MEASURING LIFECYCLE INEQUALITY

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Abstract

In this thesis, theoretically sound and empirically tractable solutions are provided to problems inherent in the traditional practice of measuring inequality in the distribution of annual income. Inequality is taken throughout to mean the extent to which society falls short of a situation in which everyone is equally well-off. The measurement of annual income inequality is inappropriate in this regard because it is consumption, not income, that produces welfare. Furthermore, individual, and therefore social, welfare depends on consumption over the lifecycle, not just in a single year. There are also problems of a less theoretical nature. Measured annual inequality includes an age-related component attributable to the shape of lifecycle income profiles. Annual inequality indices also fail to account for the effects of income mobility.

In response to these problems, two new approaches to the measurement of inequality are proposed. In the welfare approach, an improved index of inequality is sought by replacing annual income with a summary statistic of lifecycle consumption. Lifecycle inequality is then decomposed within and among age-cohorts. Intercohort inequality captures the contribution of economic growth to total inequality, while intracohort inequality is an index of pure interpersonal inequality. The decomposition approach is a compromise between the inadequacy of measuring annual income inequality and the impossibility of measuring lifecycle consumption inequality. Total inequality is measured in panel consumption data treated as a single

distribution, and then decomposed into indices of age-related, mobility-related, and pure interpersonal inequality.

Empirical implementation of the decomposition approach indicates that age-, and especially mobility-related, inequality account for substantial portions of total measured inequality. Sensitivity tests of the decomposition approach indicate that it is a robust method of measuring inequality.

Finally, the decomposition approach is applied to the problem of measuring the trend of inequality, widely observed to have been remarkably constant in the post-War period. Although the trend of measured annual inequality is constant, lifecycle inequality as measured using the decomposition approach declines over the sample period.

The principal finding of this thesis is that the decomposition approach to the measurement of inequality is essential for an accurate assessment of the level and trend of pure interpersonal inequality.

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CHAPTER ONE

Introduction

Measuring inequality in the distribution of annual income was early established as a theoretical and empirical norm. This approach has survived despite considerable evidence that annual income inequality is a poor index of the extent to which society falls short of a situation in which everyone is equally well-off. In empirical work, for example, it was discovered that the Gini coefficient is sensitive to the length of the income accounting period. Income mobility, the tendency for individuals' relative positions in a distribution to change over time, works to reduce the dispersion of incomes cumulated over several years. Since the choice of an accounting period is largely arbitrary, what ethical content might otherwise be imputed to the Gini coefficient of annual incomes is effectively destroyed.

In a similar vein, it was recognized that measured annual inequality reflects not only income differences within a population, but also its age-structure. Lifecycle profiles estimated from cross-sectional data show that income varies systematically with age, tending to rise at a decreasing rate over the working years, eventually leveling off and declining somewhat after retirement. Young people and seniors therefore predominate in the low income portions of the distribution while individuals in the prime of working life are among the majority of high income receivers. Measured annual income inequality thus includes an age-related component; a change in its value may as easily be the result of a demographic change such as the maturing of a "baby boom" generation, as a tendency for the rich to get richer at the expense of the poor.

Theoretical criticisms of the traditional practice of measuring inequality in the distribution of income have also been raised. Ideally, inequality should be measured in the distribution of welfare. A problem arises, however, because individual utility functions are known only up to a monotonic transformation. Measured inequality in the distribution of the images of individual utility functions depends on the particular functional representation of preferences, and is therefore not unique.[1] The impossibility of measuring inequality in the distribution of welfare led to its being approximated by inequality measured in the distribution of income. Only recently have objections been raised against this practice on the grounds that the distribution of welfare is more closely related to the distribution of consumption than of income. Inequality would be more accurately approximated, it has been argued, if it were measured in the distribution of consumption. Furthermore, the importance, for considerations of welfare, of lifecycle consumption has been stressed, raising further questions. For example, what summary statistic of lifecycle consumption is appropriate for use as one of the arguments in an inequality

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[1] The situation is akin to the problem pointed out by Atkinson [1970] that Dalton's [1920] measure of inequality is not invariant with respect to linear transformations of individual utility functions.

index? And how should the effects of economic growth, which puts the lifecycle consumption prospects of young people considerably above those of their elders, be taken into account in the measurement of inequality?

These are some of the problems with the theory and practice of inequality measurement which have stimulated the present research. In this introductory chapter a detailed analysis of these problems is provided within the context of a review of the literature on the measurement of inequality. This leads to the development of two new approaches to the measurement of inequality. In the welfare approach, presented in Chapter Two, summary statistics of lifecycle consumption profiles replace annual incomes as the arguments of an inequality index. While theoretically sound, this approach turns out to be impractical, and an empirically tractable alternative is presented in Chapter Three. In the decomposition approach total inequality is measured in panel consumption data and decomposed into three components, one of which may be interpreted as an index of pure interpersonal inequality. Empirical results for decomposition approach indices of inequality are presented in Chapter Four. Annual inequality indices computed from the same data set are also reported in order to evaluate and compare the performances of these two types of inequality indices. In addition, the robustness of the decomposition approach is investigated by examining the sensitivity of the computed indices to changes in the specification of certain important variables. Finally, in Chapter Five, I apply the decomposition approach to the problem of analysing the trend of lifecycle inequality. I summarize the results of my research and draw some conclusions from it in Chapter Six.[2]

The distribution of annual income has long been the centre of attention in both the theory and practice of measuring inequality. Lorenz [1905] and Gini [1912], for example, proposed methods of portraying and measuring inequality in income distributions that are still the best known and most popular techniques of inequality measurement. Dalton's [1920] pioneering theoretical work on "The Measurement of the Inequality of Incomes" provided the first insights into the social welfare foundations of the subject. Based on these influential precedents, empirical studies of inequality have concentrated on the distribution of annual income. This tradition is continued in the modern theory of inequality measurement, due primarily to Atkinson [1970], Kolm [1969], and Sen [1973].

Theoretical interest in the personal distribution of income stems in part from the classical economists' interest in the distribution of factor shares and the associated neoclassical marginal productivity theory of distribution, which are concerned with the distribution of total product or, in monetary terms, income. The predominance of the income distribution in empirical studies of inequality is largely the result of the relative availability of annual income data. The tradition of

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[2]There are two appendices attached to this thesis. In Appendix A, an alternative procedure for decomposing inequality within and among population subgroups is evaluated and compared to the one that I have employed in the decomposition approach to the measurement of inequality. A method of approximating the degree of lifetime inequality using annual data is compared to the decomposition approach in Appendix B.

measuring inequality in the distribution of annual income may thus be said to have been born of a marriage of empirical pragmatism and theoretical rationale.

Measuring inequality in the annual income distribution, however, is both theoretically and methodologically incorrect. Briefly, the methodological problems concern the failure of measured annual income inequality to take account of intertemporal and intergenerational aspects of inequality that should be distinguished from purely interpersonal inequality. In this sense, simply measuring annual income inequality is incomplete. A theoretical problem arises because the correct interpretation of inequality is the extent to which individuals in society are not equally well-off, which implies that measuring inequality in the distribution of annual income is misspecified. I will discuss these problems and what has been written about them in turn, before drawing some conclusions about how inequality should properly be measured.

Early empirical work indicated that traditional indices of income inequality are not independent of the length of the accounting period. Hanna [1948], for example, found that the incomes of a sample of Wisconsin taxpayers became more equally distributed when measured over a longer accounting period (i.e. the Lorenz curve of incomes measured over two years lay everywhere inside the average of the two Lorenz curves of annual incomes). Soltow [1965] and Kohen, Parnes, and Shea [1975] also report an inverse relationship between the Gini coefficient of incomes and the length of the accounting period. This phenomenon is the result of changes in individuals' relative positions in the income distribution over time, or income mobility, and has recently been studied in depth by Shorrocks [1978a,b].

> Those occupying the highest and lowest positions in the income hierarchy rarely remain there forever. So the aggregation of incomes over time tends to improve the relative position of those temporarily found at the bottom of the distribution, and the situation of those at the top tends to deteriorate. For this reason it is commonly supposed that inequality falls as the accounting period is lengthened. . . . (T)he little evidence available agrees with expectations (Shorrocks [1978a, p. 377]).

Measured annual income inequality thus includes a mobility-related component which should be distinguished from pure interpersonal inequality. The social significance of the degree of pure interpersonal inequality is thus overstated by measured annual income inequality.

The severity of the error inherent in measured annual income inequality depends, of course, on the degree of income mobility.

> If the income structure exhibits little mobility, relative incomes will be left more or less unaltered over time and there will be no pronounced egalitarian trend as the measurement period increases. In contrast, inequality may be expected to decrease significantly in a very (income) mobile society (Shorrocks [1978a, p. 377]).

The available evidence indicates that income mobility and mobility-related inequality are substantial. Schiller [1977] found that the United States is characterized by a very high degree of relative earnings mobility.[3] Shorrocks [1978b] quantified the effect of income mobility by charting the inverse relationship between measured inequality and the length of the

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[3]However, the subjective nature of his analysis and absence of quantitative results impair the validity of his conclusion.

accounting period. Although the results were found to be sensitive to the choice of inequality index and age, declines in measured inequality of 5 to 52 per cent compared to annual income occurred in inequality of family incomes aggregated over nine years. My own estimates, reported in Chapter Four, indicate that mobility-related inequality accounts for 21 to 39 per cent of the total.[4] It would thus seem that the failure to account for inequality attributable to income mobility represents a serious problem that has not yet been adequately solved.

Shorrocks [1978a, b] has provided the best attempt to date to deal quantitatively with the effect of income mobility on measured inequality. His suggestion is to exploit the relationship between income inequality and mobility to construct an index of mobility that reflects the extent to which incomes are equalized as the accounting period is lengthened. More specifically, he first proves that, for a large class of inequality indices which are convex functions of relative incomes and mean independent, inequality of incomes aggregated over a number of years cannot exceed a weighted average of annual income inequality, where the weights equal the proportions of aggregate income received in each year (Shorrocks [1978a], Theorem 1). The ratio of aggregate income inequality to average annual inequality is therefore bounded above by unity, which represents a situation of complete income immobility or constant relative incomes over time. Shorrocks calls this an

[4]This range reflects only the choice of inequality index (or, more specifically, the degree of inequality aversion) and would be wider still if the results were disaggregated by age.

index of income rigidity. He then defines an index of income mobility as the difference between unity and the value of the rigidity index.

Shorrocks suggests that the rigidity index be computed over a two year period, then a three year period, and so on up to the maximum number of years for which data are available.[5] Rigidity curves, showing the relationship between the value of the rigidity index and the number of years of data used to compute it, can then be plotted. The rigidity curve of a completely income immobile society will be a horizontal line, since the value of a mean independent inequality index is invariant with respect to the length of the income accounting period when relative incomes are constant (Shorrocks [1978a], Theorem 2). Income mobility will cause the value of the rigidity index to decline as the accounting period is lengthened, and the shape of the associated rigidity curve thus reflects the degree of mobility. The rigidity curve of a society in which there is little income mobility will decline only slightly and lie close to the horizontal reference line, while a more income mobile society will be characterized by a more sharply declining rigidity curve.

The shape of a rigidity curve reveals not only the degree of income mobility, but may also indicate something of the nature of the fluctuations in individual incomes over time.

For example, suppose we were to compare two groups,

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[5]Since individual incomes must be aggregated to calculate long period inequality in the numerator of the rigidity index, longitudinal data are required.

> one of which had large variations in transitory income, whilst the other experienced substantial changes in permanent incomes (but small transitory changes). Year-to-year income variations might appear to be rather similar. Yet their "rigidity curves" may be expected to be radically different. If income changes are purely due to transitory effects, relative incomes will rapidly approach their permanent values and there will then be no substantial further equalization. The rigidity curve will therefore tend to become horizontal after the first few years. This contrasts with the group with more mobility in permanent incomes, whose rigidity curves will continue to decline as the aggregation period is extended. . . . (C)alculating values of R (the rigidity index) over different aggregation periods may thus be all that is required to make the important distinction between these alternative types of income variations. (Shorrocks [1978a, p. 389])

Shorrocks [1978b] exploits this feature of rigidity curves in his empirical analysis of income stability in the United States to conclude that transitory income fluctuations predominate among the younger members of society (the 20 to 29, and especially the under 20, age groups) and among low income earning females into the middle age groups (Shorrocks [1978b, pp. 19-21]). The continual decline, over the nine year sample period, of the rigidity curves of middle aged men (aged 30-59) and all seniors indicates that income mobility in these groups is of a longer run nature. An important feature of rigidity curves is this ability to portray graphically some of the interesting characteristics of income mobility.

While income mobility is doubtless of intrinsic interest, its study is motivated primarily by the recognition of its effects on inequality: "estimates of the welfare loss due to inequality . . . tend to be biassed upwards if they are computed from short-run (i.e. annual) data" (Shorrocks [1978a, p. 388]). Thus, despite the elegance and appeal of Shorrocks' approach, it does not provide what is most needed, a method of measuring inequality free of the effects of income mobility. He does suggest that, "short run estimates of welfare losses due to inequality can be made consistent with the true long run value by reducing the short run estimate by the factor R (the value of the rigidity index)" (Shorrocks [1978a, p. 388, n. 14]). This is rather _ad hoc_, however, and results in as many estimates of long-run inequality as there are years of data in the sample. In addition, while such indices may account for intertemporal income differences and their effect on measured inequality, they ignore the equally important intergenerational income differences which should also be excluded from measured inequality.[6]

The annual incomes which an individual receives over the course of his life vary with age, giving rise to the characteristically humped shape of lifecycle income profiles. The systematic variation of income with age implies that the income differences observed in an annual income distribution are partly the result of the age-structure of the population. This intergenerational aspect of inequality is captured by indices of annual inequality, which must therefore be taken as overestimates of the degree of pure interpersonal inequality. In the extreme, if lifecycle income profiles were identical across the population, measured annual inequality will be entirely age-

[6]The intertemporal and intergenerational aspects of inequality are both accounted for in my suggested approaches to the measurement of inequality presented in the following two chapters.

related. In general the problem will not be this severe, of course, but the fact remains that inequality measured in the distribution of annual income must exclude the age-related component if it is to be a reliable estimate of pure interpersonal inequality.

This intergenerational aspect of inequality has been recognized by many, including Paglin [1975] who argued that indices of annual income inequality, "combine and hence confuse intrafamily variation of income over the lifecycle with the more pertinent concept of interfamily income variation which underlies our idea of inequality . . . " (p. 598, emphasis in original). Eschewing the use of estimated lifecycle income data or age-specific inequality indices, Paglin suggests that lifetime inequality may be approximated simply by redefining the standard of equality[7] as equality within age-cohorts rather than as equality across the population as a whole.[8]

Paglin's method is first to estimate the mean age-income profile of the population from cross-sectional (annual) data. A Lorenz curve of this distribution reflects the inequality of annual incomes that would exist, given the population age-structure, if everyone traversed the same lifecycle income profile. Paglin employs this "P-reference line" as a new standard of equality to replace the traditional 45° line of

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[7]The distribution with respect to which the social significance of inequality in the actual distribution is measured.

[8]An immediate problem is that the method has been applied only to the Gini coefficient. In Appendix B I have generalized Paglin's technique and compared the results to my own approach and to the use of age-specific indices of annual inequality.

equality; it embodies, "equal lifetime incomes, without the added constraint of a flat age-income profile" (Paglin [1975, pp. 599-600]). The actual distribution of annual income is represented by the usual Lorenz curve. The situation is illustrated in Figure I.[9]

The traditional Gini coefficient is equal to twice the area between the Lorenz curve and the diagonal,[10] and can be seen to be comprised of the sum of two parts. The shaded area between the P-line and the diagonal represents annual income inequality attributable to the mean variation of income with age over the lifecycle. The age-Gini coefficient is equal to twice this area. The unshaded area between the P-line and the Lorenz curve reflects annual income inequality excluding age-related inequality; it is measured by the Paglin-Gini which is equal to the difference between the Lorenz-Gini and the age-Gini. The Lorenz-Gini was found to overstate long-run interfamily inequality as measured by the Paglin-Gini by as much as 50 per cent in sample data. Furthermore, the trend of the Paglin-Gini revealed a 23 per cent decline in inequality in the post-war period, in sharp contrast to, "the widely accepted conclusion that there has been no significant reduction of inequality from 1947 to 1972" (Paglin [1975, p. 603]).

It will surely be agreed that Paglin has addressed an

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[9]All Figures and Tables appear at the end of the chapter.

[10]The Gini coefficient is defined as the ratio of the area between the Lorenz curve and the diagonal to the total area below the diagonal, to which the definition in the text is equivalent since the area of the square is unity.

important and difficult problem. Given the lack of observed lifecycle income data, the especial importance of Paglin's contribution lies in "reconstructing the reference line of equality to match the excellent annual income data at our disposal" (p. 599). The Paglin-Gini has, nevertheless, been subject to considerable criticism on a number of counts.

At least two authors have argued that Paglin's disaggregation of the Gini coefficient is incorrect. The usual Gini coefficient measures inequality with respect to an optimal situation in which everyone receives the population-wide mean income. Wertz [1979], accepting Paglin's argument that the optimal income should be the age-group mean, proposed an adjusted Gini coefficient which, unlike the Paglin-Gini, follows the logic of the Gini coefficient in its construction. The adjusted Gini coefficient suggested by Wertz measures non-age-related inequality with reference to the 45° line of equality. The Paglin-Gini, on the other hand, compares the Lorenz curve of the actual distribution to a redefined reference line of equality, the P-line. Paglin concedes that neither method is intrinsically superior but argues in favour of the Paglin-Gini on the grounds: (1) that the Lorenz curve corresponding to the adjusted Gini coefficient can dip below the base line of the Lorenz diagram into the negative income quadrant, and (2) that Wertz's adjusted Gini coefficient implicitly assumes zero intracohort income mobility, and thus tends to overestimate lifetime inequality. The Paglin-Gini, though not explicitly accounting for the effects of income mobility, does a better job than the adjusted Gini coefficient because the Paglin-Gini varies inversely with the mean income difference between cohorts, which is positively correlated with income mobility (Paglin [1979, p. 676]).

A second criticism of the Paglin-Gini along similar lines was made by Nelson [1977], who argued that Paglin implicitly assumed that age-group income distributions do not overlap (as would be true, for example, if families were grouped by income bracket). The difference between the Lorenz-Gini and the age-Gini calculated under this assumption, is an index of pure interpersonal inequality plus an interaction term. The degree of non-age-related inequality is thus overestimated by the Paglin-Gini according to Nelson. Paglin supports his inclusion of the interaction effect in the intra-age-group component of inequality with an argument of Battacharya and Mahalonobis [1967, p. 150]: "(a)ssuming that the means of the groups are given, it is reasonable to postulate that the between-groups component should not change simply because of the degree of within group variation."

> It follows that the between groups component in the general case is the same as the between groups component in the special case where within group variation is zero for every group. Battacharya and Mahalonobis conclude that while one cannot directly draw up a concentration curve of overall within group inequality, as one can for the between group differences, the area between the latter curve and the L-curve 'indicates the effect of within groups disparities.' (Paglin [1977, pp. 520-21])

Paglin would seem to be secure on these quite defensible grounds.

Nelson also argues, however, that the Paglin-Gini is not a pure intracohort inequality measure because it depends on cohort population and income shares as well as on inequality within cohorts. Danziger, Haveman, and Smolensky [1977] also advanced this argument in their critique of the Paglin-Gini. They investigated the contributions of intracohort inequality, cohort population shares, and cohort income shares to the trend of pure interpersonal inequality, and found that, "while all three sources contributed to the increase in inequality from 1965 to 1972, two of the sources operated to decrease the Paglin-Gini. Ironically only the changes in cohort-specific Gini coefficients contributed to the increase in Paglin-inequality over this period" (Danziger, Haveman, and Smolensky [1977, p. 508]). The problem is that the Paglin-Gini is computed by subtracting the age-Gini, which is not independent of cohort population and income shares, from the Lorenz-Gini; it is therefore sensitive to changes in these variables. More importantly, Paglin's major finding that the Paglin-Gini declines over time is seen to be the result of the trends of cohort population and income shares. Inequality within groups operated to increase Paglin inequality.

This and related problems of the Paglin-Gini stem primarily from Paglin's use of the actual cross-sectional lifecycle income profile as the basis for correcting the Gini coefficient for age-related income differences. "An inequality measure which allows for lifecycle variations is appealing. However, such a standard requires an explicit judgement on the optimum lifecycle pattern, and relying on annual observations of an arbitrarily observed pattern is unsatisfactory" (Danziger, Haveman, and Smolensky [1977, p. 512]). The Paglin-Gini's lack of any normative underpinnings is its most serious drawback. I wish briefly to discuss other criticisms of the Paglin-Gini before returning to this point.

Several writers have argued that the Paglin-Gini estimates of inequality are too low. Johnson [1977] used a simple model of income distribution to demonstrate this result. Nelson [1977] and Formby and Seaks [1980] have argued that the Paglin-Gini's underestimation of intracohort inequality results from the fact that it is not normalized to range over a [0,1] interval.

Paglin has also been faulted by Danziger, Haveman, and Smolensky [1977] for his use of full family money income in the computation of the Lorenz-, age-, and Paglin-Gini coefficients.

> (I)mplicit in Paglin's framework is a criterion for judging the effectiveness of income transfers, if the objective is to reduce inequality. An income transfer is 'Paglin-efficient' only if it reduces the variation of incomes within an age-cohort; transfers which involve intercohort redistribution are by definition 'Paglin-inefficient.' . . . In this context, it should be noted that, as calculated, the age- and Paglin-Gini coefficients incorporate transfers which are by definition Paglin-inefficient, since Paglin's income concept is census money income. . . . Consequently, Paglin's income profiles are based on an inappropriate definition of income which biases his conclusions on the trend of functional inequality in the post war period. (pp. 510-11)

In this vein Minarik [1977] reports that the trend of earned income inequality is considerably different from that of total family money income. He finds that the Lorenz-Gini rises by 8 per cent and the Paglin-Gini by 2 per cent over the period 1967-1974.

It has also been suggested that measured annual income inequality should be corrected for other factors in addition to the age-structure of the population. Minarik [1977], for example, found that, "while the Paglin-Gini, using the age-income profile for a base, finds a 2 per cent <u>decrease</u> in inequality, the adjusted Paglin-Gini, based on separate age-income profiles for groups with different schooling attainments finds a 2 per cent <u>increase</u> in inequality" (p. 515). The question here is which factors should be included in, and which excluded from, an inequality index.

> Paglin's purpose is to partition the area between the 45° line and the Lorenz curve into two parts: that inequality which <u>to him</u> is economically functional and, hence, of no concern for public policy, and the remaining . . . non-functional or policy-relevant inequality. Functional inequality in this instance reflects society's needs for varying income over the lifecycle as well as other basic facts relating to productivity, investment in human resources, and the work-leisure preferences of households, but only in an average way, insofar as these factors express themselves through the age variable. (Danziger, Haveman, and Smolensky [1977, pp. 505-6])

Kurien [1977] criticizes the Paglin-Gini along similar lines, and concludes that, "an ideal measure of income distribution will eliminate all choice-related variation (in incomes), but none of the differential opportunity-related variation" (p. 518).

The Paglin-Gini does not fail irretrievably as a result of any of the arguments just reviewed. The correct disaggregation could be derived, the resulting index could be normalized to range over a [0,1] interval, an appropriate income variable definition could be chosen, and decisions could be reached on the social significance of various sources of inequality. A more serious problem, however, which was mentioned earlier but deferred momentarily, remains. It involves Paglin's approach to the problem of separating age-related income differences from measured inequality. He has chosen to redefine the standard of equality to reflect, "equal lifetime incomes, but not the added constraint of a flat age-income profile" (Paglin [1975, p. 600]). This in itself is perfectly acceptable; it could be justified, and would probably be widely accepted, on vertical equity grounds alone. Equality "across-the-board" may not, indeed, be the answer to how income ought to be distributed. But clearly the question is a normative one, and herein lies the problem with Paglin's formulation.

> The P-reference line . . . is a normatively empty box, devoid of any ethical content. (It) confuses the peaked age-income profile thrown out by the market with the normative question of how income ought to be distributed. There is no ethical content to the prescription that the young and elderly ought to have low incomes because, on average, they do have low incomes. A meaningless age-Gini subtracted from the Lorenz-Gini results in a meaningless Paglin-Gini. (Gillespie [1979, p. 563])

An obvious solution to this problem is to replace the P-reference line with a normatively-based standard of equality which incorporates ethical consideration of how lifecycle income ought to be distributed. This may not be an easy task, however, and it would seem simpler to retain the usual ethical standard of equality, but to disaggregate inequality by source. In the case of age-related inequality, this would involve decomposing inequality within and among age-cohorts. Paglin in fact considered this alternative but found the use of age-specific inequality indices unsatisfactory because

> the empirical coefficients available are not really specific by age of family head but in fact represent broad age groups. This introduces spurious income variance by not fully eliminating the effect of the age-income profile. However, even if we had truly age-specific Gini, we would have the problem of weighting and combining fifty-some measures into one coefficient. (Paglin [1975, p. 602])

However, procedures for decomposing inequality within and among population subgroups have been proposed by Blackorby, Donaldson, and Auersperg [1981] and Shorrocks [1980], among others, and there is no reason to prevent age-groups being defined on an annual basis rather than by brackets including more than a single year.

There is a genuine problem with this suggestion, however, but it is not specific to the decomposition of inequality. It applies equally to Paglin's method, and is in fact inherent in every measure of annual income inequality because they all fail to account for the effects of income mobility.

> Mobility reduces the dispersion of lifetime incomes much below the annual income estimate. . . . While the P-Gini adjusts for average age-related inequality it also fails to catch the accompanying intracohort mobility. Until we are able to modify our static inequality coefficients by an index of mobility or collect more longitudinal household income data for an extended period of time, our estimate of inequality of lifetime incomes (or the more difficult trend of the inequality of lifetime incomes) will remain crude. (Paglin [1977, p. 527])

It would seem then that even Paglin agrees that the Paglin-Gini is a stop-gap measure for use when panel data are not available. But Paglin finds fault with this practice too, on the grounds that economic growth renders lifetime income equality an unreasonable and unattainable goal. Paglin did not suggest a Gini coefficient of lifetime income inequality based on the observed growth of real income over time. But since economic growth causes the lifetime incomes of currently young members of society to exceed the lifetime incomes of their elders, the appropriate solution would again seem to be the decomposition of lifetime income inequality within and among age-cohorts.

To recapitulate, inequality attributable to intertemporal and intergenerational income differences is confused with interpersonal inequality in indices computed from annual data. Paglin has proposed a method of excluding age-related income differences from the Gini coefficient of annual incomes, but the Paglin-Gini has no ethical foundations as a measure of non-age-related inequality. A superior method to distinguish age-related inequality from pure interpersonal inequality is to decompose total inequality within and among age-cohorts. Indices of annual inequality, however, cannot account for the effects of income mobility. Shorrocks has suggested adjusting indices of annual inequality to approximate long-run inequality. It is preferable, however, to compute long-run inequality directly from longitudinal data. To capture fully the effects of income mobility and the shape of age-income profiles requires that inequality be measured in the distribution of lifetime income. It is important to recognize that the intertemporal and intergenerational aspects of inequality do not disappear when a lifecycle perspective is adopted; they each appear in a different guise. The intergenerational problem, as was noted by Paglin, is the result of economic growth which causes the lifetime incomes of younger members of the current population to exceed those of elder members. There is thus reason to decompose lifetime income inequality within and among age-cohorts. The intertemporal problem in measuring inequality of lifetime incomes is to choose an appropriate summary statistic of lifecycle income for the purpose of measuring inequality. This is intimately related to, and will be discussed in the context of, the theoretical difficulty with measuring inequality in the distribution of income, which I take up next.

The distribution of income monopolized the attention of economists interested in distributional issues until very recently. As I suggested at the outset of this chapter, this was likely the result of a superabundance of income data and a view of inequality as the degree to which the total product of the economy is not equally shared among the population. Although not unreasonable grounds on which to justify measuring inequality in the distribution of income, its dominance in theory and practice seems curious in light of the welfare foundations of inequality measurement, originally established by Dalton [1920].

> An American writer has expressed the view that "the statistical problem before the economist in determining upon a measure of the inequality in the distribution of wealth is identical with that of the biologist in determining upon a measure of the inequality in the distribution of any physical characteristic." But this is clearly wrong. For the economist is primarily interested, not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare, which may be derived from income. (p. 348)

In this view, inequality is interpreted as the degree to which individuals in society are not equally well-off. The measurement of inequality thus involves a social evaluation of the distribution of individual welfare or utility. Dalton suggested that inequality be defined as the ratio of total welfare attainable under an equal distribution to total welfare attained under the actual distribution. Recognizing the difficulties of measuring welfare, however, Dalton argued that, "inequality, . . . though it may be <u>defined</u> in terms of economic welfare, must be <u>measured</u> in terms of income" (Dalton [1920, p. 349], emphasis in original). But welfare is derived from income only through consumption, so that there would seem to be something missing from Dalton's analysis of the problem.[11] Inequality, though defined in terms of welfare, must be measured in terms of consumption.

The significance for the measurement of inequality of the link from income through consumption to welfare was not fully appreciated until quite recently. Bentzel [1970] was the first to argue that "it is . . . this income-consumption-welfare nexus which is the reason for the great interest in the income distribution" (p. 254). That the observed inequality of incomes is not so much of intrinsic interest as it is an estimate of inequality in the distribution of well-being raises the question of how accurately the former can be expected to approximate the latter.

> For if it is . . . the distribution of welfare that is the relevant concept in political discussion, the economists' empirical analyses of income distributions will be of interest only on the assumption that there is a fairly close connection between this distribution and the corresponding welfare distribution. (Bentzel [1970, p. 254])

With this in mind, Bentzel examined the relationships between the distributions of income, consumption, and welfare with regard to the measurement of inequality. He identified three reasons for dissimilarities between the distributions of income

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[11] As he must have recognized and, indeed, hinted at: "We have to deal, therefore, not merely with one variable, but with two, or possibly more, between which certain functional relations may be presumed to exist" (p. 348, emphasis added). Dalton's injunction to measure inequality in terms of income is correct only if the functional relation between welfare and income incorporates the relationships of welfare to consumption and consumption to income.

and consumption: saving and dissaving, consumption expenditure not out of own income, and the fact that the purchasing power of incomes varies with the price level. However significant such effects might be -- the consumption distribution generally displays considerably less inequality than the distribution of income -- they pale in light of the difficulty of translating changes in the consumption distribution into their effects on the distribution of welfare. Recently observed demographic phenomena such as the "graying of society" and the increasing number of working women cause income, and to a lesser extent consumption, inequality to rise, but it is considerably more difficult to say what are their effects on the distribution of welfare. Perhaps the most difficult problem of all is accounting for, and determining the welfare effects of, public consumption. Based on his analysis of the distributions of income, consumption, and welfare, Bentzel is forced to a pessimistic conclusion regarding the prospects for learning much about the distribution of well-being from an examination of the income distribution. The situation could be improved significantly by measuring inequality in the distribution of consumption.

Interestingly, the importance of shifting attention from income to consumption for the purpose of measuring inequality is tied in with the need to extend the temporal dimension of the analysis in order to estimate inequality more accurately. Nowhere has this point been made more clearly than in the theory of consumer behaviour. Both Friedman's [1957] theory of permanent income and the lifecycle hypothesis of Modigliani and Brumberg [1954] view individual welfare as a function of lifecycle consumption which depends in turn on lifetime income.

> (T)here need not be any close and simple relation between consumption in a given short period and income in the same period. The rate of consumption in any given period is a facet of a plan which extends over . . . the individual's life, while the income accruing within the same period is but one element which contributes to the shaping of such a plan. (Modigliani and Brumberg [1954, p. 391])

The implications for the measurement of inequality have been emphasized by Friedman: "the existence of large negative savings is a symptom that the observed inequality of measured income overstates substantially the inequality of permanent income" ([1954, p. 40]).

Recent studies of inequality have thus focussed on the distribution of lifecycle consumption rather than annual income (e.g. Nordhaus [1973], Blinder [1975], and Irvine [1980]). The intertemporal and intergenerational aspects of annual income inequality, which were earlier discussed at length, reappear in different forms in the measurement of lifecycle consumption inequality. For example, the intertemporal problem is to decide upon a summary statistic of lifecycle consumption suitable for the purpose of measuring inequality. Several have been suggested in the context of measuring lifetime <u>income</u> inequality, analogues of which might be considered as possible candidates. Summers [1956] estimated individual lifetime earnings and found <u>average lifetime income</u> to be more equally distributed than annual income. Weisbrod and Hansen [1968] suggested an <u>income-net worth</u> measure of economic welfare, equal to current income plus the lifetime annuity equivalent of current net worth. Lillard [1977] measured inequality in the distribution of <u>human wealth</u> defined as the discounted present value of lifetime earnings. All of these overlook Dalton's injunction, however, that it is the welfare effects of income which are of interest in the measurement of inequality. The discounted present value of lifetime income, or its annuity equivalent, reflect the magnitude and timing of the income an individual receives over the course of his life, but not its significance in terms of economic welfare.[12]

Measuring inequality of lifecycle consumption thus requires a welfare equivalent summary statistic of the lifecycle profile, such as <u>utility equivalent annuity income</u>, suggested and employed by Nordhaus [1973] or <u>lifetime wealth</u>, proposed by Pissarides [1978]. These are, respectively, the lifetime annuity and the corresponding discounted present value that provide the same utility as the individual's chosen consumption plan. These ideas have been the subject of a recent paper by Cowell [1979], who was the first to recognize the importance of capital market conditions. His welfare equivalent summary statistics of lifecycle consumption, "wergild" and the associated "wergild annuity", are defined in terms of actual capital market conditions.[13] I follow this practice in the welfare approach to the measurement of inequality presented in the next chapter.

The intergenerational aspect of lifecycle inequality arises

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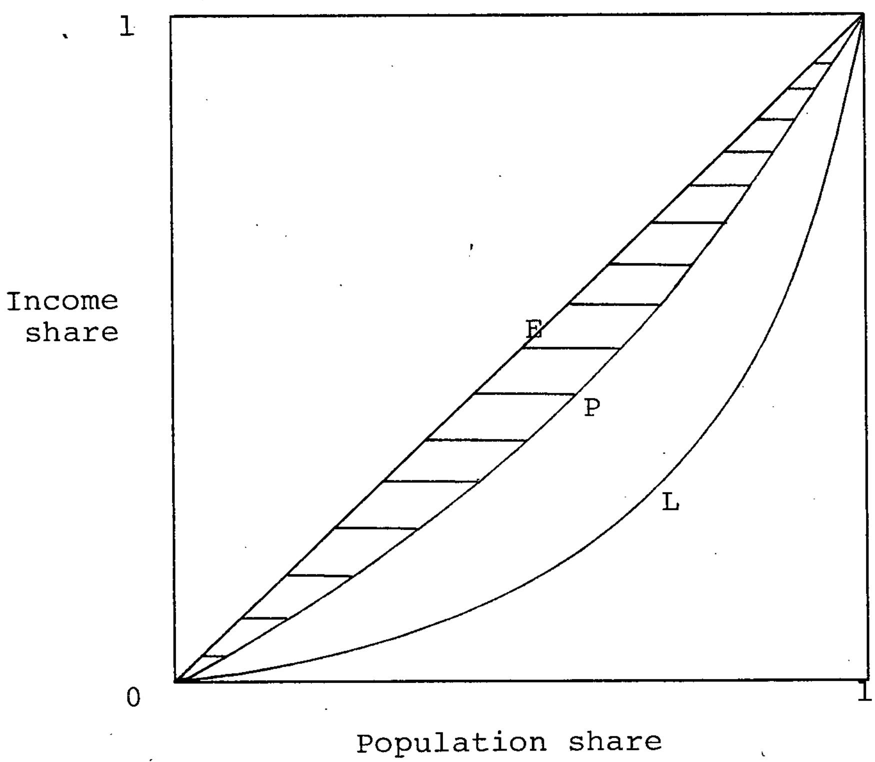
[12]Two income profiles with equal discounted present values, but differently distributed over time, will not yield equal utility to an individual without access to perfect capital markets.

[13]Recent applied work on the measurement of lifecycle inequality (e.g. Nordhaus [1973] and Irvine [1980]) has focussed exclusively on consumption plans chosen by consumers facing perfect capital markets.

because real economic growth causes consumption profiles to shift up over time. This will be reflected in the values of the wergild annuities, which will tend to be greater for later born individuals. This intergenerational inequality should be distinguished from pure interpersonal inequality, and in the welfare approach this is accomplished by decomposing lifecycle inequality within and among age-cohorts.

FIGURE I

Cumulative Income Distribution Showing Age-Related Inequality



CHAPTER TWO

A Welfare Approach to the Measurement of Inequality

The modern theory of inequality measurement, attributable primarily to Atkinson [1970], Kolm [1969], and Sen [1973], attempts to provide a sound basis for evaluating the social significance of inequality. Their work represents the most significant theoretical contribution since Dalton's [1920] pioneering article and has kindled a burst of theoretical and empirical work on inequality in the past decade. Nevertheless, their framework could be improved on a number of counts so as to strengthen its welfare underpinnings.

The major part of the work on inequality focusses on the annual income distribution. A number of writers have argued that this practice is essentially misguided, however, on the grounds that an individual's economic welfare is reflected in his consumption rather than his income. Furthermore, an accurate assessment of economic position depends on consumption levels throughout life; restricting attention to a single year tends to produce a misleading indication of well-being.

While some work on the theory and measurement of inequality has proceeded along these lines of late, it has been plagued by an apparent confusion. Inequality, whether in the distribution of annual income or lifecycle consumption, is measured in the <u>actual</u> distribution with respect to an equally distributed alternative. Attempts to date to measure lifecycle consumption inequality have uniformly assumed, however, that a unique, constant rate of interest prevails in the market for saving and borrowing. Inequality is measured in this distribution of consumption plans chosen under optimal capital market conditions with respect to an equal distribution with the same mean. This situation clearly is not representative of the actual capital market conditions under which consumption plans are chosen. Individuals face a variety of means of reallocating their income, with associated rates of interest, and differential rates for borrowing and lending. It is these actual capital market conditions which underlie observed consumption over time and which should be implicit in the measurement of lifecycle consumption inequality.

The adoption of a lifecycle perspective on the measurement of inequality introduces a new factor contributing to measured inequality that is absent when attention is limited to annual distributions. Real economic growth over time causes the lifecycle consumption opportunities of a young person to exceed those of someone older. Measured inequality in the distribution of lifecycle consumption must therefore be decomposed within and among age-cohorts so as to distinguish pure interpersonal inequality from that due to growth. Two different decomposition procedures are available for this purpose. One is inferior on both theoretical and empirical grounds, as is argued in appendix A. The other is adopted for the decomposition of per capita inequality and Atkinson-Kolm-Sen (AKS) equality indices, while a new decomposition is proposed and adopted for the decomposition of AKS indices of inequality.[14]

The approach I am proposing takes account of consumer choice exercised over a lifecycle planning horizon. It is thus possible to measure the welfare loss attributable not only to the lack of equality among persons but also to the lack of perfect means of intertemporal redistribution. That is, the welfare approach can be extended to measure the welfare loss implied by both interpersonal and intertemporal maldistribution. This is done, and a decomposition of the total is provided so that the two components and their interactive effect can be separately identified.

Dalton was the first to point out that the measurement of inequality is a question of social welfare. He, and later Atkinson and Kolm, suggested indices that measure inequality as the social welfare loss implied by departures from equality.[15] In all of their work, however, utility is made a function of income.[16] Yet economic welfare is generally taken to be a product of consumption. The principal writers on the measurement

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[14]AKS indices measure the percentage of total consumption saved by moving from the actual distribution to an equal distribution that is socially equivalent (i.e. provides the same level of social welfare). Per capita indices measure the total saving from the same move on a per capita basis.

[15]Dalton's index is not invariant with respect to linear transformations of individual utility functions. The contribution of Atkinson and Kolm was to make measured inequality independent of monotonic transformations of individual utility functions through the use of the "equally distributed equivalent" in the construction of inequality indices.

[16]Dalton alone suggested that other variables might have to be taken into account.

of inequality thus seem to have stopped short of the goal of providing a welfare foundation for the theory of inequality measurement. The problems outlined briefly above are evidence of this. In the new approach to the measurement of inequality presented in this chapter a more accurate index of individual welfare is substituted for annual income. Total inequality is decomposed within and among age-cohorts to obtain an index of pure interpersonal inequality.

This chapter begins with a discussion of the desiderata of the modern theory of inequality measurement founded by Atkinson [1970], Kolm [1969], and Sen [1973]. I then present, in several steps, a thoroughly consistent welfare approach to the measurement of inequality. Beginning with individual utility functions and lifecycle consumption profiles, I define representative lifecycle consumption as the consumption annuity that provides the individual with the same level of utility as the consumption plan that it represents. The social evaluation function is defined over these representative lifecycle consumptions and is used to derive the (population-wide) equally distributed equivalent consumption.[17] The decomposition of inequality within and among population age-groups requires that an equally distributed equivalent consumption be defined for each age-cohort. This in turn requires some separability in the social evaluation function which restricts the class of

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[17]The equally distributed equivalent of a given distribution is defined by Atkinson [1970, p. 250] as, "the level of income (consumption) per head which if equally distributed would give the same level of social welfare as the present distribution."

admissible social evaluation functions and inequality indices to certain additively separable functions. Both AKS and per capita inequality indices bear interpretation as the social saving which could be realized by moving from one distribution (of representative lifecycle consumption) to a less unequal one which is socially equivalent. The decomposition of inequality implies that inequality is eliminated in two stages: first within and then between cohorts. The individual, age-group, and population-wide equally distributed equivalents are used to define these hypothetical distributions from which AKS and per capita indices of inequality are computed. Inequality within age-groups is taken as an index of pure interpersonal inequality, while the inter-age-cohort component of the decomposition represents inequality attributable to economic growth. The decomposition procedure for indices of per capita inequality and AKS equality is due to Blackorby, Donaldson, and Auersperg [1981]. Their decomposition of AKS inequality, however, suffers from several problems, and I suggest and employ a new procedure for decomposing AKS inequality indices that is free of these problems. Having laid out the welfare approach to the measurement of interpersonal inequality, I then consider its extension to include measurement of the social significance of intertemporal maldistribution. This involves the evaluation of actual lifecycle consumption profiles with respect to a hypothetical situation in which consumption plans are arranged through perfect capital markets with a unique rate of interest. Inequality indices in the extended welfare approach can be decomposed into indices of interpersonal and "intertemporal" inequality, plus a third term which reflects the interdependency between them. Finally, the chapter closes with a discussion of the strengths and weaknesses of the welfare approach to the measurement of inequality.

I begin with lifecycle consumption data on H individuals, and posit the existence of individual intertemporal utility functions $U^h:R^{T_h}\rightarrow R^1$ with image

$$u_h=U^h(c_h) \qquad (1\leq h\leq H) \tag{2.1}$$

where $c_h=(c_{h1}, \ldots, c_{hT_h})$ is the consumption plan of person h over the T_h years of his life. Each utility function is assumed to be continuous, increasing, and quasi-concave. An individual's observed consumption plan is chosen to maximize (2.1) subject to an intertemporal budget constraint which reflects his actual lifetime opportunities for saving and borrowing.

My objective is to evaluate the social significance of differences in lifecycle consumption plans among individuals. This requires both a summary statistic of lifecycle consumption and a social evaluation function defined in terms of that summary statistic. While individual utility may seem the obvious candidate for this purpose, it is in fact unacceptable since individual utility functions are known only up to a monotonically increasing transformation. If the images of individual utility functions were used as the arguments of an inequality index, measured inequality would depend upon the particular transformation which is chosen. It would thus be possible to change the degree of inequality simply by applying a monotonically increasing transformation to individual utility functions.[18]

This problem is solved by the use of Cowell's [1979] equally distributed equivalent summary statistic of lifecycle consumption, which I call representative lifecycle consumption. It is defined as the lifecycle consumption annuity which provides the same level of utility to the individual as the consumption profile which it represents. It is invariant with respect to transformations of the utility function, and is implicitly defined by,

$$(2.2) \qquad U^h(r_h \underline{1}_{T_h}) = U^h(c_h) \qquad (1 \leq h \leq H)$$

where $\underline{1}_{T_h}$ is a unit vector of dimension T_h. The properties of $U^h(.)$ ensure that representative consumption, r_h, is unique and well-defined for every possible consumption plan, c_h. (2.2) can therefore be written as,

$$(2.3) \qquad r_h = R^h(c_h) \qquad 1 \leq h \leq H)$$

Note that r_h is an exact index of person h's well-being; that is,

$$(2.4) \qquad r_h \geq r_h' \longleftrightarrow U^h(c_h) \geq U^h(c_h') \qquad (1 \leq h \leq H)$$

The social evaluation function $W: R_+^H \rightarrow R^1$ (where R_+^H is the non-negative Euclidean H-orthant) has the image,

$$(2.5) \qquad w = W(r)$$

where $r = (r_1, \ . \ . \ . \ , r_H)$ is a vector of individuals' representative lifecycle consumptions. W(.) is assumed to be

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[18]This was the criticism of Dalton's [1920] measure of inequality which led Atkinson [1970] to propose the use of equally distributed equivalent income in the construction of inequality indices. AKS indices are sensitive to the level from which utility is measured (i.e. are scale independent), while per capita indices will vary with the units in which utility is measured (i.e. are origin independent).

continuous, increasing, and S(chur)-concave.[19] The social evaluation function (2.5) provides the ethical basis for the construction of AKS and per capita indices of inequality. <u>AKS indices measure the percentage of total consumption saved by moving from the actual distribution to an equal distribution that is socially equivalent. Per capita indices measure the total saving from the same move on a per capita basis</u>. AKS and per capita inequality indices take the following forms respectively:

(2.6) $I=1-s/m$

(2.7) $A=m-s$

These indices are constructed using $m=\sum_h (1/H)r_h$, the mean of the vector $r=(r_1, \ldots, r_H)$ of individual representative lifecycle consumptions, and s, the equally distributed equivalent of r, defined implicitly by,

(2.8) $W(s\underline{1}_H)=W(r)$

An individual's representative lifecycle consumption, r_h, reflects both the position and shape of his lifecycle consumption profile (i.e. both the magnitude and distribution of consumption during the course of his lifetime). Thus, even if all consumption profiles had exactly the same shape, the continual shifting upward of their positions, because of real growth of the economy over time, would cause r_h to be larger for the younger members of the population.

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[19] W(.) is S-concave if and only if $W(Br) \geq W(r)$ for all r in the domain of W(.) and for all bistochastic matrices B. W(.) is strictly S-concave if and only if $W(Br)>W(r)$ whenever Br is not a permutation of r. A bistochastic matrix is a square matrix of nonnegative elements whose rows and columns each sum to unity.

While measured inequality captures both intergenerational and intragenerational aspects of differences in individual welfare, it will be desirable to distinguish the former economic-growth-related inequality from the latter pure interpersonal inequality. This requires that individuals be grouped by age-cohort. That is, the population set $N=\{1, \ldots, H\}$ must be partitioned into subgroups by age, $\hat{N}=\{N^1, \ldots, N^K\}$ where N^k is the subset of the population in the kth age-cohort. The social evaluation function must be separable in the partition $\hat{N}$, in which case it can be written in the form,

$$(2.9) \qquad w=\hat{W}(W^1(r^1), \ldots, W^K(r^K))$$

where W(.) is increasing in $W^k(r^k)$ and $r^k=(\langle r_h\rangle \ \forall h \in N^k)$ is the vector of representative consumption statistics of all persons in the kth age-cohort.

The conjunction of separability and symmetry in W(.) imposes considerable structure on the social evaluation function. As Blackorby, Donaldson, and Auersperg [1981, theorem 1] have shown, these conditions imply that W(.) is additively separable; that is,

$$(2.10) \qquad w=\overset{*}{W}\left(\sum_h g(r_h)\right)$$

where W(.) is increasing in its argument and g(.) is identical for all h because of the symmetry assumption. Furthermore, S-concavity of W(.) requires that g(.) be concave; strict S-concavity requires that it be strictly concave (Berge [1963]). Thus W(.) must be quasi-concave and symmetric. In this case it can easily be shown that the equally distributed equivalent representative consumption takes the form, (2.11) $s=S(r)=g^{-1}[(1/H)\sum_h g(r_h)]$

The cohort social evaluation function $W^k:R^{n_k}\rightarrow R^1$ (where n_k is the number of people in cohort k) has the image

(2.12) $w_k=W^k(r^k)$ $(1\leq k\leq K)$

and can be picked to have the properties of W(.).[20] (2.12) can be used to define the equally distributed equivalent of r^k,

(2.13) $W^k(s_k\underline{1}_{n_k})=W^k(r^k)$ $(1\leq k\leq K)$

The properties of $W^k(.)$ ensure that representative cohort consumption, s_k, is unique and well defined for every vector r^k; thus

(2.14) $s_k=S^k(r^k)=g^{-1}[(1/n_k)\sum_{h\in N^k}g(r_h)])$ $(1\leq k\leq K)$

<u>The social evaluation function defined over individual representative lifecycle consumptions must therefore be continuous, increasing, symmetric, quasi-concave, and additively separable.</u> Only then can it provide the welfare basis for the construction of AKS and per capita indices of inequality which are decomposable within and among age-groups of the population. It is generally desirable to go a step further, however, in order to derive relative indices (which are homogeneous of degree zero in their arguments) and absolute indices (which are invariant with respect to equal absolute changes in the values of their arguments).

<u>AKS indices are relative indices if and only if the social evaluation function is homothetic</u>.[21] Thus, relative inequality

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[20]Blackorby, Primont, and Russell [1978].

[21]W(.) is homothetic if and only if it is a monotonically increasing transform of a linearly homogeneous social evaluation function.

indices are based on social evaluation functions which are continuous, increasing, symmetric, quasi-concave, additively separable, and homothetic. The class of social evaluation functions which satisfy these properties for positive representative consumptions[22] are the means of order R,

$$(2.15) \qquad W_R(r)=\overset{*}{W}[\overline{W}_R(r)] \qquad R\leq 1$$

where $\overset{*}{W}(.)$ is increasing and

$$(2.16) \qquad \overline{W}_R(r)=\begin{cases} [(1/H)\sum_h r_h^R]^{1/R} & 0\neq R\leq 1 \\ \prod_h r_h^{1/H} & R=0 \end{cases}$$

R is a free parameter determining the degree of relative inequality aversion.[23] The corresponding relative inequality indices are members of the Atkinson family of indices,

$$(2.17) \qquad I_R(r)=\begin{cases} 1-[(1/H)\sum_h (r_h/m)^R]^{1/R} & 0\neq R\leq 1 \\ 1-\prod_h (r_h/m)^{1/H} & R=0 \end{cases}$$

<u>Per capita indices are absolute indices if and only if the social evaluation function is translatable</u>.[24] Absolute indices are thus based on social evaluation functions which are continuous, increasing, symmetric, quasi-concave, additively

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[22]If the domain of W(.) is the nonnegative orthant R_+^H then we must have $0<R\leq 1$ in (2.16) and (2.17). S-concavity, additive separability, and homotheticity are not possible over R^H except in the degenerate case R=1 (Blackorby and Donaldson [1982], theorem 4).

[23]The degree of relative inequality aversion varies inversely with the value of R. As $R\rightarrow -\infty$ $\overline{W}_R(r)$ and $s=S(r)$ both go maximin; that is, $\overline{W}_R(r)=\min_h\{r_h\}=S(r)$. Thus $I_R(r)=1-\min_h\{r_h\}/m$.

[24]W(r) is translatable if and only if $W(r)=\overset{*}{W}[\widetilde{W}(r)]$ where $\overset{*}{W}(.)$ is increasing in its argument and $\widetilde{W}(r+a\underline{1}_H)=\widetilde{W}(r)+a$, for all r, $r+a\underline{1}$ in the domain of W(.).

separable, and translatable, which restricts W(.) to the Kolm-Pollak (KP) family of social evaluation functions,

(2.18) $W_G(r)=-(1/G)\ln\{(1/H)\sum_h \exp[(-G)r_h]\}$ G>0

and their corresponding absolute inequality indices,

(2.19) $A_G=(1/G)\ln[(1/H)\sum_h \exp\{G(m-r_h)\}]$ G>0

The equally distributed equivalent consumptions for an individual, an age-cohort, and the overall population are defined by (2.3), (2.14), and (2.11), respectively. By constructing reference vectors with these representative consumption statistics as elements, ethical indices of inequality can be derived by computing the social saving which could be realized by moving from one vector to another. AKS indices express this saving as a percentage of the total and per capita indices express it in per capita terms. I will compute the per capita inequality indices and the AKS indices of inequality and equality. The corresponding absolute and relative indices can be found by duplicating this procedure using the equally distributed equivalents corresponding to the social evaluation functions (2.16) and (2.18) respectively.

In order to measure intracohort inequality consider the replacement of the actual distribution of representative consumption,

(2.20) $(r_1, \ldots, r_H)$

by a socially equivalent one in which inequality is eliminated within, but not between age-cohorts. In this situation, each individual receives the equally distributed equivalent of the distribution of representative lifecycle consumption within his cohort:

(2.21) $(s_1 \underline{1}_{n_1}, \ . \ . \ . \ , s_K \underline{1}_{n_K})$

The social saving generated by the move from (2.20) to (2.21) reflects intracohort inequality. If I now replace (2.21) by a socially equivalent equal distribution,

(2.22) $(s \underline{1}_H)$

inequality between cohorts will have been eliminated and the social saving accruing from the move from (2.21) to (2.22) can be used as a measure of intercohort inequality. Notice that the saving which could be realized by moving directly from (2.20) to (2.22) measures total inequality, indicating that it will be possible to aggregate the indices of intra- and intercohort inequality into an index of total inequality.

On a per capita basis, the savings generated by the move from (2.20) to (2.21) measure intracohort per capita inequality:

$$
\begin{aligned}
A_A &= (\sum_h r_h - \sum_k n_k s_k)/H \\
&= \sum_k (n_k/H)(m_k - s_k) \qquad (2.23)
\end{aligned}
$$

where $m_k = (1/n_k)\sum_{h \in N^k} r_h$.[25] The mean social saving which results from the move between (2.21) and (2.22) is,

(2.26) $A_R = \sum_k (n_k/H)s_k - s$

which measures intercohort inequality in per capita terms. It can easily be shown that (2.23) and (2.26) sum to

$$
\begin{aligned}
A &= \sum_k (n_k/H)m_k - s \qquad (2.27) \\
&= m - s
\end{aligned}
$$

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[25]Per capita inequality in cohort k is defined, using (2.7), as,

(2.24) $A^k = m_k - s$

so that intracohort inequality can be seen to be equal to a weighted average of inequality within cohorts, with the weights being cohort population shares. That is,

(2.25) $A_A = \sum_k (n_k/H)A^k$

which is the index of total inequality measured as the per capita social saving to be realized by moving directly from (2.20) to (2.22).

Before presenting the derivation of intra- and intercohort AKS inequality indices, I wish to propose a new decomposition to replace the one suggested by Blackorby, Donaldson, and Auersperg [1981]. Their decomposition of AKS inequality indices, derived from their decomposition of AKS indices of equality,[26] has two serious drawbacks. The procedure gives different results depending on whether inequality within cohorts or inequality between cohorts is eliminated first. In either case, the decomposition lacks the simple additive aggregation of per capita inequality indices or multiplicative aggregation of AKS indices of equality.[27] My decomposition of AKS inequality indices is derived from the decomposition of per capita inequality using the property that an AKS index is equal to the corresponding per capita index normalized on the mean (representative consumption). Thus, very simply, from the decomposition of per capita inequality,

(2.28) $A=A_A+A_R$

I obtain, by dividing through by m,

(2.29) $I=I_A+I_R$

where each index in (2.29) is equal to the corresponding index in (2.28) divided by m. The decomposition (2.29) has, of course,

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[26]Using the property that AKS indices of equality and inequality sum to unity.

[27]See (2.37) below.

the same simple additive structure of (2.28) and yields a unique decomposition of inequality within and among age-cohorts regardless of whether intra- or intercohort inequality is eliminated first. While total relative inequality in (2.29) and total relative equality in (2.37) below sum to unity, however, the subindices of relative inequality in my decomposition do not retain this property.[28]

Dividing through (2.23) and (2.26) by m yields,[29]

(2.30) $I_A = (m - \sum_k (n_k/H)s_k)/m$

and,

(2.31) $I_R = (\sum_k (n_k/H)s_k - s)/m$

Intra- and intercohort AKS inequality, (2.30) and (2.31), can easily be seen to sum to,

(2.33) $I = (m-s)/m$

Finally, the construction of AKS indices of equality proceeds as follows. When a move is made from one situation to another in which consumption is less unequally distributed, the corresponding AKS equality index is computed as the ratio of total representative consumption in the latter situation to total representative consumption in the former. Thus the index

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[28]Blackorby, Donaldson, and Auersperg [1981, pp. 673-4] have shown that no aggregation of subindices of relative equality measured as percentage savings of the original distribution exists. That is, there is no decomposition of relative equality corresponding to (2.29).

[29]Again, notice that (2.30), the AKS index of intracohort inequality, is equal to a weighted average of AKS inequality within cohorts; that is,

(2.32) $I_A = \sum_k (n_k m_k/Hm)(1 - s_k/m_k)$
$= \sum_k (n_k m_k/Hm) I^k$

with the weights being the cohort shares of total representative consumption.

of intracohort AKS equality is,[30]

(2.34) $$E_A=\sum_k n_k s_k/\sum_h r_h$$
$$=\sum_k (n_k m_k/Hm)(s_k/m_k)$$

Intercohort AKS equality is given by,

(2.36) $$E_R=s/\sum_k (n_k/H)s_k$$

In this case, the product of the intra- and intercohort terms, (2.34) and (2.36), yields the index of total AKS equality:

(2.37) $$E=s/m$$

The welfare approach to the measurement of inequality presented above improves on the usual practice of measuring inequality in the distribution of annual income by focussing on the distribution of consumption and by adopting a lifecycle perspective. While some recent studies (e.g. Nordhaus [1973], Blinder [1975], Irvine [1980])have made the shift from annual income to lifecycle consumption, they have measured inequality in the <u>potential</u> distribution of representative consumption computed under the assumption of perfect capital markets. They should rather have examined the <u>actual</u> distribution of representative consumption representing lifecycle consumption opportunities obtainable under existing capital market conditions. This is accomplished in the welfare approach to the measurement of inequality by using data on actual consumption in the computation of representative lifecycle consumption defined

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[30](2.34) also demonstrates that intracohort AKS equality is equal to a weighted average of AKS equality within each cohort, which is given by,

(2.35) $$E^k=s_k/m_k \qquad (1\le k\le K)$$

with the weights being the shares of total representative consumption accruing to each cohort.

in (2.3).

AKS and per capita indices can be interpreted as inequality measures which evaluate the actual distribution with respect to a hypothetical, optimal alternative. When the social evaluation function is continuous, increasing, symmetric, and quasi-concave, average representative consumption, r, is the optimal (i.e. social welfare maximizing) distribution. Social welfare in the actual situation is represented by the equally distributed equivalent of r. AKS and per capita indices measure inequality as a function of these two statistics.

Since the welfare approach incorporates the consumer choice problem, it can be extended to measure the welfare loss attributable not only to the degree of interpersonal inequality, but also to imperfections in the means of redistributing consumption over time. In the extended welfare approach, the actual situation is unchanged but the optimal situation becomes characterized by perfect capital markets in addition to interpersonal equality. The optimal situation is thus represented by the mean of a vector of potential individual representative lifecycle consumption statistics representing consumption plans chosen under perfect capital market conditions. The situation is represented in Figure II, where variables representing the situation in which capital markets are assumed free of imperfections are denoted by a prime.

Consider first an index of per capita inequality. Total inequality is,

(2.38) $A=m'-s$

The welfare loss due to interpersonal inequality is measured by the difference between m, which reflects social welfare when interpersonal inequality is eliminated, and s, which reflects the social evaluation of the actual distribution of representative consumption. That is,

(2.39) $A^P = m - s$

This is exactly the index of interpersonal inequality which was derived in the welfare approach above, and which can be decomposed into intra- and intercohort inequality as in (2.28); that is,

(2.40) $$A^P = [\sum_k (n_k/H)(m_k - s_k)] + [\sum_k (n_k/H)s_k - s]$$
$$= A^P_A + A^P_R$$

An index of the welfare loss, measured in representative consumption dollars per capita, attributable to imperfections in capital markets can analogously be defined as the difference between s', representing social welfare when all "intertemporal" inequality has been eliminated, and s:

(2.41) $A^T = s' - s$

Finally, account must be taken of the interaction between interpersonal and "intertemporal" inequality. Improved means of borrowing and lending may yield greater benefits to some than others, altering the distribution of representative lifecycle consumption and thus changing measured interpersonal inequality. Similarly, redistribution among individuals will affect the shape of their consumption profiles and thus the social significance of existing imperfections in capital markets. Thus the interactive effect of interpersonal and intertemporal redistribution can be thought of either as the change in interpersonal inequality attributable to the elimination of capital market imperfections,

(2.42) $\quad A^{PT}=(m'-s')-(m-s)$

or as the change in "intertemporal" inequality resulting from the elimination of interpersonal inequality,

(2.43) $\quad A^{PT}=(m'-m)-(s'-s)$

These two interpretations of the interactive effect are clearly equivalent, as can be seen by comparing (2.42) and (2.43). The indices of per capita interpersonal and "intertemporal" inequality, (2.39) and (2.41), and their interactive effect, (2.42) or (2.43), can be aggregated into an index of total per capita inequality by adding them together:

$$A^{P} + A^{T} + A^{PT}$$

(2.44) $$=(m-s) + (s'-s) + [(m'-s')-(m-s)]$$

$$=m'-s$$

We may now use (2.44) to compute the corresponding AKS indices of inequality by dividing through by m'. This yields,

(2.45) $$[(m-s)/m'] + [(s'-s)/m'] + \{[(m'-s')/m']-[(m-s)/m']\}$$

$$=I^{P} + I^{T} + I^{PT}$$

Recall that the per capita index of interpersonal inequality is the same whether or not "intertemporal" inequality is measured. The AKS index of interpersonal inequality, however, is different in the extended welfare approach because total representative consumption (the basis on which AKS indices express inequality) is greater when lifecycle consumption plans are chosen under perfect capital market conditions. This is reflected in the denominator of the AKS index of interpersonal inequality, which is m' in the first term of (2.45) where it had been m in (2.33). The index of interpersonal AKS inequality in this extended welfare approach to the measurement of inequality can be decomposed within and among age-cohorts analogously to (2.29). This yields,

(2.46) $I_A^P=(m-\sum_k(n_k/H)s_k)/m'$

and,

(2.47) $I_R^P=(\sum_k(n_k/H)s_k-s)/m'$

AKS indices of equality are also easily extended to include the measurement of welfare losses due to differences between actual and potential intertemporal distributions. Total AKS equality in this case is equal to the ratio of social welfare under the actual distribution, measured in terms of representative consumption dollars, to social welfare similarly measured in a potential, optimal situation in which all inequality has been eliminated and capital markets are free of imperfections:

(2.48) $E=s/m'$

Total inequality can be decomposed into the product of three terms as follows:

(2.49) $E=[s/m]\ [s/s']\ [(s'/m')/(s/m)]$

The first term in (2.49) is the AKS index of inequality (2.37) which can be multiplicatively decomposed into the two terms given in (2.34) and (2.36). The second term measures the social cost of imperfections in the means of intertemporally reallocating income, and the third term reflects the interaction of interpersonal and intertemporal redistribution.

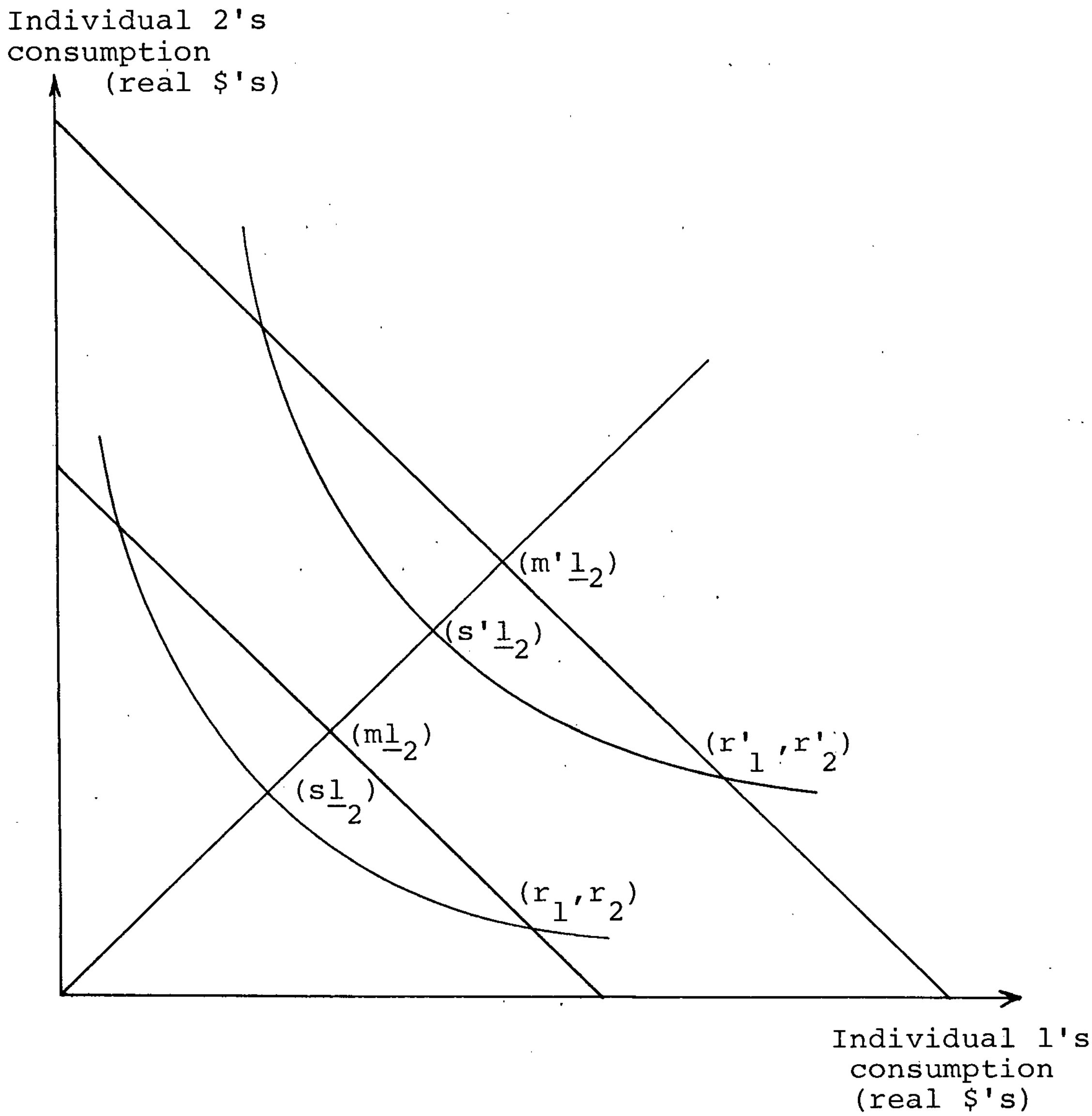
Indices of per capita inequality and the corresponding AKS indices of inequality and equality, derived in the welfare approach to the measurement of inequality and its extension to include the measurement of "intertemporal" inequality, provide theoretically sound measures of equality and inequality which have a number of very appealing characteristics. First, they are defined in terms of, and are constructed with, summary statistics of welfare which represent lifecycle consumption profiles and are calibrated in units of real consumption dollars. Second, the indices distinguish between actual and optimal, hypothetical distributions of well-being, and are carefully constructed to measure inequality in the actual distribution with reference to the optimal alternative. To this extent the welfare approach fits within the framework suggested by Atkinson [1970], Kolm [1969], and Sen [1973] which is now widely accepted as the foundation of the modern theory of inequality measurement. Third, welfare approach indices allow for the exercise of consumer choice to reallocate income streams to achieve desired consumption plans. While this may seem an obvious point to anyone familiar with economic theory, it has, in fact, largely been overlooked in the theory of inequality measurement to date. Fourth, the indices incorporate a lifecycle perspective on the measurement of inequality which is both a necessary adjunct to the explicit inclusion of consumer choice, and a great improvement on the predominant trend of measuring inequality in the distribution of annual income or consumption. Fifth, they incorporate a method of decomposing interpersonal inequality into intragenerational and intergenerational components so that they can be studied separately. And finally, welfare approach indices can be constructed so as to include the effects of both interpersonal and intertemporal inequality, and can be disaggregated so as to identify the relative magnitudes of these two sources of inequality.

These many important advantages of a welfare approach to the measurement of inequality are, unfortunately, difficult to realize in practice. Empirical implementation of the welfare approach is plagued by several problems. First, individual utility functions are required for the construction of welfare approach indices of inequality. While actual consumption paths can be observed in the data (rather than derived by the maximization of utility subject to actual market opportunities for reallocating income streams) utility functions are required to compute representative lifecycle consumption. Furthermore, estimation of the indices which I have proposed requires lifecycle income data on all members of the population. Panel data sets are rare and none covers the entire lifecycle of even one age-cohort in the population, let alone those of members of all cohorts represented in the population. To estimate welfare approach indices for the current population would require data collected over a period of roughly one hundred fifty years; that is, from the year of birth of the oldest member of the current population, to the year of death of its oldest-living member. The prospects for empirically implementing the welfare approach thus appear bleak indeed.

A situation in which the demands imposed by theory outstrip empirical resources and abilities is not unfamiliar to economists. A solution is to try to construct an alternative formulation which is empirically tractable as well as theoretically sound and attractive. This task is taken up in the next chapter.

FIGURE II

Mean and Equally Distributed Equivalent Consumption



CHAPTER THREE

A Decomposition Approach to the Measurement of Inequality

There are two theoretical problems with measuring inequality in the distribution of annual income. Their solution, I have argued, involves a shift to measuring inequality in the distribution of lifecycle consumption. But while this solution, which is the essence of the welfare approach to the measurement of inequality, may appear simple enough when put in such terms, its practice is in fact entirely precluded by lack of lifecycle consumption data and knowledge of individual utility functions. A decomposition approach to the measurement of inequality is an attempt to tread the middle ground between the theoretical rigour and empirical intractability of a welfare approach and the empirically practical but theoretically misspecified traditional approach. A decomposition approach also responds to criticisms leveled at the traditional approach to the measurement of inequality that indices of annual income inequality do not account for the effects of income mobility and the age-structure of the population.

I shall briefly review here the methodological problems of measuring inequality in the distribution of annual income. A number of empirical studies have assessed the sensitivity of popular and widely employed inequality indices, such as the Gini

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[31] See Chapter One for references.

coefficient, to the length of the accounting period.[31] All found a significant equalizing effect associated with extending the period over which income (or consumption) is cumulated. Furthermore, Shorrocks [1978a] has demonstrated that, for a large class of indices including those consistent with the approach initiated by Atkinson [1970], Kolm [1969], and Sen [1973], inequality of income aggregated over an extended accounting period cannot exceed a weighted average of measured annual inequality. In the best attempt to date to investigate the effects of income mobility, Shorrocks [1978b] has constructed and computed indices which measure income mobility in terms of the extent to which inequality is diminished by lengthening the accounting period. What his approach lacks is an explicit theoretical link between mobility and inequality indices which would allow measured annual inequality to be adjusted so as to account for the effects of income mobility. The decomposition approach eschews the mobility index approach of Shorrocks in favour of inequality indices which can distinguish pure interpersonal inequality from that attributable to income mobility.

Another widely recognized source of bias in measured annual inequality is due to the observed tendency of income streams and consumption paths to rise over the course of a lifetime.[32] Indices of annual inequality capture not only pure interpersonal inequality but also inequality related to the age-structure of

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[32] In cross-sectional data this pattern is often observed to have a declining tail after retirement.

the population when income and consumption profiles display this characteristic shape. Paglin [1975] has proposed a method for distinguishing age-related inequality from pure interpersonal inequality in the Gini coefficient computed from annual data. The Paglin-Gini has been heavily criticized[33] but no replacement has been suggested. Fortunately, however, this problem too can be solved by adopting the decomposition approach to the measurement of inequality.[34]

The decomposition approach to the measurement of inequality starts from the premise that longitudinal data are necessary if the intertemporal aspects of inequality are to be taken into account. A time-series of anonymous cross-sections is of no use for this purpose because the effects of income mobility show up only when an individuals's income is followed over time.[35] Panel data are thus required, which, even if they do not cover entire lifecycles, should at least allow the degree of mobility-related inequality to be approximated. The method of the decomposition approach is to measure inequality in the panel data treated as a single distribution. A part of this total will be due to the differences in income that an individual experiences from one

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[33]For a detailed discussion of the Paglin-Gini and the criticisms of it see Chapter One.

[34]An evaluation of Paglin's method compared to the decomposition approach is presented in Appendix B.

[35]In fact, the trend of annual inequality is remarkably stable, but this does <u>not</u> imply a low degree of income mobility because the symmetry of inequality indices means that permuting a particular distribution will not alter the degree of measured inequality. Any amount of income mobility is consistent with a stable trend of annual inequality.

year to another. By measuring the inequality attributable to these intrapersonal income differences, an index of mobility-related inequality is obtained which, in addition to its intrinsic interest, can be used to define a measure of interpersonal inequality net of the effects of income variation over time.[36] The remaining inequality is not solely interpersonal, however, but also reflects intergenerational income differences. The characteristic shape of lifecycle profiles contributes to the variation of annual income among individuals even if lifetime incomes are equally distributed. This source of inequality can also be distinguished from pure interpersonal inequality, yielding an index of age-related inequality. This index is of interest for its own sake and for use in deriving an index of pure interpersonal inequality net of the effects of intertemporal and intergenerational income differences.

The decomposition approach thus begins with panel consumption data. For each individual, h ($1 \le h \le H$), in the sample, the panel data set has a time-series of annual consumptions over a T year period, $c_h=(c_{h1}, \ldots, c_{hT})$. The entire data set can be arranged as a vector, $c=(c_1, \ldots, c_H)$, of dimension HT. In the decomposition approach to the measurement of inequality, this vector is treated as a single distribution over a population of size HT in which total inequality is measured. Inequality attributable to variation in individual incomes over

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[36]In this sense intertemporal is a synonym for intrapersonal when speaking of inequality attributable to the time paths of individual incomes.

time can then be distinguished from that due to income variation among individuals by decomposing inequality within and among population (of size HT) subgroups. The consumption vector of the members of subgroup h is c_h. Inequality is decomposed within and among population subgroups using a procedure suggested by Blackorby, Donaldson, and Auersperg [1981]. In this procedure inequality within subgroups is eliminated from the distribution by assigning each individual the equally distributed equivalent of the distribution within his own subgroup.[37] That is, letting the scalar r_h be the equally distributed equivalent of the vector c_h, intrapersonal (mobility-related) inequality is eliminated by replacing the vector $c_h=(c_{h1}, \ldots, c_{hT})$ by the socially equivalent, equally distributed alternative, $(r_h \underline{1}_T)$.[38] The equally distributed equivalent is mean consumption adjusted for inequality in the distribution it represents. Thus the equally distributed equivalent of a distribution cannot exceed its mean, and will be strictly less than the mean if there is any inequality in the distribution. Thus, total consumption in the original distribution $c=(c_1, \ldots, c_H)$ cannot be less than total consumption in the distribution in which intrapersonal

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[37]An alternative procedure for decomposing inequality has recently been proposed independently by Bourguignon [1979], Cowell [1980], and Shorrocks [1980], in which subgroup mean incomes are used to eliminate intragroup inequality. In Appendix A Shorrocks' version of this alternative procedure is compared to the Blackorby-Donaldson-Auersperg method, and their relative performances in the decomposition approach are evaluated. The conclusion reached is that the Blackorby-Donaldson-Auersperg procedure is superior for this purpose because the alternative suffers from two theoretical drawbacks which prove to be seriously damaging to its empirical performance.

[38]Where $\underline{1}_T$ is a T-dimensional unit vector.

inequality is eliminated, which is characterized by the vector $(r_1 \underline{1}_T, \ldots, r_H \underline{1}_T)$. The saving generated by a move from the former to the latter represents the social cost of mobility-related inequality.

The replacement of the vector c_h by $(r_h \underline{1}_T)$ removes all <u>intrapersonal</u> inequality, leaving only inequality due to differences between the values of r_h, $(1 \le h \le H)$. This remaining inequality is attributable to differences related to the age-structure of the population as well as to pure interpersonal differences in consumption. To distinguish these two sources from one another, inequality may be further decomposed within and among age-subgroups of the population. This is accomplished by grouping together individuals of the same age,[39] and assigning members of the same cohort the equally distributed equivalent of their cohort distribution. Letting s_k be the equally distributed equivalent of the distribution among the n_k members of cohort k, $(1 \le k \le K)$, the elimination of intracohort inequality results in the replacement of the distribution $(r_1 \underline{1}_T, \ldots, r_H \underline{1}_T)$ by $(s_1 \underline{1}_{T \cdot n_1}, \ldots, s_K \underline{1}_{T \cdot n_K})$. Again, a move between these two vectors results in a saving which reflects the social cost of inequality within cohorts, which I have called pure interpersonal inequality.

Intrapersonal and intracohort inequality having now been eliminated, only inequality between cohorts remains. This too can be eliminated by replacing the distribution

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[39]Or in the same age bracket if age-cohorts are defined over a range of years.

$(s_1 \underline{1}_{T \cdot n_1}, \ldots, s_K \underline{1}_{T \cdot n_K})$ by a distribution in which everyone receives the population-wide equally distributed equivalent, s. This third move from the distribution of cohort equally distributed equivalents to a socially equivalent equal distribution over the entire population (of size HT), $(s\underline{1}_{HT})$, implies a further social saving which is a measure of age-related inequality.

The measurement and decomposition of inequality involve the repeated replacement of distributions (over subgroups of the population) by their equally distributed equivalents. In each case, since the equally distributed equivalent is an inequality adjusted mean, there is a social saving created which reflects the social cost attributable to a particular source of total inequality. AKS indices express this saving as a proportion of total consumption, and per capita indices express it in per capita terms. Thus ethical indices measure inequality as the amount wasted on inequality.

This chapter begins with a formal discussion of the social welfare underpinnings of the decomposition approach to the measurement of inequality. The primary objective is the definition of the equally distributed equivalents, r_h, s_k, and s, which are fundamental elements of the decomposition indices that I wish to derive. This is done by successively eliminating intrapersonal, intracohort, and intercohort inequality from the original distribution. The social savings implied by moving through a succession of reference vectors measure the social costs associated with each source of inequality. AKS and per capita indices of inequality can then be constructed by expressing the social cost of inequality in percentage and per capita terms respectively. In the former case I again employ the decomposition presented in the preceding chapter which I have suggested to replace the Blackorby-Donaldson-Auersperg decomposition of AKS inequality indices.

As mentioned before, the decomposition approach can be justified on its own grounds--as a solution to the problems of measuring pure interpersonal inequality in an annual distribution--or as an empirically tractable approximation to the welfare approach. The chapter concludes with a discussion of the common ground shared by the two approaches. I show that, by adopting two simplifying assumptions, an equivalence between the two approaches can be established.

The measurement of inequality requires the existence of a social evaluation function $W:R_+^{HT} \rightarrow R^1$ with image,

(3.1) $\quad w=W(c)$

where $c=(c_{11}, \ldots, c_{1T}, \ldots, c_{H1}, \ldots, c_{HT})$ is a vector of the consumption paths of H people observed over T years. W(.) is assumed to be continuous, increasing, and S-concave.

In the decomposition approach, c is treated as a distribution of consumption over a population of size HT. I begin by decomposing this population into H exhaustive, mutually exclusive subgroups of size T, with the intention of measuring inequality in each subgroup independently of consumption of non-members. The social evaluation function must therefore be separable in these subgroups, implying that W(.) can be written as,

(3.2) $\quad w=\overline{W}(V^1(c_1), \ldots, V^H(c_H))$

where $W(.)$ is increasing in $V^h(c_h)$ and $c_h=(c_{h1}, . . . ,c_{hT})$ is a vector of the T consumptions observed for person h. The functions $V^h:R^T \rightarrow R^1$ are identical for all h, and can be used to define individual representative consumption:

(3.3) $\quad V(r_h \underline{1}_T)=V(c_h) \qquad (1 \leq h \leq H)$

The properties of $V(.)$, inherited from $W(.)$, allow r_h to be uniquely determined for every c_h so that r_h can be written as,

(3.4) $\quad r_h=R(c_h) \qquad (1 \leq h \leq H)$

Substitution of (3.3) into (3.2) yields,

$$w=\overline{W}[V(r_1 \underline{1}_T), . . . ,V(r_H \underline{1}_T)]$$

(3.5) $\quad =W(r_1, . . . ,r_H)$

This social evalution function defined over individual representative consumptions can be used as the basis of an index of interpersonal inequality free of bias due to consumption mobility. To separate intercohort effects from interpersonal inequality within age-cohorts requires that individuals be grouped by age. Let $\hat{N}=\{N^1, . . . ,N^K\}$ be a partition of the population set $N=\{1, . . . ,H\}$ into age cohorts. $h \in N^k$ means person h is a member of the k<u>th</u> age-cohort which has n_k members. r^k is a vector of representative consumptions of individuals in cohort k. Each N^k must be separable from its complement in $\hat{N}$, in which case (3.5) can be written as,

(3.6) $\quad w=W[W^1(r^1), . . . ,W^K(r^K)]$

where $W(.)$ is increasing in $W^k(r^k)$.

The conjunction of symmetry and separability in $W(.)$ implies that its structure is additive (Blackorby, Donaldson, and Auersperg [1981, theorem 1]):

(3.7) $\quad w=\overset{*}{W}[\sum_h g(r_h)]$

where $\overset{*}{W}(.)$ is increasing in its argument and g(.) is independent of h because of the symmetry assumption. S-concavity of W(.) requires that g(.) be concave; strict S-concavity requires that it be strictly concave (Berge [1963]). Thus W(.) must be quasi-concave and symmetric. The social evaluation function corresponding to decomposable AKS and per capita indices of inequality must therefore be continuous, increasing, symmetric, quasi-concave, and additively separable. Further properties are required of the social evaluation function corresponding to relative indices (which are homogeneous of degree zero in their arguments) and absolute indices (which are invariant to equal absolute changes in their arguments). AKS indices are relative indices if and only if the social evaluation function is homothetic. In conjunction with the properties listed above, this restricts the class of admissible social evaluation functions to the means of order R. Per capita indices are absolute indices if and only if the social evaluation function is translatable. This restricts W(.) to the Kolm-Pollak family of social evaluation functions.

The properties of $\hat{W}(.)$ and $W^k(.)$ are again traced back to the original social evaluation function (3.1). $W^k:R^{n_k}\rightarrow R^1$ can be used to define the representative consumption of cohort k,

(3.8) $\quad W^k(s_k \underline{1}_{n_k})=W^k(r^k)$ $\quad (1\leq k\leq K)$

which is the equally distributed equivalent of the vector of representative consumptions of all individuals in cohort k. The properties of $W^k(.)$ ensure that s_k can be explicitly defined as,

(3.9) $\quad s_k=S^k(r^k)$ $\quad (1\leq k\leq K)$

Since $W^k(.)$ has the same additive structure as W(.), with the summation being over members of the $k\underline{th}$ age-cohort only, representative cohort consumption is,

$$s_k = g^{-1}[(1/n_k)\sum_{h \in N^k} g(r_h)] \qquad (1 \le k \le K) \tag{3.10}$$

The implicit definition of cohort representative consumption, (3.8), can be substituted into (3.6) yielding,

$$\begin{aligned} w &= \widehat{W}[W^1(s_1 \underline{1}_{n_1}), \ . \ . \ . \ , W^K(s_K \underline{1}_{n_K})] \\ &= \mathring{W}(s_1 \underline{1}_{n_1}, \ . \ . \ . \ , s_K \underline{1}_{n_K}) \qquad (3.11) \end{aligned}$$

Finally, the elimination of intercohort inequality with social indifference is accomplished by assigning each person the population representative consumption implicitly defined by

$$\mathring{W}(s \underline{1}_H) = \mathring{W}(s_1 \underline{1}_{n_1}, \ . \ . \ . \ , s_K \underline{1}_{n_K}) \tag{3.12}$$

The properties of W(.) ensure that s is uniquely defined for any distribution of cohort representative consumptions, and can be written

$$s = S(s_1 \underline{1}_{n_1}, \ . \ . \ . \ , s_K \underline{1}_{n_K}) \tag{3.13}$$

While (3.13) defines s as the equally distributed equivalent of the vector $(s_1 \underline{1}_{n_1}, \ . \ . \ . \ , s_K \underline{1}_{n_K})$, it should be clear from (3.9) and (3.4) that s can also be expressed as an equally distributed equivalent of either individual representative consumptions or the actual consumption paths of each individual.

AKS indices measure inequality as the percentage of total consumption saved by moving from the actual distribution to an equal distribution that is socially equivalent. Per capita indices express the same saving in per capita terms. Both involve the social evaluation of two situations: one in which individuals receive their observed consumption paths, and another in which everyone is assigned the representative, or equally distributed equivalent, consumption.

The decomposition of inequality can be thought of as a series of situations in which various sources of inequality are eliminated in succession, and the social savings created by each move measure that part of total inequality attributable to a particular source. It is necessary first to eliminate inequality due to variation in consumption over time since summary statistics of individual consumption paths are required in order to measure interpersonal inequality. Assigning each individual his representative consumption, r_h, defined in (3.4), eliminates intrapersonal inequality and provides an exact welfare index of individual consumption paths.

Interpersonal inequality indices based on the social savings generated by a move from the distribution of representative consumption to an equally distributed equivalent distribution capture both real consumption differences among persons and inequality related to the age-structure of the population. Eliminating inequality within age-cohorts by next assigning individuals their representative cohort consumptions, s_k, leads to an index of intracohort inequality.[40] Finally, an index of age-related inequality can be based on the social saving resulting from the elimination of inequality between cohorts in the move to a situation in which everyone receives the population-wide equally distributed equivalent, s.

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[40]Blackorby, Donaldson, and Auersperg [1981] have shown that subindices of intra- and intercohort per capita inequality are invariant with respect to the order in which inequality is eliminated within and between age-cohorts. Although this is <u>not</u> true of their decomposition of AKS indices, it does apply to the decomposition of AKS inequality that I proposed in Chapter Two and that I shall employ here.

The equally distributed equivalent consumptions given by (3.4), (3.9), and (3.13) define the elements of three reference vectors which, with the vector of original consumptions, represent the four situations which characterize the successive elimination of inequality, first within each individual's consumption path over time, then within age-cohorts, and finally between age-cohorts:

(3.14) $(c_{11}, \ldots, c_{1T}, \ldots, c_{H1}, \ldots, c_{HT})$

(3.15) $(r_1 \underline{1}_T, \ldots, r_H \underline{1}_T)$

(3.16) $(s_1 \underline{1}_{T \cdot n_1}, \ldots, s_K \underline{1}_{T \cdot n_K})$

(3.17) $(s \underline{1}_{HT})$

Movements between these reference vectors are made with social indifference. The social savings which accrue as a result of such movements can be used to construct AKS inequality indices when expressed as a proportion of total consumption, or per capita inequality indices when expressed in per capita terms.

Consider, for example, the per capita saving which could be realized in a move from (3.14) to (3.15), recalling that the consumptions of H individuals received over T years are being treated as the consumptions of a population of size HT.

$$A_{AP} = (1/HT)\left[\sum_h \sum_t c_{ht} - \sum_h T r_h\right]$$

$$(3.18) \qquad = (1/H)\left[\sum_h (m_h - r_h)\right]$$

where $m_h = (1/T)\sum_t c_{ht}$ is the mean consumption of individual h during the time period covered by the data. This intrapersonal per capita inequality index, A_{AP}, is an average of $A^h = (m_h - r_h)$, $(1 \leq h \leq H)$, the per capita inequality in each individual's consumption path.

A move from (3.15) to (3.16) would produce a per capita saving of:

$$A_{AC}=(1/HT)[\sum_h Tr_h-\sum_k Tn_k s_k]$$

(3.19) $$=\sum_k(n_k/H)(m_k-s_k)$$

where $m_k=\sum_{h\in N^k}(1/n_k)r_h$ is the mean, and $s_k=S^k(r^k)$ is the equally distributed equivalent, of the vector of representative consumptions of individuals in the k<u>th</u> age-cohort, $r^k=(<r_h> \forall h\in N^k)$. It can be seen from (3.19) that per capita intracohort inequality is equal to the cohort-population-share weighted average of $A^k=m_k-s_k$, the per capita inequality within each age-cohort. This index measures pure interpersonal inequality, free of distortions attributable to consumption mobility, the age-structure of the population, and economic growth.

Finally, the per capita saving to be realized by moving from (3.16) to (3.17) is:

$$A_{RC}=(1/HT)[\sum_k Tn_k s_k-HTs]$$

(3.20) $$=\sum_k(n_k/H)s_k-s$$

which measures per capita inequality between age-cohorts as the mean of the distribution of cohort representative consumptions less its equally distributed equivalent.

The sum of the three subindices of inequality can easily be shown to be equal to the index of total per capita inequality, A=m-s,[41] which is the per capita saving that would result from a direct move from (3.14) to (3.17). Thus the decomposition of total per capita inequality is,

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[41] Where m is the mean and s is the equally distributed equivalent of the original consumption vector (3.14).

$$A = m - s$$

(3.21) $$= [(1/H)\sum_h (m_h - r_h)] + [(1/H)(\sum_h r_h - \sum_k n_k s_k)] + [\sum_k (n_k/H)s_k - s]$$

$$= A_{AP} + A_{AC} + A_{RC}$$

AKS inequality indices can easily be computed from the decomposition of per capita inequality given in (3.21). The new decomposition of AKS inequality that I presented in Chapter Two to replace the Blackorby-Donaldson-Auersperg decomposition is calculated simply by dividing through (3.21) by m, the mean of (3.14). This yields:

(3.22) $$I = I_{AP} + I_{AC} + I_{RC}$$

where the subindices of AKS inequality are defined as follows:

(3.23) $$I_{AP} = [(1/H)\sum_h (m_h - r_h)]/m$$

(3.24) $$I_{AC} = [\sum_k (n_k/H)(m_k - s_k)]/m$$

(3.25) $$I_{RC} = [\sum_k (n_k/H)s_k - s]/m$$

The AKS indices (3.23), (3.24), and (3.25) measure intrapersonal, intracohort, and intercohort inequality, respectively, as the social cost associated with these sources of consumption differences expressed as a proportion of total consumption in the original distribution. The three indices may be interpreted as measures of mobility-related inequality, pure interpersonal inequality, and age-related inequality respectively.

Finally, consider the construction of AKS indices of equality by a series of moves between successive pairs of the reference vectors (3.14) through (3.17). AKS equality indices constructed in this manner are defined as the ratio of total consumption in one vector to total consumption in the preceding vector. For the move from (3.14) to (3.15) this yields,

$$E_{AP} = \sum_h Tr_h / \sum_h \sum_t c_{ht}$$

(3.26) $$= (1/H) \sum_h r_h / m_h$$

where $m = (1/HT) \sum_h \sum_t c_{ht}$ is the mean of (3.14). Similarly, intracohort relative equality is given as total consumption in (3.16) as a proportion of total consumption in (3.15).

$$E_{AC} = \sum_k Tn_k s_k / \sum_h Tr_h$$

$$= \sum_k n_k s_k / \sum_k \sum_{h \in N^k} r_h$$

(3.27) $$= \sum_k n_k s_k / \sum_k n_k m_k$$

since $m_k = (1/n_k) \sum_{h \in N^k} r_h$. And finally, AKS intercohort equality is

$$E_{RC} = HTs / \sum_k Tn_k s_k$$

(3.28) $$= s / \sum_k (n_k / H) s_k$$

The product of these three subindices of equality yield the AKS index of total equality. Thus the decomposition of total AKS equality is,

$$E = s/m$$

(3.29) $$= [(1/H) \sum_h r_h / m] \; [\sum_k n_k s_k / \sum_h r_h] \; [s / (1/H) \sum_k n_k s_k]$$

$$= E_{AP} \; E_{AC} \; E_{RC}$$

In the decomposition of per capita inequality, (3.21), AKS inequality, (3.22), and AKS equality, (3.29), total (in)equality is expressed as a simple function of three subindices of (in)equality which measure the contributions of intrapersonal, intracohort, and intercohort (in)equality to the total. The motivation has been two-fold. The decompositions provide a solution to problems with the traditional approach of measuring inequality in annual distributions which, it has been widely argued, confuses age- and mobility-related inequality with pure interpersonal inequality. Thus, the effects on measured inequality of consumption mobility and of the typically nonconstant time path of consumption have been identified and isolated, allowing an index of pure interpersonal inequality to be constructed. To be sure, mobility and the shape of consumption profiles are sources of inequality which are of significant intrinsic interest.[42] "Inequality", however, as the phrase is commonly used both by professionals and laymen, is, I believe, meant to exclude inequality which arises from either of these two sources. And this is precisely what the decomposition approach accomplishes.

An alternative motivation for exploring the decomposition approach to the measurement of inequality is the hope that it may offer a theoretically sound and empirically tractable theory of inequality measurement. The traditional method of measuring inequality in the distribution of annual income fails on the former count. The welfare approach presented in Chapter Two provides a sound theoretical basis for measuring inequality but is incapable of empirical implementation. The decomposition approach is a successful method of measuring interpersonal inequality without confusing it with inequality arising from other sources. Dalton argued that, "the economist is primarily interested not in the distribution of income as such, but in the effects of the distribution of income upon the distribution and total amount of economic welfare" ([1921, p.348]). It should therefore be asked whether there are grounds for interpreting intrapersonal inequality in the decomposition approach as an

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[42]Witness the volume of literature written on these subjects, especially the work of Shorrocks [1978a,b] and Paglin [1975].

index of inequality in the distribution of well-being. The answer is a qualified yes. Take, for example, the decomposition of per capita inequality, (3.21). The index which I am arguing measures pure interpersonal inequality is given by the second term,

(3.19) $A_{AC} = (1/HT)[\sum_h Tr_h - \sum_k Tn_k s_k]$

In the welfare approach the corresponding index is (2.23):

$$A_A = (1/H)[\sum_h r_h - \sum_k n_k s_k]$$

In the welfare approach, representative lifecycle consumption, r_h, is the lifecycle consumption annuity between which and his actual consumption profile the individual is indifferent. It is implicitly defined by,

(2.2) $U^h(r_h \underline{1}_{T_h}) = U\ (c_{h1}, \ldots, c_{hT_h})$

In the decomposition approach, however, representative consumption is defined in terms of a social evaluation function rather than an individual utility function. That is,

(3.3) $V(r_h \underline{1}_T) = V(c_{h1}, \ldots, c_{hT})$

(2.2) and (3.3) are similar, but differ in two important respects. One, already mentioned, is that $U^h(.)$ is an individual utility function while $V(.)$ is a social evaluation function which results from the separability structure imposed on $W(.)$, from which it inherits its properties. The other difference is that the domain of $U^h(.)$ is a T_h-dimensional vector of consumption expenditures made by individual h during the course of his life, while the dimension of the domain of $V(.)$ is common to all individuals, being T observations on consumption.

Despite these differences, there are grounds for arguing that r_h in the decomposition approach is a satisfactory summary statistic of individual welfare. Inequality is measured in terms of the social cost implied by maintaining the actual distribution rather than redistributing it equally. A case might therefore by made that the arguments of an inequality index should be based on a <u>social</u>, rather than private, evaluation of the individual welfare that results from a given consumption plan. In this case V(.) may be viewed as a utility function based on the preferences of a planner rather than on individual preferences. Second, and perhaps more importantly, recall that the welfare approach was found to be empirically impractical because it requires that individual utility functions be known and that data on the lifecycle consumption paths of all members of the population are available. When other writers have met these obstacles, they have invoked the simplifying assumptions of identical utility functions and length of life across all individuals (Nordhaus [1973], Blinder [1975], Layard [1977], Irvine [1980]). Blinder [1975, pp. 31-2] has argued, furthermore, for the adoption of (the continuous-time version of) an iso-elastic form for the utility function. But this is precisely (the analogue of the discrete-time version of) the only functional form admissible as a social evaluation function in the decomposition approach to the measurement of relative inequality. Faced with the task of implementing the welfare approach, Blinder would argue for substituting the known function V(.) defined over T years for the unknown $U^h(.)$ defined over T_h years; precisely this is accomplished by adopting a decomposition approach to the measurement of inequality.

For these reasons, in the next chapter I calculate inequality indices based on the decomposition approach to the measurement of inequality. Several features of these indices will be of particular interest. Of these, perhaps the most important will be to compare intracohort inequality in the decomposition (which I take as a measure of pure interpersonal inequality) with indices of inequality in the distribution of annual consumption. Also of interest, however, will be the relative importance of the three sources of inequality which have been identified in the decomposition approach. Shorrocks [1978b] has produced some interesting empirical results on income mobility in the United States, but not in a form that allows the quantitative impact of income mobility on measured inequality to be calculated. This is possible with the decomposition approach to the measurement of inequality, as will be seen in the empirical results in Chapter Four. I shall also inquire into the sensitivity of the indices developed in the decomposition approach to the choices made regarding the size of age-cohorts brackets and the number of years of data which are employed for the computation of the indices.

CHAPTER FOUR

Empirical Implementation of the Decomposition Approach

Empirical implementation of the decomposition approach to the measurement of inequality requires consumption data observed over a number of years for a panel of individuals. This is perhaps the heaviest requirement of the decomposition approach over and above those of the traditional approach of measuring annual income inequality. I have drawn upon the best source of panel data, the Panel Study on Income Dynamics conducted by the Survey Research Center [1968] of the Institute for Social Research at the University of Michigan.[43] Currently, there are eleven years of annual data available, running from 1968 to 1978, which report a wide variety of economic and demographic variables for 6154 families and their almost 21,000 members.

I have computed the consumption variable from the point of view of the family in keeping with the idea that the family acts as a unit in making private consumption decisions and is treated as such by public transfer programs. The distribution of consumption is expressed on an individual basis, however, to exclude the effects of family size and so that inequality is measured among individuals. In going from family to individual consumption it is necessary to make use of adult equivalence scales because children do not require the same level of

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[43] Unfortunately no appropriate panel data exist for Canada.

consumption for their support as adults. There are a number of different ways to proceed.

The usual practice has been to compute family size in terms of adult equivalents. For example, treating an adult as equivalent in terms of consumption to two children, a family of five consisting of two adults and three children is equivalent to a family of three and one-half adults. If total family consumption is $17,500, per capita adult equivalent consumption is $5,000 (=$17,500/3.5). This family would then be counted, if the usual practice were followed, as three and one-half individuals each with a consumption of $5,000. I prefer to keep the number of family members at its actual value, however, so that all individuals are represented in the distribution of consumption. There are then two alternative. One is to assign the adults in this family consumptions of $5,000 and the children $2,500. This, however, creates intrafamily inequality which I do not wish to be included in measured inequality. I have therefore chosen to assign the per capita adult equivalent consumption to all family members.[44] This allows me to avoid introducing intrafamily inequality while ensuring that all individuals are represented in the consumption distribution.[45]

The consumption variable has been constructed by proceeding from market income through net income to consumption. Data

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[44]This practice is followed by Blackorby and Donaldson [1980b]. The idea is due originally to A. Sen in a private communication to Blackorby and Donaldson. See also Sen [1979, pp. 292-3].

[45]Total and therefore mean consumption are not, however, the same as in the original distribution.

limitations have prevented this from being done exactly as it should but I have attempted to compute the consumption variable to correspond as closely as possible with the theoretical ideal. Beginning with family unit money income from market sources I added the rental value of free housing which represents one of the most important non-monetary components of market income. I then added transfers and subtracted taxes to arrive at net income, and added the amount saved on food stamps to incorporate an important non-monetary component of public transfers. My estimate of family consumption was then derived by adding income from private pensions and annuities and the rental value of owner-occupied housing.[46] Since I am interested in welfare, and therefore in real rather than nominal consumption, the consumption variable has been deflated by the U. S. Consumer Price Index (1975=100) (International Monetary Fund [1980, p. 343]).

The demographic data requirements include family composition (number of adults and children) for calculating the adult equivalent per capita consumption, and age of family head, according to which families are grouped into age-cohorts. I decided to drop families that experienced a change of family head during the sample period to save on computing costs by reducing the sample size. By excluding families from the sample on this basis, the question of the age-cohort to which such

[46]The construction of the consumption variable is described in detail in Appendix C.

families should be assigned was also avoided.[47] Surprisingly, this reduced the sample size by 87 per cent.[48] My empirical work is based on this subsample, but for comparative purposes I also computed relative inequality in the whole sample. The results indicated that measured inequality is 20 to 25 per cent greater in the original sample. The relative magnitudes of the subindices of inequality, however, are very close in the two samples.[49] Thus, while my results likely underestimate the actual degree of inequality, they appear to be indicative of the differences between the traditional and decomposition approaches to the measurement of inequality.

The final sample, then, includes 797 families on which data are available over a ten year period, 1968-1977.[50] The sample mean of the distribution of adult equivalent per capita consumption among all persons in 7,970 (=797x10) households (i.e. in the pooled sample) is $4091, with a standard deviation of $2741. A histogram of the pooled sample appears as Figure III. The means and standard deviations of the annual distributions and of the pooled distribution are given in

[47]There are several possible solutions: (1)families could be reassigned to the age-cohort of their new head when a change occurs; (2)family per capita adult equivalent consumption could be calculated and then assigned to each family member in his own age-cohort; or (3)age-cohorts could be defined over longer time spans of, say, five or ten years.

[48]It is not clear that this is entirely due to changes in family head. Examination of the reported "age of family head" for some of the families dropped from the sample indicated that a part of the total might reflect reporting errors only.

[49]See Appendix D for further details.

[50]This corresponds to the 1969-1978 waves of the Panel Study on Income Dynamics since the data are collected on the previous year's income, taxes, transfers, etc. The first wave was not used because of missing data.

Table 1.

Kolm-Pollak indices of absolute inequality based on the KP family of social evaluation functions,

$$(4.1) \qquad W_G(\mathbf{c}) = -(1/G)\ln[(1/H)\sum_h \exp\{-Gc_h\}] \qquad G>0$$

and Atkinson indices of relative equality and inequality based on the means of order R social evaluation functions,

$$(4.2) \qquad W_R(\mathbf{c}) = \begin{cases} [(1/H)\sum_h c_h^R]^{1/R} & 0 \neq R \leq 1 \\ \prod_h c_h^{1/H} & R=0 \end{cases}$$

have been computed. Both (4.1) and (4.2) represent families of social evaluation functions, whose members reflect varying degrees of inequality aversion.[51] Since ethical indices of inequality measure the social significance of inequality, measured inequality in a given distribution will depend on the degree of inequality aversion which the social evaluation function exhibits.

All the indices reported in this chapter have been computed by first calculating individual, cohort, and population-wide equally distributed equivalent consumptions, the formulae for which are identical to (4.1) and (4.2) (with the limits of the summation being appropriately changed). These representative consumptions have been used to construct reference vectors corresponding to a series of situations in which various sources of inequality are successively eliminated, and the inequality

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[51] The degree of inequality aversion exhibited by members of the families of social evaluation functions (4.1) and (4.2) is reflected in the curvature of the boundaries of their level sets.

indices calculated as the per capita or percentage social saving generated by a move from one vector to the next. AKS indices of equality are computed as total representative consumption in one situation as a proportion of total representative consumption in the preceding situation in which consumption is more unequally distributed.[52]

Indices of (in)equality are reported for a number of values of the free parameters, G and R. Gini and Maximin indices have also been calculated for comparative purposes, although these indices are not additively separable and thus are not guaranteed to aggregate to total measured inequality as do the Atkinson and KP indices. The absolute inequality results are reported in Table 2. For each value of G, the degree of absolute inequality aversion, total absolute inequality in the panel and its decomposition into subindices of intrapersonal, intracohort, and intercohort absolute inequality, are given in the first four rows of the table. The value of each subindex as a percentage of the total is given in parentheses. Below these are shown, for comparative purposes, the minimum, maximum, and mean of the ten annual absolute inequality indices. The last is a population share weighted average of the ten annual absolute inequality indices, which is the index of inequality within years that would result from a decomposition of total inequality within and

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[52]The reported values of these indices should be accompanied by some measure of their statistical significance if they are to be used to infer the degree of (in)equality in the population. Beach and Davidson (Beach [1980]) have derived asymptotic standard errors for inequality indices which could be used to construct interval estimates of inequality in the population distribution.

among years. Mean annual absolute inequality is thus an appropriate index with which to compare the value of pure interpersonal inequality as measured by the index of absolute intracohort inequality.

Absolute inequality indices measure the amount of consumption per capita that is wasted on inequality. In a pure redistribution of consumption, everyone would receive the mean consumption, but when the social evaluation function exhibits a positive degree of inequality aversion, a socially equivalent distribution results when individuals receive the equally distributed equivalent consumption (mean consumption adjusted for inequality). The amount by which the equally distributed equivalent consumption falls short of mean consumption is the amount of consumption per capita wasted on inequality. This is equal to the value of the absolute inequality index. Absolute inequality can thus range between zero and the mean. The lower bound is attained when there is no inequality in the original distribution or no aversion to inequality (i.e. G=0). Absolute inequality equal to mean consumption implies that at least one individual receives zero consumption in the original distribution and that the social evaluation function exhibits an infinite degree of inequality aversion.

Notice first of all that total inequality measured in the entire panel, A, given in the first row of Table 2, is roughly equal to mean annual inequality given in row 6. This should be expected in light of the often-observed tendency of annual inequality to remain roughly constant over time. If the distribution of annual consumption were identical year after year, then these two figures would be equal, since the KP indices satisfy the principle of population replication.

It should also be noted that the magnitudes of the decomposition approach subindices of absolute inequality reported in Table 2 display considerable stability relative to one another. Each of these subindices measures inequality attributable to a particular source of consumption differences. The increase in total measured inequality as the value of G rises reflects the greater social significance of a fixed amount of consumption dispersion, objectively measured, at higher degrees of absolute inequality aversion. This is as expected, and the relative stability of the subindices may thus be interpreted as an indication that the decomposition approach is robust.[53]

The decomposition of absolute inequality reveals that intrapersonal and intracohort inequality account for most of total measured inequality. As the degree of absolute inequality aversion (G) rises, the magnitude of intrapersonal inequality rises relative to intracohort inequality, from about 34 per cent when $G=5x10^{-6}$ to about 63 per cent when $G=5x10^{-3}$. In all cases except maximin, intercohort inequality accounts for only 6 per cent of the total.

The most important comparison to make is of intracohort inequality with mean annual inequality, for the former, I have argued, measures pure interpersonal inequality, while the latter

[53]See Appendix B for a decomposition of annual inequality in which the subindices exhibit extreme variation over a range of the degree of inequality aversion.

represents the traditional method of measuring inequality among individuals. Since mean annual inequality is roughly equal to total inequality, which is equal to the sum of three nonnegative terms, it should not be surprising to find that, taken individually, these terms are less than mean annual inequality. In particular, intracohort inequality is 58 to 70 per cent of mean annual inequality. Thus, in this data set, consumption mobility and the variation of consumption over the lifecycle account for between 30 and 42 per cent of measured annual consumption inequality, which has traditionally been interpreted as an index of interpersonal inequality. Annual inequality, that is, overstates pure interpersonal inequality by 43 to 72 per cent.

I turn now to the empirical results for indices of relative inequality presented in Table 3. Relative inequality indices range over a [0,1] interval, their value representing the proportion of total consumption wasted on inequality. Relative inequality equal to 0.5 means that one-half of total consumption could be thrown away, or, equivalently, that each individual would need to receive only one-half the mean consumption, in an equal distribution that is socially equivalent to the original. Since relative inequality is equal to absolute inequality normalized on the mean, the conditions for attaining the upper and lower bounds are the same for relative and absolute indices. Zero relative inequality implies an equal original distribution or zero inequality aversion in the social evaluation function. Relative inequality equal to unity is attained when someone receives zero consumption and the degree of inequality aversion is infinite.

Total relative inequality, given in the first row of Table 3, is decomposed into the sum of its three components which are given in rows 2 to 4. Relative magnitudes of the subindices, as percentages of total inequality, are shown in parentheses. Once again, minimum, maximum, and mean annual relative inequality are given for purposes of comparison. AKS indices are reported for eight values of the degree of relative inequality aversion parameter, and the Gini and Maximin indices have also been computed.

Similar patterns emerge here as were seen in Table 2.[54] Total relative inequality is, as expected, approximately equal to average annual inequality. Of the three components into which total inequality is decomposed, intercohort inequality again accounts for the smallest part of total inequality -- in most cases less than 10 per cent. For high degrees of relative inequality aversion ($R \leq -5$) intercohort inequality is about 20 per cent of the total. Intracohort inequality is the largest of the three subindices of relative inequality for all cases. Intrapersonal inequality ranges from less than 28 per cent of intracohort inequality when $R=0.9$ (indicating a low degree of relative inequality aversion), to about 44 per cent when $R=-10$. In the case of maximin, intrapersonal inequality is 61 per cent,

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[54]In the case of the Gini coefficient, the relative magnitudes of the per capita and AKS subindices of inequality are identical because the Gini per capita and AKS inequality indices are both based on the same social evaluation function. The same is true of the maximin indices. The other inequality indices reported in Tables 2 and 3 are based on different social evaluation functions (see (4.1) and (4.2)).

and in the Gini case, 70 per cent, of intracohort inequality.

The most interesting comparison is of intracohort inequality in the decomposition approach with mean annual inequality. It can be seen that, in all cases, the index of annual inequality overstates pure interpersonal inequality as measured by intracohort inequality in the decomposition approach. At lower degrees of inequality aversion ($R \geq -1$) the magnitude of this overstatement is about one-third but thereafter it rises monotonically. When $R=-10$ mean annual inequality exceeds intracohort inequality by 65 per cent. Notice that the problem is worse for the Gini index than any of the AKS indices, and that the maximin index is the worst of all. Annual inequality overstates intracohort inequality by 76 per cent in the case of the Gini index, and by 106 per cent in the case of the maximin. These empirical results indicate the importance of adopting a decomposition approach to the measurement of inequality. To do otherwise -- that is, to measure inequality in the distribution of annual consumption -- is to risk seriously overstating the degree of pure interpersonal inequality.

Consider now the indices of relative equality reported in Table 4. One should expect certain patterns to emerge because of the fact that indices of relative equality and inequality sum to unity. This is true of the annual indices (rows 5 to 7) and of the index of total inequality (row 1), but not of the subindices (rows 2 to 4). The reason for this is that the bases on which the decomposition indices of inequality and equality express the social cost of maldistribution are different. The subindices of relative inequality are calculated as the social saving which results from eliminating some inequality, expressed as a proportion of the same base, namely total consumption in the original situation. The base on which subindices of relative equality are constructed, however, changes with each move between successive pairs of reference vectors: the base is total consumption in the more unequally distributed reference vector. It is thus necessary to examine separately the empirical results on indices of relative equality.

Once again the fact that annual relative equality does not change much from one year to the next shows up in the similarity between mean annual relative equality and total equality measured in the panel overall. The increase in measured equality that is expected when the decomposition approach is employed rather than the traditional approach of measuring annual equality is also borne out by the empirical results. For degrees of inequality aversion between 0.9 and -2 inclusive, mean annual equality is roughly 80 to 100 per cent of intracohort equality. For higher degrees of relative inequality aversion, this proportion falls to less than one-half, and in the case of the maximin index, it is a mere 22 per cent.

What the indices reported in Table 4 lack is stability in the relative magnitudes of the decomposition approach subindices which characterized the subindices of absolute and relative inequality. The intrapersonal and intercohort aspects of the distribution share the property of being least unequallly distributed. At low degrees of inequality aversion ($R \geq -2$), intercohort relative equality is greater than intrapersonal relative equality. The reverse is true for higher degrees of inequality aversion.

The results reported in the preceding three tables indicate that concern about the inadequacy of measuring annual inequality is well justified. Both mobility- and age-related inequality account for significant portions of total measured inequality and thus of annual inequality.[55] Intercohort inequality, which reflects consumption differences attributable to the age-structure of the population, runs in the neighbourhood of 5 to 10 per cent of the total. Intrapersonal inequality, which captures the effect of individual consumption mobility, averages about one-quarter to one-third of total absolute inequality, and about one-fifth of total relative inequality. Accounting for these sources of inequality and excluding them from the measurement of pure interpersonal inequality indicates that annually measured inequality overestimates pure interpersonal inequality by at least one-third, often as much as one-half, and in some cases by more than 70 per cent. Clearly, measured annual inequality cannot be relied upon to provide an accurate assessment of the social significance of pure interpersonal inequality.

There are several variables in the specification of the decomposition approach to the measurement of inequality that may differ from one application of the method to another. It is important that the decomposition approach be robust to such changes if it is to be judged a reliable method of measuring

[55]This arises because total and annual inequality are roughly equal, indicating a fairly constant trend of annual inequality.

inequality. I have thus investigated the sensitivity of the decomposition approach indices to changes in the size of age-cohort brackets and the number of years of data used in the computation of the indices.

First, age-cohorts have been defined annually thus far. The number of years included in each age-cohort (B) should affect the division of interpersonal inequality within and between age-cohorts, an increase in the size of the bracket causing inequality within cohorts to rise at the expense of intercohort inequality. The results reported in Tables 5 and 6 confirm this (only intra- and intercohort indices are reported since age-cohort bracket size does not affect total or intrapersonal inequality). Table 7 indicates that intracohort equality varies inversely, <u>ceterus paribus</u>, with the size of age-cohort brackets. This is as expected. Since the index of interpersonal relative equality is equal to the product of the intra- and intercohort indices, and relative equality indices range over a [0,1] interval, the subindices of intra- and intercohort equality must have values greater than that of the index of interpersonal relative equality. As the width of age-cohort brackets is increased, greater emphasis is placed on the intracohort component of interpersonal relative equality at the expense of the intercohort component. When B is so large that everyone is included in the same cohort, intracohort and interpersonal equality coincide (i.e. all interpersonal equality is within cohorts since there is only one cohort). Thus intracohort equality must fall as B, the number of years in each age-cohort, rises.

Although there are no restrictions on the number of years included in each age-cohort bracket, one, five, and ten years seem the most natural definitions. Tables 5 and 6 reveal that the relative magnitudes of intra- and intercohort inequality change fairly smoothly as the size of age-cohort brackets increases, with most of the effect having been felt by the time age-brackets span five years. There is only a small effect when age bracket size is increased from five to ten years. Notice that this increase in intracohort inequality as age-cohort brackets are widened reduces the discrepancy between the decomposition and traditional approaches to measuring inequality. With annual age-cohort brackets the ratio of mean annual absolute inequality to intracohort absolute inequality ranges from 1.40 to 1.72. The range of the same ratio with five year age-cohort brackets is 1.31 to 1.60, indicating that the mild quantitative effects of widening age-cohort brackets are not sufficient to alter the conclusion regarding the importance of adopting a decomposition approach to the measurement of inequality.

I have also investigated the sensitivity of the decomposition indices of equality and inequality to the number of years of data employed in their calculation. There are both theoretical and empirical reasons for doing this.

In the welfare approach, the equally distributed equivalent consumption is a summary statistic of an individual's lifecycle profile. It is used in the computation of intrapersonal inequality, which reflects that part of total inequality attributable to the shape of consumption profiles. But lifecycle data are not available, so the decomposition approach is employed to compute indices and subindices of inequality from panel data. When the number of years of data is small, however, it is difficult to claim that an individual's equally distributed equivalent consumption reflects the shape of his consumption profile. In the short run, representative consumption accounts for the effects of mobility, and intrapersonal inequality may be interpreted as an index of mobility-related inequality. These are not dichotomous interpretations, but rather the extremes of a continuous shift in the interpretations of individual representative consumption and intrapersonal inequality indices with the number of years of data. This is important for my purpose because the behaviour of the decomposition indices may provide information on the relative importance of these two aspects of the intertemporal variation of individual consumption.

With only one year of data, of course, representative consumption equals annual consumption and intrapersonal inequality is zero. As the number of years of data rises, the effects of consumption mobility are increasingly reflected in representative consumption and intrapersonal inequality. If short-run mobility were the only source of intertemporal variation it should be expected that both representative consumption and the index of intrapersonal inequality would approach limits: the former to average consumption adjusted for the degree of mobility[56] and the latter to the degree of inequality that this intertemporal distribution displays. On the other hand, if intertemporal variation in consumption is of a long-run nature, reflecting the shape of lifecycle consumption profiles, representative consumption would be less likely to approach a limit. Thus the sensitivity of the index of intrapersonal inequality to the number of years of data may indicate whether the variation in individual consumption over time is a short-run or long-run phenomenon.[57] It may well be that ten years of data are not enough to provide a clear indication of the nature of consumption variation over time, and in any event, it is likely that both effects are operative, and the distinction between them not clear cut. I leave it open to an examination of the results of this experiment to reveal what they may.

The second purpose for conducting this experiment is more for reasons of empirical practicality. It may be that the decomposition indices approach limiting values sufficiently closely when computed with less than ten years of data. For example, if the trend of annual inequality is roughly constant, the decomposition approach index of total inequality will show little variation as the number of years of data increases. If the index of intrapersonal inequality approaches a limiting

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[56]This is exactly analogous to the idea that equally distributed equivalent consumption is mean consumption adjusted for inequality.

[57]Similarly Shorrocks [1978a, p.389] argues that mobility may occur in either the transitory or permanent component of total income.

value fairly quickly, then the intracohort inequality index may also display considerable stability since the index of intercohort inequality is likely to show little sensitivity to the number of years of data.[58] In this case considerable data collection and computing cost savings could be realized because sufficiently accurate indices could be produced with less than the full ten years of data. While there is no reason a priori to expect decomposition approach indices to approach limits or display such stability, I am encouraged by Shorrocks' [1978b] finding that, in some population age-cohorts, his mobility index[59] approached a limit when computed with as few as five years of data. The results of my experiment are reported in Tables 8, 9, and 10.

Table 8 reveals that total absolute inequality varies somewhat with the number of years of data, but within a fairly restricted range. Its changing value reflects differences in annual inequality. Intrapersonal inequality, on the other hand, increases monotonically as the number of years of data is increased. This is due to the tendency for real consumption to grow over time and the fact that indices of absolute inequality are not mean independent. Both intra- and intercohort absolute

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[58]Intercohort inequality reflects the contribution of the age-structure of the population to total inequality when computed with annual, or only a few years of data. With lifecycle data it measures inequality due to economic growth. In either case it is not likely to depend much on the number of years of data used in its computation.

[59]This index is the ratio of a weighted average of annual inequality indices to an inequality index of consumption accrued over the entire time period (number of years) for which data are available.

inequality tend to decline with increases in the number of years of data used in their computation. Neither appears to approach a limiting value.

Tables 9 and 10 report the results of the experiment with the effect of the time span of the data set on indices of relative inequality and equality. Since these indices are mean independent, total measured (in)equality reflects the impact only of the distribution, and not the mean, of each additional year of data. The remarkable stability displayed by total measured relative (in)equality lends support to the widely observed tendency of annual (in)equality to remain fairly constant over time. Intrapersonal relative inequality (equality) can be seen to rise (fall) monotonically as the number of years of data is increased. It is not possible to reach a conclusion on the nature of individual consumption variation over time on the basis of these results. It certainly supports the view, however, that measuring inequality in an annual distribution is inadequate. There is some indication that the subindices of intracohort relative (in)equality do approach a limit within the ten years of the data set. If this is a general rather than data-specific property, then decomposition approach indices of pure interpersonal inequality could be computed accurately with fewer than ten years of panel data.[60] Based only on the current evidence, however, this would have to be taken as a very

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[60]This property would also prove advantageous to the measurement of the trend of inequality in the decomposition approach since, as is explained in Chapter Five, a cohort must be excluded from the population for every year of data used in the computation of the decomposition approach index of intracohort inequality.

tentative conclusion.

The empirical results presented in this chapter will, I hope, contribute to a better understanding of the decomposition approach to the measurement of inequality. The evidence indicates that the decomposition approach provides a considerably clearer picture of the dimensions of inequality by allowing indices of mobility- and age-related inequality to be computed. These indices are of intrinsic interest since they quantify the effects of two sources of consumption differences on measured inequality. In addition, of course, they allow an index of pure interpersonal inequality to be calculated. In the distribution taken from the Panel Study on Income Dynamics it was found that annual inequality overstates pure interpersonal inequality by at least one-third, and in some cases up to 75 per cent. Such magnitudes underscore the theoretical arguments in favour of adopting the decomposition approach to the measurement of inequality.

The decomposition of inequality within and among age-cohorts was shown, as expected, to depend on the width of cohort brackets. Nevertheless, the quantitative effects are not sufficiently large to undermine the conclusion regarding the superiority of the decomposition approach over the traditional practice of measuring annual inequality. The investigation of the sensitivity of decomposition approach indices to the number of years of data suggested reasonably accurate results for relative (in)equality might be obtained with fewer than ten years of data. Further empirical work is required to substantiate this tentative conclusion.

The decomposition approach to the measurement of inequality thus seems to be an accurate, efficient, and robust method of measuring inequality that can successfully be employed to solve several long-standing problems of inequality measurement. The effects of consumption mobility and the age-structure of the population confuse not only the measurement of the level of inequality, however, but also its trend. An accurate assessment of "trend" inequality clearly depends on careful and correct measurement of "static" inequality. The decomposition approach index of intracohort inequality provides such a measure of pure interpersonal inequality. In the next chapter I apply the decomposition approach to the problem of determining the trend of inequality.

FIGURE III

Histogram of the Panel Data

middle of interval	number of obs.	
500.	972	*********
1500.	4743	**
2500.	5803	***
3500.	5055	***
4500.	4153	***********************************
5500.	2699	***********************
6500.	1633	**************
7500.	1230	***********
8500.	711	******
9500.	452	****
10500.	316	***
11500.	217	**
12500.	122	**
13500.	71	*
14500.	53	*
15500.	39	*
16500.	35	*
17500.	21	*
18500.	8	*
19500.	15	*
20500.	22	*
21500.	10	*
22500.	1	*
23500.	1	*
24500.	5	*
25500.	1	*
26500.	4	*
27500.	3	*
28500.	0	
29500.	3	*
30500.	0	
31500.	0	
32500.	1	*
33500.	1	*
34500.	1	*
35500.	1	*
36500.	1	*
37500.	2	*
38500.	1	*
39500.	0	
40500.	0	
41500.	2	*
42500.	0	
43500.	2	*
44500.	0	
45500.	0	
46500.	0	
47500.	0	
48500.	0	
49500.	0	
50500.	0	
51500.	1	*

each * represents
120 observations

TABLE 1

Annual Means and Standard Deviations of the Panel Data

YEAR	MEAN	STANDARD DEVIATION
1968	3480.4	2317.3
1969	3626.2	2370.6
1970	3761.1	2487.5
1971	3922.3	2620.1
1972	4191.2	2714.3
1973	4402.2	2819.2
1974	4341.3	2953.1
1975	4300.3	2989.3
1976	4504.6	3043.9
1977	4549.9	2896.1
pooled	4091.1	2741.4

TABLE 2

Indices of Absolute Inequality

DECOMPOSITION INDICES

	$G=5x10^{-6}$	$G=5x10^{-5}$	$G=1x10^{-4}$	$G=5x10^{-4}$
A	18.61	169.17	311.12	1040.56
A_{AP}	4.49 (24.1)	42.88 (25.3)	82.00 (26.4)	316.76 (30.4)
A_{AC}	13.05 (70.1)	118.54 (70.1)	216.43 (69.6)	684.95 (65.8)
A_{RC}	1.07 (5.7)	7.75 (4.6)	12.69 (4.1)	38.85 (3.7)

ANNUAL INDICES

min	13.29	123.16	230.13	813.78
mean	18.29	166.32	306.12	1027.82
max	22.87	207.79	381.49	1243.98

DECOMPOSITION INDICES

	GINI	$G=1x10^{-3}$	$G=5x10^{-3}$	MAXIMIN
A	1362.52	1544.46	2762.01	3934.22
A_{AP}	536.85 (39.4)	503.75 (32.6)	1009.47 (36.5)	1101.81 (28.0)
A_{AC}	767.03 (56.3)	977.93 (63.3)	1591.89 (57.6)	1813.83 (46.1)
A_{RC}	58.63 (4.3)	62.78 (4.1)	160.65 (5.8)	1018.58 (25.9)

ANNUAL INDICES

min	1171.43	1235.97	2252.46	3100.60
mean	1347.90	1528.97	2745.15	3735.17
max	1527.68	1811.70	3242.13	4359.29

TABLE 3

Indices of Relative Inequality

DECOMPOSITION INDICES

	R=.9	R=.5	R=0	R=-.5	GINI
I	.0185	.0899	.1747	.2553	.3330
I_{AP}	.0038 (20.5)	.0187 (20.8)	.0372 (21.3)	.0554 (21.7)	.1312 (39.4)
I_{AC}	.0136 (73.5)	.0665 (74.0)	.1287 (73.7)	.1861 (72.9)	.1875 (56.3)
I_{RC}	.0011 (5.9)	.0048 (5.3)	.0088 (5.0)	.0137 (5.4)	.0143 (4.3)

ANNUAL INDICES

	R=.9	R=.5	R=0	R=-.5	GINI
min	.0168	.0816	.1580	.2301	.3170
mean	.0181	.0880	.1713	.2503	.3295
max	.0190	.0924	.1790	.2617	.3391

DECOMPOSITION INDICES

	R=-1	R=-2	R=-5	R=-10	MAXIMIN
I	.3321	.4761	.7956	.9076	.9616
I_{AP}	.0729 (22.0)	.1041 (21.9)	.1688 (21.2)	.2236 (24.6)	.2693 (28.0)
I_{AC}	.2391 (72.0)	.3310 (69.5)	.4640 (58.3)	.5079 (56.0)	.4434 (46.1)
I_{RC}	.0202 (6.1)	.0411 (8.6)	.1629 (20.5)	.1760 (19.4)	.2490 (25.9)

ANNUAL INDICES

	R=-1	R=-2	R=-5	R=-10	MAXIMIN
min	.2978	.4182	.6444	.7487	.8516
mean	.3260	.4692	.7370	.8395	.9130
max	.3394	.5128	.8664	.9305	.9639

TABLE 4

Indices of Relative Equality

DECOMPOSITION INDICES

	R=.9	R=.5	R=0	R=-0.5	GINI
E	.9815	.9101	.8253	.7447	.6670
E_{AP}	.9962	.9813	.9628	.9446	.8688
E_{AC}	.9864	.9323	.8663	.8029	.7842
E_{RC}	.9989	.9948	.9894	.9819	.9790

ANNUAL INDICES

	R=.9	R=.5	R=0	R=-0.5	GINI
min	.9810	.9076	.8210	.7382	.6609
mean	.9819	.9120	.8287	.7497	.6705
max	.9832	.9184	.8420	.7699	.6830

DECOMPOSITION INDICES

	R=-1	R=-2	R=-5	R=-10	MAXIMIN
E	.6679	.5239	.2044	.0924	.0384
E_{AP}	.9271	.8959	.8312	.7764	.7307
E_{AC}	.7422	.6306	.4419	.3458	.3932
E_{RC}	.9709	.9273	.5564	.3443	.1335

ANNUAL INDICES

	R=-1	R=-2	R=-5	R=-10	MAXIMIN
min	.6606	.4872	.1336	.0695	.0361
mean	.6606	.5308	.2630	.1605	.0870
max	.7022	.5818	.3556	.2513	.1484

TABLE 5

The Effect of Age-cohort Bracket Size on Intra- and Intercohort Absolute Inequality

G		NUMBER OF YEARS IN EACH AGE-COHORT BRACKET			
		B=1	B=3	B=5	B=10
$5x10^{-6}$	A_{AC}	13.05	13.73	13.94	13.95
	A_{RC}	1.07	.38	.18	.16
$5x10^{-5}$	A_{AC}	118.54	123.25	124.97	125.07
	A_{RC}	7.75	3.04	1.31	1.22
$1x10^{-4}$	A_{AC}	216.43	223.83	226.98	227.19
	A_{RC}	12.69	5.29	2.15	1.93
$5x10^{-4}$	A_{AC}	684.95	703.83	716.24	717.60
	A_{RC}	38.85	19.97	7.56	6.20
GINI	A_{AC}	767.03	796.23	808.38	811.14
	A_{RC}	58.63	29.44	17.28	14.53
$1x10^{-3}$	A_{AC}	977.93	1005.63	1026.16	1028.78
	A_{RC}	62.78	35.08	14.55	11.94
$5x10^{-3}$	A_{AC}	1591.89	1676.19	1717.32	1723.76
	A_{RC}	160.65	76.36	35.23	28.79
MAXIMIN	A_{AC}	1813.83	2081.19	2288.49	2348.19
	A_{RC}	1018.58	715.62	553.05	527.53

TABLE 6

The Effect of Age-cohort Bracket Size on Intra- and Intercohort Relative Inequality

R		NUMBER OF YEARS IN EACH AGE-COHORT BRACKET			
		B=1	B=3	B=5	B=10
.9	I_{AC}	.0136	.0143	.0145	.0145
	I_{RC}	.0011	.0004	.0002	.0002
.5	I_{AC}	.0665	.0693	.0705	.0706
	I_{RC}	.0048	.0020	.0007	.0007
0	I_{AC}	.1287	.1335	.1362	.1364
	I_{RC}	.0088	.0040	.0014	.0011
-.5	I_{AC}	.1861	.1931	.1974	.1979
	I_{RC}	.0137	.0068	.0024	.0019
GINI	I_{AC}	.1875	.1946	.1976	.1983
	I_{RC}	.0143	.0072	.0042	.0036
-1	I_{AC}	.2391	.2487	.2551	.2561
	I_{RC}	.0202	.0105	.0041	.0031
-2	I_{AC}	.3310	.3494	.3626	.3652
	I_{RC}	.0411	.0227	.0095	.0068
-5	I_{AC}	.4640	.5083	.5391	.5452
	I_{RC}	.1629	.1187	.0878	.0817
-10	I_{AC}	.5079	.5546	.5819	.5890
	I_{RC}	.1760	.1293	.1020	.0950
MAXIMIN	I_{AC}	.4434	.5087	.5594	.5740
	I_{RC}	.2490	.1749	.1352	.1289

TABLE 7

The Effect of Age-cohort Bracket Size on Intra- and Intercohort Relative Equality

R		NUMBER OF YEARS IN EACH AGE-COHORT BRACKET B=1	B=3	B=5	B=10
.9	E_{AC}	.9864	.9857	.9854	.9854
	E_{RC}	.9989	.9995	.9998	.9998
.5	E_{AC}	.9323	.9294	.9282	.9281
	E_{RC}	.9948	.9979	.9992	.9993
0	E_{AC}	.8663	.8613	.8586	.8583
	E_{RC}	.9894	.9952	.9984	.9986
-.5	E_{AC}	.8029	.7956	.7910	.7904
	E_{RC}	.9819	.9909	.9967	.9975
GINI	E_{AC}	.7842	.7760	.7726	.7718
	E_{RC}	.9790	.9893	.9937	.9947
-1	E_{AC}	.7422	.7318	.7248	.7238
	E_{RC}	.9707	.9845	.9939	.9953
-2	E_{AC}	.6306	.6100	.5953	.5924
	E_{RC}	.9273	.9856	.9823	.9871
-5	E_{AC}	.4419	.3886	.3515	.3441
	E_{RC}	.5564	.6238	.6995	.7145
-10	E_{AC}	.3458	.2857	.2505	.2414
	E_{RC}	.3443	.4168	.4754	.4933
MAXIMIN	E_{AC}	.3932	.2954	.2368	.2257
	E_{RC}	.1335	.1798	.2210	.2292

TABLE 8

The Effect of Number of Years of Data on Indices and Subindices of Absolute Inequality

G		NUMBER OF YEARS OF DATA 4	5	6	7
	A	173.25	167.16	171.64	165.91
	A_{AP}	26.83 (15.5)	29.98 (17.9)	34.50 (20.1)	36.65 (22.1)
$5x10^{-5}$	A_{AC}	136.12 (78.6)	127.30 (76.2)	127.50 (74.3)	120.43 (72.6)
	A_{RC}	10.30 (5.9)	9.88 (5.9)	9.63 (5.6)	8.82 (5.3)
	mean	172.50	165.85	170.46	164.18
	A	1055.74	1023.71	1040.84	1016.16
	A_{AP}	203.62 (19.3)	224.33 (21.9)	250.44 (24.1)	266.52 (26.2)
$5x10^{-4}$	A_{AC}	801.20 (75.9)	752.30 (73.5)	743.53 (71.4)	707.51 (69.6)
	A_{RC}	50.91 (4.8)	47.07 (4.6)	46.87 (4.5)	42.13 (4.1)
	mean	1052.26	1017.70	1035.60	1008.10
	A	2863.74	2769.26	2783.50	2723.87
	A_{AP}	628.37 (21.9)	712.92 (25.7)	795.26 (28.6)	842.96 (30.9)
$5x10^{-3}$	A_{AC}	1964.22 (68.6)	1823.33 (65.8)	1761.72 (63.3)	1690.96 (62.1)
	A_{RC}	271.15 (9.5)	233.01 (8.4)	226.52 (8.1)	189.95 (7.0)
	mean	2845.34	2754.60	2767.08	2706.53

TABLE 8 (CON'T)

G		8	9	10
		NUMBER OF YEARS OF DATA		
	A	171.21	166.46	169.17
	A_{AP}	39.42 (23.0)	40.62 (24.4)	42.88 (25.3)
$5x10^{-5}$	A_{AC}	123.34 (72.0)	117.83 (70.8)	118.54 (70.1)
	A_{RC}	8.45 (4.9)	8.02 (4.8)	7.75 (4.6)
	mean	169.28	163.84	166.32
	A	1043.90	1021.37	1040.56
	A_{AP}	289.64 (27.7)	298.08 (29.2)	316.76 (30.4)
$5x10^{-4}$	A_{AC}	713.62 (68.4)	684.61 (67.0)	684.95 (65.8)
	A_{RC}	40.64 (3.9)	38.67 (3.8)	38.85 (3.7)
	mean	1035.70	1009.53	1027.82
	A	2770.23	2713.71	2762.01
	A_{AP}	911.85 (32.9)	950.68 (35.0)	1009.47 (36.5)
$5x10^{-3}$	A_{AC}	1675.94 (60.5)	1601.11 (59.0)	1591.89 (57.6)
	A_{RC}	182.45 (6.6)	161.91 (6.0)	160.65 (5.8)
	mean	2754.48	2695.28	2745.15

TABLE 9

The Effect of Number of Years of Data on Indices and Subindices of Relative Inequality

R		NUMBER OF YEARS OF DATA 2	3	4	5	6
	I	.0843	.0852	.0865	.0870	.0877
	I_{AP}	.0060 (7.1)	.0096 (11.3)	.0109 (12.6)	.0128 (14.7)	.0140 (16.0)
.5	I_{AC}	.0729 (86.5)	.0696 (81.7)	.0696 (80.5)	.0684 (78.6)	.0681 (77.7)
	I_{RC}	.0054 (6.4)	.0060 (7.0)	.0059 (6.8)	.0058 (6.7)	.0057 (6.5)
	mean	.0841	.0847	.0860	.0862	.0870
	I	.2415	.2430	.2456	.2465	.2472
	I_{AP}	.0175 (7.2)	.0278 (11.4)	.0319 (13.0)	.0372 (15.1)	.0408 (16.5)
-.5	I_{AC}	.2078 (86.0)	.1981 (81.5)	.1970 (80.2)	.1935 (78.5)	.1906 (77.1)
	I_{RC}	.0163 (6.7)	.0171 (7.0)	.0166 (6.8)	.0158 (6.4)	.0158 (6.4)
	mean	.2406	.2413	.2442	.2442	.2453
	I	.7141	.7687	.8251	.8132	.8083
	I_{AP}	.0506 (7.1)	.0792 (10.3)	.0949 (11.5)	.1114 (13.7)	.1242 (15.4)
-5	I_{AC}	.5072 (71.0)	.4981 (64.8)	.4961 (60.1)	.4819 (59.3)	.4683 (57.9)
	I_{RC}	.1562 (21.9)	.1915 (24.9)	.2341 (28.4)	.2197 (27.0)	.2157 (26.7)
	mean	.6933	.7252	.7607	.7388	.7250

TABLE 9 (CON'T)

R		NUMBER OF YEARS OF DATA			
		7	8	9	10
	I	.0884	.0891	.0900	.0899
	I_{AP}	.0156 (17.6)	.0165 (18.5)	.0179 (19.9)	.0187 (20.8)
.5	I_{AC}	.0674 (76.2)	.0675 (75.8)	.0671 (74.6)	.0665 (74.0)
	I_{RC}	.0054 (6.1)	.0051 (5.7)	.0050 (5.6)	.0048 (5.3)
	mean	.0872	.0879	.0882	.0880
	I	.2498	.2517	.2546	.2553
	I_{AP}	.0452 (18.1)	.0484 (19.2)	.0529 (20.8)	.0554 (21.7)
-.5	I_{AC}	.1898 (76.0)	.1891 (75.1)	.1878 (73.8)	.1861 (72.9)
	I_{RC}	.0148 (5.9)	.0141 (5.6)	.0139 (5.5)	.0137 (5.4)
	mean	.2467	.2484	.2498	.2503
	I	.7994	.8010	.7937	.7956
	I_{AP}	.1373 (17.2)	.1477 (18.4)	.1598 (20.1)	.1688 (21.2)
-5	I_{AC}	.4675 (58.5)	.4645 (58.0)	.4601 (58.0)	.4640 (58.3)
	I_{RC}	.1946 (24.3)	.1887 (23.6)	.1738 (21.9)	.1629 (20.5)
	mean	.7205	.7320	.7284	.7370

TABLE 10

The Effect of Number of Years of Data on Indices and Subindices of Relative Equality

		NUMBER OF YEARS OF DATA				
R		2	3	4	5	6
	E	.9157	.9148	.9135	.9130	.9123
	E_{AP}	.9940	.9904	.9891	.9872	.9860
.5	E_{AC}	.9267	.9297	.9296	.9307	.9310
	E_{RC}	.9941	.9935	.9936	.9937	.9938
	mean	.9159	.9153	.9140	.9138	.9130
	E	.7585	.7570	.7544	.7535	.7528
	E_{AP}	.9825	.9722	.9681	.9628	.9592
-.5	E_{AC}	.7885	.7962	.7965	.7991	.8013
	E_{RC}	.9790	.9779	.9784	.9794	.9794
	mean	.7594	.7587	.7558	.7558	.7547
	E	.2859	.2313	.1749	.1868	.1917
	E_{AP}	.9494	.9208	.9051	.8886	.8758
-5	E_{AC}	.4657	.4591	.4519	.4577	.4653
	E_{RC}	.6467	.5471	.4277	.4592	.4706
	mean	.3067	.2748	.2393	.2612	.2750

TABLE 10 (CON'T)

R		7	8	9	10
		NUMBER OF YEARS OF DATA			
	E	.9116	.9109	.9100	.9101
	E_{AP}	.9844	.9835	.9821	.9813
.5	E_{AC}	.9315	.9314	.9317	.9323
	E_{RC}	.9941	.9944	.9945	.9948
	mean	.9128	.9121	.9118	.9120
	E	.7502	.7483	.7454	.7447
	E_{AP}	.9548	.9516	.9471	.9446
-.5	E_{AC}	.8013	.8012	.8017	.8029
	E_{RC}	.9806	.9815	.9817	.9819
	mean	.7533	.7516	.7502	.7497
	E	.2006	.1990	.2063	.2044
	E_{AP}	.8627	.8523	.8402	.8312
-5	E_{AC}	.4581	.4549	.4524	.4419
	E_{RC}	.5075	.5133	.5428	.5564
	mean	.2795	.2680	.2716	.2630

CHAPTER FIVE

The Trend Of Lifecycle Inequality

One of the stylized facts about inequality is that it has remained virtually constant in the post-World-War II period. All the criticisms that have been leveled at the practice of measuring inequality in the distribution of annual income, however, apply to the determination of the trend of inequality as well. An accurate assessment of the trend of inequality presumes an accurate measure of "static"[61] inequality. The development and illustration of the latter have occupied the preceding three chapters. In this chapter I consider the trend of lifecycle inequality.

As the static measurement of lifecycle inequality differs from that of annual inequality, so does the trend of lifecycle inequality differ from its annual counterpart. When inequality is measured in the distribution of annual income, its trend is simply the time series of the annual values of the index; it changes from year to year as individual incomes change. But the distribution of representative lifecycle consumption, in which lifecycle inequality is measured, is independent of time; it changes only as the population changes. Of course, changes in

[61]By "static" I mean inequality measured in a given distribution, without allowing for any change to occur. For example, annual inequality is a "static" measure, as are the indices reported in Chapter Four despite the fact that they measure lifecycle inequality.

the population occur over time, as individuals begin and end their (economic) lives, but in a lifecycle context a calendar year serves only to identify a population. Inequality is measured in the distribution of representative lifecycle consumption over the members of this population. Thus, although lifecycle inequality may be said to evolve over time, its trend is an interpersonal or interpopulation, rather than an intertemporal, phenomenon.

Measuring the trend of lifecycle inequality thus requires that measured inequality in social states with different populations can meaningfully be compared. It will therefore be necessary to replace a single social evaluation function or inequality index with a family of such functions or indices having one member for each possible population size. As a consequence, some means of linking together members of a family will be required to ensure that all functions or indices in a family reflect the same set of ethical judgments.

The chapter begins with a discussion of the nature of the trend of lifecycle inequality and the theoretical implications for fixed population social evaluation functions. I then consider the construction of variable population social evaluation functions and the ethical judgments that can be built into them, particularly as regards the evaluation of inequality in different social states. There follows a theoretical discussion of the trend of inequality in the welfare and decomposition approaches to the measurement of inequality, and, in the latter case, presentation of empirical evidence on the trend of lifecycle inequality. The chapter closes with a comparison of these results with the trend of annual inequality and some concluding comments.

In addition to "static" inequality, investigators may wish to examine the trend of inequality. When inequality is measured in an annual distribution, its trend is reflected in the time path of the inequality index, which changes year after year as individual incomes change. The trend of lifecycle inequality is essentially different, however, because representative lifecycle consumption is a summary statistic of the level of well-being that an individual is afforded by his lifecycle consumption profile; it attaches to an individual independently of time and so does not change from one year to the next. The distribution of representative lifecycle consumption changes only when the population under investigation changes. Lifecycle inequality is time-dependent only in the sense that it is over time that changes occur in the population. If the population were unchanged from one year to the next, so would be measured lifecycle inequality.[62] Typically, of course, the population does change over time, and the trend of inequality in a lifecycle context reflects the effect of this population change on inequality measured in the distribution of representative lifecycle consumption. The trend of lifecycle inequality is thus seen to be an interpersonal or interpopulation phenomenon rather

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[62]The same is not true of annual inequality, unless the new distribution is simply a permutation of the old, because individual incomes change from year to year.

than an intertemporal one.[63] Since the problem is one of measuring and comparing inequality in different populations, a solution may naturally be sought in the theory of variable population social evaluation functions. The following brief outline of the theory is drawn from Blackorby and Donaldson [1979], although I present it in terms that are relevant to the trend of lifecycle inequality (i.e. the arguments of the fixed population social evaluation functions are representative lifecycle consumptions).

The construction of a variable population social evaluation function involves two steps. First, in place of a single social evaluation function a family of (fixed population) social evaluation functions, one for every possible population size, is required. The problem of there being different people in two different social states is reduced to a problem of different population sizes by the adoption of the assumptions of anonymity[64] and a variable population analogue of welfarism.[65] Welfarism implies that social states can be fully characterized by the vector of representative lifecycle consumptions of all individuals who exist in that social state. The problem of evaluating social states is thus reduced to the need for a social ordering over all finitely-dimensioned vectors of

[63]It should also be clear that the trend of annual inequality involves both intertemporal and interpersonal effects since not only annual incomes, but the population too, change from year to year.

[64]This is the assumption that who a person is doesn't matter. It is captured by the symmetry of the social evaluation funcions and inequality indices.

[65]Welfarism is implied by the conjunction of Unlimited Domain, Independence of Irrelevant Alternatives, and Pareto Indifference.

representative consumptions. This social ordering is represented by a family of (fixed population) social evaluation functions, $W^n: R^N \rightarrow R^1$, a typical member of which has the image,

(5.1) $\quad w_n = W^n(r_n)$

where $r_n = (r_1, \ldots, r_N)$ is the vector of representative lifecycle consumptions in social state n. I assume that r_n can be represented by its equally distributed equivalent, implicitly defined by,

(5.2) $\quad W^n(s_n \underline{1}_N) = W^n(r_n)$

$W^n(.)$ is assumed to be continuous, increasing, and S-concave -- properties which guarantee the unique existence of the population-wide representative consumption.

(5.3) $\quad s_n = S^n(r_n)$

Members of the family (5.1) must somehow be linked to ensure that they all reflect the same set of ethical judgments. This is accomplished by the adoption of the <u>Population Substitution Principle</u>:[66]

If $r = (r_n, r_m)$, $r_n \in R^N$, $r_m \in R^M$, then

$s_{n+m} = S^{n+m}(r) = S^{n+m}(r_n, r_m) = S^{n+m}(s_n \underline{1}_N, r_m)$

where $s_n = S^n(r_n)$

By requiring that the equally distributed equivalent consumption be unchanged when a group's vector of representative consumptions is replaced by its equally distributed equivalent, the Population Substitution Principle ensures that the social evaluation of the group's situation is consistent with that of

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[66]The Population Substitution Principle implies some other, weaker, population principles, including Dalton's [1920] Principle of Population Replication.

the population. This implies that the members of the family of (fixed population) social evaluation functions or equally distributed equivalent functions represent the same social preferences.[67] The Population Substitution Principle also implies that the family of fixed population social evalution functions must be additively separable (Blackorby and Donaldson [1979], theorem 3.2). This should not be too surprising given that the Population Substitution Principle views two social states with different sized populations as subgroups of a single population comprised of their sum, which is exactly analogous to the structure imposed on the social evaluation function in the decomposition of inequality within and among subgroups. An additively separable functional structure results in both cases. Thus the families of indices that are admissible in the welfare and decomposition approaches to the measurement of inequality satisfy the Principle of Population Substitution.

The second step in the construction of a variable population social evaluation function is the definition of the function itself. This requires an assumption that fixed population social evaluation functions can be represented by their equally distributed equivalents.[68] The variable population social evaluation function $f:R^2 \rightarrow R^1$

(5.4) $f(s,n)$

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[67]The members of such a family are characterized by the same degree of inequality aversion. For a proof of this proposition see Appendix E.

[68]This is equivalent to assuming that a vector of the utilities of individuals in a particular social state can be represented by its equally distributed equivalent.

represents the social ordering over possible trade-offs between representative consumption and population size.

The most basic dichotomy in criteria for evaluating social states with different sized populations is whether the social ordering reflects a preference for total or per capita social utility. The adoption of additional population principles embodying either so-called classical or average population rules further restricts the structure of the variable population social evaluation function (5.4) (Blackorby and Donaldson [1979], theorems 4.1 and 5.1). I am not interested, however, in the evaluation of different social states in general, but rather in a particular aspect of these states, namely, their degree of inequality. Specifically, I am seeking a theoretical foundation for the calculation of a meaningful trend of lifecycle inequality. The adoption of the Principle of Population Substitution is sufficient for this, as it ensures that the same set of ethical judgments are being used in the measurement of the social significance of inequality regardless of the size of the population under study. Since inequality indices are functions of representative consumption, but are independent of population size, it should be clear that the measurement of inequality and its trend implicitly follows an average rule for evaluating social states with variable sized populations. The variable population social evaluation function implicit in the trend of lifecycle inequality represents a social ordering which is equivalent to ranking social states according to their degree of measured inequality.

In the welfare approach to the measurement of inequality,

inequality is measured in the distribution of representative lifecycle consumptions. This distribution changes only with changes in the population. Lifecycle inequality therefore remains constant so long as the population remains unchanged. Typically, of course, the population does change, and moreover its evolution occurs over time, so that the trend of lifecycle inequality is presented as a time series of values of an inequality index. Over time, additions to and subtractions from the population (i.e. the beginning and ending of individual economic lifecycles) result in changes to the vector of representative lifecycle consumptions which, of course, alter measured lifecycle inequality. Although in fact the population is changing continuously it is sufficient to recompute lifecycle inequality so as to record its trend, not with every economic birth or death, but rather at regular time intervals. Given annual data, the most logical choice is once a year.

Approximating the trend of lifecycle inequality in the decomposition approach is complicated by the fact that panel data collection, and the definition of the subset used in the computation of "static" inequality indices, assume an unchanging population.[69] Since population changes over time are not explicitly incorporated into the panel data set, it is necessary to simulate the evolution of the population by artificially

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[69]The Panel Study on Income Dynamics allows for the introduction of new young members of society only through the formation of households by offspring of households currently in the sample. The data subset used to compute decomposition indices of inequality in Chapter Four include no households originating after 1968.

introducing each year a new cohort of individuals who are just beginning their economic lifecycles, while removing the eldest cohort whose members it is assumed have finished their economic lives. Thus, in any given year, decomposition indices of inequality should be computed for age-cohorts in the range $\underline{k}$ to $\bar{k}$ ($1 \leq \underline{k} \leq \bar{k} \leq K$). The decomposition indices should then be recomputed for the following year using cohorts $\underline{k}-1$ through $\bar{k}-1$, to reflect (artificially) the evolution of the population during that time. With ten years of panel data, this method of simulating the effects of a changing population on lifecycle inequality results in the loss of ten cohorts from the weighted average of inequality within cohorts which comprises the index of intracohort inequality. That is, if the data set includes households with family heads aged 18 to 70 years, say, then intracohort inequality in the first year should be calculated, for the purposes of determining the trend of lifecycle inequality, as the weighted average of inequality within the 27 to 70 year old age cohorts. In the second year the age-range should be from 26 to 69 years, and so on until the final year in which the 18 to 61 year old age-cohorts should be used in the computation of intracohort inequality. Naturally, there is no theoretical ground for this exclusion of ten age-cohorts from each "static" index of lifecycle inequality, but the reduced accuracy of measured inequality that results is, in the absence of panel data that explicitly incorporates a new youngest cohort each year, an unavoidable cost associated with the calculation of the trend of lifecycle inequality.[70]

I have computed decomposition indices of relative inequality for a restricted range of age-cohorts in ten years. Table 11 contains the results of calculating the trend of lifecycle inequality in the decomposition approach, plus the trend of annual inequality, for three values of the parameter of relative inequality aversion, R. All figures which reflect an increase in measured inequality over the preceding year's figure are marked by an asterisk. The most immediately noticeable contrast between the trends of lifecycle and annual inequality is that annual inequality is far more cyclical, while lifecycle inequality tends to follow a declining trend interrupted occasionally by a one year rise in measured inequality. The total decline in lifecycle inequality over the ten year period ranges from 4.1 per cent (R=-5) to 7.4 per cent (R=-.5). This is equivalent to an average per annum reduction in lifecycle inequality of from 0.42 per cent to 0.77 per cent from the preceding year.[71] Although the method of simulating population change so as to compute the trend of inequality casts some doubt on the reliability of the results, they are, I think, indicative of the difference between the trends of annual and lifecycle inequality. Annual inequality computed from the panel data

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[70]On the other hand, if new cohorts were incorporated each year, the entire number of years of data in the panel could not be used in the computation of the trend of lifecycle inequality because of insufficient data on the most recently added cohorts.

[71]This may be contrasted with the reported 23 per cent reduction in Paglin [1975]-inequality over a 25 year period, which is equivalent to an 1.04 per cent average per annum reduction in inequality.

conforms to the stylized fact that the trend of annual inequality is constant. The moderately declining trend of lifecycle inequality, on the other hand, demonstrates that the trend of annual inequality is not a reliable indicator of the effect of economic developments and social policy on the distribution of income.

TABLE 11

The Trends Of Annual And Lifecycle Inequality

R=.5		R=-.5		R=-5	
lifecycle	annual	lifecycle	annual	lifecycle	annual
.0712	.0915	.2008	.2618	.4863	.6964
.0701	.0887	.1983	.2556	*.4907	.6909
.0701	.0870	.1974	.2442	.4875	.6444
.0691	.0858	.1943	.2427	.4812	*.7929
.0682	*.0867	.1918	*.2516	.4758	.7350
.0678	.0861	.1911	.2301	*.4804	.6531
.0671	*.0899	.1883	*.2530	.4720	*.8664
.0665	*.0910	.1868	.2506	.4687	.6552
.0663	*.0924	.1863	*.2606	.4677	*.8106
*.0664	.0863	*.1864	.2551	.4673	*.8129

CHAPTER SIX

Summary and Conclusions

In this final chapter I will briefly summarize the substance and main findings of my thesis and draw some conclusions from the theoretical and empirical results which have been presented.

I have been concerned with problems that surround the traditional practice of measuring inequality in the distribution of annual income. Throughout the thesis inequality is taken to mean the extent to which society falls short of a hypothetical situation in which everyone is equally well-off. The measurement of annual income inequality is inappropriate in this regard because it is consumption, not income, that is productive of welfare. Furthermore, welfare depends on consumption over the lifecycle, not just in a single year. Students and seniors may lie fairly far down in the distribution of annual income, but this indicates little about the extent to which they share in the fruits of society during the course of their lives. Neither does the negative annual income of a bankrupt businessman reflect his current consumption, let alone his probably enviable position in the distribution of lifecycle consumption.

Even were there not these theoretical objections to measuring inequality in the annual income distribution, there are a number of methodological problems. Annual inequality indices, that is, fail to take account of the source of income differences which results in their overstating the degree of pure interpersonal inequality. For example, the receipt of income tends to follow a rising path over much of the lifecycle. Thus, even if everyone traversed identical lifecycle income paths, measured annual inequality would be positive because of income differences attributable to the age-structure of the population. This source of inequality should properly be excluded from an index of pure interpersonal inequality. Additionally, it has been observed that measured inequality is sensitive to the length of the income accounting period. This is explained by the fact that, over time, individuals tend to change their relative positions in the income distribution. Empirical investigations have found evidence of substantial relative income mobility. The tendency for low income recipients to move up, and high income recipients to move down in the distribution, has an averaging effect on incomes as the accounting period is lengthened. This explains the often observed tendency of measured inequality to vary inversely with the length of the income accounting period. This effect should also be taken into account when measuring inequality.

In response to these problems I have proposed two new approaches to the measurement of inequality. In the welfare approach, presented in Chapter Two, inequality is measured in the distribution of a summary statistic of lifecycle consumption. Since the consumption profiles of younger members of the current population will tend to lie above those of elder members because of real growth, it is necessary to decompose lifecycle inequality within and among age-cohorts of the population. Intracohort inequality is an index of pure interpersonal inequality while intercohort inequality captures the contribution of economic growth to total measured inequality.

The welfare approach requires that individual utility functions are known (so that representative lifecycle consumption can be computed), and that lifecycle consumption data are available. Clearly neither of these conditions is met.[72] I have therefore devised an alternative method, capable of empirical implementation, which is the subject of Chapter Three. The decomposition approach is a compromise between the inadequacy of measuring annual income inequality and the impossibility of measuring lifecycle consumption inequality. The method is to treat panel consumption data on H individuals over T years as a distribution of consumption among a single population of size HT. Total inequality is then decomposed within and among various subgroups of that population.[73] First, the T observations on each individual are treated as H separate subgroups of the population, and inequality thus decomposed within and among persons. Intrapersonal inequality (attributable

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[72] Some recent work has attempted to measure lifecycle consumption inequality by assuming that individual utility functions are identical and of a particular functional form, and using estimated lifecycle consumption data computed by maximizing utility subject to an (estimated) lifetime income constraint which assumes capital markets are perfect. I have chosen a different exit from the impasse imposed by the requirements of the welfare approach to the measurement of inequality.

[73] Using a procedure suggested by Blackorby, Donaldson, and Auersperg [1981]. In Appendix A the performance of an alternative procedure for decomposing inequality, suggested by Shorrocks [1980], is evaluated. It was rejected for use in the decomposition approach to the measurement of inequality on both theoretical and empirical grounds.

to variation in individuals' consumption over time) reflects the effect on inequality of consumption mobility. Interpersonal inequality is further decomposed within and among age-cohorts. Age-related inequality is captured by the index of intercohort inequality, leaving intracohort inequality as an index of pure interpersonal inequality.

In Chapter Four I report the results of the empirical implementation of the decomposition approach to the measurement of inequality. Ten years of data drawn from the Panel Study on Income Dynamics were used to compute decomposition approach indices of absolute inequality and relative equality and inequality. The results indicate that age- and especially mobility-related consumption differences account for a substantial portion of total measured inequality in this distribution. Annual inequality overstates pure interpersonal inequality by no less than one-third, and in some cases by as much as 75 per cent. Although the magnitude of this overstatement depends in part on the width of age-cohort brackets, only a moderate reduction in the disparity between the traditional and decomposition approaches resulted from a widening of age-cohort brackets. There was also some indication that the decomposition approach index of intracohort inequality approached a limit within the ten years of the data set. If this is a general rather than data-specific property, then decomposition approach indices may be computed sufficiently accurately with as few as ten years of data.

One of the most well-established empirical observations about inequality is that its trend in the post-World-War II period has been remarkable constant. An accurate assessment of the trend of inequality, however, depends crucially on the measurement of "static" inequality. In Chapter Five I investigate the trend of lifecycle inequality. The first step is to recognize an essential difference between the trends of annual and lifecycle inequality. The former is simply the time-series of annual inequality, reflecting the fact that consumption changes from one year to the next. The trend of lifecycle inequality, however, depends not on the passage of time but on the evolution of the population. The equally distributed equivalent of an individual's lifecycle profile remains constant, by definition, and the problem, therefore, is one of measuring and comparing inequality in different populations. This requires families of social evaluation functions and inequality indices with one member for every possible population size. In addition, some means of linking together members of a family is required to ensure that they all represent the same set of ethical judgments.

Having established the theoretical basis for measuring and evaluating the trend of lifecycle inequality, I discuss how this would be accomplished in the welfare approach, and how it can be approximated in the decomposition approach. Since the members of the panel do not change over time, the evolution of the population must be simulated by allowing each year for the demise of members of the eldest cohort and the "birth" of a new cohort whose members are just beginning their economic lifecycles. The empirical results proved very interesting. The trend of annual inequality measured in the distribution is roughly constant, but lifecycle inequality declines over the ten year period. This is taken as further evidence in support of the decomposition approach to the measurement of inequality.

The principal conclusion of this thesis is that a decomposition approach to the measurement of inequality is essential for an accurate assessment of inequality. Measured annual inequality includes the effects of mobility and the age-structure of the population.[74] In the decomposition approach, indices of age- and mobility-related inequality are computed and used to construct an index of pure interpersonal inequality. This requires panel data. The collection of such data, not currently available for Canada, would be extremely useful for many kinds of economic and social research including the measurement of inequality. For this purpose, it would be useful to record both income and consumption expenditure (or assets and liabilities, which would allow consumption expenditure to be computed from income).[75] Furthermore, it would be useful for the purpose of measuring the trend of inequality if a new cohort representing individuals just beginning their economic lifecycles could be added to the panel each year. This would enable the level and trend of inequality in Canada to be

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[74]In Appendix B a procedure for separating age-related from total annual inequality is evaluated. Empirical evidence suggests that decomposing annual inequality within and among age-cohorts is a more reliable method of accounting for the effects of the age-structure of the population on annual inequality. It is recognized, however, that no approach that focusses on annual inequality can account for the effects of income mobility.

[75]The <u>Panel Study on Income Dynamics</u> reports assets, but not liabilities. The latter might prove a useful addition to the data collected by the Survey Research Center.

investigated using the decomposition approach to the measurement of inequality.

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APPENDIX A

An Alternative Procedure for Decomposing Inequality

The procedure suggested by Blackorby, Donaldson, and Auersperg [1981] for decomposing inequality employs subgroup equally distributed equivalent incomes to eliminate inequality within groups. An alternative method has recently been proposed independently by at least three authors, Bourguignon [1979], Cowell [1980], and Shorrocks [1980], who have studied the class of additively decomposable inequality indices in which subgroup mean incomes are used to eliminate intragroup inequality. In this appendix, I describe this decomposition procedure, drawing mainly on Shorrocks [1980], and compare it with the Blackorby-Donaldson-Auersperg version. I then show how it can be applied to the problem of constructing an index of pure interpersonal inequality by decomposing total inequality in panel income data within and among persons, and then further decomposing interpersonal inequality within and among age-cohorts, as was done with the Blackorby-Donaldson-Auersperg procedure in Chapter Three. Empirical results are provided to illustrate the performance of the class of indices defined by Shorrocks in both the traditional and decomposition approaches to the measurement of inequality. The latter is also compared to the decomposition approach as implemented in Chapter Four in order to evaluate the two decomposition procedures on the empirical grounds most relevant to my objective of developing an index of pure interpersonal inequality.

Shorrocks [1980] has studied the class of continuous and symmetric inequality measures, bounded below by zero (when all members of the population have the same income), which are decomposable into the form:

(A.1) $$I(y;n)=\sum_g w_g(m,n)\, I(y^g;n_g) + I(m_1 \underline{1}_{n_1}, \ldots, m_G \underline{1}_{n_G})$$

where $y=(y_1, \ldots, y_n)$ is the distribution of income over the n members of the population and m is its mean. In (A.1) (Shorrocks equation(4)), total inequality is decomposed into the sum of two terms: a weighted sum of the subgroup inequality indices, $I(y^g;n_g)$, where y^g is the vector of incomes of the n_g members of the g<u>th</u> subgroup ($1 \le g \le G$), and an index of intergroup inequality whose arguments are subgroup mean incomes. This class of additively decomposable inequality measures is also shown to satisfy the Pigou-Dalton principle of transfers (Shorrocks [1980], theorem 3). Further properties which it may be desirable for the inequality measures to satisfy narrow the class of admissible indices. For example, satisfying the principle of population replication restricts the form of the weights, $w_g(m,n)$, in the intragroup term (Shorrocks [1980], theorem 4). The assumption of mean independence (income homogeneity) imposes considerable additional structure on the form of the inequality measures (Shorrocks [1980], theorem 5). Shorrocks [1980, p. 622] shows that the class of, "additively decomposable indices satisfying both mean independence and population replication therefore comprise a one parameter family whose members are identified by the value of c:"

$$I_c(y)=(1/n)(1/c(c-1))\sum_i[(y_i/m)^c-1] \qquad ,c\neq 0,1$$

(A.2) $$I_o(y)=(1/n)\sum_i \log(m/y_i) \qquad ,c=0$$

$$I_1(y)=(1/n)\sum_i (y_i/m)\log(y_i/m) \qquad ,c=1$$

This family of inequality measures includes the square of the coefficient of variation (c=2), two indices developed by Theil [1967] from the theory of entropy (c=0 and c=1), and monotonic transformations of the entire Atkinson [1970] family of indices (Shorrocks [1980, p. 622]). As the value of the parameter c decreases, the index becomes increasingly sensitive to inequality lower down in the distribution of income. The family of inequality measures, (A.2) (Shorrocks' equation (31)), is thus similar to the Atkinson and Kolm-Pollak families of relative and absolute inequality indices, in the sense that these latter families of indices satisfy the assumptions which Shorrocks has invoked in his axiomatic derivation of the class of additively decomposable indices, with the single exception of the summary statistic of the subgroup distribution that is used to eliminate intragroup inequality.

The weights in the intragroup term of the decomposition of the indices (A.2) in the form (A.1) are:

(A.3) $$w_g(m,n)=(n_g/n)(m_g/m)^c$$

which sum to unity only when c=0 or c=1. Thus the intragroup inequality term is <u>not</u> in general a weighted average of inequality within groups, as it is in the decomposition of ethical indices of inequality. This, Shorrocks argues,

> may not be regarded as a major handicap, but Theil [1967, p. 125] has pointed out a more serious objection. It can be shown that $1-\sum_g w_g$ is proportional to the between group term in the corresponding decomposition equation. Thus, apart from the two measures proposed by Theil (c=0 and c=1), the

> decomposition coefficients are not independent of the between group contribution. ([1980, p. 624])

Clearly this problem does not arise in the procedure for decomposing inequality proposed by Blackorby, Donaldson, and Auersperg.

Decomposing inequality provides a means of quantitatively assessing the contribution of some factor to total inequality. The decomposition of inequality within and among age-cohorts, for example, determines the proportion of measured inequality attributable to the variation of income with age, and the proportion that is pure interpersonal inequality. As Shorrocks points out, however, there has been some ambiguity in the interpretation of such a procedure. That is, inequality within groups might be eliminated by assigning each individual his age-cohort mean income, which would eliminate the intragroup term of the decomposition, and leave the intergroup term as the measure of inequality due to the shape of age-income profiles. Alternatively, mean incomes might be equalized across age-cohorts without changing inequality within cohorts. This would eliminate the intercohort term of the decomposition,

> but the reduction in inequality is not simply B (the between group term of the decomposition) because, in general, changing the age group means will also affect the decomposition coefficients and hence the total within group contribution. Only when these coefficients do not depend on the subgroup means will (these alternative procedures) produce the same answer. Of the family of measures (31)(A.2 in this appendix), one alone satisfies this requirement -- the index I_0, for which the corresponding decomposition coefficients are the population shares n_g/n. For this reason, I_0 is the most satisfactory of the decomposable measures, allowing total inequality to be unambiguously split into the contribution due to differences between subgroups and the contribution due to inequality within each subgroup in such a way that total inequality is the sum of these G+1

contributions. (Shorrocks [1980, p. 625])

When the decomposition of inequality is calculated by eliminating intragroup inequality first, the original distribution is replaced by $(m_1\underline{1}_{n_1}, \ldots, m_G\underline{1}_{n_G})$. The remaining intergroup inequality is then measured in terms of (m_g/m). Intragroup inequality is calculated by subtracting the intergroup term from total inequality. This yields the decomposition given in (A.1).

If intergroup inequality is to be eliminated first the subgroup distributions are scaled, without changing inequality within the groups, so that their means are equal to the mean of the original distribution. Each income, y_i, is replaced by $x_i = y_i(m/m_g)$. According to Shorrocks, this decomposition will not yield the same result as (A.1), however, "because, <u>in general</u>, changing the age group means will also affect the decomposition coefficients and hence the total within group contribution" (Shorrocks [1980, p. 625], emphasis added). Within group inequality in this case is measured in the distribution $x = (x_1, \ldots, x_n)$ with respect to the distribution of the population-wide mean income $(m\underline{1}_n)$. The primitives of the within group inequality indices are therefore (x_i/m). But $(x_i/m) = (y_i/m_g)$, the primitives of the within group indices in (A.1) (where intragroup inequality is eliminated first). Thus while Shorrocks' argument that changing the subgroup means will affect the decomposition coefficients is true in general, it is not in this particular case where intergroup inequality is eliminated from the original distribution. This can be verified by noting that intergroup inequality is measured in terms of

$(y_i/x_i)=(m_g/m)$ which is the same as the intergroup inequality index in (A.1). Shorrocks' decomposition of inequality therefore is unique with respect to the order in which intra- and intergroup inequality are eliminated.

Blackorby, Donaldson, and Auersperg have addressed the same problem in their discussion of alternative procedures for decomposing inequality. Their decomposition of per capita inequality is independent of the order in which intra- and intergroup inequality are eliminated, but their decomposition of AKS inequality does not have this very desirable property. Fortunately, however, the decomposition of AKS inequality which I suggested in Chapter Two, and employed there and in Chapter Three, is independent of the order in which intra- and intergroup inequality is eliminated. Thus, both Blackorby, Donaldson, and Auersperg's [1981] and Shorrocks' [1980] procedures yield unique decompositions of inequality.

Members of the family of inequality measures, (A.2), shown by Shorrocks to exhaust the class of additively decomposable indices satisfying mean independence and population replication, do not range over a [0,1] interval, a property often imposed on indices of relative inequality.[76] While the indices (A.2) are bounded below by zero, their upper bound varies widely with the value of c and for $c\leq0$ they are unbounded above. For positive

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[76] Indeed, it is difficult to imagine what interpretation can be placed on a mean independent inequality index which is not confined to a [0,1] range. It is for this reason that Blackorby, Donaldson, and Auersperg [1981] argue that one of their procedures for decomposing AKS indices of inequality, which generates subindices of relative inequality that can take on values outside the [0,1] interval, is unacceptable.

values of c, the indices can be normalized to lie in the [0,1] interval, but they will then fail to satisfy the principle of population replication. For values of $c \le 0$ the family of additively decomposable indices cannot be normalized (Shorrocks [1980, p. 623, n. 7]). Ethical indices of relative inequality, on the other hand, are constructed so as to range over a [0,1] interval, and the Atkinson family of indices, which exhaust the admissible class of mean independent indices in the decomposition approach to the measurement of inequality, satisfy the principle of population replication. This is further reason to prefer the Blackorby-Donaldson-Auersperg procedure for decomposing inequality to that proposed by Shorrocks. It may prove interesting, nevertheless, to investigate the application of the class of additively decomposable inequality measures defined by Shorrocks [1980] to the problem of developing an index of pure interpersonal inequality. Recalling that the distribution in which inequality is to be measured is comprised of the annual consumptions of H individuals observed over T years,[77] the indices (A.2) are written as:

$$
\text{(A.4)}\qquad
\begin{aligned}
I_c(y) &= (1/HT)(1/c(c-1))\sum_h \sum_t [(y_{ht}/m)^c - 1] && ,c\neq 0,1\\
I_0(y) &= (1/HT)\sum_h \sum_t \log(m/y_{ht}) && ,c=0\\
I_1(y) &= (1/HT)\sum_h \sum_t (y_{ht}/m)\log(y_{ht}/m) && ,c=1
\end{aligned}
$$

where y_{ht} is the income of the hth person in the tth year. I begin by decomposing I (y), according to (A.1), into inequality

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[77]Since Shorrocks' work and the foregoing discussion of it are couched in terms of income rather than consumption, I will continue this practice throughout Appendix A in order to minimize confusion.

within the income streams of population members and inequality between individuals:

$$I_c(y)=\sum_h (T/TH)(m_h/m)^c\{(1/T)(1/c(c-1))\sum_t[(y_{ht}/m_h)^c-1]\} \\ +\{(1/H)(1/c(c-1))\sum_h[(m_h/m)^c-1]\} \tag{A.5}$$

We may now decompose interpersonal inequality, the second term in (A.5), within and among age-cohorts. This yields,

$$I_c(y)=\sum_h\{(1/TH)(m_h/m)^c(1/c(c-1))\sum_t[(y_{ht}/m_h)^c-1]\} \\ +\sum_k\{(1/H)(m_k/m)^c(1/c(c-1))\sum_{h\in N^k}[(m_h/m_k)^c-1]\} \\ +\{(1/H)(1/c(c-1))\sum_k n_k[(m_k/m)^c-1]\} \tag{A.6}$$

The decompositions of the indices (A.4) for c=0 and c=1 corresponding to (A.6) are:

$$I_0(y)=[\sum_h(1/TH)\sum_t\log(m_h/y_{ht})] \\ +[\sum_k(1/H)\sum_{h\in N^k}\log(m_k/m_h)] \\ +[(1/n_k)\sum_k\log(m/m_k)] \tag{A.7}$$

and,

$$I_1(y)=[\sum_h(1/TH)(m_h/m)\sum_t(y_{ht}/m_h)\log(y_{ht}/m_h)] \\ +[\sum_k(1/H)(m_k/m)\sum_{h\in N^k}(m_h/m_k)\log(m_h/m_k)] \\ +[(1/n_k)\sum_k(m_k/m)\log(m_k/m)] \tag{A.8}$$

In (A.6), (A.7), and (A.8), total inequality is decomposed into the sum of three subindices of inequality which measure, respectively: intrapersonal inequality, I_{AP}; intracohort inequality, I_{AC}, and intercohort inequality, I_{RC}. Values of the index and subindices of inequality computed for various degrees of inequality aversion are reported in Table 12. The indices (A.2) were also used to calculate annual inequality indices. The weights (A.3) corresponding to annually defined population subgroups were used to calculate a weighted sum of the annual inequality indices which is also reported. "Annual" inequality in Table 12 is thus the intragroup component of a decomposition within and among years of total inequality in the panel data treated as an income distribution for a single population.

Since one of my objectives is to evaluate the performance of alternative procedures in a decomposition approach to the measurement of inequality, I have, where possible, reported normalized versions of the indices in Table 12 to facilitate their comparison with the Blackorby-Donaldson-Auersperg decomposition indices of relative (i.e. mean independent) inequality reported in Table 3 of Chapter Four. Unfortunately, however, although the normalized indices range over a [0,1] interval, it appears that those indices that can be normalized (i.e. $c>0$) generally exhibit too low a degree of inequality aversion to be of much practical use. An exception is the normalized index in Table 12 for $c=1$, for which the value of the index of total inequality in the panel coincides to the third decimal place with the total inequality index value in Table 3 for $R=0.9$. The two indices are repeated together below.

	From Table 3 $R=.9$	from Table 12 $c=1$
I	.0185	.0181
I_{AP}	.0038	.0037
I_{AC}	.0136	.0133
I_{RC}	.0011	.0012

The two indices produce virtually identical results in this case, which might be taken as an indication that the two alternative decomposition procedures are substitutes for one another. But there is not sufficient evidence to warrant such a conclusion. Furthermore, certain undesirable attributes of the Shorrocks' [1980] decomposition procedure place it in an unfavourable light, compared to the Blackorby-Donaldson-Auersperg procedure, for use in a decomposition approach to the measurement of inequality.

It has already been mentioned that the range of the parameter of inequality aversion, c, over which the indices defined by Shorrocks can be normalized, overlaps only very slightly with the range of c likely to be useful in implementing the decomposition approach to the measurement of inequality. Thus, Shorrocks' indices would have to be used almost exclusively in their nonnormalized versions, a disadvantage because of the difficulty of interpreting mean-independent inequality indices that do not range over a [0,1] interval.[78]

A close inspection of the results reported in Table 12 will reveal another difficulty with Shorrocks' class of additively decomposable inequality indices. As the value of the parameter, c, falls, the degree of inequality aversion exhibited by members of the family of indices (A.2) rises. With it should rise the social significance of inequality in the distribution of

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[78]Furthermore, even the normalized versions of Shorrocks' indices do not bear the interpretation attributable to the Blackorby-Donaldson-Auersperg indices of relative inequality: that they measure the proportion of income "wasted" on inequality.

[79]All the results reported in Tables 3 and 12 measure inequality in the same distribution of income which is characterized by a certain degree of inequality "objectively" or "statistically" measured. The social significance of inequality, however, should rise with the degree of inequality aversion, indicating the greater welfare effects of a given amount of dispersion in the distribution of incomes.

income,[79] as is the case with the Blackorby-Donaldson-Auersperg decomposition indices reported in Table 3. Such is not the case, however, with Shorrocks' class of inequality indices. As the degree of inequality aversion rises (i.e. as the value of c falls), total measured inequality first falls, reaches a minimum at approximately c=.5, and increases thereafter. The strange behaviour of these indices with a low degree of inequality aversion results from the extreme importance which they place, for the measurement of inequality, on the upper end of the income distribution. As can be seen from the structure of the indices (A.2), when c is large incomes below the mean hardly affect the value of the index, while incomes above the mean, particularly the largest incomes, contribute enormously to total measured inequality. Shorrocks and others have noticed this property of the indices (A.2) with respect to their transfer properties.

> (T)he square of the coefficient of variation (corresponding to c=2) gives roughly the same weight to a transfer of $10 from a person with $10,000 to another with $2,000 as a $1 transfer from someone with $100,000 to another with $20,000 . . . (Shorrocks [1980, p. 623]). . . . This rather perverse result illustrates why the (square of the) coefficient of variation is extremely sensitive to changes in the upper tail of the distribution. In fact the transfer properties of indices corresponding to c>2 become even stranger. Although they still satisfy the principle of transfers, they show little concern for equalization except among the very rich. This has led Kolm [1976b] and Love and Wolfson [1976] to question whether they should not be eliminated from consideration as inequality measures, as would be the case if Kolm's "principle of diminishing transfers" were adopted (Shorrocks [1980, p. 623, n. 8]).

Clearly we might well follow Kolm, and Love and Wolfson, and employ only those members of Shorrocks' class of additively decomposable indices corresponding to c≤2.

The strange behaviour of these indices carries over to the decomposition of total inequality within and among subgroups. The proportions of total inequality attributable to the three subindices of intrapersonal, intracohort, and intercohort inequality vary enormously over the range of c. At its extreme values ($c \geq 5$ and $c \leq -3$), the relative magnitudes of the three subindices of inequality correspond to the order in which inequality is eliminated when computing them; that is, first intrapersonal, then intracohort, and finally intercohort, inequality. For low degrees of inequality aversion, a possible explanation of this is again the perverse behaviour of the indices (A.2) for large values of the parameter c. The computation of subindices involves the substitution of subgroup mean income for the original group distribution. This naturally reduces dispersion and with it the opportunity for the indices to take on large values because of a few observations in the upper tail of the distribution. The same is likely true for small values of c also, although in that case it would be the observations in the lower tail of the distribution which contribute to the high degree of measured inequality.[80] In either case, the replacement of actual distributions by subgroup means removes the influence of observations in one or the other tails of the distribution, thereby causing the subindices of

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[80]For $c<0$, (A.2) can be written,

$$I_c(y)=(1/n)(1/c(c-1))\sum_i[(m/y_i)^{|c|}-1]$$

from which it should be clear that when the magnitude of c is large, incomes greater than the mean would have little impact on the value of measured inequality while very small incomes would cause measured inequality to be very large indeed.

inequality to reflect more the order in which inequality is eliminated than the relative contributions of various sources of inequality to the total.

For intermediate values of the degree of inequality aversion, corresponding roughly to $-2 \le c \le 4$, intracohort inequality is the predominant contributor to total inequality, followed by intrapersonal inequality and finally intercohort inequality. In this range, the rank order of the subindices of inequality is the same as for the subindices of relative inequality, over all degrees of inequality aversion, reported in Table 3. of Chapter Four.[81] These patterns suggest that, of the class of additively decomposable indices defined by Shorrocks [1980], members displaying either high or low degrees of inequality aversion may have to be omitted if reliable estimates of inequality are to be obtained.

In conclusion, it has been shown that the class of additively decomposable inequality indices that use subgroup means to eliminate inequality within groups are not well-suited for use in a decomposition approach to the measurement of pure interpersonal inequality. In addition to departing from the practice of the modern theory of employing equally distributed equivalents to eliminate inequality, Shorrocks' decomposition indices suffer from several theoretical drawbacks from which the Blackorby-Donaldson-Auersperg indices are free.

The class of indices defined by Shorrocks [1980] do not

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[81]Furthermore, the magnitudes of the proportions of total inequality attributable to the three sources are very close for $.9 \ge R \ge -1$ in Table 3 and $1.5 \ge c \ge -.5$ in Table 12.

range over a [0,1] interval, which makes them difficult to interpret as mean-independent measures of inequality. While a subset of this class of indices is bounded above, and can therefore be normalized to range over a [0,1] interval, the results of empirically implementing the decomposition approach using Shorrocks' family of indices reveal that the degree of inequality aversion among those indices that can be normalized is too low for them to be of much practical use. In addition, estimation of the decomposition approach using Shorrocks' indices revealed strong evidence that the indices could not be deemed reliable measures of total or subgroup inequality in other than an intermediate range of the degree of inequality aversion.

These shortcomings are sufficiently serious that the class of additively decomposable inequality indices defined by Shorrocks cannot be recommended for use in the decomposition approach to the measurement of pure interpersonal inequality. Thus, in Chapters Three and Four, the decomposition approach is developed and implemented in terms of Blackorby, Donaldson, and Auersperg's procedure for decomposing inequality within and among population subgroups.

TABLE 12

Additively Decomposable Indices of Inequality

	c=5	c=5 (normalized)	c=4	c=4 (normalized)
I	3.4255	$.1051\times10^{-15}$	.8872	$.4642\times10^{-12}$
I_{AP}	1.8603	$.0571\times10^{-15}$	.3870	$.2025\times10^{-12}$
I_{AC}	1.5197	$.0466\times10^{-15}$	.4748	$.2484\times10^{-12}$
I_{RC}	.0456	$.0014\times10^{-15}$	.0254	$.0133\times10^{-12}$
annual	3.4215		.8833	

	c=3	c=3 (normalized)	c=2	c=2 (normalized)
I	.3641	$.2707\times10^{-8}$	.2252	$.1585\times10^{-4}$
I_{AP}	.1171	$.0871\times10^{-8}$	.0540	$.0380\times10^{-4}$
I_{AC}	.2299	$.1709\times10^{-8}$	.1577	$.1110\times10^{-4}$
I_{RC}	.0171	$.0127\times10^{-8}$	.0135	$.0095\times10^{-4}$
annual	.3602		.2212	

	c=1.5	c=1.5 (normalized)	c=1	c=1 (normalized)
I	.1986	$.8889\times10^{-3}$	.1861	$.1814\times10^{-1}$
I_{AP}	.0428	$.1918\times10^{-3}$	.0376	$.0367\times10^{-1}$
I_{AC}	.1433	$.0641\times10^{-3}$	.1367	$.1333\times10^{-1}$
I_{RC}	.0125	$.0056\times10^{-3}$	.0118	$.0115\times10^{-1}$
annual	.1946		.1820	

TABLE 12 (CON'T)

	c=.5	c=.5 (normalized)	c=.25	c=.25 (normalized)
I	.1841	$.4629\times10^{-1}$	.1867	$.3503\times10^{-1}$
I_{AP}	.0366	$.0921\times10^{-1}$	.0376	$.0705\times10^{-1}$
I_{AC}	.1362	$.3424\times10^{-1}$	.1380	$.2589\times10^{-1}$
I_{RC}	.0113	$.0284\times10^{-1}$	.0111	$.0208\times10^{-1}$
annual	.1780		.1826	

	c=0	c=-.5	c=-1	c=-1.5
I	.1920	.2117	.2486	.3151
I_{AP}	.0397	.0483	.0662	.1030
I_{AC}	.1413	.1527	.1718	.2015
I_{RC}	.0110	.0108	.0106	.0106
annual	.1879	.2075	.2444	.3109

	c=-2	c=-3	c=-4	c=-5
I	.4406	1.3194	8.2494	93.4406
I_{AP}	.1834	.8871	7.3821	91.3907
I_{AC}	.2466	.4214	.8558	2.0375
I_{RC}	.0106	.0109	.0114	.0124
annual	.4363	1.3149	8.2443	93.4269

APPENDIX B

Evaluation Of A Method Of Approximating Long-run Inequality

It was seen in Chapter One that measured inequality in the distribution of annual income includes inequality attributable to several different sources which should be distinguished from one another. The objective is to develop an index of pure interpersonal inequality, which requires, inter alia, that inequality due to the age-structure of the population be separated from pure interpersonal inequality. While it has been argued that this requires evaluation of actual lifecycle income streams, Paglin [1975] has proposed a revised Gini coefficient of annual income inequality which, "approximates a measure of long-run interfamily inequality" (p. 601). Such a measure, if reasonably accurate, would provide a reliable estimate of long term inequality at significant savings in data collection and computation costs. An assessment of the validity of the Paglin-Gini requires a comparison of its estimate of long run inequality with our measure of intracohort inequality in the decomposition approach. This is hampered, however, by the fact that Paglin's technique is specific to the Gini coefficient which is not included in the set of admissible inequality indices for the decomposition approach. In this appendix I first describe Paglin's revised Gini coefficient and then generalize his method to AKS and per capita indices of inequality. I am thus able to compute Paglin-type inequality measures for indices that are admissible under the assumptions of the decomposition approach, which may then be compared to the corresponding decomposition approach indices of intracohort inequality.

Annual income inequality measures, "combine and hence confuse <u>intra</u>family variation of income over the lifecycle with the more pertinent concept of <u>inter</u>family income variation which underlies our idea of inequality . . . " (Paglin [1975, p. 598], emphasis in original). It is Paglin's view that the problem is best thought of as resulting from an inappropriate standard of equality. Annual income equality implies that families not only have equal lifetime incomes, but also equal annual incomes regardless of age (of family head), a constraint which requires that all families have perfectly flat age-income profiles. Paglin argues that a more reasonable standard of equality for use with annual income data is equal lifetime incomes without the added constraint of flat age-income profiles. This is made operational by redefining the standard of equality as income equality within age-cohorts, thus allowing income variation over the lifecycle to be excluded from contributing to measured annual income inequality. Annual income inequality measured with respect to this revised standard of equality should then more closely approximate pure <u>inter</u>family inequality.

Paglin has applied his technique to the Gini coefficient as follows. He estimates the mean age-income profile of the population from annual cross-sectional data in order to construct a Lorenz curve of inequality in the distribution of cohort mean incomes. This new standard of equality, which Paglin calls a P-reference line, replaces the traditional 45° line of equality in a Lorenz diagram. The actual distribution of income is represented by the usual Lorenz curve showing the share of total income accruing to the poorest x per cent ($0 \leq x \leq 100$) of the population. The situation is illustrated in Figure IV (Paglin's Figure 1B, p. 599).

The traditional (Lorenz-) Gini coefficient is equal to the ratio of the area between the diagonal and the Lorenz curve, to the area below the 45° line of equality. In Figure IV it can be seen to include inequality due to interfamily income differences and to intrafamily income variation over the lifecycle. The latter source of inequality is represented by the shaded area between the P-reference line and the diagonal, and can be measured by a Gini concentration ratio which Paglin calls the age-Gini. The area between the P-line and the L-line represents interfamily income inequality. It too can be measured by a Gini coefficient, labelled the Paglin-Gini, which is most easily calculated as the difference between the Lorenz-Gini and the age-Gini. Paglin's method for decomposing annual income inequality is summarized in the following three steps:

(1)Calculate the mean age-income profile of the population, assign each individual his cohort mean income, and compute inequality in that distribution with respect to a reference distribution in which everyone receives the population-wide mean income. This yields a measure of age-related inequality.

(2)Compute inequality in the actual distribution of annual income with respect to the usual standard of equality.

(3)Subtract age-related from total annual income inequality, the difference being non-age-related (long-run, interfamily) inequality.

I will now apply this technique to AKS and per capita inequality indices.[82]

The population set is assumed to consist of H individuals, each belonging to one of K age-cohorts having n_k members ($1 \leq k \leq K$). The distribution of mean cohort incomes is denoted $\underline{m}=(m_1\underline{1}_{n_1}, \ldots, m_K\underline{1}_{n_K})$, where m_k is the mean income of cohort k and $\underline{1}_{n_k}$ is a unit vector of dimension n_k. I wish to measure inequality in the distribution $\underline{m}$ with respect to an equal distribution of the population-wide mean income, $(m\underline{1}_H)$.[83] This requires the existence of a social evaluation function, $W: R^H_+ \rightarrow R^1$, assumed to be continuous, increasing, and S-concave, whose image is,

(B.1) $\quad w=W(y)$

where $y=(y_1, \ldots, y_H)$ is a distribution of income among the H members of the population. I begin, following Atkinson [1970], by defining the equally distributed equivalent of $\underline{m}$:

(B.2) $\quad W(p\underline{1}_H)=W(\underline{m})$

The properties of W(.) ensure that p is unique and well-defined for every vector $\underline{m}$, so that p may be written explicitly as,

(B.3) $\quad p=P(\underline{m})$

A per capita index of age-related inequality is defined as the difference between the mean of $\underline{m}$ (=m, the mean of $(m\underline{1}_H)$) and its equally distributed equivalent, p, defined in (B.3):

(B.4) $\quad A_A=m-p$

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[82]Since Paglin's work and the foregoing discussion of it are couched in terms of income rather than consumption, I will continue this practice throughout Appendix B in order to minimize confusion.

[83]It will be important in what follows that the means of $\underline{m}$ and $(m\underline{1}_H)$ are equal.

The corresponding AKS index is defined as A divided by m:

(B.5) $I_A = (m-p)/m = 1-p/m$

Indices of total annual inequality are constructed from the mean and equally distributed equivalent of the actual distribution of income. The latter is implicitly defined by,

(B.6) $W(s\underline{1}_H) = W(y)$

The properties of W(.) ensure that s is uniquely determined for every y, allowing s to be defined explicitly as,

(B.7) $s = S(y)$

The per capita and AKS indices of total annual income inequality are given by,

(B.8) $A = m-s$

(B.9) $I = (m-s)/m = 1-s/m$

Paglin-inequality, "the long-run or lifetime degree of inequality in the economic system" (Paglin [1975, p. 601]), corresponding to the inequality measured by the Paglin-Gini, is equal to the difference between total and age-related annual income inequality. That is,

(B.10) $A_P = A - A_A = (m-s)-(m-p) = p-s$

(B.11) $I_P = I - I_A = (1-s/m)-(1-p/m) = (p-s)/m$

(B.5) and (B.11) are the components of an additive decomposition of total relative inequality, (B.9), into age-related inequality and what I will call Paglin-inequality. I have computed these indices for three values of the free parameter of the Atkinson family of inequality indices. The results are presented in Table 13. I wish to compare this approximation of interfamily lifetime income inequality with a decomposition approach index of intracohort inequality. Paglin has argued that, even if actual lifecycle income data were available, equality of lifecycle incomes is not a reasonable standard of equality when there is real growth over time because, "there will be very large differences in lifetime incomes of older workers and young workers entering the labor force . . . and there is no practical redistribution scheme which would enable the older workers to approach the probable lifetime incomes of the younger" [1975, p. 602]. This problem can be overcome, however, by decomposing interpersonal lifetime inequality within and among age-cohorts. Lifetime inequality is measured by the intracohort component of total inequality in the decomposition approach.

The empirical results in Table 13 reveal that the decomposition approach index of intracohort inequality reflects a lower degree of long-run inequality than does the index of Paglin-inequality. It appears that this is due to the inability of the Paglin-inequality index to account for the effects of income mobility.

> Mobility reduces the dispersion of lifetime incomes much below the annual income estimates . . . . While the P(aglin)-Gini (Paglin-inequality index) adjusts for average age-related inequality it also fails to catch the accompanying intracohort mobility. Until we are able to modify our static inequality coefficients by an index of mobility or collect more longitudinal household income data for an extended period of time, our estimate of inequality of lifetime incomes (or the more difficult trend in the inequality of lifetime income) will remain crude (Paglin [1977, p. 527]).

In the decomposition approach, the three components of total inequality measure inequality arising from different sources. This decomposition of inequality is not entirely clear cut, depending somewhat on the number of years of data that are used in the computation of the indices. Intrapersonal inequality, for example, picks up the effect of income variation over time. When there are only a few years of data this index largely reflects the impact of income mobility although in small part it also accounts for the effect of individuals being at different stages of the lifecycle. The latter is true because the slope of a segment of a lifecycle income profile depends on an individual's age. Early in the working life it may rise quite steeply while later, though higher, it generally levels off substantially. Where the slope is greater, intrapersonal inequality will be greater even if there is zero income mobility (i.e. variation around the lifecycle income path).

The interpretation of intercohort inequality also changes with the number of years of data. When few years of data are used to compute indices in the decomposition approach, intercohort inequality predominantly reflects the stage-of-lifecycle effect. A young cohort will have a representative income which is less than that of a middle-aged cohort precisely because of the typical shape of income profiles. Where the horizon of the data set is sufficient to cover a large part of the lifecycle of each individual, representative lifecycle income (the equally distributed equivalent of an individual's income stream) is a reasonable summary statistic of lifecycle income, and intercohort inequality reflects interpersonal (representative) income differences attributable to economic growth.

The interpretations of these inequality indices are pertinent to the comparison of them with the Paglin-inequality indices. With ten years of data the index of intrapersonal inequality may be regarded as predominantly reflecting the effects of income mobility, and, to a much lesser extent, stage-of-lifecycle effects which are primarily captured in the intercohort term. The empirical difference between the Paglin-inequality index and the decomposition approach index of intracohort inequality is due to the inability of the former to account for the effects on long-term inequality of income mobility. Thus it might be expected that the sum of intrapersonal and intracohort inequality would approximate Paglin inequality. This is, in fact, borne out by the results reported in Table 13. For each value of the free parameter, the sum of intrapersonal and intracohort inequality lies within the range of the ten annual Paglin-inequality indices. Furthermore, intercohort inequality should be approximately equal to the index of age-related inequality, with a tendency to be slightly less than that index because some part of the inequality due to the shape of lifecycle income profiles is captured by the intrapersonal inequality index, as argued above.[84] This hypothesis is confirmed for low degrees of inequality aversion.

In support of his suggested procedure for measuring long-run inequality, Paglin argued that economic growth renders lifetime income equality an unreasonable standard against which to measure inequality in the distribution of lifetime income. Paglin also criticized the use of age-specific Gini coefficients

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[84]This would also explain why the intrapersonal inequality index values tend to lie toward the upper end of the range of the annual Paglin-inequality indices.

(and, by implication, any other inequality indices) on the grounds that,

> the empirical coefficients available are not really specific by age of family head but in fact represent broad age groups. This introduces spurious income variance by not fully eliminating the effect of the age income profile. However, even if we had truly age-specific Gini, we would have the problem of weighting and combining fifty-some measures into one coefficient (Paglin [1975, p. 602]).

Paglin's first point is, in fact, most applicable to his own procedure for measuring long-term inequality and, as has been seen, represents no real difficulty for the use of age-specific inequality measures. Furthermore, the "problem of weighting and combining fifty-some measures into one coefficient" is fully resolved in the procedure for decomposing inequality within and among population subgroups suggested by Blackorby, Donaldson, and Auersperg [1981]. To investigate the performance of age-specific annual income inequality indices vis-a-vis the generalized Paglin procedure and the decomposition approach (involving the use of data covering more than a single year), I have estimated annual income inequality and decomposed it within and among age-cohorts using the Blackorby-Donaldson-Auersperg procedure. The results are reported in Table 14. Comparing the results with those of the generalized Paglin procedure reported in Table 13, it can be seen that intracohort inequality is very close to Paglin inequality, especially for low degrees of relative inequality aversion.[85] When R=-5, however, there is

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[85]For R=.5, Paglin-inequality is never more than .002 less than intracohort inequality in the same year. The corresponding figure is .005 for R=-.5 (except in the last year when Paglin-inequality exceeds intracohort inequality by .009).

significant disparity between indices of Paglin-inequality and intracohort inequality. This appears to be due to the extreme annual variation in age-related inequality at higher values of relative inequality aversion, resulting in similar instability of Paglin-inequality over time. The pattern of annual intracohort inequality exhibits a high degree of stability at all values of the free parameter, R. This I take as partial evidence of the superiority of decomposing annual income inequality within and among age cohorts to approximate long-run inequality rather than using the generalized Paglin technique developed herein.

In conclusion, where panel data are not available, the Blackorby-Donaldson-Auersperg procedure for decomposing inequality can be used to compute an estimate of long-run inequality. It appears from the empirical work to offer more reliable estimates than the generalized Paglin procedure, particularly at high degrees of relative inequality aversion. Both, however, are incapable of accounting for the effects of income mobility on measured inequality, which can be accomplished only when panel data are available. In this case the decomposition approach is the best method of measuring lifetime income inequality.

FIGURE IV

Cumulative Income Distribution Showing Age-related Inequality

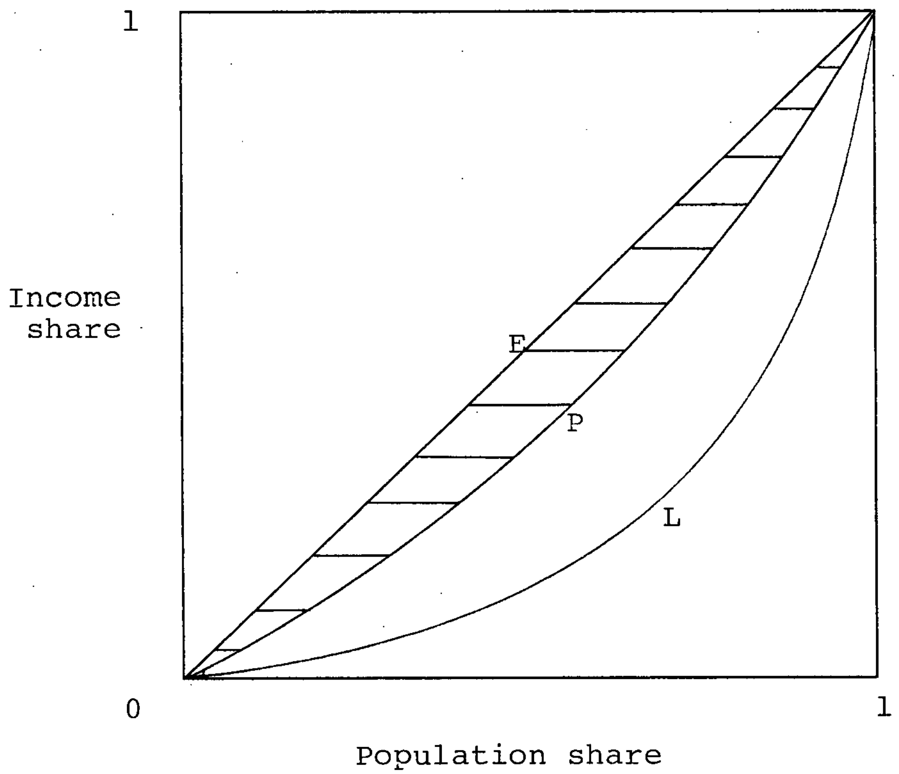

TABLE 13

Total, Age-Related, and Paglin Inequality Indices

	R=.5			R=-.5			R=-5		
YEAR	I	I_A	I_P	I	I_A	I_P	I	I_A	I_P
1968	.0917	.0081	.0836	.2616	.0231	.2385	.6964	.1364	.5600
1969	.0889	.0078	.0810	.2554	.0218	.2337	.6908	.1132	.5777
1970	.0871	.0082	.0790	.2441	.0225	.2216	.6444	.0776	.5668
1971	.0860	.0095	.0765	.2427	.0261	.2166	.7929	.0879	.7050
1972	.0869	.0071	.0798	.2514	.0198	.2316	.7350	.0695	.6655
1973	.0818	.0065	.0753	.2300	.0182	.2118	.6531	.0666	.5865
1974	.0902	.0077	.0825	.2528	.0217	.2312	.8664	.0808	.7856
1975	.0912	.0068	.0844	.2505	.0194	.2311	.6552	.0711	.5840
1976	.0925	.0060	.0866	.2604	.0169	.2436	.8106	.0665	.7441
1977	.0865	.0052	.0813	.2549	.0152	.2397	.8129	.0679	.7450

TABLE 14

Intra- and Inter-Age-Cohort Inequality Indices

YEAR	R=.5 I	I_{AC}	I_{RC}	R=-.5 I	I_{AC}	I_{RC}	R=-5 I	I_{AC}	I_{RC}
1968	.0915	.0845	.0070	.2618	.2397	.0220	.6964	.5571	.1393
1969	.0887	.0823	.0065	.2556	.2369	.0187	.6909	.5497	.1412
1970	.0870	.0801	.0069	.2442	.2260	.0182	.6444	.5271	.1173
1971	.0858	.0778	.0081	.2428	.2199	.0228	.7929	.5361	.2568
1972	.0867	.0806	.0062	.2516	.2322	.0193	.7350	.5559	.1791
1973	.0816	.0763	.0053	.2301	.2142	.0159	.6531	.5191	.1340
1974	.0900	.0833	.0067	.2530	.2315	.0215	.8664	.5283	.3381
1975	.0910	.0851	.0059	.2506	.2317	.0189	.6552	.5255	.1297
1976	.0924	.0871	.0053	.2606	.2430	.0176	.8106	.5606	.2500
1977	.0863	.0811	.0052	.2551	.2303	.0248	.8129	.5573	.2556

APPENDIX C

Data

The following variables, drawn from the eleven year family tape of the Panel Study on Income Dynamics, were used in the construction of the consumption variable.[1]

ASFS=Amount Saved on Food Stamps
FUMY=Family Unit Money Income
HV=House Value
ORPAH=Other Retirement (income), Pensions, and Annuities: Head
ORPAO=Other Retirement (income), Pensions, and Annuities: Others
RMP=Remaining Mortgage Principal
RVFH=Rental Value of Free Housing
TRHS=TRansfers: Head & Spouse
TRO=TRansfers: Others
TXHS=TaXes: Head & Spouse
TXO=TaXes: Others
TX=TaXes: total household

I began by calculating household totals for those variables that are disaggregated by Head and Others (ORPA) or by Head & Spouse and Others (TR, TX). Thus,

(C.1) ORPA = ORPAH + ORPAO (1974-77)

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[1]The eleven year tape is dated 1968-1978. Since data are collected for the previous year, this corresponds to the calendar years 1967-77. Data on transfers, TR, were not collected for 1967, so I have used only the last ten years of data, 1968-77.

(C.2) $TR = TRHS + TRO$ (1968-77)

(C.3) $TX = TXHS + TXO$ (1969-77)

I then calculated the imputed rental value of owner-occupied housing, ROOH, according to the formula prescribed in the Panel Study on Income Dynamics:

(C.4) $ROOH = .06(HV - RMP)$ (1968-77)

Since RMP is missing for the years 1972-74, I estimated its value by linear interpolation. There were two other cases of missing data: ORPA for 1968 and ASFS for 1973. These were both assigned zero values for all households.

Two further adjustments were necessary before I could compute estimates of gross income, net income, and consumption. First, the income variable on the data tape, FUMY, is defined to include total transfers, TR. Since these are not a part of gross income, I calculated Family Unit Money Income from Market Sources as,

(C.5) $FUMYMS = FUMY - TR$ (1968-77)

I then calculated gross income, YG, as,

(C.6) $YG = FUMYMS + RVFH$ (1968-77)

Second, in the data set, TR is defined to include ORPA. In keeping with my distinction between interpersonal and intertemporal redistribution, I wished to calculate net income, YN, as gross income adjusted for public transfers and private interpersonal redistribution. This required that ORPA, which represents private intertemporal redistribution, be excluded from TR in the computation of net income:

(C.7) $YN = YG + (TR - ORPA) - TX + ASFS$ (1968-77)

Finally, I calculated consumption, C, as,

(C.8) C = YN + ORPA + ROOH (1968-77)

This total household nominal consumption variable was then deflated by the U. S. Consumer Price Index (1975=100) (International Monetary Fund [1980, p.343]). The calculation of Family Adult Equivalent Consumption and the grouping of families into age-cohorts required the following demographic variables from the Panel Study on Income Dynamics:

AFH=Age of Family Head

NFU=Number in the Family Unit

NCFU=Number of Children in the Family Unit

A complete listing of the number, location, width, name, and year of the variables drawn from the data tape follows:

449	821	5	HV	1968
451	827	5	RMP	
457	843	4	RVFH	
510	976	4	ASFS	
525	1020	5	TRHS	
527	1026	5	TRO	
529	1032	5	FUMY	
532	1042	5	TX	
549	1077	2	NFU	
550	1070	2	NCFU	
1008	1829	2	AFH	
1122	2041	5	HV	1969
1124	2047	5	RMP	
1130	2063	4	RVFH	
1183	2194	4	ASFS	
1208	2244	5	TXHS	
1213	2265	5	ORPA	
1220	2284	5	TRHS	
1224	2299	5	TXO	
1225	2304	5	TRO	
1238	2347	2	NFU	
1239	2349	2	AFH	
1242	2354	1	NCFU	
1514	2706	5	FUMY	
1823	3542	5	HV	1970
1825	3548	5	RMP	
1831	3564	4	RVFH	
1884	3695	4	ASFS	
1910	3750	5	TXHS	
1915	3771	5	ORPA	

1922	3790	5	TRHS	
1926	3805	5	TXO	
1927	3810	5	TRO	
1941	3853	2	NFU	
1942	3855	2	AFH	
1945	3860	1	NCFU	
2226	4226	5	FUMY	
2423	4542	5	HV	1971
2425	4548	5	RMP	
2431	4564	4	RVFH	
2478	4680	4	ASFS	
2511	4751	5	TXHS	
2516	4772	5	ORPA	
2523	4791	5	TRHS	
2527	4806	5	TXO	
2528	4811	5	TRO	
2541	4854	2	NFU	
2542	4856	2	AFH	
2545	4861	2	NCFU	
2852	5253	5	FUMY	
3021	5542	5	HV	1972
3025	5553	4	RVFH	
3064	5638	5	TXHS	
3069	5659	5	ORPA	
3076	5678	5	TRHS	
3080	5693	5	TXO	
3081	5698	5	TRO	
3094	5741	2	NFU	
3095	5743	2	AFH	
3098	5748	2	NCFU	
3256	5976	5	FUMY	
3417	6129	5	HV	1973
3421	6140	4	RVFH	
3443	6193	4	ASFS	
3476	6261	5	TXHS	
3481	6282	5	ORPA	
3488	6301	5	TRHS	
3492	6316	5	TXO	
3493	6321	5	TRO	
3507	6368	2	NFU	
3508	6370	2	AFH	
3511	6375	2	NCFU	
3676	6618	5	FUMY	
3817	6828	6	HV	1974
3821	6841	4	RVFH	
3851	6914	4	ASFS	
3876	6965	5	TXHS	
3881	6986	5	ORPAH	
3889	7010	5	TRHS	
3893	7025	5	TXO	
3899	7050	5	ORPAO	
3905	7072	5	TRO	

3920	7123	2	NFU	
3921	7125	2	AFH	
3924	7130	2	NCFU	
4154	7437	5	FUMY	
4318	7633	6	HV	1975
4320	7640	5	RMP	
4330	7663	4	RVFH	
4364	7747	4	ASFS	
4390	7836	5	TXHS	
4396	7859	5	ORPAH	
4404	7895	5	TRHS	
4409	7911	5	TXO	
4413	7931	5	ORPAO	
4419	7961	5	TRO	
4435	8016	2	NFU	
4436	8018	2	AFH	
4439	8023	2	NCFU	
5029	8933	5	FUMY	
5217	9129	6	HV	1976
5219	9136	5	RMP	
5229	9159	4	RVFH	
5277	9282	4	ASFS	
5301	9367	5	TXHS	
5307	9390	5	ORPAH	
5316	9427	5	TRHS	
5321	9443	5	TXO	
5325	9463	5	ORPAO	
5332	9495	5	TRO	
5349	9554	2	NFU	
5350	9556	2	AFH	
5353	9561	2	NCFU	
5626	9948	5	FUMY	
5717	10129	6	HV	1977
5719	10136	5	RMP	
5727	10154	4	RVFH	
5776	10274	4	ASFS	
5800	10359	5	TXHS	
5807	10386	5	ORPAH	
5815	10419	5	TRHS	
5820	10435	5	TXO	
5825	10460	5	ORPAO	
5831	10487	5	TRO	
5849	10545	2	NFU	
5850	10547	2	AFH	
5853	10552	2	NCFU	
6173	10994	5	FUMY	

APPENDIX D

Inequality in the Original Sample

Measuring inequality in the original sample raises the question of how to treat families that experience a change of family head. When the new head is of a different age than the previous one, in which age-cohort should the family be included? For that matter, how is the family's representative consumption to be calculated: separately for each group of years corresponding to a different family head, or over the entire sample period?

Although the age-cohort to which a family is assigned depends on the age of its head, the family is not defined by its head. In the Panel Study on Income Dynamics a family retains its identity despite a change of head. In keeping with this, I have calculated each family's representative consumption over the whole sample period, ignoring changes of family head.

The distribution of families by age-cohort varies from year to year in the original sample because of changes in family heads. The choice of which distribution to use to decompose inequality within and among age-cohorts is arbitrary, but unimportant to the extent that the results are not sensitive to the decision made.

There are, therefore, two comparisons to be made. First, inequality in the original sample has been measured and decomposed according to the age-cohort distributions in different years. The results for 1968 and 1977 are reported in

Table 15, from which it can be seen that measured inequality is quite insensitive to annual differences in the age-cohort distribution.

Second, inequality measured in the original sample can be compared to the subsample results reported in Chapter Four (see Table 3). Inequality in the original sample is approximately 20 to 25 per cent greater than in the subsample. The breakdown of total inequality among the three subindices is very similar in the two samples, however, as is the degree to which annual inequality overstates pure interpersonal inequality. I conclude that the subsample results, while understating the degree of actual inequality, do provide an accurate indication of the differences between the traditional and decomposition approaches to the measurement of inequality.

TABLE 15

Relative Inequality in the Original Sample

1968 Age-cohort Distribution

DECOMPOSITION INDICES

	R=.5	R=-.5	R=-5
I	.1113	.3123	.9553
I_{AP}	.0266 (23.9)	.0784 (25.1)	.2283 (23.9)
I_{AC}	.0821 (73.8)	.2239 (71.7)	.6171 (64.6)
I_{RC}	.0026 (2.3)	.0100 (3.2)	.1100 (11.5)

ANNUAL INDICES

	R=.5	R=-.5	R=-5
min	.0969	.2821	.8454
mean	.1079	.3021	.9225
max	.1211	.3303	.9645

1977 Age-cohort Distribution

DECOMPOSITION INDICES

	R=.5	R=-.5	R=-5
I	.1113	.3123	.9554
I_{AP}	.0266 (23.9)	.0784 (25.1)	.2283 (23.9)
I_{AC}	.0802 (72.1)	.2227 (71.3)	.6089 (63.7)
I_{RC}	.0045 (4.0)	.0113 (3.6)	.1181 (12.4)

ANNUAL INDICES

	R=.5	R=-.5	R=-5
min	.0968	.2821	.8454
mean	.1079	.3021	.9225
max	.1210	.3303	.9645

APPENDIX E

A Proof

The proposition to be proved is that the members of a family of social evalution (or equally distributed equivalent) funcions reflecting "the same set of ethical judgments" are characterized by the same degree of inequality aversion. The proof[87] is in two parts.

I. With n people,

(E.1) $h^{-1}[(1/n)\sum_i h(y_i)]=g^{-1}[(1/n)\sum_i g(y_i)]$

if and only if g(.) and h(.) are cardinally equivalent.

Proof: (i)sufficiency:

g(.) and h(.) are cardinally equivalent if they are unique up to a positive affine transformation:

$$g(.)=ah(.)+b \qquad a>0$$

In this case,

$$g^{-1}(y_i)=h^{-1}[(y_i-b)/a]$$

and (E.1) can be written,

$$\begin{aligned} h^{-1}[(1/n)\sum_i h(y_i)] &= g^{-1}[(1/n)\sum_i (ah(y_i)+b)] \\ &= g^{-1}[a(1/n)\sum_i h(y_i)+b] \\ &= h^{-1}[(a(1/n)\sum_i h(y_i)+b-b)/a] \\ &= h^{-1}[(1/n)\sum_i h(y_i)] \end{aligned}$$

(ii)necessity:

Let $z_i=g(y_i)/n$. Then,

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[87] I am indebted to David Donaldson for the following proof of this proposition.

$$h(y_i)/n=h(g^{-1}(nz_i))/n=f(z_i)$$

(E.1) can then be written,

$$h^{-1}[\sum_i f(z_i)]=g^{-1}[\sum_i z_i], \text{ or}$$

(E.2) $$k(\sum_i z_i)=\sum_i f(z_i)$$

(E.2) is a Pexider equation whose solution is,[88]

$$f(t)=at+b \qquad\qquad a>0$$

Therefore,

$$\begin{aligned} h(y_i)&=nf(z_i)\\ &=n(az_i+b)\\ &=ag(y_i)+nb\\ &=ag(y_i)+\bar{b} \quad || \end{aligned}$$

II. The Population Substitution Principle implies that the equally distributed equivalent functions can be written,

$$s_n=S^n(y)=g^{-1}[(1/n)\sum_i g(y_i)] \qquad\qquad \forall n$$

Proof: The Population Substitution Principle implies that $S^n(.)$ is additively separable:

(E.3) $$S^n(y)=g_n^{-1}[(1/n)\sum_i g_n(y_i)]$$

Let m<n. (E.3) can be written

(E.4) $$S^n(y)=g_n^{-1}[(m/n)g_n\{g_m^{-1}[(1/m)\sum_i^m g_m(y_i)\}+(1/n)\sum_{m+1}^n g_n(y_i)]$$

Equating (E.3) and (E.4) and dropping the last m-n common terms yields,

$$g_n^{-1}[(1/m)\sum_i^m g_n(y_i)]=g_m^{-1}[(1/m)\sum_i^m g_m(y_i)]$$

It follows from part I that $g_n(.)$ and $g_m(.)$ are cardinally equivalent. ||

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[88]Eichhorn[1978]