EFFECTS OF ELABORATIVE PROMPT CONDITION
AND DEVELOPMENTAL LEVEL ON PERFORMANCE OF
ADDITION PROBLEMS BY KINDERGARTEN CHILDREN

by

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ABSTRACT

The purpose of the present study was to examine the possibility that an elaborative process (Rohwer, 1973) may be used by young children in the performance of addition. The effects of elaborative Prompt Condition (Concrete plus Verbal, Imaginal plus Verbal, Verbal Only) and Described Relation (Dynamic, Static) on the performance of addition word problems of the form $m+n=$ __, were examined with 108 kindergarten children at three Developmental Levels (Conservers, Transitionals, Nonconservers). Three counting models children may use to solve addition word problems, were also examined.

An analysis of the number of correct responses revealed that overall, Conservers performed significantly better than Transitionals and Nonconservers, who did not differ from each other. At each Developmental Level children performed best under the Concrete plus Verbal Prompt Condition. When performance on items where $m>n$ was compared with $m<n$, Conservers and Nonconservers performed differently under the Imaginal plus Verbal Condition as compared with Verbal Only. The effect of Described Relation was significant for two of the nine Developmental Level $\times$ Prompt Condition groups. More correct responses were observed under the Static than Dynamic Described Relation.

The results for the counting models tentatively suggested that, for all Developmental Levels, latencies under the Concrete plus Verbal Prompt Condition were related to the least efficient of the three models tested. For Conservers and Transitionals combined, latencies were related to the most efficient counting model under the Imaginal plus Verbal Prompt Condition, and to the least efficient model under the Verbal Only Prompt Condition.

The results suggest that an elaborative process may be used by kindergarten children in the solution of addition word problems. This process may be activated by different prompts, depending on the developmental level of the child. Implications of the findings for further research, as well as for the educational setting, are discussed.

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CHAPTER I

INTRODUCTION

A. Overview of the Problem

Suppes (1967, p. 5) has said the following concerning our knowledge of how children learn mathematics concepts:

"As yet, theories of learning have little to offer in providing insight into how one learns to think mathematically. The nature of abstraction, or the processes of imagery and association that are surely essential to thinking in any domain of mathematics, have as yet scarcely been studied from a scientific standpoint."

Although imagery has been raised as possibly important for mathematics learning (Skemp, 1971; Syer, 1953), very little experimental work has been done specifically relating the use of imagery to performance on mathematical tasks other than geometrical problems, (e.g. Lee, 1971).

Research in paired associate learning has established the importance of imaginal and verbal factors in noun-pair learning (Paivio, 1971; Rohwer, 1970). In Rohwer's (1973) view both are reflections of a single underlying process. Rohwer has proposed that paired associate learning is facilitated when a common referent, which creates a shared meaning for the items, is generated. This process he calls elaboration. Different types of prompts, verbal and visual, are viewed as varying along a dimension of likelihood of evoking elaboration.

The possible use of elaboration by young children in the performance of arithmetic operations remains open to empirical investigation.

The present study was an attempt to extend and apply the findings from associative learning to a more complex form of learning, that is, the arithmetic operation of addition. It is hoped that as our understanding of processes involved in performance of addition increases, educational implications for improving teaching strategies and curriculum may follow.

B.

B. Rationale

At first, young children are only able to perform addition by counting visual referents (objects or pictures). Later they are able to perform addition with number symbols. The aim of the present study was to examine how this transition may take place.

A paired-associate task has traditionally been viewed as involving stimulus differentiation (e.g. McGuire, 1961), response learning, and an association between the two components (Underwood, Runquist and Schultz, 1959). Rohwer (1973, p. 5) has proposed that "the outcome of a paired-associate task is the creation of shared meanings within each pair learned". The process which creates shared meanings consists of generating a common referent for the disparate items. This unspecified referent will vary depending on the items to be coupled. Conditions which promote the elaboration of items in a noun pair vary along a continuum, from antagonistic prompts, to minimally explicit, augmented explicit, and maximally explicit prompts, in increasing likelihood of

evoking elaboration. A maximally explicit prompt is one in which the referential event is actually enacted in the presence of the learner.

In paired-associate learning Rohwer (1973) has suggested that use of elaboration may facilitate recall of noun pairs through creating shared meaning for the items. In the context of arithmetic, before an addition operation can be performed, the meaning of the number symbols must be available in the young child's symbolic representation. It is proposed here that for young children, use of elaboration may give meaning to number symbols. In both paired-associate and addition tasks, generating a referential event may provide meaning: in the case of paired-associate learning, shared meaning for two disparate items; in the case of addition, meaning for an abstract number symbol. Initially, young children require the presence of objects or pictures (maximally explicit prompts) to give meaning to numbers. A little later children may no longer require a concrete referential event in order to understand to what units numbers refer, but may have difficulty performing an operation with number symbols in the absence of any referents. At this point, if the child can generate his own referential event to provide meaning for the number symbols, performance of arithmetic operations may be facilitated. Imagery instructions are viewed here as one type of elaborative prompt which may direct the child toward generating a referential event for himself. The way in which elaboration may provide meaning for number symbols, in the context of addition, is developed further below.

The task characteristics of solving an addition operation are more complex than a paired-associate task. For example, solving a problem of the form $x + y = z$, requires (1) understanding to what units the number symbols refer; (2) transformation of an existing state into a new state; (3) understanding an equivalence relation. It is being proposed here that elaboration may facilitate these three aspects of addition.

In the case of

(1) understanding to what units the number symbols refer:

Elaboration may play an important role in helping the child to relate to the numbers given as symbols which represent a certain number of objects. Number is a property that refers to objects, just as colour, shape and size are properties referring to objects; numbers themselves have no concrete existence. Number is a property that refers to a set of objects, e.g. no object has the property "two", a set of objects can have the property "two" (Dienes and Golding, 1966). The very young child easily learns rote recitation of numbers in sequence. A more difficult task is for the child to learn to what units numbers refer. In order to count objects, the child must learn that there is a one-to-one correspondence between each number and an object (Piaget, 1952). The child learns that in order to count he must consider each object in turn, and to stop counting when he gets to the last object. It takes quite a lot of practice before the child is able to co-ordinate

consideration of each object and the recitation so that he arrives at consistent results. For the very young child, learning about the property of "number" involves repeated instances of pairing of a number, e.g. two, with a set of two objects, and similarly x with x objects. Association may be said to be involved at this level of learning about numbers, and elaboration may facilitate this association. Young children would require a referential event in the form of concrete sets (objects or pictures) to give meaning to number symbols. Later, generation of a referential event may provide "meaning" for the number symbol.

(2) transformation of an existing state into a new state:

Relations can be established between numbers. x can be related to y : the relation is called a "function" or an "operator". An arithmetical state is represented by a set. An input state leads to an output state through the application of an operator. Examples of arithmetic operators are $+2$, -1 , etc. It is a considerable step from understanding to what units numbers refer, to performing an operation with numbers. There may be developmental changes in the nature of the addition process (Groen and Parkman, 1972). For an adult, addition may be a retrieval process of recalling an addition fact from long term memory. For a young child, addition may be a reconstructive process for which the sets must be available, at first concretely, then a little later, symbolically. It is proposed here that before an operation can be performed, the meaning of the number symbols must be available in the

young child's symbolic representation. Once the associations between sets of a given size and a particular number symbol have been well learned, children may no longer require a concrete referential event, i.e. a maximally explicit prompt, to understand to what units numbers refer, but may have considerable difficulty performing an operation with numbers in the absence of any sets. Prompts which help the child to generate his own referential event may facilitate performance of arithmetic operations during this transition period. A referential event generated in connection with a given number symbol may provide meaning for that symbol. The sets, although not available as objects or pictures, would be available symbolically to be used in a reconstructive process to perform the addition operation.

(3) understanding an equivalence relation:

In order to solve equations such as $x + y = z$, the child must "understand" an equivalence relation between sets, i.e. the $=$ sign concept. Part of transforming an existing state into a new state is forming an equivalence relation. The $=$ sign has two possible interpretations:

- (i) "results in" e.g. adding two quarts to three quarts
results in five quarts:
- (ii) "is the same as" e.g. $3 + 2$ is the same as $4 + 1$.

The "same as" interpretation is the more important one for mathematics, but the "results in" interpretation is the one which makes more sense in

working out quantitative problems (Bereiter, 1968). It is assumed here that understanding of both interpretations derives from experience with objects and pictures. If a referential event can provide the child with symbolic sets, this would help with all aspects of addition which are derived from operations with concrete sets. It is hypothesized here that an internal referential event would bridge the gap between operations performed with sets, as compared with number symbols only.

In Rohwer's (1973) view, regardless of what kind of prompts are used to activate elaboration, the underlying process is the same. In contrast to this one-process conception, Paivio (1971) proposed a two-process account, that underlying elaborative operations consist of at least two kinds of processes: imaginal and verbal. Whether a single-process or two-process position will ultimately hold the most explanatory power in various learning situations is an open question which is not of concern here. Within the context of operations on sets and on numbers by young children, the position adopted here is that an elaborative process may operate. This process can be activated in various ways; one possibility is through imagery instructions. It is being proposed here that elaboration may play an important part in the child learning to use numbers (which he has long been able to recite as a serial chain by rote) in a new way: to perform operations involving number.

Before starting school, most middle class children can count objects correctly, however they have had very limited experience involving number relations and operations. According to Piaget (1952)

most kindergarten age children are still developing a concept of number because they do not recognize that number remains invariant irrespective of spatial changes, (i.e. lack conservation). Regardless of whether one adheres to the Piagetian theory of number development, or to the view that young children's experience with number relations is limited, it is clear that most children in kindergarten are still in the process of acquiring concepts concerning the property of "number". Ultimately we want the child to perform arithmetic operations "in his head" without recourse to objects, fingers, etc. There is a transition from object counting by very young children to "mental arithmetic" during the elementary school years. Elaboration may have an important function in this transition from using concrete referents to performing abstract symbolic operations with numbers. It is postulated here that elaboration may be important during the acquisition and practice phases of learning to perform operations, i.e. until the task of combining and separating numbers is overlearned, not just until children can solve equations, but until they have had considerable practice with numbers and problems.

In the next section of this paper the review of the relevant literature is presented, and is divided into three main parts: (A) research on imagery in children's paired-associate learning, (B) research on basic mathematics learning, (C) Piaget's theoretical position on the development of the concept of number, and the development of mental symbols. The basic issues and hypotheses in the present

study are then outlined. The mathematical terms used in this paper are defined in Appendix A.

CHAPTER II

REVIEW OF RELATED LITERATURE

A. Research on Imaginal and Verbal Processes in Children's Paired-Associate Learning:

The importance of verbal and nonverbal processes in children's paired-associate learning has been demonstrated (Paivio, 1969, 1971; Reese, 1970(a); Rohwer, 1970). Noun pair learning is facilitated when a pair is linked by a verb (e.g. Rohwer, Lynch, Levin and Suzuki, 1967; Yuille 1974), or preposition (e.g. Kee and Rohwer, 1974), as compared with a conjunction or no link. Similarly, pictorial depiction of a noun pair interacting facilitates association of the nouns, as compared with side-by-side depiction of the pair (e.g. Holyoak, Hogeterp and Yuille, 1972; Rohwer, Lynch, Suzuki and Levin, 1967). Production of meaningful interactions between object pairs facilitates paired-associate learning as compared with side-by-side object pairs. (Wolff and Levin, 1972; Wolff, Levin and Longobardi, 1972; Yuille and Catchpole, 1973). In contrast to these well established findings, the development of use of elaboration is not as clear. Two areas of controversy concerning the developmental role of elaborative prompts, are examined here:

1. Preferred elaborative prompt with young children:
verbal vs. imaginal;
2. Experimenter-provided as compared with subject-provided elaboration.

1. Preferred elaborative prompt with young children: verbal vs. imaginal

There has been some controversy over the use of verbal as compared with imagery processes by young children. Rohwer (1970) suggested that

"a preference for and a capacity to make effective use of visual representation and storage develops later than is the case for verbal modes of representing and storing information."

In other words Rohwer's hypothesis was that effective use of imagery increases with age. Rohwer (1967) found that sentences plus side-by-side pictures facilitated paired-associate learning more than naming coupled with action pictures for kindergarten and grade one children. The reverse was found for children in grades 3 and 6. Milgram (1967) found that the superiority of sentence mediators as compared with picture mediators was greater for 4 year olds than for older children up to the age of 9 years. Davidson and Adams (1970), using 2nd grade children, reported that a prepositional connective with side-by-side pictures was more effective than a conjunction with joined pictures.

Paivio (1970) suggested that the interaction of type of mediator and age was an artifact due to the use of verbal recall as the dependent measure. Paivio maintains that pictorial materials do elicit imagery in the young child and that imagery can function as a mediator, but the fact that the child must respond verbally on the recall test entails decoding from an image response to a verbal response, and this may interfere with the younger children's performance. However, this

explanation is not supported by the experimental findings. Davidson and Adams (1970) used a recognition procedure to overcome this problem, and verbal elaboration was still found to be superior to pictorial. Holyoak, Hogeterp and Yuille (1972) found that a recognition procedure did not elicit a significant overall increase in the number of correct responses made by kindergarten children. This appears to rule out Paivio's hypothesis that response availability accounts for superiority of syntactic mediators.

Rohwer (1970) offered an alternative hypothesis for the findings of an increase in the effectiveness of imagery mediators with age. He said that when young children are presented with picture materials they have difficulty generating their own verbalizations, whereas older children presumably produce implicit verbalizations spontaneously. Providing a sentence along with a picture should benefit the younger children more than the older children. Holyoak, Hogeterp and Yuille (1972) tested this hypothesis and found no support for it. Lynch and Rohwer (1972) had 3rd and 6th grade children learn paired-associate picture or word lists, with or without aural labeling by the experimenter. When pictures and words were not aurally labeled, pictures led to better learning and greater gain for 6th grade. When names were provided, there was no grade x mode interaction. Providing a label did not benefit the younger children more than the older.

Many studies have failed to support Rohwer's hypothesis that verbal representation is preferred over imaginal by young children. The developmental trend was not significant in studies by Reese (1965) with children aged 3 to 7; Rohwer, Lynch, Suzuki and Levin (1967) with grades 1, 3, and 6; Rohwer (1968) with kindergarten, grades 1 and 3; Reese (1970(b)) with 3 to 5 year olds; Holyoak, Hogeterp and Yuille (1972) with kindergarten and grade 3. Levin, Davidson, Wolff and Citron (1973) found that 2nd and 5th graders benefitted from imagery and sentence strategies to the same degree. The issue of the developmental role of imaginal versus verbal representation in children's learning has not been resolved by the research so far. In conclusion, Kee and Rohwer (1974, p. 70) have suggested:

"prompt modality differences may be ephemeral phenomena, determined by very specific task and subject variables. Indeed, prompt modality differences even when detected are small in magnitude when compared to the general effect of elaborative prompts."

2. Experimenter-provided, vs. subject-provided, elaboration

Another variable of developmental significance is whether children at various ages can only use referential events provided by the experimenter, versus generating their own. This had been studied experimentally in two ways: 1. through instructions to the subject to form mental images relating the members of each pair (Yuille and Paivio, 1968); 2. providing pictures depicting an interaction between the noun pairs versus side-by-side pictures (e.g. Reese, 1970 (b);

Rohwer, 1967). Self-generated relations have been found to be more effective than experimenter-imposed relations for college subjects (Bobrow and Bower, 1969; Bower and Winzenz, 1970). First grade children (Montague, 1970) and 5 year olds (Rohwer, Ammon and Levin, 1971) appear to be able to use experimenter imposed interactions to remember picture pairs, but are unable to generate their own referential events. Rohwer (1971) found both subject-generated and experimenter-generated interactions equally effective from grades 1 to 11. There is evidence that children in upper elementary grades are capable of generating referential events from pictorial material to facilitate paired-associate learning (Levin and Kaplan, 1972). Danner and Taylor (1973) used integrated pictures of nouns, and the combination of training and integrated pictures, with noun triplets. The subjects were children in Grades 1, 3 and 6. Performance under all three conditions was significantly better than a control condition for all three grades. However in grade 6 the children who had been trained to generate their own relations between the pictured nouns recalled significantly more than those who had been shown integrated pictures. Clarkson, Haggith, Tierney and Kobasigawa (1973) found that subject generated images facilitated learning more than experimenter provided, with grades 4 and 6. It appears that there is a developmental trend in which young children can only utilize referential events provided by the experimenter, then both subject

generated and experimenter generated sources become equally effective, finally self-generated referential becomes more effective. It is not yet clear for what ages or under what conditions these transitions take place.

One purpose of the present study was to determine whether some of the findings from the paired associate studies on elaboration with children can be extended to performance of the arithmetic operation of addition in the context of word problems. The following variables which have emerged as important in paired-associate learning were included:

Use of verbal and visual elaborative prompts have both been found to be effective in facilitating paired associate learning (e.g. Rohwer, Lynch, Suzuki and Levin, 1967). In the present study an attempt was made to manipulate both types of prompts. Performance on addition word problems was compared under three elaborative Prompt Conditions: Concrete plus Verbal, Imaginal plus Verbal, and Verbal Only. The verbal stimuli differ in the Described Relation of the referents; Dynamic as compared with Static.

Interacting as compared with side-by-side stimuli: The facilitating effect of experimenter provided interacting stimuli, on learning a paired associate list, as compared with side-by-side stimuli, has been clearly demonstrated with children of early elementary school age (e.g. Rohwer, 1967). In the present study, Dynamic Described Relation,

in which the stimuli move together either visually and verbally, or just verbally, was compared with a Static Described Relation, in which the stimuli are side-by-side.

The age or developmental level at which subjects can utilize imagery instructions to generate their own referential event has been of interest in paired associate learning. The developmental level at which this capability can be observed may vary depending on the type of task employed. The type of referential event needed to give meaning to numbers may not be the same as that needed for relating noun pairs. It may be possible to generate a referential event for addition at an age at which it may be difficult to do so for a paired associate task, i.e. kindergarten. The effect of imagery instructions in prompting subjects to generate referential events was assessed in kindergarten children at each of three Developmental Levels (as determined by performance on a number conservation test). This was done by comparing performance under "imagining" instructions with performance under "listening instructions. A third condition received "watching" instructions.

Incidental Learning and Imagery

There is considerable evidence that imagery-evoking activity is associated with better recall in incidental learning tasks, as compared with non-imagery control activity with adults (e.g. Sheehan, 1973; Anderson and Hidde, 1970; Bower, 1969; Sheehan and Neisser, 1969). In incidental learning with children the role of imagery as well as other elaborative processes has not been studied to the same extent, however, there is some evidence that incidental imagery-evoking activity does facilitate learning. For example, Yarmey and Bowen (1972) investigated the effects of imagery instructions on incidental recall of picture pairs and noun pairs with children ages 8 to 13 years. They had subjects rate picture or word pairs according to vividness of images suggested by the pairs. There were three instructional conditions: intentional imagery, incidental imagery, intentional control. Performance under intentional and incidental imagery was better than under intentional control, for both nouns and pictures. Goldberg (1974) found that imagery, in the sense of nonverbal responses aroused by concrete stimulus material, facilitated incidental recall with grade 5 children. Illustrated or non-illustrated social science and science material served as incidental content for a spelling and grammar exercise. The imagery and non-imagery groups differed by the presence or absence of illustrations. Illustrated incidental material was retained better (on recognition and

recall) than non-illustrated material.

An incidental learning task was included in the present study. Following the addition word problems, each subject was asked to recall the nouns used in the problems. The reason for including this task was to check on whether the subjects under the Imaginal plus Verbal Prompt Condition had utilized the imagery instructions. If the subjects generated referential events under the Imaginal plus Verbal Prompt Condition, and not under the Verbal Only Condition, then noun recall should be better under the former as compared to the latter condition.

B. Research on Basic Mathematics Learning

1. Rules as the Basis for Mathematics Behaviour

Scandura (1966, 1967, 1972) has been concerned with developing a theoretical framework for the study of "psychomathematics":

"Mathematics is perhaps the most highly organized body of knowledge known to man. Yet in spite of its clarity of structure, most of the basic research done on mathematics learning and behaviour has been strictly empirical. To be sure, there has been a fair amount of research in the area, and the amount seems to be growing rapidly; but there has been no super-structure, no framework within which to view mathematical knowledge and mathematical behaviour in a psychologically meaningful way." (Scandura, 1972, p. 141).

Scandura (1966) maintains that the central aim of basic research in mathematics learning should be theory development. He considers the development of a precise language couched in observables, to be important for the description of psychologically relevant mathematical characteristics. Scandura proposes a set-function language (SFL) in which the critically important characteristics are formulated precisely, largely in terms of the set and function concepts of mathematics. He rejects stimulus-response language as being adequate for mathematics learning because he views the rule as the appropriate basic unit for this type of learning, rather than associations (Scandura 1966, 1972). An association is a one-to-one relationship; a concept a many-to-one relationship; and a rule a many-to-many relationship. Knowing a rule enables one to give an appropriate response in a class of responses to

any one of a class of stimuli. Scandura proposes: (1) rules are the basic building block of all mathematical knowledge; (2) all mathematical behaviour is rule governed. In addition to specifying finite rule sets, he is concerned with the importance of specifying how the constituent rules may be combined to generate behaviour. Scandura (1972) views concepts and associations as special cases of rules: concepts are rules in which each stimulus in a class is paired with a common response; associations are further restricted to a single stimulus-response pair. A rule is defined as an ordered triple (D, O, R) , where D is the set of stimulus properties which determine the response, and O is the operation or generative procedure by which the responses in R are derived from the critical properties in D . A generative procedure is a sequence consisting of at most four kinds of rules:

- "1. encoding rules by which essential properties of stimuli are put into store;
 2. transforming rules by which things in store are transformed into something else in store;
 3. decoding rules by which things are taken out of store and made observable;
 4. rules for selecting other rules, given the output of some previous rule (which is in store)."
- (Scandura, 1972, p. 144)

The following study is an example of the way Scandura has applied his framework to an experimental problem. Scandura (1966) studied the effect of verbal attribute and operation cueing on the rate of

learning a mathematical task. The study was designed to determine whether providing the relevant attributes (i.e. D, the stimulus attributes which determine the responses) or the appropriate combining operations (i.e. O), does in fact increase the rate at which arithmetical rules are discovered. The stimuli were four-tuples of numbers, e.g. (4, 8, 9, 3) and the responses were new integers which could be derived from 3 of the 4 original integers by applying some combination of two arithmetic operations. Three rules were used. The determining characteristics and operations respectively were:

P (1) A_1, A_3, A_4 ; (2) A_1, A_2, A_4 ; (3) A_2, A_3, A_4 and

Q (1) $X + Y - Z$; (2) $X \cdot Y \div Z$; (3) $X \cdot Y - Z$ where

the subscripts $i = 1, 2, 3, 4$ in A_i refer to position in the four-tuple,

and X, Y, and Z to place holders. The results were that both attribute

and operation cueing induced significantly earlier discovery of the rules.

The logical form of the rules was: If P(1) then Q(1), etc. for P (1,2,3)

and Q (1,2,3).

Scandura has made an important contribution to opening up the area of mathematics learning, not only in his experimental studies of various aspects of learning mathematical tasks, but in his emphasis on theory building. Although he does not see stimulus-response language as the best one for describing mathematics learning, or association as the basic building block, his approach is a development and extension of this tradition, rather than an opposing theoretical stand. Scandura's

approach will be fruitful, as it integrates rather than separates areas of learning. The present study was an attempt to extend findings from research on one task, i.e. paired associate verbal learning, to a task in a different cognitive domain, namely problems involving the arithmetic operations of addition. Moreover, the present study was concerned with possible algorithms, i.e. counting models, children may use to solve addition problems. Any algorithm can be viewed as a rule, or set of rules, for arriving at a solution to a mathematical task.

2. Imagery and Mathematical Tasks

Imagery has been mentioned as possibly serving an important function in mathematics learning (Syer, 1953; Suppes, 1967; Skemp, 1971). However, other than in geometrical problems, which are not of immediate concern here, very little experimental work has been done relating use of imagery to performance on a mathematical task. Hayes (1973) has carried out exploratory work with adults, concerning the use of spatial information stored in images, for solving elementary mathematical problems. For example, in Study 1 of a set of eight exploratory studies, 19 students and faculty members given problems on cards, were asked to report on their imagery during the solution process. Hayes tentative findings suggest an important function for imagery in the solution of elementary mathematics by adults. For example, generated images combined with elements of the background physical stimulus, may be used in a spatial array to store partial results during the solution process.

Lee (1971) performed an experimental study with children which explicitly relates imagery to a mathematical task. Lee studied the effects of the type and amount of visual cues on rule learning. Grade 4 children learned three linear function rules: an intercept rule (I - rule: $S = b + F$), a coefficient rule (C - rule: $S = a \cdot F$) and a complex rule (Complex Rule: $S = a \cdot F + b$). One aspect of the study was to determine whether the acquisition of these rules could be facilitated by visually represented cues,

i.e. a set-up of weighing operations involving a scale and a manipulable scale pan. The task for the subjects was to produce a mathematical sentence involving a given pair of numbers, plus an appropriate constant and a necessary operator. Lee (1971) postulated that "the imagery representations of the concrete referents of weighing operations ... remain invariant across different numerical instances, and thereby serve as mediators via keeping the imageries of the weights as well as the scale pan and division available for the presumed subprocesses." (Lee 1971, p. 132). The results indicated that presence of visually represented cues did facilitate acquisition of a mathematical rule. This finding that performance on a mathematical task was better under conditions which promoted use of imagery mediation by subjects, supports the view being put forward here as to the importance of elaboration processes in learning mathematics.

3. Acquisition of Number Concepts

A major concern in the present study was to examine possible algorithms children may use to perform the addition operation. Various models have been proposed to account for addition performance (e.g. Thomas, 1963; Suppes and Groen, 1967; Restle, 1970). Suppes and Groen (1967) formulated a counting model to account for children's responses in the addition of single digit non-negative integers. The model is based on an assumption that variations in response latencies between problems reflect some counting process the child is using. They distinguished between five possible algorithms for solving a problem of the form $m + n = \underline{\quad}$, using a counting procedure. An hypothetical counter, on which two operations are possible, was proposed: setting the counter to a certain value (which clears the previous one), and then adding a number to this current value. Suppes and Groen assumed that the addition operation is performed by successively increasing the initial value of the counter by one until the second integer has been added on. Using this counter, an addition equation of the form $m + n = \underline{\quad}$ can be solved in the following ways:

1. The counter is initially set to 0, m is added and then n.
2. The counter is set to m and then n is added.
3. The counter is set to n, and then m is added.
4. The counter is set to the minimum of m and n. The maximum is then added.

5. The counter is set to the maximum of m and n . The minimum is then added.

These five procedures will be referred to as models 1, 2 ... 5.

The setting operation was assumed to take a constant time, independent of the value to which it is set. The addition time was assumed to be proportional to the number of times the counter must be increased. Thus the total time T taken to perform the addition operation, where α is the time taken to set the counter and β the time for the counter to be increased by 1, and x the number of times the counter is increased, is:

$$T = \alpha + \beta x.$$

Suppes and Groen had first grade children solve a set of 21 visually presented addition problems of the form $m + n = \underline{\quad}$, consisting of all possible combinations of integers m and n , such that

$$m + n \leq 5$$

$$m > 0$$

$$n > 0$$

The data was best predicted by Model 5.

Groen and Parkman (1972) looked at the same counting model in the context of processes in long-term memory: reproductive processes which are concerned with the retrieval of stored facts, and reconstructive processes, which are concerned with the generation of facts on the basis of stored rules. They point out that the addition of two single digit numbers can be viewed as either a reproductive or a reconstructive process.

Traditionally it has been seen as the former: i.e. retrieval of a stored

number fact (association between two digits and their sum) which has been previously learned as part of the addition table. They point out that although a counting algorithm for addition is almost never specifically taught, it is clear that addition can be viewed as closely associated with some kind of counting process. Adding by counting is a reconstructive process involving the application of a learned rule, unlike retrieval of a number fact. Groen and Parkman, using grade 1 subjects, tested the same 5 counting models generated by Suppes and Groen (1967), but they used a wider range of addition statements, i.e. $m + n \leq 9$, and analyzed each subject's data separately in order to determine whether subjects were using different processes. They found that each subject's data was either best accounted for by model 5, or not adequately accounted for by any of the five models. The most important source of lack of fit in this study and Suppes and Groen (1967) are the "ties" (i.e. addition statements in which $m = n$). The ties always had a lower latency than any of the other statements with the same minimum addend. In the case of $2 + 2$, $3 + 3$, and $4 + 4$ the discrepancy was considerable. Groen and Parkman (1972) hypothesized that the solutions to "ties" have been memorized.

Groen and Parkman (1972) point out that by the time a person has reached adulthood, addition has been overlearned. They suggest two possibilities as to changes that occur as a result of overlearning:

- (1) There is no change in the process used except possibly a decrease in the incrementing time and setting time.

- (2) Individuals gradually cease to use the process defined by model 5. Instead the answer is memorized and it is stored in fast access memory.

It appears that ties may be the first statements to be memorized. Data on college students (Parkman and Groen, 1971) suggests that adults use a memory-hookup reproductive process, which children already use to solve ties such as $2 + 2$, and that adults occasionally have failures in retrieval and then revert back to the counting process (model 5) used by children.

Reiss (1943) discussed the development of young children from prenumerical behaviour to nominal, ordinal and cardinal use of number based on informal observation of children. Reiss distinguished three phases in learning to perform addition. Each phase is marked by a different procedure used by the child, (i.e. a different counting algorithm). The first procedure reflects nominal concept of number. For the problem "How many are 3 beads and 2 beads?"

"a child, if the objects are available, touches them or points to them successively, first to the three beads, then to the two beads, and finally to all the beads taken together, while calling off the number names, with each series started afresh with 1 In the absence of the objects themselves, fingers are used as convenient tallies . . ." (Reiss 1943, p. 153-154)

i.e. 1, 2, 3 - 1, 2
= 1, 2, 3, 4, 5

Reiss points out that if the child used 3 as a symbol of a clearly

structured group (i.e. cardinal use of number) instead of using it as a name for one of its items, he would not set out by numbering up to 3, but would immediately take 3 as a starting point. It is also striking that the child does not go on directly from the 3 in the first series to 4. The child does not yet regard an item as being at the same time the first of one set and the fourth of another.

With the change to an ordinal concept of number when the child discovers that relation between numbers and things is a matter of arbitrary arrangement, there is a change with the method of proceeding in computation. Children continue to solve computation problems by a counting procedure, but they no longer number the different sets first,

i.e. 1, 2, 3, 4, 5

When the child has acquired a cardinal concept of number, he recognizes that numbers are comprehensive group symbols, i.e. that the number which stands for the last item in a sequence is also taken to denote the group as a whole. Given the problem "add 5 and 3" he does not work up to 5 by counting, as before, but now starts directly with 5. The response pattern has changed from

1, 2, 3, 4, 5, 6, 7, 8

to

5, 6, 7, 8

He has acquired the cardinal meaning of the number 5. Reiss maintains that later still counting is no longer used.

In terms of the counting models tested by Suppes and Groen (1967) and Groen and Parkman (1972): Reiss' nominal procedure is not represented in the five counting algorithms; the ordinal procedure is equivalent to model 1, and the cardinal procedure to model 5. These counting models have not been tested with children younger than first grade. It is suggested here that development of addition operation may pass through three phases (or even four as outlined by Reiss (1943)) instead of two as suggested by Groen and Parkman (1972). Very young children may use a reconstructive process involving model 1, and at a later phase model 5. Later still as addition becomes overlearned, a reproductive process of retrieval becomes the norm. The change from using model 1 to model 5 to a retrieval process will not be uniform for all addition statements. For example addition of ties may change to a retrieval model before other combinations of integers.

It is interesting to note that Reiss (1943) suggested from her informal observations that children often resort to

"counting on their fingers or apparently to an equivalent process of visualizing concrete objects". (Reiss 1943, p.157)

Unfortunately Reiss does not elaborate on this statement. It would appear that the young child, if addition is indeed performed using a counting model, may require concrete referents in order to keep track of his counting procedure. In the absence of objects, he may use his fingers, or generate referential events in order to perform addition.

The studies considered so far in this section have been concerned with addition, and how addition may be related to counting. Next, some research on counting behaviour, and estimation of sets, will be examined. Wang, Resnick and Boozer (1971) examined the sequence in which kindergarten children acquire elementary mathematical behaviours of counting and numeration (use of written number symbols). Some of the results were as follows: the tasks which involved use of numerals in conjunction with counting appeared as the most complex behaviour in the counting task sequence, contrary to expectation. Subjects who were able to associate numerals with sets were typically able to perform all the object counting tasks. This suggests numerals are not ordinarily learned until counting is a well-established skill. Counting out a subset of a specified size is apparently learned later than counting a given set. For subjects able to count, the most typical error on counting out a subset was to continue counting out objects beyond the number specified. The difficulty of counting an ordered versus an unordered set of pictures of objects differed depending on whether 1 - 5 items were involved or 6 - 10 items. For sets of 1 - 5 items there was no difference between counting performance whether the set was ordered (in 1 or 2 straight rows) or unordered (random). However, for sets of 6 - 10 items counting was far more difficult for a random than an ordered set. A different task required counting of moveable objects instead of pictures. This task was hypothesized to be very easy since subjects could remove objects from the set as they counted,

providing a means of keeping track of which objects have been counted.

In practice most subjects treated the objects as if they were fixed in place, and the results were almost the same as for counting an unordered set of pictures of objects. The conclusion drawn was that in the absence of explicit instruction, counting is not necessarily learned as a process of successive removal of objects in a set.

There is a body of literature, in the area of perception, concerned with the immediate apprehension of number. These studies were designed to determine the number of things that can be apprehended at a single glance, referred to as 'subitizing'. For example Von Szeliski (1924) found that fields of up to 6 dots could be perceived without eye movement. Jensen, Reese, and Reese (1950) concluded that subjects subitize up to 5 or 6 dots. The main question in this line of research has been the maximum numbers of dots that can be perceived at a glance. This number has been estimated from about 4 to 7, depending upon the age of the subjects (mainly adults) and the display used.

Gelman (1972) was concerned with estimation of number by preschool children. She suggests that with the possible exception of 1 and 2, counting is the main mechanism used by young children to estimate numbers of all sizes. Gelman reports a study by Beckmann (1924) which analyzed how children aged 4 years to 6 years arrive at an accurate estimate of the number of items in an array (ranging from 2 to 6 items).

Beckman categorized the children as counters or subitizers, on the basis

of their behaviour during the task and their explanations of how they reached an answer. Children classified as counters were observed to count, and said they counted. Subjects classified as subitizers responded very quickly without giving any indication of counting, and said they could just see when asked how they knew the answer. The results were that the younger the child, the greater the tendency to count for all numbers; and the larger the number, the greater the tendency for all the children to count. Gelman concludes that children estimate a number by counting before they can subitize the same number.

Beckwith and Restle (1966) assumed that perception of small numbers is a skill developed by adults, a shortcut to counting, rather than an elementary mental event, as was the viewpoint of the classical psychological theory of perception. Beckwith and Restle had undergraduates count arrays varying in the distribution of shapes and colours. In some conditions the arrangement of shape and/or colour of the items allowed for perceptual grouping. This experiment was based upon studies concerned with additivity of cues, in which problems are constructed using two or more perceptual dimensions. The findings were that performance was best under the condition in which colour and shape co-varied, so that 2 cues were available; colour alone followed, almost as good as the first condition, and much better than shape alone. Shape had little advantage over a random condition. The results indicated that counting is significantly more rapid when an array can be grouped into sub-groups.

Beckwith and Restle concluded that subjects probably segregated the whole display into colour groups, when possible subitized the number in each group, then added these numbers to obtain a result. When no such grouping was possible, subjects had to rely on a relatively difficult and ambiguous grouping by position, or count one by one.

It was suggested in the introduction to this paper that in order to generate referential events for number, the child may need to already have acquired the concept of cardinality of number. It is quite likely that this concept is not acquired for all numbers at once, i.e. a very young child may recognize the invariance of very small numbers, e.g. 2, yet be unable to conserve number when larger sets are involved e.g. 6 elements. Indirect support for this idea that children may recognize invariance of small numbers (e.g. 1 to 5) before larger numbers (e.g. 6 to 10), is provided by Wang, Resnick and Boozer (1971). They found that counting and numeration for sets of one to five elements is acquired before sets of six to ten elements.

Siegel (1971) studied the sequence of development of Piagetian number concepts with 3 and 4 year old children. All the concepts were translated into nonverbal tasks. The conservation in this study was measured by the child's ability to choose from among four alternatives, the stimulus with the same number of objects as the sample, despite different arrangements. First each subject was administered a test of Equivalence, in which he selected from 4 alternatives the one

which was identical to the sample. There were between 1 and 9 dots in the sample and alternative. For the Conservation task the subject was first given 20 trials with stimuli similar to the Equivalence test. Five Conservation test trials were then given, with similar stimuli, except that the dots in the alternatives were closer together, so that the arrangement between the sample and the alternatives would be different. Of the 27 children aged 4 - 6 to 4 $\leq 11,448.1\%$ could do this conservation task. One reason why these children passed the Conservation task, may have been because the procedure was more similar to a training procedure, rather than an assessment of Conservation level, although the stated intention of the study was the latter.

Gelman (1972) has studied number invariance with 2 1/2 to 6 year olds, using a procedure involving two phases. First expectancies are built for two arrays of objects, then the child's reaction to surreptitious changes in one of the arrays is assessed. The results suggest that children as young as three years of age may show number invariance for very small numbers which they can estimate, e.g. sets of 2 or 3 elements. However the 2 1/2 year old children had great difficulty estimating the numerosity of arrays of two and three objects. Gelman concludes that young children (i.e. 3 years and above):

(1) know the effects of addition, subtraction and displacement on a small set;

(2) know the specific effect, on small sets, of adding or subtracting one element;

- (3) tend to count when estimating numerosity;
- (4) if they accurately estimate the numerosity of a set, they apply invariance operators to the same set.

Therefore in the present study when children are referred to as having acquired "the concept of cardinal number", or on the other hand, as lacking the concept, this is relative to the size of the sets used in the conservation pretest. In the present study sets of 6 or 7 elements were used in four tasks to assess level of conservation. Thus when a child is referred to as a Nonconservers, this means he showed no evidence of having a concept of the invariance of sets of 6 elements. Nothing is being said about his understanding of sets smaller than this; it is quite possible he may have a concept of the invariance of sets of, for example, 2 or 3 elements.

4. Concrete Materials as Aids to Arithmetic Performance

There has been research on the use of manipulative materials by elementary school children. Fennema (1972) reviewed 16 studies done between 1950 and 1966. Most of the studies used two groups: the control group was "taught in a traditional way with traditional methods (often defined no better than this)" (Fennema, 1972, p.636); in the experimental group, teaching was supplemented with some kind of concrete materials. Seven of the studies used Cuisenaire rods, which is a length representation, and will not be considered here. Of the nine studies which used other kinds of concrete aids, only two reported significant differences between the groups, both in favour of the concrete materials group. Kiernan (1969) has also criticized the studies on activity learning from 1964 to 1968, as typically being poor methodologically.

Steffe (1970) made a distinction between addition word problems in which the union of two sets is described in the problem, and problems in which the union of sets is implied by the addition question only. He called this presence or absence of a described transformation. Steffe (1970) looked at the effect of four variables on the performance of first grade children on traditional arithmetic word problems: (1) I.Q., (2) ability to make quantitative comparisons, (3) aids (presence of objects, versus pictures, versus no aids), (4) presence or absence of a described transformation in the problem. The test of quantitative

comparisons was aimed at testing number conservation, and subjects were divided into four levels on this test. There were three levels of I.Q. This resulted in 12 independent groups, with the aids and transformations variables as repeated measures. Each subject was individually given 18 addition problems: 6 with objects present, 6 with pictures present, and 6 with no aids. Three problems from each of the above 6 involved a described transformation, the remaining 3 had no described transformation. Thus the 18 problems for each subject were partitioned into 6 conditions with 3 problems in each condition. Performance on problems with objects or pictures present was significantly better than no aids; there was no difference between objects and pictures. Performance on problems which described the transformation was significantly better than those in which the transformation was not described. The variables of aids and transformations operated the same way for all groups, regardless of the I.Q. or ability to conserve number.

Steffe and Johnson (1971) found, using first grade children, that subjects who scored high on a pretest of quantitative comparisons, solved arithmetic word problems equally well in the presence or absence of manipulable objects, but only with addition, not subtraction problems. Subjects in the low scoring group on the pretest performed better with manipulable objects present. The presence or absence of a described transformation did not differentially affect problem solving, contrary to the finding of Steffe (1970).

One problem in the Steffe (1970) and Steffe and Johnson (1971) studies, is that item difficulty was confounded with experimental conditions on the repeated measures variables. The problems within experimental conditions should have been counterbalanced, to control for item difficulty. Steffe (1970) mentioned that item difficulty may have varied: "There was one particularly difficult problem that fell in the no-aids plus transformation category." (Steffe, 1970, p. 156). Presence or absence of described transformations and aids were both within subjects variables in these studies. Item difficulty may account for the contradictory findings between the two studies. Steffe's distinction between addition word problems in which the union of sets is, and is not, described, has been incorporated into the present study, and is referred to as a Dynamic vs. Static Described Relation. Described Relation and Prompt Condition were both between subjects variables in the present study.

5. Summary

It has been suggested by Reiss (1943), Suppes and Groen (1967) and Groen and Parkman (1972) that children may use a counting process in order to perform the addition operation. If this is the case, then studies concerned with counting behaviour may suggest hypotheses different from those derived from paired-associate studies, with respect to the effect of Described Relation. Wang, Resnick and Boozer (1971) found it was harder for kindergarten children to count unordered sets of 6 - 10 objects compared with 1 - 5 objects. Beckwith and Restle (1966) found that perceptually distinct groups are counted more rapidly by adults than sets which are harder to perceptually organize. In the present study, in the Static Described Relation conditions, where referents are side-by-side, the sets are smaller and more distinct. One could predict on the basis of the above findings that performance under the condition of side-by-side static sets may be better in an addition task, than the dynamic condition in which the sets are moved together, either visually and verbally or just verbally. However, these studies were directed at counting a set of a given size, not at combining sets. The described transformation of the union of discrete sets may be important for an addition task (Steffe, 1970). This may outweigh the benefits of having the sets separate for counting.

One of the purposes of the present study was to look at some algorithms children may use to solve addition word problems. In the present

study models 1, 2, and 5 of the counting models proposed by Suppes and Groen (1967) were tested. The counting models were previously assessed using integers in addition equations, not in word problems. It is proposed here that for word problems, the time it takes to initially set the counter would include the time involved for processing stimulus input.

Studies using nonverbal tasks to assess number conservation with preschool children have found that young children may show evidence of the concept of number invariance for very small numbers (e.g. Siegel, 1971; Gelman, 1972). Children may develop number conservation for small sets, e.g. less than 5, which they can estimate by subitizing, earlier than for larger numbers. In the present study number conservation was assessed with sets of 6 pairs, and 7 pairs of objects. Thus children classified as Transitionals and Nonconservers with sets of this size may already have acquired the concept of number invariance for very small numbers.

C. Piaget's Theoretical Position

Piaget's position as a stage theorist is well known (e.g. Flavell, 1963; Ginsburg and Oppen, 1969), as are general definitions of his broad developmental stages: i.e. sensori-motor, preoperational, concrete operational, formal operational, and will not be described further. Two aspects of Piaget's theory which are of particular concern for the present study are outlined below: the development of the concept of number, and the development of mental symbols.

1. Development of the Concept of Number

At present there is no definition of number which entirely satisfies all the mathematical criteria. The definition of natural number provided in the 'Mathematical Terms' section (Appendix A), proposed by Peano at the end of the nineteenth century and extended by von Neumann in 1923, is the most acceptable to mathematicians. This theory postulates that order is the basis of the number concept. The recognized limitation of this definition is that all ordered progressions can be constructed from this theory, not just natural numbers. Another theory, proposed by Russell (1908) and independently by Frege in 1884 defines a number as a class of equivalent classes. In this theory cardinality is the basis of number, not ordination. This theory of number has a serious inadequacy in that

"In order to construct numbers from classes, they have introduced numbers into classes." (Piaget, 1970, p. 37)

The concept of class is based on one-to-one correspondence between elements. This correspondence can be qualitative or quantitative.

Russell uses one-to-one correspondence in the sense that in any class, regardless of its qualities, every element counts as one, which is introducing a quantitative basis for class, i.e. presupposing the number concept which the theory is attempting to explain. Piaget's (1952, 1970) view is that number develops from a synthesis of ordinal and cardinal concepts. When mathematical criteria for a good theory are considered, Piaget's position has limitations. Brainerd (1973b) has analyzed these three theories from the mathematical and psychological viewpoints.

Although the present study is not concerned with comparing the adequacy of ordinal versus cardinal basis of number, it was considered important to place the issue of children's acquisition of number concepts within the larger framework of number theory. Piaget's position has been used in this study as a frame of reference, first because his analysis of number development in children is the most elaborated working model available; secondly because Piaget, in his writing on imagery, has explicitly related use of imagery to one-to-one correspondence and conservation of quantity (Piaget and Inhelder, 1971).

In Piaget's (1970) view, logical and mathematical knowledge is abstracted from actions or operations upon objects. The abstraction is drawn, not from the object itself, but from actions and co-ordination of actions upon objects. These co-ordinations of actions become mental

operations. An operation has four fundamental characteristics:

1. an operation is an action that can be internalized (i.e. carried out in thought);
2. an operation is a reversible action (e.g. addition/subtraction);
3. an operation always involves some conservation, some invariant;
4. no operation exists alone, it is related to a system of operations.

The focus of the present study was on the child's level of understanding of the cardinal, rather than ordinal, aspect of number, and it is that part of Piaget's theory of number development which is to be described here. One-to-one correspondence is an operation which forms the basis of cardinal number, the development of which Piaget (1952, 1970) views as follows. In a very early stage a child faced with a row of six red tokens, and asked to put as many blue tokens as there are red, will make a line of blue tokens the same length as the line of red tokens. He will pay no attention as to whether there actually is the same number of tokens. The criterion of evaluation by the child at this level is not the number of elements in the one-to-one correspondence, but perception of a global quality (Piaget 1952, p. 76). The two qualities inherent in any row of objects are the length and density of the elements.

"For the adult, a row of n spaced out elements keeps its cardinal value n if its length is diminished by closing up the elements. It is therefore the relation between the length of the row and the intervals between the elements which determines the conservation of the whole, whereas the relationships of length and density are variable. It is precisely this co-ordination, or logical composition of the two relationships in question, that the child of this level cannot make, and that is why the notion of conservation of the sets and even of one-to-one correspondence is still impossible." (Piaget, 1952, p. 77)

When children in the second stage are asked to pick out a number of elements equal to those in a model row of six, they make an optical spatial correspondence with the model. However, they no longer accept the equivalence of the two rows when the correspondence cannot actually be perceived, i.e. if one of the rows is lengthened or compressed. This level of correspondence is on an intuitive plane:

"And yet, although the child begins to make the one-one correspondence as soon as he can think simultaneously of the length and density of the rows, this correspondence does not at once lead to the notion of lasting equivalence of corresponding sets, nor to that of their quantitative constancy ... The child certainly considers the relationships of length and density simultaneously, since he can produce a copy equal in length and density to the model, but the co-ordination does not go beyond the plane of perception, i.e. as soon as the perceived figure, which made the correspondence possible, is altered, not only does the correspondence vanish, but also the co-ordination between length and density." (Piaget, 1952, pp. 80-81)

At the third stage, a child assumes the two sets to be equivalent no matter how the configuration of the elements in the sets is changed. Lasting equivalence has been achieved. One-to-one correspondence is now freed from perceptual limitations, it has become really quantifying, expressing numerical equality. Children who show lasting equivalence in their numerical judgements of sets, have acquired the concept of cardinal number.

For the purpose of the present study, performance on a modified version of Piagetian tasks of number conservation was used as an indication of the level of the child's development of the concept of cardinal number.

It is being proposed here that children who have acquired understanding of the cardinal aspect of number, i.e. Conservers, will be best able to generate referential events in order to perform the addition operation.

There has been controversy regarding the relative merits of using judgements alone, versus judgements plus explanations in evaluation of conservation concepts. Brainerd (1973a, 1974) argues that judgements are more appropriate than explanations as a basis for inferences about children's cognitive structures in Piagetian tasks, since explanations inevitably result in more Type II errors. Reese and Schack (1974) maintain that explanations may yield more Type II errors, but judgements alone yield more Type I errors, and a choice between them depends upon which type of error is considered more serious. For the purpose of the present study it was considered more important to minimize Type II error, i.e. increase the likelihood that only children who really have a firm understanding of cardinal number will be classified as Conservers. Therefore judgements plus explanations were required of the children.

2. Development of Mental Symbols

Piaget (1962) maintains that imagery is derived from imitation.

During the last stage of the sensori-motor period schemata become sufficiently independent of immediate perception and action to give rise to mental combinations. Actions become interiorized and deferred imitation appears, i.e.

"... imitation is no longer dependent on the actual action, and the child becomes capable of imitating internally a series of models in the form of images or suggestions of actions. Imitation thus begins to reach the level of representation."
(Piaget, 1962, p. 62)

His position is that the whole symbolic function is derived from action, language symbols as well as images. Children as young as 18 months perform acts of delayed imitation, i.e. sequences of actions imitating a model, at a point in time when the model is no longer present. Piaget's explanation for this is that the child must have formed a mental symbol of the action sequence and then based the deferred imitation behaviour on this symbol. Having a mental event available which represents the model's action enables the child to reproduce the behaviour at a later time. This mental symbol may or may not be conscious and may or may not involve visual imagery. It is unlikely that the language capacity of these very young children is sufficiently complex to provide an internal verbal description sufficient to be the source of delayed imitation. It is suggested that the child's mental symbols are at least in part visual images. As well as enabling deferred imitation behaviour to take place, the availability of a

mental symbol is necessary for the child to have a mature object concept. The sensori-motor child represents things by acting like them; mental symbols evolve out of this imitation. As the child becomes older he performs these imitations internally, and these abbreviated body movements constitute the mental symbol. In referring to the symbol as consisting of internal imitation, Piaget is using the term imitation in a broad sense to account for visual imagery. Mental symbols therefore initially involve the child's actions.

Gibson (1969) has criticized Piaget's view that the origin of imagery is in actions. According to Gibson, representation is derived from the discovery of distinctive features and pattern invariants. She agrees with Piaget that exploratory activity is important in detecting properties of objects, but not that this implies the activity itself is copied in an image.

According to Piaget, cognition has two aspects: the operative and the figurative (Piaget, 1964).

(1) The operative aspect is the more dynamic, it refers to actions, the result of which is some transformation or change of reality. The actions may be either internal or overt.

(2) The figurative aspect refers to actions by which the child produces a copy of reality; the focus is on the states of reality rather than its transformations. There are three subdivisions under the figurative aspect:

(i) perception

A system which functions through the senses necessitates an object be present. Through perception the child achieves a copy of things.

(ii) imitation

Through imitation the child reproduces the actions of persons or things. Imitation does involve an action, however it only produces a copy of reality, it does not transform reality.

(iii) mental imagery

refers to personal internal events which represent objects or events which are absent.

Imagery may be either static or transformational, and so may function under both aspects of cognition. Piaget's general conclusions from his empirical work on imagery (for which he used methods of verbal report, drawing, and selection of drawings from a collection provided by the experimenter) are as follows. Imagery develops over time. However unlike the evolution of cognitive operations, which displays a sequence of stages, there appears to be only one major turning point in imagery development. From the age of about 1-1/2 years to seven years the child seems to be capable of producing only static images, and even they are far from perfect. He cannot represent transformations, even the simple physical movement of an object. Piaget maintains the deficiency is due to the fact that these children focus on the initial and final states of a situation, and not the intervening event, or transformation. The major turning point corresponds to the beginning of the period of

concrete operations. At this time the child becomes able to produce images which can reproduce and anticipate transformations. His imagery has become less static, he can now imagine the initial and final states and the intervening events:

"...a flexible, anticipatory imagery may be of real aid to operational thought, indeed, may be necessary to it." (Flavell, 1963, p. 358).

The ability to produce and utilize transformational images emerges at around the same time as the child becomes capable of performing concrete operations. Ginsburg and Oppen (1969) interpret Piaget's position as to the relation between the operative and figurative aspects of thought as follows:

"...although images and operations are different sorts of entities, the presence of one assists the functioning of the other ... His ability to form images of this sort does not guarantee that he can conserve number ... Nevertheless, the child who has a correct image of the transformation is certainly ahead of the child who does not. In other words, images are a useful and necessary auxiliary to thought during the concrete operational stage. By providing relatively accurate representations of the world, images assist the process of reasoning although they do not cause it." (Ginsburg and Oppen, 1969, p. 160).

One of the purposes of the present study was to determine the child's ability to utilize imagery instructions to generate referential events at different developmental levels. A Piagetian pretest of number conservation was given to determine the developmental level of the subjects with respect to their concept of cardinal number. On the basis of this pretest the subjects were divided into three groups: Conservers,

Transitionals, Nonconservers. A child who showed consistent evidence across a number of subtasks of having acquired the concept of invariance of number would be expected to at least be entering Piaget's developmental stage of "concrete operations", at which time Piaget maintains he should be starting to show ability to generate dynamic imagery.

There is recent work which suggests Piaget's view of the development of dynamic imagery may be inadequate. Bower (1974) has found that young infants may have cognitive capacities beyond those Piaget would suggest. For example, he found evidence that infants can anticipate tactile consequences of motoric behaviour. Bryant (1974) offers evidence that young children may have the ability to make logical inferences about objects seen at different times. The experiments by Bower and Bryant suggest that infants and young children can utilize dynamic anticipatory imagery, contrary to Piaget's position.

The present study was not concerned with testing implications of Piaget's stage theory as such. A Piagetian type test of number conservation was being used here as a test of the concept of cardinal number. It was suggested in the introduction to this paper that some form of elaboration generated following imagery instructions may aid simple arithmetic problem solving, after the child has acquired the concept of cardinal number. Presumably symbolic referents for number may not be evoked in or generated by the child until associations between sets

of items and appropriate number symbols (i.e. the cardinal use of number) have been acquired. On this basis it is hypothesized that children classified as conservers (i.e. children who show evidence that they have acquired the concept of the cardinal aspect of number) should best be able to utilize the imagery instructions. The performance of the non-conserving and transitional children should be facilitated less by imagery instructions. For a different reason, the same prediction would be derived from Piaget's theory, i.e. since only the Conservers are considered to be in the stage of concrete operations, only the Conservers, and perhaps to some extent the Transitionals, should show evidence of being able to generate dynamic imagery.

CHAPTER III

BASIC ISSUES AND HYPOTHESES IN THE PRESENT STUDY

Arithmetic operations are derived from operations on sets of objects. At first children are only able to count and add numbers when visual referents (objects or pictures) are available. Later they are able to perform abstract operations with numbers. One of the aims of the present study was to attempt to determine how this transition takes place. It is suggested here that elaboration may play an important role during this transitional period when a child no longer needs totally to rely on the presence of objects or pictures as referents, but has difficulty adding number symbols. Knowledge of processes which are involved in the development of number concepts, and the ability to perform addition, is important for designing optimal teaching approaches for arithmetic.

There is considerable evidence from paired-associate learning that children are able to use elaboration of some kind in an associative learning task (Paivio, 1971; Rohwer, 1973; Yuille, 1974). In this study it is assumed that under certain circumstances elaboration can be used by children. Elaboration can be activated in various ways (Rohwer, 1973). In paired associate learning it has been established that various types of elaborative prompts, including pictorial prompts, imagery instructions, and verbal prompts may facilitate formation of associations.

It is suggested here that these findings from associative learning may extend to other tasks, such as solving addition word problems.

At some stage a child may "automatically" utilize elaboration to help him perform an arithmetic operation. However, young children who may not yet spontaneously produce their own referential events, may if instructed to do so. It is hypothesized here that children who understand the cardinal aspect of number, i.e. conserve number, will best be able to utilize the imagery instructions. In the present study performance of kindergarten children at three Developmental Levels of cardinal number concept compared on arithmetic word problems, under three Prompt Conditions: Concrete plus Verbal, Imaginal plus Verbal, Verbal Only.

One of the most consistently powerful variables which has come out of the research on noun pair learning with children is the better performance when interacting stimuli are provided as compared with side-by-side pairs (Lippman and Shanahan, 1973; Paivio, 1969; Reese, 1970a; Rohwer, 1967). Therefore it was decided to include roughly analogous conditions to see if this finding would extend to the situation of arithmetic problems. This was done by manipulating the verbal content of the arithmetic word problems. Two forms of a Described Relation were used:

- (1) Dynamic Described Relation, e.g. "7 rabbits hop to
2 rabbits here".

- (2) Static Described Relation, e.g. "7 rabbits wait there,
2 rabbits here".

The Dynamic Described Relation was intended to be analogous to the interacting stimuli used in paired associate studies, and the Static Described Relation analogous to side-by-side stimuli.

The three Prompt Conditions and two types of Described Relation result in six experimental conditions which were replicated at each of three Developmental Levels. The Prompt Conditions varied in likelihood of evoking elaboration, following Rohwer (1973). They included a maximally explicit prompt (Concrete plus Verbal), in which a referential event was actually provided, and two levels of augmented explicit prompts (Imaginal plus Verbal, and Verbal Only).

<u>Described Relation</u>	<u>Prompt Condition</u>		
	Concrete plus Verbal	Imaginal plus Verbal	Verbal Only
Dynamic	Objects moved to depict event in problem Dynamic described and enacted relation of referents	Instructions to imagine referents Dynamic described relation of referents	Dynamic described relation of referents
Static	Objects side-by-side Static described and enacted relation of referents	Instructions to imagine referents Static described relation of referents	Static described relation of referents

Based on Rohwer's (1973) viewpoint that the more likely that a given set of stimuli will evoke elaboration, the better the performance, one would make the following predictions: subjects will perform best (in terms of

number of correct responses) under the Concrete plus Verbal Prompt Condition, next best under Imaginal plus Verbal, and least well under the Verbal Only condition. The same prediction for the Prompt Condition variable can be derived from Paivio's (1971) two-process theory. The Concrete plus Verbal and Imaginal plus Verbal conditions would be viewed as involving two processes: visual and verbal. In the Concrete plus Verbal condition the visual referents were actually provided, plus the verbal input was also present. In the Imaginal plus Verbal condition, two encoding systems may be involved, but the subject must provide his own visual referents (via imagery), so that the visual system is involved to a lesser degree than in the first condition described. In the Verbal Only condition, only one potential system is activated (unless subjects spontaneously use imagery), therefore one would expect poorest performance under this condition.

The above predictions need to be modified according to developmental level. It was hypothesized above that children who have a cardinal concept of number will best be able to utilize imagery instructions to generate referential events. However it was also pointed out earlier that there is evidence (e.g. Gelman, 1972) that even very young children may recognize the invariance of small numbers, although not larger ones. The conservation test in this study used six elements as the smallest set so it is possible that some subjects who did not show evidence of number conservation with tasks involving sets of six elements

may still have a concept of invariance for smaller number.

Addition Word Problems

Hypotheses

Developmental Level and Prompt Condition

In terms of the number of correct responses:

(1) Since the Conservation Pretest gave an indication of the child's level of understanding of the cardinal use of number, overall the Conservers should perform best, Transitionals next, followed by Nonconservers.

(2) Within Each Developmental Level

When performance under Concrete plus Verbal Prompt condition is compared with the two conditions lacking concrete object referents (i.e. Imaginal plus Verbal, and Verbal Only):

i. Based on the fact that arithmetic operations are derived from operations on sets of objects, the Concrete plus Verbal Prompt condition will best facilitate performance on addition problems, for all three developmental levels.

ii. The less well developed the child's number concept, the more important the presence of concrete object referents may be: i.e. the difference between performance on the Concrete plus Verbal condition as compared with the other two (Imaginal plus Verbal and Verbal Only) will be greatest for the Nonconservers, nearly as great for the Transitionals, and not as great for the Conservers.

iii. When the Imaginal plus Verbal condition is compared to the Verbal Only, imagery instructions will facilitate performance most for the Conservers, somewhat for Transitionals, and very little if at all for Nonconservers. That is, there will be a significant difference between the Imaginal plus Verbal condition and Verbal Only, for the Conservers; but very little, if any, difference between these two experimental conditions for the Transitionals and Nonconservers. Confirmation of these hypotheses would lend support to the position presented here that the child's generating a referential event may be important in developing the capability to operate with numbers in the absence of objects or pictures, given that the child has acquired the concept of the cardinal aspect of number.

Described Relation: Dynamic vs. Static

There is no single basis for predicting which level of this variable, Dynamic or Static, should result in more correct responses. Hypotheses as to how the Described Relation variable may operate in this task of addition word problems, can be derived from paired associate or mathematical learning studies.

(1) In favour of Dynamic

i. Given the analogy suggested earlier between integrated stimuli and the Dynamic Described Relation, one may predict, based on findings

from paired associate learning, that a dynamic verbal context should facilitate performance more than a static verbal context.

ii. From the definition of addition as the union of sets, a Dynamic Described Relation, in which the union is described, may facilitate performance of addition more than a Static Described Relation.

(2) In favour of Static

If subjects are using a counting algorithm to perform addition (e.g. Suppes and Groen, 1967), the Static Described Relation may facilitate the operation relative to Dynamic. This prediction is based on Wang, Resnick and Boozer's (1971) finding that small unordered sets were easier to count than larger unordered sets.

Whether Dynamic or Static Described Relation is the more effective in facilitating performance may depend on whether the transformation or counting aspect of performing an addition operation is the more important under these conditions for these children. If the transformation aspect of addition is more important than ease of counting, the Dynamic Described Relation may result in better performance. If ease of counting is more important, the Static Described Relation may be better. The importance of each component may differ depending on the Prompt Condition and the Developmental Level involved. The effect of this variable will be examined within each Developmental Level and Prompt Condition.

Within Each Developmental Level

From paired associate studies it appears that young children are

able to utilize interacting stimuli provided by the experimenter, but have difficulty generating their own from static stimuli. Piaget (1952) maintains that children who have reached the concrete operational stage of development are able to generate their own transformational imagery. Subjects who show evidence of number conservation in the pretest may perform equally well in the Static and Dynamic conditions. That is, the performance of Transitional and Nonconserving subjects may be affected more than Conservers by the Described Relation.

Incidental Recall of Nouns in the Addition Word Problems

The predicted outcome for the incidental recall of nouns for the three Prompt Conditions was:

- (1) more nouns should be recalled under Concrete plus Verbal than Imaginal plus Verbal and Verbal Only;
- (2) more nouns should be recalled under Imaginal plus Verbal than under Verbal Only.

These predictions may be derived from Rohwer's (1973) position on the facilitative effects of elaboration on recall in paired associate tasks, with intentional learning. As outlined earlier, the Prompt Conditions differ in the degree to which they are likely to evoke elaboration. Alternatively, Paivio's (1971) dual coding hypothesis would also suggest these predictions, based on differential involvement of visual and verbal storage codes under the three Prompt Conditions. Findings

by Goldberg (1974) on superior incidental recall of illustrated versus nonillustrated materials, with grade 5 subjects, would support Hypothesis (1); and by Yarmey and Bowen (1972) of superior incidental recall of picture pairs and noun pairs under imagery instructions as compared with intentional recall under nonimagery instructions, with children aged 8 to 13 years, would support Hypothesis (2).

These hypotheses were to be tested within Developmental Levels. Hypothesis (1) should be confirmed for all three Developmental Levels. Since it was earlier predicted that Conservers should best be able to utilize the imagery instructions, noun recall may only differ significantly for Hypothesis (2), within the Conservers.

Counting Models

One purpose of the present study was to look at possible algorithms children may use, in the form of counting, to solve addition word problems. Suppes and Groen (1967) have proposed an hypothetical "counter" on which two operations are possible: setting the value of the counter to a certain value (while clearing the previous value); and, adding a number to the current value by successively incrementing by one. For addition problems of the form $m + n$

Model 1: The counter is set to 0, m is added and then n .

Model 2: The counter is set to m , n is then added.

Model 5: The counter is set to the maximum of m and n . The minimum is then added.

For example for the addition problem $2 + 7 = \underline{\quad}$:

Model 1: The counter is set to 0, 2 is added and then 7.

Model 2: The counter is set to 2, 7 is then added.

Model 5: The counter is set to 7, 2 is then added.

The word problems in the present study were presented aurally, rather than the visual presentation of an addition equation as in the Suppes and Groen study. For this reason, i.e. aural input, it was considered that temporal order of information may be important. No support for the use of Models 2, 3 and 4 was found in the Suppes and Groen study. However, Model 2 reflects temporal order (the counter is set to the first integer, then the second is incremented by ones), therefore due to the important difference in presentation mode between the two studies, this

model is included here.

For Model 1 to be used, no cardinal concept of number is required. The subject starts at 0, and then increments by one until the total of $m + n$ is reached. For Model 2 and Model 5 to be used, however, it appears that the child may need to have a cardinal concept of number, so that the counter can initially be set. With Model 2 the subject processes the information in temporal sequence. Model 5 requires the most internal manipulation of the input information out of the three models, since the child must first judge which is the larger integer, and then, if it is n , the order of processing will be opposite to the temporal order.

Hypotheses

Prompt Condition

- (1) In the Concrete plus Verbal Prompt Condition, where object referents are present, processing of the aural input may be less crucial for problem solution than in the other Prompt Conditions, because a second channel, visual input, is also providing the information necessary for performing addition. It is hypothesized here that when objects are present, Model 1 may be most frequently used by all developmental levels.
- (2) In the Imaginal plus Verbal and Verbal Only Prompt Condition, where objects are not present, it appears that Model 5 would be the most efficient process of the three being tested here.

Developmental Level

It was suggested earlier that use of Model 2 and Model 5 may require that the child have a concept of cardinal number. It is hypothesized here that there may be a developmental trend in which children with limited or no concept of cardinal number (i.e. Nonconservers) may only be able to use Model 1; ability to utilize Model 2 may follow as children are starting to develop a better ground for use of cardinal number; and use of Model 5 may not appear until cardinal number is a fairly well developed concept. Beyond this, counting models may be replaced by a fast-access long term memory system such as proposed by Groen and Parkman (1972).

Summary

The main purpose of the present study was:

- (1) to examine the effect of Prompt Condition (Concrete plus Verbal, Imaginal plus Verbal, Verbal Only) on addition word problem performance in kindergarten children at three levels of development of cardinal number concept;
- (2) to determine whether or not children at these Developmental Levels are able to utilize imagery instructions to facilitate performance of the addition operation in the absence of concrete referents;

- (3) to examine the effect on performance of the verbal context comprising a Dynamic Described Relation as opposed to a Static Described Relation;
- (4) to examine some possible algorithms children at each Developmental Level and Prompt Condition may be using to solve the addition word problems;
- (5) to determine possible directions for future research concerned with arithmetic curriculum and teacher strategies.

CHAPTER IV

METHOD

A. Design

A 3 x 3 x 2 x 2 x 2 factorial design was used. The independent variables were: Developmental Level (Conservers, Transitionals, Nonconservers); Prompt Condition (Concrete plus Verbal, Imaginal plus Verbal, Verbal Only); Described Relation (Dynamic, Static); Sex; and Trials, with repeated measures on Trials. A Latin Square design was used with 18 orders of presentation of the addition word problems; the same order over Trials 1 and 2. The Orders were represented for all variables except Sex. The sequence of the 12 test items remained the same. Order changed as follows: for Order 1, item 1 was first and item 12 last; for Order 2, item 2 was first and item 1 last, and so on up to Order 12. Orders 13 to 18 were a random sample of the 12 basic orders.

Boys and Girls within each Developmental Level were assigned randomly to one of the six experimental conditions formed by three levels of Prompt Condition and two levels of Described Relation. The verbal units which determined the Dynamic and Static Described Relation were the verb and the locative:

	<u>verb</u>	<u>locative</u>	
Dynamic:	motion	preposition "to"	e.g. "5 dogs <u>run to</u> 2 dogs here"
Static:	no motion	adverbial use of "there"	e.g. "5 dogs <u>wait there</u> 2 dogs here"

All Prompt Conditions received the verbal input, either Dynamic or Static. In the Concrete plus Verbal Prompt Condition, toy animals were present in addition to the verbal problems, and the Described Relation was paralleled by movement or lack of movement of the objects. The Imaginal plus Verbal and Verbal Only Prompt Conditions had no object referents present; the Dynamic and Static Described Relations were only presented orally.

Concrete plus Verbal:

The subject was instructed to watch the animals. Before each test item, the subject was instructed to "Now watch this". While presenting the problem orally, the experimenter placed the two sets of animal object referents on the table about 12 in. apart. In the Dynamic Described Relation condition, the experimenter moved the sets of animals together while presenting the problem. The animals were moved into two parallel rows, not into one long line. In the Static Described Relation condition, the sets of animals were not moved.

Imaginal plus Verbal:

The subject was instructed to imagine the animals. Before each test item the subject was instructed "Now imagine this". The rationale for using the word "imagine" rather than more specific instructions such as "picture in you head" was as follows. The aim was to direct the child toward generating a referential event. This

event would not necessarily involve a "mental picture" of the animals in each problem. The word "imagine" was assumed familiar to kindergarten children through exposure to children's television programs (e.g., Sesame Street, Mr. Rogers).

Verbal Only

The subject was instructed to listen about the animals. Before each test item the subject was instructed "Now listen to this."

Eight orthogonal planned comparisons were set up to test the following experimental hypotheses for each of the dependent variables:

Developmental Level

- (1) Conservers (C) will perform better than the Transitionals (T) and Nonconservers (NC).

$$\psi_1 = \bar{X}_C - (\bar{X}_T + \bar{X}_{NC}) / 2$$

- (2) Transitionals will perform better than the Nonconservers.

$$\psi_2 = \bar{X}_T - \bar{X}_{NC}$$

Prompt Condition

- (3), (4), (5) For each Developmental Level, subjects under Concrete plus Verbal (C + V) Prompt Condition will perform better than under Imaginal plus Verbal (I + V) and Verbal Only (V) combined.

$$\psi_3 = \bar{X}_{C(C+V)} - (\bar{X}_{C(I+V)} + \bar{X}_{C(V)}) / 2$$

$$\psi_4 = \bar{X}_{T(C+V)} - (\bar{X}_{T(I+V)} + \bar{X}_{T(V)}) / 2$$

$$\psi_5 = \bar{X}_{NC(C+V)} - (\bar{X}_{NC(I+V)} + \bar{X}_{NC(V)}) / 2$$

(6), (7), (8) Only the Conservers will perform better under the Imaginal plus Verbal Prompt Condition as compared with the Verbal Only. There will be no difference in performance between these two Prompt Conditions for the Transitionals and Nonconservers.

$$\psi_6 = \bar{X}_{C(I+V)} - \bar{X}_{C(V)}$$

$$\psi_7 = \bar{X}_{T(I+V)} - \bar{X}_{T(V)}$$

$$\psi_8 = \bar{X}_{NC(I+V)} - \bar{X}_{NC(V)}$$

The remaining experimental effects were tested using the appropriate univariate analysis of variance, followed by post hoc comparisons.

B. Materials

(1) Conservation Pretest:

Four subtasks, using two types of materials and two arrangements, were presented to each subject. The materials were two sets of buttons, 6 red and 6 yellow, 3/4 in. (1.9 cm.) in diameter; and two sets of 7 nuts (almonds in the shell), approximately 1 1/4 in. (3.2 cm.) in length. The arrangements were: Task 1: two sets of buttons in rows; Task 2: two sets of buttons in circles; Task 3: two sets of nuts in rows; Task 4: two sets of nuts in circles. The order of the tasks was the same for all subjects.

(2) Experimental Test:

General Warmup: 3 addition problems using the integers 1 and 2 were used in the general warmup task (i.e. $1+1$; $2+1$; $2+2$).

Addition Word Problems

The addition word problems used are presented in Table 1.

Specific Warmup: 2 addition word problems were used in this task.

Test Items: 12 addition word problems on two trials.

The addition word problems used for this study differ from the usual arithmetic word problems in that the instructions regarding the operation to be performed were not included in each problem. In order to simplify the task sufficiently for kindergarten children, it was found necessary, based on pilot testing, to include only the essential information for executing the operation in the problem itself. The instructions as to the nature of the task, and the operation to be performed, i.e. addition, were presented before the test items, in the form of general and specific warmup tasks.

Fourteen pairs of parallel word problems were used. One problem of each pair contained a Dynamic Described Relation, the other a Static Described Relation. The verbs were intransitive, and differed only in the presence or absence of motion, the location referent was "to" for Dynamic and "there" for Static. The word problems used are presented in Table 1. Problems A and B in each set of 14 items in

Table 1

Addition Word Problems

	<u>Dynamic</u>		<u>Static</u>
*A.	2 squirrels hop to 3 squirrels here	*A.	2 squirrels sit there 3 squirrels here
*B.	4 geese swim to 2 geese here	*B.	4 geese sleep there 2 geese here

(* Specific Warmup Items)

1.	2 elephants walk to 4 elephants here	1.	2 elephants stand there 4 elephants here
2.	5 goats walk to 3 goats here	2.	5 goats lie there 3 goats here
3.	3 ducks swim to 4 ducks here	3.	3 ducks sit there 4 ducks here
4.	6 cows walk to 3 cows here	4.	6 cows stand there 3 cows here
5.	3 bears run to 2 bears here	5.	3 bears sleep there 2 bears here
6.	4 pigs walk to 6 pigs here	6.	4 pigs lie there 6 pigs here
7.	7 rabbits hop to 2 rabbits here	7.	7 rabbits wait there 2 rabbits here
8.	2 horses gallop to 8 horses here	8.	2 horses stand there 8 horses here
9.	5 dogs run to 2 dogs here	9.	5 dogs wait there 2 dogs here
10.	4 chickens run to 5 chickens here	10.	4 chickens sit there 5 chickens here
11.	2 foxes run to 6 foxes here	11.	2 foxes wait there 6 foxes here
12.	7 sheep walk to 3 sheep here	12.	7 sheep stand there 3 sheep here

Table 1 are the Specific Warmup items. Each subject received 12 test items on two trials, with the same order on Trial 1 and Trial 2. The positive integers from 2 to 8 were used, with the following restrictions:

For any problem $m + n = a$, $m + n \leq 10$ and $m \neq n$

With respect to the sequence of the 12 experimental items, the following restrictions were applied: for any two consecutive problems i and j , $m_i \neq m_j$, $n_i \neq n_j$, $a_i \neq a_j$.

A foot operated Lafayette timer model number 20225 ADW, which measured response latency to .01 seconds, was used. As defined here, latency was the time elapsed from the end of the word problem presentation until the subject gave a response.

Objects: For the Concrete plus Verbal Prompt Condition, three dimensional coloured wooden animals were presented. The number of animals representing each integer in each arithmetic word problem, was glued in a row onto a strip of green cardboard. The cardboard strips were $1\frac{1}{2}$ in. (3.9 cm.) wide, and ranged in length from $2\frac{1}{4}$ in. (5.7 cm.) to $15\frac{1}{2}$ in. (39.5 cm.) depending on the number of animals in the set. The animals were pasted on their side, not standing. They ranged in height from $10/16$ in. (1.6 cm.) to $1\frac{3}{16}$ in. (3 cm.). All animals in the two sets in a given problem were identical, (e.g. all the ducks were identical, all the cows, etc.).

Scoring Procedure for Conservation Pretest

The Conservation Pretest had four tasks, each of which was scored on both judgement and explanation in Phase II.

Judgement: the subject's response was scored as conserving, if he said there was as many objects in each set; and nonconserving if he said they were not the same (i.e. one set has more).

Explanation: the subject's response was scored as conserving if he indicated he comprehended one of more of the following principles:

Invariant quantity: e.g. "Each row/circle has the same number."; "You did not add or subtract anything."

Reversibility: e.g. "If you put them back the way they were before, they would be the same."

Compensation: e.g. "This row is shorter, but the spaces between the buttons/nuts are smaller."

The subject's response was scored as nonconserving if his answer did not indicate understanding of one of the above principles: e.g. no explanation; a description of part of the procedure, e.g. "You made this row/circle bigger"; a perceptual explanation, e.g. "They look the same."

Each subject was classified as a Conserver, Transitional, or Nonconserver. The criteria for the classification were as follows:

Conserver: conserving judgements and explanations across the four tasks.

Transitional: some conserving and some nonconserving judgements or explanations; the subject indicated uncertainty about judgements or explanations by changing his mind on some tasks.

Nonconservers: nonconserving judgements and explanations across the four tasks.

These behavioural categories were considered to reflect Piaget's (1952) description of stages in number development :

Stage 1 : Global Comparisons: Children who only make judgements in terms of what was perceived.

Stage 2 : Intuitive Judgements : Children who realize the inadequacies of purely perceptual judgements, but are not able to apply operations in a consistent fashion.

Stage 3 : Concrete Operations : Children who apply operations consistently/. Details of scoring problems are presented in Appendix B.

C. Subjects

One hundred and eight kindergarten children (54 boys and 54 girls) from seven Elementary schools in the North Vancouver School District participated as subjects in the present experiment. The testing was carried out in the Spring. The schools serve a middle to upper middle class socio-economic residential area. In order to arrive at three developmental levels with equal numbers of subjects of each sex at each level, it was necessary to administer the Conservation Pretest to 166 children . The mean age of the experimental

subjects was 70.8 months, with a range of 12 months (65.0 to 77.0 months).

Following the Conservation Pretest, five children were excluded from the study for the following reasons: two due to procedural errors on the part of the experimenter; one because his knowledge of English was not adequate to perform the test; one because the experimenter was unable to elicit sufficient responses from the subject; one who failed to recognize the initial equality of the two sets in Phase I, in two of the four tasks, after help from the experimenter (this occurred with only one subject out of 166 total). This left a pool of 161 subjects for the study. Subject attrition during the experiment is described in Appendix C.

The percent of children classified as Conservers, Transitionals, and Nonconservers, on the Conservation Pretest, is presented in Table 2. From examination of Table 2, it appears that considerably more children were classified as Nonconservers, as compared to the other Developmental Levels: Conservers, 26.1%; Transitionals, 29.2%; Nonconservers, 44.7%. There were apparently no marked sex differences in the distribution across the Developmental Levels.

D. Procedure

Each subject was tested individually in a room at the school which he regular attended. The Conservation Pretest was administered on a different day (Day 1) preceding the experimental test (Day 2).

Table 2

Percentage Breakdown of Kindergarten Children by Developmental
Level and Sex

	Conservers	Transitionals	Nonconservers	
Boys	11.8	14.9	23.0	49.7
Girls	14.3	14.3	21.7	50.3
	26.1	29.2	44.7	100.0
				N = 161

Typically the experimental test followed the pretest within a week. The longest interval between the two tests was two weeks. For both sessions the subject sat opposite the experimenter at a table. Following the Conservation Pretest, each subject was classified within one of three Developmental Levels (Conservers, Transitionals, Nonconservers). Each subject was then assigned randomly to one of six experimental conditions, formed by three levels of Prompt Condition (Concrete plus Verbal, Imaginal plus Verbal, Verbal Only) and two levels of Described Relation (Dynamic, Static). Since children classified within each Developmental Level were assigned randomly to experimental conditions, variations in interval between the test days was randomly distributed across experimental conditions.

(1) Conservation Pretest:

The procedure was the same for each task. Phase I: the experimenter placed the objects on the table in one-to-one correspondence, and then asked the subject "Are there as many ... (red buttons as yellow buttons / nuts in this row as this row / nuts in the inside circle as the outside circle) ... or are there more (buttons / nuts) in one (row / circle) ?" If the subject judged the sets to be equal, the experimenter moved on to Phase II. If the subject judged the sets to be unequal, the experimenter attempted to establish the equality of the sets with the subject. Phase II commenced only after the subject agreed that the initial sets in

one-to-one correspondence were equal. Phase II: the experimenter moved each set so that, in the case of rows, one was lengthened, one was shortened. In the case of circles, what had in Phase I been two concentric circles, was changed so that the circles were side by side, one bigger than the other. The subject was then asked to judge the equality of the two new sets, with the same question as used in Phase I. The subject was then asked "Why?" he thought the sets were not equal or unequal. The experimenter recorded verbatim the subject's judgements and explanations on each task. The procedure and questions were adapted from Goldschmid and Bentler (1968). The complete instructions are presented in Appendix D.

(2) Experimental Test:

The timer was concealed to the left of the experimenter. All items were presented orally, as were the subject's responses. Corrective feedback was provided following each response; e.g. following a correct response, "Very good, X was the right answer"; e.g. following an incorrect response, "You were very close, actually it was X". The experimenter recorded the subject's response and the latency for each item. Every subject received the same general instructions which incorporated a General Warmup Task to induce a set for addition. This was followed by instructions specific to each of the three Prompt Conditions, including two practice items. The experimental items were not presented

until the subject had both practice items correct. The instructions are presented in Appendix E.

(3) Incidental Recall of the Nouns in the Addition Word Problems:

Following Trial 2 of the addition word problems, the subject was asked to recall the animals which had been in the problems. The subject was given no prior indication that this test would be included. The objects in the Concrete plus Verbal Prompt Condition were no longer in view.

CHAPTER V

RESULTS

Order of Item Presentation

A two-way analysis of variance was performed separately on the number of correct responses and latencies, on the addition word problems, to examine the effect of 18 Orders over Trials 1 and 2. The mean number of correct responses, and mean latencies, under each Order on each Trial are presented in Appendices F - 1 and F - 2. The results of the analyses of variance are presented in Summary Table form in Appendices F - 3 and F - 4. Inspection of the analysis of variance summary tables reveals that the main effect of Order was not significant, nor was the Order x Trials interaction, for either of the dependent variables. These effects were tested for significance¹ at $p < .01$.

Since the effects of Order were not significant, this variable was not given further consideration for any of the analyses.

A. Performance on the Addition Word Problems

The three dependent variables: number of correct responses, latency, and incidental noun recall, were examined separately.

Orthogonal planned comparisons were used to test the effects of Developmental Level and Prompt Condition for each dependent variable. The 8 planned comparisons were tested at a conservative overall Type I error rate of $p < .08$. Each comparison was tested at $p < .08/8$, i.e.

¹ Hereafter significance implies statistical significance.

$p < .01$ level of significance. The remaining experimental effects were tested using the appropriate univariate analysis of variance, followed by post hoc comparisons. Although this was a fully crossed design, the nature of the experimental questions suggested a simple effects analysis would be the more appropriate approach. Therefore, effects of the experimental factors were examined nested within Developmental Level, following Marascuilo and Levin (1970), and Winer (1962, p. 174). A summary of the results of the planned comparisons for each dependent variable is presented in Table 3.

1. Number of Correct Responses

The mean number of correct responses as a function of Developmental Level and experimental conditions, are presented in Table 4. Ψ_{11} was significant, confirming the hypothesis that Conservers would perform better than the Transitionals and Nonconservers combined, $F(1, 72) = 10.77$, with means of 8.25 and 6.55 respectively. Ψ_2 was not significant, therefore the hypothesis that Transitionals would differ from Nonconservers was not confirmed. Ψ_3 , Ψ_4 , and Ψ_5 were significant, confirming the hypothesis that within each Developmental Level, subjects would perform best under the Concrete plus Verbal Prompt Condition, as compared with the Imaginal plus Verbal and Verbal Only conditions combined. For the Conservers, $F(1, 72) = 14.65$, with means of 10.54 and 7.10; Transitionals $F(1, 72) = 32.31$, with means of 10.13 and 5.02; Nonconservers, $F(1, 72) = 45.58$, with means of 10.42

Table 3

F¹ Ratios for Orthogonal Planned
Comparisons for each Dependent Variable

² Source	<u>Dependent Variable</u>		
	Number of Correct Responses	Latencies	Incidental Noun Recall
$\psi_1 = \bar{X}_C - (\bar{X}_T + \bar{X}_{NC}) / 2$	10.77*	< 1	< 1
$\psi_2 = \bar{X}_T - \bar{X}_{NC}$	< 1	< 1	< 1
$\psi_3 = \bar{X}_{C(C+V)} - (\bar{X}_{C(I+V)} + \bar{X}_{C(V)}) / 2$	14.65*	< 1	3.12
$\psi_4 = \bar{X}_{T(C+V)} - (\bar{X}_{T(I+V)} + \bar{X}_{T(V)}) / 2$	32.31*	< 1	6.29*
$\psi_5 = \bar{X}_{NC(C+V)} - (\bar{X}_{NC(I+V)} + \bar{X}_{NC(V)}) / 2$	45.58*	< 1	10.18*
$\psi_6 = \bar{X}_{C(I+V)} - \bar{X}_{C(V)}$	2.71	< 1	< 1
$\psi_7 = \bar{X}_{T(I+V)} - \bar{X}_{T(V)}$	1.98	2.85	< 1
$\psi_8 = \bar{X}_{NC(I+V)} - \bar{X}_{NC(V)}$	1.36	< 1	1.40

* $p < .01$

¹For F ratios: $df = 1/72$

²Abbreviations: C Conservers, T Transitionals, NC Nonconservers;
(C+V) Concrete plus Verbal, (I+V) Imaginal plus Verbal, (V) Verbal Only

Table 4 Mean Number of Correct Responses as a Function of Developmental Level and Experimental Conditions

		<u>Prompt Condition</u>					
		<u>Concrete plus Verbal</u>		<u>Imaginal plus Verbal</u>		<u>Verbal Only</u>	
		<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>
<u>Conservers</u>							
Boys		10.17	10.83	8.50	8.83	7.33	5.50
Girls		11.33	9.83	7.17	7.33	5.50	6.67
		10.75	10.33	7.83	8.08	6.42	6.08
		10.54		7.96		6.25	8.25
		*(1.41)		(2.73)		(3.72)	
<u>Transitionals</u>							
Boys		11.33	10.83	2.17	1.67	2.83	7.00
Girls		9.33	9.00	5.83	7.50	3.67	9.50
		10.33	9.92	4.00	4.58	3.25	8.25
		10.13		4.29		5.75	6.72
		(1.51)		(3.49)		(3.72)	
<u>Nonconservers</u>							
Boys		10.00	10.83	2.83	4.17	3.67	1.17
Girls		10.67	10.17	2.17	10.67	4.50	5.67
		10.33	10.50	2.50	7.42	4.08	3.42
		10.42		4.96		3.75	6.38
		(1.31)		(3.79)		(2.66)	

*(...) standard deviations

and 4.35. It was hypothesized that the less well developed the child's number concept, the more important the presence of objects may be. The ω^2 statistic was computed to compare the proportion of variance explained by the comparison between performance in the presence as compared to absence of objects, at each Developmental Level. For Conservers $\hat{\omega}^2 = .12$, indicating that .12 of the total variance was accounted for by the comparison for Conservers. For Transitionals $\hat{\omega}^2 = .26$, for Nonconservers $\hat{\omega}^2 = .36$. These values indicated that the less well developed the children's concept of cardinal number, the larger the difference in performance when objects were present as compared with absent, supporting the hypothesis. $\Psi 6$: for the Conservers, the difference between the Imaginal plus Verbal and Verbal Only Prompt Conditions was in the predicted direction, but did not reach significance, $F(1, 72) = 2.71$, with means of 7.96 and 6.25 respectively. $\Psi 7$ and $\Psi 8$: the prediction of no significant difference between the Imaginal plus Verbal and Verbal Only Prompt Conditions, for the Transitionals and Nonconservers, was supported.

The interaction of Developmental Level and Prompt Condition was not tested. However, it is clear from an examination of the means in Table 4, that there is no difference in performance among the Developmental Levels under the Concrete plus Verbal Prompt Condition, with means of 10.54 (Conservers), 10.13 (Transitionals), and 10.42 (Nonconservers). The significantly better performance of the Conservers as

compared with the other two Developmental Levels combined, as found with planned comparison (1), must be located in the Imaginal plus Verbal and Verbal Only conditions. It appears that the difference between the means for the Conservers and the other two Developmental Levels may be greater under Imaginal plus verbal, with means of 7.96 and 4.63 respectively, than under Verbal Only, with means of 6.25 and 4.75 respectively. The difference between these means cannot be tested for significance under this nested design. However, the pattern which has emerged here appears to support the hypothesis that Conservers may be better able to utilize imagery instructions than the Transitionals or the Nonconservers.

To test the remaining effects, a $3 \times 3 \times 2 \times 2 \times 2$ analysis of variance was performed on the number of correct responses in the addition word problems. The independent variables were: Developmental Level, Prompt Condition, Described Relation, Sex and Trials, with repeated measures on Trials. Prompt Condition was nested within Developmental Level, and the remaining factors nested within Developmental Level and Prompt Condition. The results of the analysis of variance are presented in Summary Table form in Appendix G. The main effects of Developmental Level and Prompt Condition (nested within Developmental Level) have been accounted for by the planned comparisons. Each of the remaining effects, and the permissible interactions were tested at the $p < .01$ level of significance. The main

effect of Described Relation within Developmental Level and Prompt Condition, was significant, $F(9, 72) = 2.61$ with means as presented in Table 4. All other effects were nonsignificant. The effect of Described Relation was examined separately for each level of the nesting variables, and was significant for two out of the nine Developmental Level and Prompt Condition groups. Transitionals under the Verbal Only Prompt Condition had significantly more correct responses with Static, as compared with Dynamic Described Relation, $F(1, 72) = 11.63$, with means of 8.25 and 3.25 respectively. Nonconservers under the Imaginal plus Verbal Prompt Condition performed significantly better with Static as compared with Dynamic Described Relation, $F(1, 72) = 11.24$ with means of 7.42 and 2.50. There was no significant difference in performance under Dynamic as compared with Static Described Relation for the Conservers under any of the three Prompt Conditions.

2. Latencies

The same analyses performed on the number of correct responses were also performed separately on the latencies. The mean latencies, as a function of Developmental Level and experimental conditions, are presented in Table 5. None of the eight planned comparisons were significant. The results of the analysis of variance are presented in Summary Table form in Appendix H. None of the remaining effects or interactions in the analysis of variance were significant.

Table 5 Mean Latencies as a Function of Developmental Level and Experimental Conditions

		<u>Prompt Condition</u>					
		<u>Concrete plus Verbal</u>		<u>Imaginal plus Verbal</u>		<u>Verbal Only</u>	
		<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>
<u>Conservers</u>							
Boys		5.16	6.91	7.91	6.08	8.94	7.17
Girls		7.81	5.12	8.75	7.34	4.82	6.75
		6.49	6.01	8.33	6.71	6.88	6.96
		6.25		7.52		6.92	6.90
		*(2.12)		(3.10)		(3.35)	
<u>Transitionals</u>							
Boys		6.81	9.06	4.04	7.51	8.72	6.28
Girls		6.31	7.86	5.18	7.19	8.16	8.91
		6.56	8.46	4.61	7.35	8.44	7.59
		7.51		5.98		8.02	7.17
		(2.50)		(2.36)		(2.29)	
<u>Nonconservers</u>							
Boys		7.55	7.56	8.14	8.72	7.62	8.07
Girls		8.38	6.16	8.11	6.50	7.34	6.75
		7.97	6.86	8.12	7.61	7.38	7.41
		7.41		7.87		7.44	7.57
		(1.52)		(4.11)		(2.62)	87

*(...) standard deviations

3. Incidental Recall of Nouns from the Addition Word Problems

The eight orthogonal planned comparisons were also performed on the number of nouns incidentally recalled from the addition word problems. The mean number of nouns recalled, as a function of Developmental Level and experimental conditions, are presented in Table 6.

ψ_1 , was not significant, which indicated no significant difference between the number of nouns recalled by Conservers as compared with Transitionals and Nonconservers combined. ψ_2 was not significant, which indicated no significant difference in performance between Transitionals and Nonconservers. ψ_3 was not significant, i.e. for Conservers there was no significant difference in the number of nouns recalled following the Concrete plus Verbal Prompt Condition, as compared with Imaginal plus Verbal and Verbal Only combined. For the Transitionals and Nonconservers there was a significant difference in noun recall between these conditions: i.e. ψ_4 and ψ_5 were significant, $F(1, 72) = 6.29$, with means of 6.50 and 4.67, and $F(1, 72) = 10.18$, with means of 6.67 and 4.33 respectively.

The hypothesis that subjects would incidentally recall more nouns following the Concrete plus Verbal Prompt Condition than the Imaginal plus Verbal and Verbal Only combined was thus confirmed for Transitionals and Nonconservers. When performance following the Imaginal plus Verbal Prompt Condition was compared with Verbal Only, ψ_6 , ψ_7 , ψ_8 , there was no significant difference for any of the three

Table 6

Mean Number of Nouns Incidentally Recalled as a
Function of Developmental Level and Experimental Conditions

		<u>Prompt Condition</u>					
		<u>Concrete plus Verbal</u>		<u>Imaginal plus Verbal</u>		<u>Verbal Only</u>	
		<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>	<u>Dynamic</u>	<u>Static</u>
<u>Conservers</u>							
Boys		4.33	7.00	3.67	4.00	3.33	5.00
Girls		7.00	5.00	5.00	6.33	4.33	4.67
		5.67	6.00	4.33	5.17	3.83	4.83
		5.83		4.75		4.33	
		*(2.17)		(1.60)		(1.83)	
							4.97
<u>Transitionals</u>							
Boys		5.33	6.67	4.67	5.00	5.33	4.67
Girls		6.33	7.67	4.33	4.67	4.00	4.67
		5.83	7.17	4.50	4.83	4.67	4.67
		6.50		4.67		4.67	
		(2.35)		(1.61)		(2.31)	
							5.28
<u>Nonconservers</u>							
Boys		6.00	5.33	4.00	2.33	4.33	3.67
Girls		8.33	7.00	4.67	4.33	5.33	6.00
		7.17	6.17	4.33	3.33	4.83	4.83
		6.67		3.83		4.83	
		(2.15)		(1.53)		(2.08)	
							5.11

*(...) standard deviations

Developmental Levels.

To examine the remaining effects, a $3 \times 3 \times 2 \times 2$ analysis of variance was performed on the number of nouns incidentally recalled from the addition word problems. The independent variables were: Developmental Level, Prompt Condition, Described Relation, and Sex. Prompt Condition was nested within Developmental Level, and the other two factors nested within Developmental Level and Prompt Condition. The effects were tested at the $p < .01$ level of significance. The results of the analysis of variance are presented in a Summary Table in Appendix I. The main effects of Developmental Level and Prompt Condition have already been accounted for in the planned comparisons. The effects of Described Relation and Sex, and their interaction, were not significant.

B. Counting Models

The possibility that variation in response latencies between items on the addition word problems, may reflect use of different counting models was examined. Three of Suppes and Groen's (1967) counting models were tested on the latencies recorded in performance of the addition word problems. Suppes and Groen proposed that the subject sets an hypothetical counter, then increments by one, until the sum of $m + n$ is reached. x is defined as the number of times the counter is incremented:

Model 1: counter set to 0, $x = m + n$

Model 2: counter set to m , $x = n$

Model 5: counter set to maximum of m, n ; $x = \text{minimum}$

An x value for each model, for each of the 12 addition word problems, was calculated. These x values are presented in Table 7.

The use of these three Models within each Developmental Level and Prompt Condition was tested two ways in the present study:

- (1) by using mean latencies for correct responses across groups of subjects for each item for analysis;
- (2) by using individual subject's latencies for correct and incorrect responses for analysis.

1. Mean Latencies for Groups

It was of interest to compare use of Model 1, Model 2, and Model 5, within each Developmental Level and Prompt Condition, using mean latencies for correct responses only, on the addition word problems. This involved an assumption that correct responses should more precisely reflect accurate use of an algorithm than incorrect responses. The mean latency for each correct item in the addition problems was to be calculated across the 12 subjects within each Developmental Level and Prompt Condition (9 groups). However, under the Imaginal plus Verbal and Verbal Only Prompt Conditions, for some items the number of correct responses was so low that the mean latency for those items, for the group, contained the latencies of only two or three subjects. The mean

Table 7 Integers and x values for the 12 test items
in the Addition Word Problems

Item	Integers $m + n$	Model 1 $x = m + n$	Model 2 $x = n$	Model 5 $x = \min. m, n$
1	$2 + 4$	6	4	2
2	$5 + 3$	8	3	3
3	$3 + 4$	7	4	3
4	$6 + 3$	9	3	3
5	$3 + 2$	5	2	2
6	$4 + 6$	10	6	4
7	$7 + 2$	9	2	2
8	$2 + 8$	10	8	2
9	$5 + 2$	7	2	2
10	$4 + 5$	9	5	4
11	$2 + 6$	8	6	2
12	$7 + 3$	10	3	3

frequency of correct responses by Items as a function of Developmental Level and Prompt Condition is presented in Table 8. Due to the low number of correct responses for some items when object referents were not present, separate analyses for each of the 9 groups was not feasible. To examine the question of the use of the Models in relation to Developmental Level and Prompt Condition, subjects from the Imaginal plus Verbal and Verbal Only conditions were collapsed within each Developmental Level. The use of the Models was therefore examined within Developmental Levels and Prompt Conditions where objects had been present (Concrete plus Verbal) and had not been present (Imaginal plus Verbal and Verbal Only collapsed). The mean latencies for correct responses for each of the 12 items was calculated for these six groups, separately for Trial 1 and Trial 2. The number of subjects included in each mean latency varied across the items within groups, and across Trials, according to the number of correct responses available. There were significant differences in the variances across items for some of the groups. The probability associated with Bartlett's χ^2 test for homogeneity of item variances for each group is presented in Appendix J. It appears that the homoscedascity assumption for regression analysis may not have been met by these data. For this reason the results of these analyses are considered exploratory and tentative.

Table 8

Mean¹ Frequency of Correct Responses by Items
as a Function of Developmental Level and Prompt Condition

	<u>Item</u>											
	1	2	3	4	5	6	7	8	9	10	11	12
<u>Concrete Plus Verbal</u>												
Conservers	12.0	9.5	9.5	9.5	10.5	10.0	10.5	11.5	11.0	11.0	10.0	11.5
Transitionals	10.5	10.5	10.5	11.0	11.0	8.5	9.5	10.0	11.5	9.0	11.0	8.5
Nonconservers	11.0	10.5	10.5	10.5	10.5	10.0	10.0	10.5	11.5	9.0	10.0	11.0
	11.2	10.2	10.2	10.3	10.7	9.5	10.0	10.7	11.3	9.7	10.3	10.3
<u>Imaginal Plus Verbal</u>												
Conservers	8.0	8.5	7.5	8.0	10.5	5.0	8.5	8.5	9.5	7.5	6.5	7.5
Transitionals	5.0	4.5	3.5	4.0	5.5	3.0	5.5	5.0	4.0	4.5	3.5	3.5
Nonconservers	5.5	5.5	3.0	6.5	6.5	2.0	4.5	6.0	6.0	3.0	5.0	5.5
	6.2	6.2	4.7	6.2	7.5	3.3	6.2	6.5	6.5	5.0	5.0	5.5
<u>Verbal Only</u>												
Conservers	6.5	7.0	5.0	6.5	8.0	3.5	8.5	6.0	7.5	5.5	4.5	6.0
Transitionals	4.5	4.5	6.0	4.5	7.5	5.0	7.0	5.5	8.5	5.0	4.5	6.5
Nonconservers	2.5	5.0	3.5	3.5	4.5	2.5	6.5	3.5	4.0	2.0	2.0	5.5
	4.5	5.5	4.8	4.8	6.7	3.7	7.3	5.0	6.7	4.2	3.7	6.0

¹Mean across Trial 1 and Trial 2; Maximum possible mean frequency is 12.

Separate stepwise univariate multiple regression analyses, using the x values for Model 1, Model 2 and Model 5 as predictor variables, and the mean latency for each item as the dependent variable, were performed separately for Trial 1 and Trial 2 on the following 6 groups of subjects: Conservers under Concrete plus Verbal, Conservers under Imaginal plus Verbal and Verbal Only combined; Transitionals under Concrete plus Verbal, Transitionals under Imaginal plus Verbal and Verbal Only combined; Nonconservers under Concrete plus Verbal, Nonconservers under Imaginal plus Verbal and Verbal Only combined. As can be seen from examination of Table 7 (p. 90), where $m > n$, the x values for Models 2 and 5 are the same. Where $m < n$ the sum of the x values for Models 2 and 5 is equal to the x value for Model 1. In order to remove any systematic effect of item type $m > n$, $m < n$, a dummy variable representing item type was included as a predictor variable. The dummy variable was forced out first in the analyses; the remaining predictor variables (Models 1, 2, 5) were left free. The aim of these multiple regression analyses was to see if one single Model of the three would significantly predict the mean latencies for correct responses within each group, for each Trial. For each analysis $N = 12$, the number of items in the addition word problems. The F ratios were tested at $p < .01$.

The results of the stepwise multiple regression analyses are presented in Table 9. From examination of Table 9, it appears that under the Prompt Condition where objects were present, i.e. Concrete

Table 9 Results of Regression Analyses for Concrete plus Verbal Prompt Condition; and Imaginal plus Verbal, Verbal Only Combined, Within Developmental Level

Concrete plus Verbal					Imaginal plus Verbal, Verbal Only			
	Model	ΔR^2	F	p<	Model	ΔR^2	F	p<
<u>Conservers</u>								
Trial 1	D*	.046	< 1		D	.036	1.02	
	1	.478	7.50	.03	5	.610	17.42	.004
	5	.024	< 1		1	.110	3.14	
Trial 2	2	.008	< 1		2	.000		
	D	.008	< 1		D	.175	2.14	
	1	.563	11.45	.01	1	.182	2.23	
	2	.082	1.66		2	.026	< 1	
	5	.002	< 1		5	.045	< 1	
<u>Transitionals</u>								
Trial 1	D	.005	< 1		D	.134	4.57	
	1	.596	10.78	.01	5	.490	16.77	.005
	5	.012	< 1		2	.172	5.88	
Trial 2	2	.000	< 1		1	.001	< 1	
	D	.006	< 1		D	.367	10.30	.02
	1	.731	25.65	.001	5	.376	10.56	.01
	2	.021	< 1		2	.006	< 1	
	5	.042	1.47		1	.002	< 1	
<u>Nonconservers</u>								
Trial 1	D	.002	< 1		D	.313	4.30	
	1	.817	37.49	.0005	5	.171	2.35	
	5	.014	< 1		1	.007	< 1	
Trial 2	2	.015	< 1		2	.001	< 1	
	D	.090	5.42		D	.038	< 1	
	2	.758	45.64	.0003	1	.032	< 1	
	5	.027	1.60		2	.051	< 1	
	1	.010	< 1		5	.000	< 1	

R^2 : increment in R^2 , given the previous terms already entered in the equation

$F = R^2 / df_1 / (1 - R^2_{total}) / df_{error}$, where $df_1 = 1$ and $df_{error} = 7$

D*: Dummy Variable for Item Type

plus Verbal, Model 1 significantly predicted the latencies for correct addition word problems for: Conservers, Trial 2, (not Trial 1); Transitionals, Trial 1, Trial 2; and Nonconservers, Trial 1, but not Trial 2. The performance of Nonconservers on Trial 2 was significantly related to Model 2. The results for Nonconservers on Trial 2 did not support the prediction that Model 1 would be used when objects were present. The experimenter observed during testing that some of these children were distracted by the animals during the task. They were able to accurately count the number of animals, but the latencies did not reflect only the counting procedure. For the Prompt conditions where objects were not present, i.e. Imaginal plus Verbal and Verbal Only combined, the results were as follows. Model 5 significantly predicted the latencies for correct items for: Conservers on Trial 1, but not Trial 2 (on which none of the Models were significant); Transitionals, Trials 1 and 2. For Nonconservers, none of the Models reached significance.

The results reported above for Imaginal plus Verbal and Verbal Only Prompt Conditions combined, provide information with respect to the use of the counting Models for each Developmental Level, in the absence of object referents. However, it was still of interest to get some indication of the use of counting Models for the Imaginal plus Verbal and Verbal Only conditions separately. Since the numbers of correct responses for some items had been too low to do these analyses separately for each Developmental Level, as originally intended, groups

had to be recombined to get at this additional information. The Conservers and Transitionals were combined, and mean latencies for correct responses for each item, were calculated for the Imaginal plus Verbal and Verbal Only conditions separately. Stepwise multiple regression analyses were performed on Trial 1 and Trial 2 separately for these two groups. As before a dummy variable for item type was included and forced out first. The remaining variables were left free; $N = 12$; the F ratios were tested at $p < .01$. As indicated in Appendix J, there were significant differences in the item variances for the Imaginal plus Verbal group on Trial 2, and the Verbal Only group on Trials 1 and 2. A summary of the results are presented in Table 10. Since the analyses presented in Table 10 were derived from a recombination of the same subjects whose latencies were used for the analyses presented in Table 9, the F ratios in the two Tables are not independent. An examination of Table 10 indicated that under the Imaginal plus Verbal condition, Model 5 significantly predicted the latencies for correct responses for Conservers and Transitionals combined, for Trial 1. Under the Verbal Only condition for Conservers and Transitionals combined, Model 1 emerged as a significant predictor of the latencies.

2. Latencies for Individuals

For each subject, the latencies for all items, correct and incorrect, on the addition word problems, for Trial 1 and Trial 2 separately, were correlated with the x values for each item for each

Table 10

Results of Regression Analyses for
Imaginal plus Verbal and Verbal Only Prompt Conditions
for Conservers and Transitionals Combined

<u>Imaginal plus Verbal</u>				
	Model	ΔR^2	F	p<
Trial 1	D	.035	< 1	
	5	.577	10.56	.01
	2	.001	< 1	
	1	.005	< 1	
Trial 2	D	.444	12.19	.01
	5	.296	8.14	.02
	2	.005	< 1	
	1	.001	< 1	
<u>Verbal Only</u>				
	Model	ΔR^2	F	p
Trial 1	D	.093	4.47	.03
	1	.558	26.95	.001
	5	.172	8.31	.02
	2	.033	1.58	
Trial 2	D	.104	1.25	
	1	.245	2.94	
	2	.022	< 1	
	5	.047	< 1	

Model, (resulting in six correlations for each subject). Each correlation involved 12 items, i.e. $N = 12$. Each correlation was squared, resulting in a value r^2 which represents the amount of variance accounted for by each Model. Since some of the correlations between individual subjects' latencies and the x values for the Models were negative, a sign (+, -) was attached to the r^2 values¹ to reflect the direction of the correlation. These $\pm r^2$ values were then treated as a derived variable (RSQ) and analyzed further to examine the use of the Models in relation to the Developmental Levels and experimental conditions.

A $3 \times 3 \times 2 \times 2 \times 2 \times 3$ analysis of variance was performed on the RSQ values. The independent variables were: Developmental Level, Prompt Condition, Described Relation, Sex, Trials, and Models, with repeated measures on Trials and Models. Prompt Condition was nested within Developmental Level, all remaining variables were nested within Developmental Level and Prompt Condition. The mean RSQ values for each Model as a function of Developmental Level and experimental conditions are presented in Table 11. The analysis of variance Summary Table is presented in Appendix K. The question of interest in this analysis was the use of the Models in relation to the independent variables. Therefore the only factors of concern are those which reflect Models, i.e.

¹ Variance, by definition, has no sign. However, a negative correlation between latency and a given Model was conceptually uninterpretable. Therefore, for further analysis, $\pm r^2$ was used as a derived variable, rather than r^2 in order to remove the contribution of negative correlations.

Table 11

Mean RSQ as a Function of
Developmental Level and Experimental Conditions

		<u>Prompt Condition</u>																	
		Concrete plus Verbal						Imaginal plus Verbal						Verbal Only					
Model		1	2	5				1	2	5				1	2	5			
		D	S	D	S	D	S	D	S	D	S	D	S	D	S	D	S	D	S
<u>Conservers</u>																			
Boys		.28	.23	.23	.02	.22	.05	.15	-.06	.16	.21	.14	.00	.10	.16	.05	.14	.05	.14
Girls		.20	.09	.00	.09	.06	.16	.11	.31	.08	.13	.17	.14	.08	.18	.06	.29	.06	.38
		.20		.08		.12		.13		.14		.11		.13		.14		.16	
		*(.22)		-.15		-.15		-.20		-.14		-.16		-.13		-.20		-.19	
<u>Transitionals</u>																			
Boys		.72	.27	.71	.05	.14	.20	.10	-.01	.02	.01	.00	.06	.09	.10	.00	.09	.07	.00
Girls		.39	.15	.07	.05	.07	.16	.16	.02	.22	.15	.08	.21	.12	.13	.14	.10	-.07	.16
		.38		.22		.14		.07		.10		.09		.11		.08		.04	
		(.35)		(.51)		(.11)		(.12)		(.19)		(.11)		(.11)		(.13)		(.14)	
<u>Nonconservers</u>																			
Boys		.45	.29	.12	.07	.07	.04	.10	.16	.03	.08	.16	.00	.09	.03	.06	-.05	.10	.04
Girls		.26	.33	.26	.15	-.01	.04	-.06	.00	-.07	.07	-.01	.16	-.07	.20	.02	.37	.00	.08
		.33		.15		.04		.05		.03		.08		.06		.10		.05	
		(.16)		(.16)		(.08)		(.14)		(.11)		(.12)		(.14)		(.25)		(.07)	

Abbreviations: D Dynamic Described Relation; S Static Described Relation

* (...) standard deviations

the main effect for Models (nested in Developmental Level and Prompt Condition), and the first, second and third order interactions of Models with Described Relation, Trials and Sex. Each effect was tested for significance at the $p < .01$ level. The overall effect of Models within Developmental Level and Prompt Condition was significant, $F(18, 144) = 2.27$. Examination of the effect of the Models partitioned into each level of the nesting variables indicated that the significant Developmental Levels and Prompt Conditions were: Transitionals under the Concrete plus Verbal Prompt Condition, $F(2, 144) = 6.95$, with means of .38, .22, .14, for Models 1, 2 and 5 respectively; and Nonconservers under the Concrete plus Verbal Prompt Condition, $F(2, 144) = 10.30$, with means of .33, .15, .04 for Models 1, 2 and 5 respectively. Following the finding of a significant effect of Models for these levels, post hoc Sheffé comparisons were performed to locate the significant differences in use of Models, within each of these two Developmental Levels under the Concrete plus Verbal Prompt Condition. The overall error rate for the post hoc comparisons at each Developmental Level was $p < .01$. For Transitionals, the mean for Model 1, .38, differed significantly from the mean for Model 5, .14. For Nonconservers, the mean for Model 1, differed significantly from the means for Model 2, .15, and Model 5, .04. This suggests that for Transitionals, Model 1 was related to the latencies more than Model 5 under the Concrete plus Verbal Prompt Condition, but not significantly more than Model 2. For Nonconservers

under the Concrete plus Verbal Prompt Condition, Model 1 was related to the latencies significantly more than Model 2 and Model 5. None of the interactions with Models were significant.

The use of the counting Models was tested in two ways: (1) using mean latencies over groups of subjects, (2) using individual subject's latencies. Both methods have advantages and disadvantages.

(1) Using mean latencies over groups

Advantage: Only correct responses were used, which may more accurately reflect the Model used.

Disadvantages:

i. If all subjects in a group over which the means are calculated are not using the same counting Model, the correlations will be lowered.

ii. Due to the variation in the number of correct responses between experimental conditions, and across items, the number of subjects included in each mean latency varied. Moreover, for some groups there were significant differences in the variances of the items.

(2) Using individual subject's latencies

Advantage: All the data is used, 12 items on 2 trials from every subject.

Disadvantage: In order to have 12 items to correlate, latencies from incorrect items were included. Since time to an incorrect response does not reflect accurate use of an algorithm (i.e. in this

case a particular counting model), this would lower the correlations between the latencies and the x values for each Model.

C. Additional Analyses

1. Item Type

The frequency distribution of correct responses by items was examined for Trial 1 and Trial 2, within each Developmental Level and Prompt Condition. The mean frequency collapsed across Trials for each item was presented in Table 8 (p. 95). It appeared that under Imaginal plus Verbal and Verbal Only conditions, the mean frequency of correct responses was lower for some items, fairly consistently across the three Developmental Levels. In general, the data suggested that more correct responses were observed for items 5, 7, 9, 12, and fewer correct responses for items 3, 6, 10, 11. The addition problems for items 5, 7, 9, 12, were $3 + 2$, $7 + 2$, $5 + 2$, $7 + 3$, respectively; items 3, 6, 10, 11, were $3 + 4$, $4 + 6$, $4 + 5$, $2 + 6$, respectively. What the easier problems all had in common was that they were of the form $m > n$; the more difficult problems, $m < n$.

The integers for the addition problems had been selected so that in 6 of the problems $m > n$, and in 6 problems $m < n$. Although Item Type was not originally intended to be a factor in this study, it was of interest to reanalyze performance on the addition word problems for number of correct responses and latencies, with Item Type as an

independent variable. There were 6 items of Type 1 ($m > n$) and 6 items of Type 2 ($m < n$), on each Trial. Performance was collapsed over Trials, so that 12 items of each Type were available for analysis. The integers from the items of each Type are presented in Table 12.

3. (i) Number of correct responses

A $3 \times 3 \times 2 \times 2 \times 2$ analysis of variance was performed on the number of correct responses on the addition word problems. The independent variables were: Developmental Level, Prompt Condition, Described Relation, Sex, and Item Type, with repeated measures on Item Type. The mean number of correct responses for each Item Type, as a function of Developmental Level and experimental conditions, is presented in Table 13. The Summary Table for the analysis of variance is presented in Appendix L-1. The only factors of interest in this analysis were the main effect of Item Type (nested in Developmental Level and Prompt Condition), and the first and second order interactions of Item Type with Described Relation and Sex, similarly nested. All effects were tested at the $p < .01$ level of significance. The main effect of Item Type within Developmental Level and Prompt Condition was significant, $F(9, 72) = 3.41$. The effect of Item Type was examined separately for each level of the nesting variables. The Developmental Levels and Prompt Conditions which accounted for the significant main effect of Item Type were as follows: Conservers, under the Verbal Only Prompt Condition, $F(1, 72) = 8.28$, with means of 7.33 and 5.17, for Item Type 1

Table 12

Integers from the Items Used in the Addition Word Problems:
Type 1 ($m > n$), Type 2 ($m < n$)

Item	Type 1	Item	Type 2
2	$5 + 3$	1	$2 + 4$
4	$6 + 3$	3	$3 + 4$
5	$3 + 2$	6	$4 + 6$
7	$7 + 2$	8	$2 + 8$
9	$5 + 2$	10	$4 + 5$
12	$7 + 3$	11	$2 + 6$

Table 13 Mean Number of Correct Responses for Item Type 1 and 2
as a Function of Developmental Level and Experimental Conditions

	<u>Prompt Condition</u>					
	Concrete plus Verbal		Imaginal plus Verbal		Verbal Only	
	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2
<u>Conservers</u>						
Boys Dynamic	9.67	10.67	8.67	8.33	8.00	6.67
Static	10.33	11.33	9.67	8.00	5.67	5.33
Girls Dynamic	11.67	11.00	8.33	6.00	7.00	4.00
Static	10.00	9.67	8.33	6.33	8.67	4.67
	10.42	10.67	8.75	7.17	7.33	5.17
	*(1.83)	(1.23)	(3.08)	(3.49)	(4.03)	(4.02)
<u>Transitionals</u>						
Boys Dynamic	11.67	11.00	2.67	1.67	3.67	2.00
Static	10.67	11.00	1.33	2.00	7.00	7.00
Girls Dynamic	10.00	8.67	6.67	5.00	5.00	2.33
Static	9.00	9.00	7.33	7.67	10.00	9.00
	10.33	9.92	4.50	4.08	6.42	5.08
	(1.50)	(1.93)	(3.80)	(3.58)	(3.58)	(4.21)
<u>Nonconservers</u>						
Boys Dynamic	10.67	9.33	4.67	1.00	5.67	1.67
Static	11.00	10.67	4.67	3.67	.67	1.67
Girls Dynamic	10.67	10.67	2.67	1.67	6.33	2.67
Static	10.33	10.00	11.33	10.00	6.67	4.67
	10.67	10.17	5.83	4.08	4.83	2.67
	(1.23)	(1.90)	(4.09)	(3.87)	(3.74)	(2.06)

*(...) standard deviations

and 2 respectively; and Nonconservers, under Verbal Only, $F(1, 72) = 8.28$ with means of 4.83 and 2.67, for Item Type 1 and 2 respectively. These results indicate that under the Verbal Only condition both the Conservers and Nonconservers had significantly more correct responses for Item Type 1 ($m > n$), than Item 2 ($m < n$). For Transitionals, the difference in performance across Item Type was in the same direction as for the other Developmental Levels, but did not reach significance. None of the interactions with Item Type were significant. The finding of no significant difference between Item Type 1 and 2 for Conservers and Nonconservers under the Imaginal plus Verbal condition, and a significant difference between Item Types under Verbal Only, appears to indicate that the Imaginal plus Verbal condition enabled these subjects to perform as well on Items of Type 2 ($m < n$) as they did on Type 1 ($m > n$).

(ii) Latencies

A $3 \times 3 \times 2 \times 2 \times 2$ analysis of variance was performed on the latencies on the addition word problems, with the same independent variables, and testing the same effects at $p < .01$, as for the number of correct responses. The mean latencies for each Item Type as a function of Developmental Level and experimental conditions are presented in Table 14. The Summary Table of the analysis of variance is presented in Appendix L-2. The main effect of Item Type was significant, $F(9, 72)$

Table 14 Mean Latencies of Item Type 1 and 2 as a Function of Developmental Level and Experimental Conditions

		<u>Prompt Condition</u>					
		Concrete plus Verbal		Imaginal plus Verbal		Verbal Only	
		Type 1	Type 2	Type 1	Type 2	Type 1	Type 2
<u>Conservers</u>							
Boys	Dynamic	4.60	5.72	7.27	8.55	9.29	8.60
	Static	7.16	6.65	5.03	7.12	6.49	7.77
		5.88	6.19	6.15	7.83	7.89	8.18
Girls	Dynamic	7.86	7.77	8.24	9.25	4.58	5.05
	Static	4.79	5.44	6.85	7.84	3.93	9.55
		6.33	6.60	7.54	8.54	4.26	7.30
		6.10	6.39	6.85	8.19	6.07	7.74
		*(2.31)	(2.03)	(3.09)	(3.26)	(3.91)	(3.61)
<u>Transitionals</u>							
Boys	Dynamic	6.50	7.02	4.07	4.01	8.78	8.65
	Static	9.12	8.99	7.79	7.21	6.31	6.24
		7.81	8.06	5.93	5.61	7.54	7.45
Girls	Dynamic	6.25	6.20	4.62	5.74	7.45	8.87
	Static	7.36	8.31	6.51	7.85	7.84	9.97
		6.80	7.25	5.57	6.79	7.65	9.42
		7.31	7.66	5.75	6.20	7.60	8.44
		(2.66)	(2.45)	(2.60)	(2.26)	(2.00)	(2.81)
<u>Nonconservers</u>							
Boys	Dynamic	7.33	7.75	7.75	8.61	7.28	7.93
	Static	7.50	7.62	8.43	9.08	8.57	7.53
		7.42	7.69	8.09	8.84	7.92	7.73
Girls	Dynamic	7.81	8.90	8.75	7.47	5.97	8.71
	Static	5.99	6.29	5.69	7.30	5.35	8.14
		6.90	7.59	7.22	7.38	5.66	8.43
		7.16	7.64	7.65	8.11	6.79	8.08
		(1.26)	(1.92)	(4.10)	(4.31)	(2.39)	(3.38)

*(...) standard deviations

≈ 3.65 . The Sex $\times$ Item Type interaction was significant, $F(9, 72) = 2.73$. The breakdown of Item Type across the nesting variables of Developmental Level and Prompt Condition indicated that the following levels accounted for the significant main effect: Conservers under Imaginal plus Verbal, $F(1, 72) = 7.57$, with means of 6.85 and 8.19 for Item Type 1 and 2 respectively; Conservers under Verbal Only, $F(1, 72) = 11.73$, with means of 6.07 and 7.74; Nonconservers under Verbal Only, $F(1, 72) = 6.96$, with means of 6.79 and 8.08. It appears that for the Conservers under Imaginal plus Verbal, and Verbal Only Prompt Conditions, and for Nonconservers under Verbal Only, significantly more time was taken to solve problems of Item Type 2 ($m < n$) as compared with Item Type 1 ($m > n$).

The Sex $\times$ Item Type interaction was significant, ($F(9, 72) = 2.73$). The breakdown of this interaction, across the nesting variables of Developmental Level and Prompt Condition indicated significance at the following levels: Conservers under Verbal Only, $F(1, 72) = 7.99$, with the means presented in Table 14; and Nonconservers under Verbal Only, $F(1, 72) = 9.22$, with the means presented in Table 14. Appropriate post hoc comparisons for interaction, as described by Marasciulo and Levin (1970), were performed on these two significant interactions. The overall Type I error for the comparison on each interaction was $p < .01$. The comparison tested for each of these two interactions was:

$$\hat{\psi} = (\bar{X}_{11} - \bar{X}_{12}) - (\bar{X}_{21} - \bar{X}_{22})$$

This comparison was significant for each of the significant interactions. This indicates that for Conservers, and Nonconservers, under the Verbal Only Prompt Condition, there was a greater difference in the latencies between Item Types for girls than for boys. Examination of Table 14 shows mean latencies of 4.26 and 7.30 for Conserving girls on items of Type 1 and 2 respectively; and 5.66 and 8.43 for Nonconserving girls on items of Type 1 and 2 respectively. For Conserving boys, the mean latencies were 7.89 and 8.18 for items of Type 1 and 2; for Nonconserving boys, 7.92 and 7.73. This suggests that for these two Developmental Levels under the Verbal Only Prompt Condition, girls responded with a solution to problems of Type 1 ($m > n$) faster than to problems of Type 2 ($m < n$). For the boys it appears that Item Type made less, if any, difference to the amount of time taken to respond.

2. Age

A further question of interest was whether chronological age would differ significantly across Developmental Level. In other words, whether performance on the Conservation Pretest was simply a function of age, within the 12 month range represented in Kindergarten towards the end of the school year. Subjects were assigned at random to experimental conditions, once their Developmental Level had been assessed on the Conservation Pretest. Therefore no significant differences in age would be expected for the experimental factors.

A $3 \times 3 \times 2 \times 2$ analysis of variance was performed on the age (in months) of the subjects in the study. The independent variables were: Developmental Level, Prompt Condition, Described Relation and Sex; nested as for the other analyses of variance. The mean age (in months) for subjects as a function of Developmental Level and experimental conditions is presented in Appendix M-1. The analysis of variance summary table is presented in Appendix M-2. The factors were tested for significance at $p < .01$. None of the main effects or permissible interactions were significant. For the main effect of Developmental Level, $F < 1$ which clearly suggests no significant difference between the mean ages of Conservers, Transitionals and Non-conservers.

3. Incidental vs. Intentional Learning

Pearson product moment correlations were calculated between the number of nouns recalled (incidental learning task) and the number of correct responses in the addition problems, separately for Trial 1 and Trial 2, within each Developmental Level and Prompt Condition. These correlations are presented in Appendix N. None of the correlations differed significantly from 0, at $p < .01$, with $N = 12$. Therefore these results do not suggest a linear relationship between the level of performance on the incidental and intentional learning tasks used in this study.

Summary of Results

The main statistically significant findings are summarized below.

A. Performance on the Addition Word Problems

1. Number of Correct Responses:

Conservers had significantly more correct responses than Transitionals and Nonconservers. Within each Developmental Level subjects had significantly more correct responses under the Concrete plus Verbal as compared with the Imaginal plus Verbal and Verbal Only Prompt Conditions. Transitionals under the Verbal Only Prompt Condition and Nonconservers under the Imaginal plus Verbal Prompt Condition performed significantly better under the Static as compared with Dynamic Described Relation.

2. Latencies:

No significant differences were found.

3. Incidental Recall of Nouns:

For Transitionals and Nonconservers there was a significant difference in noun recall following the Concrete plus Verbal as compared with Imaginal plus Verbal and Verbal Only Prompt Conditions combined.

B. Counting Models

1. Mean Latencies for Groups:

The following models significantly predicted latencies for

correct responses, on at least one trial:

- When objects were present, Model 1 for all three Developmental Levels;
- When objects were not present, Model 5 for Conservers and for Transitionals;
- Under the Imaginal plus Verbal Prompt Condition Model 5 for Conservers and Transitionals combined;
- Under the Verbal Only Prompt Condition, Model 1 for Conservers and Transitionals combined.

2. Latencies for Individuals:

Under the Concrete plus Verbal Prompt Condition, Model 1 was related to latencies for correct and incorrect responses significantly more than Model 5 for the Transitionals, and significantly more than Models 2 and 5 for Nonconservers.

C. Item Type

1. Number of Correct Responses:

Under the Verbal Only Prompt Condition Conservers and Nonconservers had significantly more correct responses for items where $m > n$ than $m < n$.

2. Latencies:

For Conservers under the Imaginal plus Verbal and Verbal Only Prompt Conditions, and for Nonconservers under Verbal Only,

significantly more time was taken to solve items where $m < n$ than $m > n$.

For Conservers and Nonconservers under the Verbal Only Prompt

Condition girls responded with a solution to problems where $m > n$

faster than where $m < n$. For the boys Item Type made less, if any,

difference to the amount of time taken to respond.

CHAPTER VI

DISCUSSION AND CONCLUSIONS

The main purpose of the present study was to determine:

- (1) if findings from paired associate learning on the importance of
 - i. developmental level in generating a referential event,
 - ii. imagery instructions as an elaborative prompt condition,
 - iii. verbal context,could be applied to the performance of addition word problems.
- (2) whether kindergarten children may use counting models as algorithms to solve addition problems.
- (3) possible directions for further research concerned with improving instruction in arithmetic.

The developmental hypothesis that Conservers would perform significantly better than Transitionals and Nonconservers on the addition word problems in terms of number of correct responses, was confirmed. There was no significant difference between Transitionals and Nonconservers, contrary to expectation. As predicted, children of all Developmental Levels performed best under the Concrete plus Verbal Prompt Condition, and the less well developed the children's concept of cardinal number, the more important the presence of object referents appeared to be. There was clearly no difference between Developmental Levels under the Concrete plus Verbal Prompt Condition, with the mean number of correct responses for Conservers of 10.14, for Transitionals 10.13,

and for Nonconservers 10.42. The significantly better performance of the Conservers as compared with the Transitionals and Nonconservers therefore had to be located in the Imaginal plus Verbal and Verbal Only conditions.

The hypothesis that the Imaginal plus Verbal Prompt Condition would result in better performance as compared with the Verbal Only, for Conservers, was not confirmed (but the means were in the predicted direction). There was no significant difference between the Imaginal plus Verbal and Verbal Only conditions for the Transitionals and Nonconservers. However, when performance was compared across Item Type ($m > n$ vs. $m < n$), some children performed differently under the Imaginal plus Verbal Condition as compared with Verbal Only. Within each Developmental Level, subjects performed equally well in terms of number of correct responses, with items of $m > n$ and $m < n$, under the Imaginal plus Verbal Prompt Condition. Under Verbal Only, Conservers and Nonconservers found problems of Type $m < n$ to be significantly more difficult than Type $m > n$. This suggests that imagery instructions facilitated performance of items of Type $m < n$, for these two Developmental Levels. For Transitionals, the differences across Item Type were not significant, however the results were in the same direction as for the Conservers and Nonconservers.

There were no significant differences in latency of responses

across Developmental Levels or experimental conditions, for the planned analyses. However, when the data were re-analyzed in relation to Item Type, there were significant differences in latencies. For Conservers under Imaginal plus Verbal and under Verbal Only, and for Nonconservers under Verbal Only, significantly more time was taken to solve problems of Type $m < n$ than Type $m > n$. Thus it appears that under Verbal Only, even though more time was used for items where $m < n$ than for items $m > n$, when no imagery instructions were provided, items of type $m < n$ were not solved as well as $m > n$. When imagery instructions were provided there was no difference within each Developmental Level in the number of correct responses for items of both types. The fact that significantly more time was taken for items where $m < n$ than $m > n$ by Conservers under Imaginal plus Verbal, and not by the other Developmental Levels, may partly be responsible for the apparent better performance of Conservers in terms of number of correct responses. The results indicated that Conserving and Nonconserving girls under Verbal Only, respond with a solution to problems of $m > n$ faster than to problems of $m < n$. The boys appeared to use the same amount of time for both types of items. This suggests the girls were responsible for the longer responses latencies for Type 2 items for these two groups. Fennema (1974) has recently reviewed the literature on sex differences in mathematics achievement. She concluded that, in general, significant differences

have not been found between boys' and girls' mathematics achievement before entering elementary school or during the early elementary years. There is no obvious explanation for the present finding. This was the only analysis in which a significant difference in performance across sex was found. It appears that possible sex differences in response time in relation to Item Type and Prompt Condition may exist, and need to be studied further.

It was found that imagery instructions facilitated performance for the more difficult items, but not overall performance. It is possible that training children to generate referential events may be more effective than minimal imagery instructions in improving performance on addition problems for kindergarten children. Some studies in paired associate learning have found that although 5 - 6 year old children can utilize interacting stimuli provided by the experimenter, it is difficult for them to generate their own imaginal referents (e.g. Rohwer, Ammon, and Levin, 1971; Montague, 1970). Yuille and Catchpole (1974) found that following imagery training, kindergarten children could generate interactions. The results of the present study suggest it may be worthwhile to further investigate the use of elaboration by young children for arithmetic tasks, using a training procedure rather than only verbal instructions.

For the variable of Described Relation, the Static rather than Dynamic Described Relation resulted in more correct responses for

Transitionals under the Verbal Only condition, and Nonconservers under the Imaginal plus Verbal condition. There was no significant difference in performance under Dynamic as compared with Static verbal context for Conservers. This supports the prediction that Described Relation may be less important for Conservers than for the other two Developmental Levels. The superior performance found under the Static rather than Dynamic Described Relation suggests that the analogy drawn between integrated stimuli in paired associate learning, and the Dynamic verbal context provided here for addition problems, is not viable. Given that the task was addition, the ease of counting appears to have been more important than facilitating transformation of two sets into one. It was predicted that if the ease of counting was more important, the effect of this variable would be greater in Prompt Conditions where referential events of some kind may be involved, i.e. the Concrete plus Verbal and Imaginal plus Verbal conditions. However, no differences were observed for this variable under the Concrete plus Verbal condition. From the Wang, Resnick and Boozer (1971) finding that small unordered sets (1 to 5 objects) were easier for kindergarten children to count than larger unordered sets (6 to 10 objects), it was expected that the Static condition may facilitate counting objects. However, in the present study, when the sets were brought together for the Dynamic Concrete plus Verbal condition, the two rows of objects were placed side by side parallel to one another, and therefore were not unordered, except that

the rows differed in number of animals and thus also length. Moreover, since the operation required was addition of two sets, the task under the Static condition was not the same as separately counting two small sets. Therefore the finding of no significant difference under the Concrete plus Verbal condition, between counting objects under the Dynamic as compared with Static Described Relation is not surprising. Under the Imaginal plus Verbal condition, the Nonconservers did perform significantly better under Static as compared with Dynamic Described Relation, which suggests that Static Described Relation may have facilitated use of elaboration by these subjects. This would be in keeping with Rohwer's (1973) view that the verbal context can promote elaboration. There is no apparent explanation for the finding that Described Relation was significant for Transitionals under Verbal Only, rather than under Imaginal plus Verbal.

There was no significant difference in performance across Trials for either number of correct responses or latency. Some subjects improved across Trials, others had fewer correct responses on Trial 2 than Trial 1, despite corrective feedback. This was probably due to the fact that many subjects found the task too long, although with encouragement they did complete all the items.

The incidental noun recall task was included as a possible way of checking whether subjects under the Imaginal plus Verbal Prompt Condition had utilized the imagery instructions. It was expected that the

best incidental noun recall would follow the Concrete plus Verbal condition, since both visual and verbal referents had been present during the addition problems. This prediction was confirmed for Transitionals and Non-conservers. There is some evidence that imagery instructions facilitate incidental learning (Yarmey and Bowen, 1972). If elaboration was generated in the Imaginal plus Verbal condition, but not the Verbal Only, incidental noun recall should have been better under the former condition. For Transitionals and Nonconserverers there was no significant difference in noun recall between the two Prompt Conditions which lacked object referents. For Conserverers, there was no significant difference in noun recall across any of the Prompt conditions. Since noun recall for Conserverers was as good for the Imaginal plus Verbal and Verbal Only conditions as it was for Concrete plus Verbal, it would be tempting to say that perhaps Conserverers utilized elaboration under both conditions where object referents were not present. However, this interpretation is discounted by the fact that there was no significant difference overall across Developmental Level for this task. Since the Conserverers did not recall more nouns than the Transitionals and Nonconserverers, under the Imaginal plus Verbal condition, (with means of 4.75, 4.67, 3.83 respectively), or the Verbal Only condition, (with means of 4.33, 4.67, 4.83), one cannot conclude that Conserverers used elaboration under those conditions, and the Transitionals and Nonconserverers did not.

The reason that Conservers showed no difference in incidental noun recall across the three Prompt Conditions, and also performed no better than Transitionals and Nonconservers on this task, may have been because the verbal context was not intrinsic to performing the intentional learning task, i.e. the addition operation. Postman (1964) made a distinction in incidental learning between tasks in which the incidental components of the total learning task are extrinsic to the intentional components, as compared to tasks in which the incidental components are intrinsic to the intentional components. In the present study, the verbal context may have been irrelevant to performing the addition operation, given a certain level of development of cardinal concept of number. This is consistent with the finding that Described Relation was a significant variable for Transitionals and Nonconservers, but not for Conservers. In the Yarmey and Bowen (1972) study, children aged 8 to 13 years rated their imagery to noun and picture pairs. The incidental component, noun and picture recall, was therefore intrinsic to the intentional component of the task. Yarmey and Bowen did not examine developmental changes over age (they were comparing performance of retarded and normal subjects). However, there is evidence of a developmental increase in use of selective attention (e.g. Sigel and Stevenson, 1966). Hale and Taweel (1974) found a developmental improvement in the flexibility of attention, with children aged 5 to 8. There was a developmental trend toward differentiation between situations in which it is useful

to attend to several stimulus features, and situations in which it is more advantageous to attend selectively. It would appear that in the present study, the Conservers may have directed their attention to the intentional component of the learning task, i.e. the numbers, more than to the extrinsic incidental component, i.e. the nouns. The Transitionals and Nonconservers may have paid more attention to the verbal context, as compared with the Conservers. The finding that the Transitionals and Nonconservers recalled more nouns following the Concrete plus Verbal Prompt Condition, as compared with Imaginal plus Verbal and Verbal Only, supports the finding of Goldberg (1974), with grade 5 subjects, that seeing pictures in addition to verbal materials, resulted in better incidental learning, as compared with verbal materials only. It appears that the results of the incidental noun recall task were not useful for interpreting whether subjects had generated referential events under the Imaginal plus Verbal Prompt Condition, primarily because the incidental stimuli may have been extrinsic to the intentional learning task. If the nouns themselves had been intrinsic to the intentional learning task, then since Conservers had significantly more correct responses on the addition word problems than the other two Developmental Levels, they would have recalled more nouns. Presumably subjects may generate referential events for numbers without utilizing the actual nouns provided, i.e. without imagining a given number of specified animals. This would be consistent with Yuille's (1974) finding that facilitation of noun recall with children in grades 2, 4 and 6 was as great when the verb connective

changed on each trial as when it remained the same. Yuille interpreted these results to suggest that verb links may affect children's paired-associate learning in the same way as mediation instructions. The effect of the verb connective is to indicate to the child that an interaction involving the nouns is possible. The subject may not use the particular interaction denoted by the verb. A similar situation may exist for addition, especially for the Conservers. The subject may not use the event depicted in the problem, however the word problem context may indicate to the child that elaboration of some kind can be generated for numbers. The Transitionals and Nonconservers, on the other hand, may use the event provided.

Counting Models¹

These analyses were based on the assumption that variation in response latencies reflect the algorithm being used to perform the addition operation. From the three counting models (Suppes and Groen, 1967) being tested in this study, it appears that: Model 1 was used when object referents were present, (on at least one trial) by all Developmental Levels. When no objects were present, i.e. for the Imaginal plus Verbal and Verbal Only Prompt Conditions combined, Model 5 was used by

¹As mentioned in the Results chapter, it is likely that the assumptions for regression analysis have not been met by the data. Therefore all conclusions concerning the use of the Models are tentative.

Conservers on Trial 1, and by Transitionals on Trial 1 and 2. In the absence of objects none of these counting models significantly predicted latencies for Nonconservers, or for Conservers on Trial 2. Nonconservers may not be able to utilize Model 5 because of their low level of understanding of the concept of cardinal number. To try and get some indication of the use of the Models for the Imaginal plus Verbal and Verbal Only conditions, separately, latencies for Conservers and Transitionals were combined, for each of the two Prompt Conditions. Model 5 was used by Conservers and Transitionals combined, under Imaginal plus Verbal on Trial 1. Under Verbal Only Model 1 was significantly associated with the latencies. It is unfortunate that low numbers of correct responses for some items made it unfeasible to test use of the Models separately within each Developmental Level and Prompt Condition. It is not possible to tell from these findings whether both the Conservers and Transitionals separately would have shown this pattern of use of Model 5 under imagery instructions, and Model 1 with listening instructions.

The fact that Conservers and Transitionals combined were able to use Model 5 under the Imaginal plus Verbal condition, but not under Verbal Only, suggests that referential events may have been generated by subjects under the Imaginal plus Verbal condition, and this enabled them to use the most sophisticated of the three counting models. Suppes and Groen (1967) and Groen and Parkman (1972) both

found Model 5 to be the best predictor of latencies for addition. These previous studies used first grade children and visually presented addition equations. Suppes and Groen used integers of $m + n \leq 5$; Groen and Parkman used integers of $m + n \leq 9$. The present study extended the findings to kindergarten children, with aurally presented word problems with a wider range of integers, i.e. $m + n \leq 10$. However the children in the present study only used Model 5 if they had been given instructions to imagine the word problems. The findings of the previous studies that grade 1 children as a group, without objects present, used Model 5, whereas these kindergarten children only used Model 5 under certain conditions, may be due to a number of reasons, e.g. developmental level, size of integers, verbal context as compared with equations. However the most important reason may be mode of presentation, visual as compared with aural. Hayes (1973) exploratory work suggested adults may use imagery in relation to a background stimulus to keep track of steps in the solution process. It is possible that when addition equations are visually presented, as in the earlier studies, it may be easier for children to apply a solution procedure, or algorithm. For example, to apply Model 5 when a problem is visually available, the child can visually select the larger integer, then increment the smaller. This may be especially valuable when items are of the Type $m < n$. When an item where $m < n$ is aurally presented, the order of

processing the integers would be the reverse of input order. The findings of the present study support the suggestion outlined above that aural vs. visual input may be very important, since Conservers and Transitionals as a group only used Model 5 when instructions to use imagery were presented, i.e. when subjects may have generated a referential event. Moreover, when the performance on the addition word problems was compared across Item Type 1 ($m > n$) and 2 ($m < n$), Conservers and Non-conservers under imagery instructions performed as well in terms of number of correct responses, with both Item Type 1 and 2. When listening, rather than imagery instructions were presented, subjects had significantly fewer correct responses with items of Type $m < n$ as compared with Type $m > n$. Model 5 is more efficient than Model 1, especially when $m < n$, because it is difficult to remember n and to know when to stop incrementing by 1 when m has also been incremented rather than set as the initial value. Therefore a condition which facilitates use of Model 5, i.e. Imaginal plus Verbal, would be expected to result in better performance for items where $m < n$.

The group analyses of use of the Models were performed using latencies from correct responses only. Therefore the conclusion is that given correct responses, these are the Models which significantly predicted latencies, and which therefore may have been used as an algorithm to arrive at a result. The use of Model 5 under Imaginal plus Verbal, at least by Conservers and Transitionals as a group, says nothing about which

groups performed best in terms of number of correct responses. Thus the conclusion drawn is that when the response was correct, it appears that subjects used Model 1 under the Concrete plus Verbal Prompt Condition; Conservers and Transitionals as a group used Model 5 under Imaginal plus Verbal and Model 1 under Verbal Only. The performance of Nonconservers in the absence of objects was not related to any of the Models. The Conservers performed significantly better in terms of number of correct responses than the Transitionals and Nonconservers. Therefore the tentative conclusion is drawn that Conservers may be best able to utilize Model 5.

Item Type

The introduction of Item Type as a factor in the study was important, in that evidence was provided that the effect of Prompt Condition interacted with the Item Type involved, i.e. some aspects of the addition problems, possibly the position of the larger integer. The finding that the position of the larger integer, i.e. $m > n$ or $m < n$ may be important in performance of addition problems is only tentative. Other aspects of the items were not controlled, since the Item Type factor had not been part of the planned design. For example, the size of $m + n$ and the difference between m and n were not controlled across Item Type. The items $3 + 4$, $4 + 6$, $4 + 5$ may be difficult because m and n are almost of the same magnitude, as well as being of the Type $m < n$. However, Groen and Parkman (1972) found that ties, e.g. $3 + 3$, $4 + 4$ were parti-

cularly easy for grade 1 children, as compared to the other addition problems where $m+n=9$. However, ties may be a special case. A study in which the Type 1 and Type 2 items have the same integers, and only the position of the larger integer changed across Item Type, would be needed in order to determine whether position of the larger integer was the critical factor.

To summarize, the findings were :

1. (i) There was indirect evidence that imagery instructions can be used by kindergarten children to facilitate performance of addition problems.

(ii) Nonconservers as well as Conservers were able to use imagery instructions. However, Conservers performed significantly better overall than Transitionals and Nonconservers, and the difference between performance under the Imaginal plus Verbal Prompt Condition, as compared with Verbal Only, approached significance for the Conservers. This tentatively suggests Conservers may be better able to utilize imagery instructions than the other two Developmental Levels.

(iii) Verbal context did have a significant effect on performance for certain conditions. However, it was the Static, not Dynamic Described

Relation which facilitated performance for these groups in the absence of objects. This suggests the analogy between Dynamic Described Relation and integrated stimuli from paired-associate learning is untenable in the present study. Moreover the counting rather than transformation aspect of performing an addition operation appears to be more affected by verbal context.

2. This study provided evidence that kindergarten children may use counting models as algorithms to perform addition operations. It appears that these children performed addition by counting from 1 to $m + n$ (Model 1) when objects were present. In the absence of objects, some kindergarten children still attempted to add numbers using Model 1. When imagery instructions were provided, however, Model 5 was used by Conservers and Transitionals as a group. Model 5 entails selecting the larger integer, then incrementing the smaller by ones until $m + n$ is reached. This is the most efficient of the three algorithms tested here.

In this study it was proposed that there may be developmental changes in the nature of the addition process. Performance of addition by young children may depend on counting sets of objects. A little later, as some level of understanding of cardinal number is acquired, addition may be performed in the absence of objects using a more sophisticated counting procedure, i.e. by taking the larger of the two integers and incrementing the smaller by ones. Use of elaboration may help the

child to progress from object counting to a more abstract counting procedure in the absence of objects. Later still, the use of a counting model may be superceded by immediate recall of the number associated with the addition of any given pair of single integers. The findings appear to support this account of the development of addition behaviour. There was evidence for the use of Model 5 by Conservers and Transitionals, but not by Nonconservers, which supported the hypothesis that in order to use the most efficient counting model some degree of understanding of cardinal number concept would be necessary. Imagery instructions facilitated use of the most efficient of the three models by Conservers and Transitionals. This suggests that the use of elaboration, or the generation of a referential event, may assist children to transform an abstract addition problem into a form to which the most sophisticated counting algorithm can be applied. Unfortunately, clear conclusions cannot be drawn with respect to the relationship between a child's level of concept of cardinal number and use of counting models, in relation to elaborative Prompt Condition, since each Prompt Condition could not be tested separately within Developmental Level. However, the tentative findings which did emerge suggest that future investigation of the use of elaboration in arithmetic tasks by children at various developmental levels may be productive. The conclusions which may be drawn from this study alone are limited; however, it has provided

a step in extending investigation of use of elaboration from paired-associate learning to mathematics learning. Moreover, it suggests a number of research questions which warrant further investigation.

Educational Implications

The results from the present study suggest that use of elaborative prompts at various developmental levels may be important for facilitating the performance of mathematical tasks. As well as teaching the traditional areas of skills in reading, arithmetic, and so on, emphasis is now placed on listening skills with first grade children. Helping children to develop skills related to utilizing elaboration may also be important for learning. The type of elaborative prompts introduced to children would depend on their developmental level and the tasks concerned. The use of object and picture aids to facilitate acquisition of number concepts (e.g. concept of "set") has become well accepted in the school setting. Beyond use of maximal prompts to introduce number concepts, the child still has to "bridge the gap" from the concrete situation to the abstract. Developing skills for generating referential events may help the transition from concrete to abstract processes by providing meaning for the abstract symbols, in this case numbers. The abstract symbols may be reduced to a level closer to concrete experience, so that they can be operated upon. Before operations can be performed on numbers, the child has to learn to associate a given number with sets of that size. After the child has had practice with many kinds of sets in relation to number, it may be helpful to have him associate each single digit number with a fixed pattern, e.g. the domino patterns. The result would be

that when a given number was presented, a pattern would be evoked. This association between numbers and patterns may help in generating referential events. Once the sets can be represented, manipulations of the sets involved, for example, in applying Model 5 to the addition of sets, may be facilitated. Counting sets of objects is a way of performing addition. From this the child is expected to move to learning (memorizing) addition "facts". From research in paired associate learning it has become clear that a task once thought to involve associations has a large conceptual component. Learning a list of noun pairs may initially be facilitated by use of elaboration, however if the list was presented repeatedly over a long period of time until it was overlearned, the recall of the second noun, given the first, would be immediate. The same process may occur with addition. It may be helpful to the child at a certain level of learning to add numbers, to encourage use of elaboration in combination with a sophisticated counting algorithm (Model 5). This would be introduced after he has had experience with sets of objects, then patterns of dots, and understands the cardinal meaning of number. After repeated experience with two integers adding to a certain sum, that sum would be readily associated with those two integers. The recall of that result upon presentation of the addition problem would become immediate.

Three suggestions have been made: teaching elaboration as

another skill, such as is now being done with listening; introducing Model 5 as a procedure for performing addition which follows use of objects and pictures, but precedes memorizing addition "facts"; introducing use of domino patterns to facilitate use of elaboration and Model 5. Clearly, further research would be necessary before such suggestions could be implemented with confidence. These are really suggestions for directions for investigations, as much as they are for eventual practice.

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APPENDICES

Appendix A

Mathematical Terms

Sets

A set may be thought of as a "collection of objects or items" (Peterson, 1971, p. 22). The entities which comprise a set are known as its elements.

Let A and B denote two sets:

Where two different elements of A always correspond to two different elements of B, there exists a one-to-one mapping of A into B (Redei, 1967, p. 4-5). Two sets A, B are equivalent if there exists a one-to-one mapping of A onto B, (Redei, 1967, p. 14). In this case there is said to be a one-to-one correspondence between A and B.

Natural Numbers

There exists a set of natural numbers (positive integers), $1, 2, 3, \dots$, with the following basic properties (axioms):

- (1) 1 is a natural number.
- (2) Every natural number n has a definite successor n' in the set of natural numbers.
- (3) For every n , $n' \neq 1$, i.e. there is no number with 1 as its successor.
- (4) If $m' = n'$, then $m = n$, i.e. for every number there exists no number with the former as its successor.
- (5) If a set M contains the element 1, and for every element n it

also contains its successor n' , then M contains all the natural numbers. (Redei, 1967, p. 14).

Sum of Two Numbers

With every pair of numbers x, y there can be associated in exactly one way a natural number $x + y$ such that

$$x + 1 = x' \text{ for every } x$$

$$x + y' = (x + y)' \text{ for every } x \text{ and every } y$$

This definition permits us to write $n + 1$ instead of n' . The following rules or arithmetic hold:

$$(a + b) + c = a + (b + c) \quad (\text{Associative Law of Addition})$$

$$a + b = b + a \quad (\text{Commutative Law of Addition})$$

$$a + b = a + c \text{ implies } b = c \quad (\text{Van der Waerden, 1970, p. 4})$$

Ordinal Number refers to the position of an element in a set, e.g. 5th.

Cardinal Number refers to the total number of elements in a set, e.g. a, b, c, d, e , the cardinal number 5 is associated with this set.

A binary operation associates a definite number with an ordered pair of numbers. Addition is an example of a binary operation.

An algorithm is a procedure or device used to find the number that the operation associates with the given pair of numbers. For example, the operation of addition associates with the number pair 6 and 4 the number 10; the operation of subtraction associates with the same pair the number 2. The procedures used to find the numbers 10 and 2 are called algorithms.

Early instruction in addition of numbers is usually given in terms of physical objects, e.g. a teacher will speak of adding two pieces of candy to three pieces of candy to obtain five pieces of candy. The actual operation being performed is not addition, it is union of sets, since addition is an operation on numbers, not physical objects. However, addition can be defined in terms of this operation on sets, (Peterson, 1971).

A verbal arithmetic problem has an additive structure if it is an instance of the union of two or more sets. For example, the following problems have an additive structure:

- (1) "Bob has three apples and Peter has four apples. How many apples do both children have?"
- (2) "There are three cats on a rug. Four more cats run onto the rug. Now how many cats are there?"

In problem (1) the union of the two sets is implied by the question only.

In problem (2) the union of the two sets is described.

Appendix B

Scoring Problems with Conservation Pretest

There were 11 subjects classified as Conservers, whose responses required decisions, with respect to their classification as Conservers or Transitionals. On three of the four tasks these 11 subjects had all clearcut conserving responses. On one subtask they each had one response which required a decision beyond the criteria presented. Five of these 11 subjects miscounted in one subtask, but gave all conserving judgments and explanations. In response to judgment of equality of sets: two subjects said "more", followed by a spontaneous change to "same". In response to explanation: four subjects said "I don't know", followed by a spontaneous conserving explanation.

Subject Attrition

Kindergarten children from seven schools participated in the study. From the 185 children available in the first six schools: 9 had previously participated as part of a larger pilot study sample; 15 had no parental permission; 12 were absent on Pretest days. These children were therefore excluded from the study, leaving 149 potential subjects. Due to the fact that for unknown reasons there were far fewer girls than boys in many classes, it was necessary to include a seventh school where all the kindergarten girls were tested (less one who did not receive parental permission), i.e. 15, plus two boys. This resulted in a sample of 166 subjects who were given the Conservation Pretest.

Following the Conservation Pretest five subjects were omitted from the study: two due to procedural errors; one because his knowledge of English was inadequate for the test; one because the experimenter was unable to elicit sufficient responses from the subject; one who failed to recognize the initial equality of the two sets in Phase I on two of the four tasks. This left a pool of 161 subjects for the study.

One subject was absent following the Conservation Pretest, for the remainder of the test sessions in that school. Twenty-two children were excluded following the experimental test: two were unable to perform the General Warmup task; three due to failures of the timer; two due to experimenter error in presentation of instructions; 13 due to experimenter error in the order of item presentation; one who started crying during presentation of the test items. One child was omitted since the teacher had

Subject Attrition (Cont'd.)

told the experimenter she was trying to encourage the child through experiences of success. After three incorrect test items the experimenter stopped the session, to attempt to prevent interfering with the teacher's efforts with this child.

In forming experimental groups with equal numbers of boys and girls at each Developmental Level, 20 children (mainly nonconservers) had been given the Conservation Pretest, but were not needed for the experimental test, and were therefore dropped from the study.

Instructions for Conservation PretestConservation Pretest Instructions Tasks 1 and 2:

Phase I:

"I am going to ask you some questions. First watch what I do."

(Experimenter placed buttons in one-to-one correspondence.)

"Are there as many red buttons as yellow buttons, or is there more of one kind?"

(If subject responds "Same", go to Phase II; if "Not the same", continue.)

"Look, there is one red button for every yellow button. Do you see now that there are as many red buttons as yellow buttons?"

Phase II:

"Watch what I do." (Experimenter spread one set, compressed the other.)

"Now are there as many red buttons as yellow buttons, or is there more of one kind?" "Why?"

Conservation Pretest Instructions Tasks 3 and 4:

Phase I:

"Watch what I do." (Experimenter placed nuts in one-to-one correspondence.)

"Are there as many nuts in this row/circle as there are in this row/circle, or are there more in one row/circle?"

(If subject responds "Same" go to Phase II; if "Not the same", continue.)

"Look, there is one nut here for every nut here. Do you see now that there are as many nuts in this row/circle as this row/circle?"

Phase II:

"Watch what I do." (Experimenter spread one set, compressed the other.)

"Now are there as many nuts in this row/circle as this row/circle or are there more in one row/circle?" "Why?"

Instructions for Experimental Task

General Warmup:

"I bet you are really good at adding numbers. Try these:
can you tell me what is 1 and 1?"

(If subject's response correct): "That's really good, you were right,
1 and 1 is 2."

(If subject's response incorrect): "That's a good try, you were close.
Try again, 1 and 1." (If incorrect again): "You're close, 1 and 1 is 2."

"What is 2 and 1?" (Followed by appropriate feedback.)

"What is 2 and 2?" (Followed by appropriate feedback.)

"You are really good at adding. I am going to ask you some problems
with numbers of animals. I want you to tell me how many animals
there are altogether each time. Remember to add each time."

Specific Warmup:

"I bet you are really good at _____ * things. In each of the problems
I want you to _____ * the animals and then tell me how many there are
altogether. Now _____ * squirrels."

(Dynamic Described Relation): "2 squirrels hop to 3 squirrels here."

(Static Described Relation): "2 squirrels sit there 3 squirrels here."

"How many squirrels are there altogether?" (Response followed by
appropriate feedback, as above.)

"In the rest of the problems I won't ask you how many there are altogether
any more. Remember to add the numbers each time. _____ * the animals
and then tell me how many animals there are altogether each time. Now
_____ * geese:

(Dynamic Described Relation): "4 geese swim to 2 geese here."

(Static Described Relation): "4 geese sleep there 2 geese here."

Before each test item: "Now _____ * this."

Following response to each test item: correct "That's right, it was _____."
incorrect "You were close, it was _____."

* Concrete plus Verbal Prompt Condition: "watch (ing)"

Imaginal plus Verbal Prompt Condition: "imagine (ing)"

Verbal Only Prompt Condition: "listen (ing) about"

Appendix F-1

Mean Number of Correct Responses on the Addition
Word Problems for Each Order of Items Across Trials

Order	Trial 1	Trial 2
1	6.33	6.50
2	4.83	6.67
3	5.17	5.50
4	7.50	7.33
5	5.83	7.00
6	5.17	5.50
7	10.33	9.83
8	7.50	8.17
9	8.33	8.50
10	5.33	5.83
11	5.83	7.00
12	8.67	8.67
13	7.33	7.17
14	7.50	6.50
15	6.67	5.83
16	7.67	9.17
17	8.33	7.83
18	6.67	8.33

Appendix F - 2
Mean Latencies in Seconds on the Addition Word Problems
for Each Order of Items across Trials

Order	Trial 1	Trial 2
1	7.04	6.05
2	7.71	6.98
3	7.56	6.64
4	7.23	8.11
5	6.88	6.38
6	9.87	9.50
7	7.46	7.35
8	8.31	7.37
9	8.43	9.70
10	5.13	4.49
11	6.53	5.31
12	8.05	7.75
13	8.54	6.80
14	4.82	5.14
15	8.01	7.24
16	7.09	7.05
17	8.30	7.79
18	6.42	6.65

Appendix F - 3

Analysis of Variance Summary Table for Order
(Number of Correct Responses)

Source	Sums of Squares	Degrees of Freedom	Mean Square	F
Mean	10951.13	1	10951.13	362.53
Order (O)	349.20	17	20.54	< 1
Trials (T)	6.69	1	6.69	3.76
Error S(O)	2718.67	90	30.21	
Order x Trials	36.31	17	2.14	1.20
Error ST(O)	160.00	90	1.78	

Appendix F - 4

Analysis of Variance Summary Table for Order
(Latencies)

Source	Sums of Squares	Degrees of Freedom	Mean Square	F
Mean	11238.08	1	11238.08	782.44
Order	305.49	17	17.97	1.25
Trials	8.34	1	8.34	3.57
Error S(O)	1292.65	90	14.36	
Order x Trials	27.81	17	1.64	< 1
Error ST(O)	210.44	90	2.34	

Analysis of Variance Summary Table for Number
of Correct Responses on the Addition Word Problems

Source	Sums of Squares	Degrees of Freedom	Mean Square	F	p <
Mean	10936.89	1	10936.89	847.64	
Developmental Level (D)	143.29	2	71.64	5.55	.006
Prompt Condition (P) in D	1272.30	6	212.00	16.43	.000
Described Relation (R) in $D_1 P_1$	1.04	1	1.04	<1	
Described Relation in $D_1 P_2$	.38	1	.38	<1	
Described Relation in $D_1 P_3$	.67	1	.67	<1	
Described Relation in $D_2 P_1$	1.04	1	1.04	<1	
Described Relation $D_2 P_2$	2.04	1	2.04	<1	
Described Relation in $D_2 P_3$	150.00	1	150.00	11.63	.001
Described Relation in $D_3 P_1$	.17	1	.17	<1	
Described Relation in $D_3 P_2$	145.04	1	145.04	11.24	.001
Described Relation in $D_3 P_3$	2.67	1	2.67	<1	
Sex (X) in DP	280.54	9	31.17	2.42	
Trials (T) in DP	12.54	9	1.39	<1	
R x X in DP	131.71	9	14.63	1.13	
R x T in DP	13.71	9	1.52	<1	
X x T in DP	7.21	9	.80	<1	
Error S(DIMX)	929.00	72	12.90		
R x X x T in DP	21.38	9	2.38	1.14	
Error ST(DIMX)	149.67	72	2.08		

Analysis of Variance Summary Table for Latencies
on the Addition Word Problems

Source	Sums of Squares	Degree of Freedom	Mean Square	F
Mean	11238.08	1	11238.08	642.49
Developmental Level (D)	16.79	2	8.39	< 1
Prompt Condition(P) in D	76.45	6	12.74	< 1
Described Relation (R) in DP	97.05	9	10.78	< 1
Sex (X) in DP	62.51	9	6.95	< 1
Trials (T) in DP	36.98	9	4.11	1.78
R x X in DP	85.95	9	9.55	< 1
R x T in DP	11.78	9	1.31	< 1
X x T in DP	26.28	9	2.92	1.26
Error S(DIMX)	1259.39	72	17.49	
R x X x T in DP	5.08	9	.56	< 1
Error ST(DIMX)	166.46	72	2.31	

Appendix I

Analysis of Variance Summary Table for Number of Nouns
Incidentally Recalled from Addition Word Problems

Source	Sums of Squares	Degrees of Freedom	Mean Square	F	p<
Mean	2831.57	1	2831.57	661.92	
Developmental Level (D)	1.69	2	.84	<1	
Prompt Condition (P) in D	90.83	6	15.14	3.54	.004
Described Relation (R) in DP	17.08	9	4.90	<1	
Sex (X) in DP	41.08	9	4.56	1.07	
R x X in DP	22.75	9	2.53	<1	
Error S(DIMX)	308.00	72	4.28		

Appendix J

Probabilities Associated With χ^2 Tests for Homogeneity
of Item Variances

	<u>Concrete plus Verbal</u>		<u>Imaginal plus Verbal, Verbal Only</u>	
	Trial 1 p<	Trial 2 p<	Trial 1 p<	Trial 2 p<
<u>Conservers</u>	.02	.005 *	.0055 *	.04
<u>Transitionals</u>	.11	.045	.092	.017
<u>Nonconservers</u>	.0000 *	.0000 *	.0002*	.0001*

	<u>Imaginal plus Verbal</u>		<u>Verbal Only</u>	
	Trial 1 p<	Trial 2 p<	Trial 1 p<	Trial 2 p<
<u>Conservers</u> and <u>Transitionals</u> <u>Combined</u>	.10	.004 *	.003 *	.008 *

* p < .01

Analysis of Variance Summary Table for
RSQ Values for Counting Models

Source	Sums of Squares	Degrees of Freedom	Mean Square	F	p<
Mean	9.96	1	9.96	103.52	
Developmental Level (D)	.18	2	.09	<1	
Prompt Condition (P) in D	1.94	6	.32	3.36	
Described Relation (R) in DP	1.45	9	.16	1.67	
Sex (X) in DP	1.34	9	.15	1.55	
Trials (T) in DP	.41	9	.05	<1	
Models (M) in $D_1 P_1$	.16	2	.08	1.59	
Models in $D_1 P_2$	.1	2	.05	<1	
Models in $D_1 P_3$	.1	2	.05	<1	
Models in $D_2 P_1$	.72	2	.36	6.95	.001
Models in $D_2 P_2$	.01	2	.01	<1	
Models in $D_2 P_3$	.06	2	.03	1	
Models in $D_3 P_1$	1.06	2	.53	10.30	.000
Models in $D_3 P_2$	.03	2	.02	<1	
Models in $D_3 P_3$	.03	2	.02	<1	
R x X in DP	1.31	9	.15	1.52	
R x T in DP	.41	9	.05	<1	
X x T in DP	.46	9	.05	<1	
R x M in DP	1.06	18	.06	1.14	
X x M in DP	.90	18	.05	<1	
T x M in DP	.32	18	.02	<1	

Source	Sums of Squares	Degrees of Freedom	Mean Square	F
Error S(DIMX)	6.93	72	.10	
R x X x T in DP	.66	9	.07	1.27
R x X x M in DP	.97	18	.05	1.04
R x T x M in DP	.38	18	.02	< 1
X x T x M in DP	.64	18	.04	1.18
Error ST(DIMX) x Trials	4.13	72	.06	
Error SR(DIMX) x Models	7.44	144	.05	
R x X x T x M in DP	.40	18	.02	< 1
Error STR(DIMX)	4.33	144	.03	

Analysis of Variance Summary Table for Item Type
(Number of Correct Responses)

Source	Sums of Squares	Degrees of Freedom	Mean Squares	F	p<
Mean	10936.89	1	10936.89	847.64	
Developmental Level (D)	143.29	2	71.64	5.55	.006
Prompt Condition (P) in D	1272.03	6	212.00	16.43	.000
Described Relation (R) in DP	303.04	9	33.67	2.61	
Sex (X) in DP	280.54	9	31.17	2.42	
Item Type (I) in $D_1 P_1$	.38	1	.38	<1	
Item Type in $D_1 P_2$	15.04	1	15.04	4.42	
Item Type in $D_1 P_3$	28.17	1	28.17	8.29	.005
Item Type in $D_2 P_1$	1.04	1	1.04	<1	
Item Type in $D_2 P_2$	1.04	1	1.04	<1	
Item Type in $D_2 P_3$	10.67	1	10.67	3.14	
Item Type in $D_3 P_1$	1.50	1	1.50	<1	
Item Type in $D_3 P_2$	18.38	1	18.38	5.41	
Item Type in $D_3 P_3$	28.17	1	28.17	8.29	.005
R x X in DP	131.71	9	14.63	1.13	
R x I in DP	30.54	9	3.39	1.00	
X x I in DP	23.71	9	2.63	<1	
Error S(DPRX)	929.00	72	12.90		
R x X x I in DP	10.88	9	1.21	<1	
Error SI(DPRX)	245.00	72	3.40		

Analysis of Variance Summary Table for Item Type
(Latencies)

Source	Sums of Squares	Degrees of Freedom	Mean Square	F	p<
Mean	11219.42	1	11219.42	642.09	
Developmental Level (D)	17.04	2	8.52	<1	
Prompt Condition (P) in D	76.51	6	12.75	<1	
Described Relation (R) in DP	97.15	9	10.79	<1	
Sex (X) in DP	63.13	9	7.01	<1	
Item Type (I) in $D_1 P_1$	.51	1	.51	<1	
Item Type in $D_1 P_2$	10.77	1	10.77	7.57	.008
Item Type in $D_1 P_3$	16.68	1	16.68	11.73	.001
Item Type in $D_2 P_1$	.73	1	.73	<1	
Item Type in $D_2 P_2$	1.24	1	1.24	<1	
Item Type in $D_2 P_3$	4.23	1	4.23	2.97	
Item Type in $D_3 P_1$	1.40	1	1.40	<1	
Item Type in $D_3 P_2$	1.27	1	1.27	<1	
Item Type in $D_3 P_3$	9.89	1	9.89	6.96	.01
R x X in DP	86.15	9	9.57	<1	
R x I in DP	23.84	9	2.65	1.86	
X x I in $D_1 P_1$	.13	1	.13	<1	
X x I in $D_1 P_2$	.71	1	.71	<1	
X x I in $D_1 P_3$	11.35	1	11.35	7.99	.006
X x I in $D_2 P_1$	.61	1	.61	<1	
X x I in $D_2 P_2$	3.59	1	3.59	2.53	

Analysis of Variance Summary Table for Item Type
(Latencies) Contd.

Source	Sums of Squares	Degrees of Freedom	Mean Square	F	p<
X x I in D ₂ P ₃	5.28	1	5.28	3.72	
X x I in D ₃ P ₁	.27	1	.27	<1	
X x I in D ₃ P ₂	.52	1	.52	<1	
X x I in D ₃ P ₃	13.11	1	13.11	9.23	.003
Error S(DPRX)	1258.07	72	17.47		
R x X x I in DP	12.47	9	1.39	<1	
Error SI(DPRX)	102.37	72	1.42		

Appendix M- 1

Mean Age (in months) as a Function of Developmental
Level and Experimental Conditions

	Concrete plus Verbal		Imaginal plus Verbal		Verbal Only	
	Dynamic	Static	Dynamic	Static	Dynamic	Static
Conservers						
Boys	70.33	72.67	73.67	68.67	72.00	72.33
Girls	68.67	70.00	70.33	72.67	68.67	73.33
Transitionals						
Boys	67.67	71.67	71.67	71.33	71.67	69.67
Girls	72.00	68.33	70.67	70.67	69.33	74.00
Nonconservers						
Boys	71.33	70.00	72.33	65.67	68.67	74.00
Girls	69.00	72.00	70.33	73.00	69.67	72.00

Appendix M - 2

Analysis of Variance Summary Table for Age

Source	Sums of Squares	Degrees of Freedom	Mean Square	F
Mean	541875.00	1	541875.00	41358.66
Developmental Level (D)	4.22	2	2.11	<1
Prompt Condition (P) in D	24.28	6	4.05	<1
Described Relation (R) in DP	54.50	9	6.06	<1
Sex (X) in DP	89.83	9	9.98	<1
R x X in DP	218.83	9	24.31	1.86
Error S(DPRX)	943.33	72	13.10	

Correlation Between the Number of Nouns Recalled and the
Number of Correct Responses on the Addition Word Problems

	Concrete plus Verbal	Imaginal plus Verbal	Verbal Only
Conservers			
Trial 1	.23	.05	-.03
Trial 2	.14	.06	-.11
Transitionals			
Trial 1	-.30	-.04	.29
Trial 2	.09	-.21	.06
Nonconservers			
Trial 1	-.06	.25	-.06
Trial 2	-.28	.17	.14