) =$ a family firm's conditional profit function as a function of $q$ and $L_1$

$H =$ total number of hours that household's members have available for all activities

$w_2 =$ wage rate received by household members when they work for other firms.

It is assumed that the utility function $f(L_1, L_2; X_1, \ldots, X_N; T_1, \ldots, T_M)$ satisfies the following regularity conditions (Diewert, 1974):

A.1 defined and continuous from above for $X, T, L_1, L_2 \geq 0$

A.2 quasi-concave in its arguments

A.3 non-decreasing in $X$

A.4 non-increasing in $L_1$ and $L_2$.

Assumption A.1 is necessary for existence of a solution to problem (1). Condition A.2 is a standard assumption which is used in establishing duality relationships with an indirect utility function associated with problem (1). Condition A.3 is somewhat restrictive, considering that it rules out the case of "thick" indifference curves (see Debreu, 1959), but it is a necessary condition for the budget constraint to be binding.
The conditional profit function $\pi(q;L_1)$ is defined as follows:

$$\pi(q;L_1) = \max_{Q} \{ q^T Q : (Q;L_1) \in \tilde{T} \}$$

where

$q = [q_1, q_2, \ldots, q_S]$ is a column vector of net output prices

$Q = [Q_1, Q_2, \ldots, Q_S]$ is a column vector of net outputs (outputs and inputs)

and

$\tilde{T}$ is the production possibilities set, i.e., the set of all output and input combinations which the firm can produce given the state of knowledge.

It is assumed that the set $\tilde{T}$ satisfies the following conditions (Debreu):

B.1 closed, bounded from above, non-empty subset of the $S$ dimensional space,

B.2 is a convex set (non-increasing marginal rates of transformation), and

B.3 if $Q^1 \in T$, $Q^1 \geq Q^{11}$ then $Q^{11} \in T$ (free disposal condition).

Using Diewert's (1972) proof regarding the relations between the conditions on $\tilde{T}$ and on the variable profit function, it can be easily verified that if $\tilde{T}$ satisfies B then $\pi(q;L_1)$ will satisfy the following conditions:

C.1 non-negative

C.2 linearly homogeneous in $q$

C.3 convex and continuous in $q$

C.4 non-decreasing in $L_1$ and

C.5 concave and continuous in $L_1$ for fixed $q$. 
Utility maximization model (1) allows household members to have different preferences for the different types of labour services that they can provide (to their own firm or to other firms). Changes in the parameters of the model will induce changes in the distribution of labour services supplied to on-farm and off-farm activities.

Diewert (1972) has discussed a problem similar to (1) and has shown that it can be decomposed into two maximization problems:

\[ \max \{ f(L_1, L_2; X_1, \ldots, X_N; T_1, \ldots, T_M) : \sum_{m=1}^{M} T_m = H - \sum_{k=1}^{2} L_k \} \]  

(2)

and a second stage utility maximization which is

\[ \max_{L_1, L_2, X} U^*(L_1, L_2, X) \]  

(3)

s.t.  

(i) \( pX \leq \pi(q;L_1) + w_2L_2 + y \)  

(ii) \( L_1 + L_2 \leq H \)  

(iii) \( X \geq 0, L_1, L_2 \geq 0 \)

Diewert (1972) has shown that if \( f(L_1, L_2; X_1, \ldots, X_N; T_1, \ldots, T_M) \) satisfies conditions A then \( U^*(L_1, L_2, X) \) will also satisfy A. Thus a solution for problem (3) will exist (provided the constraint sets are compact in \( X, L_1 \) and \( L_2 \)) and, moreover, constraint (i) will be binding at the solution point.

Now, model (3) can be represented in a more convenient form by a further transformation in the variables of \( U^* \):

\[ U(H-L_1, H-L_2; X_1, \ldots, X_N) \equiv U^*(L_1, L_2; X_1, \ldots, X_N) \]  

(4)

The advantage of \( U(H-L_1, H-L_2; X_1, \ldots, X_N) \) is that it is defined over the non-negative orthant and that the corresponding budget constraint may be defined using non-negative prices and positive income. It is easy to
verify that if \( U^* \) satisfies conditions A then \( U \) will satisfy A.1, A.2, and A.3. Condition A.4 for \( U \) will read "nondecreasing in \( H-L_1, H-L_2 \)."

Using (4) it is now possible to reformulate model (1):

\[
\text{max U}(H-L_1, H-L_2; X) \quad \text{(5)}
\]

s.t. (i) \( pX + w_2(H-L_2) \leq Hw_2 + y + \pi(q; H-(H-L_1)) \)

(ii) \( (H-L_1) \geq 0, (H-L_2) \geq 0, X \geq 0 \)

(iii) \( (H-L_1) + (H-L_2) \geq H \)

(iv) \( (H-L_1) \leq H, (H-L_2) \leq H. \)

From now on it is assumed that constraint (iii) is not binding.

Constraint (iii) implies that the total labour the household members desire to supply cannot be greater than the total time available. Thus, this assumption implies that at all wage rates and commodity prices individuals will want to consume some leisure.

Model (5) can be significantly simplified if the production technology exhibits constant returns to scale. In representing the production technology by \( \pi(q; L_1) \) it has been implicitly assumed that there are no fixed factors of production. Therefore, if the production technology exhibits constant returns to scale and if there are no fixed factors of production then the profit function is homogeneous of degree one in \( L_1 \) and can be decomposed as follows (see proof in Appendix 1):

\[
\pi(q; H-(H-L_1)) = [H-(H-L_1)] \cdot \#(q) = L_1 \cdot \#(q) \quad \text{(6)}
\]

where \( \#(q) \) is non-negative, convex, continuous, and linear homogeneous in \( q \).

Existence of fixed or quasi-fixed factors in the short run is associated with any of the following two causes: (1) adjustment costs are important; (2) indivisibilities of durable factors or imperfections in
the capital markets (credit markets) if well developed second-hand rental markets for durable factors do not exist.

If there are indivisibilities the firm may not be able to purchase a new unit of a factor even if a fraction of it would be optimal to incorporate in the production process. Similarly, if the firm does not have access to appropriate credit sources, the firm will have to adjust upwards a factor using its own funds. Thus, cash constraints may imply that a firm can only partially adjust its durable factor stocks. However, if second-hand rental markets for durable factor services work appropriately, then adjustments of durable factors become essentially a flow problem similar to the adjustment of any non-durable, divisible input. Hence, the firm can rent or let the flow services of the factors, thus overcoming indivisibility or cash constraint problems.

Hence, in order to justify the use of (6) it is necessary to assume that adjustment costs are negligible and the existence of perfect rental markets for durable factors of production. An alternative way of justifying (6) is simply to postulate long-run equilibrium where all capital stocks have been adjusted to optimal levels.

Thus, if all factors are variable then, using (6), the utility maximization problem (5) may now be written as:

$$\max_{H-L_1, H-L_2, X} U(H-L_1, H-L_2, X)$$  \hspace{1cm} (7)

s.t.  \hspace{1cm} (i) \hspace{0.5cm} pX + \tilde{\pi}(q)(H-L_1) + w_2(H-L_2) \leq H(\pi + w_2) + y = Z \hspace{1cm} (i) \hspace{0.5cm} (H-L_1) \geq 0; \hspace{0.5cm} (H-L_2) \geq 0; \hspace{0.5cm} X \geq 0 \hspace{1cm} (ii) \hspace{0.5cm} (H-L_1) \leq H; \hspace{0.5cm} (H-L_2) \leq H. \hspace{1cm} (iii)

The advantage of using (7) rather than (5), is that (7) is a standard maximization problem with a linear constraint provided that \(\tilde{\pi}(q)\)
is known and that constraint (iii) is not binding. Thus, standard duality theory (see, for example, Diewert, 1974) can now be applied in order to derive equations for household's commodity demand, labour supply to the household's farm, and off-farm labour supply. This is so because the wage rate received by household members working on the family farm becomes independent of the household's preferences, depending only on the output and input prices which the family firm faces as well as on its production technology. Therefore, an indirect utility function \( G(p, \bar{\pi}, w_2; Z) \) can be defined in the standard manner:

\[
G(p, \bar{\pi}(q), w_2, Z) = \max_{H-L_1, H-L_2, X} \{ U(H-L_1, H-L_2, X) : \\
(i) \ pX + \bar{\pi}(H-L_1) + w_2(H-L_2) \leq Z \\
(ii) \ H-L_1 \geq 0, H-L_2 \geq 0, X \geq 0 \} \tag{8}
\]

where \( G \) will be continuous, quasi-convex in \( p, \bar{\pi} \) and \( w_2 \), nonincreasing in \( p \), nondecreasing in \( Z \) and homogeneous of degree zero in \( p, \bar{\pi}, w_2 \) and \( Z \) (see, for example, Varian, pp. 89-90).

From (8) it is possible to derive the Marshallian demand functions for \( H-L_1, H-L_2 \) and \( X \) using Roy's identity:

(i) \( H-L_1 = -\frac{\partial G/\partial \bar{\pi}(q)}{\partial G/\partial Z} = \phi(p, \bar{\pi}, w_2, Z) \)

(ii) \( H-L_2 = -\frac{\partial G/\partial w_2}{\partial G/\partial Z} = \psi(p, \bar{\pi}, w_2, Z) \) \tag{9}

(iii) \( X = -\frac{\partial G/\partial p}{\partial G/\partial Z} = \epsilon(p, \bar{\pi}, w_2, Z) \)

Furthermore, the set of conditional net supply functions can be derived from the conditional profit function using Hotelling's lemma (Hotelling).

\[
Q_i(q; L_1) = L_1 \cdot \frac{\partial \bar{\pi}(q)}{\partial q_i} \quad i=1, \ldots, S \tag{10}
\]
where $Q_i$ is the conditional net supply of commodity $i$. When commodity $i$ is an input then $z_i = -Q_i$ is defined as input $i$. Note that due to the constant return to scale assumption $L_1$ plays only a scale role in the determination of the net supply functions. The unconditional net supply functions are obtained by substituting $9(i)$ into (10):

$$Q_i(q;p,w_2,Z) = [H - \Phi(p,\tilde{\pi}(q),w_2,Z)] \frac{\partial \tilde{\pi}(q)}{\partial q_i} \quad i=1,\ldots,S \quad (11)$$

Equations (9) and (11) represent the set of supply and demand responses obtained from a model which considers consumption and production activities of the farm household within an integrated framework. Utility maximization decisions (represented by equation set (9)) and profit maximization decisions (represented by equation set (11)) are interdependent. Changes in the consumption side are transmitted to the net output supply functions via the function $\Phi(p,\tilde{\pi}(q),w_2,Z)$ in (11). For example, changes in consumer preferences will have an impact on $L_1$ which in turn will affect optimal net output supply responses (because the level of $\tilde{\pi}(\cdot)$ is changed). Similarly, changes in the production side will affect utility maximization decisions not only via $Z$ but also by changing the shadow price of $L_1$, i.e., by changing $\tilde{\pi}(q)$ in (9). Thus, if output prices increase, for example, then there will be changes in net output supply responses and also the household will reconsider its consumption and labour supply allocations because the increased output prices will imply a higher level for the shadow price of on-farm work ($\tilde{\pi}(q)$).

2.1 Conditions for Independence of Utility and Profit Maximization Decisions

In this section the conditions for independence of utility and profit maximization decisions are analyzed using model (5). In other
words, the aim is to determine under what conditions the linkages between equation system (9) and the net supply equation (11), discussed above, are disrupted. This amounts to asking under what conditions the shadow price of on-farm labour will be unrelated to the (conditional) profit function and the unconditional net output supply functions can be defined independently of the level of the household's labour supply.

The propositions presented in this section are concerned with the conditions for independence of profit and utility maximizing decisions under the maintained assumption that all outputs produced by the family firm are at least partially traded.

Thus, some propositions concerning the conditions required for the independence hypothesis are presented (see proofs in Appendix 1). The propositions make reference to Assumptions 1 and 2 which are defined in Chapter I. Assumption 1 refers to the case where households derive the same utility (or disutility) by working for their own firms or elsewhere as wage earners, even if the outside activity is entirely different from their own firm's work. Assumption 2 indicates that the family labour and the hired labour used by the family firm are perfect substitutes in production. In proving the following propositions, model (5), modified to consider the specific assumptions underlying the different assumptions associated with Propositions (1) to (3), is used.

**Proposition 1**

If Assumption 1 is true then profit maximization decisions are independent of utility maximizing decisions if the household members also work as wage earners for other firms. In this case the imputed price to household members' work in the family firm is the wage rate (assumed parametric to households) received by the household's members working as wage earners outside the
family firm. Hence the profit function will be a function of this wage rate as well as of the output and other input prices. Similarly, the indirect utility function will depend on the same wage rate which is the unique price of leisure.

Notice that if family members do not do outside work, then in general the supply shadow price of family labour will be dependent on preferences and prices of consumption goods and hence profit maximizing and utility maximizing decisions are not independent.

The situation described by Proposition 1 can be represented by using Figure 1:

In Figure 1, the curve S represents the family labour supply schedule, that is

\[ S = \frac{\partial U(H-L_1-L_2,X)}{\partial (H-L_1-L_2)} \cdot \frac{1}{\lambda} \]

which is the marginal valuation of family labour where X and the Lagrangean multiplier, \( \lambda \) are evaluated at their utility maximizing levels for any given \( H-L_1-L_2 \) value. The family labour supply schedule is unique due to Assumption 1 (if this assumption does not hold then there are two different supply schedules: on-farm and off-farm labour supply.
schedules). Curve D is the household's firm demand schedule for family labour (i.e., the shadow price function $\frac{\partial \pi(q;L_1)}{\partial L_1}$) which is downward sloping under non-constant returns to scale. Line $w_2$ is the off-farm wage rate which is assumed exogenously given. Under these conditions the equilibrium level of (total) labour supply will be OA of which OB will be on-farm work ($L_1$) and BA will be off-farm work ($L_2$). The important thing to notice is that any change on the consumption side (cost of living changes or changes in household's wealth, for example) will have no effect on on-farm work provided that household's members still work off-farm after such a change. Thus, changes in the consumption side will shift the labour supply schedule, for example, from S to $S^1$ in Figure 1. This will imply that total labour supply is reduced from OA to $OA^1$. However, the level of on-farm work will remain at the same level (OB) after the shift in S has taken place. Hence, the production sector is not affected (i.e., net output optimal supply levels remain the same) by changes on the consumption side. Similarly, a change on the production side (say, changes in q or the production technology) will shift the D' schedule but, if off-farm work still occurs after such shift, total labour supply will not be affected. A shift in D will only imply changes in the distribution of work between on-farm and off-farm activities, but it will not affect total labour supply. Given that from the viewpoint of the household as a consumer, the relevant decision variable is total labour supply rather than its distribution between on- and off-farm activities (due to assumption (1)), the shift in D will have no effect on the utility maximizing decisions. Thus, in contrast with the model represented by equation (9) and
(11), under the conditions of proposition (1) the linkages between the production and consumption sides of the model are disrupted with the only exception of the effect of total profit on household's income.\(^8\)

It is important to notice that in 1976 only 34% of all farmers in Canada and 28% of commercial farmers (those with farm sales above $2,500 per year) reported "some days" of off-farm work (Bollman, 1978). Therefore, even if assumption 1 holds, the sufficient condition for proposition (1) is satisfied by only 34% of all farmers in Canada.

**Proposition 2**

If assumption 2 is true then profit maximization decisions can be considered independent from the consumption parameters if the household firm uses hired labour. In this case the profit function will be a function of the conventional parameters, i.e., output and input prices, using the hired labour wage rate paid by the family firm as the price of all labour (family and hired) used in the production process. Similarly, demand for consumption goods and services (including leisure) will depend on consumption goods prices and on the hired labour wage rate as the unique price of leisure.

It is important to mention that according to Agriculture Canada, in 1976 less than 35% of all farms used hired labour and less than 7% employed hired labour on a yearly basis. Thus, even if it is assumed that family labour and hired labour are perfectly identical inputs it would not be satisfactory to use the conventional model based on independent consumption-production decisions in studying Canadian agriculture supply and demand responses.
Proposition 3

If assumptions 1 and 2 are both true, then profit maximization decisions can be considered independent of the consumption parameters if either one of the following situations occur: (a) the household's firm uses hired labour; (b) the household members work as salaried workers in other firms.

The following two corollaries to proposition 3 may be useful as empirically testable predictions from a model based on the joint hypothesis that assumptions 1 and 2 hold.

Corollary 3.1

If assumptions 1 and 2 hold, and if the wage rate that household's members can obtain outside the family firm is greater than the wage rate the family firm pays to its hired labour, then household's members will not work in their own firm. They will allocate their total working time outside the family firm.

Corollary 3.2

If assumptions 1 and 2 hold, and if the wage rate that household's members can obtain outside the family firm is lower than the wage rate the family firm pays to its hired labour then household's members may work outside the family firm only if the family firm does not use hired labour.

Thus, corollary 3.1 says that if one observes that household's members work in their family firm when the wage rate they obtain outside the family firm is greater than the wage the family firm pays to its hired labour, then a model based on the above two assumptions can be rejected. Similarly, when the conditions of corollary 3.2 are met and if household's members work outside the family firm and such firm does
use hired labour then one can also reject a model based on assumptions 1 and 2.

It is interesting to note that the average off-farm after tax wage rate obtained by Canadian farmers is significantly greater than the wage rate paid to hired labour used by farmers (see Appendix 2, Table A.1). Given that farmers do an important amount of on-farm work, both assumptions (1) and (2) cannot simultaneously hold for Canadian farmers according to corollary 3.1.

**Proposition 4**

If assumptions 1 and 2 do not hold then utility maximizing and profit maximizing decisions will not be independent.

This proposition is indeed a corollary to propositions 1 to 3 and can be readily proved using the first order conditions associated with a utility maximization problem similar to (5).

The above propositions show that even the two standard assumptions together, i.e., a unique price of leisure (assumption 1) and perfect substitutability between family labour and hired labour (assumption 2) are not in general sufficient to guarantee independence of the profit function from consumption parameters.

It is important to notice that the sufficient conditions for independence of profit maximization and utility maximization decisions do not depend only on tastes and technology but also on the relevant price structure faced by the household firm unit. For example, whether a firm uses hired labour or not will depend not only on production technology, tastes, etc., but also on the relative price of hired labour with respect to output, input, or other prices. An econometric model aiming to provide quantitative predictions should be valid for a relatively wide
range of relative prices. An econometric model based on the assumptions which allow propositions 1, 2, or 3 to be valid may break down if there is a sufficient price change to violate those assumptions. For instance, a labour supply model using the hired labour wage rate as the imputed price of leisure will not be valid if the relative price of hired labour reaches a level where the family firm will not use hired labour. At this price level quantitative predictions will be invalid because the price of leisure will not be the hired labour wage rate. Thus, the shortcomings of econometric models using the assumptions underlying propositions 1, 2, or 3 are serious, because those assumptions do not depend only on relatively stable characteristics of the household-family firm units, such as technology, tastes, etc. Therefore, it is not even appropriate to estimate such models for households which at given prices satisfy those assumptions if the purpose is to use them for quantitative predictions.

Thus, the importance of developing a more general model which does not rely on propositions 1, 2, or 3 is evident not only for those households which do not meet the sufficient conditions for profit maximization-utility maximization independence but also for those which do meet them at particular price levels.

2.2 Seasonality of the Self-Employment Activities

Many self-employment activities are characterized by rather strong seasonal patterns. This is particularly relevant in agricultural production (mainly for crop farms). Thus, farm households' activities in the slack season (say from November to April) are very different from the activities performed during the busy season (say May to October). Normally, in the slack season most household's time is spent on livestock
related activities, farm improvements such as construction work, machinery repairs, etc. On the other hand, the busy season involves much more hectic and diversified works including crop planting and harvesting. This seasonal pattern presents a rather serious problem because the marginal productivity schedules of labour in the two seasons may be expected to be very different. In fact, empirical studies which have considered labour used in the two seasons as different productive inputs, have shown that the two schedules differ to a large extent and that the value of the marginal product of labour (at a given level of work) is substantially lower during the slack season (Nath). Given that data on farm labour are seldom available at a disaggregated seasonal level, the seasonality of self-employment activities may be a serious problem in empirical analysis. An important question is: under what conditions is it consistent to aggregate these two labour categories by simply adding-up hours of work performed during the slack and busy seasons? In other words, under what conditions the profit functions

$$\pi(q; L_{1b}, L_{1s}) = \pi(q; L_1)$$

where $L_{1b}$ and $L_{1s}$ are hours of household's on-farm work in the busy and slack seasons, respectively, and $L_1 = L_{1b} + L_{1s}$. If the relevant price of household's labour is an exogenous wage rate then the Hicks' composite commodity theorem applies, and permits one to aggregate $L_{1b}$ and $L_{1s}$; and the fact that they have the same price means that the relevant composite commodity is simply $L_1$. However, if the price of family labour faced by the firm is not given exogenously from outside the house-hold firm (as it occurs when the conditions for propositions (1) and (2) do not hold) then a necessary condition (although not sufficient) for the composite commodity theorem to apply is that households be indifferent of working in the busy or slack seasons. To see
this, consider a utility maximization problem where busy season household's labour \((L_{lb})\) and slack season labour \((L_{ls})\) are considered different inputs in production. Thus, assuming that households are indifferent between working in the two seasons, the utility maximization problem would be:

\[
\max_{H-L_{lb}, L_{ls}, X} U(H-L_{lb} - L_{ls}, X) \tag{12}
\]

\begin{align*}
(i) \quad & pX \leq \pi(q;L_{lb}, L_{ls}) + y \\
(ii) \quad & H - L_{ls} - L_{lb} \geq 0, \ x \geq 0 \\
(iii) \quad & L_{ls} \geq 0, \ L_{lb} \geq 0 \\
(iv) \quad & L_{lb} \leq H_b, \ L_{ls} \leq H_s, \ H_s + H_b = H
\end{align*}

where \(H_b\) and \(H_s\) are the maximum number of hours that households have available for work and leisure during the busy and slack seasons, respectively and all other variables have previously been defined.

From the first order conditions of the above problem the following relation may be obtained:

\[
\frac{\partial \pi(q;L_{lb}, L_{ls})}{\partial L_{lb}} + M_b = \frac{\partial \pi(q;L_{lb}, L_{ls})}{\partial L_{ls}} + M_s = \frac{\partial U}{\partial (H-L_{lb} - L_{ls})} \cdot \frac{1}{\lambda} \tag{13}
\]

where \(M_b\), \(M_s\) and \(\lambda\) are the Lagrangean multipliers associated with constraints \(L_{lb} \leq H_b\), \(L_{ls} \leq H_s\) and the budget constraint, respectively.

Now, if \(M_b = M_s = 0\), i.e., if there exists an interior solution for both \(L_{lb}\) and \(L_{ls}\) then \(\frac{\partial \pi}{\partial L_{lb}} = \frac{\partial \pi}{\partial L_{ls}}\), and, moreover, one can consider the the shadow prices of \(L_{lb}\) and \(L_{ls}\) faced by the family firm as identical, equal to \(\frac{\partial U}{\partial (H-L_{lb} - L_{ls})} \cdot \frac{1}{\lambda}\). Thus, in this case Hicks' aggregation condition is satisfied and therefore hours of work used in the slack and busy seasons can be consistently aggregated. Hence, the necessary and
sufficient conditions for the composite commodity theorem to apply are that households be indifferent of working in the busy or slack seasons and that an interior solution for \( L_{1b} \) and \( L_{1s} \) exists. Furthermore, given that the equilibrium shadow prices of \( L_{1b} \) and \( L_{1s} \) are identical, a consistent aggregate would be \( L_1 = L_{1b} + L_{1s} \), i.e., the simple addition of hours of work of the two seasons. Hence, if households are indifferent between working in any of the two seasons and if their utility is maximized by working strictly less than the maximum possible hours of work in each season, (i.e., if leisure time is not zero in either season) then there are no aggregation biases by using data which does not distinguish between hours of work in the slack and busy seasons. In this case the conditional profit function can be written as \( \pi(q; L_1) \). Note that for simplicity of notation, off-farm work has not been considered.

However, one can verify that zero off-farm work during the busy season is not a necessary nor a sufficient condition for obtaining a corner solution in \( L_{1b} \). Thus, the fact that farmers do not do off-farm work in the busy season (as is likely to be the case), does not imply that \( L_{1b} = H_b \). Indeed the condition for an interior solution for \( L_{1b} \) and \( L_{1s} \) is rather weak; it only requires that farmers have some leisure time and/or work off-farm during the busy season and that they do some on-farm work in the slack season. Therefore, an interior solution in \( L_{1b} \) and \( L_{1s} \) is consistent with the realistic situation that farmers work only on-farm during the busy season and work off-farm and on-farm in the slack season.

Graphically, the argument may be expressed in Figure 2 where \( S = \frac{\partial U}{\partial (H - L_{1b} - L_{1s})} \cdot \frac{1}{\lambda} \) is the marginal valuation of family labour schedule along the utility maximizing path as a function of \( L_{1b} + L_{1s} \).
Curve D is the total demand for labour schedule, which indicates the quantity of $L_{1b}$ and $L_{is}$ demanded at each labour price. The intersection of curve D and the supply schedule S will give the equilibrium shadow price of family labour. In the above figure, this equilibrium shadow price is R and at this price $L^*_{ls}$ of slack season work and $L^*_{1b}$ units of busy season will be performed. Thus, the equilibrium shadow price of $L_{1b}$ and $L_{is}$ is identical equal to R. The crucial element is that under the assumption of indifference between working at different seasons of the year, there is only one supply schedule of family labour and hence that households will freely switch hours of work between the two seasons until the value of the marginal labour in the two seasons becomes identical. This is true under the assumption that equalization of the shadow prices is achieved before the maximum possible number of hours of work ($H_b$ or $H_s$) is reached in any of the seasons.

The necessity of an interior solution for $L_{1b}$ and $L_{is}$ as a condition for having an identical equilibrium shadow price for $L_{is}$ and $L_{1b}$ may be illustrated using Figure 3.

Figure 3 depicts a situation where the constraint $L_{1b} \leq H_b$ is
binding. Households would desire to allocate $L_{lb}^0$ hours in the busy season but, given their time constraint, they can only work $H_b$ hours. This means that while the equilibrium shadow price of labour in the slack season will be $R$, the shadow price in the busy season will be higher equal to $R^b$. In this case, aggregation of $L_{ls}$ and $L_{lb}$ would not be consistent and hence a profit function $\pi(q,L_1)$ where $L_1 = L_{lb} + L_{ls}$ will not exist.

2.3 Some Comparative Static Results

Using model (7) the interdependence between the consumption and production sides of the model is now examined by considering the effects of changes in net output prices on the household's allocation of its working time between on-farm and off-farm activities. Furthermore, the effects of changes in consumption parameters (such as $y$ or $p$) on supply of net outputs as well as the effects of net output prices on the net outputs supplied by the family firm are also considered.

As mentioned earlier equations (9) are the solution of the
conventional static model of the behaviour of consumer workers (Hicks, 1946) and the standard comparative static results can be derived in the usual manner. Thus, using 9(i):

\[
\frac{\partial (H-L_1)}{\partial \bar{\pi}} = \frac{\partial \phi(p, \bar{\pi}, w_2; Z)}{\partial \bar{\pi}} + \frac{\partial \phi(p, \bar{\pi}, w_2; Z)}{\partial Z} . \quad H \tag{14}
\]

Now, it is a well known result that (Samuelson, 1938):

\[
\frac{\partial \phi(p, \bar{\pi}, w_2; Z)}{\partial \bar{\pi}} = e_{\bar{\pi}} - \phi \cdot \frac{\partial \phi}{\partial Z}, \quad \tag{15}
\]

which is the Slutsky equation, \( e_{\bar{\pi}} \) is the Hicks' own substitution term, which is nonpositive and \( \frac{\partial \phi}{\partial Z} \) is the income effect.\(^9\) Hence (14) can be written as:

\[
\frac{\partial (H-L_1)}{\partial \bar{\pi}} = e_{\bar{\pi}} + L_1 \frac{\partial \phi}{\partial Z}, \quad \tag{16}
\]

which implies that the effect of \( \bar{\pi} \) on the supply of family labour to the household's firm \((L_1)\) is:

\[
\frac{\partial L_1}{\partial \bar{\pi}} = -e_{\bar{\pi}} + L_1 \frac{\partial L_1}{\partial Z}. \quad \tag{17}
\]

If leisure is not an inferior good then \( -e_{\bar{\pi}} > 0 \) and hence, although \( \frac{\partial L_1}{\partial \bar{\pi}} > 0 \), the sign of \( \frac{\partial L_1}{\partial Z} \) is unknown, which is a standard result.

The effect of a change of a net output price, \( q_1 \), on the supply of family labour to the household's firm can be now examined using (10):

\[
\frac{\partial L_1}{\partial q_1} = \frac{\partial L_1}{\partial \bar{\pi}} \frac{\partial \bar{\pi}}{\partial q_1} = \left[ -e_{\bar{\pi}} + L_1 \frac{\partial L_1}{\partial Z} \right] \frac{Q_i}{L_1}. \quad \tag{18}
\]

Thus, if \( q_1 \) is an output price then the sign of \( \frac{\partial L_1}{\partial q_1} \) will be identical to \( \frac{\partial L_1}{\partial \bar{\pi}} \) and, \( \frac{\partial L_1}{\partial q_1} \) and \( \frac{\partial L_1}{\partial \bar{\pi}} \) will have opposite signs if \( q_1 \) is an input price (recalling that in this case \( Q_i = -z_1 \)). Hence, if \( q_1 \) is an
input price then the substitution effect \( e_{\Pi L_i} = 0 \). Family labour will always appear to perform as a net complement with other inputs used in the family firm. This also holds for the relation between family labour and hired labour. Any increase in an input price will induce a fall in the demand shadow price of family labour used by the family firm. This will lead the household to decrease the allocation of family labour to its own firm and to increase labour supplied elsewhere (ignoring the income effect on leisure).

The effect of a change in the output price of family labour on the allocation of family labour outside the family firm can be analyzed in a similar manner:

\[
\frac{\partial L_2}{\partial q_1} = \frac{\partial L_2}{\partial \Pi} \frac{\partial \Pi}{\partial q_1} = \left[ -e_{\Pi W_2} + L_2 \frac{\partial L_2}{\partial \Pi} \right] \frac{Q_i}{L_2}.
\]

(19)

where \( e_{\Pi W_2} \) is the Hicks' cross substitution term.

Again \( \frac{\partial L_2}{\partial q_1} \) and \( \frac{\partial \Pi}{\partial q_1} \) will have identical signs if \( q_1 \) is an output price and opposite signs if \( q_1 \) is an input price.

Relations (18) and (19) can be represented in terms of elasticities:

\[
\varepsilon_{L_j q_1} = \frac{q_1 Q_i}{\Pi L_j}, \quad j = 1, 2
\]

(20)

where \( \varepsilon_{L_j q_1} \) is the elasticity of supply of labour service \( j \) with respect to a change in net output price \( q_1 \) and \( \varepsilon_{L_j \Pi} \) is similarly defined as the elasticity of supply of labour service \( j \) with respect to \( \Pi \).

The effect of a change in net output price \( q_1 \) on net output \( Q_j \) can be readily derived using (10):

\[
\frac{\partial Q_j}{\partial q_1} = \frac{\partial^2 \Pi}{\partial q_1 \partial q_j} L_1 + \frac{\partial \Pi}{\partial q_1} \frac{\partial L_1}{\partial q_j}.
\]

(21)
Notice that if \( i = j \), the effect of output price \( i \) on output \( i \), (21) will not be unambiguously positive as it occurs in the non-family firm case. In this case, convexity of \( \pi \) is sufficient to sign the first term on the right-hand-side of (21) as positive but the second term's sign depends on whether \( \frac{\partial L_1}{\partial q_j} \) is positive or negative. Thus, the possibility of downward sloping output supply or upward sloping input demand functions cannot be ruled out. The reason for this is that an increase in an output price may induce a net reduction in the supply of labour to the family firm if the income effect on leisure (assuming leisure is a normal good) predominates over the substitution effect. Hence, although the supply of output \( i \) per unit of family labour will always increase, it is possible that the amount of family labour used by the family firm be reduced and therefore a reduction in total output \( i \) is possible.

Equation (21) can also be expressed in elasticity terms:

\[
\varepsilon_{ij} = S_{ij} + \varepsilon_{L_{i} \pi} \frac{q_i Q_i}{\pi},
\]

(22)

where \( \varepsilon_{ij} \) is the output \( i \) elasticity with respect to \( q_j \), \( S_{ij} \) is the elasticity of \( Q_i/L_1 \) with respect to \( q_j \) and \( \varepsilon_{L_{i} \pi} \) is the elasticity of \( L_1 \) with respect to \( \pi \).

The effect of an increase in household's wealth will imply an expansion of the non-labour income available. Thus, using (7) one can obtain the effect of a change in the non-labour income, \( y \), on the net outputs \((Q_j)\):

\[
\frac{\partial Q_j}{\partial y} = \frac{\partial L_1}{\partial y} \frac{\partial Z}{\partial y} \frac{\partial \pi}{\partial q_j} = \frac{\partial L_1}{\partial y} \frac{\partial \pi}{\partial q_j}.
\]

(23)

Hence if leisure is not an inferior good, then \( \frac{\partial L_1}{\partial Z} < 0 \) and in
general one can say that $\frac{30}{\partial y} < 0$. Thus, the effect of an increase in household's wealth (and hence non-labour income) will be a reduction in productive activities. Obviously given that $L_1$ has a scale effect in productive activities, it will affect all net outputs proportionally and the net output supply elasticities with respect to $y$, $y_{1y}$ will all be identical and equal to the elasticity of labour supply $\varepsilon_{L_1y}$ with respect to $y$:

$$\varepsilon_{jy} = \varepsilon_{iy} = \varepsilon_{L_1y} \quad \text{(24)}$$

Similarly, the effect of other consumption parameters, say $p$ or $w_2$ on net output supply will be:

$$\varepsilon_{jw_2} = \varepsilon_{iw_2} = \varepsilon_{L_1w_2} \quad \text{(25)}$$

Notice that, given the assumptions used, the quantitative value of the effects of changes in the parameters of the consumption side of the model ($p, w_2, y$) on production decisions can be expected to be as important as the cross effects of $p$, $w_2$ and $y$ on $L_1$.

2.4 Review of the Literature

In this section a brief review of the literature concerning households allocation of time and household's production function is presented. Walras was one of the earlier writers to analyze the allocation of time between work and non-work activities using a utility maximization framework. He was the first to consider the case where consumers could supply many different types of labour services which have different effects on consumer's utility.

More recently, Becker, Mincer, and Lancaster have extended Walras's model to a household model stressing the production of commodities by the household (which are the elements actually entering the utility function).
using goods and household time. A typical household production activity is for example cooking, where raw food (goods) and household time are combined to produce cooked food which are the commodities which enter into the utility function.

Although Becker, Mincer, and others have extended the original Walras's model within a time and income allocation framework they implicitly assumed that individuals are qualified to offer only one type of labour service. Diewert (1971) generalized Becker's model by developing a model for individuals capable of offering many different types of labour services simultaneously. Diewert also provided a framework for the empirical estimation of the consumer-worker preferences. He defined a utility function dependent on goods and time spent in different activities which combined the parameters of the original utility function (defined in the commodity space) and the household production technology. Thus, if the major interest is on estimating the combined household technology-household preferences reduced form function then one can use Diewert's approach.

The concern of this thesis is the self-employed consumer-worker (or household) which, in general, is an analogous problem to the household production function model analyzed by Becker and others. In the household production function model households use goods and household's time to produce commodities (and leisure) which are consumed by the household. The self-employed consumer-worker uses production inputs and part of his time to produce goods and income for household's consumption. Thus, leisure and non-traded goods produced by the self-employed worker constitute the major linkages between household's preferences and the production technology. Hence, in the same way as in the model analyzed by
Becker the demand for goods and the household's time allocation reflect not only consumer preferences but also the household technology. In the case of the self-employed worker, his demand for consumer goods, his time allocation, his demand for factors of production and output supply responses jointly reflect his consumer preferences and the technology of his firm.

In the household production function model the linkage between production decisions (how much goods and time to use) and consumption decisions (how much commodities to consume) takes place because commodity (shadow) prices and leisure prices are endogenous and dependent on the household's production technology. Lancaster and Becker by (implicitly) assuming a disjoint household's production technology, constant returns to scale and that households were indifferent among alternative allocations of their time, concluded that commodity shadow prices faced by the household as consumer were exogenous to the level of commodities demanded although dependent not only on goods' prices but also the household's production technology. Also the demand for goods are dependent (up to a scale level) on the level of consumption of commodities. Thus, the importance of the Lancaster-Becker model was to emphasize that to understand market demand for goods it is necessary to consider not only goods' prices and the structure of household's preferences but also the technology of the household's production function. Pollack and Watcher have shown that if households are not indifferent among alternative allocations of their time (or if either of the assumptions regarding the production technology does not hold) then commodity shadow prices and leisure price(s) are endogenous to the levels of commodity and leisure demanded and hence commodity shadow prices are not only dependent on goods' prices.
and the household production technology but also on consumption preferences. In this case production decisions (i.e., goods' demand) are dependent on commodity demand in a more complex manner than up to a scale level.

In the case of the self-employed consumer-worker, the major linkage between consumption and production decision lies in the endogeneity of the price of the household's labour used by the family firm. As in the household production model, if some assumptions regarding the production technology are made (i.e., no fixed inputs and CRS) then the shadow price of leisure is exogenous to the level of leisure demanded although the demand for leisure (and for consumption goods) are dependent not only on net output prices but also on the production technology of the household's firm. Similarly, the supply of net outputs by the firm are dependent up to a scale level on leisure consumed (or labour supplied) by the household. If the assumptions regarding the production technology are relaxed then the shadow price of leisure is endogenous to the level of household's demand and in this case net output supply will be dependent on household's leisure demand in a more complex manner than simply to a scale level. An additional source of interdependence between the production and consumption sectors occurs if some outputs produced by the family firm are not traded and are entirely consumed by the household (the household does not buy or sell such output). In this case the shadow prices of those outputs are also endogenous dependent on the family firm technology and household's preferences.

Empirical applications of the household production model are scarce and empirical work concerning estimation of supply and demand responses of self-employed consumer-producers have ignored the interdependence
between the production and consumption sides of the model. Most of the studies on self-employed consumer-producers are related to agricultural producers which is a sector where this type of organization of production predominates. Lau, Lin, and Yotopoulos have estimated a system of expenditure functions (including leisure, agricultural commodities, and non-agricultural commodities) for farm households in Taiwan, using a utility maximization approach. Using a profit function estimated by Yotopoulos et al., for the same area, they estimated the household marketed surplus. The only source of interrelations between the consumption and production sides of the model was the effect of profit from the farm operation on the household's income. It was assumed that the price of leisure was unique and equal to the prevailing (exogenous) wage rate and that all outputs were at least partially traded. In defining a unique exogenous price of leisure it was implicitly assumed that households were indifferent among alternative allocations of their working time and that all households do off-farm salaried work. A similar approach was adopted by Barnum and Squire in analyzing consumption and production responses of farm households in Malaysia. Bollman analyzed off-farm supply of labour using Canadian farm data under similar assumptions regarding leisure price. In contrast with Lau et al., his method is more ad hoc in the sense that he does not derive the estimating equation from an explicit utility maximization framework.

The functional forms used in the Lau et al. and Barnum and Squire studies were also quite restrictive. Lau et al. assumed a homothetic to the origin preference function and a Cobb-Douglas production technology. Barnum and Squire also used a Cobb-Douglas production function but assumed a utility function which is homothetic to a point in the
commodity space which is independent of commodity prices (affine homothetic preferences).

In summary, previous empirical studies of the self-employed consumer-producer have been characterized by the following features: (1) they have failed to consider the labour choice problem (particularly choice between different working activities which may generate different disutility) which has implied that the interrelations between consumption and production activities have been restricted to one direction, from the production side to the consumption side, only via the income effect. (2) Despite the fact that a number of the studies have been concerned with farm households in underdeveloped countries where the existence of non-marketed outputs can be expected to be important, they have not considered this problem in the specification of the models. (3) The studies have either estimated total labour supply or they have concentrated on the estimation of the off-farm labour supply equation only. None of the studies have jointly estimated consumption demand, household's labour supply to the household's firm, labour supply to other firms, and the input demand and output supply equations. (4) The functional forms used for specifying preferences and production technologies have been very restrictive.

The analysis in this thesis is directed to improve upon the above shortcomings by: (1) considering labour choice and the interrelations between the production and consumption sides of the model; (2) developing a model appropriate to theoretically analyze and econometrically estimate household's supply and demand responses under conditions of failures of some output markets and general assumptions regarding the family firm's production technology (see Chapter III); (3) estimating the consumption and production branches jointly, emphasizing interrelations within and
between the two branches; (4) using more general functional forms for the production technology and households' preferences. In particular, the functional forms used for specifying preferences allow to formally test whether preferences are homothetic to the origin or whether they are homothetic to a point in the commodity space which is independent of prices.

2.5 Summary

In this chapter a general model of the farm household economic behaviour has been discussed. The model does not rely on the independence of the production and consumption sides of the model. Next by the use of some assumptions the model is simplified and transformed into a standard optimization problem which allows one to use well established duality results for comparative static and empirical estimation of the model. Additionally, the exact conditions under which the independence hypothesis would hold have been discussed. Finally, some comparative static results were derived emphasizing the interrelations between the production and consumption sectors of the model.
Footnotes

1. Real assets include the durable factors of production owned by the family firm. The returns associated with these assets correspond to their rental prices. These rental prices will be exogenous even in the short run provided there exist perfect rental markets for all durable factors of production. If such markets exist then households will rent or let a proportion of their assets until the returns obtained from those assets used by the family farm are equal to their rental prices. In Chapter III the case where such markets do not exist and hence where returns to real assets are endogenous, dependent on the amount of family labour used by the family firm and on the level of such assets, is also considered.

2. Notice that in problem (1) the variables entering the utility function have been aggregated across household's members. A more general formulation of preferences would be:

\[ f(X_{11}, X_{12}, \ldots, X_{MN}; L_{11}, L_{12}; L_{21}, L_{22}; L_{31}, \ldots, L_{M1}, L_{M2}) \]

where \( X_{ij} \) is the consumption of goods \( j \) by household's member \( i \). Similarly, \( L_{k1}, L_{k2} \) are hours of work supplied by the \( k \)th household's member to the family firm and to other firms, respectively. Now, if for any consumption goods \( j \) the prices of \( X_{ij} \) are identical for all \( i, i = 1, \ldots, M \) then Hicks' aggregation theorem can be used and the utility function may be written as:

\[ \tilde{f} = \tilde{f}(X_1, \ldots, X_N; L_{11}, L_{12}; L_{21}, L_{22}; L_{31}, \ldots, L_{M1}, L_{M2}) \]

where

\[ X_j = \sum_{i=1}^{M} X_{ij}, \quad j = 1, \ldots, N. \]

Similarly, if one assumed that prices of \( L_{k1} \) are identical for \( k = 1, \ldots, M \) and if the prices of \( L_{k2} \) are equal for \( k = 1, \ldots, M \) (indeed it is sufficient to assume that the price ratios \( \frac{w_{kl}}{w_{ml}} \) are identical for \( k = 1, \ldots, M, m = 1, \ldots, M \) across the observations) then the household's preferences can be represented as in (1):

\[ f = f(L_1, L_2; X_1, X_2, \ldots, X_N) \]

where

\[ L_1 = \sum_{k=1}^{M} L_{k1} \quad \text{and} \quad L_2 = \sum_{i=1}^{M} L_{k2} \]

These assumptions allow to substantially simplify the empirical applications of the model and are frequently used in the household's literature (see, for example, Lau, Lin, and Yotopolous).

An alternative procedure to aggregate would be to assume that \( X_{ij} \), for \( i = 1, \ldots, M \) are consumed in fixed proportions and that \( L_{k1} \) and \( L_{k2} \), for \( k = 1, \ldots, M \) are also in fixed proportion. That is to use a Leontief aggregation procedure. In this case one can define an aggregate commodity \( X_j = \min \left( \frac{1}{\alpha_1} X_{1j}, \ldots, \frac{1}{\alpha_M} X_{Mj} \right) \) for all \( j = 1, \ldots, N \) and labour supply
aggregates \( L_1 = \min\left(\frac{1}{\beta_1} L_{11}, \ldots, \frac{1}{\beta_M} L_{1M}\right) \) and \( L_2 = \min\left(\frac{1}{\rho_1} L_{12}, \ldots, \frac{1}{\rho_M} L_{2M}\right) \).

where \( \alpha_i, \beta_i, \rho_i \) (\( i = 1, \ldots, M \)) are the fixed coefficients. In equilibrium \( X_{ij} = \alpha_i X_j, L_{i1} = \beta_i L_1 \) and \( L_{i2} = \rho_i L_2 \) for \( i = 1, \ldots, M \).

Hence, the aggregate price of \( X_j \) would be \( P_j = \sum_{i=1}^{M} \alpha_i P_{ij} \) and the aggregate wage rate for \( L_1 \) would be \( w_1 = \sum_{i=1}^{M} \beta_i w_{i1} \) and the wage for \( L_2 \) would be \( w_2 = \sum_{i=1}^{M} \rho_i w_{i2} \).

This aggregation procedure was not used because, as a difference with Hicks' method, it imposes rather strong restrictions on the structure of household's preferences. It implies that there are no substitution possibilities among the consumption levels of the different individuals in the household. This assumption was judged more unrealistic than those required for the Hicks' procedure.

3 If fixed factors of production exist then the profit function would be \( \pi(q; L_1, k) \) where \( k \) is a fixed factor.

4 The assumption of perfect rental markets for durable factors does not seem too unrealistic at least in the case of agriculture in developed countries. Rental markets for land and agricultural machinery, which constitute the major durable factors used in that sector, appear to be well developed.

5 Given that in the empirical analysis cross-sectional data are used, a long-run equilibrium is postulated in order to justify the assumption that all factors of production are variable (see Chapter IV).

6 If constraint (iii) is binding then for some households \( L_1 = 0 \) or \( L_2 = 0 \). There are rather serious econometric difficulties associated with corner solutions and they have deserved the attention of a number of studies (Amemiya, Wales and Woodland, 1980). Given that the data used in the empirical analysis are aggregated, both \( L_1 \) and \( L_2 \) are greater than zero at all sample points and hence constraint (iii) is not binding at any of the observations (see Table A.4).

7 Olsen has argued that under the conditions of assumption (1) and if there is free entry and exit in the self-employment and salaried activities then, in long run equilibrium, the shadow price of on-farm work is equal to the off-farm wage rate even if household's members do not work off-farm. Hence, utility and profit maximizing decisions would be independent even if households do not work off-farm. However, many agricultural activities are subject to rather important institutional restrictions and free entry is certainly far from reality. Moreover, even if the free entry and exit conditions prevail, it is easy to verify that under realistic conditions regarding standard day hours (or week-hours) of work in the salaried activities, Olsen's conclusion is not valid.
The argument can be better illustrated using the following figure:

In the figure, \( \pi(q,L_1) \) represents the household's farm conditional profit function, MH is the exogenous wage rate prevailing in the labour market and I, II and III are indifference curves of the self-employed people. If there are no restrictions on the number of hours of work in the labour market then point A (where the shadow price of labour is less than the prevailing wage rate) would not be a long run equilibrium. In this case self-employed people would move towards the labour market becoming salaried workers. This would imply that less resources are devoted to the self-employed activities which would change net output prices \( q \) (presumably it may also lower MH) and thus causing an upward shift of \( \pi(q,L_1) \). This process would persist until the \( \pi(q,L_1) \) becomes tangent with the MH line, i.e., until the on-farm shadow price of family labour is equal to the off-farm wage rate. Thus, the relevant wage rate for the remaining self-employed households will be equal to MH regardless whether they do part-time off-farm work or not. However, this movement will not necessarily take place under the more realistic assumption that there exists a standard work hours per day. Suppose, for example, that an individual must work at least \( H-d \) hours per day (say, 8 hours) if he is to participate in the labour market at all. Then the feasible set is \( M_{dC} \) and \( edH \) (\( H \) is the zero work option). In this case individuals will maximize their utility at A and, although point B would imply a larger utility, that point is not feasible. Hence, in spite that returns to on-farm labour are lower, self-employed households will not move out of their activities because if they worked as hired labour they would be at point C which yields a lower utility. Hence point A, where the shadow price of on-farm work is lower than the off-farm wage rate, may be a long run equilibrium.

Notice, however, that the model associated with equations (9) and (11) assumes a constant return to scale production technology (CRS). Thus, CRS implies that curve \( D^- \) in Figure 1 would be a horizontal line. Hence, CRS under the conditions of proposition (1) would imply that household's members work only off-farm (if \( w_2 \) is greater than \( \hat{\pi}(q) \)) or only on-farm (if \( w_2 \) is less than \( \hat{\pi}(q) \)).
The term $e_{\bar{\pi}}$ is defined from an expenditure function

$$e(p, \bar{\pi}, w_2; \mu) = \min_{X, H-L_1, H-L_2} \{ px + \bar{\pi}(H-L_1) + w_2(H-L_2) : U(H-L_1, H-L_2, X) \geq \mu \}$$

where $\mu$ stands for utility level. The expenditure function $e(p, \bar{\pi}, w_2; \mu)$ is a positive linear homogeneous, increasing and concave function of $p$, $\bar{\pi}$, and $w_2$ and an increasing function of $\mu$ (Varian, 1978). Using function $e$ one can easily obtain the basic comparative statics results associated with the utility maximization problem (8). The term $e_{\bar{\pi}}$ corresponds to the second derivative $e$ with respect to $\bar{\pi}$, $\frac{\partial^2 e}{\partial \bar{\pi}^2}$, which is non-positive given that $e$ is concave.

See for example, the survey on labour supply by Heckman et al.

The authors used aggregated data given as average values of each of five classes for the eight agricultural regions of Taiwan for the years 1967 and 1968. The authors do not say what percentage of households in each observation engaged in off-farm salaried work. It is clear that if a sizeable proportion of the households did not work off-farm then the use of an off-farm wage rate as a unique price of household labour is not appropriate even if households were indifferent among alternative allocations of their working time (see proposition 1).
CHAPTER III

ESTIMATION OF HOUSEHOLD'S SUPPLY RESPONSES WITH FIXED FACTORS OF PRODUCTION

The model of the previous chapter was justified by assuming that all factors of production were variable and a constant returns to scale production technology. The first assumption appears reasonable in a long-run equilibrium context. In short-run analysis, however, such an assumption can only be justified if perfect rental markets for durable factors of production exist. While it might be reasonable to assume the existence of perfect rental markets in the agricultural sector of a developed economy, this assumption may not be as plausible for agriculture in developing countries. Thus, in this chapter a model appropriate to estimate the household's supply and demand relations in the short-run under conditions of imperfections in second-hand factor markets (i.e., when some inputs are fixed or quasi-fixed), which does not rely on the constant returns to scale assumption is provided.

Constant returns to scale and no fixed factors of production were assumed in Chapter II in order to obtain a linear budget constraint for the household's utility maximization problem. This allowed us to use standard duality theory in order to derive and characterize the functions which describe the optimal values of the utility maximization problem. However, if any of the above assumptions are relaxed then the budget
constraint is non-linear and hence one is faced with an optimization problem which is non-linear in both the objective and constraint functions. Conventional duality theory does not apply to this class of problems and, thus, standard duality cannot be used to specify and characterize the household's optimal responses. Moreover, the shadow price of family labour faced by the household will be dependent on hours of (on-farm) work. Thus, assuming for simplicity no off-farm employment and defining $L_1$ as on-farm work the utility maximization problem is:

$$\max_{H-L_1, X} U(H-L_1, X):$$

(i) \[ px \leq \pi(q;K,L_1) + y \]  \hspace{1cm} (26)

(ii) \[ X \geq 0, \; L_1 \geq 0; \; H-L_1 \geq 0 \]

One may write the solution of (26) as:

$$H-L_1 = L_1(p,q,K,y) \text{ and } X = X(p,q,K,y)$$  \hspace{1cm} (27)

One approach would be to define $w = \frac{\partial \pi(q,K,L_1)}{\partial L_1} = g^1(q,K,L_1)$ as the shadow price of labour and $r = \frac{\partial \pi(q,K,L_1)}{\partial K} = g^2(q,K,L_1)$ as the shadow price of capital and express the budget constraint as \[ px + w(H-L_1) \leq wh + rk + y \equiv Z. \] This approach would suggest to define an indirect utility function $G(p,w,Z)$ and obtain the demand equations in the usual manner. The estimation technique should take care, however, of the endogeneity of $w$ and $Z$ (since $Z$ is dependent on $r$ and $w$) by estimating $H-L_1(p,w,Z)$, $X(p,w,Z)$, $g^1(q,K,L_1)$ and $g^2(q,K,L_1)$ simultaneously. There are a number of inconveniences in following this approach: in the first place, the variables $w$ and $r$ are unobservable and at most one could estimate them indirectly by estimating the function $\pi(q,K,L_1)$. Secondly, it is clear that the approach is exact only under a constant returns to scale
production technology. Only under this assumption \( \pi(q, K, L_1) = wL_1 + rK \), relation which is implied by the above approach. Otherwise the "full income," \( Z \), would not be calculated correctly. Finally, even if it were possible to observe or calculate \( w \) and \( r \) their values would not necessarily correspond to the "true" values which the household actually considered in its utility maximizing decisions. In estimating household's responses one explicitly assumes that they make errors in their utility maximizing decisions. In particular, the levels of \( L_1 \) actually chosen by households also reflect possible optimization errors and, hence, given that the variables \( w \) and \( r \) are dependent on the levels of \( L_1 \), they will also be observed or measured with errors. Thus, errors made by households in choosing \( L_1 \) will affect \( w \) and \( r \) and, given that the actual \( L_1 \) used by households in calculating their \( w \) and \( r \) are unknown, the relevant shadow prices considered by households are also unknown.

The method of estimating the reduced form leisure demand and consumption demand equations (27) also presents problems if these equations need to be explicitly solved from problem (26). This is virtually impossible to do due to the complexity of the budget constraint. Thus these functions could only be arbitrarily specified and hence the connection between the theory and the estimating equations would be lost. There exists, however, an alternative which allows to estimate the reduced form equations for \( H - L_1 \) and \( X \) without explicitly solving for (27), but that preserves the linkages of the estimating equations and the theory. If one can define and characterize an indirect utility function associated with (26) and if the household's supply and demand equations are derived from such indirect utility function (say, by a generalized Roy's identity) then the restrictions implied by the utility maximization
hypothesis can be fully considered in the estimating reduced form equation. The problem of establishing duality relationships for the class of optimization problems such as (26), where both the objective function and the budget constraint are nonlinear, is nontrivial and has recently been analyzed by Epstein (1978). Epstein considered a general nonlinear optimization problem and showed the existence of a dual representation (an indirect utility function), determined general properties of the indirect utility function and derived a generalized Roy's identity. Given that Epstein's assumptions were quite general he obtained a general characterization of the indirect utility function associated with his nonlinear problem. Moreover, he did not consider the econometric problems involved in estimating the behavioural equations derived from the model. Hence, a purpose of this chapter is to show that some more specific properties of the indirect utility function can be derived and, hence, that additional comparative static results can be obtained, from a model where the nonlinear budget constraint involves a conditional profit function. A second objective is to indicate how the optimal household-firm unit's supply and demand responses associated with (26) can be derived via the indirect utility function. Finally, the problems raised when a stochastic framework for the behavioural equations is specified are discussed and an estimation procedure is proposed.

If utility maximization problem (26) is defined locally for the compact subset \( M \), then the indirect utility function associated with that problem is:

\[
G(p,q,K,y) = \max_{H-L_1,X} \{ U(H-L_1,X) : \begin{align*}
(i) & \quad px - \pi(q;K,L_1) = y \\
(ii) & \quad (H-L_1,X) \in \sum (p,q,K,y) \in \mathcal{P} \}
\]

(28)

where the attention is restricted to the set of utility levels.
M = \{u: \u \leq u \leq \bar{u}, \bar{u} < \bar{u}\} which implies that the corresponding commodity space and parameter space are compact, non-empty subsets of \(\mathbb{R}^3\) and \(\mathbb{R}^4\), respectively.

Epstein (1978) showed the existence of a duality relationship in the context of a general non-linear model of which (28) is a special case. Hence, it is not necessary to show here that such a duality relation also exists between the functions \(G(\cdot)\) and \(U(\cdot)\) for a given function \(\pi(\cdot)\). This implies that an indirect utility function \(G(\cdot)\) exists and, moreover, that a function \(U^*(H-L_1,X)\), whose behavioural implications (for a given \(\pi(\cdot)\)) are the same as \(U(H-L_1,X)\), can be retrieved from the following problem:

\[ U^*(H-L_1,X) = \min_{p,q,K,y} \{ G(p,q,K,y): \begin{align*}
& (i) \ p x - \pi(q;K,L_1) = y \\
& (ii) \ H-L_1 \in \mathbb{R} \forall p,q,K,y \in \mathcal{P} \} \] (29)

Thus, using Epstein's results a duality or one-to-one correspondence between \(U(\cdot)\) and \(G(\cdot)\) for any given function \(\pi(\cdot)\) can be established.

3.1 Properties of the Indirect Utility Function

In the linear budget constraint case an important property of the indirect utility function is that it is quasiconvex in the price space. However, if the budget constraint is nonlinear then the indirect utility function is not necessarily quasiconvex, as is shown in the following proposition (see its proof in Appendix 1):

**Proposition 5**

The function \(G\) defined by (28) will be quasiconvex in its arguments if and only if the constraint function is a concave function of the price \((p,q)\) and \(K\) variables.
The constraint function in (28) is concave in p and q (using the properties of the profit function) but it is not concave in K since \( -\pi(q;K,L_1) \) is convex in K. Therefore, function G is not quasiconvex.

Is it necessary to assume global quasiconcavity for U in problem (28)? In the linear constraint case the assumption of quasiconcavity of U is useful because it implies that all points in the commodity space (in the positive orthant) will be optimal for some set of prices and income. This insures that minimization of the indirect utility function subject to the budget constraint retrieves the original U. In the nonlinear budget constraint case the assumption of quasiconcavity of U is no longer useful in this respect unless the budget constraint is concave in its arguments. In (28) the constraint function does not satisfy this condition and hence the utility function retrieved by (29) will not necessarily be identical to the original U. Hence, the assumption of quasiconcavity of U is no longer useful and instead the following assumption, as proposed by Epstein (1978), is used:

Assumption A1.2: for all \( X^0, H-L_{01} \in \mathcal{S} \) there exists a set of parameters \( p^0, q^0, k^0, y^0 \in \mathcal{P} \) such that \( X^0, H-L_{01} \) are optimal solving utility maximization problem (28). Thus, this assumption is used instead of A.2 together with assumptions A.1, A.3, and A.4 (see Chapter II). Hence, in this chapter it is assumed that U satisfies the following properties.

Conditions A1 on U

A1.1 Defined and continuous from above.

A1.2 For all \( X^0, H-L_{01} \in \mathcal{S} \) there exist a set of parameters \( p^0, q^0, k^0, y^0 \in \mathcal{P} \) such that \( X^0, H-L_{01} \), are optimal solving (28).

A.3 Locally increasing in X.

A.4 Increasing in \( H-L_{1} \).
Epstein (1978) has shown that if U satisfies $A^{1.2}$ then the retrieved utility function obtained by minimizing the indirect utility function subject to the budget constraint will be identical to the original U. The problem with assumption $A^{1.2}$ is that it cannot be empirically verified. Epstein justifies the use of $A^{1.2}$ by the fact that if $X^0, H-L^0_1$ is not optimal for any parameter vector then $X^0, H-L^0_1$ will never be observed and hence one can define a function $U^* = U$ whose behavioural implications are exactly the same as U. If this is the case then it is irrelevant whether $U^*$ or U are retrieved from (29).

Thus, all that is needed is to test whether the second order conditions for a minimum of (29) are locally met at the solution points. These conditions will be met if the Hessian matrix of the function $G(p^*, q^*, K^*; px - \pi(q^*; K^*, L_1))$ is positive semidefinite at $p^*, q^*, K^*$. Hence, a local $U(\cdot)$ can be retrieved from $G$ if this condition is satisfied.

The following proposition summarizes the properties of $G(\cdot)$ as defined by (28).

**Proposition 6**

If U satisfies conditions $A^1$ and if the profit function $\pi$ satisfies conditions C (see Chapter II) then $G(p, q, K, y)$ defined by (28) satisfies conditions D below.

**Conditions D on G**

D.1 Defined and continuous from below.

D.2 Nonincreasing in p and increasing in q, K, y.

D.3 Homogeneous of degree zero in p, q, y.

D.4 $\psi$ vector $(p^0, q^0, K^0, y^0) \in \mathcal{P}$ a vector $(H-L^0_1, X^0, Q^0) \in \mathcal{S}$ such that $(p^0, q^0, K^0, y^0)$ is a solution to (28).
Proof

D.1 Its proof is in Epstein (1978), page 31.

D.2 Let \( B = \{ H-L_1, X : pX - \pi(q; K, L_1) \leq y \} \) and
\[
B^1 = \{ H-L_1, X : p^1X - \pi(q; K, L_1) \leq y \}.
\]
Assume that \( p^1 \geq p \), then
\[ B^1 \subseteq B \]
because \( \pi \) is decreasing in \( H-L_1 \). Hence given \( A^{1.3} \) and \( A^{1.4} \) the maximum \( U \) attainable cannot be increasing in \( p \).

The argument for showing that \( G \) is increasing in \( q, K, y \) is similar.

Note that \( \pi \) is increasing in \( q \) and \( K \).

D.3 If \( p, q \) and \( y \) are multiplied by \( t > 0 \) then the budget constraint will not change at all because \( \pi \) is homogeneous of degree one in \( q \).

Hence, the maximum \( U \) attainable will not be altered by increasing \( p, q \) and \( y \) by the same proportion.

D.4 Proof is in Epstein (1978), page 31.

3.2 Derivation of the Demand and Supply Equations

The demand and supply equations associated with problem (28) can be derived from the first order conditions of minimization problem (29). In deriving the household's supply and demand specifications one is interested in obtaining the consumption functions \( X(p, q, K, y) \), \( H-L_1(p, q, K, y) \) as well as the unconditional net output supply vector \( Q(p, q, K, y) \). Hence, it is necessary to check whether these functions can be derived from the first order conditions of problem (29).

The first order conditions of (29) are:

\[
\begin{align*}
(a) & \quad \frac{\partial G}{\partial p} + \frac{\partial G}{\partial y} X = 0 \\
(b) & \quad \frac{\partial G}{\partial K} - \frac{\partial G}{\partial y} \frac{\partial \pi}{\partial K} = 0 \\
(c) & \quad \frac{\partial G}{\partial q_i} - \frac{\partial G}{\partial y} \frac{\partial \pi}{\partial q_i} = 0 \quad i = 1, \ldots, S
\end{align*}
\]
First notice that all equations in (30) are not independent.\(^5\)

Hence equation (d) can be dropped and the \(S + 2\) remaining equations can be used in specifying the same number of supply and demand equations \((X, H-L_1\) and \(S\) net output \(Q_i\)). The demand equation for \(X\) is directly obtained from (30(a)). If the conditions of the implicit function theorem are met by the function \(\pi(\cdot)\) then one can also obtain the equation \(H-L_1(p,q,K,y)\) by solving (30(b) for \(L_1\). Using Hotelling's lemma it can be easily seen that the unconditional net output supply functions can be obtained from (30(c)):

\[
Q_i = \frac{\partial \pi}{\partial q_i} = \frac{\partial G}{\partial q_i} \quad i = 1, \ldots, S
\]

Hence, underlying the \(S\) equations in (30(c)) there are \(S\) net output supply equations \(Q_i(p,q,K,y)\). Thus, (30(a), 30(b) and 30(c) provide the utility maximizing equations for consumption goods' demand, the shadow price of fixed factors and the net output supply functions, respectively.

This approach has the interesting feature of integrating the derivation of the production and consumption household's behavioural equations and thus emphasizing the interdependence of production and consumption decisions, which is evident from the fact that both consumption and production responses are derived from the same indirect utility function.

Some properties of the behavioural functions can be directly derived from the conditions \(D\) on the indirect utility function \(G(\cdot)\). In the first place, they will be defined and continuous. Moreover, condition \(D.3\) implies that the behavioural equations for \(X, \frac{\partial \pi}{\partial K}\) and \(Q\) are homogeneous of degree zero in \(p, q,\) and \(y\). Note that the net output supply functions will not be homogeneous of degree zero in \(q\) as occurs in the
conventional model. However, the net output supply conditional on \( L_1 \) (i.e., assuming that changes in \( q \) do not affect \( L_1 \)) are homogeneous of degree zero in \( q \). The fact that the unconditional net output supply functions are not homogeneous of degree zero in \( q \) makes intuitive sense; if all \( q^j \) are multiplied by \( t \) then the optimal \( L_1 \) will change because, although the shadow price of \( L_1 \) will be unaffected, the budget constraint in (28) changes since the conditional profit level is expanded by \( t \), i.e., in (28.i) \( \pi(tq;K,L_1) = t\pi(q;K,L_1) \). The utility maximization hypothesis implies some additional restrictions which are discussed in the following section.

### 3.3 Further Implications of the Utility Maximization Hypothesis

It is possible to define an expenditure function \( e(p,q,K;u) \) associated with utility maximization problem (28):

\[
e(p,q,K;u) = \min_{X,H-L_1} \{px - \pi(q;K,L_1) : U(H-L_1,X) \geq u\} \tag{32}
\]

The solution of (32) gives the compensated or utility constant demand functions \( X^*(p,q,K;u) \) and \( H-L_1^*(p,q,K;u) \). Given that a duality relation between \( U \) and \( G \) exists then an expenditure function which is continuous twice differentiable, increasing in \( p \) and decreasing in \( q \) and \( K \) will also exist. Moreover, a maximum for the following problem will exist:

\[
\max_{p,q,K} E \equiv e(p,q,K;u) - [px - \pi(q;K,L_1)] \tag{33}
\]

Additionally, \( E = 0 \) when evaluated at its maximum. That is, the minimum expenditures \( e \) are never larger than actual expenditures and they will be identical when actual expenditures are evaluated at the solution of problem (33). Thus, the first order conditions of (33) will define the expenditure minimizer or compensated values for
consumption goods, $X^*$, and they will also implicitly define the expenditure minimizer values for on-farm work, $L^*_{1}$ as well as the net output supply values as a function of $L^*_{1}$, which is defined for a constant utility level. Thus, the first order conditions of maximization problem (33) are:

(i) \[ \frac{\partial e(p,q,K;u)}{\partial p} - x^* = 0 \]

(ii) \[ \frac{\partial e(p,q,K;u)}{\partial q} + \frac{\partial \pi(q;K,L^*_{1})}{\partial q} = 0 \]

(iii) \[ \frac{\partial e(p,q,K;u)}{\partial K} + \frac{\partial \pi(q;K,L^*_{1})}{\partial K} = 0 \]

Equation (i) is Shephard's lemma in its usual representation (except for the fact that $q$ and $K$ appear in the function $e(.)$), and directly provides the compensated demand function for consumption goods, $X^*(p,q,K;u)$. Equation (iii) implicitly defines the expenditure minimizer or compensated function for on-farm work, i.e., if the conditions of the implicit function theorem hold for the function $-\frac{\partial \pi(q;K,L^*_{1})}{\partial K}$ then one can solve (iii) for $L^*_{1}(p,q,K;u)$. Using Hotelling's lemma

\[ \frac{\partial \pi(q;K,L^*_{1})}{\partial q} = Q^*(q;K,L^*_{1}(p,q,K;u)) \]

where $Q^*$ can be interpreted as compensated net output supply functions. That is, $Q^*$ is the net output supply for a given level of $u$. Hence, using (ii) one can obtain the compensated net output supply functions in terms of the expenditure function, i.e.,

\[ Q^* = - \frac{\partial e(p,q,K;u)}{\partial q} \]

The second order conditions for a maximum require that the Hessian matrix $H$ for $E$ be negative semidefinite in $p$, $q$ and $K$. Thus, using the convention of defining $\frac{\partial e}{\partial p} = e^p$, $\frac{\partial^2 e}{\partial p^2} = e^{pp}$ and using similar notation for the other derivatives, the following comparative static matrix can be
Negative semidefiniteness of matrix $H$ implies that its diagonal elements are non-positive. That is, as in the conventional case, the expenditure function is concave in commodity prices, $p$. An additional implication of negative semidefiniteness of $H$ is that the expenditure function $e(\cdot)$ is also concave in $q$ since $e_{qq} + \pi_{qq} \leq 0$ implies that $e_{qq} \leq 0$ by convexity of $\pi(\cdot)$ in $q$. Furthermore, a testable prediction of the model is that the absolute value of $\pi_{qq}$ cannot be greater than the absolute value of $e_{qq}$. The expenditure function is not necessarily concave in $K$ because $\pi(\cdot)$ is concave in $K$. So, for example, a positive $e_{KK}$ may be consistent with negative semidefiniteness of $H$ because $\pi_{KK} \leq 0$.

Using the identities $X^*(p,q,K;u^0) = X(p,q,K;e(p,q,K;u^0))$, $L^*_1(p,q,K;u^0) = L_1(p,q,K;e(p,q,K;u^0))$ and $Q^*(p,q,K;L^*_1) = Q(p,q,K;L_1)$ where $X$, $L_1$ and $Q$ without stars denote uncompensated demand and net output supply functions, one can verify that the $3 \times 3$ matrix $D$

\[
D = \begin{bmatrix}
\frac{\partial X}{\partial p} + \frac{\partial X}{\partial y} & \frac{\partial X}{\partial q} & \frac{\partial X}{\partial K} \\
\frac{\partial Q}{\partial L_1} + \frac{\partial L_1}{\partial y} & -\frac{\partial Q}{\partial L_1} & \frac{\partial L_1}{\partial K} \\
\frac{\partial L_1}{\partial p} + \frac{\partial L_1}{\partial y} & \frac{\partial L_1}{\partial q} & \frac{\partial L_1}{\partial K}
\end{bmatrix}
\]

(36)
will be equal to the Hessian matrix $H$. Hence, matrix $D$ will be negative semidefinite and symmetric. This allows one to obtain some comparative static results; using the diagonal terms the comparative static effects on $L_1$ and $Q$ of a change in $q$ can be obtained via a modified Slutsky equation:

\[
\begin{align*}
\frac{\partial L_1}{\partial q} &= -\frac{\pi q + \pi q q}{\pi q L_1} + Q \frac{\partial L_1}{\partial y} \\
\frac{\partial Q}{\partial q} &= -\frac{\pi q + \pi q L_1}{\pi q L_1} Q \frac{\partial L_1}{\partial y}
\end{align*}
\]

where the first right-hand-side term in each equation represents the compensated (i.e., utility constant) net output price effect on labour supply and net output supply. As can be expected this effect is positive given that matrix $H$ is negative semidefinite. The second right-hand-side term in each equation is the income effect which is negative if leisure is not an inferior good. Hence, the signs of $\frac{\partial L_1}{\partial q}$ and $\frac{\partial Q}{\partial q}$ will in general be unknown. The effect of $K$ on $L_1$ can also be considered using matrices $H$ and $D$:

\[
\frac{\partial L_1}{\partial K} = -\frac{\pi K K q + \pi K K q q}{\pi K L_1} + \pi K \frac{\partial L_1}{\partial y}
\]

If $\pi_{KL_1} > 0$ then the first term on the right-hand-side is positive and the sign of the second term will be negative if leisure is a normal good. An increase in capital endowments will lead to an increase in the opportunity cost of leisure which would imply a lower demand for leisure. However, the increase in $K$ will also have an income effect which will be manifested in an expansion in the demand for leisure.

The results which are not totally obvious are those associated with the symmetry of matrix $H$. Thus, recalling that $\frac{\partial e}{\partial q} = -Q*(q;k,L_*^1)$ and
using elements (1,2) and (2,1) of (35) one may obtain the following testable symmetry relationship:

\[
\frac{\partial X^*}{\partial q} = - \frac{\partial Q^*(q;K,L^*_1)}{\partial p}
\]  

(39)

Thus, the compensated effect of a change in a net output price on the demand for consumption goods will be equal to minus the effect of a change in the consumer good price on compensated or utility constant net output supply.

Similarly, using elements (1,3) and (3,1) of (35) an additional symmetry relation becomes evident:

\[
\frac{\partial X^*}{\partial K} = - \frac{\partial \pi}{\partial K} 
\]

(40)

which implies that the effect of changes in fixed factors on \(X^*\) will be identical to minus the effect of a change in the cost of living index \((p)\) on the shadow price of capital, \(\partial \pi/\partial K\).

Finally, (2,3) and (3,2) of matrix D provide the following well known symmetry relation:

\[
\frac{\partial Q}{\partial K} = \frac{\partial \pi}{\partial K}
\]

(41)

Obviously, relations (39) to (41) can be expressed in terms of uncompensated functions using the equality of matrices D and H. It is easy to verify that if off-farm labour supply is considered \((L^*_2)\) then

\[
\frac{\partial (H-L^*_2)}{\partial q} = \frac{\partial Q(q,K,L^*_1)}{\partial w_2} \quad \text{and} \quad \frac{\partial (H-L^*_2)}{\partial K} = - \frac{\partial \pi}{\partial K} \frac{1}{\partial w_2}
\]

where \(w_2\) is the off-farm wage rate.

In summary, using the results from sections (3.1) and (3.2) the following implications of the utility maximization hypothesis are obtained.

1. The behavioural equations \(X = X(p,q,K,y)\), \(\frac{\partial \pi}{\partial K} = \psi(p,q,K,y)\) and \(Q = Q(p,q,K,y)\) are defined, continuous and homogeneous degree zero in \(p\),
q and y. In particular the net output supply equations are not homogeneous of degree zero in q.

2. The Hessian matrix of the function E as defined by (33) is negative semidefinite which implies the following empirically testable predictions:

\[ \frac{\partial L_1}{\partial q} - Q \frac{\partial L_1}{\partial y} \geq 0 \]

\[ \frac{\partial Q}{\partial q} - \pi Q \frac{\partial L_1}{\partial y} > 0 \]

\[ \frac{\partial L_1}{\partial K} - \pi K \frac{\partial L_1}{\partial y} \geq 0 \]

in addition to the standard result \( \frac{\partial X^*}{\partial p} \leq 0 \).

3. Symmetry of the Hessian matrix of the function E implies that

\[ \frac{\partial X^*}{\partial p} = \frac{\partial Q(q; K, L_1^*)}{\partial p} \]

\[ \frac{\partial X^*}{\partial K} = -\frac{\partial \pi}{\partial p} \text{ and } \frac{\partial Q}{\partial K} = \frac{\partial \pi}{\partial q} \].

Note that the first two symmetry properties relate production and consumption decisions, thus emphasizing their interdependence.

3.4 A Stochastic Specification

Using (30) and (31) one can obtain the household's demand for consumption goods, X as well as the net output supply response specifications by postulating appropriate functional forms for \( G(\cdot) \) and \( \pi(\cdot) \). With respect to \( L_1 \) it is necessary to indicate that one can only obtain an implicit representation of it using equation (30.b) and that \( L_1 \) needs to be interpreted as the equilibrium level of use of labour rather than as a labour supply schedule. This is so because labour is supplied and demanded within the individual household-firm unit and the indirect utility function \( G(\cdot) \) defined by (28) is obtained when the supply and
demand equilibrium level of $L_1$ is substituted into $U(H-L_1,X)$. This can be seen more clearly by considering the first order conditions of utility maximization problem (28): one of these conditions is that

$$- \frac{\partial U(H-L_1,X)}{\partial L_1} = \frac{1}{\lambda} \frac{\partial \pi(q,K,L_1)}{\partial L_1},$$

where the left-hand-side can be interpreted as the household's supply schedule of family labour and the right-hand-side is the farm's demand schedule for family labour. Thus, the optimal $L_1$ used in defining $G(\cdot)$ is the $L_1$ which solves the above equation, that is, the level of $L_1$ which leads to an equilibrium of supply and demand for $L_1$.

The stochastic structure of the household's equations can be specified by assuming additive disturbances with zero means and positive semidefinite variance-covariance matrix:

\begin{align*}
\text{(i)} & \quad X = - \frac{\partial G}{\partial \alpha} + e_1 \\
\text{(ii)} & \quad Q_i = \frac{\partial G}{\partial q_i} + e_2 \\
\text{(iii)} & \quad \frac{\partial \pi(q;K,L_1)}{\partial K} = \frac{\partial G}{\partial \alpha} + e_3
\end{align*}

where $e_i (i=1,\ldots,3)$ are the disturbance terms.

It is evident that equation (42.iii) cannot be estimated unless the shadow price of the fixed factor of production $K$ is observed. Unfortunately, the shadow price of $K$ is rarely observed if a rental market for factor $K$ does not exist. Although the shadow price of factor $K$, $\frac{\partial \pi}{\partial K}$, cannot be observed the variable $\pi$ ("profit") can at least be calculated; it is simply the net farm returns after payments for all variable inputs (except $L_1$) are deducted from the gross sales. Hence, given that $q$, $K$ and $L_1$ are also observed one could in principle estimate the vector of
parameters, \( \alpha \), which characterizes the profit function by estimating

\[ \pi = \pi(q, K, L_1; \alpha) + \mu \]  
(43)

where \( \mu \) is the disturbance term.

The problem of estimating (43) is that the variable \( L_1 \) may be correlated with the disturbance term and hence the estimates of \( \alpha \) would not be consistent. However, if \( \pi(\cdot) \) is linear in the parameters (as is usually the case when flexible functional forms are used) then one can use an instrumental variable technique (see, for example, Goldfeld and Quandt) and thus to obtain consistent estimates of \( \alpha \). Therefore, if an appropriate instrumental variable for \( L_1 \) exists then one can estimate equation (43) obtaining consistent estimates (\( \hat{\alpha} \)) for the parameters of the conditional profit function. Using the estimated vector \( \hat{\alpha} \) one can evaluate the function

\[ \frac{\partial \pi(q; K, L_1; \hat{\alpha})}{\partial K} \]

which is the "true" shadow price of capital, \( \frac{\partial \pi(q; K, L_1; \alpha)}{\partial K} \), measured with an error. Thus, the "true" shadow price of capital is equal to the estimated shadow price plus an error term assumed to be stochastic. That is:

\[ \frac{\partial \pi(q; K, L_1; \hat{\alpha})}{\partial K} = \frac{\partial \pi(q; K, L_1; \alpha)}{\partial K} + \mu_K \]  
(44)

Notice that equation (44) is not estimated, the \( \hat{\alpha} \) parameters are obtained by estimating equation (43) and substituted into \( \frac{\partial \pi}{\partial K} \). Equation (44) cannot be estimated because the shadow price of capital is not observed, i.e., the left-hand-side of (44) is unobservable. In other words, by estimating \( \hat{\alpha} \) in (43) one obtains a measure of the "true" shadow price of capital subject to an error, \( \mu_K \). If (44) is used in (42.iii) then one may interpret equation (42.iii) as an error of measurement.
dependent variable situation, which offers no estimation problems:

$$\frac{\partial \pi(q;K,L_1,\hat{a})}{\partial K} = \frac{\partial G(p,q,K,y)}{\partial K} / \frac{\partial G(p,q,K,y)}{\partial K} + \varepsilon_3$$

(45)

where $\varepsilon_3 = e_3 - u_K$.

Hence, if $e_3$ and $u_K$ are normally distributed and independent from $p, q, K, y$ in (42) and (44) then $\varepsilon_3$ will possess the same properties. Thus, in order to estimate (45) there is no inconvenience in using the predicted rather than the actual shadow price of $K$.

Although an explicit labour equation cannot be estimated, if the parameters of (44) and (45) are estimated then one obtains an implicit representation of $L_1$ on the left-hand-side of (45) and hence all the relevant economic information regarding household's labour use can be derived. For example, defining $\psi(q;K,L_1,\hat{a}) = \frac{\partial \pi(q;K,L_1,\hat{a})}{\partial K}$ and

$$\phi(p,q,K,y;\hat{a}) = \frac{\partial G/\partial K}{\partial G/\partial y}$$

where $\hat{a}$ are the estimated parameters characterizing $\phi(\cdot)$, the labour elasticity with respect to household's income ($e_{Liy}$) is:

$$e_{Liy} = \frac{\partial \psi/\partial y}{\partial \psi/\partial L_1} \frac{y}{L_1}.$$  

(40)

In a similar manner one can calculate the equilibrium labour supply elasticities with respect to any of the other independent variables.

Thus, the procedure involves the following steps: firstly, to postulate a function form for $G(\cdot)$ which is at least locally consistent with conditions D as specified in section 3.2 and also to postulate a functional form for the conditional profit function consistent with conditions C as discussed in Chapter II. Secondly, to estimate the function $\pi(\cdot)$ and next to jointly estimate the indirect utility function using equations (42.i), (42.ii), and (45). The utility maximization restrictions may be checked (or tested) by verifying whether the estimated
G(·) function has a positive semidefinite Hessian matrix at the observed values of p, q, K and y, whether it satisfies conditions D.2 and D.3 and if the symmetry restrictions implied by (36) are satisfied.

3.5 Non-traded Outputs

Hymer and Resnick have stressed the fact that farm households in agrarian economies devote a substantial part of their time in non-agricultural, non-leisure activities. A significant part of the household's time is spent on small manufacturing, construction, and other non-agricultural activities which are oriented to produce goods to be consumed by the same household. Thus, given that these goods are not traded, their prices are essentially endogenous to the household's decisions and constitute an additional source of interdependence between utility maximization and profit maximization decisions.

The importance of these non-agricultural activities is that they compete with the agricultural activities in the use of resources, mainly the operator and family manpower. This implies that they may have an important effect on farmers' supply responses.

Model (26) can be readily adapted to consider non-traded outputs. Assuming for simplicity no off-farm employment, (26) becomes:

\[
\max_{H-L_1, X, z} U(H-L_1, X, z) \\
\text{s.t. } pX \leq \pi(q; H-(H-L_1), z, K_1, K_2) + y \\
x, H-L_1, z \geq 0
\]  

where

\[z = \text{non-agricultural goods produced and consumed by the household}\]

\[K_1, K_2 = \text{two fixed inputs, say land and livestock.}\]

Now, one can obtain an indirect utility function \(G(p, q, y, K_1, K_2)\)
from (47) in the same manner as before. Minimizing \( G \) subject to the
budget constraint the following first order conditions are obtained:

\[
\begin{align*}
(a) & \quad \frac{\partial G}{\partial p} + \frac{\partial G}{\partial y} x = 0 \\
(b) & \quad \frac{\partial G}{\partial q_i} - \frac{\partial G}{\partial y} \frac{\partial \pi}{\partial q_i} = 0 \quad i = 1, \ldots, S \\
(c) & \quad \frac{\partial G}{\partial K_1} - \frac{\partial G}{\partial y} \frac{\partial \pi}{\partial K_1} = 0 \\
(d) & \quad \frac{\partial G}{\partial K_2} - \frac{\partial G}{\partial y} \frac{\partial \pi}{\partial K_2} = 0 \quad (48)
\end{align*}
\]

The consumption demand equations are obtained from (a), and (b) provides the supply functions of traded outputs. The equations for labour used and production of non-traded outputs are implicit in equations (c) and (d). Conditions (a)-(d) can be written as:

\[
\begin{align*}
(a^1) & \quad x = - \frac{\partial G/\partial p}{\partial G/\partial y} \\
(b^1) & \quad Q_i = \frac{\partial G/\partial q_i}{\partial G/\partial y} \quad i = 1, \ldots, S \\
(c^1) & \quad \frac{\partial \pi}{\partial K_1} = \frac{\partial G/\partial K_1}{\partial G/\partial y} \\
(d^1) & \quad \frac{\partial \pi}{\partial K_2} = \frac{\partial G/\partial K_2}{\partial G/\partial y} \quad (49)
\end{align*}
\]

Thus, by estimating the function \( \pi(\cdot) \) one can obtain the predicted values for \( \frac{\partial \pi}{\partial K_1} \) and \( \frac{\partial \pi}{\partial K_2} \) and use them in (c^1) and (d^1). The econometric framework would be similar to the one described by equations (42.i), (42.ii) and (45) with the only difference that now two rather than one shadow price equation needs to be estimated. As before, once the parameters of \( G(\cdot) \) and \( \pi(\cdot) \) are estimated one can proceed to obtain the relevant elasticities of \( L_1 \) and \( z \) with respect to the exogenous variables.

Thus, assuming appropriate functional forms for \( G(\cdot) \) and for \( \pi(\cdot) \)
one can explicitly derive a set of equations for the consumption demand functions, production responses of non-traded outputs, and supply responses for traded outputs. The advantage of the approach is that the estimating equations are explicitly derived from a utility maximization scheme and the interactions between the consumption, agricultural, and non-agricultural activities can be understood and measured.

The disadvantage of the approach is that the data requirements are high. In particular, it may be difficult to obtain data regarding production of non-traded outputs. However, a number of surveys have been carried out in underdeveloped countries which do ask questions regarding non-traded outputs and in general non-agricultural activities on the farm.
Footnotes

1 This assumption is made only for the purpose of keeping the notation simple. One may consider off-farm employment using H-L2 as one element of the vector X without changing the subsequent analysis. This can be done provided that the term H-W2 be added to the right-hand-side of (26.i). Another assumption used is that all outputs are traded. This assumption is relaxed in section 3.4.

2 Note that the solution of (26) consists not only on optimal values for X and H-L1, but also the unconditional net output supply equations are implicitly obtained. The net output equations conditional to a given level of L1, i.e., Q(q;K,L1) can be derived independently of the solution of (26) by simply differentiating π(q;K,L1) with respect to q. However, the unconditional net output supply equations, i.e., Q(q,p,K,y) = Q(q;K,L1(q,p,K,y)) are jointly determined with the optimal solutions H-L1(p,q,K,y) and X(p,q,K,y) of (26).

3 The problem analyzed by Epstein is max U(X): C(X,a) ≤ B; X ∈ S, (α,B) ∈ P where α and B are parameters and C(X,α) is jointly continuous in (X,α) and U(X) is assumed to satisfy the following properties:
(1) Defined and continuous for X ∈ S
(2) Local nonsatiation in X, and
(3) ∀ X° ∈ S ∃ (α°,B°) ∈ P such that X° is optimal.

The use of a local approach is essential in Epstein's proof of existence of duality. Assumption (3) is also necessary and hence it is assumed in problem (28) that U(*) also satisfies the assumption.

4 Notice that the function π can be written as π(q;K,H-(H-L1)) which is clearly decreasing in H-L1.

5 This can be seen by multiplying equations (a) and (c) by p and q, respectively, using ∂y = − ∂G/∂y y (where λ is the Lagrangean multiplier associated with constraint (i) in (29)) and using Euler's theorem applied to G (which is homogeneous degree zero in p, q, and y) and to π (which is linear homogeneous in q).

6 This results from a direct application of Epstein's results.

7 One could also solve the function ∂π/∂K for L1 (if a functional form for ∂π/∂K is postulated) and then proceed with the estimation of L1; if ∂π(q;K,L1)/∂L ≡ ψ(q,K,L1) then (32.iii) will become: L1 = ψ−1 [G/∂G/∂K + e,].

This method is certainly not appropriate because, given the non-linear structure of the equation, it is not in general feasible to separate an error term which is independent of the explanatory variables. Moreover, it is extremely difficult to find an explicit relationship between the new error term and the independent variables; hence the distribution of this error will be generally unknown even if e, is normally distributed.
Defining $\psi^1(q; K_1, K_2, L_1, z; \hat{\alpha}) \equiv \frac{\partial \pi}{\partial K_1}$ and $\psi^2(\cdot) \equiv \frac{\partial \pi}{\partial K_2}$ then the comparative static vector $\left[ \frac{\partial L_1}{\partial B}, \frac{\partial z}{\partial B} \right]$, where B is any independent variable, need to be obtained simultaneously by solving:

$$
\begin{bmatrix}
\frac{\partial \psi^1}{\partial L_1} & \frac{\partial \psi^1}{\partial z} \\
\frac{\partial \psi^2}{\partial L_1} & \frac{\partial \psi^2}{\partial z}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial L_1}{\partial B} \\
\frac{\partial z}{\partial B}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \phi^1}{\partial B} \\
\frac{\partial \phi^2}{\partial B}
\end{bmatrix}
$$

where $\phi^1 \equiv \frac{\partial G/\partial K_1}{\partial G/\partial y}$ and $\phi^2 \equiv \frac{\partial G/\partial K_2}{\partial G/\partial y}$. Thus, if the conditions of the implicit function theorem are satisfied by the function $\pi(\cdot)$ then it is possible to obtain the comparative static vector and hence the relevant elasticities for $L_1$ and $z$. 
CHAPTER IV

THE ESTIMATING MODEL

This chapter discusses the empirical implementation of the simplified model presented in Chapter II as applied to Canadian cross-sectional agricultural data. Four major problems are analyzed:

1. the selection of functional forms for the indirect utility function and for the conditional profit function and the derivation of explicit formulations for the supply and demand equations to be estimated;
2. the econometric model, discussing econometric problems and the procedures used to overcome them;
3. the discussion of an econometric procedure designed to formally test the hypothesis of independent utility and profit maximizing decisions;
4. the data required vis-a-vis the data available emphasizing data limitations.

4.1 Functional Forms for the Indirect Utility Function and for the Conditional Profit Function

The model to be estimated is the one described by the conditional profit function defined by equation (6) and by the indirect utility function defined by (8) in Chapter II. A major consideration in choosing functional forms for these functions is that the cross-sectional data used in the study are aggregated by census divisions (see Section 4.4). This will imply restrictions on the level of generality of the functional form postulated for the indirect utility function.
Utility maximization problem (8) defines an indirect utility function $G(\bar{n}, w_2, p; Z)$. Now, for estimation purposes it is necessary to postulate a functional form for $G(\bar{n}, w_2, p; Z)$ which is continuous, positive, non-increasing, quasi-convex function of its arguments (Diewert, 1974). Four alternatives are available given that the data available are aggregated:

1. To assume a very general utility function and that income is distributed in fixed proportions among household groups which have equal preferences. This alternative is not appropriate because it would require an assumption that there is a high correlation between the type of households' preferences and their share in total income. Obviously, this is unrealistic. Furthermore, although consistent aggregated demand functions can be obtained, they are not subject to any local restrictions except Walras's law if the number of consumers in the aggregate is greater or equal to the number of commodities (see Sonnenschein or Diewert, 1977). In particular, symmetry and negative semi-definiteness restrictions do not apply to such system of demand functions.

2. To impose some restrictions on the utility function which may lead to consistent aggregate demand functions. Specifically, homothetic and identical preferences are sufficient conditions for obtaining consistent aggregate demand functions (Chipman, 1974). Under these assumptions the system of demand functions will not only be consistent but it will also satisfy the symmetry and negative semi-definiteness restrictions. However, rather important differences in educational, race, and other variables among households lead one to consider the equal tastes assumption unrealistic. Furthermore,
homotheticity implies unitary income demand elasticities for all goods which contradicts Engel's law.

3. An approach developed by Berndt et al. (1977) allows one to assume a very general utility function identical for all households which yields consistent demand functions provided that information on income distribution is explicitly considered in the market demand equations. Thus, the market demand equation \( (X_i) \) for commodity \( i \) would be

\[
X_i(p;\phi) = M \int_0^\infty \phi(Y) \cdot x_i(p/Y) \, dY
\]

where

- \( M \) is the number of households
- \( Y \) is the income which is distributed according to a density function \( \phi(Y) \) and
- \( x_i(p/Y) \) is the household's demand function which is a function of prices, \( p \), and \( Y_i \).

This approach seems appealing mainly because it does not impose any restrictions on the household's utility function. Unfortunately, information on income distribution at the census division level is not available. In other words, the density function \( \phi(Y) \) cannot be determined given the data available and hence the procedure cannot be applied.

4. It has been shown that homotheticity to the origin of preferences is not a necessary condition for consistent aggregation (Gorman). The Gorman Polar Form (GPF) appears to be the most general restriction on preferences which allows for consistent aggregation and where the demand system satisfies the integrability conditions. Considering this a GPF for \( G(f,w_2,p;Z) \) is used in this study.
The GPF indirect utility function can be written as

\[ G(\bar{\pi}, w_2, p; Z) = \frac{Z - \Lambda(\bar{\pi}, w_2, p)}{\psi(\bar{\pi}, w_2, p)} \]  

(50)

where \( \Lambda \) and \( \psi \) are continuous, concave, nondecreasing and positively homogeneous of degree one in \( \bar{\pi}, w_2, \) and \( p \) (Blackorby et al., 1978).

In order to consider some aggregation properties of the demand functions derived from (50) it is convenient to analyze the expenditure function associated with the Gorman-Polar Form indirect utility function, which is obtained by simple inversion of \( G \) in \( Z \) (Blackorby et al., 1978):

\[ e(\bar{\pi}, w_2, p; \mu) = \mu \psi(\bar{\pi}, w_2, p) + \Lambda(\bar{\pi}, w_2, p), \]

where \( e \) is the expenditure function and \( \mu \) denotes utility level.

In general, the expenditure function is continuous, nondecreasing, homogeneous of degree one and concave in prices (see for example Varian). The Gorman-Polar expenditure function will satisfy these conditions provided the compensated demand functions, \( X, \) satisfy the following condition:

\[ X > \psi \Lambda(\bar{\pi}, w_2, p). \]

The GPF does not necessarily define preferences over the entire non-negative orthant and if \( X < \psi \Lambda(\bar{\pi}, w_2, p) \) then \( \mu \) will be negative in which case \( e(\bar{\pi}, w_2, p; \mu) \) will not necessarily be concave and increasing in \( \bar{\pi}, w_2, p. \) That \( \mu < 0 \) if \( X < \psi \Lambda(\bar{\pi}, w_2, p) \) can be easily verified using Shephard's lemma (Shephard).

If there are \( N \) households, aggregation will be possible if each household has the following expenditure function:

\[ e_h(\bar{\pi}, w_2, p; \mu) = \mu_h \psi(\bar{\pi}, w_2, p) + \Lambda_h(\bar{\pi}, w_2, p) \quad h = 1, \ldots, N \]

(51)

In this case the aggregated expenditure function will be:

\[ e(\bar{\pi}, w_2, p; \mu) = \psi(\bar{\pi}, w_2, p) \sum_{h=1}^{N} \mu_h + \sum_{h=1}^{N} \Lambda_h(\bar{\pi}, w_2, p) \]

(52)
Note that the function $\psi(\tilde{\pi}, w_2, p)$ must be identical for all consumers but $\Lambda_h$ may be different among households. The invariance of $\psi$ implies that changes in the distribution of income (and hence changes in $\mu_h$ keeping $\sum_{h=1}^N \mu_h$ constant) will not affect aggregate demand if total income does not change. Using Shephard's lemma one may obtain the compensated demand function:

$$X = \nabla e(\tilde{\pi}, w_2, p; \mu) = \nabla \psi(\tilde{\pi}, w_2, p) \sum_{h=1}^N \mu_h + \nabla \Lambda(\tilde{\pi}, w_2, p) \quad (53)$$

where

$$\Lambda(\tilde{\pi}, w_2, p) \equiv \sum_{h=1}^N \Lambda_h(\tilde{\pi}, w_2, p).$$

Now, changes in $\mu_h$ keeping $\sum_{h=1}^N \mu_h$ constant will have no effect on $X$ provided the $\psi$ function is identical for all households. Obviously, the assumption of identical $\psi$ functions can be relaxed if it is assumed that income distribution among households is approximately constant. In any case the GPF allows for quite different preferences (different $\Lambda_h$ functions) and hence it is possible to estimate a mean utility function.

So far, it has been implicitly assumed that households' characteristics such as level of education and number of family dependents do not affect preferences. A number of empirical studies, however, have concluded that these characteristics substantially affect preferences and hence the labour supply and commodity demand patterns (Huffman, 1980; Wales & Woodland, 1976). An approach commonly used has been to separate households into groups of approximately homogeneous characteristics and then to proceed with the estimation of preferences for each homogeneous group separately. The approach followed here makes use of the property
of the GPF which allows for different households' preferences via changes in the function \( \Lambda(\cdot) \) or in prices (changes in prices lead to shifts in the preference map in the commodity space). Thus, it appears reasonable to assume that households' educational level (E) and number of dependents (F) affect preferences essentially by changing the reference or base surface, i.e., by affecting \( \Lambda(\cdot) \). Thus, instead of estimating a function like \( G \) in (50) for various homogeneous households' groups, it is preferred to estimate (50) using \( \Lambda(\bar{w},w_2,p;E,F) \) considering all households at the same time. Changes in households' characteristics are thus assumed to shift the expansion path in a parallel manner. If the set

\[ B = \{X : X = \nabla \Lambda(p;E,F)\} \]

is defined as the base surface (where \( X \) is the vector of commodities and leisure consumed, \( p \) is the vector of prices), then households with different \( E \) or \( F \) will have a different set \( B \) where \( B \) and \( B^1 \) correspond to two base surfaces for households with different education, for example, and \( KK \) and \( K^1K^1 \) are the corresponding expansion paths. Note that if the function \( \psi(p) \) is independent of \( E \) and \( F \) then \( KK \) and \( K^1K^1 \) are parallel lines. The reason to assume that households' characteristics only affect the function \( \Lambda(\cdot) \) is to preserve the parallel expansion paths for those groups (or census divisions) which face same prices. If \( E \) or \( F \) vary among households within a group then the aggregation conditions

![Figure 4.](image-url)
would be violated if \( \psi(\cdot) \) was also dependent on households' characteristics. This is so, because the expansion paths of households within a group would not be parallel if these characteristics vary. Thus, it is assumed that educational levels and number of family dependents affect optimal commodity or leisure ratios consumed but that the marginal propensity to consume (when income changes) is not affected.

Blackorby et al. (1978) proposed a functional form for the GPF indirect utility function. This consists in a CES form for the \( \psi(\pi, w_2, p) \) and a generalized Leontief for the \( \Lambda(\pi, w_2, p; E, F) \) function. Accordingly, the GPF indirect utility function will be

\[
G = \frac{Z - \left[ \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \pi_i p_j + \sum_{i=1}^{3} \lambda_i p_i E + \sum_{i=1}^{3} b_i p_i F \right]}{\sum_{i=1}^{3} \alpha_i p_i^\rho} ^{1/\rho}, \quad (i,j=1,2,3) \tag{54}
\]

where \( \delta_{ij} = \delta_{ji} \), \( \lambda_i \), \( b_i \), \( \alpha_i \) and \( \rho \) are parameters to be estimated and \( p_1 \equiv \pi; \ p_2 \equiv w_2 \) and \( p_3 \equiv p \).

Using Roy's identity one can derive the demand equations in expenditure form:

\[
S_i = \frac{\alpha_i p_i^\rho \left[ Z - \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \pi_i p_j - \sum_{i=1}^{3} \lambda_i p_i E - \sum_{i=1}^{3} b_i p_i F \right]}{\sum_{i=1}^{3} \alpha_i p_i^\rho} \tag{55}
\]

\[
p_i \left[ \sum_{j=1}^{3} \delta_{ij} \left( \frac{p_i}{p_j} \right)^{\frac{1}{2}} + \sum_{i=1}^{3} \lambda_i E + \sum_{i=1}^{3} b_i F \right] \quad i = 1, 2, 3
\]

where \( S_1 \equiv p_1 (H-L_1) \)

\( S_2 \equiv p_2 (H-L_2) \)

\( S_3 \equiv p_3 X \)
Note that it is possible to test for homotheticity to the origin, by testing if all $\delta_{ij} = 0$. If $\delta_{ij} = 0$ for $i \neq j$ then preferences would be homothetic to a single point in the positive orthant and this point would be independent of relative prices.

Thus, the GPF (54) is chosen considering a number of reasons: (a) firstly, because the component $A(\cdot)$ of (54) belongs to the class of flexible functional forms, which may be interpreted as second-order approximation to any arbitrary function. (b) Secondly, the functional form (54) allows one to test for homotheticity of preferences to the origin and for homotheticity to a fixed point in the positive orthant. Given that a number of studies have estimated farm household's demand equations imposing homotheticity to the origin (Lau et al.) and homotheticity to a fixed point (i.e., Barnum & Squire) it is important to test whether those assumptions are appropriate for Canadian farm households. To test for homotheticity to the origin is particularly important because this assumption allows to avoid the use of nonlinear estimation techniques for the expenditure functions which is an expensive and difficult computational undertaking. Thus, if the hypothesis of homothetic preferences is not rejected then further studies of Canadian farm households can be undertaken using linear expenditure systems.

Given that the total expenditures cannot exceed the after tax income rather than the gross income it is necessary to modify model (55) in order to consider taxes. The budget constraint in (8) now considering taxes can be expressed as (Wales & Woodland, 1976):

$$px + \bar{p}(H-L_1) + w_2(H-L_2) \leq H(\bar{p} + w_2) + y - \tau(Y^T)$$ (56)

where $\tau(Y^T)$ are the total taxes paid as a function of the household's taxable income, $Y^T = \bar{p}L_1 + w_2L_2 + y - Ex.$, where Ex. are the tax
exemptions. The tax function can be approximated by:

\[ \tau(Y^T) = \tau_i + \beta_i (Y^T - Y^T_i) \]  \hspace{1cm} (57)

where

\[ Y^T_i = \text{smallest taxable income in tax bracket } i \]
\[ \tau_i = \text{taxes paid at income } Y^T_i \]
\[ \beta_i = \text{marginal tax rate in tax bracket } i. \]

Hence, using (56) and (57) and defining \( \bar{\pi}_i \equiv (1 - \beta_i) \bar{\pi} \) and \( w_{2i} \equiv (1 - \beta_i)w_2 \) the after tax budget constraint is:

\[ px + \bar{\pi}_i (H - L_1) + w_{2i} (H - L_2) \leq H(\bar{\pi}_i + w_{2i}) + (1-\beta_i)y + \beta_i Y^T_i - \]

\[ \tau_i + \beta_i \text{Ex.} \equiv Z_i \]  \hspace{1cm} (58)

Thus, equations (55) are estimated using the after tax values \( \bar{\pi}_i \), \( w_{2i} \), and \( Z_i \) as defined above.

In order to estimate the production side of the model it is necessary to estimate the conditional profit function of the household's firm. According to the definition provided in Chapter II (page 13), the conditional profit function is dependent on a vector of net output prices, \( q \), on the amount of family labour used by the firm and on the efficiency of production (i.e., the production technology broadly defined). The prices considered are one aggregate output price \( q_1 \) and the following factor prices: rental price of land and structures \( q_2 \), hired labour wage rate \( q_3 \), rental price of livestock capital \( q_4 \), and rental price of other forms of capital \( q_5 \). In a cross-sectional framework, efficiency differences among the observations might arise because:

(a) differences in the educational levels of farm households,
(b) regional differences in climate and soil quality, and
(c) regional differences in output composition (this factor may be
important particularly when output is aggregated).

Factor (a) may lead to improvements in productive efficiency by affecting the technology which farmers select. Education may affect farm profits and the supply of net outputs in a non-neutral way. Thus, the variable education is considered as a factor affecting profits, and hence net output supply, allowing for measuring differential effects of education on the demand for the different inputs.

Factors (b) and (c) may also affect the level of profit and the net output supply functions in a non-neutral manner, i.e., these factors may have different effects on the supply of the different net outputs at a given level of prices. For example, differences in weather conditions among regions may lead to biases towards more intensive use of some inputs and less intensive use of other resources. It was therefore decided to add dummy variables to the conditional profit for four regions.$^2$

Consequently, assuming constant returns to scale and specifying a generalized Leontief conditional profit function, which is a flexible functional form in the sense that it provides second order approximations to any local function, the profit function is:

$$\pi(q;L_1) = L_1 \left[ \sum_{i=1}^{5} \sum_{j=1}^{5} b_{ij} q_i q_j + \sum_{i=1}^{5} a_i q_i E + \sum_{i=1}^{5} \sum_{k=1}^{4} C_{ik} D_k q_i \right] \quad (59)$$

where $b_{ij} = b_{ji}$, $a_i$ and $C_{ik}$ are parameters and $D_k$ is the dummy corresponding to region $k$.

Given (59) the net output supply responses per unit of family labour can be obtained using Hotelling's lemma. Thus, the net output supply equations are:

$$\frac{Q_i}{L_1} = \sum_{j=1}^{5} b_{ij} \left( \frac{q_j}{q_i} \right)^{\frac{1}{2}} + a_i E + \sum_{k=1}^{4} C_{ik} D_k, \quad i=1,...,5 \quad (60)$$
where

\[ Q_1 = \text{Output supply} \]
\[ Q_2 = \text{Demand for land and structures} \]
\[ Q_3 = \text{Demand for hired labour} \]
\[ Q_4 = \text{Demand for animal stocks} \]
\[ Q_5 = \text{Demand for farm capital} \]

In order to estimate the conditional profit function one can proceed directly to estimate \((59)\), or, equivalently, to estimate the net supply equation system \((60)\). It is arbitrarily chosen to estimate the net supply functions \((60)\). Note that if producers are price takers then a constant returns to scale technology is sufficient to imply that the aggregation conditions are met for profit maximizing firms (Debreu).

The inclusion of operator's educational level as an explanatory variable in the profit equation deserves some further comments.³ It is well known that a decision maker's education has a positive effect on the allocative efficiency of production (Huffman, 1977). In particular, increased education is usually hypothesized to induce faster adjustments in the allocation of resources to any changes such as in prices which generate disequilibriums. In other words, an effect of education would be to increase producers' speed to adapt to changing economic conditions. However, given that the analysis is more long-run in nature, this effect of education is neglected which implies that producers in the long run are assumed to be price efficient regardless of their level of education. The interpretation given to the effect of education is as follows: at any point in time, there exist several production technologies available. Some technologies are more complex than others and an appropriate use of them requires different degrees of entrepreneurial skills. For example,
a livestock producer may choose from a large number of livestock breeds which include some rather rustic breeds and also more sophisticated animals. Although the more sophisticated breeds have a more efficient rate of conversion of feeds into meat, they require more careful management. Similarly, higher yield crop varieties coexist with low yield varieties but the former are more sensitive to cultural practices, require more fertilizers at precise periods, they require more efficient irrigation techniques, etc. Thus it is hypothesized that more educated entrepreneurs have the management abilities required to handle more complex technologies which are normally the more productive technologies. Hence, producers will choose the most efficient technology which their skills can handle. Therefore, it is assumed that producers are technically (and price) efficient in the sense that each producer will pick the best technology which has a level of complexity consistent with his skills. A less educated producer using a technology which is not the most productive one available is not inefficient because if he were to choose a potentially more productive and complex technology the end result would be a lower productivity if he is not able to handle it in the appropriate manner.

It is assumed that the diverse technologies available differ in their degree of factor augmentation and total factor productivity. The actual technology used by a farmer will depend on his skills which in turn depend upon his educational background. Thus, the level of education can be interpreted as an index of technology analogous for example, to the time trend used in time series analysis. The use of education as an index of technology in the long run equilibrium model may imply some problems if education is indeed an endogenous variable in the long run.
However, a number of authors, particularly Griliches, have argued that this is not a serious problem because, among other reasons, "the large influence of parents, the state, teachers, and classmates on the actual level of schooling achieved by an individual, only part of which can be interpreted as a result of his own ex-ante optimizing behaviour" (Griliches, p. 13). Hence, one can add terms to the original generalized Leontief function (expressed in terms of input and output prices only) to obtain a first order approximation in $E$:

$$
\bar{\pi} = \frac{5}{3} \sum_{i=1}^{5} \sum_{j=1}^{5} b_{ij} (q_{ij})^{\frac{3}{2}} + \frac{5}{3} \sum_{i=1}^{5} a_{ii} q_{i}.
$$

This form may be interpreted as a linear factor augmentation. The constant returns to scale production technology assumed is:

$$F(Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}, L_{1}) = 0$$

where $Q_{i}$, $i=1,...,5$, $L_{1}$ indicate output $i$ and on-farm operator labour in efficiency units (using as a base the least advanced technology). The efficiency level of a net output $Q_{i}$ is assumed to depend on the technology used which is determined by the farm operator's educational level ($E$), by the actual level of the net output $Q_{i}$ and by the on-farm (entrepreneurial) work by the operator ($L_{1}$). A linear form for the $Q_{i}$ functions is used:

$$Q_{i} = Q_{i} + a_{i} E L_{1}$$

and $L_{1} = L_{1} + \gamma E$. The inclusion of $L_{1}$ in the efficiency functions of all net outputs is justified by considering that the effect of education in choosing a production technology will also be influenced by the number of hours in organizational and entrepreneurial work which the farm operator is willing to provide. The $\gamma$ coefficient in $L_{1}$ would consider the "worker effect" of education which has been judged relatively small by most studies (Barichello). Hence, for simplicity it is assumed that $\gamma = 0$ and therefore, $L_{1} = L_{1}$. Thus, with
the effect of education specified in this form it can be easily seen that
the net outputs \( \frac{Q^*_i}{L^*_i} \) operator labour ratios will be

\[
\frac{Q^*_i}{L^*_i} = \frac{Q_i}{L_i} + a_i E
\]

which is the actual specification used for the net supply equations in
(60). Note that this specification of factor augmentation allows the
rate of factor augmentation to be dependent on \( Q_i \) and \( E \), i.e.,

\[
\eta_i = \frac{1}{Q^*_i/L^*_i} \frac{\partial (Q^*_i/L^*_i)}{\partial E} = \frac{a_i}{Q_i/L_i + a_i E}
\]

This is in contrast with other factor augmentation indices commonly used,
for example, \( Q^*_i = e^{a_i} Q_i \), where \( a_i = a \) is a constant independent of \( Q_i \)
and \( E \).

Finally, notice that the technical change induced by education will
have a factor augmenting effect if \( a_i \neq 0 \) for at least one \( i = 2, \ldots, 5 \).
If \( a_i = 0 \) for \( i = 2, \ldots, 5 \) and \( a_i \neq 0 \) then education will induce a neutral
effect on the \( \frac{Q_i}{L_i} (i = 2, \ldots, 5) \) factors.

In summary, a GPF functional form for the indirect utility function
and a generalized Leontief form for the conditional profit function are
chosen. The GPF is chosen considering that it is the most general form
which allows for consistent aggregation and where the demand system
satisfies the integrability conditions. The generalized Leontief func-
tional form for the profit function has been arbitrarily selected from a
number of alternative flexible functional forms.
4.2 The Econometric Model

In order to estimate the parameters of the indirect utility function and profit function it is necessary to assume a stochastic structure for (55) and (60). It is assumed that the disturbances are additive and normally distributed with zero means and positive semidefinite variance-covariance matrix $\Sigma$. Thus, if (55) and (60) are written in a more compact notation and if the disturbance terms are added then the econometric model is:

(i) $S_1 = f_1(\bar{w}, w_2, p, Z; E, F) + \mu_1$

(ii) $S_2 = f_2(\bar{w}, w_2, p, Z; E, F) + \mu_2$

(iii) $S_3 = f_3(\bar{w}, w_2, p, Z; E, F) + \mu_3$

(iv) $Q_i/L_1 = \phi_i(q; E) + \sum_{k=1}^{4} C_{ik} D_k + \nu_i, \quad i=1,...,5 \quad (61)$

Note that $\bar{w}$ is now used as a variable rather than as a function. Note that

$$\bar{w} = \sum_{i=1}^{5} \frac{Q_i}{L_1} q_i = \phi(q; E) + \nu,$$

where $\phi(q; E) = \sum_{i=1}^{5} \phi_i(q; E) \cdot q_i$

and the disturbance term $\nu = \sum_{i=1}^{5} \nu_i q_i$. The expenditure equations ($S_i$) in (61) are assumed to be dependent on the actual profit per hour of work ($\bar{w}$) rather than on the optimal or expected profit level, $\phi(q; E)$. An alternative procedure would be to specify the expenditure equations as functions of the optimal or expected profit and thus the model would be:

$$S_j = f^*_j(\phi(q; E), w_2, p, Z; E, F) + \bar{\mu}_u \quad (j = 1, \ldots, 3) \quad (62)$$

The advantage of this procedure is that the interdependence of utility and profit maximizing decisions (i.e., between the equations $S_j$
and $Q_i/L_1$ is more explicit in the econometric model because of the cross-
constraints between the parameters of the expenditure and conditional net
output supply equations. Unfortunately, estimation of a model based on
(62) was infeasible because of the extreme computational difficulties and
costs involved and, therefore, the econometric specification (61) was
used instead. Under the assumptions used (constant returns to scale and
no fixed factors of production) the variable $\bar{x}$ is exogenous, independent of
household's preferences (independent of $L_1$) and hence its use as an
explanatory variable represents no inconvenience. The interpretation of
using $\bar{x}$ as an explanatory variable in the expenditure equations is that
households are able to estimate the returns to its labour time spent on
the family farm based on information regarding output prices, input prices
and knowledge of the production technology they have available. Based on
their estimation of the returns to their work on their own farm and consid­
ering the prevailing off-farm wage rate ($w_2$), cost of living index ($p$),
and households full income they decide upon their optimal expenditures
which maximize their utility.

Another problem is the interpretation of the disturbance terms
associated with equation system (61). Given that the disturbance terms
were assumed stochastic and normally distributed, one has to interpret
the indirect utility function (54) and the conditional profit function
(59), from which the expenditure equations ($S_j$) and the net supply
functions ($Q_i/L_1$) were derived, as the true functional forms. If (54)
and (59) were interpreted as second order approximations of the true
functional forms then the disturbance terms added to the estimation
equations would also include errors of approximation. In this case,
given no information on the true functional form it is not possible to
know the nature of the disturbances and they would be non-stochastic and correlated with the explanatory variables. Thus, the disturbance terms can be seen as random errors in optimization made by the farm households.

Given the budget constraint it is clear that the covariance matrix of the disturbances is singular and hence one can drop one of the expenditure equations. It is arbitrarily chosen to drop the equation corresponding to expenditures on goods. Hence, the expenditure functions for $H-L_1$ and $H-L_2$ are estimated ($S_1$ and $S_2$, respectively).

The expenditure equations and the net supply functions in (61) need to be jointly estimated despite that there are no parameter restrictions across them. The theoretical model discussed in Chapter I and II is based on the recognition of the fact that production and consumption decisions are both taken by one individual (or household). Hence, a logical implication of such a model is that the errors made by farmers in their production decisions will be correlated with their utility maximization errors. It may be hypothesized that those farmers who make fewer errors in their production decisions will make smaller errors in their utility maximizing decisions as well. Given the recursive nature of the model and, in particular, that the expenditure equations are dependent on $\pi$ rather than on a market price (unrelated to the firm's production technology and $q$), the estimates of the expenditure equations will not only be asymptotically inefficient but also inconsistent if the production and consumption sectors are not jointly estimated.6 This represents an important difference with the conventional model (based on the hypothesis of independent production and consumption decisions) which ignores the recursive nature of the model by assuming that the shadow price of on-farm work in unrelated to the household's firm
production technology and net output prices. In the latter model the consequences of estimating the production and consumption sectors in a disjoint manner (as is usually done) are not so serious; the estimates are still consistent although some loss of efficiency occurs.\textsuperscript{7}

The system of equations (61) is jointly estimated, after dropping the consumption goods expenditure equation, using a Full-Information Maximum Likelihood Method (FIML). Using a FIML method the coefficients estimated will not depend on which equation is dropped. Thus, the parameters of the utility and profit functions which maximize the logarithm of the concentrated likelihood function, $L$, are chosen

$$
L = -\frac{kT}{2} (\ln 2\pi + 1) - \frac{T}{2} \ln |\hat{\Sigma}| + \sum_{n=1}^{T} \ln(\text{abs } |B_{N}|) \tag{63}
$$

where

- $k =$ number of equations
- $T =$ number of observations
- abs$|B_{N}| =$ absolute value of the determinant of the matrix of derivatives of the disturbances with respect to the endogenous variables, $S_1$, $S_2$, and $Q_1/L_1$.

It is important to indicate that the matrix $B_{N}$ is triangular\textsuperscript{8} and that abs$|B_{N}| = 1$ and hence the likelihood function $L$ is reduced to the first two terms of (63).

The data used on expenditures ($S_1$ and $S_2$), net output supply per unit of on-farm work ($Q_1/L_1$), farm operator's education ($E$), number of dependents ($F$), and full income ($Z$) consist of average household values by census division rather than individual household's data. On the other hand, it is assumed that prices of consumer goods, on-farm labour returns, off-farm farm wages, and net output prices are identical for
all households in any census division. It is also assumed that the variance of the individual household's disturbances are constant. However, given that the number of households varies across the different census divisions, the variances of the disturbance terms will be different for the different observations. Thus, one may expect heteroscedasticity which, as is well known, reduces the efficiency of the estimates and also invalidates some tests of statistical significance.

The left-hand-side terms of the net output supply equations are defined by

$$\sum_{k=1}^{N_t} \left( \frac{Q_{ikt}}{L_{ikt}} \right) / N_t$$

where $Q_{ikt}$ is the net output $i$ produced by household $k$ of census division $t$, $L_{ikt}$ is similarly defined, and $N_t$ is the number of households in census division $t$. Similarly, the expenditures are defined by

$$S_1 = \sum_{k=1}^{N_t} S_{1kt} / N_t$$

and

$$S_2 = \sum_{k=1}^{N_t} S_{2kt} / N_t$$

Hence, the disturbance term in equation $j$ for a given observation (or census division) $t$, will be:

$$\mu_{jt} = \frac{\sum_{k=1}^{N_t} e_{jkt}}{N_t}$$

where $e_{jkt}$ is the disturbance term corresponding to household $k$ for equation $j$ at observation $t$.

The covariance between the disturbance of equation $j$ and equation $s$ will be

$$\text{cov} \left[ \frac{N_t}{N_t} \sum_{k=1}^{N_t} e_{jkt}, \frac{N_t}{N_t} \sum_{k=1}^{N_t} e_{skt} \right] = \frac{1}{N_t^2} \sum_{k=1}^{N_t} \text{cov}(e_{jkt}, e_{skt})$$

(65)
Note that (65) holds provided the disturbances of the individual households are not correlated. Now, if the covariances between $e_{jkt}$ and $e_{skt}$ are identical for all $k$ then (65) can be written as:

$$
\text{cov} \left[ \sum_{i=1}^{N_t} e_{jkt}, \sum_{i=1}^{N_t} e_{skt} \right] = \frac{1}{N_t^2} \cdot N_t \cdot \frac{\sigma_{sj}}{N_t}
$$

(66)

where $\sigma_{sj}$ is the covariance between the disturbances of equations $s$ and $j$ associated with the individual households.

Thus, the covariances of the disturbance terms will be smaller the larger $N_t$ is and, hence, heteroscedasticity is a problem. In order to tackle this problem one needs to consider (66) in the variance-covariance matrix of the likelihood function or, equivalently, one may transform the estimating equations in such a way that the variance-covariance matrix of the transformed equations be homoscedastic. This is done by multiplying through equations (61) by the square root of $N_t$.\footnote{9} In this case the covariances for the averages will be constant equal to $\sigma_{sj}$. If (61) is multiplied through by $N_t^{\frac{1}{2}}$ then

$$
\mu_{jt} = \frac{\sum_{k=1}^{N_t} e_{jkt}}{N_t^{\frac{1}{2}}}
$$

and using expression (66) one obtains that $\text{cov}(\mu_{jt}, \mu_{st}) = \sigma_{sj}$. Thus, the above transformation allows to obtain consistent and asymptotically efficient estimates of the expenditure and profit functions.\footnote{10}

The fact that the data are aggregated implies some problems leading to the econometric complications discussed above and also to use a more restrictive functional form for the indirect utility function in order to obtain consistent and integrable demand functions as discussed in the
previous section. However, Aigner and Goldfeld have shown that under conditions of exactly off-setting measurement errors in the microvariables, a model using aggregated data will out-perform a model based on microdata. In general, the macrodata have a smaller observation error than the microdata if the correlation among the microdata errors is not perfect. There is no reason to assume that measurement errors at the microlevel will not be at least partially off-setting. Thus, although aggregated data are costly from the viewpoint of requiring more restrictive functional forms, there are also some advantages concerning smaller observation errors in using aggregated data.

4.3 Testing for Independence of Utility and Profit Maximization Decisions

The analysis of Chapters I and II suggested that the hypothesis of independence of utility and profit maximizing decisions is not likely to be appropriate for modelling farm households' supply and demand responses. It was also shown that the necessary and sufficient conditions for independence are quite strong. Based on this, a relatively more complex model which does not rely on the hypothesis of independence has been developed and estimated using Canadian farm data. A relevant question to ask is whether Canadian farm households' utility and profit maximizing decisions are independent, i.e., whether the use of a model which assumes interdependence is justified in the case of Canadian agriculture. In other words, it is necessary to test the hypothesis that Canadian farm households' utility and profit maximizing decisions are independent.

For the purpose of empirically testing the hypothesis of independence one may use a model based on the hypothesis of independence similar
to the one used by Lau et al. This model avoids the problem of inter-
dependence by assuming that households are indifferent between working
on their own farms and off-farm as wage earners. This allowed the
authors to use the off-farm wage rate as the unique exogenous price of
leisure under the implicit assumption that households do some off-farm
work. Thus, such a model is the following:

\[ \hat{G}(p, w_2, Z; E, F) = \max_{H-L_1-L_2, X} \{ U(H-L_1-L_2, X) : \]

\[ (i) \quad px + w_2(H-L_1-L_2) \leq \pi(q, w_2; E) + w_2H + y = \tilde{Z} \]

\[ (ii) \quad X \geq 0; \quad H-L_1-L_2 \geq 0; \quad L_1 \geq 0, \quad L_2 \geq 0 \] \hspace{1cm} (69)

where \( \pi(q, w_2; E) \equiv \{ \max_{Q, L_1} q^T Q - w_2L_1 : Q, L_1 \in \tilde{T}(E) \} \) is the unconditional
profit function, \( \hat{G}(\cdot) \) is the indirect utility function and all other
variables have previously been defined.

Notice that in this model the assumption of constant returns to
scale is relaxed. Relaxation of this assumption is necessary because
the hypothesis of indifference between working on-farm and off-farm is
not consistent with constant returns to scale if \( L_1 > 0 \) and \( L_2 > 0 \).
Hence, given that in the sample used \( L_1 \) and \( L_2 \) are both greater than
zero, the assumption of constant returns to scale is not used. Using
Roy's identity one can derive the estimating utility maximizing equations
from \( G(\cdot) \) and using Hotelling's lemma the unconditional net output
supply responses are obtained from \( \pi(q, w_2) \). Thus, the estimating model
is:

\[ (i) \quad H-L_1-L_2 = g^2(p, w_2, Z; E, F) + \tilde{\mu}_1 \]

\[ (ii) \quad Q_i = h^i(q, w_2; E) + \tilde{\nu}_i \quad (i = 1, \ldots, 5) \]

\[ (iii) \quad L_1 = h^6(q, w_2; E) + \tilde{\nu}_6 \]

\[ (iv) \quad X = g^3(p, w_2, Z; E, F) + \tilde{\mu}_2 \] \hspace{1cm} (70)
Model (70) is estimated using the same functional forms for the indirect utility function (Gorman Polar Form) and for the profit function (Generalized Leontief) used in estimating the model based on the hypothesis of interdependence defined by (61). As in model (61) it is necessary to drop one of the equations of the consumption side in (70). It is arbitrarily chosen to drop the equation corresponding to the demand for consumption goods (70.iv).

Before proceeding with a description of the testing procedure it is convenient to comment on the structural differences between the model based on interdependence (model (61)) and the model based on independence (model (70)). The central difference between the two models is that while in model (61) the labour supply and consumption goods' demand equations jointly reflect household's preferences and the firm's production technology, in model (70) they are solely determined by household's preferences. Furthermore, in model (61) although the net output supply conditional on \( L_1 \) are not affected by household's preferences, the implicit unconditional net output supply responses\(^1^4\) (i.e., when \( L_1 \) is considered variable) will also be jointly determined by household's preferences and the firm's production technology. This in contrast with model (70) where the unconditional net output supply equations are defined independently of the household's preferences.

More specific structural differences are the following: while in (61) labour supply on-farm and labour supply off-farm are considered two different "commodities" from the point of view of the household as a utility maximizer, in model (70) on-farm and off-farm labour supply are viewed as identical commodities. In Model (70) there is a unique relevant wage rate (\( w_2 \)) which is independent of the household's firm tech-
ology or net output price. In contrast, in model (61) there are two wage rates relevant to the household: one is the price of off-farm work and the other is the (shadow) price of on-farm work which is dependent on production technologies and net output prices. The equation for $L_1$ in (70) does not correspond to a labour supply equation. It is the demand for family labour determined at the firm level like the demand for any other input is determined. While in model (70) the level of on-farm work by household's members is entirely demand determined (i.e., it is assumed an infinitely elastic supply of household's labour and hence the household's firm labour demand determines the level of $L_1$) in model (61) is entirely supply determined. The assumption of constant returns to scale implies that the demand for $L_1$ schedule is infinitely elastic. Relaxation of the constant returns to scale assumption in model (61) would imply that the equilibrium level of $L_1$ is determined by both the supply and demand sides.

The problem in formally testing the null hypothesis of independence, i.e., that model (70) holds, against the alternative hypothesis of no independence using model (61), is that the parameter space of (70) is not contained in the parameter space of (61). Thus, if $\theta \in \Theta$ and $\tilde{\theta} \in \tilde{\Theta}$, where $\theta$ and $\tilde{\theta}$ are the vectors of estimating parameters of model (61) and (70) respectively and $\Theta$ and $\tilde{\Theta}$ represent the parameter spaces, and if $\Theta / \tilde{\Theta}$ (or $\tilde{\Theta} / \Theta$) then one is dealing with separate families of hypotheses and the standard tests cannot be employed (Goldfelt & Quandt). There are two alternative formal tests designed to discriminate between separate families of hypotheses. One is a test derived by D. R. Cox which is a modified likelihood ratio test and the other one is a test for specification error developed by Davidson and Mackinnon which in turn is a
refinement of a test originally proposed by Hoel in 1947. Cox's method is not used in spite of its rigour and elegance because the computations required turn out to be extremely difficult. The Hoel-Davidson-Mackinnon test, henceforth referred to as the HDM test, allows one to test the truth of linear or nonlinear and multivariate regression model, when there exists a non-nested alternative hypothesis. The HDM test, in contrast with the Cox's procedure, is simple and can be easily implemented using existing computer software. For this reason, the HDM procedure is used in testing the null hypothesis of independent utility and profit maximizing decisions against the alternative hypothesis of no independence.

Consider an \( N \) equation regression model, the truth of which is desired to test:

\[
H_0 : y_{ik} = z_0^k (X_i, \delta_0) + \epsilon_{ik}^0 \text{ for } k = 1, \ldots, N \quad (71)
\]

where \( H_0 \) stands for the null hypothesis, \( y_{ik} \) is the \( i \)th observation of the dependent variable of equation \( k \), \( X_i \) is a vector of observations on exogenous variables, \( \delta_0 \) is a vector of parameters to be estimated and \( \epsilon_{ik}^0 \) is the error term assumed to be normally distributed with variance-covariance \( \Sigma \).

Suppose that an alternative hypothesis suggested by economic theory is:

\[
H_A : y_{ik} = z_A^k (Z_i, \delta_A) + \epsilon_{ik}^A \text{ for } k = 1, \ldots, N \quad (72)
\]

where \( Z_i \) is a vector of observations on exogenous variables and \( \delta_A \) is a vector of estimating parameters. Assuming that \( H_A \) is not nested within \( H_0 \) and that \( H_0 \) is not nested within \( H_A \), i.e., implying that the truth of one hypothesis implies the falsity of the other, the HDM test suggest to estimate the following model:
where \( \hat{\delta}_A \) denotes the estimated \( \delta_A \) parameter vector. Since the artificial variable \( z^k_A(X_i, \hat{\delta}_A) \) is independent of \( \varepsilon_{ik} \) by the way it is constructed, (73) may be estimated like any other regression model. It is clear that if \( H_0 \) is true, one only needs to estimate (73) and test whether \( \beta_k = 0 \) for all \( k = 1, \ldots, N \).

In deriving the asymptotic properties of the test, Davidson and Mackinnon have used the following assumptions:

Assumptions E:

(E.1) Either the null hypothesis \( H_0 \) is true or the alternative hypothesis is true,

(E.2) the matrices \( X \) and \( Z \) are nonstochastic, and fixed, and

(E.3) as \( n \to \infty \), the matrices \( \frac{1}{n} [N^k_0(\delta_0)]^T [N^k_0(\delta_0)] \), \( \frac{1}{n} [N^k_A(\delta_A)]^T [N^k_A(\delta_A)] \)

and \( \frac{1}{n} [N^k_0(\delta_0)]^T [N^k_0(\epsilon_0)] \), where \( N^k_0(\delta_0) \) and \( N^k_A(\delta_A) \) are the matrices of first partial derivatives of the functions \( z^k_0 \) and \( z^k_A \) with respect to \( \delta_0 \) and \( \delta_A \), respectively, converge to well-defined finite limits for all bounded \( \delta_0 \) and \( \delta_A \).

Using assumptions E, Davidson and Mackinnon have shown that the t-statistics for \( \beta_k \) from regression (73) provide an asymptotically legitimate test for the truth of \( H_0 \) in the following sense:

1. If \( H_0 \) is true, \( \text{plim} \beta_k = 0 \) (\( k = 1, \ldots, N \)) and the variance of \( \beta_k \) is consistently estimated by (73);

2. If \( H_A \) is true then \( \text{plim} \beta_k = 1 \) (\( k = 1, \ldots, N \)), and the variance of \( \beta_k \) is overestimated by (73).

This implies that if either \( H_0 \) or \( H_A \) is true then one can test whether \( \beta_k = 0 \) (\( k = 1, \ldots, N \)) using an asymptotic t-test or a joint test such as the likelihood ratio test. Notice, however, that if neither
H_0 nor H_A is true then the asymptotic properties of the test are, in general, unknown.

In using the HDM procedure in testing the hypothesis of independence the null hypothesis is represented by equations (70.i to 70.iii) and the alternative hypothesis is embodied in equations (61.i), (61.ii), and (61.iv). It is necessary, however, to introduce some modifications into the two models in order to have the same dependent variables. Thus, the equations to be jointly estimated are the following:

(i) \[ L_1 = (1-\beta_1)h^6(\cdot) + \beta_1 \left[ H-\bar{f}^1(\cdot)/\bar{\pi} \right] + \bar{\mu}_1 \]

(ii) \[ L_2 = (1-\beta_2)\left[ H-g^2(\cdot) - h^6(\cdot) \right] + \beta_2 \left[ H-\bar{f}^2(\cdot)/\bar{\pi}_2 \right] + \bar{\mu}_2 \]

(iii) \[ Q_i = (1-\beta_{1+2})h^i(\cdot) + \beta_{1+2} \left[ \bar{f}^i(\cdot) \cdot \frac{f^1(\cdot)}{\bar{\pi}} + \text{cov} \left( \frac{\mu_1}{\bar{\pi}}, v_i \right) \right] + \bar{\mu}_{2+i} \quad (i = 1, \ldots, 5) \tag{74} \]

where a hat (\(^\cdot\)) above the functions indicates expected or predicted values.

Notice that the second terms of the right-hand-sides represent the predicted or expected values (obtained from model (61)) of \( L_1, L_2 \) and \( Q_i \) rather than of \( S_1, S_2 \) and \( Q_i/L_1 \). Thus, the null hypothesis that utility and profit maximization decisions are independent (i.e., that model (70) is the true model) is tested against the alternative hypothesis of interdependence represented by model (61) by jointly testing whether \( \beta_k = 0 \) for \( k=1, \ldots, 7 \).\(^{15,16,17} \)

The first terms of the right-hand-side correspond to model (70) modified in order to obtain a specific equation for \( L_2 \) from (70.i) and (70.iii). The interpretation of \( L_1 \) and \( L_2 \) in (74) should be carefully considered; the model based on independence does not provide two labour supply equations. It only defines one aggregated labour supply and a demand equation for \( L_1 \) determined at the firm level. Hence the equation
for $L_2$ (i.e., $H - g^2(\cdot) - h^6(\cdot)$) has been obtained from model (70) as a residual reduced form, only for the purpose of making model (70) comparable to model (61). Thus, the equation for $L_2$, obtained after some transformations of model (70) have been made, does not correspond to a household's behavioural equation. The only household's behavioural equation is the total labour supply function.

The assumption of constant returns to scale used in the alternative hypothesis may cause some problems in the interpretation of the test. If the true production technology does not approximately exhibit constant returns to scale then it is possible that neither the null hypothesis nor the alternative hypothesis are true. As indicated before, in this case the asymptotic properties of the test are generally unknown and hence it would be difficult to interpret the results of regression (74). The important thing, however, is that if $H_0$ is true then the $\text{plim} \hat{\beta}_k = 0$ (for all k) and the variance of $\hat{\beta}_k$ is consistently estimated by (73). This implies that the confidence interval for $\hat{\beta}_k$ is correctly estimated if $H_0$ is true and hence the probability of a type I error is correctly given by the level of significance chosen.  

Apart from the formal statistical test, an informal test based on comparing the estimates provided by (61) and (70) with respect to conformity of the estimates obtained with a priori knowledge is also performed. In particular, the emphasis is placed on whether the estimates of each model satisfy the quasiconvexity and monotonicity properties of the indirect utility function implied by the utility maximization hypothesis. Similarly, satisfaction of the properties of the profit function is also considered, in particular the convexity and monotonicity properties. Unfortunately, the symmetry restriction cannot be tested
because estimation of the unrestricted model increases the number of parameters to a prohibitive level.

4.4 The Data

The data used were obtained from the 1971 agricultural and population censuses. It is not possible to have access to household's data because of tax confidentiality problems. However, aggregated data are available at the census division level. There are approximately 240 agricultural census divisions in Canada and the data are available as total values per census division and given that data on number of farm households per census division are available, one can transform the data into averages per household.

All census divisions were not used, however. Some were excluded because agricultural production was negligible. More importantly, the number of census divisions corresponding to the different regions was not nearly representative of the share of the regions in agricultural production. For example, the Maritime provinces are equally represented in the original 240 census divisions as the Prairie provinces despite that the Maritimes' agricultural output was less than 15% of the Prairie provinces' output. Given that the results are expected to be approximately representative of Canadian agriculture, it was decided to use 95 census divisions. These census divisions were randomly selected from five regions in such a way that the percentage of census divisions of each region approximately correspond with the importance of the region on agricultural output and employment. Thus, the regions and their approximate share in the total sample were: Maritimes (with approximately 7% of the observations), Quebec (19%), Ontario (28%), Prairie provinces (40%), and British Columbia (5%).
The required data for this study are the number of days of off-farm work by household's members, number of days worked on-farm, off-farm wage rate, farm's net returns per day of work by household members, household's non-labour income, output and input prices faced by the household's firm, farm operator's years of schooling and the number of family dependents. An aggregated output price index and three input price indices, namely, hired labour wage rate, animal stock rental price index and a land rental price index, are needed. The price index of farm capital (machinery, implements and other intermediate inputs) is not available and is assumed constant across the observations. Farm machinery, fertilizers and spray materials in contrast with other farm inputs (such as labour, land, and livestock) are traded by large firms which operate at a national or even continental scale. It is reasonable to assume that these firms tend to charge rather homogeneous prices for their products in the different regions of the country. Thus, given that these products are traded in a national market rather than in segregated regional markets, one can expect a certain degree of price invariance throughout the country and hence the above assumption may not be too unrealistic.

The following is a brief discussion of the data sources and methods used to calculate the specific variables required in the analysis.

4.4.1 Off-farm Work and Off-farm Wages

The off-farm work (in days) of the farm operator is taken directly from the census of agriculture data. The population census provides data on off-farm wage income for each household's member. Hence, the off-farm wage rate for the farm operator is calculated by dividing the off-farm wage income by the number of off-farm days of work. It is assumed that the other male members of the household earn the same off-
farm wage rate and hence their days of off-farm work are calculated by
dividing their off-farm wage income by the calculated wage rate. For
female members it is assumed a wage rate which is a fixed proportion of
the male wage. The proportionality factor is based on estimates of
provincial average hourly wages for female and male workers (Labour Force
Survey [1970]). The actual coefficients of female wage/male wage assumed
varies from 0.65 to 0.75.

Given that a number of assumptions have been made in order to calculate wages and off-farm work for household's members other than the operator, these variables are measured with a relatively large error. One could expect that these errors would not be as large in the case of aggregated data as in the individual household's data.

4.4.2 On-farm Work, Returns to Farm Work and Non-Labour Income

The population census provides data on total number of days worked
by each household member. Given that the number of days of off-farm
work can be calculated, one can also obtain the number of days worked
on the farm by simply subtracting the off-farm work from the total number
of days worked. In this manner the number of days worked on the farm
by each household member is calculated. The variable total number of
days that household's members have available for work and non-work activi-
ties (H) was calculated by simply adding-up the total number of days
of each household member's age 13 or above, except that the women's hours
of off-farm work were weighted by the same proportionality factor used
to calculate the female members' off-farm wage rate.$^23$

In order to calculate returns to farm work it is necessary to first
calculate the net farm income. The net farm income is equal to total
farm sales less operational costs and minus the rental values associated
with the farm capital and land owned by the household. The returns to farm work are thus obtained by dividing the calculated net farm income by the total number of days worked on the farm by household's members. This variable is used as an explanatory variable \( \bar{r} \) in the expenditure equations.

The non-labour income includes two components: (a) the returns associated with financial assets (bonds, securities, etc.) owned by the household as well as government transfer payments; (b) the returns associated with real assets owned by the household which are used in the farm operation. Non-labour income component (a) is directly obtained from the population census data. Non-labour component (b) is calculated by first estimating the rental value of the total farm capital (including land, buildings, machinery, livestock, and equipment). Given that not all farm capital is owned by the household, one needs to correct the rental value obtained by an equity proportion coefficient. There are no data on equity proportions and hence an average equity proportion ratio of 0.92 as calculated by Danielson (1975) for the year 1970 is used.

4.4.3 Output and Input Prices

Data on output prices by province obtained from the Statistics Canada CANSIM data file are available. Provincial prices are available for major grains, animal products, major fruits, and vegetables crops. A divisia price index is constructed for an aggregated output price variable by province. The different provincial prices are then assigned to the census division observations according to the province where they are located.

With respect to input prices, there exist census data on total wages paid and total number of days of hired labour used by census division. Hence, one can obtain an average hired labour wage rate by
dividing total wages into the number of days of hired labour used. Data on estimated prices for farm stocks of different categories of animals are available from Statistics Canada by province. A rental price index for the animal stock as an aggregate is constructed and the procedure of assigning the corresponding provincial price to the census divisions according to the province where they are located is followed.

In order to construct an asset price index for land and structures, one can use data corresponding to total market value of land and buildings and divide it by the total number of improved acres. Thus, given that there are no data on prices of farm buildings, it is necessary to assume that improved land and constructions are in fixed proportions. Using the asset price the corresponding rental price of land is calculated using some assumptions regarding the value of depreciation rates, capital gains and discount rates (see Appendix 2).

4.4.4 Other Variables Used and Taxes

Data regarding operator's educational level and number of household's dependents are also needed. Data on schooling years of farm operators are directly available from the population census. These data are used as a proxy for operator's educational level. The number of family dependents variable is defined as the number of children age 13 or less living on the farm. These data are also directly available from the population census.

With respect to the tax calculations, it is required to know the marginal tax rate \( \beta_i \) as well as \( \tau_i \) and \( Y^T_i \) (see Section 4.1). Data on average tax paid per farm household by census division are available from the population census. Thus using these data one is able to obtain the marginal tax rate for a representative household in the
census division by consulting the 1970 federal and provincial tax table under the assumption that the representative household files a joint tax return.

4.4.5 Data Problems

The first data problem is related to the fact that there are no data available for off-farm work for household's members other than the farm operator. This made it necessary to follow the indirect procedure described above in order to calculate off-farm work, which raises doubts with respect to the reliability of these calculated variables.

Another problem is related to the calculation of a price index for land and structures. The assumption of fixed proportion between land and structures may be highly unrealistic and hence the calculated price subject to large errors. If land values are proportionally dominant over the value of the productive constructions attached to land, then the errors may not be so large. In general, the larger the proportion of land values on the total value of land and buildings, the smaller will be the error of the calculated price index.

Perhaps the most serious errors are related to the fact that in order to obtain data for a number of variables it has been necessary to use combined information from the Census of Agriculture and the Census of Population. Although both censuses asked information concerning the same year (1970) to the same households, there were no field cross-checks between the responses to the population census and agricultural census questionnaires. Consistency was checked in each census but there were no cross-checks between the two censuses. This problem may shed some doubts about the reliability of the data calculated by combining data from the two censuses and requires that the results obtained be interpreted
cautiously.

4.4.6 Was 1970 a "Normal" Year?

This is an important question to answer because if 1970 was indeed a normal year from the point of view of weather, input prices, and output prices then one can interpret the results obtained as being related to long run equilibrium responses. On the other hand, if there have been drastic price changes in that year, for example, then the long run interpretation is more questionable and it would be possible that producers be in process of adapting their expectations to the new prices or if there are adjustment lags that inputs demanded would not correspond to the long run equilibrium levels. First, considering weather, there are two characteristics which are crucial in farm production, i.e., temperatures and precipitation. Considering the period of May to October (which is the relevant crop season) average temperatures per month in 1970 as measured in 25 experimental stations, spanning the major crop growing area of Canada, were within a 10% range of the average monthly temperature for the 1960-70 decade (Danielson). The only exception was the month of June which in 1970 showed an average temperature 45% above the average for the decade. A similar situation occurs with respect to monthly precipitations and again only the month of June appears to have been well above the average precipitation for the decade. Monthly data on temperature and precipitation show only slight variations with respect to the previous two or three years with the only exception of the month of June. Thus, in general it seems that as far as the weather pattern is concerned the year 1970 appears to be fairly normal in comparison with previous years with the exception of the month of June which shows above normal temperatures and precipitations. With respect to input prices, the nominal average farm wage rate in
1970 was slightly above the average farm wage rate prevailing in the previous two years although it had jumped 8.5% in 1968 (Lopez). Average land prices in 1970 remained essentially at the same level as in 1969 but the nominal land price had increased by 7% in 1969 and 3.5% in 1968. Average farm machinery nominal prices increased at a very constant rate of approximately 3.5% per year between 1965 and 1970. Feed and fertilizer prices both decreased slightly in 1970 and, in fact, they had been slowly decreasing since 1967. With respect to output price, one can use the agricultural wholesale price index of farm products, which remained in 1970 essentially at the same level as in 1969 although it had increased by approximately 4% in the previous years. Thus, in general input and output prices in 1970 remained approximately at the same levels of the previous years or they continued increasing at similar historical rates.

With respect to the grain industry however, the situation was not normal. In 1969 a rather large grain inventory was accumulated and in 1970 the federal government implemented a program oriented to divert land from grain production. Given that changes in inventories are not considered in the income data, the rather important changes in grain inventories which took place in the years 1969 and 1970 may imply that the net farm income variable used in the estimation is subject to important errors in those census divisions where grain production is dominant. Moreover, the acreage diversion program may have had an important effect in grain farmers' production decisions which were not considered in the empirical study. The use of dummy variables for different regions might partially capture this problem given that grain production tends to be concentrated in specific regions, in particular the Prairie provinces.

In general one can conclude that the major abnormalities in 1970
were related to the grain industry which is indeed a very important sector in Canadian agriculture. With respect to the rest of the agricultural industry, one may indicate that although 1970 was not a perfectly normal year in relation to the previous 5 to 10 year period, it is at least possible to indicate that this year cannot be singled out as a notoriously abnormal year. Hence, the interpretation of the results obtained as long run equilibrium supply and demand responses may not be considered totally inappropriate.
1. The empirical work could have been developed using the model presented in Chapter III. The main advantage of that model with respect to the model actually used is that it is more general, allowing for fixed factors of production and non-constant returns to scale. However, given that the data used are cross-sectional it is appropriate to postulate long-run equilibrium (see, for example, Nadiri & Rosen) and hence it is not a problem to assume that all factors are variable. Moreover, the use of the model of Chapter III would have implied that all estimating equations would be highly non-linear. This is in contrast with the model in Chapter II which allows to estimate linear conditional net output supply equations (see section 4.1). Thus, only the labour supply equations are non-linear and, hence, the estimation procedure is less expensive and less complex by using the model of Chapter II. Finally, the model of Chapter II allows to disentangle the production technology from households' preferences and also provides an explicit equation for on-farm work. In contrast, the model of Chapter III does not allow us to do this. Thus, there are some advantages in using the simple model of Chapter II although a major cost is the necessity of using the assumption of constant returns to scale, which not only imposes restrictions on modelling farm households' demand and supply responses but also leads to some problems in interpreting the results of the test for independence of production and consumption decisions (see section 4.3).

2. The four regions considered were: (a) the Prairie provinces; (b) the Maritimes; (c) Quebec; and (d) the rest of the country. It was felt that each of these regions were more or less homogeneous from the standpoint of weather conditions and output composition. Given that the use of dummies is an ad-hoc procedure, they will be considered in the final model only if they are statistically significant and/or improve the specification of the model.

3. Notice that the inclusion of \( E \) in the conditional profit function is not contradictory with the assumption of no fixed factors. The level of education of the operator is seen as a variable which affects the production technology rather than as a production factor. Thus, considering education, the conditional profit function can be defined by:

\[
\pi(q;L_1,E) \equiv \{ \max Q : F(Q;L_1,E) = 0 \}
\]

where \( F(Q;L_1,E) \) is a transformation function whose characteristics are dependent on \( E \). It can be shown that \( \pi(q;L_1,E) \) is homogeneous degree one in \( L_1 \) provided \( F \) exhibits constant returns to scale in all factors of production (excluding \( E \)).

\[
\pi(q;\lambda L_1,E) \equiv \{ \max Q : F(Q;\lambda L_1,E) = 0 \}
\]

\[
= \{ \max Q : F(\lambda Q;\lambda L_1,E) = 0 \}
\]

\[
= \{ \max Q : \lambda F(Q;L_1,E) = 0 \}
\]
$$= \lambda \{ \max_{q} q^T Q : F(q; L_1, E) = 0 \}$$

$$\pi(q; \lambda L_1, E) = \lambda \pi (q; L_1, E).$$

Hence, the profit function will be homogeneous of degree one in $L_1$.

Certainly, the selection of a technology also depends on decision maker's farm experience and extension work usually made by the government in making new techniques available to farmers. Unfortunately there are no data on farmers' experience and extension expenditures and hence they are ignored.

It might exist, however, a problem given the method used to calculate $\bar{\pi}$. The procedure used was to divide total net returns (sales minus total expenditures, excluding expenditures on family labour) into the total number of household's on-farm work, i.e., $\pi/L_1$. Since the actual $L_1$ used involves a stochastic component related to the disturbance term $\mu_1$, the calculated $\bar{\pi}$ will be correlated with $\mu_1$, and hence the explanatory variable $\bar{\pi}$ in (61.i) would be correlated with the disturbance term. This problem is considered in the estimation method described below.

Note that the estimates of the net output-supply equations would be consistent but not asymptotically efficient. The inconsistency of the estimates of $f_j (j = 1, \ldots, 3)$ can be verified by noting that the explanatory variable $\bar{\pi}$ in the $j$ equations in (61) is correlated with the disturbance terms $\mu_j$; recalling that $\bar{\pi} = \phi(q; E) + \nu$ where

$$\nu = \sum_{i=1}^{\infty} v_i q_i$$

and $v_i$ are the disturbance terms associated with the net output-supply equations:

$$E(\bar{\pi} \cdot \mu_j) = E[\phi(q; E) + \nu] \cdot \mu_j] \neq 0$$

Thus, if $\nu$ and $\mu_j$ are correlated then $\text{cov}(\nu, \mu_j) \neq 0$ and hence $\bar{\pi}$ and $\mu_j$ are correlated.

Another important difference with the conventional model is that, although there are no parameter restrictions across the production and consumption sectors in model (61), all the cross symmetry restrictions between the profit maximizing and utility maximizing equations discussed in Chapter III are implicit in it. Indeed, these cross restrictions are imposed by the structure of the model rather than by parameter restrictions. Thus, in model (61) the compensated constant utility effect of a net output price increase on $L_2$ is

$$\frac{\partial L_2}{\partial q_i} = \frac{\partial L_2}{\partial \bar{\pi}} \bigg|_{\bar{\pi}=0} = \frac{\partial L_2}{\partial \bar{\pi}} \bigg|_{\bar{\pi}=0} \cdot \phi^i(q; E)$$

and the effect on net output supply ($Q_i$) of a change in $w_2$ is

$$\frac{\partial Q_i}{\partial w_2} = \frac{\partial \bar{\pi}}{\partial q_i} \bigg|_{\bar{\pi}=0} = \phi^i(q; E) \cdot \frac{\partial L_1}{\partial w_2} \bigg|_{\bar{\pi}=0}$$

but by the well known symmetry restriction between utility maximizing equations:
\[
\frac{\partial L_2}{\partial \pi} \bigg|_{du=0} = \frac{\partial L_1}{\partial w_2} \bigg|_{du=0}
\]
and, therefore, using this in the above equations:
\[
\frac{\partial L_2}{\partial q_i} \bigg|_{du=0} = \frac{\partial Q_i}{\partial w_2} \bigg|_{du=0}
\]
which is a cross symmetry restriction between the consumption and production sectors. Similarly, it can be shown that in (61) that
\[
\frac{\partial X}{\partial q_i} \bigg|_{du=0} = \frac{\partial Q_i}{\partial p} \bigg|_{du=0}
\]
which is the other cross symmetry restriction discussed in Chapter III. Thus, a major distinctive feature of a model which assumes interdependence of utility and profit maximization is the consideration of these cross symmetry restrictions. These restrictions may be incorporated via parametric restrictions or imposed by the structure of the model as occurs in model (61).

8 The \( 7 \times 7 \) \( B_N \) matrix is:
\[
B_N = \begin{bmatrix}
1 & 0 & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
0 & 1 & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
where \( a_{13} = \frac{\partial u_1}{\partial (Q_1/L_1)} = \frac{\partial u_1}{\partial \pi} \cdot q_1 \neq 0 \), and in general \( a_{k1} = \frac{\partial u_k}{\partial (Q_{i-2}/L_1)} \cdot q_{i-2} \neq 0 \)

9 This transformation can be done given that the averaged variables Z, E, and F enter in the right-hand-side of (61) in a linear form when the values of the other explanatory variables are fixed at their corresponding census division levels.

10 In maximizing the likelihood function I the Fletcher algorithm as described in the UBC:NLMON (1975) write-up as well as the non-linear version of Shazam (1979) were utilized.

11 An alternative would be to test a model which avoids interdependence by assuming that operator and hired labour are perfect substitutes in production and thus using the hired labour wage rate as the price of leisure, under the assumption that some hired labour is used by the households. This alternative is not considered because previous empirical studies have shown that hired labour and operator labour are not perfect substitutes in production.
Or using Olsen's argument that in the long run equilibrium the on-farm labour returns are identical to the off-farm wage rate (see footnote 6 in Chapter II).

An additional advantage in relaxing the constant returns to scale assumption is that if this was not done and if the model based on independence were rejected the doubt would persist whether the rejection was due to the imposition of such an assumption or because of the assumption of independence.

The unconditional net output supply responses are not directly estimated mainly because it would lead to rather serious econometric problems associated with the nature of the error structure. However, the unconditional net output supply responses to any exogenous variable are calculated ex-post using the estimates for (61.i) and the estimates of the conditional net output equations (see Chapter V).

The expected value of \( Q_i \) is obtained from the model based on interdependence in the following way:

\[
E(Q_i) = \frac{Q_i}{L_1} \cdot L_1 + E\{(L_1 - E(L_1)) \cdot (\frac{Q_i}{L_1} - E(\frac{Q_i}{L_1}))\} = \frac{\hat{f}_i(\cdot)}{\pi} \cdot \hat{f}_i(\cdot) + \text{cov} (\frac{1}{\pi}, \frac{1}{\pi})
\]

To be consistent with the model based on interdependence (equations 55), an exploratory estimation of the model based on independence (equations 70) using expenditure rather than quantity functions for (70.i) and (70.iii) was performed. Consequently, the test for independence was made using expenditure equations for \( L_1 \) and \( L_2 \) in (74.i) and (74.ii). Given that in this model the results were less consistent with economic theory and that the test for independence was rejected by a very wide margin, it was preferred to use a model based on independence which estimates quantity demand equations which provided more reasonable results. That is, in testing for independence the best specification for the model based on independence has been chosen.

The roles of the alternative and null hypotheses can also be reversed, thus using interdependence as the null hypothesis. This test is also performed although one should indicate that this is indeed a joint hypothesis of interdependence-constant returns to scale. Hence, it can be rejected even if interdependence holds if the technology does not approximately exhibit constant returns to scale.

The power of the test may, nevertheless, be affected, i.e., the probability of not rejecting \( H_0 \) when it is indeed false may increase because of the constant returns to scale assumption in \( H_A \).
The problem of regional representativity of the sample is important when one recognizes that there are important differences in weather, soil quality, distance to markets, etc., among regions which may affect farm supply and demand responses. An effort has been made to capture part of these differences using regional dummy variables for the intercept coefficients of the estimating net output supply equations. However, this may not be sufficient given that regional differences may affect not only intercept terms but also slope coefficients. One could also use dummies for the slope coefficients but this would greatly increase the number of estimating parameters. Thus, recognizing the existence of regional differences which are not entirely considered in the estimating model one has to accept as a second best alternative that the estimated coefficients would reflect some sort of average technology. If this interpretation is used then one needs to select a set of observations which is approximately representative of the Canadian agricultural sector.

There are only 40 observations for the Prairie Provinces. This would have allowed one to use a maximum of 100 observations if the Prairie Provinces are to be represented by 40% of the total sample. Using only 95 observations allowed us to obtain a sample of 38 observations from the Prairie Provinces in a random manner.

See Appendix 2 for a more extensive description of the data and methods used in transforming the data.

The econometric problems raised by this procedure of calculating labour returns are neglected (Aigner).

This implies the additional assumption that the shadow prices of leisure, households' activities, and on-farm working activities are the same for each household's member. This assumption allows to use Hicks' aggregation condition for the variable H.

For a description of the rental value calculations for land, livestock, and other farm capital see Appendix 2.

In calculating aggregate output as well as input price indices a quadratic mean of order one (Diewert, 1977c) is used as a discrete approximation of a divisia price index.
CHAPTER V

EMPIRICAL RESULTS

In this chapter the empirical results obtained from the estimation of the expenditure equations (55) and the net output supply equations (60) using Canadian farm census data are reported. The results of the different hypothesis tests are also provided. All tests have been performed using restrictions on equations (55) and (60) with the exception of the test for independence which uses equations (74) as discussed in section 4.3. Additionally, the relevant supply (and demand) response elasticities corresponding to the production and consumption sides of the model are presented.

5.1 Hypothesis Testing

The main hypothesis tested is that utility and profit maximization decisions are independent, i.e., that $\beta_k = 0$ for $k = 1, \ldots, 7$ in (74). A related hypothesis considered is the one concerned with the correlation of the errors of the labour supply and conditional net output supply equations in (61). Other hypotheses are restrictions on the functional form of preferences, on the effects of education and on the effects of household's dependents. First, the hypothesis of affine homotheticity is tested. This hypothesis implies the restrictions that $\delta_{ij} = 0$ for all $i \neq j$ in (55). Next, homotheticity to the origin is tested if the affine homotheticity assumption is not rejected. This implies that
\( \delta = 0 \) for all \( i, j \) in (55). A further hypothesis considered is that the indirect utility function is independent of the level of education, that is, that labour supply responses are not affected by education. This hypothesis requires that \( \ell_i = 0 \) for \( i = 1, 2, 3 \) in (55). A fifth hypothesis is that the level of education exerts a neutral effect on the demand for factors of production, that is, that \( a_i = 0 \) for \( i = 2, \ldots, 5 \) in (59). Finally, the hypothesis that the number of family dependents does not affect the indirect utility function and hence labour supply is also tested. This hypothesis needs that \( b_i = 0 \) for \( i = 1, 2, 3 \) in (55).

The importance of the hypotheses regarding restrictions on the functional form of preferences is evident considering that a number of studies have used them in analyzing labour supply and consumption responses (i.e., Lau et al.; Barnum & Squire; etc.). The effect of the decision maker's education may be especially important in the family farm where production decisions are usually made by the owner himself without the assistance of professional management personnel as normally occurs in larger corporate firms. The scale of operation of the family farm does not usually allow hiring of professional management expertise. Hence, it is important to formally test whether this variable has indeed an important effect on production decisions.

In order to carry out the above hypothesis tests, asymptotic likelihood ratio tests were performed. The likelihood ratio is the ratio of the maximum of the likelihood function under the null hypothesis to the maximum of the likelihood function under the alternative hypothesis. Minus twice the logarithm of a likelihood ratio has asymptotically a chi-square \((\chi^2)\) distribution where the number of degrees of freedom is equal to the number of restrictions imposed by the null hypothesis.
Table 1 shows the estimated $\chi^2$ values calculated at 5% and 1% level of significance (LOS) for the corresponding degrees of freedom.

The first row of Table 1 shows the $\chi^2$ for the null hypothesis that utility and profit maximizing decisions are independent against the alternative hypothesis of interdependence, i.e., that $\beta_k = 0$ for all $k = 1, \ldots, 7$ against the alternative hypothesis that not all $\beta_k$ coefficients are zero. The calculated $\chi^2$ is 127.20 which is higher than the critical values at 5% and even 1% LOS. Hence, the hypothesis that production and consumption decisions are independent is categorically rejected. The hypothesis of zero correlation among the errors of the labour supply and conditional net output supply equations is also rejected although the rejection is not as strong as in the previous hypothesis. Rejection of this latter hypothesis implies that, even though the constant returns to scale assumption allows to estimate conditional net output supply equations which do not have cross-constraints with the labour supply equations, these equations need to be jointly estimated. Thus, there exist significant gains in explanatory power and efficiency by estimating the consumption and production sectors jointly.

The hypothesis of affine homothetic preferences can be rejected at 5% LOS but is not rejected at 1% LOS and homotheticity to the origin is categorically rejected at 1% LOS. Thus, one can conclude that Canadian farm households' preferences are not homothetic to the origin and hence that its imposition may induce serious specification errors which lead to inconsistent estimates. Therefore, preferences are better specified if one allows the reference expenditures to be a function of commodity prices.

Hypotheses (5) and (6) in Table 1 are related to the effect of
<table>
<thead>
<tr>
<th>Null Hypotheses</th>
<th>$\chi^2$ Value</th>
<th>Degrees of Freedom</th>
<th>5% LOS</th>
<th>1% LOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Independence of production and consumption decisions</td>
<td>127.20**</td>
<td>7</td>
<td>14.07</td>
<td>18.48</td>
</tr>
<tr>
<td>2. Zero correlation among the errors of labour supply and conditional net output supply equations</td>
<td>24.60**</td>
<td>10</td>
<td>18.31</td>
<td>23.20</td>
</tr>
<tr>
<td>3. Affine homotheticity</td>
<td>9.51*</td>
<td>3</td>
<td>7.81</td>
<td>11.34</td>
</tr>
<tr>
<td>4. Homotheticity to the origin</td>
<td>202.76**</td>
<td>6</td>
<td>12.59</td>
<td>16.81</td>
</tr>
<tr>
<td>5. No effect of education on labour supply</td>
<td>28.63**</td>
<td>3</td>
<td>7.81</td>
<td>11.34</td>
</tr>
<tr>
<td>7. No effect of number of family dependents on labour supply</td>
<td>62.86**</td>
<td>3</td>
<td>7.81</td>
<td>11.34</td>
</tr>
</tbody>
</table>

Note: * denotes significance at 5% LOS  
** denotes significance at 1% LOS
education on labour supply and in production decisions, respectively. Both hypotheses are rejected at 1% LOS which implies that education significantly affects labour supply decisions (and hence it affects the indirect utility function) and that education plays a non-neutral role in determining optimal factor demands. Finally, hypothesis (7) confirms the results obtained in previous studies (Barichello; Huffman, 1980; Lau et al.; etc.) regarding the importance of the number of family dependents on labour supply decisions.

5.2 Supply and Demand Responses

The structural parameter estimates obtained by the joint estimation of the consumption and production sides of the model are presented in Table 2. The asymptotic standard errors of the coefficients are presented in brackets under the coefficients. Most coefficients in the consumption and production sectors appear to be significant with the exception of the $y_{12}$, $b_{24}$, $b_{34}$, and $b_{35}$ parameters. There is one degree of freedom in the parameters of the CES function which can be exhausted by any suitable normalization (Blackorby et al., 1978). The normalization chosen is that the share parameter $a_2$ is equal to one.

The coefficients of the regional dummy variables used turned out to be insignificant and, moreover, the inclusion of these variables distorts the values of other coefficients. Given that the use of dummies is essentially an ad-hoc procedure, it was decided not to use them in the model actually reported. The results obtained when the regional dummies were used were less consistent with economic theory and, in general, provided elasticity estimates which appeared to be quite unlikely. A reason for the lack of significance of the regional dummies may be that
### TABLE 2
Parameter Estimates of the Consumption and Production Equations (Equations 55 and 60)

<table>
<thead>
<tr>
<th>( \delta_{ij}, \xi_i, b_i )</th>
<th>Leisure 1</th>
<th>Leisure 2</th>
<th>Consumption Goods</th>
<th>Education</th>
<th>Number of Family</th>
<th>Number of Dependents</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Consumption Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.980</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.886</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>1.124</td>
<td>41.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.222)</td>
<td>(10.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure 1 (H-L_1)</td>
<td>612.5</td>
<td>-9.111</td>
<td>4.749</td>
<td>-14.83</td>
<td>160.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.591)</td>
<td>(3.746)</td>
<td>(9.603)</td>
<td>(3.205)</td>
<td>(7.149)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure 2 (H-L_2)</td>
<td>-829.3</td>
<td>60.86</td>
<td>-24.88</td>
<td>166.3</td>
<td>0.812</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(15.94)</td>
<td>(6.16)</td>
<td>(2.534)</td>
<td>(5.835)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditures</td>
<td>-</td>
<td>-2418</td>
<td>-2.812</td>
<td>42.76</td>
<td>105.5</td>
<td>(1.078)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10.59)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. Production Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_{ij}, a_i )</td>
<td>Output</td>
<td>Land</td>
<td>Hired Labour</td>
<td>Animal Stocks</td>
<td>Farm Capital</td>
<td>Education</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>Output Supply</td>
<td>113.6</td>
<td>147.4</td>
<td>-99.09</td>
<td>-39.61</td>
<td>-233.17</td>
<td>-38.77</td>
<td>0.835</td>
</tr>
<tr>
<td></td>
<td>(7.044)</td>
<td>(2.562)</td>
<td>(7.455)</td>
<td>(2.755)</td>
<td>(2.858)</td>
<td>(2.276)</td>
<td></td>
</tr>
<tr>
<td>Demand for Land</td>
<td>-147.4</td>
<td>-160.1</td>
<td>68.71</td>
<td>-2.584</td>
<td>150.2</td>
<td>15.56</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>(2.562)</td>
<td>(1.969)</td>
<td>(4.562)</td>
<td>(1.683)</td>
<td>(4.366)</td>
<td>(1.414)</td>
<td></td>
</tr>
<tr>
<td>Demand for hired labour</td>
<td>99.09</td>
<td>102.6</td>
<td>-37.01</td>
<td>-88.86</td>
<td>-9.124</td>
<td>0.801</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.455)</td>
<td>(22.21)</td>
<td>(4.702)</td>
<td>(9.743)</td>
<td>(1.199)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand for animal stocks</td>
<td>39.61</td>
<td>-</td>
<td>7.518</td>
<td>2.359</td>
<td>1.011</td>
<td>0.645</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.755)</td>
<td></td>
<td>(4.795)</td>
<td>(3.499)</td>
<td>(0.346)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand for farm capital</td>
<td>233.17</td>
<td>-</td>
<td>-</td>
<td>235.68</td>
<td>-32.48</td>
<td>0.874</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.858)</td>
<td></td>
<td></td>
<td>(4.674)</td>
<td>(1.783)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Generalized \( \tilde{R}^2 = 0.994 \)
a substantial proportion of the regional differences in weather and soil quality are captured in land price differences amongst the different regions. For example, the average rental price of improved land in the Fraser Valley of British Columbia was approximately 15 times as large as the average rental price of improved land in Saskatchewan. This price differential reflects to a large extent the better weather conditions and, in general, better land fertility in the Fraser Valley, which the dummy variables are supposed to capture. A reason for the distortion of the estimates of other coefficients induced by the dummy variables may be due to the negative effect on the econometric efficiency associated with the inclusion of redundant variables. Thus, a reason for obtaining results which are less consistent with economic theory might be that the inclusion of the dummies decreases the precision of the estimates.

The goodness-of-fit measure used is the "generalized $R^2" which was originally proposed by Baxter and Cragg. In systems of equations without intercepts such as the one estimated, the conventional $R^2$ measure is no longer appropriate and hence the generalized $R^2$ measure was used. This coefficient is defined as follows:

$$\tilde{R}^2 = \{1 - \exp \left[2(L_0 - L_{\text{max}})/N\right]\}$$

where $L_0$ is the value of the logarithm of the likelihood function when all parameters are constrained to zero, $L_{\text{max}}$ is the maximum when all coefficients are allowed to vary and $N$ is the total number of observations. The $\tilde{R}^2$ coefficient obtained is very close to 1 indicating that the goodness-of-fit of the estimation was very good. Raw-moment $R^2$ coefficients are provided for the individual equations as complementary information.
In order for the estimated function $G(\cdot)$ to be a valid indirect utility function, it must be monotonically decreasing and quasiconvex in prices. This implies that the functions $\Lambda(\cdot)$ and $\pi(\cdot)$ should be concave and monotonically increasing in prices. These properties were checked using the estimated coefficients. The monotonicity property was satisfied by both functions at each sample point. The function $\psi(\cdot)$ is globally concave, that is for all $p \geq 0$, since all $\alpha_i$ coefficients estimated are positive. Unfortunately, the function $\Lambda(\cdot)$ is not globally concave and moreover it does not satisfy this property at 62% of the observation points. Therefore, the quasiconvexity property of the indirect utility function is satisfied at only 38% of the observations. However, the calculated matrix of elasticities of substitution exhibits negative own substitution effects at approximately 55% of the observations.

The conditional profit function reported in Section 2 of Table 1 should also possess certain properties which are checked at each of the sample points. First, the conditional profit function has the correct gradients with respect to prices, that is, conditional profit increases with increases in the output price and it is decreasing in input prices. Secondly, the estimated conditional profit function is positive at each of the sample points. Thirdly, the estimated conditional profit function should satisfy the required convexity property. The sign of the determinants of the principal minors associated with the Hessian matrix of the estimated conditional profit function were checked. Although this matrix was not convex at approximately 60% of the observations, its diagonal elements were all positive at more than 80% of the sample points, which implies that the own price net output supply elasticities
show the correct sign when evaluated at most of the observations.

Another aspect interesting to note in Table 2 is that the reference expenditures which depend on the $\delta_{ij}$, $l_1$ and $b_1$ coefficients are positive for H-L$_1$ and H-L$_2$. However, the reference expenditures in consumption goods are negative at all sample points. This precludes the interpretation of $VA(p)$ as subsistence quantities. It is important to note, however, that these reference expenditures are less than actual expenditures at all the observations and hence the indirect utility levels are positive at all sample points.

Table 3 contains the on-farm and off-farm labour supply elasticities

<table>
<thead>
<tr>
<th>Labour Supply Elasticities</th>
<th>(at mean values of the variables)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>On-farm Labour returns</td>
</tr>
<tr>
<td>On-farm labour supply</td>
<td>0.119</td>
</tr>
<tr>
<td>Off-farm labour supply</td>
<td>-0.259</td>
</tr>
<tr>
<td>Total labour supply</td>
<td>0.043</td>
</tr>
</tbody>
</table>

with respect to on-farm returns to farm household labour, off-farm wage rate received by household's members and household's non-labour income. The own wage elasticities of labour supply are both positive when evaluated at mean values being the off-farm labour supply elasticity substantially larger than the on-farm elasticity. However, the on-farm supply elasticity was negative at 8% of the observations and the off-farm
elasticity was negative at 19%. These estimates are not comparable with previous studies because previous estimates were obtained for aggregate labour supply. One could calculate the total elasticity of labour supply with respect to a simultaneous change in the on-farm labour returns and the off-farm wage rate for the purpose of comparison. This elasticity is approximately 0.024 which is substantially lower than labour supply elasticities obtained by Lau et al. using farm household's data from Taiwan (0.16), by Barnum and Squire who used similar data from Malaysia (0.08). Huffman (1980) using U.S. farm households' data obtained off-farm labour supply elasticities of 0.33 for husbands and -0.06 for wives. On the other hand, Wales and Woodland (1976) using a sample of U.S. households also found small positive supply elasticities for some households and negative elasticities for others. The average supply elasticities for husbands was 0.11 for those on the upward section of the supply curve and -0.32 for those on the downward part. It is important to indicate that the labour supply elasticities are very sensitive to whether the variable education is used in the estimating labour supply equations or not. When education is not included, the total elasticity of labour supply rises to 0.38 which is somewhat larger than previous estimates obtained.

Table 3 also shows the cross-wage effects on labour supply. A 1-percent increase in the off-farm wage rate induces a 0.1% decrease in the number of days worked on their own farm by the household's members. The effect of on-farm labour returns on off-farm work is stronger. A 1-percent increase in farm labour returns will induce a 0.25 percent decrease in the off-farm supply of labour. The estimated elasticities of on-farm and off-farm labour supply with respect to household's non-labour income are -0.162 and -0.539, respectively. The effect of non-labour
income on total labour supply (on and off-farm) is approximately -0.23. This estimate can be compared with previous studies. For example, Ashenfelter and Heckman found elasticities of -0.112 for males and -0.594 for females using U.S. cross-sectional household data, and Horney and McElroy estimates were -0.213 and -0.097 respectively (Heckman et al.), also using household data.

The effect of education on both off-farm and on-farm labour supply is positive but its effect on off-farm work is substantially larger than on on-farm work. In fact, while a 1-percent increase in formal schooling training (measured in years of schooling) induces a 0.35% expansion of on-farm work, a similar increase in education will lead to a 1.25% expansion in off-farm work. Thus, the effect of education on labour supply appears to be quantitatively very strong with a bias towards off-farm activities.

The effect of education on off-farm work can be compared with Huffman's estimated elasticity of off-farm work with respect to farm operator educational level which was 1.03.

The on-farm and off-farm labour supply elasticities with respect to number of family dependents were both negative (-0.082 and -0.241, respectively). This result is not consistent with a priori expectations. One may be inclined to expect a positive effect and in fact most previous studies have obtained positive elasticities (Lau et al., for example, obtained an elasticity of 0.20 for total labour supply, and Huffman's estimate was 0.659 using children under 5 years of age as a proxy). Thus, it is very difficult to rationalize this result and it appears to be erroneous given that it contradicts not only reasonable a priori expectations but also previous quantitative estimates.

Table 4 contains the estimated compensated price elasticities of
demand for leisure and goods

\[ u_{ij} = \eta_{ij} + E_{iz} \frac{p_{ix}}{Z} \]

where

\( \eta_{ij} \) is the uncompensated price elasticity of demand, and

\( E_{iz} \) is the elasticity of demand for commodity \( i \) with respect to full income \( Z \).

**TABLE 4**

Compensated Demand Elasticities
(at mean values)

<table>
<thead>
<tr>
<th>On-farm</th>
<th>Off-farm</th>
<th>Price of Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour Returns</td>
<td>Wage Rate</td>
<td>Goods</td>
</tr>
<tr>
<td>Leisure 1 (H-L1)</td>
<td>-0.056</td>
<td>0.040</td>
</tr>
<tr>
<td>Leisure 2 (H-L2)</td>
<td>0.097</td>
<td>-0.158</td>
</tr>
<tr>
<td>Goods</td>
<td>0.038</td>
<td>0.026</td>
</tr>
</tbody>
</table>

The compensated price elasticities in Table 4 are less than unity in absolute value. The diagonal elements (the own compensated price elasticities) indicate that the leisure associated with off-farm work, H-L2, is the most price responsive and H-L1 is the least price responsive. The off-diagonal elements indicate that the two types of leisure and consumption goods are all net substitutes. The largest substitution possibilities take place between the two types of leisure, and leisure 2 tends to exhibit a larger substitutability with goods than leisure 1. The result that goods (income) and leisure are net substitutes is consistent with previous empirical studies. For example, Wales and Woodland (1976) found that both husband's leisure and wife's leisure were
substitutes with income.

Next, the cross-effects of changes in the production sector on the labour supply responses are considered. As discussed before, a model based on independence of consumption and production decisions imposes a model structure which does not allow any direct cross-effects from the production to the consumption sector except through the income effect. The model presented in this thesis, on the other hand, does allow for cross-effects and Table 5 shows the quantitative relevance of these.

TABLE 5
Labour Supply Elasticities with Respect to Net Output Prices
Calculated using Equation (20)

<table>
<thead>
<tr>
<th></th>
<th>Output Price</th>
<th>Land Price</th>
<th>Hired Labour Wage Rate</th>
<th>Animal Stock Price</th>
<th>Farm Capital Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-farm Labour Supply</td>
<td>0.390</td>
<td>-0.046</td>
<td>-0.027</td>
<td>-0.015</td>
<td>-0.145</td>
</tr>
<tr>
<td>Off-farm Labour Supply</td>
<td>-0.849</td>
<td>0.101</td>
<td>0.059</td>
<td>0.033</td>
<td>0.315</td>
</tr>
</tbody>
</table>

Table 5 presents the estimated labour supply elasticities with respect to output and input price changes evaluated at mean values. These elasticities have been calculated using equation (20) from Chapter II. As can be expected changes in the aggregate output price index have the largest effect on off-farm and on-farm labour supply in absolute values. A 1-percent increase in output price increases the on-farm labour supply by 0.39% and decreases the off-farm supply of labour by approximately 0.85%. Among the input prices the smallest effect is the one associated with changes in livestock prices which have labour supply elasticities of minus 0.015. On the other hand, changes in farm capital prices do have
important effects on both on-farm and off-farm labour supply.

Notice the quantitative magnitude of the cross-effects from the production sector on both on-farm and off-farm labour supply. The effects of output prices are larger than the effects of on-farm labour returns or off-farm wage rates on labour supply. The magnitude of these cross-effects suggests that a model which does not consider the interdependence between the production and consumption sectors neglects quantitatively important effects which may be even larger than the direct wage rate effects on labour supply.

Table 6 presents the supply and demand elasticities conditional on $L_1$ for outputs and inputs evaluated at the mean prices. The conditional elasticities $(CS_{ij})$ are defined as

$$CS_{ij} = \frac{\partial Q_i / L_1}{\partial q_j Q_i / L_1}.$$  

These elasticities can be interpreted as net output supply responses assuming that operator and family labour remain constant after a net output price has changed. The diagonal elements in Table 6 show the own price elasticities which are positive for output and negative for all inputs. The off-diagonal elements are the conditional cross-elasticities of supply of output and demand for inputs.

All inputs, except land, are positively affected by output prices (as reflected in the positive elasticities of column one or in the negative values of row 1), being the demand for hired labour the most responsive to price changes. The largest negative effect of an input price increase on output supply is when the farm capital prices increase. A land price increase has a positive effect on output levels. This may lead one to believe that land is an inferior input which does not seem a very plausible situation.
TABLE 6
Conditional Net Output Supply Elasticities
(at mean values)

<table>
<thead>
<tr>
<th>Prices</th>
<th>Output</th>
<th>Land</th>
<th>Hired Labour</th>
<th>Animal Stocks</th>
<th>Farm Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.332</td>
<td>0.113</td>
<td>-0.126</td>
<td>-0.049</td>
<td>-0.269</td>
</tr>
<tr>
<td>Land</td>
<td>-0.912</td>
<td>-0.418</td>
<td>0.458</td>
<td>-0.016</td>
<td>0.888</td>
</tr>
<tr>
<td>Hired Labour</td>
<td>1.557</td>
<td>0.797</td>
<td>-0.420</td>
<td>-0.600</td>
<td>-1.334</td>
</tr>
<tr>
<td>Animal Stocks</td>
<td>1.103</td>
<td>-0.053</td>
<td>-1.107</td>
<td>-0.006</td>
<td>0.063</td>
</tr>
<tr>
<td>Farm Capital</td>
<td>0.626</td>
<td>0.298</td>
<td>-0.260</td>
<td>0.005</td>
<td>-0.660</td>
</tr>
</tbody>
</table>

TABLE 7
Unconditional Net Output Supply Elasticities
Calculated using Equation (22) (at mean values)

<table>
<thead>
<tr>
<th>Prices</th>
<th>Output</th>
<th>Land</th>
<th>Hired Labour</th>
<th>Animal Stocks</th>
<th>Farm Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.732</td>
<td>0.066</td>
<td>-0.153</td>
<td>-0.064</td>
<td>-0.414</td>
</tr>
<tr>
<td>Land</td>
<td>-0.522</td>
<td>-0.464</td>
<td>0.430</td>
<td>-0.031</td>
<td>0.743</td>
</tr>
<tr>
<td>Hired Labour</td>
<td>1.947</td>
<td>0.750</td>
<td>-0.447</td>
<td>-0.666</td>
<td>-1.479</td>
</tr>
<tr>
<td>Animal Stocks</td>
<td>1.493</td>
<td>-0.099</td>
<td>-1.134</td>
<td>-0.021</td>
<td>-0.082</td>
</tr>
<tr>
<td>Farm Capital</td>
<td>1.016</td>
<td>0.251</td>
<td>-0.287</td>
<td>-0.010</td>
<td>-0.835</td>
</tr>
</tbody>
</table>
Table 7 contains the unconditional supply and demand elasticities as defined by equation (22) in Chapter II. These elasticities measure the actual market net output supply responses after the effects of output or factor price changes on family and operator labour supply have been considered.

The output supply elasticity obtained is 0.73 which is somewhat lower than supply elasticities obtained by previous studies for agriculture. For example, Tweeten and Quance, using different procedures, obtained estimates of 0.31, 1.79, and 1.52 for long-run aggregate output supply elasticities in U.S. agriculture. The effects of factor price changes on output supply are generally small with the exception of farm capital prices. A 1-percent increase in farm capital prices induces a 0.4 percent decrease in output supply. Changes in land prices have a small effect on the demand for all inputs with the only exception of hired labour. Factor demands are not very responsive to changes in their own prices. All factors present a rather inelastic demand schedule and the demand for animal inputs appears to present the least elastic with an elasticity coefficient of only -0.006. These estimates can be compared with previous results for U.S. and Canadian agriculture. Binswanger's (1974) own factor demand elasticity estimates for U.S. agriculture were -0.34 for land, -0.91 for labour, -1.089 for machinery, and -0.95 for fertilizers. Lopez estimates for Canadian agriculture were -0.52 for labour, -0.35 for farm capital, -0.42 for land, and -0.41 for intermediate inputs. Thus, although the results are not entirely comparable because the inputs disaggregation is different and because these studies estimated compensated price elasticities, (i.e., for a constant level of output), the general pattern of inelastic factor
demands is consistent in the three studies.

The sign structure of the cross-elasticities of demand is of considerable interest. An increase in the hired labour wage rate leads to a decrease in output and a reduction in the demand for all other inputs except land. The largest depressing effect of a wage rate increase is on the demand for animal inputs. This is consistent with the fact that production of animal outputs is more labour intensive than other activities. An increase in the price of farm capital (say machinery) will cause a reduction in output production and an increase in the demand for land but a decrease in the use of all other factors, being the effect on hired labour the largest. Similarly, an increase in livestock prices causes a decrease in the demand for all inputs, with the largest depressing effect on hired labour demand.

The effect of farm operator's education on the structure of the net output vector can be considered by inspecting the coefficients associated with the variable education in Table 2. The effect of education on the structure of production can be analyzed, as previously discussed, as a proxy for technical change. Changes in technology may have a neutral effect on the allocation of resources, i.e., the marginal rate of substitution between factor i and factor j stays constant (at a constant factor i/factor j ratio) or may also have a biased effect on resource allocation if the marginal rates of substitution do change when education changes. Accordingly, one may use the convention of defining a factor i positive bias of education if it induces an increase in the factor i cost share and a factor j negative bias if it induces a reduction in factor j cost share. Neglecting the effect of education on labour supply, it can be shown that the cost share of factor i will always increase if the
coefficient associated with education \( (a_j) \) is positive. However, the converse is not necessarily true, i.e., a negative \( a_j \) coefficient does not necessarily imply that the cost share of factor \( j \) decreases with education. If there are more than one negative \( a_j \) coefficients then all one can say is that education induces a negative bias for at least one of the factors which exhibits negative \( a_j \). When there are more than one negative \( a_j \), the effect of education on those factor shares will depend on prices and cost shares. In Table 2 there is a negative effect of education on the shares of two factors with negative \( a_j \) coefficients when its effect is evaluated at mean values of prices and factor demands. Hence, given the joint significance of the variable education one may conclude that the effect of operator's education on resource allocation is non-neutral and biased towards livestock forms of capital and land and against all other factors. The negative effect of education on output levels is quite surprising. It implies that farm operator's increased education leads to reduce the scale of production and farm expenditures perhaps towards other investment opportunities outside agriculture. Education would allow farmers to consider alternative, perhaps more profitable, sources of investment. Thus, the main effect of education would be to induce cost savings rather than an output expansion. It is important to note, however, that the negative effect of education is on conditional output supply and that in order to calculate the total effect one needs to consider the effect of education on on-farm work which is positive. This effect reduces the size of the negative effect but is not sufficient to reverse it. A 1-percent increase in education induces a 0.09% reduction in farm output when this effect is evaluated at mean values. In any case, these results should
be interpreted cautiously because the variable education used as a proxy for technical change may not be entirely appropriate, considering that other variables such as extension expenditures and farm operator's experience have been neglected.  

Finally, the cross effects of changes in some parameters of the consumption sector on the scale of production are considered. Thus, using results obtained in Chapter II (equation (24)), the elasticity of the scale of production with respect to the non-labour income variable is identical to the elasticity of on-farm labour supply with respect to the same variable. Hence, a 1-percent increase of non-labour income would lead to reduce on-farm labour supply and the scale of production by 0.16% (see Table 3). Thus, increasing farmer's assets which yield higher non-labour returns lead to quite an important contraction in the scale of agricultural production (including output supply and input demand). This effect has been ignored in previous studies which have assumed that changes on the consumption side have no effects on net output supply. To assess the importance of the error made by neglecting these effects one can compare the effect of changes in non-labour income on output supply with the effect of input prices on output supply; the non-labour income (or wealth effect) is indeed larger in absolute terms than any of the input price effects, except farm capital, on output supply.

Similarly, in Chapter II it was shown that the impact of the off-farm wage rate on the scale of production is identical to its effect on on-farm labour supply (equation (25)). Thus, the elasticity of on-farm labour supply with respect to the off-farm wage rate is -0.107 and, therefore, the elasticity of net output supply with respect to a change in \( w_2 \) will be the same. Hence, a 1-percent increase in off-farm wages
received by farmers will cause a contraction in net output supply of approximately 0.1%. Although this effect is not as large as the effect of non-labour income it is by no means negligible and is more important than the effect of land prices and animal inputs on output supply.

5.3 Further Implications of the Results

The most important result of this chapter is the categoric rejection of the hypothesis of independence of consumption and production decisions when a model based on interdependence is used as the alternative hypothesis. The wide margin by which the null hypothesis has been rejected is certainly quite striking. It would be interesting to repeat this test using household's microdata and relaxing the assumption of constant returns to scale imposed in the model based on interdependence used as the alternative hypothesis. Intuitively, one may argue that imposition of the constant returns to scale assumption on the alternative hypothesis, while this restriction is not imposed on the null hypothesis, increases the probability of accepting the null hypothesis. Thus, if this is true, relaxation of constant returns to scale should lead to an even stronger rejection of the null hypothesis.

Apart from the formal econometric rejection of the hypothesis of independence it is important to note that the estimates of the model based on independence conform less with a priori knowledge than the model based on interdependence. In particular, the estimated indirect utility function under independence does not satisfy the required quasi-convexity properties at any of the sample points although it does satisfy monotonicity. The estimated profit function shows the expected gradients with respect to the net output prices but it is convex at only 45% of the
sample points and, in contrast, with the main model estimated in this thesis, the own price net output supply elasticities have the correct signs at only 32% of the sample points. Thus, the empirical evidence shows that a model based on independence of production and consumption decisions should be statistically rejected and also that its estimates conform less with economic theory than a model based on interdependence.

Another result which is interesting is the one related to the test for affine homotheticity. The fact that this hypothesis cannot be rejected at 1% LOS suggests that the hypothesis of affine homothetic preferences, which is so convenient in estimating aggregated labour supply and commodity demand relations via linear expenditure systems, might not be as bad an approximation of Canadian farmers' preferences after all. However, the strong rejection of homotheticity to the origin implies that econometric models based on such hypothesis should be discarded.

With respect to the quantitative results of the consumption side, it appears that the large majority of Canadian farmers are still on the upward part of both the on-farm and off-farm labour supply schedules. Hence, one can expect a positive on-farm and off-farm labour supply response to wage related incentives. A result consistent for all observations is that off-farm work is more responsive to off-farm wages than on-farm work is to on-farm labour returns. However, while farmers' total labour supply (on-farm plus off-farm) response to changes in the off-farm wage is negative, total labour supply is positively affected by an increase in on-farm labour returns. Farmers' non-labour income has a very strong negative effect on both on-farm and off-farm work. However, the negative effect on off-farm work is substantially larger than
the effect on on-farm work.

An important issue discussed in Chapter I and II was whether farmers view on-farm and off-farm work as perfect substitutes. If this were the case one would expect an infinite Allen elasticity of substitution between H-L₁ and H-L₂. The fact that the compensated cross demand elasticities between H-L₁ and H-L₂ (see Table 4) are so low is in indication that indeed farmers do not regard on-farm and off-farm work as perfect substitutes. Also, the average Allen elasticity of substitution between H-L₁ and H-L₂ was only 0.12 and, moreover, its largest value when evaluated at any sample point was less than 0.2.

The labour supply responses estimated may have important implications for the implementation of policies aimed to raise incomes of rural families (which has been an explicit policy objective in Canada). Farmers' incomes can be increased essentially via three mechanisms: (1) to increase on-farm returns to their work (i.e., to increase the profit level conditional to hours of on-farm work, $\bar{h}$); (2) to subsidize off-farm work, i.e., to increase the effective off-farm wage, $w₂$; (3) to increase the amount of direct government transfers, that is, to increase the non-labour income received by farm families. Each of these mechanisms has a direct impact on total farm household's income and also an indirect effect via induced changes on labour supply off-farm and on-farm. An important policy question is which of these three mechanisms is more effective in raising rural families' income. Using the estimated coefficients it can be shown \(^{10}\) that for each dollar spent by the government via mechanism (1) the average farm family increases its income by $1.10. If mechanism (2) is used instead then one dollar spent will yield only 94 cents increase in the farm family income. Finally, if
mechanism (3) is used then income will increase by 98 cents. The reason for this is that total labour supply decreases when mechanisms (2) and (3) are used while it increases when mechanism (1) is used. Thus, on the basis of purely the cost to the government of raising farm families income, mechanism (1) should be preferred.

Using the estimates of the indirect utility function one can also determine which of the above three mechanisms should be chosen if the goal is to improve farm families welfare rather than merely their income. One may rank the effect of one dollar spent via each of the mechanisms on the level of \( G(h, w_2, p; Z) \), i.e., which of the above mechanisms will yield the highest increase in the indirect utility function of the average farm household. The smallest effect on family welfare takes place if mechanism (2) is chosen while mechanism (1) and (3), being substantially more effective than (2), yield very similar results. This pattern was consistent when the three mechanisms were compared at all sample points. Thus, mechanism (2) is the least efficient using either the income or welfare criterion and hence it should not be used. Mechanisms (1) and (3) are equally effective in improving welfare but it is reasonable to argue that mechanism (3) might be relatively simpler to administrate. Hence, it appears that if the purpose is to increase farm families welfare one might prefer to increase the amount of direct government transfers rather than subsidizing labour returns.

The effects of an income tax cut on farm households' labour supply and on the scale of farm production can also be analyzed using the estimated coefficients. A 1% reduction in the marginal tax rate (i.e., a 1% reduction in the \( \beta_1 \) coefficient in (58)) will induce the average farm household to decrease both on-farm and off-farm labour supply.
effect on off-farm work, however, is substantially larger than the effect on labour supply to farming activities. Thus, while a 1% tax cut would decrease off-farm labour supply by 0.155%, a similar tax cut would reduce on-farm work by only 0.036%. The effect on the scale of production is also negative and equal to the effect on on-farm labour supply. It is important to point out that the effect of an income tax cut on labour supply is consistently negative when evaluated at all sample points and at all tax brackets.

A reason for the negative effect of a reduction in the tax rate on labour supply is that the income effect of the tax cut is stronger than the effect of the induced increase in the after tax returns to on-farm and off-farm work. A tax cut will induce an expansion of "full income" which will have a strong depressing effect on labour supply which will more than off-set the positive effect associated with the increase in after tax returns to labour. The small reduction in the scale of production is due to the contraction of on-farm work. Thus, an income tax cut will have an important depressing effect on labour supply, affecting mainly the levels of off-farm work, and it will also induce some negative effects on farm production.

In connection with the production sector, it is important to note the quantitative differences between the conditional and unconditional net output supply equations (Tables 6 and 7, respectively). The impact of on-farm households' labour supply response on output supply, demand for animal inputs and farm capital is quite impressive; the output supply elasticity is doubled and the own demand elasticity for animal inputs is more than tripled when the effect of output and input price changes in labour supply is considered. On the other hand, the effect of
households' labour on land demand and hired labour demand is very small as reflected by the small changes in their own conditional and unconditional elasticities.

The generally inelastic output supply and input demand patterns are rather surprising considering that, as discussed in the previous chapter, the estimates can be considered as long-run elasticities. In particular, the low elasticity of demand for hired labour suggests that observed trends towards farm workers unionization in Canada may lead to higher returns to farm workers with a relatively small decrease in hired labour employment, and hence to increase their share in total farm income.  

The fact that the elasticity of demand for land with respect to the output price is negative is quite surprising and may suggest that land is an inferior input. One interpretation could be that as output prices increase and hence as output levels are expanded, the output composition changes towards outputs which are less intensive users of land, i.e., from crops to poultry and hog production. Thus, although the pure output scale effect may be positive, the negative effect on demand for land due to changes in the composition of outputs might predominate. It is also possible that the positive effect of land prices on output supply (which is equivalent to say that output prices induce a negative effect on land demand) may be due to the fact that land quality differences among the observations were ignored in the estimation process. Thus, a substantial part of the land price variability may be associated with changes in land quality. Higher land prices may also imply better land and, hence, this may have a positive effect on output supply.

The results obtained as well as their policy implications discussed above should be interpreted only as a preliminary approximation of the
problem of measuring supply and demand responses of the self-employed producer-consumer. The major limitations of the analysis are the following.

1. Apart from the data limitations discussed in Chapter IV one needs to consider the problems implied by the fact that lack of data on output prices at the census division level made it necessary to allocate provincial prices to the different census divisions according to their geographical location. It is clear that many farmers belonging to census divisions located in bordering areas might have sold their products in the neighboring province rather than in their own province. This would imply that an additional source of error is generated by allocating commodity prices according to the province where the census divisions are located.

2. The fact that all outputs are aggregated into one category is also a problem, particularly in cross-sectional analysis. Output bundles are rather heterogeneous in the different regions of the country. The divisia price index approximation used may consider these differences in output composition via changes in the weights of the individual outputs. In any case, it is well known that the restrictions on production technologies (i.e., separability or fixed proportions) or on relative prices (i.e., Hicks' aggregation condition) required for consistent commodity aggregation, are very strong. A better procedure would have been to disaggregate outputs into, for example, grains, animal products, and other commodities. Unfortunately, this would have greatly increased the number of parameters to be estimated in an already complex model.

3. The assumption of constant returns to scale is also restrictive. Moreover, this assumption combined with the assumption of no fixed factors led to an empirical model in which the interdependence between utility
and profit maximization decisions are not as prominent (although still important) as in a model where either of those assumptions are relaxed.

4. The assumption regarding the existence of a consistent household utility function, although widely used, also requires rather strong and unrealistic conditions. By postulating a household utility function it is assumed that the household makes consistent centralized decisions. This is an additional restriction of the empirical analysis which is important to consider in evaluating the results and policy implications.

5. Finally, another limitation of the approach is the lack of consideration of changes in the quality of inputs among the observations. This problem is particularly important for the land input but it could also be relevant for other inputs such as hired labour. Thus, part of the observed price variability on inputs may be due to differences in quality of the inputs among census divisions.

In summary, the above limitations suggest that the results obtained might be substantially affected by the rather unrealistic assumptions used and, consequently, they should be considered only as a first approximation to the problem and should be cautiously interpreted.
It is important to point out that all the $\beta_k$ coefficients, except those associated with the demand for hired labour and demand for farm capital equations, were significant at 5% LOS when the t-statistic values were calculated for each coefficient.

However, when the roles of the null and alternative hypotheses were reversed in (74), i.e., when the null hypothesis was interdependence under constant returns to scale, the calculated $\chi^2$ value was not sufficiently large to reject it at 1% LOS.

Given that at many sample points the determinants of most minors were very small negative numbers, one could expect that a statistical test would not reject the null hypothesis that convexity holds. A test for convexity using a method suggested by Lau (1978) which uses the fact that any positive semidefinite Hessian matrix has a Cholesky factorization was tried. Unfortunately, given the large number of parameters being estimated, the added computational burden implied by the method made the estimation very difficult and expensive and convergence could not be obtained.

The means, standard deviations and the extreme values of the elasticities when evaluated at the different sample points are presented in Appendix 4. This information is provided in Appendix 4 not only for the labour supply elasticities but also for all other elasticities estimated.

The average off-farm labour supply elasticity was -0.126 for those on the downward part of the supply curve and 0.231 for those on the upward side. The average on-farm elasticities were -0.017 and 0.165 for those on the downward and upward segments, respectively.

This can be seen more clearly by inspecting the Allen cross-partial elasticities of substitution,

$$A_{ij} = \frac{Z \cdot (\partial x_i^*/\partial p_j)}{x_i \cdot x_j}$$

where $\left(\partial x_i^*/\partial p_j\right)$ denotes compensated or utility constant change in $x_i$ when $p_j$ changes. The estimated Allen elasticities are $A_{12} = 0.120$, $A_{13} = 0.011$ and $A_{23} = 0.054$.

Certainly this result may also be reflecting unaccounted improvements in the quality of land as its price increase across the observations.

In compact notation one may write the factor demand equation as:

$$Q_i = L_1 \cdot \phi(q) + a_{1} L_1 E$$

where $a_{1}$ is the coefficient associated with education.

The cost share equations are:
s_i = \frac{q_i Q_i}{c(q, E)} = \frac{q_i L_1 \phi(q)}{c(q, E)} + \frac{a_i L_1 E q_i}{c(q, E)}

where

c(q, E) is the total cost evaluated at optimal output level Q^*_i(q, E).

Differentiating with respect to E

\frac{\partial s_i}{\partial E} = \frac{a_i L_1 p_i - \frac{\partial c}{\partial E} \cdot s_i}{c(q, E)}.

Given that \frac{\partial c}{\partial E} \leq 0, i.e., it is assumed that increased education does not increase cost, if a_i > 0 then \frac{\partial s_i}{\partial E} > 0. Thus, in this case education is factor i biased. However, if a_i < 0 the effect of E on s_i is ambiguous and will depend on price levels, educational levels and on the factor share.

9 Education may also be negatively correlated with farm operator's age, and hence it may also reflect the effect of younger farmers on production decisions. This may be consistent with the interpretation of education as a proxy for technical change if one can reasonably argue that younger farmers are more receptive to new technologies.

10 The total income of a farm household (Y) can be defined as follows:

Y = \pi L_1 + w_2 L_2 + y

Hence an increase in Y (\Delta Y) can be decomposed in the following way:

\Delta Y = \pi \Delta L_1 + L_1 \Delta \pi + w_2 \Delta L_2 + L_2 \Delta w_2 + \Delta y

Alternative (1) implies to spend one dollar in increasing on-farm returns, \pi. Given that the average farm family works 320 days on-farm, it implies that \Delta \pi = \frac{1}{320} for the average farm family. Hence, using the estimates for \frac{\partial L_1}{\partial \pi} and \frac{\partial L_2}{\partial \pi}, \Delta Y = \frac{\pi}{\Delta \pi} \Delta L_1 + L_1 + w_2 \Delta L_2 = 1.10.

Thus, the effect of an increasing of \pi by $\frac{1}{320}$, i.e., of spending $1 in subsidizing \pi is an increase in the average family's income of $1.1. Similarly, the effect of subsidizing w_2 can be calculated:

\frac{\Delta Y}{\Delta w_2} = \frac{\Delta L_1}{\Delta w_2} + w_2 \frac{\Delta L_2}{\Delta w_2} + L_2 = 0.94

and, finally, the effect of increasing non-labour income by $1 is:

\frac{\Delta Y}{\Delta y} + \frac{\Delta L_1}{\Delta y} + w_2 \frac{\Delta L_2}{\Delta y} + 1 = 0.98.
Perhaps the easiest way of subsidizing \( \pi \) is by subsidizing output prices. If one dollar per family is spent on subsidizing the output price received then the average family income will increase by \$8.8. This effect is substantially larger because in this case not only labour supply increases but also the level and composition of output and inputs are optimally changed.

Notice, however, that these results do not take into account possible effects of the subsidy on wages and prices faced by the farm families.

The effect of increasing the net on-farm labour returns on the utility level of the average farm family is:

\[
\frac{\partial U}{\partial \pi} = \frac{\partial G}{\partial \pi} + \frac{\partial G}{\partial Z} \frac{\partial Z}{\partial \pi}
\]

where \( U \) is utility level. Thus, \( \frac{\partial U}{\partial \pi} \) reflects the increase on welfare when \( \pi \) increases by \$1. If \$1 per family is spent on raising on-farm labour returns then, given that the average family works 320 days on-farm, it means that \( \pi \) increases by \( \frac{1}{320} \). Thus the effect of spending \$1 per family on increasing on-farm labour returns is \( \frac{\partial U}{\partial \pi} \frac{1}{320} \).

Similarly, using \( \frac{\partial U}{\partial \omega_2} = \frac{\partial G}{\partial \omega_2} + \frac{\partial G}{\partial Z} \frac{\partial Z}{\partial \omega_2} \) and \( \frac{\partial U}{\partial \gamma} = \frac{\partial G}{\partial Z} \frac{\partial Z}{\partial \gamma} \) one can calculate the effects of mechanisms (2) and (3) on the level of utility.

Defining \( \varepsilon_{L_1} = \frac{\partial L_1}{\partial \beta_i} \) as the rate of change in \( L_1 \) due to a 1% increase in the marginal tax rate, \( \beta_i \), one can verify that:

\[
\varepsilon_{L_1} \beta_i = \left[ \frac{\partial L_1}{\partial \pi} \frac{\partial \pi}{\partial \beta_i} + \frac{\partial L_1}{\partial \omega_{2i}} \frac{\partial \omega_{2i}}{\partial \beta_i} + \frac{\partial L_1}{\partial \gamma} \frac{\partial \gamma}{\partial \beta_i} \right] \frac{\beta_i}{L_1}
\]

Also

\[
\frac{\partial \pi}{\partial \beta_i} = - \pi = - \frac{\pi_i}{1-\beta_i}
\]

\[
\frac{\partial \omega_{2i}}{\partial \beta_i} = - \omega_2 = - \frac{\omega_{2i}}{1-\beta_i}
\]

and using the definition of \( Z_i \) in equation (58):

\[
\frac{\partial Z_i}{\partial \beta_i} = \frac{Y_T}{1} - y - H(w_2 + \pi) + Ex.
\]

then

\[
\varepsilon_{L_1} \beta_i = \left[ - \varepsilon_{L_1} \pi_i - \varepsilon_{L_1} \omega_{2i} + \varepsilon_{L_1} Z_i \right] \frac{\beta_i}{\frac{Y_T}{1} - y - H(w_2 + \pi) + Ex.}
\]
where $\varepsilon_{L1\tilde{\pi}_1}$ is the on-farm labour supply elasticity with respect to $\tilde{\pi}_1$ and $\varepsilon_{L1w_2}$ and $\varepsilon_{L1Z_1}$ are similarly defined. Using the same procedure one can find that

$$\varepsilon_{L2}\beta_1 = \left[ -\varepsilon_{L2\tilde{\pi}_1} - \varepsilon_{L2w_2} + \varepsilon_{L1Z_1} \frac{Y^T - y - H(w_2 + \tilde{\pi}) + Ex.}{Z_1} \right] \frac{\beta_1}{1 - \beta_1}$$

Substituting the values for the elasticities evaluated at mean prices and using the mean levels for $Y^T, y, H, w_2, \tilde{\pi}, Ex$ and $Z_1$ one obtains that $\varepsilon_{L1\beta_1} = 0.036$ and that $\varepsilon_{L2\beta_1} = 0.155$.

If the change in hired labour wage rate does not induce any change on output and input prices then it is easy to calculate the effect of an increase in the wage rate ($w$) due to workers unionization on their share in the total value of output, defining the labour share by:

$$S = \frac{wL}{qQ}$$

where
- $L =$ hired labour used
- $q =$ output price
- $Q =$ output levels

then

$$\frac{\partial S}{\partial w} = \frac{L}{qQ} (1 + \varepsilon_{Lw}) - \frac{1}{qQ} \varepsilon_{Qw}$$

where $\varepsilon_{Lw}$ is the own wage elasticity of labour demand and $\varepsilon_{Qw}$ is the output elasticity with respect to $w$. Given that $\varepsilon_{Qw} < 0$ then a sufficient condition for $\frac{\partial S}{\partial w}$ to be positive is that $\varepsilon_{Lw} > -1$. Given that $\varepsilon_{Lw}$ is approximately $-0.25$, then $\frac{\partial S}{\partial w}$ is unambiguously positive.

The problems involved in postulating a household utility function are discussed by Samuelson (1956).
CHAPTER VI

SUMMARY AND CONCLUSIONS

In this final chapter a summary of the study and the main conclusions are presented.

The main purposes of this research have been to analyze the economic behaviour of self-employed farm producers, to provide an empirically feasible model to estimate farm households supply and demand responses and to formally test the often used hypothesis of independence of production and consumption decisions. It has been shown that an important peculiarity of self-employed households is that the allocation of resources takes place with reference to market prices for some commodities and to internal non-market prices for other commodities. In particular, the household's labour allocated to its own firm (which is normally the most important factor of production used by this type of firm) and the existence of non-traded goods produced and entirely consumed by the household are perhaps the most important examples of commodities which are traded within individual households and which do not have a "visible" price. The shadow price of these commodities are internally generated and may be endogenous with respect to the households. The importance of these internally generated prices is that they certainly affect the allocation of resources of not only labour and non-traded goods but the allocation of resources and commodities which do have exogenous market prices as well. Another implication is that production and consumption decisions are not likely to
be independent and hence in order to estimate households' supply and demand responses it is necessary to adapt the conventional household's model and to modify the conventional econometric framework which is mainly oriented to estimate the consumption and production equations in a disjoint manner.

In Chapter II a general static model for the self-employed household was developed and, using some simplifications based on assumptions which appear reasonable for Canadian farm households, a model appropriate to empirically analyze supply and demand responses of self-employed households was derived. In Chapter III a model which does not rely on the above simplifications and that can be used in estimating preferences and production technologies of self-employed producers under general conditions was discussed. The use of a dual representation of preferences for a given non-linear budget constraint which contains the production sector embodied in a conditional profit function was discussed. Moreover, some specific comparative static effects were analyzed and an econometric framework for the estimation of the supply and demand relations was proposed.

In Chapter IV an econometric framework to estimate the simplified model developed in Chapter II was developed and a procedure to formally test the hypothesis of independence of production and consumption decisions was discussed. In Chapter V the empirical results obtained when the above model was used to estimate preferences and production technologies of Canadian farmers have been presented. The major empirical findings were:

1. The hypothesis of independence between utility maximizing and profit maximizing decisions was categorically rejected. Moreover, it was
shown that there are important gains in explanatory power by estimating the production and consumption sectors jointly. These gains occur despite that the use of the constant returns to scale assumption allowed to estimate conditional net output supply equations which had no parameter cross-constraints with the consumption sector.

2. The cross-effects between the unconditional net output supply equations and the labour supply responses were quantitatively very strong.

3. The frequently used hypothesis of homotheticity of preferences has been rejected. However, the test of the hypothesis of affine homotheticity provided less conclusive results; affine homotheticity is rejected at 5% level of significance but not at 1%.

4. It has been shown that farm operator's educational level has a significant non-neutral effect on demand for inputs. Moreover, education also had a significant effect on labour supply responses and induced a re-allocation of household's labour from on-farm to off-farm work.

5. Finally, the number of family dependents also had an important effect on labour supply responses.

Additionally, it is noted that the model estimated explains farm households' consumption and production decisions reasonably well. The model generates results which are consistent with economic theory on the consumption side. And, although there are some problems with the production side of the model, one may claim that it represents a substantial improvement with respect to previous studies. In the empirical implementation of the model, a major problem was the use of aggregated rather than household's data. A number of detailed farm households surveys have been recently carried out in Canada and elsewhere yielding data which are
already available or which will be available in the near future. This may allow a more precise estimation of farmers' preferences and farm production technologies.

The results obtained may be used in deriving policy implications. The effect of a tax cut (or negative income tax schemes) on labour supply decisions and the derived cross-effect on output supply and factor demands has been analyzed. Similarly, the results can be used to analyze the effects of changes in government transfer payments to farm families on labour supply and production decisions. This can be done by using the labour supply elasticities with respect to non-labour income conveniently adjusted by the weight of such transfer payment on total non-labour income. Another policy implication associated with the large differences in labour supply responses to changes in on-farm labour returns and off-farm wage rates was discussed in Chapter V. The fact that the total labour supply elasticity with respect to on-farm labour returns is positive while the total labour supply elasticity with respect to off-farm wages is negative, led to the conclusion that a policy oriented to increase on-farm returns may be more effective and less costly in increasing farmers' income than one oriented to expand off-farm labour returns.

In closing this chapter it is worthwhile to identify the specific theoretical and empirical contributions of this thesis. (1) Theoretical contributions: Using recent developments in duality theory, a set of empirically testable predictions have been obtained for the self-employed household producer-consumer. The problem of obtaining testable predictions for the farm-household as a unit of production and consumption has been frequently analyzed by authors concerned with agriculture in developing economies. Their focus of attention, however, has centered almost
exclusively in obtaining predictions for output supply responses (see, for example, Sen or Hymer and Resnick). Given that output supply responses to price changes are in general ambiguous in the theoretical models, it has been very difficult to empirically test the validity of the farm-household model. A theoretical contribution of this thesis has been to show the existence of other predictions which can be used to empirically test the farm household model. In particular, a constant-utility output supply expression has been derived and it has been shown that, in contrast with the variable-utility (or "Marshallian") output supply response, it is unambiguously non-negative. Similarly, constant-utility comparative static expressions for changes in the equilibrium level of (on-farm) household work with respect to net output prices and fixed capital have been derived and shown to be non-negative. It was also proved that the behavioural equations for net output supply, shadow price of farm capital, and demand for consumption commodities are homogeneous of degree zero in consumption good prices, net output prices, and non-labour income. In particular, the net output supply equations are no longer homogeneous of degree zero in net output prices as in the conventional case and the equation for the equilibrium level of household work (on-farm), in general, does not satisfy any homogeneity condition. Finally, all symmetry conditions prevailing in the conventional models of the household and of the firm were found to hold in the farm-household model. Moreover, it was also proved that cross-symmetry conditions between the production and consumption sectors constitute an additional distinctive feature of the farm-household model. In particular, the compensated effect of a change in net output price on demand for consumer goods is identical to the constant-utility effect of a change in consumer good
prices on net output supply. Similarly, compensated changes in consumer
goods due to changes in fixed capital are identical to minus the constant-
utility change in the shadow price of fixed capital associated with changes
in consumption good prices. (2) Empirical Contributions: Two non ad-
hoc$^3$ empirically estimable models of the farm-household behavioural equa-
tions have been derived. A simplified model which relies on the assump-
tions of constant returns to scale and no fixed factors was estimated
using Canadian farm census data. The advantages of this model with re-
spect to a more general model which do not use these assumptions are that
standard duality theory can be used in deriving the estimating behavioural
equations of the farm-household, that it allows to disentangle household
preferences from the farm production technology and that it is relatively
easier to estimate since the equations associated with the production
sector are all linear in the parameters. A problem of this model is
that, although some basic linkages between the utility and profit maximiza-
tion equations exist, these linkages are less important than in a model
which does not rely on the constant returns to scale and/or no fixed fac-
tors assumptions.

A more general model which does not rely on any restrictive assump-
tions regarding production technologies or household preferences was also
developed and a feasible procedure to estimate it was indicated. Major
advantages of this model with respect to the previous one are that the
interdependences between production and consumption decisions become now
more apparent and that it is more general.

It was shown that even the simplified model appears to be more appro-
priate to estimate consumption and production technologies than the conven-
tional model based on independence of production and consumption decisions.
A formal econometric test of the hypothesis of independence was performed resulting in a categoric rejection of such hypothesis. Moreover, the simplified model was used to jointly estimate the equations of labour choice between on-farm and off-farm employment and the conditional net output supply functions of Canadian farm-households.

In summary, the major empirical contributions of this thesis have been to provide alternative empirically estimable models of the farm-household which explicitly account for interdependence of production and consumption decisions, to estimate one of these models using Canadian census data, thus quantifying labour choice responses, net output supply responses and the cross effects of changes in the production sector on consumption decisions and vice-versa.

An additional empirical contribution has been to formally test the hypothesis of independence of production and consumption decisions. To the best of the author's knowledge, no one of the above-mentioned analyses have previously been performed and hence they may be regarded as original contributions.
Danielson, for example, estimated a profit function for Canadian agriculture assuming independence of production decisions from consumption decisions. His estimated profit function did not satisfy the convexity property at any sample points. Moreover, he obtained the "correct" sign for the own price elasticity for two out of three outputs and the corresponding negative sloping demands for only one out of four inputs considered.

When this thesis was at a late stage, data from a farm household's survey carried out in the province of Saskatchewan were made available.

The term non ad-hoc is used here to signify that direct linkages between the theoretical model and the estimating equations exist. In particular, the restrictions implied by the optimization hypothesis considered are fully used either by imposing them on the estimating equations or by testing them. Furthermore, the functional forms specified for the estimating equations are explicitly derived from plausible functional forms representing household preferences and farm production technologies.

The author is not aware of any non ad-hoc empirical study on labour choice at least for farm producers.
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APPENDIX 1

Proof of Propositions

1. Proof of Proposition 1

Proposition 1 refers to the case when (a) the household members are indifferent between working in the family firm or elsewhere, and (b) family labour and hired labour are not identical factors of production. Thus, one may formulate the problem in the following manner:

\[
\max_{X,L_1,L_2} U(H-L_1-L_2,X)
\]

\[\begin{align*}
(i) & \quad px \leq \pi(q;L_1) + w_2L_2 + y \\
(ii) & \quad L_2 \geq 0 \\
(iii) & \quad L_1 \geq 0, X \geq 0
\end{align*}\]

where all symbols are defined in Chapter II.

Assuming differentiability and an interior solution for \(X\) and \(L_1\), the Kuhn-Tucker first order conditions are:

\[
\begin{align*}
U_1 - \lambda \frac{\partial \pi(q;L_1)}{\partial L_1} &= 0 \\
U_1 - \lambda w_2 - \mu &= 0 \\
U_2 - \lambda p &= 0 \\
L_2 &\geq 0; \quad \mu L_2 = 0; \quad \mu \geq 0 \\
-px + \pi(q;L_1) + w_2L_2 + y &= 0
\end{align*}
\]

Where \(\lambda\) and \(\mu\) are the Lagrangean multipliers associated with constraints (i) and (ii), respectively.

Now, if \(\mu = 0\), i.e., if \(L_2 > 0\), outside salaried work takes place then using (a) and (b) one obtains that

\[
\frac{\partial \pi(q;L_1)}{\partial L_1} = w_2.
\]

Thus, the shadow price of household's members working in the family firm is equal to the exogenous wage rate received by family members while working outside the family firm, \(w_2\). Hence, the unconditional profit function \(\pi^0(q,w_2)\), can now be defined as:

\[
\pi^0(q,w_2) \equiv \{\max_{L_1} \pi(q;L_1) - w_2L_2\}
\]

which implies that \(\pi^0(q,w_2)\) is independent of consumer preferences, consumption good prices and non-labour income.

To show that when \(L_2 > 0\) the consumption side of the model is independent from the production side except through the income effect, an indirect utility function \(\mathcal{G}\) can be defined:
\[ G(p,w_2;Z^0) \equiv \{ \max_{H-L_1-L_2,X} U(H-L_1-L_2,X) \} \]

(i) \( px + w_2(H-L_1-L_2) \leq w_2H + y + \pi^0(q,w_2) \equiv Z^0 \)

(ii) \( H-L_1-L_2 \geq 0; \quad X \geq 0 \).

Thus, given that \( L_1 \) and \( L_2 \) have the same price, \( w_2 \), the household maximizes for \( H-L_1-L_2 \) rather than for \( L_1 \) and \( L_2 \) as it occurs when their prices differ. Hence, the only effect of the production sector on the consumption decisions takes place through the income effect via \( Z^0 \).

If \( L_2 = 0 \) and hence \( \mu > 0 \) then

\[ \frac{\partial \pi(q;L_1)}{\partial L_1} = w_2 + \frac{\mu}{\lambda}; \]

i.e., the shadow price of \( L_1 \) is in general greater than \( w_2 \). Therefore, \( w_2 \) cannot be used as the price of \( L_1 \) and hence the profit function would be:

\[ \pi(q,w_2 + \frac{\mu*}{\lambda*}) \equiv \{ \max_{L_1} \pi(q;L_1) - (w_2 + \frac{\mu*}{\lambda*})L_1 \}. \]

Where \( \mu* \) and \( \lambda* \) represent optimal values. However, \( \mu* = \mu*(p,q,y,w_2) \) and \( \lambda* = \lambda*(p,q,y,w_2) \) and hence the profit function \( \pi(q,w_2,p,y) \) will not be independent of the consumption decisions.

On the other hand, using the first order conditions (a) to (e) it is clear that when \( L_2 = 0 \) then

\[ \lambda \frac{\partial \pi}{\partial L_1} = \lambda w_2 + \mu \]

and the household solves for \( H-L_1(p,q;y), X(p,q,y), \lambda(p,q,y) \) using equations (a), (c) and (e). Hence the indirect utility function will now be \( G(p,q,y) \) and the effect of changes on the production side of the model on the consumption side will not only be through the income effect. Note that the actual function \( G \) will not only depend on the structure of the utility function but also on the structure of the profit function \( \pi(q;L_1) \).

2. **Proof of Proposition 2**

Proposition 2 refers to: (a) household's members are not indifferent between working in their own firm and elsewhere, (b) family labour is an identical input as hired labour. Now the problem would be:

\[ \max_{H-L_1,H-L_2,X} U(H-L_1,H-L_2,X): \quad \text{(i) } px \leq \hat{\pi}(q_0;L_0+L_1)+w_2L_2 + y - w_0L_0 \quad \text{H-L_1,H-L_2,X} \]

\[ \text{(ii) } L_0 \geq 0, \quad \text{(iii) } L_1,L_2;X \geq 0 \]

where \( L_0 = \) hired labour used by the family firm

\( \hat{\pi} = \) profit function conditional on \( L_0 \) and \( L_1 \)

\( q_0 = \) net output price vector excluding \( w_0 \).

Assuming that \( L_1 > 0, L_2 > 0, X > 0 \); the Kuhn-Tucker first order conditions are
A + e = 0 (c')

156.

\( T \) = 0 (a')

\( w_2 + \mu = 0 \) (b')

\( \lambda \left( \frac{\partial \pi(q_0;L_0+L_1)}{\partial L_0} - w_0 \right) + e = 0 \) (c')

\( L_0 \geq 0; \ L_0 \epsilon = 0; \ e \geq 0 \) (d')

\( U_2 - \lambda p = 0 \) (e')

\( -px + \hat{\pi}(q_0;L_0+L_1) + w_2L_2 + y - w_0L_0 = 0 \) (g')

where \( \epsilon \) is the Lagrangean multiplier associated with constraint (ii). Notice that

\[ \frac{\partial \pi(q_0;L_0+L_1)}{\partial L_0} = \frac{\partial \pi(q_0;L_0+L_1)}{\partial L_1} \]

and hence if \( L_0 > 0 \), then \( \epsilon = 0 \) and hence using (c')

\[ \frac{\partial \hat{\pi}}{\partial L_0} = w_0 = \frac{\partial \hat{\pi}}{\partial L_1}. \]

Hence, the unconditional profit function will be \( \pi(q_0,w_0) \), independent from the consumption sector of the model. Also, the indirect utility function \( G \) will be:

\[ G(p,w_0,w_2;Z) = \{ \max_{H-L_1,H-L_2,X} U(H-L_1,H-L_2,X): (i) \ p_x \leq \pi(q_0;L_1+L_0) - w_0L_0 + y + w_2L_2 \]

\[ \text{and} \]

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\[ \text{and} \]

\[ \text{and} \]

\[ \text{and} \]
\[ U_1 - \lambda \frac{\partial \pi}{\partial L_1} + \eta = 0 \]  
(a'')

\[ U_1 - \lambda w_2 + \mu = 0 \]  
(b'')

\[ U_2 - \lambda p = 0 \]  
(c'')

\[ \lambda \left( \frac{\partial \pi}{\partial L_0} - w_0 \right) + \varepsilon = 0 \]  
(d'')

\[ L_0 \geq 0; \quad \varepsilon L_0 = 0; \quad \varepsilon \geq 0 \]  
(e'')

\[ L_2 \geq 0; \quad \mu L_2 = 0; \quad \mu \geq 0 \]  
(f'')

\[ L_1 \geq 0; \quad \eta L_1 = 0; \quad \eta \geq 0 \]  
(g'')

\[ px - \pi(q; L_1 + L_0) + w_0 L_0 - y - w_2 L_2 = 0 \]  
(h'')

where \( \eta \) is the Lagrangean multiplier associated with the constraint \( L_1 \geq 0 \). From (a'') and (b'') we have that

\[ \frac{\partial \pi}{\partial L_1} = w_2 + \frac{\mu - \eta}{\lambda} \]

and from (d'') we have that

\[ \frac{\partial \pi}{\partial L_0} = w_0 + \frac{\varepsilon}{\lambda} \]

Note that by assumption \( \frac{\partial \pi}{\partial L_0} = \frac{\partial \pi}{\partial L_1} \). If \( L_0 > 0 \) and \( L_1 > 0 \), but \( L_2 = 0 \), then \( \varepsilon = \eta = 0 \) and hence

\[ \frac{\partial \pi}{\partial L_0} = \frac{\partial \pi}{\partial L_1} = w_0 = w_2 + \frac{\mu}{\lambda} \]

Thus the profit function (unconditional) will be \( \pi(q_0, w_0) \) and the indirect utility function will be \( G(p, w_0, z) \) being \( \pi \) and \( G \) independent.

If \( L_2 > 0 \) and \( L_1 > 0 \) but \( L_0 = 0 \) then \( \eta = \mu = 0 \) and hence the unconditional profit function will be \( \pi(q_0, w_2) \) and the indirect utility function will be \( G(p, w_2, z) \) again being \( \pi \) and \( G \) independent.

\section*{4. Proof to Corollary 3.1}

From (a''), (b'') and (d'') in the proof of proposition (3) one obtains that

\[ w_2 = w_0 + \frac{\eta - \varepsilon - \mu}{\lambda} \]

hence if \( w_2 > w_0 \) then \( \eta - \varepsilon - \mu > 0 \). Given that \( \varepsilon \geq 0 \) and \( \mu \geq 0 \) it is necessary that \( \eta > 0 \). This implies that \( L_1 = 0 \) (using g'').

\section*{5. Proof to Corollary 3.2}

Again using the relation

\[ w_2 = w_0 + \frac{\eta - \varepsilon - \mu}{\lambda} \]

If \( w_2 < w_0 \) then \( \eta - \varepsilon - \mu < 0 \). Now, suppose that the firm uses hired labour and the household's members do work outside the family farm, i.e., that \( L_0 > 0 \) and \( L_2 > 0 \). Equation (e'') and (f'') imply that \( \varepsilon = \mu = 0 \) and given that \( \eta \geq 0 \) one obtains that \( \eta - \varepsilon - \mu \geq 0 \), which
contradicts the original assumption that $w_2 < w_0$. Hence, if $L_0 > 0$ and $L_2 > 0$ then $w_2$ cannot be less than $w_0$.

Proof of Proposition 5

Define $G(p,y) = \{\max U(x) : E(x,p) \leq y\}$

where

$E(x,p) \leq y$

is a non-linear budget equation. It is assumed that $E(x,p) = y$ at the optimum. The variables $x, p, y$ are defined as follows:

$x =$ consumption bundle

$p =$ a price vector

$y =$ "income"

Proposition 5 says that $G(p,y)$ will be quasi-convex in $p$ if and only if $E(x,p)$ is concave in $p$.

Define

$B \equiv x : E(x,p) \leq y$

$B' \equiv x : E(x,p') \leq y$

$B'' \equiv x : E(x,p'') \leq y$

where

$p'' = tp + (1-t)p'$ and $0 < t < 1$.

It is necessary to show that $G(p,y) \leq k$ and $G(p',y) \leq k$ will imply that $G(p'',y) \leq k$ if and only if $E(x,p)$ is concave in $p$. This is equivalent to show that if $E(x,p)$ is concave in $p$ then $B''$ will be contained in either $B$ or $B'$ or both, i.e., $B'' \subseteq BUB'$.

Suppose $E(x,p)$ is concave in $p$ but $B'' \not\subseteq BUB'$ then

$E(p'',x) = E[pt + p'(1-t),x] \leq y$ but some of the $x$ will not be contained in $BUB'$, hence:

(i) $E(p,x) > y$

(ii) $E(p',x) > y$.

Now multiplying (i) by $t$ and (ii) by $(1-t)$ and adding:

(i) $tE(p,x) > ty$

(ii) $(1-t)E(p',x) > (1-t)y$

(iii) $tE(p,x) + (1-t)E(p',x) > y$.

But if $E(p,x)$ is concave in $p$ then:

$tE(p,x) + (1-t)E(p',x) \leq E(tp + (1-t)p',x)$.

Hence

$E(tp + (1-t)p',x) = E(p'',x) > y$

which contradicts our original assumption (it contradicts A). This implies that if $E(p,x)$ is concave in $p$ then $B'' \subseteq B'UB'$, i.e., that $G(p,y)$ is quasi-convex.

Proposition 7. If the production technology exhibits constant returns to scale and if there are no fix inputs then the conditional profit function is homogeneous of degree one in $L_1$ and hence it can be written as

$\pi(q,w_A; L_1) = L_1 \hat{\pi}(q,w_A)$. 


Proof. 

\[ \pi(q, w_A; L_1) = \{ \max_{Q,z} qQ - w_A z : F(Q, z; L_1) = 0 \} \]

\[ \pi(q, w_A; tL) = \{ \max_{Q,z} qQ - w_A z : F(Q, z; tL_1) = 0 \} \]

\[ = \{ \max_{tQ,tz} t(qQ - w_A tz) : F(tQ, tz; tL_1) = 0 \} \]

\[ = \{ \max_{Q,z} t(qQ - w_A z) : tF(Q, z; L_1) = 0 \} \]

\[ = t\{ \max_{Q,z} qQ - w_A z : F(Q, z; L_1) = 0 \} \]

\[ = t\pi(q, w_A; L_1). \]

Hence \( \pi(q, w_A; L_1) \) is homogeneous of degree one in \( L_1 \) and can be written as \( L_1 \cdot \pi(q, w_A) \).
APPENDIX 2

Data Transformations

1. Rental Price Calculations

In calculating rental prices of durable factors of production one may follow Diewert's (1972) method which in turn is based on Walras' approach.

1.1 Rental Price of Land and Buildings

\[ p_T = p_T - \frac{\delta_T \hat{P}_T}{1 + r} \]  \hspace{1cm} (A.1)

where

- \( p_T \) = rental price of land
- \( P_T \) = current asset price of land
- \( \hat{P}_T \) = expected price of land at the end of the period
- \( \delta_T \) = depreciation rate of land
- \( r \) = interest rate

Following Danielson (1975) it is assumed that \( r = 0.05 \) and \( \delta_T = 0 \). In this case A.1 can be written as

\[ p_T = \frac{P_T - \hat{P}_T + r}{1 + r} \]

and defining

\[ \rho_T = \frac{\hat{P}_T - P_T}{P_T} \]

where \( \rho_T \) is the expected rate of growth of the asset price. Thus

\[ p_T = \frac{r - \rho_T}{1 + r} P_T. \]  \hspace{1cm} (A.2)

Barichello (1979) has estimated the rate of growth of land prices in 1970 at 3%. Hence, it is assumed that \( \rho_T = 0.03 \).

1.2 Rental Price of Animal Stocks

It is assumed that the expected rate of change in the asset price of livestocks is zero. Furthermore, a long run steady-state equilibrium, in the sense that producers are satisfied with their animal stocks and do not intend to expand it, is assumed. Finally, it is assumed that each census division is entirely self-sufficient in producing the animals to replace those which are sold. In this case the animal stock rental price \( (q^*_A) \) formula is simply:
where $P_A$ is the asset price of the animal stock.

1.3 Rental Value of Machinery and Equipment

It is assumed that the rental price of machinery and equipment is constant across the observations given that there are no data on asset prices. However, in order to calculate net profit it is necessary to estimate rental values of machinery and equipment. Given that there are data on the asset value of machinery and equipment one can calculate its rental value, $v_M$. Thus, assuming that the asset prices are not expected to increase,

$$v_M = \frac{(1 + \delta_M)}{1 + r} V_M$$

where

$v_M \equiv P_M \cdot M$ is the rental value of machinery and equipment

$V_M = P_M \cdot M$ is the asset value of machinery and equipment

$\delta_M = \text{depreciation rate.}$

A depreciation rate $\delta_M = 0.13$ as calculated by Danielson is used.

2. Data Available (Data Code)

$X_1$ = Number of farm households in each census division
$X_2$ = Operator's off-farm labour income
$X_3$ = Spouse's off-farm labour income
$X_4$ = Other household's members off-farm labour income
$X_5$ = Operator's off-farm work (in days)
$X_6$ = Operator's total number of days worked (on-farm and off-farm)
$X_7$ = Spouse's total number of days worked (on-farm and off-farm)
$X_8$ = Other household's members total number of days worked
$X_9$ = Total non-labour income for all household's members (interests, dividends, government transfers, excludes returns to farm capital)
$X_{10}$ = Total number of people age 13 above living in the farm
$X_{11}$ = Total taxes paid by the household's members
$X_{12}$ = Total value of farm sales
$X_{13}$ = Total operational costs per farm: fertilizers, spraying materials, machinery and land rentals, fuel, seed and wages
$X_{14}$ = Total value of land and building at 1970 estimated market prices
$X_{15}$ = Total value of machinery and equipment at 1970 estimated market prices
$X_{16}$ = Total value of livestocks on the farm at 1970 estimated market prices
$X_{17}$ = Number of improved acres of land
$X_{18}$ = Output prices by province in 1970 (major grains, animal products, and fruit and vegetables)
$X_{19}$ = Estimated prices of major livestocks on the farm (cattle except cows, milk cows, hogs, and sheep) by province
$X_{20}$ = Total hired labour (in days) used by the farms
X21 = Total wages paid to hired labour
X22 = Operator's schooling years
X23 = Average distance to urban centers (miles to the closest metropolitan area with a population of 100,000 or more)
X24 = Total value of sales by each census division.

Notes: 1. All data correspond to the year 1970 and are available by average farm household per census division.
2. Variables X2, X3, X4, X5, X6, X7, and X8 are also disaggregated by household's members sex.

4. Data Required

Inspecting equation (55) in Section (4.1) one can verify that the following data are required:
1) Total number of days of on-farm work for all household's members (L1).
2) Total number of days of off-farm work for all household's members (L2).
3) After tax household's "full income" (\(Z_1\)).
4) Net returns to household's work on the farm (\(\hat{\pi}\)).
5) Off-farm wage rate (per day) earned by the household's members who work off-farm (\(w_2\)).

Using the data code of section 3 of this appendix the above variables are defined in the following way:

The on-farm work is:
\[L_1 = (X6 - X5) + \frac{X3}{w_0} + \frac{X4}{w_0}\]  
(A.5)
where \(w_0 = \frac{X2}{X5}\) is the operator's off-farm wage rate.

For female household's members we use \(w\) instead of \(w_0\), where \(\tilde{w} = \varepsilon \cdot w_0\), where \(\varepsilon\) is a factor of proportion of average women's wage/men's wage, which is calculated by province using data from the Labour Force Survey for average wage of males and females.

The off-farm work is calculated:
\[L_2 = X5 + \frac{X3 + X4}{w_0}\]  
(A.6)
where we also use \(\tilde{w}_0\) instead of \(w_0\) for female members.

The net returns to household's members' work on the farm or equivalently the farm's profit per unit of family labour is defined by:
\[\hat{\pi} = \frac{X12 - X13 - V}{L_1}\]  
(A.7)
where \(V\) is the rental value of all durable factors of production (land and structures, livestock capital, and machinery and equipment), i.e.,
\[V = \left(\frac{r - \rho_T}{1 + r}\right) V_T + \left(\frac{r}{1 + r}\right) V_A + \frac{r + \delta_M}{1 + r} V_M\]  
(A.8)
where
- \(V_T\) is the asset value of land and buildings
- \(V_A\) is the asset or stock value of animal stocks on the farm
- \(V_M\) is the asset value of the machinery and equipment stocks on the farm.
The average off-farm wage rate received by household's members is calculated as a weighted average of the male and female wage rates where the weights are allocated according to the proportion of off-farm wage income of males and females.

The after tax "full income" is (see equation (48) in Chapter IV):

$$Z_i = 365 \cdot X_{10}(1+\nu_2) + (1-\beta_i)(X_9+\gamma \cdot V) + \beta_i Y_i^T - T_i$$

where $\gamma$ is the proportion of farm capital owned by the household and $\beta_i$, $Y_i^T$, and $T_i$ stand for the marginal tax rate at tax bracket $i$, the smallest taxable income at tax bracket $i$ and taxes paid at income $Y_i^T$, respectively.

4.2 Data for the Conditional Net Output Supply Equations

The data required for estimating the conditional net output equations are the following (see equation (60) in Chapter IV):

1. Aggregate output price index ($q_1$).
2. Rental price for land and buildings ($p^r$).
3. Wage rate paid by the household to the hired labour used by the farm ($q_2$).
4. Rental price of livestock capital ($q_5$).
5. Operator's schooling years ($E$).
6. Net output supply per day of on-farm work ($Q_{L_1}, Q_{L_2}, Q_{L_3}, Q_{L_4}, Q_{L_5}$).

An aggregate price index by province is calculated and then these prices are assigned to the different census divisions according to the province where they are located. The output price index is calculated as follows: a quadratic mean of order one price index, which is a superlative index number, is used. This index is exact for a generalized Leontief cost function as has been shown by Diewert (1977c):

$$q_1 = \left( \sum_{i} \frac{q_{1i}^0 y_{1i}^0}{TV_{1i}^0} \right)^{1/2} \left( \sum_{j} \frac{q_{1j}^0 y_{1j}^0}{TV_{1j}^0} \right)^{1/2}$$

(A.10)

where

- $y_{1i}$ are the different outputs considered
- $q_{1i}$ are the prices of the outputs
- $TV_{1i}$ stands for the total value of output in each province.

The superscript 0 indicates the value of the variables in the benchmark province (where $q_1 = 1$) and the superscript $j$ stands for the value of the variables in each of the other provinces.
The specific outputs \( Y_{11} \) considered were wheat, barley, oats, other grains, various fruit crops, vegetables, poultry, eggs, dairy products, beef, hogs and sheep and lambs, corn, potatoes, and tobacco. Data on prices and outputs per province were obtained from different Statistics Canada publications, including catalogue numbers 23-203, 23-201, 21-513, 23-202, and 23-203.

In order to calculate the rental price of land and buildings, the asset price of land, \( P_T \), was first estimated:

\[
P_T = \frac{X_{14}}{X_{17}}.
\] (A.11)

Next \( P_T \) is used in (A.2) in order to obtain \( p_T \), i.e., the rental price of land and buildings.

The wage rate (per day) paid for hired labour is directly obtained from the census data:

\[
q_2 = \frac{X_{21}}{X_{20}}.
\] (A.12)

For calculating the rental price index of livestocks first the rental prices for the different livestock categories are calculated using (A.3). The following livestock categories are considered: cows, cattle except cows, pigs, sheep and lambs. Provincial prices obtained from Statistics Canada publications nos. 21-514, 21-513, and 23-203 are used. Next an aggregate price index is calculated using a formula equivalent to (A.10) where now the \( q_{5i} \) correspond to the rental prices of the different livestock categories and the \( Y_{5i} \) correspond to the quantity of animals per category. Thus, an aggregate price index per province is calculated and then the corresponding provincial price is assigned to the census divisions.

In calculating the net output supplies per day of on-farm work the following procedure was used: the total value of output sales by census division \((\times 24)\) is divided by the total number of on-farm days of work \((L_1)\) per census division as estimated by (A.5). This operation yielded the total value of output per day of on-farm work per census division. Next the value of output \( L_1 \) was divided by the output price index as calculated in (A.10), which gave a quantity index of output per day of on-farm work. Finally, this index was transformed in terms of average farm household by dividing by the total number of farms in each census division. A similar method was used in obtaining the quantity indexes for demand for land and structures, hired labour, livestocks on farm and farm capital. The variables \( X_{14}, X_{15}, X_{16}, \) and \( X_{13} \) were used for this purpose.

Finally, the variable operator's schooling years are directly obtained from census data (variables \( X_{22} \) and \( X_{23} \), respectively).

4.3 Summary Statistics of the Data Used

Table A.1 contains the mean values, standard deviations and extreme values of the most important variables considered in the analysis. It is interesting to note the large differentials between the mean values of the off-farm wage rate received by farmers and their on-farm labour returns. Off-farm wage rates are substantially larger than on-farm
TABLE A.1

Mean, Standard Deviation and Extreme Values of Some Important Variables Considered

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of farm households per census division</td>
<td>2090.00</td>
<td>1492.00</td>
<td>30.00</td>
<td>6362.00</td>
</tr>
<tr>
<td>Before tax returns to family labour farm work ($ per day)</td>
<td>22.20</td>
<td>13.60</td>
<td>5.50</td>
<td>93.00</td>
</tr>
<tr>
<td>Before tax off-farm wage rate ($ per day)</td>
<td>50.90</td>
<td>15.30</td>
<td>23.60</td>
<td>101.00</td>
</tr>
<tr>
<td>Days of on-farm work by the operator and his family</td>
<td>320.40</td>
<td>29.50</td>
<td>253.40</td>
<td>380.90</td>
</tr>
<tr>
<td>Days of off-farm work by the operator and his family</td>
<td>80.60</td>
<td>18.50</td>
<td>53.70</td>
<td>122.60</td>
</tr>
<tr>
<td>Value of output per farm ($)</td>
<td>16707.50</td>
<td>7819.00</td>
<td>7877.00</td>
<td>49683.00</td>
</tr>
<tr>
<td>Rental value of land per farm ($)</td>
<td>2016.00</td>
<td>1274.00</td>
<td>530.00</td>
<td>7050.00</td>
</tr>
<tr>
<td>Hired labour expenditures per farm ($)</td>
<td>1193.50</td>
<td>1556.00</td>
<td>214.00</td>
<td>7286.00</td>
</tr>
<tr>
<td>Rental value of animal stocks ($ per farm)</td>
<td>651.50</td>
<td>300.60</td>
<td>102.00</td>
<td>1747.00</td>
</tr>
<tr>
<td>Rental value of farm capital ($)</td>
<td>6240.00</td>
<td>2505.00</td>
<td>2439.00</td>
<td>16281.00</td>
</tr>
<tr>
<td>Rental price of land ($ per acre)</td>
<td>6.03</td>
<td>8.41</td>
<td>1.24</td>
<td>49.50</td>
</tr>
<tr>
<td>Hired labour wage rate ($ per day)</td>
<td>12.65</td>
<td>2.69</td>
<td>6.32</td>
<td>20.10</td>
</tr>
<tr>
<td>Farm operator's years of schooling</td>
<td>8.6</td>
<td>1.05</td>
<td>6.00</td>
<td>10.20</td>
</tr>
<tr>
<td>Distance to urban centres (miles)</td>
<td>20.90</td>
<td>12.00</td>
<td>5.00</td>
<td>148.40</td>
</tr>
</tbody>
</table>
labour returns in all census divisions. Moreover, the differences are statistically significant at 95% even if it is assumed that transportation costs from the farm to the off-farm working place are 25% of the off-farm daily wage rate. This is another indication that off-farm work and on-farm work are not substitutes in consumption, i.e., that there are utility differences between the two works.

Table A.2 and A.3 show the different expenditures as a proportion of family labour farm returns and the cost shares of the various inputs. An important implication of these data is the large weight of family labour in the cost structure. Thus, land costs are less than 30% of the family labour (and operator) costs and moreover, land costs, hired labour costs, and livestock costs together are less than 50% of the operator and family labour costs. The only input which approximates the importance of family labour (88% of its cost) is farm capital which includes machinery, equipment, and all intermediate inputs. Family and operator labour constitutes 41% of total production costs and hence the importance of the cross effects between production and consumption decisions may be expected to be considerable.
### TABLE A.2

**Different Expenditures as a Proportion of Family Labour Farm Returns**

<table>
<thead>
<tr>
<th>Expenditure</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm Sales</td>
<td>2.35</td>
</tr>
<tr>
<td>Land Costs</td>
<td>0.28</td>
</tr>
<tr>
<td>Hired Labour Costs</td>
<td>0.168</td>
</tr>
<tr>
<td>Livestock Costs</td>
<td>0.092</td>
</tr>
<tr>
<td>Farm Capital Costs</td>
<td>0.879</td>
</tr>
</tbody>
</table>

### TABLE A.3

**Cost Shares of the Different Factors**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land</td>
<td>0.117</td>
</tr>
<tr>
<td>Hired Labour</td>
<td>0.069</td>
</tr>
<tr>
<td>Livestock</td>
<td>0.038</td>
</tr>
<tr>
<td>Farm Capital</td>
<td>0.363</td>
</tr>
<tr>
<td>Family and Op. Labour</td>
<td>0.413</td>
</tr>
</tbody>
</table>
APPENDIX 3

The Data Used

The transformed data set used in the study are provided in Tables A.4 and A.5. The variables are identified with the same symbols used in the text (see chapter IV). Table A.4 presents the net output price indexes ($q_1$ to $q_4$), the after tax on-farm and off-farm returns to labour ($\tilde{w}_1$ and $w_{2i}$, respectively), the levels of expenditures ($S_1$ and $S_2$) and farm operator's education ($E$). Table A.5 shows the net outputs per unit of family labour for each of the five net outputs considered.
<table>
<thead>
<tr>
<th>q1</th>
<th>q2</th>
<th>q3</th>
<th>q4</th>
<th>( \frac{\pi}{\gamma} )</th>
<th>( w_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

**TABLE A.4**

Price Indexes of Net Outputs and After Tax Labour Returns, Expenditures and Farm Operator's Schooling Years
<table>
<thead>
<tr>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( q_3 )</th>
<th>( q_4 )</th>
<th>( q_5 )</th>
<th>( v_{21} )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>9.0455</td>
<td>1.4910</td>
<td>12.742</td>
<td>11.758</td>
<td>0.76024</td>
<td>3.6163</td>
<td>542.05</td>
<td>351.18</td>
</tr>
<tr>
<td>60</td>
<td>9.0455</td>
<td>1.4530</td>
<td>12.176</td>
<td>11.752</td>
<td>0.62761</td>
<td>2.7461</td>
<td>413.67</td>
<td>251.35</td>
</tr>
<tr>
<td>64</td>
<td>9.4896</td>
<td>1.4204</td>
<td>11.997</td>
<td>12.401</td>
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APPENDIX 4

Dispersion of the Estimated Elasticities Across the Sample Points

In this appendix the means, standard deviations, minimum and maximum values of the various elasticities when evaluated at the sample points are provided. The purpose of presenting this information is to give an indication of how the calculated elasticities change when they are evaluated at the different sample points.

The symbols used in table A.4 are defined as follows: $E_{L_1}$ is the supply elasticity of on-farm work with respect to $x$, $E_{L_2}W_2$ is the supply elasticity of off-farm work with respect to $w_2$ and in general the $E$ symbols indicate labour supply elasticities, the first subscript represents the dependent variable and the second subscript stands for the independent variable. Similarly, $CS_{Q_1}q_1$, for example, represents the conditional net output, $Q_1$, supply elasticity with respect to the price of output, $q_1$. In general, the subscripts can be interpreted in the same way as the subscripts of the labour supply elasticity.
### TABLE A.6
Mean Elasticities, Standard Deviations, Minimum and Maximum Values

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