SIMPLE PION-NUCLEON AND PION-NUCLEUS POTENTIALS

WITH APPLICATIONS OF THE DOORWAY MODEL

TO PION-NUCLEUS SCATTERING

by

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ABSTRACT

The pion-nucleon total cross section is reproduced in the pion kinetic energy range of zero to 250 MeV by a simple Yukawa potential with the depth and width as free parameters. This potential is then averaged over a nuclear matter distribution in an attempt to produce a pion-nucleus potential for pion-$^{12}\text{C}$ scattering. The model is explored by varying several parameters in the potential and observing their effect on the differential cross section for pion-$^{12}\text{C}$ scattering with the pion kinetic energy equal to 50 MeV. No completely satisfactory fit to the data is found. It is suggested that treating this system as a pion-nucleon system interacting in the average field of the residual nucleus may give better results and some ideas are presented as to how this might be done. Applications of such a treatment of pion-nucleus interactions to the description of pion-nucleus "doorway" states (so far, apart from the $N^*$, unobserved) are discussed.
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I. INTRODUCTION

Most of the work that has been done in the theoretical treatment of pion-nucleus scattering is characterized by a formal, mathematical approach. Many approximations are made, and one is left with an uneasy feeling because it is very difficult to picture in simple terms what is actually happening in the maze of equations. What we intend to do here is to present a simple picture of pion-nucleus scattering, and to see what one can learn while avoiding as much formalism as possible. We will find that we cannot fit the data, which is not surprising, but we wish to point out that the formal approaches to this problem often fare no better than we.¹

II. CONSTRUCTION OF A PION-NUCLEON POTENTIAL

We will first review the method of obtaining cross sections from a given potential.

The wave equation which describes the pion's interaction with a nucleon is the Klein-Gordon equation with Coulomb and nuclear potentials added. The equation is derived by replacing the momentum p of the particle by the quantum mechanical operator $-i\hbar \nabla$ in the equation

\[ E^2 = p^2c^2 + m^2c^4 \]

The Coulomb potential may be included in a covariant manner by the substitutions

\[ E + E' - e\phi \quad \text{and} \quad cp + cp' - eA \]
Here $e$ is the charge of the particle, $\phi$ is the electrostatic potential and $A$ is the vector potential. We will take $A = 0$ here. To include a strong interaction potential in (1) we should know its Lorentz transformation properties. However, since these are not known we have no way of determining which part of (1) is the one into which the potential should be placed. It is a common assumption that the strong potential, which we call $v$, can be included with the energy term:

(3) \[ E \rightarrow E - e\phi - v \]

The Klein-Gordon equation is thus

(4) \[ \{-\hbar^2 c^2 \nabla^2 + m^2 c^4\} \psi = (E - e\phi - v)^2 \psi \]

The terms $2e\phi v$ and $v^2$ are usually dropped from the right-hand side of (4). This cannot be justified on any physical grounds (unless $E \gg e\phi >> v$ which is not the case in what follows) but since the location of the potential is already somewhat arbitrary we will accept this approximation. Equation (4) then becomes

(5) \[ \{-\hbar^2 c^2 \nabla_r^2 + \frac{e^2 L^2}{r^2} + m^2 c^4\} \psi = (E^2 - 2e\phi E + e^2 \phi^2 - 2Ev) \psi \]

where $\nabla^2_r$ is the radial part of the Laplacian:

(6) \[ \nabla^2_r \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) \]

and $L^2$ is the square of the orbital angular momentum operator.
If we define
\[ \psi_{\ell}(r) \equiv \frac{u_{\ell}(r)}{r} Y_{\ell m}(\theta, \phi); \quad U_{N} \equiv \frac{e\varphi}{\hbar c}; \quad \xi \equiv \frac{E}{\hbar c}; \]
(7)
\[ U_{N} \equiv \frac{v}{\hbar c}; \quad k^{2} \equiv \left( \frac{p}{\hbar} \right)^{2} = \frac{E^{2} - m^{2}c^{2}h}{\hbar^{2}c^{2}} \]
we can write (5) for a state of definite \( \ell \) as

(8) \[ u_{\ell}'' + \left( k^{2} + U_{c}^{2} - 2\ell(U_{c} + U_{N}) - \frac{\ell(\ell + 1)}{r^{2}} \right) u_{\ell} = 0 \]

Equation (8) is integrated outward from the origin to some cut-off radius \( R \), where \( U_{N} \) can be set equal to zero for all \( r>R \). Thus for \( r>R \), \( u_{\ell} \) will be a solution to the pure Coulomb scattering problem which, for non-relativistic kinematics, is a linear combination of the irregular and regular Coulomb wave functions \( F_{\ell} \) and \( G_{\ell} \):

(9) \[ u_{\ell} = \exp(i\delta_{\ell})(F_{\ell}\cos \delta_{\ell} + G_{\ell}\sin \delta_{\ell}) \]
\[ = \frac{1}{2i}(\exp(2i\delta_{\ell}) - 1)F_{\ell} + \frac{1}{2i}(\exp(2i\delta_{\ell}) - 1)G_{\ell} \]
\[ r \geq R \]

For a description of the properties of \( F_{\ell} \) and \( G_{\ell} \) see reference 2. The quantity \( \delta_{\ell} \) is the phase shift due to \( U_{N} \). Then the continuity requirement together with equation (9) give

(10) \[ \exp(2i\delta_{\ell}) = \frac{D_{\ell}(F_{\ell} + iG_{\ell}) - (F_{\ell} + iG_{\ell})}{(F_{\ell} + iG_{\ell}) - D_{\ell}(F_{\ell} - iG_{\ell})} \]
\[ r=R \]

where \( D_{\ell}(r) = u'_{\ell}(r)/u_{\ell}(r) \) for \( r \geq R \). The use of non-relativistic in place of relativistic kinematics greatly simplifies the analysis of the Coulomb wave functions without introducing significant errors, as has been checked.
The pion nucleon interaction depends on the total angular momentum $J$ and the total isospin $I$ of the pion-nucleon system as well as on $\ell$.

Since the pion has spin zero and the nucleon has spin $\frac{1}{2}$, $J$ can have only two values for each $\ell$: $J = \ell \pm \frac{1}{2}$. The pion isospin is $I_\pi = 1$ and the nucleon isospin is $I_N = \frac{1}{2}$ so $I$ can be either $\frac{1}{2}$ or $3/2$.

The complete expression for the differential cross section $\sigma(\theta)$ is

$$\sigma(\theta) = |f(\theta)|^2 + |g(\theta)|^2$$

where

$$f(\theta) = f_c(\theta) + \sum_{\ell=0}^{\infty} \exp(2i\sigma_\ell) \left\{(\ell+1)f_{\ell+} + \ell f_{\ell-}\right\} P_\ell (\cos \theta)$$

$$g(\theta) = \sum_{\ell=0}^{\infty} \exp(2i\sigma_\ell) \left\{f_{\ell+} - f_{\ell-}\right\} \sin \theta P_\ell'(\cos \theta)$$

The Coulomb amplitude $f_c(\theta)$ is

$$f_c(\theta) = \frac{-\eta}{2k \sin^2(\theta/2)} \exp\{2i(\sigma_0 - \eta \ln(\sin\theta/2))\}$$

(11)

$$\eta = \frac{Z_\pi Z_N m_\pi}{\hbar c}$$

$$\sigma_\ell = \arg \Gamma(\ell+1+i\eta)$$

where $Z_\pi$ is the pion charge, $Z_N$ the nucleon charge, $m_\pi$ the relativistic center of mass frame pion mass, $\theta$ the center of mass scattering angle.

The nuclear amplitudes $f_{\ell \pm}$ are defined in terms of individual amplitudes $f_{\ell \pm}^{2I}$ for each isospin channel. For the case of scattering in an $I = 3/2$ channel (e.g. $\pi^+ p$ or $\pi^- n$) we have $f_{\ell \pm} = f_{\ell \pm}^3$ while for $\pi^- p$ or $\pi^+ n$ scattering $f_{\ell \pm} = (f_{\ell \pm}^3 + 2f_{\ell \pm}^1)/3$.

The $f_{\ell \pm}^{2I}$ are defined in terms of a phase shift $\delta_{\ell \pm}^{2I}$ as follows:

$$f_{\ell \pm}^{2I} \equiv \frac{1}{2ik} \left(\exp 2i\delta_{\ell \pm}^{2I} - 1\right)$$
The phase shifts are determined from equation (10) where $D_\lambda$ must now be considered as depending on $J$ and $I$ as well as $\lambda$, i.e. a different potential $v$ is required in eqn. (5) for each combination of $\lambda$, $J$ and $I$.

Since the $l=1, J=I=3/2$ resonance (the $(3,3)$ resonance) dominates pion-nucleon scattering at low energies, we will assume that the corresponding potential $v_{P33}$ is the only one that is non-zero here. This simplifies the cross section formula considerably:

$$f(\theta) = f_c(\theta) + 2\exp(2i\sigma_1)f_{1+}^3 P_1(\cos \theta)$$

$$g(\theta) = \exp(2i\sigma_1)f_{1+}^3 \sin \theta P_1'(\cos \theta)$$

We shall consider $\pi^-n$ scattering so that all Coulomb effects disappear: $f_c=\sigma_1=0$. Then, using $P_1(\cos \theta)=\cos \theta$ and $P_1'=1$ we have

$$\sigma(\theta) = |f(\theta)|^2 + |g(\theta)|^2 = |f_{1+}^3|^2(3\cos^2 \theta+1)$$

The total cross section is then

$$\sigma_T = 2\pi \int_0^\pi \sigma(\theta) \sin \theta \, d\theta = \frac{4\pi}{k^2}(1-\cos^2 \delta_{1+})$$

If Coulomb scattering is present, as in a pion-nucleus interaction, the total cross section becomes infinite. Differential cross sections can still be calculated and compared with data, however. The relevant formulas are \(^5\) (for a spin-zero, isospin-zero nucleus such as $^{12}$C):

$$\sigma(\theta) = |f_c(\theta) + f_N(\theta)|^2$$

$$f_N(\theta) = \frac{-i}{2k} \sum_{\lambda} (2\lambda+1)\exp(2i\sigma_\lambda)(\exp(2i\delta_\lambda) - 1)P_\lambda(\cos \theta)$$

where the Coulomb amplitude $f_c$ and Coulomb phase shifts are defined in (11).
Let us now try to find a simple potential which reproduces the observed pion-nucleon cross section at low energies.

For the pion-nucleon potential $v_{p33}(r)$ (henceforth called $v(r)$), we will assume a Yukawa interaction with two free parameters:

\begin{equation}
(14) \quad v(r) = -V_0 \frac{\exp(-r/r_n)}{r/r_n}
\end{equation}

The depth parameter $V_0$ is positive and has dimensions of energy; the width parameter $r_n$ has dimensions of length. We will attempt to adjust $V_0$ and $r_n$ to fit the existing $(3,3)$ resonance data for cross sections.

Because $v(r)$ is real, $\delta^3$ is real and $|S^3| = 1$. What this means physically is that the scattering is purely elastic. This is a very good approximation for the $(3,3)$ resonance since the branching ratio for decay into the elastic channel is $> 0.99$.

In order to calculate values of $V_0$ and $r_n$ which give an acceptable fit to the $\pi N$ scattering data the program PIRK$^3$ was modified to calculate the phase shift for the potential $v(r)$ in (14). The quantity $\sigma_T/4\pi$ was calculated from the phase shift $\delta_{1+}^3$ using (12) for various values of $V_0$ and $r_n$ in the energy range $0 < T_\pi < 350$ MeV where $T_\pi$ is the lab kinetic energy of the incoming pion.

In order to compare the calculation with the data, an analysis of $\pi N$ scattering data by Rowe, Salomon and Landau$^7$ was used. We will summarize the analysis here.

The $\pi N$ S-matrix is factorized into a non-resonant part $S_b$ (b for background) and a resonant part $S_r$: $S = S_bS_r$. Using a relativistic Breit-Wigner form for the resonant part and an effective range approximation for the non-resonant part one finds the equation
Here $q$ is the center of mass pion or nucleon momentum, $x$ is the branching ratio for the decay of a $\pi N$ resonance into the elastic channel ($x$ depends on $\ell$ and $I$), $\omega_0$ is the total center of mass energy at resonance, $q_0$ is the center of mass momentum of the pion at resonance, and $\omega$ is the total center of mass energy corresponding to momentum $q$:

\[
\omega = \sqrt{q^2 + m_\pi^2} + \sqrt{q^2 + m_N^2}
\]

The parameters $a$, $b$, and $c$ are obtained by fitting (15) to experimentally determined phase shifts.

For our purposes only the $J=3/2$, $I=3/2$ P-wave phase shift (denoted by $\delta^{3/2}_{3/2}$) is needed. The values of $a$, $b$, and $c$ obtained in the least squares fit (15) together with the resonance parameters are given in Table I.

\[
\begin{align*}
\text{TABLE I} \\
\text{PARAMETERS USED IN PHASE SHIFT CALCULATION} \\
a &= 4.03 \times 10^{-8} \text{ (MeV/c)}^{-3} & x &= 0.99 \\
b &= -2.79 \times 10^{-13} \text{ (MeV/c)}^{-5} \\
c &= 7.26 \times 10^{-19} \text{ (MeV/c)}^{-7} \\
\Gamma_0 &= 116 \text{ MeV} \\
\omega_0 &= 1233 \text{ MeV}
\end{align*}
\]

It was found that $V_0 = 2110$ MeV and $r_n = 0.52$ fm gave good agreement with the least squares fit for $T_\pi < 250$ MeV. In Figure 1 the quantity $(1 - \cos 2\delta)/k^2$ is plotted against $T_\pi$ for both calculations.

In order to check that this potential:

\[
v(r) = -2110 \exp(-r/0.52 \text{ fm}) \text{ MeV}
\]
FIGURE 1

COMPARISON OF YUKAWA WELL CROSS SECTION WITH DATA

\[ \frac{\sigma_T}{4\pi} = \frac{(1-\cos 2\delta_1)/k^2}{k^2} \]

Yukawa Well

Least Squares Fit to Data

\( T_\pi \) (MeV)
has the (3,3) resonance as its first resonance rather than some higher resonance the wave function $u_1(r)$ (as in (8) with $\ell = 1$) was calculated for $T_n = 190$ MeV, the resonance energy. It was found that there is one node for $r<0.52$ fm (the one for $r = 0$) indicating that this is indeed the first resonance.

However, the potential (17) does NOT give the correct phase shifts for S-wave scattering. There is a broad S-wave resonance which would if included, greatly increase the $\pi N$ cross section. If one wished, one could attempt to find a new set of parameters $(V_0$ and $r_n)$ to produce a more accurate representation of the S-wave scattering, but there is little value in doing this here since the actual S-wave cross section at this energy is much less than the P-wave. What the calculation shows is that if one tries to reproduce $\pi N$ scattering using a simple local potential the potential depends on $\ell$.

### III. CONSTRUCTION OF A PION-NUCLEUS POTENTIAL

The model for pion-nucleus scattering that we will try is the following: we assume a pion nucleus potential of form

$$V(r) = \int \rho(r') v(|r-r'|) d^3r'$$

Here $\rho(r)$ is the density of nuclear matter in the target nucleus and $v$ is the pion-nucleon potential (17). This potential makes several assumptions:

(i) It assumes that the nucleus is a static matter distribution. This is not correct since the individual nucleons have a Fermi motion due to the exclusion principle which, especially for low energy pion scattering, can be greater than the energy of the pion. However, if the pion-nucleus potential is not momentum dependent the Fermi motion does not matter,
since the nucleus as a whole is at rest. The energy dependence of the cross section then depends entirely on the positions of the resonances in a fixed potential.

(ii) The pion is always a free particle in this description. We will see later that this a bad approximation. It is here that the doorway model can improve matters by treating the N* separately from the average pion-nucleus interaction.

(iii) Since $V$ is real, only elastic scattering is permitted. Absorption is known to occur within the nucleus and this requires the addition of an imaginary term to the potential. This will be done to show the effect of absorption on the differential cross section.

(iv) $v(r)$ was derived for the $l = 1$ partial wave only and is known to be incorrect for $l = 0$. A proper calculation of pion-nucleus scattering requires the inclusion of all $l$. We can attempt to compensate for this by varying the depth of the potential. The rationale for this is that to produce a minimum in the S-wave cross section for pion-nucleon scattering we would have to vary the depth $V_0$ of the potential (14). Thus to account for the presence of both S and P waves in the pion-nucleus potential (18) we could adjust its depth.

Given all these drawbacks one would hardly expect this approach to reproduce the data well and in fact it does not as we will see. But the interesting thing is that the potential (18) (with absorption added) gives qualitatively just as good (or bad) a curve as several standard potentials (Kisslinger, Laplacian, etc.). Figure 2 (obtained from reference 8) shows the cross sections obtained for $\pi^+$ on $^{12}$C at $T_\pi = 50$ MeV from three standard potentials. The parameters used in the potentials were obtained from pion-nucleon phase shifts. Using a least squares proc-
Pion-nucleus cross sections predicted by some standard potentials compared with the data from reference 8. (LMM = Londergan-McVoy-Moniz). For details of the calculation of these curves see the references contained in reference 8.
edure, it is possible to fit the data with these potentials, but the resulting parameters are unphysical. Further details can be found in reference 8 and references contained therein.

We now describe the calculation of (18). The pion-nucleon potential will be

\begin{equation}
\nu(|r-r'|) = -V_0 \frac{\exp\left(-\sqrt{r^2+r'^2-2rr'\cos \theta}/r_n\right)}{\sqrt{r^2+r'^2-2rr'\cos \theta}/r_n}
\end{equation}

where $\theta$ is the angle between $r$ and $r'$.

For the nuclear density we will take the Fermi form

\begin{equation}
\rho(r') = \frac{P}{1 + \exp((r'-c)/t)}
\end{equation}

c and $t$ are radius and thickness parameters for the target nucleus. A plot of the density function is given in Figure 3 for $^{12}$C. $P$ is a normalization constant determined from the requirement that

\[ \int \rho(r')d^3r' = A \]

where $A$ is the mass number of the nucleus. This gives for $P$:

\begin{equation}
P = \frac{A}{4\pi} \left( \frac{c^3 + ct^2\pi^2}{3} - 2t^3 \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3} \exp(-nc/t) \right)^{-1}
\end{equation}

Substituting (19) and (20) into (18) gives after doing the angular integration:

\[ V(r) = 2\pi V_0 r_n^2 \int_0^\infty \frac{r'/r}{1+\exp((r'-c)/t)} \left[ \exp(-|r-r'|/r_n) - \exp(-(r+r')/r_n) \right] dr' \]

or
FIGURE 3

NUCLEAR DENSITY (EQ. (20))

Parameters:
\[ P = 0.0176 \quad (A = 10) \]
\[ c = 4.5 \text{ fm} \]
\[ t = 1 \text{ fm} \]
\begin{align*}
V(r) &= -2\pi V_0 P r_n^2 \frac{\exp(-r/r_n)}{r} \left[ \int_0^{\infty} \frac{r' \sinh(r'/r_n)}{1 + \exp((r'/c)/t)} \, dr' + \right. \\
&\hspace{2cm} \left. (\exp(2r/r_n)-1) \int_0^{\infty} \frac{r' \exp(-r'/r_n)}{1 + \exp((r'/c)/t)} \, dr' \right]
\end{align*}

The second integral can be given finite limits by the transformation $x = 1/r'$, so the final expression for $V(r)$ is

\begin{align*}
V(r) &= -2\pi V_0 P r_n^2 \frac{\exp(-r/r_n)}{r} \left[ \int_0^{\infty} \frac{r' \sinh(r'/r_n)}{1 + \exp((r'/c)/t)} \, dr' + \right. \\
&\hspace{2cm} \left. (\exp(2r/r_n)-1) \int_0^{1/r} \frac{1/x \exp(-1/x r_n)}{x^3(1 + \exp(1/xt - c/t))} \, dx \right]
\end{align*}

Despite the apparent $1/r$ behaviour of $V(r)$ for small $r$ it is actually finite for $r = 0$. Note that the first integral vanishes for $r = 0$ while the second term behaves like $2r/r_n$ multiplied by a constant (the integral). Thus

\begin{align*}
V(0) &= -4\pi V_0 P r_n \int_0^{\infty} \frac{\exp(-1/x r_n)}{x^3(1 + \exp(1/xt - c/t))} \, dx
\end{align*}

A graph of $V(r)$ is given in Figure 4 for a particular set of parameters.

Now that we have the potential we will see how close to the data the resulting cross sections are. According to the arguments above, we can vary the depth of the potential somewhat because of the omission of the $nn$ S-wave component. We will do this by varying $P$, the normalization of the nuclear matter density. Since we have also ignored the isospin dependence of the pion-nucleus potential, this parameter is already rather arbitrary. If we consider only $I=3/2$ scattering in $\pi^+$ scattering then $P$ should be normalized to include the protons plus only one third of the neutrons (due to Clebsch-Gordon algebra). If we assume $V(r)$ to be isospin
FIGURE 4

PION-NUCLEUS POTENTIAL (EQ. (22))

Parameters:
$V_0 = 2110$ MeV
$A = 10$ nucleons ($P = 0.0176$ fm$^{-3}$)
$r_n = 0.52$ fm
$c = 4.5$ fm
$t = 1$ fm
independent then \( P \) should be normalized to \( A \). Thus by varying \( P \) we are using it as a garbage dump to dispose of the uncertainties in \( V(r) \).

The reader may be feeling rather uneasy at this point. We have made a huge number of approximations to get this far—many of them physically unrealistic. But the point of what we are doing is not to derive the pion-nucleus interaction from first principles—only a theory of the strong interaction could do that—we are starting with a very simple idea and just seeing how close to reality we can make it come. Many attempts have been made to fit pion-nucleus data but none have been completely successful, and all attempts involve approximations that are very difficult to understand physically. This is not to say that the approximations are wrong—it is just that it is difficult to say exactly what part of the physical system we are ignoring when we make an approximation in a mathematical expression that is several steps removed from physical intuition.

What we present in the next few figures are several plots of differential cross sections for \( \pi^+\text{-}^{12}\text{C} \) elastic scattering. In each graph we present three curves as well as the data points (obtained from reference 8). The three curves in each plot show the effect of varying one parameter in (22) while all others are held fixed. In all curves the \( \pi^+ \) is incident with a lab kinetic energy of 50 MeV. All cross sections and scattering angles are measured in the center of momentum frame.

In most of the plots absorption has been added. This consists of adding a purely imaginary term to (22) of one of two forms:

(i) The standard Kisslinger form

\[
V_{\text{abs}}(r) = -A k^2 b_o \rho(r) + A b l V^*(\rho(r)V)
\]

(ii) The local Laplacian form
Here $b_0$ and $b_1$ are imaginary parameters.

The calculations were done by modifying the program PIRK to accept the potential (22) as well as a Coulomb potential appropriate to a charge distribution of the form (20). For reference this Coulomb potential is:

\[
V_c(r) = \frac{1}{r} \int_0^r \frac{4\pi e^2 P_z r'^2 dr'}{1+\exp((r'-c)/t)} + \int_r^\infty \frac{4\pi e^2 P_z r' dr'}{1+\exp((r'-c)/t)}
\]

For $r<c$

\[
V_c(r) = 4\pi e^2 P_z \left(\frac{c^2}{2} - \frac{r^2}{6} + \frac{r^2 t^2}{6} + \sum_{n=1}^\infty (-1)^n \left[\left(\frac{2t^3}{n^3 r} + \frac{t^2}{n^2}\right) \cdot \exp(n(r-c)/t) - \frac{2t^3}{n^3 r} \exp(-nc/t)\right]\right)
\]

For $r>c$

\[
V_c(r) = 4\pi e^2 P_z \left(\frac{c^3}{3r} + \frac{c\pi^2 t^2}{3r} + \sum_{n=1}^\infty (-1)^n \left[\left(\frac{2t^3}{n^3 r} + \frac{t^2}{n^2}\right) \cdot \exp(n(c-r)/t) - \frac{2t^3}{n^3 r} \exp(-nc/t)\right]\right)
\]

where $P_z$ is given by (21) with $Z$ in place of $A$.

The cross sections were calculated using equations (12). The details of the calculation were as follows.

The integration of (8) was done using a step size of 0.06 fm and the cutoff radius was 12 fm for a total of 200 integration points. Two of the parameters in the potential (22) were the same in all plots.
These are $V_0 = 2110$ MeV and $r_n = 0.52$ fm. In addition the sum in equation (12) for $f_N(\theta)$ was calculated up to $\ell = 7$.

In Figure 5 the effect of adding an imaginary (absorptive) term to the potential is shown. The parameters in (22) were $A = 10$ (used in (21)), $c = 4.5$ fm and $t = 1$ fm. The imaginary part is of form (24) with the values of $b_0$ and $b_1$ shown in the figure. The units of $b_0$ and $b_1$ are MeV-fm$^{-5}$.

Absorption is seen to reduce the cross section and smooth out the peaks and valleys. It also causes the minima to occur at smaller angles. The curve in Figure 5 with $b_0 = -30i$ is about the best approximation to the data that was found. The minimum at scattering angle $60^\circ$ agrees with the experimental angle and the qualitative shape of the curve is correct, but the cross section is too large by a factor of about three. Also, all curves rise much too steeply at small angles.

In Figure 6 the effect of changing the radius of the nucleus is shown. Here the parameters are the same as for the $b_0 = -30i$ line in Figure 5 however absorptive potential (25) was used this time, with $b_0 = -55i$ and $b_1 = 110i$.

Notice that the first minimum moves in as $c$ is increased. This is the sort of behaviour that a diffraction pattern in optics shows: the larger the diffracting object the smaller the pattern. In fact Blair$^9$ derives an expression for the cross section for scattering from a black sphere of radius $c$, i.e. one which absorbs all particles which strike it. This is

$$\sigma(\theta) = (kc^2)^2 \frac{J_1(kc\theta)}{kc\theta}$$

(29)

This has minima (actually zeros) at the zeros of the Bessel function $J_1$;
FIGURE 5

EFFECT OF ABSORPTION ON CROSS SECTION
FIGURE 6

EFFECT OF NUCLEAR RADIUS ON CROSS SECTION

\[ \sigma_{cm}(\theta) \text{ (mb/sr)} \]

\[ \theta_{cm} \text{ (deg)} \]
the first such case occurs when $k c \theta = 3.83$. Now for $T_\pi = 50$ MeV, $k = 0.639$ fm$^{-1}$ so that the first minimum should occur when

\begin{equation}
\theta = \frac{6}{c} \text{ (radians)}
\end{equation}

For our cases we have the results shown in Table II.

<table>
<thead>
<tr>
<th>$c$ (fm)</th>
<th>$\theta_{\text{Blair}}$ (deg)</th>
<th>$\theta_{\text{Rowe}}$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>115</td>
<td>80</td>
</tr>
<tr>
<td>4.5</td>
<td>75</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>50</td>
</tr>
</tbody>
</table>

Considering the crudeness of both models even this much agreement is surprising, but it illustrates that very simple notions can be important in complex problems. Some of the discrepancies in Table II could be due to the facts that (i) the nucleus is not a rigid sphere; it has a fuzzy edge. This edge takes up a greater percentage of the nucleus for smaller values of $c$ hence the discrepancy should be largest there, which it is. Also (ii) the nucleus is not a perfect absorber.

The first point is illustrated in Figure 7 which shows the effect of varying $t$, the nuclear skin thickness. Here the absorption potential is (25) and all other parameters are as for the $c = 4.5$ line in Figure 6.

Increasing $t$ moves the minimum further in and also increases the cross section considerably. This could be due to the thinning out of the nuclear matter so that the effects of absorption are less pronounced. The case where $t = 0.5$ fm has a minimum at $\theta = 70^\circ$, closer to the diffraction
value of 75° than for t = 1 fm. The structure has largely disappeared from the plot and the cross section is generally smaller.

Figure 8 shows the effect of changing the depth of the potential. This was done by varying \( A \) in (21). The absorption potential is (24) and all other parameters are the same as for Figure 5, \( b_0 = -30i \) line. Varying the potential depth does not have a significant effect on the cross section curve over the range of \( A \) which we have chosen.

As mentioned earlier, the only real problem with all these cross sections is that they are too large. We will present here some possible explanations of this problem.

In the medium and large angle region, the best calculated cross section is about a factor of three greater than the data. This is most likely due to the contributions to the scattering amplitude in partial waves other than \( \ell = 1 \). Recall that the \( \pi N \) potential resulted in a large S-wave contribution to the cross section which was ignored in calculating the \( \pi N \) cross section but was included in the pion-nucleus calculations. One may think of expanding \( v(r) \) in a partial wave series before doing the integral (18) and including only the \( \ell = 1 \) term in the integral. However, it is not possible to do this and still have a pion-nucleus potential which is local.\(^10\) If one wishes to expend as much effort as would be required to carry through this non-local potential calculation one would be better advised to approach the problem from a different viewpoint; possibly the doorway model described in the next section.

The huge cross sections at small angles are a different matter, however. As can be inferred from Figure 2, other theoretical calculations show a minimum at angles near 15° (since Coulomb effects cause the cross section to increase as \( \theta \rightarrow 0 \)). A detailed study of one particular pion-nucleus ...
FIGURE 8

EFFECT OF POTENTIAL DEPTH ON CROSS SECTION

\[ \sigma_{cm}(\theta) \text{ (mb/sr)} \]

\[ \theta_{cm} \text{ (deg)} \]

\( A = 20 \)
\( A = 10 \)
\( A = 5 \)
nucleus calculation at $T_\pi = 50$ MeV by Thomas and Landau\textsuperscript{10} shows that this is due to interference between the Coulomb and nuclear amplitudes and it turned out that such interference does not exist in our calculation.

The reason, it turns out, is that our potential has two bound pion-nucleus states (for $A=10$): one in the S-wave and one in the P-wave. Thomas and Landau's potential is repulsive in S-waves and possesses no P-wave bound states.

There are two arguments to suggest that our potential has bound states, the second of which also provides an explanation for the large cross sections at small angles.

Firstly, if one approximates our potential by a square well of depth 60 MeV and width 4.5 fm then one can show\textsuperscript{11} that this well will have one bound S-wave and one bound P-wave state but no others.

Secondly, if one calculates (using PIRK) the S-matrix for very low energy pions ($T_\pi = 0.5$ MeV) one finds that $\sin 2\delta$ is small and negative while $\cos 2\delta = 1$ for $\ell = 0$ and 1 but that $\sin 2\delta$ is positive for $\ell = 2$ (for $A=10$). As one decreases $A$ with $T_\pi$ constant, first the sign of $\sin 2\delta$ changes for $\ell = 1$ and, as $A$ is decreased further, $\sin 2\delta$ for $\ell = 0$ becomes positive.

Now according to Levinson's theorem\textsuperscript{12}

$$\delta_\ell(T_\pi=0) - \delta_\ell(T_\pi=\infty) = n_\ell \pi$$

where $n_\ell$ is the number of bound states in partial wave $\ell$. Since $\delta_\ell(\infty)$ is taken to be zero in the usual definition of phase shift we see that $\delta_\ell(0) = \pi$ for one bound state, but $\delta_\ell(0) = 0$ for no bound states. Thus
for a small $T_\pi$, $\delta_\phi$ would be slightly less than $\pi$, explaining why
$\sin 2\delta_\phi < 0$ here. As the well depth is decreased, the bound states become
resonances, so for low $T_\pi$, $\delta_\phi$ would be small and positive, as would $\sin 2\delta_\phi$.

The presence of bound states will therefore change the sign of the
nuclear amplitudes so that they will interfere constructively rather
than destructively with the Coulomb amplitude.

It should be noted that there is no reliable data at low angles
so that it is unknown whether such Coulomb-nuclear interference actually
takes place.

IV. MOTIVATION FOR A DOORWAY MODEL IN PION-NUCLEUS SCATTERING

In order to describe the pion-nucleus interaction one would, of
course, like to have an explicit theory which describes the process,
i.e. one would like to have a strong interaction potential. Since such
a theory does not yet exist, one must resort to some form of approximation
to the true interaction if one wishes to tackle the problem at all.

In order to see what a reasonable approximation might be, let us
write down an equation describing pion-nucleus scattering symbolically
and ask how much we know about each part of this equation. We will use
non-relativistic quantum mechanics. This is not accurate for a description
of pion-nucleus scattering near the energy of the N* resonance (190 MeV)
because the pion here is moving at relativistic speeds. The generalization
to relativity can be made later, after we understand the physical ideas
of the model.

The Schrödinger equation describing the pion-nucleus system as a
whole is
Here particle 0 is the pion, particles 1 to A are the nucleons in the target nucleus, T is the kinetic energy, \( v_{0i} \) is the pion-nucleon interaction and \( v_{ij} \) is the nucleon-nucleon interaction. This form for \( H \) assumes that all forces are two body forces, i.e. that the presence of a third particle does not affect the force between two other particles.

It may seem odd to describe the pion-nucleus system by a set of eigenfunctions corresponding to discrete energies, each energy corresponding to one of the resonance energies of the system. (The parameter \( \lambda \) in (31) has the values 0, 1, 2, ... with \( \lambda = 0 \) corresponding to the ground state of the pion-nucleus system.) After all, a continuous range of energies is allowed whenever the total energy of the system is above all potential barriers. The rationale is the following.

We have assumed that the exact resonant eigenfunctions \( \psi_\lambda \) form a basis for the pion-nucleus space. Being a basis the exact eigenfunction for any energy can be expanded in terms of the \( \psi_\lambda \). That is, if \( \psi_E \) represents a pion-nucleus state with some energy \( E \) which is not a resonant energy we can write

\[
(32) \quad \psi_E = \sum_\lambda a_{\lambda E} \psi_\lambda
\]

for some set of constants \( a_{\lambda E} \). Actually the \( a_{\lambda E} \) are continuous functions of the energy. They must satisfy the condition that when \( E = E_\mu \), the resonant energy corresponding to the state \( \psi_\mu \), then \( a_{\lambda E_\mu} = \delta_{\lambda \mu} \).

It would be possible to use the entire set of all pion-nucleus state functions \( \psi_E \) to describe the pion-nucleus space, in which case the expansions
(32) would not be needed. It is a little easier for purposes of visualization however, to consider only the set of $\Psi_\lambda$ as the pion-nucleus basis.

Equation (31) may not appear to help—after all, we don't know the explicit forms of either $v_{oi}$ or $v_{ij}$. But if we are only interested in what the pion does, then $v_{oi}$ is all that matters since $v_{ij}$ doesn't even mention the pion. So let's try replacing $H$ by another hamiltonian $H_o$ in which the $v_{oi}$'s are replaced by a potential $V + iW$ where $V$ and $W$ are real and depend only on the pion's co-ordinates

\begin{equation}
    H_o = \sum_{i=0}^{A} T_i + V + iW + \sum_{i>j}^{A} v_{ij} = H + V + iW - \sum_{i=1}^{A} v_{oi} = H - H'
\end{equation}

What does all this mean? We know that $H$ gives all the possible states of the pion in the field of $A$ nucleons. $H_o$ however, contains only one potential $V + iW$ connecting the pion with the nucleus. In other words, $H_o$ describes what happens when a pion encounters a potential obtained by treating the nucleus as one particle—not necessarily a point particle, but a "particle" which represents the average of $A$ smaller particles, the nucleons.

Suppose $H_o$ satisfies

\begin{equation}
    H_o \psi_n = \epsilon_n \psi_n \quad n = 0, 1, 2, ...
\end{equation}

Then the ground state $\psi_o$ of $H_o$ should have the same energy $\epsilon_o$ as the average over nucleon co-ordinates of $H$ in the state $\psi_o$. That is

\begin{equation}
    \langle \psi_o | H | \psi_o \rangle = \epsilon_o
\end{equation}

where the operation $\langle \psi_o | A | \psi_o \rangle$ means an average of the operator $A$ over
all nucleon and pion co-ordinates. From (33) and (34), (35) implies

\[ \langle \psi_0 | H' | \psi_0 \rangle = 0 \]

which, if the \( v_{oi} \) are taken to be real gives

\[ \langle \psi_0 | V | \psi_0 \rangle = \langle \psi_0 | \sum v_{oi} | \psi_0 \rangle; \quad \langle \psi_0 | W | \psi_0 \rangle = 0 \]

To see what the function \( W \) represents we consider first \( H_0 - iW \). Suppose this hamiltonian has a resonance of the Breit-Wigner form

\[ \sigma_{\text{TOT}} \propto \left| \frac{1}{E - E' - i\Gamma/2} \right|^2 \]

with \( E \) as the resonance energy where \( \sigma \) reaches its maximum and \( \Gamma \) is the full width at half-maximum (assuming \( \Gamma \) to be independent of \( E \) which it usually isn't, but this is only a qualitative argument). It would seem reasonable to assume that adding an imaginary part \( iW \) to the potential would affect the width but not position of a resonance; specifically we might expect the width to be increased by about \( 2W \) (since the imaginary part of the resonant energy in (38) is half the width, not the whole width).

But what is \( W \) physically? It is certainly not the same type of line broadening as \( \Gamma \) since \( \Gamma \) would appear using even the exact hamiltonian \( H \). \( W \) appears because we are using an approximate hamiltonian.

Consider the case of a resonance due to \( H \). Since a resonance is not a stable particle, that is, its lifetime can be localized to within a finite time interval, its energy cannot be a precisely determined quantity. The time-energy uncertainty principle tells us that \( \Delta t \Delta E \gg h \).

Thus given that \( \Delta t \ll \infty \), \( \Delta E \) must be non-zero. We can therefore only measure the probability that the energy of the resonance is any particular value.
A graph of this probability versus energy is, in fact, what a cross section plot represents. One way of visualizing this is as follows.

Suppose we had a machine that produced a constant current of particles, say 100 per second. Suppose also that the beam's energy could be continuously varied from zero to infinity. We project this beam onto a target and measure how many particles pass through. We assume that any particle which interacts with the target is never detected. We can now irradiate the target for one second at each energy from zero to infinity (if we have lots of beam time) and draw a graph of \((100 - \text{number of particles detected at energy } E)\) versus \(E\). If we observe a peak in this graph we call it a resonance with energy \(E'\), which is the energy at which the graph reaches its highest point, and width \(\Gamma\). Since this graph is a probability graph we see that \(\Gamma\) is analogous to a standard deviation of the resonance energy. \(\Gamma\) is due to the fact that the resonance is unstable: if the resonance was stable, \(\Gamma\) would be zero.

Suppose now that we have a hamiltonian \(H\) which gives rise to a series of stable states (or resonances with zero width if you like) and that we decide to use an approximate hamiltonian \(H_0\) to describe one of these states, say \(\psi_0\). In general the approximate state \(\psi_0\) will not equal \(\psi_0\), but can be expanded in terms of the complete set of eigenstates \(\psi_\lambda\) of \(H\). In other words \(\psi_0\) can be expanded in terms of a set of functions each of which represents a different energy so that the approximation of \(\psi_0\) by \(\psi_0\) is a state with non-zero width. The probability of finding the approximate state at a given energy is proportional to the square of the coefficient of the exact state that represents that energy. This is the sort of width that \(W\) represents. When one's exact hamiltonian has eigenstates with non-zero width, then an approximate hamiltonian \(H_0\) will have genuine
Γ-type widths as well as W-type ones arising from the approximation being made.

A measure of the W-type width introduced by using $H_0$ in place of $H$ is given by the second moment $M_2$ defined by

$$M_2 = \sum_{\lambda} (E_{\lambda} - \varepsilon_0)^2 C_{\lambda 0}^2$$

where the $E_{\lambda}$ are the eigenvalues of the exact states as in (31) and $\varepsilon_0$ is the eigenvalue of the approximate state $\psi_0$. The $C_{\lambda m}$ are the expansion coefficients of $\psi_m$ in terms of the $\psi_{\lambda}$

$$\psi_m = \sum_{\lambda} C_{\lambda m} \psi_{\lambda} ; \quad \psi_{\lambda} = \sum_{m} C_{\lambda m} \psi_m$$

and have the properties

$$\sum_{\lambda} C_{\lambda m} C_{\lambda n} = \delta_{mn} ; \quad \sum_{\lambda} C_{\lambda n} C_{\mu n} = \delta_{\lambda \mu} ; \quad \sum_{\lambda} C_{\lambda m}^2 = 1$$

In view of what has been said above it is reasonable to assume that

$$\langle \psi_0 | \hat{W}^2 | \psi_0 \rangle = M_2$$

It is also true that

$$M_2 = \langle \psi_0 | H^{'2} | \psi_0 \rangle$$

as we now see.

$$H^{'2} | \psi_0 \rangle = (H-H_0)(H-H_0) | \psi_0 \rangle$$

$$= (H-H_0)(H | \psi_0 \rangle - \varepsilon_0 \langle \psi_0 | \psi_0 \rangle)$$

$$= (H-H_0) \sum_{\lambda} C_{\lambda 0} E_{\lambda} | \psi_{\lambda} \rangle - \varepsilon_0 \langle \psi_0 | \psi_0 \rangle)$$
Using the orthonormality of $|\psi_m\rangle$ and (41) we obtain

$$\langle \psi_o | H^{1/2} | \psi_o \rangle = \sum_{\lambda,\lambda'} C_\lambda^\lambda C_{\lambda'}^\lambda E_{\lambda} E_{\lambda'} |\psi_m\rangle^2 = M_2$$

Q.E.D.

Now that we have explored the meaning of the replacement of $H$ by $H_o$, we should ask if it is a reasonable approximation. When the pion is outside the nucleus we would expect the nucleus to appear as a smeared potential, so here we are all right. But once we get inside the nucleus things are not so simple. We know that a pion interacts very strongly with a nucleon at energies of around 190 MeV so it is certainly likely that a pion-nucleon interaction would occur as the pion travels through the nucleus. To get some idea how likely, let us compare the lifetime of the $N^*$ to the mean time the pion spends between collisions with nucleons in nuclear matter.

The $N^*$ lifetime is roughly

$$\frac{\hbar}{\Gamma} \approx \frac{6.6 \times 10^{-22} \text{ MeV-sec}}{110 \text{ MeV}} = 6 \times 10^{-24} \text{ seconds}$$

The mean free path for a pion in nuclear matter consisting of 60% neutrons and 40% protons at a total density of $1.7 \times 10^{44}$ nucleons per cubic meter (this assumes a nuclear radius of $1.12 A^{1/3}$ fm) can be found from experimental cross sections: $\sigma_{\pi^+p} \approx 200 \text{ mb}$ at resonance while $\sigma_{\pi^+n} \approx 70 \text{ mb}$. Thus
If pions travel at \( v = c \) then the mean time between collisions is \( \lambda / c \) or roughly \( 1.6 \times 10^{-24} \) sec. At resonance then, we expect the pion to spend more time as an \( N^* \) than as a free particle. The simple model \( H_0 \) is not appropriate inside the nucleus.

Since a pion spends more of its time as an \( N^* \) than as a free pion the next logical approximation is to single out one of the nucleons from the nucleus (the nearest one to the pion since the pion-nucleon interaction is known to be short range) and treat this interaction separately from the general pion-nucleus one. The method we will suggest is in analogy to one that has been tried successfully in nucleon-nucleus interactions.

It has been found that in nucleon-nucleus interactions several levels of complexity can be observed in the scattering data. With crude energy resolution one obtains the so-called "single-particle giant resonances" which are shown schematically in Figure 9. These resonances are caused by the formation of a quasi-stable system consisting of the incident nucleon interacting with an average potential due to the entire target nucleus (hence the name "single-particle"). With better energy resolution one obtains what is shown in Figure 10.

The finer peaks are caused by the incident nucleon interacting with one of the nucleons in the nucleus in particular; this two nucleon system is then treated as being in an average potential due to the rest of the nucleus.

More detailed pictures are possible (e.g. a nucleon interacts with two or more nucleons) but it turns out that the widths of the resonances from these interactions are greater than their spacing so that no structure is observable. The ultimate level of detail is that in which the energy

\[
\lambda = \frac{1}{n_n \sigma \pi^{-n} + n_p \sigma \pi^{+p}} = 0.49 \times 10^{-15} \text{ meters}
\]
FIGURE 9 - An oversimplified nucleon-nucleus scattering cross section with poor energy resolution. This plot illustrates the single particle giant resonance.

FIGURE 10 - The same cross section as in Figure 9 with better energy resolution. The finer resonances may be interpreted as evidence for doorway states.
from the incident nucleon is spread over all the nucleons in the nucleus, resulting in an enormously complicated diagram of resonances called the compound nucleus resonances.

In the pion-nucleus system one may describe the processes in a similar way.

The simplest model is one in which the incident pion interacts with one of the nucleons in the nucleus, forming the N* resonance. (There is also a non-resonant pion-nucleus interaction at all energies but we are only considering resonances here.) The N* is coupled to two channels: the entrance channel and the compound nucleus channel. In the first case, the N* decays into a pion-nucleon system, the nucleon rejoins the nucleus and the pion escapes. In the second case, the N* gives off its energy to the remaining nucleons with the pion having been absorbed in the process.

The simplest doorway hypothesis, which is the one employed by reference 14, is that these compound nucleus states can be formed only through the N*, i.e. the N* system is a doorway through which the pion-nucleus system must pass before compound nucleus states can be formed.

In analogy with the nucleon system one asks if there are other methods of coupling from entrance channel to compound nucleus. Such postulated alternate states may be called hallway states and consist of a pion interacting with more than one nucleon at a time (e.g. a pion-alpha particle interaction). At this point one can assume practically whatever one wishes. However, in order for the hallway states to be of any importance they must satisfy these two conditions: (i) their widths must be less than their spacing in order that they be observable and (ii) their postulated existence must not interfere with accepted scattering data, i.e. for poor enough resolution the hallway theory must reduce to the old "giant res-
Symbolically reference 14 does what is shown in Figure 11 as a kind of flow chart. Two modifications to consider are illustrated in Figure 12. The first diagram here would consist of replacing the N* in Figure 11 by a hallway state (i.e. a direct coupling of the pion to more than one nucleon), while the second involves inserting an extra step into the doorway process.

The first would probably be easier to start with. In more conventional diagrams, Figure 13 shows two possibilities that this could lead to: elastic scattering and nuclear breakup. The R* is the analog of the N* (R for either "Rowe" or "resonance" depending on the reader's interpretation of the author's ego), i.e. it is the intermediate state consisting of the pion and more than one nucleon. The mechanism for what happens inside the circles in Figure 13 is anybody's guess at present.

For the special case of the pion-nucleus doorway the argument may go something like this. If we concentrate on nucleon n the new approximate hamiltonian would be

\[
H_0 = \sum_{i=1}^{A} T_i + \sum_{i>j \neq n} v_{ij} + V_\Delta + iW_\Delta + v_{0n}
\]

where v_{0n} is the pion nucleon interaction while \( V_\Delta + iW_\Delta \) represents the average interaction of the center of mass of the N* with the residual (A-1)-nucleon nucleus.

\( v_{0n} \), in exact form, would represent the interaction of the two quarks in the pion with the three in the nucleon (assuming, of course, that quarks exist). This interaction would presumably give the experimental cross section exactly. Such a calculation is not practical at present however, as the quark-quark interactions are not well known and five-body
FIGURE 11

FIGURE 12
Residual Nucleus

Elastic Scattering

Residual Nucleus

Nuclear Breakup

FIGURE 13
problems are very difficult to solve meaningfully. Rather we consider $v_{0\alpha}$ to be a pion-nucleon optical potential: we average over the three nucleon quarks and the two pion quarks separately.

If $H_\alpha$ has an eigenstate $\psi_\alpha$ with resonance energy $\epsilon_\alpha$ then we can expand $\psi_\alpha$ as in (40) and use relation (36) (where $H'$ is now $H - H_\alpha$ with $H_\alpha$ given by (46)) to see the relation of $\epsilon_\alpha$ to the energies of the exact resonances:

$$0 = \langle \psi_\alpha | H - H_\alpha | \psi_\alpha \rangle = \langle \psi_\alpha | H | \psi_\alpha \rangle - \epsilon_\alpha$$

$$= \sum_{\lambda,\mu} \langle \psi_\alpha | E_\lambda \psi_\alpha \rangle - \epsilon_\alpha$$

$$= \sum_{\lambda,\mu} \langle \psi_\alpha | E_\lambda \psi_\alpha \rangle - \epsilon_\alpha$$

or

$$\epsilon_\alpha = \sum_{\lambda} C_{\lambda}^2 E_\lambda$$

so that $\epsilon_\alpha$ is the weighted average of the exact energies $E_\lambda$.

Before we actually start writing computer programs however, we should have some evidence that such intermediate states exist, i.e. we should see some kind of fine structure in the pion-nucleus cross section analogous to Figure 10. Since this has not yet been seen we must be content to stop here and await future developments.
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