FREGE'S ONTOLOGY: A CRITICAL EXAMINATION OF ITS FOUNDATIONS

by

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Abstract

There are two contentious ontological claims that Frege makes. Firstly, that predicates have reference and secondly, that their reference is 'unsaturated'. It is the primary aim of this essay to examine the latter claim.

Following an introductory exposition of the Fregean concepts crucial to this essay, the claim that the references of predicates are 'unsaturated' is considered as a response to the problem of the unity of the proposition. As a special case of this problem, F.H. Bradley's attack on relations is discussed. It is argued that once one understands that relations are unsaturated, Bradley's problem, and by extension the problem of the unity of the proposition, disappears.

But a crucial semantic principle is shown to emerge from this attempted solution to these problems: Any language capable of talking about relations and concepts cannot refer to these by means of proper names.

This principle, it then appears, renders us incapable of specifying in a given instance which concept a predicate refers to. Two recent attempts to avoid this problem are discussed and rejected. Insight gained from this discussion leads to a third, purely Fregean solution to the problem. This solution, which is forced upon Frege by his own principles, is seen to involve him in a vicious circle. It appears that Frege's ontological doctrines are ill-founded - that they cannot solve the very problems for which they were designed.
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Bibliography
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Chapter 1

Introduction

The main purpose of this opening chapter is to familiarize the reader with those Fregean concepts which feature in the remainder of this essay. Only the briefest introduction to those concepts is intended. A more detailed and extensive survey of Frege's doctrines can be found elsewhere (for example, Furth's introduction to Bib. (20)). Most of the issues dealt with in this chapter are developed in greater depth there. A reader who is reasonably well acquainted with Frege's ontological and semantic theories may omit the expository portion of the first chapter (ie. all but section 1.45). A system of cross reference has been employed such that each key idea developed in this introductory chapter is sequentially numbered. Where that idea features crucially in later chapters, the reader is referred by these numbers to the appropriate section in the first chapter. Thus if a claim made in later chapters appears to be based on an unusual interpretation of Frege's doctrines, the reader will be able to refer immediately to that portion of the first chapter where that unusual interpretation is defended.
(1.01) Frege's fundamental ontological principle is this: Everything is either a function or an object and nothing is both. But Frege repeatedly claims that the terms 'function' and 'object' are logically simple and therefore impossible to define. He writes:

(1.02) Now something logically simple is no more given us at the outset than most of the chemical elements are; it is reached only by means of scientific work. If something has been discovered that is simple, or at least must count as simple for the time being, we shall have to coin a term for it, since language will not originally contain an expression that exactly answers. On the introduction of a name for something logically simple, a definition is not possible; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the words as is intended.

("Concept and Object" Bib. (28) p43)

(1.03) It turns out that we can reach a clearer understanding of this ontological principle via two other Fregean distinctions - the proper name/function name distinction and the sense/reference distinction. These further distinctions in turn are perhaps best explained by first considering Frege's category of proper names.

(1.04) Proper Names

Frege extends considerably the grammarians notion of proper names. Thus for example, apart from such obvious proper names as 'Pegasus', 'John Smith' and 'Moses', he includes definite descriptions such as 'The winged horse that was captured by Bellerophon'.

1 Due to Quine.
and 'The author of Waverley'.\textsuperscript{2} Sentences too fall into the class of proper names. Frege nowhere attempts a formal classification of this class of names though he frequently remarks that as a rule, the definite article followed by a common noun in the singular, forms a proper name. As a sample of other kinds of proper names consider: 'Caesar's death', 'Humanity', 'Charity', 'Killing', and 'His late arrival'.

(1.05) Let us now see how the sense/reference ('Sinn'/ 'Bedeutung') distinction applies to proper names. An expression is said to denote its reference and express its sense. In the case of proper names, the reference of a proper name is simply that entity referred to by the proper name. So, for a definite description, the entity referred to is just that entity which fits the definite description. Thus 'the author of Waverley' refers to Scott, the author of the Waverley novels. (1.06) Crudely put, the sense of an expression is what one needs to grasp in order to locate its denotation.\textsuperscript{3} In the case of what grammarians call proper names, it might be thought doubtful whether they have senses at all so it is best at this stage to consider definite descriptions. Thus the sense of the expression 'the author of Waverley' is roughly, what one needs to grasp in order to determine that its reference is Scott himself.

\textsuperscript{2} Due to Russell.
\textsuperscript{3} I use 'Denotation' and 'Reference' interchangeably.
(1.07) In addition to its sense and reference, an expression can have an idea associated with it. Frege introduces the notion of the associated ideas of expressions primarily to contrast associated ideas of expressions with their senses. An idea associated with an expression consists roughly in the images and perhaps feelings which the expression conjures up in the reader or hearer. It is entirely subjective - never the same for two people. On the other hand the sense of an expression is an objective existent entity which can be grasped by any skilled user of the language in which that expression is employed. Any two skilled language users will grasp the sense of an expression (assuming the expression to be unambiguous). Frege invokes the analogy of a telescope to elucidate the relationship between the sense, reference and associated ideas of an expression. If we train a telescope onto the moon, an image of the moon appears within the telescope which reflects onto the retina of the observer's eye. The moon itself is likened to the reference of an expression, the image within the telescope to the sense (it can be perceived by any number of observers), and the retinal image to an associated idea. For this image is never the same for two observers.

(1.08) Sentences are somewhat special proper names for Frege. He countenances two truth values, which he
called 'The True' and 'The False' and a sentence is said to denote one of these truth values accordingly as it is true or false. (1.09) The sense of a sentence is a proposition or thought.  

(1.10) Frege considered it a defect of natural languages that there are a) names which do not denote, b) names which denote more than one entity, and c) names which are ambiguous - have more than one sense. He stipulated than in a logically perfect language each name must express exactly one sense and denote exactly one reference.

(1.11) Frege introduced the sense/reference distinction in order to explain the informativeness of true identity statements such as 'Hesperus = Phosphorus'. The question arising is why such statements are informative while statements such as 'Hesperus = Hesperus' are not. The solution Frege proposed was that while 'Hesperus' and 'Phosphorus' have the same denotation they do not express the same sense. (1.12) An entity may be named by several expressions and the senses of these expressions may differ. Furthermore one and the same sense may be expressed by different expressions (either of the same language or of different languages). But Frege regarded it as a fault of natural languages that an expression

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4 Frege's term 'Gedanke' has been variously translated as either 'proposition' or 'thought'. I shall from here on use 'thought'.

may express a sense without denoting anything. (Though of course, if it does denote something then it must express a sense.)

(1.13) Before turning to the discussion of function names, it is worth ending the discussion of sense and reference by noting Frege's compositional theory of sense and reference:

i) If a component of a denoting expression is replaced by an expression having the same denotation as that component, the denotation of the entire expression is unchanged. Furthermore, if any component of an expression lacks a reference then the entire expression lacks a reference.

ii) If a component of an expression is replaced by an expression having the same sense as that component, the sense of the entire expression is unchanged. Furthermore, if any component of an expression fails to express a sense then the entire expression also fails to express a sense.

(1.14) Function Names

Given the general notion of a proper name, it is possible to introduce the notion of a function name. Frege claimed to have adopted the notion from mathematics and to have extended it considerably for his own purposes by applying it to the natural languages. It seems appropriate then to trace
this development. Consider the expressions:

\[ 2.1^3 + 1' \]
\[ 2.4^3 + 4' \]
\[ 2.5^3 + 5' \]

Each one of these expressions is for Frege a proper name denoting a number. Yet we recognise a common structure insofar as each expression can be seen as formed by successively placing '1', '4' and '5' into the gaps of the expression

\[ 2.( )^3 + ( )' \]

where in this case we indicate the position of the gaps by means of parentheses. Frege calls the expression

\[ 2.( )^3 + ( )' \]

a function name. In general, the removal of a proper name from a proper name containing that name as a proper part results in a function name. Applying this now to a natural language, consider the proper name

'The author of Waverley'.

If we remove from this expression the proper name 'Waverley' and mark the position from which it is removed with parentheses, the resulting expression,

'The author of ( )'

is a function name. (1.15) The result of inserting
the proper name 'Waverley' into the gap of this function name ('Waverley' then being the name of an argument of the function) yields a proper name whose denotation is. Scott is said to be the value of the function for the argument Waverley.

(1.16) Frege frequently employs the term 'ungesättigt' (translated variously as 'unsaturated' or 'incomplete') to describe function names. For example, in "What is a Function?", writing about mathematical functions, he notes:

"... the sign for a function is unsaturated; it needs to be completed with a numeral, which we then call the argument-sign."

(Bib. (23) see (28) p113/4)

(1.17) Instead of brackets, Frege usually employs small Greek letters to indicate the gaps in function names. Thus he would write the last mentioned function name as:

'The author of \$\xi\$'.

We will see shortly that there are function names of higher levels than the above which require a different kind of argument name for their completion and different Greek letters serve to mark the appropriate type of argument name.

(1.18) Now just as proper names express their sense and denote their reference, so too with function names. The senses of function names will be discussed a little
later. (1.19) For the moment let us restrict ourselves to considering the denotation of function names, i.e. functions. Frege provides a clear statement of the conditions under which a function name denotes, and although there is a circularity involved in the combination of this statement and his statement of the conditions under which a proper name denotes (we can only know whether a proper name denotes if we already know which function names denote and we can only know whether a function name denotes if we already know which proper names denote), it is worth noting. Frege writes:

"A name of a first-level function of one argument has a denotation (denotes something, succeeds in denoting) if the proper name that results from this function name by its argument place being filled by a proper name always has a denotation if the name substituted denotes something."

(Bib. (20) p84)

(1.20) What kind of entity a function is, of course, is not yet clear. But perhaps if the notion of a function name is clear and if the relationship between a proper name and its denotation is reasonably clear, then by viewing a function as standing in an analogous relationship to its name, some insight into its nature is gained. Frege himself resorts to metaphor. Attempting to explain mathematical functions, he writes:

(1.21) "The peculiarity of functional signs, which we here called 'unsaturatedness', naturally has something answering to it in the functions themselves."
They too may be called 'unsaturated' and in this way we mark them out as fundamentally different from numbers. Of course this is not a definition; but likewise none here is possible. I must confine myself to hinting at what I have in mind by means of metaphorical expression, and here I rely on my reader's agreeing to meet me half-way."

(Bib. (23) see (28) p115)

(1.22) We are now better poised to appreciate Frege's distinction between function and object. Proper names denote objects ie. saturated entities while function names denote functions ie. unsaturated entities. Everything is such that it is appropriately named either by a proper name or by a function name. Nothing is appropriately named by both a proper name and a function name. Thus the ontological division between function and object is precisely tied to the linguistic division between proper names and function names.

(1.23) Concepts

Functions of one argument which always take truth values as their values are called 'concepts'. Since sentences denote truth values, a concept name can be formed by omitting a proper name from a declarative sentence. Thus:

'ξ wrote Waverley'

names a concept whose value is the True for Scott as argument and the False for anything else as argument.

(1.24) Concept names are for Frege predicates and he repeatedly stresses the predicative nature of concepts -
that they, like functions, are the references of incomplete expressions. He writes:

"What I call here the predicative nature of the concept is just a special case of the need of supplementation, the 'unsaturatedness' that I gave as the essential feature of a function."

(Bib.(21) see (28) p47n)

(1.25) If the value for a given object as argument of a concept is the True, that object is said to fall under the concept. (1.26) Frege also stressed that "a concept must be everywhere defined":

"It must be determinate for every object, whether it falls under the concept or not; a concept word which does not satisfy this requirement on its reference, is denotationless . . . "

(Bib.(13) pl33)

Clearly this requirement is just Frege's condition for a first level function name having a denotation applied to concepts.

(1.27) It is worth noting two criteria Frege mentioned for distinguishing concept names. The indefinite article 'a' followed by a common noun eg. 'a horse' in 'a horse is an animal' usually indicates a concept name. Plural forms of common nouns eg. 'horses' in 'horses are animals' also usually indicate concept names.

(1.28) It is also worth noting that Frege held that the distinction between proper names and predicates and thus the distinction between objects and concepts apply universally to all languages.
(1.29) Before considering second level concepts, it is convenient here to fulfil an earlier promise of saying something about the senses of predicates and function names in general.

(1.30) In the early stages of commentary on Frege's ontology, a controversy arose as to whether Frege intended his sense/reference distinction to apply to function names. Some commentators (e.g. Marshall — see Bib. (31) ) argued that Frege's use of 'Bedeutung' in connection with function names was not to be taken in the strict technical sense which it has in connection with proper names but was intended to mean something like the English 'meaning'. In which case the 'Bedeutung' of a function names was much like the 'Sinn' of a proper name and the notion of the 'Sinn' of a function name simply did not apply.

The publication of Frege's 'Nachgelassene Schriften' (Bib. (13) ) ended this debate. Therein he wrote, regarding the distinction which he had drawn in "Sinn und Bedeutung" between the sense and reference of a proper name:

"The same distinction can also be made with regard to concept words. There is some danger that the classification into concepts and objects will be mixed up with the distinction between sense and reference, so that sense and concept on the one hand and reference and object on the other will be fused. To every concept word or proper name corresponds, as a rule, a sense and a reference . . ."

(Bib. (13) p128)

Further research revealed that the senses of function names
are also unsaturated. Thus the symmetry of unsaturatedness between a sign and its reference is preserved between a sign and its sense. The sense of a function name is unsaturated while the sense of a proper name is saturated.

(1.31) Second level concepts

If from a sentence a proper name which is a proper part of that sentence is omitted, the resulting expression is, as we have seen, a concept name or predicate. The concept named by such an expression, Frege calls a first level concept. If however the grammatical predicate is dropped from a sentence, the resulting expression is a name of a second level concept which takes first level concepts as arguments. (Provided of course that the conditions for being a concept are met by the entity so named.) For example,

'Scott φ'

denotes a second level concept whose value is the True for the concept ξ wrote Waverley as argument. (1.32) Frege uses the letters 'ξ' and 'ζ' as gap holders for proper names and 'φ' and 'ψ' as gap holders for first level function names.

(1.32) The most ubiquitous second level concept in Frege's system is the universal quantifier. The expression

§ see H. Jackson Bib.(29)
used to denote this concept is:

\( \neg \phi(a) \).

This expression names a second level concept whose value for a particular first level function as argument is the True if and only if the value of that function is the True for every argument. (1.34) If the result of completing a second level concept with a first level concept yields the True, the first level concept is said to fall within the second level function.

(1.35) Relations

Functions of two arguments which always take truth values as their values are called relations. If for example we drop the proper name 'Waverley' from the concept name

'\( \xi \) wrote Waverley'

the resulting expression

'\( \xi \) wrote \( \zeta \)'

denotes a relation. I will call the arguments of a relation its terms. Relations, like concepts, can be of first or second level i.e. some are completed by objects and some by first level functions. (1.36) But some relations have the further property of relating terms of different levels e.g. a relation may take an object as one of its terms and a first level function as the other. Such relations are called unequal levelled relations.
Relations, both of whose terms are objects or functions of the same level are called equal levelled relations. The level of an unequal levelled relation is one higher than its highest levelled argument.

(1.37) Identity conditions for functions

We now turn to a crucial question: How are Frege's functions individuated? Or considering just first level concepts? Are concepts $\Phi$ and $\Psi$ identical if exactly the same objects fall under each? Commentators have disagreed. Rulon Wells for example, claims Frege

"... is prepared to admit that there can be functions that are different from each other even though they have precisely the same values for the same arguments ... This is tantamount to rejecting so-called extensionality of functions. We can only say that Frege has not laid down for us conditions under which different expressions identify the same functions."

(Bib. (45) see (30) pl1)

On the other hand, Furth maintains that:

"... in Frege's theory, functions and concepts are also extensional, they are not distinguished from courses of values and the extension of concepts on this ground, but on the ground that courses of values and the extension of concepts are objects whereas functions and concepts are not."

(Bib. (20) pxxxix)

(1.38) I hasten to add an explanation of Frege's notion of course-of-values of functions. Consider an example stated in the terms of set theory. The course-

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6 Frege uses capital Greek letters where modern authors might use metalinguistic variables ranging over first level functions.
of-values of the concept \( \xi^2 = 4 \) (Frege uses the expression \( \xi (\xi^2 = 4) \) to denote this course-of-values) can be seen as the set of ordered pairs \((x,y)\) such that \(y\) is the value of this concept for \(x\) as argument. The course-of-values of a relation can similarly be viewed as the set of ordered triples \((x,y,z)\) where \(z\) is the value of the relation for \(x\) and \(y\) as arguments. In the case of concepts/relations, the second/third member of each member of the set of ordered pairs/triples constituting the course of values of the concept/relation, is a truth value.

(1.39) The question of the identity conditions of concepts can now be restated. Are two concepts the same when they have the same course-of-values? The answer is a qualified yes. Qualified, because as Frege noted, the relation of equality or sameness cannot strictly be said to hold between concepts. He wrote:

(1.40) "... the relation of equality, by which I mean complete coincidence, identity, is also conceivable only in connection with objects, not with concepts ..."

(Bib. (13) p130/1)

and again,

"We have now recognised that the relation of equality between objects cannot also be conceived between concepts, and that there is however a corresponding relation. The word 'same', which is used for the designation of the former relation between objects, cannot consequently properly serve also for the designation of the latter."

(Bib. (13) p132)
Perhaps this point is better expressed if we depict the relation of identity thus:

\[ \xi = \zeta \]

Clearly, this relation can only be completed by objects. Any filling of the argument places of this relation name by concept names yields not the name of a truth value but the name of a new relation.

(1.41) If we wish to express the thought that the concepts \( \Phi \) and \( \Psi \) stand in that relation (mentioned above by Frege) corresponding to the relation of identity between objects, we may say:

\[ \neg \Phi(a) = \Psi(a) \]

About this relation, Frege is quite explicit. He sums up:

"But even if the relation of equality is only conceivable in connection with objects, a similar one occurs in connection with concepts which, as a relation between concepts I call a relation of second order while I call the former equality a first order relation. We say, an object \( a \) is equal to an object \( b \) (in the sense of complete coincidence), if \( a \) falls under every concept under which \( b \) falls, and conversely. We obtain something corresponding to this for concepts, if we let the concept and object interchange their roles. We could say then, the relation mentioned above occurs between the concept \( \Phi \) and the concept \( \Psi \), if every object which falls under \( \Phi \) also falls under \( \Psi \) and conversely."

(Bib.(13) p131)

(1.42) Identity of courses-of-value is for Frege, a necessary and sufficient condition for this relation holding between concepts. (1.43) But concepts are still to be distinguished from their courses-of-values - the
former are unsaturated, the latter saturated. This principle naturally extends to relations and to functions in general.

(1.44) In the course of this essay I shall have occasion to enquire whether relation A is the same as relation B. Owing to the above considerations, the reader should understand this to mean, 'Does relation A have that relation to B which corresponds to the relation of identity for objects? Obscure though this question may seem, the answer is readily available once we know the courses-of-values of A and B.

(1.45) This is the end of the brief exposition of those Fregean notions with which this essay is concerned. But the reader may already have noticed a problem which has infected this exposition and which will in fact plague anyone attempting such an exposition. During the course of my exposition, I have said such things as:

"ξ wrote Waverley has the truth value the True for Scott as argument."

But this expression is not a sentence by Frege's criteria, for it is incomplete in the same way that the first level predicate 'ξ wrote Waverley' is incomplete. If anything it denotes a first level concept. The difficulty is that a concept (and in general a function) can only be referred to within the context of a complete name. But it seems that in order to say the things about concepts which any expo-
sition of Frege's doctrines requires saying, we need to refer to concepts on their own, so to speak - not in the context of complete names. Frege was well aware of this difficulty. He commented:

"It is clear that we cannot present a concept as independent, like an object; rather it can occur only in connection. One may say that it can be distinguished within, but that it cannot be separated from the context in which it occurs. All apparent contradictions that one may encounter here derive from the fact that we are tempted to treat a concept like an object, contrary to its unsaturated nature. This is sometimes forced upon us by the nature of our language. Nevertheless it is merely a linguistic necessity."

(Bib (19) p 34n)

But this problem is really twofold and goes deeper than Frege will allow. Predicates, according to Frege, exhibit that feature corresponding to concepts themselves - they are unsaturated. We attempted to exhibit this feature by indicating the argument place by means of Greek letters or parentheses. But our standard way of referring to these expressions - by enclosing the entire expression in quotation marks - refers to just those signs within the quotation marks and these signs noticeably bear no trace of unsaturatedness. Thus for example, the quotation mark name,

'£ wrote Waverley'

refers to an object - the group of signs within the quotation marks. Another way of seeing this is that quotation mark names are complete names and thus on Frege's principles cannot denote an unsaturated entity.
And this problem remains no matter how one chooses to indicate the argument place of a predicate. Worse still, even if we could form names of predicates which were themselves incomplete, they would be no use whatsoever. For suppose,

'! \xi w\text{rote Waverley}!'\]

were such a name. Then,

'! \xi w\text{rote Waverley! denotes a function of one argument}'

is exactly on par with,

'\xi w\text{rote Waverley has the truth value the True for Scott as argument}.'

insofar as it is incomplete and hence not a sentence.

Perhaps it is considerations such as these which have prompted Dummett to claim that predicates for Frege, are not linguistic expressions at all - contrary to the way we have been treating them throughout this exposition. Dummett writes:

"The incompleteness of a predicate . . . resides in the fact that it is not, in general, a separate piece of the sentence but is rather, a feature of the way in which the sentence is constructed. A predicate is thus something which literally cannot be exhibited separately." (Bib (10)vol 3, p 230/1)

Thus there appears to be an insuperable obstacle to explaining Frege's theories. Clearly, the unsaturated/saturated division is at the heart of this obstacle. Why
did Frege employ such a distinction which effectively prevented him from stating his own theory? It is the purpose of this essay to answer this question and ultimately to show that the function/object or saturated/unsaturated distinction is ill-founded - that it cannot solve the very problems for which it was designed. For this purpose however, I must set aside for the time being the problems just discussed and I shall continue referring to concepts and predicates in the same way as I have been referring to them throughout this exposition.
Chapter 2

Introduction

There are two contentious ontological claims that Frege makes. Firstly that predicates have reference and secondly that their reference is 'unsaturated' or 'incomplete'. This chapter concerns itself with the nature and justification of these claims.

From Frege's analysis of the relationship between language and thought, an argument is produced in support of the view that every component of a sentence must have a sense. An analogous argument is considered in support of the claim that every component of a sentence (and hence predicates) must have a reference. But this argument is shown to be based on an assumption which Frege himself later repudiated.

Frege's claim that the reference of a predicate is 'unsaturated' is then considered as a response to the problem of the unity of the proposition. As a special case of this problem, we consider Bradley's problem of relations and construct a Fregean response to this problem in some detail. It is argued that once relations are seen to be explained as that which relational expressions refer to, Bradley's problem and the general problem of which it is a special case, disappear. It is characteristic of relations that they can only be explained in the above manner and it is this characteristic which
best elucidates the Fregean metaphors of 'unsaturatedness' and 'incompleteness'. But - it is argued - a crucial semantic principle emerges from this discussion: We cannot refer to a concept or relation (and in general a function) by using a proper name. Furthermore, any language capable of talking about relations and concepts at all, must be governed by this principle.
Dummett has pointed out that nowhere in Frege's published writings does Frege explicitly argue for his position that function words have 'Bedeutung'.\(^7\) (1.18, 1.19, 1.30) However, a manuscript of 1906, reports Dummett,\(^8\) contains two statements approaching such an argument. Frege writes:

"It is altogether improbable that a proper name should be so different from the remaining part of a singular sentence that it should be important for it alone to have 'Bedeutung'... It is unthinkable that there could be a 'Bedeutung' only in the case of proper names and not in the remaining part of the sentence."

However, this statement is just tendentious unless we assume that a sentence denotes a truth value and we can adduce some support for the principle that if part of an expression lacks 'Bedeutung' then so does the entire expression.

Frege's second statement is not much more convincing. It proceeds along these lines: Consider the relational sentence 'Jupiter is larger than Mars'. This states that a certain relation holds between Jupiter and Mars. But surely such a relation must subsist in the same realm as Jupiter and Mars - namely the realm of reference. For the being-larger-than, if it is to hold between two

\(^7\) Of course, Frege probably arrived at his position from his analysis of number. However Resnik has argued convincingly (Bib. (36) p333/4) that one cannot appeal to this analysis in support of the view that function words have 'Bedeutung' without begging the question.

\(^8\) see Bib. (9) p205/6
entities of the objective world, must itself be a part of that world.

This argument only begins to be convincing if we find the move from 'Jupiter is larger than Mars' to 'There is a relation which holds between Jupiter and Mars' unobjectionable. But it is precisely this move which anyone who doubts that relation words have reference is likely to find objectionable. The move is from an ontologically innocent relational statement to a statement explicitly asserting the existence of a relation. If we accept this move we will much more readily accept that relation words have 'Bedeutung'. But for the present it is not obvious that we must accept the move.

There are two contentious claims that Frege makes regarding function words. Firstly that they have reference and secondly that their reference is 'unsaturated', 'incomplete'. (1.14,1.16,1.18,1.19,1.21) This second claim I propose receives its strongest support from Frege's analysis of the relationship between language and the world. Problems involving this relationship arise which are seen to require for their solution the view that the references of connectives are unsaturated. Resnik has argued\(^9\) that the first claim too is grounded upon similar concerns. In this section I discuss the metaphysics which purportedly supports these claims.

\(^2\) see Bib. (36)
I think the two claims can profitably be separated although the overall theory is clearly linked. But I shall indicate in due course where such linkage occurs. Resnik's argument is based upon an analogy with Frege's analysis of thoughts and the sentences whose senses they are. (1.09) We begin then by considering this analysis.

Some problems with which Frege's theory of sense explicitly deals are the following:

1) How do we succeed in communicating new thoughts?
2) How with only a few syllables in our language are we capable of expressing a huge number of thoughts?
3) How do we succeed in communicating thoughts at all?

The answers to all these questions will become apparent if we succeed in answering just the first. Let us then concentrate on this one.

Frege's approach to these questions is in the style of the descriptive metaphysician - we know that new thoughts are communicated, what then are the conditions which make this possible? He writes:

"It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even a thought grasped by a human being for the very first time can be put into a form of words which will be understood by someone to whom the thought is entirely new. This would be impossible, were we not able to distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence serves as an image of the structure of the thought ... If then, we look upon thoughts as composed of simpler parts, and take these, in turn to correspond to the simple parts of sentences, we can understand how a few parts of
sentences can go to make up a great multitude of thoughts."
(Bib. (15) see (30) p537/8)

Let us attempt a reconstruction in some detail of the conditions which make the transmission of a new thought possible. Suppose I one day grasp a thought never before grasped and I wish to communicate it to another. I perceive the structure of the thought, that is I perceive its components and the way in which these components combine to constitute the whole thought. (This last I shall call the ordering of the components of the thought.) Now the structure of the thought corresponds to the structure of a piece of language, in particular a sentence whose sense the thought is. ("The world of thoughts has a model in the world of sentences, expressions, words, signs. To the structure of the thought there corresponds the compounding of words into a sentence and here the order is in general not indifferent."\textsuperscript{10}) I then locate those expressions in my language the senses of which constitute the components of the thought. I then arrange these expressions in that ordering corresponding to the ordering of the components of the thought. I have now formed a complete sentence whose sense is the thought I wish to communicate. It remains to be seen how the sense of this sentence is grasped by another individual who has never before grasped this sense. This individual is presented with a series of expressions all of whose

\textsuperscript{10} Bib. (17) see (28) p123
senses we may assume he grasps. All that is new to the other person is the ordering of the senses of the components of my sentence and he is presented with a model of this in the ordering of the component expressions of my sentence since as we have seen, this ordering mirrors the ordering of the thought components. He is thus in a position to grasp the new thought.

Looking back on this now we see what is essential to the communicating of a new thought.

1) A skilled language user can locate, given a component sense of a thought, an expression whose sense it is.

2) There are objective senses of expressions which can be grasped by a skilled language user.

3) Thoughts have a perceivable structure which is mirrored in the structure of the sentences expressing those thoughts.

The important point here is (3). Success in communicating new thoughts requires that "the structure of the sentence serves as an image of the structure of the thought." There is nothing odd about this assumption. We might have assumed that there is no expression in my vocabulary which is not in that of the individual and this is quite compatible with my grasping a thought which has never been grasped by this individual. Certainly, if there is a component of my newly formed sentence which is not understood by the other person, then he will not understand the sense of the entire sentence. For this reason we assume that he grasps the senses of all components of the sentence.

This statement is incompatible with Frege's stated view that the sentences 'A' and 'A & A' express the same sense. For these sentences clearly have different structures and the senses they express would need to be different if the structures of the sentences were to mirror the structure of the thoughts they express. I ignore this problem here.
so that to each component of the sentence there corresponds a component of the thought. (If this were not so then presenting the ordering of expressions by say, uttering a sentence, would not present the hearer with a model of the ordering of the components of the thought we wish him to grasp.) Now this entails that every component of every sentence expressing a thought has a sense and thus we derive Frege's principle that if a part of a sentence lacks a sense then so does the entire sentence. (1.13)

It is interesting to speculate whether Frege might at one time have thought that an analogous argument dealing with reference could be adduced in support of his view that functional expressions have reference. For he once held the view (which he later abandoned) that to the structure of a definite description there corresponds the structure of the object it names and if we ask the question 'How do we succeed in forming definite descriptions of things never before named?', we might be tempted to give the following response:

'We perceive the structure of the object to be named i.e., its components and their ordering. We then locate those expressions whose references are the components of the object to be named and arrange these expressions in that ordering which models the ordering of the components of the object. Crucial to our success in this is that the structure of the definite description corresponds to the structure of its reference and this entails that
each component of our definite description names a component of the named object. Since definite descriptions contain functional expressions, this further entails that functional expressions have reference."

However, Frege abandoned the view that objects were composed of parts which are denoted by parts of the definite description denoting the object, when faced with having to defend the ludicrous position that Denmark is a part of Copenhagen. For 'Denmark' is a component of 'the capital of Denmark' and the latter expression denotes Copenhagen.

The upshot of this is that while the theory we explored in relation to sense and thought can coherently be held to support the view that every component of a sentence expressing a thought has a sense, the same style theory relating to reference cannot coherently be employed to support the view that functional expressions have reference. We turn now to consider the advantages of positing incomplete or unsaturated entities.

The problems concerning communication of thought, we have argued, are effectively resolved by Frege's views on the relationship between language and thought from which emerge certain of his semantic principles. Central to these views is his belief that sentences have a structure, that they can be decomposed into components such that the manner of decomposition mirrors the manner in which the thought which the sentence expresses is decomposed into its components. That the thought can be so comp-
posed provides the basis for Frege's solution to the abovementioned problems. But how from its components a single thought is formed gives rise to another range of problems. This second type of problem Frege attempted to solve by postulating that the sense of a predicate is unsaturated or incomplete. Similar problems arising at the level of reference induce Frege to postulate that the reference of predicates is likewise unsaturated.

The problems referred to here, exercised some of the greatest minds of this century including Russell, Moore and Wittgenstein and cannot indeed be neatly divided into those which relate to sense and those which relate to reference. This division of these problems unmistakeably bears Frege's stamp and of course those who worked with these problems usually did not adhere to such Fregean distinctions as sense and reference. A Russellian proposition of 1903 for example, in no way resembles a Fregean thought. But to appreciate the force of Frege's attempted solution to these problems it is useful to view them from the perspective of a non-Fregean ontology (in particular, an ontology corrsponding roughly to Russell's of 1903) and once we have done this we can then divide the problem into its two Fregean components and see how Frege's attempted solution applies to the problems so divided.

Consider the relational proposition expressed by 'Brutus killed Caesar'
If we attempt to list the constituents of this Russellian proposition we would mention Brutus, the relation of killing and Caesar. But these constituents are also present in the proposition expressed by

'Caesar killed Brutus'

and the two propositions are clearly distinct. In order to capture the difference, we might say (as Russell at one time said) that a further constituent of the first proposition is a particular three termed relation relating Brutus, killing and Caesar and that this relation is not expressed by any of the words of the sentence but by a relation among the words of the sentence. But - so a counter-argument runs - if the relation killing is not sufficient to relate Brutus and Caesar in the proposition and a second relation is required, then this relation too will not be sufficient to relate Brutus, killing and Caesar and a third relation will be required. But this process is endless. Simple relational propositions it is argued, break down on analysis into an infinite number of relations.

Chief among the proponents of this type of argument was F.H. Bradley who invoked it in his attack against pluralism. If the world is composed of many things, then these things must be in some relation to each other or else we could know nothing of the world. If we can know nothing of the world then pluralism is self-defeating.
But Bradley's argument purports to show that the notion of relations is self-contradictory and thus too is the notion of a pluralistic universe. Before considering the details of his argument, it should be pointed out that the problem of the unity of the proposition, the problem of how the elements of a proposition create a unity which is greater than the sum of its parts, is of course not restricted to relational propositions. The Russellian proposition expressed by the simple subject/predicate sentence

'Cassius is ill'

has as its constituents, Cassius, the property of being ill and some relation between them. But the mere listing of these entities fails to attribute illness to Cassius—a crucial feature of the proposition. And no matter how many more relations one might list as constituents of the proposition, this essential ingredient of the proposition will be missing from the list. With this in mind we turn to Bradley's argument against relations. His central argument is as follows:

"But how the relations can stand to the qualities is, on the other side, unintelligible. If it is nothing to the qualities, then they are not related at all, and if so, as we saw, they have ceased to be qualities, and their relation is a non-entity. But if it is to be something to them, then clearly we shall require a new connecting relation. For the relation hardly can be the mere adjective of one or both of its terms; or at least, as such it seems indefensible. And being something itself, if it does not itself bear a relation to the terms, in what intelligible way will it succeed in being anything to them? But here again
we are hurried off into the eddy of a hopeless process, since we are forced to go on finding new relations without end. The links are united by a link, and this bond of union is a link which also has two ends; and these require each a fresh link to connect them with the old. The problem is to find how the relations can stand to its qualities and this problem is insoluble. If you take the connection as a solid thing, you have got to show and you cannot show, how the other solids are joined to it. And if you take it as a kind of medium or unsubstantial atmosphere, it is a connection no longer." (Bib. (3) p32/3)

To modern eyes there is much that is vague in this passage. What for example does it mean for a relation to "succeed in being something to" its terms? What is meant by a "solid thing"? Still, perhaps the crux of the argument is clear enough: If a relation is an entity connecting two terms then there must be a further relation linking that relation with each of its terms or else the sentence will not say anything. But the same difficulty arises for the new relation and so on ad infinitum and we are led into an infinite regress.

Interestingly, this argument recurs in more modern authors, notably Dummett. He argues that Bradley's infinite regress results from viewing relations as special kinds of objects i.e. as nameable by proper names. I paraphrase:

If we try to consider the relational expression in an atomic sentence as standing for an object, we shall be at a loss to account for the unity of the sentence.

13 Bib. (9) p256
We shall have to think of the sentence as saying that the objects named by the two proper names stand in a particular relation to the object referred to by the relational expression. We shall be in a quandary how to explain the character of this peculiar relationship, which all our assertions involve. If we regard this relationship as an object we embark on an infinite regress. If we regard it as a function in Frege's sense then we might as well have explained the initial relation as such a function. In any case we are trapped in a circle. Any explanation of a relation must involve reference to the mysterious relation which relates objects to the (object-) relation. But we will be unable to explain what this mysterious relation is for this reason: The explanation of any relation must presuppose knowledge of what kinds of entities can serve as terms of the relation, and this cannot be presupposed in this case since one of the terms of the relation to be explained is itself only explained with reference to this relation.

Russell discussed the problem of the unity of the proposition in The Principles of Mathematics. Therein he discussed the difference between the propositions expressed by 'Socrates is human' and 'Humanity belongs to Socrates', the difference between verbs and their nominals (eg. 'kills' and 'killing') and the relationship between declarative sentences and their nominals (eg. 'Caesar died' and 'the death of Caesar'). He argued that any attempt to
distinguish the references of verbs or adjectives used in predicates from the references of the corresponding nominals of these expressions results in a contradictory state of affairs. He noted:

"It might be thought that a distinction ought to be made between a concept as such and a concept used as a term, between eg. such pairs as *is* and *being*, *human* and *humanity*, *one* in such a proposition as "This is one" and 1 in "1 is a number". But inextricable difficulties will envelop us if we allow such a view."

(Bib. (41) p45)

The difficulties Russell is referring to are akin to those discussed in 1.45. In order to deny that *human* is the same as *humanity* one is forced to use the expression 'human' not in a predicate position but in a subject position and how then could its reference differ from the reference of 'humanity'? Such considerations were sufficient to lead Russell to his view that each pair of underlined expressions above simply contains different names of the same entity.

But this view also has its difficulties. For one thing one cannot then explain why a listing of the elements of a proposition fails to express that proposition itself. Russell was well aware of this. He remarked:

"A proposition, in fact, is essentially a unity and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. The verb when used as a verb embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of this distinction."

(Bib. (41) p50)
Now Frege of course took exactly that path which Russell claimed was contradictory. The reference of 'humanity' is for Frege an object - something quite distinct from the reference of the predicate 'is human'. (1.22) A relation is something quite distinct from its terms where its terms are objects. (1.35) All difficulties which result from this position are purely linguistic, Frege declared. They are rooted in our grammatical structures and not founded on the nature of things. He noted:

"... the obstacle is essential, and founded on the nature of our language; ... we cannot avoid a certain inappropriateness of linguistic expression; and there is nothing for it but to realize this and always take it into account." (Bib. (21) see (28) p55)

It is interesting that Russell himself, well after The Principles of Mathematics, expressed a similar point of view. Concerning the sentence 'Caesar loves Brutus' he wrote:

"There is in the above sentence a relation which is symbolised by a relation, not by a word; this is the three termed relation of love to Caesar and Brutus. This is symbolised by the order of the words i.e. by a three term relation. But in order to mention this relation, it is necessary to treat 'love' grammatically as a substantive which tends to confuse the distinction between a substance and a relation. However it is not very difficult to avoid the false suggestions due to this peculiarity of language when once the danger of them has been pointed out." (Bib. (40) p243)

No doubt more could be said about Dummett's and Bradley's arguments, but our main purpose here is merely to sketch the problems which led Frege to his view of unsaturated
entities and to enhance our understanding of the crucial metaphor. Suffice it to note for the moment that Bradley partly anticipates a Fregean response to his argument. The infinite regress develops from taking relations to be entities of the same kind as the terms they relate. Frege's reply, briefly, is that relations are entities of a completely different kind from the objects they relate. Bradley anticipates this by arguing that they are then 'unsubstantial' and hence incapable of performing the function we attribute to them. This argument appears to be based on the presupposition that if a relation is to 'be anything' to its terms, it must be something solid i.e. of the same kind as its terms. Frege's answer to this is that it is simply a mistake. For a relation to relate two objects indeed implies that it itself must belong to the realm of reference. It is not necessary however, that this realm be uniform i.e. consist only of entities of the same kind. Rather this realm consists of two distinct kinds of entities, the saturated and the unsaturated.

Let us now view in detail Frege's attempted solution to the problems under discussion. We have already seen a crucial part of it - that the references of predicates are absolutely distinct from the references of proper names. Precisely because one constituent of a Russellian proposition is a concept (or relation), it is impossible to list all the constituents of such a proposition; for a concept or relation can only be referred to within the context of a sentence. (1.45)
One can understand what the constituents of such a proposition are but one cannot list them. Nor is there any difficulty understanding how the elements of a proposition hang together once we understand a concept/relation to be that which a concept word/relation word refers to. In general, by tying the ontological distinction between functions and objects to the linguistic distinction between function names and proper names (1.22), Frege is able to avoid such problems as the unity of the proposition. For once we understand what proper names are, we can understand the workings of functional expression (1.14) and once functions are explained as that which functional expressions refer to, (1.18, 1.19, 1.20) our understanding of how functions and objects join up to form complete entities is tied to our understanding of how functional expressions and proper names join up to form complete expressions, and in this there is no difficulty.

Bradley's problem can now be seen as arising from the same source of confusion as Russell's - viewing relations as special kinds of objects. Once one understands relations as being what relational expressions refer to, (1.35) one understands that a relation requires a link to connect it with its terms no more than a relational expression requires a link to connect it with the expressions filling its argument places. As Dummett expresses this point: 14

14 My discussion of Frege's proposed solution to the problems under discussion owes much to Dummett. See especially Bib.(9).
"A concept and an object, or a relation and two objects, need no glue to fit them together; they fit together naturally, in a way we can think of as analogous to that in which a predicate and a proper name, or a relational expression and two proper names, fit together to form a sentence. And this will seem to us natural and unproblematic as soon as we grasp that we can think of a concept only as the referent of a predicate, of a relation only as the referent of a relational expression."

What the unsaturatedness of concepts and relations does for Frege then is to eliminate the need for a further relation connecting concept (or relation) and subsumed object(s) in a proposition. Although Frege frequently writes as though the relation of subsumption is a genuine relation between for example, a concept and an object, in his more careful treatment of the subject he urges that we are deceived in thinking that there is such a relation by the inaccuracy of our grammatical expressions by means of which we attempt to analyse sentences. It is this inaccuracy which:

"... erweckt auch den Anschein, als ob die Beziehung der Subsumption ein Drittes wäre, was zu dem Gegenstande und dem Begriffe hinzukomme. Das ist nicht der Fall, sondern die Ungesättigkeit des Begriffes bewirkt, dass der Gegenstand, indem er die Sättigung bewirkt, unmittelbar an dem Begriffe haftet, ohne dass es eines besonderen Bindesmittels bedürfte."\(^{15}\)

Once this relation is abolished, the first step into Bradley's regress is blocked. Yet the difference between

15 My rendition: "... creates the impression that the relation of subsumption is a third thing which attaches to the object and concept. That is not the case, but rather the unsaturatedness of the concept brings it about that the object in which it effects the saturation adheres directly to the concept, without needing a special binding agent."
the proposition expressed by 'Brutus killed Caesar' and 'Caesar killed Brutus' can still be explained as the result of completing a relation in two different ways though indeed with the same objects in both cases.

In a word then, Frege's attempted solution to the problem of the unity of the proposition, and to the problem of Bradley's infinite regress is that concepts and relations are unsaturated - they can be explained only as the referents of predicates and relational expressions. It is this attempted solution which will be the main concern of the remainder of this essay.

But let us now return to a Fregean ontology and split the problems we have discussed above into two in order to see how Frege deals with them first at the level of sense and then at the level of reference.

At the level of sense, we are faced with this problem: A sentence expresses a thought and each of its components a sense. (1.13) But this thought is not just a mere series of senses - it is a single communicable thought. How do the components of a thought combine to produce such a unity? Or as Frege puts it:

"But the question now arises how the thought comes to be constructed, and how its parts are so combined together that the whole amounts to something more than the parts taken separately . . ."  

(Bib. (15) see (30) p537/8)

The answer is that the sense of the predicate in a sentence is unsaturated (1.30) in just the sense explained
above. It is this unsaturated sense which provides the bonds, so to speak, which enables the senses to coalesce into a single thought. The sense of a simple sentence is not just a series of senses but a series of senses which attains a unity by virtue of its having an unsaturated component which is saturated by saturated component(s).

Frege writes:

"... not all the parts of a thought can be complete; at least one must be 'unsaturated' or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept prime number' without a link. We supply such a link in the sentence 'The number 2 falls under the concept prime number'; it is contained in the words 'falls under', which need to be completed in two ways - by a subject and an accusative; and only because their sense is thus 'unsaturated' are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought. I say that such words or phrases stand for a relation."

(Bib. (18) see(28) p54/5)

The reference of a sentence is for Frege, as we have seen, a truth value. (1.08) Since functional-, predicative- and relational expressions are all denoting expressions for Frege, it is incumbent upon him to explain how for example, by referring to two objects and a relation, we have succeeded in referring to a truth value. How do we succeed for example, in naming the True by uttering 'Cain is the brother of Abel' when according to Frege's analysis we have named Cain, a certain relation and Abel in that order. On this issue, Frege wrote:
"Take the proposition 'Two is a prime number'. . .
The first constituent, 'two', is a proper name of a certain
number; it designates an object, a whole that no longer
requires completion. The predicative constituent 'is a
prime number', on the other hand, does require completion
and does not designate an object. I also call the first
constituent saturated; the second, unsaturated. To this
difference in the signs there of course corresponds an
analogous one in the realm of references: to the proper
name there corresponds the object; to the predicative
part, something I call a concept. This is not supposed
to be a definition; for the decomposition into a saturated
and an unsaturated part must be considered a logically
primitive phenomenon which must simply be accepted and
cannot be reduced to something simpler. I am well aware
that expressions like 'saturated' and 'unsaturated'
are metaphorical and only serve to indicate what is meant
- whereby one must always count on the cooperative
understanding of the reader. Nevertheless, it may
perhaps be made a little clearer why these parts must be
different. An object, eg. the number 2, cannot logically
adhere to another object, eg. Julius Caesar, without some
means of connection. This, in turn, cannot be an object
but rather must be unsaturated. A logical connection
into a whole can come about only through this, that an
unsaturated part is saturated or completed by one or more
parts."

(Bib. (19) p32/3)

Once again the answer is the same. Once one realizes
that a sentence names a truth value, and that concepts and
relations are that which are referred to by predicates
and relational expressions, the problem disappears. It
is the very nature of concepts to adhere to objects to
form truth values and similarly for first level relations -
the relation is the mapping of objects onto truth values.

From Frege's attempted solution to the problems we
have been discussing, a crucial semantic principle emerges:
A) We cannot refer to a concept (or indeed a relation or
any function) by means of a proper name.

The problems discussed above are avoided by explaining
a concept and first level relation as that entity
which a predicate refers to, a relation as that which a relational expression refers to. By tying the notion of a concept directly to the notion 'denoted by a predicate', the problem of the unity of subject/predicate sentences disappears. Again, by tying the notion of a relation directly to the notion 'denoted by a relational expression', Bradley's infinite regress cannot be generated. As Furth notes, these notions are:

"... related by that strictest of equivalences of meaning which connects explicandum and explicans."

(Bib. (24) p22)

Relating this to functions in general, the principle emerges that for a given language:

B) If X denotes a function then X is an incomplete expression. Clearly, one cannot retain this analysis and allow that a proper name can denote a function. To allow a proper name to denote a function is to reject that analysis which provides the escape from Bradley's infinite regress and the larger problem of the unity of the proposition;

Nor is principle (A) restricted to the language in which this discussion has taken place. For Frege, as we have seen (1.28), the distinction between proper names and predicates applies universally to all languages. Since we can only understand a function as being that which a functional expression refers to, any language capable of talking about (in particular) concepts and relations,

15 I use 'talk about' in a wide sense, eg. in the sense in which 'Socrates is wise' is as much about the concept referred to by 'is wise' as it is about Socrates.
must be governed by principle (B) and hence by the principle that concepts and relations cannot be referred to by using proper names. To put this in another way: Since the notions of a concept and a relation are directly tied to the notions 'denoted by a predicate' and 'denoted by a relational expression' respectively, we simply could not conceive of a language in which concepts and relations were referred to by some other means.

It is not that every language must talk about relations and concepts - though a little reflection will be sufficient to see that a language which cannot talk about relations and concepts must be a most impoverished language indeed - but that if a language does talk about relations and concepts, it can only employ relational- and predicate expressions to refer to them. We shall see now that this result has dire consequences for Frege's ontology.
Chapter 3

Introduction

Milton Fisk's attempt to derive a paradox from Frege's principle that a concept can only be referred to by a predicate is discussed and criticized. Even though Fisk's paradox is ill-formed as it stands, it seems that the maxim derived in the last chapter - that any language capable of talking about relations and concepts cannot employ proper names to refer to them - renders us in principle incapable of specifying in a given instance which concept a predicate refers to. Two recent attempts to manoeuvre Frege out of this difficulty are discussed and criticized in some detail. It is shown that the problem of specifying predicate reference remains.

Nevertheless, from the second attempt (Dummett's), valuable insights are salvaged. The relation of denotation between a predicate and a concept is one of second level and it is necessary to coin a new relational expression to refer to this relation. Accordingly I coin the expression 'prenote' and exhibit its usage. Once this expression is in our vocabulary, there is no longer any difficulty in specifying predicate reference. Unfortunately however, our attempt to escape Bradley's problem of relations and the larger problem of the unity of the proposition, has come full, vicious circle.

Bradley's problem was avoided in the first
place by viewing relations as 'unsaturated' - as explained as that which relational expressions denote. But the use of 'denote' here is inappropriate - a relational expression prenotes. But we can understand what this means only if we can understand what this prenoting relation is. Now crucial to understanding any relation, it is argued, is a knowledge of the kind of entities which can serve as terms of the relation. But one of the terms of the prenoting relation is itself a relation - precisely that entity which we explained as being that which a relational expression prenotes. We are blocked in principle from explaining the very relation which was to pave the way for the escape from Bradley's regress.

Thus it appears that the better we understand Frege's ontological doctrines, the clearer it becomes that they are ill-founded - that they cannot solve the very problems for which they were designed.
If one wishes to say of an object - say Vesuvius - that it is an object, one natural way is to form the following sentence:

'The object Vesuvius is an object.'

Analogously, if one wishes to say of a concept that it is a concept one naturally forms a sentence such as:

'The concept horse is a concept.'

Unfortunately this will not do. The three words 'the concept horse' clearly do not constitute a grammatical predicate but a proper name (1.04) and hence cannot on Frege's principles denote a concept. (1.22) If in fact 'the concept horse' denotes at all then

'The concept horse is not a concept but an object.'

is a true though admittedly paradoxical sentence. Frege recognised this difficulty. He noted:

"The essence of the concept is a great hindrance for appropriate expression and for understanding. If I want to speak of a concept, language forces an unsuitable expression on me with almost inescapable might . . . If I say 'the concept equilateral triangle' then one should assume by grammatical analogy that I thereby designate a concept, just as I surely name a planet, when I say 'the planet Neptune'. But this is not the case; for the predicative nature is lacking. Therefore the reference of the expression 'the concept equilateral triangle' (so far as one exists) is an object."

(Bib. (13) p130)

When Frege first noted this difficulty in his published writings, (in "Concept and Object") he regarded it as an unavoidable quirk of language. The inappropriateness of expression is embedded in our language and "there
is nothing for it but to realize this and always take it into account." One cannot allow that concepts can be referred to by proper names for this would undermine their unsaturated nature and it is this feature of concepts which as we saw in the preceding chapter provides an escape from Bradley's infinite regress and attendant problems.

Now Frege's theory of functions and objects can be seen as providing a set of formation rules for a language. We could choose to provide the same rules purely formally. Thus for example, instead of speaking of the denotation of 'ξ is a horse' as determining how this predicate is used, we could just say, "'ξ is a horse' is a first level predicate" and fix the behaviour of the expression by forming rules governing first level predicates. Presenting the formation rules thus, we would avoid such paradoxical sentences as

'The concept horse is not a concept.'

but we have seen in the previous chapter some of the considerations which may have led Frege to express his theory in the material mode. It is interesting to note however that Frege seemed to consider that a formal treatment failed to avoid the above-mentioned difficulties. In a footnote to his admission of these difficulties, he wrote:

"A similar thing happens when we say as regards the sentence 'this rose is red': The grammatical predicate
'is red' belongs to the subject 'this rose'. Here the words "the grammatical predicate 'is red'" are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property." (Bib. (21) see(28) p46n)

But Frege is wrong here. There is nothing paradoxical about a grammatical predicate (eg. 'is red') being the reference of the grammatical subject of a sentence.

Now I think that Frege was correct in regarding 'the concept horse is not a concept' as a mere peccadillo. It is true that it goes against an accepted principle governing definite descriptions but there is no reason why we could not tolerate an exception to this principle. We could for example consider the underlined expression following 'the concept' as forming a special context to which the principle in question should not be applied, and retain the principle for all other contexts.

But unfortunately we cannot rest with this. What above has appeared as just a quirk of language has deeper roots and, it has been widely argued, blossoms into a genuine paradox. The paradox which I shall discuss here is the one proposed by Milton Fisk. There are variants of this paradox and indeed they have been thoroughly discussed in the secondary literature. Nevertheless,

16 Namely that principle which informs us that for any common noun $\beta$ and name $\alpha$, 'the reference of 'the $\beta\alpha'$ must be a $\beta'$ is true.
17 Bib. (12)
the one to be discussed here provides as useful a starting point as any for delving to the heart of the matter.

Fisk's argument is as follows:

1) 'the concept horse' is a proper name.
2) 'is a horse' is a predicate.
3) If 'the concept horse' is a proper name, the concept horse is an object.
4) If 'is a horse' is a predicate, 'is a horse' refers to the concept horse.
5) If 'is a horse' is a predicate, 'is a horse' does not refer to an object.
6) 'is a horse' does not refer to an object. (from 2 & 5)
7) The concept horse is an object. (from 1 & 3)
8) 'is a horse' refers to the concept horse. (from 2 & 4)
9) 'is a horse' refers to an object. (from 7 & 8)

(1) follows from Frege's criterion that the definite article followed by a singular term always introduces a proper name. (1.04) (2) is uncontroversial. (3) to (5) Fisk claims, follow from Frege's principles that the reference of a proper name is an object, that the reference of a predicate is a concept and that no concept is an object. (1.22, 1.23, 1.24) Now (6) clearly contradicts (9) which suggests that the above-mentioned principles are inconsistent. Fisk himself takes this paradox as contradicting Frege's view of "the predicative nature of the concept" - the view that concepts can only be referred to by predicative expressions and not by proper names. In "On Concept and Object" Frege noted regarding the sentence 'the concept horse is not a concept':

"One would expect that the reference of the grammatical subject would be the concept, but the concept as such cannot play this part, in view of its predicative nature..."

Bib. (20) see (30) p46)
If we reject "the predicative nature of the concept", we reject accordingly step (3) of the argument without which the paradox cannot be derived. For to reject the view that concepts can only be referred to by predicates is to accept the view that they may be referred to by proper names (1.22) and hence even though 'the concept horse' is a proper name, it no longer follows from this that the concept horse is an object. But we need not be quite so hasty in rejecting what we have seen to be so crucial to Frege's metaphysics. Fisk's argument is ill-formed as it stands. Step (4) is just false. 'is a horse' does not refer to the concept horse because the concept horse is an object and 'is a horse' refers to a concept since it is a predicate. Nevertheless we have here a fundamental difficulty. Whatever name we substitute for 'the concept horse' in (4) will, if it is a proper name, refer to an object. In which case (4) will be false. What we need is a name of a concept ie. a predicate. But substituting a predicate for 'the concept horse' in the consequent of (4) yields not a true sentence but a senseless one. eg.

"'is a horse' refers to is a horse;"

It is apparent that a fundamental asymmetry exists between the relationship between a proper name and the object it denotes on the one hand and a predicate and the concept it denotes on the other. In the case of proper
names, it is always possiblemeaningfully and truly to complete the schema

'A denotes x'

where quoted proper names substitute for 'A' and names of the referents of these proper names substitute for 'x'. eg.

"'Charles Dickens' denotes Charles Dickens."

If we take seriously Frege's view that function names must always carry with them their argument places, we see that completion truly of the above schema for predicate expressions is always impossible. Consider:

"'F(ξ)' denotes x."

If in place of 'x' we put a proper name we get a false sentence. If in place of 'x' we put a function name, we get an incomplete and hence senseless sentence. What made (1) true in the above argument was Frege's criterion that the definite article followed by a singular term always introduces a proper name. We see now that difficulties beset Frege's doctrines independently of the validity of this criterion. Indeed, these doctrines render us in principle incapable of specifying in a given instance which concept a predicate refers to.

Could we nevertheless adhere to the view that predicates refer to concepts and admit the impossibility of specifying in a given instance which concept a predicate refers to? This last resort is indeed consistent but depicts
a highly improbable state of affairs, for the following reason: We arrived at the notion of the relation of denotation between proper names and objects by observing the use of the predicate 'denotes' in such sentences as:

"'Charles Dickens' denotes Charles Dickens."

This is not to say that this is the only way we could have arrived at the idea of such a relation, only that as a matter of fact we did arrive at it in this way. Now if we cannot in principle form such sentences as the above for predicates then we will be faced with the difficulty of providing a justification for supposing that the relation of denotation holds between predicates and concepts. What the inability of specifying predicate reference does then, is to cast doubt upon the very notion of a predicate denoting a concept, a notion which we have seen is crucial to Frege's ontology and metaphysics.

We are nearer now to reaching an understanding of the causes of the difficulties which we have been discussing. Before presenting what I consider to be the resolution of these difficulties - a resolution which is fatal to Frege's doctrine - it is instructive to consider two suggestions for manoeuvring out of these difficulties. Both suggestions provide valuable insight into the problem and will render my proposed resolution almost obvious. I intend first to consider a suggestion attributed by Furth to, among others, Richard Montague and David Kaplan.18

18 Bib (24)
The suggestion is this: We specify denotation of an object language expression, say '\( \xi \) is a horse' in a metalanguage. Thus for example,

"'\( \xi \) is a horse' denotes the concept horse."

is a sentence of the metalanguage. To argue that this sentence is false because 'the concept horse' is a proper name and hence denotes an object is to make the straightforward mistake of applying rules governing the object language to expressions in the metalanguage. 'the concept horse' in the above sentence is an expression of the metalanguage and by the rules governing this language, it may well denote a concept. 'the concept horse' may not occur at all as an expression in the object language or if it does it could have a completely different denotation from its metalinguistic counterpart.

Before considering the merits of this argument, it is interesting to see what it implies for Fisk's argument. We rejected Fisk's argument because of the faulty premise (4) but this suggestion would seem to restore that premise. Is then the paradox Fisk attempts to derive genuine? No, because (7) is a statement in the object language while (8) is a statement in the metalanguage and so (9) cannot be derived from (7) and (8). For it is clear that (9) follows from (7) and (8) only if the expression 'the concept horse' denotes the same entity in (7) as it does in (8). But 'the concept horse' in (8) is an expression of the metalanguage and the same phrase
may occur in the metalanguage and the object language with different denotations. Since the expression may denote distinct entities in its occurrences in (7) and (8), the truth of (7) and (8) does not guarantee the truth of (9). One merit of the suggestion we are here discussing then, is that it permits the premises of Fisk's argument but effectively blocks the conclusion.

But difficulties remain. Whatever comes after

"'ξ is a horse' denotes . . . ."

in a metalinguistic specification of denotation must either be a proper name or a predicate expression. Thus for example we might let 'the concept horse' be a proper name of the metalanguage and complete the above specification of denotation thus:

"'ξ is a horse' denotes the concept horse."

Or we might let 'ξ is a horse' be a predicate of the metalanguage and complete it thus:

"'ξ is a horse' denotes is a horse."

The first completion is obviously unsatisfactory. We saw in the previous chapter that any language capable of talking about relations (and the metalanguage in question must have this capability for it to talk about the relation of denotation) cannot employ proper names to refer to such relations. The second completion is less obviously

19 As we have seen, Frege held that these categories apply universally to all languages. (1.28)
unsatisfactory. The chief difficulty is making sense of it. If the metalanguage is English, it is clearly senseless. If the metalanguage is some other language, one way of making sense of it would be to translate it into English. This however seems like a thankless undertaking given that the object language in this case is English and the problem arose precisely because of the impossibility of specifying the denotation of a predicate in this object language.

The second suggestion I wish to consider is due to Dummett, and I wish to consider it in some detail.

We have seen that Frege's semantic and ontological principles lead to the intolerable view that we cannot specify for any given predicate which concept it refers to. Dummett does not accept this conclusion and attempts to provide a mode of expression for specifying predicate reference which is fully in accord with Frege's doctrines. On the other hand, Dummett accepts as genuinely paradoxical the sentence

"The concept horse is not a concept!"

which we regarded as tolerably irksome. Dummett's proposed solution then, attempts to solve both these difficulties.

20 Bib (9) p211-222
Dummett's proposal

The problem arises, he argues, with the predicate 'ξ is a concept'. The word 'concept' is a common noun and hence, argues Dummett, the predicate formed from it is one of first level. Since according to Frege's formation rules only object names (proper names) can fill the argument places of first level predicates, every appropriate completion of 'ξ is a concept' will yield a false sentence. The predicate 'ξ is a concept' is totally unsuited for the work we want it to do, ie. say of some concept that it is a concept. What we need is a predicate that will operate with respect to concepts as the predicate 'ξ is an object' operates with respect to objects ie. yields a true sentence upon every appropriate completion. What we need then is a second level predicate (1.31) which is appropriately completed only by first level predicates and such that every such completion yields a true sentence.

In the natural language, relative pronouns and such words as 'something', 'someone', have a double use. We may characterise this use loosely by saying that such words introduce clauses which sometimes refer to objects and sometimes to properties. eg. 'what you gave me yesterday' and 'what you are and I have just become'. The former refers to some object while the latter refers to some property eg. impecuniousness. Clauses such as the latter can be converted into first level predicates by
prefixing a copula to them, eg.

'John is what you are and I have just become.'

But these clauses have a further use. They can be converted into second level predicates in the same way. When thus used they are completed by what Dummett calls a 'predicative expression', eg.

'Impecunious is what you are and I have just become.'

The predicative expression here is 'impecunious'. To see generally which expressions Dummett takes to be predicative expressions, consider the simplest subject/predicate sentences which are formed by completing a first level predicate with a proper name. Now, says Dummett:

"... if the grammatical predicate consists of the copula together with an adjectival phrase or one whose main constituent is a common noun preceded by the indefinite article, the predicative expression is formed by simply dropping the copula; if the main verb is anything other than the copula, the predicative expression is formed by converting the main verb into the participial form of the same tense."

(Ibid. p215)

What we need then as a replacement for the illicit predicate 'is a concept' is a predicate which is completed by predicative expressions and such that every completion yields a true sentence. Taking advantage of Frege's requirement that a concept must be everywhere defined, (1.26) Dummett proposes the following:

"... is something which everything either is or is not."
(This should be understood as 'For all $a, a$ is $\emptyset$ or $a$ is not $\emptyset$' where '$\emptyset$' represents the argument place. cf.(1.33)

This solves half the problem. We still need a predicate expression which will refer to the concept horse. Our natural choice in referring to the concept in question was 'the concept horse' but we will have to choose again because 'the concept horse' is not a predicative expression. Dummett proposes:

"what ' $\xi$ is a horse' refers to;"

He argues that we are misled into viewing this as a proper name by the double use of relative pronouns such as 'what' but that we must view this expression as standing for a concept not an object. I will examine this argument in due course but for the moment let us accept its conclusion. Thus we view

"Kentucky Kid is what ' $\xi$ is a horse' stands for;"

as parallel in construction to

'Kentucky Kid is four legged.'

We are now in a position to express the thought we found ourselves incapable of expressing with the sentence

'The concept horse is a concept.'

We now say:

"What ' $\xi$ is a horse' refers to is something which everything either is or is not."

By eliminating the false predicate ' $\xi$ is a concept' we have succeeded in eliminating such paradoxical
constructions as

'The concept horse is not a concept.'

We have however yet to see how all this enables us to specify which concept a particular predicate refers to. Dummett concludes the above discussion thus:

"Once pseudo-predicates like 'ξ is a concept' and 'ξ is a relation' have been extruded, there is no longer any means of constructing paradoxical sentences like 'The concept horse is not a concept.' nor is there any difficulty in saying what in particular any given predicate stands for. We can, for example, say 'A philosopher is what "ξ is a philosopher" stands for', or more informatively, 'What "ξ is a philosopher" stands for is what Socrates and Plato both were.' "

(p217)

It is at this point that doubts arise about Dummett's proposed solution. His first offering:

1) "A philosopher is what 'ξ is a philosopher' stands for."

is a permissable construction only if we view

"is what 'ξ is a philosopher' stands for"

as a second level predicate and (1) as the completion of this predicate by the predicative expression 'a philosopher'. But in that case, all that (1) asserts is that one concept (namely the concept philosopher) falls within another (namely the concept what 'ξ is a philosopher' stands for). Loosely, we may say that (1) asserts that the property of being a philosopher has the property of being referred to by the expression 'ξ is a philosopher'. But this is hardly a specification of the reference of 'ξ is a philosopher'.

21 Dummett uses 'stands for' where I have been using 'denotes'.

philosopher'. Certainly (1) can be seen as a statement about the concept *what 'ξ is a philosopher' stands for* in the same way that 'Socrates is wise' can be viewed as a statement about the concept *wise* but it cannot be construed as informing us which concept 'ξ is a philosopher' denotes. Nor is

2) "What 'ξ is a philosopher' stands for is what Socrates and Plato both were."

more informative. This would be more informative if Socrates and Plato had just one property in common but this is hardly the case. This objection may be put more formally as follows: A minimum requirement of successfully specifying reference in the case of predicates is that in the event that two predicates denote the same concept we be able to recognise this fact from identical specifications of their references. This requirement phrased for proper names is clearly met. For example, from

"The reference of 'Cicero' is the man who did \( \gamma \)"

and

"The reference of 'Tully' is the man who did \( \gamma \)"

we can conclude that 'Cicero' denotes the same object as 'Tully'. But from

"What 'ξ is a philosopher' stands for is what Socrates and Plato both were."

and
"What 'ξ is a Greek' stands for is what Socrates and Plato both were."

We cannot conclude that 'ξ is a philosopher' and 'ξ is a Greek' stand for the same concept since they obviously do not. Dummett's solution to this part of the difficulties clearly doesn't work. The intolerability of not being able to specify a predicate's reference remains.

There is more that is dubious about Dummett's proposal. Much of his enterprise depends on our construing the clause "What 'ξ is a horse' stands for" as a predicative expression. This construal is probably inspired by the following remark of Frege's:

"Strictly speaking the expression 'the reference of the concept word A' is to be rejected, because the definite article before 'reference' relates to an object and revokes the predicative nature of the concept. It would be better no doubt to say 'what the concept word A refers to' for this is in any case to be used predicatively: 'Jesus is what the concept word 'man' refers to (bedeutet)' in the sense of 'Jesus is a man.'" 

(Bib. (13) p133)

Now it should be noticed that in all the contexts in which the expression "what 'ξ is a horse' stands for" or others like it are to feature, we could substitute a proper name without loss of sense. Thus consider the sentence:

"Jesus is what the concept word 'man' signifies."

It makes perfectly good sense to say:

"Jesus is the Son of God."
As another example consider:

"What 'ξ is a philosopher' refers to is what Plato and Socrates both are."

We have already seen that the predicate '___ is what Plato and Socrates both are' is ambiguous between first and second level construal. Thus it is meaningful to say:

'Aristotle is what Plato and Socrates both are.'

Now we can normally distinguish predicate expressions from proper names precisely because the latter cannot be substituted for the former in certain contexts, eg.

'Swimming is my favorite pastime.'

Here 'swimming' is not meaningfully substituted for by some proper name except perhaps such artificial ones such as 'the activity of swimming'. What grounds do we have for regarding "What 'ξ is a horse' stands for" as a predicative expression? Perhaps appreciating these difficulties, Dummett feels constrained to offer an argument for his view:

"The two expressions 'Mount Everest' and 'what "Mount Everest" stands for' are completely interchangeable: the conditions for their having a reference are the same, and if they have one, it must be the same for each. This is just to say that the expression 'what "Mount Everest" stands for' must, if it stands for anything, stand for what 'Mount Everest' stands for; . . . By analogy, it follows that 'what "ξ is a horse" stands for' ought, if it stands for anything, to stand for what 'ξ is a horse' stands for. Hence in particular, it cannot be construed as a proper name (singular term), for then it would stand for an object, and hence could not have the same reference as 'ξ is a horse'."

(p213/4)
This argument is unconvincing. Exactly the same considerations will show that 'the reference of "ξ is a horse"' is not a proper name, which contradicts Frege's formal criterion regarding the use of the definite article. (1.04) This points to the error in the above argument. One cannot argue by analogy that because a particular mode of expression adapted to proper names can be used to refer to objects, that same mode of expression adapted to predicates must refer to concepts. In particular, one cannot argue that because 'what "Mount Everest" refers to' refers to what 'Mount Everest' refers to, so must 'what "ξ is a horse" refers to' refer to what 'ξ is a horse' refers to. One cannot argue this because it is precisely due to a disanalogy between the way we can refer to objects and the way we can refer to concepts that the seeming paradox arose in the first place. We attempted to refer to the reference of 'ξ is a horse' by the means most natural to referring to the reference of a proper name ie. 'the concept horse'. It seems that we must accept that we simply do not have the means for picking out particular concepts which we have for picking out particular objects and if we attempt to pick out concepts with the same type of expression we employ to pick out objects, we will fail in our task.

This it seems, must conclude this discussion unless independent grounds can be advanced for regarding such expressions as "What 'ξ is a horse' stands for" as predicative.
Nevertheless we can salvage some valuable insights from Dummett’s discussion of these problems. The predicate ‘\( \xi \) is a concept’ is at best useless, as Dummett remarks, for we can never obtain a true sentence by completing it. But just as this predicate is useless so is the relational expression ‘\( \xi \) denotes \( \zeta \)’ when its first argument place is filled with a name of a predicate expression. For the relation of denotation between a proper name and its referent is not supposed by Frege to be the same relation as that between a predicate and its referent. The terms of the first mentioned relation are both objects while the terms of the last mentioned relation are one an object and the other an unsaturated entity. Two relations are the ‘same’ for Frege if they have the same values for the same arguments. (1.37-1.42) Now since the two relations in question cannot take the same values, they cannot be the ‘same’ relation. No wonder then that it is in principle impossible to complete

‘is a horse’ denotes ______

for we have used the name of an equal-, first levelled relation where we needed a name of an unequal-, second levelled relation taking as first argument an object and as second argument a concept, something unsaturated. (1.36)

What we need then is a new relational expression which

22 This is perhaps not strictly true in view of (1.45) but since the conclusion of this argument holds irrespectively, (that the relations under discussion are not the same) I will continue to ignore such complications.
names this unequal levelled relation. That there is no such expression in our vocabulary should not surprise us, since before Frege, no such relation was thought of. We have discovered a new relation and there is nothing for it but to coin a new expression. By doing so, we make a useful addition to our natural language. Let us use the expression 'prenote' to refer to this relation. We can now say:

"'ε is a horse' prenotes is a horse"

and any failure to see this as a well formed sentence of our language is a failure to understand the use of the relational expression 'prenotes'. We can illustrate its usage thus:

'ε prenotes φ (ε)'

In fact, the realization that this new relational expression is required, spawns an entire range of new expressions if we wish to extend the facility of speech which we have for talking about objects to talking about functions. We saw that a quite general obstacle stood in our way if we wanted to talk about a specific concept. For example, in saying that the sentence 'Blue Peter is a horse' says something about both Blue Peter and the concept horse we miss the thought we wish to express. We can now view this failure as the result of using the inappropriate expression 'says something

23 Here the reader may balk at the insouciant use of the expression 'prenotes' - which clearly contravenes conventional syntactic rules. But as will soon become apparent, I use this expression merely to illustrate a point and do not advocate its employment as serious reform.
about'. This expression denotes an equal levelled relation when we required an unequal levelled relation.

As regards the oddity,

'The concept horse is not a concept'

we can either retain the useless predicate 'is a concept' and regard 'the concept horse' as embedded in an exceptional context or we can devise a new second level predicate which replaces 'is a concept', and which takes names of concepts eg. '✌ is a horse' in its argument place, or both.

The expressions 'the reference of A' or 'the prenotation of A' will have to be banned and Frege in fact insisted on this too in a letter to Russell. But we can get by without this expression simply by using the name of the concept when we wish to say something about that concept. Just as the expression "the reference of 'Vesuvius'" is redundant once we have the expression 'Vesuvius', so too is the corresponding expression for quoted predicates.

Of course, once we have posited such new relations we still have to provide some justification for thinking them to exist, as was suggested earlier. We would also need to investigate their properties. For example we would need

\[24\] After all, this predicate does not on Frege's principles denote a concept since this concept will be everywhere defined. (1.26)

\[25\] Dated June 29th, 1902. This is mentioned by Resnik (see Bib. (37) p246)
to compare the properties of the prenoting relation to the denoting relation to see if these properties were sufficiently alike to even warrant talking of functions, concepts and relations as we talk of objects. But for the moment let it suffice to see how the positing of such new relations enhances our understanding of Bradley’s infinite regress.

We can now see that Bradley’s problem arose from the mistaken view that relational expressions denote. If we view relational expressions as denoting then strictly, we must view the entities they denote as objects. And once we do this we are beset immediately with the well known difficulties explored in the preceding chapter. But once we understand that relations prenote, then we can have no difficulty in understanding how relations connect their terms and all the difficulties which hinged upon this disappear.

Unfortunately however, we have come full, vicious circle.25 To recapitulate: Bradley’s problem arose in the first place from viewing relations as objects. This we now see, is equivalent to viewing relational expressions as denoting. Initially we avoided Bradley’s problem by explaining a relation as that which a relational expression denotes. We argued that once a relation is understood by way of this explanation, there can be no greater difficulty in understanding how a relation connects its terms than

26 The reader should notice the similarity of the argument which follows with that of Dummett’s paraphrased on p35/6.
there is in understanding how a relational expression functions in a sentence. But our enhanced understanding of the denotation relation forces us to modify this argument for we can no longer explain a relation as that which a relational expression *denotes* for this would imply that relations are objects. We now need to argue that Bradley's problem disappears once a relation is understood to be that which a relational expression *prenotes*.

But we also need to explain what this prenoting relation is. Indeed our success in explaining what a relation is will depend entirely on the success of this explanation. But a minimum requirement for being able to explain what any given relation is, is an ability to explain what kind of entities can serve as terms of the relation i.e. what kind of things the relation relates. For example, in order to explain what the grandfather relation is we would begin by explaining that it is a relation which relates people, and then proceed to explain specifically which people are related in this way. But it is obvious that this explanation is useless unless it is already understood what people are. Now one of the terms of the prenoting relation is itself a relation and thus if our explanation of the prenoting relation is to succeed we must assume a prior understanding of what a relation is. But – and this is the crucial point – we cannot assume such prior understanding for as we have seen, an understanding of what a relation is, is reached only
via an explanation that it is what a relational expression prenotes. Assuming a prior understanding of what a relation is, is assuming a prior understanding of what the prenoting relation is. We are caught in a trap. We cannot explain a relation as that entity which a relational expression denotes and we cannot explain a relation as that entity which a relational expression prenotes because this latter explanation makes reference to a relation which we cannot understand unless we already know what a relation is. The explanans presupposes knowledge of the explanandum.

Thus it is that the better we understand Frege's doctrines, the less convincing is his proposed solution to those problems which prompted the formation of those doctrines in the first place. The problem of the unity of the proposition, Frege would have argued, results from viewing concepts and relations as special kinds of objects. Frege attempted to avoid this and related problems by arguing that concepts and relations are unsaturated - that is that they can only be explained as that which predicate- and relational expressions denote. But we see now that this explanation is incorrect as it stands and the required revision invokes a notion whose explication presupposes a prior understanding of the notion. The ontological division between saturated and unsaturated simply cannot be made out and hence cannot solve the problems for which it was designed.

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