AEROELASTIC GALLOPING OF TALL STRUCTURES IN SIMULATED WINDS

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ABSTRACT

This thesis studies the effects of model aspect ratio on the static forces and galloping vibrations of bluff shapes when exposed to a turbulent boundary layer similar to the atmosphere. Previous investigations have analyzed the galloping oscillations of finite prismatic bodies exposed to a turbulent shear flow on the basis of the quasi-steady theory and the assumption of an average lateral force. Herein consideration is given to the variation of lateral forces with height and the galloping oscillations of two finite square towers are predicted.

The turbulent boundary layer was grown over a long fetch of roughness and at the location of the static and dynamic tests was 28" deep and had properties similar to a suburban or forested full scale exposure. The geometric scale of the models found from an analysis of velocity spectra was about 1/500.

For the height to width ratios examined, aspect ratio had little effect on the average static forces for small angles of attack. The local static forces, measured from the pressure distribution, had a wide variation over the height of the model. For the finite sections examined the response predicted from the local forces gave higher amplitudes for the same reduced velocity as compared to the response found from the average forces. The results of the dynamic tests agreed with the galloping response predicted from the local sectional forces indicating that the three-dimensional effects are important in the consideration of the galloping phenomenon.

The measurements of velocity spectra in the wake of the rigid
28" model indicate that the Strouhal shedding frequency varies along the span of the model. Similar velocity spectra behind the galloping 28" model did not exhibit a discernible peak at the stationary value of the Strouhal number.
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<td>a</td>
<td>tip amplitude of the model</td>
</tr>
<tr>
<td>( \bar{a} )</td>
<td>dimensionless amplitude = ( a/h )</td>
</tr>
<tr>
<td>a*</td>
<td>dimensionless amplitude = ( \bar{a} \eta/\beta )</td>
</tr>
<tr>
<td>( \bar{A}_i )</td>
<td>average aerodynamic constants</td>
</tr>
<tr>
<td>( A_{i,j} )</td>
<td>matrix of aerodynamic constants incorporating local changes</td>
</tr>
<tr>
<td>b</td>
<td>longitudinal dimension of the model</td>
</tr>
<tr>
<td>B_i</td>
<td>numerical coefficients</td>
</tr>
<tr>
<td>c</td>
<td>viscous structural damping</td>
</tr>
<tr>
<td>( C_i )</td>
<td>function involving the velocity profile and mode shape of the structure</td>
</tr>
<tr>
<td>( \bar{C}_D )</td>
<td>average drag coefficient</td>
</tr>
<tr>
<td>( C_{fy} )</td>
<td>local lateral force coefficient</td>
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<tr>
<td>( \bar{C}_F )</td>
<td>average lateral force coefficient</td>
</tr>
<tr>
<td>( \bar{C}_L )</td>
<td>average lift coefficient</td>
</tr>
<tr>
<td>( C_{PU} )</td>
<td>pressure coefficient on the upper surface of the model</td>
</tr>
<tr>
<td>( C_{PL} )</td>
<td>pressure coefficient on the lower surface of the model</td>
</tr>
<tr>
<td>d</td>
<td>distance from the base of the model to the point of rotation</td>
</tr>
<tr>
<td>D</td>
<td>total drag force</td>
</tr>
<tr>
<td>( D_{i,j} )</td>
<td>function involving the velocity profile, mode shape and aerodynamic constants ( A_{i,j} )</td>
</tr>
<tr>
<td>f</td>
<td>natural frequency of the model</td>
</tr>
<tr>
<td>f_y</td>
<td>local lateral force</td>
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<tr>
<td>F_y</td>
<td>total lateral force</td>
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<tr>
<td>h</td>
<td>lateral dimension of the model</td>
</tr>
<tr>
<td>I</td>
<td>inertia of the rotating assembly about the axis of rotation</td>
</tr>
<tr>
<td>k</td>
<td>spring stiffness</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>length of the model</td>
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<tr>
<td>L</td>
<td>total lift force</td>
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<tr>
<td>( x_{LV} )</td>
<td>turbulent length scale of the longitudinal velocity component</td>
</tr>
<tr>
<td>n</td>
<td>frequency of velocity fluctuation</td>
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P_L - pressure on the lower surface of the model
P_U - pressure on the upper surface of the model
Re - Reynolds number
S - Strouhal number = \( \frac{nh}{V} \)
S(n) - power spectral density of the longitudinal velocity component
t - time
U_\xi - reduced velocity at the height of the model = \( \frac{V_\xi}{\omega_n h} \)
U - local mean velocity
V - mean velocity at the height of the model
V_{rel} - relative mean velocity seen by the vibrating model
v(z) - function describing the velocity variation with height
v' - total RMS velocity fluctuation of the longitudinal velocity component
W - total work done by the damping forces
x - along-wind direction
y - cross-wind direction
\dot{y} - velocity of vibrating model
y_n - mode shape of the structure
z - vertical direction
z_s - distance from the point of rotation to the point of attachment of the springs

\alpha - angle of attack
\beta - fraction of critical damping = \( \frac{C}{C_{cr}} \)
\delta - boundary layer thickness
\gamma - power law exponent
\eta - mass parameter for a square section = \( \rho/4 \rho_m \)
\theta - angular rotation of the model
\rho - density of the fluid
\rho_m - density of the model
\omega_n - natural circular frequency
The author wishes to thank Dr. G.V. Parkinson whose amiable attitude made this work a pleasant experience.

Sincere thanks must go to Bob Strachan whose expertise in computer science made the acquisition of large amounts of data a bearable experience. The author is most grateful to the numerous graduate students who assisted in this work.
CHAPTER 1

INTRODUCTION

1.1 Background An elastically mounted structure may vibrate when exposed to a fluid flow. The causes of such motion may be random buffeting by turbulence or they can be coherent instabilities arising from the interaction between the structure and the wind. The latter instabilities are usually oscillatory and are caused by the separating shear layers from an aerodynamically bluff shape. One such instability results from the two separated shear layers which are unstable and roll up to form discrete vortices which result in an oscillatory pressure loading on the afterbody of the structure. When this periodic loading occurs at a frequency close to the natural frequency of the structure a resonant vibration can occur. The resonant vibrations are termed vortex-induced and are characterized by displacements of the order of the width of the structure. They can only occur over a discrete wind speed range defined by the Strouhal number.

A second class of oscillatory instabilities is termed galloping. Galloping is typically a low-frequency high amplitude motion in a single uncoupled mode of vibration in a plane perpendicular to the wind direction. Self-excited oscillations of the galloping type are caused by the aerodynamic instability of the cross-section of the body so that the motion generates forces which increase the initial amplitudes. A continuous increase in steady-state amplitudes with increasing wind speed is characteristic of a galloping phenomenon.
There have been numerous studies made of galloping instability. Smith (1) investigated extensively the galloping mechanism of a two-dimensional square prism in smooth flow. Using Parkinson's (2) quasi-steady assumption of forces excellent agreement was found between experiment and theory for the square section. Later Santosham (3) under similar test conditions to Smith's showed that the same quasi-steady approach could be applied to the 2/1 rectangle under the condition that the onset velocity for galloping is much higher than the velocity at which vortex-induced resonance occurs. Laneville (4) investigated the effects of turbulence intensity and scale on the nature of galloping oscillations. This study of two-dimensional rectangular cylinders shows the quite surprising result that an increasing turbulence intensity can completely change the stability characteristics of a section. Higher levels of turbulence made those sections which behave as soft oscillators in smooth flow more stable in a turbulent stream. An opposite trend was observed for those sections which are stable at rest in smooth flow, i.e. they became more unstable with an increased level of turbulence. The scale of the turbulence, within the range tested, showed no marked influence on the stability characteristics.

Novak in a series of papers (5,6,7,8,9) has examined the galloping oscillations of long prismatic bodies, typical of a tall structure, when exposed to atmospheric and grid-generated turbulence. Novak (5) first extended the quasi-steady approach to continuous elastic systems, exposed to a turbulent shear flow, on the basis of an energy consideration. Later studies (6) investigated the effects of turbulence
on the general character of galloping oscillations. The conclusion was that turbulence generally reduces the amplitudes of oscillation but has no severe effect on the onset of the oscillation for a square section. Other studies (7, 8) have shown that turbulence can change the stability characteristics of prismatic bodies, and that galloping oscillations can arise with sections which formally do not obey Den Hartog's stability criterion. Novak also proposed a universal response curve which would permit the prediction of galloping characteristics from a single dynamic test of a particular bluff body.

1.2 Purpose Investigations into the aeroelastic galloping of structures is important because strong lateral self-excited oscillations can develop at a certain wind speed as a result of the lateral force component. The onset velocity is usually high but the constantly decreasing specific weight, damping and stiffness of tall buildings, typical of modern practice, enhance the possibility of this aeroelastic instability. The tendency to galloping at velocities lower than the onset velocity produces a negative damping which reduces the inherent positive structural damping and results in an increased response to lateral wind gusts.

The purpose of this study is to investigate the galloping behavior of a finite vertical structure of square cross-section which has mechanical properties similar to a tall building, and which is situated in a turbulent flow representative of the atmosphere. The effects of building aspect ratio (i.e. the ratio of height to width of the structure) on the galloping characteristics are to be examined also. Previously Novak (6, 7, 8) assumed an average force coefficient was
applicable and computed the galloping responses using the quasi-steady theory. Herein consideration is given to the fact that the lateral force coefficients are variable with height in a boundary layer and a comparison is made between the responses predicted by an "average" and a variable force coefficient. In addition some of the simpler aspects of the complex problems of flow around a three-dimensional bluff body are considered.
2.1 Quasi-steady assumption  

For the mathematical description of the oscillations due to the aerodynamic instability of bluff cylinders the quasi-steady approach is assumed to be valid. The quasi-steady theory assumes that the forces experienced by the vibrating cylinder are the same forces exerted on a static model which is at an angle of attack equal to the apparent angle of attack seen by the vibrating cylinder, as shown by Fig. 1. Under two-dimensional, smooth flow conditions the quasi-steady assumption leads to a weakly non-linear differential equation which can be solved for both the steady and transient amplitudes of vibration (2). In some practical considerations galloping can occur with finite vertical structures which are exposed to a sheared turbulent boundary layer flow. Novak (5, 6, 7) has examined such systems and using the quasi-steady average forces has solved for the amplitudes of steady vibration on the basis of an energy balance. In the present experiments conditions were three-dimensional, as in Fig. 1, and a similar energy approach was used in the solution of the problem.

In a boundary layer there is a velocity gradient between the wall and the free stream. This velocity variation can be expressed by the simple relation

\[ V(z) = V_x v(z) \]

where \( V_x \) is the velocity at the reference point, here the top of the structure, and \( v(z) \) is a function describing the velocity profile.
Figure 1. Finite vertical structure vibrating in a turbulent boundary layer flow
Dealing with atmospheric boundary layers it is most convenient to consider $v(z)$ to be a power law profile whose shape depends on the roughness geometry (11, 12).

Turbulent flow past a bluff body implies that there are fluctuating components of velocity and force. Since the time for developing a steady galloping oscillation is hundreds of cycles (1) the overall effect of the velocity fluctuations is small and can be accurately ignored. To dispose of the lateral fluctuating force components is more difficult but is accurate if the onset velocity is much higher than the velocity at which vortex resonance occurs (3, 6). Therefore treating only time average values the local mean force in the $y$-direction is given in coefficient form by the expression

$$F_y(z, \alpha) = C_{fy}(z, \alpha) \frac{\rho h}{2} V^2(z)$$

At present there is no adequate theory which gives $C_{fy}$ as a function of the angle of attack, $\alpha$, or as a function of the vertical dimension, $z$. The lateral force coefficient can only be determined experimentally through force or pressure measurements, as in Appendix 1. The necessary empiricism is then introduced by assuming the lateral force coefficient can be represented by a polynomial curve fit of the data in the general form

$$C_{fy}(z, \alpha) = \sum A_i(z) \tan \alpha^i$$

where now the aerodynamic constants, $A_i$, are variables with height.

For symmetrical prismatic sections $C_{fy}$ is an odd function of
tan \alpha and as such even powers of tan \alpha should vanish. Preserving the even powers however, results in a smoother approximating function and is accomplished by properly considering the absolute value of tan \alpha (5). Introducing the absolute value signs in all even powered terms, and using the quasi-steady implication that

\[ \tan \alpha = \frac{\dot{y}(z)}{V(z)} \]

results in the general equation for the lateral force coefficient as

\[ C_{fy}(z, \alpha) = \sum_{r=1}^{2r-1} A_{2r-1}(z) \left( \frac{\dot{y}(z)}{V(z)} \right)^{2r-1} + \sum_{s=1}^{2s} A_{2s}(z) \left( \frac{\dot{y}(z)}{V(z)} \right)^{2s} \frac{\dot{y}(z)}{|\dot{y}(z)|} \]

The above expression represents a force coefficient which is analogous to the force coefficient obtained for two-dimensional smooth flow conditions. The main difference is that the expression in equation 3 is some complex function of the vertical dimension.

2.2 Energy approach  The only net exchange of energy between the mechanical and aerodynamic forces, over a period of vibration, is that due to the dissipative forces. Therefore considering a structure with idealized viscous damping, c, the total dissipative force acting over a differential length \( dz \) is

\[ F(z, \dot{y})\, dz = (C_{fy}(z, \dot{y}) \frac{\rho}{2} h V^2(z) - c \dot{y}(z))\, dz \]

Steady vibrations exist when the total work done by the damping forces, over a period of vibration, is identically zero. Thus the equation determining the steady amplitudes of vibration is
\[ W = 0 = \int_0^L \int_0^t F(z,\dot{y}) \, dz \, \dot{y} \, dt \]

Noting the similarity between the three-dimensional system and the two-dimensional single degree of freedom system, Novak (5) assumed that the structure would have a response similar to that of a free vibration. The assumed motion which is accurate to the first approximation is given by

\[ y(z, t) = a y_n(z) \cos \omega_n t \]

Here \( a \) is the amplitude at the reference point, the tip of the structure, \( y_n(z) \) is the normalized mode shape and \( \omega_n \) is the natural circular frequency. Previously Novak (6, 7, 8) applied an average lateral force coefficient over the height of the structure and then computed the steady galloping amplitudes using the above equations. If this two-dimensional assumption of forces is accurate the general algebraic equation describing the amplitudes of steady vibration, resulting from the integration of equation 5 is:

\[ \frac{1}{\bar{U}} = \sum_{i=1}^{l} \overline{A}_i \overline{B}_i \overline{C}_i \left( \frac{a^*}{\bar{U}} \right)^{i-1} \]

where now \( \bar{U} \) and \( a^* \) are dimensionless wind velocity and amplitude given by

\[ \bar{U} = \frac{n \cdot V}{\beta \omega_n h} \quad ; \quad a^* = \frac{na}{\beta h} \]

Here \( \overline{A}_i \) are average coefficients found from equation 2, \( \overline{B}_i \) are numerical coefficients, for odd \( i = r \) are
\[ B_r = 2 \frac{1 \cdot 3 \cdot 5 \ldots r}{2 \cdot 4 \cdot 6 \ldots (r+1)} \]

and even \( i = s \) are

\[ B_s = \frac{4}{\pi} \frac{2 \cdot 4 \cdot 6 \ldots s}{1 \cdot 3 \cdot 5 \ldots (s+1)} \]

and \( C_1 \) are coefficients describing the vibration mode and wind profile given by

\[
C_1 = 2^{-i} \int_0^L v(z) y_n(z) \left| y_n(z) \right|^{i+1} dz \\
\int_0^L y_n(z) dz
\]

If a two-dimensional average force is not accurate an alternate lateral force coefficient can be considered. To do so requires that the average aerodynamic constants, \( \bar{A}_1 \), in equation 2 be replaced by a function which expresses their dependence on the vertical dimension, \( z \).

The polynomial expression used here was

\[
A_1(z) = \sum_{j=0}^{L} A_{1,j} \left( \frac{z}{L} \right)^j
\]

If equation 8 is substituted for \( \bar{A}_1 \) in all previous equations the expression describing the steady galloping amplitudes is

\[
\frac{1}{U} = \sum_{i=1}^{L} B_i D_i \left( \frac{a^*}{U} \right)^{i-1}
\]

where all terms are as before except the coefficients \( D_i \) replace \( \bar{A}_1 C_1 \) and are given by
By examining equation 7 or 9 it is observed that the galloping response predicted is universally valid. Thus, galloping oscillations of all elastic systems having the same cross-section, height and mode shape should collapse onto a single universal response curve when exposed to the same wind profile for all mass and damping configurations. This fact should enable the direct determination of the universal response curve for a particular structure by measuring the galloping characteristics of a single arbitrary elastic model in the wind tunnel.

The amplitudes of stationary oscillation can be found from equation 7 or 9 but some of these amplitudes may be unstable. Parkinson (2) and Novak (7) have examined in detail the stability of galloping amplitudes. For the analysis of stability, the first derivative of equation 5 is needed and in the sense of orbital stability amplitude \( a_s \) is stable when

\[
\left. \frac{dW}{da} \right|_{a_s} < 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 10
\]

and with the opposite sign the motion is unstable. Performing the differentiation in equation 10 leads to the general equation predicting the stability of a particular amplitude which is

\[
\sum_{i=1}^{i+1} \frac{1}{Z} \frac{v(z)2-i}{A_iB_iC_i} \left( \frac{a^*}{U} \right)^{i-l} - \frac{1}{U} < 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 11
\]
Once again the coefficients $\bar{A}_1 C_1$ can be replaced by coefficients $D_1$ for those cases in which the lateral force coefficient is not constant with height.

In summary, using the quasi-steady assumption of forces the galloping response of a vertical structure can be predicted. In those cases where an average lateral force coefficient is an adequate approximation the galloping response is that given by equation 7. For three-dimensional situations where the lateral force coefficient varies with height the galloping oscillation is predicted by equation 9. To analyze the stability of the steady amplitudes the relationship given by equation 11 should be used.
3.1 **Outline of experiments conducted** The purpose of this study is to investigate the galloping behavior of a finite vertical structure exposed to a simulated atmospheric boundary layer flow. To this end experiments were conducted as follows:

a) **Velocity measurements** Measurements were made of the boundary layer's velocity profile, turbulence intensity and spectral distribution of energy. This data was used to define a characteristic length scale so that model data can be compared to full scale information.

b) **Force and pressure measurements** Once the properties of the turbulent flow had been sufficiently characterized their effects on the static behavior of finite square cylinders were investigated. Force measurements were made on square cylinders of four different aspect ratios at various angles of attack. Later pressure measurements were conducted on two prisms to obtain local lateral force information.

c) **Response measurements** The dynamical behavior of two elastically mounted square towers, placed in the same turbulent flow as for the force measurements, was investigated for various combinations of model damping and frequency. The variation of tip amplitude with wind speed was recorded and compared with the theoretical predictions, which utilized the force and pressure readings made
d) **Wake measurements**  Additional information about the galloping behavior of the square towers was obtained by examining the spectra of velocity fluctuations in the wake behind the rigidly and elastically mounted 28" model.

### 3.2 Wind tunnel
All experiments were conducted in the industrial aero-dynamics wind tunnel at U.B.C.. The wind tunnel is an open circuit, blower type tunnel 8' wide and initially 5.17' high with a test section 80' long. The area contraction ratio is 4:1 and a constant speed, variable pitch fan blows air through the test section at speeds between 7 and 70 ft/s. The test section roof can be adjusted to maintain ambient pressure in the tunnel. Pressure taps located at 8' intervals along the back wall of the tunnel were connected to a multitube manometer to accurately set the pressure gradient to zero. The velocity profile and turbulence characteristics of the boundary layer are determined by the roughness of the floor covering. The turbulent boundary layer for this study was created by covering the entire test section with roughness elements 1.5" high, 0.75" wide and 0.041" thick, 6" apart in staggered rows. A view of the roughness can be seen in Fig. 2.

### 3.3 Velocity measurements
The important properties of the turbulent boundary layer were measured with a single hot-wire with linearized response. The hot-wire system used was a DISA type 55D01 anemometer. The signal from the anemometer was fed into a DISA linearizer, type
Figure 2. Elastic 20" model and upstream surface roughness
55D10, and frequencies higher than 10KHz were eliminated with a DISA 55D25 filter. Using the linearized hot-wire signal mean and RMS measurements were made of the boundary layer's profile and turbulence intensity at three different wind speeds.

In order to determine the scales of the turbulence the spectrum of the longitudinal velocity component was analyzed. The spectrum was measured with a Bruel and Kjaer, type 1614, 1/3 octave band filter. Digital readout was accomplished by using a Schlumberger Time Domain Analyzer with real time averaging. Spectra of the longitudinal velocity component were computed at several different heights in the boundary layer.

3.4 Static models The force and pressure measurements were conducted on a 2" by 2" square plexiglass tower. The tower was composed of 4" tall segments which could be assembled to form a tower of the desired aspect ratio. The local forces were determined by the integration of the local pressures, and to this end two 4" long sections were built and fitted with rows of pressure taps. Each row consisted of seven pressure ports of 0.025" in diameter. A view of the model can be seen in Fig. 3.

3.5 Force measurements One of the necessary inputs to the galloping theory of Chapter 2 is the determination of the lateral forces. It was most convenient to measure the lift and drag forces and then compute an average lateral force via equation 3 in Appendix 1.

To measure the lift and drag, an Aerolab pyramidal strain gauge balance was employed. The balance is designed to support a model in the
Figure 3. Test models (left to right 20" and 28" elastic models and variable height static model)
wind tunnel and vary its angle of yaw over a 360° range with a precision of 0.1°. Links separate the individual force or moment components so that each can be measured individually. Since the models were mounted vertically in the wind tunnel the lift force was measured with the side force unit with angles of attack being replaced by angles of yaw. The electrical signals coming from the drag and side force load cells were then fed into a PDP 11/10 minicomputer to be digitized. The sample period at each angle of attack was approximately 40 seconds, and three runs were made at each aspect ratio. Average drag and lift coefficients were then computed.

The reference dynamic pressure was measured from a pitot static tube with an inclined Lambrecht manometer. The reference pitot tube was located 45" above the tunnel floor and left of the tunnel centerline. This dynamic pressure was measured to be 6% higher than the dynamic pressure measured at a height of 28" above the tunnel centerline. Thus dynamic pressures measured from the reference pitot at 45" were reduced by 6% to yield the actual dynamic head at the height of 28", i.e. the location in the wind tunnel where the models were tested.

3.6 Pressure measurements To obtain the local lateral forces acting on the model the pressures at a particular height were measured and then integrated. Since measuring and recording pressures at 14 taps for 30 or more angles of attack at several different locations required a large number of readings the pressure measuring system was automated. The system used was quite effective in obtaining and storing the large quantity of pressure data accumulated.
Pressure taps across a face of the model were connected to a Scanivalve multiport scanner whereupon the pressure signal was converted into a voltage by a Barocel, type 511, pressure transducer. The electrical signal was further amplified and conditioned by a Datametrics Electric Manometer, type 1018B, and was then input into the PDP 11/10 minicomputer to be digitized and stored. The multiport pressure scanner was driven by an electrical impulse from the computer, so after each 30 second sample the scanner was advanced and a new tap was sampled and converted into pressure coefficient form. The pressure coefficients calculated were non-dimensionalized by the dynamic pressure at the particular height of the row of pressure taps. Once all the pressure coefficients had been calculated a cubic spline curve fit and then a Simpson's rule integration routine were used to perform the calculation of the local lateral force coefficient given by equation 9 in Appendix 1.

3.7 Elastic models and mounting

To verify the theoretically predicted galloping responses dynamic measurements were conducted on two elastically mounted square towers. A model together with its mounting is shown in Fig. 4. The basic model was attached to a vertical 1/2" diameter hollow steel rod which in turn was fixed to a 1" diameter thin walled aluminium tube which was supported by two cylindrical air core bearings. The 1/2" steel rod after passing through the aluminium tube was flexibly connected to rigid steel legs by two horizontal helical tension springs.

The air core bearings providing the model support were similar to those designed by Smith (1). To prevent any motion from occurring in the along-wind direction the air bearings were drilled and fitted with end
Figure 4. Dynamic balance and elastic test model
plates. The model was thus capable of rigid body rotation in a single degree of freedom in a plane perpendicular to the wind direction. The air supply to the bearings and end plates came directly, via a flexible hose, from the compressed air line available in the lab and was kept constant throughout a test.

The two models tested were each built of 1/2" thick balsa wood and measured 2" by 2" and were 20 and 28 inches tall. A varnish was used to protect the surfaces and corners of the models and this resulted in a smooth exterior finish.

In addition to the damping forces already inherent in the pivoting system, eddy current damping was also employed. The dissipative forces due to the eddy currents are almost entirely equivalent to viscous damping which was desirable in this analysis. To this end a thin aluminium disk was attached to the bottom of the steel rod to provide eddy current damping as it moved between the poles of a G electromagnet as shown in Figs. 4 and 7. The current powering the electromagnet was provided by a D.C. power supply and was controlled by a variable resistance. After each test any residual magnetism was removed from the magnet by switching the current over to a slowly decreasing A.C. supply.

The helical tension springs were made by cutting the required number of coils from a known spring and then were calibrated by a simple load deflection test. The two springs had a combined stiffness of 14.5 lb/in.

Changes equivalent to changes in model density were produced by varying the vertical distance from the point of rotation to the point of
attachment of the springs. Varying this distance effected a change in frequency and a consequent change in average 'effective density' of the model.

The complete air bearing model support system was built on a rigid steel frame approximately 21" high, which in turn was supported under the wind tunnel by a heavy table. After the model was aligned to zero angle of attack the steel frame was firmly clamped to the support table. The model was fixed to the air bearing system through a hole in the tunnel floor, and an inch space separated the tunnel floor and the air bearing system to reduce possible effects due to tunnel vibrations. The entire system exhibited no perceptible motion at even the highest amplitudes of model vibration.

3.8 Deflection measurements and calibration The amplitudes of steady vibration due to the mechanism of galloping are known to be large and occur at a frequency close to the natural frequency of free vibration. These characteristics, the large amplitudes and the frequency of vibration, dictated the type of deflection measuring instrumentation. Strain gauges are often used in dynamic systems of the type described here (10) but non-linearity in the strain gauges due to the large amplitudes could be a problem. The deflection of the model was instead measured by a Bruel and Kjaer, type 4332, accelerometer mounted inside and quite near the top of the model. The accelerometer used was fairly large, it had a mass of 30 grams, but also had a high sensitivity about 46 mV/g and a flat frequency response to about 1 Hz. Since the desired quantity was displacement not acceleration the high impedance output.
from the accelerometer was fed into a Bruel and Kjaer 2625 preamplifier where the signal was integrated and amplified. The resulting low impedance signal had a D.C. offset of 13 volts which was blocked by a capacitor before being fed into a Krohn Hite low pass filter which removed signals above 160 Hz. The filtered output was then displayed on an oscilloscope, plotted on a Honeywell visicorder oscillograph and digitized by the PDP 11/10 computer. RMS data were measured and 20 second samples were taken over a one minute period after the flow had stabilized in the wind tunnel. Longer sampling periods were used if the amplitudes fluctuated a great deal.

A slight disadvantage in using the accelerometer to measure the model displacement was that the calibration of the model for deflection had to be done in a dynamic test. To determine the displacement versus voltage characteristics of the accelerometer a very thin wire was attached to the tip of the model and fixed to the side of the wind tunnel. The resulting horizontal deflection was then measured with a pair of vernier calipers, using a rigid stand mounted next to the model as a reference point. The visicorder was set to a known speed and the wire was sharply cut. The original displacement was taken as the peak of the first oscillation cycle of the oscillograph trace. Knowing the displacement versus voltage response of the visicorder the model deflection could then be converted into a voltage. Calibration of the model for deflection was performed before each test and conducted at five different initial displacements. The constant obtained by plotting voltage versus displacement, Fig. 5, was used as input to the computer program which
Figure 5. Typical calibration curve for accelerometer output versus model deflection.
converted the electrical signals into RMS displacements.

3.9 **Damping measurements**  The damping of the model for a particular current in the electromagnet was obtained by plucking the model in the cross wind direction and recording the output onto the visicorder oscillograph. The decay curve was repeated for three different initial displacements and was measured and plotted on a semi-log graph as shown in Fig. 6. The log decrement used was the average for the three trials. Decay traces were taken before each dynamic test and the log decrement was correspondingly calculated. This procedure incorrectly includes the still-air aerodynamic damping of the model itself, but this is relatively small and is partly compensated for by the higher values of non-aerodynamic damping actually occurring during galloping.

Later, after performing the damping calibrations it was found that the percentage of critical damping already present in the pivoting system due to friction was a significant amount. Depending on the frequency the fraction of critical damping, $C/C_{cr}$, due to the pivoting system was between 0.0045 and 0.008. Since the onset velocity is directly proportional to the damping present tests were often performed with the electromagnet not present, so as to keep the wind speed within a reasonable range. The damping in the pivoting system was then assumed to be entirely viscous and was calculated in the same manner as previously outlined.

3.10 **Frequency measurements and density calculation**  The model frequencies were calculated from the oscillograph traces resulting from the
Figure 6. Typical decay plot for damping calibration
damping calibration. A 1 cycle per second triangular wave from a function generator served as the time base for the calculation of the frequency. The frequency measurements were repeatable, but were checked before each dynamic test.

One of the necessary inputs to the galloping theory which was computed directly from the frequency measurements was the determination of the average density, $\rho_m$, of the model. To determine the effective density of the model the moment of inertia of the pivoting system had to be calculated. The inertia of the rotating assembly, i.e. model, steel rod, aluminium shaft, springs damping plate and accelerometer, about the horizontal axis of rotation was obtained from the equation of free vibration. Given the geometry in Fig. 7, if the moment of inertia about the axis, 0, is $I$ the equation of free vibration is given by

$$I\ddot{\theta} + c\dot{\theta} + 2kz_s^2\theta = 0$$

and the model inertia can be calculated directly from the expression

$$I = \frac{2kz_s^2}{(2\pi f)^2}$$

where $2k$ = total spring stiffness

$f$ = frequency

$z_s$ = vertical distance between the point of rotation and the springs

$\theta$ = angular rotation

Now for a rectangular prism rotating about the axis 0 the inertia is known and the effective density can be computed from
Figure 7. Elastic model and mounting rig for dynamic tests
\[ \rho_m = \frac{1}{V_m} \left[ \frac{h^2}{12} + \left( \frac{h+d}{2} \right)^2 \right] \]

where

\( h \) = length of the model
\( z \) = lateral dimension of the model
\( d \) = distance from the base of the model to the point of rotation, here \( d = 2.4" \)
\( V_m \) = volume of the model

The dynamic tests were conducted on two square towers for various combinations of frequency and damping. A summary of the model properties for each particular test are presented in Table 1. (See nomenclature for definition of the symbols.)

**TABLE I**

<table>
<thead>
<tr>
<th>Height (in)</th>
<th>( f ) (c/s)</th>
<th>( z_s ) (in)</th>
<th>( \rho_m ) (lb/ft(^3))</th>
<th>( \beta \times 10^{-3} )</th>
<th>( \eta \times 10^{-3} )</th>
<th>( \beta/\eta )</th>
</tr>
</thead>
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<td>28</td>
<td>3.79</td>
<td>5.00</td>
<td>11.4</td>
<td>6.6</td>
<td>1.250</td>
<td>5.28</td>
</tr>
<tr>
<td>28</td>
<td>3.58</td>
<td>5.00</td>
<td>12.7</td>
<td>7.3</td>
<td>1.150</td>
<td>6.35</td>
</tr>
<tr>
<td>28</td>
<td>3.79</td>
<td>5.00</td>
<td>11.4</td>
<td>7.9</td>
<td>1.320</td>
<td>5.98</td>
</tr>
<tr>
<td>28</td>
<td>4.41</td>
<td>6.00</td>
<td>12.1</td>
<td>8.0</td>
<td>1.210</td>
<td>6.61</td>
</tr>
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<td>20</td>
<td>4.65</td>
<td>4.31</td>
<td>14.1</td>
<td>4.5</td>
<td>0.950</td>
<td>4.74</td>
</tr>
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<td>20</td>
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<td>4.31</td>
<td>14.1</td>
<td>5.2</td>
<td>0.942</td>
<td>5.52</td>
</tr>
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<td>13.7</td>
<td>7.3</td>
<td>0.913</td>
<td>8.00</td>
</tr>
<tr>
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<td>5.38</td>
<td>5.00</td>
<td>14.1</td>
<td>7.4</td>
<td>0.913</td>
<td>8.11</td>
</tr>
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CHAPTER 4

RESULTS AND DISCUSSION

4.1 Velocity measurements  In a neutrally stable atmospheric boundary layer the properties of the mean flow are known to be almost entirely dependent on the roughness of the surface (11,12). Simply put this means that the rougher the surface, the greater the drag force at the surface, turbulence intensity, the Reynolds stresses, the gradient height and the retardation at the surface. In (11, 12) Davenport has catalogued some of the properties of typical atmospheric boundary layers and their corresponding surface roughness. The intent here was to make velocity measurements of the model boundary layer's properties and compare these to the full scale information in (11, 12).

The variation of the mean velocity and the RMS turbulence intensity with height are shown respectively by Figs. 8 and 9. From Fig. 8 it can be seen that for the particular roughness used the boundary layer thickness, 6, is approximately 28" and the profile exponent, γ, in the equation

\[
\frac{V}{V_{28"}} = \left(\frac{z}{\delta}\right)^{\gamma}
\]

is 0.264. The power law exponent was obtained by plotting \(\log \left(\frac{V}{V_{28"}}\right)\) versus \(\log (z/\delta)\). The velocity profile and the turbulence intensity were measured for three different gradient wind speeds and both properties were found to be reasonably invariant with \(R_e\).
Figure 8. Variation of velocity with height in the boundary layer
Figure 9. Variation of turbulence intensity with height in the boundary layer.
An important characteristic of a turbulent boundary layer is the distribution of energy with frequency. Spectral measurements of the longitudinal velocity component were made at several heights in the boundary layer at a gradient wind speed of 37 ft/s. The spectra measured at 2/3 of the model height for the 28" and the 20" models, Fig. 10, are given as dimensionless power versus non-dimensional frequency. To determine the turbulent length scale, $L_v$, the measured spectra were compared with Von Karman's theoretical distribution of energy, i.e.

$$\frac{n S(n)}{v'^2} = \frac{4 n L_v^4}{V} \left[ 1 + 70.7 \left( \frac{n L_v}{V} \right)^2 \right]^{5/6}$$

where:
- $n$ = the frequency
- $L_v$ = the length scale
- $V$ = the mean local velocity
- $v'$ = the total RMS velocity fluctuation

Matching the measured spectra to the theoretical curve was done over the moderate frequency range.* For the spectra taken at 18.67" the characteristic length scale was 1.03' and at 13.33" the scale was found to be 0.942'.

The power law exponent, the distribution of turbulence intensity and the spectra all scale to what Davenport calls a suburban or forested exposure. For the 28" model corresponding full scale data has a typical eddy length of 560' and for the 20" model the atmospheric scale of turbulence is 475'. Taking the geometric scale as the ratio of full scale turbulence to the scale of wind tunnel turbulence implies that for the 28" model the scale is 1/540 and for the 20" model the scale is 1/500.

*(This corresponds to dimensionless frequencies, in Fig. 10, between 1.0 and 1.8.)
Figure 10. Power spectrum of the longitudinal velocity component
Thus both models represent tall towers exposed to a "suburban" wind.

4.2 Average force measurements

4.2.1 Drag coefficient

By examining Fig. 11 it is seen that for angles of attack between 0 and 30 degrees the effect of lowering the model's aspect ratio results in a lower average drag coefficient. For small angles of attack ($\alpha \leq 13^\circ$), the drag coefficients tend to become independent of wind orientation as the aspect ratio decreases. When compared to Laneville's (4) two-dimensional results the drag coefficients measured here are lower and the minimum drag occurs at a slightly higher angle of attack.

Vickery in (13) measured the drag coefficients, at zero angle of attack, of several finite square bluff shapes exposed to a flow of 10% turbulence intensity. His results indicate that for a decrease in aspect ratio from 15 to 2 a reduction in the mean drag coefficient could be as much as 30%. In Vickery's data a marked feature of the variation of $C_D$ with aspect ratio was the attainment of a maximum value of drag coefficient at a finite value of aspect ratio. For the range of height to width ratios examined here the drag coefficients were always found to increase with increasing aspect ratio.

The reduction in drag for the shapes tested is most likely the end product of two mechanisms, one being the increased flow over the tip of the model and two the overall higher level of turbulence intensity approaching a lower aspect ratio model. Both mechanisms serve to reduce
Figure 11. Variation of average drag coefficient with angle of attack for four aspect ratios.
the drag coefficients by increasing the base pressure behind the model.

4.2.2 Lift coefficient As shown by Fig. 12 the effect of decreasing the model aspect ratio is to progressively reduce the maximum negative lift, leaving the initial trend at small angles of attack unchanged. The invariance of lift coefficient at small angles of attack for different turbulence intensities was also observed by Laneville. Comparison of the two-dimensional and three-dimensional lift coefficient curves reveals that in the latter situation the slopes at the origin are not nearly as steep indicating that the square section is more stable in a three-dimensional flow.

The invariance of the lift curve slopes for small angles for aspect ratios between 14 and 6 was unexpected. The first appearance of a tip flow seems to cause a discrete jump in the initial slope of the lift coefficient curve with further increases in model three-dimensionality having negligible effect on the initial trends of the lift curve slope. A decrease in aspect ratio does appear to reduce the angle at which the maximum lift occurs. This result is difficult to isolate from Fig. 12. since a reduction in aspect ratio corresponds to an increased turbulence intensity which is known to reduce the angle at which the maximum negative lift occurs. More experiments will have to be done to explain the unexpected behavior of the lift curve slope with model aspect ratio.

4.2.3 Lateral force coefficient The average lateral force coefficients calculated from the lift and drag measurements, as in Appendix 1, are
Figure 12. Variation of average lift coefficient with angle of attack for four aspect ratios.
plotted in Fig. 13 for several different model aspect ratios. As a comparison the lateral force coefficients from (4), obtained under two-dimensional isotropic turbulent flow conditions are plotted in Fig. 14.

Comparing the two-dimensional and the three-dimensional results reveals that in the latter situation the initial slope at the origin is much reduced from the two-dimensional case. The reduced slope of the lateral force coefficient in the three-dimensional situation is indicative of the lower values of lift coefficient measured. As can be seen though $C_{Fy}$ for a finite square section even in a turbulent shear flow exhibits a positive slope for small angles indicating the section is unstable. The reduced slope is evidence that the finite square sections tested should all gallop but at a higher reduced velocity than under two-dimensional conditions.

From an examination of Fig. 13 it is observed that for small angles of attack the lateral force coefficient is almost completely independent of the particular model's aspect ratio. The invariance of the lateral force coefficient is due to the measured lift coefficients which had little dependence on the model's height to width ratio for small angles of attack. In general decreased aspect ratio and increased turbulence intensity only serve to reduce the value of the maximum lateral force and the angle at which it occurs. If the assumption of an average lateral force is adequate the characteristics of these lateral force coefficients should be reflected in the model's galloping response. Finite square towers having an aspect ratio between 14 and 6 should all have reasonably the same onset velocity but gallop with
Figure 13. Variation of average lateral force coefficient with tanθ for four aspect ratios.
Figure 14. Effect of turbulence intensity on $C_{FY}$ for square section re.

- $C_{FY}$
- Turbulence intensity

- 3-D slope fits 13
- $0.125$
- $0.15$
- $0.091$
- $0.07$

Lateral force coefficient
lower amplitudes as the model aspect ratio decreases.

4.3 Local lateral force coefficient In a turbulent boundary layer the local drag and lift coefficients can be expected to vary along the height of the model. Naturally the local lateral force coefficient will reflect these changes and will be dependent upon the vertical dimension, \( z \), in some manner. To ascertain the variation of local lateral force with \( z \), local pressures were measured and integrated as in section 3.6.

As shown by Figs. 15 and 16 the dependence of the local lateral force upon the dimensionless height, \( z/\lambda \), for both the 28" and the 20" models is quite striking. The lateral force coefficient for the 28" model basically goes through three regions. Over the bottom third of the model the lateral force coefficient is negative indicating the flow is well reattached and according to Den Hartog's criterion is stable for all wind speeds. The force coefficient is negative over this region due most likely to the high levels of turbulence and the complex manner in which the fluid separates in a sheared flow. In the middle section of the model the lateral force coefficient curves again have positive slopes at the origin and are correspondingly unstable. The slope at the origin, the maximum lateral force and the angle at which it occurs all gradually increase as the vertical dimension approaches 70% of the model height. Near this point the lateral force coefficient curve has a slope and a maximum value both approximately 2/3 of the corresponding values obtained under two-dimensional conditions of 9% turbulence intensity, (Fig. 14). The variation of the lateral force coefficient
Figure 15. Variation of local lateral force coefficient along the span of the 28" model.
Local lateral force coefficient $C_{fy}(z)$

Figure 16: Variation of local lateral force coefficient along the span of the 20" model.
with angle of attack at this location however, is distinctly different than under two-dimensional conditions as the maximum force occurs at a higher angle of attack and the force is positive over a broader range of angles. After 70% of the model height has been reached the slope of the force coefficient curve and the maximum value both begin to fall off as the influence of the model's tip comes into consideration.

For the 20" model the local lateral force coefficients obtained at three dimensionless heights have basically the same variation with height and angle of attack as the local lateral force coefficients measured on the 28" model. The major difference between the measured coefficients for the 28" and the 20" model is that for the latter the lateral force coefficients all attain their maxima at lower angles of attack. Though the force coefficients on the 20" model were not measured at exactly the same dimensionless height as for the 28" model, it can be seen that for corresponding heights the initial slopes of the lateral force coefficient curves are nearly identical.

In summary the measurements of local lateral force coefficients have shown that there is a definite variation of lateral force along the span of the model and that the forces on two models of different aspect ratios are distributed in the same way.* The major effect of aspect ratio and turbulence intensity is to reduce the maximum mean lateral force and the angle at which it occurs. Since the lateral force coefficient exhibits distinctive changes depending on the height it is to be expected that the galloping response utilizing the local lateral force coefficients will be different than the galloping response found

*(Although not shown it has been verified that the pressure measurements can be integrated to obtain the average force coefficients in Fig. 13.)
from the average lateral force coefficients.

4.4 Theoretical galloping response

4.4.1 Response using average lateral force coefficients  The galloping response of the model towers was predicted using the theory outlined in Chapter 2. The aerodynamic constants, $\bar{A}_1$, in equation 7 were obtained from a curve fit of the experimental average lateral force coefficient data. The curve fit is a fifth order polynomial including the even terms and was performed with a least squares orthogonal polynomial computer routine (14). The average aerodynamic constants, $\bar{A}_1$, for the 28" model and the 20" model can be found in Appendix 2.

In Fig. 17 the variation of tip amplitude with tip wind speed is plotted for models of three different aspect ratios. The response curves are universally valid; that is, for all configurations of model frequency, damping and density the variation of tip amplitude with tip wind speed is given by these curves (5). The theoretical responses are calculated assuming a rigid body rotation about the horizontal axis so that the normalized bending mode corresponds to

$$y_n(z) = \frac{z}{\lambda}$$

The velocity variation is taken as a power law profile with the exponent equal to 0.264.

By examining Fig. 17 it can be seen that for a given wind speed as the model height decreases so does the amplitude of oscillation. The onset velocity for all three models is basically the same and is greater than in the two-dimensional cases (1,4). The invariance of onset velocity
with model aspect ratio results from the lateral force coefficient curves having nearly the same slope for small angles of attack. The asymptote of the response curves tends to decrease as the aspect ratio decreases, indicative of the reduced lateral force coefficients.

4.4.2 Response using local lateral force coefficients

The variation of lateral force coefficient with angle of attack exhibits distinctive changes with height, Figs. 15 and 16. To incorporate this three-dimensional effect into the galloping theory the aerodynamic constants, $\bar{A}_i$, have to be treated as functions of the model height, $z/\lambda$. In this analysis the variation of local lateral force coefficient with angle of attack is curve fitted as in the average coefficient case. Each resulting aerodynamic constant, $A_i$, is then approximated by a second order polynomial in powers of $z/\lambda$ across the height of the model. Thus the aerodynamic characteristics of each particular bluff shape are described by a matrix of 15 elements. The matrix of constants for the 28" and the 20" models are given in Appendix 2.

Using the aerodynamic coefficients obtained from the local lateral force data the galloping responses, that is the variation of tip amplitude with tip wind speed, of the 28" and the 20" models are computed from equation 9 and plotted in Figs. 18 and 19. The bending mode and the velocity profile functions are the same as in the average force coefficient case. As is obvious from Figs. 18 and 19 the galloping response calculated using the local lateral force data is distinctly different than the response calculated from the average force coefficients. The responses predicted from the local forces give higher amplitudes for the same reduced
Figure 18. Comparison of theoretical and experimental galloping amplitudes
Figure 19. Comparison of theoretical and experimental galloping amplitudes.
velocities and exhibit higher asymptotes than the responses utilizing the average force data. The difference in the galloping responses is due entirely to the fact that the forces are not constant along the height of the model.

The effect of bending mode on the character of a galloping response of the 28" model is computed for two modes,

1) \( y_n(z) = \frac{z}{\ell} \)
2) \( y_n(z) = \frac{z^2}{\ell^2} \)

The first corresponds to a rigid body rotation about a horizontal axis and the second represents a cantilevered motion where the base is fixed. From Fig 20. it can be seen the galloping oscillation is dependent upon the mode shape with differences increasing with wind speed. The onset velocity remains nearly the same for both mode shapes. Similar characteristics were also observed by Novak (5, 7).

4.5 Response measurements To verify the theoretical predictions made in section 4.4 two elastically mounted square towers were placed in the wind tunnel and exposed to the same turbulent boundary layer flow as for the force measurements. Changes were made in the model's frequency, damping and density with RMS tip displacements and gradient wind speeds being recorded, (a summary of the model properties for each test can be found in Table I). The results of the dynamic tests on the 28" and 20" square towers are plotted in Figs. 18 and 19. The experimental points are plotted in the universal coordinates \( a_n/\beta \) and \( U_k \bar{h}/\beta \) whereby all experiments on the same section can be compared. The theoretical
Figure 20. Effect of mode shape on the theoretical galloping response computed from the local lateral force coefficients.
predictions treat the amplitudes as being purely harmonic, and to make the RMS experimental amplitudes comparable to the theoretical values they are correspondingly multiplied by \( \sqrt{2} \).

Tests on the 28" model, Fig. 18, indicate that the onset velocity for this model is somewhere between 0.4 and 0.5. This value is slightly higher than the two-dimensional smooth flow value of 0.372 and is much lower than the onset velocity predicted by the quasi-steady theory using the average or local forces. As can be seen though, the experimental points at higher velocities collapse reasonably well on to the galloping curve computed from the local lateral forces. The greatest discrepancy between experiment and theory occurs for the model configuration having the lowest ratio of \( \beta/\eta \). The experimental amplitudes for this model arrangement tend to be lower than the predicted values which is a trend similar to what Santosham (3) observed for the 2/1 rectangle. Santosham explained the discrepancy between the galloping theory and the measured values as being due to the wake vortices, that is having the velocity at which vortex resonance occurs close to the onset velocity of galloping oscillations. A similar effect is most likely occurring here since in a turbulent flow the square section is known to behave as a section with increased afterbody length (4). Another possible explanation is that for this configuration the damping was due entirely to the frictional forces inherent in the pivoting system which perhaps changed with large amplitudes.

The model with the highest ratio of \( \beta/\eta \) had a distinctly different pattern to its galloping cycle. The model did not start galloping
at the same velocity as the others but continued to remain relatively quiet until the reduced velocity, $U_\alpha \eta/\beta$ reached a value of 1.0. The model then galloped vigorously and continued to do so as the wind speed increased. Lowering the wind speed produced a hysteresis in the cycle as the model continued to gallop at reduced velocities down to about 0.75. From Fig. 18 the observed hysteresis in the galloping cycle is seen to be similar to the theoretical prediction.

Dynamic tests on the 20" model, Fig. 19, had similar features compared to the tests on the 28" model. One of the distinctive differences was that the motion was noticeably more random indicative of a higher turbulence level. The model configurations having the two lowest values of $\beta/\eta$ have lower amplitudes than the predicted values. The values of damping in this instance were as low or lower than for the 28" model and consequently the wake vortices exert a more pronounced effect on the galloping characteristics. No oscillation hysteresis existed for the highest levels of $\beta/\eta$ tested.

The dynamic tests have given evidence that to properly apply the quasi-steady theory to tall structures requires that consideration be given to the variation of lateral forces along the span of the model.

4.6 Wake measurements To understand some of the effects of the wake on a galloping bluff cylinder a hot-wire was positioned behind both the rigidly mounted and elastically mounted 28" model. The location of the hot-wire probe depended upon the magnitude of the RMS fluctuation. Although difficult to locate, the region of greatest RMS longitudinal fluctuation was sought after as the most advantageous location to do a spectral analysis. The spectra were computed as in section 3.3.
In Fig. 21 are plotted the non-dimensional power spectra of velocity fluctuations versus the dimensionless frequency based on the local velocity for the rigidly mounted model. The spectra were computed at four different heights along the model because of the phenomenon observed by Maull in (15). Maull measured the velocity fluctuations behind a finite bluff body in a non-turbulent shear flow and these results indicated that the frequency at which the vortices were shed depended upon the vertical location of the hot-wire. Essentially what were observed were discrete jumps in Strouhal number as the vertical distance was varied. A close examination of Fig. 21 reveals that the peaks of the spectra for this bluff section occur at different dimensionless frequencies depending upon the relative height of the hot-wire, but there are no sudden jumps as observed by Maull (15). Near the bottom of the model the peak of the spectrum occurs at a Strouhal number of 0.105. This Strouhal number gradually increases with height until at 0.643 of the model height the peak of the spectrum occurs at a Strouhal number of 0.125. Under the influence of the tip the location of the spectral peak again reverts to a lower dimensionless frequency. Another noticeable trend is an increase in the magnitude of the peak as the distance from the floor increases. It is perhaps interesting to note that the Strouhal number and the local lateral forces exhibit similar variations along the height of the model.

Vickery in (13, 16) deduced values of average Strouhal number by measuring the spectra of load fluctuations on finite and infinite square shapes in turbulent and smooth flow. For the finite square shapes tested in a turbulent stream of 10% intensity in (13), the values
of S varied between 0.10 and 0.12 all of which are lower than the two-dimensional smooth flow value of 0.135 measured by Smith (1). For the same velocity vortices are shed at a lower frequency in a turbulent flow than in smooth flow. The lower shedding frequency indicates that in a turbulent flow the square section as Laneville proposed, has characteristics of a section which has an increased afterbody length, (i.e. ratio of b/h > 1).

If the assumptions in the quasi-steady theory are correct then approximately the same value of shedding frequency should be measured behind the galloping 28" model as for the rigidly mounted section (1). In Fig. 22 are plotted the power spectra of velocity fluctuations versus the dimensionless frequency based on the velocity at .25". As can be seen the peak of the spectrum moves depending on the velocity of the wind. The peak of the spectrum does not occur at the stationary value of the Strouhal number but occurs at the natural frequency of the model. The stationary Strouhal number is not readily discernible in any of the spectra over the velocity range tested. These results indicate that most of the power in the wake comes from the motion of the model. The wake dynamics resulting from the motion of the model obviously have an important effect on the galloping behavior of a bluff section but are not readily comparable to the wake dynamics behind a rigidly mounted section.

In (17) the authors concluded that the quasi-steady formulation of "negative aerodynamic damping" did not have a significant effect on the level of cross-wind response of tall rectangular buildings for
Figure 22. Power spectra of the longitudinal velocity fluctuations in the wake of the balloting 28" model.
reduced velocities, \((V_\ell/f_h)\), up to 10. Part of the basis for their conclusion was the analysis of wake spectra in which they observed prominent peaks at the stationary value of the Strouhal number indicating that the building motion is primarily wake excited. However, from Fig. 22 it is obvious that for reduced velocities greater than 10 the level of motion does have an effect on the wake dynamics as the wake becomes predominantly model-excited.
CHAPTER 5

SOME EFFECTS OF ASPECT RATIO ON THE FORCE COEFFICIENTS

The galloping characteristics of a bluff section are determined by the variation of the lift and drag coefficients with angle of attack. Thus basic investigations into the parameters effecting the drag and lift coefficients will yield knowledge about the galloping mechanism of a bluff body. The purpose here is to investigate the surprising effects of aspect ratio on the mean lift coefficients measured in Chapter 4. Two simple experiments were conducted to see if a change in aspect ratio would effect a change in the lift coefficients.

5.1 First experiment The first experiment was conducted in the industrial aerodynamics wind tunnel using the same roughness as for all previous investigations. Lift and drag coefficients were measured on two square towers of height 36 and 44 inches using the instrumentation as previously outlined in Chapter 3. The towers which had aspect ratios of 18 and 22 extended into the smooth flow well above the turbulent boundary layer on the wind tunnel floor. The tunnel height at the location of the tests was 74" so the models tested were still three-dimensional in character.

The variation of lift coefficient with angle of attack for the two models is presented in Fig. 23, and as a comparison the lift coefficients obtained for the 28" model are presented in the same figure. The average lift coefficient plotted in Fig. 23 was obtained from the
Figure 23. Variation of average lift coefficient with angle of attack for three models of high aspect ratio.
expression

\[ C_L = \frac{L}{h \int_0^h \frac{\rho}{2} V(z)^2 dz} \]

where now the function \( V(z) \) is a power law variation for the first 28" and is taken as smooth flow from 28" to the model height. As can be seen from Fig. 23, the only effect of increased aspect ratio observed here is to increase the maximum lift coefficient and the angle at which it occurs. The initial trend at small angles of attack remains invariant with aspect ratios up to at least 22. The results here emphasize the proposition that turbulence merely serves to reattach the flow at a lower angle of attack, but does not otherwise alter the lift coefficients of square cylinders.

5.2 Second experiment  The second experiment was conducted in the U.B.C. low speed, low turbulence, return-type wind tunnel which has an overall turbulence level of < .1%. The test section is 9' long with a cross section 27" high by 36" wide. The wind speed was measured with a Betz micromanometer located at the end of the tunnel contraction. Other pertinent details concerning this wind tunnel can be found in (3).

Lift and drag coefficients for various angles of attack were measured on four 2" by 2" square sections. The force measuring instrumentation was as used in Chapter 3. The square sections mounted vertically in the wind tunnel had heights of 12", 24", 26" and one section completely spanned the 27" test section giving two-dimensional conditions. The graphs of drag, lift and lateral force coefficients versus angle of
attack for the four models can be found respectively in Figs. 24, 25 and 26. The data is presented uncorrected for wind tunnel wall effects.

The drag and lift coefficients were defined in their standard forms as

\[ C_D = \frac{D}{\frac{1}{2} \rho V^2 h \ell} \quad ; \quad C_L = \frac{L}{\frac{1}{2} \rho V^2 h \ell} \]

and the lateral force coefficient \( C_{Fy} \) was computed from the quasi-steady assumption as in equation (3) in Appendix 1.

The variation of drag coefficient with angle of attack for the four different models exhibits the same trends as the drag coefficients measured in the turbulent boundary layer in Chapter 4. From Fig. 24 it can be seen that the drag coefficients fall rapidly as the aspect ratio decreases and it appears that the angle at which the minimum drag occurs increases slightly. The value of the drag coefficient at \( \alpha = 0 \) for the two-dimensional model is representative of what others have measured for a square section (18).

The lift coefficient variation is plotted in Fig. 25. As can be seen the model must be nearly two-dimensional to effect a change in the lift curve slope. The 26" model spans 0.96 of the tunnel height yet the lift curve slope in this case is only slightly steeper than the lift curve slope for the 24" and 12" models. Comparing the lift curve slope of the 24" and 12" models to the same curves measured for models in the turbulent boundary layer, Figs. 12 and 23, it is observed that for small angles the slopes are nearly identical. The important three-dimensional effects on the lift coefficient slope must occur suddenly with a change from two-dimensional to three-dimensional conditions. The main effect of aspect ratio away from the tunnel roof is to reduce the maximum lift
Figure 24. Effect of aspect ratio on the drag coefficient of square prisms in smooth flow
Figure 25. Effect of aspect ratio on the lift coefficient of square prisms in smooth flow
Lateral force coefficient

$C_{Fy}$

0.1
0.2
0.3
0.4

Figure 26. Effect of aspect ratio on the lateral force coefficient of square prisms in smooth flow.
coefficient leaving the angle at which it occurs relatively the same.

The lateral force coefficient versus $\tan \alpha$ is plotted in Fig. 26. The variation of lateral force coefficient with angle of attack for the two-dimensional model is nearly the same as what Smith (1) measured. The other three curves bear evidence of the reduced lift coefficients under three-dimensional smooth flow conditions. By examining Fig. 26 it is observed that the lateral force coefficient curve for the 12" model has a steeper slope at the origin as compared to the same curve for the 24" model which is zero or slightly negative for small angles. Thus the 12" model should be more unstable than the taller 24" model under these flow conditions. This surprising result comes about because the drag coefficient falls as the aspect ratio decreases and the lift coefficient remains relatively the same.
6.1 Full scale interpretation of model results  

Interpretation of the model results for a full scale structure is shown in Table II. The geometric length scale is taken as the ratio of full scale turbulence to wind tunnel turbulence at 2/3 of the model height (10, 11, 12). Using this length as a scaling parameter both models are seen to represent tall slender structures between 800 and 1300 feet high.

Under practical considerations the most important design quantity is the onset velocity of galloping. The quasi-steady theory alone does not always accurately predict the onset velocity, due to the vortex shedding mechanism. However, a conservative design calculation can be made by assuming from Figs. 18 and 19 that the onset velocity is close to a dimensionless reduced velocity, $U^*_\lambda \sqrt{h/\beta}$ equal to 0.4. Thus the onset velocity at the height of the structure, for a square section, is given by the simple equation.

$$V_* = \frac{(.4)(2\pi) fh^4 \beta m}{\rho}$$

Now from reference (19) an approximate formula for the first natural frequency of a building in terms of height and lateral dimensions is

$$f = \frac{\sqrt{h}}{0.05\lambda}$$

where $h$ and $\lambda$ are in feet. Thus the onset velocity can be easily found.
<table>
<thead>
<tr>
<th></th>
<th>Suburban or Forested Exposure</th>
<th>Model</th>
<th>Full Scale</th>
<th>Model</th>
<th>Full Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Wind Profile Exponent</td>
<td></td>
<td>0.264</td>
<td>0.28</td>
<td>0.264</td>
<td>0.28</td>
</tr>
<tr>
<td>Scale of Turbulence at Height $2/3$</td>
<td></td>
<td>0.942 ft.</td>
<td>475 ft.</td>
<td>1.03 ft.</td>
<td>560 ft.</td>
</tr>
<tr>
<td>Intensity of Turbulence at Height $2/3$</td>
<td></td>
<td>12.5%</td>
<td>13.4%</td>
<td>9.8%</td>
<td>10%</td>
</tr>
<tr>
<td>Tower Height</td>
<td></td>
<td>1.67 ft.</td>
<td>833 ft.</td>
<td>2.33 ft.</td>
<td>1260 ft.</td>
</tr>
<tr>
<td>Frequency °/s</td>
<td></td>
<td>0.219</td>
<td></td>
<td>0.153</td>
<td></td>
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<tr>
<td>Onset Velocity mph*</td>
<td></td>
<td>87.4</td>
<td></td>
<td>65.0</td>
<td></td>
</tr>
</tbody>
</table>

*Computed from equation 12 with $\beta = .005$ and an average density of $\rho_m = 9 \text{ lb/ft}^3$. Length scales are as given above.
from the equation

\[ V = 201 h \frac{3/2 \rho}{2 \rho_m} \]  

The designer is thus left with two unknown building properties the average density and the fraction of viscous damping. The average density can usually be accurately determined and can vary between 9 and 20 lb/ft\(^3\) depending upon the construction material. The fraction of critical damping however, is usually relatively unknown. Full scale measurements (20, 21) indicate that most modern buildings are very lightly damped with the fraction of viscous critical damping varying between 0.005 and 0.05. It should also be noted that often structural damping is not entirely viscous but involves some hysteretic action (21). In (22) the effects of a non-linear hysteretic damping on the galloping oscillation of a 2/1 rectangle were investigated. Due to the nature of the damping term considered there was no effect on the onset velocity but at higher reduced velocities increasing non-linearity caused the galloping amplitudes to decrease.

Obviously low values of damping and density are required for galloping to be a consideration. It should be mentioned that the velocity predicted by equation 12 is the velocity at the height of the structure which corresponds to a much lower value of ground level wind.

6.2 Conclusions

1) The turbulent boundary layer produced with this set of roughness has a velocity profile, turbulence intensity distribution and spectral distribution of energy similar to a suburban or forested full-
scale exposure. The 28" and 20" models have a geometric scale of about 1/500.

2) The mean lift curve slope of finite square prisms appears to be unaffected by model aspect ratio in smooth and turbulent flow for height to width ratios between 22 and 6.

3) The lateral force coefficient which characterizes the galloping behavior of a bluff section exhibits distinctive changes along the span of a model in a sheared turbulent flow.

4) Galloping responses computed from the local lateral forces yield distinctly higher amplitudes than the response computed from the simpler average lateral forces. To properly apply the quasi-steady theory to finite bluff shapes in turbulent flow should include the local variation of lateral forces.

5) The experimental galloping amplitudes for different damping ratios collapse reasonably well on to the predicted response calculated from the local forces. The quasi-steady assumption applies best when the reduced velocity at which galloping occurs is appreciably higher than the velocity at which vortex induced resonance takes place.

6) The Strouhal number based on the local velocity, obtained from the peaks of wake velocity spectra, has a slight variation along the height of the rigidly mounted 28" model. The values of Strouhal number measured are lower than the two-dimensional smooth flow values for a square section.
BIBLIOGRAPHY


Force calculations  To calculate the lateral force from the measurements of lift, \( L \), and drag, \( D \), consider the geometry in figure (a). By taking the proper components of lift and drag the total lateral force, \( F_y \) is given by
\[
F_y = -L \cos \alpha - D \sin \alpha \quad \ldots \ldots \ldots \ldots \ldots \quad (1)
\]
In a boundary layer flow the velocity varies with height and the need arises to define an "average" force coefficient. The average lift and drag coefficients used here, non-dimensionalized by the relative dynamic pressure are respectively
\[
\bar{C}_L = \frac{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz}{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz} ; \quad \bar{C}_D = \frac{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz}{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz} \quad \ldots \ldots \ldots \ldots \ldots \quad (2)
\]
The average lateral force coefficient is similarly defined as
\[
\bar{C}_{Fy} = \frac{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz}{\int_0^b \phi \frac{v^2}{2} \text{rel}(z) \, dz}
\]
Noting the fact that
\[
V(z) = V_{\text{rel}}(z) \cos \alpha
\]
it is a simple matter to show that in coefficient form equation (1) becomes

\[ C_{Fy} = -C_L \sec \alpha - C_D \tan \alpha \sec \alpha \]  

(3)

The average lift and drag coefficients defined by equation (2) were calculated by assuming the velocity variation in the boundary layer was described by a power law profile of the form

\[ V = V_G \left( \frac{z}{\delta} \right)^\gamma \]  

(4)

where \( \gamma, \delta \) and \( V_G \) are respectively the power law exponent, the boundary layer thickness and the velocity at the top of the boundary layer. For this analysis a more convenient reference point is the tip of the model, i.e. \( z = \ell \). The velocity at the reference height being

\[ V_{\ell} = V_G \left( \frac{\ell}{\delta} \right)^\gamma \]

and at all other heights is

\[ V = V_{\ell} \left( \frac{z}{\ell} \right)^\gamma \]  

(5)

Substitution of (5) into the expressions for the lift and drag coefficients, equation (2), and subsequent integration yielded the average lift and drag coefficients used throughout this study.

**Force calculations from the pressure measurements** In order to determine the variation of the lateral force coefficient along the height of the model local pressures were measured and then integrated. Considering the geometry in figure (b) the lateral force per unit length is
\[ \text{fy}(z) = \int_0^b (P_U - P_L) \, dx \] \hspace{1cm} (6)

The pressures on the upper and lower surfaces of the model were put into coefficient form as

\[ C_{PU,L} = \frac{P_{UL} - P_\infty}{\frac{\rho}{2} V_{rel}^2(z)} \] \hspace{1cm} (7)

and the lateral force coefficient per unit length was defined as

\[ C_{fy}(z) = \frac{fy(z)}{\frac{\rho}{2} V_{rel}^2(z) h} \] \hspace{1cm} (8)

Substitution of equations (7) and (8) into (6) yields the equation for the local lateral force coefficient as

\[ C_{fy}(z) = \sec^2 \alpha \int_0^1 (C_{PU} - C_{PL}) \, dx \] \hspace{1cm} (9)

The pressure coefficients were measured at seven locations on both the upper and lower faces of the model. A cubic spline curve fitting routine was used to smooth the discrete points together and a form of Simpson's rule was then implemented to perform the integration in equation (9).
APPENDIX 2

The average aerodynamic constants, $\overline{A}_i$, in the equation

$$C_{Fy} = \sum_{i=1}^{5} \overline{A}_i \tan \alpha_i$$

for the 28" and 20" models are given in the following table

<table>
<thead>
<tr>
<th>Model</th>
<th>$\overline{A}_1$</th>
<th>$\overline{A}_2$</th>
<th>$\overline{A}_3$</th>
<th>$\overline{A}_4$</th>
<th>$\overline{A}_5$</th>
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<tr>
<td>28&quot;</td>
<td>1.37</td>
<td>-9.13</td>
<td>110</td>
<td>-561</td>
<td>772</td>
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<tr>
<td>20&quot;</td>
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<td>-278</td>
<td>718</td>
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</table>

The matrix of constants $A_{i,j}$ found for the local lateral force coefficients, i.e.

$$C_{Fy} = \sum_{i=1}^{5} \tan \alpha_i \sum_{j=0}^{2} A_{i,j} \left(\frac{z}{\lambda}\right)^j$$

are given in the following two tables

**TABLE IV**

**CONSTANTS FOR THE 28" MODEL**

<table>
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<td>2</td>
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<td>4</td>
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<tr>
<td>5</td>
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### TABLE V

**CONSTANTS FOR THE 20" MODEL**

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