THE RELATIONSHIP BETWEEN VARIOUS PUPIL CHARACTERISTICS AND PERFORMANCE ON MATHEMATICS LABORATORIES

by

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Abstract

The purpose of this study was to examine the relationship between certain pupil characteristics and performance on mathematics laboratories. The four classes of grade six students involved in the study were classified by sex, field-dependence-independence, reflective-impulsive tempo, past performance in mathematics, present performance in mathematics, and intelligence. Eight mathematics laboratories designed and used in the study were categorized topically into number theory or geometry laboratories. Each laboratory activity was designed to allow pupils to manipulate materials while exploring an idea and collecting data. In part two of a laboratory activity, which included a test section, pupils were required to analyze data, make a prediction, and verify the prediction using manipulative materials before extending a pattern or rule. Laboratories were randomly assigned to classes.

Results showed that all the selected characteristics except sex had a significant relationship with performance on mathematics laboratories. Sex showed a significant relationship only to the geometry laboratories. An analysis of covariance was performed using past achievement as the covariate. The results indicated that there was a significant difference in performance only on the geometry laboratories between boys and girls and between field-dependent and field-independent students. The differences were found to be in favour of the girls and the field-independent students.
The results of this study suggested that further research is necessary to determine the most effective means of using mathematics laboratories.
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CHAPTER I
THE PROBLEM

Background of the Study

In the present North American educational system, many different instructional methods are advocated and used. This variety of methods can be partially attributed to the desire of educators to respond to what they perceive as changes in the demands of society. Many educators believe that the present demands of society require that the products of the educational system should be able not only to respond automatically to a known situation, but also to apply their knowledge to new situations (Reys & Post, 1973, P. 6).

In mathematics such a perceived demand has caused a shift in emphasis away from a heavy concentration upon computational efficiency and toward an 'understanding' of the underlying structures of mathematics (Scott, 1966, p. 193).

Educators have also advocated a variety of teaching methods because of their desire to respond to individual differences. Differences in mental ability, previous achievement, social and emotional maturity, personality, and environmental background have influenced the placement of pupils into special classes or groups and have caused teachers to try a variety of teaching methods (Jones & Pingry, 1960, pp. 121-148).

Brown concluded that:

At present teachers do not know which students should be exposed to what
methods. The successful teacher tries many methods until he finds one that seems to work with a particular student or group of students (1968, p. 411).

He also suggested that there is "no one answer to all problems with all students" (Brown, 1968, p. 411).

Currently there are several teaching methods which attempt to promote 'discovery' on the part of the student. Since discoveries are often the outcome of experiment, it has become common to use a setting which simulates that of a laboratory.

Bruner (1962, p. 85) is convinced that "the greater the participation in the learning process on the part of the student, the greater will be the transfer of training and the more likely will be the development of intuitive thinking". In addition, Bruner supported the use of a mathematics laboratory when he hypothesized four major benefits from using a discovery approach to teach mathematics. Bruner (1961) claimed that there will be an "increase in intellectual potency, ... the shift for extrinsic to intrinsic rewards, ... learning the heuristics of discovering, ... and an aid to memory processing" (p. 23).

Barson (1971, p. 565) also supported the use of mathematics laboratories when he observed that a "mathematics laboratory is activity centered; the child is placed in a problem-solving situation and through self-exploration and discovery provides a solution based on his experience, needs, and interests".
The Problem

It is the purpose of this study to explore the relationship between some pupil characteristics and the performance of pupils in mathematics laboratories. Selected characteristics of pupils were sex, previous achievement in mathematics, present achievement in mathematics, intelligence, reflective-impulsive tempo, and field-dependence-independence.

Definition of Terms

Mathematics Laboratory shall mean a context in which a pupil is presented with a mathematics problem which requires manipulation of physical objects to generate data, and in which, after the collection and organization of data, the pupil is expected to detect a mathematical relationship, extend the relationship by interpolation and extrapolation, and try to state a generalization.

Reflective-Impulsive Tempo is a facet of conceptual tempo which involves the slow or fast decision-time of students.

Field-Dependence-Independence is a facet of cognitive style known as perceptual style. In a field-dependent mode, perception is strongly influenced by the organization of the field. In a field-independent mode, parts of the field are perceived as discrete from the organization of the field.

Justification for the Study

Control, as a goal of science, is defined by Van Dalen (1966, p. 43) as "The process of manipulating certain of the
essential conditions that determine an event so as to make the event happen or prevent it from occurring". Control, as a goal in education, is the process of manipulating the method of instruction for particular students. This analogy suggests that investigations should be undertaken concerning the type of student that will benefit from a particular instructional method. An advocate of this kind of research is Cronbach (1966, pp. 91-92) who suggested that "we have to explore a five-fold interaction -- subject matter, with type of instruc­tion, with timing of instruction, with type of pupil, with outcome". The outcome of research that explores these aspects should provide sufficient information to enable teachers to say "that a fourth grader with one profile of attainment needs discovery experience, whereas another will move ahead on all fronts if teaching is didactic" (Cronbach, 1966, pp. 90-91).

Additional support for this type of research is given by Brousseau (1973) when he stated that:

There is a need for knowledge of not only the methods involved, but also of classifying and coding the educational and learning characteristics of pupils in order that one might fit the method of instruction to the individual student (p. 103).

Statistical Hypotheses

Nine hypotheses were tested in this study:
H₁: There will be no significant relationship between the sex variable and performance in mathematics laboratories.

H₂: (a) There will be no significant relationship between Verbal I.Q. and performance on mathematics laboratories.
   (b) There will be no significant relationship between Non-verbal I.Q. and performance on mathematics laboratories.

H₃: (a) There will be no significant relationship found between past achievement in mathematics and performance on mathematics laboratories.
   (b) There will be no significant relationship found between present achievement in mathematics and performance on mathematics laboratories.

H₄: There will be no significant relationship between field-dependency-independency and performance on mathematics laboratories.

H₅: There will be no significant relationship between reflective-impulsive tempo and performance on mathematics laboratories.

H₆: (a) The mean score for girls on number theory laboratories will not be significantly higher than for boys when I.Q. is controlled.*
   (b) The mean score for boys on geometry laboratories will not be significantly higher than for girls when I.Q. is controlled.*

* See page 36.
(c) The mean score for boys and girls will not be the same on all laboratories when I.Q. is controlled.*

\[ H_7: \] The mean score for impulsive girls will not be different from the mean score for reflective girls on mathematics laboratories.

\[ H_8: \] The mean score for impulsive field-dependent students will not be different from the mean score for reflective field-dependent students on mathematics laboratories.

\[ H_9: \] The difference between the means of field-dependent boys and field-dependent girls will not be significantly different from the difference between the means of field-dependent boys and field-dependent girls when I.Q. is controlled.*

* See page 36.
CHAPTER 2
REVIEW OF THE LITERATURE

Introduction

In the literature of mathematics education, the term 'mathematics laboratory' is not well-defined. The two most popular connotations, however, seem to be as follows:

(i) a particular methodology for learning.
(ii) a particular place in which learning occurs.

Reys & Post (1971, p. 9-10) stated that the term laboratory encompasses the physical facilities and educational intent, but should be extended beyond the use of materials and facilities to incorporate the aspects of inquiry through exploration and discovery and to create a problem solving atmosphere.

General Opinions on Mathematics Laboratories

One call for the initiation of mathematics laboratories was given by Moore in 1902 when he addressed the American Mathematical Association. Moore suggested that:

this program of reform calls for the development of a thorough-going laboratory system of instruction in mathematics and physics, a principal purpose being as far as possible to develop on the part of every student that true spirit of research, and an appreciation, practical as well as theoretic, of the fundamental methods of science .... (p. 250)
It seems, however, to have taken over fifty years for the use of the laboratory method of teaching mathematics to take hold in North America.

An article by Davidson and Fair (1970) indicated not only a possible structure for a laboratory, but also it suggested some of the possible outcomes from using a mathematics laboratory. Their suggestions to pupils, used while trying to establish the tone for their laboratory, indicate that they hold these beliefs:

(i) the use of manipulative materials can act as a vehicle for learning mathematics;

(ii) the mathematics laboratory method involves investigation, exploration, hypothesizing, and looking for patterns;

(iii) hypotheses can be checked by using manipulative materials;

(iv) laboratory experiences can be related to specific mathematics concepts, to problem-solving techniques, or to modes of mathematical thinking. (p. 107)

Other outcomes suggested by users of mathematics laboratories usually include items such as improvement of pupils' attitudes towards mathematics, increased interest in mathematics, increased self-confidence, and improvement shown in areas of concepts and concept applications (Beuthel & Meyer, 1972; Schaeffer & Mauthe, 1970; Vance & Kiernan, 1971). One further
suggested learning outcome from the use of mathematics laboratories is the development of problem-solving skills (Kidd, Myers & Cilley, 1970). Kidd, Myers and Cilley (1970) expanded their opinion concerning the development of problem-solving skills by stating that:

The surroundings of physical objects aid him in sizing up the problem and in selecting sound and feasible methods of attack. He also receives training in selecting data relevant to the question that has been posed and to his methods of attack. Furthermore, there are often built-in verifications that give him feedback on the soundness of his method, the accuracy of his data, and the correctness of his computations. (p. 28)

Most mathematics laboratories seem to begin by having students manipulate physical materials and then abstracting concepts from their manipulation (Fitzgerald, 1972, pp. 2-3). Copeland (1974) described a mathematics laboratory as "a classroom designed to allow children to individually perform the necessary physical manipulations or concrete operations that are necessary for real learning of mathematical concepts" (pp. 327-328). He concluded that he sees the mathematics laboratory as contributing to the "nature of learning as envisioned by Piaget" (p. 360).
Kiernan and Vance (1971) suggested that the activities used in a mathematics laboratory could be used not only to develop concepts, but also to permit discovery of a relationship.

Laboratory activities are designed to lead to the development of a concept or the discovery of a relationship. The concrete materials serve not only to create interest and motivate learning but to provide a real-world setting for the problem to be solved or the concept to be investigated. The students use physical objects or manipulative devices to perform experiments and collect data relating to a problem, and they record this information in a table or on a graph if possible. On the basis of these observations, hypotheses are formulated and tested. Finally, generalizations are stated. The newly discovered rule might then be used to answer additional questions or to do practice exercises for the purpose of consolidating learning. (p. 586)

Davidson and Walter (1972) defined the mathematics laboratory as

... an approach to learning materials rather than a particular place in a building. Such an approach encompasses exploring,
investigating, hypothesizing, experimenting, and generalizing. It means that students are actively involved in "doing" mathematics at a concrete level. It provides abundant opportunities for them to manipulate objects, to think about what they have done, to discuss and write about their findings, and to build necessary skills. Problems are related to the children's own experiences and often emerge from the natural surroundings. The teacher acts as a catalyst and resource person, becoming an active investigator along with the students. (p.222)

Research Relevant to Mathematics Laboratories

Within the field of mathematics education, the recent trend has been toward involving students actively in learning. Shipp and Deer (1960), upon investigating the percent of class time spent on developmental activities and on practice work, found that there was a trend toward higher achievement at all levels of ability when the time spent on developmental activities was increased (pp. 117-121). Also it seemed that more than fifty percent of class time should be spent on developmental activities. In a later study, Shuster and Pigge (1965) investigated the effects of devoting twenty-five percent, fifty percent, and seventy-five percent of class time on
developmental-meaningful activities. They concluded that fifty to seventy-five percent of class time has to be spent on developmental activities to show an increase in achievement (pp. 24-31).

The results of a study done by Kuhfitting (1974) involving manipulative aids seemed to imply that:

the benefits derived from using the type of aids that students can manipulate themselves tend to be lost when the expository instructional method is employed, while independent work with such aids may lead to discovered insights. (p. 108)

He also suggested that if this finding is substantiated by further studies, the increased use of the mathematics laboratory is to be highly recommended (p. 108).

Wilkinson (1970) reported that he found that students who were taught geometry by a laboratory method did as well as students taught by the conventional teacher-textbook approach on a geometry achievement test. In a study comparing the laboratory method and individualized instruction when teaching metric geometry, Whipple (1972) found that the mathematics laboratory classes did better both on a conventional test and in the computing of areas and volumes which required the use of actual objects. Silbaugh (1972) found significant differences between criterion scores for groups using laboratories, groups not using laboratories when laboratory facilities
were in the school, and groups not using laboratories when laboratory facilities were not available in the school.

Cohen (1970) compared laboratory and conventional methods to teach fractions to underachievers. He found that the students using the conventional method did better. He noted also that the laboratory method required more time than the conventional method.

Ropes (1970) was able to use a laboratory method with pupils once a week for fourteen weeks. He reported that there was no significant change in attitude on the part of the students in the laboratory setting. The findings also showed that the lab students demonstrated no greater competency than the non-participants in the ability to solve problems or in the ability to classify and formulate class concepts. The lab students, however, did score as well as the non-participants on a standardized test even though the lab students had spent 20% less time under conventional instruction.

Nowak (1972) also did a study which compared the effects of mathematics laboratories on achievement and attitudes. She found that attitude changes were non-significant, that fifth and sixth grade students did better in a laboratory than in a non-laboratory program, that an individualized laboratory program was more effective than an individualized non-laboratory program, and that fourth grade students did better in the conventional mathematics classes than those in the laboratory setting.
Research Relevant to Pupil Characteristics

Previous studies relevant to this study include those which relate general mental ability and mathematics performance. As a variable, mental ability is frequently employed in studies that consider what type of student profited by the training. Results of the studies utilizing the general mental ability variable usually show either that the low-ability students performed as poorly as before and the high-ability students did very well, or that the low-ability students did exceptionally well and the high-ability students performed about as well as usual. Because of such results, Anderson (1967) suggested that:

interactions, more expressly correlations, between aptitude measures and performance after training, contain information that can be employed to improve training by matching the kind of training to the kind of person in one of several possible ways, or by modifying the training so that those with low aptitude scores achieve better.

(p. 87)

A study by Jarvis (1964), which demonstrates these typical findings, compared ability differences and performance in elementary school mathematics. Jarvis found that:

1. The bright boys were found to be superior to their peer group girls in both reasoning and fundamentals.
2. All classifications of male students excelled the female students in their ability to perform arithmetic reasoning functions.

3. All classifications of girls were superior to boys in their ability to execute the arithmetic fundamental operations with the exception of the bright group. (p. 659)

In the above study, general mental ability was claimed to be an individual difference which affects pupil performance.

The second variable to be considered in this study is one involving conceptual tempo. A conceptual tempo known as the reflective-impulsive dimension refers to the slow or fast decision-time when response uncertainty is present.

Tests have been designed and considerable research has been done by Jerome Kagan and his associates at the Fels Institute using the reflective-impulsive tempo variable. From the studies undertaken, Kagan (1965) has suggested that the tendency:

for reflection increases with age, is stable over periods as long as 20 months, manifests pervasive generality across varied task situations, and is linked to some fundamental aspects of the child's personality organization. (p. 134)

Also he suggested that the tendency "to show fast or slow decision times was not highly related to verbal ability" (p. 140)
and "to reflect over alternative hypotheses generalizes not only across tasks where all response alternatives are given but also shows generality on tasks where the child must generate his own alternatives" (p. 141). The observed behavioral characteristics of impulsive students are also of interest.

Impulsive students were likely to display momentary lapses of attention during involvement in a school task; they were more likely to look out a window, gaze at a peer, or orient to a sound during a period when they were working at an academic task. These behavioral observations are in complete agreement with the problem-solving behavior displayed by children in the laboratory setting.

(Kagan, 1964, p. 29)

A further study done at the Fels Institute examined the relationship of the reflective-impulsive conceptual tempo and persistence. The findings of this study revealed that:

reflective boys spent more time with the hard tasks than impulsives and this difference was greater for low-verbal than for high-verbal boys .... Among girls, verbal skill was a more critical variable in determining free play performance, and the brighter girls spent more time with
hard tasks. As with boys, the impulsive girls with low verbal ability spent less time on hard tasks, but the impulsive bright girls spent the longest time on the difficult tasks. (Kagan, 1964, p. 158)

Kagan realized that many teachers were not aware of the difference in conceptual tempo between children.

Most teachers are not attuned to this behavioral aspect of the child. Teachers are apt to categorize the child as bright or dull, obedient or disobedient, timid or outgoing; but rarely do they notice whether the child is impulsive in his conceptual approach to problems. When they do acknowledge this dimension there is a tendency to classify the reflective child as 'slow' and less bright than the impulsive, quick child with the same intelligence score and social-class background. (pp. 158-159)

Kagan also suggested that reflective-impulsive tempo could have an effect on pupil performance in subjects where the program utilizes the discovery method of instruction (Kagan, Pierson & Welch, 1966, p. 594).

Whenever reports of research are examined, one frequently finds reference made to the difference in performance between
the sexes. As already seen, both Kagan and Jarvis in their studies made direct reference to the different levels of attainment achieved by boys and girls. Fennama (1974), after exploring the depth the literature since 1960 concerning mathematics learning and the sexes, remarked that:

although generalizing results from so many divergent studies that analyzed different aspects of mathematics learning hides subtle and important variations, it appears safe to conclude that in overall performance on tests measuring mathematics learning that there are no significant differences that consistently appear between the learning of boys and girls in the fourth to ninth grade. There appears to be a trend, however, that if a difference does exist, girls tend to perform better in tests of mathematics computation ... and boys tend to perform better in tests of mathematical reasoning... (p. 135)

Further studies which demonstrate the analysis of sex differences in learning are those done by Witkin concerning the field-dependence-independence mode of perception. Witkin (1962) found that "this dependence of females on the visual field was apparent at all ages from the adult level to the eight year old level and indicates that from an early age females are less analytical than males" (p. 14). Waetjen
(1962) suggested that if girls are less analytical than boys, then schools need to be aware of this variable. Since girls are more field-dependent and more sensitive to people than boys, it would seem that they should fare better in the usual classroom where the teacher and the curriculum content are major part of the field. ... being more analytical, boys may create difficulties for themselves by making too many decisions on their own rather than responding to suggestions and directions from the teacher (Waetjen, 1962, p. 14). Waetjen's conclusions suggested that boys will perform better in a setting where they can make many of their own decisions. If this is so, then boys may be more successful in lessons using the laboratory method where they are able to work more independently.

Research of Specific Relevance of the Problem

Two studies were found which considered the reflective-impulsive tempo and performance in mathematics. A study by Cathcart and Liedtke (1969) which considered the reflective-impulsive conceptual tempo and achievement in mathematics found that reflective students obtained higher mathematics achievement scores in the dimension of basic facts and problem
solving, but showed no significant difference in the application of concepts (p. 594). Upon examination of the reflective-impulsive tempo and performance in elementary school mathematics, Rebhun (1973) found that reflective children generally did obtain higher scores than did impulsive children on each of the Metropolitan Mathematics Tests involving computation, concepts, and problem solving.

After examining predictors of seventh grade achievement in mathematics, Fuys (1974) recommended that a measure of prior achievement be used for selecting students for a unified modern mathematics program. He also found that a measure of prior achievement and the Chapter One test for the program predicted success in the program as effectively as all six of his predictors.

Hollis (1972), who used a diagnose-prescribe model of a mathematics laboratory, found that both slow and gifted students at the fifth and sixth grade levels showed a slightly increased academic achievement in mathematics when compared to the corresponding control groups. Also, the laboratories facilitated an increased positive attitude toward mathematics with the significant increase occurring in the school located in a deprived area. There was no significant difference in the achievement scores between laboratory and control groups.
Summary of Research Findings

After reviewing the available literature concerning mathematics laboratories, it appears that although mathematics laboratories are perhaps no better than conventional methods of teaching mathematics, the achievement ratings of pupils involved in research studies tend to suggest that the pupils taught using mathematics laboratories perform as well as those students taught using conventional methods.
CHAPTER 3

DESIGN AND PROCEDURE

Design

The present study employed mathematics laboratories and four classes of students. All subjects performed all of the laboratories. Information was gathered for each subject concerning individual characteristics as follows: sex, Verbal IQ, Non-verbal IQ, performance in mathematics during the previous and the current school year, field dependence-independence, and the reflective-impulsive tempo.

Controls

Precautions were taken to control teaching style and Hawthorne-type effects.

No attempt was made to select teachers with a similar teaching style. However, upon questioning the teachers, it was found that none had had previous experience in the laboratory approach when teaching mathematics.

Although the novelty of doing mathematics laboratories was a factor in the experiment, all students experienced the same effect, and therefore it can be assumed that all students were affected equally.

Subjects

The population chosen was Grade Six students from some
British Columbia schools in the lower mainland. Four classes were selected in two schools which would provide the study with pupils of varying ability and with different teachers. Laboratories were assigned on a random basis to each class. A necessary restriction on the assignment of the laboratories was that no two classes could do the same laboratory on the same day due to the necessity of sharing equipment.

Laboratories

Ideas for the laboratories were selected from number theory and geometry. An attempt was made to choose topics which would, in all probability, be unfamiliar to the pupils. This was done in order to avoid the need to pretest for prior knowledge. Each laboratory had two parts. The first part was designed to allow pupils the opportunity of using manipulative materials to explore the idea and to collect data. Teachers were permitted to assist pupils in handling the materials in a manner appropriate to the problem. In the second part, which was used as the test section, pupils were required to analyze their data, make a prediction, and verify the prediction by using manipulative materials before extending the pattern or rule discovered. (See Appendix I)

Initially the laboratories were prepared for a pilot study using two sixth grade classes. After completion of the laboratories, part two of each laboratory was scored. A reliability coefficient was then obtained for each laboratory
using the Kuder-Richardson Formula (KR20). Since each laboratory could be related to either number theory or geometry, the laboratories with their reliability coefficients are listed in the two categories.

Table 1

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Reliability Coefficient</th>
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<tbody>
<tr>
<td>Divisors of a Number</td>
<td>.52</td>
</tr>
<tr>
<td>Figurate Number</td>
<td>.54</td>
</tr>
<tr>
<td>Squares in a Square</td>
<td>.74</td>
</tr>
<tr>
<td>(sums of square numbers)</td>
<td></td>
</tr>
<tr>
<td>Primes and Composites</td>
<td>.82</td>
</tr>
<tr>
<td>Common Divisors</td>
<td>.86</td>
</tr>
<tr>
<td>Least Common Multiple</td>
<td>.93</td>
</tr>
</tbody>
</table>
Table 2

Reliability Coefficients for Geometry Laboratories

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Reliability Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Networks</td>
<td>.49</td>
</tr>
<tr>
<td>Traceable Networks</td>
<td>.54</td>
</tr>
<tr>
<td>Euler's Formula</td>
<td>.67</td>
</tr>
<tr>
<td>Classification of Triangles</td>
<td>.66</td>
</tr>
<tr>
<td>Triangles in Polygons</td>
<td>.69</td>
</tr>
<tr>
<td>Making Polygons using Triangles and Squares</td>
<td>.73</td>
</tr>
<tr>
<td>Pic's Theorem</td>
<td>.77</td>
</tr>
<tr>
<td>Diagonals of Polygons</td>
<td>.85</td>
</tr>
</tbody>
</table>

In each category the four laboratories with the highest reliability coefficients were selected for use in the experiment. (See Appendix I) For reference when marking laboratories in the main study, a record was kept of the responses of patterns and generalizations that were accepted in the pilot.

Measurements of Individual Characteristics

Sex and Mathematics Performance

Some measures were gathered from each subject's Permanent Record Card. Schools in British Columbia are required by the Department of Education to keep this record card. The card
contains not only information about a student and his family, but also a record of the results of all standardized tests given to the student and the results of his achievement in school subjects at each grade level. The core subjects, which include Language, Spelling, Reading, Social Studies, Arithmetic, and Science, are graded on a seven point letter grade system. This Permanent Record Card was used to check the sex and age of each subject and to find the subject's achievement level in Mathematics for Grade 5 and Grade 6. The Grade 6 achievement level, later referred to as present performance, was an extremely current rating since the record cards were in the process of being completed for the school year at the time data were gathered.

IQ Scores

Verbal and Non-verbal IQ scores were obtained from the Canadian Lorge-Thorndike Intelligence Test which was administered and scored by the experimenter.

Field-Dependence-Independence

Scores for this characteristic were obtained from the Group Embedded Figures Test administered and scored by the experimenter. This test was designed by Phillip K. Oltman, Evelyn Raskin, and Herman A. Witken (1971) and published by the Consulting Psychologists Press, Inc.
Reflective-Impulsive Tempo

Subjects completed a questionnaire which was intended to indicate the reflective-impulsive tempo of each pupil. This questionnaire was a simplified version of Part C of a questionnaire used in Kilpatrick (1967) "Analyzing the Solution of Word Problems in Mathematics: An Exploratory Study".

Procedure

Before the experiment began the experimenter met with the teachers involved to explain their role. Emphasis was placed on the need to help students on the first part of the laboratories only. Arrangements were made for delivering and collecting the laboratories each day and for administering the tests and questionnaire following the laboratory sessions.

Each day during the experiment the laboratories and required materials were delivered to the teachers. Time was provided for the teacher to try some of the questions in the lab and to ask any questions concerning the lab or the procedure. The previous day's laboratory and equipment was picked up at this time.

Following the completion of all eight laboratories the teachers administered the reflective-impulsive tempo questionnaire. Three additional days were then required for the administration of the Lorge-Thorndike IQ Test and the Group Embedded Figures Test, by the investigator.
Part two of each laboratory was marked. The answers relating to patterns and generalizations were marked with reference to the acceptable responses made during the pilot study. Subjects' scores were kept in the two categories of number theory and geometry. To make the summing of scores possible, the scores on each lab were translated to normalized standard scores. Due to absence, some of the subjects did not complete all the labs. The total score for each category was calculated only for those subjects who had completed all the laboratories in the category. Also a grand total score was calculated for those students who had completed all eight laboratories.

The reflective-impulsive questionnaire, the Lorge-Thorndike IQ Test, and the Group Embedded Figures test were hand marked and scored by the investigator. Results were recorded for each pupil. Again, due to absence, not all subjects wrote all tests.

**Statistical Procedure**

The analysis to follow is considered in terms of seven pupil characteristics and the scores on the mathematics laboratories. The pupil characteristics were Verbal IQ, Non-verbal IQ, reflective-impulsive tempo, field-dependence-independence, mathematics performance in Grade 5, mathematics performance in Grade 6, and sex. The mathematics laboratories were used as a whole unit and were subdivided into the two
categories of number theory oriented labs and geometry oriented labs.

Since not all tests were validated on groups similar to that of the study, the reliabilities of the laboratories, the Group Embedded Figures Test, and the reflective-impulsive tempo questionnaire were calculated in terms of a Cronbach Alpha. The Cronbach Alpha was utilized in order to decide whether it would be reasonable to sum the individual laboratory scores to get Number Theory and Geometry subtest scores.

All data were analyzed at the University of British Columbia Computing Centre using a reliability of test program, a correlation program, and an analysis of covariance program.
CHAPTER 4
RESULTS OF THE STUDY

Results

Reliability figures in terms of a Cronbach Alpha are given in Table 3 for all the laboratories, the Number Theory laboratories, the Geometry laboratories, the Group Embedded Figures test (GEFT), and the reflective-impulsive questionnaire (RI Quest).

Table 3

<table>
<thead>
<tr>
<th>Test</th>
<th>Cronbach Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Laboratories</td>
<td>.77</td>
</tr>
<tr>
<td>Number Theory Labs</td>
<td>.67</td>
</tr>
<tr>
<td>Geometry Labs</td>
<td>.53</td>
</tr>
<tr>
<td>GEFT</td>
<td>.85</td>
</tr>
<tr>
<td>RI Questionnaire</td>
<td>.02</td>
</tr>
</tbody>
</table>

Because of the low reliability of the RI Quest (.02), no analysis was performed involving the reflective-impulsive tempo characteristic. Although some of the laboratory reliabilities appear to be low, it was decided to continue the planned analysis, based on Davis' interpretation of reliabilities. Davis suggested that reliabilities as low as
.5 are serviceable when measuring characteristics of class-size groups, and even lower reliabilities may yield useful information when the group is larger (Davis, 1964, p. 24).

The correlation between performance on the Number Theory labs and the Geometry labs was .67. Therefore it was decided to consider these attributes separately as well as in total.

The appropriate correlation between each pupil characteristic and the mathematics laboratories is shown in Table 4.

### Table 4

Correlations Between Pupil Characteristics and Mathematics Laboratories

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>No. Theory Labs</th>
<th>Geometry Labs</th>
<th>All Labs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>-.07</td>
<td>.22*</td>
<td>.10</td>
</tr>
<tr>
<td>Verbal IQ</td>
<td>.41**</td>
<td>.52**</td>
<td>.49**</td>
</tr>
<tr>
<td>Non-verbal IQ</td>
<td>.47**</td>
<td>.48**</td>
<td>.51**</td>
</tr>
<tr>
<td>Math Performance Grade 5</td>
<td>.51**</td>
<td>.48**</td>
<td>.58**</td>
</tr>
<tr>
<td>Math Performance Grade 6</td>
<td>.49**</td>
<td>.34**</td>
<td>.49**</td>
</tr>
<tr>
<td>GEFT</td>
<td>.43**</td>
<td>.47**</td>
<td>.51**</td>
</tr>
</tbody>
</table>

* Significant at the .05 level  
** Significant at the .01 level
The table shows that all pupil characteristics except sex were significant at the .01 level.

Since the Lorge Thorndike IQ test was administered after the completion of the laboratories, this variable could not be used as a covariate. Past achievement, therefore, was selected as the covariate. The results of the analysis of covariance follow in Tables 5 to 8.

Table 5
Means and Adjusted Means

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Laboratories</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No. Theory</td>
<td>Geometry</td>
<td>All Labs</td>
</tr>
<tr>
<td>Boys</td>
<td></td>
<td>198.3</td>
<td>190.5</td>
<td>389.9</td>
</tr>
<tr>
<td></td>
<td>*Adjusted Mean</td>
<td>191.7</td>
<td>180.5</td>
<td>372.1</td>
</tr>
<tr>
<td>Girls</td>
<td></td>
<td>200.9</td>
<td>203.7</td>
<td>408.8</td>
</tr>
<tr>
<td></td>
<td>*Adjusted Mean</td>
<td>204.9</td>
<td>200.6</td>
<td>407.7</td>
</tr>
<tr>
<td>Field-Dependent</td>
<td></td>
<td>194.2</td>
<td>190.5</td>
<td>389.1</td>
</tr>
<tr>
<td></td>
<td>*Adjusted Mean</td>
<td>196.8</td>
<td>200.6</td>
<td>406.2</td>
</tr>
<tr>
<td>Field-Independent</td>
<td></td>
<td>204.9</td>
<td>203.7</td>
<td>409.6</td>
</tr>
<tr>
<td></td>
<td>*Adjusted Mean</td>
<td>205.0</td>
<td>206.8</td>
<td>411.5</td>
</tr>
</tbody>
</table>

*Means were adjusted by using previous mathematics achievement as a covariate.
Table 6

Analysis of Covariance
Sex vs Field-Dependence-Independence
on Number Theory Laboratories

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>1</td>
<td>0.252</td>
<td>0.623</td>
</tr>
<tr>
<td>GEFT</td>
<td>1</td>
<td>3.523</td>
<td>0.062</td>
</tr>
<tr>
<td>Sex X Geft</td>
<td>1</td>
<td>0.242</td>
<td>0.630</td>
</tr>
<tr>
<td>Error</td>
<td>58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>61</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7

Analysis of Covariance
Sex vs Field-Dependence-Independence
on Geometry Laboratories

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>1</td>
<td>5.199</td>
<td>0.025*</td>
</tr>
<tr>
<td>GEFT</td>
<td>1</td>
<td>5.010</td>
<td>0.028*</td>
</tr>
<tr>
<td>Sex X Geft</td>
<td>1</td>
<td>1.443</td>
<td>0.233</td>
</tr>
<tr>
<td>Error</td>
<td>57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* p < .05
Table 8

Analysis of Covariance
Sex vs Field-Dependence-Independence
on All Mathematics Laboratories

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>F.</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>1</td>
<td>3.308</td>
<td>0.072</td>
</tr>
<tr>
<td>GEFT</td>
<td>1</td>
<td>3.368</td>
<td>0.070</td>
</tr>
<tr>
<td>Sex X Geft</td>
<td>1</td>
<td>2.101</td>
<td>0.150</td>
</tr>
<tr>
<td>Error</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion of the Results

The data presented in Table 4 indicated that all the pupil characteristics except the sex attribute was statistically significant at the .01 level. This level of significance was attained whether the mathematics laboratories were taken as a whole or subdivided into the categories of number theory laboratories and geometry laboratories. The sex characteristics showed significance at the .05 level with respect to the geometry laboratories only. Since the correlations at the .01 level of significance were \(0.34 \leq r \leq 0.58\), the variance attributed to these characteristics ranged from \(0.12 \leq r^2 \leq 0.34\).

The statistical hypotheses results follow.
Hypothesis $H_1$ which predicted no significant sex relationship in performance on mathematics laboratories had mixed results. With respect to number theory laboratories or all laboratories, the hypothesis was not rejected. But with respect to geometry laboratories, the hypothesis was rejected at the .05 level of significance. This difference in performance on geometry laboratories was in favour of the girls.

Hypothesis $H_2(a)$ which predicted no significant relationship between Verbal IQ and performance on number theory laboratories, geometry laboratories, or all laboratories was rejected at the .01 level of significance. Predictably this was in favour of higher Verbal IQ pupils.

Hypothesis $H_2(b)$ which predicted no significant relationship between Non-verbal IQ and performance on number theory laboratories, geometry laboratories, or all laboratories was rejected at the .01 level of significance. This was in favour of pupils with higher Non-verbal IQ.

Hypothesis $H_3(a)$ which predicted no significant relationship between general mathematics performance during the past year and performance on number theory laboratories, geometry laboratories, and all laboratories was rejected at the .01 level of significance. This difference was in favour of higher general mathematics performance.

Hypothesis $H_3(b)$ which predicted no significant relationship between general mathematics performance during the current year and performance on number theory laboratories was rejected
at the .01 level of significance. This difference was in favour of high general mathematics performance.

Hypothesis $H_4$ which predicted no significant relationship between field-dependence-independence and performance on number theory laboratories, geometry laboratories, and all laboratories was rejected at the .01 level of significance. This difference was in favour of the field-independent pupils.

Hypothesis $H_5$ which predicted no significant relationship between a measure of reflective-impulsive tempo and performance on laboratories was not tested because the reliability of the reflective-impulsive questionnaire was too low.

Hypothesis $H_6(a)$ which predicted no significant difference between the mean score for girls and boys on the number theory laboratories when past achievement was controlled was not rejected.

Hypothesis $H_6(b)$ which predicted no significant difference between the mean score for girls and boys on the geometry laboratories when past achievement was controlled was rejected at the .05 level of significance. This difference was in favour of the girls.

Hypothesis $H_6(c)$ which predicted no significant difference between the mean score for girls and boys on all laboratories when past achievement was controlled was not rejected.

Hypotheses $H_7$ and $H_8$ could not be tested because both hypotheses required the use of the reflective-impulsive tempo questionnaire which was unreliable.
Hypothesis $H_9$ which predicted no significant difference between the means of field-dependent boys and field-dependent girls and the means of field-independent boys and field-independent girls when past achievement is controlled on all mathematics laboratories was not rejected.
CHAPTER 5
SUMMARY and CONCLUSIONS

The Problem

The purpose of this study was to find some characteristics of pupils that related well to their performance on mathematics laboratories. The characteristics selected were sex, intelligence, field-dependence-independence, previous mathematics achievement, and current mathematics achievement.

The Findings

The results of the data analysis indicated that, in general, there was a relationship between the characteristics of intelligence, field-dependence-independence, previous mathematics achievement, and current mathematics achievement. The range of the correlations was $0.34 \leq r \leq 0.58$. All these correlations were statistically significant at the .01 level. The better performance on the mathematics laboratories was related to higher intelligence, greater field-independence, and higher mathematics achievement.

When the laboratories were subdivided topically into number theory laboratories and geometry laboratories, the sex characteristic showed significance only on the geometry laboratories. The correlation of .22, which was statistically significant at the .05 level, was in favour of the girls.
Implications

Although most of the correlations between the selected pupil characteristics and the performance on mathematics laboratories were statistically significant, the correlations were too low for many implications to be drawn as a consequence of this study.

However, the low correlations between both Verbal and Non-verbal IQ and the performance on the mathematics laboratories indicate that only approximately 25% of the variance is accounted for by the intelligence characteristics. Therefore, grouping on the basis of intelligence for mathematics laboratories may be inappropriate.

It was also noted that the correlation between pupil achievement and performance on mathematics laboratories related to geometry dropped from .48 with past achievement to .34 with present achievement. Some of this difference may be attributed to the increase in computational work in the curriculum between years five and six. As a result of the increased load with respect to computation in year six, geometry is forced to play a more minor role, and the achievement rating for this year was probably mainly a reflection of the pupils' computational skill. If geometry is intended to be a integral part of the elementary mathematics program, the teachers need to be encouraged to place more emphasis on geometry at the year six level.
Limitations

The limitations of this study fall into two categories: those that the investigator found unavoidable and those that appeared while the study was in progress. The unavoidable limitations involve the population, the materials, the timing and teacher characteristics. Limitations which were unexpected but appeared while the study was in progress involved variations in the teachers' ability and attitude, teacher-pupil interaction, and previous learning experiences of pupils.

The population sample was restricted. Although four classed of students were used, these students came from only two schools in similar economic areas. Clearly a more representative sample of the population could have produced different results.

Because the mathematics laboratories were experimenter-designed and because the reliabilities of the tests varied considerably, the accuracy of the scores on the laboratories may be questionable.

Laboratory assignments could have been scheduled more effectively in order to achieve maximum psychological effect. Each day the pupils were given a lab involving a different topic. The pupils received no feedback on the previous laboratory related either to the acceptable responses or to their own performance. This created anxiety for some pupils and may have affected the performance of pupils on the laboratories.
Two areas of weakness with respect to the participating teachers were found. First, the teachers generally were unskilled in using manipulative materials with students. Although informal inservice training did take place, formal inservice training involving the use of manipulative materials could conceivably have influenced the results of the study. Second, it was found that half of the pupils involved in the study had teachers who felt that using manipulative materials was a waste of time. The performance scores on the laboratories for these pupils were probably affected by the attitude of the teachers.

The structure of the laboratories allowed for no discussion either between teacher and pupil or between pupil and pupil during the time that the pupils were attempting to find a pattern and trying to formulate a generalization. This loss of interaction was probably too severe for pupils considering their age and their previous experience at generalizing in mathematics. Again, the scores were probably affected.

Many pupils had difficulty in formulating generalizations. Even the initial steps were beyond the capability of some pupils. If the data were naturally organized by the format of the laboratory, many pupils were able to detect the pattern and extend it. If, however, the collected data required organization, some pupils were unable to organize it and the data therefore, were unlikely to recognize the pattern. For pupils who were able to organize the data, detect a pattern, and extend it, many seemed unable to decode the pattern which would allow
for the expression of a generalization. Even the simple decoding skills of looking at differences or renaming numbers were not obvious to many pupils. Pupils who were able to decode the pattern then had difficulty expressing what they had discovered either in statement form or in a mathematical sentence.

Further Research

Directions for further research are suggested by some of the limitations of this study. Certainly it would be desirable to have a broader sampling of the population. Perhaps the sample could span ages as well as economic background. In addition, an inservice program for teachers involving the use of manipulative materials would be beneficial. This would not only improve the manner in which the materials were used, but also provide the experimenter with an opportunity to improve teacher attitude concerning manipulative materials. After seeing the weakness in the skills of organizing data, processing data, and communicating a generalization, it appears that some pre-teaching of these skills should be done. Scandura (1971) examines the skills for detecting regularities and suggests techniques for teaching pupils to describe ideas (pp. 6-40). If these skills were pre-taught, then a criterion level test could be incorporated into the investigation.

Considerable scope exists for variables which could be correlated with performance on mathematics laboratories. Some
possible characteristics that could be used are transience of students, learning environment, mixed laterality, convergent-divergent thinking, family size and birth order, and type of family housing.

Certainly the structure of the laboratories could be changed, and they could be classified differently. Laboratories could be related to a topic in the curriculum rather than the topics being extensions of the curriculum. The laboratories could all be related to a single topic, or, if time permitted, be related to a number of topics. Classifying the laboratories by the type of generalization involved or by the processing skills involved in detecting the generalization are two other possible methods of categorizing.

The laboratories could be used in a different manner and preferably over a longer period of time. Groups using the laboratories could be compared to groups unexposed to this method of teaching. Groups taught using only a laboratory method could be compared to groups where the laboratories are used only when introducing a topic or compared to groups where laboratories were interspersed with lessons using various other teaching methods. Another possible study using group comparison is one which involves groups of differing ability and groups of similar ability. One further possibility is a study comparing groups where the mathematics laboratory has been used with an entire class, with small groups, and as independent study work.
Clearly more investigation is needed to enable educators to determine the value of using a laboratory method when teaching mathematics. The manner in which laboratories can be used most effectively should be examined more intensively. Thus, it would appear that considerable scope exists for useful research involving the mathematics laboratory.
BIBLIOGRAPHY


Barson, A. The mathematics laboratory for the elementary and middle school. The Arithmetic Teacher, 1971, 18(8), 565-567.


APPENDIX I
A NUMBER PATTERN

Materials: Graph Paper, Ruler, Pencil, Cubes

Part I:

1. A square 1 x 1 drawn on the graph paper shows exactly one square shape.

2. Draw a 2 x 2 square on the graph paper. How many different square shapes can you see in the drawn square? Record this information on the table below.

3. Repeat question 2 for squares that are 3 x 3, 4 x 4, and 5 x 5. Record the information for each size of square on the table below.

<table>
<thead>
<tr>
<th>Size of Square</th>
<th>No. of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1</td>
<td></td>
</tr>
<tr>
<td>2 x 2</td>
<td></td>
</tr>
<tr>
<td>3 x 3</td>
<td></td>
</tr>
<tr>
<td>4 x 4</td>
<td></td>
</tr>
<tr>
<td>5 x 5</td>
<td></td>
</tr>
</tbody>
</table>

Part II:

1. Without using the graph paper, add to the table the information for a 6 x 6 square.

Now check your result using graph paper.

2. Now add to the table the information for a 7 x 7 square and an 8 x 8 square.

3. Examine the column in the table that gives the number of squares.
   (a) What would the 20th terms of this sequence be? 
   (b) Write a mathematical sentence for the pattern of this sequence.
4. Use cubes to help you find the sequence of numbers for the number of cubes in a cube. Write the first five numbers in the sequence on the line below.
PRIMES AND COMPOSITES

Materials: 30 Counters

Part I:

1. A particular number of counters can be arranged into different rectangular arrays. Sixteen counters can be arranged as shown below.

<table>
<thead>
<tr>
<th>1 by 16</th>
<th>2 by 8</th>
<th>4 by 4</th>
<th>8 by 2</th>
<th>16 by 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>xxxxxxxxxxxxxxx</td>
<td>xxxxxxx</td>
<td>xxx</td>
<td>xx</td>
<td>x</td>
</tr>
<tr>
<td>xxxxxxxxxx</td>
<td>xxxrix</td>
<td>xx</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>xxxxxxxx</td>
<td>xxx</td>
<td>xx</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>xx</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>xx</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>xx</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>xx</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Record on a table like the one below information about the rectangular arrays for each whole number from one to thirty.

<table>
<thead>
<tr>
<th>Number</th>
<th>Arrays</th>
<th>Number of Arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 by 1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1 by 4, 2 by 2, 4 by 1</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Look at your table and record below any numbers that have
   (a) only one array
   (b) only two arrays
   (c) more than two arrays
   (d) an "□ by □" array

A number having only two arrays is called a PRIME number.
A number having more than two arrays is called a COMPOSITE number.
A number having an "□ by □" array is called a SQUARE number.

Part II:
1. Find the first prime number after 30. ______
2. Find the first composite number after 30. ______
3. Find the first square number after 30. ______
4. Think of the square number greater than 1.
   Are they prime or composite? ______
5. Is 42 a prime or composite number? ______
   Why ___________________________
6. List all the arrays for 50.
   __________________________________
   __________________________________
   __________________________________
   __________________________________
C. D.'s

Materials: 1/2" Grids, Scissors, Ruler, Pencil, Coloured Pencils

Part I:

1. From one sheet of gridded paper cut out some 1 x 1, 2 x 2, 3 x 3, 4 x 4, and 5 x 5 squares. These squares will be called tiles.

2. On another sheet of gridded paper draw a rectangle 4 x 8 to represent a floor plan for a kitchen nook. Without cutting any of the tiles, can you completely cover the floor plan with your 1 x 1 tiles?

3. Record on the chart below the tile size that could be used for this kitchen nook.

4. Complete the chart below for the other room sizes given. Use your gridded paper and cut tiles to help you. Perhaps you can find the reason why only some tile sizes work.

<table>
<thead>
<tr>
<th>Floor Size</th>
<th>LENGTH of side of each size of square tile used</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 8</td>
<td></td>
</tr>
<tr>
<td>6 x 8</td>
<td></td>
</tr>
<tr>
<td>7 x 12</td>
<td></td>
</tr>
<tr>
<td>10 x 15</td>
<td></td>
</tr>
<tr>
<td>10 x 4</td>
<td></td>
</tr>
<tr>
<td>18 x 12</td>
<td></td>
</tr>
<tr>
<td>11 x 7</td>
<td></td>
</tr>
</tbody>
</table>

Part II:

1. The number pairs given below could be thought of as room sizes. Without using your tiles and gridded paper, give all the different sizes of tiles that could be used for tiling these floors.

   (a) 9, 15  
   (b) 5, 7  

Now check your results using the gridded paper and tiles.
2. The sizes of tiles that can be used are the divisors that are the same for both numbers. Divisors that are the same for two or more numbers are called COMMON DIVISORS.

Find the common divisors for the pairs of numbers below.

(a) 9, 18
(b) 30, 24
(c) 25, 18
(d) 48, 20

3. Find the common divisors for the two groups of numbers below.

(a) 12, 36, 16
(b) 50, 30, 15
L.C.M.


Part I:
1. Cut out several rectangles from the gridded paper that are 4 x 2, 1 x 3, 2 x 1, 3 x 4, 5 x 10, 2 x 3, 9 x 6, 5 x 3, and 4 x 6.

2. Take the cut out rectangles that are 4 x 2. Arrange the rectangles to form a square. Find the smallest sized square that you can. Record the size of the smallest square that you can make on the chart below.

Repeat the above for each set of rectangles that you have made. Record the size of the smallest square that you can find for each set.

<table>
<thead>
<tr>
<th>Size of Rectangle</th>
<th>Size of Smallest Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 2</td>
<td></td>
</tr>
<tr>
<td>2 x 1</td>
<td></td>
</tr>
<tr>
<td>3 x 4</td>
<td></td>
</tr>
<tr>
<td>5 x 10</td>
<td></td>
</tr>
<tr>
<td>2 x 3</td>
<td></td>
</tr>
<tr>
<td>9 x 6</td>
<td></td>
</tr>
<tr>
<td>5 x 3</td>
<td></td>
</tr>
<tr>
<td>4 x 6</td>
<td></td>
</tr>
</tbody>
</table>

Part II:
1. Predict the smallest size of square that can be made using rectangles 2 x 5.

Check your answer using gridded paper.

The smallest square that can be made from a set of rectangles the same size is the square whose edge length is the smallest number that both dimensions of the rectangle will divide.

Example: For a rectangle 4 x 6, the smallest sized square is 12. The smallest square is 12 because 12 is the smallest number that both 4 and 6 will divide. 12 is called the **LOWEST COMMON MULTIPLE** for 4 and 6.
2. Find the lowest common multiple (smallest sized square) for the pairs of numbers below.

(a) 1, 3
(b) 4, 8
(c) 5, 6
(d) 8, 12
(e) 6, 3
(f) 7, 2

3. There is a lowest common multiple for a group of three or more numbers also. Find the lowest common multiple for the groups of numbers below.

(a) 3, 2, 4
(b) 5, 20, 8
(c) 12, 8, 3
TRIANGLES IN POLYGONS

Materials: Set of Plane Figures, Ruler, Pencil

Part I:

1. Draw lines to cut the polygon into only triangular pieces. Find the least number of triangular pieces that a shape can be cut into. Record this information on the chart below.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part II:

1. What is the least number of \( \triangle \) for an 11-sided polygon? _____

Test your answer by drawing an 11-sided polygon on the back and show the lines.

2. Complete your table.

3. Predict the least number of triangles for a 48-sided polygon. _____

4. Can you find a pattern for the least number of triangles for a polygon? If you can, write a number sentence about it. _____

5. Would your sentence hold for the two polygons below? _____

Draw in the necessary line segments on the polygons below to show the least number of triangular pieces.
MAKING POLYGONS

Materials: Patterns for two right isosceles triangles and two squares, pencil, straight edge, scissors, and large pieces of newsprint.

Part I:
1. Cut out the shapes on the back page to use as patterns.
2. You are going to be asked to make polygons by tracing around two or more of your patterns. A rule that must always be followed is that when two shapes are fitted together, the edges that meet must be the same length.

This would be all right. This would not be right.

3. Using just two triangles, we can fit them together to make another triangle ...........

Try to make a triangle by fitting together just (a) 3 triangles (b) 4 triangles (c) 5 triangles. Show your results on the large sheet of newsprint.

4. Try to make a square using just (a) 2 triangles (b) 3 triangles (c) 4 triangles (d) 5 triangles.

5. Try to make a rectangle using just (a) 2 triangles (b) 3 triangles (c) 4 triangles (d) 5 triangles.

6. Try to make a square using just (a) 2 squares (b) 3 squares (c) 4 squares (d) 5 squares (e) 6 squares.

7. Try to make a rectangle using just (a) 2 squares (b) 3 squares (c) 4 squares (d) 5 squares (e) 6 squares.

8. Make as many different shapes as you can using just (a) 1 triangle and 1 square (b) 2 triangles and 1 square (c) 2 triangles and 3 squares.

Part II:
1. Show two different ways of fitting squares and triangles together to make the shape shown at the right ...

(Use your straight edge and pencil to draw lines on each to show how you would do it.)
2. By making one straight cut on each of the figures on the left, you could fit the resulting pieces together to make the figures on the right. Use your straight edge and pencil to draw a line on the left figure to show where you would cut it if you had to.

Where would you cut this to make this

(a)  

(b)  

3. Examine the figure below closely.

Using only the line segments given, count and record the maximum number of each named polygon that you can see in the diagram.

(a) triangle  
(b) square  
(c) rectangle  

4. Use your straight edge and pencil to draw lines on each diagram to show how you would cut it if you had it, to make what is asked for. You must use all of the shape given in the diagram.

(a) How would you cut this rectangle to make 3 triangles?

(b) How would you cut this quadrilateral to make a rectangle and 2 triangles?

(c) How would you cut this rectangle to make a square and a rectangle?
PIC'S THEOREM

Part I:

1. Construct a polygon with 4 pegs on its boundary and no pegs inside.
   The area of this polygon is ________________.
   Construct other polygons with 4 pegs on their boundaries and no pegs inside. What is the area of each polygon? ________________.

2. Construct three different polygons each with 5 pegs on their boundaries and no pegs inside. What is the area of each polygon? ________________.

3. Fill in the table below, where $\Delta$ means number of pegs on the boundary and $\Box$ means area (in square units).

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Box$</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. If two students each make a polygon that has 8 pegs on its boundary and no pegs inside, would the area of the two polygons be the same? ________________.

Part II:

1. Predict the area for a polygon with 10 pegs on its boundary and no pegs inside. ________________ Check your prediction using the geoboard.

2. If you construct a polygon with fifteen pegs on its boundary and no pegs inside, what will be the area of this polygon? ________________.

3. Add the information about polygons with 10 to 15 pegs on their boundaries to your table above.

4. Examine the table carefully. Which number sentence below gives the relationship between the $\Delta$ (number of pegs on the boundary) and $\Box$ (area)? Circle the number sentence.

   (a) $\Delta = 2 + \Box$   (b) $\frac{1}{2} \Delta - 2 = \Box$   (c) $\frac{1}{2} \Delta - 1 = \Box$

5. Now construct five polygons on your geoboard that have exactly 1 peg inside its boundary. Count the pegs on the boundary of each and find the area of each polygon.
6. Record the information about the polygons in question 5 on the table below. Remember that \( \triangle \) means number of pegs on the boundary and \( \Box \) means the area of the polygon.

\[
\begin{array}{cccccc}
\triangle & & & & & \\
\Box & & & & & \\
\end{array}
\]

7. Write a number sentence using \( \triangle \) and \( \Box \) to show the relationship between the number of pegs on the boundary and the area when the polygons have 1 peg inside. ________
DIAGONALS OF POLYGONS

Materials: Cardboard, Pins, String, Pencil

Part I:
1. Make a quadrilateral on the piece of cardboard by marking the vertices with pins. Use the pieces of string to mark the diagonals.

Find the (a) number of sides for this polygon (b) the number of diagonals from one vertex (c) the total number of diagonals.

Record this information about the quadrilateral on the chart below.

2. Repeat part #1 for the triangle, pentagon (5 sides), hexagon (6 sides), and heptagon (7 sides).

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>No. of Diagonals from 1 vertex</th>
<th>Total number of Diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heptagon</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part II:
1. Predict the total number of diagonals for an octagon (8 sides). Now check using your cardboard, pins and string. Add this information to your chart.

2. Add to your chart the polygons nonagon (9 sides) and decagon (10 sides). Complete the information about these polygons.

3. How many diagonals would a 20-gon have? 

4. Write a mathematical sentence for finding the number of diagonals for a 20-gon.
APPENDIX II
Table 1 - Sex vs Number Theory Labs

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>249.0</td>
<td>249.0</td>
</tr>
<tr>
<td>227.0</td>
<td>227.0</td>
</tr>
<tr>
<td>205.0</td>
<td>205.0</td>
</tr>
<tr>
<td>183.0</td>
<td>183.0</td>
</tr>
<tr>
<td>161.0</td>
<td>161.0</td>
</tr>
<tr>
<td>139.0</td>
<td>139.0</td>
</tr>
</tbody>
</table>

Table 2 - Sex vs Geometry Labs

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>258.0</td>
<td>258.0</td>
</tr>
<tr>
<td>230.0</td>
<td>230.0</td>
</tr>
<tr>
<td>202.0</td>
<td>202.0</td>
</tr>
<tr>
<td>174.0</td>
<td>174.0</td>
</tr>
<tr>
<td>146.0</td>
<td>146.0</td>
</tr>
<tr>
<td>118.0</td>
<td>118.0</td>
</tr>
</tbody>
</table>
Table 3 - Sex vs All Labs

Table 4 - Verbal IQ vs Number Theory
Table 5 - Verbal IQ vs Geometry

Table 6 - Verbal IQ vs All Labs
Table 7 - Nonverbal IQ vs Number Theory

Table 8 - Nonverbal IQ vs Geometry
Table 9 - Nonverbal IQ vs All Labs

Table 10 - Math Performance (Gr.5) vs Number Theory
Table 11 - Math Performance (Gr. 5) vs Geometry

Table 12 - Math Performance (Gr. 5) vs All Labs
Table 13 - Math Performance (Gr.6) vs Number Theory

Table 14 - Math Performance (Gr.6) vs Geometry
Table 15 - Math Performance (Gr.6) vs All Labs

Table 16 - GEFT vs Number Theory
Table 17 - GEFT vs Geometry

Table 18 - GEFT vs All Labs
QUESTIONNAIRE
ON
PROBLEM SOLVING

Name ________________________ Teacher ________________________

For each of the following statements put an x in the space on the line which shows the degree to which you agree or disagree with the statement.

1. I usually guess the answer to a problem before working it out in detail.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

2. Talking about ideas in mathematics is much more enjoyable to me than working on arithmetic problems.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

3. I would rather ask for help on a difficult problem than work on it alone.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

4. I would rather respond slowly to a question my teacher has asked me in mathematics than to gamble on my answer.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

5. I have always thoroughly enjoyed working on mathematical games and puzzles.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>
6. I can tell without checking whether an answer I have gotten is correct.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

7. I have found that a period of silence before answering a question does not help me.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

8. It is more important to be able to talk about ideas in mathematics than to be able to work a variety of mathematical problems.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

9. I would rather give a wrong answer than no answer at all.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

10. The most important reason for learning mathematics is its usefulness.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

11. The best way to learn how to solve mathematics problems is to solve lots of them.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

12. My first choice of a subject to study in high school would be mathematics.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>
13. Frequently the answer to a problem occurs to me after I have gone on to do something else.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

14. I have a hard time concentrating on my mathematics homework.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

15. I like to discover several different ways to do the same problem.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

16. I prefer doing to thinking.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

17. I only raise my hand when I know I have the correct answer.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

18. I would rather ask for help on a difficult mathematics problem than have someone tell me the answer.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

19. I don't like to discuss the problems on a test after I've finished it.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>
20. It's better to be slower and right all of the time than to be faster and right part of the time.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>

21. I always guess on a multiple-choice test when I don't know the right answer.

<table>
<thead>
<tr>
<th>agree strongly</th>
<th>agree moderately</th>
<th>agree a little</th>
<th>disagree a little</th>
<th>disagree moderately</th>
<th>disagree strongly</th>
</tr>
</thead>
</table>