

AN OPTIMIZATION MODEL OF BRITISH COLUMBIA'S
GEORGIA STRAIT CHINOOK AND COHO SALMON FISHERY

by

MICHAEL JAMES STALEY

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ABSTRACT

A computational procedure for optimization of large multidimensional models is presented. The procedure is applied to a model of the Georgia Strait sport and commercial fisheries of chinook (Oncorhynchus tshawytscha) and coho (O. kisutch) salmon. Optimal seasons for these fisheries are calculated and compared to current regulations. Differences, in form and performance, between the optimal seasons and present seasons are minimal and insignifigant. However, inorder to match present age structure, population levels and harvests a value of near zero must be placed on fish left in the water at the end of the season.

The computational requirements of the optimization are proportional to those of the model. In the case study in this thesis the optimization required approximately eight to ten times the computer time of the model.

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1. INTRODUCTION

The sheltered waters of British Columbia's Georgia Strait support viable commercial and sport fisheries on stocks of chinook salmon (Oncorhynchus tshawytscha) and coho salmon (O. kisutch). In the 1950s, the commercial troll fishery caught the majority of fish taken by hook and line in the Georgia Strait. Ten year averages for the years 1953 to 1962, inclusive, show that the commercial trollers caught 125,000 chinook while sportsmen angled 92,700 chinook in Georgia Strait. Average catches on the entire British Columbia coast by hook and line in saltwater totalled 949,100 chinook annually. Coho average catches were 307,200 commercial and 190,800 sport in the Georgia Strait as compared with a coastwide total of 3,131,400 coho (Milne 1964).

The past 15 years have witnessed a dramatic change in the distribution of catch between the commercial and sport fishermen. Increases in human population, wealth, leisure time, and advances in gear efficiency accompanied a significant growth in effective sport fishing effort. Subsequently, the sport catch has risen to 348,000 chinook and 464,000 coho. Meanwhile the commercial troll catch has declined to 99,000 coho and 181,000 chinook per year. (Argue, Coursley, and Harris 1977).

Lately, concern has arisen about the ability of the many populations of chinook and coho salmon that make up the Georgia Strait stocks to withstand high fishing pressure. Spawning escapement estimates have not shown a general decline, but, the amount of effort expended by the fisheries department to find and estimate spawning populations has increased significantly during the past few years (A.W. Argue and F.E.A. Wood pers. comm.). It is the position of the Department of Fisheries that "this increase and more efficient enumeration has produced an artificially stable situation in the escapement trend."

The Fraser River gillnet catch, as an indicator of escapement, has shown a considerable decline in chinook (Fig. 1) and a moderate decline in coho (Fig. 2). Another indication that chinook populations may have become a dangerous conservation problem is the age structure of the catch. The average weight of a troll caught chinook has declined (Fig. 3), indicating a shift in age composition towards younger age classes (ocean age 2 and 3) (Anon. 1978).

Historically, the commercial trollers have caught the majority of the fish. They have also borne the brunt of regulations intended to ensure the conservation of chinook and coho salmon. The restrictions have taken on three major forms: season closures - restricting fishing to summer and fall months (Milne 1964), limited entry (Mitchell 1977) and size limits. To date, the sport fishery has been subject to size limit and to bag limit regulations. Now, given the increased importance of the sport catch and the concern over conservation, the

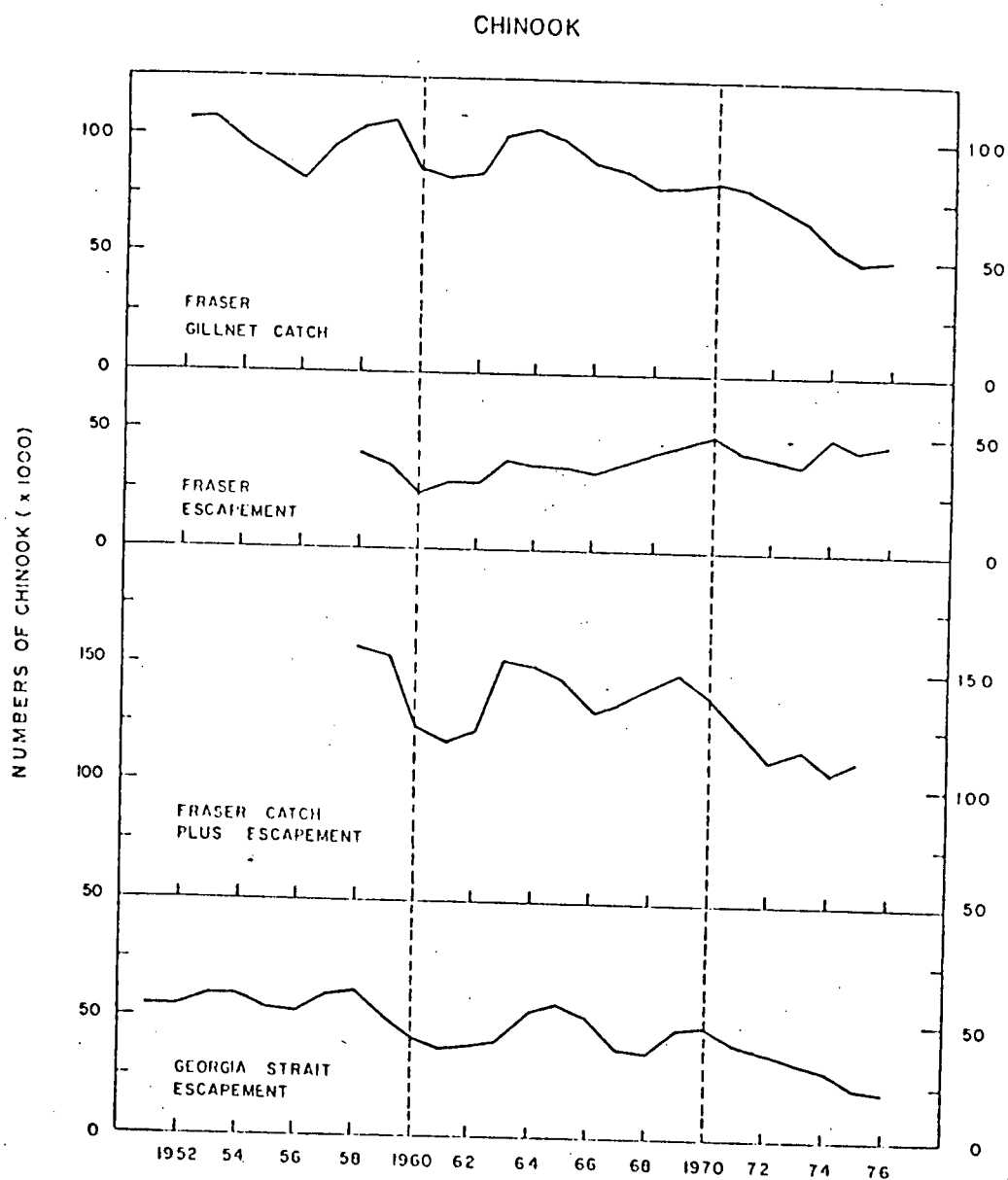


Figure 1: Georgia Strait, Fraser River Escapement and Fraser River Gillnet Catch of Chinook Salmon.

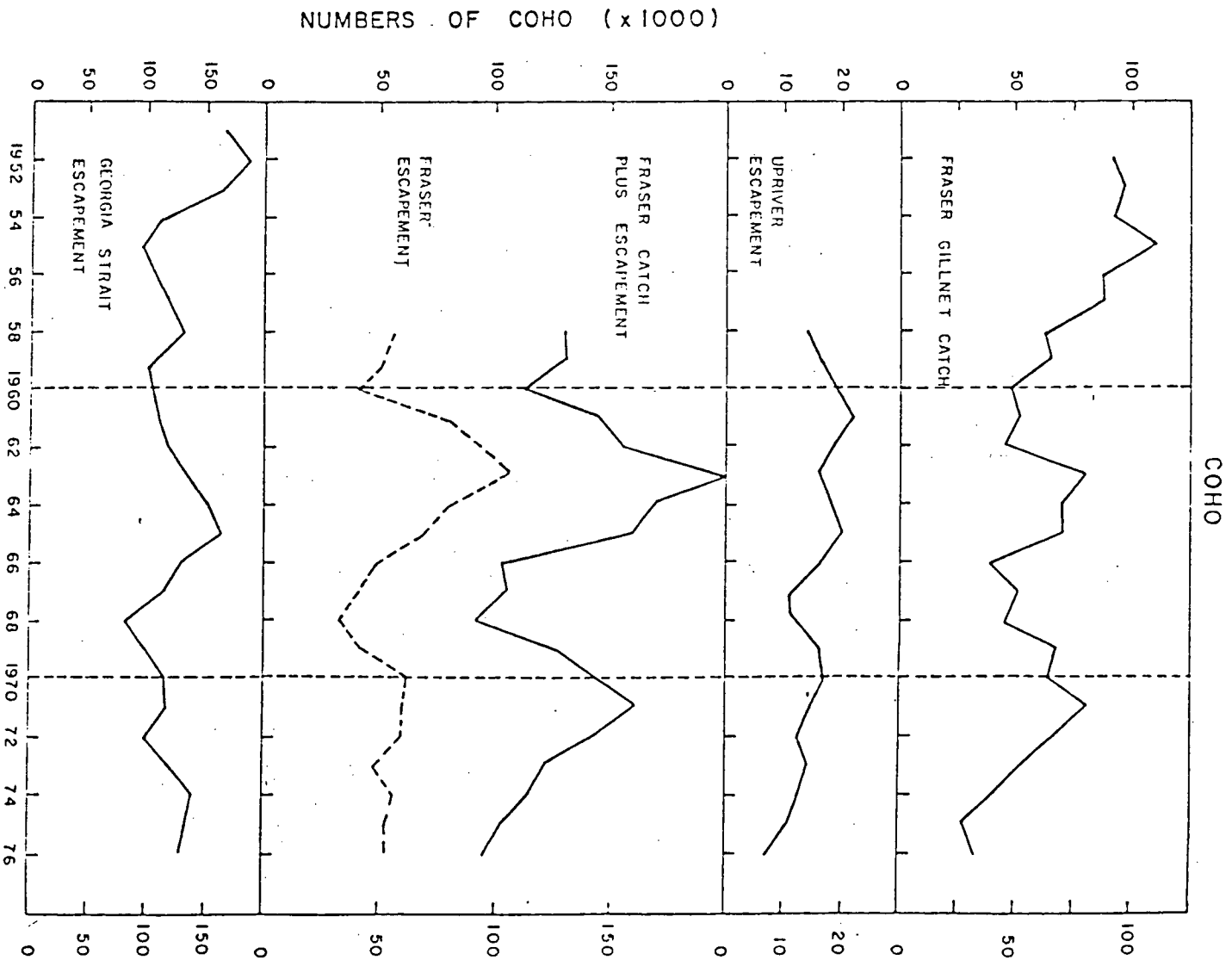


Figure 2: Georgia Strait, Fraser River Escapement and Fraser River Gillnet Catch of Coho Salmon.

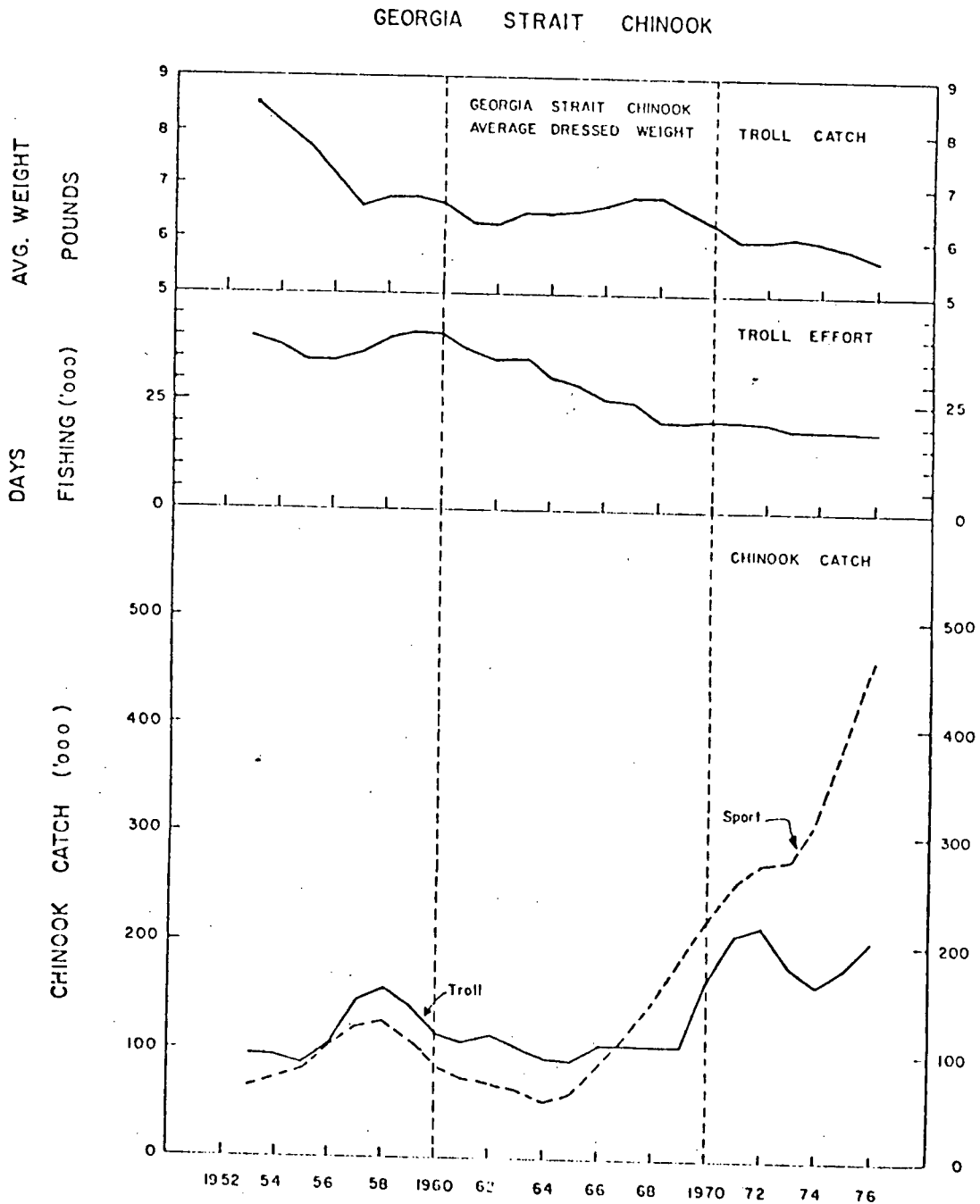


Figure 3: Georgia Strait Troll and Sport Catch, Troll Effort and Average Weight of Chinook Salmon.

Department of Fisheries has begun to investigate new regulations for the sport fishermen, and changes in the commercial troll regulations.

To assist in the assessment of new management policies, the author has worked with a team of scientists, under the direction of Dr. C.J. Walters at the University of British Columbia, to develop a computer simulation model (Wiegert 1975, Walters 1971, Holling et al. 1978). The model has been used to test the effectiveness of various restrictions on the Georgia Strait chinook and coho fisheries.

The simulation model is designed to predict population numbers, fishing effort, catch, and escapement on a bi-monthly basis for a single fishing area representing the Georgia Strait. Both in its attention to the details of fish and fishermen dynamics and the extent to which it mimics real catch and effort statistics from the Georgia Strait, the model represent a "realistic" account of the fishery. This realism makes the model an attractive test bed for new and powerful methods of analysis. In particular, the model can be used to test policies designed by techniques of optimization (Walters, Hilborn 1978).

This thesis contributes to the development of a new computational framework for some large scale resource management optimization problems. The optimization is embedded in the context of the simulation model and maintains most of the detail of the simulation model. The coupling of the optimization model with the simulation model affords the optimization model a modest degree of realism and, I hope, some

degree of practicality.

The main problem of this thesis is to find optimal fishing seasons with respect to changing fish stocks, effort levels, prices, and benefits for both the sports and commercial fishermen, while meeting target escapement levels from the fisheries. The computed seasons for "current" conditions, as predicted by the simulation model, are compared with regulations now controlling the fishery. Optimal seasons are computed under a variety of assumptions about values of parameters for which there is a great deal of uncertainty. Also discussed are the effects of increased abundance of some populations, on optimal seasons, as may result from the "Salmonid Enhancement Program" (MacLeod 1977, Larkin 1974).

The problems with analysis and optimization in this exercise are not unique to the Georgia Strait fishery. Most resource exploitation problems are complex and multi-dimensional in the exploitation regime and in the objectives of management. Therefore, they present major difficulties for optimization. The methodology developed below is capable of dealing with most of the complexity and dimensionality of the Georgia Strait fisheries, and may also be useful for other problems. I hope that the apparent "realism" of the simulation model and optimization model in conjunction with the immediate pressure for the conservation of Georgia Strait chinook and coho will serve to enhance the practicality of this thesis.

2. COMPONENTS

The optimization work to be discussed in chapter three is based upon a simulation model. At present there is no document describing the Georgia Strait model. Therefore, to lay the foundations for the task of this thesis, it will be necessary to describe some of the important ingredients of the simulation model.

There are four major components in the model: (1) population dynamics of fish (life history, mortality, growth, migration, etc.); (2) dynamics of fishing effort; (3) a set of controls available to management; (4) estimates of how benefits flow from the fish to the fishermen and managers.

There are many more facets to the actual management problem. These four components form a kernel around which new regulations can be developed and then examined in a broader, intuitive context, while still maintaining a resemblance to the real world. The broader context would not overtax analytic methods now available.

2.1. Biology

The biology of the pacific salmon (Oncorhynchus spp.) is very complex. In spite of years of work by many investigators, there is still much uncertainty surrounding some very important relationships. A lot of information exists on the reproductive biology and early life stages of some species of salmon. However, due to the difficulty of ocean studies, very little is known for certain about the marine stage. The problem addressed by this thesis deals exclusively with the juvenile and adult ocean life of coho and chinook salmon. As a consequence, there are biological parameters used in this model about which there are very little "solid" data to substantiate particular values. Although some information about each parameter used in the model exists, much of it is tangential to the parameter in question and, often concerns geographical areas and species which are not the focus of this model. Much of the "data" used in the model is a product of the experience and "wisdom" of two members of the Department of Fisheries, A.W. Argue and D. Anderson. These two gentlemen interpreted the available information and made informed estimates on many of the parameters. It is hoped that these estimates reflect the current state of the Georgia Strait chinook and coho fishery.

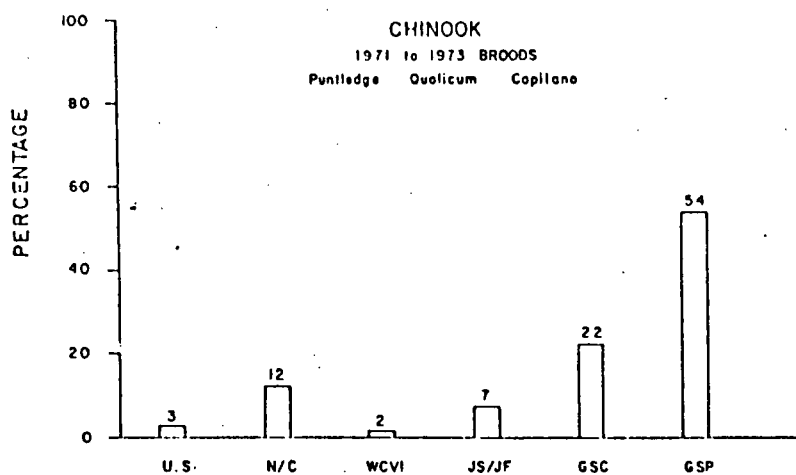
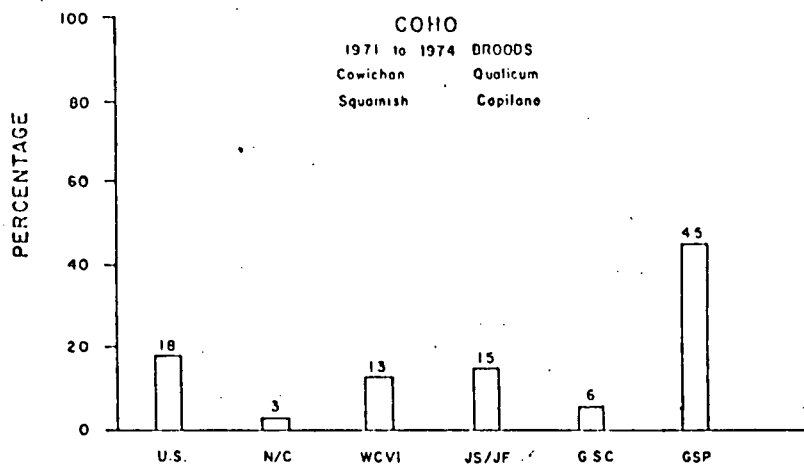
2.1.1. Life History

Like the other commercial salmon species, chinook and coho are anadromous. The eggs are laid in rivers and lakes. The young fish spend some time in freshwater before migrating to the ocean. When mature, they return to freshwater to spawn and die. It is believed that a fraction of the chinook and coho spend most of their ocean life in coastal waters while the other fish spend their ocean life on the high seas. Tag returns from hatchery-reared fish indicate that as much as 76% of the chinook and 51% of the coho produced in Georgia Strait reside in Georgia Strait for some period of time (Fig. 4) (Anon. 1978). It is the portions of the stocks resident in the Georgia Strait which are the subjects of this analysis.

Coho eggs are laid in the fall and hatch the next spring. The smolts generally migrate to the ocean the following spring where they spend one winter and return to spawn the following fall. By the late summer of their first year at sea, coho are first caught by the sport fishery and, by the next spring, they are caught by the commercial trollers. In the model, coho are assumed to be recruited to the fishery by August 1 (A.W. Argue pers. comm.). During the summer and fall of the last year at sea, coho move into freshwater to spawn. The model uses the timing of the Fraser River gillnet catch (Fig. 5) to approximate the migration timing (Ledbetter and Hilborn 1978).

Chinook have a more flexible life history. Ocean migration may take place in the spring through to fall after hatching, or after one or two winters in freshwater. The model assumes that

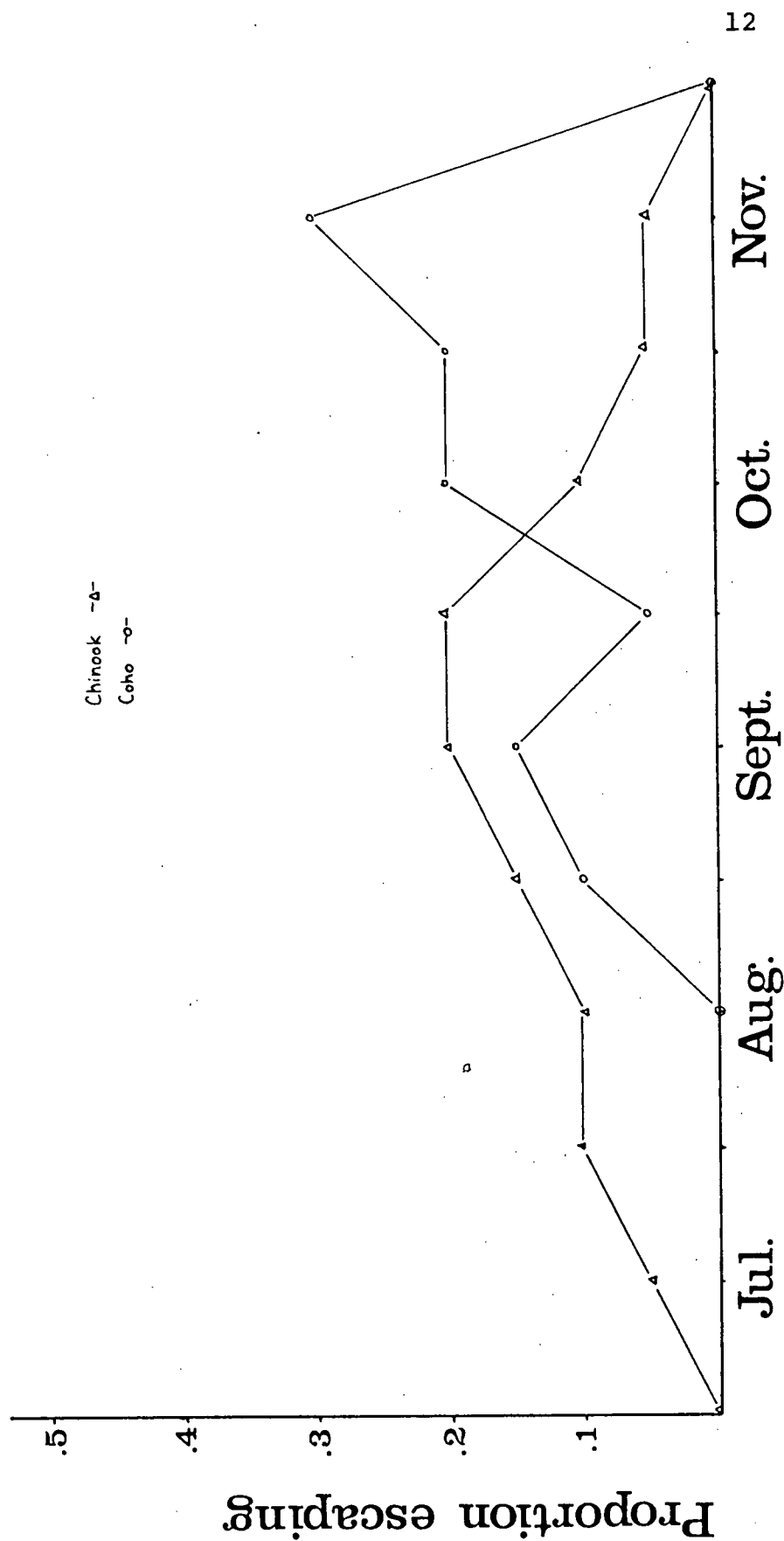
PERCENTAGE CATCH HARVESTED BY CATCH REGION



U.S. - UNITED STATES
 N/C - NORTH CENTRAL B.C.
 WCVI - WEST COAST VAN. IS.
 JS/JF - JOHNSTONE STR. - JUAN DE FUCA STR.
 GSC - GEORGIA STRAIT COMMERCIAL
 GSP - GEORGIA STRAIT SPORT

Figure 4: Tag Recoveries of the Georgia Strait Coho and Chinook Salmon.

Figure 5: Escapement Timing for Georgia Strait Chinook and Coho Salmon.



chinook are recruited to the fishery by the beginning of October of their first year at sea. In general, maturation takes place during the second through fifth year at sea. For this model, an estimate of the proportion of fish maturing and returning to spawn for each age class was needed. An analysis of tag returns in the Fraser River gillnet versus the Georgia Strait sport and troll fishery led to the following estimates (A.W. Argue pers. comm., Argue 1976).

Ocean age two	3% mature
Ocean age three	40% mature
Ocean age four	80% mature
Ocean age five	100% mature

As with coho, the run timing of mature fish was estimated from the Fraser River gillnet catch (Fig. 5) (Ledbetter and Hilborn 1978).

A feature more important for chinook than coho is the tendency for Georgia Strait fish to migrate out of the Strait before maturing. Again, through an analysis of tag returns inside and outside the Strait, estimates of the net migration rates from inside to outside were derived (A.W. Argue pers. comm.) :

Ocean year one	40%
Ocean year two	25%
Ocean year three	15%
Ocean year four	10%
Ocean year five	0%

Fish moving outside are assumed not to return to the gulf until their rapid spawning migration, and thus to be relatively invulnerable to the gulf sport and troll fisheries.

2.1.2. Natural Mortality

Natural mortality of fish at sea is a very difficult process to study. Experiments generally include small numbers of fish and are prone to large observation errors. Environmental variability often confounds experimental errors, resulting in biases for which there is no information concerning direction or magnitude (Ricker 1976).

Parker (1960) estimated the mean bi-monthly non-catch mortality rates for chinook salmon of age three through five to be 0.0175. Henry's (1978) estimates were 0.026. It is generally thought that natural mortality rates decrease with increased size and age (Ricker 1976). The parameters chosen for the model are meant to reflect natural mortality only, not mortality caused by fishing. (A.W. Arque pers. comm.). The instantaneous natural mortality rates (per 15 days) in the model are:

Ocean age 1	0.035 for 3 months
Ocean age 2	0.015 for 12 months
Ocean age 3	0.0075 for 12 months
Ocean age 4	0.0075 for 12 months
Ocean age 5	0.0075 for 10 months
Average	0.011

Ricker (1976) estimates the instantaneous rate of non-catch ocean mortality for coho to be 0.04 for a half month period. The values used in the model for natural mortality alone are:

Ocean age 1	0.04 for 5 months
Ocean age 2	0.02 for 10 months
Average	0.027

Much better estimates of natural mortality will likely be available soon, through analysis of the many tagging studies conducted during the 1960's (Walters, pers. Comm.).

2.1.3. "Shaker" Mortality

Non-catch ocean mortality is made up of two components - natural mortality and mortality caused by fishing, but not included in catch statistics. Mortality of the second kind includes salmon caught that are less than legal size, or caught during a closed period for the species in question. These fish are discarded dead or mortally injured - either after being boated, or by shaking them from the gear as it is hauled up. Shaker mortality caused by trolling for coho and chinook averages about one fish killed for every two that are boated (Ricker 1976). The effect of various regulation changes on

shaker mortality and the effect of various assumptions about shaker mortality rates on the consequence of regulation changes is a major concern of the simulation model. In the model, a shaker mortality rate of 50% is used for trollers (Ricker 1976). Little empirical work has been done on sport shaker mortality. It is a general belief that sportsmen kill more fish than commercial fishermen because of inexperience or lack of concern. Therefore, the shaker mortality rate in sport fishery is assumed to be 80%.

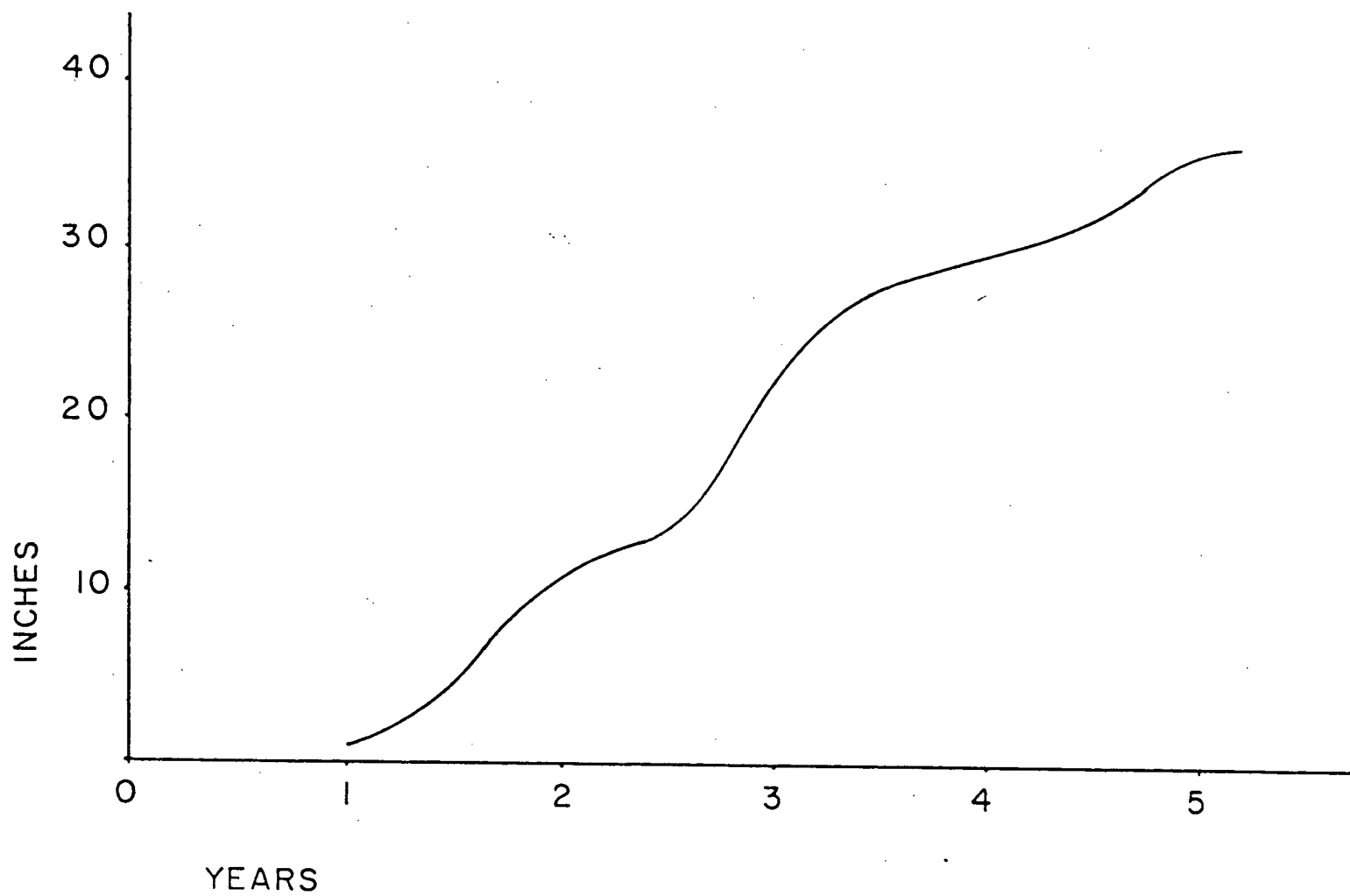
2.1.4. Growth

In the model, the size of a fish is assumed to be a function of its age and the time of year. An estimate of the growth of a chinook or coho at sea (Fig. 6) was made using data on the size, age, and time of capture in the Georgia Strait troll fishery. A length-weight relation was also constructed from these data (Argue and Marshall 1976).

2.2. The Dynamics of Fishing Effort

There have been several dynamic models of fishing effort proposed (Gatto, Rinaldi, and Walters 1976; Clark and Munro 1975; Clark 1976). These models have studied the theoretical behavior of capital or fixed inputs in and out of fishing fleets, with little or no empirical basis. Hilborn and Ledbetter (1978) document the allocation of salmon seiners among fishing areas on the British Columbia coast. Their conclusion is that the key determinant of boat movement is

Figure 6: Size at Age of Chinook and Coho Salmon.



availability of fish. A major ingredient of the simulation model and the optimization model of this thesis is the response of fishing effort to the success of fishing.

2.2.1. Sports Effort Response

Motivation of sport effort can be assumed, in part, to be associated with the capture of fish. Consequently, a change in the number of successful captures experienced by fishermen will likely change the amount of fishing they do. Furthermore, it is probable that an increase in the average success per angler in the sport fishery will motivate an increase in fishing effort. Undoubtedly, many other factors, besides fishing success, determine the amount of sport effort. Weather, holiday time, fuel prices, and the general state of the economy, among other things, may affect the motivation of the sportsman. The fishery manager does not have jurisdiction over the economy, nor the environment. He must, therefore, pursue management plans based upon relationships under his control, i.e. the success of fishing.

The responsiveness of sport effort to catch per unit effort (CPUE) is indicated in Figures 7 through 15. Each graph represents catch and effort data from the Georgia Strait (Statistical areas 13-18, 28, 29) for one month from the years 1968 to 1975 (Anon. B.C. Sports Catch Statistics 1968-1975). It appears for most months that higher CPUE is associated with increased effort. For the simulation, it is assumed that effort is simply proportional to recent success as measured by CPUE.

Seasonal variation in the ratio of effort to catch per effort is shown in Figure 16 for the same data set. This variation indicates that the responsiveness of the fishermen is much greater in the summer than the winter.

Consideration of the behavior of fishermen is important to evaluate the impacts of regulation changes intended to increase the abundance of fish significantly. For example, an increase in the success of the fishermen may stimulate enough effort so as to cancel any intended increase in spawning escapement. The relationship included in the model is based upon the number of fish caught. It is very likely that the size of fish caught is more important in motivating effort than mere numbers. In such a case, regulations intended to reduce the catch of small fish (size limits) may increase the average size of fish caught. Sport effort may increase, due to the larger and older fish, and cancel any intended increase in spawner escapement. The analysis of this thesis is based upon the relationship of numbers rather than weight. Therefore, any conclusions should be weighed by the uncertainty in the relationship of effort to success of fishing.

In the model, sport effort is assumed not to saturate or reach an upper limit at high levels of success. Some upper limit must exist, but, it has apparently not been reached in recent years (Figs 7-15). Unlike the commercial fishery with its licence program, which limits the number of vessels able to participate, there is no legal limit to the numbers of people who can participate in sport fishing.

Figure 7: Effort and CPUE in the Georgia Strait Sport Fishery
January to April 1968 - 1975.

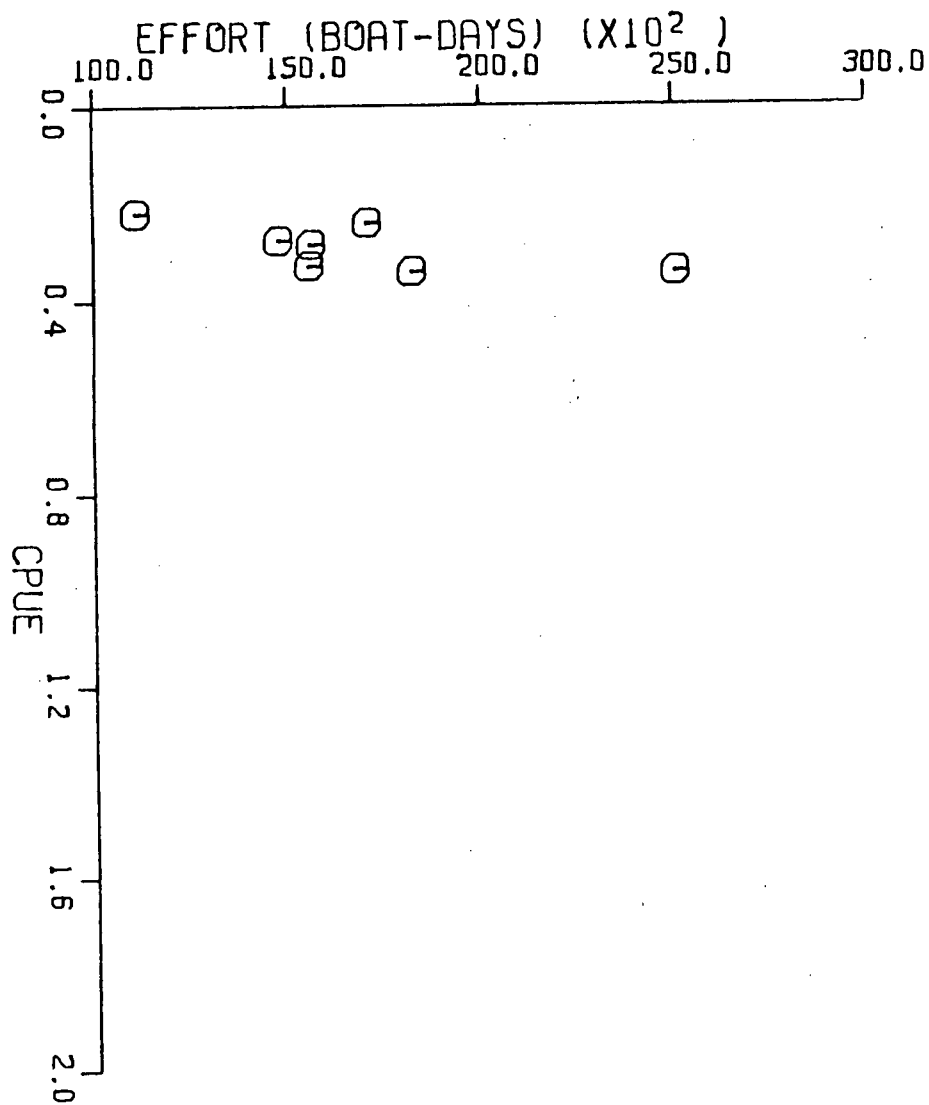
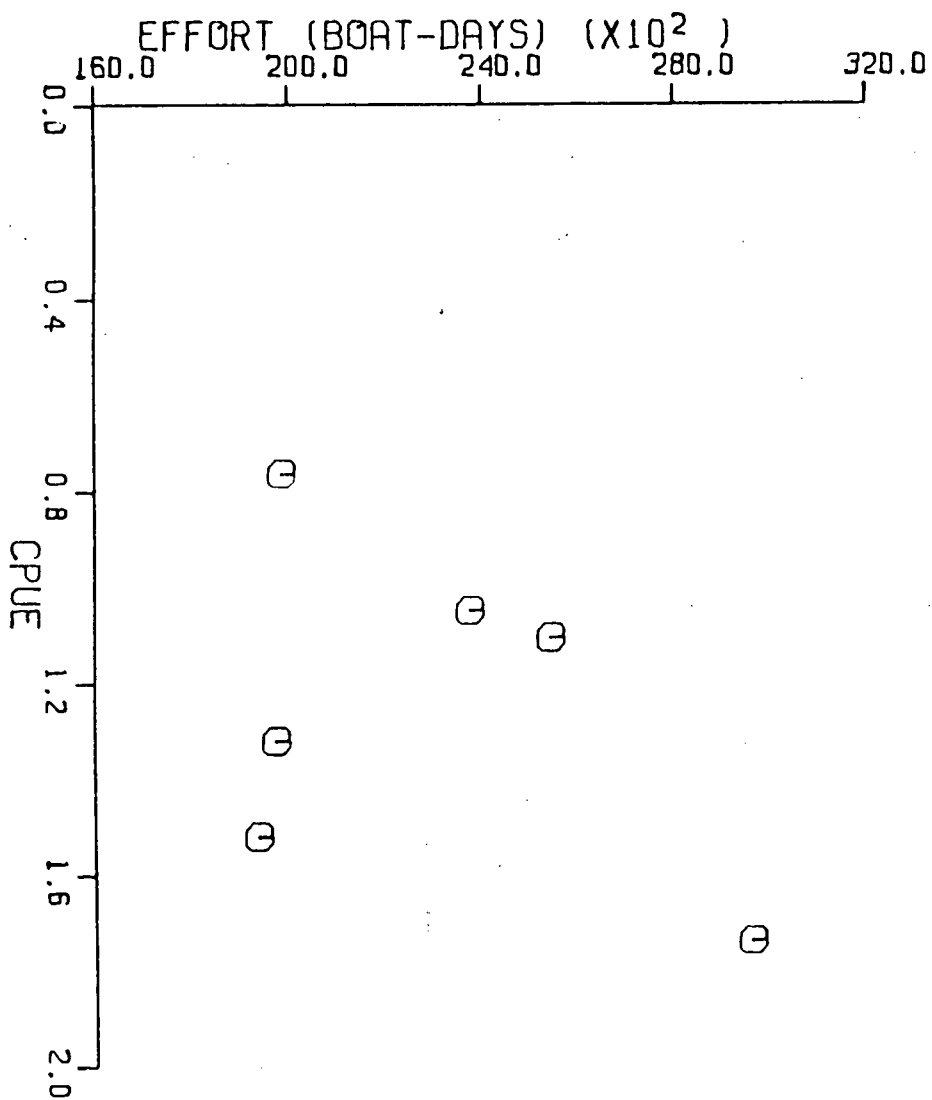


Figure 8: Effort and CPUE in the Georgia Strait Sport Fishery
May 1968 - 1975.



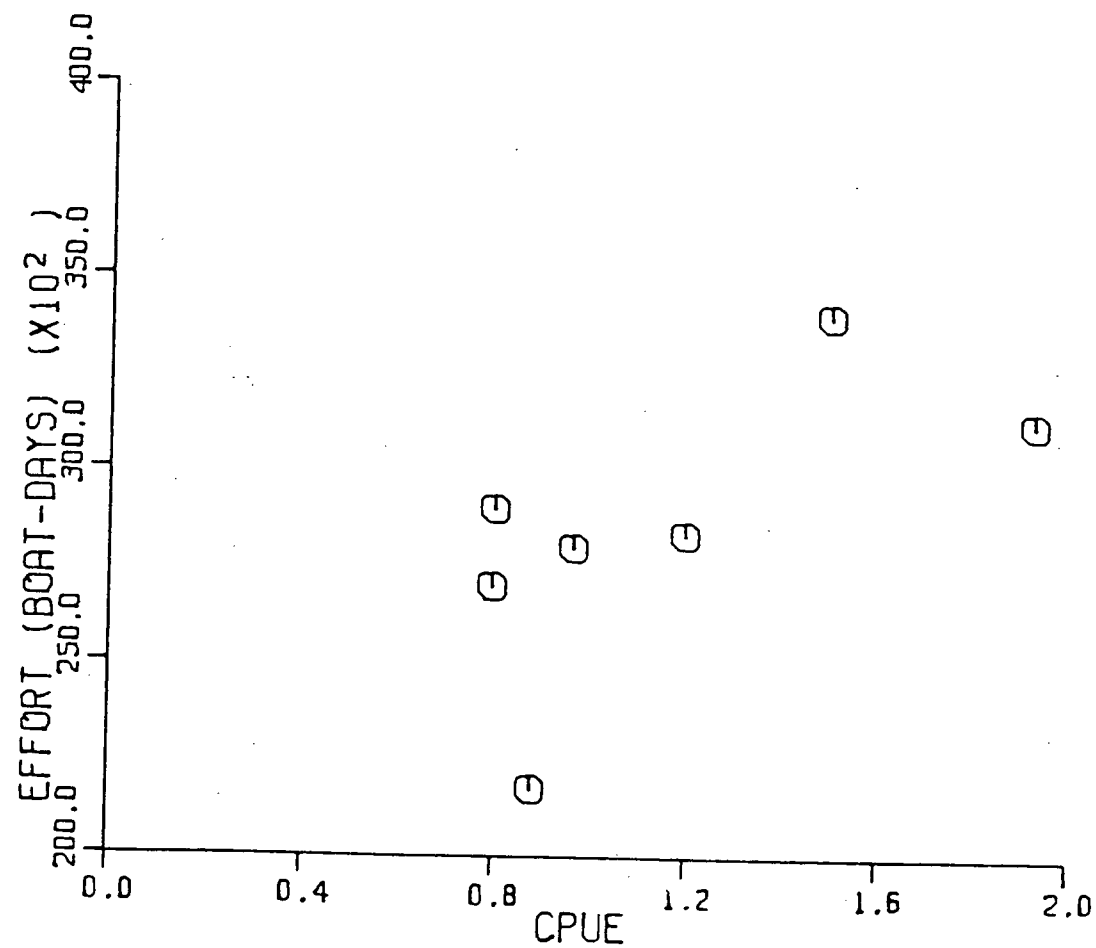


Figure 9: Effort and CPUE in the Georgia Strait Sport Fishery
June 1968 - 1975.

Figure 10: Effort and CPUE in the Georgia Strait Sport Fishery
 July 1968 - 1975.

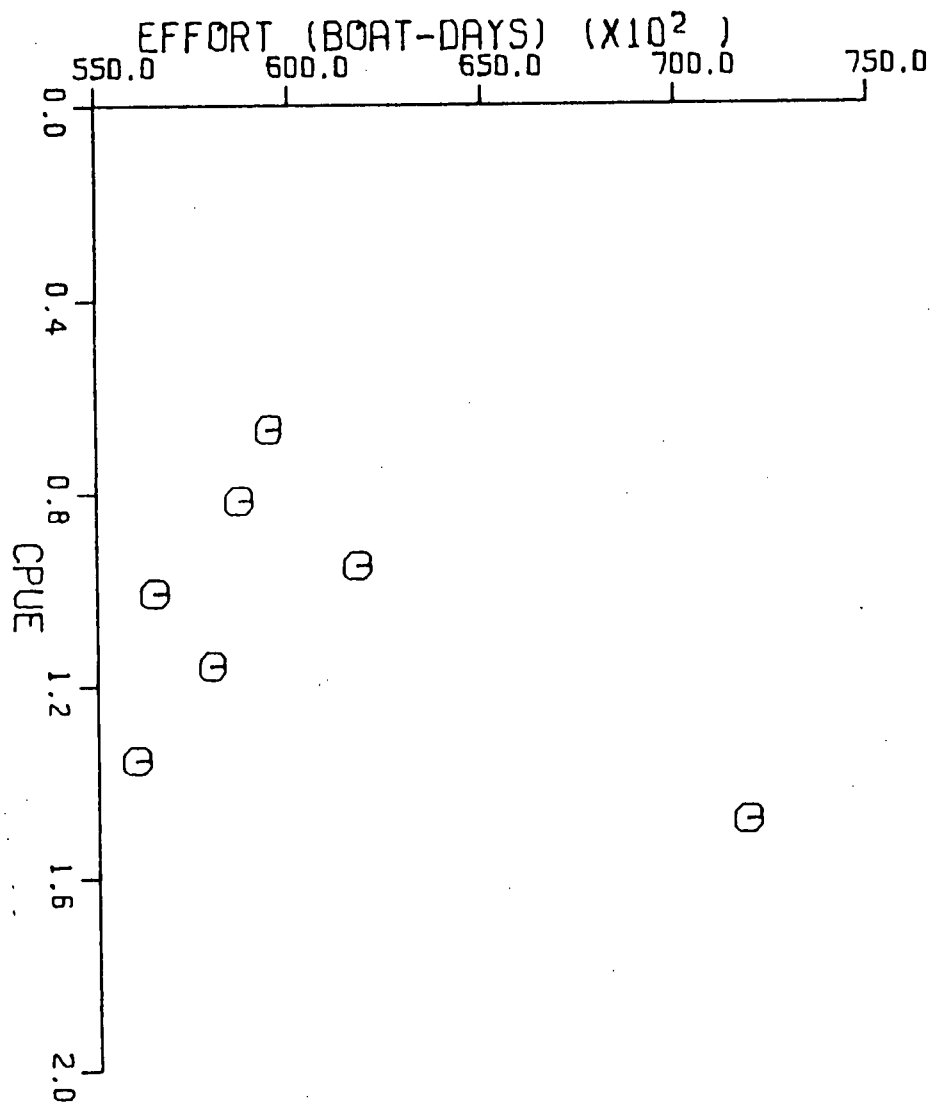


Figure 11: Effort and CPUE in the Georgia Strait Sport Fishery
August 1968 - 1975.

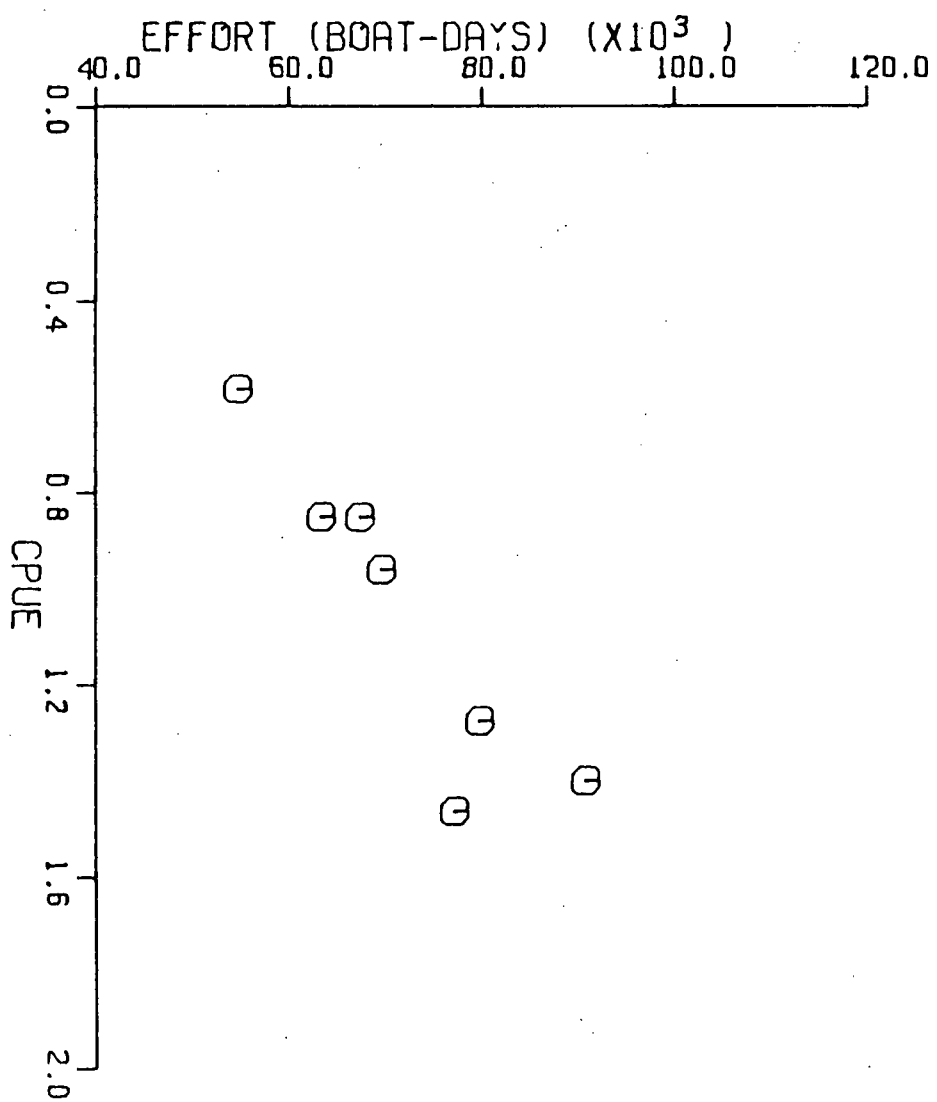


Figure 12: Effort and CPUE in the Georgia Strait Sport Fishery
September 1968 - 1975.

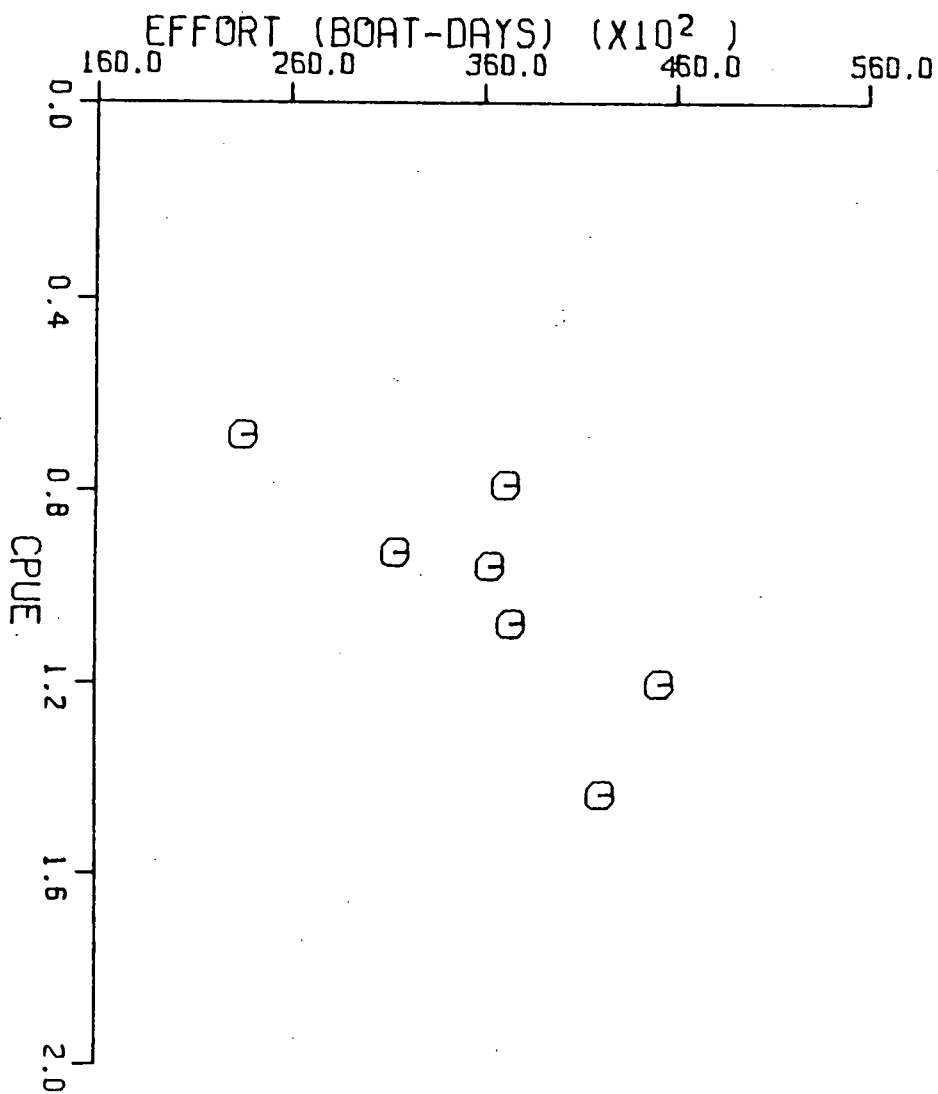
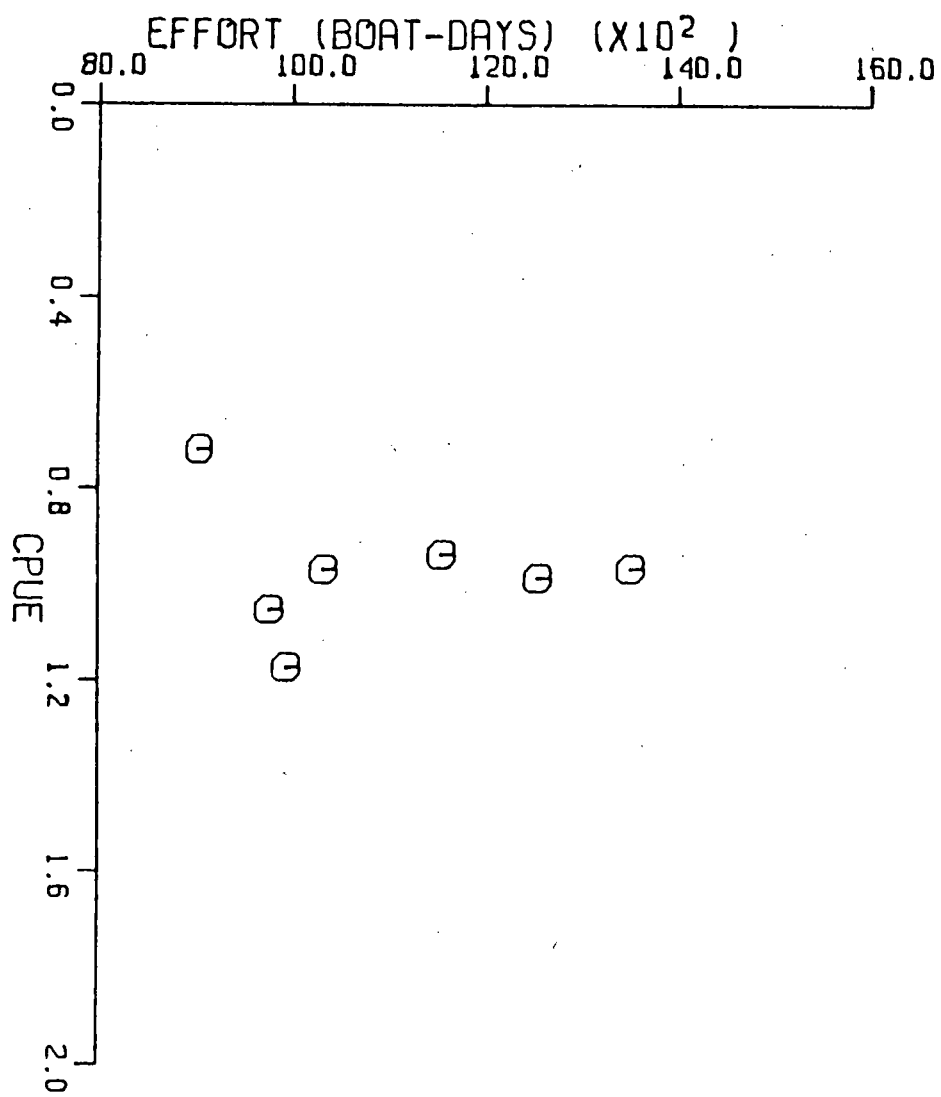


Figure 13: Effort and CPUE in the Georgia Strait Sport Fishery
October 1968 - 1975.



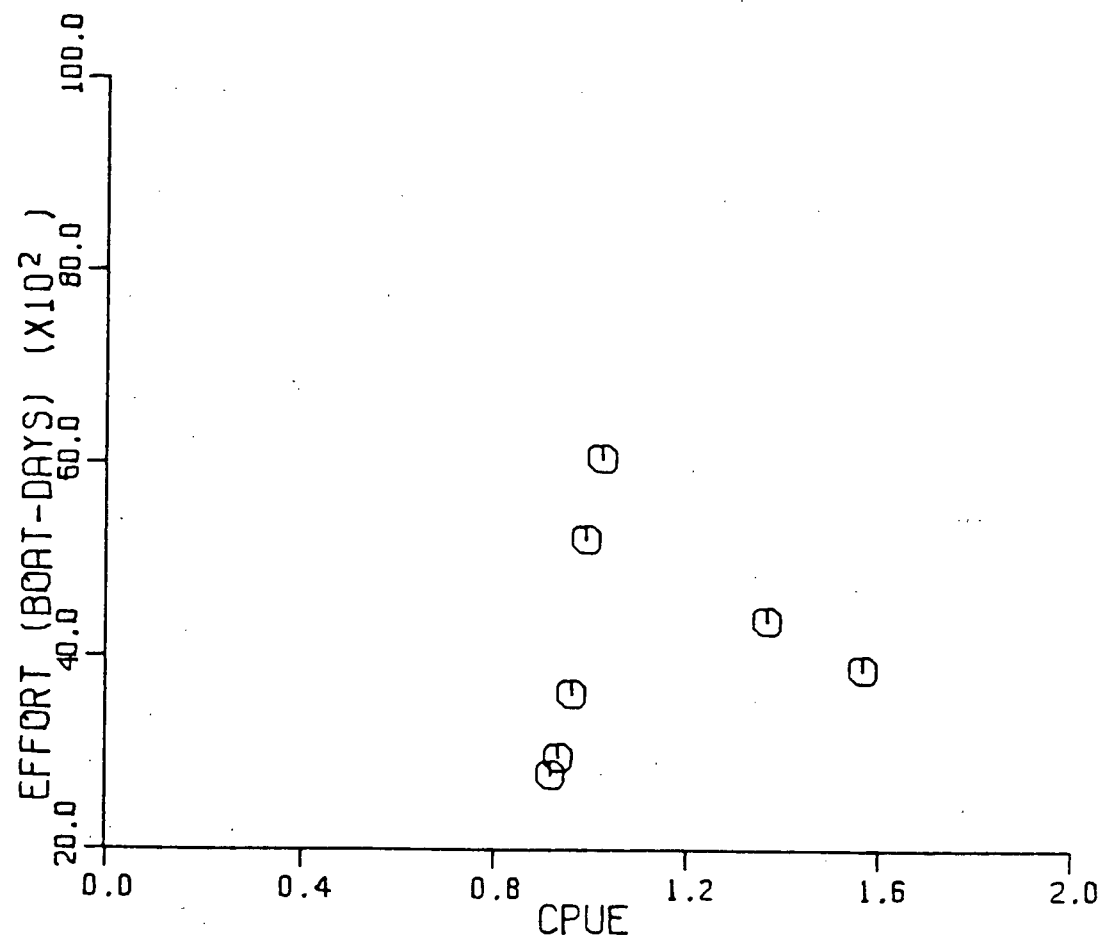


Figure 14: Effort and CPUE in the Georgia Strait Sport Fishery
November 1968 - 1975.

Figure 15: Effort and CPUE in the Georgia Strait Sport Fishery
December 1968 - 1975.

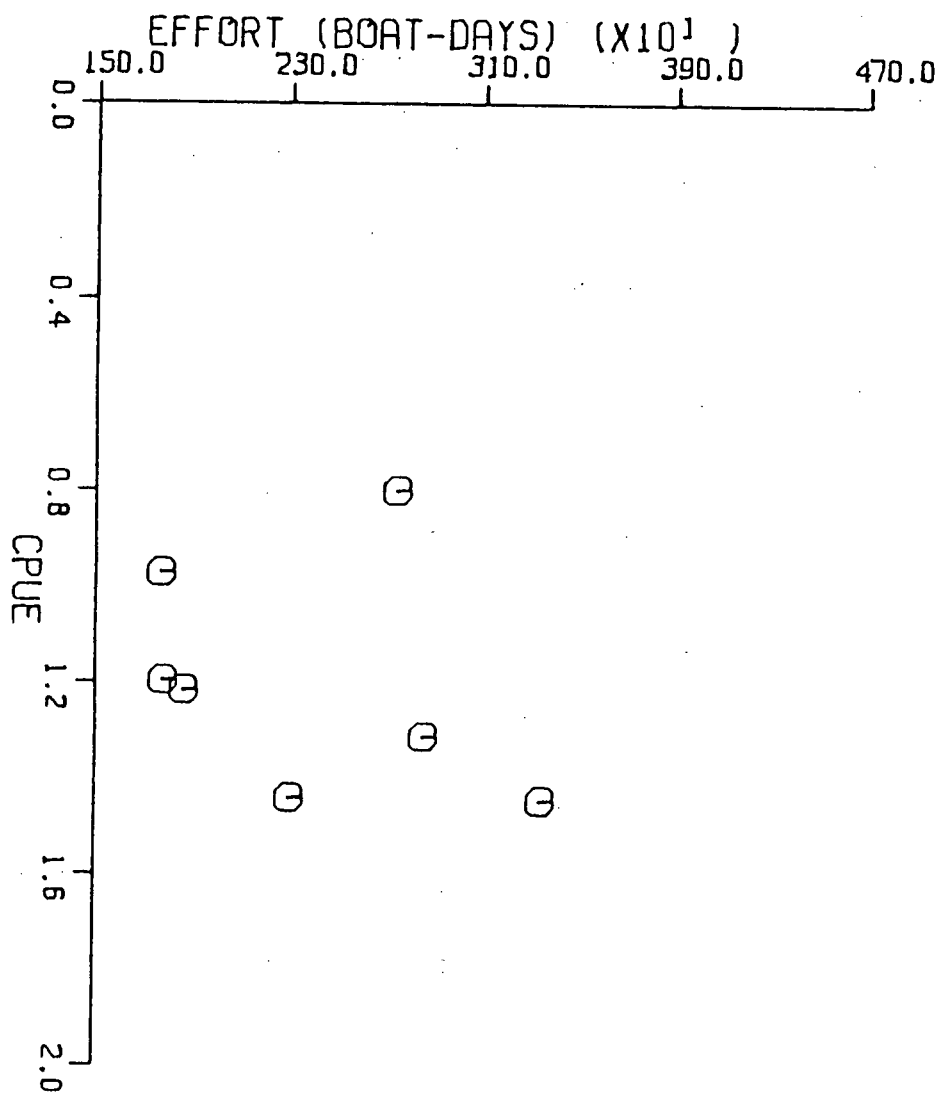
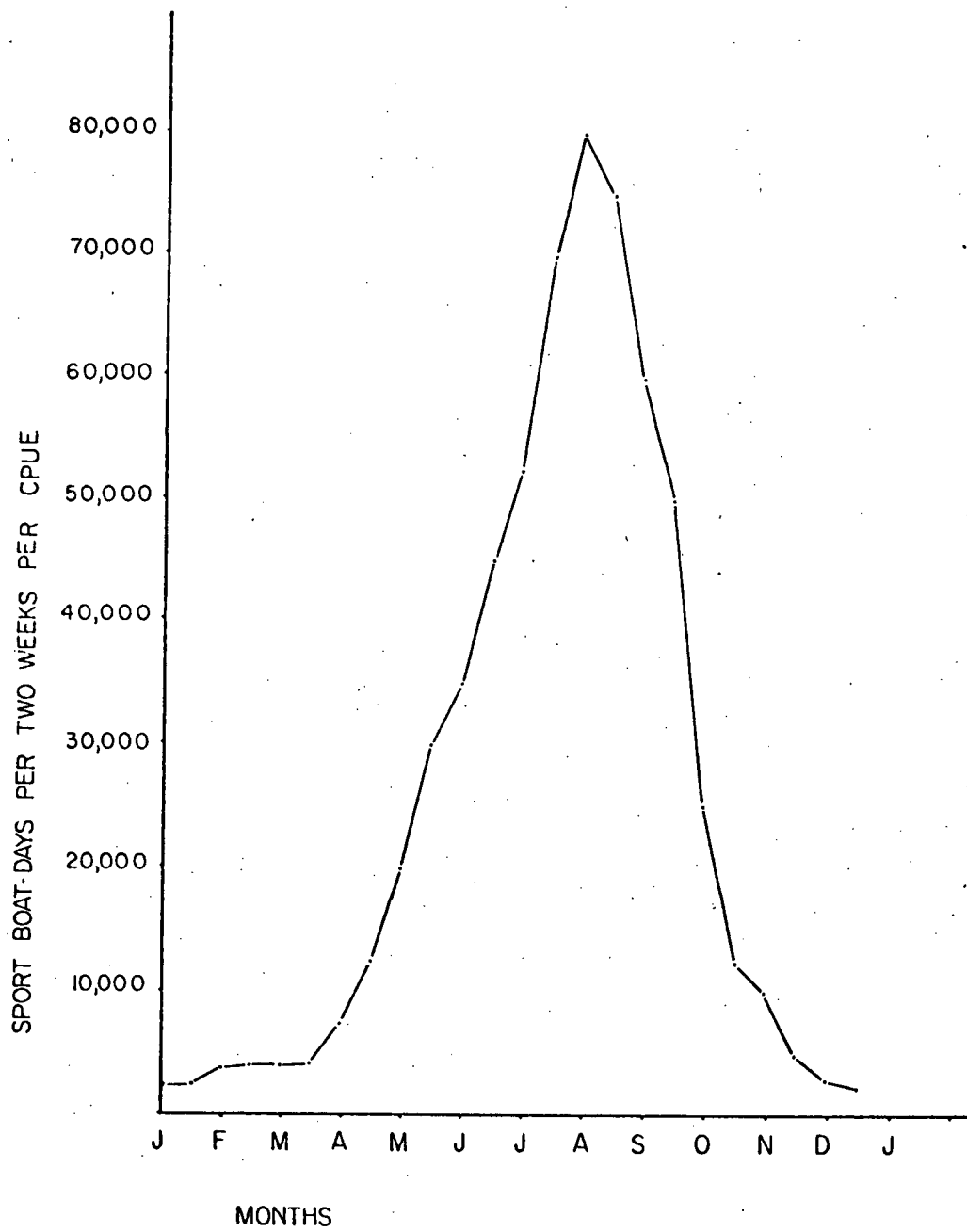


Figure 16: Ratio of Effort to CPUE in the Georgia Strait Sport Fishery.



There is a great deal of variability in the data depicted by Figures 7 through 15. However, predictions that ignored the behavior of fishermen are surely more misleading than predictions that take some account of this numerical response process.

2.2.2. Commercial Effort Response

Motivation of the commercial fisherman is less difficult to define than that of the recreational fisherman. For the most part, the commercial fisherman is out to make money. The more potential to catch fish there is in an area, the more fishermen will participate. Unlike the sport fleet, the commercial fleet can brave the elements outside the sheltered waters of Georgia Strait if the fishing is good enough to warrant it. Therefore, the commercial effort in Georgia Strait is affected by the abundance of fish inside, as well as the fishing opportunities outside Vancouver Island and along the north coast. Our concern is with the inside fishery. Therefore, the relationship between effort and CPUE is constructed with Georgia Strait data alone. Figures 17 through 23 show effort and CPUE data in the Georgia Strait troll fishery for the months of April through October for the years 1968 to 1975. There is a definite limit to the number of boats in the commercial troll fleet because of both the licence program and the number of boats that are not equipped to handle the rigors of the more lucrative outside fishery. Therefore, the relationship of effort to CPUE is assumed to saturate. Figure 24 indicates the seasonal variation

in the maximum troll effort assumed in the model and shows the level of CPUE at which half saturation of effort is reached.

2.2.3. Catchability

An important piece of any fisheries modeling exercise is the coupling of fish and fishing effort. The assumption about capture used in this model is the traditional one that catch per effort is proportional to abundance (Ricker, 1940). In this model, the catchability coefficient is the proportion of the available fish that is caught by a single unit of effort in a unit of time. Catchability is assumed to vary as a function of age and species of the fish as well as time of year. The variability may be due to environmental changes, seasonal changes in the behavior of the fish or seasonal changes in the efficiency and composition of the effort. For example, sport fishing effort in the summer may be made up of a higher proportion of inexperienced fishermen than the ardent fishermen of the winter months.

Using the "calibration phase" methodology developed by Johnson 1975, the simulation model has been used to reconstruct catchability coefficients for chinook and coho in the Georgia Strait. Average escapement by age were used to start a backward calculation of abundance using catch by age and gear type. The escapement of the oldest age class of fish was added into the pool fishery according to the maturation schedule and run timing discussed before. Progressing backwards in time, at each period the catch and natural mortality at the oldest age was

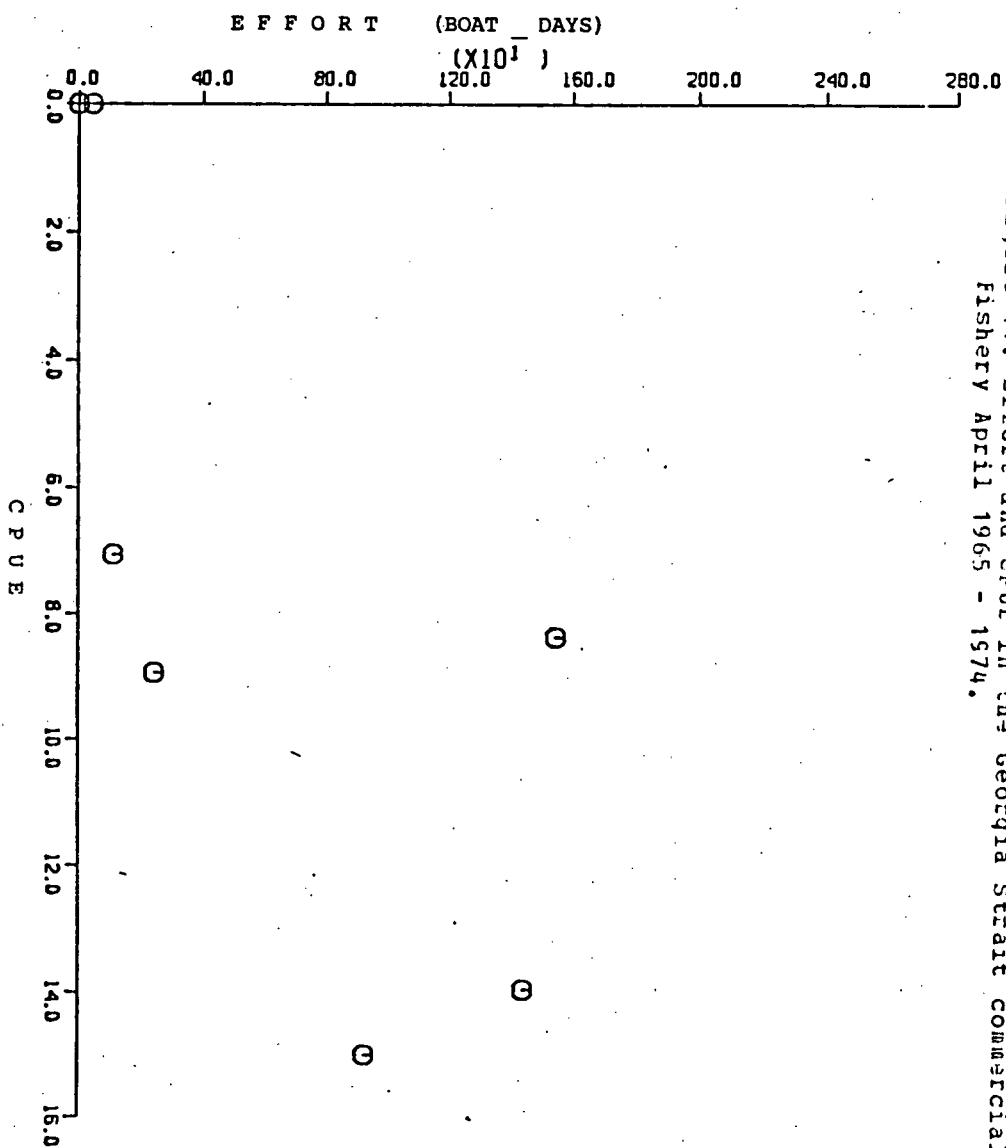


Figure 17: Effort and CPUE in the Georgia Strait commercial fishery April 1965 - 1974.

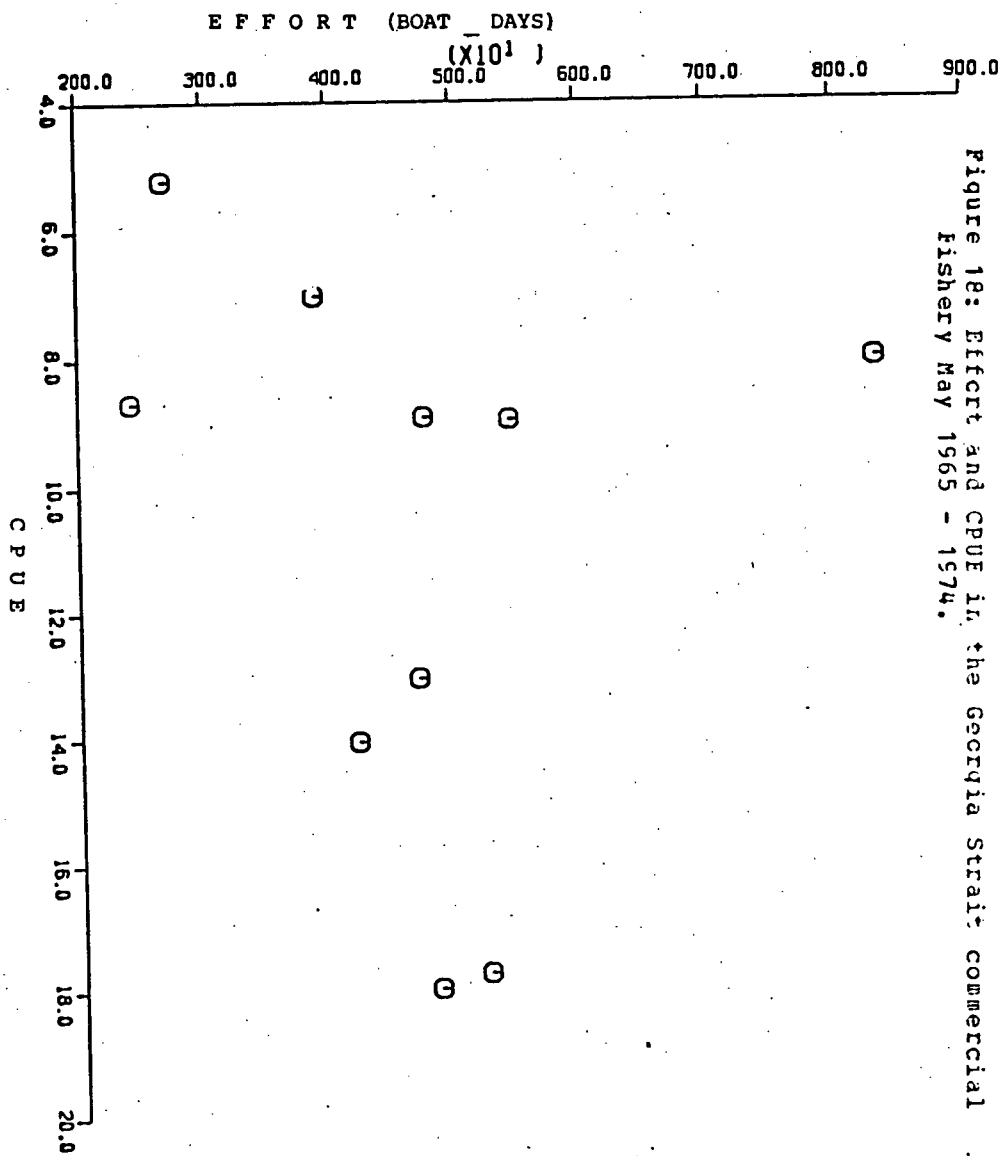
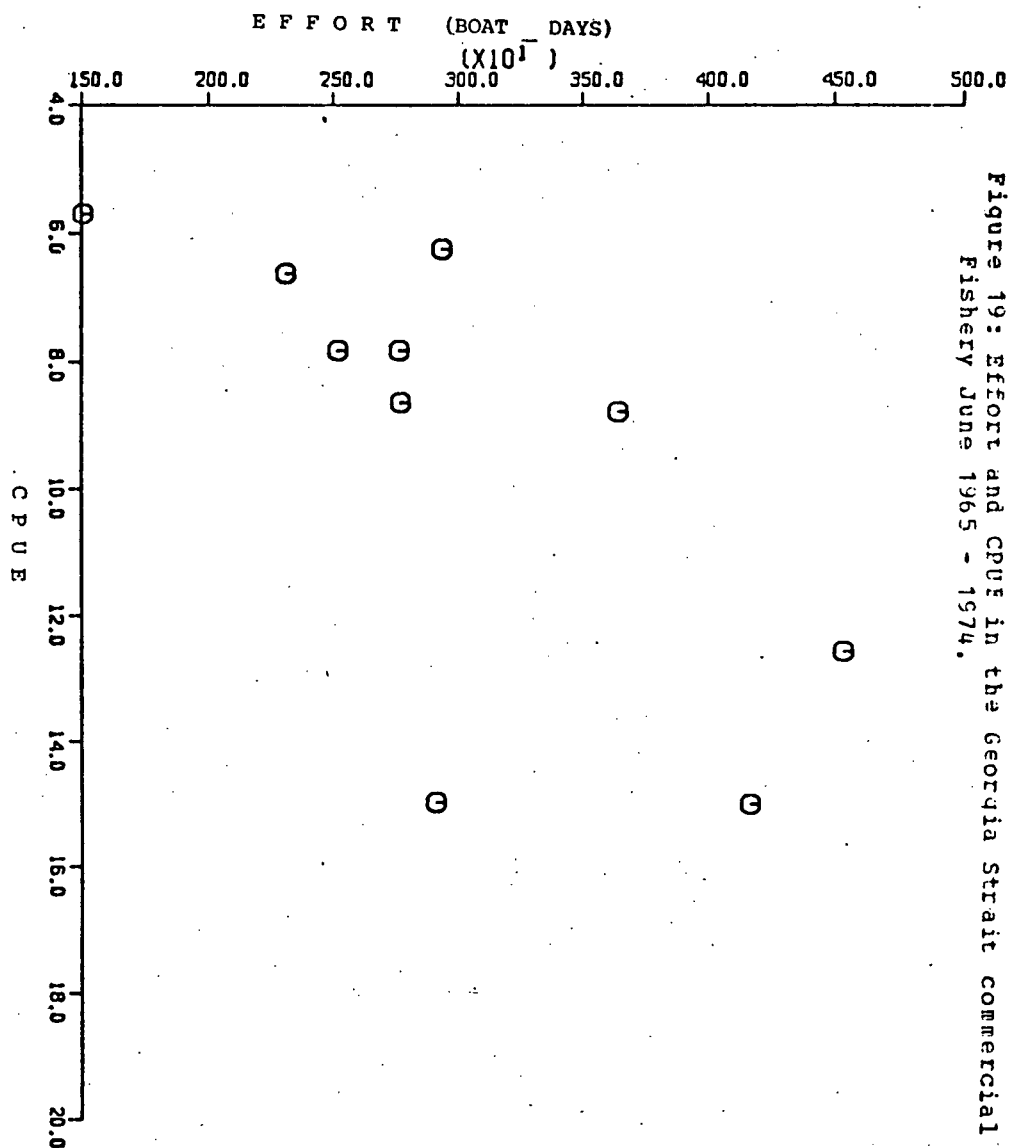


Figure 18: Effort and CPUE in the Georgia Strait commercial fishery May 1965 - 1974.



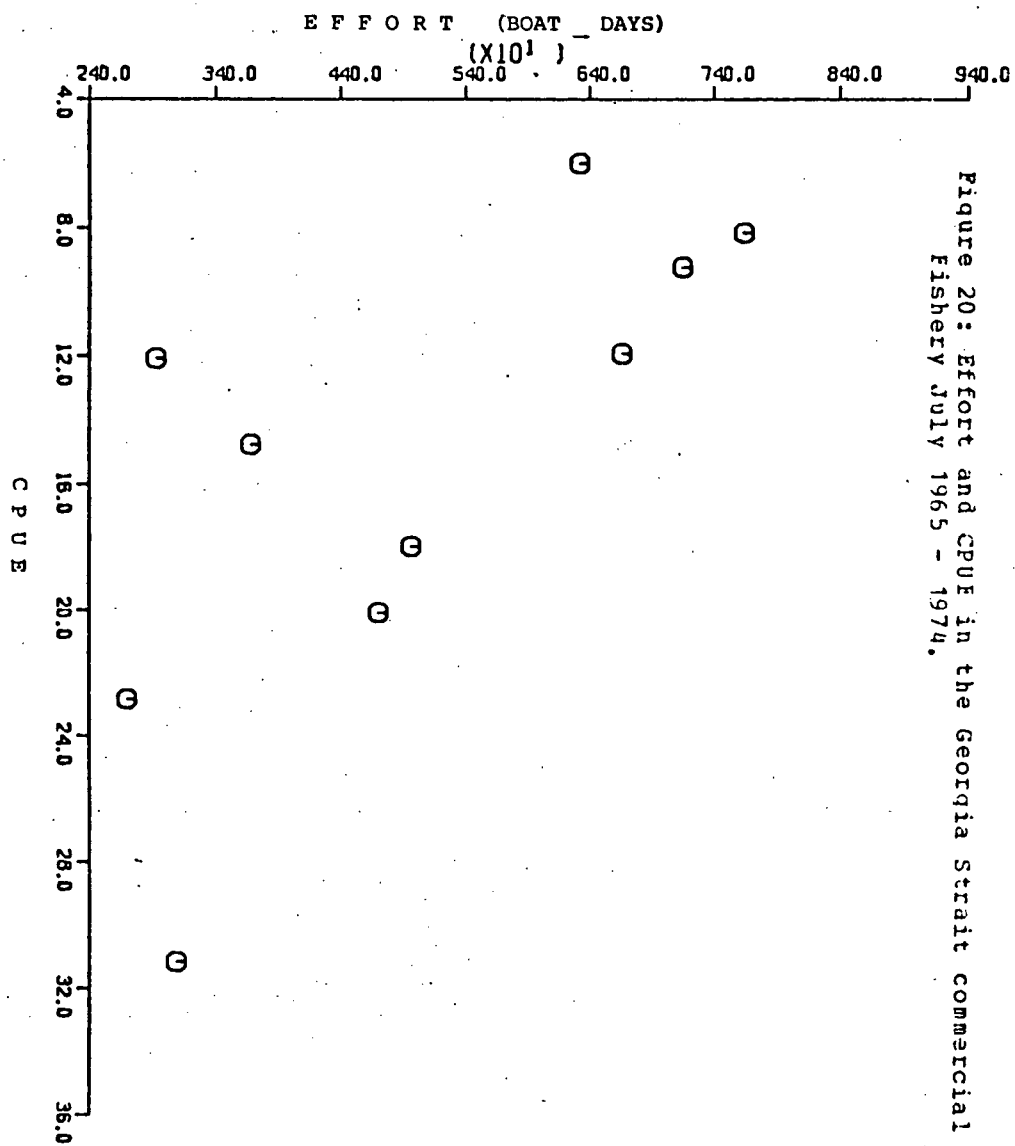


Figure 20: Effort and CPUE in the Georgia Strait commercial fishery July 1965 - 1974.

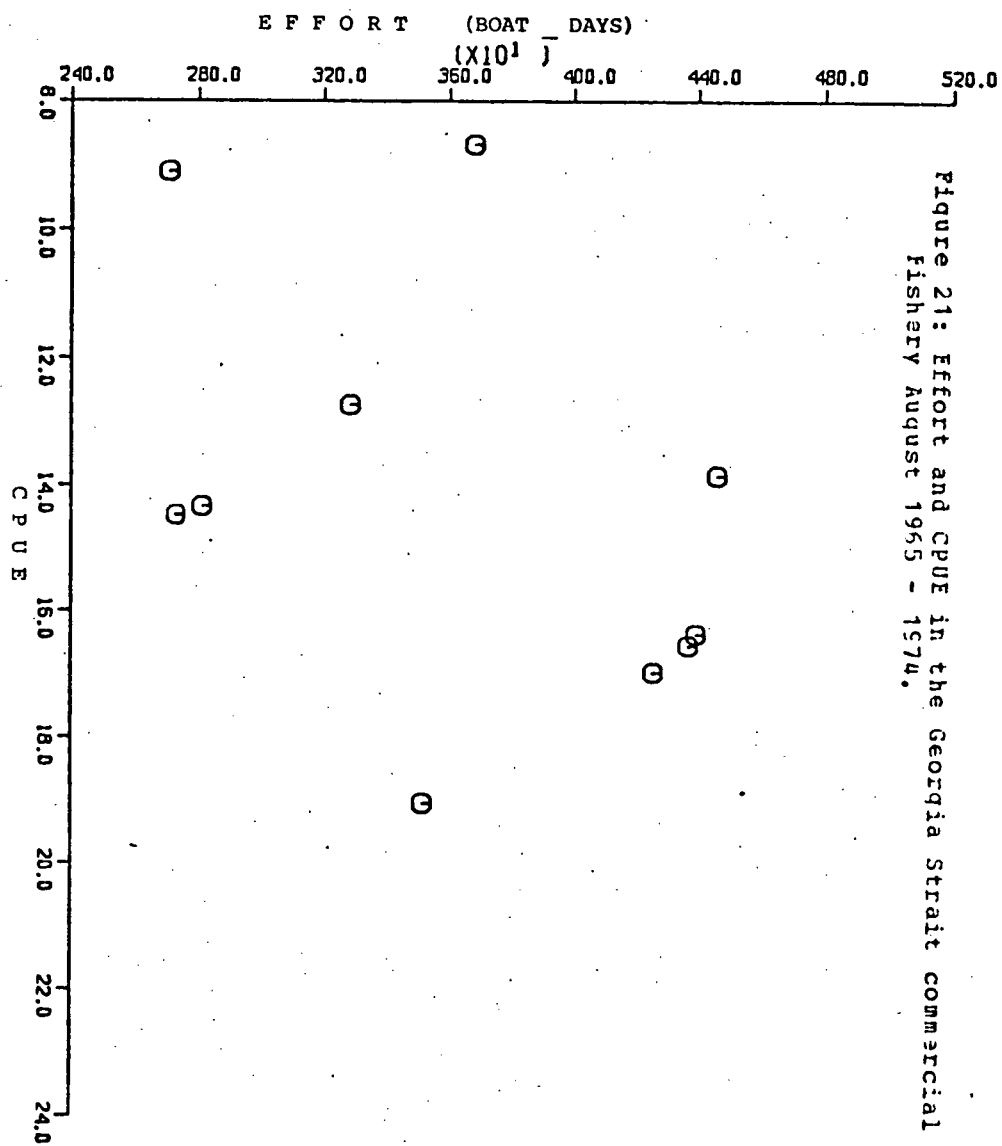
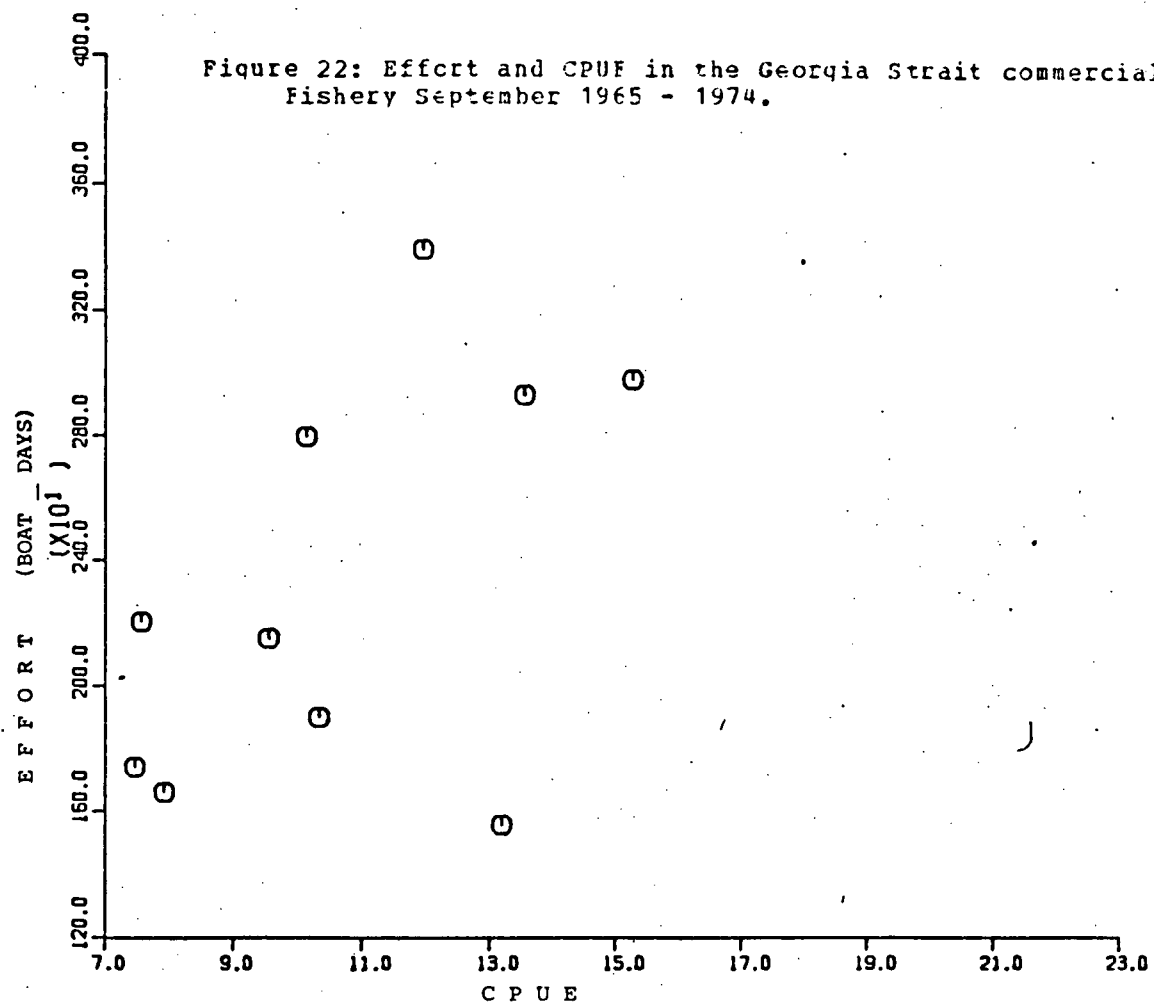


Figure 21: Effort and CPUE in the Georgia Strait commercial fishery August 1965 - 1974.



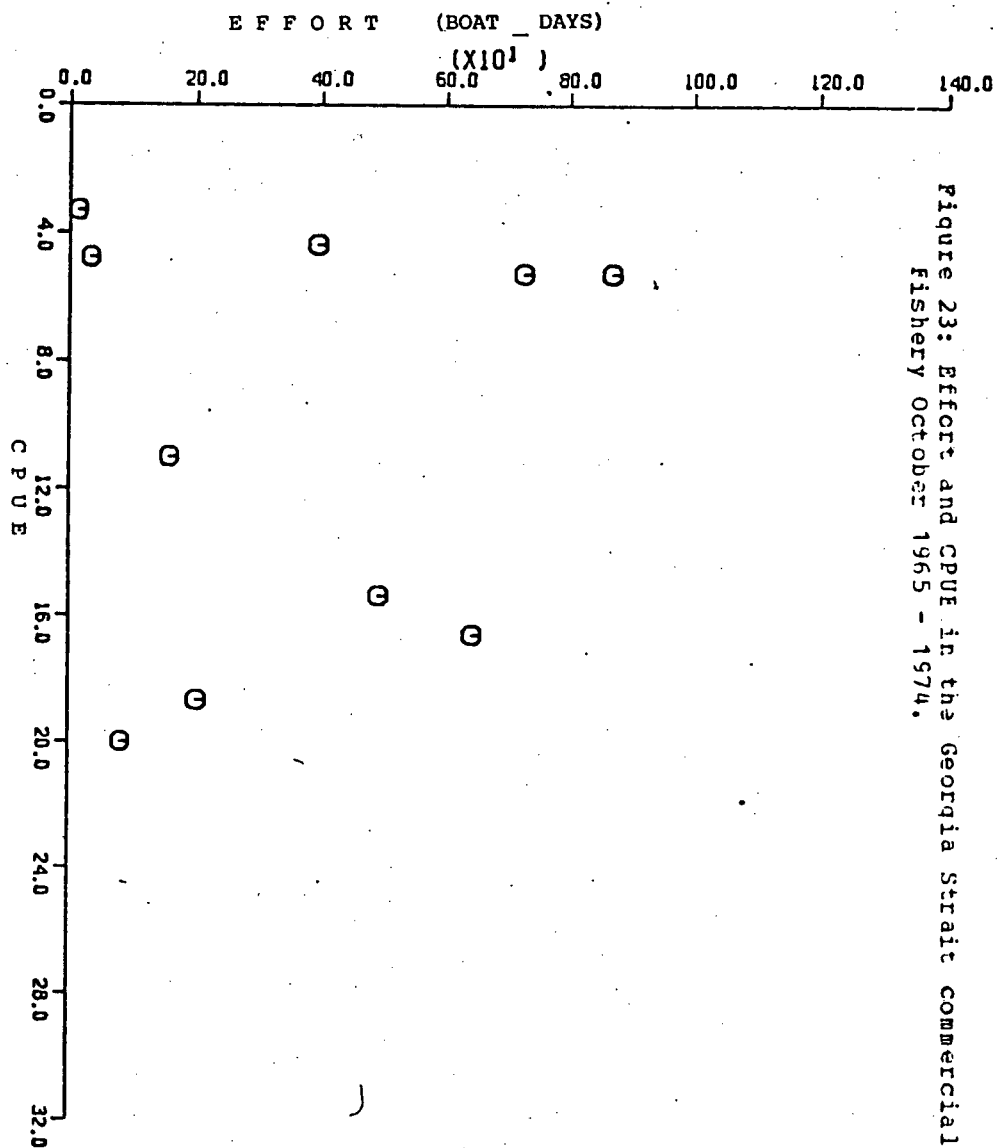


Figure 23: Effort and CPUE in the Georgia Strait commercial fishery October 1965 - 1974.

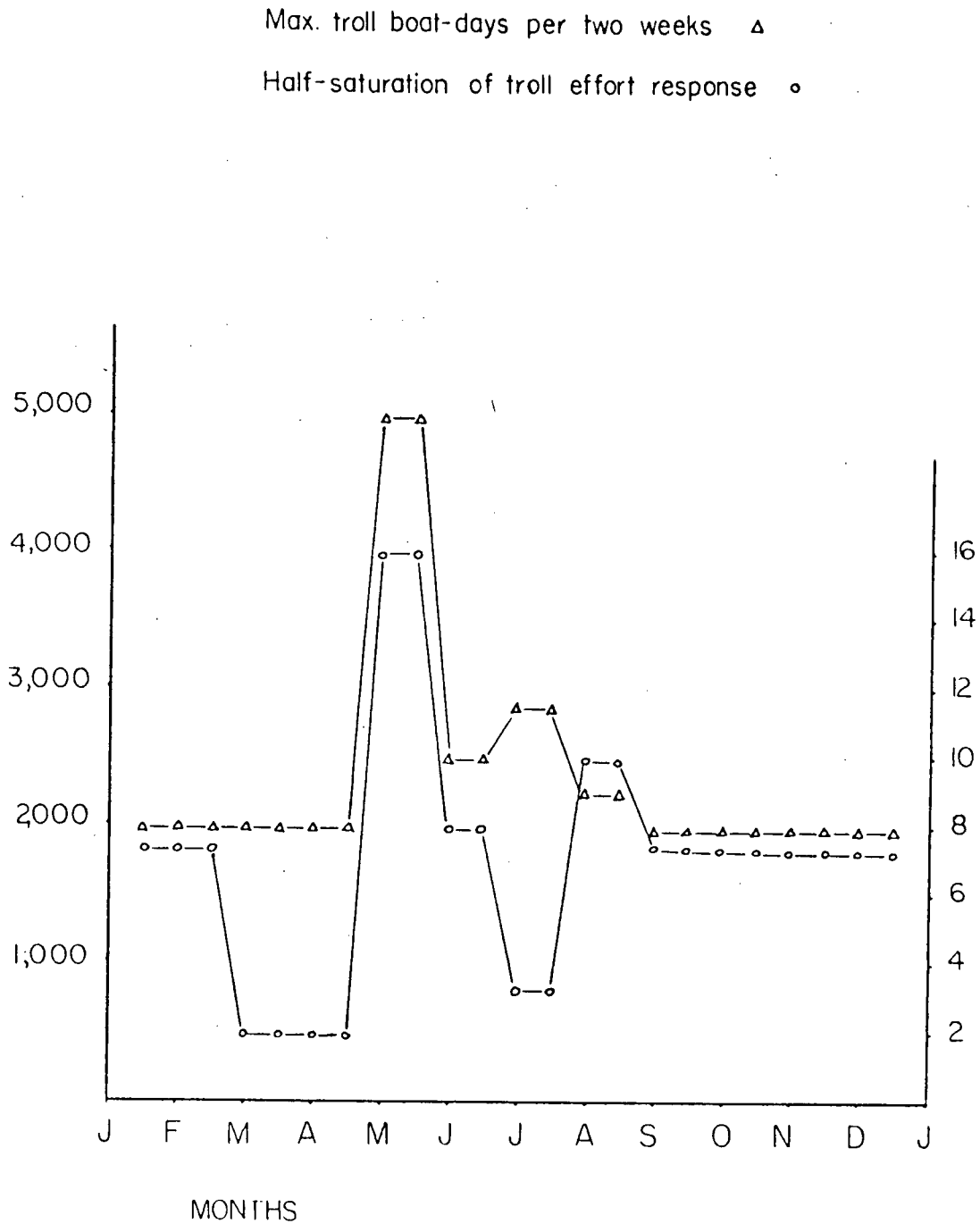


Figure 24: Parameters of the Troll Effort Response Model.

added to the pool, then the proportion taken by the sport and troll effort present formed the basis for the calculation of catchability. At the beginning of the year, the estimated abundance of the older age class represented the residual for the younger age class and the backward calculation continued for the younger age classes. This procedure provides an exact estimate of time varying catchability provided the natural mortality schedule is known.

The escapement and catch at age used in the backward calculation is presented in Table 1. The resulting catchability estimates are presented in Figures 25 and 26.

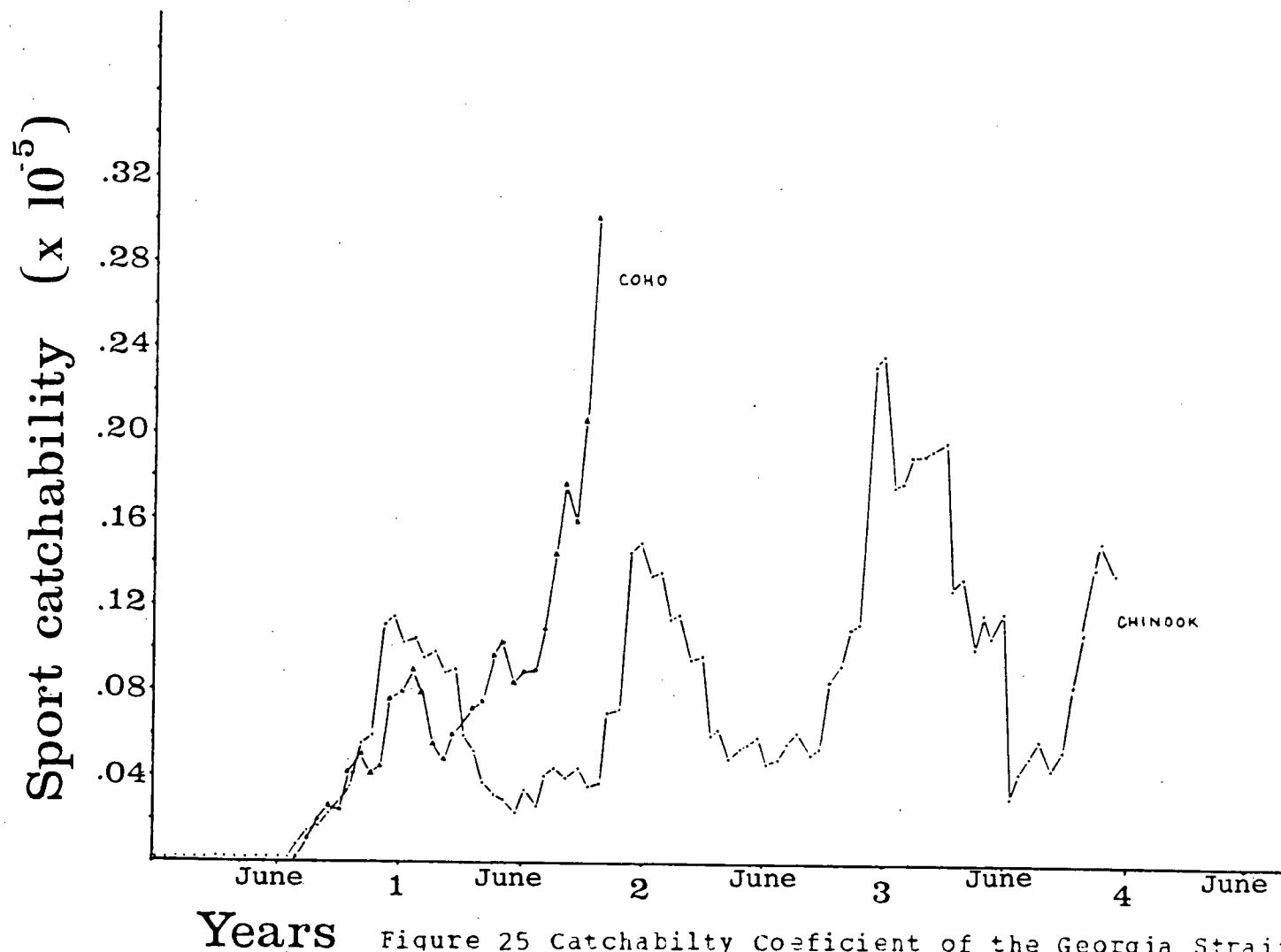
2.3. Controls

A central issue in the design of management strategies is the set of admissible controls. There are five important control options open to the managers of the Georgia Strait fishery. They can regulate the size of the fish that may be kept if caught, the gear with which the fish are caught, the number caught or landed per trip (bag-limit), the area in which fish may be caught, or the times when fishing is allowed.

One control option not considered feasible is to restrict participation in salt water angling. Limiting the number of participants or charging for the privilege of fishing, so as to restrict effort, would be met with considerable disapproval by the sport fishermen (Sport Fish Advisory Committee 1977 pers. comm.). In British Columbia, salmon fishing is considered to be every resident's right. Limitation of

Table 1: Current Catch and Effort Data for Georgia Strait by Month.

	January	February	March	April	May	June	July	August	September	October	November	December
Sport Effort	7407.	7407.	7407.	22961.	65921.	35179.	169618.	202949.	110362.	37775.	14073.	9629.
Troll Effort	0.	0.	0.	1786.	4693.	3287.	4560.	2394.	2280.	0.	0.	0.
Sport CPUE	1.5	1.3	1.4	1.2	1.3	1.1	1.1	1.3	1.0	1.0	1.0	1.8
Troll CPUE	0.0	0.0	0.0	15.9	15.5	10.6	19.7	11.8	11.0	0.0	0.0	0.0
Chinook Sport Harvest Pieces												
Age 1	1.	0.	0.	0.	0.	0.	0.	1.	1.	4.	15.	76.
Age 2	119.	127.	202.	517.	2294.	6372.	35280.	61697.	29717.	8205.	5582.	7742.
Age 3	5200.	4353.	3514.	6693.	15030.	18160.	26528.	32776.	12018.	5953.	2423.	3374.
Age 4	1881.	1966.	1985.	3869.	8069.	8109.	5020.	6291.	2165.	892.	361.	208.
Age 5	207.	220.	225.	402.	975.	1431.	1018.	710.	221.	45.	0.	0.
Chinook Sport Harvest Weight in Pounds												
Age 1	0.	0.	0.	0.	0.	0.	1.	1.	1.	8.	46.	162.
Age 2	95.	102.	182.	517.	2753.	10195.	81144.	172752.	95158.	28719.	20653.	30194.
Age 3	21320.	18718.	16164.	33465.	87174.	119856.	193654.	252375.	100951.	51196.	21322.	30366.
Age 4	17305.	19070.	21041.	46428.	108932.	117581.	77303.	101235.	36372.	15342.	6189.	3765.
Age 5	3767.	4026.	4118.	7397.	18429.	28191.	21378.	16046.	5238.	1107.	0.	0.
Chinook Troll Harvest Pieces												
Age 1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
Age 2	0.	0.	0.	170.	655.	2334.	4990.	3301.	8722.	0.	0.	0.
Age 3	0.	0.	0.	22264.	50628.	24564.	13724.	4181.	4255.	0.	0.	0.
Age 4	0.	0.	0.	5793.	20877.	7596.	3086.	813.	1060.	0.	0.	0.
Age 5	0.	0.	0.	170.	582.	348.	88.	84.	99.	0.	0.	0.
Chinook Troll Harvest Weight in Pounds												
Age 1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
Age 2	0.	0.	0.	594.	2096.	7501.	15968.	10993.	29565.	0.	0.	0.
Age 3	0.	0.	0.	111320.	293642.	162122.	100185.	32194.	35742.	0.	0.	0.
Age 4	0.	0.	0.	69516.	281840.	110142.	47524.	13049.	17838.	0.	0.	0.
Age 5	0.	0.	0.	3128.	11000.	6856.	1849.	1932.	2346.	0.	0.	0.
Coho Sport Harvest Pieces												
Age 1	0.	0.	0.	0.	0.	0.	1187.	6494.	5960.	3400.	1970.	3177.
Age 2	3704.	2963.	4444.	16073.	59329.	59625.	117546.	155865.	60257.	19265.	3659.	2600.
Coho Sport Harvest Weight in Pounds												
Age 1	0.	0.	0.	0.	0.	0.	356.	2598.	2980.	2380.	1576.	2859.
Age 2	3704.	3259.	5333.	22502.	142190.	220613.	505448.	732566.	283208.	92472.	17563.	12480.
Coho Troll Harvest Pieces												
Age 1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
Age 2	0.	0.	0.	0.	0.	0.	67944.	19870.	10944.	0.	0.	0.
Coho Troll Harvest Weight in Pounds												
Age 1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
Age 2	0.	0.	0.	0.	0.	0.	292159.	93389.	51437.	0.	0.	0.

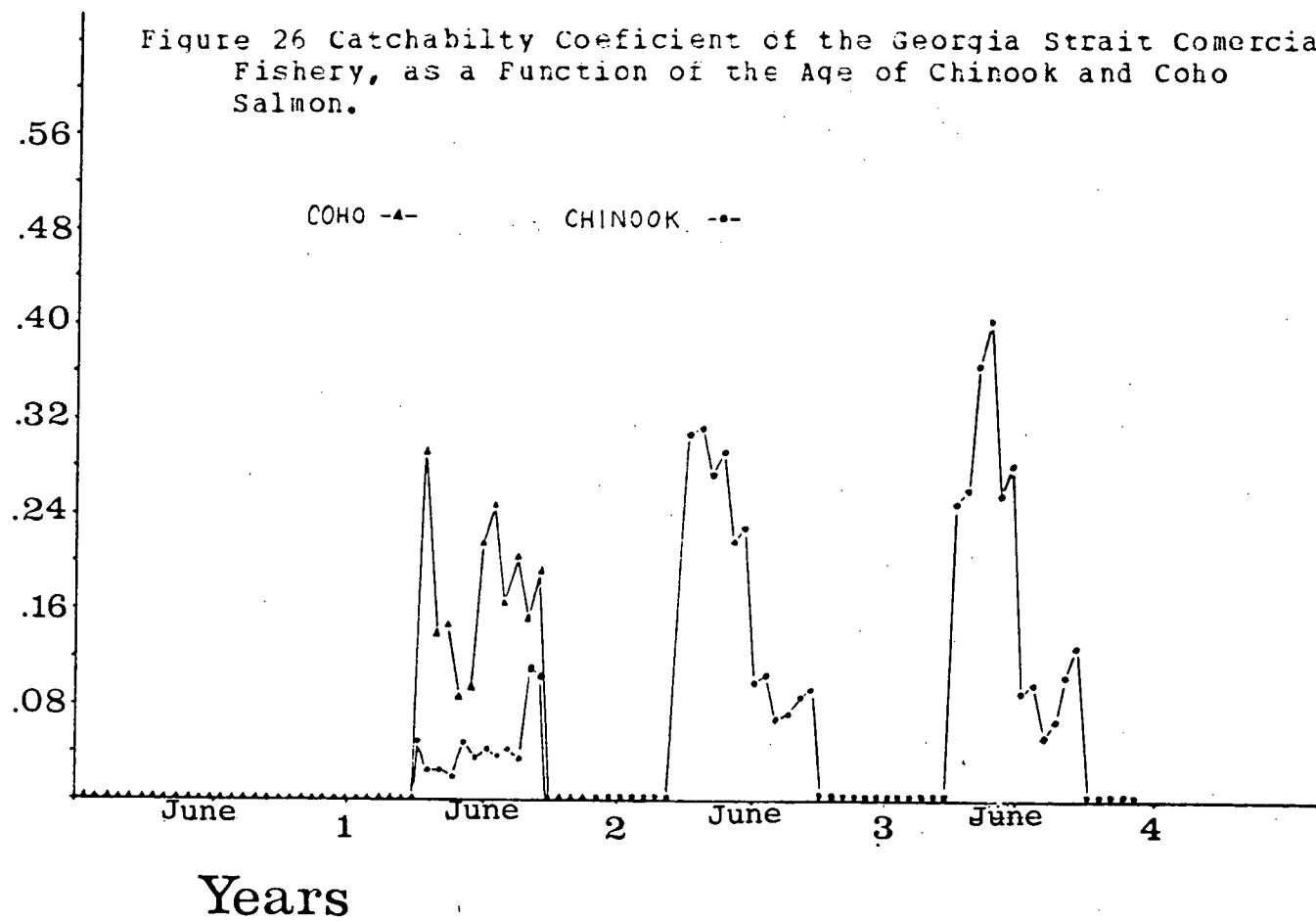


Years

Figure 25 Catchability Coefficient of the Georgia Strait Sport Fishery, as a Function of the Age of Chinook and Coho Salmon.

Troll catchability ($\times 10^{-4}$)

Figure 26 Catchability Coefficient of the Georgia Strait Commercial Fishery, as a Function of the Age of Chinook and Coho Salmon.



commercial effort is more palatable and is in effect for the entire coast.

The set of controls currently in effect are as follows: The size limit for the commercial troll fleet is three pounds dressed weight or approximately 18 inches. There is no bag limit on the commercial fishery. Many areas, particularly inlet and river mouths, are closed to commercial trollers and fishing is restricted to the summer and fall months.

The current size limit is 13 inches for sport caught fish. All lines must be hand reeled. There are virtually no restrictions on where fishing takes place in marine waters and the sportsman is allowed to fish year round. The bag limit is four salmon per angler-day.

2.4. Value

The model outlined above produces a hopelessly complex set of performance indicators. Simpler value measures are necessary for optimization. The simplification used here is the landed value of the catch for the commercial troll fishery and the value of a recreational boat-day.

2.4.1. Commercial Fishery

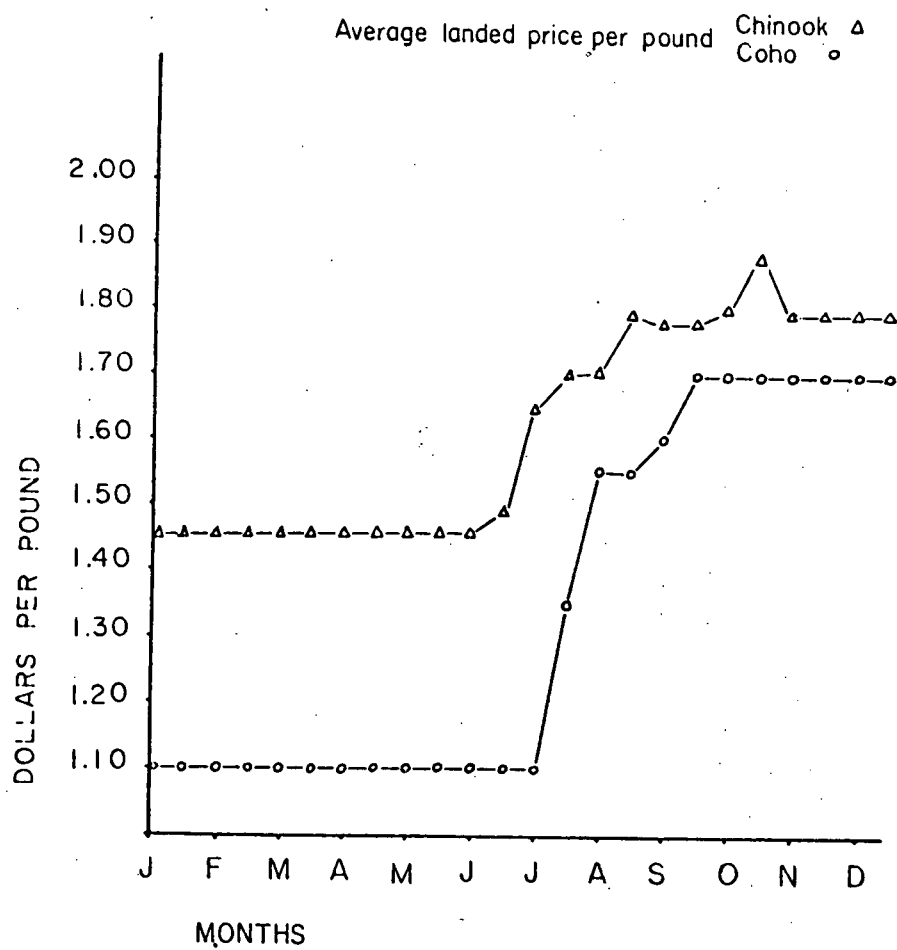
In the model, it is assumed the value to the commercial trollers is generated by the number of pounds of salmon landed. The management agency may have more general concerns, such as the health of the processing or marketing sector, but a good index of the prosperity of the entire industry is the landed value of the catch. The price per pound varies with the time of year and the size of the fish.

An average price per pound is used in the optimization (Fig. 27). The price paid for a landed salmon is assumed to be independent of the total number of salmon landed from the Georgia Strait. Georgia Strait catches may influence local prices to some degree. However, the Georgia Strait represents a small catch compared with the entire coast including Alaska, British Columbia, Washington, Oregon, and California and would therefore have little influence on the general market price.

2.4.2. The Benefits of the Sport Fishery

The sport fishery has a significantly different way of generating value. There are many sport fishermen who like to eat fresh fish, and a fish caught in the sport fishery does substitute for other food. More important, however, is the value of the act of fishing. Bryan (1974), in a questionnaire study of sport fishermen, concluded that the eating of fish ranked significantly lower than other attributes of fishing, such as the outdoor experience, in motivating sport fishermen. The task of putting a dollar value on the recreational aspects

Figure 27: Landed Price per Pound of Troll Caught Chinook and Coho Salmon.



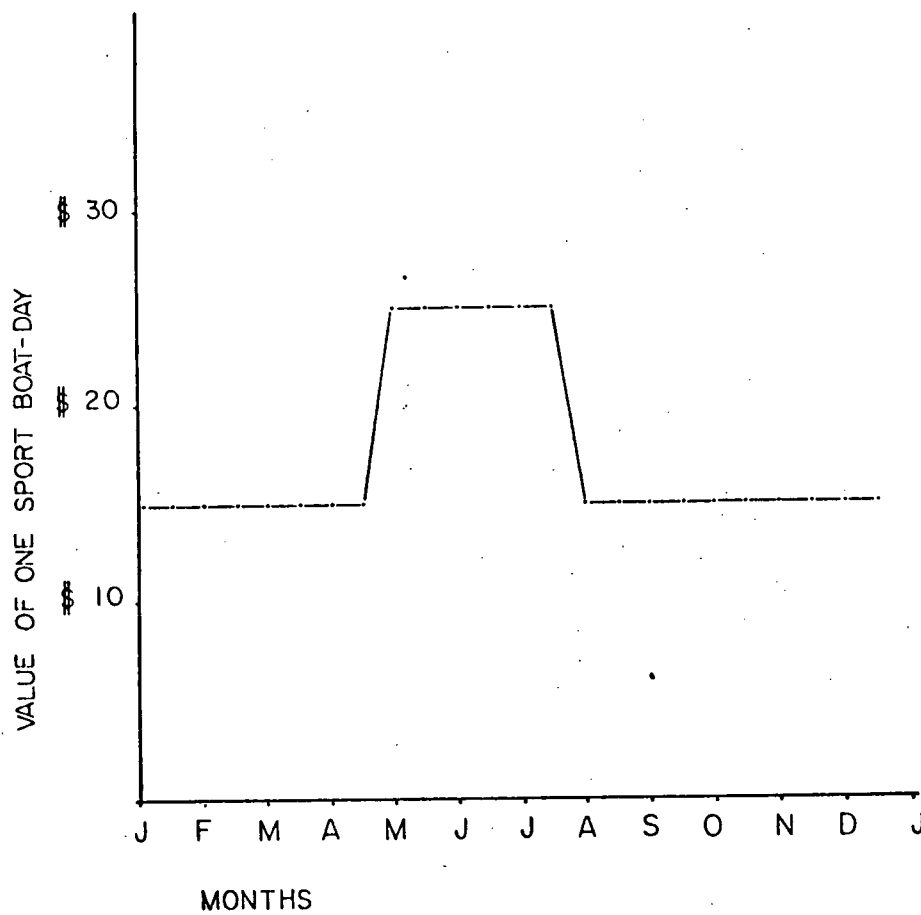
of the sport fishery is difficult: There is a need for a common denominator between the sport and commercial fishery. The value of commercial fishing is readily accountable in the value of the catch. If the fish are to be allocated between the two user groups, an equivalent metric is needed for the sport fishery.

Many methods have been suggested for the evaluation of the benefits from the sport fishery. The suggestions range from accounting the costs of using the resource to questions about how much a user would be willing to pay to fish. Stevens (1966) reviews the major schemes.

One of the most interesting approaches to evaluating sports fishing was conducted in the Washington State Fishery. In 1967, Mathews and Brown (1970) asked the question "For what minimum price would you be willing to sell your right to salmon fish for a year" The resulting value was \$20.00-\$60.00 per fishing trip, depending strongly upon the area fished and the quality of the fishing experienced. In the model of the Georgia Strait fishery, the value of \$15.00 in the winter and \$25.00 in the summer per boat day (Fig. 28) is used (Masse and Peterson 1977).

Once a metric has been established for measuring the performance of each fishery, the numbers from the two fisheries must be combined into an overall measure. The simplest approach and the one to be used in this analysis is to sum the dollar benefits from the two fisheries. Another approach may be to transform the benefits from a particular fishery into a utility function (Hilborn and Peterman 1977). In the case of the sport fishery, an added unit of sport effort at an already high

Figure 28: Recreational Value of a Boat-Day of Sport Effort in the Georgia Strait.



effort level may have a quite different appeal to a manager than that same unit of effort at a low level of effort. Also, the way in which the benefits, or utility, from the two fisheries is combined may take on a quite different form (Hilborn and Walters 1977, Keeney and Raiffa 1976, Keeney 1977). It is apparent that the choice of objective functions will affect the form of the optimal policies. It may also be that the study of optimal policies can be used to determine better objectives.

3. OPTIMIZATION MODEL

Much of the present theory on optimal exploitation of natural fish populations has been developed assuming that recruitment is independent of stock size, leading to the concept of yield per recruit (Beverton and Holt 1967, Ch. 18, 19; Ricker 1958, Ch. 10; Clark 1976, Ch. 8). Another popular approach is to assume that growth, mortality, and reproduction can be pooled into a gross model of population change (Scheafer 1957; Clark 1976, Ch. 2; Clark, Edward, and Friedlander 1973).

Single-stage stock recruitment models have proven useful when life cycle length is fixed (Ricker 1954). Walters (1975), using dynamic programming, confirmed that a fixed escapement policy is optimal in the presence of environmental variability. Other approaches include solving for an optimal age structure given an arbitrary total population (Beddington 1974, Beddington and Taylor 1973). Walters (1969) developed a general simulation model of fish populations, and tested various harvest schemes on the model. Other concerns have been raised about overexploitation in multiple stock fisheries (Paulik, Hourston, and Larkin 1966; Hilborn 1976). To date, there is no general solution to the optimal exploitation problem of age structure population when recruitment depends upon stock size.

The source of much of the difficulty with optimization is the "curse of dimensionality" (Bellman 1961). For the mixed fishery problem considered in this thesis, one has to deal with all possible combinations of abundance of two age classes of coho and five age classes of chinook. Unfortunately, some reduction in the problem is needed in order to apply current technology.

The simplifications used in the model is to assume that recruitment is constant and that there exists an initial and a target escapement abundance for each age class of each species. The optimization problem is to determine an optimal plan of within season operation that maximizes total benefit from the fishery and maintains target escapement populations. The model assumes each age class of each species is separate from the others in its biology. The only interaction is in the common exploitation of all the fish.

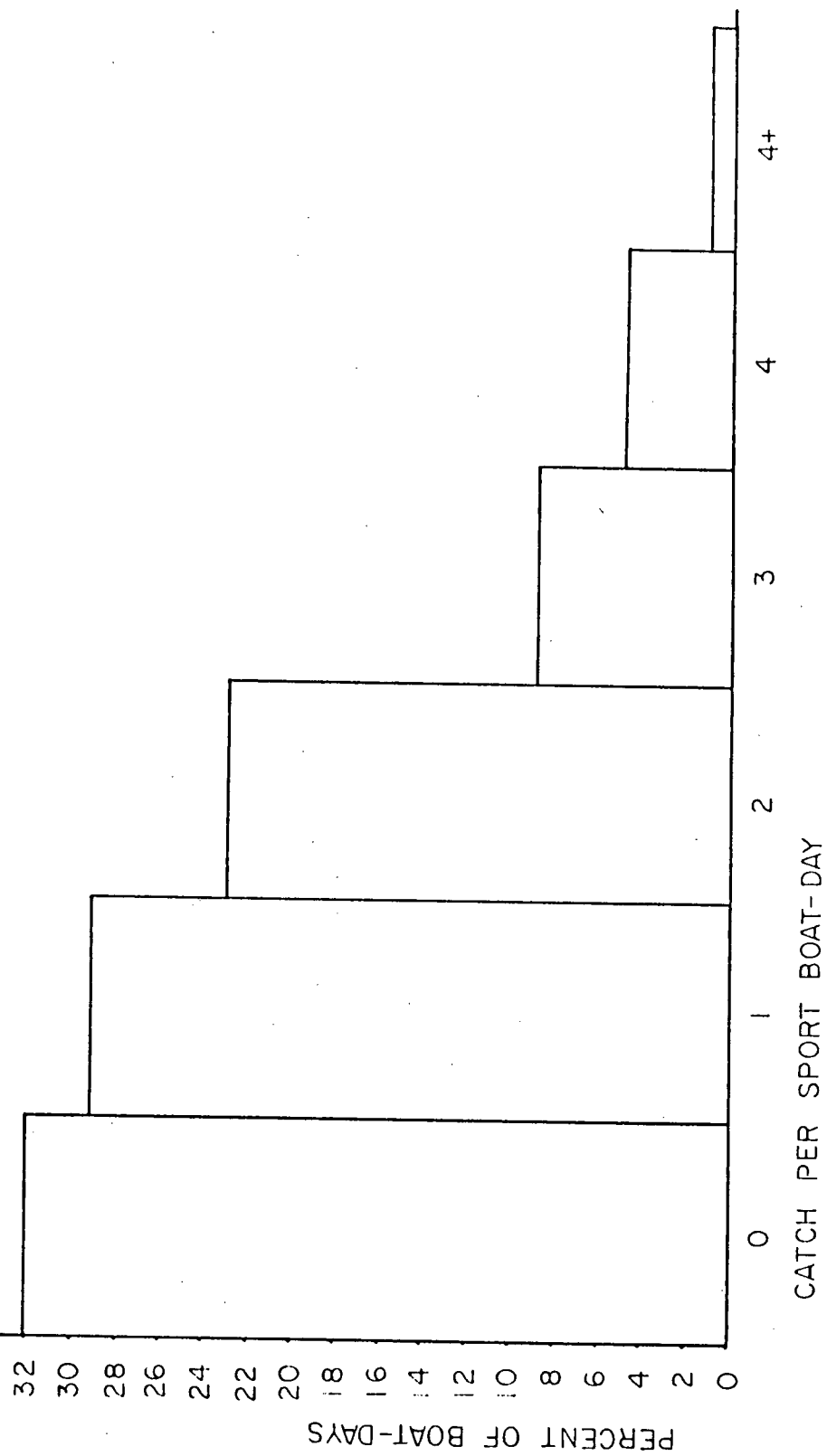
Another complication in the analysis of this fishery is the diversity of the available controls. The task of designing a complete portfolio of controls with a single computational procedure is very difficult. The only optimization to be done in this thesis is to determine seasons of fishing for both fisheries. Optimal seasons will be studied under various size limit regulations and under a variety of catchability coefficients representing a variety of possible gear restrictions.

Bag limit changes and area closures for the sport fishery are not considered as control options. Figure 29 shows the frequency of bags per boat-day for sport fishermen who keep log books for the Department of Fisheries (A.W. Argue pers. comm.). A bag limit reduction to one fish per angler-day corresponds to a limit of two and one half per boat-day. This reduction in bag could produce an approximate 15% reduction in immediate catch, but would result in an ultimate reduction in catch of only 2% to 10% (Allen 1955). The consequence of a severe bag limit on spawning escapement could be considered minimal at best. A less severe bag limit of one chinook, two salmon total per angler-day has been suggested and was found completely unacceptable by the sport fishermen (Sport Fish Advisory Committee Meeting, June 23, 1978).

Area restrictions on the sport fishermen have been discounted on the basis of the inequities they would generate within the tourist industry. If some areas were closed during peak tourist times, the disparity among resort operators in the affected areas would be unacceptable.

The following sections describe the four major components of the optimization model: fish dynamics, effort dynamics, controls, and revenues.

Figure 29: Distribution of Bags in the Georgia Strait Fishery.



3.1. Fish Dynamics

The fish model is a standard random encounter model. The change in a class of fish during period k is governed by the following ordinary differential equation:

$$dx(i,t)/dt = -[m(i,k) + (l(i,k,c) + (1-l(i,k,c))v(c))$$

$$u(k,c)E(k,c)q(i,k,c)$$

$$+ (l(i,k,s) + (1-l(i,k,s))v(s))$$

$$u(k,s)E(k,s)q(i,k,s)]x(i,t)$$

$$k \leq t \leq k+1, \quad k=1, \dots, N, \quad i=1, \dots, 7$$

(1)

where: $m(i,k)$ is the sum of natural mortality rate, migration rate out of Georgia Strait, and mature run timing for age and species i during period k ;

$l(i,k,c) = 1$ if age and species fish i is over the commercial size limit;

$l(i,k,c) = 0$ if age and species fish i is under the commercial size limit;

$l(i,k,s) = 1$ if age and species fish i is over the sport size limit;

$l(i,k,s) = 0$ if age and species fish i is under the sport size limit;

$v(c)$ and $v(s)$ are shaker mortality rates for commercial and sport fisheries respectively;

$u(k,c)$ and $u(k,s)$ are the controls for the commercial and sport fisheries respectively during period k ;

$E(k,c)$ and $E(k,s)$ are the effort levels for the commercial and sport fisheries respectively during period k ;

$g(i,k,c)$ and $g(i,k,s)$ are the catchability coefficients for age and species i , in the commercial and sport fisheries respectively;

$x(i,t)$ - the number of age and species i fish available at time t ;

$x(i,k)$ - the number of age and species i fish present at the beginning of period k ;

$x(i,1)$ - the number of age and species i fish present at the beginning of the year;

N - the 24 bi-monthly periods in a year.

The seven classes of fish are two ages of coho and five of chinook.

The assumption of random encounter is quite tenuous. Lack of information on the process of encounter and capture in most fisheries precludes the use of other possible forms of the model. In the Georgia Strait fishery, fishing takes place at relatively localized "hot spots". It is not clear whether success is a product of the aggregation of the fish or of different behavior of fish in the fishery areas. On the other hand, success may be due to the behavior of fishermen at particular sites and may have nothing to do with the fish over the range of stock sizes normally present. Because there is little known about the process underlying successful capture, the number of tenuous assumptions required increases with the complexity of the model. It is therefore more prudent to make one bad assumption (random encounter) than many worse ones.

Integrating Equation 1 we get:

$$y(i,k+1) - y(i,k) = -[m(i,k) + l(i,k,c)]$$

$$\begin{aligned}
& + (1-l(i,k,c))v(s))u(k,c)E(k,c)q(i,k,c) \\
& + (l(i,k,s) + (1-l(i,k,s))v(s)) \\
& u(k,s)E(k,s)q(i,k,s)]
\end{aligned}$$

(2)

where: $y(i,k) = \ln(x(i,k))$

At this point, we may define

$$F(i,k) = y(i,k+1) - y(i,k)$$

3.2. Effort

The random encounter model (Ricker, 1940) relates instantaneous rate of catch, effort, and population as follows:

$$\text{catch} = \text{effort} \times \text{catchability coefficient} \times \text{population}$$

Therefore, catch per unit effort (CPUE) may be written:

$$\text{CPUE} = \text{catch/effort} = \text{catchability coefficient} \times \text{population}$$

or

$$\text{CPUE} = qx$$

Sport effort is assumed to be proportional to CPUE. Therefore,

$$E(k,s) = c(k) \sum_{i=1}^7 q(i,k,s) x(i,k) l(i,k,s) \quad (3)$$

where: $c(k)$ is the time varying coefficient shown in figure 16, it may be viewed as the marginal effort generated by a unit increase in CPUE. Commercial troll effort is assumed to saturate with CPUE.

$$\begin{aligned}
E(k,c) = a(k) & \sum_{i=1}^7 q(i,k,c) x(i,k) l(i,k,c) / [b(k) + \\
& \sum_{i=1}^7 q(i,k,c) x(i,k) l(i,k,c)]
\end{aligned}$$

where: a and b are depicted in Figure 24 (Note: response is assumed for legal sized fish only).

3.3. Controls

The only controls for which optimization is done in this thesis are the seasons of sport and troll fishing. Therefore:
 $u(k,c)=1$ if commercial fishery is open in period k ;
 $u(k,c)=0$ if commercial fishery is closed in period k ;
 $u(k,s)=1$ if sport fisheries fishery is open in period k ;
 $u(k,s)=0$ if sport fisheries fishery is closed in period k .
 Note that $u=1$ does not imply that fishing will actually occur; equations (3) and (4) may produce low fishing pressure if the stock size is low or, in the sport fishery, if it is winter.

3.4. Revenue

Revenue is generated from the commercial fishery in the form of catch and in the sport fishery in the form of effort. Sport effort is dependent upon catch, so for the purpose of optimization, revenue from the sport fishery can also be represented in terms of catch. The average CPUE in the sport fishery is approximately one fish per boat-day. Therefore, the price of a sport caught fish can be assumed the same as the value of a boat-day of effort. The revenue expression is taken to be the sum of the components due to fishing on the right hand side of Equation 2, weighted by the price and size of fish

caught.

$$\begin{aligned}
 R(k) = & u(k, c) E(k, c) p(k, c) \sum_{i=1}^7 l(i, k, c) w(i, k) q(i, k, c) x(i, k) \\
 & + u(k, s) E(k, s) p(k, s) \sum_{i=1}^7 l(i, k, s) q(i, k, s) x(i, k)
 \end{aligned}
 \tag{6}$$

where: $w(i, k)$ is the weight of age and species i during period

k (Fig. 6 and length weight relationship);

$p(k, c)$ is the price per pound of commercially caught fish during period k (Fig 27);

$p(k, s)$ is the price per fish in the sport fishery (Fig. 29).

4. OPTIMIZATION

With the ingredients defined in the previous chapter, the optimal control problem is to find $u(k,c), u(k,s), x(i,k)$ $k=1, \dots, n$ $i=1, \dots, 7$ (Equation 1) that maximizes $R(k)$ (Equation 6). In addition, the population levels after period N ($x(i, N+1)$) should not depart dramatically from target levels ($X(i)$). The later condition can be formalized by a terminal payoff function.

$$G(x(i, N+1)) = -d(i)(x(i, N+1) - X(i))$$

where: $X(i)$ is the target level of age and species i ;

$d(i)$ is a parameter used to weigh the terminal condition against the within season benefits.

The problem is formulated in the method of "Lagrange" multipliers (Kolman and Trench 1971, p. 224; Clarke 1976, p. 250).

4.1. Mathematical Formulation

To ease the notation and differentiation, the formulation will be carried out in $y(i)$ rather than $x(i)$. Recall that $y(i) = \ln x(i)$, hence $x(i) = \text{EXP}(y(i))$. The problem may then be expressed as

$$\text{Maximize} \left\{ \sum_{k=1}^N R(k) + \sum_{i=1}^7 G_i \right\}$$

(8)

subject to

$$y(i, k+1) - y(i, k) = F(k) \quad k=1, \dots, N$$

$$0 \leq u(k, c), u(k, s) \leq 1$$

(9)

The "Lagrangian" for this problem (Clark, 1976) is:

$$L = \sum_{k=1}^N [R(k) - \sum_{i=1}^7 z(i) (y(i, k+1) - y(i, k) - F(i, k))] + G$$

(10)

Where: z is the Lagrange multiplier.

A necessary and sufficient condition for optimality of a policy $u(.,.)$ is that L must be maximized with respect to all y 's and u 's. Differentiating with respect to y and rearranging terms produces

$$z(i, k-1) - z(i, k) = dR(k)/dy(i, k) + z(i, k) dF(i, k)/dy(i, k) \quad k=2, \dots, N$$

(11)

$$z(i, N) = dG/dy(i, N+1)$$

(12)

From Equations 2 and 6, note that L is linear in the u 's. Therefore, to maximize L with respect to u , the following conditions must be satisfied:

$$p(k, c) \sum_{i=1}^7 w(i, k) l(i, k, c) \leq \sum_{i=1}^7 z(i, k) [l(i, k, c) + (1-l(i, k, c))v(c)] \quad u(k, c)=1$$

$$p(k, c) \sum_{i=1}^7 w(i, k) l(i, k, c) \geq \sum_{i=1}^7 z(i, k) [l(i, k, c) + (1-l(i, k, c))v(c)] \quad u(k, c)=0$$

$$p(k, s) \sum_{i=1}^7 w(i, k) l(i, k, s) \leq \sum_{i=1}^7 z(i, k) [l(i, k, s) + (1-l(i, k, s))v(s)] \quad u(k, s)=1$$

$$p(k, s) \sum_{i=1}^7 w(i, k) l(i, k, s) \geq \sum_{i=1}^7 z(i, k) [l(i, k, s) + (1-l(i, k, s))v(s)] \quad u(k, s)=0$$

(13)

4.2. The Algorithm

Some methods exist to handle this problem. One could solve for the seven times 23 x 's and z 's and two times 24 u 's. This approach would mean solving a system of 370 non-linear equations for 370 unknowns, and would be prohibitively expensive. Fortunately for this study, it was possible to derive a much simpler procedure, similar to "policy iteration" (Howard, 1960). The procedure involves the following steps:

Step 1: Pick a set of u 's.

Step 2: Solve Equation 1 forward to time N using current best estimate of the optimal u 's.

Step 3: Determine $z(N)$ and solve equation (11) backwards from N to 1 using current u 's.

Step 4: Using z 's, determine a new set of u 's according to equation (13).

Step 5: If, during two iterations, all x 's, z 's, and u 's remain unchanged, then stop; else go to Step 2.

No theory exists concerning the numerical properties of this algorithm. Puterman and Brunelle (1976) have compared policy iteration to the "Newton" method of non-linear programming. Convergence of the algorithm is not guaranteed. However, if the algorithm stops, then a solution to the "Lagrange" problem has been reached and it is guaranteed that the resulting policy is locally optimal.

5. OPTIMIZATION RESULTS

The optimization procedure described in the preceding section requires initial population sizes ($x(i,1)$) and target population sizes ($X(i)$). The population sizes used in this study were taken from the simulation model. An initial population structure is used to start the simulation then, as a consequence of constant recruitment an equilibrium population structure is reached after six years. The final population structure of the simulation model is likely the best available estimate of "current" conditions in the Georgia Strait. The optimization uses these values (Table 2) as the $x(i,1)$.

An important parameter in the optimization is $d(i)$ (Eq. 7). The magnitude of $d(i)$ determines the degree to which $X(i)$ is met. Differentiating G with respect to $y(i)$, leaves $d(i)$. Therefore, $d(i)$ is the terminal value of $z(i)$ and the basis for calculating the rest of the z 's. The z 's have an interesting interpretation. In economics, they are known as the "shadow" prices (Intriligator 1971). Ignoring size limits and shaker mortality, system (13) states that if the shadow price of a fish left in the sea is greater than the price of the landed fish, then the optimal decision is to leave the fish in the water. On the contrary, if the fish is more valuable in the boat than in the sea, then the fishermen should be allowed to fish.

TABLE 2. Initial and target population sizes

Coho

Ocean year 1	Initial 1,800,000	Target 1,080,000
Ocean year 2	Initial 1,080,000	Target 0

Chinook

Ocean year 1	Initial 1,300,000	Target 980,000
Ocean year 2	Initial 980,000	Target 490,000
Ocean year 3	Initial 490,000	Target 142,000
Ocean year 4	Initial 142,000	Target 17,000
Ocean year 5	Initial 17,000	Target 2,500

The z 's represent the value of leaving the fish in the sea to be caught later when they are larger and more valuable. The z 's also include the cost of violating target escapements. The value chosen for $d(i)$ can be interpreted as the value of leaving a fish in the water for future benefits. These benefits may come from the offspring if the fish is allowed to reach the spawning grounds, or from the value of the fish in next years catch.

For the optimization in this thesis, independent estimates of the d 's were not available. Therefore, suitable values had to be found by other means. The approach used was to assume that the d 's should be proportional to the landed value in the commercial fishery of a fish of age and species i in the last period of the year.

$$d(i) = r p(i, N, c) w(i, N)$$

(14)

A search for the value of r which generated a policy that met the terminal values (X 's) most closely resulted in r equal to 0.07 and left an average trivial discrepancy of 0.71% from the target X 's.

5.1. Finding an Optimal Policy (Convergence)

The method described in the last section found solutions very efficiently in all cases. Convergence was obtained in four or five iterations of the algorithm. The calculation included solving equation (1) and equation (11) five times. This computational requirement is equivalent to running the

simulation model eight to ten years.

Table 3 shows intermediate policies obtained as the method converged on an optimal policy for "current" conditions. The initial policy used in all cases included sport fishing all year and no commercial fishing. This policy underharvested the populations leaving an average of 59% too many fish. The next policy computed by the algorithm closed up both fisheries all year around. This policy over harvested the population leaving an average 17% too few fish. The next policy closed the sport fishery for the months of January through March and closed the commercial fishery from January until June 1, leaving an average of 0.5% too few fish. The final policy left the sport season unchanged and opened the commercial fishery one half month later.

5.2. Optimal Policy

The first optimization result obtained was using nominal or "best guess" parameter estimates and the assumption that management will attempt to maintain current age structure and escapement levels. The optimal policy for this case is presented in Table 3, final iteration.

There are three major differences between the optimal policy and current management practices: 1) The optimal policy closes the sport fishery during the winter. 2) The optimal policy opens the commercial fishery June 15 as opposed to the current practice of April 15 opening for chinook and July 1 opening for coho. 3) The optimal policy leaves the commercial

TABLE 3. Intermediate policies for computation of optimal policy under "current" conditions.

FIRST ITERATION

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Sport	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
Troll	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1

Average deviation from target populations - 17%

SECOND ITERATION

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Sport	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
Troll	0 0	0 0	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1

Average deviation from target population - .5%

THIRD ITERATION

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Sport	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
Troll	0 0	0 0	0 0	0 0	0 0	0 1	1 1	1 1	1 1	1 1	1 1	1 1

Average deviation from target population .71%

1 - fishery open
 0 - fishery closed

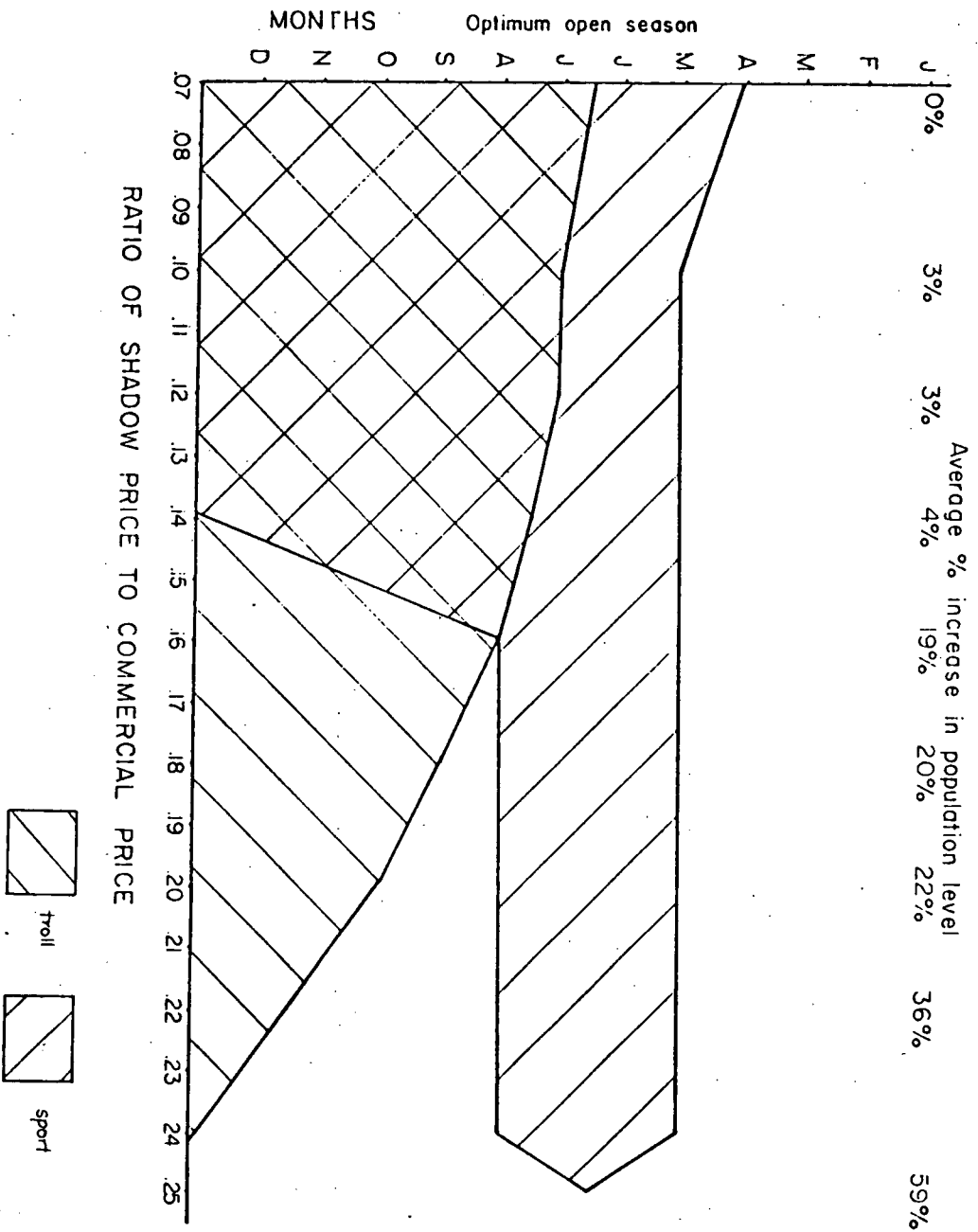
fishery open during November and December. In practice, the fishery closes in October. The major reason for these differences is the higher value of commercially caught fish during the latter months of the year. Both the rising price per pound (Fig. 27) and increased weight per fish, due to growth, contribute to a very high value per fish in the commercial fishery during the later months. Under these conditions, it is better to leave the fish in the water during the early months and to harvest them later when they are more valuable.

The optimal policy was compared to current seasons using the simulation model. Closing the winter sport fishery resulted in a 140,000 dollar decrease in sport benefits, which was offset by a 140,000 dollar increase in the commercial landed value. The net change in value using the optimal policy was insignificant.

5.3. Increased Escapement

Much of the impetus for the development of the Georgia Strait simulation model and this optimization exercise has come from concern over conservation of the fish stocks. This concern can be interpreted as an increased value of fish left in the water after the fishing season. A series of optimal policies were developed using increased values of the r parameter in Equation 14. Figure 30 shows the effect of increased value of a fish in the water at the end of the year on the optimal fishing seasons. As r is increased, the length of the seasons decrease. The sport fishery is confined to the summer months when it has

Figure 30: Optimal Seasons for Sport and Troll Fishing in the Georgia Strait with Respect to Increased Escapement and Increased Shadow Price.



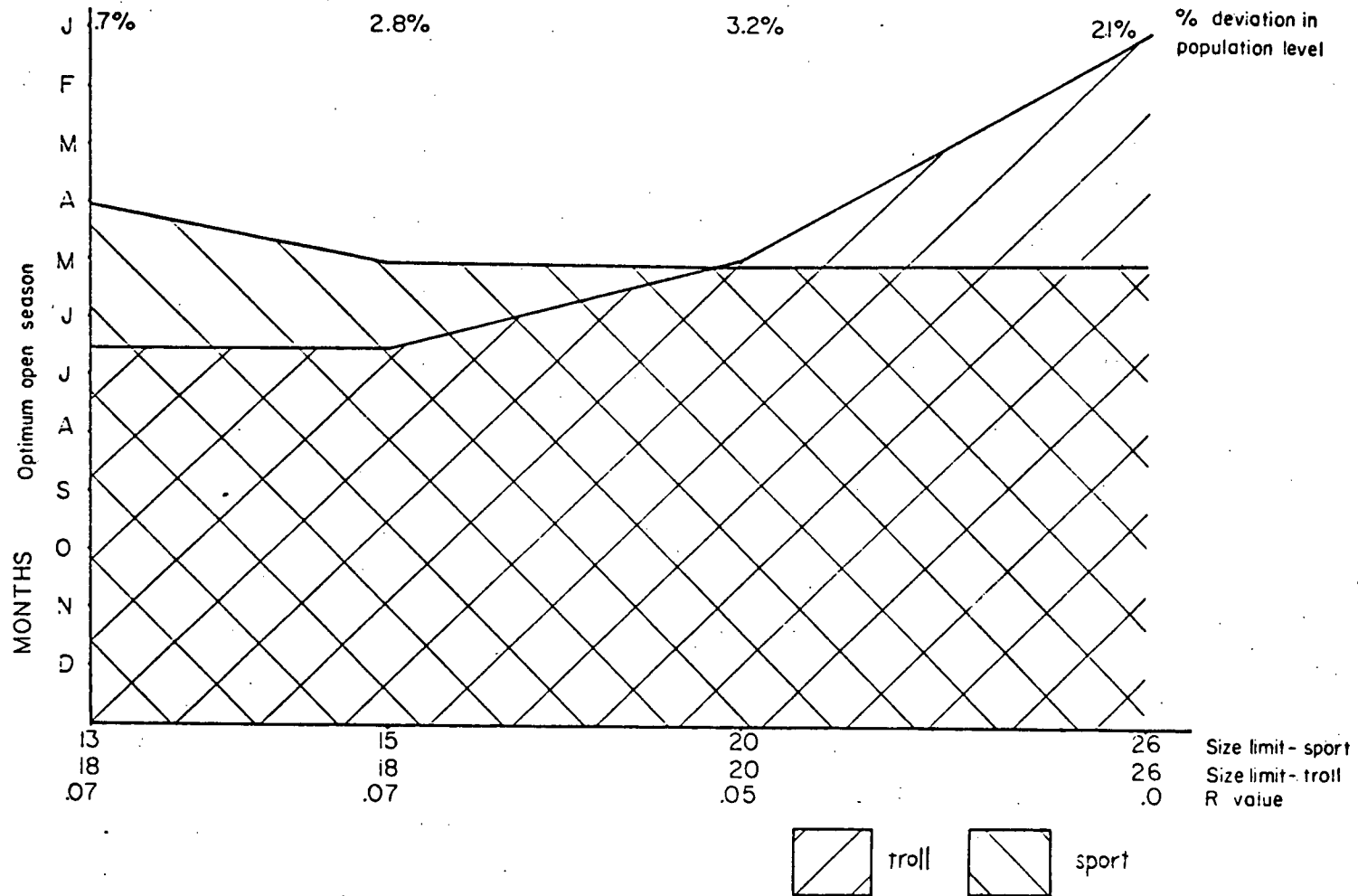
the highest value, and commercial fishing is delayed to the later months when the fish have the highest value. The abrupt change in the sport season where r is changed from 0.14 to 0.16 reflect the "square" nature of the sport fishery value curve (Fig. 28). The sport fishery value curve is surely smoother than the curve assumed in this model. However, its "real" shape is still uncertain.

5.4. Size Limits

Optimal fishing seasons were computed for various size limits. Figure 31 shows the resulting policies. Increased size limits had little effect upon the sport fishing season. The sport fishermen are assumed to respond to legal sized fish only. Therefore, the harvest of undersized fish contributes to the kill-through shaker mortality, but does not contribute to the value of the fishery. The results show that, for size limits at least up to 20 inches, it is never optimal to have a winter sport fishery. Increased size limits, on both the sport and commercial fisheries, from present to 20 inches allowed an expansion of the commercial fishery to the same season as the sportsman's.

While increasing the size limit, it was found necessary to decrease the value of r so as not to overshoot target population levels. The results for the 26 inch size limit are puzzling. At 26 inches and a value of 0 for r , the optimal policy is to open the commercial fishery the year round, but not to include a winter sports fishery. This policy resulted in

Figure 31: Optimal Seasons for Sport and Troll Fishing in the Georgia Strait with Respect to Increased Size Limits.



an average of 21% too many fish in the water at the end of the year. Opening both fisheries all year round with a 26 inch size limit does not violate the target population. However, r must be negative for this policy to be optimal. Interpretation of the shadow price being negative is difficult. However, one could speculate that it is an artifact of the "current" age and species structure.

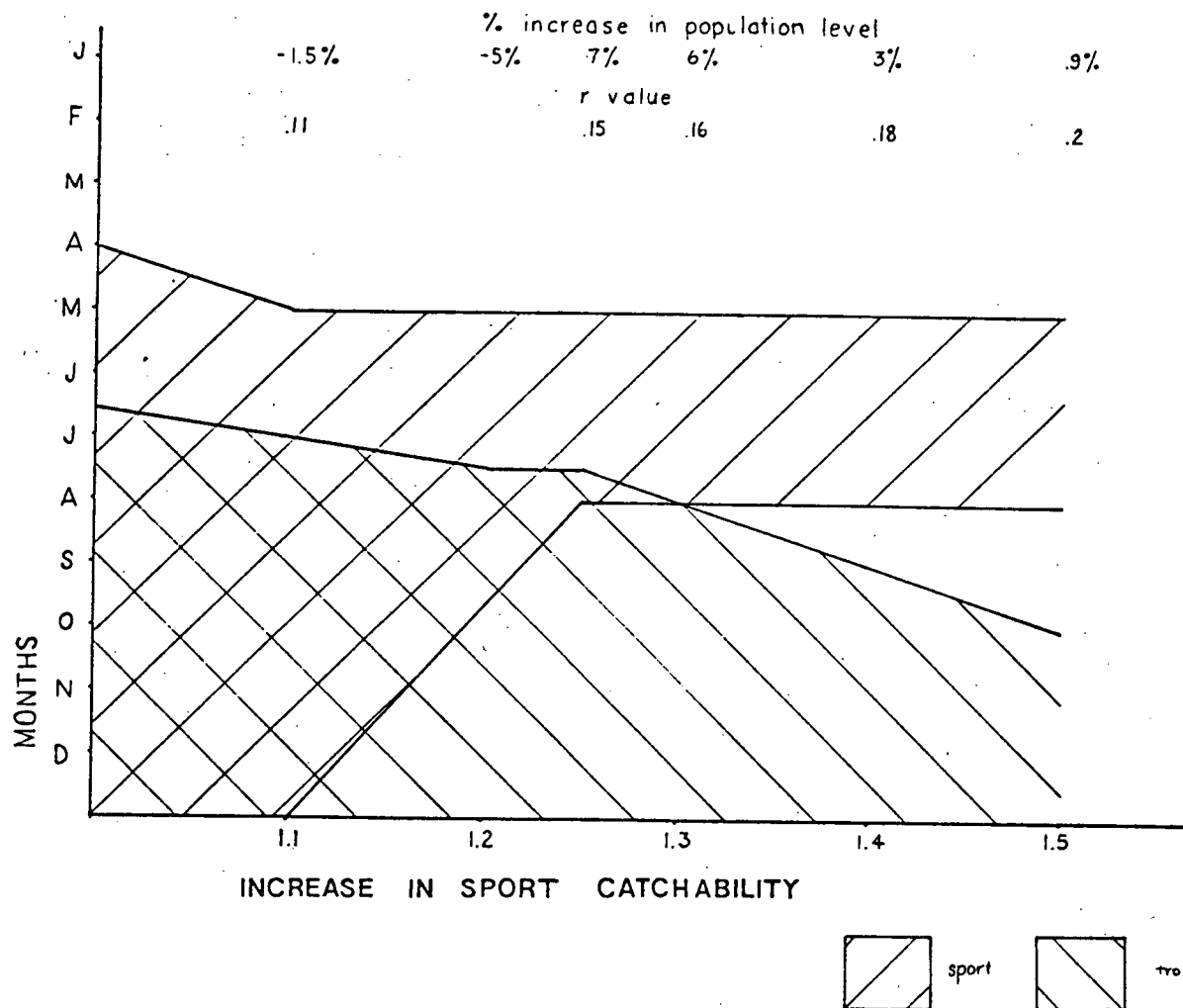
5.5. Increased Sport Efficiency

The catchability coefficient is the most uncertain parameter in the model. The uncertainty arises because catchability is computed from natural mortality rates, catch, effort, and escapement, all of which are prone to error. Catch and effort data for the commercial fishery is reasonably sound. However, sport fishing statistics are based upon small samples of the sport fishing fleet, and are believed to be badly biased (Argue Coursley and Harris 1977). Catchability also enters into the calculation of catch per unit effort, from which the sport effort response is predicted.

The efficiency of the sport fleet is increasing and will likely continue to do so. Therefore, optimal policies were computed for increased sport catchability to reflect both uncertainty and future increases in sport efficiency (Fig. 32).

As the sport catchability increases, the amount of fishing allowed in both fleets is reduced. Again, the sport season is confined to the summer and the commercial season to the end of the year. Increased sport catchability results in both an

Figure 32: Optimal Seasons for Sport and Troll Fishing in the Georgia Strait with Respect to Increased Efficiency in the Sport Fleet.



increase in the amount of effort and an increase in the impact of any unit of effort on the fish stocks. As sport efficiency increases, the value of a fish left in the water ($d(i)$) must also increase to provide for enough fish at the end of the season.

5.6. Enhancement

Enhancement was represented by increasing the initial abundance of the two age classes of coho (Tab. 4). The effect of doubling the number of coho on the optimal seasons was not dramatic, but the direction was significant. The optimal policies were to delay the opening of the sport season until the beginning of May and start the troll season the first of July. More severe restrictions in fishing were required in part to offset over harvest. The average departure from target conditions was 8% more fish than needed. All the chinook ages were overharvested. The second and fifth year chinook were the most severely overfished (15% and 16% respectively). Overfishing of the chinook was compensated by 96% more coho present than necessary at the end of the year. Exactly this type of result is now occurring in the Georgia Strait, with large excess escapement of coho to hatcheries such as Capilano. Judging from public reaction to government decisions to sell these excess fish, it might be a good idea to model the terminal cost term G (equation 7) to reflect a penalty on high as well as low deviations from desired terminal stock sizes.

Table 4. Optimal season with enhanced coho

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Sport	0 0	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
Troll	0 0	0 0	0 0	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1

Deviation from target populations

Coho	Chinook	Age 1	Age 2	Age 3	Age 4	Age 5
96%		-.4%	-15%	-1%	-2%	-16%

Average deviation from target population - 8%

This result may have serious implications for enhancement. Enhanced fish, by stimulating fishing effort, may cause the overexploitation of unenhanced stocks. Restrictions on fishing may need to be more severe. Also, a poor decision regarding the opening or closing of any fishery may, by stimulating large amounts of fishing effort, be very costly in terms of overexploitation of unenhanced stocks.

6. DISCUSSION AND CONCLUSION

Conclusions can be drawn in three areas: the management of this particular fishery, the computational scheme proposed, and the importance of optimization in resource management.

6.1. Georgia Strait Fishery

One of the main purposes for building simple models and determining optimal policies under a simple set of assumptions is to see how sensitive these models and policies are to parts of the system about which there are large uncertainties. In the model presented here, there is uncertainty about all the parameters; there is also very little known about the abundance of fish in pre-spawning age classes. On examination, there are two components which enter the model quadratically: the catchability coefficient and the stock abundance. In the single stock case:

$$\text{rate of catch} = (a + bq_x)q_x$$

Errors in estimates of the q 's will result in a square effect on predicted catch. Errors in assessing the abundance of a cohort will also result in large errors in predicting catch from all cohorts. Optimal policies are in turn very sensitive to the catch. The amount of time that the fisheries are closed reflects the potential for the catch to violate target escapements. Therefore, the policies of closures are very

sensitive to the unknown quantities catchability and abundance.

The major conclusion regarding the Georgia Strait fishery is that, under the assumed price schedule in both the sport and commercial fisheries, there were no cases tested where a winter sport fishery is optimal. There is little confidence in the particular price schedule used in this thesis. However, the general trend of sport fishery being more valuable in the summer than in the winter is likely a good assumption. The analysis shows that fish should be left in the water in the winter so they can be harvested during the more valuable summer sports fishery. The analysis also shows that season closure can be used as a tool to insure escapement from the Georgia Strait fishery. Dramatic increases in the number of fish left in the water after the fishing season best obtained with a summer sport fishery and a fall commercial fishery.

Allen (1954) discusses the use of size limits as a fisheries regulation. "Size limits may be used either to maintain a sufficient breeding stock, or to promote the maximum catch of the desired kind." Maximum catch can take on three different forms:

- (1) The maximum total numbers of fish, independent of size;
- (2) The maximum number of large fish; and
- (3) The maximum total weight.

In this analysis, the sport fishery is assumed to operate under the first objective (maximum number). In such a case, Allen concludes "... no size limit should be applied so that anglers can be allowed to take as many fish as possible before they die from natural causes". Size limits in the Georgia Strait fisheries have been defended upon the basis of maintaining breeding stocks.

It is true that one of the reasons for size limits in the fishery is conservation. However, the commercial fishery is operated with an objective resembling (3) above. Size limits in both fisheries are in fact mechanisms for distributing benefits between the two users of the resource. For example, a larger size limit in the commercial than in the sport fishery allows more smaller fish for two purposes. One purpose is to allow the small fish to grow bigger and more valuable for the commercial fisherman. The other purpose is to make available to the sportsmen more of the smaller and more abundant fish. The analysis shows size limits can be used as instruments to increase escapement. However, the current policies of size limits and seasons appear to be aimed at distributing the harvest and not at preserving the stocks.

The computed optimum policy does not differ greatly from the present practiced policy in either appearance or performance (Table 5). Earlier results indicated that to match current conditions one must maximize present within season benefits and attribute near zero value to stocks left in the water. The current policies and their resulting allocation between commercial and sport fisheries, have evolved due to

Table 5. A comparison of the optimal policy with a variety of other policies.

Policy	Chinook Escapement	Coho Escapement	Troll landed Value	Sport landed Value
(1) Present seasons	34,879	124,861	\$2,300,000	\$12,640,000
(2) Optimal seasons	36,577	124,660	\$2,440,000	\$12,500,000
(3) Optimal seasons with 20" size limit both species, both fisheries	49,572	177,390	\$2,900,000	\$ 8,180,000
(4) Present seasons 20" sport chinook size limit. Troll effort con- stant at 40% present maximum	50,000	149,000	\$1,400,000	\$12,000,000

political and economic pressures from both groups. The relative value of sport versus commercial fisheries used in this thesis is likely a manifestation of these pressures. By submitting to these pressures the management has put forward an objective of maximizing present benefits rather than conserving for the future.

The focus of this thesis has been the within-season management of the Georgia Strait fishery. A technique for the development of seasonal management plans designed to meet annual goals has been presented. The long term goals and consequence of policies have not been the subject of this analysis. The time is right for a look at the fishery, its objectives, and dynamics from a long time perspective.

6.2. The Methodology

The evaluation of policy design methods can be approached from several points of view. The predictions and policies generated from any form of analysis must be intuitively clear, for if there is no reasonable and intuitive path by which one can reach similar conclusions, then great doubt should be placed upon the computed answer. Conversely, reasonable results from most techniques of policy design could, in hindsight, have been developed without the mathematical and computational trappings. The computer becomes necessary when the arithmetic becomes too cumbersome for pencil and paper. Furthermore, it is unlikely that the human alone can find the right combination and sequence of steps in a finite period of time to arrive at a

correct and reasonable solution to a management problem.

A promising approach to policy design and analysis is simulation modeling. Modeling allows the synthesis of data and known processes with the not so well known and guessed at processes. The known and the uncertain are glued together to form a dynamic representation of the "real" system. A laboratory world is created into which one can make management interventions and observe predicted results. Alternate policies can be evaluated for their performance in the model world without risks of damaging the real world. There are several ways to use a simulation model for policy design. One method involves the use of the simulation model to exhaustively search for best control policies (Peterman 1975, 1977). Others use more formal optimization techniques on simpler models and then apply the control policies to the larger models (Winkler 1975, Holling and Dantzig 1978). The method presented in this thesis preserves all of the components of the simulation with the exception of a longer term perspective, and even long term effects are partially accounted for through the specification of desired terminal stock sizes. A formal optimization technique is used on the complex and detailed simulation model. This fact makes the described methodology a most powerful optimization procedure for dealing with large scale, complex, and multidimensional models.

The Georgia Strait problem was a good test bed for the procedure. The computation was very efficient, requiring little more computing resources than the simulation model. The computational requirements are only proportional, not

geometrical to the dimensionality of the model like methods such as dynamic programming (Walters 1975). It is hoped that the described method will prove useful on other resource management problems.

6.3. Optimization and Resource Management

The final area of discussion relates to the propriety of optimization in resource management problems. Modeling of resource systems has been of great help in pointing to uncertainties and gaps in our understanding of behavior (Holling et al., 1978). Models, however, are only simplified characterizations of how we believe the world operates. A great deal of caution must be used when extrapolating the model to the real world. Optimization as an extension of the modeling process allows us to judge the importance of uncertainties and of ignorance with respect to the way in which we value the world (Walters and Hilborn 1978). For example, the sensitivity of the optimal policy of season closures to uncertainties about harvest efficiency and fish abundance suggests that increased understanding of harvesting and pre-spawner abundance may increase the benefits of the Georgia Strait salmon resources.

Another function of optimization is to identify uncertainties about how we value the outputs of complex systems. The optimization of a simple model under a particular objective shows us how we should behave if we value the world in a particular way. If the optimal policy generated under one objective function is unacceptable, then the optimization

exercise has demonstrated a discrepancy between the explicit statement of our value system and the way in which we value the world. Such discrepancies can arise through uncertainty about the way benefits flow from a resource or by an omission of some critical component of the value of the resource. For example, one major concern to the managers of the Georgia Strait fisheries is the allocation of catch between the commercial and the sport fisheries. A critical assumption has been that a fish caught by a sportsman is always more valuable than a fish caught by the commercial fishery. Past policies have always suggested the complete elimination of the commercial fishery before any control of the sportsman. Many people would argue, particularly the fishermen's union, that the sport fishermen should accept some of the burden of conservation and that elimination of the commercial trawler is unacceptable. Thus, a discrepancy between the stated value system and a real value system has been pinpointed for rational discussion. Disparities may also arise from additional amenity or utility, over and above the landed value of the fish. These values may be associated with employment or tradition, and predominate the benefits at small catch levels.

Optimization can be used in a similar way to models. Modeling may help to clearly define areas of uncertainty about the behavior of a resource system. Optimization then helps to define uncertainties and conflicts about how the benefits of resource systems are perceived.

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APPENDIX 1

The results and predictions from the large simulation model are numerous and varied. The presentation of the predictions has taken on two forms. Nomograms (Peterman, 1977) have been used extensively to allow managers to observe the effects of various combinations of control actions. The other form of presentation has been to examine in detail a variety of specific management actions in a tabular form. Tables 4 through 7 illustrate model predictions under extreme assumptions of shaker mortality and sport effort response. Within the tables a variety of actions are pursued to test the model under extreme management actions and to determine the affect of some admissible regulations other than sport season closures. The tables present a set of indicators which are thought to be important and of interest to the people involved in decision making.

Escapement of spawners is thought to be of utmost importance at present. One of the managers' objectives was to find a policy that would double escapement levels, or at least return them to historical levels.

Indicators of the commercial fishery are catch, effort, CPUE and a variety of attributes of monetary value. Similar indicators are presented for the sport fishery. Shaker mortality and the average weight of chinook in both fisheries are listed.

The first management action on the tables represents the model predictions under current regulations. It is not intended to be an accurate account, but, to form the basis from which to evaluate departures. The next three actions represent extreme regulations (rows 1-3). Note that the objective of doubled escapement would require complete elimination of the sport fishery. This prediction suggests that the objective was unreasonable. Actions four and five are increased size limits on commercially caught salmon. Action six is meant to emulate a restriction of movement of commercial trollers inside and outside Vancouver Island. It is assumed that forty percent of the observed maximum effort is from boats which would choose to operate exclusively in Georgia Strait and that the other sixty percent would fish outside and be excluded from the inside fishery. Actions eight through ten are increases in the size limit of sport caught fish. Action eleven is an increase in the size limit of chinook for part of the season. Actions twelve through fourteen are combinations of the other actions. Finally action fifteen simulates a one chinook per day bag limit in the sport fishery.

Table 6: Predictions from the Georgia Strait Simulation Model
Under the Assumptions of 50 Percent Shaker mortality in the
Troll Fishery, 80 Percent Shaker mortality in the Sport
Fishery and Sport Effort Response.

MANAGEMENT ACTIONS	INDICATORS																			
	Escapement		Commercial Troll								Sport				Shaker Mortality			Average Weights		
	Chinook Escapement	Coho Escapement	Total Troll Catch	Chinook Troll Catch	Coho Troll Catch	Troll Effort	Troll CUPE	Troll Net Value	Troll Land Value	Troll Value Boat-hwy	Total Sport Catch	Chinook Sport Catch	Coho Sport Catch	Sport Effort	Sport CUPE	Sport Value	Chinook Shaker Mortality	Coho Shaker Mortality	Average Weight Troll Chinook	Average Weight Sport Chinook
	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$		$\times 10^6$	$\times 10^6$		$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$		$\times 10^6$	$\times 10^3$	$\times 10^3$		
(0) Present	34.	130.	252.	159.	93.	19.	13.	2.2	2.3	123.	753.	308.	445.	679.	1.11	13.0	456.	589.	7.45	5.45
(1) No sport fishery	73.	321.	450.	294.	156.	22.	21.	4.4	4.6	211.	0.	0.	0.	0.	1.78	0.0	241.	121.	8.26	6.14
(2) No troll fishery	52.	142.	0.	0.	0.	0.	0.	0.0	0.0	0.	914.	405.	510.	747.	1.22	14.3	664.	556.	0.0	6.30
(3) No sport fishery No troll fishery	143.	394.	0.	0.	0.	0.	0.	0.0	0.0	0.	0.	0.	0.	0.	2.29	0.0	0.	0.	0.0	7.60
(4) Troll 24" chinook	39.	130.	171.	95.	86.	17.	10.	2.0	1.9	113.	784.	331.	452.	693.	1.13	13.3	1126.	652.	10.90	5.68
(5) Troll 26" chinook	40.	129.	147.	63.	84.	16.	9.	1.7	1.6	105.	795.	340.	455.	698.	1.14	13.4	1152.	761.	12.39	5.77
(6) Troll effort constant at 40% of present max	42.	135.	131.	84.	47.	9.	14.	1.2	1.3	134.	832.	354.	478.	714.	1.17	13.6	910.	573.	7.67	5.67
(7) Troll June 1 chinook July 1 coho	42.	126.	178.	85.	93.	13.	14.	1.4	1.5	121.	805.	351.	453.	703.	1.14	13.5	917.	571.	7.65	5.64
(8) Sport 20" both species	46.	181.	321.	206.	115.	20.	16.	2.9	1.1	152.	332.	135.	158.	445.	0.75	8.4	1539.	556.	7.74	8.60
(9) Sport 20" chinook	39.	144.	282.	182.	100.	13.	15.	2.5	2.6	136.	575.	164.	411.	592.	0.97	11.3	1475.	823.	7.58	8.72
(10) Sport 24" chinook	42.	153.	296.	193.	103.	20.	15.	2.7	2.8	143.	452.	101.	332.	549.	0.90	10.5	1648.	936.	7.65	11.53
(11) Sport 24" chinook Oct 1 to June 1	37.	133.	273.	176.	57.	15.	14.	2.4	2.5	132.	677.	241.	435.	642.	1.05	12.3	1105.	662.	7.55	6.33
(12) Sport 20" chinook Troll 26" chinook	46.	143.	166.	74.	51.	16.	10.	2.1	1.9	118.	612.	150.	422.	612.	1.00	11.7	1740.	967.	12.52	9.03
(13) Sport 20" chinook Troll same 6	50.	149.	143.	54.	49.	9.	15.	1.3	1.4	150.	652.	204.	448.	630.	1.03	12.0	1481.	832.	7.83	9.17
(14) Sport 20" chinook Troll same as 7	49.	139.	197.	57.	100.	13.	15.	1.6	1.7	134.	623.	202.	421.	618.	1.01	11.9	1476.	825.	7.62	9.22
(15) Sport 1 chinook per day bag-limit	37.	138.	268.	172.	57.	19.	14.	2.4	2.5	130.	648.	221.	426.	630.	1.03	12.0	1246.	734.	7.55	5.60

Table 7: Predictions from the Georgia Strait Simulation Model
Under the assumptions of 30 Percent Shaker mortality in the
Troll Fishery, 30 Percent Shaker mortality in the Sport
Fishery and Sport Effort Response.

MANAGEMENT ACTIONS	INDICATORS													
	Commercial Troll							Sport						
	Escapement													Average Weight
	Chinook Escapement	Coho Escapement	Total Troll Catch	Chinook Troll Catch	Coho Troll Catch	Troll Effort	Troll CUPE	Troll Net Value	Troll Land Value	Troll Value Boat-Day	Total Sport Catch	Chinook Sport Catch	Coho Sport Catch	Sport Effort
	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^6$	$\times 10^6$	$\times 10^6$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$
(0) Present	35. 132. 265. 169. 57. 19. 14. 2.3 2.4 127. 805. 331. 474. 702. 1.15 13.5 417. 232. 7.43 5.43													
(1) No sport fishery	34. 122. 457. 300. 156. 22. 21. 4.5 4.6 210. 0. 0. 0. 0. 1.33 0.0 145. 73. 6.27 6.17													
(2) No troll fishery	33. 144. 0. 0. 0. 0. 0. 0.0 0.0 0. 576. 412. 545. 773. 1.26 14.8 340. 219. 0.0 6.26													
(3) No sport fishery No troll fishery	33. 394. 0. 0. 0. 0. 0. 0.0 0.0 0. 0. 0. 0. 0. 2.20 0.0 0. 0. 0.0 7.60													
(4) Troll 24" chinook	41. 131. 182. 52. 50. 17. 11. 2.1 2.0 115. 246. 362. 484. 720. 1.17 13.3 526. 311. 10.89 5.72													
(5) Troll 26" chinook	43. 131. 156. 64. 08. 16. 10. 1.9 1.8 111. 559. 372. 487. 726. 1.18 13.9 547. 323. 12.39 5.60													
(6) Troll effort constant at 40% of present max	44. 127. 135. 67. 48. 5. 14. 1.2 1.3 139. 681. 380. 511. 733. 1.21 14.1 374. 235. 7.65 5.86													
(7) Troll June 1 chinook July 1 coho	44. 128. 186. 89. 57. 13. 14. 1.5 1.6 125. 661. 377. 483. 727. 1.19 14.0 383. 230. 7.62 5.81													
(8) Sport 20" both species	49. 204. 354. 229. 125. 21. 17. 3.2 3.4 105. 335. 156. 228. 478. 0.81 9.0 715. 425. 7.75 6.88													
(9) Sport 20" chinook	44. 144. 314. 210. 104. 20. 16. 2.6 3.0 149. 632. 192. 440. 621. 1.02 11.9 654. 360. 7.60 8.72													
(10) Sport 24" chinook	49. 151. 338. 231. 107. 20. 17. 3.1 1.3 162. 542. 124. 418. 577. 0.94 11.1 738. 401. 7.72 11.97													
(11) Sport 24" chinook Oct 1 to June 1	39. 134. 293. 192. 101. 20. 15. 2.6 2.6 140. 730. 265. 464. 667. 1.09 12.8 485. 285. 7.57 6.36													
(12) Sport 20" chinook Troll 26" chinook	54. 142. 185. 90. 53. 17. 11. 2.4 2.2 132. 655. 231. 454. 648. 1.06 12.4 836. 460. 12.54 9.59													
(13) Sport 20" chinook Troll same as 7	54. 149. 156. 105. 50. 9. 17. 1.5 1.6 166. 724. 243. 432. 665. 1.09 12.7 635. 354. 7.85 9.20													
(14) Sport 20" chinook Troll same as 7	55. 139. 214. 110. 104. 11. 16. 1.8 1.9 146. 692. 280. 452. 652. 1.06 12.5 641. 355. 7.85 9.24													
(15) Sport 1 chinook per day bag-limit	42. 110. 292. 191. 101. 20. 15. 2.6 2.0 141. 703. 248. 455. 657. 1.07 12.6 547. 317. 7.62 5.72													

Table 8: Predictions from the Georgia Strait Simulation Model
Under the Assumptions of 50 Percent Shaker mortality in the
Troll Fishery, 80 Percent Shaker mortality in the Sport
Fishery and Fixed Sport Effort Pattern.

INDICATORS

Escapement

Commercial Troll

Sport

Shaker
Mortality

Average
Weights

Chinook Escapement

Coho Escapement

Total Troll Catch

Chinook Troll Catch

Coho Troll Catch

Troll Effort

Troll CPUE

Troll Net Value

Troll Land Value

Troll Value
Boat-Day

Total Sport Catch

Chinook Sport Catch

Coho Sport Catch

Sport Effort

Sport CPUE

Sport Value

Chinook Shaker
Mortality

Coho Shaker
Mortality

Average Weight
Troll Chinook

Average Weight
Sport Chinook

MANAGEMENT ACTIONS

50% shaker mortality troll
80% shaker mortality sport
fixed sport effort pattern

	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$		$\times 10^6$	$\times 10^6$		$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$		$\times 10^6$	$\times 10^3$	$\times 10^3$		
(0) Present	33.	122.	246.	156.	90.	19.	13.	2.1	2.2	120.	783.	317.	466.	729.	1.07	14.1	933.	604.	7.33	5.38
(1) No sport fishery	73.	321.	450.	294.	156.	22.	21.	4.4	4.6	211.	0.	0.	0.	0.	1.73	0.0	24.1	121.	8.26	6.14
(2) No troll fishery	54.	146.	0.	0.	0.	0.	0.	0.0	0.0	0.	500.	396.	504.	729.	1.24	14.1	813.	522.	0.0	6.38
(3) No sport fishery No troll fishery	143.	354.	0.	0.	0.	0.	0.	0.0	0.0	0.	0.	0.	0.	0.	2.20	0.0	0.	0.	0.0	7.63
(4) Troll 24" chinook	38.	123.	168.	84.	84.	16.	10.	2.0	1.8	112.	805.	337.	468.	729.	1.11	14.1	1135.	666.	10.89	5.64
(5) Troll 26" chinook	40.	124.	145.	63.	82.	16.	9.	1.7	1.6	105.	813.	344.	469.	729.	1.12	14.1	1163.	704.	12.33	5.74
(6) Troll effort constant at 40% of present max	42.	133.	131.	85.	46.	9.	14.	1.2	1.3	134.	840.	354.	485.	729.	1.15	14.1	899.	564.	7.69	5.89
(7) Troll June 1 chinook July 1 coho	42.	121.	176.	65.	92.	13.	14.	1.4	1.5	120.	820.	354.	465.	729.	1.12	14.1	924.	576.	7.66	5.91
(8) Sport 20" both species	34.	132.	257.	164.	93.	19.	14.	2.2	2.4	125.	448.	183.	266.	729.	0.62	14.1	2233.	1369.	7.45	6.51
(9) Sport 20" chinook	34.	122.	254.	164.	90.	19.	13.	2.2	2.3	124.	649.	183.	466.	729.	0.25	14.1	1714.	952.	7.46	6.51
(10) Sport 24" chinook	35.	122.	258.	168.	90.	19.	14.	2.3	2.4	126.	532.	116.	466.	729.	0.80	14.1	2052.	1124.	7.49	11.55
(11) Sport 24" chinook Oct 1 to June 1	34.	122.	250.	160.	90.	19.	13.	2.2	2.3	122.	724.	258.	466.	729.	0.99	14.1	1294.	758.	7.45	6.24
(12) Sport 20" chinook Troll 26" chinook	41.	124.	149.	66.	83.	16.	9.	1.8	1.7	108.	676.	207.	469.	729.	0.93	14.1	1924.	1066.	12.40	6.33
(13) Sport 20" chinook Troll same 6	44.	133.	134.	88.	46.	9.	14.	1.2	1.3	139.	703.	217.	486.	729.	0.97	14.1	1654.	923.	7.72	8.99
(14) Sport 20" chinook Troll same as 7	44.	121.	181.	99.	92.	11.	14.	1.5	1.6	124.	683.	218.	465.	729.	0.94	14.1	1676.	934.	7.69	9.01
(15) Sport 1 chinook per day bag-limit	34.	122.	250.	160.	90.	19.	13.	2.2	2.3	123.	708.	241.	466.	729.	0.97	14.1	1358.	759.	7.47	5.44

Table 9: Predictions from the Georgia Strait Simulation Model
Under the assumptions of 30 Percent Shaker mortality in the
Troll Fishery, 30 Percent Shaker mortality in the
Fishery and fixed Sport Effort Pattern.

	INDICATORS																			
	Escapement				Commercial Troll				Sport				Shaker Mortality		Average Weight					
	Chinook Escapement	Coho Escapement	Total Troll Catch	Chinook Troll Catch	Coho Troll Catch	Troll Effort	Troll CUPE	Troll Net Value	Troll Land Value	Troll Value Boat-Day	Total Sport Catch	Chinook Sport Catch	Coho Sport Catch	Sport Effort	Sport CUPE	Sport Value	Chinook Shaker Mortality	Coho Shaker Mortality	Average Weight Troll Chinook	Average Weight Sport Chinook
	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^6$	$\times 10^6$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$	$\times 10^3$
(0) Present	35. 127. 263. 158.	95. 15. 14. 2.3	2.4 127. 820. 334. 457. 729. 1.13 14.1 416. 252. 7.44 5.10																	
(1) No sport fishery	74. 122. 457. 300. 156. 22. 21. 4.5 4.6 214. 0. 0. 0. 1.43 0.0 156. 73. 8.27 6.17																			
(2) No troll fishery	56. 153. 0. 0. 0. 0. 0. 0.0 0.0 0. 940. 414. 526. 729. 1.29 14.1 310. 199. 0.0 6.40																			
(3) No sport fishery No troll fishery	143. 354. 0. 0. 0. 0. 0. 0.0 0.0 0. 0. 0. 0. 0. 0. 2.23 0.0 0. 0. 0.0 7.60																			
(4) Troll 24" chinook	41. 129. 193. 93. 70. 17. 11. 2.2 2.0 120. 848. 359. 489. 729. 1.16 14.1 522. 308. 10.91 5.70																			
(5) Troll 26" chinook	43. 129. 156. 70. 08. 16. 10. 1.9 1.8 1.3. 857. 368. 490. 729. 1.18 14.1 541. 319. 12.42 5.81																			
(6) Troll effort constant at 10% of present max	45. 139. 137. 89. 46. 5. 15. 1.3 1.3 142. 650. 372. 508. 729. 1.21 14.1 365. 226. 7.89 5.51																			
(7) Troll June 1 chinook July 1 coho	44. 127. 188. 91. 97. 13. 15. 1.5 1.6 126. 858. 372. 486. 729. 1.19 14.1 382. 234. 7.66 5.53																			
(8) Sport 20" both species	40. 147. 310. 200. 109. 20. 16. 2.7 2.9 145. 515. 211. 304. 749. 0.71 14.1 67. 575. 7.51 6.16																			
(9) Sport 20" chinook	40. 127. 296. 199. 96. 20. 15. 2.6 2.8 142. 698. 212. 486. 729. 0.56 14.1 729. 400. 7.51 6.57																			
(10) Sport 24" chinook	45. 127. 314. 216. 97. 20. 16. 2.9 3.0 151. 630. 144. 485. 729. 0.86 14.1 695. 480. 7.62 11.65																			
(11) Sport 24" chinook Oct 1 to June 1	37. 127. 278. 183. 96. 19. 14. 2.4 2.6 134. 767. 281. 487. 729. 1.05 14.1 549. 316. 7.50 6.33																			
(12) Sport 20" chinook Troll 26" chinook	51. 129. 175. 66. 45. 17. 11. 2.3 2.1 127. 736. 247. 489. 729. 1.01 14.1 865. 485. 12.47 6.55																			
(13) Sport 20" chinook Troll same 6	51. 139. 151. 103. 46. 9. 16. 1.4 1.5 160. 762. 254. 508. 729. 1.05 14.1 679. 376. 7.79 9.09																			
(14) Sport 20" chinook Troll same as 7	52. 127. 204. 166. 95. 13. 14. 1.7 1.8 140. 740. 254. 489. 729. 1.01 14.1 697. 384. 7.76 9.13																			
(15) Sport 1 chinook per day bag-limit	37. 127. 280. 184. 96. 19. 14. 2.5 2.6 136. 751 264. 486. 729. 1.03 14.1 577. 334. 7.57 5.61																			

APPENDIX 2

The Lagrange system in one dimension:

$$\text{Max}_{Y, U} \sum_{k=1}^N R(y_k, u_k) + G_{N+1}$$

Subject to the constraint of the state dynamics equation:

$$y_{k+1} = y_k + F(y_k, u_k)$$

Maximize the "Lagrangion" with respect to $Y(y_k: k=1 \dots N+1)$ and $U(u_k: k=1 \dots N+1)$.

$$\text{Max}_{Y, U} L = \sum_{k=1}^N [R(y_k, u_k) - Z_k (y_{k+1} - y_k - F(y_k, u_k))] + G_{N+1}$$

Maximize over Y :

$$\frac{dL}{dy_k} = dR/dy_k + Z_k + Z_k dF/dy_k - Z_{k-1}$$

$$dG/dy_{N+1} - Z_N$$

Set equal to zero:

$$Z_{k-1} = Z_k + dR/dy_k + Z_k dF/dy_k$$

$$Z_N = dG/dy_{N+1}$$

Maximize over U :

R and F are assumed linear in U therefore

$$u_k = U_{\max}(1) \text{ or } U_{\min}(0)$$

The Lagrangian system has three sets of simultaneous equations:

$$y_{k+1} = y_k + F(y_k, u_k) \quad k = 1 \dots N$$

$$y_1 = y^*$$

$$Z_{k-1} = Z_k + dR/dy_k + Z_k dF/dy_k \quad k = 1 \dots N$$

$$Z_N = dG/dy_{N+1}$$

$$u_k = U_{\max} \text{ or } U_{\min} \quad k = 1 \dots N$$