FLUCTUATIONS IN THE PRICES OF CANADIAN COMMON STOCKS AND THE RANDOM WALK HYPOTHESIS

by

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ABSTRACT

Chairman: Professor G. David Quirin

A stock market speculation scheme proposed by Sidney S. Alexander was tested and was found to be more profitable than would be predicted on the assumption that stock prices perform a random walk with a Gaussian distribution and linear trend. The difference between empirical and theoretical gains was found to be statistically significant.

By means of an F-ratio, it was demonstrated that the statistical estimate of the variance of the population of price fluctuations tends to increase as the size of sample is increased. This is consistent with the assumption that the population actually has an infinite variance. but is not consistent with the behavior which would be predicted by any variety of Gaussian random walk.

The sophisticated quantity theory of money was found to be unable to account for any non-random movements in stock market price indices.

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CHAPTER I

INTRODUCTION

This thesis investigates the ability of the random walk hypothesis to account for fluctuations in the prices of Canadian common stocks.

If the random walk hypothesis were true, past changes in the price of a common stock would have no power whatever to predict future price changes, and any speculation scheme wherein the timing of stock market transactions is based solely on past price levels or fluctuations would be worthless, even on a before-commission basis. This is consistent with the assumption of a perfect market.

In Chapter II, the actual gains produced by a speculation scheme proposed by Sidney S. Alexander are compared to the gains predicted on the assumption of a random walk with Gaussian distribution and linear trend. In Chapter III, an empirical investigation is made to determine whether the probability distribution of common stock price changes is Gaussian. In Chapter IV, the ability of the sophisticated quantity theory of money to explain any possible success of the Alexander scheme is tested.

Professional stock market analysts generally believe that there are certain currently knowable facts which generate stock price trends, so that a speculator who knows these facts and how to interpret them will likely be guided into profitable transactions.

The "fundamental" school[1] of analysts seeks to obtain the necessary facts from sources outside the stock market itself, usually studying general business conditions and attempting to forecast the prospects in various industries.

The "technical" school[2] hopes to acquire enough of the necessary facts from immediate past movements in the price of the stock or commodity under study. Both schools assume that the market does not immediately re-assess the intrinsic value of a security as new information becomes available, but instead adjusts rather slowly.

Economists generally have not accepted the existence of even partly systematic changes in the prices of stocks. They tend to view the stock market as one of the real phenomena which come closest to fulfilling the assumptions of their theoretical model, the perfect market. They tend to believe that stocks trade at a price such that the expectation of gain to the buyer (in excess of his opportunity costs) is zero. Since security prices are, according to this theory, priced in such a way that the expected gains are zero, price changes will come about only as new information becomes available, altering the market's expectations. Such information, if truly "new", is unpredictable in content. Therefore the market's reactions cannot be predicted before-hand.

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1. See. for example, Benjamin Graham, David L. Dodd, and Sidney Cottle, Security Analysis (4th ed.; New York: McGraw-Hill Book Co. Inc., 1962).

2. See, for example, Robert D.Edwards and John Magee, Technical Analysis of Stock Trends (4th ed.,Springfield,Mass: by John Magee,1958).

As Alexander points out,

> The professional analysts would certainly not subscribe to the notion that the best picture of the future movements of prices can be gained by tossing a coin or a set of coins. Yet that is just what the academic students of speculative markets have come to say is the best way. The academic students of speculative markets have come to deny the very existence of trends in speculative prices, claiming that where trends seem to be observable, they are merely interpretations, read after the fact, of a process that really follows a random walk. A price can be said to follow a random walk if at any time the change to be expected can be represented by the result of tossing a coin, not necessarily a 50-50 coin, however. In particular, a random walk would imply that the next move of the speculative price is independent of all past moves or events.[1]

The most general statement of the random walk model is that the price of any security follows a first-order Markov process.[2]

Not all academic students of the stock market, however, accept the random walk model. For example, Cootner contends that one of the pre-requisites of a perfect market, equal knowledge among all buyers and sellers, is not even approximately attained.

> Many investors are. . .engaged in other occupations in which they have a comparative advantage, so it is very costly, at least in terms of opportunity cost per unit of relevant information uncovered, for them to devote time to the relevant kind of stock market research. . . Now let me introduce another group of investors and speculators who specialize in the stock market. As professionals, their opportunity cost of research is much less than that of the uninformed (largely because they know <u>what</u> to look for and where),

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1. Sidney S. Alexander, "Price Movements in Speculative Markets: Trends of Random Walks?" <u>Industrial Management Review</u>, II (May, 1961), pp.7-26; cited in Paul H. Cootner (ed.), <u>The Random Character of Stock Market Prices</u> (Cambridge: The M.I.T.Press, 1964), p.200.

2. In a Markov process a variable can assume any one of several values. The process is said to be first-order if the probabilities of transition from the current value to each possible value are all specified entirely by the current value of the variable, i.e., there is no "residue of influence" from values the variable has assumed in the past.

> but it is, nevertheless, non-negative. They do have an idea of what is going to happen, but they cannot profit from it unless the current price deviates enough from the expected price to cover their opportunity costs. Their profits will come from observing the random walk of the stock prices produced by the non-professionals until the price wanders sufficiently far from the expected price that they can expect future surprises to force prices toward their mean more often than not. Competition among these professionals will tend to restrict the potential profit to the opportunity costs. Furthermore, they must recognize the possibility of error in their own forecasts and must recognize that even if they buy the stock at a favourable price the actual rate at which the stock appreciates or depreciates will be governed in fact by the random rate of approach to the expected price. . . Prices will behave as a random walk with reflecting barriers.[1]

It is conceivable that the non-professional investors and speculators might, for psychological reasons, play a game of "follow-the-leader" as it were, so that within the reflecting barriers, price movements would not be a random walk. In such a case, Alexander's scheme, as outlined on pp. 7-8 would be expected to be profitable on a before-commission basis, especially for small "filters."[2]

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1. Paul H. Cootner, "Stock Prices: Random vs Systematic Changes," Industrial Management Review, III (Spring, 1962), pp. 24-45; cited in Paul H. Cootner (ed.), The Random Character. . .,pp.233-234.

2. The term "filter" has been borrowed from communications engineering see, for example, R.B.Blackman and J.W.Tukey, The Measurement of Power Spectra. (New York: Dover Publications, Inc., (1958).

The simplest random walk model asserts that if today's price of a stock, P_t, is known, then tomorrow's price P_{t+1} is given by

$$P_{t+1} = P_t + u \qquad (1a)$$

where u is a random Gaussian[1] variable with mean zero. The price at time t+T will be given by

$$P_{t+T} = P_t + v\sqrt{T} \qquad (1b)$$

where v is a random Gaussian variable with the same mean and variance as u.

The model can be altered by assuming that the expected price of the stock in the future is a linear, increasing function of time. In this case,

$$P_{t+1} = P_t + g + u \qquad (1c)$$

$$P_{t+T} = P_t + gT + v\sqrt{T} \qquad (1d)$$

where g is the expected rate of growth of the price of the stock in dollars per unit time.

Other random walk models have been proposed, for example,

$$P_{t+1} = P_t(1 + w) \qquad (2)$$

where w is a Gaussian variable with a mean of zero; and

$$\log(p_{t+1}) = \log(p_t) + x \qquad (3)$$

where x is also a Gaussian variable whose mean is zero.

The model represented by equations (1c) and 1d) is to be tested.

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1. The term "Gaussian" and "normal" are synonomous.

Bachelier[1] conducted the first known scientific study of stock market behavior. He tested the hypothesis that stock prices were generated by a purely random mechanism. He found that the data fitted this hypothesis very well. He speculated that the statistical population from which stock price changes are randomly selected would have a normal distribution.

Kendall[2] believed the random walk hypothesis to be false and tested a number of time series--wheat futures, cotton futures, etc., and found, to his surprise, that the random walk hypothesis worked well, except for one series, New York spot cotton prices. Alexander[3] demonstrated that the anomalous result in the New York spot cotton spot prices series was due to the use of time-averaged data, and that mutatis mutandis the random walk hypothesis was completely vindicated.

Cowles and Jones[4] used a different method of testing for non-randomness. A price rise followed by a price rise or a price fall followed by a price fall was designated a sequence; and a rise followed by a fall, or vice versa was designated a reversal. It was concluded that the ratio of reversals to sequence was significantly greater than

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1. M.Louis Bachelier, Theorie de la Speculation (Paris: Gauthier-Villars, 1900)

2 M.G.Kendall,"The Analysis of Economic Time Series - Part I: Prices," Journal of the Royal Statistical Society (Series A), XCVI (1953),pp.11-25.

3. Alexander, op.cit.

4. Alfred Cowles, and Herbert E. Jones, "Some A Posteriori Probabilities in Stock Market Action," Econometrica, V (July, 1937), pp.280-94.

would be expected on a random walk basis. However, in a later paper Cowles[1] realized that in the previous work the error of using time-averaged data (as Kendall did in working with New York spot cotton prices) had been made. Modifying their results accordingly, they still found them to be statistically significant. Alexander[2] pointed out that Cowles had failed to adjust for trend, and that mutatis mutandis the new results were no longer significantly different from predictions based on the random walk model. Thus the random walk was again vindicted.

Mandelbrot[3] produced evidence to indicate that the error term in equations (1c) and(1d)does not have a Gaussian distribution. He concluded that the probability distribution of the error term was stable Paretian--a distribution with infinite variance, for which no statistical methods of hypothesis testing have been developed. The logarithms of the characteristic function of Pareto distribution is given by

$$\log f(t) = \log \int_{-\infty}^{+\infty} \exp(iut)\, d\, P(\tilde{U} < U)$$
$$= i\delta t - \gamma |t|^{a} \left[1 + i\, b\, (t / |t|) \tan(a\pi/2)\right]$$

where a, b, γ and δ are four parameters whose value specify the member of the Pareto family of distributions. An explicit expressions for the probability density functions have never been derived, except for special cases. Mandelbrot goes further, and attacks the Gaussian assumption made almost universally throughout econometric work. If he is correct,

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1. Alfred Cowles, "A Revision of Previous Conclusions Regarding Stock Price Behavior", Econometrics, XXVIII (October,1960) pp.909-915.

2. Alexander, op.cit.

3. Benoit Mandelbrot, "The Variation of Certain Speculative Prices," Journal of Business, XXXVI (October,1963),pp.394-419.

nearly all work in econometrics will probably have to be revised. The Paretian hypothesis is invoked to account for the frequently observed phenomenon of leptokurtosis.

Alexander[1] speculated that while the evidence makes it abundantly clear that stock prices are random with respect to time, they may nevertheless be non-random with respect to the "move." He postulated that when the price of a stock has risen to $K there may be a slight amount of inertia which would make it possible that the price will attain a level of $(K + x) before it attains the level $(K - x). Similarly, if a price drop has taken place, to $L, it is probable that the price $(L - x) will occur sooner than the price $(L + x). He devised a scheme which would take advantage of this inertia.

Suppose that a speculator owns shares in a stock which has dropped y% in value from a previous peak. He will, following this scheme, sell his holdings and sell short an equal number of shares. He will continue in his short position until the stock's price hits a trough and then rises y%. It is now time for the speculator to terminate his short position and buy in again. He will continue indefinitely in this manner. Hereafter, the size of y will be designated the "filter size."

On testing this scheme originally, Alexander was immensely pleased with the results. His findings indicated before-commission returns on one filter of the order of thirty-six per cent annually.[2]

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1. Alexander, op. cit.

2. .5% filter, 1929-1959.

However, in a subsequent paper,[1] he revised his findings to eliminate certain biases inherent in his original approach, and found that while the scheme was considerably less profitable than suggested earlier, it was still more profitable than buying and holding, on a before-commission basis, for small filters. The data used were Dow-Jones and Standard and Poor's industrial averages, treated as if a single stock.

McElroy[2] tested the scheme on active Toronto penny mining stocks and found that it yielded a positive return on a before-commission basis. However, no attempt was made to compare the actual returns with those which would be predicted by some random walk model. No evidence has been brought forth to determine whether the profitability of the Alexander scheme is attributable to non-random movements of the stock market as a whole, or whether the non-random behavior of the indices merely reflects independent non-random components of the fluctuations in the prices of individual stocks. If one attributes the success of the scheme to inertia, one would subscribe to the latter view.

If the former view is correct, then one should expect that the "causes" of the profitability of Alexander's scheme would be external and work on stock prices as a whole. This would support the views of the "fundamental" analysts. On the other hand, if the Alexander scheme

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1. Sidney S. Alexander, "Price Movements in Speculative Markets; Trends or Random Walks, No. 2," The Random Character. . ., ed. Paul H. Cootner, pp. 338-72.

2. R. Duke McElroy, "A Study of the Price Action of an Active Mining Stock On the Toronto Stock Exchange" (unpublished B. Comm. graduating essay, Faculty of Commerce and Business Administration, University of British Columbia, 1963).

works just as well on individual stocks as on price indices, the position of the "technical" analysts would appear to be strengthened, since non-randomness would appear to be an intrinsic factor in the movements of individual stocks, and price movements in the immediate past would be shown to provide an indication as to the direction of stock price movements in the immediate future.

CHAPTER II

AN EMPIRICAL TEST OF THE RANDOM WALK HYPOTHESIS.

Of all the Canadian common stocks listed on stock exchanges which, according to the Financial Post, were selling at a price of forty-five or more dollars per share at the end of 1954, twelve were chosen at random. Of these, one had to be dropped because the required information was not readily available. Also, one other, Canadian Westinghouse Co., Ltd., was studied only from 1959 on, as it acquired a listing on the Toronto Stock Exchange only in that year, and data on previous prices were not readily available on a daily basis.

In order to determine buy and sell points for the Alexander scheme, the price range of each week for each stock was obtained from the Financial Post. Occasionally there was a certain ambiguity when it was desirable to know whether the weekly high price preceded or followed the weekly low price of a stock in time. Whichever of the two was closer to the closing price was assumed to have occurred later; for example, if the quotation were High - $55, Low - $50, Close - $53; it would be assumed that the low price preceded the high price. This rule does not remove all ambiguity. For a comprehensive discussion of the conventions used to resolve ambiguities, see Appendix A <u>infra</u>. For purposes of locating buy and sell points, only transaction prices were considered--if no transaction took place during a certain week, then no buy or sell point was recognized, regardless of the level of the bid and ask quotations. The prices at which the speculator's transactions

are assumed to take place are the closing price on the Monday following the buy or sell point, as reported in the Toronto Globe and Mail, or, if no shares were traded on that Monday, the average of the quoted bid and ask prices was assumed to be the speculator's transaction price. If one is to attempt to determine whether the price of a relatively illiquid stock exhibits a random walk over time, one must first decide what is meant by "the price" of a stock on a day when none has been traded; here it is defined as the average of the reported bid and ask price. The scheme was tested for the years 1955 to 1963 inclusive, on a before-commission basis.

Since brokerage fees are much smaller proportionately for high-priced stocks than penny stocks, the perfect market's prerequisite of an exchange where transactions can take place without commissions comes closer to being attained. Therefore, one might expect the random walk model to give better predictions of empirical results.

The gain on a buy-and-hold basis over the period of the scheme, and the number of weeks in which the speculator maintained a long position on a particular stock (or a short position) are regarded as autonomous variables in a statistical test to be derived. The object of the test is to predict the profitability of any speculation scheme involving long and short positions whatever, on the basis of the random walk model with linear trend and variance independent of current price, given the values of the exogenous variables. Formulae for the expected gain and the variance thereof are to be derived.

The particular random walk model chosen is represented by the equation

$$P_{t+T} = P_t + gT + v\sqrt{T} \qquad (1d)$$

The expected value of P_{t+T} is given by

$$E(P_{t+T}) = P_t + gT + E(v)\sqrt{T}$$
$$= P_t + gT$$

Similarly, if the speculator had sold short at time t, his accrued losses by time t + T would be gT. In general, the expected gains can be written as KgT over time interval T, where K takes the value 1 for a long position, and -1 for a short position. If T_i denotes the time interval between the ith and the (i+1)st transaction, then the expected gain for n+1 transactions can be written as

$$\sum_{i=1}^{n} g K_i T_i = g \sum_{i=1}^{n} K_i T_i = g(T_{long} - T_{short}) \qquad (4)$$

where T_{long} is the total amount of time in which the speculator held a long position between the first and the (n+1)st transaction, and T_{short} is the balance of the elapsed time.

The rate of growth, g, an autonomous variable, can be computed from the equation

$$g = \frac{P_{n+1} - P_1}{\sum_{i=1}^{n} T_i} = \frac{P_F - P_I}{T_{long} + T_{short}} \qquad (5)$$

where P_{n+1} is understood to be the price of the (n+1)st transaction. Substituting into (4), the expected gain is found to be

$$\frac{T_{long} - T_{short}}{T_{long} + T_{short}} (P_F - P_I) \qquad (6)$$

over the duration of the scheme, Note that $P_F - P_I$ is the other autonomous variable, the gain on a buy-and-hold basis.

It is necessary to introduce a new notation in order to derive the formula for variance. The quantity $u_{i,k}$ is the value of u as defined in equation (1c) for the time interval between k and k+1 time units (weeks) after the origin, where the origin is understood to be the time of occurence of the ith transaction. The quantity v_i is the value of v as defined in equation (1d) for the time interval between the ith and the (i+1)st transaction. T_i is the amount of time elapsing between the ith and the (i+1)st transaction. Thus equation (1a) becomes

$$P_{i,k+1} - P_{i,k} = \delta + u_{i,k}$$

Obviously, $P_{i,T_i} = P_{i+1}$, so summing over k, the result is

$$\sum_{k=0}^{T_i-1} (P_{i,k+1} + P_{i,k}) = \delta T_i + \sum_{k=0}^{T_i-1} u_{i,k} \qquad (7)$$

In this notation, equation (1d) becomes

$$P_{i+1} - P_i = v_i \sqrt{T_i} \qquad (8)$$

From (7) and (8)

$$v_i \sqrt{T_i} = \sum_{k=0}^{T_i - 1} u_{i,k} \qquad (9)$$

Over any one time unit, the actual earnings exceed the predicted earnings by $K_i u_{i,k}$. Thus in determining the dispersion of forecasting error, a convenient statistic to use is s_{Ku}^2, an estimate of the variance. The computational formula is to be derived. From (9)

$$\overline{u_i} = \frac{\sum_{k=0}^{T_i-1} u_{i,k}}{T_i} = \frac{v_i\sqrt{T_i}}{T_i} = \frac{v_i}{\sqrt{T_i}}$$

$$\overline{K_i u_i} = \frac{K_i v_i}{\sqrt{T_i}} \qquad (10)$$

Rearranging (8) one obtains

$$v_i = \frac{P_{i+1} - P_i - gT_i}{\sqrt{T_i}}$$

so that

$$\overline{K_i u_i} = \frac{K_i}{\sqrt{T_i}} \times \frac{p_{i+1} - p_i - gT_i}{\sqrt{T_i}} = K_i\left(\frac{p_{i+1} - p_i}{T_i} - g\right)$$

If X is a random variable with a Gaussian distribution, and known mean but unknown variance, and the data provide only the means of various empirical samples of X, then the Variance of X can be estimated as $S_X^2 = \frac{\sum_{i=1}^{n} m_i \left[\bar{X}_i - E(X)\right]^2}{n}$

where m_i is the number of elements in sample i and E(X) is the mean of X.

The statistic of interest is s_{Ku}^2 given data for K_i, v_i, and T_i and the hypothesis that E(Ku) = 0. Thus,

$$S_{Ku}^2 = \frac{\sum_{i=1}^{n} T_i\left(\frac{K_i d_i}{T_i} - K_i g - 0\right)^2}{n} = \frac{\sum_{i=1}^{n} \frac{K_i^2 d_i^2}{T_i} - \sum_{i=1}^{n} T_i K_i g^2}{n}$$

$$= \frac{\sum_{i=1}^{n} \frac{d_i^2}{T_i} - \frac{(P_F - P_I)^2}{T_{long} + T_{short}}}{n}$$

where d_i is defined to be $P_{i+1} - P_i$. It should be noted that g was not obtained independently, so we have one less degree of freedom. The denominator becomes n-1, the number of degrees of freedom so that

$$S_{Ku}^2 = \frac{\sum_{i=1}^{n} \frac{d_i^2}{T_i} - \frac{(P_F - P_I)^2}{T_{long} + T_{short}}}{n-1}$$

What is desired is a measure of the standard error of estimate for the random walk's predicted gains over the whole scheme, or of the statistic x where $x = \sum_{i=1}^{n} \left(\sum_{k_i=0}^{T_i-1} u_{i,k} \right)$

It can be shown[1] that s_x is equal to $\sqrt{s_K^2 \, u \sum_{i=1}^{n} T_i}$

The hypothesis that the expected value of the gain on any stock is in fact equal to the value predicted by the random walk can be tested by the use of the t - distribution; accept the hypothesis if $x - t_{.025} s_x < \mu < x + t_{.025} s_x$ where x is the observed gain and μ is the predicted gain, using a 95 per cent confidence interval.

If there are enough degrees of freedom, t can be approximated by z, of the Gaussian distribution. Since $z_{.025} = 1.96$, the confidence interval becomes

$$x - 1.96\, s_x < \mu < x + 1.96\, s_x$$

During the period the Alexander scheme was tested, there were two mergers of firms studied. The Imperial Bank of Canada was merged with the Canadian Bank of Commerce to form the Canadian Imperial Bank of Commerce, and Western Grocers was bought out by George Weston.

It turned out that of the eleven stocks, six were traded virtually every day, while five were traded no more than perhaps three out of four days. The stocks have been classed as "liquid" and "illiquid", respectively.

1. $\Sigma X = n\bar{X}$; $S^2_{\Sigma x} = S^2_{\overline{nx}} = n^2 S^2_{\bar{x}}$; $S^2_x = n\, s^2_{\bar{x}}$; $S^2_{\Sigma x} = n\, s^2_x$; $S_{\Sigma x} = \sqrt{n\, s^2_x}$

The names of the stocks studied are: Algoma Steel Corp.Ltd., Imperial Bank of Canada,/Canadian Imperial Bank of Commerce, Canada Wire and Cable Co. Ltd. (Class B), Canadian Westinghouse Co., Ltd., Hinde and Dauch Ltd., International Nickel Co. of Canada Ltd., Hiram Walker - Gooderham and Worts Ltd., Western Grocers Ltd.,/ George Weston Ltd. (Class B), and McIntyre Porcupine Mines Ltd.

TABLE I

ILLIQUID STOCKS

Stock	Filter %	Gain on Buy-and-Hold	Gain on Buy-and-Hold ÷ Initial Price.	Adjusted Final Price	Initial Price	Gain on Scheme ÷ Initial Price	Gain on Scheme
Canada Wire & Cable B	3	-$ .50 *	- .00735	$236.00	$ 68.00	.294	$ 20.00
	4	- .50	- .00735	236.00	68.00	1.594	108.38
	5	- .50	- .00735	236.00	68.00	3.570	242.75
	8	- 5.50	- .0735	236.00	73.00	1.022	74.62
Canadian Westinghouse	3	- 17.75	- .378	29.25	47.00	- .0745	- 3.50
	4	- 17.75	- .378	29.25	47.00	- .492	- 23.12
	5	- 17.75	- .378	29.25	47.00	.112	5.25
	8	- 19.25	- .397	29.25	48.25	- .562	- 27.25
Crow's Nest Pass Coal	3	17.50	.156	130.00	112.50	.547	61.50
	4	17.50	.156	130.00	112.50	1.34	151.25
	5	17.50	.156	130.00	112.50	.564	63.50
	8	18.00	.161	130.00	112.50	.196	22.00
Goodyear Tire	3	- 25.00	.157	134.00	159.00	.604	96.00
	4	- 25.00	.157	134.00	159.00	1.932	307.12
	5	- 25.00	.157	134.00	159.00	.473	75.25
	8	- 11.00	.0759	134.00	145.00	- .383	- 55.50
Hinde and Dauch Paper	3	1.75	.0315	57.25	55.50	- .309	- 17.12
	4	- 2.75	- .0458	57.25	60.00	- 1.102	- 66.12
	5	- 2.75	- .0458	57.25	60.00	- 1.133	- 68.00
	8	- 2.75	- .0458	57.25	60.00	- 1.120	- 71.75
Western Grocers & George Weston B	3	21.83	.223	119.83	98.00	1.569	153.75
	4	21.83	.223	119.83	98.00	.492	48.25
	5	21.83	.223	119.83	98.00	.189	18.50
	8	21.83	.223	119.83	98.00	- .353	34.62

* Losses are indicated by -.

TABLE II

LIQUID STOCKS

Stock	Filter %	Gain on Buy-and-Hold	Gain on Buy-and-Hold + Initial Price	Adjusted Final Price	Initial Price	Gain on Scheme + Initial Price	Gain on Scheme
Algoma Steel	3	$184.75	3.60	$236.00	$51.25	1.620	$83.00
	4	184.75	3.60	236.00	51.25	1.044	53.50
	5	184.75	3.60	236.00	51.25	.448	23.00
	8	184.75	3.60	236.00	51.25	3.492	179.00
Imperial Bank/Cdn Imperial B.of.Comm.	3	28.83	.625	74.96	46.12	.537	24.79
	4	28.83	.625	74.96	46.12	.848	39.10
	5	28.83	.625	74.96	46.12	.628	28.98
	8	28.83	.625	74.96	46.12	- .583	- 26.88
International Nickel	3	72.00	1.274	128.50	56.50	- .686	- 38.75
	4	72.00	1.274	128.50	56.50	- 2.232	-126.12
	5	66.00	1.056	128.50	62.50	.888	55.50
	8	72.00	1.274	128.50	56.50	5.270	297.75
Hiram Walker - Gooderham & Worts	3	115.38	1.703	183.12	67.75	- 1.594	-108.00
	4	115.38	1.703	183.12	67.75	- 1.166	- 79.00
	5	111.62	1.561	183.12	71.50	- .126	- 9.00
	8	111.62	1.561	183.12	71.50	- 3.290	-235.25
George Weston B	3	- 2.25	- .0638	33.00	35.25	.744	26.25
	4	- 2.25	- .0638	33.00	35.25	.145	5.12
	5	- 2.25	- .0638	33.00	35.25	.773	27.25
	8	- 2.25	- .0638	33.00	35.25	- .248	- 8.75
McIntyre Porcupine Mines	3	97.00	1.366	168.00	71.00	1.299	92.25
	4	97.00	1.366	168.00	71.00	4.993	70.50
	5	96.00	1.333	168.00	72.00	.635	45.75
	8	96.00	1.333	168.00	72.00	.483	34.75

TABLE III

EXPECTED AND OBSERVED GAINS - 8% FILTER

Stock	T_{long}	T_{short}	Gain on Buy-and-Hold	Expected gain	Actual gain	Actual-Expected ÷ Initial Price	n
Algoma	325	142	$184.75	$58.15	$179.00	2.553	49
Imperial	272	195	28.83	4.75	- 26.88	-1.097	41
Inter.Nic.	307	160	72.00	22.66	297.75	4.869	40
Walker	341	108	111.62	59.44	-235.75	-4.129	29
Weston	236	185	- 2.25	- .27	- 8.75	- .241	58
McIntyre	245	204	96.00	8.77	34.75*	.361*	55

* It turned out that for the 8% filter, all of the stocks had transactions listed in the Toronto Globe and Mail for every day a transaction was required by the Alexander scheme, with only one exception. There was one day when no shares of McIntyre Porcupine were traded. The table assumes that the transaction could be executed at the average of the bid and ask prices. If, however, the convention were adopted that sales would be executed at bid prices and purchases would be executed at ask prices, the "actual gain" for McIntyre would be $31.75, and the quantity "(actual-expected) ÷ initial" would be .319.

A stock (International Nickel) was chosen at random, and the value of $s_{Ku}{}^2$ was calculated. It turned out to be 20.06 and s_x turned out to be $96.78. The 95% confidence limits are from - $167.03 to $212.35. The actual gain was $297.75, which is well outside the 95% confidence limits. Thus the random walk model represented by equations (1c) and (1d) must be rejected.

One might be somewhat skeptical of a statistical test based on what turned out to be the most lucrative stock of the six, even though it was

chosen randomly. However, it can be shown that the probability is .95 that none of six randomly selected samples from a population with a mean of 22.66 and a standard deviation of 96.78 will exceed 253.96. The observed gain on International Nickel, $297.75 was well outside this limit, so the random walk hypothesis must clearly be rejected.

A measure of the extent to which the random walk's predictions fall short of observed gains is to be derived. It is necessary to deflate all stocks by dividing data by the initial price to make the measurements commensurate. Furthermore, not all stocks were studied for the same number of weeks, so the results for each stock should be weighted by the number of weeks of observation. This leads to the statistic

$$\sum_{j=1}^{6}\left(\sum_{i=1}^{n_j} T_{ij}\right)\left(\frac{x_j-\mu_j}{P_{x_j}}\right)\Big/\sum_{j=1}^{6}\sum_{i=1}^{n_j} T_{ij}$$

where $\sum_{j=1}^{6}$ is a summation over all six stocks. The value of this statistic turns out to be .427, i.e. the actual gains over the period exceeded the expected gains by 42.7 % of the initial price of the stock on the average. The annual discrepancy is actually larger than 42.7 ÷ 9 (=4.74%), since the average period of study was less than nine years.

CHAPTER III

A TEST OF THE GAUSSIAN ASSUMPTION

If a probability distribution actually has an infinite variance but is assumed by a statistician to be Gaussian, the statistician will generally find that the larger his samples become, the larger is his estimate of the variance of the population from which they were drawn. It can easily be shown that s_{Ku}^2 is equal to s_u^2.

In order to determine the estimate of σ_u^2 increases, as sample size increases, the previously derived value of s_u^2 can be compared to an estimate of σ_u^2 obtained by using data from all six stocks, rather than just from International Nickel. Again, in order to make comparisons among stocks meaningful, it is necessary to deflate by dividing data by the initial price of each stock.

To test the equality of two variances, s_1^2 and s_2^2, one may use the statistic

$$F = \frac{s_1^2}{s_2^2}$$

The quantity s_1^2 is equal to the estimate about to be derived, and s_2^2 is the previously obtained value of s_u^2 for International Nickel, divided by the square of the initial price. It must be assumed that the large observed deviations from behavior predicted by the random walk model will not be such as to cause serious distortions in the value of F.

If the means of various samples of variable Y are known, the variance of Y can be estimated by the formula

$$S^2_Y = \sum_{j=1}^{r} m_{\cdot j}(\bar{Y}_j - \bar{Y}_{\cdot\cdot})^2 \Big/ (r-1)$$

where $\bar{Y}_{..} = \sum_{j=1}^{r} m_{.j} \bar{Y}_{.j} \Big/ \sum_{j=1}^{r} m_{.j}$ and $m_{.j}$ is the number of elements in sample j, Here $\sum_{i=1}^{n_j} T_{ij}$ corresponds to m_j and $(x_j - \mu_j) / p_{Ij} \sum_{i=1}^{n_j} T_{ij}$ corresponds to $\bar{Y}_{.j}$. The formula simplifies to

$$\frac{\sum_{j=1}^{r} \left[\left(\frac{x_j - \mu_j}{p_{xj}}\right)^2 \Big/ \sum_{i=1}^{n_j} T_{ij} \right] - r \sum_{j=1}^{r} \left(\frac{x_j - \mu_j}{p_{xj}}\right) \Big/ \left(\sum_{j=1}^{r} \sum_{i=1}^{n_j} T_{ij} \right)^2}{r-1}$$

the computed value of s_1^2 is .0207.

The value of s_2^2 is s_u^2 for International Nickel. This quantity was found to be .0064. Thus

$$F = .0207 \ / \ .0064 \cong 3.23$$

The numerator has five degrees of freedom, and the denominator, thirty-nine. Tables of the F-distribution list the following critical values:

$$F_{.05;\ 5,\ 38} = 2.46; \quad F_{.05;\ 5,\ 40} = 2.45; \quad F_{.01;\ 5,\ 38} = 3.54;$$
$$F_{.01;\ 5,\ 40} \cong 3.51$$

Thus the hypothesis that s_1^2 is not significantly different from s_2^2 is to be rejected at the five per cent level of confidence, but accepted at the one per cent level.

The data certainly lend weight to the contention that stock prices fluctuations are drawn from a population whose variance is infinite and thus to the stable Paretian model of Mandelbrot. Also, this casts doubt upon the validity of the statistical test in which the random walk was rejected.

Note that the large value of F cannot be attributed to the possibility that the wrong random walk model was used. If some other random walk model gave a better fit, it would produce a lower value for the denominator of the F-ratio. Then F would turn out to be even larger.

CHAPTER IV

A TEST OF THE QUANTITY THEORY APPROACH

The empirical results in Chapter II indicated that the Alexander scheme worked better than was anticipated by the random walk model with linear trend and Gaussian distribution. However, it was not as lucrative, even on a before-commission basis, as buying and holding. Alexander's second paper indicates that his scheme was more profitable on a before-commission basis than buying and holding for the period 1928 to 1961, although he does not give figures for different segments of this period. When he adjusted for trend, he found "that almost the entire profit on the detrended series is made prior to 1940."[1] Unfortunately, he did not give figures for the profitability of the scheme before detrending as compared to buying and holding for the two sub-periods January 3, 1928 to September 27, 1940 and September 28, 1940 to December 29, 1961. The latter period would have corresponded somewhat more closely to the period 1955-63 chosen for this study, and would have made possible a comparison of the scheme's profitability on stock price indices with its profitability on individual stocks, to determine whether, in addition to non-random movements in individual stock prices, there are also non-random movements in stock prices in aggregate. Since there is evidence of non-randomness, a question which arises is whether the non-randomness is related to other factors in the economy, or to the business cycle.

The quantity theory of money predicts from a theoretical framework that changes in the money supply should be a leading economic indicator.[2]

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1. Alexander, op.cit.

2. See, for example, Milton Friedman, "The Demand for Money; Some Theoretical and Empirical Results," Journal of Political Economy, LXVII (August, 1959) pp. 327-51.

Quantity theorists contend that the reason that stock prices are such a good leading indicator of the reference cycle is that both respond to changes in the money supply, but stock prices do so sooner. Thus changes in the rate of increase in the money supply should lead changes in stock price indices.

If the quantity theory is correct, one would expect non-random movements in stock prices in the aggregate, since trends in stock prices should tend to follow exogenous changes in the rate of increase in the rate of increase in the money supply. Since monetary policies are said to respond to economic data only after a long lead time, a quantity theorist would expect that there would be a series of different long trends historically in the rate of increase of the money supply. Thus if the money supply was contracted in one month, it should be probable that it would be contracted further in the following month. Since a quantity theorist contends that changes in the rate of increase in the money supply cause corresponding changes in stock prices, he might well expect the Alexander scheme to work. Inertia in stock price changes would result at least in part from inertia in the rate of increase in the money supply. The Alexander scheme would presumably work better on stock averages than on individual stocks, since random fluctuations which might swamp systematic price movements in individual stocks would largely cancel out in an aggregate index.

Macesich[1] has found that the rate of change in the Canadian monetary stock is a leading indicator of the reference cycle. The money stock was defined to include "(1) noted of chartered banks, Dominion notes, and Bank of Canada notes in public hands, plus (2) demands and "notice" deposits in chartered banks in public hands, plus (3) subsidiary coin in public hands. The term 'in public hands' is used to indicate that each series is adjusted so as to exclude the holdings by government and banks."[2]

The lead times were determined by comparing the cycle peaks and troughs, as defined by the National Bureau of Economic Research, for the money stock and the reference cycle. The National Bureau criteria are reviewed and discussed in Appendix B.

To determine whether the money stock was a leading indicator of stock prices, peaks and troughs were located in the monthly series of Dominion Bureau of Statistics investors security price index numbers for the years 1940 to 1958. National Bureau criteria were used, with two exceptions: data were not available on an end-of-month basis, so averages of Thursday closing prices for each month were used; and the indices were not de-seasonalized. Raw, rather than deseasonalized data

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1. George Macesich, "The Rate of Change in Money Stock as a Leading Canadian Indicator." Canadian Journal of Economics and Political Science, XXVII (August, 1962), pp. 424-30.

2. Ibid., p. 429

were used, on the grounds that Morgenstern and Granger[1] have established that New York stock prices do not exhibit seasonal fluctuations.

TABLE IV

TURNING POINTS IN THE CANADIAN MONEY SUPPLY AND STOCK PRICES.

	Money Supply*		Stock Prices**		Lead (+) or Lag (-)	
Trough				4/1942		
Peak	5/1942		4/1946		+47	
Trough		2/1947		12/1946		- 3
Peak	10/1948		6/1948		- 4	
Trough		1/1951		6/1949		-19
Peak	12/1951		1/1952		+13	
Trough		10/1953		10/1953		0
Peak	12/1954		5/1957		+29	
Trough		3/1957		1/1958		+10
Peak	8/1958		10/1958		+ 2	
Average					+17.4	- 3.0

* Source: Macesich, op. cit.

** Source: Index numbers taken from Canada, Dominion Bureau of Statistics, Canadian Statistical Review.

As can be seen from Table IV, there is a strong association between the rate of increase in the money supply and the level of the stock prices. However, the money supply led the stock prices on five occasions, coincided with them on one occasion, and trailed on three

1. Oskar Morgenstern and Clive W.J.Granger, "Spectral Analysis of New York Stock Market Prices," Kyklos, XVI (1963), pp.1-27.

occasions. This is not the sort of pattern one would expect from a causal relationship. The rate of change in the money stock certainly would not satisfy the National Bureau criteria as a leading indicator of common stock prices. It does not lead the stock indices two-thirds of the time, for example, and it certainly does not establish confidence that its future behavior will closely resemble its past behavior.

As far as can be determined, no exactly parallel study has been done using American data. However, Sprinkel[1] has done rather similar work. His treatment of the money supply differs from that of Macesich in two ways: Money is defined to include holdings by the public of coin, currency, and bank demand deposits, but not the time deposits of commercial banks; the series was smoothed by means of a six-month moving average of month-to-month percentage increases in the money supply, expressed as an annual rate. Sprinkel did not explain whether he smoothed the end-of-month Standard and Poor's Industrial Averages series before attempting to locate turning points.

Sprinkel reports that the average lead time for peaks is fifteen months and, strangely, that the average lead time for troughs is two months. As shown in Table V, the lead time for troughs is on the average only one-ninth of a month - clearly indicating a coincident, not a lagging indicator. It should be noted that lead times for peaks in Sprinkel's data are more nearly equal than the comparable figures in Table IV. It is not clear whether the improvement is attributable to

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1. Beryl Wayne Sprinkel, Money and Stock Prices, (Homewood, Ill: Richard D. Irwin, Inc., 1964).

superiority in Sprinkel's method of defining turning points, or to differences between the American and Canadian economies. It seems strange to assert that changes in the money stock "cause" downturns but not upturns in stock prices, although the money stock peaks consistently led the stock market indices in the United States and led the Canadian stock market indices in four out of the five peaks.

Sprinkel's most convincing evidence is a speculation scheme wherein an investor purchases the Standard and Poor's Industrial Averages in August, 1918; selling fifteen months after every peak in the smoothed money supply series; and buying back two months after every trough in the smoothed money stock series. The fund, originally of $100,000 in August, 1918, would have risen in value to $13,433,424 by February, 1960 -- a compound rate of return of 14.3 per cent per annum, neglecting dividends. The trouble is that, except as mentioned supra, the method of locating turning points in the money stock was essentially that of the National Bureau of Economic Research, which, as explained in Appendix B, employs hindsight extensively. It is not possible in practice to determine whether a trough as defined by Sprinkel occurred only two months ago. For example, in attempting to determine what transactions would have been made if a speculator did not have the benefit of hindsight, it appeared as if stock would have been bought in July, 1929, and not sold until January, 1931. A few disasters like this would greatly reduce the profitability of the scheme. Sprinkel's method is certainly not, as he claims, completely objective. In any event, the money stock results cannot reasonably be said to explain the profitability of the Alexander scheme. However, the final word on this question is yet to be spoken.

The issue has not been resolved beyond reasonable doubt.

TABLE V

TURNING POINTS IN THE AMERICAN MONEY SUPPLY

AND STOCK PRICES*

	Money Supply		Stock Prices		Lead (+) or Lag (-)	
Peak	12/1918		10/1919		+10	
Trough		6/1921		8/1921		+ 2
Peak	6/1922		3/1923		+ 9	
Trough		4/1924		10/1923		- 6
Peak	10/1924		2/1926		+16	
Trough		12/1926		4/1926		- 8
Peak	10/1927		9/1929		+23	
Trough		3/1932		6/1932		+ 3
Peak	6/1935		3/1937		+21	
Trough		12/1937		5/1938		+ 5
Peak	1/1945		5/1946		+16	
Trough		2/1949		6/1949		+ 4
Peak	1/1952		1/1953		+12	
Trough		11/1953		8/1953		- 3
Peak	2/1955		7/1956		+17	
Trough		1/1958		12/1957		- 1
Peak	11/1958		7/1959		+ 8	
Trough		5/1960		10/1960		+ 5
Average					+14.2/3	+ 1/9

*Source: Sprinkel, op.cit.

CHAPTER V

CONCLUSIONS.

It has been established that stock price fluctuations do not perform over time a random walk with a Gaussian distribution and linear trend. Indeed, it has been shown that no random walk model with a Gaussian distribution is consistent with the evidence. There is some indication that the variance assocuated with price fluctuations is infinite. This conclusion has been verified by Fama[1] in a recent study of each of the thirty stocks comprising the Dow-Jones Industrial Averages. Unfortunately, Fama's article was not available soon enough to be discussed in the previous chapters. Fama found that the stocks' price fluctuations did tend to follow a stable Paretian distribution, of infinite variance. If this is so, it is not possible to determine whether the Alexander scheme is more profitable than might reasonably be attributed to chance, since no hypothesis test has been devised for samples from populations with infinite variance. Since individual stocks were tested, it has been established either that the success of the scheme is attributable partly, if not entirely, to intrinsic non-random movements in the prices of individual securities; or that the statistical population of price fluctuations has a probability distribution which is not only non-Gaussian, but not well-behaved in the mathematical sense. Whether there are also non-random movements in stock prices in aggregate

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1. Eugene F. Fama, "The Behavior of Stock-Market Prices," Journal of Business, XXXVIII (January, 1965), pp. 34-105.

could not be determined. Even if such non-random movements do exist, the sophisticated quantity theory of money was found to be incapable of explaining them. Non-random movements in the prices of individual securities could be accounted for by a modification of the hypothesis of Cootner as cited on pages 2-3 supra. For psychological reasons, non-professional investors tend to buy and sell stocks in such a way as to play a game of "follow-the-leader," as it were, within limits. As soon as the price of a stock wanders sufficiently far from what professional investors judge to be its intrinsic value, they will execute transactions which will force the market value of the stock back toward its supposed intrinsic value. However, within narrow limits, the price is free to wander according to the whims of the non-professional investors. On this basis, the smallest filters would be expected to be most profitable on a before-commission basis, as was found in Alexander's second paper.

Fama studied the performance of the Alexander filtering technique for approximately the period from the end of 1957 to September 26, 1962, on the thirty Dow-Jones industrial stocks; which is similar to the period covered in Chapter II supra, 1955 to 1963, inclusive. He lists the gains and losses for the Alexander scheme for 100 shares of each stock, on both before-commission and after-commission bases, the figures given being the average gains and losses for a large number of filter sizes ranging from 0.5 per cent up to 50 per cent.[1] Buy-and-hold gains and losses are also tabulated. Fama noted

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1. op.cit., p.84

that of the thirty stocks, sixteen produced before-commission gains and fourteen produced before-commission losses with the Alexander scheme. On this basis, Fama concluded that the random walk model was vindicated, since the number of losses nearly equaled the number of gains.

If he had only totalled his columns, Fama might well have reached a different conclusion! Although the most profitable stock for the Alexander scheme (A. T. & T.) gained only $16,577.26, and the least profitable stock (Standard Oil of California) lost $3,639.79; a speculator who had used the Alexander scheme on 100 shares of each of the thirty stocks would have gained $74,774.27 on a before-commission basis--over four times the gain on the most lucrative stock. This figure compares rather well to the buy-and-hold gain of $117,506.10 on a portfolio consisting of 100 shares of each of the thirty stocks. The relative magnitudes of the over-all gain on the Alexander scheme and the over-all gain on buy-and-hold are very similar to those of the corresponding figures for the "liquid" stocks of Table III <u>supra</u>, with an 8 per cent filter. One might assume that if the statistical tests applied to the liquid Canadian stocks were applied to the thirty Dow-Jones industrial stocks, the random walk hypothesis would again be rejected. However, it is impossible to work out expected gains on a random walk basis for the Alexander scheme from the data Fama has presented. Despite Fama's assertions to the contrary, his data can be taken to be a corroboration of the findings in this thesis that the random walk hypothesis should be rejected, although it is conceded

that this conclusion cannot be confirmed by statistical inference once the Gaussian assumption is relaxed.

There are no important implications for investment policy. Since the existence of inertia has been verified, one might use this finding as a basis for timing the purchase of securities--buying only when the price of a security has been rising. Perhaps the best prospect is that since the evidence indicates that stock prices are non-random, it may be possible to devise a mechanical scheme which will more efficiently take advantage of the non-random movements in common stock prices, and be more profitable on an after-commission basis than buying and holding. However, if the modification of Cootner's hypothesis is the true explanation for the profitability of the scheme, no mechanical scheme which is more lucrative on an after-commission basis than buying and holding could be possible.

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APPENDIX A.

In determining the buy and sell points for the Alexander scheme, whenever it was important to decide whether the weekly low price preceded or followed the high price, the following conventions were used:

If the closing price for the week was not equal to the average of the high and low prices, then whichever was further from the closing price was assumed to occur sooner.

If the closing price was equal to the average of the high and low prices, but not equal to the previous week's closing price, then whichever price (high or low) was closest to the preceding week's closing price was assumed to occur sooner. If both the closing price of a given week and the previous week's closing price were equal to the average of the week's high and low price, then the high and low were assumed to occur in the same order as the high and low of the previous week.

Where the above conventions failed to resolve the issue, the presumed order of precedence of the high and low prices was settled by the toss of a coin.

These conventions are, of course, highly artificial. However, they do not involve hindsight, and if the random walk hypothesis were true, would be expected neither to increase nor to decrease the profitability of the Alexander scheme. At the start of the study of

each stock, the initial movement in price (from low to high or high to low) was assumed to be the current price trend. No ex-dividend corrections were made. Fama has shown that this does not materially affect the results.[1]

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1. op.cit., p.46

APPENDIX B

Burns and Mitchell[1] describe the National Bureau of Economic Research criteria for dating the troughs and peaks of specific cycles:

> To determine whether a time series has specific cycles, and if so, to fix the dates when each cycle began, culminated, and ended, we plot the data, both in their original form and after adjustment for seasonal variations, upon a semi-logarithmic chart and study the whole record in this graphic form . . .When specific cycles are made doubtful by random movements, we smooth the data by moving averages and base judgments upon the curve of moving averages . . .We do not recognize a rise and fall as a specific cycle unless its duration is at least fifteen months, whether measured from peak to peak or from trough to trough . . .Once the specific cycles have been distinguished we proceed to date their turning points. When the cycles are clear in outline, our practice is to take the lowest and highest points of the plotted curves as the dates of the cyclical turns . . .If the series is especially choppy . . .moving averages are used . . .We aim especially to disregard such extreme isolated values as we know are associated with strikes, tariff changes, or other random events.[2]

Later, they concede:

> There is ample opportunity for vagaries of judgment. At times our rules failed to yield a clear-cut decision. At times the members of our statistical staff disagree in their efforts to apply the rules to a given series.[3]

In a book review, Koopmans comments:

> The book is unbendingly empirical in outlook . . .This decision greatly restricts the benefit that might be secured from the use of modern methods of statistical inference. The pedestrian character of the statistical devices employed is directly traceable to the authors' reluctance to formulate explicit assumptions,however general,concerning the probability distribution of the variables . . .[4]

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1. Arthur F.Burns and Wesley C.Mitchell, Measuring Business Cycles (New York: National Bureau of Economic Research, 1947).

2. Ibid., pp.56-59

3. Ibid., p.64

4. Tjalling, C.Koopmans, "Measurement without Theory," Review of Economics and Statistics, XXIX (August, 1947), p.172.

The National Bureau criteria can be shown to find "cyclical" behavior in a random walk. As was admitted by Burns and Mitchell, Hindsight is employed extensively. The widespread use of these criteria is attributable mainly to their simplicity. The method of determining leads or lags between two time series which is currently regarded by economists as conceptually correct is cross-spectral analysis.[1]

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1. See, for example, Morgenstern and Granger, op. cit.