AN EVALUATION OF QUADRATIC PROGRAMMING AND THE MOTAD MODEL AS APPLIED TO FARM PLANNING UNDER UNCERTAINTY

by

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B.Sc., University of Chile, 1973

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

(Department of Agricultural Economics)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

July, 1977

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ABSTRACT

AN EVALUATION OF QUADRATIC PROGRAMMING AND THE MOTAD MODEL AS APPLIED TO FARM PLANNING UNDER UNCERTAINTY.

by

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The objective of this thesis was to study the efficiency of three methods used in farm planning under uncertainty. The first method considered was the QP-VAR method which minimizes the variance of activity returns subject to a minimum income level using a quadratic programming algorithm. The second method is the MOTAD method which minimizes the mean absolute deviation of activity returns subject to a minimum income level using a linear programming algorithm. The third method is the Semivariance method which minimizes the negative semivariance of activity returns subject to a minimum income level. The main elements used to evaluate the efficiency of these methods were the magnitude of the biases and the dispersion of the estimates of the income-risk frontier obtained using each method.

In order to achieve this objective, a research procedure comprising a theoretical and an empirical study was developed. The theoretical study included an analysis of the measures of risk used by each

method and of the assumptions underlying the use of such measures. Furthermore, the plausibility of these assumptions was thoroughly discussed. Using the conclusions drawn from the theoretical study, a set of experiments (the empirical study) was designed to test the efficiency of the methods as estimators of income-risk frontiers. The purpose of these experiments was to test the performance of the methods when applied using sample data of relatively small size rather than complete frequency distributions of activity returns. Two trivariate normally distributed populations (one with high and the other with low degrees of correlation among activity returns) and two trivariate gamma distributed populations (one with high, the other with low degrees of correlation among activity returns) representing activity returns data were generated using a random number generator. Using these populations as data bases, three points on the "true" income-risk frontiers were determined applying the appropriate method in each case. Estimates of the income-risk frontiers were obtained using randomly drawn samples from the populations and the mean risk estimates obtained using each method were compared to establish bias. The degree of dispersion of the estimates as provided by each method was also compared. If two methods were unbiased, the method with the smallest dispersion of its estimates was considered more efficient.

A general conclusion drawn from this thesis was that there is not an optimal method to be used in all cases. In order to choose the best method, it is necessary to consider the nature of the farm decision-

maker's utility function and the frequency distribution of activity returns. However, the QP-SEMIV method appears to be appropriate under a wider range of empirical situations than the QP-VAR and MOTAD methods.

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ACKNOWLEDGEMENTS

I would like to thank my advisor, Professor John Graham and the members of my Thesis Committee, Professor Rick Barichello and Professor George Eaton, for their numerous suggestions which led to substantial improvements in the final product.

CHAPTER I

INTRODUCTION

1.1 The Problem

Agricultural production may be considered a risky activity. Wide variations in yields and prices over time are common among agricultural commodities. Certain economic characteristics such as relatively price inelastic demand functions and the dependence of agricultural production on climate might be mentioned as factors which contribute to explain the greater variability of returns from agriculture as compared to other sectors.

Given this situation, it is necessary to consider the uncertainty of returns as an important factor influencing farm decisions. Farm planning techniques which use only deterministic tools have considerable limitations. Attention is increasingly being directed to studies that allow for uncertainty, both in a theoretical and empirical framework (1, 9, 16).

The method which minimizes the variance of activity returns subject to a minimum expected income level using a quadratic programming algorithm (the QP-VAR method) and the method which minimizes the mean total absolute deviations of activity returns subject to a minimum expected income level using a linear programming algorithm (The MOTAD

method) have been two widely applied procedures (1, 9, 16). Although the Semivariance method which minimizes the negative semivariance of activity returns subject to a minimum expected income level has not been used frequently, it has, nevertheless, been considered an important method (23, 25). Despite some theoretical discussions about the reliability of these methods (6, 15, 21, 29) it is not clear how to evaluate their performance under different empirical situations. How do the results differ when QP-VAR, MOTAD and Semivariance methods are applied? Under what circumstances can the relative efficiency of these methods be considered similar? A systematic evaluation of the absolute and relative efficiency of the methods under different empirical circumstances is missing.

An answer to this problem is important since knowledge of the limitations of these methods may be important for researchers who wish to decide which (if any) of the methods should be applied under certain specific empirical conditions. In any research the quality of the results is going to depend, among other factors, on the adequacy with which the method used is appropriate for the empirical situation.

1.2 Objectives

A general objective of this thesis is to study the efficiency of the QP-VAR, MOTAD and Semivariance methods as estimators of the income-risk frontier when applied to different frequency distributions of the activity returns*. In studying the efficiency of the methods two main

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* A detailed description of these methods and of the income-risk frontier is provided in Chapter II.

elements are considered, namely the magnitude of the bias and the variance of the sampling estimates of the income-risk frontier.

This primary objective may be specified in a number of more specific sub-objectives as follows:

1. (a) To review some theoretical concepts concerning the utility function of a decision maker when the expected income and risk are considered to be variables.

(b) To review the concept of an efficient income-risk frontier as defined by the QP-VAR, MOTAD and Semivariance methods.

(c) To analyze situations under which these three methods are applicable given certain assumptions regarding decision makers' utility functions and the probability distribution of the returns.

2. (a) In order to extend the theoretical concepts outlined in (1) above, a small farm planning model consisting of three activities and six constraints will be employed. Assuming that the activity returns of the three variables are normally distributed and that the degree of correlation among them is low, an income-risk frontier will be derived using the complete population of activity returns as the data base.

(b) Step (a) as defined above will be repeated but in this instance the degree of correlation among activity returns will be assumed to be relatively high.

(c) In most empirical situations the complete set (population) of observations is not available, but a selected sample thereof is. Therefore, income-risk frontiers will be estimated using the QP-VAR and MOTAD methods

applied to randomly drawn sample data of activity returns, rather than to the population data. The income-risk frontiers will be estimated for samples drawn from the same populations as defined in (a) and (b).

(d) The income-risk frontier as determined for the population data and that estimated by using the randomly drawn sample data will be compared to determine the degree of departure (extent of bias) of the latter from the former.

(e) A measure of the degree of dispersion of the income-risk sample estimates will provide an additional element with which to evaluate the efficiency of the methods.

3. Under conditions where the distribution of activity returns is non-normal, in addition to the QP-VAR and MOTAD methods, the Semivariance method will be used as an estimator of the efficient income-risk frontier. Steps (a), (b), (c), (d) and (e) as defined in 2. above, will be repeated.

4. In order to illustrate the performance of these methods, data from an actual farm of the Peace River District of British Columbia will be used in order to estimate an income-risk frontier using each of the methods. Sample observations for this farm (the last eight years' records of yields, prices and costs) do not allow one to affirm that the activity returns are normally distributed and the data indicate that the activity returns are fairly highly correlated..

1.3 Important Hypotheses to be tested

The following four hypotheses are stated in order to allow for a closer specification of the objectives of this study. These hypotheses

are based on observations made by the author on various theoretical studies in the area. Chapter II provides a theoretical basis supporting these hypotheses*.

1. The QP-VAR approach as applied to small sample data provides an unbiased estimate of the actual population income-risk frontier if the activity returns are normally distributed, regardless of the degree of correlation among the activity returns.

2. The MOTAD method provides an unbiased estimate of the actual population income-risk frontier only if the following two conditions are satisfied.

   (a) The activity returns are normally distributed, and

   (b) The correlation coefficients among the activity returns are close to zero.

3. If activity returns are non-normally distributed, the QP-VAR method and the MOTAD method yield unbiased estimates of the actual population income-risk frontier.

4. When activity returns are non-normally distributed, the Semivariance method will provide unbiased estimates of the actual population income-risk frontier.

Other hypotheses related to comparisons of the degree of dispersion of the estimates provided by the methods should also be included. However, because there is no a priori knowledge regarding the dispersion of the

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* The reader will notice that the exception is hypothesis 3. As may be seen in Chapter II, the theoretical study supports the alternative hypothesis implicit to hypothesis 3 rather than the null hypothesis formulated.

estimates these hypotheses are not formulated. Nevertheless, the dispersion of the estimates provided by the methods will be an important element to consider in the evaluation of the methods.

1.4 Research Procedure

The research procedure includes four steps:

1. A theoretical study of the basic assumptions and characteristics of the QP-VAR, MOTAD and Semivariance methods, thus providing qualititative knowledge of the performance of these methods in estimating income-risk frontiers.

2. An experimentation phase where a random number generator is used to generate different sets of populations as required by the assumptions regarding probability distributions of returns and the degree of correlation among activity returns.

3. A "solving of the models" phase, where the QP-VAR, MOTAD and in some instances the Semivariance method will be used to solve models for different sets of population data and for a number of samples drawn from each type of population generated.

4. An analysis of the results where the mean income-risk frontier* as estimated using each method will be compared to the actual population income-risk frontier (determined with the appropriate method) attempting to establish if there are significant differences between the two frontiers, i.e., whether the estimate is biased or not. The dispersion of the estimates (variance) provided by the methods will also be compared. If two methods

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* The "mean income-risk frontier" is the income-risk frontier obtained from mean values of the numerous sample solutions.

are unbiased, the method with smaller variance of its estimates will be considered more efficient. A limited trade-off between bias and variance of the estimates will be considered in evaluating methods with different degrees of dispersion and bias.

1.5 Organization of the Study

Chapter 1 includes the statement of the problem, the objectives and the basic methodology to be followed.

Chapter 2 is devoted to a theoretical study of the Variance, Total Absolute Deviation and Semivariance as risk indicators which may be used in farm planning. The discussion centers around the accuracy with which these indicators may represent risk.

The next chapter describes the procedures used to generate populations, the sampling process and the models used when each method is applied.

The fourth chapter reports on the analysis of the results and the testing of the hypotheses. It examines the performance of the QP-VAR, MOTAD and Semivariance methods in four possible experimental empirical situations and their relative efficiency as estimators of the income-risk frontier.

Chapter 5 describes an application of the QP-VAR, Semivariance and MOTAD methods to data obtained from a case farm of the Peace River District of British Columbia.

Finally, Chapter 6 summarizes the study and provides basic conclusions.

CHAPTER II

THEORETICAL REMARKS

The purpose of this chapter is to discuss some theoretical aspects of the farm planning methods used in this study. The ability of these methods to adequately generate estimates of income-risk frontiers and the closely related problem regarding farmers' utility functions will be considered in an evaluation of the methods.

2.1 The Income-Risk Efficient Frontier

As a preliminary step it is necessary to define the concept of an efficient plan. Markowitz (23) notes that a plan is efficient if it is not possible to obtain a higher expected income with the same variability of income (risk), or if there is no other plan with a smaller variability of income for the same level of expected income.

Risk may be measured as the degree of variability of return. One or a combination of the following measures of variability may be used as risk indicators: variance, semivariance , absolute deviation, skewness and kurtosis. Each of the possible measures of variability has different characteristics and represent certain specific aspects of risk differently.

An iso-risk curve represents all possible combinations of activities which yield the same risk level as measured by any of these measures. It has commonly been assumed that the iso-risk curves

are elliptical when represented diagramatically for the case of two possible activities (16).

FIGURE 2.1 Income Iso-Risk Curves Showing Different Degrees of Correlation between the Activity Returns

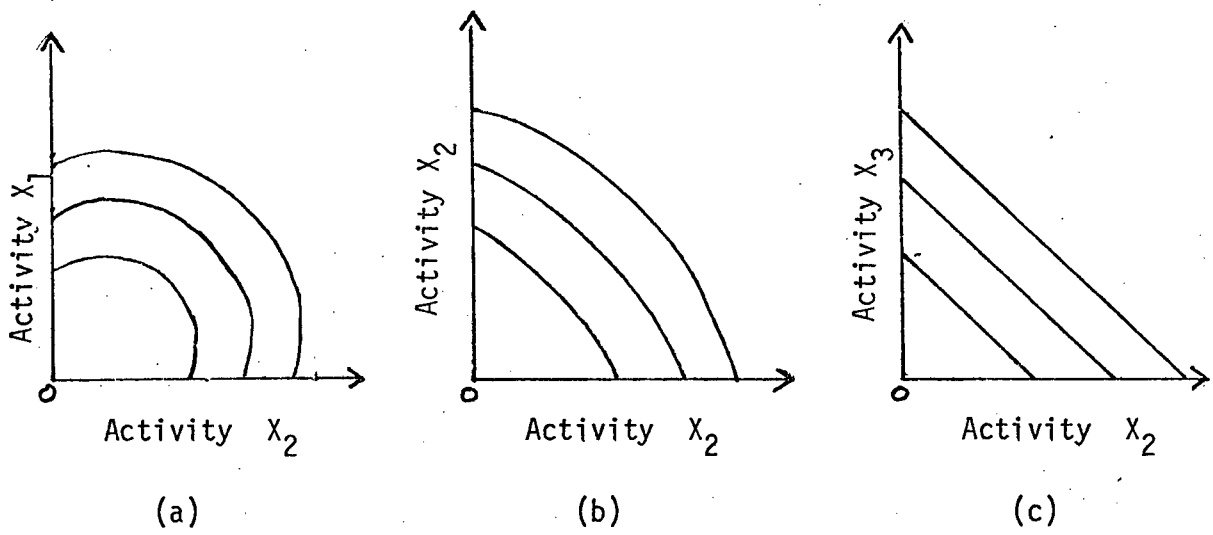

Figure 2.1 shows three families of income iso-risk curves, where risk levels are increasing from the origin. Figure 2.1(a) shows a family where there is a strong negative correlation coefficient between the returns of activity X_1 and X_2 (close to -1.0). In this case for a given level of income there is a decrease in the level of risk through producing a combination of the two activities rather than producing just one. As the correlation coefficient approaches zero or becomes positive the iso-risk curves are less concave to the origin (figure 2.1 (b)), meaning that diversification does not reduce risk greatly. Finally,

when the correlation coefficient is equal to +1.0, the iso-risk curves are straight lines as shown in Figure 1(c). In this case there are no benefits from diversification as far as the risk situation is considered.

By introducing the familiar concept of iso-income (or iso-revenue) lines, an income-risk expansion line may be defined.

FIGURE 2.2 Income-Risk Expansion Line for a Two Activity Situation

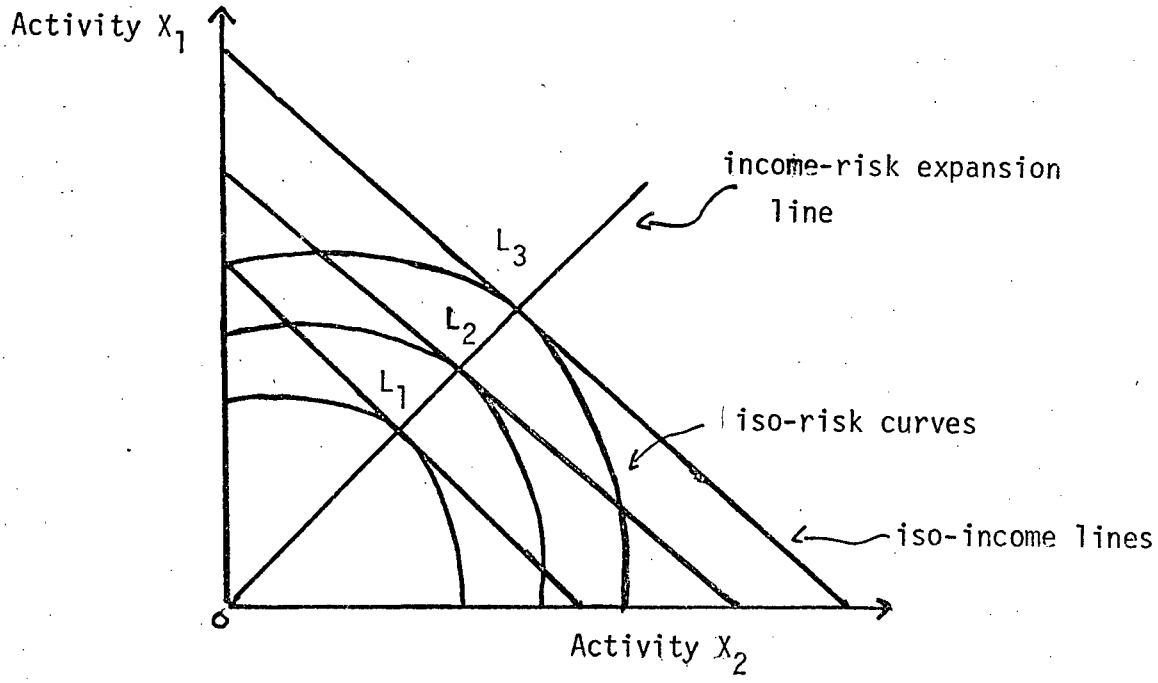

Figure 2.2 shows the income-risk expansion line for the two activity case. The income-risk expansion line is defined as the locus of points which define the minimum risk level at each level of expected income, represented by the points at which the iso-income lines are tangential to the iso-risk lines. The income-risk expansion line shows the efficient

combinations of the activities X_1 and X_2 which a firm will produce if the maximum income is to be achieved at the minimum level of risk. It is important to note that there is a unique combination of activity levels (L_1, L_2, L_3) which generate any efficient plan *. The income - risk line will be a straight line given the following assumptions: (1) The risk function (risk as a function of X_1 and X_2) is an homothetic function. This implies that the iso-risk curves will be parallel (they all have the same shape) throughout all levels of risk: (2) The relative prices of X_1 and X_2 remain constant at all levels of production, i.e., the firm is not able to alter the slope of the iso-income lines when it expands its production (perfect competition). Income-risk expansion lines will be straight lines in most farm planning situations since these assumptions are generally met.

If points L_1, L_2, L_3 of Figure 2.2 which represent different levels of income at the smallest risk levels are graphed in an income-risk plane, Figure 2.3 is obtained. This is defined as an efficient income-risk frontier, which shows the maximum level of income obtainable at each level of risk.

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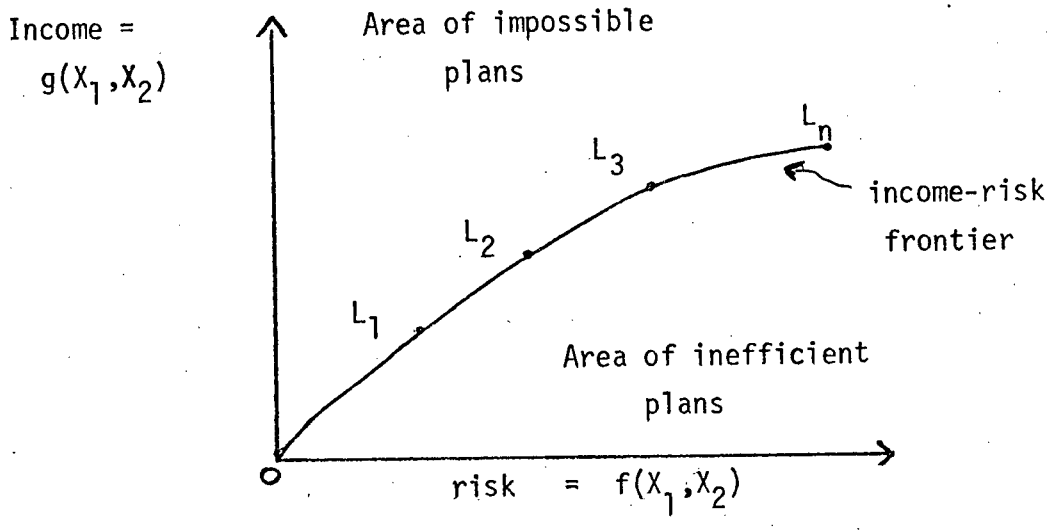
* This implies that in evaluating a particular method it is sufficient to consider either the risk level estimated at a certain level of income or the activity levels estimated. This conclusion is important in the design of the experiments described in the following chapter.

FIGURE 2.3 The Income-Risk Frontier



In many studies it has been assumed that risk levels increase at an increasing marginal rate when the expected income is expanded further than a certain level (15, 16). This fact is corroborated by a number of empirical estimations of the income-risk frontier (1, 15). Figure 2.3 shows an income-risk frontier of this type and point L_n represents the maximum possible income which can be obtained given a limited resource base. Any plan chosen along the income-risk frontier is efficient and the actual point or plan chosen will depend upon the specific utility function of the decision maker.

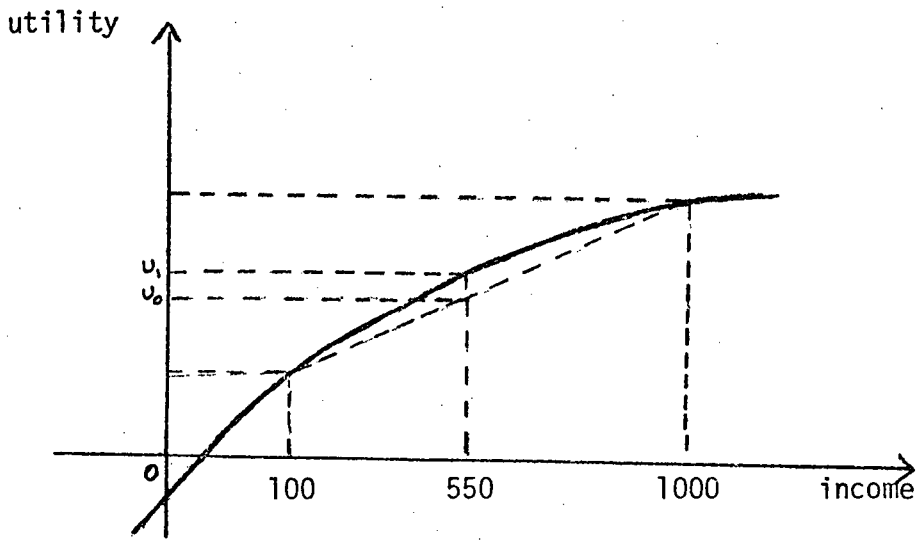
2.2 The Expected Utility Function

Until a number of decades ago, it was assumed that the proper objective of an individual when faced with uncertain situations was to maximize expected monetary return (23). It was later found that this objective does not reflect reality. Instead, the expected utility rule

was proposed as a substitute to the expected return rule. This approach assumed that the individual tries to maximize his expected utility rather than his expected returns. Expected utility is a function of the expected returns but this functional relation is not necessarily linear. Furthermore, expected utility is not only a function of the expected returns but also a function of the degree of risk involved in trying to pursue such returns.*

Markowitz (23) accepted the hypothesis of decreasing marginal utility as the level of income increases. Figure 2.4 shows this relationship.

FIGURE 2.4 Utility Curve for a Risk Averse Individual



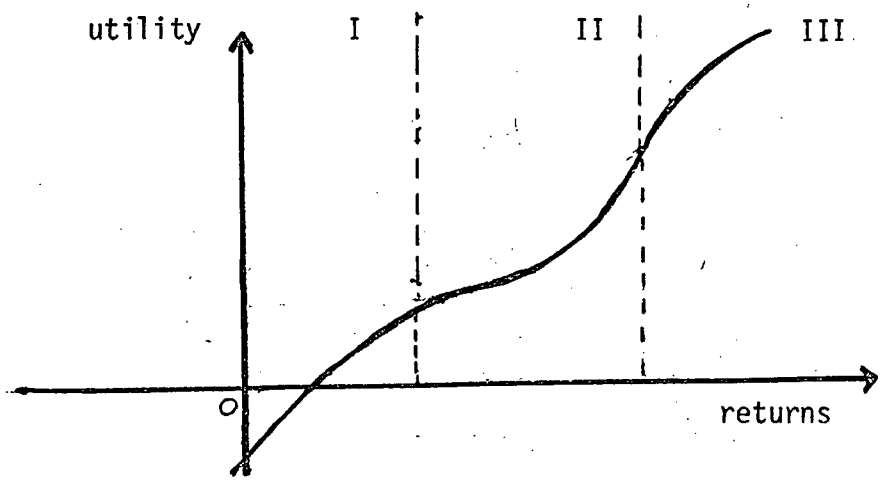
* The expected utility can also be a function of other factors such as prestige of the decision-maker and others. This study will be concerned only with expected returns and risk as factors affecting the utility level.

For an individual who possesses a strictly concave utility function the gain in utility from winning a dollar will be less than the loss in utility from losing a dollar (11). This individual will never participate in a "fair" game of chance. For example, given a game in which he has an equal chance of winning and losing $450 as shown in Figure 2.4, the expected utility of such a game is smaller than the utility of certainty of $550, i.e., $U_1 > U_0$ in the graph.

The strictly concave utility function may explain why individuals are willing to take insurance against big losses even if the insurance company makes a profit. Friedman and Savage (11) have pointed out that given the diminishing marginal utility assumption, individuals would always have to be paid to induce them to bear risk. However, this statement is clearly contradicted by actual behaviour. People not only engage in fair games of chance, they engage freely in such unfair games as lotteries. People enter risky occupations and make risky investments that yield even smaller average returns than relatively safe investments. This problem is still more serious considering that many individuals insure against damage and simultaneously they buy lottery tickets or invest in highly risky activities with average returns.

Friedman and Savage (11) hypothesized that the shape of the utility function for most individuals is similar to the one shown in Figure 2.5, with two concave stages and an intermediate convex stage. The convex stage is a transitional phase which is relevant when the outcome may imply losses or gains sufficiently large to transfer the individual from one socio-economic position to a qualitatively lower or higher one.

FIGURE 2.5 Typical Shape of Utility Curve



The utility function as shown in Figure 2.5 has three stages; two concave (stages 1 and III) and one convex (stage 11). The shape of the function assumed in this study will be strictly concave as shown in Figure 2.4 and it is thereby assumed that the farmer's socio-economic status is not changed in the short run by the outcome of production decisions that he makes. Only in very rare occasions will the outcome of the production decisions at the farm level be enough to move a farmer into lower or higher qualitative socio-economic positions (say from an average to a rich farmer).

Considering utility to be a function of income and risk, a utility function may be written as:

$$U = F(E,R), \qquad (2.1)$$

where

U = Level of Utility

E = Expected Income

R = Level of Risk

The utility level will increase/decrease when expected income increases/decreases and will decrease/increase for increasing/decreasing levels of risk for all levels of income and risk, hence:

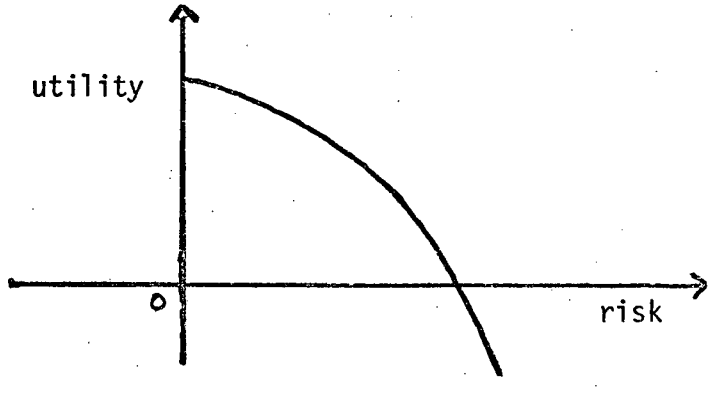
$$\frac{\partial U}{\partial E} > 0 \qquad \text{and} \qquad \frac{\partial U}{\partial R} < 0 \,. \qquad (2.2)$$

Since utility is a strictly concave function of the returns and utility decreases at an increasing rate with higher levels of risk:

$$\frac{\partial U^2}{\partial^2 E} < 0 \qquad \text{and} \qquad \frac{\partial U^2}{\partial^2 R} < 0 \,. \qquad (2.3)$$

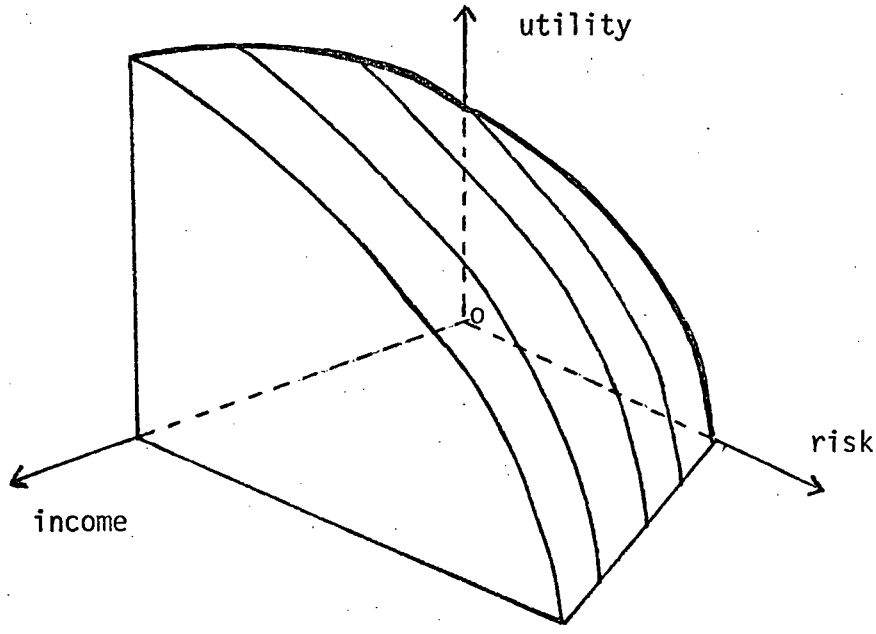
Graphically, the relation between U and R given a fixed level of E can be represented as in Figure 2.6

FIGURE 2.6 Utility as a Function of the Risk Levels



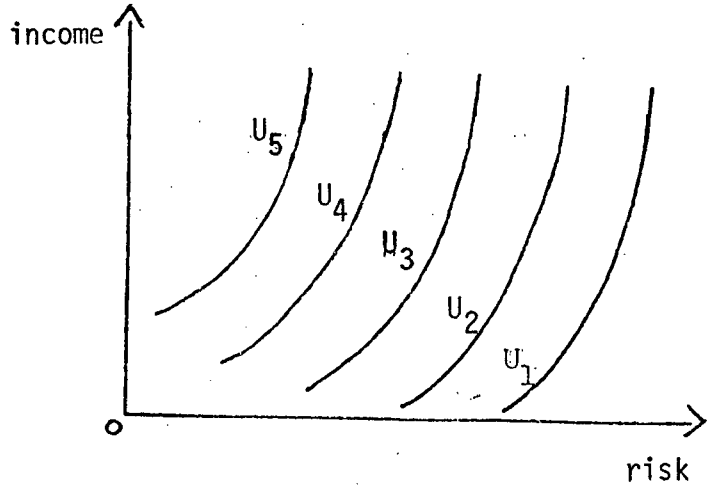
Utility as a simultaneous function of both income and risk levels is shown in Figure 2.7. The negative part of the utility function is not shown because of difficulties in drawing it, but obviously it should be extended to the negative area.

FIGURE 2.7 Utility as a Function of Income and Risk



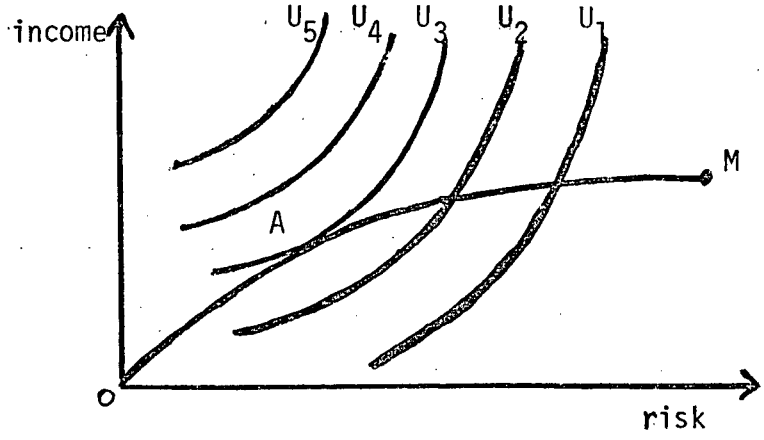
As defined by equations 2.3, this function is strictly concave and it is possible to draw convex iso-utility curves as shown in Figure 2.8 where $U_5 > U_4 > \ldots > U_1$. The iso-utility curves represent combinations of income and risk that provide a constant level of utility.

FIGURE 2.8 Iso-Utility Curves



Given an efficient income-risk frontier, i.e., knowing the optimum income-risk combinations, it is possible to find the plan chosen by an individual who maximizes his particular utility function. Figure 2.9 shows a family of iso-utility curves corresponding to a specific utility function and an efficient income-risk frontier. Point A indicates the levels of income and risk which maximize utility for the given income-risk frontier.

FIGURE 2.9 Determination of the Optimal Level in an Income-Risk Frontier



The optimum point depends on the income risk frontier and on the utility function of the decision maker, as determined by the degree of risk aversion. Point M in Figure 2.9 corresponds to that plan which maximizes profit for given resources and only in very rare occasions will a plan represented by M be chosen. Lin, Dean and Moore (20) tested the expected utility versus the profit maximization criterion in predicting actual decisions of a number of California farmers. It was shown that the utility formulations provided greater accuracy in predicting actual and planned crop patterns. Thus, plans obtained by using simple Linear

Programming may be considered unrealistic by the majority of farmers and therefore for farm planning purposes it may be a better procedure to present farmers a set of efficient plans (the income-risk frontier), where they may choose one which maximizes their expected utility. To derive the proper income-risk frontier it is necessary to use an appropriate method, and, as will be shown later, in order to choose the appropriate method some characteristics of the decision maker's utility function and of the distribution of returns have to be considered.

2.3 Risk Aversion

Pratt (26) has defined a measure of the absolute risk aversion (r) as follows:

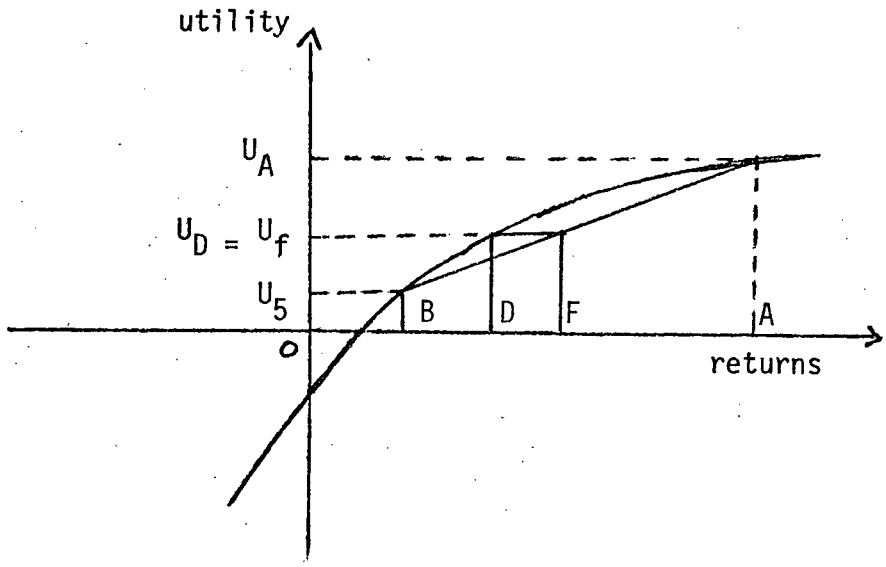
$$r = \frac{-U''}{U'}, \qquad (2.4)$$

where U' is the first derivative of utility, U, with respect to income and U" is the second derivative.

Neither the slope of U (U') nor the change of the slope (U") are appropriate measures of risk aversion. The risk aversion coefficient is the rate of change of the slope, rather than the absolute change of the slope. It is important to understand the negative sign assigned to U". If U" is negative the utility function is strictly concave and therefore, the risk aversion coefficient must be positive since U' is positive. But if the utility function is convex, i.e., the decision maker is a risk taker, r will be negative.

If a decision maker has a high degree of risk aversion, he will be willing to pay a high premium to avoid risk and as r increases this premium will also increase. The maximum amount of risk premium which an individual would be willing to pay is illustrated in Figure 2.10. In Figure 2.10, Point F corresponds to the mean or expected return of a combination of returns A and B with given probabilities. Distance DF corresponds to the maximum risk premium (5).

FIGURE 2.10 Risk Premium in a Concave Utility Function



The risk aversion function as defined in equation 2.4 is a measure of the absolute risk aversion. Pratt has also defined a relative risk aversion measure (r*) :

$$r^* = r.E \quad , \qquad (2.5)$$

Most authors agree (4,6,10,30) that an appropriate utility

function for risk averse individuals should have the following basic properties:

(a) $U' > 0$, i.e. marginal utility of income is positive

(b) $U'' < 0$, i.e. decreasing marginal utility of income

(c) $\frac{d(r)}{dE} = r' \leq 0$, i.e., absolute risk aversion should if anything, decrease when the income increases.

(d) $\frac{d(r^*)}{dE} \geq 0$, i.e., the relative risk aversion should, if anything increase when the income increases.

It is important to mention that these properties are not totally met by quadratic utility functions; neither by any other polynomial function (3, 30).

2.4 The Utility Function and the Income-Risk Frontier

In Section 2.1 the iso-utility curves and the income-risk frontier were represented in a two-dimension space. A relevant question to ask is whether risk can be represented by a single parameter (say the standard deviation or the absolute deviation) or by a more complex concept which is the resultant of two or more parameters. Furthermore, is the decision maker able to make consistent decisions when he is faced with an income-risk frontier based on only two parameters, the expected income and a single parameter representing risk?

First, consider a utility function $U(E)$ and expand it into a Taylor's series around the mean income, $\bar{E}$, in such a way to transform it to a polynomial.

$$U(E) = U(\bar{E}) + U'(\bar{E}).\ (E-\bar{E}) + \frac{U''(\bar{E})}{2!}.\ (E-\bar{E})^2 + \frac{U'''(\bar{E})}{3!}.\ (E-\bar{E})^3 + \ldots\ldots + \frac{U^{(n)}(\bar{E})}{n!}.\ (E-\bar{E})^n + R_{n+1}, \tag{2.6}$$

where $R_{n+1} = \frac{U^{(n+1)}(E)\,p}{(n+1)!}.\ (E-\bar{E})^{n+1}$

and p is some point between E and $\bar{E}$.

Then, the expected utility will be:

$$E\left[U(E)\right] = U(\bar{E}) + \frac{U'(\bar{E})}{2!}\bar{m}_2 + \frac{U'''(\bar{E})}{3!}\bar{m}_3 + \ldots\ldots\ldots\ldots + \frac{U^{(n)}(\bar{E})}{n!}\bar{m}_n + E\left[R_{n+1}\right], \tag{2.7}$$

where $\bar{m}_2$, $\bar{m}_3$, ... $\bar{m}_n$ are the second, third and successive higher moments with respect to the mean. If the series is convergent R_{n+1} can be neglected. It is important to note that the first term of the right hand side of equation 2.7, $U(\bar{E})$, is the utility corresponding to the mean or expected income $(\bar{E})$, $\bar{m}_2$ is the variance, $\bar{m}_3$ the skewness and so forth. It is also noted that the first moment with respect to the mean vanished in 2.7. Therefore, expected utility is not merely a function of two parameters, say expected income and variance but, it is a function of the expected income and n-1 parameters representing risk.

The actual value of n, i.e., the number of parameters which the decision maker will consider, will depend on the number of consecutive derivatives which can be obtained from the original utility function. If the decision maker's utility function is linear then $U''(\bar{E}) = U'''(\bar{E}) = \ldots\ldots\ldots = U^{n}(\bar{E}) = 0$. An individual with such utility function will only consider expected income $U(\bar{E})$ in his decisions. If the utility function is quadratic $U'''(\bar{E}) = \ldots\ldots \ldots\ldots = U^{(n)}(\bar{E}) = 0$, the decision maker will consider the expected income and the variance $\bar{m}_2$ in his decisions. But if the utility function is of a higher order the decision maker will have to account for more parameters in his decisions. Alternatively, higher order terms in equation 2.7 may vanish if some of the $\bar{m}$ elements become 0. This is clearly related to the frequency distribution of income. In symmetric distributions, say the normal distribution, $\bar{m}_3 = 0$ and $\bar{m}_4, \bar{m}_5 \ldots \bar{m}_n$ are in a fixed relation with $\bar{m}_2$. Therefore, the expected utility will depend only on the expected income and $\bar{m}_2$, even if $U'''(\bar{E}), \ldots U^{(n)}(\bar{E}) \neq 0$. In any skewed distribution $\bar{m}_2 \neq 0$ and $\bar{m}_3 \neq 0$ and hence, the expected utility will depend at least on 3 parameters $(E, \bar{m}_2, \bar{m}_3)$ when the utility function is neither linear nor quadratic. Thus, the actual number of parameters which should be considered are equal to the number of times the utility function is differentiable or to the number of independent parameters which characterizes the distribution of returns, whichever is smaller:

$$n = \text{Min}\,(d_E, P_s), \tag{2.8}$$

where d_E represents the number of times which the utility function is differentiable and P_s is the number of parameters which define the random distribution of returns.

Referring back to the question posed earlier, it may be said that risk should be represented in terms of a single parameter only when the utility function is quadratic or when the returns are normally distributed. At this point, it is important to remember that the income-risk frontier, provides the set of efficient plans from which farmers choose that plan which maximizes their expected utility. If this set of alternatives has not been determined considering the appropriate number of parameters as defined by formula 2.8, the decision maker may take erroneous decisions. For instance, if returns are not normally distributed (as may be expected in many cases) the researcher should present the income-risk frontier based on two parameters (expected income and variance) only when he is certain that the utility function is quadratic. If the utility function is not quadratic, the plan chosen might not maximize the decision maker's utility function.

To sum up, the nature of the utility function is important in judging the suitability of different methods used in farm planning under uncertainty, even if these methods are used only to determine the set of efficient plans (the income-risk frontier) leaving to the farmer to pick one of them. The indicators of risk used in estimating the income-risk frontier must be consistent with the risk indicators implicitly considered in the decision maker's utility function. The nature of the decision maker's utility function and the type of frequency distribution of the activity returns are the main elements to be considered in determining which method, if any, may be used in order to derive an income-risk frontier.

2.5 Some Methods Used in Farm Planning Under Uncertainty

In the early sections of this chapter the discussion was centered on the concepts of an efficient income-risk frontier, the utility function, risk aversion and on the determination of an optimum efficient plan. In section 2.4 the discussion focused on the number of parameters required to express risk in order to provide the decision maker with the necessary information to maximize his expected utility. The relation between this problem and the nature of the utility function assumed was also stressed.

This section is devoted to an evaluation of the QP-VAR, MOTAD and Semivariance methods using the framework provided in sections 2.1 to 2.4. Are the risk indicators used by these three methods adequate to allow for maximization of expected utility? What utility functions should be assumed in order to apply these methods? Are the utility functions assumed consistent with the basic conditions enumerated in section 2.3 ? What frequency distribution of the returns should be assumed in order to apply each method? Are the variance, semivariance or the total absolute deviation, when used as a single measure of risk, sufficient to express it? An answer to these questions will be attempted in this section given the analytical framework developed in the past sections.

2.5.1 Quadratic Programming: The Variance Approach

Markowitz (23) first formulated the risk problem in a mathematical programming model. He used a Quadratic Programming method with the variance of the total returns as a risk measure (QP-VAR). Total

returns (E) may be defined as folows:

$$E = \sum_{j}^{n} c_j x_j , \qquad (2.9)$$

where c_j is the return per unit activity and x_j is the activity level. The total variance, V, corresponding to that expected return is calculated as follows:

$$V = \sum_{i}^{m} \sum_{j}^{n} x_i \sigma_{ij} x_j, \qquad (2.10).$$

where σ_{ii} is the variance of return c_i for activity x_i and $\sigma_{ij} = \sigma_{ji}$ is the covariance between the returns of x_i and x_j.

Defining equation (2.10) in matrix notation,

$$V = X' QX, \qquad (2.11)$$

where Q is the variance-covariance matrix, X is a column vector of activities and X' is the transpose vector of X.

The problem of determining the efficient set of plans using variance as a risk indicator is often tackled through a Quadratic Programming model, which may be stated as follows (16):

$$\text{Min} \quad V = X' QX \qquad (2.12)$$

s.t.

$$AX \leq b$$

$$cX = \lambda$$

$$X \geq 0 ,$$

where A is a matrix of technical coefficients, b is a vector of resource restraints, c is a vector of activity returns and λ is a parameter of total expected return which is parameterized for n different values. The solution to this problem yields an income-risk frontier, which is presented to the farmers in order to allow them to choose a plan according to their utility functions (14).

It is important to note some characteristics of the objective function $V = X' QX$:

1. It is a positive definite or semidefinite function since the variances are all positive. This is a desirable feature of the variance approach since it means that any local minimum will be a global minimum (since the feasible set is convex).

2. Matrix Q includes not only the individual variability of the activity returns but also the correlated variations of the activity returns (co-variances).

The QP-VAR method assumes that risk may be represented as a single parameter, the variance. In other words, this method neglects the influence of other higher order moments with respect to the mean. Recalling from section 2.4, if the utility function of the decision maker is quadratic, then $U'''(\bar{E}) = U^{iv}(\bar{E}) =, \ldots\ldots = U^{(n)}(\bar{E}) = 0$. This implies that the decision maker only considers the expected income $(\bar{E})$ and the variance in his decisions. Thus, if the efficient set of plans is presented to him in an income-variance plane he will have all the information required to maximize his expected utility. Hence, the variance method is appropriate in this case. Even if the utility function is not

quadratic the variance may still be a good indicator of risk if the outcomes (the total income) are normally distributed. In this case, $\bar{m}_3 = 0$ and higher order moments are either in a fixed relation with $\bar{m}_2$ or vanish. This implies that although the decision maker would like to consider other higher moments, he may make his decisions based only on the mean and variance because all other moments are at a zero level or in a fixed relation with $\bar{m}_2$. Thus, the QP-VAR method may be used only when at least one of the two following conditions occur:

a. The utility function of the decision maker is quadratic, or

b. The distribution of returns is normal.

If these conditions do not hold the QP-VAR method could lead to misleading results. Assume for instance, that a decision maker with a non-quadratic utility function chooses from an income-variance set of possible plans and that the distribution of the outcomes is skewed with $\bar{m}_2 \neq 0$, $\bar{m}_3 \neq 0$ and assume that all other terms in the right-hand side of equation 2.7 can be neglected. The decision maker must choose a plan from the income-variance frontier, which maximizes his expected utility. But, the income-variance frontier does not consider $\bar{m}_3$, although it is different from zero and $U'''(\bar{E})$ does not vanish. Once the decision maker chooses a plan from the income-variance frontier without considering $\bar{m}_3$ he may make a mistake, in the sense that the effect of $\frac{U'''(\bar{E})\bar{m}_3}{3!}$ in equation 2.7 has not been considered and hence, it is possible that there are other feasible plans which provide a greater expected utility, that is, plans which have a smaller negative $\bar{m}3$. This difference might

more than compensate for the difference in the second term, $\frac{U'(\bar{E})\bar{m}_2}{2!}$. The magnitude of the mistake of not considering $\frac{U'''(\bar{E})\bar{m}_3}{3!}$ depends on the utility function (the value of $U''(\bar{E})$) and the skewness of the distribution. This is a quantitative problem over which a number of authors, Tsiang (30), Borch (4), Feldstein (10) and others have argued. It seems clear that it is not possible to make any general conclusion about the magnitude of the higher order terms. Hence, unless the researcher is certain that the value of $U'''(\bar{E})\bar{m}_3$ is small relative to $U(\bar{E}) + \frac{U''(\bar{E})\bar{m}_2}{2!}$ for all E he should not use the QP-VAR method. It is seldom possible to make this estimation and therefore, if conditions (a) or (b) are not satisfied the mean-variance analysis should not be applied. Unfortunately, these conditions are very restrictive. A quadratic utility function is not generally accepted (4, 5, 10, 30) because it does not satisfy the four conditions which should characterize a risk averse individual mentioned in Section 2.3. The absolute risk aversion coefficient r, in the quadratic utility function is increasing throughout all levels of income contradicting condition (c) as enumerated in section 2.3. To illustrate, assume any quadratic utility function:

$$U = E - bE^2 \qquad (2.13)$$

then

$$\frac{dU}{dE} = U' = 1 - 2bE \qquad (2.14)$$

and

$$\frac{dU^2}{d^2E} = U'' = -2b. \qquad (2.15)$$

Using the risk aversion measure as stated in equation 2.4, section 2.3, and substituting 2.14 and 2.15 in 2.4,

$$r = \frac{-U''}{U'} = \frac{2b}{1-2bE} \qquad (2.16)$$

and the derivative of r with respect to E will be:

$$\frac{dr}{dE} = r' = \frac{4b^2}{(1-2bE)^2} . \qquad (2.17)$$

As it can be seen r' is positive at all levels of income, E. This is quite absurd since it would mean for instance, that an individual would be willing to pay more insurance against the same absolute risk as his total income increases; this contradicts one of the properties which the utility function of a risk averse individual should have.

The assumption that the returns are normally distributed is also difficult to sustain. Hazell (15) tried to justify this assumption for a multiactivity farm based on the Central Limit theorem, but Chen (8) showed that this application of the Central Limit theorem was incorrect. Hazell tried to demonstrate that if a farm produces several activities the distribution of the total returns obtained from all these activities should be normal regardless of the distribution of the individual activity returns. This is true only when the activity returns

are independent among each other; obviously this is not a general situation. Thus, when the activity returns are correlated the total return will not be normally distributed unless the individual activity returns are all approximately normally distributed. However, what results may one expect if the correlation coefficients among activity returns are relatively low? Would this lead to approximately normally distributed total returns and hence, would the magnitude of the error be negligible? This is a quantitative problem and a theoretical analysis may not provide clear cut conclusions.

In summary, the QP-VAR method presents inconveniences when it is applied to empirical situations which do not correspond to either one of the two conditions mentioned above (quadratic utility function or normal distribution of returns). If at least one of these conditions occurs, the QP-VAR provides an appropriate representation of the income-risk frontier. Unfortunately, these conditions do not seem to be present frequently in empirical situations. A question not answered by using only analytical concepts is the following: What is the magnitude of the error in estimating in income-risk frontier using the QP-VAR method when neither of the conditions described above are met? One of the purposes of the set of experiments reported in the following chapters is to provide some indications regarding this problem.

2.5.2 The Semivariance Approach

One of the limitations of the variance approach is that it considers as equally undesirable negative and positive fluctuations of income around the expected income. It is clear that farmers are interested in minimizing negative variations of their income but not positive deviations. It is reasonable to sacrifice part of the expected income

in order to diminish negative variations of the income but it would be foolish to do so in order to diminish positive variations.

The Semivariance approach considers this problem. Markowitz (23) studied semivariance as an indicator of risk, being mainly concerned with the negative semivariance. The negative semivariance for one activity may be expressed as follows:

$$S_E = \frac{1}{T-1} \sum_{h}^{T} (\min. \{(c_{hi} - \bar{c}_i), 0\})^2, \qquad (2.18)$$

where T is the total number of years, c_{hi} is the actual income during year h of the activity i and $\bar{c}_i$ is the expected or average income of activity i. As can be seen, the Semivariance method is only concerned with variations of the income below its expected value.

The semivariance corresponding to the total income, the income-semivariance, may be calculated in a similar way to the variance.

$$S = \sum_{ij}^{mn}\sum x_i \, s_{ij} \, (t_g) \, x_j \quad (\text{for all } g=1, ----, K), \qquad (2,19)$$

where S is the total semivariance and t_g (1, ----, K) are the years in which the plan implies returns below the expected income and s_{ij} is defined as follows:

$$s_{ij}(t_g) = \frac{1}{T-1} \sum_{ij}^{m}\sum (c_i - \bar{c}_i)_{tg} \cdot (c_j - \bar{c}_j)_{tg}. \qquad (2.20)$$

The income-semivariance frontier can be obtained in the same way as using the variance, but instead of minimizing the total variance, equation 2.19 should be minimized, subject to the same restraints as the variance minimisation problem as shown in equation 2.12.

Another formulation of the semivariance which could be used is the following:

$$S = \sum_{i}^{m}\sum_{j}^{n} x_i \, s_{ij} \, x_j , \qquad (2.21)$$

where

$$s_{ij} = \frac{1}{T-1} \sum_{h}\sum_{i}\sum_{j} \min. (c_{hi} - \bar{c}_i), 0\} \, . \, \min. \{(c_{hj} - \bar{c}_j), 0\},$$

where

T is the total number of years

c_{hi} is the actual return during year h of activity i.

c_{hj} is the actual return during year h of activity j.

This formulation could be called the Activity Semivariance since it considers the semivariance of the individual activities (and the cosemivariances among the activities) as a criterion to choose the optimal plan, namely the optimal combination of activities. It is important to stress that this formulation is only an approximation of the income semivariance. But it has the advantage that it can be minimized using a quadratic programming algorithm, whereas the income semivariance can only be solved through simulation techniques, thus also providing an approximate solution. The Activity Semivariance will be referred to as the QP-SEMIV method. Using the QP algorithm, it is just necessary to minimize equation 2.12, substituting s_{ij} in matrix Q for σ_{ij} as calculated in equation 2.21. This matrix Q will keep its desirable characteristics as in the variance model. It is important to note that the activity cosemivariances can never have a negative sign, their smallest possible value is zero.

If total income is normally distributed the semivariance is exactly one-half of the variance at all levels of expected income and variance. Thus, in this case the semivariance follows all changes of the variance and if plan A is considered better, equivalent or worse than plan B according to the variance criterion it will also be considered better, equivalent or worse respectively, if the semivariance is used as a criterion. If the income is not normally distributed, the semivariance will not necessarily follow the variations of the variance.

It should be noted that if two distributions are compared, one with a low skewness and the other with a greater skewness, the differences in the semivariance of the two distributions will be greater than differences in the variance value. In other words, the semivariance is sensitive to changes in the skewness but the variance is not. If the skewness assumes a higher positive value the semivariance will tend to diminish even if the variance does not decrease. If the skewness becomes more negative the semivariance will tend to increase even if the variance does not increase. To illustrate, assume a skewed distribution where $\bar{m}_2$, $\bar{m}_3$ are greater than zero and assume that the utility function is not quadratic. For simplicity also assume that all terms higher than $\bar{m}_3$ can be neglected. Then the expected utility will be:

$$E\{U(E)\} = U(\bar{E}) + \frac{U''(\bar{E})\bar{m}_2}{2!} + \frac{U'''(\bar{E})\bar{m}_3}{3!} . \qquad (2.23)$$

Assuming a risk averse individual, $U''>0$ and also assume that $U'''>0$. Applying the income variance criterion, the third term of the right-hand side would be neglected, which obviously would alter the value

of E {U (E)}. The important point, however, is that $\frac{U'''(\bar{E})\bar{m}_3}{3!}$ may alter the relative order of a set of plans. Thus, if the variance is substituted for the semivariance in $\bar{m}_2$, the variations of $\bar{m}_3$ are also taken into consideration, despite that $\bar{m}_3$ does not appear explicitly. If $\bar{m}_3$ becomes more positive, $\bar{m}'_2$ measured by the semivariance will be smaller and ther term $\frac{U''(\bar{E})\bar{m}'_2}{2!}$ which is negative recalling that $U''(\bar{E}) < 0$, will be less negative which tends to compensate the neglected effect of the greater positive value of $\frac{U'''(\bar{E})\bar{m}_3}{3!}$. If another plan has a negative skewness, then the semivariance will be larger and hence $U''(\bar{E})\bar{m}'_2$ will become more negative than in the former example, which accounts for the greater negative value of $\frac{U'''(\bar{E})\bar{m}_3}{3!}$. Therefore, despite the fact that the Semivariance method does not formally consider the effect of moment $\bar{m}_3$, it is implicitly considering the combined effect of $\bar{m}_2$ and $\bar{m}_3$, which leads to a better approximation than the variance method of the true ordinal classification of the plans. The compensation effect may not be sufficient in comparisons among plans where the distribution of outcomes is extremely skewed or when the value of $U'''(\bar{E})$ is extremely large as compared to the absolute value of $U''(\bar{E})$.

An example may help to understand the ideas just mentioned. Assume three different plans (I, II, III) with the following characteristics:

Plan I, which will give:

0 with probability 0.5

or 4 with probability 0.5

Here: Mean ($\bar{E}$) = 2 semivariance (SV) = 2

Variance (V) = 4 skewness (SK) = 0

Plan II, which will give:

-2 with probability 0.2

or 3 with probability 0.8

Here: $\bar{E}$ = 2 SV = 3.2

V = 4 SK = -12.0.

Plan III, which will give:

0.7 with probability 0.68

or 5.0 with probability 0.32

Here: $\bar{E}$ = 2 SV = 1.14

V = 4 SK = 7.16.

These three plans have the same mean and variance and hence they would be considered equally efficient from the point of view of the variance analysis. However, according to equation 2.23 considering V and SK, plan III should give a higher E{U(E) } than plans I and II and plan I should give a greater E{U($\bar{E}$) } than plan II. All plans have the same mean $\bar{E}$, and the same variance but plan III is positively skewed, skewness of plan I is zero and plan II is negatively skewed. This holds regardless of the actual value of $\frac{U''(\bar{E})}{2!}$ and $\frac{U'''(\bar{E})}{3!}$. The mean-semivariance

analysis provides the same ordering without explicitly considering skewness. Plan III is better than plan II and plan I because plan III has a smaller semivariance and plan I is ranked higher than plan II because plan I has a smaller semivariance.

It is important to note, however, that if the utility function of the decision maker were quadratic these three plans would be equally desirable to him. If this is the case, the variance analysis would provide correct results and the Semivariance would not. Hence the Semivariance method should not be used in situations where the decision maker's utility function is quadratic.

2.5.2.1 The Ordinal Classification of the Expected Utility

As was pointed out before, the aim of using income-risk methods is to obtain an ordinal classification of alternative plans which allows the decision maker to choose a plan which maximizes his expected utility. In order words, a method is suitable if it can provide all the information necessary to order alternative plans on a relative scale.

Most of the discussion among Borch (5), Feldstein (10), and Tsiang (30), with regard to the validity of income variance analysis, has centered around whether the terms in the expected utility neglected by the income variance analysis are large or not. The first two authors concluded that the terms corresponding to the higher order moments in the expected utility function may be large, even larger than the variance term, and hence the income variance analysis would be valid only when the utility function is quadratic or when the outcomes are normally distributed. Tsiang (17) concluded that under frequent circumstances the

higher order moment terms may be neglected and therefore the income variance analysis is valid even when the utility function is not quadratic and the outcome distribution is not normal. In connection with this discussion it is interesting to note the following remarks:

1. These authors agree that if $\bar{m}_3, \bar{m}_4, \ldots \bar{m}_n$ are large enough, the income variance analysis is not valid (assuming a non-quadratic utility function), for example, if the outcome distribution is very skewed. However, this argument is not always true. The coefficient of skewness, β, is defined:

$$\beta = \frac{\bar{m}_3}{\bar{m}_2} \quad . \tag{2.24}$$

If β does not change substantially among the different alternative plans, the income variance analysis will provide the correct ordering of the set of plans, because there is a constant monotonic relation between $\bar{m}_2$ and $\bar{m}_3$. Hence, it is sufficient to classify the plans according to any of the two measures and therefore, the normal distribution requirement is not strictly necessary for the validity of the income variance analysis. This method is also valid for skewed (even very skewed) distributions whose coefficient of skewness, β, does not abruptly change among the different possible plans. Even if $\bar{m}_3$ changes substantially, there is no problem in applying the income variance analysis if β remains approximately constant.

2. A similar argument may be made in connection with higher order moments say $\bar{m}_4$. A coefficient of Kurtosis, γ, is defined as follows:

$$\gamma = \frac{\bar{m}_4}{\bar{m}_2} . \qquad (2.25)$$

If γ does not substantially change, again the income variance analysis provides the correct ordering of plans regardless of $\bar{m}_4$ which might be very large and fluctuates greatly among the alternative plans.

It is interesting to note that $\bar{m}_4$ is not zero in the normal distribution, but all the above-mentioned authors agreed that when the outcomes are normally distributed, the income variance analysis is correct. The coefficient of Kurtosis for the normal distribution is always equal to 3 and thus, there is a fixed relationship between the variance and $\bar{m}_4$.

3. The advantage of the income-semivariance analysis is that it is more closely related to the skewness than the variance. Even if the coefficient of skewness varies among plans, the relation coefficient between the semivariance and $\bar{m}_3$ might not change. Indeed, there is a functional relation between the semivariance and the skewness, which is stable.

2.5.2.2. Conclusions Regarding the Semivariance Method.

(a) The semivariance may be considered a better indicator of risk than the variance when the frequency distribution of the returns is not normal and if the decision maker's utility function is non-quadratic.

(b) The semivariance may provide closer representations of the income-risk frontier when applied to non-normal data if the kurtosis coefficient does not change much among the althernative plans.

(c) Since the Total Income Semivariance method needs to be approximated through non analytical methods, the Activity Semivariance method is proposed as an approximation of the Total Income Semivariance method. The advantage of the Activity Semivariance method is that it can be estimated using an analytical method such as the Q.P algorithm.

2.5.3 The MOTAD Model

Many efforts have been devoted to develop adequate linear indicators of risk as alternatives to the quadratic (variance) and semi-quadratic (semivariance) indicators reviewed in the former sections. This is due mainly to the fact that linear indicators of risk allow the use of Linear Programming algorithms which are better known than QP algorithms and also cheaper from the point of view of computational costs. One of the linear models most commonly used is the MOTAD model devised by Hazell (15).

The risk criterion used in the MOTAD model is the Total Absolute Deviation of the income (T.A.D.) for a set of t observations over n activities which may be calculated as follows:

$$T.A.D. = \sum_{h}^{t} \sum_{j}^{n} (c_{hj} - \bar{c}_j) x_j , \qquad (2.26)$$

where c_{hj} is the h observation of income of the jth activity and $\bar{c}_j$ is the expected income of the activity X_j. Thus, the minimization of the T.A.D. can be cast in an LP problem as follows:

$$\text{Min} \sum_{h}^{t} \sum_{j}^{n} (c_{hj} - \bar{c}_j) x_j \qquad (2.27)$$

s.t.

$$\sum_{j}^{n} a_{ij} x_j \leq b_i \quad \text{(For all } i = 1, \ldots, m\text{)}$$

$$\sum_{j}^{n} c_j x_j = \lambda$$

$$x_j \geq 0 \quad ,$$

where a_{ij} is the ith resource required for the jth activity

b_i is the ith resource capacity and

λ is any level of expected income which can be parameterized.

As Hazell proposed it, it is possible to minimize only the negative part of the T.A.D. if the expected returns of the activities, $\bar{c}_j$, are the sample mean income. Thus, when $\bar{c}_j$ complies with this requirement the minimization of the negative T.A.D. is equivalent to minimize the T.A.D.

The MOTAD model bases its estimation of risk on the first moment with respect to the mean (Mean absolute deviation, MAD). As can be seen in equation 2.7, MAD is not considered in the determination of the expected utility; despite that MAD is one of the determinants of the levels of utility, it is not an element determining the expected utility. Thus, the MOTAD model classifies plans using an indicator of risk which is not even considered by the decision maker in the process of maximization of expected utility. Hence, this procedure would be appropriate only if the utility function is linear, i.e., the decision maker does not consider $\bar{m}_2, \bar{m}_3 \ldots \bar{m}_n$. However, Thomson and Hazell (29) have pointed out that there is a constant relation between the variance and the mean absolute deviation when the outcomes are normally

distributed;

$$\bar{m}_2 = \frac{11}{7(n-1)} \cdot (MAD)^2, \qquad (2.28)$$

where n is the total number of observations in the sample. Given n, it is possible to calculate $\bar{m}_2$ from MAD and the decision maker may choose from a set of plans based on the mean variance criterion, having the same advantages and limitations as using the income variance analysis. Further, any ordinal classification of the alternative plans using the mean and MAD as a criterion would be equivalent to a classification using the mean and $\bar{m}_2$ as a criterion.

It is important to note that formula 2,28 applies only to estimations of variance calculated from single activity mean absolute deviations. It is not appropriate for calculations of the variance of the total income generated by the combined effect of a number of activities, unless the correlation coefficient among the activities is zero. Therefore, the mean absolute deviation is not a good estimator of the income variance when the correlation coefficients among the activities is significantly different from zero.

However, MAD has some further disadvantages with respect to $\bar{m}_2$. As Thomson and Hazell (25) recognized, there are differences in the relative statistical efficiency of MAD with respect to $\bar{m}_2$. This efficiency is dependent on the sample size n. If n is small the relative efficiency of MAD with respect to $\bar{m}_2$ will be small and as n increases the efficiency increases asymptotically to 88% of the sample variance in

estimating the population variance.

Thus, the absolute deviation may be seen as an indirect estimator of risk which may be used when the outcomes are normally distributed, when the correlation coefficients among the activities are close to zero and when samples are large. In other words, two further restrictions are added to those which affect the applicability of the mean variance analysis. If any of these restrictions is not met the MOTAD model will provide unreliable results.

2.6 Conclusions

The following conclusions may be drawn from this discussion:

1. The QP-VAR method provides an appropriate representation of the income risk frontier for an individual who possesses a quadratic utility function regardless of the frequency distribution of activity returns.

2. The QP-VAR method provides an appropriate representation of the income risk frontier if the distribution of activity returns is normal, regardless of the decision maker's utility function.

3. The QP-VAR method yields an unreliable representation of the income risk frontier if the activity returns are non-normally distributed and if the decision maker's utility function is not quadratic.

4. The MOTAD method provides appropriate representation of the income risk frontier only if the following conditions are simultaneously satisfied.

(a) The activity returns are normally distributed or the decision maker's utility function is quadratic,

(b) The correlation coefficients among activity returns are close to zero and

(c) The sample size is large.

5. The QP-SEMIV method is proposed as a good indicator of the income - risk frontier when the utility function of the decision maker is not quadratic or linear, for any distribution of activity returns where moments higher than $\bar{m}_3$ are not important.

6. In this analytical section conclusions regarding the ability of the methods to determine an income risk frontier were obtained under the implicit assumption that they were applied using the complete frequency distribution of activity returns as data base. Little was said about situations when the data base consists of relatively small samples rather than the complete frequency distribution and therefore, it is necessary to determine whether the conclusions drawn from this discussion are also valid to the estimates of the income-risk frontier using samples as data bases. Furthermore, it is also important to compare the dispersion of the estimates provided by the methods in order to evaluate their relative efficiency. An attempt to clarify these points is made in Chapter III.

CHAPTER III

THE EMPIRICAL MODEL

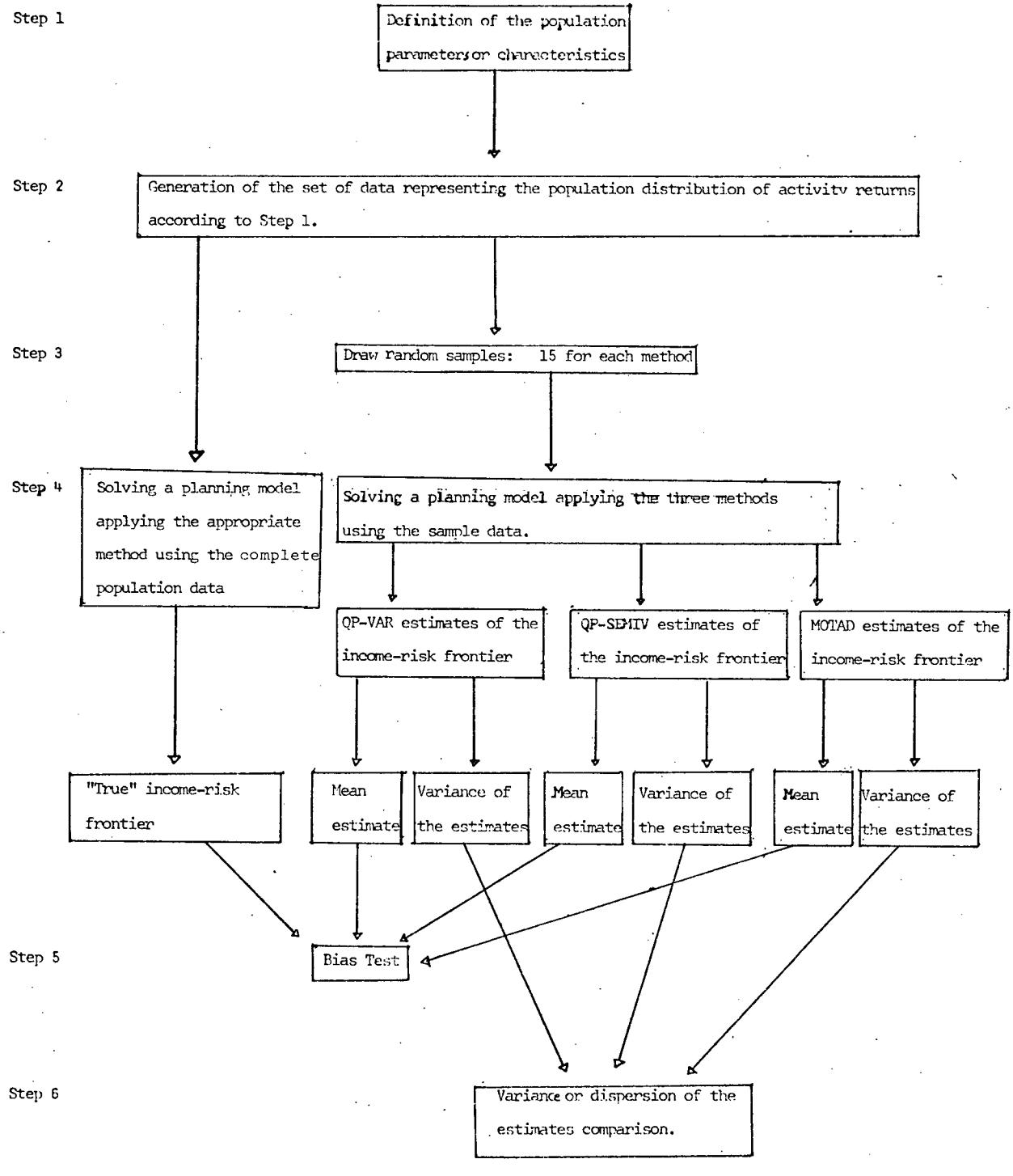
As it was pointed out in the introductory chapter, the main elements to be considered in the evaluation of the methods were the magnitude of the bias and the dispersion of the estimates of the income-risk frontier. The analytical study from Chapter II provided a qualitative evaluation of the methods under the assumption that the complete frequency distribution of activity returns was used as the data base. However, in order to test the hypotheses more precisely, quantitative information regarding bias and dispersion is required for the case where the data base consists of samples rather than complete frequency distributions of activity returns. The purpose of this chapter is to describe the procedure used in evaluating the performance of the methods when applied in a planning model using randomly drawn samples from normal and non-normal distributions. The fact that the methods are applied in a planning model using data consisting of relatively small samples allows one to evaluate their performance under conditions which are similar to those prevailing in field research applications.

3.1 General Overview of the Research Procedure

Decision makers choose from alternatives involving different returns and different degrees of risk. In order to examine this situation a model of a firm with three production activities is hypothesized; the

upper levels of the activities are limited by a number of constraints. Under most circumstances a decision maker has limited information regarding the frequency distributions of activity returns. That is, the complete frequency distribution (population) is seldom known and therefore it is assumed that the information available to the decision maker may be represented by a sample of limited size which is randomly drawn from the populations. The three methods are used to estimate income-risk frontiers, using the sample data provided. This procedure is repeated a number of times (using different samples) in order to obtain statistically verifiable results. The same problem is solved using the complete population distribution of activity returns as a data source by applying the appropriate method depending on the character of the distribution (normal or non - normal). The income risk combinations thus obtained are considered to be the "true" income risk frontiers. The solutions obtained from the sample data are compared to the true income-risk frontier in order to establish bias and relative efficiency measures of the methods. Figure 3.1 presents a general view of the research procedure. Step 1 in Figure 3.1 involves defining the population (normal or non - normal) and the parameters which define it. The magnitude of these parameters will depend on the empirical situation which is simulated. Step 2 involves the process of generating populations according to the characteristics defined in Step 1. Step 3 involves the drawing of samples from the populations generated. Step 4 considers the models to be solved in order to obtain two types of results;

FIGURE 3.1 AN OVERVIEW OF THE RESEARCH PROCEDURE



(a) The "true" income risk frontier, which is obtained when a method judged appropriate is applied directly to the complete population data:

(b) The estimated income risk frontiers, which are obtained when the methods are applied to the samples data.

In Step 5 the mean income risk frontier as estimated by each method is compared to the "true" income risk frontier in order to establish whether the different methods provide biased estimates. Finally, in Step 6, the variances of the estimates obtained with each method are compared.

The proceedure outlined in Figure 3.1 is repeated four times. In each instance the "true" income-risk frontier is compared with the mean income-risk frontier as estimated by each method, but the assumptions underlying the distribution of activity returns differed. In the following section these steps will be elaborated upon.

3.2 Generation of the Populations

In order to evaluate the methods under study, it is assumed that a complete set of frequency distribution of activity returns is known. It is also assumed that if the appropriate method is used to derive an income-risk frontier using the complete frequency distribution as it data source, then this income-risk frontier may be considered as the "true" one. For this reason randomly distributed sets of data were generated thus representing complete frequency distributions of returns for three activities. Four trivariate distributions of returns were required, two normal distributions with high and low correlation among activity returns and two non-normal distributions with high and low degree of correlation among activity returns.

The non - normal population distributions chosen were of the gamma type. The gamma distribution has two characteristics that are of importance to this study, namely, it is positively skewed and its values cannot be negative. The use of a positively skewed distribution may be justified because it is expected that historical series of activity returns will be positively skewed because of continuous technological improvements in agriculture, provided that real price changes do not offset such trends. Luttrell and Gilbert (22) measured the degree of skewness which characterized yield distributions during the last 41 years for a number of crops in numerous states of the United States. In all cases, the skewness coefficients calculated were positive although not all were statistically significant.

The use of a distribution whose values cannot be negative is justified when risk is measured using gross return rather than net returns. Many empirical studies have used gross returns (12, 15, 16) because it is difficult to calculate net returns for each activity for all years of a historical series, since this necessitates knowing variable and overhead costs for each activity over that period. Thus, it appears that the gamma distribution closely approximates the possible distribution of gross activity returns which obviously cannot be negative.

In order to generate the different populations their means and variance-covariance matrices of activity returns were predefined. Table 3.1 shows the mean activity returns for the four populations generated and Table 3.2 presents the variance-covariance matrices.

TABLE 3.1 Mean Activity Returns, Mean Correlation Coefficients and Skewness Coefficient of the Populations Generated

	Type of Distribution	Mean Gross Returns			Mean Correlation Coefficient	Skewness Coefficient
		X_1	X_2	X_3		
Population I	Normal	6.0	10.0	9.0	0.2	0
Population II	Normal	6.0	10.0	9.0	0.6	0
Population III	Gamma	8.9	17.0	13.5	0.2	1.1
Population IV	Gamma	8.9	17.0	13.5	0.6	1.4

Table 3.1 also shows the mean correlation coefficient among activity returns which is calculated from the variance-covariance values and the skewness coefficients. In Table 3.2 the variances are the diagonal elements of each matrix and off diagonal elements are the covariances.

TABLE 3.2 Variance-Covariance Matrices of the Activity Returns Corresponding to the Different Populations Generated.

	Normal I			Normal II			Gamma III			Gamma IV		
	X_1	X_2	X_3	X_1	X_2	X_3	X_1	X_2	X_3	X_1	X_2	X_3
X_1	0.5	0.18	0.15	0.5	0.46	0.63	23.1	14.3	7.5	49.8	96.1	56.1
X_2		1.0	0.30		1.0	0.76		222.1	22.5		515.1	179.8
X_3			1.5			1.5			58.2			175.3

The semivariance-cosemivariance matrices were calculated for the gamma distributed populations according to equation 2.22 from Chapter II. Table 3.3 shows the semivariance-cosemivariance matrices of the gamma populations

TABLE 3.3 Semivariance-Cosemivariance Matrices of the Activity Returns Corresponding to the Gamma Populations

	Gamma (III)			Gamma (IV)		
	X_1	X_2	X_3	X_1	X_2	X_3
X_1	7.9	12.2	7.1	19.2	90.0	49.8
X_2		72.6	16.6		197.1	154.2
X_3			22.4			72.6

It is important to note that the data representing each population consisted of a discrete set of 500 observations. This made it possible to calculate the activity semivariances and cosemivariances from the gamma populations. As will be seen in section 3.4, mean activity returns, semivariance-cosemivariance matrices and variance-covariance matrices will be used in solving the QP-SEMIV and QP-VAR methods.

3.3 The Sampling Process

A number of samples (size 12) are randomly drawn from each

population defined above. Each sample represents information regarding activity returns provided by historical records over a number of years in an empirical setting. The size of 12 was judged to be a reasonable approximation of the number of years data that would normally be available to a decision maker in a real world environment.

Thirty samples were randomly drawn from each normal population. Fifteen of these were used to obtain solutions using the QP-VAR method (solved 15 times) and the remaining fifteen samples were used in solving for the MOTAD method (the same number of times). Forty five samples were drawn from each gamma population distribution and these were used in solving for the QP-SEMIV, QP-VAR and MOTAD methods, fifteen times each. This allowed sufficient income-risk estimates for statistical testing.

From each of the samples drawn from the normal populations, the means, variances and covariances were calculated. This information was used in the objective function of the QP-VAR method. The MOTAD method used the complete sample distribution to minimize the absolute deviation from the mean or expected income. For the samples drawn from the gamma populations the same parameters were calculated and in addition the semivariance and cosemivariance matrices were obtained. The mean activity returns and the variance-covariance matrix were used in the QP-VAR method and the semivariance- cosemivariance matrix in addition to the mean activity returns were used in the QP-SEMIV method. Table 3.4 shows an example of the parameters calculated from one sample drawn from the normal population (II) and used in solving the QP-VAR

method. It is noted that the data presented in Table 3.4 represent the parameters of one of the numerous samples obtained.

TABLE 3.4 Variance-Covariance Matrix and Mean Activity Returns Calculated from a Sample Drawn from a Normal Population (II): An Example

	Variance-Covariance Matrix			Mean Activity Returns
	X_1	X_2	X_3	
X_1	0.41	0.38	0.60	6.3
X_2		0.63	0.46	9.7
X_3			1.50	9.5

Table 3.5 shows the semivariance-cosemivariance matrix and mean activity returns as calculated from a sample drawn from the Gamma I population. This information is used in solving the QP-SEMIV using each of the samples drawn.

TABLE 3.5 Semivariance-Cosemivariance Matrix and Mean Activity Returns Calculated from a Sample Drawn from a Gamma (1) Population

	Semivariance-Cosemivariance Matrix			Mean Activity Returns
	X_1	X_2	X_3	
X_1	5.6	9.9	7.0	10.3
X_2		44.2	20.4	13.4
X_3			20.5	20.5

As may be expected, the parameters calculated for the samples approximated the population parameters, and the mean values obtained from all samples were similar to the population parameters in each case.

3.4 Solution of the Models

This section describes the process of determining the "true" income risk frontier and the sample estimates of the income risk frontier for the hypothetical firm situation discussed earlier. A small model characterizing the set of constraints for this firm given an objective of minimizing risk for certain levels of expected income was developed. This model was solved using each of the methods under study.

3.4.1. Description of the General Model

The general model minimizes risk (measured differently according to the method used) subject to seven linear constraints, six simulating

resource constraints and one a minimum expected return.

The general model used was the following:

$$\text{Min.} \quad R = \emptyset \ (X)$$

Subject to (3.1)

$$AX \leq b$$

$$\bar{c}X \geq \lambda$$

$$X - 0,$$

where X is the column vector of activity levels, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

R is the risk level as a function of the activity levels,

$\bar{c}$ is a row vector of expected activity returns,

λ is the total expected return,

A is a 6 x 3 matrix of technical coefficient defined as follows *:

$$A = \begin{bmatrix} 2.0 & 0.4 & 0.3 \\ 2.5 & 1.2 & 1.0 \\ 1.0 & 0.7 & 1.2 \\ 4.0 & 3.9 & 3.0 \\ 1.5 & 2.4 & 2.5 \\ 3.0 & 0.3 & 0.2 \end{bmatrix}$$

---

* The actual matrix A and vector b were chosen so as to generate a smooth production possibility frontier allowing for a large number of boundary solutions. The basic consideration in determining matrix A and vector b was to avoid corner solutions which would have diminished the sensitivity of the results to the different methods used.

and b is a column vector of resource constraints given as follows:

$$b = \begin{bmatrix} 9.0 \\ 14.5 \\ 9.0 \\ 33.5 \\ 20.5 \\ 10.5 \end{bmatrix}$$

The risk function, Ø, depends on the method used. The constants of the model (matrix A and vector b) remain the same for all methods. The value λ is parameterized for three levels of expected income, obtaining three solutions for each method.

In solving for the QP-VAR method the objective function represented the total variance of the income:

$$V = [\, x_1, x_2, x_3 \,] \cdot [Q] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

where Q is the variance-covariance matrix calculated from the activity returns data.

The same quadratic objective function was used in solving the QP-SEMIV method, but matrix Q is substituted by a semivariance-cosemivariance matrix as calculated from the activity returns data.

In solving the MOTAD method, the objective function represented the total absolute deviation of the activity returns with respect to

their means.

3.4.2 The 'True' Income Risk Frontiers.

As shown in Chapter II, the QP-VAR method provides a 'true' approximation of the income risk frontier when applied to normally distributed data. In order to determine the true income risk frontier for normally distributed population, the QP-VAR method was applied using the parameters which characterize these normally distributed populations.

The model used in solving the QP-VAR method applied to the normal population (1) data is the following:

$$\text{Min.} \quad V = \left[x_1, x_2, x_3\right] \cdot \begin{bmatrix} 0.50 & 0.18 & 0.15 \\ 0.18 & 1.00 & 0.30 \\ 0.15 & 0.30 & 1.50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Subject to :

$$AX \leq b \qquad (3.2)$$

$$\bar{c}X \geq \lambda ,$$

where x_1, x_2, x_3 are the activity levels.

A is the matrix of technical coefficients shown in equation 3.1,

b is a column vector of resource constraints shown in equation 3.1,

λ is the expected total income to be parameterized at three levels,

and $\bar{c}$ is a row vector of mean activity returns which for the normal population (1) is defined as follows (see Table 3.1):

$$c = \left[6.0 \ 10.0 \ 9.0\right] .$$

The matrix shown in the objective function of equation 3.2 is the variance-covariance matrix of the normal population (1) as presented in Table 3.2.

The model is solved for three levels of expected income, thus representing three points of the true income risk frontier. In determining the true income risk frontier for the normal population (11), the same model is used except that the variance-covariance matrix used in the objective function is calculated from the normal population (11).

The QP-SEMIV method was used in determining the true population income-risk frontiers for the gamma populations, because, as may be recalled from Chapter II, the semivariance provides appropriate reprsentations of risk for skewed distributions. Instead of using the variance-covariance matrix in the objective function, the semivariance-cosemivariance matrix, as defined for the gamma populations, was used (see Table 3.5). Additionally, the mean activity returns were those defined for the gamma populations (see Table 3.1).

3.4.3 The Income Risk Frontier Estimates.

Considering that the samples represent limited information available to the decision maker in real situations, the estimates of the income-risk frontier using this sample data were used to evaluate the three methods. In other words, the methods were evaluated considering the departure from the true income-risk frontier (calculated according to Section 3.4.2) of their estimates. In order to obtain these estimates, the three methods were solved using the same general model described in Section 3.3.1 but in this instance sample data rather than the complete population data was used to calculate the parameters of

the objective function and the expected activity returns. The income risk frontier was estimated at three levels of expected income using each method. The QP-VAR and MOTAD methods provided fifteen estimates of the income risk frontier for each of the four distributions generated. The QP-SEMIV method also provided fifteen sample estimates of the income-risk frontier for each of the gamma distributions. This method was not applied to the normally distributed data because its estimates are equivalent to those of the QP-VAR method as was shown in Chapter II.

The model used to estimate the income risk frontier, using the QP-VAR and QP-SEMIV methods, was similar to that used in determining the true population income risk frontier as shown in equations 3.1 and 3.2. The model used for the MOTAD method as applied to one sample drawn from a normal population (I) is presented in Table 3.6. The objective function of the MOTAD model consists of 12 variables which account for the total negative absolute deviation of income for each observation. The resource constraints and their maximum levels are defined by the values shown in matrix A and vector b in the general model as stated in equation 3.1. The values of the expected income constraint depend on the mean activity returns for the samples, parameterized for the same levels of income as in the determination of the true income-risk frontier. The absolute deviations were the functional part of the objective function; the negative absolute deviations with respect to the mean activity returns are summed in the objective function. Thus, the model is designed to choose that level of activities which minimizes the total absolute negative deviation given an expected level of income subject to the resource constraints.

Table 3.6 The MOTAD Model as Applied to a Sample Obtained From the Normal Population II

		X_1	X_2	X_3	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}	Y_{12}	RHS
Objective Function					1	1	1	1	1	1	1	1	1	1	1	1	Minimize
Resource Constraints		2.0	0.4	0.3													≤ 9.0
		2.5	1.2	1.0													≤ 14.5
		1.0	0.7	1.2													≤ 9.0
		4.0	3.9	3.0													≤ 33.5
		1.5	2.4	2.5													≤ 20.5
		3.0	0.3	0.2													≤ 10.5
Expected Income Constraint		5.9	10.1	8.9													≥ λ
Absolute Deviations Constraints	T1	0.1	0.4	-1.5	1												≥ 0
	T2	-0.2	0.9	0.6		1											≥ 0
	T3	0.5	1.8	1.7			1										≥ 0
	T4	0.3	0.0	-0.4				1									≥ 0
	T5	-0.5	0.4	-2.9					1								≥ 0
	T6	0.3	0.1	- .5						1							≥ 0
	T7	-0.9	-2.3	1.0							1						≥ 0
	T8	0.6	-0.5	2.5								1					≥ 0
	T9	-0.4	-0.9	-0.4									1				≥ 0
	T10	0.3	-0.1	0.8										1			≥ 0
	T11	0.1	0.5	-1.1											1		≥ 0
	T12	-0.2	-0.2	0.3												1	≥ 0

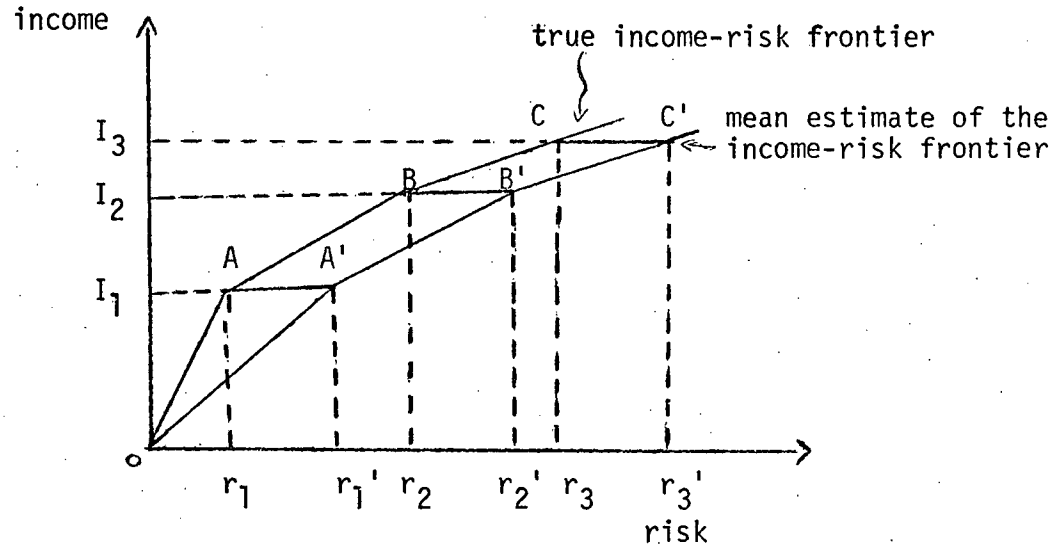
In order to compare the QP-VAR and MOTAD estimates of the income-risk frontier with the true income-risk frontier for normally distributed data, the levels of risk are measured as the magnitude of the standard deviation of the total income (calculated ex-post from the MOTAD solutions). This procedure was chosen by assuming that risk may be adequately represented by the variance when the activity returns are normally distributed (see Section 2.4). In Chapter II the semivariance was shown to be an appropriate measure of risk in skewed distributions. Thus, the square root of the total semivariance was used to measure risk when comparing the QP-SEMIV, QP-VAR and MOTAD solutions for the gamma distributions. The semivariance was calculated ex post for the QP-VAR and MOTAD solutions.

3.4.4 Analysis of the Solutions

This subsection is concerned with steps 5 and 6 shown in Figure 3.1. The main criterion used to evaluate the results obtained using the three methods were the following:

(1) Bias of an income-risk frontier mean estimate; establishing if the level of risk estimated by the methods at each level of income are significantly different from the true risk levels. In Figure 3.2 the true income-risk frontier and an estimated income-risk frontier are shown.

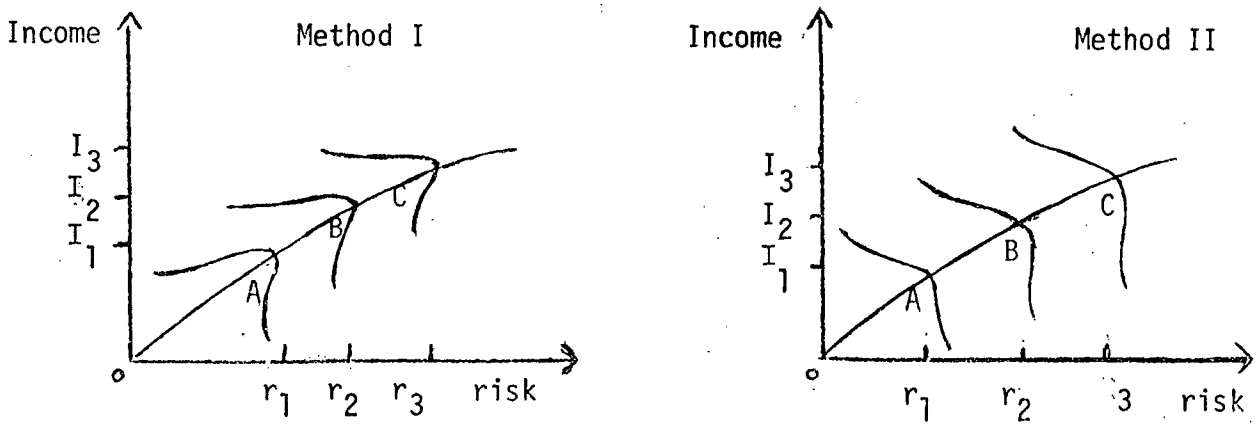
FIGURE 3.2 Bias in the Income-Risk Frontier Estimator



If the differences shown as AA', BB' and CC' are significant (at 5% level of significance), the method used to estimate that income-risk frontier is considered biased.

(2) Dispersion of the estimates; the variances of the estimates as provided by the methods will be compared, as illustrated in Figure 4.2.

FIGURE 4.2 A Dispersion Comparison Between the Estimates Provided by Two Methods



In Figure 4.2 two unbiased estimates of the income-risk frontier as provided by any two methods are shown. Method I is more efficient than method II if the variance of the estimates provided by method I is smaller than the variance of the estimates of method II.

It is also possible to compare activity levels as estimated by the methods to the actual level of activities underlying the true income risk frontier. Given that there is only one activity combination which minimizes risk at each level of expected income (see Chapter II, figure 2.2), the results of this comparison will be the same as those considering the risk levels criterion. Thus comparing the income risk estimates it is a sufficient criterion to judge the relative efficiency of the methods.

3.5 The QP-SEMIV Method as a Substitute of the Income Semivariance Method

In chapter II it was shown that the Total Income Semivariance is an adequate measure of risk when the data is non - normally distributed. On the other hand, the QP-SEMIV method, which uses the Activity Semivariance as a risk measure, was proposed as an approximation of the Income Semivariance method, which uses the Total Income Semivariance as a risk measure. This section provides a description of two tests designed to evaluate the QP-SEMIV method as a substitute of the Income Semivariance Method:

1. From the solutions provided by the three methods, the Total Income Semivariance (as defined in Equation 2.15) was calculated *ex post*, in order to see if the QP-SEMIV method provided the smallest income semivariance of the methods as expected *a priori* according to the discussion in Chapter II.

If so, this would imply that the QP-SEMIV provides the best solution of the methods under study when the distribution of the activity returns is skewed (gamma).

2. A number of gamma populations of different degrees of skewness were generated. The QP-VAR and QP-SEMIV methods were applied to each of these populations and their solutions were compared. The purpose of this experiment was to determine the differences among the solutions presented by each method when the degree of skewness changes. A priori one may expect that as the degree of skewness increases, the differences in the solutions provided will become more apparent. In other words, there should be a positive correlation between the divergence of the solutions and the degree of skewness of the populations.

3.6 Summary

A set of experiments was designed to test the ability of the methods to generate unbiased and efficient estimates of true income-risk frontiers. Four populations representing activity returns data were generated and using these as data bases three points on an income-risk frontier were determined. Estimates of the income-risk frontier were obtained using randomly drawn sample data from the populations and the mean risk estimates obtained with each method were compared to the true levels of risk to establish bias. The degree of dispersion of the estimates as provided using each method was also compared.

Two assumptions were implicitly made throughout this analysis: (1) activity returns may be considered randomly distributed and (2)

the decision maker's utility function is non-quadratic and his expected utility is not strongly affected by moments higher than the skewness moment. The first assumption is widely accepted and it has been used implicitly or explicitly in most theoretical and applied studies of risk (1, 21, 22,23, 28). The second assumption is relevant for the analysis as applied to gamma distributions. If a quadratic utility function is assumed, the QP-SEMIV method as applied to the population data cannot generate a true income-risk frontier (see Chapter II, Section 2.5). However, as may be seen in sections 2.3 and 2.4 a large number of decision makers may not possess a quadratic utility function.

CHAPTER IV

THE RESULTS

The purpose of this chapter is to report on the results obtained from the experiments described in Chapter III. As may be recalled from Chapter III, the methods were tested for four different populations where the frequency distributions and degree of correlation among activity returns varied. Hence, four sets of results corresponding to these four situations will be reported in the following sections. In presenting the results obtained for each situation, three sets of information will be shown, namely the mean levels, variances and ranges of the risk estimates provided by each method at three income levels. The complete set of sample estimates of risk may be found on Tables A.1 to A.10 of the Appendix.

4.1 The Normal Case with Low Degree of Correlation Among Activity Returns

This section presents the results corresponding to the first situation analyzed, i.e., normally distributed activity returns with a low degree of correlation among activity returns. Table 4.1 shows the QP-VAR and MOTAD mean estimates of risk measured by the standard deviation of the total income. Bracketed figures beside the mean estimates of risk are the value of the t statistics calculated in order to establish whether there are significant differences between the estimated and true values.

Table 4.1 Mean Risk[1] Levels as Estimated by QP-VAR and MOTAD Methods and the True Population Values for Three Levels of Expected Income. Normal Distributions with Low Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	5.7	4.0	2.6
QP-VAR Estimates	5.9 (1.42)[2]	4.1 (1.11)	2.7 (1.66)
MOTAD Estimates	6.0 (2.00)	4.1 (1.00)	2.7 (1.24)

(1) Risk is measured by the standard deviation of the total income.

(2) Figures between brackets are the t statistic values.

As may be seen in Table 4.1, the QP-VAR and MOTAD mean estimates are not significantly different to the true risk value at 1% or even 5% level of signicance (LOS). Thus the differences between the true and the mean estimates are no larger than those that would arise from sampling error. The mean estimates of risk were calculated from fifteen estimates of risk obtained when the methods are applied to the same number of samples randomy drawn from the population (see Tables A.1 & A.2 of the Appendix.)

With respect to the dispersion of the QP-VAR and MOTAD estimates, the variances of the QP-VAR estimates were always smaller than the variances of the MOTAD estimates as it is shown in Table 4.2.

Table 4.2 Variances and Mean Variability Coefficient of the MOTAD and QP-VAR Estimates of Risk at Three Levels of Expected Income

	Levels of expected Income			Mean Variability Coefficient
	High	Medium	Low	
QP-VAR estimates	0.32	0.13	0.07	0.10
MOTAD estimates	0.38	0.18	0.10	0.12

The Mean Variability Coefficient (MVC) is calculated as follows:

$$MVC = \sum_{i=1}^{3} \frac{SD_i}{MR_i},$$

where SD_i is the standard deviation of the estimates at an income i and MR_i is the mean risk level estimated at income i. The mean variability coefficient is also larger in the case of the MOTAD estimates. However, none of the differences between QP-VAR and MOTAD variance of their estimates was significant at 5% LOS when the F statistic test was applied. Most of the F values (1.18, 1.38 and 1.43 for low, medium and high levels of expected income, respectively) were significant only at 25% LOS.

The degree of dispersion of the results is in general satisfactory for both methods. Table 4.3 shows the range values of the estimates with 95% and 68% probability as compared to the true values of risk. For example, at a high level of income, 95% of the estimates made using the QP-VAR method fall between 4.8 to 7.0 with a true value of 5.7 .

Table 4.3 Range Levels of the Risk Estimates Provided by QP-VAR and MOTAD Methods as Compared to the True Risk Values. Normal Distribution, Low Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	5.7	4.0	2.6
QP-VAR Risk Range:			
95% Probability	4.8 - 7.0	3.4 - 4.8	2.2 - 3.2
68% Probability	5.3 - 6.5	3.7 - 4.5	2.4 - 3.0
MOTAD Risk Range			
95% Probability	4.8 - 7.2	3.3 - 4.9	2.1 - 3.3
68% Probability	5.4 - 6.7	3.5 - 4.7	2.3 - 3.1

As may be seen in Table 4.3 the degree of dispersion of both estimates is reasonably low. None of the estimates are more than 26% different from the true risk value with 95% probability. This means that in 19 out

of 20 estimates of risk the magnitude of the error was less than 26%. In approximately 14 out of 20 estimates the magnitude of the error when compared to the true risk level was less than 16%.

In summary, when the returns are normally distributed with a low degree of correlation among activity returns the QP-VAR and MOTAD methods may be considered unbiased estimators of the income-risk frontier. Furthermore, statistical evidence was not sufficient to demonstrate categorically that one method is more efficient than the other. It should be noted that these results may change for samples of smaller size, since as may be recalled from Chapter II, Equation 2.28 is not valid for small samples. However, the sample size used in the study approximates the amount of observations available in applied problems. Only on rare occasions will a researcher work with less than 8 years data or with more than 15 years data in this type of analysis (1, 12, 16).

4.2 The Normal Case with a High Degree of Correlation Among Activity Returns

This section reports on the results obtained when the QP-VAR and MOTAD methods were applied to normally distributed data with high degree of correlation among activity returns. Table 4.4 shows the mean levels of risk estimated at three levels of expected income by the QP-VAR and MOTAD methods as compared to the true population values of risk.

TABLE 4.4 Mean Risk (1) Levels as Estimated by QP-VAR and MOTAD Methods and the True Risk Values for Three Levels of Expected Income Normal Distribution with High Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	6.4	5.1	3.4
QP-VAR Estimates	6.5 (0.38)	4.8 (1.57)	3.2 (1.3)
MOTAD Estimates	7.6 (4.28**)	5.8 (3.68**)	3.9 (2.3*)

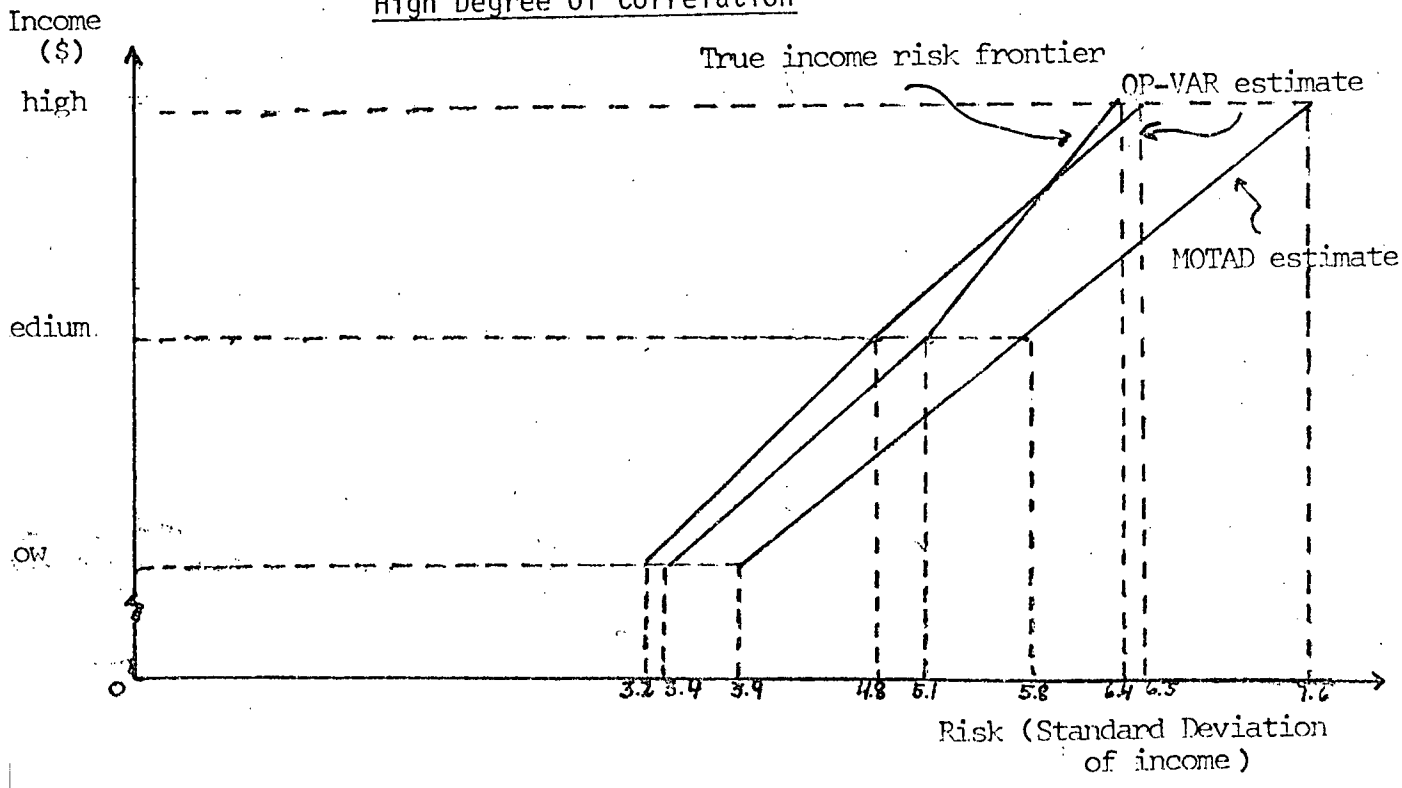
(1) Risk is measured by the standard deviation of the total income. Figures between brackets are the t statistic values.

* Significant at 5% level of significance.

** Significant at 1% level of significance.

The data shown in Table 4.4 may be graphed in an income risk plane as illustrated in Figure 4.1.

Figure 4.1 The True Population Income-risk Frontier and the Income-Risk Frontier as Estimated by QP-VAR and MOTAD. Normal Distribution. High Degree of Correlation



The t tests applied showed that the differences between the QP-VAR estimates and the true levels of risk were not significant at 1% or 5% LOS (see Table 4.4). The MOTAD estimates were all significantly different to the true risk levels at 1% LOS except for the low level of income which was signicant at 5%. Hence, the MOTAD estimates of risk may be considered biased estimates of the true risk levels. As may be seen in Table 4.4, the MOTAD estimates are positively biased and the magnitude of the bias fluctuated from approximately 13% (at medium income level) to 19% (at the high level of income).

The MOTAD method estimates had a larger variance than the QP-VAR estimates at all levels of income as may be seen in Table 4.5. The differences in the variances as provided by the methods were significant at 5% (LOS) for the medium and low level incomes when the F statistic was applied (the F values were 1.12, 2.42 and 3.36 for the high, medium and low income levels respectively). Table 4.5 also includes the mean variability coefficient calculated as indicated in Table 4.4.

Table 4.5 Variances and Mean Variability Coefficient of the MOTAD and QP-VAR Estimates of Risk at Three Levels of Income. Normal Distribution, High Degree of Correlation

	Levels of Expected Income			Mean Variability Coefficient
	High	Medium	Low	
QP-VAR Estimates	1.06	0.58	0.36	0.17
MOTAD Estimates	1.19	1.39	1.21	0.21

As may be seen in Table 4.5, the variances of the MOTAD estimates more than doubled the QP-VAR estimates except at the high level of income *. These figures are also more dispersed than the QP-VAR estimates.

Table 4.6 shows the risk ranges of the QP-VAR and MOTAD estimates at 95% level of probability.

Table 4.6 Range Levels of the Risk Estimates Provided by QP-VAR and MOTAD Methods as Compared to the True Risk Values. Normal Distribution, High Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	6.4	5.1	3.1
QP-VAR Risk Range:			
95% Probability	4.5 - 8.6	3.3 - 6.3	2.0 - 4.4
68% Probability	5.5 - 7.6	4.0 - 5.6	2.6 - 3.8
MOTAD Risk Range:			
98% Probability	5.4 - 9.8	3.4 - 8.2	1.7 - 6.1
68% Probability	6.5 - 8.7	4.6 - 7.0	3.8 - 5.0

It is interesting to note in Table 4.6 that at the 68% probability level the risk range of MOTAD estimates does not even include the true value at the high and low levels of expected income. This means that approxi-

* A reason for this may be that the combinations of activities which yield the high income level are fewer than those at lower levels of income.

mately 14 out of 20 estimations of risk made using the MOTAD method will not include the true values of risk. Furthermore, the risk range of the MOTAD estimates are wider than the QP-VAR estimates, which implies that the probability of errors in the estimates is smaller in the QP-VAR method.

It is also important to note that both methods are less efficient in this case than in the normal distribution with low correlation coefficient case. This is reflected in larger variances and larger mean variability coefficients of the estimates. A reason for this may be that the larger the correlation coefficients among activity returns, the larger are the fluctuations on the risk levels due to a given change in activity levels. Thus, small variations on activity levels which occur when the methods are applied using different samples, generate larger fluctuations on risk levels when the correlation coefficients are high.

In summary, when returns are normally distributed with a high degree of correlation among them, the QP-VAR method may be considered an unbiased estimator of the income-risk frontier. The MOTAD method has shown to be inadequate in this situation since it provides biased estimates of the income-risk frontier and the dispersion of its estimates is larger than that of the QP-VAR estimates. Additionally, the levels of efficiency of both methods are lower than in the normal - low correlation case.

4.3 Gamma Distributions and Low Degree of Correlation Among Activity Returns

This section presents the results obtained when the methods were applied to gamma distributed data with low correlation among activity

returns. Table 4. 6 shows the MOTAD, QP-VAR and QP-SEMIV mean estimates of risk (measured as the square root of the semivariance) as compared to the true levels of risk.

Table 4.7 Mean Risk Levels (1) as Estimated by QP-SEMIV, QP-VAR and MOTAD Methods and the True Values of Risk for Three Levels of Expected Income. Gamma Distribution with Low Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	44.4	27.2	18.1
QP-SEMIV Estimates	46.3 (0.78)	29.1 (1.57)	19.6 (1.32)
QP-VAR Estimates	64.4 (7.69**)	39.7 (7.96**)	27.1 (7.14**)
MOTAD Estimates	68.7 (6.53**)	45.7 (6.63**)	29.9 (6.70**)

(1) Risk is measured as the square root of the semivariance of the total income.

Figures between brackets are the t statistic values.

* Significant at 5% LOs

** Significant at 1% LOS.

The differences between the QP-SEMIV mean estimates of risk and the true levels were not significant at 1% or 5% as may be seen in Table 4.7. The QP-VAR estimates and MOTAD method estimates were significantly different to the true value at 5% and 1% LOS at the different levels of expected income. The QP-VAR and MOTAD estimates were positively biased

and the magnitude of the QP-VAR bias fluctuated between 45% at the high and medium income levels and 49% at the low level of income. The bias of the MOTAD estimates were larger and fluctuated between 54% at the high level of income and 68% at the medium income.

Table 4.8 shows the variances of the QP-SEMIV, QP-VAR and MOTAD estimates of risk at each level of expected income.

TABLE 4.8 Variances of the QP-SEMIV, QP-VAR and MOTAD Estimates of Risk at Three Levels of Expected Income. Gamma Distribution, Low Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
QP-SEMIV	88.6	22.1	19.4
QP-VAR	102.0	37.2	24.0
MOTAD	207.4	68.9	42.2

The variances of the QP-VAR estimates were not significantly different from the variances of the QP-SEMIV estimates at 5% LOS or even at 10% when the F test was applied. The MOTAD and QP-SEMIV variances were all significantly different at 10% except at the low level of income.

Table 4.9 shows the range of the risk levels estimated by each method as compared to the true risk values. The ranges of the MOTAD estimates are wider than the QP-SEMIV and QP-VAR ranges of their estimates.

Additionally, the risk ranges of the MOTAD and QP-VAR estimates do not even include the true values with 68% probability. The range of the QP-SEMIV estimates is narrower and it includes the true values of risk at all levels as expected income.

Table 4.9 Range of the Risk Levels as Estimated by QP-SEMIV, QP-VAR and MOTAD Methods as Compared to the True Risk Values. Gamma Distributions, Low Degree of Correlation.

	Levels of Expected Income		
	High	Medium	Low
True risk values:	44.4	27.2	18.1
QP-SEMIV Risk Range:			
95% Probability	27.9 - 64.7	19.7 - 38.5	10.8 - 28.4
68% Probability	37.1 - 55.5	24.4 - 33-8	15.2 - 24.0
QP-VAR Risk Range:			
95% Probability	44.2 - 84.6	27.5 - 51.9	17.3 - 36.9
68% Probability	54.3 - 75.5	33.6 - 45.8	22.2 - 32.0
MOTAD Risk Range:			
95% Probability	40.1 - 97.7	29.1 - 62.3	15.7 - 41.7
68% Probability	54.5 - 83.3	37.4 - 54.0	22.3 - 35.2

Thus the QP-SEMIV method is the only method which provides unbiased estimates of the true income risk frontier. Furthermore, the dispersion of the QP-SEMIV estimates is significantly smaller than the MOTAD estimates, but not smaller than the dispersion of the QP-VAR esti-

mates. The results suggest that under conditions of gamma distribution of returns and small correlation coefficient among activity returns the QP-SEMIV method provides good estimates of the income risk frontier. The QP-SEMIV method is clearly the most efficient method followed by the QP-VAR method. The MOTAD method may be considered the least efficient since the magnitude of its bias and the dispersion of its estimates as measured by the variance is the largest of all methods.

4.4 Gamma Distributions and High Degree of Correlation Among Activity Returns

This section presents the results obtained when the methods were applied using gamma distributed data with high degree of correlation among activity returns. Table 4.10 shows the QP-SEMIV, QP-VAR and MOTAD mean estimates of risk as compared to the true population levels of risk at three levels of expected income.

Table 4.10 Mean Risk Levels (1) as Estimated by QP-SEMIV, QP-VAR and MOTAD Methods and the True Population Values of Risk for Three Levels of Expected Income. Gamma Distribution with High Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
QP-SEMIV	45.3 (1.12)	29.6 (1.18)	24.2 (0.86)
QP-VAR	76.1 (5.63**)	52.9 (6.72**)	38.7 (7.75**)
MOTAD	89.8 (6.51**)	60.2 (6.63**)	43.3 (6.70**)
True Values	49.8	32.6	22.5

Legend:[1] Risk is measured by the square root of the semivariance of the total income.

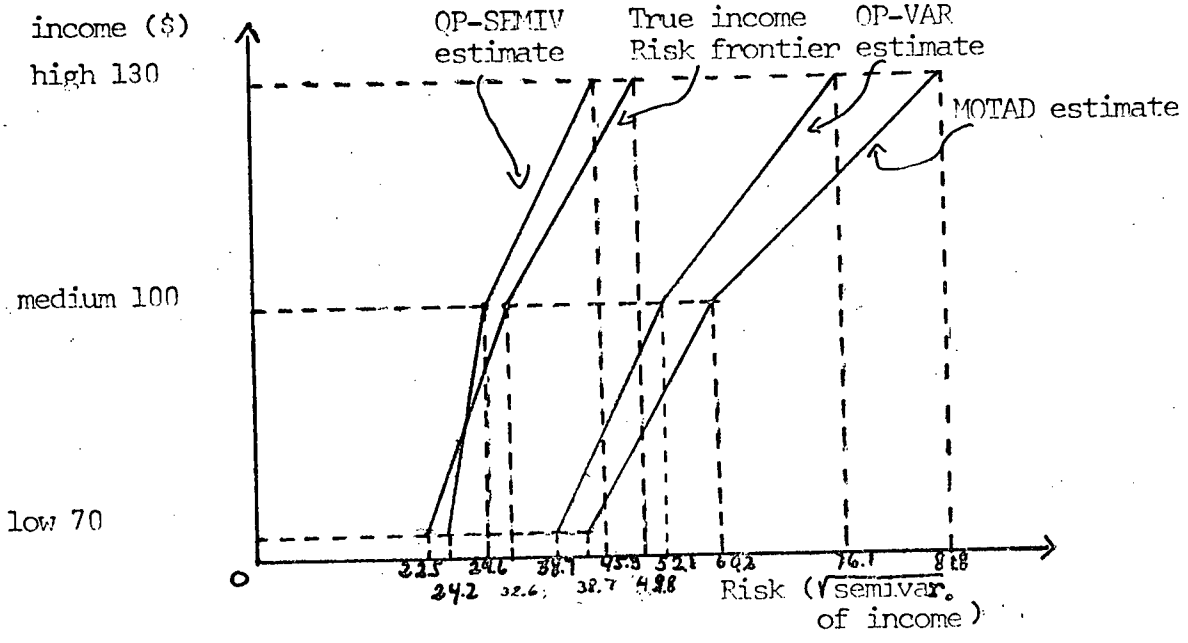
Figures between brackets are the t statistic calcualted.

* significant at 5% LOS

** significant at 1% LOS

The data shown in Table 4.10 may be graphed in an income risk plane as illustrated in Figure 4.2.

Figure 4.2 The True Population Income-Risk Frontier and the Income-Risk Frontier as Estimated by QP-VAR, MOTAD and QP-SEMIV Methods. Gamma Distribution, High Degree of Correlation



The t test applied showed that the differences between the QP-SEMIV mean estimates and the true values of risk were not significant at 1% LOS (see Table 8, Appendix). The MOTAD and QP-VAR mean estimates were significantly different to the true values at 1% LOS (see Tables 9 and 10, Appendix) and hence, MOTAD and QP-VAR estimates may be considered

biased. The magnitude of the QP-VAR bias fluctuated from 52% at the high level of expected income to 72% at the low level of expected income. The MOTAD bias fluctuated between 80% for the high level of income and 92% for the low level of income.

Table 4.11 shows the variances of the QP-SEMIV, QP-VAR and MOTAD estimates of risk at each level of expected income.

Table 4.11 Variances of the QP-SEMIV, QP-VAR and MOTAD Estimates of Risk at Three Levels of Expected Income. Gamma Distribution, High Degree of Correlation

	Levels of Expected Income		
	High	Medium	Low
QP-SEMIV	240.2	96.1	57.8
QP-VAR	327.6	136.8	65.6
MOTAD	561.7	259.2	144.0

As may be seen in Table 4.11, the QP-SEMIV estimates have the smallest variance of all three methods, and the MOTAD estimates have the largest variance at the three levels of expected income. The differences between the variances of the QP-SEMIV and MOTAD estimates were all significant at 5% LOS when the F test was applied. There were no significant differences when the QP-SEMIV variances were compared to the QP-VAR variances. These results suggest that the QP-SEMIV is the only unbiased method and that the dispersion of its estimates is smaller than the dispersion of the MOTAD

estimates and not larger than the dispersion of the QP-VAR estimates. Thus, the QP-SEMIV may be considered an efficient estimator of risk and a better one than the QP-VAR and MOTAD methods for gamma distributed returns with high degree of correlation among them.

It is interesting to observe that as occurs in the normal distribution case, the level of efficiency of all methods is lower when the degree of correlation among activity returns is high than when it is low. The reason for this may be similar to that discussed for the normally distributed returns case.

Finally, Table 4.12 shows the range of the risk levels estimated by the QP-SEMIV, QP-VAR and MOTAD methods.

Table 4.12 Range of the Risk Levels as Estimated by QP-SEMIV, QP-VAR and MOTAD Methods as Compared to the True Risk Values, Gamma Distributions, High Defence of Correlation

	Levels of Expected Income		
	High	Medium	Low
True Risk Values	49.8	32.6	22.5
QP-SEMIV Risk Range:			
95% Probability	14.3 - 76.3	10.0 - 49.2	9.0 - 39.7
68% Probability	29.8 - 60.8	19.8 - 38.8	16.6 - 31.8
QP-VAR Risk Range			
95% Probability	39.9 - 112.3	29.5 - 76.3	22.5 - 54.9
68% Probability	58.0 - 94.2	41.2 - 64.6	30.6 - 46.8

.. table, cont'd.

	Levels of Expected Income		
	High	Medium	Low
MOTAD Risk Range			
95% Probability	42.4 - 137.2	27.8 - 92.4	19.3 - 67.3
68% Probability	66.1 - 113.5	44.1 - 76.3	31.3 - 55.3

Risk ranges are wider in this case than for the case of gamma distributions with low correlation among activities. This fact implies that the method are less precise when the degree of correlation among activity returns is high.

4.5 Two Validations of the QP-SEMIV Method

The QP-SEMIV method has been used to determine the true income-risk frontier for the gamma case. Doubts may be raised regarding the ability of this method to approximate the true minimum income semi-variance. Two validations of this method are provided here. These verifications may not be absolutely conclusive but may provide an indication of the aptitudes of the QP-SEMIV method to minimize income semivariance.

The income semivariance was calculated ex post with the solutions provided by the three methods as applied to samples from the gamma population. In all cases, the QP-SEMIV method provided solutions with the smallest income semivariance. Table 4.13 shows the mean income semi-variance as calculated ex post for the three methods in the case of

gamma populations with low degree of correlation among the activities.

TABLE 3.13 The Mean Income Semivariance as Calculated Ex Post with the QP-SEMIV, QP-VAR and MOTAD Solutions. Gamma Distributions, Low Degree of Correlation

	Levels of Expected Income		
Method Used:	High	Medium	Low
QP-SEMIV	1415.2	650.3	226.2
QP-VAR	1623.0	680.3	289.9
MOTAD	1704.1	763.4	334.1

These observations indicate that the QP-SEMIV method provides plans with smaller levels of income semivariance than the QP-VAR and MOTAD methods. Hence, the QP-SEMIV method is more efficient since it provides plans with lower risk levels.

A second verification was obtained by generating a set of populations of different degrees of skewness. The QP-SEMIV and the QP-VAR methods were applied to these populations in order to minimize risk at three levels of income. A simple regression between the degree of skewness and the total absolute differences in the activities as proposed by the

two methods was performed. The fitted regression line obtained was the following:

$$TD = 0.04 + 0.73\ SK\ , \qquad (4.1)$$

where

TD is a measure of the magnitude of the divergence between the QP-VAR and QP-SEMIV solutions and SK is the skewness coefficient.

The correlation coefficient obtained was 0.78 and this coefficient and the slope coefficient were significant at 99%. Thus, there is a rather strong correlation between the divergences in the results provided by the methods and the skewness coefficient. The greater the degree of skewness the greater the divergences. When the skewness is zero, there are practically no differences between the solutions provided by both methods. Exactly the same is expected to occur if the Income-Semivariance method were used. Hence this is another indication that the QP-SEMIV method provides good approximations to the minimum income semivariance, at least closer results than the QP-VAR.

4.6 Testing the Hypothesees

Four hypotheses were stated in Chapter I. This section reports on the tests and implications of these hypotheses. It is important to recall that the conclusions from Chapter II regarding the efficiency of the methods in determining income-risk frontiers are valid assuming that the methods were applied using complete frequency distributions of activity returns as data bases. It was not clear whether these conclusions still

hold if the methods were used to estimate (rather than to determine) an income risk frontier using sample information (rather than the complete population data). Since in real world problems the methods are applied using relatively small sample data and only in very rare ocassions using complete frequency distributions, the hypotheses were stated in terms of the properties of the methods when applied using sample data. Thus, in order to test the hypotheses it was necessary to develop a set of experiments as described in Chapter III where the methods were applied using numerous samples randomly drawn from hypothetical population. The results of these experiments and the conclusions obtained from Chapter II have been used to test the hypotheses.

4.6.1 Hypothesis I

"The QP-VAR approach as applied to sample data provides an unbiased estimator of the actual population income risk frontier, if the activity returns are normally distributed, regardless of the degree of correlation among the activity returns."

A priori it was shown (Chapter II) that the QP-VAR method provides an appropriate representation of the income risk frontier when applied using normally distributed population data. As it was shown in Sections 4.1 and 4.2 the QP-VAR method also provides unbiased estimates of the income risk frontier when applied using samples drawn from normally distributed populations of activity returns. In other words, the QP-VAR not only provides appropriate representations of the income risk frontier using normally distributed population data but also the estimates obtained using sample data were unbiased and therefore hypothesis I may be accepted in full.

Furthermore, the QP-VAR method was shown to be not only unbiased but also an efficient method since the variance of its estimates was not larger than the variance of the MOTAD estimates when the correlation among activity returns was low and it was significantly smaller when the degree of correlation was high.

4.6.2 Hypothesis 2

"The MOTAD method provides an unbiased estimator of the actual population income-risk frontier only if the following two conditions are satisfied:

a) The activity returns are normally distributed and

b) The correlation coefficients among the activity returns are close to zero".

It was shown in Chapter II that the MOTAD method only provides close approximations to the income risk frontier when applied using normally distributed data with low degree of correlation among activity returns. It was also shown that if these conditions are not met the MOTAD solutions are not appropriate. The results of the experiments confirmed these conclusions for the estimates of the income risk frontier obtained using sample data. The MOTAD estimates were significantly biased in all situations except when the samples were drawn from normally distributed-low correlation populations. Therefore, hypothesis 2 is accepted. The MOTAD method was not only biased but also the method that provided the more dispersed estimates under all situations. Thus the MOTAD method was the least efficient of the methods when applied to highly correlated activity returns and/or gamma

distributed data (considering that in this later case its bias was larger than the bias of the QP-VAR estimates).

4.6.3 Hypothesis 3.

"If the activity returns are non-normally distributed, the QP-VAR method and the MOTAD method yield unbiased estimators of the actual population income-risk frontier."

Conclusions obtained from Chapter II pointed out that the QP-VAR method does not provide appropriate representations of the income risk frontier when applied using non-normally distributed data. Sections 4.3 and 4.4 reported on the QP-VAR solutions obtained using sample data drawn from gamma populations. The QP-VAR estimates were biased when the degree of correlation of activity returns was high and also when it was low. Hence hypotheses 3 should be rejected in its part corresponding to the QP-VAR estimates.

The conclusions of the analytical discussion from Chapter II and the fact that the MOTAD estimates were also biased when applied to gamma distributed data allow one to reject the second part of hypothesis 3 as well.

4.6.4 Hypothesis 4.

"When the activity returns are non - normally distributed the semivariance method will provide unbiased estimates of the actual population income risk frontier."

Two basic conclusions were obtained from the discussion in Chapter II: (1) The income semivariance method may provide appropriate representations of the income-risk frontier when applied to non-normally distributed frequency distributions (provided that kurtosis and higher order moments are not important and that the utility function of the decision maker is not quadratic); (2) The QP-SEMIV was proposed as an approximation of the Income-Semivariance method. As reported in Sections 4.3 and 4.4 the QP-SEMIV method provided unbiased estimates of the income risk frontier when applied using samples randomly drawn from gamma populations. Thus, hypothesis 4 is accepted. Furthermore, the QP-SEMIV may be considered the most efficient of the three methods in situations where they are applied using gamma distributed data.

4.7 Summary

This chapter has reported on the results obtained from the experiments described in Chapter III. Results have been reported for the four cases regarding frequency distribution of activity returns considered, i.e., normal distribution with low and high degree of correlation among activity returns. Using these results and the conclusions drawn from the theoretical study, the four hypotheses as stated in Chapter I were tested. All hypotheses except hypothesis 3 were accepted. Furthermore, two tests designed to validate the QP-SEMIV method yield results which provide

an additional evidence to sustain the assertion that the QP-SEMIV method is a close substitute to the Income-Semivariance Method.

Throughout the process of reporting the results it was observed that the efficiency of the methods tend to be lower when the degree of correlation among activity returns is high as compared to when it is low. This fact may have a direct practical implication since it would mean that the higher the degree of correlation among activity returns, the larger must be the sample size. Thus, a greater number of activity records are required in order to keep the efficiency of the methods at acceptable levels.

CHAPTER V

A CASE STUDY FARM

In order to illustrate the performance of the methods in a more realistic model, data from the Peace River District of B.C. was used to estimate income-risk frontiers. The farmer chosen was mainly a crop producer with no lifestock activities. The following crop production activities were considered in the model:

(1) Wheat grown after summer fallow; (2) Wheat grown after stubble;
(3) Barley grown after summer fallow; (4) Barley grown after stubble;
(5) Oats grown after summer fallow; (6) Oats grown after stubble;
(7) Rapeseed; (8) Fescue seed; (9) Alsike seed; (10) Alfalfa.

The main constraints considered were arable land owned by the operator, arable land available for renting, cash capital owned by the farm operator, cash borrowing capacity and family labour available. Figure 5.1 outlines a general overview of the model used. The objective function varied with the method used. It is intended to minimize the total absolute deviation (MOTAD), total variance (QP-VAR) or total semivariance (QP-SEMIV). The capital control rows consider the flow of operating and overhead capital into the model through submatrices $D_2^{-1}{}_{,1}$ and $D_2^{-1}{}_{.2}$ and this capital may be used for various activities such as costs of owning land $(A_{2,3})$ renting land $(A_{2,}^{+a}{}_{6})$ or buying other variable inputs for producing crops such as fertilizers, seed, renting machinery and so forth $(A_{2,}^{+a}{}_{8})$. All other control rows may be interpreted in the same way keeping in mind the legend in Figure 4.7.

FIGURE 5.1: Structure of the Models

ACTIVITIES	Own Capital	Borrowed Capital	Own Land	Rented Land	Family Labour	Hired Labour	Cropping Activities	Variable Inputs	R.H.S.
Constraints									
Objective Function	Measure of risk which depends on method used								Minimize
Capital Control	$D^{-1}_{2,1}$	$D^{-1}_{2,2}$	$A^{+a}_{2,3}$	$A^{+a}_{2,4}$	$A^{+a}_{2,5}$	$A^{+a}_{2,6}$		$A^{+a}_{2,8}$	≤ 0
Land Control			$D^{-1}_{3,3}$	$D^{-1}_{3,4}$			$A^{+1}_{3,7}$		≤ 0
Labour Control					$D^{-1}_{4,5}$	$D^{-1}_{4,6}$	$A^{+a}_{4,7}$		≤ 0
Cropping Activity Resource Use.							$A^{+a}_{5,7}$	$D^{-1}_{5,8}$	≤ 0
Expected Net Income Summary Row	$A^{-a}_{6,1}$	$A^{-a}_{6,2}$	$A^{-a}_{6,3}$	$A^{-a}_{6,4}$	$A^{-a}_{6,5}$	$A^{-a}_{6,6}$	$A^{+a}_{6,7}$	$A^{-a}_{6,8}$	$\geq R$
Structural Bounds	B^m_1	B^m_2	B^m_3	B^m_4	B^m_5	B^m_6			

Legend: (Superscripts, when shown, represent the type of non zero element in the submatrix.)
A is a general submatrix with some non zero elements,
B^m is a structural upperbound, D is a diagonal submatrix.

All submatrices in the net income summary row are row vectors. Vectors $A_{6.1}$ and $A_{6.2}$ account for the costs of using different types of capital. The vectors $A_{6.3}$ (own land costs) $A_{6.4}$ (land rental costs), $A_{6.5}$ (family labour costs), $A_{6,6}$(hired labour costs), $A_{6.7}$ (gross returns from crops and $A_{6.8}$ (variable costs for crop production) all account for income and expenses for the different activities. Negative coefficients indicate costs and the positive coefficients, returns. The expected net income is not equivalent to farm profit because the depreciation costs of fixed capital and interest charged on capital other than those of land and livestock have not been included.

The model as solved for the MOTAD method has 38 rows, approximately 28 columns and 10 structural bounds. The QP-VAR and QP-SEMIV models have 34 rows and 28 columns.

The case farm considered had 370 acres of arable land and the possibility existed for renting another 80 acres. The farm operation had $5,000 available in cash capital plus a cash borrowing capacity estimated at $25,000 per year. A total of 1500 hours of family labour was considered to be available in addition to 1000 hours of hired labour.

The basic inputs used were the last eight years of records regarding yields per acre, prices and costs of the different production activities considered. This data was used to calculate the variances, semi-variances or absolute deviations of the activity returns to be used on the QP-VAR, QP-SEMIV and MOTAD methods respectively.

All methods were solved for minimizing risk under the same constraints and at the same levels of net income. The maximum net income attainable from this farm considering the expected return of the activities was approximately $6,000 per year. The model was solved to minimize risk at $6,000, $5,000 and $4,000 expected net income per year.

Table 5.1 shows the minimum risk levels as estimated by the methods at three levels of expected net income.

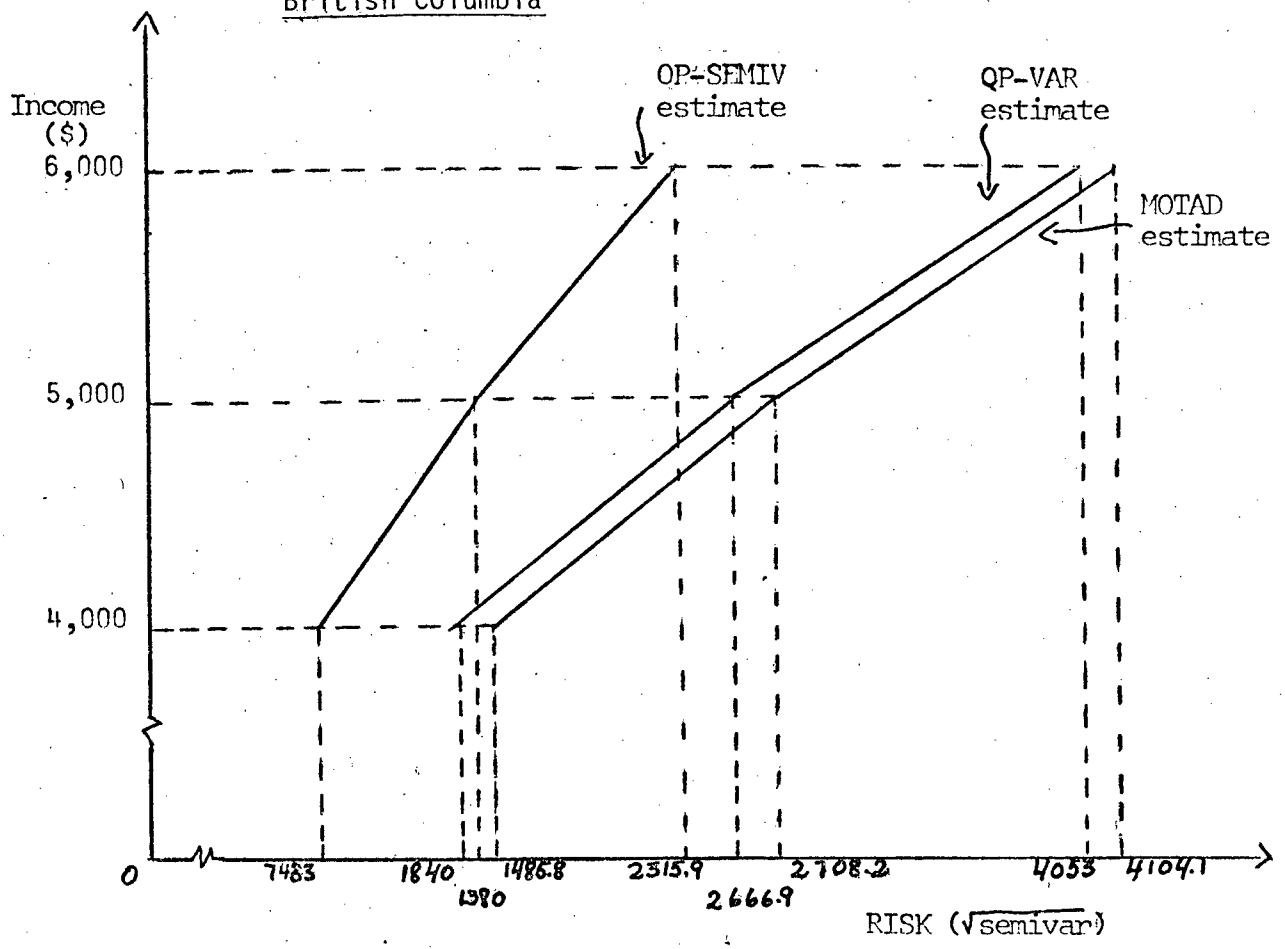
TABLE 5.1: Risk Levels (Expressed as the Square Root of the Semi-variance) as Provided by the QP-SEMIV, QP-VAR and MOTAD Model Solutions for a Farm in the Peace River District of British Columbia

	LEVELS OF NET INCOME		
METHOD USED	$6,000	$5,000	$4,000
QP-SEMIV	2315.9	1380.9	748.3
QP-VAR	4053.0	2666.9	1340.4
MOTAD	4114.1	2708.2	1485.8

The total semivariance was calculated *ex post* from the solutions provided by the QP-VAR and MOTAD methods.

Figure 5.2 shows the efficient income-risk frontier as estimated by the three methods. As in Table 5.1, risk is expressed as the square root of the semivariance.

Figure 5.2 The Income-Risk Frontier as Estimated by QP-SEMIV, QP-VAR and MOTAD Methods for a Farm in the Peace River District of British Columbia



At all levels of income the plans recommended by the QP-SEMIV method implied the smallest levels of risk (measured as the square root of the semivariance). The MOTAD method always provided solutions slightly worse than the QP-VAR method. Table 5.2 shows the level of production activities as recommended in the solutions provided by the methods at six thousand dollars expected net income level.

TABLE 5.2 Levels of Activities as Proposed by MOTAD, QP-SEMIV and QP-VAR Models for a Typical Small Farm in the Peace River Area of British Columbia (Net Income = $6000)

MODEL USED	ACTIVITIES PROPOSED						
	Acres of WAF	Acres of WAS	Acres of BAF	Acres of BAS	Acres of OAS	Acres of FES	Acres of ALF
MOTAD	77	-	5	-	164	-	61
QP-SEMIV	-	42	82	34	-	-	91
QP-VAR	62	75	20	-	-	57	58

* WAF = Wheat After Fallow; WAS = Wheat After Stuble; BAF = Barley After Fallow; BAS - Barley After Stubble; OAS = Oat After Stubble; FFS = Fescue; ALF = Alfalfa.

As may be seen in Table 5.2., the level of activities proposed by each of the three methods differs substantially. Only three production activities do not appear in any solution, oats after fallow, rapeseed and alsike. All other activities are at least in one of the solutions and only two activities are in the three solutions, barley after fallow and alfalfa.

Thus, the three methods considered in this study have provided very different solutions when they have been applied to a Peace River district farm. It is interesting to note that most correlation coefficients among the activity returns are relatively large. Only the returns of fescue and alsike appear to be weakly correlated with any other activity return and in

some cases having negative sign (see Table A.11 Appendix). The mean correlation coefficient among all other activities is approximately 0.66. This fact would explain why the QP-VAR and MOTAD solutions are so different. The eight year gross return records suggest that most activity returns are skewed distributed, which would explain the apparently better plans provided by the QP-SEMIV method.

Thus, this example illustrates the magnitude of the error which may occur if the wrong method is used. Considering the relatively high degree of correlation of the activities and the fact that the data appears to be non-normally distributed the QP-SEMIV method may be considered as providing the best solution. To use the MOTAD method instead of the QP-SEMIV method implied that plans generated were on the average 80% more risky and the QP-VAR method provided plans which were 70% more risky than the QP-SEMIV method. This example is useful in illustrating the importance of choosing the appropriate method in farm planning under uncertainty conditions. Thus, the phase of deciding which method should be used is crucial for the results (as shown by the important differences in the results provided by the methods in this example), and hence, it is worthwhile to devote a great deal of resources and time in order to be a able to select the most appropriate method according to the specific nature of the problem.

CHAPTER VI

SUMMARY, CONCLUSIONS AND RECOMENDATION FOR FURTHER STUDIES

This chapter presents a review of the study and principal findings. Given the limitations of the approach further research is proposed in order to be able to assess the empirical relevance of these findings.

6.1 Summary and Conclusions

The basic objective of this study was to evaluate the performance of three methods used in farm planning under uncertainty, namely, the QP-VAR, MOTAD and Semivariance methods. The work was developed in two parts, a theoretical or conceptual section and a set of experiments comprising an empirical section. The theoretical study was concerned with the characteristics of the methods under the assumption that the population distribution of activity returns was known. The main criterion used to evaluate the methods was their ability to provide the information required by the decision maker to maximize his expected utility. That is, to provide an ordinal classification of alternative plans consistent with the level of expected utility which each plan implies to a decision maker. To meet this requirement, a necessary condition was that the method should provide a set of efficient plans, i.e., an income-risk frontier which would enable a decision maker to choose the plan which maximizes his expected utility.

The empirical work tested the methods' ability to provide income-risk frontiers as applied using sample data of limited size rather than complete frequency distribution of activity returns. Under a situation of perfect knowledge, the theoretical analysis indicated which methods provide appropriate representations of the income-risk frontier. In the empirical work section, the income-risk frontiers provided by such methods as applied to complete frequency distributions data were considered the "true" ones. The estimates of the income-risk frontier obtained using sample data were compared to the "true" income-risk frontier in order to measure bias and dispersion of the estimates. Given the results, conclusions were drawn regarding the relative efficiency of the methods as estimators of the true income-risk frontier.

A general conclusion which may be drawn from this study is that unless activity returns are assumed to be normally distributed (which may be an unrealistic assumption) planning under uncertainty needs to consider the nature of the utility function of the decision maker. This increases the complexity to these studies given the difficulties involved in knowing the utility functions of decision makers. Thus, there is not an optimal method to be used in all cases and hence there is not an easy rule to be applied in farm planning under uncertainty.

More specific conclusions regarding the three methods are:

1. If the decision maker's utility function is quadratic the QP-VAR method may be used, irrespective of the distribution of activity returns.

2. The QP-SEMIV method is proposed as the most suitable method if the following conditions are met:

   a. The decision maker's utility function is not quadratic nor linear
   b. The frequency distribution of activity returns is non - normal, but moments of order higher than the skewness moment are negligible.

3. The MOTAD method is in general not recommended as being useful because it is biased and less efficient than the QP-SEMIV method. The only situation where the MOTAD method may be used is when activity returns are normally distributed and the degree of correlation among activity returns is low.

4. No one of the three methods was found to be appropriate if the following conditions occur simultaneously:

   a. A decision maker's utility function is non-quadratic
   b. Fourth and higher order derivatives of the utility function do not vanish
   c. The frequency distribution of the activity returns is non-normal and moments higher than skewness moments are not negligible.

5. As important as the actual results obtained is the general procedure used to evaluate the different methods. To explain, there is

a recognition that the most important feature of a method is its performance when applied using sample data rather than complete frequency distribution for in empirical work, the data source is normally a relatively small sample drawn from the population of activity returns. Indeed, the fact that a method provides an appropriate representation of the income-risk frontier under a perfect knowledge situation is not a sufficient nor a necessary condition for such a method to generate equally appropriate estimates when using small sample data. This view of the problem required an objective evaluation of the methods considered. The criterion used was the concept of efficiency of estimators as defined in the statistics sense. Bias and dispersion of the estimates were used to judge the solutions provided by the different methods. Using these concepts it becomes clear why the appropriateness of the income-risk frontier derived when using complete frequency distribution data is not sufficient nor necessary for efficiency as estimators of the income-risk frontier. Just as the standard error measure is a biased estimator of the standard deviation ($E(\hat{\sigma}) \neq \sigma$), a method may provide biased estimates of the income risk frontier despite providing a "true"one when applied to the complete frequency distribution. Similarly, a method may be slightly biased but if it has a small dispersion it may be more efficient than an unbiased method which provides highly dispersed estimates. Thus, the study provides a research procedure to test different methods which may be used in farm planning under uncertainty. The research procedure used is based on the recognition that, conclusions from analytical discussions based on the assumption of full knowledge of the frequency distribution of the activity returns are not directly applicable in the practical process of deciding which method should be used. However,

the analytical results were highly important in designing the experiments to test the different methods and the majority of the steps followed in chapter three were based on the conclusions obtained from the theoretical study developed in the preceding chapter.

6.2 Recommendations for further studies

It has been shown that the efficiency of the different methods used in farm planning under uncertainty depends on the nature of the utility function and on the frequency distribution of activity returns. Hence, it would be advisable to investigate the presence of any sort of regularity in the nature of the utility function of farmers as decision makers. In other words, it is important to know whether certain utility functions can be ruled out for the majority of farmers and whether there are some families of utility functions which are peculiar to them. It would also be important to determine if there are some frequency distributions which characterize better the distribution of the gross activity returns. Accordingly, further studies should be orientated mainly in the following directions:

1.- Empirical studies to establish whether quadratic utility functions are indeed unusual among farmers as theoretical studies suggest. This is important because in such a case the QP-SEMIV method could have wider applications. But if the quadratic utility function occurs frequently among farmers, before choosing the method to be used, it would be necessary to determine the utility function of the decision maker in each case (unless there is certainty that returns are normally distributed).

2.- Empirical studies could investigate the frequency distribution of gross activity returns for different crops and livestock enterprises. Two basic points need investigation: In the first place, it would be necessary to establish whether gross activity returns for most important crops and livestock enterprises produced have been approximately normally distributed during the last decades. In the second place, if the returns appear to be non - normally distributed it would be necessary to obtain some idea regarding the sign of skewness of the distributions and the magnitude of higher order moments relative to the skewness moment. If the distributions appear to be approximately normal the QP-VAR method may be used and the QP-SEMIV method would not be necessary. If this is not the case the QP-SEMIV method would be useful (provided non-quadratic utility functions).

3.- Additionally, it would be important to study how the performance of these methods is affected by changes in the sample size. It can be expected that results become more accurate as the sample size increases, but these improvements are not necessarily proportional to the rate of increase in the sample size. Given that to increase sample size has a cost (more information regarding gross returns over time is necessary). It would be possible to find an optimal sample size (or an optimal range) to be used in empirical studies.

4.- It is also necessary to test the performance of these methods in prediciting actual behaviour of farmers. It is quite clear that the

linear programming results may be very different from the actual plans which farmers' make. It would be important to study the plans obtained using QP-VAR and specially QP-SEMIV methods, and compare these to farmers' actual plans. A priori, it could be expected that the QP-SEMIV method would provide the closest approximation to the farmers' actual production plans. If the approximations provided by this method are better, the QP-SEMIV method could be used as a tool in predicting production, price and farm income fluctuations at the macro-economic level.

REFERENCES

1. Anderson, J.R. "Programming for Efficient Planning Against Non - normal Risk." Australian Journal of Agricultural Economics, 19: 107 - 115, 1975.

2. Arrow, K., Aspects of the Theory of Risk Bearing, North Holland, Helsinski, 1964.

3. Bierwag, G., "The Rationale of the Mean-Standard Deviation Analysis: Comment", American Economic Review, 64: 431-433, 1974.

4. Borch, K., The Economics of Uncertainty, O. Morgenstern et al, Princeton, New Jersey, 1968.

6. Borch, K., "The Rationale of the Mean-Standard Deviation Analysis: Comment ", American Economic Review, 64: 428-433, 1974.

7. Candler, W., "An Introduction to Quadratic Programming", Unpublished, Purdue University Paper, 1972.

8. Chen, J.T., "A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty: Comment ", American Journal of Agricultural Economics 53: 662-663, 1971.

9. Driver, H., "Risk Programming as a Tool for Selecting Alternatives for Agricultural Stability", Canadian Journal of Agricultural Economics, Proceedings of the 1976 Workshop, 106-121, 1976

10. Feldstein, M.D., "Mean Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection ", The Review of Economic Studies, 36: 5-11, 1969.

11. Friedman, M. and Savage, L., "The Utility Analysis of Choices Involving Risk", in Readings in Price Theory, American Economic Association Series, George Allen and Murrin Ltd., London, 1953.

12. Graham, J.D. and Lopez, R., Farm Planning Under Uncertainty - An application of MOTAD Programming to the Peace River Block of B.C., Department of Agricultural Economics, UBC, Vancouver, B.C. 1976

13. Hadley, G., Introduction to Probability and Statistical Decision Theory, Holden-Day, Inc., San Francisco, California, 1967

14. Hadley, G., Non Linear and Dynamic Programming, Addison Wesley Publishing, Massachusetts, 1964.

15. Hazell, P.B. "A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty", American Journal of Agricultural Economics, 53: 53-62, 1971.

16. Hazell, P.B. and How, R.B., "Obtaining Acceptable Farm Plans Under Uncertainty", paper submitted to the Contributed Papers section of the International Association of Agricultural Economists at Minsk, U.S.S.R., 1970.

17. Intriligator, M., Mathematical Optimization and Economic Theory, Prentice-Hall, New York, 1971.

18. Kmenta, J., Elements of Econometrics, MacMillan Publishing Co., New York, 1971.

19. Levy, H., "The Rationale of the Mean-Standard Deviation Analysis: Comment", American Economic Review, 64: 343-441, 1974.

20. Lin, W. Dean, G. and Moore, C., "An Empirical Test of Utility vs. Profit Maximization in Agricultural Production", American Journal of Agricultural Economics, 56: 497-508, 1974.

21. Lopez, R.E., "Linear and Non Linear Indicators of Risk as Applied to Farm Planning under Uncertainty", Unpublished, Department of Agricultural Economics, U.B.C., 1976.

22. Luttrell, C. and Gilbert, R., "Crop Yields: Random, Cyclical, or Bunchy?" American Journal of Agricultural Economics, 58: 521-531, 1976.

23. Markowitz, H., Portfolio Selection: Efficient Diversification of Investments, John Wiley and Sons, Inc., New York, 1959.

24. McMillan, C. and Gonzales, R., Systems Analysis a Computer Approach to Decision Models, Richard Irwin Inc., Homewood, Illinois, 1968.

25. Mood, A. and Graybill, F. Introduction to the Theory of Statistics, McGraw-Hill, New York, 1963

26. Pratt, J.W., "Risk Aversion in the Small and the Large", Econometrica, 32: 122-136, 1964.

27. Raiffa, H., Decision Analysis, Addison-Wesley, Reading, Massachusetts, 1968

28. Scott, J. and Baker, C., "A Practical Way to Select an Optimum Farm Plan Under Risk", American Journal of Agricultural Economics 54: 657-660, 1972.

29. Thomson, K., and Hazell, P., "Reliability of Using the Mean Absolute Deviation to Derive Efficient E.V. Farm Plans." American Journal of Agricultural Economics, 54: 503-506, 1972

30. Tsiang, S.C., "The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference and the Demand for Money", The American Economic Review, 62: 354-371, 1972.

31. Wolf, F.L. Elements of Probability and Statistics, McGraw-Hill Inc., New York, 1974.

APPENDIX

TABLE A.1 Estimates of Risk* as Obtained Using the QP-VAR Method as Applied to Fifteen Samples Randomly Drawn from a Normally Distributed Population. Low Degree of Correlation Among Activity Returns.

Data source used	Levels of Expected Income		
	High	Medium	Low
Sample 1	6.0	4.0	2.5
Sample 2	5.8	4.0	3.1
Sample 3	6.1	4.1	2.7
Sample 4	6.0	4.2	2.7
Sample 5	5.7	3.9	3.2
Sample 6	6.6	4.7	3.1
Sample 7	7.1	4.6	2.9
Sample 8	5.8	4.0	2.6
Sample 9	5.8	4.0	2.6
Sample 10	6.2	4.4	2.8
Sample 11	6.1	4.3	2.8
Sample 12	4.6	3.3	2.2
Sample 13	5.9	4.2	2.6
Sample 14	5.9	4.2	2.8
Sample 15	5.3	3.7	2.4
Mean Estimates	5.9	4.1	2.7
Variance	0.32	0.13	0.07

* Standard Deviation of Total Income

TABLE A.2 Estimates of Risk* as Obtained Using the MOTAD Method as Applied to Fifteen Samples Randomly Drawn from Normal Population with Low Degree of Correlation

Data source used	Levels of Expected Income		
	High	Medium	Low
Sample 1	6.5	4.3	2.8
Sample 2	6.0	4.1	2.6
Sample 3	6.1	4.2	2.8
Sample 4	5.9	4.0	2.6
Sample 5	5.9	4.1	2.7
Sample 6	6.7	4.6	3.3
Sample 7	6.7	4.6	3.0
Sample 8	5.1	3.2	2.2
Sample 9	6.2	4.3	2.9
Sample 10	4.8	3.4	2.2
Sample 11	6.0	4.1	2.7
Sample 12	6.9	4.7	3.2
Sample 13	5.8	3.9	2.4
Sample 14	6.0	4.1	2.8
Sample 15	4.9	3.6	2.4
Mean Estimates	6.0	4.1	2.7
Variance	0.38	0.18	0.10

TABLE A.3 Estimates of Risk* as Obtained Using the QP-VAR Method as Applied to Fifteen Samples Randomly Drawn from a Normal Population with High Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	6.5	5.2	2.6
Sample 2	6.6	5.0	3.3
Sample 3	6.4	5.1	3.9
Sample 4	5.8	4.4	3.0
Sample 5	6.2	4.8	3.5
Sample 6	4.5	3.5	2.3
Sample 7	6.7	4.6	2.9
Sample 8	7.6	5.5	3.5
Sample 9	5.4	3.9	2.4
Sample 10	5.1	3.4	2.1
Sample 11	10.2	6.3	4.2
Sample 12	6.9	5.1	3.9
Sample 13	7.1	5.3	3.3
Sample 14	6.6	4.8	3.2
Sample 15	6.8	5.1	3.3
Mean Estimates	6.5	4.8	3.2
Variance	1.06	0.58	0.36

* Standard Deviation of Total Income.

TABLE A.4 Estimates of Risk* as Obtained Using the MOTAD Method as Applied to Fifteen Samples Randomly Drawn from a Normal Population with High Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	10.5	9.3	7.4
Sample 2	7.1	5.0	3.1
Sample 3	8.4	6.4	4.1
Sample 4	7.7	5.9	4.0
Sample 5	7.3	5.3	3.6
Sample 6	6.6	5.0	3.3
Sample 7	8.7	6.5	4.5
Sample 8	7.0	5.0	3.3
Sample 9	7.6	5.5	3.5
Sample 10	5.7	3.9	2.4
Sample 11	7.5	5.7	3.9
Sample 12	7.9	6.0	4.2
Sample 13	7.5	5.8	3.9
Sample 14	6.6	5.0	3.2
Sample 15	7.7	6.2	4.1
Mean Estimates	7.6	5.8	3.9
Variance			

* Standard Deviation of Total Income.

TABLE A.5 Estimates of Risk* as Obtained Using the QP-SEMIV Method as Applied to Fifteen Samples Randomly Drawn from a Gamma Population with Low Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	44.2	23.1	15.0
Sample 2	34.7	23.9	15.6
Sample 3	37.2	27.3	18.8
Sample 4	56.7	32.1	19.6
Sample 5	43.9	29.0	19.1
Sample 6	29.1	25.1	18.3
Sample 7	45.5	24.3	14.6
Sample 8	38.1	26.8	19.3
Sample 9	47.3	25.6	17.4
Sample 10	49.7	26.5	19.1
Sample 11	50.3	30.8	21.3
Sample 12	58.4	31.3	29.2
Sample 13	57.8	30.2	15.4
Sample 14	59.2	35.6	28.6
Sample 15	42.1	37.1	22.2
Mean Estimates	46.3	29.1	19.6
Variance	88.6	22.1	19.4

* Square root of semivariance of total income.

TABLE A.6 Estimates of Risk* as Obtained Using the QP-VAR Method Applied to Fifteen Samples Randomly Drawn From a Gamma Population with Low Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	54.5	31.9	25.3
Sample 2	62.9	41.9	30.2
Sample 3	10.8	49.8	29.3
Sample 4	56.5	32.2	16.2
Sample 5	56.8	35.1	28.6
Sample 6	55.7	36.2	24.8
Sample 7	58.2	33.1	24.7
Sample 8	55.4	39.7	23.7
Sample 9	73.2	4.15	24.4
Sample 10	78.6	48.9	35.7
Sample 11	59.6	38.7	22.4
Sample 12	66.7	37.8	24.9
Sample 13	73.7	51.1	34.7
Sample 14	60.9	39.3	29.8
Sample 15	62.8	38.6	28.7
Mean Estimates	64.4	39.7	27.1
Variance	102.0	37.2	24.0

* Square Root of Semivariance of the Total Income.

TABLE A.7 Estimates of Risk* as Obtained Using the MOTAD Method Applied to Fifteen Samples Randomly Drawn from a Gamma Population with Low Degree of Correlation

Data ource Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	77.5	47.7	33.4
Sample 2	66.4	44.0	28.4
Sample 3	44.0	32.0	18.5
Sample 4	94.0	52.3	32.9
Sample 5	66.2	42.5	28.1
Sample 6	94.3	58.0	42.1
Sample 7	81.2	54.4	28.7
Sample 8	75.6	51.6	29.1
Sample 9	68.0	54.7	37.2
Sample 10	69.4	45.9	27.6
Sample 11	61.2	35.2	19.9
Sample 12	69.7	49.1	33.6
Sample 13	47.1	29.0	19.4
Sample 14	56.4	40.6	24.6
Sample 15	62.8	45.8	26.5
Mean Estimates	68.7	45.7	29.9
Variance	207.4	68.9	42.2

* Square Root of Semivariance of the total income.

TABLE A.9 Estimates of Risk* as Obtained Using the QP-VAR Method Applied to Fifteen Samples Randomly Drawn from a Gamma Population with High Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	62.1	44.5	35.0
Sample 2	116.2	85.4	59.8
Sample 3	66.1	51.9	40.1
Sample 4	71.0	45.9	33.7
Sample 5	67.8	49.0	35.3
Sample 6	69.3	49.5	35.9
Sample 7	52.6	37.2	29.8
Sample 8	108.4	75.2	49.8
Sample 9	89.9	54.8	41.4
Sample 10	90.6	62.8	47.0
Sample 11	79.0	52.7	38.0
Sample 12	60.1	45.8	32.4
Sample 13	78.6	49.1	38.8
Sample 14	61.2	40.7	29.2
Sample 15	67.8	50.3	38.0
Mean Estimates	76.1	52.9	38.7
Variance	327.6	136.8	65.6

* Square Root of Semivariance of Total Income.

TABLE A.10 Estimates of Risk* as Obtained Using the MOTAD Method Applied to Fifteen Samples Randomly Drawn from a Gamma Population with High Degree of Correlation

Data Source Used	Levels of Expected Income		
	High	Medium	Low
Sample 1	120.2	84.3	59.0
Sample 2	65.2	39.3	30.1
Sample 3	92.7	63.0	45.2
Sample 4	60.6	40.9	35.7
Sample 5	102.4	73.5	47.8
Sample 6	84.1	59.3	44.5
Sample 7	123.0	70.5	57.8
Sample 8	112.1	66.3	42.5
Sample 9	92.5	83.5	41.0
Sample 10	81.7	53.7	32.3
Sample 11	56.7	36.7	32.4
Sample 12	121.4	88.6	69.3
Sample 13	101.6	74.0	51.1
Sample 14	64.0	45.0	32.6
Sample 15	69.1	49.6	28.6
Mean Estimates	89.8	60.2	43.3
Variance	561.7	259.2	144.0

* Square root semivariance of total income.

TABLE A.11 Variance-Covariance Matrix of the 8 Year Activity Return Data Corresponding to a Case Farm in the Peace River District of British Columbia

Activities	WAFC	BAFC	OAFC	RAFC	FESC	ALFALFA	ALSIKE
WAFC	981.7	806	564.2	502.9	-443	244.4	43.2
BAFC		1276.6	774.0	833.4	61.2	443	1 27.4
OAFC			681.2	517.1	-167.8	219.2	220.1
RAFC				629	10.7	351.1	3.8
FESC					1836.1	497.1	-361.3
ALFALFA						400	-147.2
ALSIKE							235