AN INVESTIGATION TO DETERMINE THE EFFECTS
OF TEACHING ELEMENTARY LOGIC TO TENTH-
GRADE GEOMETRY STUDENTS.

by

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ABSTRACT

The purpose of this investigation was to evaluate the effects on achievement in and attitude towards mathematics of teaching certain elementary logic concepts to high school mathematics students. To achieve this purpose, four classes of tenth-grade geometry students were selected from a single school of the Vancouver School District. Two of the classes served as the experimental group for this investigation. Both the experimental and control groups were taught by the investigator. The program for the experimental group involved a one-week introduction to elementary logic concepts followed by a two-week study of "Similarity" concepts. The control group's program involved only the two-week study of the "Similarity" concepts.

The students were evaluated at the beginning and end of the treatment period and again three weeks later. Most of the instruments administered were developed by the investigator and consequently not standardized.

The mean test scores obtained were statistically analyzed for significance of differences using t-statistics. The null hypothesis was tested at the five percent level of confidence. Analysis of the data collected showed that the null hypothesis is accepted at the high, medium, and low ability levels. The acceptance of the null hypothesis implied that the teaching of logic concepts to tenth-grade geometry students had no significant effects on achievement in mathematics or attitude towards mathematics.

W. E. Hall

Approved by:
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# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ........................................ ii

Chapter

I. INTRODUCTION ........................................ 1
   The Problem ........................................... 2
   Statement of the Problem ............................... 2
   Hypothesis ............................................. 2
   Significance of the Problem ............................ 3
   Limitations ............................................. 5
   Clarification of Terms ................................ 5
   Prospectus of the Experimental Design ............... 6

II. REVIEW OF THE RELATED LITERATURE .................... 7
   Historical Background ................................ 7
   Learning Theory ....................................... 7
   Curriculum Studies .................................... 8
   REVIEW OF THE RELATED RESEARCH ..................... 9
   Introduction ........................................... 9
   Research at the Elementary School Level .............. 10
   Research at the Secondary School Level ............... 11
   Research at the University Level ...................... 13
   Summary and Implications .............................. 14

III. DESIGN OF THE STUDY ................................... 16
   Problem ............................................... 16
   Procedure ............................................ 16
   Subjects .............................................. 17
   Data-Gathering Instruments ............................ 18
   Treatment and Nontreatment Programs .................. 20
   Statistical Procedure ................................ 20
TABLE OF CONTENTS—Continued

<table>
<thead>
<tr>
<th>Chapter IV</th>
<th>RESULTS OF THE STUDY</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Preliminary Test Results</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Table 1: Means and Standard Deviations</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Table 2: t-Values and Degrees of Freedom</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Tests of the Hypothesis</td>
<td>27</td>
</tr>
<tr>
<td>Hypothesis 1</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>Hypothesis 3</td>
<td></td>
<td>28</td>
</tr>
<tr>
<td>Hypothesis 4</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>Hypothesis 5</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Hypothesis 6</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>Hypothesis 7</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>Hypothesis 8</td>
<td></td>
<td>33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>V. SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>35</td>
</tr>
<tr>
<td>Conclusions</td>
<td>35</td>
</tr>
<tr>
<td>Implications</td>
<td>36</td>
</tr>
<tr>
<td>Recommendations</td>
<td>37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIBLIOGRAPHY</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LIST OF APPENDICES</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix A</td>
<td>41</td>
</tr>
<tr>
<td>Appendix B</td>
<td>47</td>
</tr>
<tr>
<td>Appendix C</td>
<td>52</td>
</tr>
<tr>
<td>Appendix D</td>
<td>54</td>
</tr>
<tr>
<td>Appendix E</td>
<td>57</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

In recent years there has been increased concern among mathematics educators to improve the academic calibre of students graduating from our high school mathematics program. This concern has led to many attempts to improve the mathematics curriculum. One of these, the Commission on Mathematics, was formed to make recommendations for the modernization, modification, and improvement of secondary school mathematics. Among the Commission's recommendations was the following: "Understanding of the nature and role of deductive reasoning—in algebra as well as in geometry". Several other curriculum study groups, which will be reviewed in Chapter II, have made similar recommendations.

An extensive examination of research relative to the effects of teaching logical-deductive reasoning on achievement in or attitude towards mathematics yields very little conclusive evidence. The purpose of this investigation is to provide experimental evidence to allow evaluation of some of these effects. The tenth-grade modern geometry program has been chosen as the setting for this investigation since there exists considerable agreement among mathematics educators that one of the principal purposes of the program is to illustrate logical-deductive reasoning. The basis of deductive reasoning is found in

logic. The nature of logical-deductive reasoning can be summarized as follows: Certain statements are logical consequences of other statements solely on the basis of accepted rules of logic and the logical form of the statements.

A cursory survey of available texts for the modern high school geometry program indicates that very few authors have attempted to include logic concepts in their presentations. Many authors inductively illustrate deductive reasoning. The student is asked to study and attempt a multitude of examples of deductive arguments. From this study the student is expected to learn to reason deductively.

Can the high school student be taught to reason deductively? Will teaching him logic concepts increase his ability to reason deductively in other settings? What effects will teaching him logic concepts have on his ability to write deductive geometric proofs?

It is the purpose of this study to gather data to allow statistical testing of these and other related questions, which are of prime concern to mathematics educators.

THE PROBLEM

Statement of the Problem. The present investigation will attempt to answer the following questions: (1) What are the effects on mathematical achievement and attitude toward mathematics of teaching certain elementary logic concepts to tenth-grade geometry students? (2) What are the relationships between these effects and the student's mathematical achievement level? (3) What effect will the passage of time have on the results of learning the elementary logic concepts?

Hypothesis. In order to answer the research questions, the following
null hypothesis will be statistically tested:

1. There is no significant difference, between the treatment (receiving instruction on elementary logic concepts) and nontreatment groups, in mean mathematical achievement posttest scores.

2. There are no significant differences, between the treatment and nontreatment groups, in mean mathematical achievement posttest scores at three mathematical ability levels.

3. There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores on geometric proofs at three mathematical ability levels.

4. There is no significant difference, between the treatment and nontreatment groups, in mean posttest scores of attitude towards mathematics.

5. There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores of attitude towards mathematics at three mathematical ability levels.

6. There is no significant difference, between the treatment and nontreatment groups, in mean mathematical achievement posttest scores three weeks after the treatment period.

7. There are no significant differences, between the treatment and nontreatment groups, in mean mathematical achievement posttest scores three weeks after the treatment period at three mathematical ability levels.

8. There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores on geometric proofs three weeks after the treatment period at three mathematical ability levels.

**Significance of the Problem.** Modern mathematics programs require students to understand algebraic and geometric proofs. A familiar example is the present tenth-grade geometry program. In it modern geometry is taught in a logical-deductive manner. The principles of deductive reasoning are discussed as they arise during the logical-deductive method of presentation. This method of presenting the principles of deductive reasoning and proof, however, is probably not sufficient to illustrate the logical-deductive nature of mathematical reasoning. What is required is a study of materials in which the major emphasis

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is on the rules of reasoning, rather than the "facts" of mathematics. Such materials are beginning to appear in modern mathematics texts.

4 Carl Allendoerfer summarizes the need for a study of the logical-deductive nature of mathematical reasoning by high school students as follows:

Since the deductive method is an essential part of modern mathematical thinking, the teacher should use every opportunity to illustrate it in every aspect of her work. Illustration, however, is probably not enough to teach the students the essential structure of a deductive system. At some stage in the high school mathematics curriculum there should be a serious discussion of deductive systems per se, and later applications of this to mathematics and to nonmathematical situations should be used to reinforce the understanding of students about deductive methods. Perhaps, the tenth grade is the place for this, but no firm statement of this kind should be made until more experimental teaching has been carried out.

In recent high school mathematics texts attempts are present to include elementary logic concepts in the mathematics curriculum. However, there appears to be an absence of experimental evidence indicating that a study of these concepts will facilitate achievement in mathematics. More definitive research in this area is required.

If it could be shown that teaching logic-concepts to high school students has a significant effect on achievement in mathematics, the implications would be pertinent to the British Columbia mathematics curriculum. The present B. C. mathematics curriculum does not require high school students to formally study the nature of deductive reasoning. Since modern mathematics programs have been taught for six years and continue to be taught in B. C. high schools, research is required to determine the effects of including this teaching.

LIMITATIONS

In any experimental study of this type in which the treatment period is relatively short it is extremely difficult to identify and control the numerous factors which could significantly affect the results. Ideally, the students would be handled individually and randomly assigned to treatments. In the high school setting it is usually not possible to obtain this ideal. Instead, classes must be assigned randomly to treatment classifications and treated intact. Also confounding the results are initial student differences in mathematical ability and attitude towards mathematics due to prior teaching.

CLARIFICATION OF TERMS

By "modern" geometry is intended high school geometry in which the emphasis is on the axiomatic, deductive, and abstract nature. Such geometry programs have been developed for the secondary schools during the last decade.

The logic program was developed by the investigator based on materials presented in the sixth chapter of the MacLean, Mumford, et al text.\(^5\)

The similarity program was based on materials presented in the first half of the twelfth chapter of the Moise and Downs text.\(^6\) At the time of the investigation the students had studied materials from the first eleven chapters of the text.


It has been assumed that geometric proofs require deductive reasoning. The writing of geometric proofs has been emphasized as an important part of the modern geometry program. Typically, the student is required to state given and required information followed by the application of one or more mathematical generalizations and the desired conclusion.

PROSPECTUS OF THE EXPERIMENTAL DESIGN

In the spring of 1968, four tenth-grade geometry classes were selected from a single school of the Vancouver School System. The classes selected were similar in mathematical ability. The four classes were regularly taught by two mathematics teachers, each teacher teaching two of the classes. The investigation involved about 120 students. The students had studied modern geometry for eight months using the Moise and Downs text. The four classes were randomly assigned to the treatment and nontreatment conditions.

Following the administration of pretests on attitude, logic and mathematics, the treatment groups were given a program of study by the investigator. The treatment group's program involved one week of instruction on logic concepts followed by two weeks of instruction on similarity concepts. The nontreatment group's program consisted of only the two weeks of instruction on similarity concepts.

At the conclusion of the treatment periods, posttests were given to allow evaluation of the student's understanding of the logic concepts, achievement in geometry, and attitude towards mathematics. Three weeks later, another posttest was administered to both groups to measure retention of the similarity concepts. The data was processed according to the procedures described in Chapter III.
CHAPTER II

REVIEW OF RELATED LITERATURE

Historical Background. Since the days of Euclid it has been generally recognized that mathematics is in some way connected with logic. Euclid, without explicit formulation, utilized laws of logic in many of his geometric proofs. For well over 2000 years his *Elements* has served as a model of logical reasoning. The *Elements* is one of the first attempts to present geometry in an organized, logical fashion, starting with a few simple assumptions and definitions and developing a complex mathematical system. This development requires the use of logical-deductive reasoning.

Learning Theory. We have, for centuries, believed in the ability of mathematics to train our minds for logical thinking in other areas of knowledge. This statement is testified to by the fact that mathematics has continued to occupy such an important place in secondary school and university curricula. The basis of this belief was the theory of automatic transfer of training. About the beginning of the twentieth century experimental evidence mounted against this theory. The reaction to this evidence culminated in the law of specificity of training. This law rejected mathematics as the irreplaceable preparation for clear thinking and logical reasoning. The truth likely falls somewhere between these two extreme theories. Bruner\(^1\) in his book, *The Process of Education*, states;

formal discipline was poorly stated in terms of the training of faculties, it is indeed a fact, massive general transfer can be achieved by appropriate learning even to the degree that learning properly under optimum conditions leads one to "learn how to learn".

Curriculum Studies. One of the first major attempts to develop a new mathematics curriculum was begun during the mid-1950's by the University of Illinois. The University of Illinois Curriculum Study in Mathematics (UICSM) suggested changes in both the content and methodology of secondary school mathematics. The study advocated extensive use of the "discovery" approach to teaching and learning mathematics. Gertrude Hendrix, a UICSM member, asserts that students can acquire an introduction to logical-deductive reasoning through a structural treatment of elementary algebra or an axiomatic study of geometry.

A second attempt to reorganize the secondary school mathematics curriculum was made by the Commission on Mathematics (1955) of the College Entrance Examination Board. The Commission reviewed the existing mathematics curriculum and made recommendations for its modernization, modification and improvement. A study of the logical-deductive nature of mathematical reasoning was recommended by the Commission.

The Ball State Teacher's College Experimental Program (1954) developed a new geometry program based on a simplified version of the Hilbert Postulates. Elementary logic was included in this program as an aid to understanding of the logical-deductive development of the program.

The School Mathematics Study Group (1958) developed a new mathematics program containing several topics which were new to the school or grade curriculum. In addition SMSG completely reorganized their approach to

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3 College Entrance Examination Board, op. cit., p. 22.
traditional topics of mathematics. Their approach to curriculum organization involves strong emphasis on the principles of mathematics. The basic principles of mathematics are used as the logical framework within which mathematical facts and skills can be developed. Their geometric materials emphasize the need for precise definitions of terms and statements of theorems as well as a logical-deductive development of geometry.

The Accelerated Learning of Logic Project (1960) of Yale University was developed to provide learning materials for elementary school students. It was hoped that the ALL materials would help develop a favorable attitude towards symbol-manipulating activities and provide practice in abstract thinking. This objective was approached through games designed to teach some mathematical logic. As a result of the ALL Project a set of programmed materials and a series of games requiring applications of the programmed materials were developed.

REVIEW OF THE RELATED RESEARCH

Introduction. During the last half-century there have been numerous appeals for the inclusion of the concepts of elementary logic in the high school curriculum. Nathan Lazar published an extensive paper in 1938 suggesting the importance of logic laws and concepts in the study of geometry. Since then there have been numerous journal articles written in support of an explicit study of the nature of deductive reasoning in mathematics and

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particularly in geometry. However, there have been very few investigations which report the effects of teaching the nature of deductive reasoning.

Research at the Elementary School Level. In the early-1960's Suppes and Binford of Stanford University reported pertinent research findings at the elementary level. They developed an experimental program of elementary logic to be taught to school children of ages 10, 11, and 12. The purpose of the program was described as follows:

... to deepen and extend the mathematical experiences of the able elementary school child at the broadest level of mathematics, the level of methodology and the theory of proof. The approach is through a study of modern mathematical logic, in particular that portion of it which is concerned with the theory of logical inference or the theory of deduction.

The specific objective of the research project was to determine the effects of teaching mathematical logic to academically talented fifth- and sixth-grade children. Of primary interest was the evaluation of the capacity of children in this age group to handle deductive arguments. Also of interest was the evaluation of their ability to transfer acquired skills of analysis and logical reasoning to other areas of knowledge. On the basis of the experimental evidence obtained from this study, Suppes and Binford concluded that:

(1) The upper quartile of elementary school students can achieve a significant conceptual and technical mastery of elementary mathematical logic. The level of mastery is 85 to 90 percent of that achieved by comparable university students.

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7 P. Suppes and F. Binford, op. cit., p. 194.
(2) This mastery of the subject matter by elementary school students can be accomplished in an amount of study time comparable to that needed by college students if study is allocated over a longer period of time and if the students receive considerable more direct teacher supervision.

(3) The more dedicated and able elementary school teachers can be adequately trained in five or six semester hours to teach classes in elementary mathematical logic. It is probably essential that this teacher-training program be very closely geared to the actual program of instruction the teacher will follow in the classroom.

(4) Anecdotal evidence from teachers suggests that there is some carryover in critical thinking and attitude into other fields, especially arithmetic, reading, and English. More explicit behavioral data on carry-over in critical thinking would be desirable.

(5) The work with the special summer class in 1963 indicates that able elementary school students who have received prior training in mathematical logic can make rapid progress in other parts of modern mathematics organized on a deductive basis.

Smith in commenting on the Suppes-Binford study agreed with Suppes as to the importance of teaching elementary logic-concepts but disagreed with the conclusions drawn by Suppes and Binford. Smith enumerated three reasons for objection to Suppes' and Binford's conclusions: (1) Suppes generalized his results to the upper quartile of the elementary school population from a specially chosen sample, (2) the composition of the control group used did not allow valid comparisons, and (3) Smith found unacceptable Suppes' definitions of "equal time" and "equal content".

Research at the Secondary School Level. In 1959, Corley designed a study to evaluate the ability of students in grades six through ten to study successfully an introduction to modern geometry. In addition to her study was


designed to determine the ability of students in this age group to understand the logical-deductive nature of mathematical reasoning. Based on statistical analysis of the collected data, Corley was able to conclude:

(1) The ability of pupils to learn geometric terms and concepts is quite well developed at the sixth grade level. This ability improves at a slow, approximately uniform rate as the pupils progress into the tenth grade.

(2) The ability of pupils to understand the three methods of reaching general conclusion and apply them in either geometric or nongeometric situations is moderately well developed at this level. At the seventh grade level a stage of much slower growth had been entered. This slower rate of increase continues for the next two or three years.

(3) The logical structure of geometry and the proof of theorems in geometry are too complex for comprehension by all but very few in the sixth grade. The ability to understand this study is much improved at the seventh grade level, after which the rate of improvement is much slower up to the tenth grade.

Retzer and Henderson reported an experimental study designed to measure the effects of teaching select elementary-logic concepts on the ability of college-capable junior-high school students to verbalize discovered mathematical generalizations. Also studied was the dependence of achievement in the elementary-logic unit on ability level. To this end the treatment group completed a unit on elementary logic, the nontreatment group did not. Both groups were led to discover generalizations about vectors by means of a programmed unit within which each student was asked to verbalize his discoveries. The authors reported the following conclusions:

(1) The treatment group did significantly better than the control group in verbalizing generalizations precisely.

(2) The gifted students verbalize significantly better than the other college-capable group.

(3) The verbalization of the gifted was aided more by studying logic concepts than that of the others.

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Research at the University Level. Elton of the University of Kentucky reported a study carried out on sophomores of his university during the fall of 1963. The stated purpose of the study was to evaluate the validity of the assumption that logic-training might affect a student's score on a test of reasoning ability. To this end all students enrolled in an elementary logic course, an introductory course devoted to the principles underlying logical-deductive reasoning, were pretested using the Valentine Reasoning Test. Concomitantly, students enrolled in a sophomore course in applied psychology were given the same test. During the last week of classes (after 16 weeks) the two groups were again tested using the Valentine Reasoning Test. Analysis of the test data indicated that there were no significant differences, in terms of reasoning ability, between the two treatment groups as a result of the logic-training.

Jensen reported research designed to determine the effectiveness of presenting a unit on logical-deductive nature of mathematical reasoning to beginning college students, using computer-assisted instruction. That is, the computer was programmed to "give individual and immediate guidance to a student as he constructs the steps of a simple deductive proof." After comparison of the data obtained from the computer group and the control group the author came to the conclusion that there were no significant differences between the experimental groups on any measure except the mean pretest and posttest scores,


indicating only that a significant amount of learning took place in each group as a result of the instruction.

**Summary and Implications.** The findings of the Suppes experiment indicate that capable elementary school students are capable of mastering logic subject matter. In addition, Suppes reports evidence of some transfer of this logic training to other settings. Corley suggests, as a result of her investigation, that the majority of the secondary school students are capable of studying modern geometry by the ninth or tenth grade. Retzer reports in favour of teaching select elementary-logic concepts to college-capable secondary school students. He found that this teaching increased the ability of college-capable students to verbalize discovered mathematical generalizations. Also significant was his finding that the higher ability students were aided more by the logic study than were the others. In apparent opposition to these studies were Elton's findings. His study reports that logic-training had no significant effect on the reasoning ability of his subjects. Jensen's investigation suggests that teaching the logical-deductive nature of mathematical reasoning to college students did not significantly improve their performance on deductive reasoning.

This review of related research indicates that secondary mathematics students are capable of understanding the nature of deductive reasoning if it is appropriately presented. The high school geometry program would appear to be a suitable area in which to carry out this study since it purposes to be largely deductive in nature. The important issue is whether a study of the nature of deductive reasoning will facilitate understanding and achievement in other areas. There is a definite need for more experimental evidence at the high school level in order to resolve this issue. In particular, the
effect of teaching the nature of deductive reasoning on achievement in high school geometry requires further study.

It is the purpose of this research to augment the conclusions of the studies reported in this review.
CHAPTER III

DESIGN OF THE STUDY

Problem. The purpose of this study is to determine the effects on mathematical achievement and attitude towards mathematics of teaching logic concepts to tenth-grade geometry students. The relationship between these results and the student's mathematical achievement level and the effect of the passage of time on these results are also studied.

Procedure. During the third week of April, 1968, after the tenth grade geometry students had studied modern geometry for approximately eight months, the present study was initiated. Four comparable tenth grade geometry classes were selected as the sample for the study. The classes were randomly assigned to the experimental and control groups and pretested to measure their attitude towards mathematics and their understanding of the logical-deductive nature of mathematical reasoning. An achievement test given to all tenth grade geometry students prior to the Easter recess was used as a pretest evaluation of achievement in geometry. Each of the pretest instruments will be discussed more thoroughly later in this chapter.

The treatments administered to the experimental (treatment) and control (nontreatment) groups were developed by the investigator and designed to teach both groups introductory similarity concepts. Preceding their introduction to the similarity concepts, the experimental group was given a one-week program on logical-deductive reasoning. In order to facilitate concurrent testing of the similarity concepts it was necessary
to begin the experimental group's treatment before the control group's. The investigator administered all treatments. The content of the experimental group's one-week logic unit is outlined in Appendix A. Both the experimental and control groups were then given a two-week program in which introductory concepts of similarity were presented. An outline of the two-week similarity program appears in Appendix B. Great care was exercised to ensure "equal time" and "equal content" conditions between the groups for the similarity program.

At the conclusion of the treatment period all students were given three posttests. The first posttest was designed to measure attitude towards mathematics, the second to measure understanding of logic concepts, and the third to measure achievement in mathematics on the similarity concepts. Three weeks later, a second posttest of achievement in mathematics on the similarity concepts was administered. Adequate time was provided for writing all pre- and post-tests.

The test scores obtained from these tests were statistically analyzed for significance of differences between means, both within and between groups, using t-statistics. The data was processed on the University of British Columbia's IBM 7044 computer.

Subjects. The subjects were selected from a single high school of the Vancouver School System based on the recommendations of the chairman of the thesis committee, the principal of the school selected, and the mathematics teachers whose classes were to be used. The sample consisted of four classes of tenth grade geometry students most of whom were studying modern geometry for the first time. All classes were heterogeneously grouped
according to scholastic ability. Two of the four classes selected had been taught by one teacher and the remaining two by another teacher. One class from each teacher was randomly assigned to the experimental and control classifications.

In order to control absence effects, it was necessary to delete from the study persons missing any part of the logic program or more than one hour of the similarity program. At the conclusion of the investigation, the experimental and control groups each contained exactly 50 useable subjects.

**Data-Gathering Instruments.** Control of initial differences and evaluation of final differences requires careful and extensive pre- and post-testing. Seven data-gathering instruments were administered for this investigation. All instruments were given to both the experimental and control groups. None of the instruments used was standardized. Information regarding their validity and reliability was limited. Valid and reliable standardized instruments suitable for the present investigation were not available.

The pretest of mathematical achievement used was constructed and administered by the regular mathematics teachers. The content validity of the instrument can be assumed since the test items were chosen by agreement of all geometry teachers involved. The purpose of the test was to evaluate the achievement level of all tenth-grade geometry students in the school. The material covered by the test consisted of the geometry concepts studied during the first five months of the school year. As a result of the reluctance of the school to publish this test, a copy of the test cannot be made available.
The Aiken and Dreger Opinionnaire,\(^1\) *Attitude Toward Mathematics*, was selected as the pre- and post-test of mathematics attitude. A review of available mathematics attitude scales revealed very few instruments appropriate for use in the high school. The co-authors of the scale selected reported\(^2\) a test-retest reliability of 0.94. No evaluation of validity is reported. The Aiken and Dreger Opinionnaire consisted of twenty items and required the student to reply: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), or Strongly Agree (SA) to each item. The items were scored by assigning numerical values from 1 to 5 for each item. A numerical evaluation of attitude was obtained by summing the assigned values. A copy of the Opinionnaire appears in Appendix C.

The pre- and post-test of logic concepts used was constructed and administered by the investigator. The instrument was designed to measure knowledge of logic concepts required for understanding of the logical-deductive nature of mathematical reasoning. The students were not told after the pretesting that they would be posttested on the logic concepts. Any improvement in performance on the posttest as a result of "practice" was assumed identical for the experimental and control groups. The logic pre- and post-test is included in Appendix D.

The posttest evaluations of mathematical achievement on the "similarity" concepts were developed by the investigator. Two forms of the


achievement posttest were developed. The purpose of Form A was to evaluate mathematical achievement on the similarity concepts immediately following the three week treatment period. The purpose of Form B was to evaluate retention on related concepts. Each form of the achievement posttest consisted of two sections. The second section of each form required the student to write three geometric proofs. The first sections did not require the student to display this competency. It was not assumed for the purposes of this study that the two forms of the achievement test or their related sections were equivalent. The investigator did attempt to make Form A and Form B parallel. Thirty-five minutes was allowed for completion of each achievement posttest. Form A and Form B of the achievement posttest are included in Appendix E.

Treatment and Nontreatment Programs. The treatment (experimental) group's program consisted of a one-week (three hour) study of elementary logic concepts followed by a two-week (six hour) study of similarity concepts presented in Chapter 12 of Geometry by Moise and Downs. The nontreatment (control) group's program consisted solely of the two-week (six hour) study of the similarity concepts.

Included in Appendix A is a complete outline of the one-week logic program given to the treatment group. Appendix B contains an outline of the two-week program given to both groups. All theorem numbers and page references pertain to the related sections of this text.

Statistical Procedure. The mean pre- and post-test scores of the treatment and nontreatment groups were statistically tested for significance
of differences. Since the mean pretest scores of the treatment and non-treatment groups were not significantly different at the five percent level of confidence, all hypotheses were tested using t-tests. The five percent level of confidence was required in all tests of the null hypothesis.

For hypotheses 2, 3, 5, 7, and 8 it was necessary to assign subjects to mathematical ability levels on the basis of their pretest scores of mathematical achievement. Cut-off points based on the pretest scores of mathematical achievement were calculated such that each ability classification would contain approximately thirty-five students. Comparisons of the treatment and nontreatment group's mean posttest scores at three levels of mathematical ability were then possible.
CHAPTER IV

RESULTS OF THE STUDY

Introduction. In brief, this chapter presents the results of the statistical procedures outlined in the last section of Chapter III. The format of presentation will be as follows: statement of the hypothesis, statistical analysis of the data collected, and the decision to accept or reject the hypothesis. Tables I and II have been embodied in this chapter providing convenient reference to the collected data.

Preliminary Test Results. In order to evaluate initial "between-group" differences, it was necessary to statistically analyze the pretest scores of achievement in mathematics, achievement on logic-concepts, and attitude towards mathematics. Table II reports the t-values of the differences between these scores to be -1.41 (94 degrees of freedom), 1.87 (94 degrees of freedom), and -1.06 (96 degrees of freedom), respectively. None of these t-values are statistically different at the five percent level of significance. Thus, the treatment and nontreatment groups were not statistically different on any of the three pretest measures. Based on these results it was decided that t-statistics could be used for further analysis of the data. However, all analyses were substantiated using analysis of covariance. Analysis of covariance revealed findings identical to those presented in the remainder of this chapter.
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>GROUP</th>
<th>N</th>
<th>MEAN</th>
<th>S.D.</th>
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<td>57.35</td>
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# N.T. = Nontreatment Group
## T. = Treatment Group
TABLE II

**t-VALUES AND DEGREES OF FREEDOM**

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<tr>
<th>VARIABLE</th>
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<td>6.56</td>
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### TABLE II (continued)

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<td>0.16</td>
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<td>-0.57</td>
<td>32</td>
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</tbody>
</table>

* Indicates means are significantly different at the five percent level of confidence.
Tests of the Hypothesis.

Hypothesis 1.

There is no significant difference, between the treatment and non-treatment groups, in mean mathematical achievement posttest scores.

Table I reports the mean pretest and posttest scores. The mean mathematical achievement posttest scores for the treatment and nontreatment groups were 15.67 and 14.87, respectively. Table II describes the t-values and degrees of freedom obtained by the group analyses. The reported t-value of -0.75 with 94 degrees of freedom does not exceed the critical value (±1.99) for statistical significance of difference at the five percent level of confidence. Therefore, the null hypothesis can be accepted. Interpretation of this result suggests that students who have been given the one-week program on elementary logic concepts do not obtain significantly higher scores on the test of achievement in geometry than do their counterparts who have not been given the logic program.

Hypothesis 2.

There are no significant differences, between the treatment and nontreatment groups, in mean mathematical achievement posttest scores at three mathematical ability levels.

(a) High Ability Group. Table I reports the means of the scores on the mathematical achievement posttest for the high ability groups to be 18.26 for the treatment group and 19.82 for the nontreatment group. Table II gives the t-value to be 1.02 with 28 degrees of freedom. This t-value does not exceed the critical value of ±2.05 for the five percent level of confidence. Therefore, the null hypothesis can be accepted at the five percent level of confidence.
(b) **Medium Ability Group.** The reported means of the mathematical achievement posttest scores for the medium ability groups were 16.50 and 15.21 for the treatment and nontreatment groups, respectively. The t-value obtained was -0.75 with 31 degrees of freedom. Again, the null hypothesis can be accepted at the five percent level of confidence since the obtained t-value does not exceed the critical value of ±2.04.

(c) **Low Ability Group.** The mean scores obtained by the low ability groups on the posttest of mathematical achievement were 11.88 for the treatment group and 11.29 for the nontreatment group. Table II reports the t-value to be -0.39 with 31 degrees of freedom. The critical value of ±2.04 was not exceeded by this value. Therefore, the null hypothesis can again be accepted at the five percent level.

At each ability level the differences in mean posttest scores obtained for mathematical achievement were not statistically significant at the five percent level of confidence.

**Hypothesis 3.**

There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores on geometric proofs at three mathematical ability levels.

(a) **High Ability Group.** The mean posttest scores on geometric proofs of the high ability treatment and nontreatment groups are given in Table I to be 11.37 and 12.45, respectively. The t-value of 0.94 with 28 degrees of freedom given in Table II does not exceed the critical value of ±2.05 for the five percent level of confidence. Therefore, the null hypothesis can be accepted for the high ability group.

(b) **Medium Ability Group.** The reported means of the geometric
proofs posttest scores for the medium ability groups were 10.64 for the treatment group and 8.63 for the nontreatment group. Table II reports the t-value to be -1.36 with 31 degrees of freedom. This t-value does not exceed the critical value of ±2.04 for the five percent level of confidence. The null hypothesis can again be accepted.

(c) Low Ability Group. The mean scores obtained by the low ability groups on the geometric proofs posttest were 6.56 and 6.06 for the treatment and nontreatment groups, respectively. The t-value obtained was -0.37 with 31 degrees of freedom. Again, the null hypothesis must be accepted at the five percent level of confidence since the obtained t-value does not exceed the critical value (±2.04).

The difference in mean scores on the geometric proofs posttest between the treatment and nontreatment groups were not statistically significant at any of the three ability levels. Thus, Hypothesis 3 can be accepted at the five percent level of confidence.

Hypothesis 4.

There is no significant difference, between the treatment and nontreatment groups, in mean posttest scores of attitude towards mathematics.

Table I reports the mean posttest scores of attitude towards mathematics to be 62.12 for the treatment group and 56.92 for the nontreatment group. Table II reports the t-value for this comparison of means to be -1.47 with 98 degrees of freedom. The reported t-value does not exceed the critical value (±1.99) for statistical significance of difference at the five percent level of confidence. The null hypothesis can be accepted. Consequently, students who have been given a one-week program on elementary
logic-concepts do not obtain significantly different scores on a test of attitude towards mathematics than do their counterparts who have not been given the logic program.

Hypothesis 5.

There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores of attitude towards mathematics at three mathematical ability levels.

(a) High Ability Group. Table I reports the mean posttest scores of attitude towards mathematics for the high ability treatment and nontreatment groups to be 65.75 and 64.45, respectively. Table II gives a t-value of -0.21 with 29 degrees of freedom. This t-value does not exceed the critical value ($t^{2.05}$) for the five percent level of confidence. Therefore, the null hypothesis can be accepted for the high ability groups.

(b) Medium Ability Group. The mean posttest scores of attitude towards mathematics of the medium ability treatment and nontreatment groups are given in Table I to be 68.36 and 61.35, respectively. The t-value of -1.11 with 32 degrees of freedom reported in Table II does not exceed the critical value ($t^{2.04}$) for the five percent level of confidence. Hypothesis 5 can be accepted for the medium ability group.

(c) Low Ability Group. The mean scores reported for the low ability groups on the posttest of attitude towards mathematics were 52.13 for the treatment group and 47.89 for the nontreatment group. Table II reports the t-value to be -0.82 with 33 degrees of freedom. This t-value does not exceed the critical value of $t^{2.04}$ indicating that the posttest means for the low ability treatment and nontreatment groups were not significantly different at the five percent level of confidence.
The differences in mean scores on the posttest of attitude towards mathematics obtained by the treatment and nontreatment groups were not statistically significant at the three mathematical ability levels. Thus, Hypothesis 5 can be accepted at the five percent level for the high, medium and low ability groups.

**Hypothesis 6.**

There is no significant difference, between the treatment and non-treatment groups, in mean mathematical achievement posttest scores three weeks after the treatment period.

The mean mathematical achievement posttest scores for the treatment and nontreatment groups, three weeks after the treatment period, are reported in Table I to be 13.45 and 14.04, respectively. Table II reports the t-value of the difference in means to be 0.55 with 93 degrees of freedom. The reported t-value does not exceed the critical value of ±1.99 for statistical significance of difference at the five percent level of confidence. Hypothesis 6 can be accepted. Thus, students who have been given the one-week program on elementary logic concepts show no significant difference in mean mathematical achievement, three weeks after the treatment period, from their counterparts who have not been given the one-week program.

**Hypothesis 7.**

There are no significant differences, between the treatment and non-treatment groups, in mean mathematical achievement posttest scores three weeks after the treatment period at three mathematical ability levels.

(a) **High Ability Group.** The mean posttest scores for the treatment and nontreatment groups in mathematical achievement, three weeks after the treatment period, are given in Table I to be 14.26 and 16.82, respectively.
Table II reports the t-value for the difference between these means to be 1.47 with 28 degrees of freedom. This t-value does not exceed the critical value ($\pm 2.05$) for significance of difference at the five percent level of confidence. Therefore, Hypothesis 7 can be accepted for the high ability groups at the five percent level.

(b) **Medium Ability Group.** Table I reports the mean posttest scores in mathematical achievement, three weeks after the treatment period, to be 15.50 for the medium ability treatment group and 16.24 for the medium ability nontreatment group. The t-value reported for this difference in means is 0.45 with 29 degrees of freedom. This t-value does not exceed the critical value of $\pm 2.05$ for the five percent level of confidence. The null hypothesis can be accepted at the five percent level for the medium ability groups.

(c) **Low Ability Group.** The mean scores reported for the low ability groups on the posttest of mathematical achievement, three weeks after the treatment period, were 10.69 for the treatment group and 10.28 for the nontreatment group. Table II reports the t-value for the difference in these means to be $-0.24$ with 32 degrees of freedom. Since the obtained t-value does not exceed the critical value of $\pm 2.04$, the null hypothesis can be accepted at the five percent level for the low ability groups.

The differences in mean posttest scores of mathematical achievement, three weeks after the treatment period, between the treatment and nontreatment groups were not statistically significant at any of the three ability levels. Consequently, the null hypothesis can be accepted at the five percent level of confidence.
Hypothesis 8.

There are no significant differences, between the treatment and nontreatment groups, in mean posttest scores on geometric proofs three weeks after the treatment period at three mathematical ability levels.

(a) High Ability Group. Table I reports the mean posttest scores on geometric proofs of the high ability treatment and nontreatment groups, three weeks after the treatment period, to be 7.58 and 9.55, respectively. The t-value of this difference is reported in Table II to be 1.37. The t-value reported does not exceed the critical value of $±2.05$ for significance of difference. Thus, the null hypothesis can be accepted at the five percent level of confidence for the high ability groups.

(b) Medium Ability Group. The mean posttest scores on geometric proofs of the medium ability group, three weeks after the treatment period, are given in Table I. The treatment group mean score was 9.14. The nontreatment group mean score was 9.06. Table II reports the t-value of the difference of these means to be $-0.06$ with 29 degrees of freedom. This value does not exceed the critical value ($±2.05$) for significant difference at the five percent level of confidence. The null hypothesis can be accepted for the medium ability groups at the five percent level.

(c) Low Ability Group. The low ability groups' mean scores were reported in Table I for the posttest on geometric proofs, three weeks after the treatment period, to be 5.69 for the treatment group and 4.89 for the nontreatment group. The t-value of $-0.57$ with 32 degrees of freedom reported in Table II does not exceed the critical value of $±2.04$ required to reject the null hypothesis. Therefore, Hypothesis 8 can be accepted for the low ability groups at the five percent level of confidence.
The differences in mean posttest scores obtained on geometric proofs, three weeks after the treatment period, were not statistically significant between the treatment and nontreatment groups at the five percent level of confidence for any of the three ability groups. The null hypothesis can be accepted for all ability groups at the five percent level.
CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

Summary. The purpose of this investigation was to evaluate the effects on achievement in and attitude towards mathematics of teaching certain logic concepts to tenth-grade geometry students. To this end, four classes of grade ten modern geometry students were selected and assigned to the treatment conditions. The treatment group was given a one-week introduction to logic concepts and a two-week study of similarity concepts. The non-treatment group was given only the two-week similarity program. Both groups were given three sets of tests. The pretreatment testing involved administration of the math attitude opinionnaire and a test of achievement in mathematics. The first posttreatment testing consisted of a re-administration of the attitude measure followed by an achievement test in mathematics. The second posttesting, three weeks after the first, involved administration of another achievement test in mathematics. The collected data was statistically analyzed for significance of differences between means using t-tests. Statistical testing of the hypothesis was carried out at the five percent level of confidence.

Conclusions. Comparisons of the mean pretest and posttest scores of the treatment and nontreatment groups on the test of attitude towards mathematics indicated no significant differences at the five percent level of confidence. Also not significant were the differences between the mean scores of the treatment and nontreatment groups on the first and second
posttests of achievement in mathematics. At the three mathematical ability levels used, no significant differences were found between the treatment and nontreatment groups in achievement in or attitude towards mathematics as a result of the one-week logic program.

The analysis of the gathered data suggests that teaching the one-week logic program to tenth-grade geometry students has no significant effects on achievement in or attitude towards mathematics as measured by the instruments used. Also suggested by this analysis is that the logical-deductive nature of mathematical reasoning can be taught to tenth-grade geometry students. However, such teaching does not seem to increase their ability to reason deductively in geometry as measured by performance on tests of deductive geometric proofs.

**Implications.** Because of the lack of common ground for comparison, the findings of this study may not legitimately be compared with other research findings. However, if the variations among the studies can be ignored, several comparisons can be made. The findings of this study suggest no advantages for the high ability group over the lower ability groups. This finding opposes the Suppes-Binford and Retzer results. Also in opposition to the Suppes-Binford findings but corroborated by the Elton study were the transfer of logic-training results. No convincing evidence was obtained from this investigation to suggest that logic-training would affect student achievement in geometry. This finding endures even when achievement on deductive geometric proofs only was considered. This may imply that proofs in modern geometry, although deductive in nature, do not require deductive reasoning. The students may be learning a "ritual" from
which deductive proofs result. This "ritual" may not be facilitated by logical-deductive reasoning. If this is true, tenth-grade geometry students should not be required to write geometric proofs.

In addition to its limited effects on achievement in geometry, the logic-training program appears to have minimally affected general attitude towards mathematics as measured by the Aiken and Dreger instrument. These results may be attributable to the relatively short treatment period and may not be conclusive. Since the logic-training program has exhibited no significant effects on achievement or attitude as measured by the instruments used in this study, there appears to be few benefits to be gained by modern geometry students from such a program.

Suggested Studies. Before concluding that a study of the logical-deductive nature of mathematical reasoning offers no benefits to the high school student, further research is required. Research is required to determine the effects on attitude and achievement of a prolonged logic program in which better control of treatment conditions can be obtained. Settings other than the grade ten geometry course should be considered. Individualized programmed and/or computer-assisted treatments should be utilized. Students should be assigned randomly to treatment classifications. Standardized tests should be used. A valid and reliable mathematics attitude scale suitable for use in the high school setting must be developed. The effects of a study of logic-concepts should be determined using a smaller sample with individualized treatments and stringent control of treatment conditions.


## LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendices</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Outline of One Week Logic Unit</td>
<td>41</td>
</tr>
<tr>
<td>B. Outline of Two Week Similarity Unit</td>
<td>47</td>
</tr>
<tr>
<td>C. Attitude Towards Mathematics Opinionnaire</td>
<td>52</td>
</tr>
<tr>
<td>D. Logic Achievement Test</td>
<td>54</td>
</tr>
<tr>
<td>E. Mathematics Achievement Tests - Form A and B</td>
<td>57</td>
</tr>
</tbody>
</table>
APPENDIX A

DAY 1

Short Study of the Nature of Deductive Reasoning

Introduction:

Two important types of reasoning in mathematics

i) inductive reasoning
   —method of making probable inference or conjecture from an
   examination of particular cases.
   ie. study millions of cases and observe
   that always: \( a + b > c \) \( \Rightarrow \) it is
   probably true.

ii) deductive reasoning
   —method by which facts in mathematics can be proved from
   general cases.
   ie. deductive reasoning on one general
   case proves \( \triangle ABD \cong \triangle ACD, \angle B = \angle C, \angle 3 = \angle 4, \ldots \) are
   definitely
   true.

—based on simple logic ideas.

DAY 2

Logic:

defn: A "sentence" in logic is a statement which is either T or F,
but not both.
defn: A "sentential connective" connects two simple sentences to
produce a compound sentence.
ex. simple sentence connective compound sentence

It is raining. and It is raining and it is
cold.

It is raining. or It is raining or it
is cold.
APPENDIX A—Continued

<table>
<thead>
<tr>
<th>simple sentence</th>
<th>connective</th>
<th>compound sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is raining.</td>
<td>if, then</td>
<td>If it is raining, then it is cold.</td>
</tr>
<tr>
<td>It is cold.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>It is raining.</td>
<td>not</td>
<td>It is not raining.</td>
</tr>
</tbody>
</table>

The basic forms of compound sentences are:

i) (....) and (.....)  \(\text{conjunction of 2 sentences}\)
ii) (....) or (.....)  \(\text{disjunction of 2 sentences}\)
iii) If (....), then (.....)  \(\text{implication of one sentence by another}\)
iv) Not (....)  \(\text{negation of a sentence}\)

In logic capital letters are used to represent simple sentences:

i) \(P\) and \(Q\)
ii) \(P\) or \(Q\)
iii) If \(P\), then \(Q\)  \(P\) implies \(Q\)  \(P \rightarrow Q\)
iv) Not \(P\)  \(\neg P\)

\text{defn:}  The process of making necessary conclusions from accepted statements by applying accepted rules of logic is called "deductive reasoning".

\text{ex.}  major premise : If it is raining, then it is cold.
minor premise : It is raining.

logical consequent : It is cold.

In symbols the argument is represented as:

major premise : \(P \rightarrow Q\)
minor premise : \(P\)
logical consequent : \(\therefore Q\)

The rule of logic which permits us to make this conclusion from these statements is called the "Law of Detachment".

\text{ex}  If logic is easy, then he will master it.
Logic is easy._
\(\therefore\) He will master it.

If I pass my examinations, I am very happy.
I am very happy._
\(\therefore\) No conclusion.

\text{Classwork:}

Day 2 Exercises - to be handed in during the hour and corrected by the instructor.
1. Which of the following are sentences in logic? Why are they or are they not sentences?

1. A triangle is a polygon.
2. $3 + 2 = 5$.
3. If $x + 3 = 7$, then $x = 5$.
4. An isosceles triangle has two sides having the same length.
5. A quadrilateral is a square.

2. Classify each of the following compound sentences in terms of conjunction, disjunction, implication, or negation (or some combination).

1. If $m$ is parallel to $n$ and $n$ is perpendicular to $l$, then $m$ is perpendicular to $n$.
2. If a triangle is not equilateral, it is not equiangular.
3. Dick is sick or Mel is not swell.
4. If $\square ABCD$ has opposite sides parallel or congruent, then $\square ABCD$ is a parallelogram.
5. $x + 3 \neq 6$.

3. Using deductive reasoning state the logical consequent, if there is one, for each of the following sets of premises:

1. i) If $x - 3 = 0$, then $x = 3$
   ii) $x - 3 = 0$
2. i) $(AD \perp BC) \rightarrow (m \parallel \overline{ADC} = 90)$
   ii) $(AD \perp BC)$
3. i) $(\overline{AD} \cong \overline{DB}$ and $\overline{DE} \parallel \overline{BC}) \rightarrow (\overline{AE} \cong \overline{EC})$
   ii) $\overline{AE} \cong \overline{EC}$
4. i) $(M \text{ or } N) \rightarrow O$
   ii) $(M \text{ or } N)$
5. i) $A \rightarrow B$
   ii) $B \rightarrow C$
   iii) $C \rightarrow D$
   iv) $A$
APPENDIX A—Continued

DAY 3

Deductive Proofs:

Most deductive arguments consist of more than one application of the Law of Detachment.

ie. Premises: 1. If it is snowing, then it is cold.
2. If it is cold, then I will stay home.
3. It is snowing.

Conclusion: I will stay at home.

The formal proof is written as follows:

Hypothesis: \( P \rightarrow Q \)
\( Q \rightarrow R \)
\( P \)

Conclusion: \( R \)

Proof:

1. \( P \rightarrow Q \)  
   | Hypothesis 
2. \( P \)  
   | Hypothesis 
3. \( Q \)  
   \( \therefore \) \( R \)  
   | Law of Detachment \((1,2)\) 
4. \( Q \rightarrow R \)  
   | Hypothesis 
5. \( \therefore R \)  
   | Law of Detachment \((3,4)\)

ex. Symbolize the following sentences and write a proof to justify the conclusion.

Hypothesis: 1. If humid air rises, it cools.
2. When it cools, clouds form.
3. Humid air rises.

Conclusion: Clouds form.

Hypothesis: \( A \rightarrow B, B \rightarrow C, A \)

Conclusion: \( C \)

Proof:

1. \( A \rightarrow B \)  
   | Hypothesis 
2. \( A \)  
   | Hypothesis 
3. \( \therefore B \)  
   | Law of Detachment \((1,2)\) 
4. \( B \rightarrow C \)  
   | Hypothesis 
5. \( \therefore C \)  
   | Law of Detachment \((3,4)\)
Syllogisms:

defn: A "syllogism" is a form of logical argument involving:

i) Major Premise: a general implication

ii) Minor Premise: a particular case of i

iii) Conclusion: the logical consequent of the general and particular statements

ex. Major Premise: All birds fly.
Minor Premise: Polly is a bird.

Conclusion: Polly flies.

ex. Major Premise: All right \( \infty \)'s are \( \infty \).
Minor Premise: \( \infty A \) and \( \infty B \) are \( \infty \).

Conclusion: No Conclusion.

ex. Major Premise: If a man is a policeman, then he must be 5'10" tall.
Minor Premise: John Law is a policeman.

Conclusion: John Law is 5'10" tall.

Classwork:

Day 3 Exercises – to be handed in during the hour and corrected by the instructor.
APPENDIX A—Continued

DAY 3 Exercises

1. Complete the following deductive proofs:

   i) Hypothesis: A, A → (¬B), (¬B) → C
      Conclusion: C
      Proof:
      1. A
      2. A → (¬B)
      3. (¬B)
      4. (¬B) → C
      5. C

   ii) Hypothesis: A → B, B → C, A, C → D, D → E
       Conclusion: E

   iii) Hypothesis: P → (Q and R), P, (Q and R) → S
        Conclusion: S

   iv) Hypothesis: (¬F) → Q, Q → (¬R), (¬F)
       Conclusion:

   v) Hypothesis: (a = b and b = c), (a = b and b = c) → (a = c),
      (a = c) → (c = a)
      Conclusion:

   vi) Hypothesis: (¬C) → D, C → (¬D), (¬E) → (¬C), D → B.
       E → C, E
       Conclusion: B
APPENDIX B

Day 1:

1. Introduction to Similarity

Congruent $\triangle$'s must have same size and shape.

° corresponding sides are $\cong$, and corresponding angles are $\cong$.

Similar $\triangle$'s must have same shape only.

° corresponding angles are $\cong$.

What can we say about corresponding sides?

$\triangle ABC \sim \triangle A'B'C' \iff \angle A \cong \angle A', \angle B \cong \angle B', \angle C \cong \angle C$ and $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

2. Proportionality

Defn: Given two sequences $a, b, c, \ldots$ and $p, q, r, \ldots$ of positive numbers. If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} \ldots$, then the sequences $a, b, c, \ldots$ and $p, q, r, \ldots$ are "proportional".

ie. $1, 2, 3, 4 \ldots$ and $3, 6, 9, 12, \ldots$ are proportional sequences since $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \ldots$

ie. $2, 1, 8, 3$ is proportional to $\frac{1}{2}, \frac{1}{2}, 2, 3$ since $\frac{2}{1} = \frac{1}{2} = \frac{8}{2} = \frac{3}{3/4}$.

ie. $3, 4, 5$ and $\frac{3}{5}, \frac{4}{5}, 1$ are proportional.

ie. $2, 4, 8$ and $2, 4, 8$ are proportional.

ie. $1, 5$ and $5, 25$ are proportional.

Proportionalities involving only 4 numbers are called "proportions".

47
APPENDIX B—Continued

What are some of the properties of proportions?

i) If \( \frac{a}{p} = \frac{b}{q} \), then \( a \cdot q = b \cdot p \). \hspace{1cm} \text{(ratio test)}

ii) If \( \frac{a}{p} = \frac{b}{q} \), then \( \frac{a}{b} = \frac{p}{q} \). \hspace{1cm} \text{(inversion property)}

iii) If \( \frac{a}{b} = \frac{p}{q} \), then \( \frac{a}{p} = \frac{b}{q} \). \hspace{1cm} \text{(alternation property)}

iv) If \( \frac{a}{b} = \frac{p}{q} \), then \( a + p = b + q \).

v) If \( \frac{a}{b} = \frac{p}{q} \), then \( a - p = b - q \).

vi) If \( \frac{a}{b} = \frac{p}{q} \), then \( b^2 = a \cdot c \) or \( b = \sqrt{a \cdot c} \) and \( b \) is called the "geometric mean" between \( a \) and \( c \).

Exercises: Text pp. 324 #2,3,5,6,7,10,11.

Day 2:

3. Similarities Between Triangles

\[
\begin{align*}
&\text{If } \angle A \cong \angle A', \quad \angle B \cong \angle B', \quad \angle C \cong \angle C' \quad \text{and } \quad \frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}, \\
&\text{then } \Delta ABC \sim \Delta A'B'C'.
\end{align*}
\]

\text{defn: If corresponding } \angle \text{'s are } \cong \text{ and corresponding sides are proportional, then the correspondence is a "similarity".}

The natural question to ask ourselves is "Is it necessary to satisfy all six conditions of the definition to show triangles are similar?"

4. The Basic Proportionality Theorem and Its Converse

Consider \( \Delta ABC \sim \Delta ADE \)

It looks like \( \Delta ABC \sim \Delta ADE \)

To prove this requires the following theorem:
APPENDIX B—Continued

Theorem 12-1: "The Basic Proportionality Theorem"

If \( \overline{DE} \parallel \overline{BC} \), then \( \frac{AD}{AB} = \frac{AE}{AC} \).

Proof:

(see the text p. 330)

The converse of Theorem 12-1 can now easily be proved.

Theorem 12-2: "Converse of B.P. Theorem"

If \( \frac{AD}{AB} = \frac{AE}{AC} \), then \( \overline{DE} \parallel \overline{BC} \).

Proof:

(see the text p. 331)

Exercises: Text pp. 332 #1a-c, 2,3,4a,7

Day 3:

5. The Basic Similarity Theorems

Theorem 12-3: "The AAA Similarity Theorem"

If \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \), then \( \triangle ABC \sim \triangle DEF \).

Proof:

(see the text p. 336)

Corollary 12-3.1: "The AA Corollary"

If \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \), then \( \triangle ABC \sim \triangle DEF \).
Corollary 12-3.2:

If \( \overline{DE} \parallel \overline{BC} \), then \( \triangle ABC \sim \triangle ADE \).

Exercises: Text pp. 338 #1,3,4,6,8

Day 4:

Theorem 12-4: "The Transitive Property of Similar Triangles"

If \( \triangle ABC \sim \triangle DEF \) and \( \triangle DEF \sim \triangle GHI \), then \( \triangle ABC \sim \triangle GHI \).

Proof:

(follows from the defn. of similar triangles)

Theorem 12-5: "The SAS Similarity Theorem"

If \( \frac{AB}{DE} = \frac{AC}{DF} \) and \( \angle A \cong \angle D \), then \( \triangle ABC \sim \triangle DEF \).

Proof:

(see the text p. 342)

Theorem 12-6: "The SSS Similarity Theorem"

If \( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \), then \( \triangle ABC \sim \triangle DEF \).

Proof:

(see text p. 343)

Exercises: Text pp. 344 #1,2,4
Day 5:

Review and Analysis of Several Proofs

i) Review of Similarity Theorems

ii) Prove that if $\triangle ABC \cong \triangle DEF$, then $\triangle ABC \sim \triangle DEF$.

iii) Prove that if $\overline{AC} \parallel \overline{BD}$, then $\triangle ACE \sim \triangle BDE$.

iv) Prove that if $X$ is the midpoint of $\overline{PQ}$ and $Y$ is the midpoint of $\overline{PR}$, then $\triangle PXY \sim \triangle PQR$.

v) Mathematics Opinionnaire.

vi) Posttest on Logic.

Day 6:

Posttest of Achievement in Geometry on the Similarity Material Studied during the Treatment Period.
APPENDIX C

Math Attitude Opinionnaire

Directions: Please write you name in the space provided. Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are:

SD  Strongly Disagree
D   Disagree
U   Undecided
A   Agree
SA  Strongly Agree

You are to encircle the letter(s) which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU.

1. I do not like mathematics. I am always under a terrible strain in a math class. SD D U A SA
2. I do not like mathematics, and it scares me to have to take it. SD D U A SA
3. Mathematics is very interesting to me. I enjoy math courses. SD D U A SA
4. Mathematics is fascinating and fun. SD D U A SA
5. Mathematics makes me feel secure, and at the same time it is stimulating. SD D U A SA
6. I do not like mathematics. My mind goes blank, and I am unable to think clearly when working math. SD D U A SA
7. I feel a sense of insecurity when attempting mathematics. SD D U A SA
8. Mathematics makes me feel uncomfortable, restless, irritable, and impatient. SD D U A SA
9. The feeling that I have toward mathematics is a good feeling. SD D U A SA
10. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out. SD D U A SA
APPENDIX C—Continued

11. Mathematics is something which I enjoy a great deal.  SD D U A SA

12. When I hear the word math, I have a feeling of dislike.  SD D U A SA

13. I approach math with a feeling of hesitation—hesitation resulting from a fear of not being able to do math.  SD D U A SA

14. I really like mathematics.  SD D U A SA

15. Mathematics is a course in school which I have always liked and enjoyed studying.  SD D U A SA

16. I don't like mathematics. It makes me nervous to even think about having to do a math problem.  SD D U A SA

17. I have never liked math, and it is my most dreaded subject.  SD D U A SA

18. I love mathematics. I am happier in a math class than in any other class.  SD D U A SA

19. I feel at ease in mathematics, and I like it very much.  SD D U A SA

20. I feel a definite positive reaction to mathematics; it's enjoyable.  SD D U A SA
APPENDIX D

Logic Achievement Test

Classify the following sentences as involving conjunction (C), disjunction (D), implication (I) or negation (N) or any combination of C, D, I, and N.

1. If he is certain, then he will not answer.
2. If \( x^2 = 4 \) and \( x \) represents a natural number, then \( x = 2 \).
3. A person who doesn't understand logic will not understand geometry.
4. If I think logically, I can solve this problem.
5. It will not rain when skies are clear.

Rewrite the following sentences using capital letters to represent simple sentences. ie. \( P \rightarrow Q \)

1. Logic is easy.
2. If \( x = 3 \), then \( 3x - 2 = 7 \)
3. If skies are clear, it will not rain.
4. If Phil is a pill and Mary is a fairy, then Jane is a pain.
5. If today is Saturday or Sunday, there is no school.

Write the logical consequent, if there is one, for each of the following:

1. Major Premise: \( (S \text{ or } R) \rightarrow (T \text{ or } U) \)
   Minor Premise: \( R \)
   Consequent: 

2. Major Premise: \( (\text{ ABCD is a } II \text{ogram}) \text{ (AB} \parallel \text{ DC and BC} \parallel \text{ AD}) \)
   Minor Premise: \( \text{ ABCD is a } II \text{ogram} \)
   Consequent: 

3. Major Premise: \( \angle A + \angle B = 90 \rightarrow \angle A \text{ and } \angle B \text{ are complementary} \)
   Minor Premise: \( \angle A = \angle B \)
   Consequent: 

54
APPENDIX D—Continued

4. Major Premise: \( \angle A \) and \( \angle B \) form a linear pair \( \rightarrow m \angle A + m \angle B = 180 \)
   Minor Premise: \( m \angle A + m \angle B = 180 \)
   Consequent: 

5. Major Premise: \( \angle A \) and \( \angle B \) form a linear pair \( \rightarrow m \angle A + m \angle B = 180 \)
   Minor Premise: \( \angle A \) and \( \angle B \) do not form a linear pair.
   Consequent: 

Write a proof to justify the following logical conclusions:

1. Hypothesis: \( A \rightarrow B, \ C \rightarrow A, \ C \)

   Conclusion: \( B \)

   Proof:
   1.
   2.
   3.
   4.
   5.
   6.
2. Hypothesis: \( C \rightarrow D, \ E \rightarrow C, \ E \rightarrow D, \ D \rightarrow B, \ E \)

   Conclusion: \( B \)

   **Proof:**
   
   1. 
   2. 
   3. 
   4. 
   5. 
   6. 
   7. 

3. Hypothesis: \( (A \ or \ B), \ (A \ or \ B) \rightarrow D, \ (E \ and \ F) \rightarrow (G \ or \ H), \ D \rightarrow (E \ and \ F) \)

   Conclusion: \( G \ or \ H \)

   **Proof:**
   
   1. 
   2. 
   3. 
   4. 
   5. 
   6.
Achievement Test in Geometry - Form A

I. Solve each of the following as directed and write your answer in the appropriate blank to the right.

1. Solve for \(a\): \(a + 2 = 20\)  
   \[\frac{3a}{30}\]  
   1. \(a = \) __________

2. Solve for \(x\): \(14 \cdot x = 21\)  
   \[\frac{x}{3}\]  
   2. \(x = \) __________

3. Solve for \(p\): \(\frac{4q - 3}{5} = \frac{p}{7}\)  
   \[\frac{q}{7}\]  
   3. \(p = \) __________

II. Circle T if the statement is True, F if the statement is False.

4. All right triangles are similar 4. T F

5. All squares are similar 5. T F

6. All rectangles are similar. 6. T F

7. If a line intersects two sides of a triangle and cuts off segments proportional to these two sides, then the line is parallel to the third side of the triangle. 7. T F

8. If the measures of two angles of a triangle are 45 and 60 and the measures of two angles of another triangle are 45 and 75, the triangles are similar. 8. T F

III. Solve each of the following, simplifying answers whenever possible.

Given the figure with \(\overline{MK} \parallel \overline{RS}\)

9. If \(PH = 7\), \(PK = 5\) and \(HG = 3\), what is \(KS\)? 9. \(KS = \) __________

10. If \(PR = 12\), \(MR = 4\) and \(PK = 6\), what is \(PS\)? 10. \(PS = \) __________
APPENDIX E—Continued

Complete the following proofs in the space provided:

#1. In the trapezoid ABCD, prove that $\triangle AEB \sim \triangle DEC$.

![Diagram of trapezoid ABCD with intersecting diagonals]

Proof:

1. 
2. 
3. 
4. 
5. 

#2. Prove that two isosceles triangles are similar if they have a base angle of the first equal to a base angle of the second.

![Diagram of two isosceles triangles]

Proof:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
APPENDIX E—Continued

#3. Prove that the triangle formed by joining the midpoints of the sides of \( \triangle ABC \) is similar to \( \triangle ABC \).

Proof:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
APPENDIX E—Continued

Achievement Test in Geometry—Form B

I. Solve each of the following as directed and write your answer in the appropriate blank to the right.

1. Solve for \( x \) : \( \frac{x + 3}{4x} = \frac{14}{35} \).
   \[ x = \square \]

2. Solve for \( \frac{x}{5} \) : \( 8x = 20 \).
   \[ x = \square \]

3. Solve for \( \frac{r}{s} \) : \( \frac{2a}{5} = \frac{4r}{6} \).
   \[ r = \square \]

II. Complete each sentence with the correct symbol(s) or word(s).

4.-5. A correspondence between two triangles is a similarity if the corresponding (4) are proportional and the corresponding (5) are congruent.
   4. 
   5. 

6. If "m" is the geometric mean between the two positive numbers "r" and "s", then m = ___.

III. Answer the following statements with "TRUE" if the statement is true and "FALSE" if the statement is false.

7. Two triangles are similar if any pair of angles of the first triangle is congruent to any pair of angles of the second. 7. ___

8. Two right triangles are similar if an acute angle of the first triangle is complementary to an acute angle of the second triangle. 8. ___

IV. Given the figure with \( \overline{AC} \parallel \overline{PQ} \)

9. If \( AT = 5 \), \( AP = 2 \), and \( CQ = 3 \), what is \( TC \)?
   9. ___

10. If \( PT = 33 \), \( AP = 12 \), and \( DB = 8 \), what is \( TD \)?
    10. ___
APPENDIX E—Continued

COMPLETE THE FOLLOWING PROOFS IN THE SPACE PROVIDED:

1. Given the parallelogram LMNO with diagonals LN and MO
   Prove that $\triangle KLM \sim \triangle KNO$.
   Proof:
   1. 
   2. 
   3. 
   4. 
   5. 
   6. 
   7. 

2. Given isosceles triangles, $\triangle ABC$ and $\triangle XYZ$, with their vertex angles congruent
   i.e. $\angle A \cong \angle X$
   Prove that $\triangle ABC \sim \triangle XYZ$.
   Proof:
   1. 
   2. 
   3. 
   4. 
   5. 
   6. 
   7. 
   8. 
   9. 

3. Given the quadrilateral ABCD with midpoints L, M, N, and O
   of the sides,
   Prove that $\triangle OLM \sim \triangle MNO$.
   Proof:
   1. 
   2. 
   3. 
   4. 
   5. 
   6. 
   7. 
   8. 
   9. 