AN EXPERIMENTAL INVESTIGATION OF THE
AUTOROTATION OF A FLAT PLATE

by

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We accept this thesis as conforming to the
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ABSTRACT

Autorotation measurements were made for a 4 inch chord thin plate under approximately two-dimensional flow conditions in a low speed wind tunnel. To determine the instantaneous angular velocity and angular acceleration of the plate, a new technique based on the principle of an angular displacement transducer was developed. The closely linear variation of tip speeds with wind speed and the non-linear characteristics of the build-up time and angular acceleration were investigated in the wind speed range from 10.9 fps to 37.3 fps. To examine the instantaneous aerodynamic loading on the autorotating plate as a function of angular position of the plate, surface fluctuating pressure measurements were made, with the aid of a pressure transducer and a dynamic pressure seal, and correlated with displacement transducer readings at the wind speed of 26.3 fps. Instantaneous aerodynamic torque during one autorotation cycle was estimated from the integration of the fluid moments resulting from the instantaneous surface pressure fluctuations. Some discussion of the aerodynamic phenomena is given.
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## SYMBOLS

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<tr>
<td>$\Delta_{\text{max}}$</td>
<td>Maximum breadth of the model at all angles of attack</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Instantaneous magnetic flux in the primary winding</td>
</tr>
<tr>
<td>$\phi_{\text{max}}$</td>
<td>Amplitude of the magnetic flux in the primary winding</td>
</tr>
<tr>
<td>$N_S$</td>
<td>Number of turns of the secondary coil</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>Instantaneous magnetic flux linking the secondary coil</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency of the carrier signal</td>
</tr>
<tr>
<td>$w$</td>
<td>Circular frequency of the carrier signal</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angular position of model and also angle between the magnetic axis of the primary winding and secondary winding</td>
</tr>
<tr>
<td>$e_S$</td>
<td>Induced emf in secondary winding</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Rotor Winding #1 of the Angular displacement transducer</td>
</tr>
<tr>
<td>$S_1$</td>
<td>Stator Winding #1 of the angular displacement transducer</td>
</tr>
<tr>
<td>$S_2$</td>
<td>Stator winding #2 of the angular displacement transducer</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Rotor winding #2 of the angular displacement transducer</td>
</tr>
<tr>
<td>$\ddot{\theta}$</td>
<td>Angular acceleration</td>
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<tr>
<td>$\omega$</td>
<td>Instantaneous angular velocity</td>
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<tr>
<td>$\omega_{\text{auto}}$</td>
<td>Autorotation speed</td>
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<tr>
<td>$U$</td>
<td>Actual wind velocity</td>
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<tr>
<td>$V$</td>
<td>Chart speed of visicorder oscillograph</td>
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<tr>
<td>$y_0$</td>
<td>Amplitude of the carrier signal</td>
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<tr>
<td>$p'$</td>
<td>Fluctuating Pressure</td>
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\( \alpha \) Speed of sound

\( \Omega \) Circular frequency of piston

\( \delta \) Calibration piston amplitude

\( \rho \) Air density

\( S \) Writing speed of visicorder oscillograph

\( C_p' \) Fluctuating pressure coefficient \((p'/2\rho U^2)\)

\( C \) Chord of model

\( T \) Torque per unit span

\( C_T \) Torque coefficient \((T/2\rho U^2 C^2)\)

\( x \) Distance along the model chord

\( k \) Amplitude of the elliptic integral

\( \phi \) Velocity Potential
1. INTRODUCTION

The phenomenon of autorotation of bodies of certain shapes in a transverse wind presents challenging problems in fluid mechanics and engineering practice. Fluid mechanicians look on the problem as an interesting separated flow phenomenon on which little work has been done as yet, while design engineers have encountered the annoying problem that flat antenna elements sometimes exhibit rotation induced by wind. A possible engineering application is the use of a rotating flap as a high-lift or control device for the conventional airplane (1) (2).

The term autorotation is applied to the steady, aerodynamically-excited rotation of bodies in an air stream. At least two forms of the phenomenon have been observed. The major difference between them is in the direction of the air flow relative to the axis of rotation of the body.

In the first form of autorotation, the axis of rotation is aligned with the wind, and the bodies are bluff cylindrical propellers. The aerodynamic toy "aerial tourbillion" described by Lanchester (3), Den Hartog (4), and in a recent paper by Parkinson (5) offers a good example. This form of autorotation is not considered in the present research program. It is, in any case, fairly well understood. In the second form of autorotation, a thin flat or cambered plate is free to rotate about a spanwise mid-chord axis in its own plane of symmetry in the presence of a uniform wind at right angles to the axis. The system is stable at rest, no matter how hard the wind is blowing. But if the plate is given an initial spin in either direction,
it will accelerate to a higher steady rotational speed -- the so-called autorotation speed. The theory of fluid motion associated with autorotation presents tremendous difficulties, since it deals with unsteady separated flow. It may be surmised that the shape of the cylinder causes flow separation and the shedding of vortices in the wake. The resulting asymmetrical flow field during each revolution produces instantaneous fluid forces and moments which are conducive to the "free-wheeling" of the cylinder. Measurement of these forces and moments is one of the objects of the present investigation.

Surprising though it may seem, some aerodynamic properties of autorotation were examined even before the era of flight. Maxwell (1835) demonstrated and tried to explain why an oblong card, while falling freely in the air, tended to rotate about its longitudinal axis. Crabtree (1) and Neumark (2) summed up some earlier research in Europe up to World War II and further presented some analysis based on potential flow theory. It was pointed out that an isolated rotating wing could develop circulation and high lift in the mainstream but that a more practical development was a fixed mainplane with a rotating flap just below the trailing edge. A mathematical two-dimensional model of the flow was developed by assuming a potential vortex to be positioned below the trailing edge of a thin airfoil. The theoretical prediction of the optimum relative position of the airfoil-rotating flap combination was confirmed by the experimental results.

As to the investigation of the rotating wing alone, only a few crude experiments were attempted until recent years. James and Stone (6)
measured the autorotation speed and forces (lift and drag) produced in autorotation on a three component balance. Their models were flat and cambered thin plates of various aspect ratios. A simple circuit, consisting of a capacitor, a D.C. power supply, and a spring contact switch built in one end of the model, was set up to estimate the times between successive half revolutions. Acceleration and torque during the build-up period were then derived from the movie films of records taken from a cathode-ray oscilloscope. A final attempt was the visualization of airflow at all parts of the cycle during autorotation, by means of fumes of titanium tetrachloride. In a similar experiment, Baird and Pick (7) studied the autorotation speed as a function of wind speed and of size and shape of a series of rectangular plates.

Although the review of past work just described is admittedly far from complete, it indicates that the mechanism of autorotation of thin plates is not yet understood.

Accordingly, the following research project is a fairly elaborate experiment to investigate, for a thin flat plate under approximately two-dimensional conditions, the critical wind speed to cause autorotation, the dependence of tip speed on wind speed, the time and acceleration during the build-up period, and the angular velocity and acceleration during one autorotation cycle. A final scheme is to examine the amplitude, modulation, frequently, and phase of fluctuating pressure on the surface of an autorotating flat plate, and thereby determine the instantaneous aerodynamic torque as a function of angular position in the autorotation cycle.
II. INSTRUMENTATION

2.1 General Outline

Wind tunnel measurements were made for an autorotating thin plate. The plate was mounted between one set of end plates (section 2.5) so that approximately two-dimensional flow conditions existed over most of the span. Also the plate was supported at each end on coaxial spindles. The upper spindle was rigidly attached to an angular displacement transducer (section 2.6) so that synchronization with the angular motion could be achieved. Into the lower spindle was introduced a dynamic pressure seal (section 2.10) to effect the "relay" of the pressure signal from the surface taps to a pressure transducer (section 2.8). Fig. 1 shows the experiment setup in the wind tunnel.

2.2 Wind Tunnel

The wind tunnel used in this programme is a low speed, low turbulence, return type. Velocities can be varied through the range 4 feet per second to 150 feet per second with turbulence level of less than 0.1%. The pressure differential across the contraction section of 7:1 area ratio is measured on a Betz micromanometer which can be read to 0.2 millimeter of water; the test section velocity is calibrated against the above pressure differential. The test section is rectangular in cross-section (36 inches by 27 inches) with 45° corner fillets. Variation of the corner fillets from 6 inches by 6 inches to 4 3/4 inches by 4 3/4 inches compensates for boundary layer growth. The spatial variation of velocity is less than 0.25%. Tunnel power is supplied by a 15 horsepower direct current motor driving a commercial axiflow fan with a modified Ward-Leonard system of speed control.
aerodynamic outline of the tunnel is shown in Fig. 2.

2.3 Model

In view of the elaboration involved in machining, only one autorotation model was constructed. The nominal dimensions were 4 inch chord, 12 inch span, and 3/16 inch thickness. To simulate the geometric shape of a thin flat plate, leading and trailing edges were chamfered off to yield 12 degree sharp angles and a symmetrical trapezoidal cross section. It was argued that the flat surface approximated the flat plate while the other surfaces provided the necessary rigidity.

Since one of the purposes of the programme was to examine the fluctuating pressure on the surface of the model, it was intended to convey the pressure signal from surface taps via small diameter tubing to an externally mounted pressure transducer. (Section 2.8) Surface taps with internal tube connections leading to the bottom end of the model greatly complicated the design. Two sheets of acrylic plastic, which were hollowed out with lightening holes, furnished the upper and lower parts of the framework (ribs and spars). A pressure block was drilled to effect 90 degree bends for convenient tube connections and was incorporated in the upper framework. On the cambered surface of the lower framework, a spanwise groove was milled out to accommodate the bundle of pressure tubing. Fig. 3 shows the detailed arrangement together with the aluminum tabs.

To provide surface finish and torsional resistance, the whole framework was covered with 0.020 inch thickness aluminum skin. Over the flat
surface of the skin at mid-span were distributed 13 pressure taps (no pressure tap on the cambered surface). Fig. 4 gives the distribution of the pressure taps on the model surface. Pressure tap holes were 0.018 inch in diameter and were drilled to match individual tube connections, after an epoxy adhesive was applied to bond the aluminum skin to the framework. Polyethylene tubing used to transmit the pressure from surface taps was 0.066 inch inside diameter, 0.095 inch outside diameter, and 4 feet in length. Fig. 5 shows the finished model.

2.4 Model Supporting System

Earlier investigation (7) showed that a two-dimensional model, spanning the tunnel test section (27 inches), experienced considerable bending and vibration and produced inconsistent results. It was thus decided to construct a model of shorter span and to eliminate the end effects by a set of end plates (Section 2.5). In consequence, the model supporting system was mounted inside the tunnel and minimum blockage to the air flow was a major consideration. In addition, the scheme to measure the surface fluctuating pressure of an autorotating model brought up at least three tricky problems in the design. First, pressure tube connections from surface taps to the pressure transducer naturally became an integral part of the mounting system. Since it was only possible to take pressure readings from individual taps one at a time, ease of switching tube connections was essential. Second, the use of a pressure transducer which is stationary to measure a revolving pressure source through rotating pressure tubes strongly suggested that some sort of dynamic pressure seal be adopted to
effect the "relay" of the pressure signal. It was then necessary to
incorporate such an arrangement of pressure seal (Section 2.10) into the
mounting system. Third, it was intended to set up a synchro type of angular
displacement transducer to indicate the phase of the surface fluctuating
pressure (Section 2.6). The problem of accurately matching the mechan­
cal angle of the model to the electrical angle of the synchro output
presented no less difficulty than the previous two problems (Section 2.7).

Emphasis was also placed on accurate alignment of spindles and
constraint of motion to purely one degree of freedom. Because of these
requirements, and that of small friction damping, it was decided that spindles
mounted on ball bearings with a cylindrical bearing block be adopted as a
construction technique. Four SKF 6202 ball bearings were selected and
used in the design of the upper and lower autorotation stands. It must be
realized that satisfactory performance of the revolving unit could be obtained
only by careful and difficult alignment of four bearings.

A two inch outside diameter hollow cylinder provided the body of the
bearing block of the upper autorotation stand. It was counterbored from each
end to suitable diameters and depths so that one end permitted seating of
the ball bearings and the other end provided housing for the angular dis­
placement transducer. At the transducer end of the spindle, a small hole
was drilled to accommodate the transducer shaft; thus the spindle and the
transducer shaft could be tightened up by a socket head setscrew from outside.
At the model end of the spindle, a deep groove was milled out diametrically
to allow a tongue-and-groove joint of the model tab. The base of the
bearing block was held firmly by a steel disk, which in turn was bolted to the flange of the tunnel wall. Fig. 6 shows the drawing of this design.

The lower autotrotation stand was similar in construction. The spindle was mounted in a tight fit on two ball bearings and a counterbored hollow cylinder served as the bearing block. However, the pressure tube connection and the pressure seal unit further complicated the design. (Figs. 7 and 8).

The lower model tab was permanently attached by two pins to a hollow cylindrical coupling, the inside diameter of which was determined by the space necessary for 13 pressure tubes. The model end of the spindle was threaded externally (9/16-12NC). The coupling and the spindle were then linked together by a locking nut when desired. The locking nut had ordinary threads (9/16-12NC) at the spindle end and special threads of the same pitch (1-12NC) at the coupling end so that, when the spindle was held stationary, it could be screwed up to connect the coupling and screwed down to disconnect the coupling with little effort. Switching of tube connections was carried out while the locking nut was dismantled from the coupling. (Fig. 9).

The lower end of the spindle extended through the tunnel floor and made up the revolving element of the pressure seal unit. The stationary element of the pressure seal unit was bolted onto a semi-annular spacer in such a way that vaseline seal could be applied constantly. In addition, the spindle was drilled concentrically all the way through. Into this hole a
piece of polyethylene tubing was introduced. The upper end of this tubing connected the pressure tubes from surface taps one at a time in a plexiglas tube; the lower end connected to the revolving part of the pressure seal unit. Fig. 10 shows the model and the upper and lower autorotation stands in the test section.

2.5 End Plates

Many wind tunnels make use of end plates which fit inside the normal test sections to provide approximately two dimensional flow for shorter span models. The need arises, for instance, when it is intended to increase the model breadth without causing too much tunnel blockage or structural problems.

A considerable amount of literature is available on the use of end plates in conjunction with unstalled streamlined sections. Very few references, however, could be found to their use with models experiencing separated flow. In an investigation of the fluctuating forces acting on a 1 1/8 inch stationary circular cylinder, Keefe (8) studied the effect of a pair of 5 1/2 inch circular disks as end plates. It was found that the end plates were rather ineffective at large spacing between them but that local two-dimensionality existed at a small spacing equal to three times the cylinder diameter through the Reynolds number range from 20,000 to 100,000. In a later report, Cowdrey (9) investigated the minimum dimensions of rectangular end plates for use with a 4 inch square cylinder. Flow visualization showed that, when the end plates were too small, entrainment of the flow around the end of the model into the wake caused strong spanwise flow over the rear surfaces
of the model. The function of the end plates was to close the ends of this "cavity". Accordingly it was suggested that the minimum values of the width and of the downstream dimension of the end plates could be taken as $2 \Delta_{\text{max}}$ and $2 \ 1/2 \ \Delta_{\text{max}}$ respectively, where $\Delta_{\text{max}}$ was the maximum breadth of the model at all angles of attack.

Wind tunnel end plates made of 1/2 inch thick birch were designed and used in this programme. The criteria just described served as a rough guide in the design. To determine the dimensions of the end plates, the 4 inch autorotating plate was considered (for this purpose only) to be replaced by a 4 inch rotating circular cylinder. The spacing between the end plates was chosen as 12 inches (the span of the model was made the same accordingly). The plates were 16 inches long and 18 inches wide with the model axis 12 inches upstream from the trailing edge. To allow quick mounting of the model between the end plates, a 4 1/4 inch circular hole with a suitable recessed shoulder was drilled and centered on the model axis, both for the upper and lower end plate. Circular disks with a 4 1/4 inch boss were cut diametrically into two halves and were installed around the model spindles to make up part of the smooth parallel walls of the end plates after mounting. Each end plate was bolted to three vertical struts positioned as the vertices of an isosceles triangle. Accurate alignment of the end plates was achieved by independently adjusting the differential screws, which were tapped internally to accommodate the struts and threaded externally to match the flanges bolted onto the tunnel walls and whose pitch equaled the difference between those of the external and internal threads. Small gaps
between the ends of the model and the end plates were always kept less than 1/32 inch. Fig. 11 gives the drawing of this design and Fig. 12 shows the end plates with the removable circular disks.

Two sets of end plates of slightly different design were constructed. Most dimensions were made the same. The difference between them was that the edges of one set were chamfered off while those of the other set were rounded. Only one set of end plates was selected for use in the fluctuating pressure measurements. The decision was based on the spatial uniformity of the air flow between the plates. (Section 3.1.1)

2.6 Angular Displacement Transducers

2.6.1 Use of the Transducer

An angular displacement transducer was used to indicate the phase of the autorotating model, yielding a continuous history of rotation at constant speed or during acceleration. Once such a time history at a certain wind speed was recorded, many interesting features of autorotation would come to light. Among some of the possible investigations are the dependence of tip speed on wind speed, the time and acceleration during the build-up period, and the angular displacement, velocity, and acceleration during one autorotation cycle. Another function is to correlate the phase angle of the fluctuating pressure on the surface of the model to that of the angular position of the model. This correlation will help explain the mechanism of autorotation by the aerodynamic forces.
2.6.2 Requirements of an Ideal Transducer

1) low friction damping and small moment of inertia—negligible compared to that of the autorotation assembly.
2) linear response or known non-linear response such as a sine or cosine function.
3) output independent of angular velocity or angular acceleration.
4) high accuracy.

2.6.3 Principles of Transducers

Two types of angular displacement transducer were considered in this project. The first try was a variable-resistor transducer; the second attempt, a successful one, was a mutual-inductance transducer.

Resistance wire wound around an insulating ring with three terminals, represents the simplified form of a potentiometer. It gives continuous conversion of rotational displacements into electric signals. As expected, the response proved to be fairly linear even for large angles (up to 300°). However, the resistance in such potentiometers increases in steps rather than continuously when the contact moves from one turn of the resistance wire to the next. This stepwise variation limits the resolution of the potentiometer.

Another disadvantage was that the last few turns of the resistance wire on either side of the contact travel were sometimes not accessible for resistance variation. The range of angular movement through which a
variation of the electrical output takes place (electrical angle) was sometimes smaller than the mechanical angle through which the potentiometer shaft can move. In view of the high accuracy desired, this variable-resistor transducer was abandoned.

The second attempt was a synchro resolver -- a mutual-inductance transducer used for rotary displacements. The electrical circuit is shown in Fig. 13. The assembly and components are shown in Fig. 14. A synchro is in principle a transformer. The only difference from a conventional transformer is that the primary winding can rotate with respect to the secondary winding. As in a transformer, an alternating current in the primary winding results in a magnetic flux in the vicinity,

$$\phi_p = \phi_{\text{max}} \cos 2\pi ft$$  

(1)

It is seen that the magnetic flux changes sinusoidally in intensity and direction with the frequency of the alternating current. If a secondary winding is placed in the same magnetic field, a voltage is induced in the secondary winding and a current will flow through a load. From Faraday's law, the magnitude of this induced emf is given by

$$e_s = -N_s \frac{d\phi_s}{dt}$$  

(2)

where \(N_s\) = number of turns of the secondary coil

\(\phi_s\) = magnetic flux linking the secondary coil.

If the same flux links both the primary winding and the secondary winding as in an ideal transformer (i.e. the coil axes line up with each other),

$$e_s = -N_s \frac{d\phi_p}{dt}$$
\( (\phi_s)_{\text{max}} = \phi_p = \phi_{\text{max}} \cos 2\pi ft \) \( \quad (3) \)

and \( (e_s)_{\text{max}} = E_{\text{max}} \sin 2\pi ft \)

where \( E_{\text{max}} = N (2\pi f)\phi_{\text{max}} \)

Now consider the general case when the magnetic axes of the windings are at an angle \( \theta \) to each other. Since only a portion of the total magnetic flux effectively links the secondary coil, the output depends on the angular displacement and is proportional to the cosine of that angle.

\[ i.e. \quad e = E_{\text{max}} \sin 2\pi ft \cos \theta \] \( \quad (5) \)

It follows that, for a given angular position, there is only one corresponding secondary voltage. If the position of the primary winding is varied at some constant rotary speed, the amplitude of the secondary voltage will be modulated sinusoidally at the rotation frequency of the primary. In other words, the envelope of the amplitude modulated signal during 180° rotation will take a sinusoidal form. This is the basis of dynamic calibration of the angular displacement transducer to be discussed in Section 2.7.

2.6.4 Construction of the Transducer

A system used to transmit mechanical shaft angles to a remote location by means of electrical voltages is known as a synchro system. Synchros (10) are small motorlike components with multiple outputs. The basic structure consists of a wound rotor with connections made through slip rings and a wound stator carrying two or three pairs of windings. In the conventional
use, an ac voltage is applied to the rotor, the primary winding; the flux induces voltages in the stator, the secondary winding.

The angular displacement transducer under investigation falls under the category of a synchro resolver. Most synchro resolvers have two-phase stator windings with coil axes displaced 90 degrees, and two-phase rotor windings with coil axes also at 90 degrees. Thus the induced stator voltages are sine and cosine functions of the displacement angle.

\[ e_{s1} = E \sin(2\pi ft) \cos \theta \max \]
\[ e_{s2} = -E \sin(2\pi ft) \sin \theta \max \]

Among the commercially available synchro resolvers, the autosyn synchro resolver AY-221-3-B was found satisfactory. It has a primary coil impedance of 224 ohms and a secondary coil impedance of 52 ohms at 500 c.p.s. excitation, as measured by a General Radio Impedance Bridge.

The rotor winding \( P_2 \) of the angular displacement transducer was left open circuit. The rotor winding \( P_1 \) was excited by a Heathkit audio generator model 1G-72. The induced secondary voltage \( e_{s1} \) was monitored with a Tektronix model 564 storage oscilloscope or alternatively was recorded continuously by a Minneapolis Honeywell 906c visicorder oscillograph. The induced secondary voltage \( e_{s2} \) was connected across a dummy load of 35 ohms. This arrangement was found necessary to eliminate unwanted fluctuation of the applied voltage in the primary. The reason is to provide a balanced two-phase load such that there is always a constant reaction (constant \( T_R \)) on the primary. This is similar to the case when an ordinary static trans-
former has a fixed load. Thus the input impedance on the primary power are independent of shaft angle. Fig. 13 shows the electrical circuit of the angular displacement transducer.

As seen from the basic relations derived earlier, the induced secondary voltage depended upon such free parameters as the amplitude and frequency of the excited source. Theoretically the transducer could be working at any arbitrary excited voltage, so long as the induction coils were not saturated. However, the visicorder oscillograph narrowed the choices of the excited power.

The essential element of a visicorder is a sub-miniature galvanometer. The technical number of our galvanometer is Heiland M1000. It has a coil resistance of 35 ohms and requires a source resistance of 3 to 100 ohms as its damping resistance. Its range of flat frequency response is from 0 to 600 c.p.s. The amplitude of the excited voltage was so chosen that the maximum induced secondary voltage across the galvanometer coil caused exactly 4 inches peak to peak deflection on the photographic paper. The frequency of the excited voltage was chosen as 500 c.p.s. so that the required writing speed of the visicorder was below the maximum writing speed of 10,000 inches per second. (Appendix A). An additional advantage is that the modulated signal automatically provides 2 milli-second time markers on the photographic paper.

Another point is perhaps worth mentioning. The signal from the angular displacement transducer was not demodulated. An unsatisfactory attempt
was made to demodulate the output with a diode ring demodulator followed by an RC filter. It was found that at its rated excitation frequency the transducer was successfully demodulated only with a large capacitor. This large capacitance in turn means a large time constant or slow dynamic response of the circuit. The dynamic calibration showed considerable distortion of the sinusoidal envelope. Since demodulation sacrificed the degree of accuracy, it was abandoned.

2.7 Calibration Equipment of Angular Displacement Transducer

The purpose of the calibration of the angular displacement transducer was to establish a known relation between the mechanical angle input and the electrical voltage output. To this end, a calibration system was set up on the bench. Fig. 15 gives the flow diagram of this calibration system.

To check the dynamic response, the angular displacement transducer was coupled to a d-c shunt motor. The armature of the motor was connected to the d-c terminals of a diode ring rectifier, the a-c terminals of which were powered by a transformer. Coarse speed control of the motor was achieved by adjusting the transformer voltage across the rectifier. To allow fine speed control, the circuit of the field coil was modified to connect across the d-c terminals of a second diode ring rectifier similar to the first one. A potentiometer in series with a field coil provided fine adjustment of speed control. The transformer, the two rectifiers, and the potentiometer were enclosed in a cabinet (Fig. 16). Fig. 17 gives the circuit diagram. In addition, the angular velocity of the motor transducer assembly was
regulated by a six-inch steel flywheel, and interpreted by a General Radio strobotac. Figs. 18 and 19 show the actual calibration equipment.

In the course of calibration considerable difficulty was encountered in matching the zeros of the mechanical and electrical angles. The mechanical angle was zero when the horizontal indicator pointed to zero degree on the dial. The electric angle was zero when the rotor winding was perpendicular to the stator winding. Pains were taken to line up these two angles.

One obvious solution was to fix the transducer on the motor shaft to zero electric angle first and then adjust the dial to zero mechanical angle manually. When the electrical angle, mechanical angle, and the angle indicator were all in proper position, the setscrew of the flywheel was tightened. Slippage of the transducer shaft on the setscrew was observed to cause 2 or 3 degrees of error.

To ensure positively no slippage, the previous scheme was revised. A hole on the transducer shaft was drilled to fit the pointed end of a setscrew. The shaft of the transducer and that of the motor were locked permanently together by the setscrew. The matching of the mechanical and electrical angles was achieved by adjusting the position of the stator relative to the rotor of the transducer.

2.8 Pressure Transducer

The desire to measure fluctuating acoustic level pressures in the present programme suggested that use be made of a transducer developed by Ferguson (11). Readers are referred to his thesis for a detailed
Ferguson's design is a resistance-change type of pressure transducer. It utilizes the dynamic response of a flexible rubber diaphragm to the pressure fluctuation in a cavity. A shutter was mounted on a rubber diaphragm and placed half way between an ordinary flashlight bulb and a light dependent resistance (Phillips type No. B8731 03). The displacement of the shutter due to diaphragm deflection intercepted a portion of the light beam shining through and thus caused a substantial resistance change in the light dependent resistance. To convert this change in resistance to a voltage signal, the light dependent resistance was included in one arm of a two-external-arm bridge circuit and connected to a bridge amplifier and meter (Ellis Associates BAM-1). However, it was found that, to match the allowable arm resistance of the above instrument, it was necessary to shunt across the LDR a 2,000 ohm strain gauge in a temperature compensating circuit. A dummy arm of similar arrangement was set up in the bridge circuit to eliminate the thermal drift problem. The output from the bridge amplifier and meter was displayed on a cathode ray oscilloscope. The sensitivity of the transducer was found to be 0.0005 psi. Fig. 20 gives the circuit diagram and the components of the pressure transducer.

2.9 Pressure Transducer Calibration Apparatus

The calibration technique for the pressure transducer was developed by Ferguson (11) and was modified to include calibration data down to 1 c.p.s. A diagramatic layout of the calibration apparatus is shown in Fig. 21.
The cavity of the pressure transducer was connected to one end of a polyethylene tube. Into the opposite end a piston of fine bore was introduced. The piston was rigidly attached to a mild steel cantilever beam. The free end of the beam was driven by a Goodmans vibration generator; on the fixed end of the beam were mounted four strain gauges in a bridge circuit to indicate the piston amplitude and phase relation. Fluctuating pressures in the tube were generated by piston oscillations. Quantitative pressure fluctuations were calculated from the solution of the boundary value problem of inviscid acoustic wave propagation in a semi-infinite 1-dimensional tube (Appendix B). According to the theory of gasdynamics, the fluctuating pressure produced at the piston by its oscillations was a function of both piston amplitude and frequency. The control of these two parameters was provided by the chosen setting of the function generator. Since the function generator was essentially a voltage source, it was necessary to include a power amplifier following the function generator and preceding the vibration generator in the circuit. The purpose was to effect the impedance matching and to supply the driving force for the vibration generator. A three stage push-pull type of transistorized power amplifier was specially designed and found satisfactory even at very low frequency (e.g. 1 c.p.s.). Fig. 22 gives the circuit diagram while Fig. 23 shows the actual amplifier.

2.10 Pressure Seal

As mentioned earlier, the measurement of the fluctuating pressure on the surface of an autorotating model was part of the present investigation. At least two different approaches could be considered as feasible solutions
to this scheme. The first approach suggested that a pressure sensor be incorporated in the revolving unit and the pressure signal be converted to electric voltage and picked up by slip rings. An alternative was the use of a dynamic pressure seal to relay the pressure signal from a revolving pressure tap to a stationary pressure transducer. The latter method was given prior consideration because of the problems of inertia and miniaturization involved in the former method.

In addition to the function of transmitting the pressure signal to the pressure transducer, the pressure seal was required to provide a leak-proof sealing so that no communication across the pressure differential could be possible. The pressure differential in this case was the small pressure fluctuation with respect to the atmosphere and was in the order of magnitude from 0.0005 to 0.05 psi, so that only a light seal was needed. However, any leakage in the pressure transmission line would mean the critical reading was being contaminated and could impair the true pressure readings to such an extent that it would be almost impossible to calibrate the loss. Consideration was also given to the friction damping existent in the pressure seal unit, as small friction damping was essential in the investigation of autorotation.

A careful survey of all these factors suggested that the design of a concentric journal and sleeve bearing be adopted as the revolving and stationary elements of the pressure seal unit. The necessary clearance between them was filled with a thick film of lubricant to effect an airtight pressure seal.
A piece of 2 inch diameter teflon rod, backed up by a hollow steel disk, made up the body of the sleeve bearing. A 0.095 inch central hole was drilled to serve as part of the pressure tube leading to the pressure transducer and was aligned with a similar central hole in the journal. The journal was made from a 3/8 inch teflon tube and part of it was buried in the lower model spindle so that it was (indirectly) supported on remotely mounted ball bearings instead of on its sleeve bearing. (Fig. 6) The idea was to preserve the concentricity at all times and to minimize the friction and wear and leakage due to surface contact and lateral motion of the journal.

To check the effectiveness of the pressure seal unit, the calibration apparatus of the pressure transducer (Section 2.9) was adopted for the bench test. The first practical problem encountered was the lack of a known pressure source which was revolving. To cope with this difficulty, two pressure seal units sharing the common revolving element were constructed in the test equipment. At the input end of a first pressure seal unit, the oscillating piston was used to generate a known fluctuating pressure; the amplitude and phase shift of the same signal at the output end of a second pressure seal unit were examined by the pressure transducer. (Section 2.8) The revolving element was driven by an adjustable speed d-c motor (Fig. 17) through pulleys and rubber belt. Figs. 24 and 25 show the setup of this pressure seal test equipment.

2.11 Photography

The storage capabilities of a Tektronix Type 564 Storage Oscilloscope enabled the experimental data to be displayed for qualitative observation
and to be recorded on film for later detailed analysis. Satisfactory results were obtained by the use of an Asahi Pentax, single lens reflex, 35 mm camera with a No. 3, 49 mm close up lens attachment. The camera was mounted on the oscilloscope by means of a specially designed mounting bracket. Film used was Kodak Plus-X Panchromatic and best exposure values were found to be f2.8 at 1/30 sec. Enlargements (4 inches by 5 inches) were printed. Over two hundred data shots were taken and the majority of the processing was performed in the dark room of the Department of Mechanical Engineering.
III. EXPERIMENTAL PROCEDURES

3.1 Calibration Procedures

3.1.1 Static and Dynamic Pressures Between the End Plates

Measurements of the three-dimensional static pressure distribution were made at the wind speed of 26.3 feet per second. A static pressure probe, a Lambrecht micromanometer, and a traversing gear were set up for this purpose. It was found that, except for the region about 1/2 inch from the walls, the static pressure distributions in the longitudinal direction became quite uniform after a distance of 2 inches downstream of the leading edge for both sets of end plates. However, the chamfered end plates gave more uniform static pressure distribution both in the lateral and transverse directions. It was thus decided to pick the chamfered plates for better flow conditions in the fluctuating pressure measurements.

In addition, the effect of the end plates on the dynamic pressure was investigated. A pitot-static tube mounted midway between the end plates registered 5% increase of wind velocity in the range from 6 to 50 feet per second. The velocity profile along the model mid-chord axis was found very uniform; the spanwise variation in velocity was in the order of 1% at the wind speed of 26.3 feet per second. Results of this investigation are shown in Figs. 26 and 27.

3.1.2 Static and Dynamic Calibrations of Angular Displacement Transducer

Extensive bench tests showed that the dynamic response of the transducer was instantaneous below 2,000 rpm. This is well above the estimated upper limit of the autorotation speed. Also, the outputs of
induced secondary voltage remained unchanged either at constant speed or
during acceleration. As expected, the envelope of the amplitude modulated
signal coincided with a known sinusoidal signal of the same frequency and of
the same amplitude from the function generator. Data of the dynamic cali-
bration are included in Fig. 28. It must be noted, however, that, as the
motor speed was not 100% regulated, a phase relation existed between the
transducer output and the sinusoidal signal. There were times, however,
when the phase difference did vanish as indicated by arrows on the recording
paper.

The dynamic calibration described above showed great promise. Since
the angular motion has no effect whatsoever on the output characteristics
of the induced secondary voltage, the data of the static calibration apply
equally well to any angular speed. This performance greatly simplified the
process of calibration and expedited the analysis of the data later on.

The object of the static calibration was then to make the most of the
available instrumentation. Since the optimum frequency and amplitude
of the excitation signal were established (paragraph 2.6.4), the static cali-
bration was well on the way. By turning a 360° protractor fastened to the
flywheel past a fixed horizontal indicator, the angular displacements of the
transducer were calibrated. The corresponding voltage outputs were
recorded by the visicorder photographic paper. Data of this calibration are
plotted on Fig. 29. The solid curve is a cosine function.
3.1.3 Pressure Transducer Calibration

For a constant value of fluctuating pressure amplitude at a given frequency and a tube of given length and diameter, the amplitude and phase of the transducer signal was governed by two factors: bridge amplifier gain setting and light dependent resistance value.

Calibration of the amplifier gain was achieved by two strain gauges mounted in a temperature compensating two arm bridge circuit. After connections had been made to the bridge amplifier and meter, the bridge balance obtained, an internal resistance (0.5M ohm) in the above instrument was shunted across one arm of the strain gauge circuit. The unbalance caused a current to flow in the bridge and a meter reading was registered. Adjustment of the gain control varied this reading and, for convenience, it was set at 70 on the upper scale of the meter face.

On the other hand, the light dependent resistance value varied with the light intensity or the power input to the light bulbs. The extreme sensitivity of the transducer output to light intensity suggested that a consistent way of setting light intensity be adopted. A known pressure, generated by the oscillating piston at 10 c.p.s., was introduced to the transducer cavity by means of a 4 foot, 0.066 inside diameter, polyethylene tube. Adjustment of the light circuit potentiometer controlled the transducer output and allowed a standard signal to be established. The above procedures were repeated at the beginning of each series of tests and proved to be satisfactory in producing consistent results. Calibration curves for a range of frequencies 1 - 12 cycles/second are shown in Fig. 30.
3.1.4 Pressure Seal Calibration

Initial bench testing showed that a small amount of vaseline was sufficient to seal the gap between the teflon journal and bearing but that excessive vaseline would block the pressure transmission line. If the latter unfortunately were the case, the pressure signal would be lost completely in the pressure seal unit. Also it was found that the pressure seal was most efficient in the static condition. The amplitude and phase shift at the output end of the pressure seal of a known pressure signal were compared with the calibration data of the pressure transducer. No discrepancy was noticeable. However, the pressure seal when operating in dynamic conditions introduced a new problem. The fluctuating pressure arising from the rotation of the pressure tubing proved to cause amplitude modulation of the pressure signal at the frequency of rotation (Fig. 31). The theory (Appendix B) did not take into account this factor and it was difficult to eliminate completely in experiment. A final resort was to measure the effect of rotating tubing on individual pressure taps in the fluctuating pressure measurement and to subtract this amount from the resulting pressure signal. (paragraph 3.2.2)

The effect of a leaking pressure tube was also investigated. A hole purposely pierced through the tubing caused much attenuation of the pressure signal. It was thus decided, in case of doubt, to claim the maximum pressure fluctuation for individual taps as the final experiment results.

3.2 Test Procedures

3.2.1 Kinematic Measurements

Kinematic measurements of the autorotating plate in the wind tunnel
included the autorotation speed, the time and acceleration during the build-up period, and the angular displacement, velocity, and acceleration during the autorotation cycle. With the exceptions of the d-c motor and its speed control box, the calibration equipment for the angular displacement transducer (Fig. 19) was used in this measurement.

As before, the rotor winding $P_1$ of the transducer was excited at 500 c.p.s. by the audio generator. But the rotor in this case was driven by the model spindle instead of by the motor shaft. The corresponding transducer output was fed into the visicorder oscillograph. Four out of the seven recording channels in the above instrument were set up. Channel 1 was used to record the output of the stator winding $S_1$ or the angular displacement of the model. The galvanometer coil of channel 6 was used as a dummy load for the stator winding $S_2$ (paragraph 2.6.4). Since the record from channel 1 was sufficient to indicate the angular motion of the model, the output of channel 6 was not needed. Horizontal adjustment of the galvanometer mirror removed the reflected light beam away from the recording plane so that the signal of channel 6 did not show up on the recording paper. To provide two millisecond time markers positioned near the edges of the recording paper, channels 4 and 5 were connected in parallel to the function generator. The signal from the function generator was a 500 c.p.s. triangular wave. Typical oscillograph records are shown in Fig. 32 (the time markers did not show up because of the limited space in the photograph).

In all autorotation measurements, the angular displacement of the
model was interpreted by the transducer amplitude. In order to produce consistent results in subsequent tests, calibration of the transducer amplitude was carried out before taking any quantitative measurements. The transducer outputs of the two most important angular positions, i.e. zero and 90° angular position, were checked carefully from time to time. The calibration data were then applied to all other angular positions.

The angular position of the model was adjusted manually to zero angular position with the flat surface of the model facing the front side panel of the tunnel. Since the shaft of the rotor was rigidly attached to the model spindle, the magnetic axis of the rotor winding accordingly took up a definite orientation. By turning the bearing block of the upper autorotation stand on the tunnel flange, the angular position of the stator could be adjusted to correspond to 90° electrical angle or minimum output position. This calibration procedure was preferably carried out when the wind was off.

The calibration for 90° angular position or maximum output position was less tricky and could even be performed during autorotation. It has been shown that, for a given angular position, there is only one transducer output (paragraph 2.6.3) and that the transducer outputs are independent of angular motion (paragraph 3.1.2). It follows that the maximum transducer amplitude at any autorotation speed always represents the occurrence of 90° angular position. By controlling the amplitude setting of the audio generator in such a way that the maximum transducer output caused 4 inches peak to peak maximum deflection on the recording
paper, the transducer amplitude at 90° angular position was calibrated.

Kinematic measurements of the autorotating plate were made under two different-flow conditions: model mounted between chamfered end plates and without end plates. Wind velocity was increased by small increments until the critical wind speed to initiate autorotation was reached. From then on wind velocity was increased in steps by 5 feet per second.

Over the tested wind speed range from 0 to 35 feet per second, the model was excited from rest, with no exceptions, by a stick. Shortly before the model was excited manually, the record drive switch of the visicorder was triggered by a remote control switch to allow a complete history of angular motion to be recorded from the beginning of the build-up period to steady autorotation. The recording speed was 25 inches of recording paper per second. Over eight hundred feet of such records were obtained from the visicorder in these measurements.

3.2.2 Surface Fluctuating Pressure Measurements

Quantitative surface fluctuating pressure measurements were made under approximately two-dimensional conditions, i.e. model mounted between the chamfered end plates, at the wind speed of 26.3 feet per second. The pressure transducer output was allowed to build up on the storage screen of the Tektronix Type 564 Oscilloscope. To correlate the phase of the fluctuating pressure to the angular position of the model, a signal from the angular displacement transducer was displayed simultaneously on the same screen. The storage capabilities of the Oscilloscope
enabled such data to be recorded on film for later detailed analysis. Fast sweep speeds provided data on frequency and phase relation of the pressure signal, while slow sweep speeds enabled the amplitude modulation to be observed. Correlation of surface pressure and model angular displacement was obtained for both the transient build-up period and at steady state rotation. The build-up region was investigated by triggering the CRO sweep while the model was stationary, i.e. before the model was excited manually. Finally, the effect of the rotating tube on the pressure signal was examined by blocking off individual pressure taps with Scotch tape. The pressure transducer output was recorded on the storage screen during the build-up period and during autorotation as before. The above procedures were repeated for 13 pressure taps, the pressure readings being taken from individual taps one at a time. The switching of pressure tube connections was carried out when the wind was off and when the model was dismantled from the lower autorotation stand and the upper autorotation stand was lifted up by two metal wedges outside the tunnel ceiling. (Fig. 9)

The experimental setup for the fluctuating pressure measurements is shown in Fig. 33.
IV. EXPERIMENTAL RESULTS

4.1 Angular Velocity Measurements

The angular displacement transducer described in section 2.6 provided a useful means to determine the instantaneous angular velocity of the autorotating model. As seen from Fig. 32, the kinematic measurements produced continuous (amplitude-modulated) signals of angular displacement on the visicorder oscillograph paper. The instantaneous angular positions of the model were then interpreted from the corresponding transducer amplitudes. By measuring the distances between two consecutive nodes and dividing this value by the time scale of 25 inches per second, the period and thus the average rotational speed during 180 degrees of rotation were determined. This new technique of measuring angular velocity has advantages over the conventional stroboscope in that it could operate at extremely low rotational speeds and could even detect the instantaneous angular velocity during acceleration.

4.2 Dependence of Tip Speed on Wind Speed

4.2.1 Model With End Plates

Repetitive trials indicated the existence of a critical wind speed below which the phenomenon of autorotation was not observed. At each wind speed above this value, there were two equilibrium rotational speeds. The lower rotational speed of the two was an unstable one, giving the required rpm for autorotation, while the higher rotational speed was the stable autorotation speed. The results of this measurement are displayed in the form of tip speed vs wind speed in Fig. 34. In the absence of reliable data for
the time-dependent separated flow problems, none of the results presented here are corrected for tunnel blockage.

The autorotation tip speed is seen to increase nearly linearly with wind speed, with values of the order of 50% of the wind speed. The scatter in the data is small. The initial tip speed, however, decreases with wind speed, indicating that a smaller initial spin was required for autorotation at higher wind speeds and conversely a larger initial spin was required at lower wind speeds. The wind speed for which the two tip speeds coincide was the critical wind speed to initiate autorotation.

Thus the plot of tip speed vs wind speed enables the upper and lower limit cycles of the autorotation of the flat plate to be established. In the wind speed range below the critical wind speed, there is no possibility of autorotation. In the region confined by the abscissa, the critical wind speed, and the initial tip speed curve, the initial rpm is not sufficient for the wind to take over and the angular motion will be damped out completely after a few cycles. Only in the region above the critical wind speed and above the initial tip speed curve is autorotation possible. This implies that, if the plate was given any initial speed greater than or equal to the minimum initial speed, it would undoubtedly accelerate to its final autorotation speed, reached when an energy balance was obtained over one complete cycle.

4.2.2 Model Without End Plates

The initial and autorotation tip speed curves in this case show exactly the same trend and agree fairly well in the low wind speed range.
with those obtained with end plates. The effects of the end plates are to lower the critical wind speed and to raise both the autorotation and initial speeds under similar damping conditions and at the same wind velocity. Results of this measurement are plotted on Fig. 34 for comparison.

4.3. Time, Angular Velocity, and Angular Acceleration in the Build-up Period

In the process of reducing the autorotation speed from the visicorder oscillograph paper, it was found that the transition from the transient build-up period to the steady state rotation was not clearly defined. It was thus decided to plot the instantaneous angular velocity vs time for each wind speed and identify the region below 98% of the final rotational speed as the build-up period.

Results of this measurement are presented in Figs. 35 and 36. All such curves start with a dotted section covering the period of the manually applied initial spin and all but the curve for the critical wind speed show the feature that the angular velocity increases almost linearly at the beginning of the build-up period and approaches the autorotation speed gradually and smoothly. Also it is noted that the effect of a high wind speed is a rapid increase of angular velocity over a short period of time. For example, at the wind speed of 37.3 fps where the initial speed differs most widely from the autorotation speed (the initial speed lowest and autorotation speed highest), the build-up time is only 3.0 seconds, while at the wind speed of 15.8 fps where the initial speed is much closer to the autorotation speed, it takes 5.3 seconds or nearly twice as much time for the same model to go through the build-up period. Results of the build-up time at
different wind speeds are presented in Fig. 37.

All these results indicate the fact that the excitation torque is appreciable at high wind speed and is barely enough to overcome the friction damping at low wind speed. Qualitatively, this implies that the mean aerodynamic torque and angular acceleration would be much greater at high wind speed. Since the data giving the instantaneous angular velocity vs time were already tabulated, quantitative results of angular acceleration could be derived from simple graphical or numerical differentiation. Direct graphical measurement of the slopes of the angular velocity - time curves was used, and the results are plotted in the form of angular acceleration $\ddot{\theta}$ vs the dimensionless angular velocity $\omega/\omega_{\text{auto}}$ where $\omega_{\text{auto}}$ is the autorotation speed, on Fig. 38.

All the acceleration curves confirm the previous two observations. First, the rate of change of angular velocity declines from an approximately constant value at the beginning of the build-up period until the steady state is reached, where the mean angular acceleration is zero. Second, the mean angular acceleration and thus the resultant torque are greater at higher wind speeds. Since a high mean angular acceleration leads to a low build-up period, this agrees with the conclusion of Fig. 37 that the build-up period is longer at lower wind speeds.

4.4 Angular Velocity and Angular Acceleration During One Autorotation Cycle

The instantaneous angular velocity was examined by studying the output of the angular displacement transducer during one autorotation cycle;
the angular acceleration was derived from the instantaneous angular velocity vs time curve. As shown in Fig. 32 the transducer output is an amplitude-modulated signal at the frequency of rotation. A smooth envelope was drawn very carefully and the height of the envelope was measured to 0.01 inch at time intervals of 2 milliseconds. It was found that the wave form of such an envelope was very closely sinusoidal and that the discrepancy was of the same order of magnitude as that of the static calibration data of the transducer. (Fig. 29). According to the theory of the transducer (paragraph 2.6.3), the amplitude modulated signal takes a sinusoidal wave form only when the angular displacement is increasing at a constant rate, i.e. constant angular velocity during one complete cycle. It follows that, within the accuracy of the present investigation, the angular velocity remains constant and the angular acceleration remains zero during one autorotation cycle.

4.5 Surface Fluctuating Pressures

Surface fluctuating pressure measurements were made both for the transient build-up period and for steady state rotation at the wind speed of 26.3 fps. Typical oscilloscope traces of fluctuating pressure signals are shown in Fig. 39. In each photograph the upper signal represents the output from the pressure transducer; the lower signal represents the output from the angular displacement transducer. The frequency of a fluctuating pressure signal was obtained by counting the number of complete cycles within a definite period of time. The phase angle of a fluctuating pressure signal was correlated to that of the instantaneous angular position of the model. A constant time lag of 10 milliseconds was taken into account in the
analysis of the fluctuating pressure during steady state rotation. (Fig. 30b). The magnitude of the pressure signal was determined by the (vertical) deviation from the reference line of zero output (third horizontal line from top) with suction giving upward displacements, and this reading was converted to pressure units according to Fig. 30a. Correction in the order of 10% (Fig. 39e) due to (rotating) tube effect was made to individual pressure signals (paragraph 3.1.4 and 3.2.2).

4.5.1 Fluctuating Pressure During the Build-up Period

It is interesting to note that during the build-up period the frequency of the fluctuating pressure always equals the frequency of rotation. Fig. 39a shows that the frequency builds up from 2 cps to 10 cps in about 28 cycles. Although one might anticipate at least two relevant frequencies, representing the wake vortex formation and the model angular motion, the latter appears to be the predominant factor.

The amplitude of the fluctuating pressure grows gradually as the model accelerates. Irregular, unorganized wave forms of the pressure signal were observed at the initiation of autorotation (Fig. 39b). However, this phenomenon is temporary and lasted only for the first few cycles or before the amplitude increased to the steady state value, about 5 times the initial value. It must be pointed out that, although the pressure transducer signal appears to be larger in some cases at the initiation of autorotation, the true magnitude of the fluctuating pressure is actually smaller than that obtained during steady state rotation. The apparent decrease in fluctuating pressure amplitude with increasing frequency is a result of the increas-
ing attenuation of the transducer signal. Therefore care must be taken to choose the right calibration curve (Fig. 30) in the reduction of data during the build-up period.

In short, the transient build-up period is characterized by the simultaneous increase of both the frequency and amplitude of the pressure fluctuations.

4.5.2 Pressure Coefficient vs Angular Position During One Autorotation Cycle

Just as in the transient build-up period, the frequency of the surface fluctuating pressure coincides with the frequency of rotation. Since the rotational frequency is stable during autorotation, the frequency of the fluctuating pressure remains stable. The smooth wave form of the pressure signal repeats itself very faithfully for every 360° rotation (Fig. 39c and 39d). Little or no amplitude modulation was observed. However, the wave form is definitely not symmetrical with respect to the reference line. The pressure coefficients for all 13 taps vs angular position are given in Fig. 40. It is noted that for taps No. 2, No. 5 and No. 6 the negative pressure peaks are very slightly lower than the positive pressure peaks and that for the remaining taps the negative peaks are higher. The difference is biggest for the taps close to the leading edge, i.e., taps No. 13, No. 12, No. 11 and No. 10, and is smallest for the taps close to the trailing edge; i.e. taps No. 1 and No. 2. The taps positioned in the central part of the chord give differences of peak readings falling in between these extremes. This suggests that the leading edge pressures
make more contribution to the (clockwise) aerodynamic torque than the trailing edge pressures. This observation was later confirmed by the torque calculation. (Section 4.6).

With regard to the phase of the surface fluctuating pressure, it is almost impossible to draw any simple conclusions. Nearly all the taps reach their zero, maximum, and minimum at different angular positions. For example, taps No. 13, No. 12, No. 11, No. 10 and No. 9 reach their minimum pressure around 75° and taps No. 8, No. 7, No. 6, No. 5, No. 4 No. 3, No. 2 and No. 1 around 86°. The angular position where the taps reach their maximum is less well defined and spreads over the fairly wide range from 220° to 296°. A final remark is that with the exception of taps No. 13, No. 12, No. 11, and No. 10, the pressure reading is negative over the greater part of the cycle.

4.6 Torque Coefficient vs Angular Position

Aerodynamic torque during one autorotation cycle was obtained by the indirect, tedious method of integration of the instantaneous moments of the surface pressures.

The distribution of instantaneous fluctuating pressure over the plate for 10 degree increments of angular position was derived from Fig. 40 for all 13 taps. It is recalled that due to construction difficulties only the flat surface has pressure taps. It was therefore assumed that the flow pattern repeated every half cycle and that the image taps, after displacing 180°, could register the pressure readings on the cambered surface at the same angular position. This assumption was justified experimentally by rotating
the model in the reverse direction. As expected, a tap registered the pressure reading of its image tap when the model was rotating in the usual direction. Typical results for 0°, 30°, 60°, 120°, and 150° are presented in Fig. 41.

As seen from Fig. 41, there is unfortunately some scatter of data of the $C_p'$ values. Also the pressure distribution is unknown close to the leading and trailing edges, where it is impossible to arrange a pressure tap. To cope with this problem and to simplify the computations, the following assumption was made. It was assumed that the pressure coefficient could be considered as constant over short intervals along the chord. In other words, the area under the $C_p'$ curve was approximated by the sum of a series of rectangles. The differential moment was thus the product of the rectangular area and its corresponding moment arm from the center tap; the resultant moment at a certain angular position was the summation of these differential moments.

The resulting torque coefficient vs angular position is presented in Fig. 42. It turns out to be a fairly symmetrical curve with zero torque at 14° and 104°, maximum driving torque at 60°, and maximum resisting torque at 150°. A planimeter was used to establish the mean torque coefficient, which was found to be $2.1 \times 10^{-2}$ or equivalent to a sectional torque of $1.91 \times 10^{-3}$ lb-ft/ft. Clearly the excitation torque must balance the mean damping torque of the bearings and rotating seal, and the order of magnitude seems reasonable, since the present system, with its greater constraints, would be expected to have higher damping torque than the tour-
billion apparatus, described in Section 1, and this was reported to have damping torque of the order of $10^{-4}$ lb-ft. (5).

It is interesting to note that the torque coefficient curve is very closely sinusoidal.

4.7 Wake Survey

A short qualitative wake survey was carried out with a DISA 55A01 constant temperature hot wire anemometer. A few observations were made during the steady state rotation at the wind speed of 26.3 fps. It was found that the sense of rotation of the model had a definite influence on the wake geometry. If the position of the hot wire probe was fixed downstream and to one side of the model, the anemometer output gave clean and well-organized harmonic signals while the model was rotating in one direction, but messy and erratic signals while the model was rotating in the opposite direction. This is positive indication that in the former case the wake was deflected away from the probe and the position of the probe was outside the wake while in the latter case the wake was deflected towards the probe and the position of the probe was inside the wake. This observation was supported by the previous argument that rotation of the plate could develop circulation. As a result of the additional velocity field due to this circulation, the wake exhibited angular deflection similar to the deflection of streamlines in the problem of uniform flow past a circular cylinder with circulation in potential flow theory.
V. DISCUSSION

5.1 Autorotation Speed vs Wind Speed

Fig. 34 shows that the autorotation speed increases almost linearly with the wind speed. However, the autorotation curves, if extended, would not pass through the origin but would intersect the abscissa at about 6 feet per second. This displacement is almost certainly a function of the amount of mechanical damping torque in the revolving elements. Thus, the autorotation speed during the pressure measurements of section 4.5 at a wind speed of 26.3 fps was 9.5 cps, corresponding to a tip speed of 10.0 fps. This gives a point lying to the right of the autorotation curves of Fig. 34, although the aerodynamic conditions were the same as for the curve with end plates. Presumably the explanation is the increased mechanical damping caused by the rotating pressure seal of the tube connections. Also the ratio of tip speed to wind speed is about 0.5 for the case of the model with end plates and is about 0.45 for the case of the model without end plates. These results agree generally with the data quoted by Crabtree (1) and compare favourably with the measurements by James and Stone (6) and by Baird and Pick (7), both using the stroboscope technique.

The increase of autorotation speed for the same model when mounted between end plates was also observed in (1).

5.2 Initial Tip Speed vs Wind Speed

No reference to the measurement of the initial speed required for autorotation could be found in the technical literature. This is probably because the measurement of instantaneous angular velocity during acceleration is beyond the capability of a stroboscope.
It is interesting to note that the general shape of the initial speed curve is inclined downward and indicates that the higher the wind speed, the lower the initial tip speed. It is argued that at higher wind speeds more energy could be extracted from the stream. The excitation torque in this case would be greater and thus the tendency to autorotate about the axis of symmetry increases with wind speed.

Perhaps equally interesting is the comparison of the present tip speed vs wind speed curves with those of the 'aerial tourbillion' worked out by Parkinson (5). It is noted that these two forms of autorotation yield the same trend in the autorotation curves and show an increase of autorotation speed with wind speed. However, the initial tip speed curves show completely reversed trends -- the initial tip speed curves of the 'aerial tourbillion' increase with wind speed. This seems to be a contradictory result at first glance.

It must be recalled, however, that the mechanism of autorotation of the 'aerial tourbillion' is completely different and is attributed to the sectional flow separation and reattachment characteristics of a D-section cylinder. A 'very considerable initial spin' is required to produce a relative wind velocity at a sufficiently high angle of attack to overcome the adverse pressure gradient and cause flow reattachment on the afterbody in the wake. Static force measurements by Santosham (13) showed that the D-section cylinder changes its reattachment characteristics on the afterbody in the angle of attack range from $36^\circ$ to $60^\circ$. It follows that the initial tip speed had to be greater than half of the wind speed to allow a large apparent angle
of attack of the same order of magnitude. Hence the initial tip speed must increase with wind speed.

5.3 Stability of Autorotation

The results of the kinematic measurements show closely linear characteristics of initial and autorotation speeds and nonlinear characteristics of the build-up time and angular acceleration during the transient build-up period. At high wind speeds, the initial speed is low, the autorotation speed is high, the build-up time is short, and the angular acceleration is large. Conversely, at low wind speeds, the initial speed is high, the autorotation speed is low, the build-up time is long, and the angular acceleration is small. It follows that autorotation is more stable at high wind speeds and is less stable at low wind speeds. If the damping level is constant, the wind velocity is the dominant factor in the stability of autorotation.

5.4 Some Aerodynamic Arguments on Autorotation

The flow pattern during one autorotation cycle is fairly complicated and is not fully understood as yet. However, the fluctuating pressure measurements, the aerodynamic torque distribution, and the wake survey indicate the following:

Vortices were undoubtedly formed from the shear layers separating from the plate leading and trailing edges during the autorotation cycle. The leading edge vortices exerted a large influence on the fluctuating pressures; the effect of the trailing vortices was less noticeable in the fluctuating pressure measurements, as the trailing vortices were swept downstream.
in a rearward, downward direction. Near $\theta = 0^0$ the fluid particles adjacent to the plate experienced a velocity component normal to the plate as a result of the angular motion of the plate. The relative velocity was therefore at a negative angle of attack to the surface of the plate near the leading edge. The flow therefore remained attached to the upper surface as $\theta$ increased to appreciable positive values; flow separation was delayed. As $\theta$ increased further, the flow was unable to remain attached to the surface while passing around the sharp leading edge to the upper surface and tended to form a separated shear layer. This shear layer then rolled up, due to the reversed flow effect in separation, and formed a separation bubble or localized region of laminar separated flow near the leading edge. The characteristic of this region was high velocity and low pressure as indicated by the high suction for taps No. 13, No. 12, and No. 11 from $\theta = 33^0$ to $75^0$ (Fig. 40a). The size of the separation bubble seemed to grow rearward, as $\theta$ increased further. The pressure finally reached its highest suction peak as indicated by the pressure readings of taps No. 13, No. 12, No. 11, No. 10, and No. 9 for $\theta = 75^0$. At this instant, the separation produced very strong interaction with the plate as shown by the large pressure coefficient of the order of minus 5. For still larger values of $\theta$ the suction peak decreased fairly rapidly as shown by taps No. 13, No. 12 and No. 11. This seemed to be an indication that the separation bubble had grown until the upper surface flow was completely separated. The circulation of the flow then carried the trapped vortex rearward and downward.
5.5 Fluctuating Pressure Coefficient

As seen from Fig. 40, the fluctuating pressure coefficient has its largest peak to peak variation for leading edge taps (No. 13 and No. 12) and smallest for trailing edge taps (No. 1 and No. 2). The very high suction peak occurring at about 75° was explained by the flow separation and formation of vortices (Section 5.4). The very high positive pressure coefficient was probably due to the contribution of the time-dependent local acceleration term in the equation of motion of the fluid for unsteady flow. Thus, the corresponding equations for irrotational steady flow Bernoulli's equation

\[ p_{\infty} + 1/2 \rho U^2 = p + 1/2 \rho U^2 \]

and

\[ C_p' = \frac{p'}{1/2 \rho U^2} = 1 - \frac{U^2}{2 U^2} \]

evaluating the pressure coefficient in terms of the velocities, no longer hold for an unsteady flow problem. An additional term \( \rho \frac{\partial \phi}{\partial t} \) which is an unknown function of time should be added to the right hand side of Bernoulli's equation and \( C_{p'} \) is modified accordingly. Also it must be pointed out that the instantaneous velocity of the particles near the plate was the vector sum of the free stream velocity and the velocity induced by the angular motion of the plate. The occurrence of the highest positive pressure coefficient was for tap No. 13 at \( \theta = 300° \). As the rotational speed was 9.5 cps, the velocity relative to the plate was about 140\% of the free stream velocity. Thus a pressure coefficient based on this relative wind velocity would be reduced to the order of 1.8.
VI. SUMMARY OF RESULTS

Results of this investigation are summarized as follows:

1. The autorotation speed and initial speed show nearly linear variation with wind speed. The autorotation speed increases with wind speed; the initial speed decreases with wind speed.

2. The build-up time is shorter and the mean angular acceleration is larger for higher wind speeds; the build-up time is longer and the mean angular acceleration is smaller for low wind speeds.

3. During the steady state autorotation cycle, the angular velocity remains constant to the accuracy of measurement possible in this investigation; i.e. about 5%.

4. During the build-up period, the frequency and amplitude of the surface fluctuating pressure increase simultaneously as the rotational speed builds up immediately after the manual excitation.

5. During the steady state rotation, the frequency of the surface fluctuating pressure is equal to the frequency of autorotation. The amplitude is the largest for leading edge taps and is the smallest for the trailing edge taps. There is no simple phase relation. Little or no amplitude modulation is observed.

6. The instantaneous distribution of aerodynamic torque is fairly symmetrical and results in a small positive mean torque of \(1.91 \times 10^{-3}\) lb-ft/ft during 180° of rotation.

7. The wake exhibited angular deflection as a result of the circulation developed during autorotation.
8. Flow separation is delayed during autorotation. Leading edge vortices are developed in a separation bubble from about $33^\circ$ to $75^\circ$ and peel off above $90^\circ$. 
APPENDIX A

ESTIMATE OF THE REQUIRED WRITING SPEED OF VISICORDER

FOR 500 CYCLES PER SECOND CARRIER SIGNAL

Light Beam Displacement \( y = y_o \sin wt \)  \( (1) \)

Chart Displacement \( z = Vt \)  \( (2) \)

Hence \( y = y_o \frac{wz}{V} \)  \( (3) \)

\[
\frac{dy}{dz} = y_o \frac{w}{V} \cos \frac{wz}{V} \quad (4)
\]

The distance traveled by the light beam during one complete cycle

can be expressed by

\[
S = \int_0^{2\pi V \over w} \sqrt{1 + \left( \frac{dy}{dz} \right)^2} \, dz \quad (5)
\]

Substituting (4) into (5)

\[
S = \int_0^{\pi V \over 2w} \sqrt{1 + y_o^2 \left( \frac{w \over V} \right)^2 \cos^2 \frac{wz}{V}} \, dz
\]

Changing variables from \( {wz \over V} \) to \( \phi \)

\[
S = \frac{4V}{w} \int_0^{\pi/2} \sqrt{1 + y_o^2 \left( \frac{w \over V} \right)^2 \cos^2 \phi} \, d\phi
\]

\[
= 4 \sqrt{2 + \left( \frac{V \over w} \right)^2} \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \phi} \, d\phi \quad \text{where} \quad k = \sqrt{2 \over V^2 + y_o \left( \frac{w \over V} \right)^2} \]

\[
= 4 \sqrt{y_o^2 + \left( \frac{V \over w} \right)^2} E(k, \pi/2)
\]

This is elliptic integral of the second kind.
To evaluate this integral numerically we recall

\[ \omega = 2\pi f = 2\pi(500) = 1000\pi \text{ rad/sec} \]

\[ y_0 = 2 \text{ inch (4 inches peak to peak deflection)} \]

\[ v = 25 \text{ inch/sec} \]

Therefore the amplitude of the elliptic integral is \( \pi/2 \), and the modulus is

\[ k = \frac{\sqrt{4(10^6)\pi^2}}{\sqrt{25^2+4(10^6)\pi^2}} = 1 \]

\[ \therefore E(k, \pi/2) = 1 \]

from (5)

\[ s = 4 \sqrt{2^2 + \frac{25^2}{10^6\pi^2}} = 8 \text{ inch/cycle} \]

This result is closely checked for high frequency signals since the travel distance during each quarter cycle can be approximated by the amplitude (2 inches) of the sine curve without too much difference.

For 500 cps signal the light beam travels

\[ 500 \text{ cycle/second} \times 8 \text{ inch/cycle} = 4000 \text{ inch/second} \]

This is well within the range of allowable writing speed of 10,000 inches per second for standard collector lens.
The pressure developed by the piston oscillations (section 2.9) at the input end of the tube was calculated from one dimensional acoustic theory (12).

At the piston, \( p'(t) = a \rho \Omega \delta \cos \Omega t \)

where piston displacement = \( \delta \sin \Omega t \).

The effect of phase lag and attenuation due to viscous dissipation at the output end of the tube were avoided by conducting the calibration of the transducer with the same length of tubing as was used in the actual experiment.
BIBLIOGRAPHY


CLOSE-UP OF MODEL, SUPPORTING SYSTEM, AND END PLATES INSIDE THE TUNNEL

Fig. 1
AERODYNAMIC OUTLINE OF WIND TUNNEL

Fig. 2
UNFINISHED MODEL

Fig. 3
<table>
<thead>
<tr>
<th>Tap No.</th>
<th>Distance from Center Tap (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 11/16</td>
</tr>
<tr>
<td>2</td>
<td>1 7/16</td>
</tr>
<tr>
<td>3</td>
<td>1 1/8</td>
</tr>
<tr>
<td>4</td>
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<td>1 11/16</td>
</tr>
</tbody>
</table>

PRESSURE TAP POSITIONS FOR
MODEL AT DEFINED ZERO ANGULAR DISPLACEMENT

Fig. 4
FINISHED MODEL

Fig. 5
Fig. 6  UPPER AUTOROTATION STAND AND ANGULAR DISPLACEMENT TRANSUCER
Fig. 7  LOWER AUTOROTATION STAND AND PRESSURE SEAL UNIT
1. Tunnel Flange
2. Bearing Block
3. Model Spindle
4. Plexiglas Tube Connection
5. Locking Nut
6. Cylindrical Coupling
7. Semi-annular Spacer
8. Stationary Element of the Pressure Seal Unit

LOWER AUTOROTATION STAND

Fig. 8
1. Model
2. Cylindrical Coupling
3. Locking Nut
4. Bearing Block

SWITCHING OF PRESSURE TUBE CONNECTION

Fig. 9
MODEL AND ITS SUPPORTING SYSTEM—LOOKING DOWNSTREAM INTO WIND TUNNEL TEST SECTION

Fig. 10
SUPPORT AND ADJUSTING ARRANGEMENT OF END PLATE STRUT

Fig. 11
ELECTRICAL CIRCUIT OF ANGULAR DISPLACEMENT TRANSUDER

Fig. 13
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a. Components

b. Assembly

ANGULAR DISPLACEMENT TRANSDUCER

Fig. 14
CALIBRATION EQUIPMENT OF ANGULAR DISPLACEMENT TRANSUCER

Fig. 15
SPEED CONTROL BOX

Fig. 16
CIRCUIT OF THE SPEED CONTROL BOX OF A DC MOTOR
ARROW A IS COARSE CONTROL. ARROW B IS FINE CONTROL.

Fig. 17
1. Motor
2. Flywheel
3. 360° Protractor
4. Horizontal Angle Indicator
5. Angular Displacement Transducer

ASSEMBLY OF THE STATIC CALIBRATION OF ANGULAR DISPLACEMENT TRANSDUCER

Fig. 18
1. Function Generator
2. Visicorder Oscillograph
3. Remote Control Switch
4. Stroboscope
5. Transducer
6. Audio Generator
7. Motor
8. Speed Control Box
9. Storage Oscilloscope

ANGULAR DISPLACEMENT TRANSDUCER CALIBRATION EQUIPMENT

Fig. 19
2000 ohm (Strain Gauges)

LDR - Light Dependent Resistance

a. BRIDGE CIRCUIT

2000 ohm (Strain Gauges)

LDR - Light Dependent Resistance

To Bridge Amplifier and Meter

b. PRESSURE TRANSDUCER - COMPONENTS

Fig. 20
Diagrammatic Layout of Pressure Transducer Calibration Apparatus

Fig. 21
CIRCUIT OF POWER AMPLIFIER

Fig. 22
LOW FREQUENCY POWER AMPLIFIER

Fig. 23
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2. Revolving Element of Pressure Seal
3. Ball Bearing Blocks
4. Pulley and Rubber Belt
5. Pressure Seal Unit (output)

PRESSURE SEAL TEST EQUIPMENT

Fig. 24
1. Function Generator
2. Amplifier
3. Vibration Generator and Piston
4. Pressure Seal Units
5. Pressure Transducer
6. BAM-1
7. Storage Oscilloscope
8. Motor
9. Speed Control Box

CALIBRATION APPARATUS OF THE PRESSURE SEAL UNIT

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CALIBRATION OF THE ACTUAL WIND VELOCITY BETWEEN THE END PLATES

Fig. 26
VELOCITY PROFILE FOR 25° CHAMFERED END PLATES ALONG THE MODEL AXIS

Fig. 27
a. DYNAMIC CALIBRATION DATA OF ANGULAR DISPLACEMENT TRANSUDER AT ROTATIONAL SPEED OF 20 c.p.s.

Fig. 28
b. DYNAMIC CALIBRATION DATA OF ANGULAR DISPLACEMENT TRANSUDER AT ROTATIONAL SPEED OF 30 c. p. s.

Fig. 28
Fig. 29

MECHANICAL INPUT ANGLE (DEGREES)

RECORDING PAPER DEFLECTION (INCHES)

ANGULAR DISPLACEMENT TRANSDUCER CALIBRATION DATA
Fig. 30 TRANSDUCER CALIBRATION DATA FOR USE WITH A TUBE 4.0 FT LONG, 0.066 INCH DIAMETER
Pressure Signal at the Output End of a Static Pressure Seal

Piston Displacement

Pressure Signal at the Output End of a Dynamic Pressure Seal at 10 c.p.s.

Piston Displacement

Pressure Signal at the Output End of the Same Dynamic Pressure Seal at 20 c.p.s.

Piston Displacement

CALIBRATION DATA OF PRESSURE SEAL UNITS

Fig. 31
SAMPLE EXPERIMENTAL RECORD OF THE KINEMATIC MEASUREMENTS

Fig. 32
EXPERIMENTAL SETUP OF THE FLUCTUATING PRESSURE MEASUREMENTS

Fig. 33
TIP SPEED vs WIND SPEED

Fig. 34
Fig. 35 INSTANTANEOUS ANGULAR VELOCITY DURING ACCELERATION (with end plates)
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(measurement without end plates)
BUILD-UP TIME vs WIND SPEED

Fig. 37
Fig. 38

ANGULAR ACCELERATION vs DIMENSIONLESS ANGULAR VELOCITY (with end plates)
TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS

Fig. 39

a. Transient build-up (Tap No. 8)
b. Transient build-up (Tap No. 8)

TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS

Fig. 39
b. Tap No 13 (during steady state rotation)

**TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS**

Fig. 39
TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS

Fig. 39
TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS

Fig. 39
Unsuccessful build-up (Tap No. 8)

TYPICAL OSCILLOSCOPE TRACES OF FLUCTUATING PRESSURE MEASUREMENTS

Fig. 39
Fig. 14.0 a. PRESSURE COEFFICIENT vs ANGULAR POSITION FOR INDIVIDUAL TAPS
Fig. 40 b. PRESSURE COEFFICIENT vs ANGULAR POSITION FOR INDIVIDUAL TAPS
Fig. 41 a.
THE CHORD AT DIFFERENT ANGULAR POSITIONS
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Fig. 41 b.
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