SUBSURFACE CURRENTS IN THE STRAIT OF GEORGIA,
WEST OF STURGEON BANK

by

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ABSTRACT

Spectral characteristics of sub-surface current and temperature records spanning 310-533 days from the Strait of Georgia near Sturgeon Bank are examined. Of particular interest are low-frequency fluctuations having a characteristic period of 30 days. Typically, the energy of the low-frequency and tidal currents are equal at 50m (each about 40% of the total), and 10% and 70%, respectively, at 200m.

Low-frequency currents are observed to be baroclinic and possess a lateral length scale less than 10km. Their generation mechanisms are not identified. The low-frequency fluctuations in sea-level and current have poor-to-fair coherence, while those in air-pressure and current have poor coherence. Low-frequency fluctuations in wind and current have fair coherence at the eastern mooring, and poor coherence at the remaining two moorings.

Fluctuations in water temperature have a characteristic period of 300 days. Water temperatures at different locations generally have poor coherence except at the lowest frequencies where they are fair-to-good. Water temperature and current at the same location have poor coherence.

Tidal currents are found to be ellipses, with the largest constituents being M_2 and K_1 . The magnitude, shape and orientation of the tidal current ellipses are time-dependent. Internal tides are suggested as an explanation.

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Chapter 1

INTRODUCTION

This thesis deals with the analysis of subsurface water currents and temperatures west of Sturgeon Bank in the Strait of Georgia. A shortened version appears in Chang et al. (1976).

The Strait of Georgia (see Figure 1) is adjacent to a large centre of population and industry. It is the commercial seaway to the Port of Vancouver and is an important fishing area. Its use as a recreational area is still growing. For these reasons, the water properties of the Strait of Georgia have been relatively well studied.

Hutchinson and Lucas (1931) described the water properties, the nutrient concentrations and phytoplankton populations in Strait of Georgia during the four summers of 1926-1929. A more extensive study was conducted by Tully and Dodimead (1957) using data collected by N.M. Carter and J.F. Tully in 1931 and 1932. Waldichuk (1957) gave a comprehensive treatment of the physical oceanography of the Strait of Georgia, based mainly on data taken during 1949-1953. Though not examined in detail, there are oceanographic data for the Strait of Georgia and Juan de Fuca Strait collected regularly during 1968 at intervals of approximately 5 weeks (Crean and Ages, 1971).

Information on the circulation in the Strait of Georgia is more sparse, due to the fact that current measurements are more difficult to make than customary oceanographic measurements, and require much more time, effort, and resources. The surface circulation, being more easily measured, was the first to be examined. Waldichuk (1958) described the surface circulation in the Strait of Georgia based on drift bottle data collected by W. Clemens during the five summers of 1926-1929 and 1931. Much more recently, Tabata (1972) deduced the surface circulation near the Fraser River mouth from aerial photographs collected in June, 1950. The behavior of the Fraser River plume was examined by Giovando and Tabata (1970) for the summers of 1966 and 1967, and by Tabata et al. (1971b) during the spring and summer of 1968. Most recently, Cordes (in progress) at the Institute of Oceanography, U.B.C., conducted an intensive survey of the plume behavior in the spring of 1974.

The subsurface circulation in the Strait of Georgia was first examined in the summer of 1953 by Waldichuk (1957). Measurements were done over a 25 hour period from an anchored vessel using an Ekman current meter at a number of scattered locations in the Strait. Pickard (1956) measured the surface and bottom currents for 25 hours at each of several stations on a line from Galiano Island to Point Roberts. Vertical profiles of the current using a Savonius rotor from an anchored ship were obtained during 1967-1969 by Tabata et al. (1970a), (1970b), (1970c), (1971b). Measurements down to 50m were

made; stations were occupied for as long as 90 hours; anchorage sites were in the region between Point Grey and Point Roberts.

The development of reliable self-recording current meters radically increased the time and space coverage of current measurements. Self-recording current meters have been available for some time, but they were either unreliable or the data processing was very time consuming. Huggett (1966) was perhaps the first to report on the usage of such equipment in the Strait of Georgia. He employed these meters from moored cables at three stations between Nanaimo and Sechart for periods up to 30 days. The Department of the Environment (1972a), (1972b), (1972c), (1973) continued these measurements in the years 1969-1972. Measurements were taken with moorings that were positioned along lines running across the Strait of Georgia. These lines of moorings were placed as far south as Point Roberts and as far north as Texada Island.

In 1968 Tabata and Stickland (1972d) initiated a program to provide an extensive time coverage of the subsurface currents west of Sturgeon Bank. The program attempted to gather water current and temperature data over a period of 18 months at each of six locations. Tabata et al. (1971a) reported on the success of this program and on the performance of the self-recording current meters used. The data themselves were presented in the form of histograms and progressive vector diagrams by Tabata and Stickland (1972a), (1972b), (1972c).

The supplementary analysis of this body of data is the subject of this thesis. Definitions and mathematical tools

used are presented in the next chapter. Examples, details and peculiarities of the data are shown and discussed in Chapter 3. In the succeeding chapter, the frequency representation of the observations is examined. Attention is then focussed on the variations contained in specific frequency bands. Chapter 5 deals with the variations having periods from two days to about one year. The tidal oscillations are examined separately in Chapter 6. A summary of the results is given in the final chapter.

Chapter 2

DEFINITIONS AND METHODS

The functions to be calculated for the spectral analysis are defined in this chapter. The theory is not comprehensive and the reader should turn to the references cited for more details. The material in the first section is largely taken from Moccers (1973). Jenkins and Watts (1968) and Bendat and Piersol (1971) give a much more complete treatment of stochastic processes. A very cursory presentation of harmonic analysis by the least-squares method is contained in Chapter 6. The theoretical background of least-squares estimation can be found in Chapter 4 of Jenkins and Watts (1968). Godin (1972) discusses in depth the harmonic analysis by least-squares methods and by other methods. Brigham (1974) describes the Fast Fourier Transform algorithm. He also gives an excellent explanation of the differences between Fourier transforms of continuous functions defined over all time, and discrete frequency-limited Fourier transforms of discrete functions defined at a finite number of time points.

2.1 Frequency Representation of Time-Varying Functions

Continuous and Discrete Fourier Transforms

The time-varying horizontal current vector is resolved

into its real-valued components, $\underline{u}(t)$ and $\underline{v}(t)$. Positive $\underline{u}$ and $\underline{v}$ denote the true east and true north components, respectively. The components are regarded as superpositions of sinusoids having frequencies f :-

$$\begin{aligned}\underline{u}(t) &= \int_{-\infty}^{\infty} [\underline{B}_1(f) \cdot \cos(2\pi ft) + \underline{B}_2(f) \cdot \sin(2\pi ft)] df \\ &= \int_{-\infty}^{\infty} \underline{U}(f) \cdot \exp(i2\pi ft) df \\ \underline{v}(t) &= \int_{-\infty}^{\infty} [\underline{B}_3(f) \cdot \cos(2\pi ft) + \underline{B}_4(f) \cdot \sin(2\pi ft)] df \\ &= \int_{-\infty}^{\infty} \underline{V}(f) \cdot \exp(i2\pi ft) df\end{aligned}\quad (2.1)$$

where $i^2 = -1$ and $-\infty \leq t \leq \infty$. The $\underline{B}_i(f)$'s are real-valued while $(\underline{U}(f), \underline{V}(f))$ are called the Fourier transforms of $(\underline{u}(t), \underline{v}(t))$:-

$$\begin{aligned}\underline{U}(f) &= \int_{-\infty}^{\infty} \underline{u}(t) \cdot \exp(-i2\pi ft) dt \quad (-\infty \leq f \leq \infty) \\ \underline{U}(\pm f) &= [\underline{B}_1(f) \pm \underline{B}_2(f)]/2 \quad (0 \leq f \leq \infty)\end{aligned}\quad (2.2)$$

and similarly for $\underline{V}(f)$ and $\underline{V}(\pm f)$. The $\underline{B}_i(f)$'s are defined only over positive frequencies (one-sided transform) while $\underline{U}(f)$ and $\underline{V}(f)$ are defined over all frequencies (two-sided transforms).

In practice, one is usually interested in functions which are known only over a finite period of time, and which are defined to be zero elsewhere. To this end, the function $g(t)$ is introduced. $g(t)$ is unity over the region $|t| < T/2$ and zero elsewhere, while $\underline{u}(t)$ is defined to be $\underline{u}(t) \cdot g(t)$. The Fourier transform, $\underline{U}(f)$, of $\underline{u}(t)$ is $\underline{U}(f) * G(f)$, where the symbol $*$ denotes the convolution operator:-

$$\underline{U}(f) * G(f) = \int_{-\infty}^{\infty} \underline{U}(f') \cdot G(f-f') df' \quad (2.3)$$

and $G(f) = 2T \cdot \sin(\pi Tf) / (\pi Tf)$ is the Fourier transform of $g(t)$.

In general, $\underline{U}(f) \neq \underline{U}(f)$ and one must use caution in the choice of T so that $\underline{U}(f)$ is an usable approximation of $\underline{U}(f)$. As a result of the convolution with $G(f)$, the (amplitude,width) of peaks in $\underline{U}(f)$ are (smaller,larger) relative to those in $\underline{U}(f)$. One consequence is that two lines of equal amplitude, $\delta(f-f_1)$ and $\delta(f-f_2)$ (δ denotes the Dirac delta function), in $\underline{U}(f)$ will not be resolved in $\underline{U}(f)$ unless $(f_1-f_2)^{-1} < T$. If one of the lines has a much larger amplitude than the other, then T must be much larger than $(f_1-f_2)^{-1}$ for the two lines to be resolved in $\underline{U}(f)$. Another consequence is that a low-frequency line $\delta(f-f_3)$ ($f_3 \approx 0$) will not be resolved in $\underline{U}(f)$ unless $f_3^{-1} < T$.

Computationally, one deals with finite sets of numbers. Thus both the time-varying function and its transform are discrete functions. Consider the finite discrete time-series $u(r\Delta t)$ of length $T=2n\Delta t$ which is a superposition of sinusoids:-

$$\begin{aligned} u(r\Delta t) &= \sum_{m=0}^n [E_1(m\Delta f) \cdot \cos(\dots) + B_2(m\Delta f) \cdot \sin(\dots)] \\ &= \sum_{m=-n}^n U(m\Delta f) \cdot \exp[i(\dots)] \end{aligned} \quad (2.4)$$

where $(r=-n, \dots, 0, 1, \dots, n-1)$, $\Delta f = T^{-1}$ and $(\dots) = (2\pi \cdot m\Delta f \cdot r\Delta t)$. $U(m\Delta f)$, the discrete complex-valued Fourier transform of $u(r\Delta t)$, is defined over both positive and negative frequencies, while the real-valued $B_i(m\Delta f)$'s are defined only over positive frequencies:-

$$\begin{aligned} U(\pm m\Delta f) &= (2n)^{-1} \sum_{r=-n}^n u(r\Delta t) \cdot \exp[\mp i(\dots)] \\ &= [B_1(m\Delta f) \mp iB_2(m\Delta f)]/2 \end{aligned} \quad (2.5)$$

where $(m=1, \dots, n)$, $(\dots) = (2\pi \cdot m\Delta f \cdot r\Delta t)$, and $U(0) = B_1(0)$. Similar

relations hold for the component $v(r\Delta t)$ and its discrete Fourier transform $V(\pm m\Delta f) = [B_3(m\Delta f) \mp iB_4(m\Delta f)]/2$.

One does not choose the sampling interval Δt in a haphazard manner. Let $u(t)$ denote the continuous time-series from which $u(r\Delta t)$ is derived by sampling at $t=r\Delta t$ ($r=-n, \dots, 0, 1, \dots, n-1$). If there are variations in $u(t)$ which have frequencies higher than $f_n = (2\Delta t)^{-1}$, aliasing (appearance of high frequency energy at lower frequencies) occurs because:-

$$\exp[i2\pi(kf_n \pm f)t] = \exp(\pm i2\pi ft)$$

where

$$\begin{aligned} k &= 2, 4, 6, 8, \dots \\ f &= m\Delta f; \quad m=1, 2, \dots, n-1 \\ t &= r\Delta t; \quad r=-n, \dots, 0, 1, \dots, n-1 \end{aligned} \quad (2.6)$$

Hence to prevent aliasing, the highest frequency in $u(t)$ must be less than $f_n = (2\Delta t)^{-1}$, the Nyquist frequency.

In equations (2.4) and (2.5), it appears that $u(r\Delta t)$ is only defined for $|t| \leq n\Delta t$, and that its discrete Fourier transform $U(m\Delta f)$ is only defined for $|f| \leq f_n$. In fact, the discrete Fourier transform implicitly assumes that $u(r\Delta t)$ is periodic and defined over $-\infty \leq t \leq \infty$, and that one complete period is contained in $-n\Delta t \leq t \leq (n-1)\Delta t$. In fact, the discrete Fourier transform is also periodic and defined over $-\infty \leq f \leq \infty$, and that one complete period is contained in $-f_n \leq f \leq +f_n$. Brigham (1974) explains clearly the necessity of such definitions.

The Elliptical Interpretation

The time-varying current vector may be considered as a

complex horizontal velocity $w(r\Delta t)$:-

$$\begin{aligned} w(r\Delta t) &= u(r\Delta t) + iv(r\Delta t) \\ &= \sum_{m=-n}^n W(m\Delta f) \cdot \exp(i2\pi \cdot m\Delta f \cdot r\Delta t) \end{aligned} \quad (2.7)$$

where the imaginary axis points to true north and where $W(m\Delta f)$ is the complex discrete Fourier transform of $w(r\Delta t)$:-

$$\begin{aligned} W(m\Delta f) &= \sum_{r=-n}^{n-1} w(r\Delta t) \cdot \exp(-i2\pi \cdot m\Delta f \cdot r\Delta t) \\ &= U(m\Delta f) + iV(m\Delta f) \end{aligned} \quad (2.8)$$

Examination of equation (2.7) indicates that $w(r\Delta t)$ is a superposition of a stationary vector $W(0)$, and n rotating vectors $E(r\Delta t, m\Delta f)$, each of whose tip traces cut an ellipse in the horizontal complex plane. Any of these ellipses can be written as:-

$$E(r\Delta t, m\Delta f) = W(m\Delta f) \cdot \exp[i(\dots)] + W(-m\Delta f) \cdot \exp[-i(\dots)] \quad (2.9)$$

where $(m=1, \dots, n)$, where $(r=-n, \dots, 0, 1, \dots, n-1)$ and where $(\dots) = (2\pi \cdot m\Delta f \cdot r\Delta t)$. Each vector $E(r\Delta t, m\Delta f)$ traces cut an ellipse in a period of $(m\Delta f)^{-1}$. Each of these vectors may be regarded as a superposition of a clockwise and anti-clockwise rotating vector. This is emphasized by writing:-

$$W(m\Delta f) = \begin{cases} A(m\Delta f) \cdot \exp[i\phi(m\Delta f)] & m=1, \dots, n \\ C(m\Delta f) \cdot \exp[-i\theta(m\Delta f)] & m=-n, \dots, -1 \end{cases} \quad (2.10)$$

The (amplitude, phase) of the anti-clockwise and clockwise vectors are (A, ϕ) and (C, θ) , respectively. These are defined at positive and negative frequencies, respectively.

Each vector $E(rat, m\Delta f)$ traces out an ellipse with a semi-major axis length of $A+C$ and a semi-minor axis length of $|A-C|$. The sign of $(A-C)$ determines the sense of rotation. $E(rat, m\Delta f)$ rotates clockwise or anti-clockwise (as seen from above) if the sign of $(A-C)$ is negative or positive, respectively. The semi-major axis has an orientation of $\alpha = (\phi - \theta)/2$ relative to the real x-axis. At $rat=0$, the two rotating vectors with amplitudes of (A, C) have an orientation of $(+\beta, -\beta)$ relative to the semi-major axis, where $\beta = (\phi + \theta)/2$.

The components (u, v) of $w(rat)$ are resolved using the coordinate system $(\hat{i}, \hat{j})$, where $\hat{i}$ and $\hat{j}$ are unit vectors directed towards true east and true north, respectively. The parameters A , C and β are invariant under rotation of the coordinate system (i, j) . Although α is dependent on the coordinate system used to resolve $w(rat)$, the semi-major axis orientation, relative to true north, is invariant under rotation of $(\hat{i}, \hat{j})$.

It is the $B_i(m\Delta f)$'s of equation (2.4) that are computed with the Fast Fourier Transform algorithm (Singleton, 1969). From these A , C , ϕ and θ are calculated according to:-

$$\begin{aligned} A^2 &= [(B_1 + B_4)^2 + (B_3 - B_2)^2] / 4 \\ C^2 &= [(B_1 - B_4)^2 + (B_3 + B_2)^2] / 4 \\ \tan(\phi) &= (B_3 - B_2) / (B_1 + B_4) \\ \tan(-\theta) &= (B_3 + B_2) / (B_1 - B_4) \end{aligned} \tag{2.11}$$

where ϕ and θ are restricted to the range $(-\pi, \pi)$, and where the arguments $(m\Delta f)$ of the parameters have been dropped.

Power and Cross Spectrum

The complex vector cross spectrum between the two horizontal complex velocities, $w_1(t)$ and $w_2(t)$, is $S_{12}(f)$:-

$$S_{12}(f) = \langle W_1^* W_2 / \Delta f \rangle \quad (2.12)$$

$$= \begin{cases} \langle A_1 A_2 \cdot \exp(-i\Delta\phi_{12}) / \Delta f \rangle & (0 < f < f_n) \\ \langle C_1 C_2 \cdot \exp(i\Delta\theta_{12}) / \Delta f \rangle & (-f_n < f < 0) \end{cases}$$

where $W_1(f)$ is the discrete Fourier transform of $w_1(t)$ as defined in equation (2.8), and where $*$ denotes the complex conjugate, $\Delta f = T^{-1} = (2n\Delta t)^{-1}$, $\Delta\phi_{12} = \phi_1 - \phi_2$ and $\Delta\theta_{12} = \theta_1 - \theta_2$. It is understood that the symbols f and t stand for discrete frequencies and time, respectively. The $\langle \cdot \rangle$ operator denotes an average over a number of consecutive frequencies. Because of the $\langle \cdot \rangle$ operator, $S_{12}(f)$ is defined over frequencies which are fewer in number than the original $2n+1$ ones. The cross spectrum may also be written as a combination of two real-valued functions:-

$$S_{12}(f) = Cc_{12}(f) + iQ_{12}(f)$$

where

$$Cc_{12}(f) = \begin{cases} \langle A_1 A_2 \cdot \cos(\Delta\phi_{12}) / \Delta f \rangle & (0 < f < f_n) \\ \langle C_1 C_2 \cdot \cos(\Delta\theta_{12}) / \Delta f \rangle & (-f_n < f < 0) \end{cases}$$

$$Q_{12}(f) = \begin{cases} \langle -A_1 A_2 \cdot \sin(\Delta\phi_{12}) / \Delta f \rangle & (0 < f < f_n) \\ \langle C_1 C_2 \cdot \sin(\Delta\theta_{12}) / \Delta f \rangle & (-f_n < f < 0) \end{cases} \quad (2.13)$$

$Cc_{12}(f)$ and $Q_{12}(f)$ are the co-spectrum and quadrature spectrum, respectively. The function $|S_{12}(f)|$ shows whether the $w_1(t)$ amplitude of motions in a particular frequency band is

associated with a large or small $w_2(t)$ amplitude in the same frequency band. This association can be further divided into associations of an in-phase component, $Co_{12}(f)$, and a quadrature component (90° phase difference), $Q_{12}(f)$.

The complex-valued function, $S_{12}(f)$, can be alternatively represented by two other real-valued functions, the vector coherency-squared spectrum, $\gamma_{12}^2(f)$:-

$$\begin{aligned}\gamma_{12}^2(f) &= |S_{12}(f)|^2 / [S_{11}(f) \cdot S_{22}(f)] \quad (-f_n < f < f_n) \\ &= [Co_{12}^2(f) + Q_{12}^2(f)] / [S_{11}(f) \cdot S_{22}(f)]\end{aligned}\quad (2.14)$$

and the vector phase spectrum, $\chi_{12}(f)$:-

$$\tan[\chi_{12}(f)] = -Q_{12}(f) / Co_{12}(f) \quad (-f_n < f < f_n) \quad (2.15)$$

whose range is $(-\pi, \pi)$. The normalized $|S_{12}(f)|$ spectrum is just the $\gamma_{12}(f)$ spectrum which is restricted to the range $(0, 1)$. The interpretation of $\gamma_{12}(f)$ and $|S_{12}(f)|$ is identical but the former may be easier to work with because of its limited range.

The real-valued vector power spectrum, $S_{jj}(f)$, of $w_j(t)$ is:-

$$\begin{aligned}S_{jj}(f) &= \langle |W_j(f)|^2 / \Delta f \rangle \quad (-f_n < f < f_n) \\ &= \begin{cases} \langle A_j^2(f) / \Delta f \rangle & (0 < f < f_n) \\ \langle C_j^2(f) / \Delta f \rangle & (-f_n < f < 0) \end{cases}\end{aligned}\quad (2.16)$$

The $S_{jj}(f)$ spectrum is defined in terms of a $|W_j(f)|^2$ average over b consecutive frequencies (the zero frequency is not one of these). Since $W_j(f)$ is defined over $2n+1$ frequencies spaced

Δf apart, $S_{jj}(f)$ is defined over $2k+1=(2n/b)+1$ frequencies (f_p) spaced $b\Delta f$ apart (it is assumed that n/b is also an integer):-

$$f_p = (pb+1)\Delta f/2 \quad p = -k, \dots, 0, 1, \dots, k \quad (2.17)$$

so that:-

$$\sum_{p=-k}^k S_{jj}(f_p) b\Delta f = \text{variance}(u_j) + \text{variance}(v_j) \quad (2.18)$$

where it is understood that $w_j(t) = u_j(t) + iv_j(t)$ has a zero mean. Hence, $S_{jj}(f)$ indicates how the combined variance of the $w_j(t)$ components is distributed with frequency.

The ultimate resolution of the $S_{jj}(f)$ spectrum is $(2n\Delta t)^{-1} = T^{-1}$ with no averaging ($b=1$). This resolution of T^{-1} arises because the $S_{jj}(f)$ spectrum is defined in terms of the discrete Fourier transform $W_j(f)$, which also has a resolution of T^{-1} . There are other methods of defining the $S_{jj}(f)$ spectrum which yield better resolutions than T^{-1} . Lacroix (1971) discusses methods which have resolutions proportional to $T^{-3/2}$ and T^{-2} .

As stated previously, an horizontal two-dimensional coordinate system is used where its unit vectors $(\hat{i}, \hat{j})$ are pointed (true east, true north). The functions (u_1, v_1) are the components of the current vector w_1 resolved along $(\hat{i}, \hat{j})$. The (real, imaginary) axes of the complex plane also coincide with $(\hat{i}, \hat{j})$. Rotation of $(\hat{i}, \hat{j})$ implies an equal rotation of the complex plane axes. It is assumed that the same coordinate system is used to measure (u_1, v_1) and (u_2, v_2) . The parameters A_j , C_j , $\Delta\phi_{12}$ and $\Delta\theta_{12}$ in equations (2.12) and (2.13) are not affected by a rotation of $(\hat{i}, \hat{j})$. Hence Co_{12} , Q_{12} , S_{12} , S_{jj} ,

χ_{12} and X_{12} are invariant under a rotation of $(\hat{i}, \hat{j})$.

2.2-----Stochastic Processes

Random Variables

In the majority of cases, the functions under study are not deterministic. That is, one is unable to exactly predict their future values from their past values. With this in mind, the observed value of $w_j(t_0)$ can be regarded as one member from an infinite set of possible values. The observation $w_j(t_0)$ is described by the random variable $w_j(t_0)$ which has an associated probability distribution giving the probability of observing the value $w_j(t_0)$.

The complex velocity $w_j(t)$ is described by the ordered infinite set of random variables $\{w_j(t); -\infty < t < \infty\}$ which is called a stochastic process. The complex velocity $w_j(t)$ is regarded as a realization of the stochastic process $\{w_j(t)\}$.

If the stochastic process $\{w_j(t)\}$ is stationary, then a realization, $w_j(t)$, is in 'statistical equilibrium'. That is, the average properties (mean, variance, power spectra, etc.) of $w_j(t)$ during two different time intervals are not significantly different. Significant differences are those that are larger than the expected differences inherent in statistical sampling. Furthermore, $w_j(t)$ contains no trends over the time interval considered.

Theoretical Spectral Functions

The sample (calculated) functions $W_j(f)$ and $S_{jj}(f)$, may also be described by the ordered sets of random variables, $\{W_J(f)\}$ and $\{S_{JJ}(f)\}$ respectively. The expected value of the random variable $\{S_{JJ}(f)\}$ is denoted by $E\{S_{JJ}(f)\}$. It is the average power spectrum, averaged over the spectra of all possible realizations $w_j(t)$ with record length T . The theoretical vector power spectrum, $\Gamma_{JJ}(f)$, is defined as:-

$$\Gamma_{JJ}(f) = \lim_{T \rightarrow \infty} E\{S_{JJ}(f)\} \quad (2.19)$$

It is the average power spectrum, averaged over the spectra of all possible realizations $w_j(t)$ with infinite record length. The theoretical spectrum $\Gamma_{JK}(f)$ corresponding to $S_{jk}(f)$ is defined in a similar manner. The theoretical spectra corresponding to $C_{jk}(f)$, $Q_{jk}(f)$, $\gamma_{jk}^2(f)$ and $\chi_{jk}(f)$ are defined in terms of $\Gamma_{JJ}(f)$, $\Gamma_{KK}(f)$ and $\Gamma_{JK}(f)$.

Confidence Intervals

Regardless of the probability distribution of the stationary stochastic process $\{w_J(t)\}$, the $\{S_{JJ}(f)\}$ process does tend (for large ν) to be distributed according to a χ_ν^2 distribution with $\nu=2b$ degrees of freedom ($S_{jj}(f)$ is a $|W_j(f)|^2$ average over b consecutive frequency bands). This is an indirect result of the Central Limit Theorem. Hence the confidence limits for $S_{JJ}(f)$ can be constructed with the aid of χ_ν^2 tables.

The construction of confidence limits for the theoretical coherency and phase spectra is not as straight forward. Let $\gamma_{JK}(f)$ and $\chi_{JK}(f)$ denote the random variables describing the realizations, $\gamma_{jk}(f)$ and $\chi_{jk}(f)$ respectively. The confidence limits for the theoretical coherency spectrum is a function of frequency. However, the random variable $Y_{JK}(f) = \text{arctanh}[\gamma_{JK}(f)]$ has a variance of $(2b)^{-1}$ which is independent of frequency. With the reasonable assumption that $Y_{JK}(f)$ has a Normal distribution, confidence limits for the theoretical coherence spectrum can be constructed. The confidence limits for the theoretical phase spectrum is dependent on the theoretical coherency spectrum. The 95% confidence limits for the theoretical phase spectrum are given in Figure 9.3 of Jenkins and Watts (1968), which is reproduced later in Figure 13 (page 50). According to Granger and Hatanaka (1964), these confidence limits are crude approximations.

Chapter 3

THE DATA

This chapter is concerned with the data records, their peculiarities and their preliminary treatment. The current and water temperature records were collected by Tabata and Stickland (1972a), (1972b), (1972c). The winds, sea elevation and air pressure were also obtained from Tabata who obtained them from the Atmospheric Environment Service. The Fraser River discharge data were supplied by the Water Survey of Canada (Inland Waters Directorate, Pacific Region).

3.1 The Currents and Water Temperatures

The locations of the current meters are shown in Figure 2, and the time coverage in Figure 3. Most of the data records were collected with the Aanderaa (Bergen) Model 4 self-recording current meter, and several with the Geodyne Model 850. The Aanderaa meters record the current and water temperature every 10 minutes on magnetic tape, while the Geodyne meters are only equipped to sample the current every 15 minutes. For the Geodyne each value every 15 minutes consists of an average of 15 values sampled every 5 seconds. A subsurface buoy mooring was used until the latter part of April 1970, after which a surface buoy taut rope mooring was used so

Figure 1. Map of Vancouver Island and vicinity.

Figure 3. Schematic showing time coverage of the current and water temperature records. Geodyne current meters recorded only the current vector every 15 minutes, while Bergen current meters recorded both the current vector and water temperature every 10 minutes. Types of mooring employed are also indicated (after Tabata *et. al.*, 1971a).

that the near surface currents could be measured. Tabata et al. (1971a) give details on the mooring systems, the accuracy of the meters and their performance.

Briefly, the preliminary treatment of the data consisted of the following steps. Suspect data values in the original data records were corrected to yield the edited data records. These edited data records have sampling intervals of 10 or 15 minutes. The edited data records were time-averaged to yield the half-hourly data records, records with a sampling interval of 30 minutes. All the half-hourly data records for the same location were then joined to yield a single half-hourly time-series of much greater length.

Editing

The original data records are those that have already undergone one step of translation and processing to convert them into a computer-compatible digital format. Every original record was machine plotted and visually inspected. This procedure is necessary to determine whether the instrument did function properly. For some of the records taken with the Aanderaa instruments, only the current or temperature data are available. In these cases, the temperature channel malfunctioned while the current velocity channel continued to function, or vice versa.

Many of the original records required editing. Examples of original and edited records of current and

temperature are given in Figure 4 and Figure 5. Positive v and u-component values (of currents and winds) correspond to true north and true east, respectively. In the process of editing suspect data values were replaced by linearly interpolated ones. Two points on either side of the suspect values (or several consecutive suspect values) were used in the linear interpolation. For the currents, either the speed or direction was edited when only one of these parameters was suspect. When both these parameters were suspect, the current components were edited instead.

The editing was done manually (rather than with a computer program) by visual inspection of the plotted data records and references to a listing of their tabulated values. Data values are suspect because they appear to be unreasonably large values, while other data values are suspect when the time-derivatives of the parameter have unreasonably large magnitudes (see Figure 4 and 5). A magnitude in the time-derivative is deemed unreasonably large by comparison to those of the neighbouring points.

Suspect data values also arise from intermittent errors. These errors most probably originate at the time the values are recorded on magnetic tape, or at the time the tape is read for translation into a computer-compatible digital form.

CURRENTS

Figure 4. Portions of the original and edited 50m current records from the central mooring. The sampling interval is 10 minutes.

H26-140M WATER TEMPERATURES

Figure 5. Portions of the original and edited water temperature records from the 140m eastern location. The sampling interval is 10 minutes.

The Half-hourly Time-series

The instruments were started at random times. Hence the observations do not have times corresponding to a multiple of 10 or 15 minutes. Mechanical clocks were utilized in both the Aanderaa and Geodyne instruments to keep time. These clocks have relatively low accuracy and over a period of 30 days, the time has an error of 4 hours, in the worst case. For the Aanderaa instruments, the specified sampling interval of 10 minutes was not used. Instead, the sampling interval was calculated from the start and stop times of the instrument and from the number of values in the record. Hence some of the time intervals (these are rounded to the nearest minute in the original records) between consecutive values in the original and edited records may be 9 or 11 minutes. The sampling interval for the two Geodyne records is assumed to be exactly 15 minutes. Since the Geodyne meters use the same basic clock, the Geodyne records may also have timing errors. The magnitudes of these errors were unknown at the time the records were edited.

Data records with values every half-hour were derived by averaging three consecutive values of those edited records having a sampling interval of 10 minutes. These half-hourly data records have values whose times are on the hour or half-hour. The three consecutive values used for averaging were picked such that the middle value has a time that is within ± 5 minutes from the hour or half-hour. The current components of the edited records are averaged to yield the current components

of the half-hourly records. Thus the time-average is a vector average.

Two half-hourly data records from 40m and 50m at the central mooring were derived from edited records (Geodyne) having a sampling interval of 15 minutes. The half-hourly record from the earlier (Dec/1969) Geodyne record at 40m was derived by simply picking every second value. This was possible because the sampling times of the edited record are 14, 29, 44 and 59 minutes after the start of each hour. The other half-hourly record from the later (Jan-Feb/1970) Geodyne record at 50m was derived by vector-averaging two consecutive values. This was done because the sampling times of the edited record are 9, 24, 39 and 54 minutes after the start of each hour.

All the half-hourly data records for the same location were joined to yield a single time-series of much greater length. In the construction of the time-series at 50m of the central mooring, the half-hourly Geodyne record (Dec/1969) at 40m was used because the meter at 50m failed. The reliability of this meter was suspect and another meter was placed at 40m as a back-up. For those periods where there were no available data, the average value (of temperature and current components) over all the available data was used in the construction of the single time-series for each location. As shown in Figure 3, these data gaps may be quite substantial. Usually there is a short gap of a few hours between the end of one record and the start of the next. These data gaps correspond to the interval when the instruments are brought on board ship for servicing.

The single half-hourly time-series for each location are the subject of the analysis in Chapter 4 and 6. Chapter 5 examines the daily low-passed time-series for each location.

3.2 The Supplementary Data

Portions of the data records of the winds, sea elevation and air pressure (at sea level) are given in Figure 6 and Figure 7. Refer to Figure 1 for locations. All these data records have a sampling interval of one hour. Except for the winds, these data records span the two years of 1969-70. The wind record for Entrance Island covers the periods October 10, 1969 to December 31, 1969 and April 1, 1970 to December 31, 1970, while the wind data for Sand Heads span the period from January 20, 1969 to September 1, 1970.

Figure 6. Portions of the surface wind records. The sampling interval is one hour.

AIR PRESSURE (MBAR)

SEA LEVEL ELEVATION (METERS)

Figure 7. Portions of the sea-level and air-pressure records from various locations around Vancouver Island. The sea-levels are relative to the reference bench marks while the air-pressures are corrected so that they correspond to those measured at mean sea-level. The air-pressure axis refers to that at Vancouver. Air-pressure signals are separated from each other by 5mb. The sampling interval is one hour for all records.

Chapter 4

THE FREQUENCY REPRESENTATION

The frequency representation of the time-series is examined in this chapter. A given time-series may be the observed result of several processes occurring simultaneously. However, each of these processes may occur in separate and distinct frequency regions. When this is so, the processes may be isolated in the time domain by filtering, even though they are proceeding simultaneously.

4.1 Power Spectrum and Data Gaps

The vector power spectrum (S) is the frequency (f) distribution of the variance in the time-series under examination. This distribution is presented in two graphical formats. The first are graphs of S versus f , and the second are graphs of $f \cdot S$ versus $\log f$. With both formats, the area under the graphs is proportional to variance, and the total area (integral) is equal to the sum of the u -component variance and the v -component variance. Each point of a graph has an associated ν (number of degrees of freedom). The parameter ν is proportional to the smoothing of S , which is effected by averaging in the frequency domain. The points of the first graphical format have a constant ν , while those of the second

have a γ that increases rapidly with increasing $|f|$, and a confidence interval which decreases with increasing $|f|$. The second graphical format is used primarily to present the vector power spectrum over the complete frequency range.

A scalar time-series (temperature say) is converted into a pseudo-vector time-series whose u-component is equal to the scalar time-series and whose v-component is zero. Such a conversion is necessary for a cross-spectral analysis between a vector and scalar time-series. The vector power spectrum of a pseudo-vector time-series is an even function of frequency. Because of this symmetry, such spectra are frequently presented for positive frequencies only. Henceforth, the vector power spectrum will be referred to as the power spectrum or more simply as the spectrum.

The power spectra presented in this chapter are based on a time-series length of $N = N_d + N_z$ points. The time-series contain N_d data points and a total of N_z points that correspond to one or more gaps in the time-series. As discussed in the previous chapter, these gaps are filled with the mean derived from the N_d data points. This is equivalent to filling the gaps in a time-series having a zero mean with zeroes. The total variance of the vector time-series should be computed according to:-

$$\text{var}_d = N_d^{-1} \sum (u^2 + v^2) \quad (4.1)$$

where u and v are the components (having zero means) of the vector time-series and the summation is taken only over the N_d

data points. However, the integral of the power spectrum gives a total variance cf:-

$$\text{var}_S = (N_d + N_z)^{-1} \sum (u^2 + v^2) < \text{var}_d \quad (4.2)$$

where the summation is now taken over both the N_d data points and the N_z points, which are zeroes. Hence, the power spectrum gives a variance, var_S , that is lower than the most appropriate estimate, var_d . In this chapter, all the power spectra shown have been multiplied by a factor $(N_d + N_z)/N_d$ because:-

$$\text{var}_d = \text{var}_S \cdot (N_d + N_z) / N_d \quad (4.3)$$

The hypothetical total variance is defined as:-

$$\text{var}_h = (N_d + N_z)^{-1} \sum (\underline{u}^2 + \underline{v}^2) \quad (4.4)$$

where $\underline{u}$ and $\underline{v}$ are the components of the hypothetical time-series which would have been observed had there been no equipment malfunctions, etc., which produced the data gaps. In this thesis it is implicitly assumed that $\text{var}_d = \text{var}_h$. This assumption is exact only if:-

$$N_d^{-1} \sum (\underline{u}^2 + \underline{v}^2) = N_z^{-1} \sum (\underline{u}^2 + \underline{v}^2) \quad (4.5)$$

where the left and right summation are taken over the data points in $(\underline{u}, \underline{v})$ that correspond to the N_d data points in (u, v) , and to the N_z points in (u, v) , respectively.

For purposes of discussion regard N_z as being fixed and on the order of $(N_d + N_z)/10$ (this in fact is the situation with the data being analyzed). Depending on the nature of the power

spectrum, equation (4.5) may or may not be a good approximation to the real situation. Equation (4.5) closely approximates the real situation when all the variance is concentrated in very narrow frequency bands, and when the periods of these oscillations are small when compared to a period corresponding to N_3 . Variance concentrated in very narrow frequency bands represents sinusoidal oscillations, or at worst, quasi-steady sinusoidal-like fluctuations. Good examples of such fluctuations are tidal currents. Equation (4.5) is also a good approximation for broad-band variance having a characteristic period that is small compared to N_3 . However, equation (4.5) can be a potentially poor approximation for variances whose characteristic periods are on the order of N_3 . Generally, the smaller the characteristic period (relative to N_3) of the variance under consideration, the better the approximation. This statement stems from the stationary assumption, which ensures that the variance of the time-series, calculated over an appropriate length, is independent of time. An appropriate length is one that contains many characteristic periods of the fluctuations under consideration.

4.2 Spectra of Original and Edited Records

Shown in Figure 8 are the power spectra of the original and edited ($\Delta t = 10 \text{ min.}$) current records which appear in Figure 4, and the power spectra of the original and edited temperature records which appear in Figure 5. The spectrum of the edited

Figure 8. Spectra ($f \cdot S$) of original and edited currents, and of original and edited temperatures. Based on 8 day time-series of (current, temperature) at (50m, 140m) of the (central, eastern) mooring starting (July 12 1970, July 16 1969), which are shown in Figure (4, 5).

current record differs only slightly from that of the original current record. As expected, the high-frequency variance of the edited record in the frequency interval (3,72cpd) is reduced from that of the original record. Comparing the two power spectra, there is also a slight tendency for the variance to be shifted from one region to another in the frequency domain. Generally though, the main features in the edited current spectrum closely resemble those in the original current spectrum.

In contrast, the power spectra of the original and edited temperature records are very different. The high-frequency (3,72cpd) variance of the original record is much greater than that of the edited record. The low-frequency spectral features of the original record have been severely distorted when compared to those of the edited record. The differences between the two temperature spectra are reasonable, considering the severe differences between the original and edited temperature records.

The conversion of edited records having a sampling interval (Δt) of 10min to those having a Δt of 30min is discussed in the previous chapter. During this conversion, the spectra of the records with $\Delta t=30$ min can be badly distorted by the high-frequency (24,72cpd) variance in the records with $\Delta t=10$ min. From the spectra of the edited records shown in Figure 8, this variance is 2% and 4% of the total variance for the edited current and edited temperature records, respectively. By inspection of the 95% confidence intervals

for a frequency of 24cpd, it is concluded that these low percentages are typical of other edited records with $\Delta t=10\text{min}$. Hence, the spectral distortion of the time-series ($\Delta t=30\text{min}$), due to the aliasing associated with the conversion from $\Delta t=10\text{min}$ to $\Delta t=30\text{min}$, is negligible.

4.3 Spectra of Currents

The power spectra of the current time-series are shown in Figure 9 and Figure 10. These results are based on time-series of 533 1/3 days starting April 16, 1969. The normalized 95% confidence interval (y_l, y_u) is also indicated. The lower and upper limits of the confidence interval for a point $S(f_0)$, are $y_l \cdot S(f_0)$ and $y_u \cdot S(f_0)$, respectively. The six spectra have the same general features. All have relatively broad-band variance at very low frequencies and narrow-band variance at or near $\pm 1\text{cpd}$ and $\pm 2\text{cpd}$.

Shown in Table I are the variances of the currents calculated for selected frequency bands. They are given as a percentage of the total variance, that found in the complete frequency range (0.002, 24cpd). The variance has been separated into the u-variance, v-variance, C-variance and A-variance. The former two are the vector component variances, while the latter two are the rotary (clockwise, anti-clockwise) variances. In any frequency band, the sum of the component variances is identical to the sum of the rotary variances. The total u-variance and total v-variance are variances

Figure 10. Spectra of 140m and 200m currents. Based on time-series of 533 days starting April 16 1969. All spectra have 18 degrees of freedom. Numbers above the off-scale peaks indicate their values.

TABLE I — Frequency distribution of the variance in the currents **

FREQUENCY RANGE (CYCLES/DAY)	WESTERN MOORING					CENTRAL MOORING					EASTERN MOORING					
	U	V	U+V	C	A	U	V	U+V	C	A	U	V	U+V	C	A	
.002 - .05	2.59	15.08	17.68	8.14	9.60	15.87	6.52	22.39	8.64	13.82	0.94	21.92	22.85	12.30	10.55	50m
.05 - .10	2.11	5.46	7.57	2.90	4.70	2.93	2.43	5.36	2.15	3.23	0.67	14.26	14.92	8.60	6.32	
.10 - .15	1.48	2.86	4.34	1.85	2.51	1.96	1.35	3.31	1.20	2.12	0.69	4.78	5.47	3.34	2.13	
.15 - .20	0.92	0.90	1.82	1.21	0.61	0.72	0.94	1.66	0.94	0.73	0.24	1.37	1.62	0.90	0.72	
.20 - .25	0.40	0.48	0.89	0.50	0.39	0.38	0.42	0.80	0.39	0.41	0.21	1.02	1.23	0.69	0.54	
.002 - .25	7.51	24.78	32.29	14.61	17.81	21.85	11.66	33.51	13.32	20.30	2.75	43.34	46.09	25.82	20.25	
.25 - .50	1.33	1.13	2.46	1.50	0.97	0.96	1.13	2.10	1.01	1.10	0.65	1.77	2.43	1.33	1.09	
.50 - .80	0.75	0.87	1.62	0.93	0.69	0.73	0.86	1.59	1.00	0.60	0.47	1.08	1.55	0.81	0.74	
.80 - 1.1	2.87	14.67	17.54	8.31	9.29	4.14	12.61	16.75	6.70	10.10	1.01	9.95	10.96	5.22	5.73	
1.1 - 1.8	3.27	2.87	6.15	5.05	1.11	3.20	2.85	6.05	5.12	0.95	1.96	2.20	4.17	3.18	0.98	
1.8 - 2.1	4.31	24.78	29.09	12.24	16.96	3.07	27.45	30.52	15.25	15.37	3.71	20.48	24.19	13.18	11.00	
2.1 - 3.0	1.70	1.78	3.48	2.61	0.89	1.39	1.63	3.02	2.27	0.76	2.12	1.78	3.90	3.02	0.88	
3.0 - 4.0	1.04	0.99	2.03	1.47	0.56	1.05	0.93	1.98	1.44	0.54	1.19	0.99	2.18	1.56	0.62	
4.0 - 24.0	2.56	2.86	5.41	3.37	2.06	2.29	2.25	4.55	2.75	1.81	2.41	2.21	4.62	2.90	1.71	
.002 - 24.0	51.27	151.34	202.61	100.67	101.18	72.63	115.26	187.89	91.13	96.15	21.48	110.81	132.29	75.45	56.90	Western, Central 200m Eastern 140m
.002 - .05	0.27	2.36	2.63	1.14	1.50	1.01	4.16	5.17	2.81	2.36	0.86	11.17	12.02	6.30	5.71	
.05 - .10	0.30	2.43	2.74	1.44	1.31	0.42	2.40	2.82	1.69	1.13	0.60	13.45	14.06	7.20	6.85	
.10 - .15	0.14	0.64	0.78	0.31	0.48	0.37	1.78	2.15	1.17	0.98	0.29	5.46	5.74	3.11	2.63	
.15 - .20	0.13	0.55	0.68	0.37	0.31	0.50	1.18	1.67	0.97	0.70	0.20	1.58	1.77	1.04	0.73	
.20 - .25	0.06	0.31	0.37	0.21	0.17	0.19	0.35	0.54	0.26	0.29	0.10	1.69	1.78	1.01	0.77	
.002 - .25	0.91	6.29	7.20	3.46	3.76	2.49	9.87	12.36	6.89	5.46	2.04	33.34	35.37	18.65	16.70	
.25 - .50	0.19	0.56	0.75	0.40	0.35	0.94	1.19	2.13	1.41	0.72	0.39	1.37	1.76	0.99	0.77	
.50 - .80	0.34	0.52	0.86	0.47	0.39	0.71	0.74	1.45	0.78	0.67	0.30	1.11	1.42	0.80	0.62	
.80 - 1.1	3.21	22.18	25.39	18.81	6.64	2.38	27.02	29.40	13.52	15.87	2.34	20.12	22.45	9.62	12.82	
1.1 - 1.8	1.50	1.52	3.02	2.36	0.67	1.80	2.27	4.06	2.98	1.08	1.28	2.23	3.51	2.21	1.29	
1.8 - 2.1	17.08	36.53	53.61	43.53	10.22	9.86	32.60	42.46	25.48	16.95	6.27	17.58	23.85	14.60	9.24	
2.1 - 3.0	1.80	1.49	3.28	2.49	0.80	1.29	1.93	3.22	2.31	0.91	2.79	2.22	5.01	3.58	1.42	
3.0 - 4.0	0.74	1.21	1.86	1.30	0.57	0.70	0.93	1.62	1.08	0.54	1.18	1.47	2.66	1.73	0.92	
4.0 - 24.0	1.94	2.16	4.10	2.62	1.49	1.53	1.87	3.41	2.10	1.30	1.97	2.13	4.10	2.55	1.55	
.002 - 24.0	40.47	105.77	146.24	109.61	36.28	30.05	108.78	138.83	78.51	60.39	21.37	94.13	115.50	63.21	52.36	

** Variances summed over the complete frequency range (6^{cpd} , 24)cpd are in $cm^2 sec^{-2}$. All other variances are given as a % of the total variance (total u + total v). This table is based on time-series lengths of 533 days starting April 16 1969.

contained over the complete frequency interval $(0.002, 24\text{cpd})$, and the total variance is the sum of the total u-variance and total v-variance. All variances contained in Table I have been weighted by the factor $(N_d + N_z)/N_d$ discussed earlier.

As Table I shows, the low-frequency variance in the interval $(0.002, 0.25\text{cpd})$, and the diurnal and semi-diurnal variances account for typically 80% of the total variance, while the variance above 2.1cpd accounts for only 10% of the total variance. It is seen that the total variance varies from location to location. The total variance of the three 50m locations and that of the three deeper locations increases from east to west. At each of the three stations, the total variance decreases with depth. It is seen that the anti-clockwise and clockwise variances $(0.002, 24\text{cpd})$ at five locations are roughly equal. At the 200m western location, the clockwise variance $(0.002, 24\text{cpd})$ is about three times that of the anti-clockwise variance.

The low-frequency $(0.002, 0.25\text{cpd})$ variance at most locations contributes a significant amount to the total variance. As Table I shows, the low-frequency variance of the two 200m locations are 7% and 12% of the total variance, while that of the remaining four locations range from 32% to 46% of the total variance. The low-frequency variances of the three 50m time-series are approximately equal. Most of the low-frequency variance at all six locations is concentrated in the region $|f| < 0.25\text{cpd}$.

The low-frequency spectral details shown in Figure 9

and Figure 10 require careful interpretation. For all six locations, there are spectral peaks between 0 and ± 0.2 cpd. To 95% confidence, these peaks are not statistically significant, and are ignored. It is concluded that the spectra, for the 140m and 50m locations, increases with decreasing frequency in this low-frequency range, while those of the 200m locations have a tendency to level off as the frequency approaches zero. These conclusions can also be made from inspection of the low-frequency variances listed in Table I. However, these conclusions are to be viewed with some caution. Except for the 50m central location, all the time-series contain sizable data gaps (see Figure 3) which may have distorted the low-frequency region of their spectra.

The narrow-band variances concentrated near or at ± 1 cpd and ± 2 cpd represent the tidal currents. The major tidal constituents, which are associated with these currents, are identified in Figure 9 and Figure 10. Because of leakage of variance to adjacent frequencies, these spectral peaks appear to be much broader than they are. At all six locations, the variance of the $(K_1 + P_1)$ constituent is larger than the O_1 constituent, and the M_2 constituent is the largest of the three semi-diurnal constituents. At all but one of the locations (140m eastern location near -2 cpd), the variance of the $(K_2 + S_2)$ constituent is larger than (or approximately equal to) the N_2 constituent.

The spectra for the 50m locations reveal that the M_2 constituent at -1.932 cpd appears to have been broadened. This

broadening results (at some locations) in the absence of the spectral valley between the M_2 and N_2 spectral peaks near -2 cpd. The time-domain description is that the M_2 clockwise rotating vector has an amplitude which is slowly changing with time, instead of one which is relatively constant with time.

Inspection of Table I shows that, at all six locations, the $(u+v)$ variance of the semi-diurnal currents is larger than that of the diurnal ones. Both the diurnal and semi-diurnal currents have a v -variance that is two to several times that of the u -variance. The A -variance is approximately equal to the C -variance for both diurnal and semi-diurnal currents, at five locations. Hence at the majority of the locations, the tidal currents are more or less rectilinear. At the 200m western location, the tidal motions tend to be clockwise rotating, the C -variance at 1cpd and 2cpd being 3 and 4 times that of the corresponding A -variance. At the 200m locations, the tidal motions account for about $3/4$ of the total variance, while those at the four remaining locations range from 36% to 46% of the total variance.

The graphs of $f \cdot S$ versus $\log f$ for the currents are shown in Figure 11 and Figure 12. These graphs show the low-frequency ($|f| < 0.7$ cpd) spectral structure at the expense of statistical confidence. The high-frequency ($|f| > 2$ cpd) regions of the spectra have low resolution and high statistical confidence. For all six locations, the spectral valley between ± 0.1 cpd and ± 1 cpd is statistically significant to 95% confidence. Generally, the low-frequency variance is

Figure 11. Spectra (f·S) of 50m currents. Based on time-series of 533 days starting April 16 1969. Numbers above the off-scale peaks indicate their values.

Figure 12. Spectra ($f \cdot S$) of 140m and 200m currents. Based on time-series of 533 days starting April 16 1969. Numbers above the off-scale peaks indicate their values.

concentrated between $\pm 0.01\text{cpd}$ and $\pm 0.1\text{cpd}$, with a spectral peak in or near this frequency region. The variance is seen to increase as $|f|$ increases from $.01\text{cpd}$ upwards.

The variance increase with frequency, the low-frequency spectral peak, and the spectral structure for $|f| < .1\text{cpd}$ are to be viewed with caution. For most locations, these features are not statistically significant to 95% confidence. For other locations, the low-frequency spectral peak is statistically significant, but the presence of the data gaps have introduced an unknown amount of low-frequency distortion. This distortion may affect the total low-frequency variance, it may shift variance from a frequency region to neighbouring frequency regions, and even from negative frequency regions to positive ones (and vice-versa). Subject to this qualification, it appears that the current time-series are long enough to resolve the broad low-frequency peak, at each of the six locations. Some of the individual peaks are not statistically significant, but their average is statistically significant to 95% confidence.

The two tidal peaks at 1cpd and 2cpd are statistically significant. It is seen that the variance decreases as $|f|$ increases from 3cpd upwards. This feature is statistically significant to 95% confidence. The high-frequency variance ($|f| > 3\text{cpd}$) contributes typically 10% to the total variance, and this low contribution is also statistically significant. For all six locations, the high-frequency clockwise variance is larger than the high-frequency anti-clockwise variance.

4.4 Coherence and Phase Functions

The calculation of the vector coherence squared function, γ_{12}^2 , involves a time-series 1 and a time-series 2, both of length N . Originally there are data gaps in both time-series. The data gaps in time-series 1 differ in number and length from those in time-series 2, and generally the gaps of the two time-series do not coincide. Before performing the cross-spectral analysis, additional gaps are introduced in both time-series. After this operation, the data gaps of both time-series are identical in number and length, so that the gaps exactly coincide in the time domain. The net result is that the limits of the γ_{12}^2 function are still $(0,1)$.

The vector coherence squared function is henceforth referred to as the coherence function. It is a measure of the resemblance between the fluctuations of time-series 1 and time-series 2, in a particular frequency region. If all the fluctuations of both time-series are associated with a single linear process, the coherence function is unity. The coherence function becomes less than unity if that single process is non-linear.

Consider the situation where all the fluctuations (in a certain frequency region) of time-series 1 are associated with a single linear process, and where the major portion of the fluctuations in time-series 2 are also associated with the same linear process. A second linear process now contributes partially to the fluctuations in time-series 2. The coherence function again becomes less than unity. The larger the

contribution to the fluctuations of time-series 2 by the second linear process, the closer the coherence function approaches zero. In this case, that portion of the fluctuations in time-series 2 due to the second linear process is 'noise', and the remaining portion due to the first linear process is termed the coherent portion.

In all cases, the coherence function is plotted against a linear frequency scale with each and every point having a constant ν (degrees of freedom). Often the coherence function is plotted using an $\text{arctanh}(Y_{12})$ vertical scale. With such a scale, a single confidence interval suffices for each and every point of the function. According to Jenkins and Watts (1968) the parameter, $Y_{12} = \text{arctanh}(Y_{12})$, has approximately a normal distribution with a variance of ν^{-1} . Thus the 95% confidence intervals of Y_{12} are $\pm 1.96/\sqrt{\nu}$. According to Bendat and Piersol (1971) however, Y_{12} is normally distributed subject to the conditions that $0.35 \leq Y_{12}^2 \leq 0.95$ and that $\nu \geq 20$. When these conditions do not hold, it is assumed in this thesis that Y_{12} is still approximately normal, and that the confidence interval so calculated is a crude but tolerable approximation.

A light horizontal line appears in all the graphs of the coherence function. Groves and Hannan (1968) call it the 95% level of the null hypothesis, but henceforth, it is referred to as the 95% noise level. It is given by:-

$$Y_n^2 = 1 - \alpha^M \quad (4.6)$$

where $1 - \alpha = 0.95$ and $M = 2/(\nu - 2)$. The 95% noise level, Y_n^2 , is not

related in any manner to the coherence function under scrutiny, but to a hypothetical coherence-function estimate between two time-series which have zero coherence. The 95% noise level is the value that would be exceeded 100 α =5% of the time by this hypothetical coherence-function estimate. It is not identically zero because the two time-series used for analysis are of finite length. In contrast, the 95% confidence interval is related to the coherence function being examined.

The coherence function under scrutiny is compared with the 95% noise level, and when the former falls below the latter, the coherence between the two time-series may be judged to be essentially zero. The 95% confidence interval is to be used independently of the 95% noise level. The confidence interval is used to determine whether a particular feature of the coherence function under examination is statistically significant.

The vector phase function, ϕ_{12} , is frequently referred to as the phase function. A positive value of the phase function means that the rotary component (clockwise or anti-clockwise) in time-series 1 is leading, in time, that of time-series 2. In all discussions of γ_{12}^2 and ϕ_{12} , time-series 1 is always identified before time-series 2.

When the coherence function is much less than unity, the phase function may still be meaningful. A coherence function value smaller than unity indicates that only a portion of the fluctuations in both time-series is coherent, and the coherent portions have a constant phase difference given by the

phase function. Unfortunately, the confidence intervals for the phase function are dependent on ν and on the coherence function. The 95% confidence intervals, $\pm\Delta\phi$, for the phase function are given in Figure 13, which shows the dependence of $\Delta\phi$ on ν and the coherence function. For a fixed ν , it is seen that the confidence interval increases to $\pm\Delta\phi=\pm180^\circ$ as the coherence function approaches zero. Thus in regions where the coherence values are low, information contained in the phase function can potentially be obscured by the scatter in the latter. This scatter arises because of the large amounts of noise present in both time-series, the large amounts of noise being indicated by the low coherence values.

4.5 Current-Current Relations

Shown in Figure 14 and Figure 15 are the coherence and phase functions between the currents from vertically and horizontally separated locations, respectively. The regions of greatest interest are the low-frequency and tidal regions, since the major portion of the variance is found there. The presence of the data gaps can be expected to increase the low-frequency coherence values by a slight amount, while their effect on the phase function is difficult to predict.

Coherence

Generally, there are poor coherences between currents

Figure 13. The 95% confidence intervals of the phase spectrum (after Jenkins and Watts, 1968).

Figure 14. Coherence and phase spectra between currents from vertically-separated locations. All have 48 degrees of freedom. Parts (A,B,C) are based on 360 day time-series starting (April 17, June 17, July 11) 1969.

Figure 15. Coherence and phase spectra between currents from horizontally-separated locations. Parts (A,B,C) are based on (50m, 50m, 200m) currents from the (central-eastern, central-western, central-western) mooring. Spectra in parts (A,B,C) have (48,64,50) degrees of freedom using (360,432,333) day time-series starting (April 17, July 11, July 11) 1969.

from different locations in the low-frequency ($|f| \leq 0.25 \text{ cpd}$) region, the highest observed value of γ^2 being 0.28. There is no doubt that the low-frequency current fluctuations have not been spatially well resolved by the instrument array used.

In the low-frequency region, it is seen that the highest coherence values are obtained with currents from the vertically-separated eastern locations. The coherences between the clockwise fluctuations are higher than those of the anti-clockwise fluctuations, but both lie above the 95% noise level. The coherence function for the horizontally-separated 200m locations have low-frequency features very similar to the coherence function just discussed. The remaining four coherence functions are observed to have coherence values that, on the average, lie below the 95% noise level. All six coherence functions lack any low-frequency features which are statistically significant. That is, the 95% confidence intervals allow one to draw a horizontal line connecting the points of the coherence function from the lowest frequency to about 0.8cpd.

The only statistically significant (95%) peaks that appear in the coherence functions are the ones representing the diurnal and semi-diurnal tidal currents. The tidal fluctuations from different locations are highly coherent, the lowest observed value of γ^2 being 0.55. These high values are obtained because a major portion of the tidal fluctuations at each location can be associated with a single process. This process is the forcing of the fluid motions in the Strait of

Georgia, by the tidal motions of the Pacific Ocean surface at the mouth of Juan de Fuca Strait, and at the northern end of Vancouver Island. Comparing the semi-diurnal currents, it is observed that the coherence between the clockwise component is consistently lower than the anti-clockwise component. With one exception, this is also true of the diurnal currents. The exception occurs with the diurnal currents from the vertically-separated western locations.

Phase

In Figure 14 and Figure 15, the highest low-frequency ($|f| \leq 0.25 \text{ cpd}$) coherences are obtained with the currents from the vertically-separated eastern locations. It is observed that the 50m low-frequency fluctuations lag those at 140m by 45° . The next highest low-frequency coherences are obtained with the currents from the horizontally-separated 200m locations. In this case, the fluctuations of the central location lead those of the western location by about 160° . The sharp decrease of the phase function at $+0.15 \text{ cpd}$ is statistically significant to 95% confidence.

In three of the remaining four cases, the phase functions in the low-frequency region have a relatively constant value or a smooth variation with frequency. This behaviour is inconsistent with the coherence functions, which fall below the 95% noise level in the low-frequency region, implying that the fluctuations have zero coherences. With two

time-series having a zero coherence, a phase function having large scatter is expected. The phase function of two such time-series is not expected to have a relatively constant value or a smooth variation. In this situation, it is concluded that the coherences are non-zero, even though they lie below the 95% noise level.

In the low-frequency ($|f| \leq 0.2 \text{ cpd}$) region, the phase between the fluctuations from the vertically-separated eastern locations has a relatively constant value of -45° . Using the 50m currents from the central and eastern locations, the phase has a smooth variation at negative frequencies ($|f| \leq 0.2 \text{ cpd}$) and a relatively constant value of -90° at positive frequencies. The 50m low-frequency current fluctuations from the central and western locations are also observed to have relatively constant phase differences.

The smallest 95% confidence intervals for the phase functions are found at the tidal frequencies, since the highest coherence values are found at these frequencies. The phases between the tidal currents from different locations do not show a consistent behaviour, and this inconsistency in the phases cannot be explained by their statistical uncertainty. It is observed that the rotary tidal current components are in phase to within $\pm 45^\circ$.

A relatively constant value of 0° is observed for two phase functions over a wide frequency range, from $|f| = 0.8 \text{ cpd}$ to $|f| = 2.5 \text{ cpd}$. The constant phase differences are found at positive frequencies for the 50m central and western locations,

and at negative frequencies for the 200m central and western locations. The corresponding coherence functions fall below the 95% noise level at many places in the frequency domain. However, it is concluded (from the behaviour of the phase functions) that the two pairs of time-series being examined do in fact have non-zero coherences.

4.6 Spectra of Temperatures

The graphs of $f \cdot S$ versus $\log f$ for the water temperatures are shown in Figure 16. These spectral results are based on time-series all of length $533 \frac{1}{3}$ days starting April 16, 1969. As Figure 3 shows, there are more (or as many) gaps in the temperature time-series, as there are in the current time-series. The frequency distribution of the variance in the temperature time-series is given in Table II. All variances in this table have been multiplied by the factor $(N_d + N_z)/N_d$ which appears in equation (4.3).

From Table II, it is seen that the total variance decreases with depth at each of the three stations. The central 50m location has the largest total variance of the three 50m locations, while the central 200m location has twice the total variance of the western 200m location. It is observed that typically 95% of the total variance is low-frequency ($f < 0.25 \text{ cpd}$) variance. The low-frequency variance of the temperature time-series is concentrated in a much narrower frequency region than that of the current time-series. Table I

Figure 16. Spectra of water temperatures. Based on 533 day time-series starting April 16 1969.

Table II - Frequency Distribution of the Variance in the Water Temperatures**

Frequency Range (Cycles/Day)	Western Station		Central Station		Eastern Station	
	50m	200m	50m	200m	50m	140m
.002 - 0.01	74.2	72.3	65.1	64.1	65.2	66.5
0.01 - 0.02	12.0	7.61	19.1	14.7	20.4	15.8
0.02 - 0.03	3.22	3.72	1.67	3.98	2.16	2.98
0.03 - 0.04	0.77	2.11	2.26	2.21	1.18	1.25
0.04 - 0.05	0.89	2.18	0.92	2.83	1.14	2.50
0.05 - 0.10	3.23	6.02	2.17	2.49	3.52	3.01
0.15 - 0.20	1.06	1.50	1.94	1.48	0.90	1.28
0.20 - 0.25	0.60	0.39	1.05	0.84	0.96	1.15
.002 - 0.25	96.5	96.5	94.9	93.4	95.9	94.8
0.25 - 0.50	1.04	0.83	1.65	1.34	1.10	0.82
0.50 - 0.80	0.34	0.43	0.52	0.82	0.48	0.30
0.80 - 1.10	0.31	0.68	0.68	1.13	0.64	1.44
1.10 - 1.80	0.47	0.32	0.56	0.85	0.57	0.42
1.80 - 2.10	0.29	0.46	0.32	0.67	0.28	1.08
2.10 - 3.00	0.31	0.27	0.44	0.55	0.34	0.52
3.00 - 4.00	0.20	0.21	0.27	0.38	0.23	0.24
4.00 - 24.0	0.63	0.55	0.74	1.08	0.55	0.47
.002 - 24.0	57.83	8.42	64.23	15.59	44.32	36.29

** The total variances (0.002-24 cpd) are given in units of $10^{-2} \text{ }^{\circ}\text{C}^2$. All other variances are given as a % of the total variance. Based on time-series lengths of 533 days, starting April 16, 1969.

and Table II shows that a major fraction of the low-frequency variance in the temperature and current time-series is found in the regions, $f \leq 0.03 \text{ cpd}$ and $|f| \leq 0.15 \text{ cpd}$, respectively. Typically 0.5% to 1% of the total variance is found in the frequency range (4,24) cpd.

The low-frequency spectral structures of the graphs in Figure 16 are not statistically significant to 95% confidence. Furthermore, it appears that the time-series used are too short to resolve the low-frequency fluctuations. Spectral peaks at 1 cpd and 2 cpd are found in all six spectra. These peaks are statistically significant to 95% confidence, except for the 1 cpd peak of the central 50m location. The peaks in the temperature spectra are associated with the tidal currents, but they are much broader than the tidal peaks in the current spectra. At some locations, the peaks in the temperature spectra are not located exactly at the frequencies of the tidal currents.

4.7 Temperature-Temperature Relations

The coherence and phase functions for temperatures from vertically-separated locations are shown in Figure 17. The starting dates and lengths of the temperature time-series used are also indicated. These are picked so the time-series contain only short data gaps. The low-frequency ($f \leq 0.20 \text{ cpd}$) regions of the coherence and phase functions are the regions of greatest interest, since almost all the variance is found

Figure 17. Coherence and phase spectra between water temperatures from vertically-separated locations. Parts (A,B,C,D) have (30,32,52,40) degrees of freedom and are based on (100,107, 260,130) day time-series starting (April 16, August 28, July 10, April 16) 1969.

there.

In all four coherence functions, high values are observed in the low-frequency region (0,0.2)cpd. The increase of the coherences in the low-frequency region is a statistically significant (95%) feature. None of the peaks found elsewhere in the frequency domain are statistically significant. Over much of the frequency domain, the coherence values lie below the 95% noise level. The temperature fluctuations at 1cpd and 2cpd are not coherent.

The fluctuations from the vertically-separated western locations are 180° out of phase at the lowest frequency. In contrast, the fluctuations of lowest frequency from the vertically-separated central and eastern locations tend to be in phase. There are two phase functions for the temperatures from the vertically-separated eastern locations. These are based on time-series which span different periods in the time domain. The phase function for the earlier (April 1969) time-series pair has a relatively constant value of 0° over the complete frequency range. That for the later (Aug 1969) time-series pair shows large scatter. Hence the phase functions for the vertically-separated eastern locations are time-dependent.

On the 50m plane, the temperatures from the central location are compared with those from the eastern location during two periods in 1969. The results are shown in Figure 18. During the earlier period, the fluctuations of lowest frequency are highly coherent and in phase. During the later period, the coherence is poor at the lowest frequency. Hence,

the coherence at the lowest frequency is time-dependent. In the comparisons of temperatures (50m) from the central and western location, the coherence at the lowest frequency is also time-dependent.

In the comparison between temperatures, relatively high coherences at the lowest frequency are the only features which appear above the 95% noise level, and which are statistically significant to 95% confidence. However, the 95% confidence intervals are not valid when the coherence function is time-dependent. Its time-dependency indicates that the temperatures time-series do not represent stationary stochastic processes. The temperature time-series analyzed are too short to satisfy the stationary criterion. The spectra of the temperatures indicate that the variances are found in the region 50-500 days, giving a characteristic period of 200 days (say). The coherences between temperatures are based on time-series lengths from 90-260 days, which are approximately the duration of a characteristic period, and which are too short to satisfy the stationary criterion.

A temperature time-series length of 1000 days (5 characteristic periods) may be sufficiently long to satisfy the stationary criterion for the low-frequency fluctuations. In view of this demanding requirement of the time-series length, the classical methods (such as isentropic analysis) of physical oceanography (Waldichuk, 1957) appear to be more appropriate for the analysis of the temperature time-series, than the methods of time-series analysis.

4.8 Current-Temperature Relations

The currents and temperatures from the same location are compared and the results are shown in Figure 19 and Figure 20. The comparisons for the 50m locations appear in the first figure. Generally, the currents and temperatures at a location are incoherent. In several instances, the coherence does rise above the 95% noise level, but in all cases these peaks are not statistically significant to 95% confidence. The currents have narrow-band variance at the tidal frequencies, but these fluctuations are not coherent with those in the temperatures. At the central 50m location, the current and temperature for two periods are compared, showing that the coherence at the lowest frequency is time-dependent.

The comparisons of current and temperature for the deeper locations appear in Figure 20. Generally, the currents and temperatures at the deep locations are also incoherent over a major portion of the frequency range. The coherence for the deep central location shows slightly higher coherences than those at the other locations. The peak at 0.7cpd is above the noise level, and statistically significant to 95% confidence. Probably though, this feature is time-dependent, making invalid the assumption on which the confidence interval is based.

The coherence between current and temperature is a parameter not easily related to the physical situation. Consider an infinitely long canal of uniform width and depth running in the x-direction. The fluid in the channel moves as a rigid mass with velocity $U(t)$ which is only dependent on

Figure 19. Coherence and phase spectra between current and temperature at the 50m locations. Parts (A,B,C,D) have (72,34,16,64) degrees of freedom and are based on (480,225,100,432) day time-series starting (April 16 1969, April 16 1969, June 14 1970, July 10 1969).

time. The temperature of the water is a function of p (the p -axis origin moves along with the fluid):-

$$\Theta(p) = \sum_{k=0}^{\infty} a_k p^k \quad (4.7)$$

Relative to the p -axis, Θ is time-independent. The origin of p moves along with the fluid with velocity $U(t)$, so that:-

$$p = x - \int_{-\infty}^t U(t') dt' = x - d(t) \quad (4.8)$$

is the relation between the origin of p and x . Hence the temperature moving with a fluid element is constant because molecular diffusion, heat sinks and sources have been neglected. A sensor at a fixed location $x=x_0$ would measure a temperature variation of:-

$$\left. \frac{\partial T(t)}{\partial t} = \frac{\partial \Theta(x,t)}{\partial t} \right]_{x=x_0} = -U(t) \cdot \left. \frac{\partial \Theta(x,t)}{\partial x} \right]_{x=x_0} \quad (4.9)$$

$$= -U(t) \cdot \sum_{k=1}^{\infty} k a_k [x_0 - d(t)]^{k-1}$$

and

$$\mathcal{F}\{T(t)\} = i/(2\pi f) \cdot \mathcal{F}\{U(t)\} * \mathcal{F}\left\{\sum_{k=1}^{\infty} k a_k [x_0 - d(t)]^{k-1}\right\} \quad (4.10)$$

where $\mathcal{F}$ and $*$ denote the Fourier transform and convolution operators, respectively.

Even for a situation as simple and unrealistic as the one discussed, the power spectrum of temperature may be very

different (because of the convolution with the last transform) from that of the currents, giving rise to a low coherence between current and temperature. It is doubtful that a simple relation between $\mathcal{F}\{T\}$ and $\mathcal{F}\{U\}$ exists for the case of a more realistic situation.

4.9 Spectra of Sea-Level, Air-Pressure, Winds

The power spectra of sea-level (Pt. Atkinson), air-pressure (Vancouver) and winds (Sand Heads) are shown in Figure 21. The sea-level has significant low-frequency variance in the region $(0, 0.3)$ cpd. This variance decreases sharply for increasing frequency, a feature which is statistically significant to 95% confidence. The small peak at 0.07cpd is not statistically significant. The major portion of the sea-level variance lies at the tidal frequencies near or at 1cpd and 2cpd.

The air-pressure (Figure 21b) also has low-frequency variance but in a larger region $(0, 0.5)$ cpd. The fluctuations in the spectrum below 0.5cpd is not statistically significant. The only statistically significant feature in this region is the increasing variance as the frequency decreases to zero. The air-pressure spectrum has sharp peaks at 1cpd and 2cpd, both of which are statistically significant. The peak at 2cpd is too small to be seen in the graph.

A major portion of the wind energy (Figure 21a) is contained in the region $(0, 1.5)$ cpd. The peak at 1cpd is

Figure 21. Spectra of wind (Sand Heads), air-pressure (Vancouver) and sea-level (Pt. Atkinson). All have 22 degrees of freedom and all are based on 600 day time-series starting January 03 1969. Numbers in brackets indicate the values of the off-scale peaks.

statistically significant, while the peaks at lower frequencies are not. The only statistically significant low-frequency feature is the increase of energy for decreasing $|f|$.

The distribution of variance is plotted on a log frequency scale in Figure 22. The sea-level contains a large annual oscillation. However, the spectral features below 0.1 cpd are not statistically significant. The decrease of variance as the frequency is increased from 0.1cpd to 0.5cpd, and the increase of variance as the frequency is further increased, are statistically significant. A small percentage of the sea-level variance is found at frequencies greater than 2.1cpd. The peak at 5cpd is statistically significant. The low-frequency sea-level fluctuations have periods 2-300 days.

In Figure 22b, it is seen that the air-pressure has essentially zero variance at frequencies above 2cpd. In contrast to the sea-level, the air-pressure does not have a large annual variation. The decrease of variance as the frequency increases upward from 0.2cpd is statistically significant, and this rate of decrease is much larger than the corresponding decrease in the sea-level spectrum. The air-pressure fluctuations have periods 2-100 days.

The wind has a relatively broad spectrum (Figure 22a). The wind energy is found from 0.015cpd to the Nyquist frequency of 12 cpd. The peaks at ± 1 cpd are the only statistically significant ones. The decrease of variance as $|f|$ increases upward from 1cpd, and as $|f|$ decreases downward from 0.1cpd, are statistically significant features. It is seen that the

Figure 22. Spectra ($f \cdot S$) of wind (Sand Heads), air-pressure (Vancouver) and sea-level (Pt. Atkinson). All are based on 600 day time-series starting January 03 1969. Numbers near the off-scale peaks indicate their values.

TABLE III - Frequency Distribution of the Variance in Sea-Level, Air-Pressure, and Wind**

Frequency Range (Cycles/Day)	Sea-Level Point Atkinson	Air Pressure Vancouver	Winds Sand Heads				
			U	V	U+V	C	A
.002 - 0.05	1.16	38.50	8.86	3.95	12.80	6.69	6.11
0.05 - 0.10	0.29	19.00	6.93	3.95	10.90	6.18	4.70
0.10 - 0.15	0.11	12.10	6.73	3.72	10.50	6.31	4.14
0.15 - 0.20	0.05	8.22	3.63	2.41	6.04	3.31	2.73
0.20 - 0.25	0.07	7.79	4.07	2.40	6.48	3.65	2.83
0.25 - 0.30	0.04	3.63	2.52	1.04	3.56	2.07	1.48
0.30 - 0.35	0.02	1.91	2.68	0.97	3.65	1.83	1.82
0.35 - 0.40	0.03	2.34	2.63	0.88	3.51	2.14	1.36
0.40 - 0.45	0.02	1.65	2.84	0.87	3.70	2.12	1.59
0.45 - 0.50	0.02	1.18	1.98	1.01	2.99	1.72	1.27
0.50 - 0.60	0.02	1.04	2.41	1.11	3.52	2.00	1.52
0.60 - 0.70	0.03	0.90	2.52	0.86	3.38	1.54	1.84
0.70 - 0.80	0.03	0.38	1.85	0.78	2.63	1.32	1.31
0.80 - 0.90	0.59	0.26	1.70	0.67	2.36	1.28	1.08
0.90 - 1.10	58.10	0.74	3.34	2.15	5.49	2.88	2.61
1.10 - 1.45	0.10	0.15	2.54	1.64	4.18	2.37	1.81
1.45 - 1.80	0.05	0.05	1.54	1.02	2.56	1.45	1.11
1.80 - 2.10	39.30	0.09	0.96	0.75	1.71	0.99	0.72
2.10 - 3.00	0.02	0.03	1.48	1.40	2.88	1.72	1.16
3.00 - 4.00	0.01	0.02	0.88	0.98	1.86	1.08	0.78
4.00 - 12.0	0.02	0.04	2.36	3.07	5.43	2.92	2.50
.002 - 12.0	11,360	53.31	2.214 $\times 10^5$	1.222 $\times 10^5$	3.436 $\times 10^5$	1.909 $\times 10^5$	1.528 $\times 10^5$

** The total variances (0.002-12 cpd) of sea-level and air-pressure are given in units of cm^2 and mb^2 , respectively. All other variances of sea-level and air-pressure are given as a % of the total variance. The wind variances in the range (0.002-12 cpd) are given in units of $\text{cm}^2\text{sec}^{-2}$. All other wind variances are given as a % of the U+V variance in the range (0.002-12 cpd). Variances of all parameters are based on time-series lengths of 600 days, starting January 3, 1969.

wind spectrum tends to level off at $\pm 10\text{cpd}$, indicating the wind has energy above 12cpd , and that some aliasing has occurred. It is certain that a portion of the high-frequency variance is digitization noise. The wind directions are only digitized to the nearest 22.5° . It is judged that the aliasing has not seriously distorted the wind spectrum.

The gross distribution of variance in the sea-level, air-pressure and winds are given in Table III. Examination of the spectra and of Table III indicates that these three parameters have variance in different and overlapping frequency regions. For example, half of the wind energy is found above and below 0.3cpd (approximately). The approximate corresponding frequencies of the air-pressure and low-frequency sea-level fluctuations are 0.1cpd and 0.07cpd , respectively. Hence, the wind energy is found in a higher and overlapping frequency region than the air-pressure variance, which in turn is found in a higher and overlapping frequency range than the low-frequency sea-level variance.

4.10 Relations Between Sea-levels

The power spectra of sea-level at Victoria, Tofino and Alert Bay (see Figure 1 for locations) were also calculated, but not shown since they are very similar to that at Pt. Atkinson. The comparisons of sea-level at Pt. Atkinson with those at the three other locations appear in Figure 23. Note that the coherence functions are plotted on a linear scale.

Figure 23. Coherence and phase spectra between sea-level at Pt. Atkinson with those at other locations. All have 52 degrees of freedom and all are based on 533 day time-series starting April 16 1969.

The coherences at some frequencies are close to unity, a value which is located at infinity on the arctanh scale.

The 95% statistically significant features of the coherence function for Pt. Atkinson and Victoria are now described. Starting at the lowest frequency, the coherence function decreases to a local minimum at 0.8cpd. From this minimum it increases to a broad peak centred at 1cpd, after which it again decreases at 1.15cpd. Between 1.15cpd and 1.8cpd, the coherence function has a constant value of 0.75 (the local minimum centred at 1.45cpd is not statistically significant). The two peaks at 1.93cpd and 2.02cpd are statistically significant. These peaks near or at 1cpd and 2cpd represent the diurnal and semi-diurnal tidal fluctuations in the sea-level. From the semi-diurnal peak, the coherence function decreases to a small value at 2.85cpd. The two peaks at 2.95cpd and 3.85cpd are statistically significant while that at 3.14cpd is not. Between the former two peaks, the coherence function has a constant value of about 0.15.

Negative values of the phase function correspond to a time lag of the fluctuations at Pt. Atkinson relative to those at Victoria. The phase appears to have a constant value of zero from the lowest frequency to about 1.1cpd. From 1.1cpd, the phase decreases smoothly and almost linearly to a value of -155° at 2.95cpd. The phase is observed to be -166° at 3.85cpd. The broad maximum in the phase function centred at 3.4cpd is a statistically significant feature.

The high coherences found at 2.95cpd and 3.85cpd are

associated with the shallow water tidal constituents. These constituent do not appear in the gravitational potential. Rather, they are produced by non-linear interactions between the constituents which are in the gravitational potential. The peak at 2.95cpd is associated with the constituent MK_3 and perhaps with SO_3 . The constituent MK_3 has a frequency which is the sum of the frequencies of M_2 and K_1 . The peak in the coherence function at 3.85cpd is associated with the shallow water constituent, M_4 .

The coherence function for Pt. Atkinson and Tofino are in many ways similar to that for Pt. Atkinson and Victoria. The 95% statistically significant features are now described. Starting from the lowest frequency, the coherence function for Pt. Atkinson and Tofino remains at a relatively constant value until at 0.3cpd, it takes a step decrease to another constant value. From this latter constant value, the coherence function decreases to a local minimum at 0.8cpd. After increasing to a maximum near 1cpd, another local minimum is again reached at 1.15cpd. The very broad peak centred at 1.93cpd is also statistically significant. Again, the peaks at 2.95cpd and 3.85cpd are statistically significant, while the features between these two peaks are not.

Starting from a value of 10° at the lowest frequency, the phase function decreases almost linearly until at 0.35cpd, it reaches a constant value of about -35° . A relatively sharp decrease appears at 1.15cpd. The phase function remains at a constant value of -150° at frequencies greater than 1.15cpd.

However, the peak in the phase function at 2.95cpd is statistically significant.

The coherence function for Pt. Atkinson and Alert Bay is very similar to that for Pt. Atkinson and Tcfinc. The only differences are that the former does not have a peak at 3.85cpd while the 2.95cpd peak now appears at 3cpd. Furthermore, starting from the lowest frequency, the coherence function decreases smoothly to a local minimum at 0.75cpd, with no indication of a step decrease.

The phase function for Pt. Atkinson and Alert Bay is constant at the two lowest frequencies. A decrease from the second point to the third point (0.12cpd) is observed. From 0.12cpd upwards to 1cpd the phase function remains at a constant value of -25° . The relatively sharp decrease from -25° to -130° is observed to be centred at 1.15cpd. Except for the perturbation centred at 2.95cpd, the phase function remains at a constant value of -130° for frequencies above 1.5cpd. It is seen that the phase function for Pt. Atkinson and Alert Bay is very similar to that for Pt. Atkinson and Tcfinc.

The coherence function for Pt. Atkinson and Victoria shows high values of 0.5 and larger between 1cpd and 2.5cpd. When the sea-level at Pt. Atkinson is compared with that at Alert Bay, high values are observed between 1.4cpd and 2.7cpd. High values are observed between 1.25cpd and 2.85cpd in the coherence function for Pt. Atkinson and Tcfinc. In these frequency regions (excluding tidal frequencies), the power spectra of the sea-levels are observed to be relatively flat,

indicating that these are very broad-band fluctuations. To give such high coherences, the processes associated with these fluctuations are essentially the same at each of the four widely separated locations. These non-tidal sea-level fluctuations appear to be barotropic with spatial scales as large as the length of Vancouver Island.

4.11 Relations Between Air-Pressures

The power spectra of air-pressures at Victoria, Comox, Tofino and Alert Bay (see Figure 1 for locations) were calculated. Being very similar to the power spectrum of air-pressure at Vancouver, they are not shown. The air-pressure at Vancouver is compared with those at the other four locations and the results exhibited in Figure 24. Note that the coherence functions are plotted on a linear scale.

The coherence functions all have 95% statistically significant peaks at 2, 3 and 4cpd. Only the two comparisons of air-pressure at Vancouver with that at Victoria and with that at Comox, have 95% statistically significant peaks at 1cpd. The relatively high coherences (peaks) at 1, 2, 3 and 4cpd are associated with atmospheric tides, and the diurnal heating and cooling of the air. The air-pressure also has a diurnal variation since it is dependent on the air-temperature. This process occurs at each location, giving rise to high coherences when the air-pressures at different locations are compared. The diurnal variation in air-pressure is not purely sinusoidal,

Figure 24. Coherence and phase spectra between air-pressure (at mean sea-level) at Vancouver with those from other locations. All have 52 degrees of freedom and all are based on 533 day time-series starting April 16 1969.

giving rise to the higher harmonics at 2,3 and 4cpd. It is not known why some of the coherence functions have statistically significant peaks at 2,3, and 4cpd, but not at 1cpd.

In the frequency regions not at 1,2,3 and 4cpd, the air-pressure fluctuations are associated with another process, the passage of weather systems. This process gives rise the following behaviour of the coherence functions. Against the background of the peaks, the four coherence functions have relatively high values at the lowest frequency, and which decrease smoothly with increasing frequency. The decrease with frequency indicates that high frequencies are associated with high wavenumbers (small scales), which are not as coherent as the low wavenumbers (low frequencies, large scales) because of the distance separating the locations.

Ignoring the peaks for the moment, the coherence functions decrease smoothly until they drop below the 95% noise level. The frequencies at which this occurs are 1.3, 2.1, 2.5 and 3.3cpd for the comparisons of air-pressure at Vancouver with those at Port Hardy, Tofino, Comox and Victoria, respectively. For a fixed frequency (length scale), the fluctuations at Vancouver are most coherent with those at Victoria and least coherent with those at Port Hardy. The geographical relations of the four locations relative to Vancouver are not inconsistent with the observations made from the four coherence functions.

The air-pressure fluctuations at Vancouver are in phase with those at the two closest locations, Victoria and Comox.

The air-pressure fluctuations at Vancouver lag those at the two farthest locations, Tofino and Port Hardy. From a value of 0° at the lowest frequency, these phase lags increase with increasing frequency. The slope of the phase functions for (Vancouver, Tofino) and (Vancouver, Port Hardy) appear to be identical. Evidently, the pressure field arrives at Port Hardy and Tofino simultaneously, before arriving at Vancouver, Victoria and Comox, simultaneously.

4.12 Relations Between Winds

The wind record from Entrance Island is not a continuous one. The wind at Sand Heads is compared with those at Entrance Island for two different periods, and the results shown in Figure 25. For the 80 day comparison period, the coherence function shows an average value of 0.7 for the region $0 < |f| < 0.5$. None of the peaks are statistically significant to 95% confidence. The only statistically significant feature is that the coherence decreases smoothly as $|f|$ increases, reaching the 95% noise level at $\pm 1 \text{ cpd}$. The phase function shows that the coherent portion of the fluctuations is in phase, when the inherent scatter in the phase function is ignored.

The coherence function for the 150 day comparison period is smoother for it has better statistics. In the region $(0, -0.5) \text{ cpd}$, the coherence function has an average value of 0.8, while in the region $(0, +0.5) \text{ cpd}$, a lower value of 0.75 is

Figure 25. Coherence and phase spectra between wind at Sand Heads and those at Entrance Island. Parts (A,B) have (16,30) degrees of freedom and are based on (80,150) day time-series starting (October 10 1969, April 1 1970).

observed. The peak at -2.15cpd is statistically significant to 95% confidence. The diurnal fluctuations of the wind at different locations are not coherent. Again, the coherence function decreases smoothly as $|f|$ increases, but reaching the 95% noise level at $\pm 1.5\text{cpd}$ instead of the $\pm 1\text{cpd}$ observed in the 80 day comparison. The phase function shows that the coherent portion of the fluctuations for this period is also in phase.

The coherence function decreases for increasing frequencies, or for increasing wavenumbers in the wind field. The high wavenumbers (smaller length scales) propagating past each location have less resemblance than the lower wavenumbers (larger length scales), because the former are affected by the topography to a greater extent than are the latter.

4.13 Relations Between Sea-Level, Air-Pressure, Winds

The sea-level at Pt. Atkinson is compared with the air-pressure at Vancouver and with the wind at Sand Heads in Figure 26. The sea-level fluctuations are shown to be coherent ($\gamma_{12}^2 = 0.7$) with the air-pressure fluctuations in the region $(0, 0.5)\text{cpd}$ (Figure 26a). Upwards from 0.5cpd , the coherence function decreases smoothly reaching the 95% noise level at 0.8cpd . The peaks at 1cpd and 2cpd are statistically significant to 95% confidence. It is observed that the peak at 2cpd is above the noise level to 95% confidence. The sea-level and air-pressure fluctuations are out of phase by 180° in the region $(0, 0.8)\text{cpd}$. In this region, the sea-level fluctuations

Figure 26. Coherence and phase spectra among wind (Sand Heads), air-pressure (Vancouver) and sea-level (Pt. Atkinson). All have 60 degrees of freedom and are based on 600 day time-series starting January 03 1969.

are caused by the air-pressure fluctuations (inverse barometer effect). Amidst the scatter in the phase, it is seen that the fluctuations at 1cpd and 2cpd are also out of phase by 180° .

At each location, a large portion of the sea-level fluctuations are caused by the air-pressure fluctuations. The comparison of air-pressures between four pairs of locations show that sea-levels from a pair of locations should also be coherent, as is observed. To a fair approximation, the air-pressure field acts as a rigid lid on the waters around Vancouver Island in the low-frequency region (0,0.5)cpd.

The comparison between sea-level at Ft. Atkinson and wind at Sand Heads is now discussed (Figure 26b). Over much of the frequency range, the coherence between sea-level and wind fluctuations is below the 95% noise level. In the region (0, -0.5)cpd, the coherence function has an average value of 0.25. In this region, the phase (sea-level is leading) starts at 160° before levelling off at about 90° . The coherence function has an average value of 0.41 in the region (0,+0.1)cpd, before dropping very sharply (at 0.1cpd) below the 95% noise level. The phase starts at a value of -165° (wind leading) in this region. A sharp transition of the phase is observed (at 0.1cpd) from -165° to about 95° , and this corresponds to the sharp decrease in the coherence function. None of the peaks are statistically significant to 95% confidence except for the peak at 1cpd. Here, the coherence value is 0.38 while the phase is 165° .

The air-pressure at Vancouver is compared with the wind

at Sand Heads in Figure 26c. The coherence function is seen to be fairly symmetrical for frequencies close to zero. The average coherence values are 0.18 and 0.27 for the regions $(0, -0.1)$ cpd and $(0, +0.1)$ cpd, respectively. With increasing $|f|$, the coherence function drops below the 95% noise level at ± 0.1 cpd. Corresponding to the decrease in coherence at ± 0.1 cpd, the phase makes a sharp transition (at ± 0.15 cpd) from 60° to -165° . In contrast, the phase for negative frequencies, is seen to start at 0° before decreasing smoothly to a constant value near -90° . This behaviour (smooth variation and sharp transition for negative and positive frequency, respectively) of the phase function is very similar to the phase function between sea-level and wind. The similarity is explained by the fact that there is a high coherence between sea-level and air-pressure fluctuations in the region $(0, 0.5)$ cpd. The peaks at ± 1 cpd in the coherence functions are statistically significant to 95% confidence. These peaks are associated with the sharp peaks at ± 1 cpd found in the power spectra of air-pressure and of wind. The peak at -1.7 cpd is considered a spurious one.

Consider a linear system where the air-pressure fluctuations contribute to the sea-level fluctuations, but where the wind fluctuations do not. Then:-

$$\hat{\gamma}_{SW}^2 = \gamma_{PW}^2 \cdot \gamma_{SP}^2 \quad (4.11)$$

where s, p and w denote sea-level, air-pressure and wind, respectively. The subscripts indicate the coherence function

resulting from the comparison of a pair of these parameters. This equality is not observed to hold, indicating that the wind fluctuations are contributing to the sea-level fluctuations to some extent.

The comparison of $\hat{\gamma}_{sw}^2$ of equation (4.11) and γ_{sw}^2 is shown in Figure 27. The functions shown have large scatter, indicating that better statistics (longer time-series) should be used for more confident conclusions. Contributions by the wind fluctuations to the sea-level fluctuations are indicated by gross deviations between $\hat{\gamma}_{sw}^2$ and γ_{sw}^2 . Of course, the comparison is only valid in regions where a significant portion of the sea-level fluctuations are caused by the air-pressure fluctuations. The coherence function between sea-level and air-pressure indicate that these regions are $(0, 0.6)$ cpd and possibly a narrow region near 1cpd.

Examination of Figure 27 indicates that the wind fluctuations (clockwise components) are contributing to the sea-level fluctuations in the region $(0, -0.5)$ cpd and not at -1 cpd. In contrast, the wind fluctuations (anti-clockwise components) contribute to the sea-level fluctuations in the region $(0, +0.1)$ cpd and also at +1cpd. It is stressed that the confidence of these conclusions is only fair.

4.14 Relations Between Currents and Sea-Level, Air-Pressure, Winds

The coherence and phase functions of the currents with sea-level, air-pressure and wind are examined in this section.

Figure 27. Comparison of observed coherence (solid line) between sea-level (Pt. Atkinson) and wind (Sand Heads), and the coherence between the same parameters (dashed line) as calculated by equation (4.11). Observed coherence values needed for this comparison come from Figure 26.

These may provide some ideas about the generation of the currents. The low, diurnal and semi-diurnal frequencies are of the greatest interest, since a major portion of the variance in the currents is found there.

Currents and Sea-Level

The currents on the 50m plane are compared with the sea-level at Pt. Atkinson in Figure 28. The only statistically significant (95%) peaks for the comparison of the eastern location currents with sea-level, are the ones at the tidal frequencies near or at $\pm 1\text{cpd}$ and $\pm 2\text{cpd}$, and then only the higher of the two peaks at $\pm 1\text{cpd}$ is statistically significant. Two peaks appear at $\pm 1\text{cpd}$ because the O_1 constituent and the (K_1, P_1) combination have been resolved, while only one peak appears at $\pm 2\text{cpd}$, which is associated with the (M_2, K_2, S_2, N_2) combination. The peaks at $\pm 0.05\text{cpd}$ are not statistically significant. The coherence between the eastern 50m location currents and sea-level are rather low in regions away from the tidal frequencies.

The comparisons of the currents at the other two 50m locations give similar results: no statistically significant (95%) peaks except at the tidal frequencies, and poor coherences in regions away from the tidal frequencies. Because of better resolution for the central location currents and sea-level, two statistically significant peaks appear at the semi-diurnal frequencies. It is observed that the highest low-

Figure 28. Coherence and phase spectra between 50m currents and sea-level at Pt. Atkinson. Parts (A,B,C) have (46,52,44) degrees of freedom and are based on (469,520,432) day time-series starting (April 16, April 16, July 10) 1969.

Figure 29. Coherence and phase spectra between (140m,200m) currents and sea-level at Pt. Atkinson. Parts (A,B,C) have (38,42,38) degrees of freedom and are based on (375,417,375) day time-series starting (April 16, April 16, July 10) 1969.

frequency (± 0.05 cpd) coherences are obtained when comparing the currents at the 50m eastern location with sea-level.

Some consistency is observed in the phase functions for the 50m currents and sea-level. The phase decreases smoothly from -0.02 cpd to -0.2 cpd. This smooth decrease is also observed in the region $(0.02, 0.2)$ cpd in one of the comparisons. Although the three phase functions have large scatter, some tendencies in the phase are observed. The clockwise rotating current fluctuations (that portion which is coherent with sea-level) have a tendency to lag the sea-level fluctuations by 60° , while the anti-clockwise rotating fluctuations have a tendency to lead the sea-level fluctuations by 150° . It is seen that these tendencies are obeyed at the tidal frequencies where the coherences are highest, and the confidence intervals on the phase estimates are smallest.

The currents at the three deeper locations are compared with the sea-level at Pt. Atkinson in Figure 29. Generally, the results of these three comparisons are similar in many ways to the three comparisons formerly discussed. It is seen that the only statistically significant (95%) peaks are at the tidal frequencies and that away from these frequencies, the coherence values are poor. The highest low-frequency (± 0.05 cpd) coherences are obtained when comparing the currents at the two side locations with sea-level.

The phase functions in the low-frequency region ($|f| < 0.4$ cpd) are observed to decrease smoothly. Tendencies in the phase functions are again observed. In two of the

comparisons, the clockwise-rotating deep current fluctuations (that portion which is coherent with sea-level fluctuations) have a tendency to lag the sea-level fluctuations by about 45° , while the anti-clockwise deep fluctuations have a tendency to lead the sea-level fluctuations by about 140° . The remaining phase function (western 200m location current, sea-level) is dissimilar to the former two phase functions. The clockwise-rotating current fluctuations appear to lag the sea-level fluctuations by greater and greater amounts as the frequency decreases to more negative values. The anti-clockwise current fluctuations have a tendency to be 180° out of phase with the sea-level fluctuations.

Currents and Air-Pressure

The currents at the three 50m locations are compared with the air-pressure at Vancouver in Figure 30. The coherences between the current and air-pressure fluctuations are seen to be poor over much of the frequency range. The only statistically significant (95%) peaks are all at $\pm 2\text{cpd}$ (in one comparison the peak at -2cpd is absent). In all three comparisons, larger coherence values are observed at 2cpd than at -2cpd . The peaks at $\pm 1\text{cpd}$ are not statistically significant. The highest diurnal peaks are found with the central 50m location current and air-pressure. All three phase functions are observed to have large scatter and no tendencies.

The currents at the three deeper locations are compared

Figure 30. Coherence and phase spectra between 50m currents and air-pressure at Vancouver. Parts (A,B,C) have (46,52,44) degrees of freedom and are based on (469,520,432) day time-series starting (April 16, April 16, July 10) 1969.

Figure 31. Coherence and phase spectra between (140m,200m) currents and air-pressure at Vancouver. Parts (A,B,C) have (38,42,38) degrees of freedom and are based on (375,417,375) day time-series starting (April 16, April 16, July 10) 1969.

with the air-pressure at Vancouver in Figure 31. In the comparison using the currents at the western 200m location, statistically significant peaks are found at ± 2 cpd. Using the currents at the central and eastern locations, statistically significant peaks are found only at $+2$ cpd and -2 cpd, respectively. No other statistically significant peaks are found and the coherences are below the 95% noise level over much of the frequency range. A large amount of scatter is again observed in all three phase functions.

Currents and Wind

The currents at the three 50m locations are compared with the winds at Sand Heads in Figure 32. With one exception, there are no statistically significant features in the coherence functions which show poor coherences everywhere. The highest low-frequency ($|f| < 0.3$ cpd) coherences are obtained using the currents at the eastern 50m location. However the high values in the region $(0, -0.2)$ cpd are not statistically significant. The peak at $+0.1$ cpd is also not statistically significant. What is statistically significant (95%) is the smooth decrease in the coherence function as the frequency increases from zero to more positive values. It is also observed that higher coherences are found at $+1$ cpd than at -1 cpd. A large amount of scatter is observed in the three phase functions.

Similar results are obtained when comparing the

Figure 33. Coherence and phase spectra between (140m, 200m) currents and wind at Sand Heads. Parts (A, B, C) have (38, 42, 38) degrees of freedom and are based on (375, 417, 375) day time-series starting (April 16, April 16, July 10) 1969.

currents at the three deeper locations with the wind at Sand Heads (Figure 33). The highest low-frequency ($|f| < 0.3 \text{ cpd}$) coherences are found using the currents at the eastern 140m location. In this comparison, a statistically significant (95%) feature of the coherence function is the smooth decrease found in the region $(0, 0.3) \text{ cpd}$. The high coherences (using the currents at the eastern 140m location) in the region $(0, -0.3) \text{ cpd}$ are not statistically significant, but the peak at -0.9 cpd is statistically significant. Everywhere else, the coherences are poor for the eastern location currents and winds.

No statistically significant features in the coherence functions are found using the currents at the deep central and western locations. The coherence functions generally lie below the 95% noise level. Relatively high coherence values ($\chi^2_{12} > 0.25$) are found near -0.35 cpd when the currents at the western 200m location are compared with the wind at Sand Heads.

The three phase functions for the deep currents and wind again show a large degree of scatter. However, two of the three phase functions (the two obtained with the side location currents) show consistent behaviour in the low-frequency region $(0, \pm 0.3) \text{ cpd}$. In this region, the two phase functions show a smooth decrease with increasing $|f|$.

4.15 Summary

The low-frequency $(0, 0.25 \text{ cpd})$ and tidal portions of the 50m and 140m currents are approximately equal. Together they

account for 80% of the total energy in these currents. The low-frequency and tidal portions contribute about 10% and 70%, respectively, to the total energy of the 200m currents. The coherence functions reveal that the horizontal and vertical resolutions of the low-frequency currents are relatively poor, while that of the tidal currents are relatively good.

A large portion of the temperature variance is found in the low-frequency region $(0, 0.03)$ cpd. Typically 1% of the temperature variance is found at frequencies above 4cpd. The temperature and current spectra are quite dissimilar. The coherence functions between temperature, together with the temperature spectra, suggest that the temperature time-series are too short to be considered stationary, because they contain fluctuations having periods on the order of the time-series length.

The tidal oscillations contribute 97% to the total variance of the sea-level. The remaining portion is found in the region $(0, 0.9)$ cpd. The low-frequency $(0, 0.25)$ cpd fluctuations contribute 85% to the total variance of air-pressure. It is found that the low-frequency sea-level and air-pressure fluctuations are highly coherent. The wind energy is found in a relatively wide frequency region. Only about 50% of the wind energy is found at frequencies below 0.25cpd. It is found that the wind fluctuations do contribute to the low-frequency sea-level fluctuations.

The tidal currents are found to be highly coherent with the tidal oscillations in the sea-level. The 2cpd current and

air-pressure fluctuations have fair coherences. Pccr coherences are found between the low-frequency currents and the low-frequency fluctuations in sea-level and air-pressure. Pcor-to-fair coherences are observed between the low-frequency fluctuations in the currents and wind. The highest low-frequency coherences are observed when comparing the eastern location currents with the wind. This suggests that an investigation on the generation of currents at the eastern location by the wind may be fruitful.

Chapter 5

LOW-FREQUENCY FLUCTUATIONS

In this chapter, the low-frequency fluctuations are examined in more detail. The (u,v) vector components are taken to be positive in the true (east,north) directions. The directions follow the convention, 0° for true north, 90° for true east, etc. The mean speeds and directions, unless otherwise stated, are computed from the means of the vector components. Thus these mean speeds represent the magnitudes of the vector 'mean' velocities. For the overall average, they are the magnitude of the vector mean. In the case where smoothed or filtered components are used, they represent the magnitude of what is essentially a running vector mean. The mean directions are the resultant directions of these vector-averaged data.

5.1 Low-Pass Filtering

The low-frequency fluctuations are examined in the time domain. To this end, the data are low-passed using the filter function, $H(f)$, shown in Figure 34a. This function has a sloping side. Problems arise with a filter function having a much steeper side. A time-series, low-passed with such a steep-sided filter function, would exhibit unnatural

Figure 34. Filter function employed to obtain the low-passed daily time-series (A), and that corresponding to a running daily mean (B).

oscillations or 'ringing'. These oscillations in the low-passed time-series are the strongest in the vicinity of fast changes or large 'jumps'. With the filter function, $H(f)$, this ringing is essentially absent in the resulting low-passed time-series.

The low-pass filtering of a time-series involves four steps. The time-series is Fourier transformed into the frequency domain. Then the Fourier coefficients are multiplied by $H(f)$. Both the Fourier cosine and sine coefficients are multiplied by the same factor. Hence the phases of the resulting low-passed time-series remain unchanged, relative to those in the original time-series. Thirdly, the coefficients are Fourier transformed back into the time domain. Finally, the time-series is resampled at the start of each day. A vector time-series is low-pass filtered by performing the operation on the (u,v) components individually.

The last step above produces a daily time-series from a half-hourly time-series. Aliasing would occur at this stage if the half-hourly time-series contain fluctuations having frequencies greater than the Nyquist frequency (0.5cpd) of the daily time-series. The filter function, $H(f)$, is chosen to have a cut-off frequency of 0.44cpd. Thus no aliasing occurs because fluctuations having frequencies greater than 0.5cpd, are completely removed from the half-hourly time-series before they are resampled at the start of each day.

The low-passed daily time-series of currents, water temperature, wind, air-pressure and sea-level, as derived using

the filter function of Figure 34a, are the subject of this chapter. All results presented in this chapter use these low-passed daily time-series as the starting point of the analyses.

The low-passed filtering described above is somewhat similar to the method of daily averages, but the former has better characteristics. The latter method consists of averaging 48 points of the half-hourly time-series to obtain a single point of the daily time-series. The filter function, $H_1(f)$ for the method of daily averages is:-

$$H_1(f) = \sin(48\pi f \Delta t) / \{48 \cdot \sin(\pi f \Delta t)\} \quad (5.1)$$

where $\Delta t = (1/48)$ day⁻¹ (Gcclin, 1972, page 62), and its graph appears in Figure 34b.

As can be seen, the characteristics of this filter (H_1) are not as good as those of the one used. The cut-off in the range 0-0.5cpd is not as sharp, and aliasing from frequencies 0.5cpd to 24cpd can occur because these frequencies are not completely suppressed. In particular, the O_1 , N_2 and M_2 tides are not well removed, and a significant portion of these tides would remain as aliases in the daily time-series.

5.2 Low-Frequency Time-Series

Shown in Figure 35 and Figure 36 are the low-frequency current component fluctuations, as seen through the frequency window of Figure 34a. These low-frequency fluctuations are aperiodic in character. This is to be expected since their

Figure 35. Low-passed daily time-series of the 50m currents.

Figure 36. Low-passed daily time-series of the 140m and 200m currents.

power spectra show them to have broad-band variance. It is seen that the currents at one location do not closely resemble those at any other location. This is not surprising since the coherence functions between currents yield a low value when they are averaged over the region $(0, 0.4)$ cpd.

The low-frequency fluctuations of the water temperature are exhibited in Figure 37. Poor resemblance is observed between any time-series pair. The temperature fluctuations at the two 200m locations are much smaller than those at the other locations. The temperature and current fluctuations at a location are dissimilar. It is seen that the fluctuations, having time scales of a week to a few months, dominate the current time-series but not the temperature time-series. Inspection shows that the fluctuations in the temperature time-series are contained in a lower frequency region than those in the current time-series, a result confirmed by their power spectra. The temperature time-series are dominated by fluctuations having such low frequencies that they appear as trends in Figure 37.

The low-frequency air-pressure and sea-level fluctuations, as seen through the frequency window of Figure 34a, appear in Figure 38. The air-pressure fluctuations at one location are very similar to those at the other locations. This is also true for the sea-level fluctuations. It is seen that the sea-level fluctuations are not dominated by the fortnightly and monthly tidal oscillations. The air-pressure and sea-level fluctuations are the largest during fall and

Figure 37. Low-passed daily time-series of water temperature.

Figure 38. Low-passed daily time-series of air-pressure and sea-level.

Figure 39. Low-passed daily time-series of wind.

Figure 40. Daily time-series of Fraser River discharge at Agassiz, British Columbia ($49^{\circ}14.7' \text{ N}$, $121^{\circ}45.5' \text{ W}$).

winter. At any location, the air-pressure and sea-level fluctuations are very similar and 180° out of phase, the inverse barometer effect. On the open sea, a rise of 1mb in air-pressure depresses the sea-level by 1cm through the inverse barometer effect (Wunsch, 1972). However, in the vicinity of Vancouver Island a rise of 1mb in air-pressure depresses the sea-level by about 2cm, indicating amplification of the inverse barometer effect. Finally, to a fair approximation, the air-pressure acts as a rigid lid on the waters around Vancouver Island.

Shown in Figure 39 are the low-frequency wind fluctuations, as seen through the frequency window of Figure 34a. Inspection of Table III shows that the low-frequency fluctuations exhibited in Figure 39 represents about 50% of the wind variance. The wind fluctuations are aperiodic in character. The low-frequency fluctuations at Sand Heads and Entrance Island show a fair-to-good resemblance.

The Fraser River discharge (at Agassiz, British Columbia) is presented in Figure 40. This time-series is obtained in the form of daily averages, so that further low-pass filtering is unnecessary. In both years shown, the freshet occurred during May and June.

5.3 U-V Scatter Diagrams

The U-V scatter diagrams of the currents at each of the six locations are shown in Figure 41. In these diagrams, each

Figure 41. U-V scatter diagrams of the low-passed currents.

current observation (u,v) of the daily time-series is plotted as a point on the u - v plane. The points inserted in the gaps of the time-series are not plotted. For each scatter diagram, there is an associated correlation coefficient, c , defined as:-

$$c = \overline{(u-U)(v-V)} / \sqrt{\text{var}(u) \cdot \text{var}(v)} \quad (5.2)$$

where (U,V) are the means of the daily (u,v) component time-series, where $\text{var}(\cdot)$ denotes the variance about the mean of the indicated parameter, and where the overbar indicates a time average. Points inserted in the gaps of the time-series are not used in the calculation of the variance and time averages. The coefficient, c , has a range $(-1,1)$. It has a value of ± 1 when all the points on the u - v plane fall on a straight line having a $(\pm)$ slope. The coefficients (c) for the U - V scatter diagrams are listed in Table IV.

It is seen that the current motions are aligned in a NS direction at the two eastern locations. The currents at the deep eastern location have a predominately northward v -component. The currents at the central 50m location exhibit the most scattering. The overall strength of the currents is seen to decrease with depth.

The currents need not be resolved into components that are parallel to the (x,y) axes, where the y -axis is oriented towards true north. The currents may be resolved into the (u',v') components that are parallel to the (x',y') axes, so that the correlation coefficient, (c') based on the (u',v') components, is zero. Hence the Reynolds stress, $\overline{(u'-U')(v'-V')}$

$\overline{V'})$, as measured in the (x',y') coordinate system is zero. There is a coordinate system (x'',y'') where the correlation coefficient (and Reynolds stress) has a maximum value, c_m . The coordinate system (x',y') coincides with (x'',y'') if the former is rotated 45° in an anti-clockwise direction. Generally the y' -axis is not parallel to the mean vector (U,V) . The y' -axis direction and the maximum correlation coefficient, c_m , for each location are given in Table IV.

It is seen that the y' -axis is within 13° of true north at five locations. The currents at the central 50m location have a y' -axis whose direction is 68° .

The Boundaries

The depth contours are aligned north-south in the region being studied. To the north and south of this region, the depth contours are aligned approximately northwest-southeast. The horizontal distances in both the NS and EW directions, from each of the six locations to the nearest boundaries, are listed in Table IV. These boundaries are relevant to the study of the low-frequency current motions.

Most of the boundaries are the side boundaries of the Strait of Georgia, but some are not. Relative to the western 200m location, there is a submarine ridge 4km to the south. This ridge is aligned in a NNW direction and its peak lies at a depth of 170m. Relative to the central 200m location, there lies the steep wall of a submarine plateau 3.5km to the east.

TABLE IV - Long-Term Means and FKE

	H06 (Western)		H16 (Central)		H26 (Eastern)	
	50m	200m	50m	200m	50m	140m
Days of Available Data	419	389	533	346	412	319
Distance to Boundary (E or W)	5.5 (W)	2.6 (W)	10.8 (E)	3.5 (E)	2.3 (E)	0.8 (E)
(km)						
Distance to Boundary (N or S)	10.4 (S)	4.0 (S)	20.3 (N)	12.6 (N)	15.8 (N)	10.6 (S)
Mean magnitude of current vector (cm/s)	3.0 $\pm$.5	1.6 $\pm$.8	4.3 $\pm$.5	1.4 $\pm$.5	3.0 $\pm$ 1.1	7.4 $\pm$.5
(with 95% confidence limits)						
Mean direction of current vector (deg.true)	355 $\left(\begin{smallmatrix} 4 \\ 345 \end{smallmatrix} \right)$	346 $\left(\begin{smallmatrix} 353 \\ 330 \end{smallmatrix} \right)$	108 $\left(\begin{smallmatrix} 116 \\ 81 \end{smallmatrix} \right)$	21 $\left(\begin{smallmatrix} 44 \\ 0 \end{smallmatrix} \right)$	9 $\left(\begin{smallmatrix} 15 \\ 5 \end{smallmatrix} \right)$	7 $\left(\begin{smallmatrix} 14 \\ 0 \end{smallmatrix} \right)$
(with 95% confidence limits)						
Mean of current speeds (cm/s)	7.65	3.16	8.23	3.56	6.98	8.40
u-component	8.3 $\left(\begin{smallmatrix} 14.0 \\ 5.5 \end{smallmatrix} \right)$	0.7 $\left(\begin{smallmatrix} 1.2 \\ 0.5 \end{smallmatrix} \right)$	20.5 $\left(\begin{smallmatrix} 45.7 \\ 11.6 \end{smallmatrix} \right)$	2.0 $\left(\begin{smallmatrix} 2.8 \\ 1.5 \end{smallmatrix} \right)$	2.1 $\left(\begin{smallmatrix} 2.8 \\ 1.6 \end{smallmatrix} \right)$	1.3 $\left(\begin{smallmatrix} 2.2 \\ 0.8 \end{smallmatrix} \right)$
Low Frequency Fluctuating						
Kinetic Energy per unit						
mass (ergs/gr) (with 95%	25.4 $\left(\begin{smallmatrix} 53.6 \\ 14.7 \end{smallmatrix} \right)$	4.8 $\left(\begin{smallmatrix} 7.4 \\ 3.4 \end{smallmatrix} \right)$	11.3 $\left(\begin{smallmatrix} 22.3 \\ 6.8 \end{smallmatrix} \right)$	7.2 $\left(\begin{smallmatrix} 10.8 \\ 5.1 \end{smallmatrix} \right)$	28.9 $\left(\begin{smallmatrix} 48.5 \\ 19.1 \end{smallmatrix} \right)$	19.0 $\left(\begin{smallmatrix} 32.5 \\ 12.5 \end{smallmatrix} \right)$
confidence intervals)						
v-component						
u+v	33.7 $\left(\begin{smallmatrix} 58.8 \\ 21.8 \end{smallmatrix} \right)$	5.5 $\left(\begin{smallmatrix} 8.0 \\ 4.1 \end{smallmatrix} \right)$	31.8 $\left(\begin{smallmatrix} 58.0 \\ 20.1 \end{smallmatrix} \right)$	9.1 $\left(\begin{smallmatrix} 13.1 \\ 6.7 \end{smallmatrix} \right)$	30.9 $\left(\begin{smallmatrix} 50.0 \\ 21.0 \end{smallmatrix} \right)$	20.3 $\left(\begin{smallmatrix} 33.4 \\ 13.6 \end{smallmatrix} \right)$
Correlation coefficient (c)	0.30	0.27	0.30	-0.29	0.11	-0.64
Maximum correlation coefficient (c _m)	0.57	0.76	0.41	0.61	0.87	0.93
y-axis orientation for zero						
Reynolds stress (y') (deg. true)	13	7	68	349	2	10
Mean water temperature (°C)	8.47	8.82	8.81	8.70	8.48	8.45

This wall, which is aligned in a NS direction, extends a considerable distance to the south but only about 2km to the north. The edge of this plateau lies 180m below the water surface. From this edge, the plateau slopes gradually upward towards the east.

5.4 Monthly Means

The monthly vector means of the low-frequency currents are shown in Figure 42. Monthly vector means are derived from sections of the low-passed daily time-series all having a duration of one month. The mid-point time-origin of consecutive sections are separated by 10 days. Monthly vector means are plotted against the mid-point time-origin of the one month section from which they are derived.

At the eastern mooring, the 140m current directions are always north. There, the 50m current directions are mostly north, although south directions are observed when the current magnitudes are below 4 cm/s. The magnitude variations of the 50m and 140m currents at the eastern mooring are somewhat similar. The relevant coherence functions of the previous chapter show that the highest vertical coherence are found at the eastern mooring.

At the central mooring, the highest 50m current magnitudes are observed during March, October, and December of 1969 and during July 1970. During the three periods in 1969, the 50m current has directions which are east. During July

Figure 42. Magnitude and direction of the monthly vector means for the low-passed currents versus time.

1970 however, the 50m current has a SW direction. The 50m current during July of the previous year is directed south. These southward directions of the 50m current coincide with the times of the Fraser River freshet. However, there is no way of knowing with only two events whether or not there is any relationship. The 50m current is observed to flow in a WNW direction with a relatively small magnitude of 3 cm/s during February 1970.

The 200m current at the central mooring has directions varying from N to NE during periods of relatively high current magnitudes. Other directions are observed for current magnitudes less than 1 cm/s. The magnitudes of the 50m and 200m currents at the central mooring are not well correlated.

At the western mooring, the highest magnitudes of the 50m current are observed to be 7 cm/s. At such times the current is directed N or NNW. During February 1970, a relatively high 50m current magnitude of 5 cm/s is found to be directed south. At the western mooring, the 200m current direction is often north. During December 1969 and January 1970, the 200m current is directed to the W and WNW. A 200m current direction of SW is observed during June and July 1970. The magnitude variations of the 200m current at the western location do not correlate with those of the 50m current at the same mooring, or with that of the 200m current at the central mooring.

The magnitude variations of the 50m currents at the three moorings tend to be correlated with each other. High 50m

current magnitudes are observed during March, October, and December of 1969, and during July 1970 at all locations where there is available data. It is seen that the highest 50m current magnitudes at the western mooring tend to occur half a month earlier than those at the central mooring.

The monthly vector means of wind at Sand Heads, the monthly means of air-pressure at Vancouver, and that of sea-level at Pt. Atkinson are given in Figure 43. Between May 1969 and February 1970, the wind directions are mostly found in the range W to NW. During this period, the largest wind magnitudes of 3 m/s are associated with a WNW direction. A relatively high wind magnitude of 2 m/s is directed approximately NE in April 1970.

The influence of air-pressure on sea-level is easily discerned. Most peaks and valleys in air-pressure correspond to valleys and peaks in sea-level. However, the air-pressure variations cannot account for all the sea-level variations. The large 20cm drop in sea-level during March 1970 is not accounted for by the air-pressure. This sea-level drop is also not associated with the wind, which has relatively small magnitudes in March 1970. The increase of sea-level in June 1970 is attributed to the Fraser River freshet. Waldichuk (1964) gives ample evidence pertaining to the sea-level changes in the Strait of Georgia due to the Fraser River freshet. During June 1969, the sea-level peak is caused by the Fraser River freshet as well as by a drop in air-pressure. The monthly means of sea-level and wind do not correlate too well.

Figure 43. Magnitude and direction of the monthly vector means for the low-passed wind (Sand Heads), and the monthly means of the air-pressure (Vancouver) and the low-passed sea-level (Pt. Atkinson), all versus time.

At the two eastern locations, there is some tendency for high current magnitudes to occur when the wind magnitudes are low. However, this tendency is not observed during November and December of 1969, and the data record is too short to be sure that this apparent possible relationship is not just a coincidence. The monthly means of sea-level, air-pressure, or wind correlate poorly with those of the currents at any of the four remaining locations. These conclusions could have been predicted from the low-frequency regions of the coherence functions presented in the previous chapter.

5.5 Monthly FKE

The fluctuating kinetic energy per unit mass (ergs/gw) shall be denoted by FKE. The u-FKE is half the variance of a u-component time-series, and similarly for the v-FKE. The total FKE is the sum of the u-FKE and v-FKE. The calculation of FKE proceeds after removal of the long-term component means from the component time-series. Long-term means are the means derived using all the available data from a particular location.

Consecutive monthly FKE's are derived from consecutive one month sections of the low-passed daily time-series whose mid-point time-origins are separated by 10 days. A monthly FKE is plotted against the mid-point time-origin of the section from which it is derived.

The monthly FKE of the current components and the

monthly total FKE are presented in Figure 44. The monthly variances of the wind at Sand Heads, of the air-pressure at Vancouver, and of the sea-level at Pt. Atkinson are presented in Figure 45. These monthly variances are also calculated from the low-passed daily time-series after removal of the long-term means.

The u-FKE's for the currents from the three deep locations do not exceed 5 ergs/gm at any time. On the 50m plane, the u-FKE at the central location is usually much larger than those at the side locations. u-FKE's above 40 ergs/gm are found during February and June of 1970 at the 50m central location. The large u-FKE of June 1970 is not present during June of the previous year. The u-FKE's of the 50m central and western locations are not well correlated.

Among the three deep locations, the highest v-FKE's are always found at the eastern location. Among the three 50m locations, the highest v-FKE's are also found at the eastern location, while the lowest v-FKE's are observed at the central 50m location. At the central 50m location, high v-FKE's tend to be associated with high u-FKE's. This is not true of the FKE's at the western 50m location. The v-FKE's of the three 50m locations are not well correlated. The v-FKE's of the vertically-separated eastern locations also show poor correlation.

The total FKE's of the deep locations are not well correlated with each other. This is also true of the total FKE's at the 50m locations. During July 1970, low values of

Figure 44. Monthly FKE of the low-passed current components versus time.

Figure 45. Monthly variance of the low-passed wind components (Sand Heads), low-passed air-pressure (Vancouver) and low-passed sea-level (Pt. Atkinson), all versus time.

the total FKE are observed at the central 50m location. One year later, very high values of the total FKE are found at this location. There may be seasonal variations in the monthly total FKE's, but much longer time-series are required to perceive them.

There is a fair correlation between the u-variance and v-variance of the wind. The highest wind variances are observed during April 1970. The monthly variance of air-pressure show a broad maximum during fall-winter. Very low values of the sea-level variance coincide with those of air-pressure variance. The peak in the sea-level variance during April 1970 coincides with one in the total wind variance. However, neither the total wind variance nor the air-pressure variance appear to be responsible for the large peak in sea-level variance during December 1969.

One may compare the total wind variance, the air-pressure variance, or that of sea-level with the total FKE of the currents at any of the six locations. In all comparisons, poor correlations are obtained. This is also true when comparisons are made with the u-FKE and v-FKE of the currents. The wind, air-pressure, and Fraser River discharge may well be important parameters in the generation of these low-frequency currents. The coupling of these parameters to the sub-surface currents may well be a very complex coupling, explaining the poor correlations observed. However, these parameters do not constitute a complete list of important parameters. The water density is an important parameter (because the currents are

baroclinic) that is not available.

5.6 Long-Term Means

The long-term means are those derived from all the available data in a time-series (excluding data gaps). The long-term current component means are given in Table IV as magnitudes and directions. The directions of the current means at the four side locations are all within 16° of true north. They are essentially parallel to the depth contours. At the central mooring, the direction of the 200m current mean is NNW, while that of the 50m current mean is ESE. The highest mean current magnitude of 7.4 cm/s is found at the deep eastern location. Equal mean current magnitudes are observed at the two 50m side locations. The magnitudes found at the pair of 200m locations are also approximately equal. A relatively high mean current magnitude of 4.3 cm/s is observed at the 50m central location.

These long-term current means may not be used in the calculation of transport. Previous results show that the current motions are poorly resolved both in the vertical and horizontal. The moorings were placed 10km apart. Moorings placed 2km apart might be sufficient to yield long-term current means suitable for transport calculations. The two instruments on the eastern mooring were separated by 90m, a separation too large to resolve the vertical structure of the low-frequency current motions.

The Department of the Environment (1972b), (1972c), placed lines of moorings within 10-20 km of the present line being studied. The moorings in these lines were placed 4km apart and data collected from these instruments span approximately a month. Possibly, examination of this data base may yield a better idea of the horizontal instrument separation required to resolve the low-frequency current motions well.

One may wonder about the dependence of the long-term means on the length of the time-series. The magnitudes and directions of the current means versus averaging time are presented in Figure 46. For comparison, the wind vector mean, the air-pressure mean, and the sea-level mean are plotted versus averaging time in Figure 47. For these calculations, data gaps in a low-passed daily time-series are removed and the remaining sections are shifted in time as necessary to join them end to end. An average over N days is then derived from the first N points in the low-passed daily time-series of the components, and the resultant magnitude and direction are plotted against the averaging time, N.

The mean current magnitudes at the three deep locations (1-2 cm/s at 200m) are constant to within 0.4 cm/s for averaging times of 8 months or longer. Those at the three 50m locations (about 3 cm/s) vary within a range of 1 cm/s for averaging times of 10 months or longer. The mean current directions at the two eastern locations and that at the western 200m location are constant to within 5° for averaging times of 8 months or longer. Those at the remaining locations are less

Figure 46. Magnitude and direction of the vector means for the low-passed currents versus averaging time. Each graph of direction has an arrow indicating its vertical scale.

Figure 47. Magnitude and direction of the vector mean for the low-passed wind (Sand Heads), and the means of the low-passed air-pressure (Vancouver) and low-passed sea-level (Pt. Atkinson), all versus averaging time.

constant. They vary within a range of 16° for averaging times of 8 months or longer.

The mean wind direction is constant to within 20° while its magnitude (about 100 cm/s) is constant to within 50 cm/s, for averaging times of 6 months or longer. The mean air-pressure varies within a range of 1mb for averaging times of 6 months or longer. There is an inverse correlation between the fluctuations of mean sea-level and air-pressure because of the inverse barometer effect. For averaging times of 14 months or longer, mean sea-level has a trend while mean air-pressure does not. The power spectrum of sea-level shows that fluctuations with a period of 300 days have significant variance. Thus the behaviour of mean sea-level for averaging times of 14 months or longer is intuitively expected.

Overall, the current vector means appear to converge towards stable values with increasing averaging time. These means will continue to converge (for averaging times of 2 years and longer) provided that the currents are truly stationary. It is impossible with the available data to conclude that the currents are truly stationary or non-stationary. However, it appears that the currents are stationary (with respect to their means) during the limited period 1969-70.

Confidence Intervals for the Means

Suppose that an infinite stationary time-series, $u(t)$ has mean μ and variance σ^2 . Its autocorrelation function $r(\tau)$

is:-

$$r(\tau) = \overline{[u(t) - \mu] \cdot [u(t+\tau) - \mu]} / \sigma^2 \quad (5.3)$$

where the overbar indicates a time average. This function is an even function of τ and it has the limits $(-1, 1)$. One can only obtain estimates of the autocorrelation function, $r(k)$, with a discrete time-series, u_i :-

$$r(k) = [\overline{u^2} \cdot (N-k)]^{-1} \cdot \sum_{n=1}^{N-k} (u_n - \bar{u}) \cdot (u_{n+k} - \bar{u}) \quad (5.4)$$

where the subscripts are time indices, N is the time-series length, and $(\bar{u}, \overline{u^2})$ are the (mean, variance) of the time-series.

Consider means, U_T , that are derived from finite sections of $u(t)$ all having length T . The distribution of U_T (for large T) about the expected value (μ) is Gaussian with a variance:-

$$\text{var}(U_T) = (2\sigma^2/T) \int_0^T [1 - (\tau/T)] \cdot r(\tau) d\tau \quad (5.5)$$

a result given by Tennekes and Lumley (1972) (section 6.4, 6.5).

The 95% confidence intervals for the means (U_T, V_T) of the current components are $\pm 1.96 \cdot [\sqrt{\text{var}(U_T)}, \sqrt{\text{var}(V_T)}]$, which are listed in Table V for various values of T . These confidence intervals are themselves estimates since the calculation of $\text{var}(U_T)$ proceeds after $r(\tau)$ has been replaced by $r(k)$, and (μ, σ^2) have been replaced by $(\bar{u}, \overline{u^2})$. As expected, the confidence intervals for the current component means decrease with increasing averaging time, T . If the time-series

TABLE V - Variances (according to equation 5.5) and 95% confidence intervals of current component means derived from time-series of length, T.

Location	T (days)	Var(U_T) (cm/sec) ²	Var(V_T) (cm/sec) ²	95% CONFIDENCE INTERVALS	
				U(cm/sec)	V(cm/sec)
EASTERN 50m	10	1.09	27.1	2.05	10.2
	20	0.587	18.4	1.50	8.40
	30	0.399	13.7	1.24	7.26
	40	0.307	10.7	1.09	6.41
	50	0.233	8.23	0.946	5.62
	60	0.191	6.60	0.856	5.04
	370	0.00145	0.317	0.0746	1.10
EASTERN 140m	10	0.984	13.0	1.94	7.06
	20	0.738	7.37	1.68	5.32
	30	0.667	5.38	1.60	4.55
	40	0.632	4.37	1.56	4.10
	50	0.600	3.57	1.52	3.70
	60	0.566	3.08	1.48	3.44
	180	0.198	0.0283	0.873	0.329
CENTRAL 50m	10	26.1	11.1	10.0	6.54
	20	17.9	8.08	8.28	5.57
	30	12.2	6.65	6.85	5.05
	40	8.98	5.86	5.87	4.75
	50	6.91	5.16	5.15	4.45
	60	5.41	4.57	4.56	4.19
	480	0.0215	0.0874	0.287	0.579
CENTRAL 200m	10	1.27	4.80	2.20	4.29
	20	0.817	2.20	1.77	2.91
	30	0.639	1.29	1.57	2.22
	40	0.561	1.01	1.47	1.97
	50	0.506	0.722	1.39	1.67
	60	0.442	0.580	1.30	1.49
	300	0.0628	0.0229	0.491	0.297
WESTERN 50m	10	5.06	26.7	4.41	10.1
	20	2.84	17.4	3.30	8.17
	30	1.80	13.7	2.63	7.25
	40	1.23	11.6	2.17	6.67
	50	0.918	10.3	1.88	6.29
	60	0.755	9.08	1.70	5.91
	290	0.0458	0.0511	0.419	0.443
WESTERN 200m	10	0.395	3.15	1.23	3.48
	20	0.269	1.75	1.02	2.59
	30	0.230	1.38	0.940	2.31
	40	0.205	1.26	0.888	2.20
	50	0.188	1.09	0.850	2.05
	60	0.171	0.961	0.811	1.92
	330	0.00161	0.147	0.0786	0.751

TABLE VI - Comparisons between the theoretical variances of Table V and the corresponding empirical variances calculated from the current component time-series of length, T.

Location	T (days)	Var(U_T) (cm/s) ²		Var(V_T) (cm/s) ²	
		Empirical	Equation 5.5	Empirical	Equation 5.5
EASTERN 50m	30	0.370	0.399	14.1	13.7
	60	0.136	0.191	6.16	6.60
	90	0.0870	0.121	3.73	4.34
EASTERN 140m	30	0.749	0.667	5.75	5.38
	60	0.709	0.566	3.33	3.08
	90	0.554	0.470	1.99	2.05
CENTRAL 50m	30	12.5	12.2	6.83	6.65
	60	5.70	5.41	4.68	4.57
	90	2.80	2.60	3.81	3.55
CENTRAL 200m	30	0.614	0.639	1.17	1.29
	60	0.278	0.441	0.541	0.580
	90	0.172	0.290	0.330	0.349
WESTERN 50m	30	2.07	1.80	14.3	13.7
	60	0.864	0.755	9.41	9.08
	90	0.528	0.445	5.72	6.64
WESTERN 200m	30	0.244	0.230	1.49	1.38
	60	0.176	0.171	0.887	0.961
	90	0.134	0.123	0.595	0.723

is truly stationary, one can obtain an estimate (U_T) that is arbitrarily close to the theoretical value (μ), by choosing a sufficiently long averaging time, T .

The 95% confidence intervals for the long-term current means, listed in Table IV, are derived from the entries in Table V which have the maximum value of T . The largest 95% confidence intervals for the (magnitude, direction) of the long-term vector means are approximately (± 1 cm/s, $\pm 25^\circ$). Hence, if the currents are truly stationary, vector means based on future time-series of comparable length will probably be within 2 cm/s and 50° of the values listed in Table IV. As stated earlier though, it is not possible with the available data to conclude that the currents are truly stationary or non-stationary.

In Table VI the variances of the component means, as derived from equation (5.5), are compared with the empirically derived values, for $T=30, 60, 90$ days. The empirically derived values for $T=30$ days are calculated using the time-series of monthly current means presented earlier. There is seen to be fairly close agreement.

5.7 Long-Term Variances

The long-term total FKE's (Table IV) of the currents does vary consistently with depth, as shown in Figure 48a. It is tempting to state that the total FKE in a water column is dependent only on depth, provided that the total FKE is measured outside any boundary layers. The 140m eastern

Fig. 48. Long-term total FKE of the low-passed currents versus depth (A), and maximum correlation coefficient (C_m) and ratio of v-FKE/u-FKE, both versus distance to the nearest lateral boundary (B).

location is situated 25m from the bottom and 0.8km from the lateral boundary. After locating the total FKE for this location in Figure 48a, it is tempting to state that this location is outside the boundary layers.

Whatever the behaviour of the total FKE in a water column, it must be zero at maximum depth. One may naively expect the total FKE to decrease with depth. Most of energy sources are located at the surface while an energy sink (friction) is located at the bottom. That the total FKE should decrease linearly with depth (if it does), requires a more sophisticated explanation.

The partition of the total FKE into the u-FKE and v-FKE is influenced strongly by the lateral boundaries. Figure 48b shows the dependence of v-FKE/u-FKE on the separation of the locations from the nearest lateral boundary. Also plotted are the maximum correlation coefficients, c_m , (Table IV) of the low-frequency current U-V scatter diagrams. The coefficient, c_m , may be interpreted as one measure of the tendency towards rectilinear motion. This parameter is more useful in that it is bounded (0,1), and that it is independent of any coordinate system. A value of unity indicates completely rectilinear motion, while a value of zero indicates completely disorganized (horizontally isotropic) motion.

It is intuitively reasonable to expect the tendency towards rectilinear motion (c_m) to increase as one approaches a lateral boundary. Figure 48b confirms this expectation. It is observed that c_m varies most rapidly at a location-to-boundary

separation of 2.5 km. The exact significance of this number is unknown. Perhaps, it will ultimately be useful as a lateral length scale, or as a characteristic length of some boundary layer for the low-frequency current motions.

The component and total current FKE's versus averaging time are presented in Figure 49. These are derived from the low-passed-daily time-series. For comparison, the wind variance at Sand Heads, the air-pressure variance at Vancouver, and the sea-level variance at Pt. Atkinson are plotted versus averaging time in Figure 50. These calculations proceed only after the long-term means are removed from each low-passed time-series.

The total current FKE of the three deep locations are constant to within 3 ergs/gm for averaging times over 4 months. That of the two 50m side locations vary within a range of 3 ergs/gm for averaging times over 8 months. The total FKE of the central 50m location varies within a range of 7 ergs/gm for averaging times over 10 months. The corresponding component FKE's are constant to within the same specified ranges (if not less) for averaging times greater than the specified times.

The component and total wind variances are constant to within 15 (m/s)^2 for averaging times over 6 months. The air-pressure and sea-level variances vary within a range of 5 m^2 and 15 cm^2 , respectively, for averaging times over 10 months.

The component FKE's of the 50m currents at the central mooring do not appear to converge towards stable values even for averaging times of 16 months and longer. However, this

Figure 49. FKE of the low-passed current components versus averaging time.

Figure 50. Variance of the low-passed wind components (Sand Heads), low-passed air-pressure (Vancouver), and low-passed sea-level (Pt. Atkinson), all versus averaging time.

behaviour does not mean that the current time-series is non-stationary. The FKE (second order statistic) of a truly stationary time-series having a Gaussian distribution converges more slowly than its mean (first order statistic). This is readily seen if one compares the 95% confidence intervals for the mean and FKE. It is not possible to conclude that the low-frequency currents are truly stationary or non-stationary. However, it appears that they are stationary with respect to the FKE during the limited period 1969-70.

Confidence Intervals for the FKE

Suppose that an infinite stationary time-series, $\underline{u}(t)$, has zero mean and variance σ^2 . Consider estimates $(\sigma_T^2)_u$ that are based on different sections of $\underline{u}(t)$ which all have length T . The estimates $(\sigma_T^2)_u$ are clustered about the theoretical value (σ^2) with a variance of:-

$$\text{var}(\sigma_T^2)_u = (4\sigma^4/T) \int_0^T [1 - (\tau/T)] \cdot r^2(\tau) d\tau \quad (5.6)$$

a result given by Bendat and Piersol (1971) (section 6.2). The estimates $u\text{-FKE} = (\sigma_T^2)_u / 2$ has a variance of $\text{var}(\sigma_T^2)_u / 4$. The primary assumption made is that $\underline{u}(t)$ is a stationary Gaussian stochastic process. The variances of σ_T^2 for the current components are given in Table VII for various values of T . As expected the variance is a decreasing function of T .

The variance of the parameter $(\sigma_{T+V}^2) = (\sigma_T^2)_u + (\sigma_T^2)_v$ is given by :-

TABLE VII - Theoretical Variance (equation 5.6) of
Current Component FKE's Based on Record
Length, T

Location	T	$\text{Var}(\sigma_T^2)_u$	$\text{Var}(\sigma_T^2)_v$
	(days)	(cm/s) ⁴	(cm/s) ⁴
EASTERN 50m	10	5.91	2200.
	20	3.11	1240.
	30	2.13	874.
	40	1.63	680.
	50	1.33	571.
	60	1.12	494.
	420	0.385	182.
EASTERN 140m	10	2.97	771.
	20	1.80	429.
	30	1.41	303.
	40	1.21	235.
	50	1.07	194.
	60	0.957	166.
	320	0.417	83.6
CENTRAL 50m	10	1510.	313.
	20	917.	187.
	30	656.	136.
	40	511.	109.
	50	419.	90.7
	60	356.	77.7
	480	97.2	22.3
CENTRAL 200m	10	5.36	90.8
	20	2.93	49.0
	30	2.04	34.2
	40	1.62	27.0
	50	1.36	22.6
	60	1.18	19.6
	300	0.448	7.35
WESTERN 50m	10	109.	1860.
	20	57.3	1040.
	30	39.4	733.
	40	30.4	572.
	50	24.8	473.
	60	21.1	403.
	400	7.58	133.
WESTERN 200m	10	0.788	45.3
	20	0.456	24.6
	30	0.341	17.3
	40	0.282	13.8
	50	0.245	11.5
	60	0.216	9.94
	330	0.0985	3.55

TABLE VIII - Comparison Between Theoretical and Empirical Variances
of Current Component FKE's Based on Record Length, T.

Location	T (days)	Var($\sigma_{T_u}^2$) (cm/s) ⁴		Var($\sigma_{T_v}^2$) (cm/s) ⁴		Cov [$(\sigma_{T_u}^2), (\sigma_{T_v}^2)$]
		Empirical	Equation 5.6	Empirical	Equation 5.6	
EASTERN 50m	30	4.18	2.13	1000.	874.	7.81
	60	1.81	1.12	578.	494.	4.82
	90	1.01	0.793	387.	358.	2.14
EASTERN 140m	30	0.761	1.41	237.	303.	2.29
	60	0.427	0.957	87.1	166.	0.978
	90	0.532	0.743	87.0	127.	1.57
CENTRAL 50m	30	1020.	656.	208.	136.	92.3
	60	720.	357.	135.	77.7	71.2
	90	525.	252.	110.	54.9	57.1
CENTRAL 200m	30	3.98	2.04	36.8	34.2	2.22
	60	2.49	1.18	19.4	19.6	1.57
	90	1.62	0.858	14.2	14.5	1.08
WESTERN 50m	30	55.9	39.4	482.	733.	-2.51
	60	29.7	21.1	235.	403.	-4.77
	90	23.6	15.0	112.	280.	-2.97
WESTERN 200m	30	0.654	0.341	29.1	17.3	0.244
	60	0.373	0.216	14.1	9.94	0.245
	90	0.281	0.166	8.12	5.14	0.280

$$\text{var}(\sigma_{T,u+v}^2) = \text{var}(\sigma_{T,u}^2) + \text{var}(\sigma_{T,v}^2) + 2 \cdot \text{cov}[(\sigma_{T,u}^2), (\sigma_{T,v}^2)] \quad (5.7)$$

where cov denotes the covariance between the two indicated parameters. The empirical values of these covariances (T=30, 60, 90 days) are found in Table VIII. At three locations, the covariances are about 1% of $[\text{var}(\sigma_{T,u}^2) + \text{var}(\sigma_{T,v}^2)]$. The highest covariances are observed at the central 50m location where they are 10% of $[\text{var}(\sigma_{T,u}^2) + \text{var}(\sigma_{T,v}^2)]$.

The distributions of the estimates $(\sigma_{T,u}^2)$, $(\sigma_{T,v}^2)$, and $(\sigma_{T,u+v}^2)$ are unknown. Being positive-valued variables, these estimates are fitted to an $a \cdot \chi_\gamma^2$ distribution where:-

$$\gamma = 2\sigma^4 / \text{var}(\sigma_{T,u}^2) \quad a = \text{var}(\sigma_{T,u}^2) / (2\sigma^2) \quad (5.8)$$

a procedure used by Jenkins and Watts (1968) (section 3.3.5). The estimates u-FKE are fitted to $(a/2) \cdot \chi_\gamma^2$ distribution.

The values of $\text{var}(\sigma_T^2)$ derived from equation (5.6) are compared to the empirical values in Table VIII. Empirical values for T=30 days are calculated using the time-series of the monthly current FKE's. It is seen that the empirical values do disagree, by as much as a factor of two, with those values derived from equation (5.6). It is conjectured that the disagreement stems from the assumption of $\underline{u}(t)$ following a Gaussian distribution.

The 95% confidence intervals for the long-term current FKE's are listed Table IV. These are derived using the entries (having the largest T) from Table VII, and by fitting the FKE estimates to $(a/2) \cdot \chi_\gamma^2$ distribution. Note that the variances for the component FKE's from this table have been doubled,

prior to the calculation of the confidence intervals for the central and western 50m locations. This has been done because Table VIII indicates that the variances, derived from equation (5.6) for these two locations, are low. The confidence intervals for the monthly FKE's may be similarly derived using the entries (with $T=30$ days) in Table VII:

The variances of the long-term total FKE estimates for the four side locations are calculated by assuming that $\text{cov}[(\sigma_T^2)_u, (\sigma_T^2)_v]$ is approximately zero. For the two central locations, the covariance has been approximated as 10% of $[\text{var}(\sigma_T^2)_u + \text{var}(\sigma_T^2)_v]$. These procedures are taken with reference to the empirical covariance values in Table VIII.

Chapter 6

TIDAL CURRENTS

6.1-----Harmonic Analysis

The results of this chapter are based on harmonic analyses of 60 day sections of the half-hourly current time-series. A harmonic analysis is essentially a least-squares fit to the current components using functions of the form:-

$$\begin{aligned} u'(t) &= \sum_{k=1}^n [B_1(f_k) \cos(2\pi f_k t) + B_2(f_k) \sin(2\pi f_k t)] + a + bt \\ v'(t) &= \sum_{k=1}^n [B_3(f_k) \cos(2\pi f_k t) + B_4(f_k) \sin(2\pi f_k t)] + c + dt \end{aligned} \quad (6.1)$$

where a , b , c , d and the $B_i(f_k)$'s are the unknowns. Each oscillation (tidal constituent) with frequency, f_k , has been given a name by previous workers. These names and their associated frequencies appear in Table IX. With the exception of the Mm constituent, the set of constituents in this table is identical to the set used by the Canadian Hydrographic Service for their analyses on 29 day time-series.

In general, each tidal constituent is represented by a vector whose tip traces out a complete ellipse (in the horizontal plane) in a time given by $1/f_k$. The ellipse may be described by its semi-major axis length (A_k), its semi-minor axis length (a_k), and the orientation of the semi-major axis

TABLE IX - Tidal Constituents Used For Harmonic Analysis

k	f_k (cpd)	Constituent	$\{ L /M_{kk}\}^*$
10	0.0362 9165	M_m	---
11	0.0732 0220	M_f	---
1	0.9295 3571	O_1	2850.61
2	0.9972 6209	P_1	883.09
3	1.0027 3791	K_1	879.17
4	1.8959 8197	N_2	2857.65
5	1.9322 7361	M_2	2835.00
6	2.0000 0000	S_2	883.08
7	2.0054 7582	K_2	878.61
8	3.8645 4723	M_4	2879.30
9	3.9322 7361	MS_4	2879.30

* Term appears in equation (6.6)

(θ_k) . These are related to the $B_i(f_k)$'s according to:-

$$\begin{aligned} A_k &= A_{C_k} + C_k & \theta_k &= (\alpha_k + \beta_k) / 2 \\ a_k &= A_{C_k} - C_k \end{aligned} \quad (6.2)$$

where:-

$$\begin{aligned} A_{C_k} &= \sqrt{(B_1 + B_4)^2 + (E_3 - E_2)^2} / 2n_k \\ C_k &= \sqrt{(E_1 - E_4)^2 + (B_3 + B_2)^2} / 2n_k \\ \tan(\alpha_k) &= (E_3 - E_2) / (E_1 + B_4) \\ \tan(-\beta_k) &= (B_3 + E_2) / (E_1 - B_4) \end{aligned} \quad (6.3)$$

and where the arguments of the $B_i(f_k)$'s have been dropped. Positive and negative values of a_k correspond to anti-clockwise and clockwise rotation, respectively, of the vector when viewed from above. The node factors, n_k , are actually sinusoidal functions with a period of 18.67 years (Shureman, 1958). However, they are taken to be constants over a 60 day period.

Uncertainties

The discrete Fourier transform has a resolution of $1/T$ where T is the length of the time-series. Unlike the Fourier transform, the least-squares fit of a harmonic analysis has a resolution given approximately by $1/(T \cdot \sqrt{SN})$, where SN is the signal-to-noise ratio (Munk and Hasselmann, 1964). In actual fact, the resolution capability of a harmonic analysis is phrased in a different manner: What is the largest uncertainty in the $B_i(f_k)$'s that can be tolerated?

Let the variance of the noise be denoted by:-

$$\text{var}(u) = N^{-1} \sum_{j=1}^N \{u(t_j) - u'(t_j)\}^2 \quad (6.4)$$

and similarly for $\text{var}(v)$. Here, N is the total number of points in the east-west current component time-series $u(t_j)$, while $u'(t_j)$ is the discrete form of $u'(t)$ appearing in equation (6.1). According to Godin (1972), the uncertainty or variance in $B_1(f_k)$ and $B_2(f_k)$ is:-

$$\Delta_u^2(f_k) = \text{var}(u) \cdot M_{kk} / |L| \quad (6.5)$$

and similarly for $\Delta_v^2(f_k)$, the variance or uncertainty in $B_3(f_k)$ and $B_4(f_k)$. Here, $|L|$ is the determinant of the 9×9 matrix (L) whose elements are given by:-

$$(L)_{kk} = \sin\{\pi N \Delta t (f_k - f_\ell)\} / \sin\{\pi \Delta t (f_k - f_\ell)\} \quad (6.6)$$

where the frequencies $(f_k, f_\ell : k, \ell = 1, 2, \dots, 9)$ are identified in Table IX, where M_{kk} is the determinant of the matrix formed by deleting the k (th) row and k (th) column from the matrix (L) , and where Δt is the sampling interval of the time-series. Also tabulated in Table IX are the values of $|L|/M_{kk}$ for $T=60$ days and $\Delta t=30$ min.

It is assumed that the $B_i(f_k)$'s follow a Gaussian distribution and that $B_i(f_k)$ and $B_j(f_k)$ are uncorrelated. Thus, terms such as $(B_1 + B_4)$ in equation (6.3) also follow a Gaussian distribution with a variance of $\Delta^2(f_k) = \Delta_u^2(f_k) + \Delta_v^2(f_k)$. The 95% confidence limits for such terms as $(B_1 + B_4)$ are given by $(B_1 + B_4) \pm 1.96 \cdot \Delta(f_k)$. These limits, in conjunction with equations (6.2) and (6.3), may be used in calculating the 95% confidence limits for A_k , a_k and θ_k . All the confidence limits

given in this chapter are the 95% confidence limits derived in this manner.

Let $S_u(f)$ denote the power spectrum of the noise so that:-

$$\text{var}(u) = \int_{-\infty}^{\infty} S_u(f) df \quad (6.7)$$

The derivation of equation (6.5) assumes that the noise is white, that $S_u(f)$ is a constant function of frequency. Invariably, this will not be the case. Munk and Hasselmann (1964) give more general expressions which replace equation (6.5) for non-white noise.

The terms a, b, c and d in equation (6.1) take care of any non-zero means and trends which may be present in the current vector time-series. The presence of these terms serve to decrease the variance of the noise and the uncertainty in the calculated ellipse parameters. The least-squares fit of equation (6.1) includes the M_m and M_f constituents, the monthly and fortnightly tidal oscillations in the current. As seen in the previous chapter, these oscillations are buried in the low-frequency broad-band fluctuations whose origins are other than tidal forcing. Because of this, the results for the M_m and M_f constituents are not presented. Again, the presence of these constituents in equation (6.1) serve to decrease $\text{var}(u)$ and $\text{var}(v)$, and to decrease slightly the uncertainty in the calculated ellipse parameters. Hindsight gives an alternate and preferred method of minimizing $\text{var}(u)$ and $\text{var}(v)$. This is to high-pass filter the half-hourly time-series, to remove the

broad-band low-frequency energy before performing the harmonic analysis.

6.2-----Time Variations

Central 50m Location

Harmonic analyses were performed on 60 day sections of the half-hourly current time-series at the central 50m location. These were chosen so that the time origins (mid-point) of consecutive sections are separated by 15 days. The ellipse parameters, calculated from equations (6.2) and (6.3), are plotted against the time origins of the 60 day sections. In addition, the 95% confidence limits for the ellipse parameters are also calculated (as described previously) and plotted. The time variations of the ellipse parameters for nine constituents appear in Figure 51 through to Figure 59.

An ellipse parameter is considered constant with time if a horizontal line can be drawn which lies completely between the 95% confidence limits of that parameter. Otherwise, the ellipse parameter is said to be time-dependent.

The most interesting ellipse parameter is the semi-major axis length (A_k). It is observed that the A_k for the K_2 , M_4 and MS_4 constituents are time-invariant. The A_k for the remaining six constituents are time-dependent. These variations in A_k have periods of 3-6 months. The M_2 constituent has the greatest range, varying from 7 cm/s to 12 cm/s. The A_k variations of one constituent do not correlate

Figure 51. Time variations of the O_1 tidal current ellipse and 95% confidence intervals. Parameters A, a, and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 52. Time variations of the P_1 tidal current ellipse and 95% confidence intervals. Parameters A, a, and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 53. Time variations of the K_1 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 54. Time variations of the N_2 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 55. Time variations of the M_2 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 56. Time variations of the S_2 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 57. Time variations of the K_2 tidal current ellipse and 95% confidence intervals. Parameters A, a, and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 58. Time variations of the M_4 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

Figure 59. Time variations of the MS_4 tidal current ellipse and 95% confidence intervals. Parameters A , a , and θ are the semi-major length, semi-minor length, and orientation of the major ellipse axis, respectively.

well with those of any other constituent.

The sign of the semi-minor axis length (a_k) determines the rotational sense of the constituent ellipse. Anti-clockwise and clockwise rotation (as viewed from above) are represented by positive and negative values, respectively. It is seen that the a_k for the O_1 , S_2 , K_2 , M_4 and MS_4 constituents are time-invariant. In addition, the rotational sense of the S_2 , M_4 and MS_4 constituent ellipses is ambiguous. That is, horizontal lines can be drawn which lie completely between the 95% confidence limits, and which lie both above and below the zero value. Of the nine constituents, the M_2 constituent has the largest $|a_k|$ value of 3 cm/s.

The rotational sense of the N_2 and M_2 ellipses is time-dependent. During June and July of both years, the a_k of these two ellipses attain their most positive values. These events coincide with the Fraser River freshet. With the exception of these two constituents, the a_k variations of one constituent correlate poorly with those of any other constituent.

There is an ambiguity of 180° in θ_k , the orientation of the semi-major axis. In most cases, values of θ_k are chosen which lie in the half-circle from east to west, and which includes north. As A_k decreases, the confidence limits for θ_k increase. It is seen that the orientations of the O_1 , P_1 , M_4 and MS_4 ellipses are time-invariant. That of the remaining five constituents are time-dependent. The range of the θ_k changes for the K_1 ellipse, which is 33° , is larger than that for the M_2 ellipse, which is 19° . Probably, the range for the

K_1 ellipse is greater because the A_k for this ellipse is smaller. The K_1 ellipse orientation is predominantly NNW, while that for the M_2 ellipse is predominantly north. For those constituents with a time-varying orientation, the θ_k changes of one ellipse do not correlate well with those of any other ellipse.

Generally, it appears that if there are interactions in the currents between the tidal constituents, these interactions do not appear as obvious correlations in the ellipse parameters of any two constituents.

In the previous chapter, the monthly means of the current (at the central 50m location), wind, air-pressure and sea-level are presented. The variations in the ellipse parameters are poorly correlated with these parameters presented in the previous chapter. In addition, the variations in the ellipse parameters are also poorly correlated with the variations in the monthly current (at the central 50m location) and with the variations in the monthly wind, sea-level, and air-pressure variances. The semi-minor axis parameter (a_k) of the N_2 and M_2 ellipses are the only exceptions to the previous statement. It is observed that the a_k for these two ellipses are most positive during June and July of both years, and these features coincide with the low summer variances observed in both air-pressure and sea-level.

It appears that if there are interactions between the low-frequency currents (Chapter 5) and the tidal currents, these interactions do not appear as obvious correlations

between the parameters discussed in the preceding paragraph.

Five Other Locations

The current time-series for central 50m location is the longest continuous one. Those for the five other locations contain significant gaps. Harmonic analyses were also performed on 60 day sections of the half-hourly current time-series at the five other locations. The time origin is again taken to be the mid-point of each 60 day section. For the five other locations, the time origins of consecutive 60 day sections are separated by 45 days whenever possible, and not by 15 days as for the central 50m location. The 45 day separation is used to decrease the amount of computational labour. The ellipse parameters along with their 95% confidence limits are tabulated in Appendix I. In this appendix, the directions of (east,north,west) are represented by $(0^\circ, 90^\circ, 180^\circ)$, while south is represented by -90° . This convention for the directions is the standard mathematical one.

As before, an ellipse parameter is considered to be time-invariant if a horizontal line can be drawn (on a graph of the parameter versus time) which completely lies between the 95% confidence limits. Examination of Appendix I shows that the semi-major axis length (A_k) of the M_2 ellipse at the eastern 50m location is time-invariant. The A_k 's of the M_2 ellipse at the remaining four locations are time-dependent. It is observed that the A_k of the K_1 ellipse is time-dependent

only at the deep eastern and central locations. The A_k of the O_1 ellipse is seen to vary at three of the five locations. Those of the remaining ellipses are seen to vary at two or less of the five locations.

The semi-minor axis length (a_k) of the M_2 ellipse is time-dependent at all five locations. However, its rotational sense is only observed to vary at the deep eastern location. The a_k of the K_1 ellipse is time-dependent only at the central 200m location. At three of the five locations, the a_k of the O_1 ellipse is time-dependent. The a_k of the remaining six constituents are time-invariant at all five locations.

The orientation (θ_k) of the M_2 ellipse is time-dependent at all five locations. That of the K_1 ellipse is time-dependent only at the western 50m and eastern 200m locations. At two of the five locations, the θ_k of the N_2 ellipse is time-dependent. The θ_k of the remaining six ellipses are time-dependent at only one or none of the five locations.

The M_2 ellipse parameters at any pair of locations may be plotted against time for comparison. The simultaneous time coverage of the data for a given pair of locations may be small. However, the coverage is sufficient to suggest that the M_2 ellipse parameters at one location correlate poorly with the corresponding M_2 ellipse parameters at any other location. This is also true for comparisons between corresponding parameters of the other constituents which are time-dependent. The only exception to these poor correlations appears to be the

A_k parameter for the M_2 constituent at the western mooring. There, the A_k of the M_2 ellipse at the 200m location decreases with time, while that at the 50m location increases with time. In general however, the variations of the ellipse parameters at any one location do not appear to be highly coherent with the variations of the corresponding ellipse parameters at any other location.

6.3 Time-Averaged Parameters

The ellipse parameters are time-averaged and tabulated in Table X. For the central 50m location, they are derived from the graphs appearing in Figure 51 through to Figure 59. For the other locations, the time-averaged values are derived from the values in Appendix I. The confidence limits are ignored.

At all six locations, the M_2 and K_1 ellipses have the largest and second largest amplitudes (A_k), respectively. According to equilibrium tide theory, the three diurnal constituents in order of decreasing amplitude are (K_1, O_1, P_1). This order is observed at all the locations except the eastern 50m location. The four semi-diurnal constituents in order of decreasing amplitude are (M_2, S_2, N_2, K_2). This order is observed at all locations except the eastern 140m location. At all six locations, the M_4 amplitude is larger than the MS_4 amplitude.

At all three moorings, both the O_1 and K_1 amplitudes increase with depth. Both the N_2 and M_2 amplitudes increase

TABLE X - Average Tidal Current Ellipses

Depth	Western Station				Central Station				Eastern Station			Depth
	A (cm/s)	a (cm/s)	θ (deg.)		A (cm/s)	a (cm/s)	θ (deg.)		A (cm/s)	a (cm/s)	θ (deg.)	
50m	2.91	.22	106	O_1	2.83	.37	119		1.20	.24	87	50m
200m	3.10	-.97	97		3.63	.35	96		2.90	.50	79	140m
50m	2.55	.52	116	P_1	2.29	.32	106		2.45	-.14	99	50m
200m	1.78	-.59	86		2.84	-.18	98		1.94	.08	109	140m
50m	5.88	.53	110	K_1	5.47	.69	117		3.06	.59	89	50m
200m	5.94	-1.58	97		7.52	-.05	97		5.10	.56	76	140m
50m	2.25	.03	112	N_2	2.21	.04	95		1.60	.02	92	50m
200m	2.63	-1.12	120		1.90	-.27	111		.97	-.02	87	140m
50m	8.52	1.67	109	M_2	9.31	.73	95		6.70	.26	92	50m
200m	11.10	-3.97	120		8.70	-.39	123		6.31	-.55	111	140m
50m	3.02	-.04	86	S_2	2.70	-.23	101		1.65	.33	116	50m
200m	2.64	-.95	120		2.62	-.19	117		2.19	-.31	38	140m
50m	.95	-.33	92	K_2	1.53	-.67	112		.93	-.33	94	50m
200m	.96	-.35	104		1.53	-.24	88		1.80	.17	16	140m
50m	.36	-.12	102	M_4	.61	-.25	111		.49	-.16	142	50m
200m	.62	-.15	89		.47	-.04	91		.86	-.02	50	140m
50m	.29	-.10	105	MS_4	.47	-.15	130		.26	-.04	157	50m
200m	.55	-.09	74		.31	.01	93		.61	.04	69	140m

with depth at the western mooring, but both decrease with depth at the remaining two moorings. At all three moorings, both the M_4 and MS_4 amplitudes increase with depth.

At the western 200m location, all constituent ellipses are rotating clockwise, while at the central 200m location seven out of nine ellipses are rotating clockwise. The O_1 ellipse through to the M_2 ellipse rotate anti-clockwise at the western 50m location. At the same location, the remaining ellipses rotate clockwise. The preceding two statements also apply to the constituent ellipses at the central 50m location. The rotational sense of the N_2 and M_2 ellipses change with depth at all three moorings.

At all three moorings, the orientation of the M_2 ellipse increases with depth, while those of the O_1 and K_1 ellipses decrease with depth.

6.4 Internal Tides

One may expect internal (baroclinic) tides in a system where there exists a significant density stratification and rapid changes in bottom topography (Forrester, 1974). These conditions exist in the vicinity of the three moorings.

The observed depth variations in the ellipse amplitude (A_k) support the hypothesis of internal tides. For example, at the eastern mooring during March 1969, the K_1 amplitude decreases with depth. This difference in the K_1 amplitudes cannot be explained by the 95% confidence limits. The same is

true of the K_1 amplitude differences at the central mooring during May 1970, and of the M_2 amplitude differences at the western mooring during August 1969.

The time-varying current ellipse parameters cannot be explained by the variations in the constituent amplitudes for the sea surface. Fourier transforms were performed on nine 60 day sections of sea-level at Pt. Atkinson. The mid-point time origins of these sections ranged from May 16, 1969 to September 8, 1970. After performing the node factor corrections, the M_2 amplitudes are observed to vary from 89.35 cm to 94.60 cm. Relative to the average value of 91.64 cm, this represents a 6% variation. However, the amplitudes of the M_2 current ellipse range from 7 cm/s to 12 cm/s. Relative to the average value of 9 cm/s, this represents a 56% variation.

Internal tides are the most plausible explanation of the time-varying ellipse parameters. The character of the internal tides is expected to change with changes in the density stratification, and these are known to exist in the vicinity of the three moorings (Crean and Ages, 1971). Unfortunately, there are no time-series of density profiles for 1969 and 1970, profiles which may or may not be well correlated with the time-varying ellipse parameters.

Chapter 7

SUMMARY

Sub-surface currents in the Strait of Georgia near Sturgeon Bank have been analysed. The currents, collected at six locations on three moorings, span 319-533 days. There are two main partitions of energy: broad-band low-frequency and narrow-band tidal motions. Together they account for 80% of the energy in the currents. The low-frequency motions are located in peak centred at 30 days. They are neither predominantly clockwise nor predominantly anti-clockwise.

The currents at the three 50m locations contain equal amounts of low-frequency energy. This energy is also approximately equal to the tidal energy. The evidence suggests that the low-frequency energy may decrease linearly with depth. Close to the side boundaries of the Strait, the low-frequency motions are constrained to be highly rectilinear. At five locations, the current means are northward, parallel to the depth contours. The mean current is directed ESE at the central 50m location.

Moorings were positioned laterally across the Strait at 10km intervals. The low-frequency motions are essentially incoherent, indicating that they have a lateral length scale much smaller than 10km. In addition, these motions are very baroclinic. Low-frequency motions at the eastern mooring are

the least baroclinic.

Wind, air-pressure and sea-level also have broad-band low-frequency fluctuations with characteristic time scales of 3, 10 and 30 days, respectively. Both sea-level and air-pressure have horizontal length scales as least as large as the length of Vancouver Island. The wind has a smaller length scale, but it is still much larger than those of the low-frequency currents. Low-frequency sea-level and air-pressure fluctuations are highly coherent. Wind and air-pressure variations are much less coherent, as are the wind and sea-level variations.

Generation mechanisms of the low-frequency currents are not identified. Low-frequency fluctuations of currents and air-pressure are not coherent. Those of currents and sea-level exhibit poor-to-fair coherence. At the eastern mooring, those of currents and wind have fair coherence. This lends encouragement to a further study of the low-frequency currents and wind in the vicinity of this mooring.

Tidal currents at 50m and 200m represent 40% and 70% of the energy. The largest components are the M_2 and K_1 constituents. Tidal currents are highly coherent both in the horizontal and vertical. The parameters describing the tidal current ellipses are time-dependent. Time-varying parameters for different constituents are poorly correlated, as are those for different locations. Generally, the ellipse parameter variations are poorly correlated with those in the monthly means and variances of the low-frequency currents, wind, sea-

level and air-pressure.

Variations of the tidal ellipse parameters cannot be explained by variations of the tidal constituents for sea-level, which are relatively small. Internal tides are hypothesized. The changing ellipse parameters are then connected to the changing character of the internal tide, which is effected by the changing density stratification.

Time-series of water temperature span 307-411 days. These temperature fluctuations are broad-band with a characteristic time scale of 300 days. The time-series are too short to be considered stationary. Water temperatures at different locations generally have low coherence, as have water temperature and current at the same location.

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APPENDIX I

Results of harmonic analyses on 60 day sections of the half-hourly current time-series are given here. Dates correspond to the mid-point of each time-series section. Ellipse parameters A , a and θ are the semi-major length, semi-minor length, and orientation of the major axis, respectively. Subscripts l and u are the lower and upper limits of the 95% confidence intervals whose calculation is explained in Chapter 6. Ellipse parameters A and a are in cm/s. Values of θ of (0,90,180,-90,-180) correspond to true (east, north, west, south, west), respectively.

APPENDIX I continued

WESTERN MOORING 50m

		A	A ₁	A _u	a	a ₁	a _u	θ	θ ₁	θ _u
O ₁	1969 Aug 14	2.97	2.39	3.58	0.10	-0.48	0.71	104.9	92.5	115.8
	Sep 28	3.03	2.47	3.62	0.39	-0.19	0.96	102.7	91.5	114.8
	Nov 12	2.54	1.80	3.32	0.22	-0.54	0.98	109.6	92.8	126.9
	Dec 27	2.83	2.15	3.55	0.86	0.15	1.55	109.2	92.4	123.7
	1970 Feb 10	3.19	2.65	3.75	-0.49	-1.04	0.06	103.5	93.6	114.6
P ₁	1969 Aug 14	2.20	1.08	3.51	0.36	-0.85	1.58	128.0	91.0	167.2
	Sep 28	2.33	1.10	3.67	0.77	-0.56	2.00	112.8	69.6	156.5
	Nov 12	2.18	0.98	3.85	1.37	-0.26	2.60	160.1	106.7	244.1
	Dec 27	3.58	2.21	5.12	-0.10	-1.53	1.37	63.4	38.7	87.5
	1970 Feb 10	2.49	1.29	3.78	0.20	-1.00	1.49	117.6	89.5	147.5
K ₁	1969 Aug 14	5.40	4.27	6.59	-0.02	-1.20	1.12	110.1	98.4	122.9
	Sep 28	5.76	4.67	6.91	0.80	-0.30	1.94	101.3	88.7	112.1
	Nov 12	6.07	4.55	7.61	1.49	-0.05	3.01	94.4	80.1	110.1
	Dec 27	6.04	4.66	7.48	0.66	-0.75	2.08	132.6	120.2	147.1
	1970 Feb 10	6.14	5.03	7.28	-0.31	-1.45	0.81	111.3	101.0	122.1
N ₂	1969 Aug 14	2.33	1.56	3.12	0.24	-0.54	1.03	118.8	99.5	137.5
	Sep 28	2.14	1.43	2.92	0.36	-0.40	1.09	113.9	94.6	135.4
	Nov 12	1.99	1.14	2.91	0.03	-0.89	0.89	116.5	88.3	143.2
	Dec 27	2.38	1.59	3.23	-0.09	-0.90	0.74	103.1	80.1	123.7
	1970 Feb 10	2.39	1.75	3.10	-0.39	-1.05	0.30	106.3	89.5	124.0
M ₂	1969 Aug 14	7.64	6.92	8.37	2.43	1.72	3.16	114.9	108.7	121.3
	Sep 28	7.78	7.01	8.56	2.25	1.48	3.04	106.1	99.7	112.4
	Nov 12	8.68	7.77	9.60	1.47	0.55	2.38	112.3	106.3	118.7
	Dec 27	9.44	8.61	10.30	1.45	0.59	2.28	109.3	104.3	114.7
	1970 Feb 10	9.05	8.39	9.72	0.73	0.06	1.40	103.8	99.5	108.0
S ₂	1969 Aug 14	3.43	2.19	4.76	-0.19	-1.43	1.14	64.1	40.7	85.7
	Sep 28	2.92	1.81	4.22	0.17	-1.05	1.36	85.5	55.8	109.4
	Nov 12	3.03	1.68	4.60	-0.52	-1.95	0.97	82.5	51.1	118.3
	Dec 27	2.71	1.27	4.25	0.66	-0.77	2.21	108.4	73.9	141.0
	1970 Feb 10	3.03	1.76	4.35	-0.32	-1.62	0.97	87.9	61.6	109.2
K ₂	1969 Aug 14	1.37	0.56	2.37	-0.59	-1.49	0.33	86.3	23.0	143.0
	Sep 28	1.06	0.15	2.09	-0.14	-1.08	0.86	62.0	0.2	131.2
	Nov 12	1.11	0.39	2.39	-0.74	-1.55	0.45	26.9	-74.7	90.1
	Dec 27	0.63	0.00	1.86	0.18	-0.85	1.02	153.3	44.4	262.3
	1970 Feb 10	0.60	0.06	1.63	-0.37	-1.00	0.57	129.7	10.6	211.2
M ₄	1969 Aug 14	0.36	0.00	1.19	-0.03	-0.61	0.57	77.3	-76.0	160.4
	Sep 28	0.58	0.13	1.37	-0.29	-0.80	0.45	118.0	44.7	235.1
	Nov 12	0.24	0.00	1.24	-0.10	-0.66	0.58	28.4	-71.9	176.1
	Dec 27	0.20	0.00	1.16	-0.01	-0.58	0.58	171.5	48.9	300.5
	1970 Feb 10	0.44	0.00	1.21	-0.16	-0.68	0.53	116.8	9.4	219.9
MS ₄	1969 Aug 14	0.09	0.00	0.89	0.02	-0.44	0.45	174.4	49.2	309.4
	Sep 28	0.35	0.00	1.16	-0.13	-0.65	0.52	153.8	26.8	261.2
	Nov 12	0.41	0.00	1.36	-0.11	-0.74	0.62	132.4	-2.0	226.6
	Dec 27	0.17	0.00	1.11	-0.06	-0.58	0.53	31.0	-69.1	186.2
	1970 Feb 10	0.43	0.00	1.19	-0.21	-0.71	0.49	32.4	-113.8	111.7

APPENDIX I continued

WESTERN MOORING 200m

		A	A ₁	A _u	a	a ₁	a _u	θ	θ ₁	θ _u
O ₁	1969 Aug 14	3.24	2.94	3.55	-0.65	-0.95	-0.34	99.7	94.2	105.5
	Sep 28	3.00	2.64	3.36	-0.88	-1.24	-0.52	87.6	80.6	95.5
	Nov 12	3.67	3.21	4.13	-0.72	-1.18	-0.26	90.8	83.2	98.0
	Dec 27	3.19	2.82	3.59	-1.07	-1.45	-0.68	88.8	80.1	95.9
	1970 Feb 10	2.81	2.43	3.20	-1.22	-1.60	-0.84	98.3	89.2	109.0
	Mar 27	2.68	2.26	3.13	-1.26	-1.69	-0.81	113.6	100.6	126.0
P ₁	1969 Aug 14	2.21	1.57	2.93	-0.42	-1.10	0.27	90.3	74.1	112.1
	Sep 28	1.47	0.79	2.24	-0.60	-1.30	0.15	74.6	33.8	120.1
	Nov 12	1.71	1.01	2.54	-0.68	-1.43	0.09	89.4	46.1	132.5
	Dec 27	2.03	1.17	2.99	-1.07	-1.93	-0.11	75.5	29.4	118.4
	1970 Feb 10	1.56	0.89	2.36	-0.43	-1.15	0.33	82.7	45.1	121.8
	Mar 27	1.72	0.82	2.71	-0.32	-1.22	0.67	105.2	67.3	142.3
K ₁	1969 Aug 14	5.79	5.09	6.49	-1.31	-2.01	-0.61	100.1	92.7	107.2
	Sep 28	5.62	5.05	6.24	-1.86	-2.45	-1.26	91.7	84.1	98.1
	Nov 12	6.63	5.75	7.51	-1.91	-2.80	-1.03	93.4	85.3	101.8
	Dec 27	6.16	5.47	6.90	-1.31	-2.02	-0.59	95.9	88.5	103.1
	1970 Feb 10	5.76	4.95	6.58	-1.32	-2.14	-0.51	97.7	89.3	106.3
	Mar 27	5.70	4.92	6.53	-1.79	-2.58	-0.98	102.9	94.4	112.4
N ₂	1969 Aug 14	3.01	2.60	3.43	-1.13	-1.55	-0.71	115.5	106.0	124.9
	Sep 28	2.87	2.43	3.31	-1.30	-1.75	-0.86	119.4	107.8	130.5
	Nov 12	2.81	2.32	3.32	-1.03	-1.54	-0.54	116.0	103.6	127.7
	Dec 27	2.63	2.16	3.14	-0.94	-1.41	-0.44	123.0	109.7	136.1
	1970 Feb 10	2.33	1.89	2.81	-0.85	-1.30	-0.38	107.3	95.4	121.6
	Mar 27	2.11	1.68	2.60	-1.45	-1.89	-0.97	137.0	101.7	171.3
M ₂	1969 Aug 14	12.15	11.72	12.58	-3.74	-4.17	-3.31	120.3	118.2	122.6
	Sep 28	11.74	11.31	12.17	-3.94	-4.37	-3.51	117.3	115.0	119.6
	Nov 12	11.35	10.85	11.85	-4.05	-4.55	-3.55	116.4	113.4	119.4
	Dec 27	10.85	10.32	11.38	-3.98	-4.52	-3.45	119.2	115.8	122.5
	1970 Feb 10	10.70	10.18	11.22	-4.85	-5.38	-4.33	126.2	122.7	129.7
	Mar 27	9.84	9.25	10.43	-3.24	-3.83	-2.65	123.2	119.5	127.1
S ₂	1969 Aug 14	2.30	1.73	2.95	-0.33	-0.93	0.28	131.8	113.2	150.2
	Sep 28	2.60	1.85	3.41	-0.98	-1.74	-0.18	99.3	77.0	117.1
	Nov 12	2.10	1.36	2.95	-0.22	-1.02	0.57	128.0	103.7	155.9
	Dec 27	3.30	2.45	4.22	-1.40	-2.26	-0.49	99.5	77.1	119.6
	1970 Feb 10	2.45	1.72	3.29	-1.28	-2.05	-0.48	141.0	112.6	170.5
	Mar 27	3.07	2.13	4.09	-1.50	-2.44	-0.48	118.9	94.3	145.4
K ₂	1969 Aug 14	0.97	0.54	1.48	-0.43	-0.89	0.06	92.2	40.9	140.9
	Sep 28	0.82	0.33	1.41	0.02	-0.52	0.56	165.8	116.9	213.8
	Nov 12	1.24	0.59	1.96	-0.44	-1.08	0.28	155.8	114.5	201.7
	Dec 27	0.84	0.40	1.62	-0.73	-1.18	0.04	103.7	32.8	188.0
	1970 Feb 10	1.10	0.50	1.77	-0.09	-0.70	0.57	80.1	45.4	118.3
	Mar 27	0.77	0.23	1.55	-0.45	-1.01	0.31	25.7	-42.7	105.5
M ₄	1969 Aug 14	0.75	0.39	1.18	-0.43	-0.81	-0.02	53.2	-4.4	109.5
	Sep 28	0.57	0.26	1.04	-0.34	-0.68	0.10	50.4	-36.5	107.9
	Nov 12	0.56	0.16	1.13	-0.33	-0.74	0.24	102.7	35.8	184.4
	Dec 27	0.57	0.05	1.17	-0.06	-0.61	0.51	130.5	79.4	247.9
	1970 Feb 10	9.58	0.16	1.10	0.14	-0.33	0.61	114.7	32.8	169.7
	Mar 27	0.70	0.12	1.32	0.12	-0.50	0.70	82.0	37.6	154.6
MS ₄	1969 Aug 14	0.34	0.06	0.78	-0.12	-0.44	0.28	71.3	-5.4	173.4
	Sep 28	0.38	0.03	0.81	0.03	-0.36	0.42	47.9	-30.0	139.6
	Nov 12	0.37	0.06	0.90	0.17	-0.31	0.53	58.1	-31.8	157.9
	Dec 27	0.65	0.31	1.21	-0.41	-0.80	0.10	113.1	58.7	207.6
	1970 Feb 10	0.81	0.36	1.31	-0.20	-0.66	0.30	69.8	25.5	109.4
	Mar 27	0.73	0.21	1.32	-0.02	-0.59	0.52	81.0	35.6	125.3

APPENDIX I continued

CENTRAL MOORING 200m

			A	A ₁	A _u	a	a ₁	a _u	θ	θ ₁	θ _u
O ₁	1969 Sep 28		3.13	2.72	3.55	0.65	0.23	1.06	103.5	95.4	111.9
	Dec 27		3.69	3.25	4.15	0.29	-0.16	0.74	102.2	94.9	109.0
	1970 Feb 10		4.04	3.69	4.39	-0.22	-0.57	0.13	101.2	96.3	106.3
	Mar 27		3.88	3.38	4.38	0.30	-0.20	0.80	90.3	83.1	98.0
	May 11		3.43	2.67	4.25	0.71	-0.08	1.50	82.1	66.0	96.8
P ₁	1969 Sep 28		2.43	1.57	3.39	-0.33	-1.26	0.57	117.4	93.8	139.2
	Dec 27		2.24	1.30	3.24	-0.19	-1.15	0.79	89.6	68.6	117.0
	1970 Feb 10		2.54	1.70	3.41	-0.63	-1.50	0.21	102.1	83.0	122.6
	Mar 27		2.00	1.05	3.05	-0.38	-1.36	0.64	86.4	57.1	120.7
	May 11		5.01	3.55	6.68	0.66	-0.92	2.21	92.5	69.3	115.0
K ₁	1969 Sep 28		6.30	5.49	7.13	0.75	-0.06	1.58	101.7	93.5	108.9
	Dec 27		6.34	5.44	7.24	-0.17	-1.06	0.74	104.2	96.0	112.2
	1970 Feb 10		8.14	7.52	8.79	-0.95	-1.59	-0.31	95.4	91.0	100.3
	Mar 27		6.81	5.83	7.79	-0.44	-1.42	0.54	89.5	81.1	97.5
	May 11		10.01	8.15	11.88	0.54	-1.33	2.41	91.8	81.2	102.4
N ₂	1969 Sep 28		1.84	1.35	2.39	-0.30	-0.82	0.23	105.8	89.4	123.5
	Dec 27		2.14	1.67	2.67	-0.47	-0.96	0.04	122.4	107.2	136.6
	1970 Feb 10		2.08	1.65	2.54	-0.06	-0.51	0.39	106.5	94.2	119.0
	Mar 27		1.88	1.25	2.52	0.51	-0.13	1.14	88.7	68.3	108.0
	May 11		1.55	0.70	2.73	-1.03	-1.88	0.15	132.2	40.6	183.4
M ₂	1969 Sep 28		10.15	9.58	10.71	-1.48	-2.04	-0.91	110.8	107.5	113.9
	Dec 27		7.96	7.37	8.55	0.29	-0.30	0.87	126.5	122.2	130.6
	1970 Feb 10		7.90	7.40	8.40	0.17	-0.33	0.67	122.8	119.2	126.5
	Mar 27		8.50	7.98	9.04	-0.35	-0.88	0.19	122.8	119.2	126.4
	May 11		9.02	7.86	10.18	-0.57	-1.73	0.60	130.2	123.0	137.7
S ₂	1969 Sep 28		3.65	2.76	4.61	-0.45	-1.35	0.50	110.1	95.1	125.8
	Dec 27		2.63	1.79	3.58	-0.47	-1.37	0.42	128.9	105.2	146.5
	1970 Feb 10		1.99	1.27	2.81	0.81	0.03	1.57	140.0	109.0	167.9
	Mar 27		3.32	2.40	4.29	-0.25	-1.21	0.68	119.0	100.7	135.8
	May 11		1.52	0.07	3.58	-0.59	-2.08	1.43	87.6	-24.9	156.3
K ₂	1969 Sep 28		0.77	0.12	1.54	-0.25	-0.91	0.51	23.7	-39.2	93.7
	Dec 27		1.16	0.50	1.91	-0.32	-1.00	0.41	122.2	76.4	168.2
	1970 Feb 10		1.69	1.07	2.35	-0.36	-1.00	0.28	90.5	71.3	115.5
	Mar 27		1.11	0.49	1.87	-0.11	-0.78	0.59	82.0	33.6	133.4
	May 11		2.91	1.68	4.36	-0.16	-1.51	1.18	120.1	89.0	149.7
M ₄	1969 Sep 28		0.46	0.01	1.04	0.01	-0.51	0.53	148.1	46.8	228.2
	Dec 27		0.54	0.02	1.15	-0.12	-0.64	0.49	33.0	-50.9	90.2
	1970 Feb 10		0.51	0.12	1.00	-0.15	-0.58	0.30	172.4	97.0	239.6
	Mar 27		0.41	0.00	1.06	0.03	-0.52	0.54	36.3	-69.6	152.2
	May 11		0.44	0.00	1.63	0.01	-0.82	0.81	63.9	-42.7	193.9
MS ₄	1969 Sep 28		0.29	0.00	0.88	-0.10	-0.50	0.39	104.4	-35.3	203.4
	Dec 27		0.25	0.00	0.83	0.06	-0.39	0.44	54.1	-33.1	196.5
	1970 Feb 10		0.43	0.11	0.92	-0.18	-0.54	0.27	26.1	-80.5	90.7
	Mar 27		0.28	0.00	0.90	0.01	-0.45	0.45	143.1	62.5	295.7
	May 11		0.30	0.00	1.47	0.26	-0.61	0.86	137.3	-17.1	231.0

APPENDIX I continued

EASTERN MOORING 50m

		A	A ₁	A _u	a	a ₁	a _u	θ	θ ₁	θ _u
O ₁	1969 May 16	0.97	0.46	1.51	0.28	-0.25	0.80	84.1	56.4	117.9
	Sep 28	1.70	1.28	2.17	0.21	-0.24	0.65	101.0	84.0	116.2
	Nov 12	1.27	0.74	1.84	0.24	-0.32	0.78	97.4	75.5	124.7
	1970 Mar 27	0.93	0.42	1.48	0.06	-0.49	0.57	69.3	32.1	106.7
	May 11	1.16	0.54	1.88	0.39	-0.30	1.05	80.8	33.0	127.8
P ₁	1969 May 16	2.03	1.02	3.11	-0.70	-1.78	0.31	115.8	82.8	147.5
	Sep 28	2.30	1.46	3.30	-0.14	-1.06	0.78	90.2	63.4	120.3
	Nov 12	2.39	1.23	3.59	-0.75	-1.91	0.45	91.6	60.1	117.7
	1970 Mar 27	1.30	0.15	2.55	0.21	-1.03	1.38	83.4	38.6	174.6
	May 11	4.26	2.79	5.82	0.70	-0.83	2.21	111.4	91.6	132.9
K ₁	1969 May 16	2.07	1.18	3.08	0.82	-0.15	1.75	68.4	36.2	106.3
	Sep 28	3.03	2.01	4.06	0.27	-0.76	1.30	94.0	74.3	112.1
	Nov 12	3.02	2.10	4.02	0.43	-0.56	1.37	102.4	83.5	123.8
	1970 Mar 27	3.83	2.94	4.83	0.08	-0.88	1.01	86.4	69.8	99.3
	May 11	3.33	2.21	4.66	1.38	0.13	2.58	91.6	59.0	123.0
N ₂	1969 May 16	1.97	1.40	2.59	-0.69	-1.27	-0.08	64.5	42.0	86.9
	Sep 28	1.74	1.16	2.36	0.49	-0.10	1.10	149.2	126.4	168.8
	Nov 12	1.55	0.89	2.24	0.03	-0.65	0.70	93.5	72.0	119.8
	1970 Mar 27	1.23	0.66	1.89	-0.03	-0.66	0.58	105.8	75.7	140.1
	May 11	1.49	0.58	2.43	0.33	-0.61	1.25	44.2	13.8	80.4
M ₂	1969 May 16	7.05	6.56	7.56	-0.29	-0.79	0.21	87.8	83.6	91.9
	Sep 28	6.86	6.25	7.48	0.94	0.32	1.55	108.4	103.4	113.7
	Nov 12	6.64	5.99	7.31	0.11	-0.55	0.76	95.5	90.0	101.4
	1970 Mar 27	6.29	5.62	6.97	-0.68	-1.35	0.00	92.9	86.4	98.8
	May 11	6.67	5.76	7.59	1.25	0.33	2.16	75.1	66.8	83.0
S ₂	1969 May 16	0.91	0.01	2.04	-0.03	-1.04	0.99	158.1	65.2	242.8
	Sep 28	2.47	1.48	3.58	0.62	-0.46	1.64	129.2	99.5	154.4
	Nov 12	1.67	0.61	2.86	0.02	-1.13	1.12	72.9	35.9	113.0
	1970 Mar 27	1.24	0.32	2.49	0.63	-0.61	1.56	145.8	53.1	200.9
	May 11	1.93	0.57	3.49	0.42	-1.01	1.91	74.4	-18.6	121.1
K ₂	1969 May 16	1.28	0.72	2.15	-0.97	-1.55	-0.12	54.6	-43.6	102.2
	Sep 28	0.53	0.00	1.38	-0.00	-0.70	0.68	153.5	61.6	266.8
	Nov 12	0.57	0.00	1.50	0.08	-0.71	0.79	49.4	-17.8	199.5
	1970 Mar 27	1.09	0.65	1.98	-0.90	-1.40	-0.07	146.8	58.9	212.8
	May 11	1.16	0.20	2.34	0.15	-0.94	1.20	65.7	4.7	176.3
M ₄	1969 May 16	0.41	0.04	1.10	-0.27	-0.69	0.37	124.1	9.2	206.0
	Sep 28	0.70	0.15	1.37	-0.20	-0.79	0.44	19.5	-34.6	98.5
	Nov 12	0.53	0.04	1.22	-0.08	-0.64	0.54	160.0	79.7	265.8
	1970 Mar 27	0.42	0.00	1.11	0.03	-0.54	0.58	125.3	26.5	232.7
	May 11	0.39	0.00	1.31	-0.28	-0.77	0.54	103.2	-19.7	203.4
MS ₄	1969 May 16	0.12	0.00	0.78	-0.05	-0.41	0.36	127.2	-0.9	252.4
	Sep 28	0.42	0.02	1.07	-0.16	-0.62	0.43	22.5	-78.1	119.2
	Nov 12	0.42	0.00	1.08	0.01	-0.55	0.54	15.6	-78.4	124.8
	1970 Mar 27	0.28	0.00	0.96	0.03	-0.46	0.50	125.7	20.1	251.5
	May 11	0.09	0.00	1.01	-0.01	-0.51	0.50	134.0	7.3	269.5

APPENDIX I continued

EASTERN MOORING 140m

		A	A ₁	A _u	a	a ₁	a _u	θ	θ ₁	θ _u
O ₁	1969 May 16	2.96	2.38	3.58	0.22	-0.38	0.82	75.3	63.9	87.3
	Sep 28	2.73	2.32	3.18	-0.06	-0.49	0.37	87.6	77.3	97.4
	Nov 12	3.53	2.97	4.11	0.62	0.06	1.20	75.7	66.5	85.8
	1970 Feb 10	2.87	2.31	3.44	0.83	0.26	1.39	79.7	66.7	90.9
	Mar 27	2.41	1.94	2.93	0.88	0.38	1.37	78.4	65.6	94.0
P ₁	1969 May 16	2.34	1.27	3.62	0.87	-0.34	2.01	93.4	48.5	142.7
	Sep 28	1.80	0.77	2.93	-0.47	-1.51	0.64	107.4	60.9	151.2
	Nov 12	1.23	0.26	2.46	-0.15	-1.24	0.96	171.0	98.9	248.4
	1970 Feb 10	1.62	0.76	2.75	-0.81	-1.77	0.23	100.7	30.7	156.0
	Mar 27	2.73	1.72	3.87	0.97	-0.11	2.04	73.4	46.9	102.2
K ₁	1969 May 16	6.89	5.79	8.05	0.59	-0.56	1.70	75.7	65.8	85.4
	Sep 28	5.24	4.15	6.33	0.68	-0.41	1.77	93.5	81.9	105.9
	Nov 12	4.57	3.64	5.60	-0.06	-1.04	0.93	84.2	72.1	98.5
	1970 Feb 10	3.73	2.74	4.79	0.38	-0.68	1.38	58.3	43.7	75.8
	Mar 27	5.06	4.02	6.14	1.23	0.16	2.29	68.1	54.8	80.2
N ₂	1969 May 16	0.89	0.17	1.74	-0.19	-0.96	0.61	120.4	38.2	170.6
	Sep 28	0.90	0.27	1.60	-0.18	-0.83	0.51	13.4	-28.0	70.6
	Nov 12	0.81	0.12	1.57	-0.11	-0.80	0.65	81.0	15.2	133.1
	1970 Feb 10	1.18	0.57	1.85	0.06	-0.61	0.68	106.6	72.8	142.3
	Mar 27	1.09	0.48	1.77	0.32	-0.32	0.98	114.3	77.1	164.0
M ₂	1969 May 16	5.44	4.72	6.19	-0.12	-0.87	0.60	105.7	97.9	113.6
	Sep 28	6.55	5.84	7.26	-2.44	-3.14	-1.73	95.2	88.1	102.4
	Nov 12	6.93	6.21	7.67	-1.58	-2.32	-0.86	103.9	97.6	110.5
	1970 Feb 10	6.54	5.83	7.25	0.38	-0.33	1.09	130.6	124.5	137.0
	Mar 27	6.11	5.44	6.79	1.02	0.36	1.70	121.8	115.4	128.6
S ₂	1969 May 16	3.39	2.16	4.75	0.12	-1.21	1.38	53.4	32.3	79.2
	Sep 28	3.00	1.95	4.15	-1.19	-2.25	-0.05	58.4	28.7	86.9
	Nov 12	0.86	0.01	2.20	-0.28	-1.24	0.96	165.8	60.0	265.7
	1970 Feb 10	1.88	0.97	3.00	0.56	-0.47	1.56	0.0	-47.2	45.1
	Mar 27	1.80	0.79	3.00	-0.73	-1.81	0.40	89.8	42.1	150.2
K ₂	1969 May 16	2.54	1.49	3.64	0.22	-0.83	1.31	33.6	11.8	58.8
	Sep 28	1.64	0.81	2.55	0.38	-0.53	1.21	33.4	-2.1	68.2
	Nov 12	0.85	0.00	1.87	0.08	-0.88	0.99	149.1	67.8	258.1
	1970 Feb 10	2.21	1.44	3.07	0.04	-0.79	0.85	30.8	6.4	53.1
	Mar 27	1.76	1.00	2.64	0.12	-0.70	0.95	14.2	-18.2	43.5
M ₄	1969 May 16	0.93	0.25	1.76	-0.09	-0.84	0.67	61.5	-1.0	123.8
	Sep 28	0.95	0.32	1.65	0.22	-0.48	0.85	59.2	8.0	107.6
	Nov 12	0.88	0.31	1.57	0.10	-0.54	0.72	45.0	-16.5	107.9
	1970 Feb 10	0.96	0.27	1.70	-0.22	-0.92	0.51	45.8	3.3	103.1
	Mar 27	0.60	0.05	1.35	-0.12	-0.73	0.57	36.8	-19.7	170.7
MS ₄	1969 May 16	0.69	0.05	1.50	-0.05	-0.77	0.68	63.1	-25.5	144.5
	Sep 28	0.42	0.00	1.12	0.19	-0.46	0.66	81.5	9.0	229.4
	Nov 12	0.91	0.28	1.65	-0.12	-0.81	0.56	67.4	4.7	116.6
	1970 Feb 10	0.64	0.04	1.32	0.06	-0.61	0.67	65.6	-1.8	170.0
	Mar 27	0.38	0.00	1.09	0.14	-0.48	0.62	66.6	-45.1	170.6