CAPACITOR COMPENSATION EFFECTS ON THE
PERFORMANCE OF SYNCHRONOUS MACHINES CONNECTED
TO LONG TRANSMISSION LINES

by

AMADU MUSTAPHA CONTEH

A THESIS SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in the Department of
Electrical Engineering

We accept this thesis as conforming to the
required standard.

Research Supervisor .........................
Members of the Committee ..................
 ........................................
Head of the Department ...................

Members of the Department
of Electrical Engineering

THE UNIVERSITY OF BRITISH COLUMBIA
August, 1966
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Electrical Engineering
The University of British Columbia
Vancouver 8, Canada
Date 23rd August 1966
ABSTRACT

The increase in length, size and voltages of transmission lines and the development of high voltage capacitors have re-vitalized interest in recent years in the study of the performance characteristics of series or shunt capacitor compensated power systems. Most of the investigations are performed, however, either on the synchronous machine or on the transmission line. Major contributions were made by Butler, Bodine, Dineley, Peterson and others, as outlined in chapter 1.

The thesis is to develop a general analytical method for the study of the compensation problem including the effects of both the salient pole synchronous machine and the transmission line with various power factor loads connected at the receiving end. The equations for the series capacitor compensated power system with balanced load are derived in chapter 2. The transmission line is represented by a three phase nominal-T network. The transmission line equations are transformed into Park's cross field reference frame which is employed for the formulation of equations for the synchronous machine. The equations obtained can be applied to both transient and steady state analyses. Chapter 3 also studies the series capacitor compensated system except that the load is unbalanced.

The equations for the study of the shunt capacitor compensated system are developed in chapter 4. Both balanced
and unbalanced loads are included.

Two numerical examples of practical systems are presented to illustrate the applications of the method. The synchronous machine internal voltage against load current characteristics and those of the power angle between the internal voltage and the receiving end voltage versus load current, balanced and unbalanced, at various power factor, and with different degrees of series or shunt compensation of the system are investigated.

The numerical results from digital computation confirm that the series capacitor compensation is more effective than the shunt capacitor compensation and that both the series and shunt capacitor compensations increase the transmission capacity.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td></td>
<td>vi</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td></td>
<td>viii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1.1 General</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1.2 Review of Literature</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>1.3 Outline of the Project</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>2. DERIVATION OF EQUATIONS FOR SERIES CAPACITOR COMPENSATED POWER SYSTEMS WITH BALANCED LOAD</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>2.1 General Equations for Series and Shunt Capacitor Compensated Power System</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>2.2 Series Capacitor Compensated Power System Equations and Co-ordinate Transformations</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>2.3 Steady State Analysis of Series Capacitor Compensated System</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>3. SERIES CAPACITOR COMPENSATED POWER SYSTEMS WITH UNBALANCED LOAD</td>
<td></td>
<td>21</td>
</tr>
<tr>
<td>4. DERIVATION OF EQUATIONS FOR SHUNT CAPACITOR COMPENSATED POWER SYSTEMS WITH BALANCED AND UNBALANCED LOADS</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>4.1 General System Equations</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>4.2 Steady State Analysis of Shunt Capacitor Compensated System</td>
<td></td>
<td>33</td>
</tr>
<tr>
<td>4.3 Shunt Capacitor Compensated Power Systems with Unbalanced Load</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>5. NUMERICAL EXAMPLES OF PRACTICAL SYSTEMS</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>5.1 The Series Capacitor Compensated 600 Mile System</td>
<td></td>
<td>41</td>
</tr>
</tbody>
</table>
5.2 The Series Capacitor compensated 250 Mile System .......................... 42
5.3 The Shunt Capacitor compensated 600 Mile System .......................... 43
5.4 The Shunt Capacitor compensated 250 Mile System .......................... 43
5.5 Summary of Results ................................................. 44
6. CONCLUSIONS AND REMARKS ............................................. 63
APPENDIX I List of Symbols ............................................. 65
APPENDIX II Transformation of Equation 2-9 .......................... 69
APPENDIX III Derivation of the Impedance Matrix $Z_0(\Theta)$ and Evaluation of $E_0$, $E_d$ and $E_q$ ............................................. 78
APPENDIX IV Illustrative Numerical Examples .......................... 82
 a) 230KV, 250 Mile Transmission System .......................... 82
 b) 500KV, 600 Mile Transmission System .......................... 86
REFERENCES ................................................................. 87
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Block Diagram of System under Study</td>
<td>6</td>
</tr>
<tr>
<td>1.2</td>
<td>Simplified Circuit Representation of System Under Study</td>
<td>7</td>
</tr>
<tr>
<td>2.1</td>
<td>Schematic for a Series Capacitor Compensated Power System</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Phasor Diagram for the Receiving End Current and Voltage</td>
<td>19</td>
</tr>
<tr>
<td>4.1</td>
<td>Star Connected Shunt Compensation and Lumped Line Capacitors</td>
<td>27</td>
</tr>
<tr>
<td>5.1</td>
<td>Excitation Characteristics of the Series Compensated 600 Mile Power System, 0.85 PF Lagging Load</td>
<td>45</td>
</tr>
<tr>
<td>5.2</td>
<td>Excitation Characteristics of the Series Compensated 600 Mile Power System, Unity PF Load</td>
<td>46</td>
</tr>
<tr>
<td>5.3</td>
<td>Excitation Characteristics of the Series Compensated 600 Mile Power System, Zero PF Leading Load</td>
<td>47</td>
</tr>
<tr>
<td>5.4</td>
<td>Torque Angle Characteristics of the Series Compensated 600 Mile Power System, 0.85 PF Lagging Load</td>
<td>48</td>
</tr>
<tr>
<td>5.5</td>
<td>Torque Angle Characteristics of the Series Compensated 600 Mile Power System, Unity PF Load</td>
<td>49</td>
</tr>
<tr>
<td>5.6</td>
<td>Torque Angle Characteristics of the Series Compensated 600 Mile Power System, Zero PF Leading</td>
<td>50</td>
</tr>
<tr>
<td>5.7</td>
<td>Excitation Characteristics of the 250 Mile Series Compensated Power System, 0.85 PF Lagging Load</td>
<td>51</td>
</tr>
<tr>
<td>5.8</td>
<td>Excitation Characteristics of the 250 Mile Series Compensated Power System, Unity PF Load</td>
<td>52</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.9</td>
<td>Excitation Characteristics of the 250 Mile Series Compensated Power System, Zero PF Load</td>
<td>53</td>
</tr>
<tr>
<td>5.10</td>
<td>Torque Angle Characteristics of the Series Compensated 250 Mile Power System, 0.85 PF Lagging Load</td>
<td>54</td>
</tr>
<tr>
<td>5.11</td>
<td>Torque Angle Characteristics of the Series Compensated 250 Mile Power System, Unity PF Load</td>
<td>55</td>
</tr>
<tr>
<td>5.12</td>
<td>Torque Angle Characteristics of the Series Compensated 250 Mile Power System, Zero PF Leading Load</td>
<td>56</td>
</tr>
<tr>
<td>5.13</td>
<td>Excitation Characteristics of the Shunt Compensated 600 Mile Power System, 0.85 PF Lagging Load</td>
<td>57</td>
</tr>
<tr>
<td>5.14</td>
<td>Excitation Characteristics of the Shunt Compensated 600 Mile Power System, Unity PF Load</td>
<td>58</td>
</tr>
<tr>
<td>5.15</td>
<td>Torque Angle Characteristics of the Shunt Compensated 600 Mile Power System, 0.85 Lagging and Unity PF Loads</td>
<td>59</td>
</tr>
<tr>
<td>5.16</td>
<td>Excitation Characteristics of the Shunt Compensated 250 Mile Power System, 0.85 PF Lagging Load</td>
<td>60</td>
</tr>
<tr>
<td>5.17</td>
<td>Excitation Characteristics of the Shunt Compensated 250 Mile Power System, Unity PF Load</td>
<td>61</td>
</tr>
<tr>
<td>5.18</td>
<td>Torque Angle Characteristics of the Shunt Compensated 250 Mile Power System, 0.85 Lagging and Unity PF Loads</td>
<td>62</td>
</tr>
<tr>
<td>A3-1</td>
<td>A Four Winding Salient Pole Synchronous Machine</td>
<td>78</td>
</tr>
<tr>
<td>A4-1</td>
<td>One Line Diagram of the 250 Mile Power System</td>
<td>85</td>
</tr>
<tr>
<td>A4-2</td>
<td>One Line Diagram of the 600 Mile Power System</td>
<td>85</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The author gratefully acknowledges the assistance and guidance of the supervising professor Dr. Yao Nan Yu, throughout the course of the research. The foundation of the thesis was established through formal lectures presented by him. This help was very much appreciated. The author would like to express his gratitude to the members of the Department of Electrical Engineering and especially Dr. R. W. Donaldson for reading the manuscript.

The author wishes to thank the B. C. Hydro and Power Authority for providing the data employed in the illustrative examples of the thesis. His thanks are equally due to Miss H.M. Klassen for typing the thesis and to Mr. A. MacKenzie for drawing the graphs.

The author is indebted to the Canadian Universities Foundation for the award of a Commonwealth Fellowship (1964 to 1966) and to the National Research Council of Canada, whose financial support made the project possible.
I. INTRODUCTION

1.1 General

A modern electrical power system consists of a great number of different components combined together to form an entity. During system operation all the elements interact with each other, thus acting as a single unit to produce, transmit, distribute and consume electrical energy. An essential difference exists between the functioning of the system as a whole and that of the individual components. It is natural that the complete power system must be analysed as a single unit instead of dealing with its components separately.

The increase in size, length and complexity of modern power systems demands re-examination of some of the existing results of analysis. One of the typical problems, resulting from the design of large power systems is the capacitive effects on the steady state and transient performance of alternators connected to long distance EHV transmission lines. The usefulness of series and shunt capacitor compensation has been demonstrated in Sweden, Canada, and the United States. Capacitor installations are particularly effective on transmission lines where problems of transmission capacity, voltage regulation and stability are dominant. Large economic benefits, both in initial capital cost and operational expenditure, can be achieved by the proper use of series and shunt capacitors.
The exact analysis of a complete system entails considerable amount of difficulties. Directly solving the differential equations of a large power system comprising synchronous generators and compensated transmission lines is not quite feasible at the present time, particularly if an exact representation of the complete system is desired. Alternative methods of analysis and system representations employed by various authors are described briefly in the following section.

1.2 Review of Literature

Star and Evans investigated the use of series capacitors to increase the permissible loadings of long high voltage ac transmission lines by setting up and studying a laboratory model. The handicap of this method is that system parameters cannot be varied with ease.

Butler, Paul, and Schroeder studied the steady state and transient stability of transmission systems with series capacitors with the aid of a network analyser. Critical switching times were obtained with the application of equal-area criterion.

Bodine, Concordia, and Kron studied the self-excited oscillations of capacitor compensated long distance transmission systems. They developed an equivalent circuit method and used an ac network analyser. The equivalent circuit method has the advantage that the system parameters can be easily varied.

Dineley and Glover, applying Park's equations and ignoring the effects of saturation and armature resistance, studied the terminal voltage rise of a synchronous generator.
switched on to a capacitive load directly connected to the machine terminals.

Chen and Duesterhoeft analyzed transient voltages and currents of an alternator with a balanced capacitive load suddenly applied to the terminals. Clarke's\textsuperscript{8,9} \( \alpha, \beta, \phi \) co-ordinates and Laplace transform techniques were applied. Armature resistance and saturation effects were neglected and closed form solutions were obtained.

Mikhail and Keener\textsuperscript{10} derived expressions for transient analysis of sudden application of a capacitive load to a generator without damper windings. Park's\textsuperscript{11,12} equations modified by Crary were applied. While the complementary solution gave the free transient, the particular solution was obtained by using Rudenberg's\textsuperscript{13} method to account for the effect of saturation.

Peterson\textsuperscript{14,15} et al using an analogue computer investigated the electromechanical transient phenomena of a parallel ac–dc power system. Park's transformation was applied to the separate components comprising an equivalent synchronous machine, a series capacitor compensated ac transmission line, a dc transmission line, converters, rectifiers and a load.

In summary, most of the authors studied the capacitor problem either by considering the transmission line without actually considering the generators, or by lumping the capacitors directly to the terminals of the synchronous generator without considering the transmission line.

1.3 Outline of the Project

The aim of this project is to investigate the effect of
series and shunt capacitor compensation on the performance of
a salient pole synchronous machine connected to long trans-
mission lines. The project consists mainly of two parts.
The first part gives the derivation of the fundamental equations
of the system for both transient and steady state analyses.
The second part carries out the steady state analysis in detail.
Two different load conditions are considered, the balanced
and the unbalanced cases. The system under study is shown in
a block diagram in Fig. 1.1. The simplified circuit repre-
sentation of the composite sections of the system is shown
in Fig. 1.2. There are four sections, the generator, the
sending end transmission line, the receiving end transmission
line, and the load. The T-equivalent circuit configuration is
used for the representation of the transmission line.

The general procedures are as follows:—

a) Equations for each composite section of the system
are first written in a-b-c co-ordinates.

b) Park's transformations are applied to change the
current and voltage equations in the phase co-ordinates into
those in the direct and quadrature axis co-ordinates.

c) Equations of all the sections are combined in accord-
ance with the system connections and analyses are carried out
in Park's d-q co-ordinates.

d) The final solutions are transformed back into a-b-c
phase co-ordinates.

Two numerical examples of practical systems are given to
illustrate the application of the results.
The equations are formulated in matrix form and a digital computer is employed for the computation.

Throughout the thesis the per unit system based on the M.K.S. units is used for the computations unless otherwise specified.
SYNCHRONOUS GENERATOR
STEP UP TRANSFORMER
TRANSMISSION LINE
SERIES OR SHUNT CAPACITOR UNITS
TRANSMISSION LINE
STEP DOWN TRANSFORMER
LOAD

FIG. 1-1 BLOCK DIAGRAM OF SYSTEM UNDER STUDY
FIG. 1-2 SIMPLIFIED CIRCUIT REPRESENTATION OF THE SYSTEM UNDER STUDY
The equations for a series compensated power system are developed in this chapter. From the standpoint of operation and protective relaying it is desirable to insert a large number of capacitors of small units, although not economically feasible.\textsuperscript{16,19}

Johnson et al\textsuperscript{17} and Crary and Saline\textsuperscript{18} demonstrated in their papers that the best location for the series capacitor is at the middle of the line and not at the receiving end or at the sending end. Jancke and Akerstrom\textsuperscript{19} observed in a Swedish Power system that the effect of the series capacitor would be slightly less when placed at the receiving end of the line. If power is to be transmitted in one direction only it is also practical to insert the capacitor at the receiving end. For this reason the series capacitor for compensation in this study is assumed to be located at the receiving end of the line. A schematic for the series capacitor compensated power system is shown in Fig. 2-1.
FIG. 2.1 SCHEMATIC FOR A SERIES CAPACITOR COMPENSATED POWER SYSTEM
2.1 General Equations for Series and Shunt Capacitor Compensated Power System

According to Fig. 1.2, the voltage equations of the generator, the sending end section, the receiving end section and the current equations at the T-junction of the transmission line may be written as follows:

\[
\begin{align*}
    e &= Z_0(p) i_1 + v_1 \\
    v_1 &= Z_{SE}(p) i_1 + v_2 \\
    v_2 &= Z_{RE}(p) i_3 + v_3 \\
    i_1 &= Y_I(p) v_2 + i_3
\end{align*}
\]

The equations are written in a general form convenient for both series and shunt compensation studies. The series compensation capacitance can be included in the operational impedance matrix \( Z_{RE}(p) \) and the shunt operational admittance matrix \( Y_I(p) \) of the transmission line itself can be expanded to include the shunt compensation capacitance. The synchronous machine operational impedance matrix \( Z_0(p) \) is given in Appendix III. For notations see Appendix I.

The internal induced voltage \( e \) of the salient pole synchronous machine in terms of the voltage and current at the receiving end, \( v_3 \) and \( i_3 \), is to be found next. Differentiations of, and substitution into the first three equations in (2-1) leads to

\[
\begin{align*}
    p e &= p Z_0(p) i_1 + pv_1 \\
    pv_1 &= (R_{SE} + L_{SE} p^2) i_1 + pv_2
\end{align*}
\]
\[
\begin{align*}
\mathbf{pv}_2 &= \left\{ R_{\text{REP}} + L_{\text{REP}}^2 + \left( \frac{1}{C_s} \right) \right\} \mathbf{i}_3 + \mathbf{pv}_3 \\
\mathbf{i}_1 &= C_l \mathbf{pv}_2 + \mathbf{i}_3
\end{align*}
\]

where \( Y_l(p) \) is replaced by \( C_l p \). The transmission line tower leakage conductance has been neglected although it can be easily included. Next the substitution of \( \mathbf{pv}_2 \) and \( \mathbf{i}_1 \) into \( \mathbf{pv}_1 \) in (2-2) results in the equation

\[
\begin{align*}
\mathbf{pv}_1 &= \left\{ \left( R_{\text{SEP}} + L_{\text{SEP}}^2 \right) C_l + \mathbf{u} \right\} \left\{ \left[ R_{\text{REP}} + L_{\text{REP}}^2 + \left( \frac{1}{C_s} \right) \right] \mathbf{i}_3 \\
&\quad + \mathbf{pv}_3 \right\} + \left\{ R_{\text{SEP}} + L_{\text{SEP}}^2 \right\} \mathbf{i}_3
\end{align*}
\]

Further substitutions of these results into \( \mathbf{pe} \) in (2-2) yields

\[
\begin{align*}
\mathbf{pe} &= \left\{ \mathbf{pZ}_o(p) + \left( R_{\text{SEP}} + L_{\text{SEP}}^2 \right) + \mathbf{pZ}_o(p) \right\} \left\{ C_l \left[ R_{\text{REP}} + L_{\text{REP}}^2 + \left( \frac{1}{C_s} \right) \right] + \mathbf{u} \right\} \left[ R_{\text{REP}} + L_{\text{REP}}^2 + \left( \frac{1}{C_s} \right) \right] \mathbf{i}_3 + \left\{ \mathbf{pZ}_o(p) C_l + \left[ R_{\text{SEP}} + L_{\text{SEP}}^2 \right] \right\} \mathbf{pv}_3
\end{align*}
\]

### 2.2 Series Capacitor Compensated Power System Equations and Co-ordinate Transformations

Since the effect of the line shunt capacitance is relatively small compared with that of the series capacitance, the former will be neglected. Equation (2-4) becomes,

\[
\begin{align*}
\mathbf{pe} &= \left\{ \left( R_o + R_{\text{SE}} + R_{\text{RE}} \right) p + \left( L_o + L_{\text{SE}} + L_{\text{RE}} \right) p^2 + p^2 L_2(\Theta) \\
&\quad + \left( \frac{1}{C s} \right) \right\} \mathbf{i}_3 + \mathbf{pv}_3
\end{align*}
\]

where the operational impedance matrices are expanded into \( R, L \), and \( C \) matrices.

Let the voltage and current vectors in Park's d-q co-ordinates be denoted by \( \mathbf{e}, \mathbf{v} \) and \( \mathbf{i} \) with primes, and that in a-b-c phase co-ordinates without, and let the transformations be written as
\[ e = A_{a,d} e' \]
\[ v_3 = A_{a,d} v_3' \]  \hspace{1cm} (2-6)
\[ i_3 = A_{a,d} i_3' \]

where

\[ A_{a,d} = \begin{vmatrix} 
\sqrt{2/3} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{vmatrix} \]

\[ C = \cos \Theta, \quad C' = \cos(\Theta - 120), \quad C'' = \cos(\Theta + 120). \]  \hspace{1cm} (2-7)
\[ S = \sin \Theta, \quad S' = \sin(\Theta - 120), \quad S'' = \sin(\Theta + 120). \]

Equation (2-7) is a transformation matrix based on Park's original work and modified to the unitary transformation by Yu. \(^{20,21}\) Substitution of (2-6) into (2-5) and premultiplication by \( A_{d,a}^* \), the transpose of \( A_{a,d} \) which in this case equals the inverse, yields

\[ A_{d,a}^* p_{a,d} e' = A_{d,a} \left( R_0 + R_{SE} + R_{RE} \right) p_{a,d} i_3' \]
\[ + A_{d,a} \left( L_0 + L_{SE} + L_{RE} \right) p_{a,d}^2 i_3' \]
\[ + A_{d,a} \left( \theta A_{a,d} i_3 \right)' + A_{d,a} \left( 1/C_3 \right) A_{a,d} i_3' \]
\[ + A_{d,a} p_{a,d} v_3' \]  \hspace{1cm} (2-9)

Carrying through the transformation results in equation (2-10). The details of the transformation are given in Appendix II.
<table>
<thead>
<tr>
<th>p</th>
<th>0</th>
<th>0</th>
<th>( e_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>p</td>
<td>( \dot{\theta} )</td>
<td>( e_d )</td>
</tr>
<tr>
<td>0</td>
<td>( \dot{\theta} )</td>
<td>p</td>
<td>( e_q )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{1}{C_s} + R_T p + L_A p^2 &= 0 \\
\frac{1}{C_s} + R_T p - L_B \dot{\theta}^2 + L_B p^2 &= 0 \\
-\frac{3}{2} L_2 \dot{\theta}^2 + 3 L_2 \ddot{\theta} p + 3/2 L_2 p^2 &
\end{align*}
\]

\[
\begin{align*}
R_T \dot{\theta} + L_B \ddot{\theta} + 2 L_B \dot{\theta} p + 3/2 L_2 \dot{\theta}^2 &
\end{align*}
\]
\[
\begin{array}{ccc}
V_{30} & V_{3d} & V_{3q} \\
0 & -\theta & p \\
0 & p & \theta \\
p & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
\delta_{30} & \delta_{3d} & \delta_{3q} \\
0 & -R_\theta - I_{B,\theta} - 2I_{B,\theta} & \frac{1}{c} + \frac{1}{c} - I_{\eta,\theta} + I_{\eta,\theta}^2 \\
\frac{1}{c} + \frac{1}{c} - I_{\eta,\theta} + I_{\eta,\theta}^2 & \frac{1}{c} + \frac{1}{c} - I_{\eta,\theta} + I_{\eta,\theta}^2 & \frac{1}{c} + \frac{1}{c} - I_{\eta,\theta} + I_{\eta,\theta}^2 \\
\end{array}
\]
2.3 Steady State Analysis of Series Capacitor Compensated System

Equation (2-10) permits transient as well as steady state analysis. For steady state solutions the differential operator \( p \) in a machine's equation may be replaced by zero instead of \( jw \) because the exciting functions (i.e. applied voltages and currents) for normal operations are constant when referred to the \( d-q \) axes. Since there is no change in the flux linkages in the \( d \) and \( q \) axes of a synchronous machine in the steady state, no transformer voltages are induced. On the other hand as the conductors move relative to the field in the \( d \) and \( q \) co-ordinates, speed voltages are generated. The preceding statements also apply to the transmission line because their equations are also expressed in the same \( d-q \) co-ordinates.

For the steady state analysis the angular velocity of the synchronous machine \( \dot{\phi} \) is constant. In the symmetrical case the zero axis current vanishes. The matrix equation (2-10) reduces to equation (2-11).

Dividing equation (2-11) by \( \dot{\phi} \) yields

\[
\begin{bmatrix}
-e_q \\
e_d
\end{bmatrix}
= \begin{bmatrix}
B & -R_T \\
R_T & D
\end{bmatrix}
\begin{bmatrix}
i_{3d} \\
i_{3q}
\end{bmatrix}
+ \begin{bmatrix}
-v_{3q} \\
v_{3d}
\end{bmatrix}
\]

(2-12)

where

\[
B = \frac{1}{w_C} - wL_e - \frac{3}{2}wL_2
\]

\[
= \frac{1}{w_C} - 2w(L_{aa} - L_{ab}) - \frac{3}{2}wL_d
\]
\[ (\theta - \frac{3}{2}L\Phi^2) + \frac{1}{C}\Phi - I_3 \Phi = \frac{1}{3} \]

\[ \frac{1}{C}\Phi = -3 \]

\[ \frac{1}{C} - I_3 \Phi = -1 \]

\[ \frac{1}{C} - I_3 \Phi = 0 \]

\[ \frac{1}{C} - I_3 \Phi = 0 \]

\[ \frac{1}{C} - I_3 \Phi = 0 \]

\[ \frac{1}{C} - I_3 \Phi = 0 \]
\[ D = \frac{1}{wC_g} - wL_B + \frac{3}{2}wL_2 \]
\[ = \frac{1}{wC_g} - 2w(L_{aa} - L_{ab}) - \frac{3}{2}wL_q \]
\[ L_B = \frac{3}{2}L_{os} + 2(L_{aa} - L_{ab}) \]
\[ L_{os} = \frac{L_a + L_q}{2}, \quad L_{2S} = \frac{L_d - L_q}{2} \]

(2-13)

Since power engineers are rather interested to know the relation between the generator excitation and the receiving end voltages and currents, all of which can be easily measured, the d-q components are transformed to phase co-ordinates as follows:

For the symmetrical case

\[ K_d = \sqrt{2/3} \quad \begin{array}{ccc}
C & C' & C'' \\
S & -S & -S''
\end{array} \]

(2-14)

\[ K_q = \sqrt{2/3} \quad \begin{array}{ccc}
K_a & K_b & K_c
\end{array} \]

\[ K = \text{magnitude of the steady state positive sequence current or voltage.} \]

and

\[ \phi = \alpha \text{ for the receiving end current and } \beta \text{ for the receiving end voltage.} \]

Application of equations (2-14) and (2-15) to the first term in the right hand side of equation (2-12) results in
Application of (2-15) to the second term in the right hand side of (2-12) yields

\[
\begin{align*}
\sqrt{3/2} V_{m3} & = \sqrt{3/2} V_{m3} \cos \beta \\
\sqrt{3/2} V_{m3} & = \sqrt{3/2} V_{m3} \sin \beta
\end{align*}
\]  

Therefore the voltages in (2-12) become

\[
\begin{align*}
-E & = (B \sin \alpha + R_T \cos \alpha) \sqrt{3/2} I_{m3} + \sqrt{3/2} V_{m3} \cos \beta \\
E_d & = (R_T \sin \alpha - D \cos \alpha) \sqrt{3/2} I_{m3} + \sqrt{3/2} V_{m3} \sin \beta
\end{align*}
\]
Fig. 2.2 Phasor Diagram for the Receiving End Current and Voltage.

Finally, the internal voltage of the machine $E_q$, which is a function of excitation, can be expressed in terms of receiving end voltage, receiving end current and power factor as follows.

Let the power factor angle at the receiving end as depicted in Fig. 2.2 be $\phi_r$ and let $\alpha = \beta - \phi_r$. According to equation (A3-7), Appendix III, one may set $E_d = 0$ in equation (2-18).

\[
\sqrt{3/2} \left[ R_T \sin(\beta - \phi_r) - D \cos(\beta - \phi_r) \right] I_{m3} + \sqrt{3/2} V_{m3} \sin \beta = 0
\]

or

\[
(R_T I_{m3} \cos \phi_r - D I_{m3} \sin \phi_r + V_{m3}) \sin \beta - (R_T I_{m3} \sin \phi_r + D I_{m3} \cos \phi_r) \cos \beta = 0
\]

hence

\[
\tan \beta = \frac{x}{y}
\]

\[
\sin \beta = \frac{x}{\sqrt{x^2 + y^2}}
\]

\[
\cos \beta = \frac{y}{\sqrt{x^2 + y^2}}
\]

(2-19)

where $x = I_{m3} R_T \sin \phi_r + I_{m3} D \cos \phi_r$

and $y = I_{m3} R_T \cos \phi_r - D I_{m3} \sin \phi_r + V_{m3}$

(2-20)
Substitution of (2-19) into $E_q$ of (2-18) results in

$$-E_q = \sqrt{3/2} [B \sin(\beta - \phi_r) + R_T \cos(\beta - \phi_r)] I_m^3 + \sqrt{3/2} V_m^3 \cos \beta$$

$$= \sqrt{3/2} \left( [B \cos \phi_r + R_T \sin \phi_r] I_m^3 \sin \beta + [I_m^3 R_T \cos \phi_r - I_m^3 B \sin \phi_r] + V_m^3 \right) \cos \beta$$

(2-21)
If the load is unbalanced, the d-q transformation is of the form:

\[
\begin{vmatrix}
K_d = \sqrt{2/3} & \cos\theta & \cos(\theta-120) & \cos(\theta+120) & K_m \sin(wt+\phi) \\
K_q = \sqrt{2/3} & -\sin\theta & -\sin(\theta-120) & -\sin(\theta+120) & \sin(wt-120+\phi)
\end{vmatrix}
\]

\[
= C C' C''
\]

\[
= \begin{vmatrix}
\sin(wt+\phi) \\
\sin(wt-120+\phi) \\
\sin(wt+120+\phi)
\end{vmatrix}
\]

(3-1)

or alternately:

\[
\begin{vmatrix}
K_d = \sqrt{2/3} & C & C' & C'' \\
K_q = \sqrt{2/3} & -S & -S' & -S''
\end{vmatrix}
\]

\[
= \begin{vmatrix}
\sin(wt+\phi) \\
\sin(wt-120+\phi) \\
\sin(wt+120+\phi)
\end{vmatrix}
\]

(3-2)

where

- \(\phi = \alpha\) for the receiving end current and \(\beta\) for the receiving end voltage.
- \(K\) = magnitude of the steady state instantaneous positive sequence current or voltage.
- \(K_m\) = a factor different from unity.
- \(\varepsilon = 1-K_m\)

Application of the equations above to the receiving end current transformation and premultiplication of the results by the impedance matrix of the complete system leads to
\[
\begin{align*}
B & \quad -R_T \\
R_T & \quad D
\end{align*}
\]

\[
\begin{align*}
\sqrt{3/2} I_{m3} & \quad \sqrt{3/2} I_{m3}^R R_T Cosa \\
R_T Sina - D Cosa & \quad -\sqrt{3/2} I_{m3}^R R_T Cosa \\
\end{align*}
\]

\[
\begin{align*}
B & \quad -R_T \\
R_T & \quad D
\end{align*}
\]

\[
\begin{align*}
\text{Coswt Sin(wt+\alpha)} & \quad I_{m3} \\
-\text{Sin wt Sin(wt+\alpha)}
\end{align*}
\]

\[
\sqrt{3/2} I_{m3} B Sina + \sqrt{3/2} I_{m3}^R R_T Cosa \\
- \sqrt{3/2} I_{m3}^R R_T Cosa \\
= \frac{B Cosa}{2} \text{Sin 2wt} + \frac{Sin}{2} (1 + \text{Cos 2wt}) \\
+ \frac{R_T Cosa}{2} (1 - \text{Cos2wt}) + \frac{Sin}{2} \text{Sin 2wt}
\]

\[
\sqrt{3/2} I_{m3} R_T \text{Sin} - \sqrt{3/2} I_{m3}^D Cosa \\
- \sqrt{3/2} I_{m3}^D Cosa \\
= \frac{R_T Cosa}{2} \text{Sin 2wt} + \frac{Sin}{2} (1 + \text{Cos 2wt}) \\
- \frac{D Cosa}{2} (1 - \text{Cos 2wt}) + \frac{Sin}{2} \text{Sin 2wt}
\]

A similar transformation can be carried out for the receiving end voltage,

\[
\begin{align*}
\sqrt{2/3}
\begin{array}{ccc}
S & S' & S'' \\
C & C' & C''
\end{array}
\end{align*}
\]

\[
\begin{align*}
\text{Sin(wt+\beta)} & \quad 0 \\
\text{Sin(wt-120+\beta)} & \quad 0 \\
\text{Sin(wt+120+\beta)} & \quad 0
\end{align*}
\]
Adding equation (3-3) to (3-4) and multiplying through by \( \sqrt{3/2} \), gives

\[
\sqrt{3/2} V_m^3 \cos \beta \quad \sqrt{2/3} \epsilon \, \frac{V_m^3}{2} (1 - \cos 2\omega t) \cos \beta
\]

\[
- \sqrt{2/3} \epsilon \, \frac{V_m^3}{2} (1 + \cos 2\omega t) \sin \beta
\]

\[
\sqrt{3/2} V_m^3 \sin \beta \quad - \sqrt{2/3} \epsilon \, \frac{V_m^3}{2} \sin 2\omega t \cos \beta
\]

\[
\sqrt{2/3} \epsilon \, \frac{V_m^3}{2} (1 + \cos 2\omega t) \sin \beta
\]

\[
\sqrt{3/2} \epsilon \, \frac{V_m^3}{2} (1 - \cos 2\omega t) \cos \beta
\]

\[
\epsilon \sin \omega t \sin (\omega t + \beta)
\]

\[
\epsilon \cos \omega t \sin (\omega t + \beta)
\]

\[
\epsilon \sin \omega t \cos (\omega t + \beta)
\]

\[
\epsilon \cos \omega t \cos (\omega t + \beta)
\]

\[
\epsilon \sin \omega t \sin (\omega t + \beta)
\]

\[
\epsilon \cos \omega t \sin (\omega t + \beta)
\]

\[
\epsilon \sin \omega t \cos (\omega t + \beta)
\]

\[
\epsilon \cos \omega t \cos (\omega t + \beta)
\]

\[
(3-5)
\]

\[
- \sqrt{3/2} \epsilon_d = \frac{k}{2}\left(I_m R T \sin \alpha - I_m D \cos \alpha + V_m^3 \sin \beta\right)
\]

\[
+ \frac{\epsilon}{2}\left(I_m R T \sin \alpha - I_m D \cos \alpha - V_m^3 \sin \beta\right) \sin 2\omega t
\]

\[
- \frac{\epsilon}{2}\left(I_m R T \sin \alpha + I_m D \cos \alpha + V_m^3 \sin \beta\right) \cos 2\omega t
\]

\[
(3-6)
\]

where \( k = (3/2 - \epsilon/2) \)

Taking the root mean square value of \( \epsilon_d \) and setting it equal to zero yields the following
\[
3/2E_d = W(Sin^2\beta + Cos^2\beta) + k^2I_{m3}^2 Sin^2\beta
\]
\[
+ k^2I_{m3}^2 \left[ R_T^2 Sin^2\alpha - 2R_T D Sin\alpha Cos\alpha + D^2 Cos^2\alpha \right]
\]
\[
- 2k^2(I_{m3}^2 DV_{m3} Cos\alpha Sin\beta - I_{m3}^2 R V_{m3} Sin\alpha Sin\beta)
\]

where
\[
W = \frac{g^2}{8} \left[ I_{m3}^2 (D^2 + R_T^2) + V_{m3}^2 \right]
\]
\[
+ \frac{g^2}{4} \left[ I_{m3}^2 DV_{m3} DV_{m3} Sin\phi_r + I_{m3}^2 R V_{m3} Cos\phi_r \right]
\]

(3-7)

Substituting \( \alpha = \beta - \phi_r \) and collecting terms yields
\[
K_1 Sin\beta Cos\beta + K_2 Cos^2\beta + K_3 Sin^2\beta = 0 \quad (3-8)
\]

where
\[
K_1 = k^2 \left\{ (-R_T I_{m3}^2 + D^2 I_{m3}^2) Sin 2\phi_r - 2R_T D I_{m3}^2 Cos2\phi_r - 2R_T I_{m3}^2 V_{m3} Sin\phi_r - 2V_{m3} I_{m3}^2 D Cos\phi_r \right\}
\]
\[
K_2 = W + k^2 (R_T Sin\phi_r + D Cos\phi_r) I_{m3}^2
\]
\[
K_3 = W + k^2 \left\{ V_{m3}^2 + (R_T Cos\phi_r - D Sin\phi_r) I_{m3}^2 - 2V_{m3} I_{m3}^2 D Sin\phi_r + 2R_T I_{m3}^2 V_{m3} Cos\phi_r \right\}
\]

(3-9)

Division of (3-8) by \( Cos^2\beta \) gives
\[
K_3 Tan^2\beta + K_1 Tan\beta + K_2 = 0 \quad (3-10)
\]

The solution can be also put in the form
\[
Tan\beta = x_r/y \quad r = 1, 2 \quad (3-11)
\]
\[
Cos\beta = y/\sqrt{x_r^2 + y^2} \quad (3-12)
\]
\[
Sin\beta = x_r/\sqrt{x_r^2 + y^2}
\]
\[
y = 2K_3
\]
Although there are two values of \( p_r \), only one is valid. The correct choice is determined by comparing the results of the above computations with those of the balanced case.

Substitution of \( \alpha = \beta - \phi_r \) in equation (3-6) results in

\[
\frac{3}{2} E_q^2 = k^2 \left[ (R_T \text{Im}_3 \sin \phi_r + BI_{m3} \cos \phi_r) \sin \beta 
+ \left\{ R_T \text{Im}_3 \cos \phi_r - BI_{m3} \sin \phi_r + V_{m3} \right\} \cos \beta \right]^2
+ \frac{E_q^2}{8} \left[ (R_T \text{Im}_3 \sin \phi_r - BI_{m3} \cos \phi_r) \sin \beta 
+ \left\{ R_T \text{Im}_3 \cos \phi_r + BI_{m3} \sin \phi_r + V_{m3} \right\} \cos \beta \right]^2
+ \frac{E_q^2}{8} \left[ (R_T \text{Im}_3 \cos \phi_r + BI_{m3} \sin \phi_r + V_{m3}) \sin \beta 
+ \left\{ - R_T \text{Im}_3 \sin \phi_r + BI_{m3} \cos \phi_r \right\} \cos \beta \right]^2
\]

(3-14)

Equation (3-14) gives the internal voltage in terms of receiving end current, receiving end voltage and power factor for a series capacitor compensated system with unbalanced load.
4. DERIVATION OF EQUATIONS FOR SHUNT CAPACITOR COMPENSATED POWER SYSTEMS WITH BALANCED AND UNBALANCED LOADS

In this chapter equations of the shunt capacitor compensated power system are derived. The details of Section 2 of the system in Fig. 1.2 are shown in Fig. 4.1. No series capacitors are connected to the transmission line in this case. The shunt compensation capacitor units are star connected with the neutral connected to earth.

4.1 General System Equations

Transforming equation (2-1) into d-q co-ordinates and eliminating \( v_1, v_2, \) and \( i_1 \) gives the equation

\[
e' = \left[ u + \left( Z'_o(p) + Z'_SE(p) \right) Y(p) \right] v_3' \\
+ \left[ Z'_RE(p) + \left( Z'_o(p) + Z'_SE(p) \right) \left( u + Y(p) Z'_RE(p) \right) \right] i_3'
\]

(4-1)

where \( u \) is a unit matrix, \( e' \) is the transformed internal voltage vector of the synchronous machine, \( v_3' \) the transformed receiving end voltage vector, \( i_3' \) the transformed receiving end current vector, \( Z'_o(p) \) the transformed operational impedance matrix of the synchronous machine, \( Z'_SE(p) \) and \( Z'_RE(p) \) respectively are the transformed operational impedance matrices of the sending end and receiving end of the transmission line, and \( Y(p) \) is the transformed total operational shunt admittance matrix consisting of \( Y'_sh(p) \), the admittance matrix due to shunt compensation capacitors and \( Y'_l(p) \), that due to the line shunt capacitance. All quantities are expressed in d-q co-ordinates.
FIG. 4-1 STAR CONNECTED SHUNT COMPENSATED AND LUMPED LINE CAPACITORS
The transformed operational impedance and admittance matrices are given in equations (4.2) through (4.7).

\[
\begin{align*}
\mathbf{Z}_0(p) &= \\
&= \begin{bmatrix}
R_0 + pL_o & 0 & 0 \\
0 & R_o + 3/2L_d p & -3/2L_q \dot{Q} \\
0 & 3/2L_d \dot{Q} & R_o + 3/2L_q p
\end{bmatrix} \\
\text{(4-2)}
\end{align*}
\]

where \( L_d = L_{os} + L_{2s} \) and \( L_q = L_{os} - L_{2s} \)

\[
\begin{align*}
\mathbf{Z}_SE'(p) &= \\
&= \begin{bmatrix}
R_{SE} + (L_o - L_o)p & 0 & 0 \\
0 & R_{SE} + L_1 p & -L_1 \dot{Q} \\
0 & L_1 \dot{Q} & R_{RE} + L_1 p
\end{bmatrix} \\
\text{(4-3)}
\end{align*}
\]

\[
\begin{align*}
\mathbf{Z}_RE'(p) &= \\
&= \begin{bmatrix}
R_{RE} + (L_o - L_o)p & 0 & 0 \\
0 & R_{RE} + L_1 p & -L_1 \dot{Q} \\
0 & L_1 \dot{Q} & R_{RE} + L_1 p
\end{bmatrix} \\
\text{(4-4)}
\end{align*}
\]

where \( L_o' = L_o + L_{aa} + 2L_{ab} \)

and \( L_1 = L_{aa} - L_{ab} \)
\[ Y(p) = Y_{sh}(p) + Y_{l}(p) \]

\[
\begin{array}{ccc}
C_{1p} & 0 & 0 \\
0 & C_{1p} & -C_{1} \dot{\phi} \\
0 & C_{1} \dot{\phi} & C_{1p}
\end{array}
\]  

(4-5)

where

\[ C_{0} = C_{aa} + C_{sh} - 2C_{ab} \quad \text{and} \quad C_{1} = C_{aa} + C_{ab} + C_{sh} \]

\[
\begin{array}{ccc}
C_{sh} p & 0 & 0 \\
0 & C_{sh} p & -C_{sh} p \\
0 & C_{sh} \dot{\phi} & C_{sh} p
\end{array}
\]

(4-6)

\[
\begin{array}{ccc}
(C_{aa} - 2C_{ab})p & 0 & 0 \\
0 & (C_{aa} + C_{ab})p & -(C_{aa} + C_{ab}) \dot{\phi} \\
0 & (C_{aa} + C_{ab}) \dot{\phi} & (C_{aa} + C_{ab}) p
\end{array}
\]

(4-7)

The procedure of the transformation of \( Z_{0}'(p) \) is as follows:--
As in equation (A2-9) the operational impedance matrix of the synchronous machine can be written in three parts,

\[ Z_o(p) = R_o + pL_o + pL_2(\theta) \quad (4-8) \]

The transformations are carried out using transformation matrices \( A_{a,d} \) and \( A_{d,a} \) as given in equations (A2-15) and (A2-16). Since the transformation is unitary

\[ A_{d,a} R_o A_{a,d} = R_{o} s A_{d,a} A_{a,d} = R_o \quad (4-9) \]

The transformation of the second term in the R.H.S. of equation (4-8) can be expressed as

\[ A_{d,a} p(L_o A_{a,d}) = A_{d,a} L_o A_{a,d} p + A_{d,a} \frac{\partial}{\partial \theta} (L_o A_{a,d}) \hat{\theta} \quad (4-10) \]

and the transformation of the third term as

\[ A_{d,a} p(L_o A_{a,d}) = A_{d,a} L_o A_{a,d} p + A_{d,a} \frac{\partial}{\partial \theta} (L_o A_{a,d}) \hat{\theta} \quad (4-11) \]

The results are summed up in equation (4-2).

The same procedure is applied to the transmission line operational impedance and admittance matrices. The transmission line inductance matrices \( L_{SB} \) and \( L_{RE} \) are given in (A2-13) and the line shunt capacitance matrix \( C_L \) is given in (A2-14). The operational admittance matrix for shunt capacitor compensation is given as

\[
\begin{array}{c|c|c}
 & C_{shP} & \\
\hline
Y_{sh}(p) = & C_{shP} & C_{shP} \\
\hline
& C_{shP} & \\
\end{array}
\quad (4-12)
\]
The transmission line operational impedance matrices can be separated into the resistive and the inductive parts. Since the resistance matrix is diagonal it remains unchanged after the unitary transformation. The transmission line inductance matrix transformation takes the same form as equation (4-10), namely \( A_d, a_p(\text{L}_{SE} A_d) \) and \( A_d, a_p(\text{L}_{RE} A_d) \). Similarly for line and compensation capacitances; \( A_d, a_p(\text{C}_1 A_d) \) and \( A_d, a_p(\text{C}_{sh} A_d) \).

On making the necessary substitutions and expanding the coefficients of \( v_3 \) and \( i_3 \) of equation (4-1) one obtains the following:

\[
Z'(p) + Z_{SE}'(p) =
\begin{array}{ccc}
R_{os} + R_s + L_o p & 0 & 0 \\
0 & R_{os} + R_s + (L_1 + 3/2L_d)p & -(L_1 + 3/2L_q)p \\
0 & (L_1 + 3/2L_d)p & R_{os} + R_s + (L_1 + 3/2L_q)p \\
\end{array}
\]

(4-13)

\[
u + [Z'(p) + Z_{SE}'(p)] Y(p)
\]

\[
\begin{array}{ccc}
1 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{array}
\]

(4-14)
\[
\begin{array}{|c|c|c|}
\hline
\{ (d^L_T + H^+_R)^0 + \theta(\bar{\Psi}_T + \theta^+_R)^0 \} \times \\
\{ (d^L_T + \theta^+_R)^0 + \theta(\bar{\Psi}_T + \theta^+_R)^0 \} - \\
\{ (d^L_T + \theta^+_R)^0 + \theta(\bar{\Psi}_T + \theta^+_R)^0 \} \times \\
\{ (d^L_T + \theta^+_R)^0 + \theta(\bar{\Psi}_T + \theta^+_R)^0 \} \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+_R)^0 + S_R + S_O \} - \\
\{ d(p_T + \xi + H^+}
4.2 Steady State Analysis of Shunt Capacitor Compensated Systems

For balanced steady state operation p=0 and the zero-axis relation may be omitted. Hence equation (4-1) can be reduced to the following form

\[
\begin{align*}
\begin{array}{c|c|c}
\text{eq} & \text{ed} & \text{F} \\
\hline
\text{vd} & \text{H} & -F \\
\hline
\text{v3d} & \text{v3q} & \text{K} & -M_1 \\
\hline
\text{M_2} & \text{J} & \text{i3d} & \text{i3q}
\end{array}
\end{align*}
\]

(4-17)

where

\[
H = 1 - (L_q + 3/2L_d)C_1 \dot{\theta}^2
\]
\[ F = (R_0 + R_s)C_1 \dot{\phi} \]
\[ G = 1 - (L_1 + 3/2L_d)C_1 \dot{\phi}^2 \]
\[ K = (R_0 + R_s)(1 - C_1 L_1 \dot{\phi}^2) - (L_1 + 3/2L_q)C_1 R_R + R_R \]
\[ J = (R_0 + R_s)(1 - C_1 L_1 \dot{\phi}^2) + (L_1 + 3/2L_q)C_1 R_R \dot{\phi}^2 + R_R \]
\[ M_1 = (R_0 + R_s)C_1 R_R \dot{\phi} + L_1 \dot{\phi} + (L_1 + 3/2L_q)(\dot{\phi} - C_1 L_1 \dot{\phi}^3) \]
\[ M_2 = (R_0 + R_s)C_1 R_R \dot{\phi} + L_1 \dot{\phi} + (L_1 + 3/2L_q)(\dot{\phi} - C_1 L_1 \dot{\phi}^3) \]

When a transformation similar to equation (2-14) is applied to the first and second terms on the right hand side of equation (4-17), the result is as follows:

\[
\begin{array}{c|c}
H & -F \\
F & G \\
\end{array}
\]
\[
\begin{array}{c|c}
K & -M_1 \\
M_2 & J \\
\end{array}
\]

\[
\begin{align*}
H \sin \beta + F \cos \beta & = \sqrt{3/2} \cdot V_{m3} \\
F \sin \beta - G \cos \beta & = \sqrt{3/2} \cdot I_{m3} \\
K \sin \alpha + M_1 \cos \alpha & = \sqrt{3/2} \cdot I_{m3} \\
M_2 \sin \alpha - J \cos \alpha & = \sqrt{3/2} \cdot I_{m3}
\end{align*}
\]

The voltages in equation (4-17) now become

\[
\begin{align*}
\sqrt{3/2} & \begin{bmatrix}
E_d \\
E_q
\end{bmatrix} = (H \sin \beta + F \cos \beta) V_{m3} + (K \sin \alpha + M_1 \cos \alpha) I_{m3} \\
& + (F \sin \beta - G \cos \beta) V_{m3} + (M_2 \sin \alpha - J \cos \alpha) I_{m3}
\end{align*}
\]
Substitution of $\beta - \phi_r$ for $\alpha$ in the first equation of (4-21) and setting $E_d = 0$ according to (A3-7) gives

$$(H_{m_3} + K_I m_3 \cos \phi_r + M I_m \sin \phi_r) \sin \beta$$

$$+ (F_{m_3} - K_I m_3 \sin \phi_r + M I_m \cos \phi_r) \cos \beta = 0 \quad (4-22)$$

Let $x_r = K_I m_3 \sin \phi_r - F_{m_3} - M I_m \cos \phi_r$ and

$$y = V_{m_3} H + K_I m_3 \cos \phi_r + M I_m \sin \phi_r$$

One has

$$\tan \beta = x_r / y \quad r=1,2$$

$$\cos \beta = y / \sqrt{x_r^2 + y^2} \quad (4-23)$$

$$\sin \beta = x_r / \sqrt{x_r^2 + y^2}$$

Substitution of $\beta - \phi_r$ for $\alpha$ in the second equation of (4-21) gives

$$\sqrt{2/3} E_q = (V_{m_3} F + I_m^2 M \cos \phi_r - I_m^2 m_3^2 \sin \phi_r) \sin \beta$$

$$- (V_{m_3} G \cos \beta + M I_m \sin \phi_r + J I_m \cos \phi_r) \cos \beta \quad (4-24)$$

This is the equation to be used for the computation of the internal voltage of the synchronous machine for a given receiving end current, receiving end voltage and power factor of a shunt capacitor compensated system with balanced load.

**4.3 Shunt Capacitor Compensated Power Systems with Unbalanced Load**

Corresponding to equation (3-3) of the series capacitor compensated power system with unbalanced load, the first term in the right hand side of equation (4-17) can be written as
\[ H \sin \beta + F \cos \beta \]

\[ \sqrt{\frac{2}{3}} \hat{e}_m \]

\[ \sqrt{\frac{2}{3}} \hat{e}_m \]

\[ F \sin \beta - G \cos \beta \]

\[ \begin{array}{c|c|c}
\text{H} & \text{F} & \text{Cos}wt \sin(wt+\beta) \\
\hline
\text{F} & \text{G} & -\sin wt \sin(wt+\beta) \\
\end{array} \]

\[ \begin{align*}
\sqrt{\frac{3}{2}} \hat{e}_m & \left( H \sin \beta + F \cos \beta \right) \\
- & \sqrt{\frac{2}{3}} \hat{e}_m \left( F \sin \beta - G \cos \beta \right) \\
= & \sqrt{\frac{3}{2}} \hat{e}_m \left( K \sin \alpha + M_1 \cos \alpha \right) \\
& - \sqrt{\frac{2}{3}} \hat{e}_m \left( M_2 \sin \alpha - J \cos \alpha \right) \\
\end{align*} \]

The second term on the right-hand side of equation (4-17) can be expressed as

\[ \begin{array}{c|c|c|c}
\text{K} & \text{M}_1 & \text{Cos} \alpha \sin 2wt + \frac{\sin \alpha}{2} \\
\hline
\text{M}_2 & \text{J} & \frac{(1+\cos 2wt)}{2} \\
\end{array} \]
\[
\begin{align*}
\sqrt{3/2} I_{m3}^2 K \sin \alpha + & \frac{3}{2} I_{m3}^2 M_1 \cos \alpha \\
- \sqrt{2/3} \varepsilon I_{m3} \left\{ K \left[ \frac{\cos \alpha}{2} \sin 2\omega t + \frac{\sin \alpha}{2} (1 + \cos 2\omega t) \right] \\
+ M_1 \left[ \frac{\cos \alpha}{2} (1 - \cos 2\omega t) + \frac{\sin \alpha}{2} \sin 2\omega t \right] \right\} = \\
\sqrt{3/2} M_2 I_{m3} \sin \alpha - & \sqrt{3/2} JI_{m3} \cos \alpha \\
- \sqrt{2/3} \varepsilon I_{m3} \left\{ M_2 \left[ \frac{\cos \alpha}{2} \sin 2\omega t + \frac{\sin \alpha}{2} (1 + \cos 2\omega t) \right] \\
- J \left[ \frac{\cos \alpha}{2} (1 - \cos 2\omega t) + \frac{\sin \alpha}{2} \sin 2\omega t \right] \right\} \tag{4-26}
\end{align*}
\]

Combining the results in equations (4-25) and (4-26) yields

\[
\begin{align*}
(3/2-\varepsilon/2) I_{m3} (K \sin \alpha + M_1 \cos \alpha) \\
+ (3/2-\varepsilon/2) V_{m3} (H \sin \beta + F \cos \beta) \\
+ \varepsilon/2 (-I_{m3} K \cos \alpha - I_{m3} M_1 \sin \alpha - V_{m3} H \cos \beta - V_{m3} F \sin \beta) \sin 2\omega t \\
+ \varepsilon/2 (-I_{m3} K \sin \alpha + I_{m3} M_1 \cos \alpha - V_{m3} H \sin \beta + V_{m3} F \cos \beta) \cos 2\omega t \\
= \\
(3/2-\varepsilon/2) I_{m3} (M_2 \sin \alpha - J \cos \alpha) \\
+ (3/2-\varepsilon/2) V_{m3} (F \sin \beta - G \cos \beta) \\
+ \varepsilon/2 (-M_2 I_{m3} \cos \alpha + J I_{m3} \sin \alpha - F V_{m3} \cos \beta + G V_{m3} \sin \beta) \sin 2\omega t \\
- \varepsilon/2 (M_2 I_{m3} \sin \alpha + I_{m3} J \cos \alpha - F V_{m3} \sin \beta + G V_{m3} \cos \beta) \cos 2\omega t \tag{4-27}
\end{align*}
\]
The rms value of $e_d$ must be zero.

$$(3/2-\epsilon/2)^2\left[K_{m_3} m_3^2 \sin \alpha + M_{m_3} m_3^2 \cos \alpha + H V_{m_3} \sin \beta + F V_{m_3} \cos \beta \right]^2$$

$$+ \frac{\epsilon^2}{8}\left[M_{m_3} m_3^2 \sin \alpha + K_{m_3} m_3^2 \cos \alpha + F V_{m_3} \sin \beta + H V_{m_3} \cos \beta \right]^2$$

$$+ \frac{\epsilon^2}{8}\left[-K_{m_3} m_3^2 \sin \alpha + M_{m_3} m_3^2 \cos \alpha - H V_{m_3} \sin \beta + F V_{m_3} \cos \beta \right]^2$$

$$= 0 \quad (4-28)$$

This equation can also be expressed as

$$U + k^2\left[H^2 \sin^2 \beta + F^2 \cos^2 \beta + 2HF \sin \beta \cos \beta \right] V_{m_3}^2$$

$$+ k^2\left[(K^2 \sin^2 \alpha + M^2 \cos^2 \alpha + 2K_{m_3} \sin \alpha \cos \alpha) I_{m_3}^2 \right]$$

$$+ 2(KH \sin \alpha \sin \beta + KF \sin \alpha \cos \beta + M_{m_3} \cos \alpha \sin \beta +$$

$$+ M_{m_3} \cos \alpha \cos \beta) V_{m_3}^2 I_{m_3}^2 \right] = 0 \quad (4-29)$$

where $U = \frac{\epsilon^2}{8}\left\{\left(M_{m_3}^2 + K_{m_3}^2\right) I_{m_3}^2 + \left(H^2 + F^2\right) V_{m_3}^2 \right\}$

$$+ \frac{\epsilon^2}{4}\left\{(KF - M_{m_3} H) \sin \phi_r + (KH + M_{m_3} F) \cos \phi_r \right\} V_{m_3}^2 I_{m_3}^2 \right\}$$

and $k = (3/2-\epsilon/2)$

Substitution of $\beta - \phi_r$ for $\alpha$ in the preceding equation results in an equation of the form

$$K_4 \sin \beta \cos \beta + K_5 \cos^2 \beta + K_6 \sin^2 \beta = 0 \quad (4-30)$$

Dividing throughout by $\cos^2 \beta$ gives

$$K_6 \tan^2 \beta + K_4 \tan \beta + K_5 = 0 \quad (4-31)$$

where

$$K_4 = 2k^2\left\{(HFV_{m_3}^2 \right. + \left[1/2(M_{m_3}^2 - K_{m_3}^2) \sin^2 \phi_r + K_{m_3} \cos^2 \phi_r \right) I_{m_3}^2 \right\}$$

$$+ \left[(M_{m_3} F - KH) \sin \phi_r + (KF + M_{m_3} H) \cos \phi_r \right] V_{m_3} I_{m_3}$$

$$K_5 = k^2\left\{(K \sin \phi_r - M_{m_3} \cos \phi_r) I_{m_3}^2 \right\}$$

$$+ \left(4-32)$$
\[ + 2(M_1 \cos \phi_T - K \sin \phi_T)F V_{m3} I_{m3} + F^2 V_{m3}^2 \} + U \]

\[ K_6 = k^2 \{ (K \cos \phi_T + M_1 \sin \phi_T)^2 I_{m3}^2 \]

\[ + 2(K \cos \phi_T + M_1 \sin \phi_T)H V_{m3} I_{m3} + H^2 V_{m3}^2 \} + U \]

Let \( p_r = -K_4 \pm \sqrt{K_4^2 - 4K_6 K_5} \) \( r=1,2 \)

\[ q = 2K_6 \]

Solving equation (4-25) results in

\[ \tan \beta = \frac{p_r}{q} \]

and hence

\[ \cos \beta = \frac{q}{\sqrt{p_r^2 + q^2}} \]  \hspace{1cm} (4-33)

\[ \sin \beta = \frac{p_r}{\sqrt{p_r^2 + q^2}} \]

Although there are two values of \( p_r \), only one is valid. The correct choice is determined by comparing the results of the above computations with those of the balanced case.

On finding \( \beta \) one has the rms value of \( e_q \) as

\[ k^2 \{ [(M_2 \cos \phi_T - J \sin \phi_T) \sin \beta - (M_2 \sin \phi_T + J \cos \phi_T) \cos \beta] I_{m3} \]

\[ + (F \sin \beta - G \cos \beta) V_{m3} \}^2 \]

\[ + \frac{e}{8} \{ [(J \cos \phi_T - M_2 \sin \phi_T) \sin \beta - (J \sin \phi_T + M_2 \cos \phi_T) \cos \beta] I_{m3} \]

\[ + (G \sin \beta - F \cos \beta) V_{m3} \}^2 \]

\[ + \frac{e}{8} \{ [(M_2 \cos \phi_T + J \sin \phi_T) \sin \beta + (J \cos \phi_T + M_2 \sin \phi_T) \cos \beta] I_{m3} \]

\[ + (F \sin \beta + G \cos \beta) V_{m3} \}^2 = \frac{3}{2} E_q^2 \]  \hspace{1cm} (4-34)

Equation (4-14) gives the internal voltage in terms of the receiving end current, the receiving end voltage and power factor for a shunt capacitor compensated system.
5. NUMERICAL EXAMPLES OF PRACTICAL SYSTEMS

In this chapter, two power systems are analysed. One is already in existence, and the other planned as described in Appendix IVA and IVB. The first is a 500 kv, 2400 MVA and 600 mile transmission system. The other is a 250 kv, 200 MVA and 250 mile transmission system. In both cases the effects of the generator are included.

This thesis is concerned with the steady state, although the theory developed also applies to transient analysis. Computations are carried out for the internal voltage of the synchronous generator connected to a compensated transmission line. The analysis is important for the choice of excitation capacity of the generator, and for the power angle between the internal voltage and the transmission line receiving end voltage. The latter is maintained constant. Both series and shunt capacitor compensated systems are investigated. Compensations of 0, 25, 50 and 75 per cent are used.

Eighty five per cent power factor lagging and unity power factor loads are considered for both series and shunt capacitor compensation studies. A zero power factor leading load is also included for the case of series compensation. The former studies take care of normal operations. The latter are for the rare cases, including light loads. The results are summarised in the following paragraphs.
5.1 The Series Capacitor Compensated 600 Mile System

The results of the study of the series capacitor compensated 600 mile system are shown in Fig. 5.1 through 5.6.

Fig. 5.1 shows the relation of the machine internal voltage versus load current. The receiving end voltage is maintained constant. The balanced case is shown by solid lines and the unbalanced case by broken lines. The degree of series compensation is varied. The internal voltage for a line with 50 per cent compensation at full load which corresponds to 0.24 per unit current is 2.93 per unit for balanced load and 2.75 per unit for the unbalanced case.

Fig. 5.2 includes the result of similar studies except that the power factor is unity instead of 0.85 as in Fig. 5-1.

Fig. 5.3 describes similar studies save that the power factor is zero leading. It is interesting to note the V-shaped characteristic of the internal voltage with respect to load current. It also reveals the important effect that, unless the excitation of the generator is appropriately reduced, the receiving end voltage can be excessively high for a capacitive load. The voltage rise may occur during the initial charging of, or the load rejection from, a transmission line.

Figs. 5.4 through 5.6 show the characteristics of the power angle between the receiving end voltage and the generator internal voltage with respect to the receiving end load current. At zero power factor, excessively large power angles are observed for certain load currents.
5.2 The Series Capacitor Compensated 250 Mile System

Figs. 5.7 through 5.12 show the results of the investigation of a series capacitor compensated 250 mile system.

Fig. 5.7, like Fig. 5.1, shows the plots of machine internal voltage as a function of receiving end current for both balanced and unbalanced load conditions. The balanced case is shown by solid lines and the unbalanced case by broken lines. The degree of compensation is varied from zero per cent to 75 per cent at intervals of 25 per cent. The internal voltage for the system with 50 per cent compensation and 0.85 PF lagging is 4.25 per unit for a balanced full load and 4.0 per unit for an unbalanced load. The full load current in this case is 0.2 per unit. It is evident, as in Fig. 5.1, that the no-load internal voltage is lower for the balanced case than for the unbalanced case.

Fig. 5.8 gives similar plots of excitation characteristics for unity power factor. The pattern of the curves is practically the same as those of Fig. 5.2.

The excitation characteristics for the system with zero power factor leading load are presented in Fig. 5.9. The statement in section 5.1 which says that "unless the excitation of the generator is appropriately reduced, the receiving end voltage can be excessively high" is also valid in this case. The dominant feature in Fig. 5.9 is that for certain values of load current, negative values of internal voltage must be maintained, which implies negative excitation.
Figs 5.10 through 5.12 demonstrate the plots of the power angle between the receiving end voltage and the generator internal voltage as a function of load current. As in Fig. 5.6 excessively large power angles are observed for particular load currents in the zero power factor study.

5.3 The Shunt Capacitor Compensated 600 Mile System

Figs. 5.13 and 5.14 show two sets of curves of the internal voltage plotted as a function of load current at 0.85PF lagging and unity PF for the 600 mile shunt compensated system. The balanced case is shown by solid lines and the unbalanced case by broken lines. The line shunt capacitance is neglected and the shunt capacitor compensation is varied from zero to 75 per cent at intervals of 25 per cent. The pattern of the two sets of curves is similar.

Fig. 5.15 summarises the results obtained from the study of the power angle characteristics of the shunt compensated 600 mile system at 0.85 power factor lagging and unity power factor plotted against receiving end current.

5-4 The Shunt Capacitor Compensated 250 Mile System

The results obtained for the shunt capacitor compensated 250 mile system similar to that of the 600 mile system study in the previous section are summarised in Figs. 5.16 through 5.18.
5.5 Summary of Results

As expected, the internal voltage of the synchronous machine which decides the excitation capacity in either the series or shunt compensated system is smaller for unity power factor loading than that for 0.85 PF lagging loading.

It is obvious from the excitation characteristic studies that the 600 mile system definitely needs series or shunt compensation and it is also desirable for the 250 mile transmission line to have such compensation.

The results also reveal that a high degree of compensation is desirable as long as it is economically feasible. Alternatively for a given excitation system the capacity of the transmission line can be increased by the use of series or shunt capacitor compensation.

For the 600 mile system the series capacitor compensation is more effective than the shunt capacitor compensation around the full load operating point, in terms of excitation capacity. There is little difference in series and shunt compensation, for the 250 mile system.
RECEIVING END CURRENT PER UNIT

FIG. 5-1 EXCITATION CHARACTERISTICS OF THE SERIES COMPENSATED 600 MILE POWER SYSTEM, 0.85PF LAGGING LOAD
FIG. 5-2 EXCITATION CHARACTERISTICS OF THE SERIES COMPENSATED 600 MILE POWER SYSTEM, UNITY PF LOAD
FIG. 5-3 EXCITATION CHARACTERISTICS OF THE SERIES COMPENSATED 600 MILE POWER SYSTEM, ZERO PF LEADING LOAD
FIG. 5-4  
TORQUE ANGLE CHARACTERISTICS OF THE 
SERIES COMPENSATED 600 MILE POWER SYSTEM, 
0.85 PF LAGGING LOAD
Fig. 5-5 Torque Angle Characteristics of the Series Compensated 600 Mile Power System Unity PF Load
Fig. 5-6 Torque Angle Characteristics of the Series Compensated 600 Mile Power System, Zero PF Leading Load.
FIG. 5-7 EXCITATION CHARACTERISTICS OF THE 250 MILE SERIES COMPENSATED POWER SYSTEM, 0.85 PF LAGGING LOAD
FIG 5-8 EXCITATION CHARACTERISTICS OF THE 250 MILE SERIES COMPENSATED POWER SYSTEM UNITY PF LOAD
FIG 5-9 EXCITATION CHARACTERISTICS OF THE 250 MILE SERIES COMPENSATED POWER SYSTEM ZERO PF LEADING LOAD
Fig 5-10: Torque Angle Characteristics of the Series Compensated 250 Mile Power System, 0.85 PF lagging load.
FIG. 5-11 TORQUE ANGLE CHARACTERISTICS OF THE SERIES COMPENSATED 250 MILE POWER SYSTEM, UNITY PF LOAD
FIG 5-12 TORQUE ANGLE CHARACTERISTICS OF THE SERIES COMPENSATED 250 MILE POWER SYSTEM, ZERO PF LEADING LOAD
FIG 5-13 EXCITATION CHARACTERISTICS OF THE SHUNT COMPENSATED 600 MILE POWER SYSTEM, 0.85 PF LAGGING LOAD
FIG. 5-14 EXCITATION CHARACTERISTICS OF THE SHUNT COMPENSATED 600 MILE POWER SYSTEM, UNITY PF LOAD
FIG 5-15, 5-15B, TORQUE ANGLE CHARACTERISTICS
OF THE SHUNT COMPENSATED 600 MILE POWER SYSTEM
0.85 LAGGING AND UNITY PF LOADS
FIG 5-16 EXCITATION CHARACTERISTICS OF THE SHUNT COMPENSATED 250 MILE POWER SYSTEM, 0.85 PF LAGGING LOAD
FIG 5-17 EXCITATION CHARACTERISTICS OF THE SHUNT COMPENSATED 250 MILE POWER SYSTEM, UNITY PF LOAD
RECEIVING END CURRENT PER UNIT

FIG. 5-18, 5-18A, TORQUE ANGLE CHARACTERISTICS OF THE SHUNT COMPENSATED 250 MILE POWER SYSTEM, 0.85 LAGGING AND UNITY PF LOADS
6. CONCLUSIONS AND GENERAL REMARKS

The major objectives of the thesis are to derive the basic equations for a capacitor compensated power system for steady state and transient analyses and to find the internal voltage $E_q$ for the synchronous machine in terms of line parameters, load parameters and series and shunt compensation capacitance.

Using Park's transformation, equations for the synchronous machine and transmission line are transformed into the d-q co-ordinates using the same reference frame. The equations for the complete system have been obtained through the elimination of unwanted variables and the final results of currents and voltages are expressed in a-b-c phase co-ordinates.

The equations developed in the thesis are particularly suitable for the study of EHV long transmission lines transferring power from generators at one end to the load centre at the other end. This type of system is quite common in Sweden, Canada, and the United States.

The manifest advantage of the method is the inclusion of the important parameters ($X_d$, $X_{df}$, $X_{ab}$, $X_{aa}$, $X_q$ etc) of both the transmission line and the salient pole synchronous generator in the equations, enabling the power engineer to make a proper choice of parameters in the design of the system.

From studies on typical systems it is found that the series capacitor compensation is more effective than the shunt capacitor compensation. For example, with 50 per cent compensation
and with a 0.85 power factor lagging symmetrical full load, the series capacitor compensated 600 mile line requires 2.93 per unit internal voltage, while the shunt compensated 600 mile system needs 3.80 per unit internal voltage.

Another result from the excitation characteristic studies is that the series and shunt capacitor compensation increases the transmission capacity. For example, for a 3.0 per unit internal voltage the 600 mile transmission system with 0.85 lagging power factor load may transmit 0.12 per unit power without compensation, 0.168 per unit power with 25 per cent series compensation and 0.252 per unit with 50 per cent series compensation, maintaining the receiving end voltage at 1 per unit.

The present thesis is confined to the steady state study of the series and shunt capacitor compensated systems including the effect of the synchronous machine with linear parameters. The system equations (2-4) and (2-5) for the series capacitor compensated system and (4-1) through (4-7) for the shunt capacitor compensated system, indeed, are quite general. They can be applied for the transient studies. Further developments may include the effects of magnetic saturation on the reactances and voltages of the generator and main transformer and the amortisseur windings of the generator for the transient as well as dynamic stability studies.
APPENDIX I

List of Symbols

\( p = \frac{d}{dt} = \text{time derivative} \)

\( \varphi = \text{instantaneous angular position of the rotor} \)

\( p\varphi = \dot{\varphi} = \text{angular velocity of machine} \)

\( \omega = \text{rated angular velocity} \)

\( f = \text{rated system frequency} \)

\( v_{3d} = \text{direct-axis component of receiving end voltage} \)

\( v_{3q} = \text{quadrature-axis component of receiving end voltage} \)

\( v_{30} = \text{zero-sequence component of receiving end voltage} \)

\( v_l = \text{sending end voltage matrix} \)

\( v_2 = \text{T-junction voltage matrix} \)

\( v_3 = \text{receiving end voltage matrix} \)

\( V_{m3} = \text{maximum value of voltage per phase at the receiving end} \)

\( e_d = \text{direct-axis component of internal voltage} \)

\( E_d = \text{rms value of direct-axis component of internal voltage} \)

\( e_q = \text{quadrature-axis component of internal voltage} \)

\( E_q = \text{rms value of quadrature-axis component of internal voltage} \)

\( e_0 = \text{zero sequence component of internal voltage} \)

\( E_0 = \text{rms value of zero sequence component of internal voltage} \)

\( e = \text{machine internal voltage matrix} \)

\( I_f = \text{machine field current (dc)} \)

\( i_l = \text{sending end current matrix} \)
\[\begin{align*}
  \mathbf{i}_2 &= \text{current matrix for T-junction} \\
  \mathbf{i}_3 &= \text{receiving end current matrix} \\
  \mathbf{i}_{3d} &= \text{direct-axis component of receiving end current} \\
  \mathbf{i}_{3q} &= \text{quadrature-axis component of receiving end current} \\
  \mathbf{i}_{30} &= \text{zero sequence component of receiving end current} \\
  \mathbf{I}_{m3} &= \text{maximum value of receiving end current} \\
  \mathbf{X}_d &= \text{direct-axis synchronous reactance} \\
  \mathbf{X}_q &= \text{quadrature-axis synchronous reactance} \\
  \mathbf{L}_d &= \text{direct-axis synchronous inductance} \\
  \mathbf{l}_o &= \text{armature leakage inductance} \\
  \mathbf{L}_f &= \text{field self-inductance} \\
  \mathbf{L}_{df} &= \text{direct axis magnetising inductance} \\
  \mathbf{L}_{aa} &= \text{half of self inductance of the line per phase} \\
  \mathbf{L}_{ab} &= \text{half of mutual inductance between each pair of lines per phase} \\
  \mathbf{L}_T &= \text{receiving end and sending end transformer leakage inductance per phase.} \\
  \mathbf{L}_{SE} &= \text{sending end section inductance matrix} \\
  \mathbf{L}_{RE} &= \text{receiving end section inductance matrix} \\
  \mathbf{R}_{0S} &= \text{armature resistance per phase} \\
  \mathbf{R}_F &= \text{field winding resistance} \\
  \mathbf{R}_S &= \text{sending end section resistance per phase} \\
  \mathbf{R}_R &= \text{receiving end section resistance per phase} \\
  \mathbf{R}_0 &= \text{armature resistance per phase} \\
  \mathbf{R}_{SE} &= \text{sending end section resistance matrix} \\
  \mathbf{R}_{RE} &= \text{receiving end section resistance matrix}
\end{align*}\]
\( Z(p) \) = Operational impedance matrix of synchronous machine without damper windings

\( Z_0(p) \) = Operational impedance matrix of the armature windings of a salient pole synchronous machine

\( Z_0 \) = impedance matrix of the armature windings of machine

\( Z_{SE}(p) \) = sending end section operational impedance matrix

\( Z_{SE} \) = sending end section impedance matrix

\( Z_{RE}(p) \) = receiving end section operational impedance matrix

\( Z_{RE} \) = receiving end section impedance matrix

\( C_{l} \) = Line shunt capacitance matrix

\( C_{sh} \) = Shunt compensation capacitance matrix

\( C_{aa} \) = Line mutual capacitance between a pair of lines

\( C_{ab} \) = Line mutual capacitance between a pair of lines

\( C_{s} \) = series compensation capacitance matrix

\( Y_{l}(p) \) = operational admittance matrix of the line

\( Y_{l} \) = admittance matrix of the line

\( Y_{sh}(p) \) = shunt compensation operational admittance matrix

\( Y_{sh} \) = shunt compensation admittance matrix

\( Y(p) \) = total operational admittance matrix

\( Y \) = total admittance matrix

\( \alpha \) = receiving end current angle with respect to the direct-axis.
\[ \beta = \text{receiving end voltage angle with respect to the direct-axis} \]

\[ \phi_r = \text{phase angle of the load} \]

Primes indicate transformed quantities in \( d-q \) co-ordinates.
APPENDIX II

Transformation of Equation (2-9)

The second term on the right hand side of equation (2-9) which involves the second derivative can be expanded in the following way:

\[ p^2A_{ad}i_3'' = p(\dot{A}_{ad}\dot{\phi} + A_{ad}\ddot{p})i_3' \]  \hspace{1cm} (A2-1)

where

\[ pA_{ad}(\dot{\phi}i_3') = [\ddot{A}_{ad}\dot{\phi} + A_{ad}\ddot{p}](\dot{i}_3') \]

\[ = [\ddot{A}_{ad}\dot{\phi}^2 + A_{ad}\ddot{p}(\dot{\phi}i_3')] \]  \hspace{1cm} (A2-2)

\[ = [\ddot{A}_{ad}\dot{\phi}^2 + A_{ad}\ddot{\phi} + A_{ad}\ddot{p}]i_3' \]

and

\[ pA_{ad}(p'i_3') = [\ddot{A}_{ad}\ddot{\phi} + A_{ad}\ddot{p}]p'i_3' \]  \hspace{1cm} (A2-3)

hence

\[ p^2A_{ad}i_3'' = [\dddot{A}_{ad}\ddot{\phi} + \dddot{A}_{ad}\ddot{\phi} + 2\dddot{A}_{ad}\ddot{\phi}p + A_{ad}\dddot{p}]i_3' \]  \hspace{1cm} (A2-4)

and

\[ A_{da}p^2A_{ad}i_3'' = [A_{da}A_{ad}\dddot{\phi}^2 + A_{da}A_{ad}\dddot{\phi} + 2A_{da}A_{ad}\dddot{\phi}p + A_{da}A_{ad}\dddot{p}]i_3' \]  \hspace{1cm} (A2-5)

Similarly the third term in equation (2-9) which also involves the second derivative can be expressed as

\[ A_{da}p^2L_2(\theta)A_{ad}i_3'' = [A_{da}B_{ad}\dddot{\phi}^2 + A_{da}B_{ad}\dddot{\phi} + 2A_{da}B_{ad}\dddot{\phi}p + A_{da}B_{ad}\dddot{p}]i_3' \]  \hspace{1cm} (A2-6)

In order to carry through the transformation the details of the current and voltage vectors, impedance and transformation matrices must be given.
The current and voltage vectors are

\[
\begin{bmatrix}
    i_3'
\end{bmatrix} = \begin{bmatrix}
    i_{30} \\
    i_{3d} \\
    i_{3q}
\end{bmatrix}, \quad
\begin{bmatrix}
    e'
\end{bmatrix} = \begin{bmatrix}
    e_0 \\
    e_d \\
    e_q
\end{bmatrix}, \quad
\begin{bmatrix}
    v_3'
\end{bmatrix} = \begin{bmatrix}
    v_{30} \\
    v_{3d} \\
    v_{3q}
\end{bmatrix}
\]

(A-27)

For brevity the following short hand notations will be adopted:

\[
S = \sin \theta, \quad S' = \sin(\theta - 120), \quad S'' = \sin(\theta + 120).
\]

\[
C = \cos \theta, \quad C' = \cos(\theta - 120), \quad C'' = \cos(\theta + 120).
\]

\[
d = \cos 2\theta, \quad d' = \cos(2\theta - 120), \quad d'' = \cos(2\theta + 120).
\]

The operational impedance matrix of the synchronous machine is a function of \( \theta \) and can be written in three parts as follows:

\[
Z_0(p) = R_o + pL_o + pL_2
\]

(A2-9)

\[
\begin{array}{ccc}
    a & b & c \\
    \hline
    a & l & 0 \\
    b & 1 & 0 \\
    c & 0 & 1
\end{array}
\]

(2-10)

\[
\begin{array}{ccc}
    a & b & c \\
    \hline
    a & l & -\frac{1}{2} \\
    b & -\frac{1}{2} & l \\
    c & -\frac{1}{2} & -\frac{1}{2} \\
\end{array}
\]

(A2-11)
The inductance matrix of the sending end and receiving end halves of the balanced T-circuit is given as

\[
L_{RE} = L_{SE} = \begin{pmatrix}
L_{aa} & L_{ab} & L_{ab} \\
L_{ab} & L_{aa} & L_{ab} \\
L_{ab} & L_{ab} & L_{aa}
\end{pmatrix}
\]  

(A2-13)

where \( L_{aa} = L_{aa}^{'} + L_{T} \)

\( L_{aa}^{'} \) = half of line self inductance per phase

\( L_{T} \) = transformer leakage inductance per phase

The line shunt capacitance matrix of the equivalent T transmission line, although not needed in the series compensation study, is given here together with parameters of other components of the system for convenience.

\[
C_{l} = \begin{pmatrix}
C_{aa} & -C_{ab} & -C_{ab} \\
-C_{ab} & C_{aa} & -C_{ab} \\
-C_{ab} & -C_{ab} & C_{aa}
\end{pmatrix}
\]  

(A2-14)
Park's transformation matrices are written as

\[
A_{ad} = \sqrt{2/3} \cdot \begin{bmatrix}
0 & \frac{\sqrt{2}}{2} & 0 \\
\frac{\sqrt{2}}{2} & C & -S \\
0 & 0 & 0
\end{bmatrix}
\]

\[
A_{da} = \sqrt{2/3} \cdot \begin{bmatrix}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
C & C' & C'' \\
-S & -S' & -S''
\end{bmatrix}
\]

Their derivatives are

\[
\dot{A}_{ad} = \sqrt{2/3} \cdot \begin{bmatrix}
0 & -S & -C \\
0 & -S' & -C' \\
0 & -S'' & -C''
\end{bmatrix}
\]

\[
\ddot{A}_{ad} = \sqrt{2/3} \cdot \begin{bmatrix}
0 & -C & S \\
0 & -C' & S' \\
0 & -C'' & S''
\end{bmatrix}
\]
Define

\[ B_{ad}(\theta) \triangleq L_2(\theta) A_{ad} \]

From this the following matrices are obtained

\[ B_{ad}(\theta) = \sqrt{\frac{3}{2L_{2s}}} \]

\[
\begin{array}{ccc}
0 & C & S \\
0 & C' & S' \\
0 & C'' & S''
\end{array}
\] (2-19)

The derivatives are

\[ B_{ad}'(\theta) = \sqrt{\frac{3}{2L_{2s}}} \]

\[
\begin{array}{ccc}
0 & -S & C \\
0 & -S' & C' \\
0 & -S'' & C''
\end{array}
\] (2-20)

\[ B_{ad}''(\theta) = \sqrt{\frac{3}{2L_{2s}}} \]

\[
\begin{array}{ccc}
0 & -C & -S \\
0 & -C' & -S' \\
0 & -C'' & -S''
\end{array}
\] (A2-21)

With the preceding information available, the transformation operations of equation (2-9) may be easily performed.
The results are given as follows. The left hand side of equation (2-9) becomes

\[ A_{da} p A_{ad} \varepsilon' = \]

\[
\begin{array}{ccc|c}
  p & 0 & 0 & e_{30} \\
  0 & p & -\dot{\theta} & e_{3d} \\
  0 & \dot{\theta} & p & e_{3q}
\end{array}
\]

(A2-22)

The first term of the right hand side is given as

\[ A_{da} [R] p A_{ad} i_3' = R_T A_{da} [A_{ad} \Theta + A_{ad} p] i_3' \]

\[ = R_T \]

\[
\begin{array}{ccc|c}
  p & 0 & 0 & i_{30} \\
  0 & p & -\dot{\theta} & i_{3d} \\
  0 & \dot{\theta} & p & i_{3q}
\end{array}
\]

(A2-23)

where \( R_T = R_{SE} + R_{os} + R_{RE} \)

With reference to equation (A2-5) the components of the second term of equation 2-9 are

\[ A_{da} L A_{ad} \Theta^2 = \]

\[
\begin{array}{ccc|c}
  0 & 0 & 0 & 0 \\
  0 & -L_B \dot{\Theta}^2 & 0 \\
  0 & 0 & L_B \dot{\Theta}^2 & 0
\end{array}
\]

(A2-24)
\[ A_{da} \mathbf{\hat{A}}_{ad} \hat{\Theta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_B \Theta \\ 0 & L_B \Theta & 0 \end{bmatrix} \]  
\text{(A2-25)}

\[ 2A_{da} \mathbf{\hat{A}}_{ad} \hat{\Theta}_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2L_B \Theta_p \\ 0 & 2L_B \Theta_p & 0 \end{bmatrix} \]  
\text{(A2-26)}

\[ A_{da} \mathbf{\hat{A}}_{ad} \mathbf{p}^2 = \begin{bmatrix} L_A \mathbf{p}^2 \\ L_B \mathbf{p}^2 \\ L_B \mathbf{p}^2 \\ L_B \mathbf{p}^2 \end{bmatrix} \]  
\text{(A2-27)}

where

\[ L = \begin{bmatrix} L_I & L_{II} & L_{II} \\ L_{II} & L_I & L_{II} \\ L_{II} & L_{II} & L_I \end{bmatrix} \]  
\text{(A2-28)}
\[ L = L_0 + L_{RE} + LSE \]
\[ L_I = L_{os} + 2L_{aa} \]
\[ L_{II} = (-1/2)L_{os} + 2L_{ab} \]
\[ L_{os} = (L_d + L_q)/2 \]

and

\[ A_{da} L = \sqrt{2/3} \cdot \begin{array}{ccc}
\sqrt{\frac{1}{2}} L_A & \sqrt{\frac{1}{2}} L_A & \sqrt{\frac{1}{2}} L_A \\
 c L_B & c' L_B & c'' L_B \\
-s L_B & -s' L_B & -s'' L_B
\end{array} \]

where

\[ L_A = L_I + 2L_{II} \]
\[ L_B = L_I - L_{II} \]

The third term of the right hand side of equation (2-9) expanded as in the form of equation (A2-6) consists of

\[ A_{da} \ddot{B}_{ad}(\theta) \dot{\theta}^2 = \begin{array}{ccc}
0 & 0 & 0 \\
0 & -3/2L_{2s} \dot{\theta}^2 & 0 \\
0 & 0 & 3/2L_{2s} \dot{\theta}^2
\end{array} \]

\[ A_{da} B_{ad}(\theta) \ddot{\theta} = \begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 3/2L_{2s} \dot{\theta} \\
0 & 3/2L_{2s} \ddot{\theta} & 0
\end{array} \]
\[ 2A_{da} B_{ad} \dot{\phi}_p = \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 3I_{2s} \dot{\phi}_p & 0 \\ 0 & 0 & -3I_{2s} \dot{\phi}_p \end{array} \quad (A2-32) \]

\[ A_{da} B_{ad} (\theta) p^2 = \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 3/2I_{2s} p^2 & 0 \\ 0 & 0 & -3/2I_{2s} p^2 \end{array} \quad (A2-33) \]

And the results of the last two terms are

\[ A_{da} (1/C_s) A_{ad} i_3' = \begin{array}{ccc} 1/C_s & 0 & 0 \\ 0 & 1/C_s & 0 \\ 0 & 0 & 1/C_s \end{array} \quad i_{30} \quad (A2-34) \]

\[ A_{da} p A_{ad} v_3' = [A_{da} \dot{A}_{ad} \ddot{\phi} + A_{da} A_{ad} \ddot{\theta}] v_3' \]

\[ = \begin{array}{ccc} p & 0 & 0 \\ 0 & p & -\ddot{\phi} \\ 0 & \ddot{\phi} & p \end{array} \quad v_{30} \quad v_{3d} \quad (A2-35) \]

Sum up all the terms results in equation (2-10)
For a steady state analysis the effect of amortisseur windings may be neglected leaving a 3Ø symmetrical winding on the stator and a single field winding on the rotor.

From Yu's formula\textsuperscript{20,21} the impedance matrix can be written as equation (A3-1). 

Let the internal induced voltages be

\[
\begin{array}{c|ccc}
 & e_a & e_b & e_c \\
\hline
e_a & - pL_d F \cos \theta \\
e_b & - pL_d F \cos (\theta - 2/3\pi) \\
e_c & - pL_d F \cos (\theta - 4/3\pi)
\end{array}
\]

\[ I_f \quad (A3-2) \]
<table>
<thead>
<tr>
<th>Z(p)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_0 + p(L_\text{os} + L_2 \cos \theta) )</td>
<td>( p \left{ - \frac{L_\text{os} + L_2 \cos (2\theta - 120)}{2} \right} )</td>
<td>( p \left{ \frac{L_\text{os} + L_2 \cos (2\theta - 240)}{2} \right} )</td>
<td>( pL_d \cos \theta )</td>
</tr>
<tr>
<td>b</td>
<td>( p \left{ - \frac{L_\text{os} + L_2 \cos (2\theta - 240)}{2} \right} )</td>
<td>( + p \left{ L_\text{os} + L_2 \cos (2\theta - 240) \right} )</td>
<td>( p(- \frac{L_\text{os} + L_2 \cos 2\theta}{2}) )</td>
<td>( pL_d \cos(\theta - 120) )</td>
</tr>
<tr>
<td>c</td>
<td>( p \left{ - \frac{L_\text{os} + L_2 \cos (2\theta - 240)}{2} \right} )</td>
<td>( + p \left{ \frac{L_\text{os} + L_2 \cos (2\theta - 120)}{2} \right} )</td>
<td>( + p \left{ L_\text{os} + L_2 \cos (2\theta - 120) \right} )</td>
<td>( pL_d \cos(\theta + 120) )</td>
</tr>
<tr>
<td>F</td>
<td>( pL_d \cos \theta )</td>
<td>( pL_d \cos(\theta - 120) )</td>
<td>( pL_d \cos(\theta + 120) )</td>
<td>( R_F + pL_F )</td>
</tr>
</tbody>
</table>

(A3-1)
With reference to Fig. 1.2 the synchronous machine voltage equation may be written as

\[ e = Z_0(p)i_1 + v_1 \]  \hspace{1cm} (A3-3)

where

\[
\begin{align*}
e &= \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \\
v_1 &= \begin{bmatrix} v_{1a} \\ v_{1b} \\ v_{1c} \end{bmatrix}
\end{align*}
\]

and

\[
\begin{array}{ccc}
  \text{a} & \text{b} & \text{c} \\
  \hline
  R_{os} & \frac{L_0 + L_2 \cos \theta}{2} & \frac{L_0 + L_2 \cos (2\theta - 120)}{2} \\
  + p(L_0 + L_2 \cos (2\theta - 240)) & + p(L_0 + L_2 \cos (2\theta - 240)) & + p(L_0 + L_2 \cos (2\theta - 120)) \\
  Z_0(p)_a & \p(\frac{L_0 + L_2 \cos (2\theta - 240)}{2}) & \p(\frac{L_0 + L_2 \cos (2\theta - 120)}{2}) \\
  Z_0(p)_b & \p(\frac{L_0 + L_2 \cos (2\theta - 240)}{2}) & \p(\frac{L_0 + L_2 \cos (2\theta - 120)}{2}) \\
  \hline
  R_{os} & \p(\frac{L_0 + L_2 \cos (2\theta - 120)}{2}) & \p(\frac{L_0 + L_2 \cos (2\theta - 120)}{2}) \\
  & + p(L_0 + L_2 \cos (2\theta - 240)) & + p(L_0 + L_2 \cos (2\theta - 120))
\end{array}
\]

For the steady state operation, \( p=0 \)
\( \Theta = wt, \Theta = \omega \) and \( I_f \) is constant. Differentiating and substituting these conditions into (A3-2) yields

\[
\begin{array}{c|c}
   e_a & \frac{\omega}{\sqrt{2} L_d F} \sin \Theta \\
   e_b & \frac{\omega}{\sqrt{2} L_d F} \sin(\Theta - 2\pi/3) \\
   e_c & \frac{\omega}{\sqrt{2} L_d F} \sin(\Theta + 2\pi/3)
\end{array}
\]

(A3-6)

Applying d-q co-ordinate transformations one has

\[
\begin{array}{c|c|c|c}
   e_o & \sqrt{2/3} & \sqrt{2/3} & \sqrt{2/3} \\
   e_d & \cos \Theta & \cos(\Theta - 120) & \cos(\Theta + 120) \\
   e_q & -\sin \Theta & -\sin(\Theta - 120) & -\sin(\Theta - 120)
\end{array}
\]

Therefore

\[
\begin{array}{c|l}
   E_o & 0 \\
   E_d & 0 \\
   E_q & \sqrt{3/2} (\frac{\omega}{\sqrt{2} L_d F}) I_f
\end{array}
\]

(A3-7)

It is noted that the zero and d-components of the internal voltages are always zero for a synchronous machine with symmetrical phase windings.
APPENDIX IV

a) Illustrative Numerical Example A
230KV, 250 Mile Transmission Line

Two practical examples are given to demonstrate the applications of the theory developed in chapters 2, 3, and 4. Figure A4-1 depicts one of the practical systems investigated.

i) Machine Parameters

The electric power of the system is generated by four salient pole hydro-electric generators. Each generator has the following ratings

- Generator MVA rating = 50MVA
- Rated armature voltage = 13.8KV
- Power factor = 0.9PF

The per unit values of the parameters of the generator as referred to the machine's rated voltage and apparent power are as follows

\[
\begin{align*}
    d\text{-axis synchronous reactance} &\quad X_d = 0.876pu \\
    q\text{-axis synchronous reactance} &\quad X_q = 0.523pu \\
    \text{Armature resistance} &\quad R_o = 0.00227pu \\
    \text{Potier reactance} &\quad X_p = 0.29pu \\
\end{align*}
\]

The four synchronous machines are combined into a single equivalent synchronous machine. Based on 1000MVA and 13.8KV the equivalent machine has the following parameters.

\[
\begin{align*}
    d\text{-axis synchronous reactance} &\quad X_d = 4.38pu \\
    q\text{-axis synchronous reactance} &\quad X_q = 2.62pu \\
    \text{Armature resistance} &\quad R_o = 0.0114pu \\
    \text{Potier reactance} &\quad X_p = 1.45pu \\
\end{align*}
\]
ii) **Transmission line parameters**

The example considered is the Bridge River-Prince George line of B.C. Hydro Power Commission. It is a single circuit 230KV overhead transmission line with a rated capacity of 200MVA.

The line has a horizontal configuration. Its chief dimensions are as follows:

- **Conductors:** 927 MCM (thousand circular miles) ASC (Aluminium Stranded Conductor)
- **Spacing between Phase Conductors:** 18 ft
- **Ground Wires on top:** Two
- **Mean Height of Conductors:** 70 ft
- **Length of Line:** 250.4 Mile

iii) **Transformers**

The sending end transformers are oil immersed forced air cooled type with three phase star and grounded neutral connection. Each transformer has the following ratings

- **Capacity:** 50MVA
- **Voltage:** 230/13.8KV

Each transformer has 9 per cent reactance referred to its own MVA base. There is only one receiving end transformer which has the same rating equivalent to the four transformers at the sending end.
iv) Transmission Line Parameters of the
Three Phase Equivalent-T Network

\[
\begin{align*}
\begin{array}{ccc}
\text{b} = & 2.71 & -0.94 & -0.867 \\
-0.94 & 2.79 & -0.94 \\
-0.867 & -0.94 & 2.71 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{ccc}
\mathbf{[r]}_\text{SE} = \mathbf{[r]}_\text{RE} = \frac{1}{125.2} & 0.1135 & 0 & 0 \\
0 & 0.1135 & 0 \\
0 & 0 & 0.1135 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{ccc}
\mathbf{[x]}_\text{RE} = \mathbf{[x]}_\text{SE} = \frac{1}{125.2} & 127 & 10.8 & 11.4 \\
10.8 & 127 & 21.6 \\
11.4 & 10.8 & 127 \\
\end{array}
\end{align*}
\]

The line parameters based on 1000MVA and 230KV are

\[
\begin{align*}
\begin{array}{ccc}
\mathbf{R}_\text{SE} = \mathbf{R}_\text{RE} = & 0.269 & 0 & 0 \\
0 & 0.269 & 0 \\
0 & 0 & 0.269 \\
\end{array}
\end{align*}
\]
FIG. A4-1 ONE LINE DIAGRAM OF THE 250 MILE POWER SYSTEM

FIG. A4-2 ONE LINE DIAGRAM OF THE 600 MILE POWER SYSTEM
\[ X_{SE} = X_{RE} = \begin{array}{ccc} 
2.4 & 0.211 & 0.211 \\
0.211 & 2.4 & 0.211 \\
0.211 & 0.211 & 2.4 \\
\end{array} \text{pu} \]

\[ W_{c1} = \begin{array}{ccc} 
0.1446 & -0.0479 & -0.0479 \\
-0.0479 & 0.1446 & -0.0479 \\
-0.0479 & -0.0479 & 0.1446 \\
\end{array} \text{pu} \]

The transmission line is transposed and hence electrically symmetrical.

b) ILLUSTRATIVE NUMERICAL EXAMPLE B

500KV, 600 Mile Transmission Line

Fig. A4-2 shows a 2390MVA equivalent synchronous machine connected to an infinite bus bar through a 500KV, 600 mile double circuit transmission line and transformers. The diagram is a simplified version of B.C. Hydro and Power Commission planned Peace River EHV transmission system.\(^{22}\) The per unit parameters of each of the ten machines based on 239MVA and 13.8KV are given as

- \( R_o \) armature resistance = 0.00247pu
- \( X_p \) Potier reactance = 0.165pu
- \( X_q \) q-axis synchronous reactance = 0.55 pu
- \( X_d \) d-axis synchronous reactance = 0.973pu

The per unit values of an equivalent single machine based on 10,000MVA and 13.8KV are
\[ R_o = 10.247 \times 10^{-4} \text{pu} \]
\[ X_p = 6.845 \times 10^{-2} \text{pu} \]
\[ X_q = 22.8217 \times 10^{-2} \text{pu} \]
\[ X_d = 40.3737 \text{pu} \]

The reactance of the equivalent single transformer at the sending end or receiving end is 0.542 per unit.

The conductor assembly per phase for each transmission line in Fig. A4-2 consists of four ACSR conductors in an 18 inch square bundle. Each transmission line has the following dimensions:

- Sub-conductor size: 583.2MCM ACSR
- Sub-conductor d-c resistance: 0.1764 ohm/mile
- Conductor diameter (including 0.948 in stranding factor)
- Average Phase Spacing: 40 ft
- Average Height above ground: 54 ft

The lines are transposed and each has the following parameters.

\[ L_1 = \begin{bmatrix}
0.283 & 0.283 & 0.283 \\
0.283 & 1.63 & 0.283 \\
0.283 & 0.283 & 1.63 \\
\end{bmatrix} \text{ mh/mile} \]
Since two lines are connected in parallel the equivalent inductance and capacitance are given as

\[ L = \frac{1}{2L_1} \quad \text{and} \quad C = 2C_1 \]

Hence the per unit reactance

\[ X_{RE} = X_{SE} = \]

and the per unit admittance
Resistance of one bundle = 0.0441 /mile
Resistance per phase = 0.0221 /mile
Hence the resistance matrix in per unit value

\[
\begin{bmatrix}
0.2094 & -0.0284 & -0.0284 \\
-0.0284 & 0.2094 & -0.0284 \\
-0.0284 & -0.0284 & 0.2094 \\
\end{bmatrix}
\]

\[
R_{SE} = R_{RE} = 
\begin{bmatrix}
0.2652 & 0 & 0 \\
0 & 0.2652 & 0 \\
0 & 0 & 0.2652 \\
\end{bmatrix}
\]

pu
REFERENCES


9. Ibid., December 1938 p 545.


