THE DEPENDENCE OF THE STATIC COEFFICIENT OF FRICTION ON THE TIME OF STATIONARY CONTACT

by

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ABSTRACT

The dependence of the static coefficient of friction on the time of stationary contact has been determined using stick-slip vibration to provide a periodic time of stationary contact between two metallic bodies.

To describe this time dependence a theory based on the creep by diffusion for metals in contact has been developed. This approach is similar to the adhesion theory presented by many authors as representative of junction strength in friction.

Eleven friction-couples were studied; ten couples being very pure metals run against an annealed steel disk. The eleventh couple consisted of a hardened steel slider against the annealed steel disk. The results indicated that, fundamentally, very little difference in static friction values was apparent between the friction-couples. Because of the difficulty in obtaining material properties it was not possible to compare the experimental results with purely theoretical predictions. However, within reasonable experimental accuracy, the shape of the friction-time curves agreed with those predicted by theory.

By varying the system parameters it was found that, in general, the static coefficient of friction was load and area independent but seemed to be very dependent on surface finish and the micro-structure of the friction-couples. Notable exceptions to this rule were indium and silver which showed excessive creep under load thus directly affecting the static coefficients of friction.

It is important to stress that quantitative results for the friction-growth curves are not applicable since the system parameters affect the results greatly. However, a qualitative idea of the fundamental problems
that exist allows a reasonable prediction of friction values to be made.
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CHAPTER I

1.1 INTRODUCTION

In the theory of stick-slip vibration the importance of the dependence of the static coefficient on the time of stationary contact has been realized and several mathematical relations have been proposed to predict the shape of this curve, yet no clear explanation of this phenomenon has been presented although several attempts have been made by various authors. The fact that the variation of static friction with time of stick determines the critical velocity point at which stick-slip vibration ceases and pure sliding commences suggests that a study of the mechanism of this phenomenon is important. Hence an experimental apparatus was built which utilizes stick-slip vibration to determine the time dependence of static friction. Since stick-slip vibration is a periodic function of saw-tooth form where the increasing slope is the stick portion of the cycle and the vertical
slope is the slip portion of the cycle, a definite time of stick, even of very short duration, can be duplicated readily and controlled easily simply by changing the parameters of the system such as the surface velocity or the stiffness of the spring.

A general theory of friction induced vibration has been presented by Cameron (12). The solution of the differential equations involved in stick-slip vibration was accomplished by phase plane methods. For the experiment conducted by the author the static coefficient of friction was determined by:

\[ \mu_s = \frac{K\delta}{W} \]

where \( K \) is the spring constant, \( W \) the normal load and \( \delta \) the maximum distance travelled by the specimen in the stick cycle from the equilibrium position. Also the kinetic coefficient of friction could be obtained in the stick-slip regime by the equation:

\[ \mu_k = \frac{K\delta_{\min}}{2W} + \frac{\mu_s}{2} \]

where \( \delta_{\min} \) is the minimum distance travelled by the specimen from the equilibrium position of the spring.

1.2 BACKGROUND

From the time that two surfaces were first moved at some relative velocity it has been noticed that often other than smooth sliding resulted. This intermittent motion, when the relative speeds are low, has been termed "stick-slip" vibration or relaxation oscillation. Although stick-slip vibration was common no attempt to give a good theoretical reason for its
existence was made until the early 1900's.

In 1929 Wells (1) observed stick-slip while attempting to measure the kinetic coefficient of friction at low sliding velocities and proposed that this phenomenon could occur only if the static coefficient were larger than the dynamic coefficient of friction. Various authors attempted to explain stick-slip vibration by correlating the system parameters, such as load, spring force, damping etc., to the observed magnitude and frequency of stick-slip. Since the two surfaces in contact were stationary for some length of time during the stick portion of the cycle it was evident that the static coefficient of friction would influence the stick-slip vibration especially if it were time dependent. In 1939 Bowden and Leben (11) stated that the static coefficient of friction depended on the breakdown of the welded junctions of the asperities in contact between the two rubbing surfaces and this welding process would be time dependent.

Dokos (3) found that the static coefficient varied inversely with the frequency of stick-slip and his data indicated that the $\mu_s - t_s$ curve was linear when plotted logarithmically although he never postulated exactly how this occurred.

Later work by Rabinowicz (2) supported this theory of adhesion of metallic asperities. Rabinowicz found that the shear forces at the asperities built up as a result of imposed tangential micro-displacement and this indicated that the static friction coefficient would build up as a result of the time of stationary contact.

Howe, Puddington and Benton (9) suggested that the relation,

$$\mu_s = \mu_k + (\mu_{s\infty} - \mu_k)(1 - e^{-Ct_s})$$

where $\mu_{s\infty}$ is the static coefficient of friction for a very long time of
contact and C is some arbitrary constant, would be representative of the \( \mu_s - t_s \) curve. However this curve was derived for glass in contact with glass and the theory was based on Van der Waal's electrostatic forces and hence is not strictly applicable to metals. At this time an experimental study of the problem was presented by Spurr (6) and his data conflicted with the predicted theoretical curve by Howe et al. Whereas Howe's theory predicts an asymptotic curve as the time of stationary contact increases Spurr's data shows no such flattening of the \( \mu_s - t_s \) curve.

Another possible curve for the time dependence of the static coefficient of friction was presented by Derjagin, Push and Tolstoi (13). They proposed that:

\[
\mu_s = \mu_k + \frac{Ct_s^2}{k + t_s}
\]

where \( k \) is some arbitrary constant. The argument for this shape of curve was based on the idea that as the sliding velocity falls to zero just before stick is imminent, the time of interaction between opposing asperities increases, which leads to an increase in contact area. However, in a reply to the above presentation Rabinowicz proposed that:

\[
\mu_s = \mu_k + K_2 t^\beta
\]

where \( K_2 \) and \( \beta \) are constants and \( \beta \) is less than unity, based on the data from Dokos and from his own experimental work.

Kosterin and Kraghelsky (10), after observing that the static friction force grew intensively in the first moments of stationary contact, agreed that the \( \mu_s - t_s \) curve would resemble in form that presented by Howe et al. In a later article by Kosterin and Kraghelsky (7) an extensive theoretical analysis based on the theory of visco-elasticity and plastic
deformation and using the rheological properties of the materials in contact, 
showed that the exponential form of the $\mu_s - t_s$ curve was correct. Their 
experimental data, however, was not conclusive since it did not follow the 
form of the predicted curve. Further agreement with the exponential $\mu_s - t_s$ 
curve as predicted first by Howe was given by Cameron (12). In all of the 
experimental evidence given to support the exponential $\mu_s - t_s$ curve the 
time of stationary contact was relatively small so that predicting the exact 
form of the curve tended to be hazardous.
CHAPTER II

II.1 THEORY

A discussion of the mechanism of friction must inevitably start with the fact that in reality even the most carefully prepared surface has a roughness which, compared to molecular distances, makes the surface appear as a mountainous terrain and when the surfaces are placed in contact the regions which actually touch make up an extremely small proportion of the apparent area of contact. Hence to move one of these surfaces relative to the other will obviously require some distortion of the asperities in contact either by shearing or by a ploughing action. The requirements for motion are the same whether the surfaces are moving relative to one another continuously or are starting from rest. It would appear then that the difference between the static coefficient and the dynamic coefficient of friction is one of magnitude of forces and not of mechanism of deformation.
Since the asperity seems to be the dominating factor in friction it becomes obvious that certain physical factors will affect the type and size of the asperities. Such variables as hardness of surface and contaminants, atmospheric conditions, and relative activities of the friction couples will affect the friction force. Kosterin and Kraghelsky (7) attempted to predict the magnitude of the friction force using the rheological properties of the materials in contact, but their results indicated that the analysis was not comprehensive enough for general use. Bowden and Tabor (5) and Rabinowicz (4) have presented an extensive list of factors which influence friction in addition to those previously mentioned.

The classical laws of friction, as first proposed by Amontons in 1699, stated that the friction force is essentially proportional to the load and is independent of the area in contact. A later addition to these laws was that friction is independent of velocity. Also recognized was the fact that the friction force required to start motion was greater than that required to maintain the motion. However, it is widely recognized today that these classical laws are a gross over-simplification. In order to understand the limitations of these laws and to predict the friction phenomenon in some manner, a theory dependent on the asperities of two surfaces in contact and their interaction must be formulated.

Assuming that friction forces result when asperities are sheared it is necessary to decide whether the asperities have joined by adhesion, cohesion, self-diffusion or are simply being ploughed or torn. It seems reasonable that all of these modes of shearing exist but experiments presented in Bowden and Tabor (5) show that in fact adhesion is the main method of forming junctions between asperities for metals. Hence the junction between two asperities in contact will have some shear strength
dependent on the conditions of formation and on the physical properties of the metals.

A typical junction may resemble Fig. 1 except that the vertical scale has been magnified greatly. When metals are placed in contact they touch only at the tips of the asperities

![Diagram of an Asperity](image)

**Fig. 1**

Diagram of an Asperity

where the pressure is always high enough to cause plastic deformation. The metal flows plastically until a sufficient area is formed to support the normal load. If the normal load is $W$ and the yield pressure of the metal is $P_0$, assuming two similar metals are in contact, the true area of contact is:

$$A_0 = \frac{W}{P_0}$$

Also if the bulk shear strength of the adhered junction is $s_0$ then the force required to shear the junction becomes:

$$F = s_0 A_0 = \frac{s_0 W}{P_0}$$
Noting that the coefficient of friction is defined as:

\[ \mu = \frac{F}{W} = \frac{s_0}{p_0} \]

It is obvious that the coefficient of friction is independent of load and area of contact and is simply the ratio of two physical properties of the metals in contact, the shear strength and yield pressure. This simple model also demonstrates Amontons law that the friction force is proportional to the load and independent of the area of contact. No statement has been made as to whether the model demonstrates the static or dynamic coefficient of friction.

The problems of such a model are obvious in that the shear strength and yield pressure are related to one another and are not too much different and yet coefficients of friction have a large range, some being greater than unity, depending on the physical conditions. The first modification would be that surface contaminants tend to give a lower shear stress value. Secondly, and most important, the analysis does not include the effect of the applied shear stress on the plastic deformation.

To consider the effect of the shear stress apply Von Mises yield criterion to a two-dimensional case as done by Bowden and Tabor (5) in their junction growth theory.

\[
(\tau_{11} - \tau_{22})^2 + (\tau_{22} - \tau_{33})^2 + (\tau_{33} - \tau_{11})^2 + 6(\tau_{12}^2 + \tau_{13}^2 + \tau_{23}^2) = 6k^2
\]

where \( \tau_{11} = p \) (the normal pressure) and \( \tau_{12} = s \) (the applied shear stress) and \( k = s_0 \) (the yield strength in shear). The other stresses are zero. Hence:

\[
p^2 + 3s^2 = 3s_0^2
\]

For a three-dimensional case no exact solution exists but the form remains the same as the two-dimensional case. (Ref. 5)
\[ p^2 + \alpha s^2 = \alpha s_0^2 \]

Under combined stress the area increases from the initial \( A_0 \) to some new value \( A \) and the following relations now hold.

\[
\begin{align*}
p &= \frac{W}{A} \\
s &= \frac{F}{A}
\end{align*}
\]

If the asperity interface has a shear strength \( s_i \) which is lower than the bulk shear strength of the material \( s_o \) then:

\[ s_i = ks_0 \quad \text{k<1} \]

The criterion which determines plastic yielding will be the interface shear strength \( s_i \). For lower values of shear stress than \( s_i \) the combined stress contributes to junction growth. The condition of gross sliding is determined by:

\[ p^2 + \alpha s_i^2 = \alpha s_0^2 \]

Rearranging and substituting:

\[
\begin{align*}
p^2 &= \alpha s_i^2 \left( \frac{1}{k^2} - 1 \right) \\
\therefore \frac{s_i}{p} &= \frac{k}{\sqrt{\alpha}} \sqrt{\frac{1}{1-k^2}}
\end{align*}
\]

From this expression it is obvious that the ratio \( \frac{s_i}{p} \) is very sensitive to the value of \( k \). Noting that the friction force is:

\[ F = s_i A \]

and the normal pressure:

\[ p = \frac{W}{A} \]

then:
The above expression for the coefficient of friction is very dependent on the surface shear stress of the metals in contact. (Ref. 5)

If the asperities in contact are plastically deformed then it is reasonable to assume that the materials will creep under load. Evidence to this effect has been reported by Bowden and Tabor (5), and Pomey et al., (8) where hardness tests showed that the indentation hardness of metals decreased if the loading time increased. The process was described by Bowden and Tabor as obeying the viscous creep phenomenon in tensile tests where the following relation holds.

\[
\frac{de}{dt} = C_0 n e^{-\frac{Q}{RT}}
\]

Here \( C \) and \( n \) are constants, \( Q \) is the activation energy for self-diffusion, \( R \) is the universal gas constant and \( T \) is the temperature.

The real area of contact for a junction may be estimated in terms of strain by assuming that the asperities are truncated cones. The area would be:

\[
A_r = \pi r^2
\]

where \( r \) is the radius of the truncated cone. If the apex angle of the cone is \( 2\alpha \) and the vertical strain is \( \varepsilon \), the area becomes:

\[
A_r = \pi (h \tan \alpha)^2 = k_3 \varepsilon^2
\]

where \( h \) is the asperity height. Kraghelsky and Kosterin (7) suggest that the real area of contact obeys the relation:

\[
A_r = A_a b \varepsilon^\nu
\]

where \( b \) and \( \nu \) are constants, which is the same form as that obtained from
the simple model. The apparent area of contact is $A_a$.

The normal stress at the junction interface becomes:

$$
\sigma = \frac{W}{A_r} = \frac{W}{A_a} e^\nu
$$

$$
\therefore \varepsilon = \left( \frac{W}{A_a} \right)^{1/\nu} (\sigma)^{-1/\nu}
$$

Differentiating:

$$
\frac{d\varepsilon}{dt} = \left( \frac{W}{A_a} \right)^{1/\nu} \left( -\frac{1}{\nu} \right) \sigma \frac{d\sigma}{dt}
$$

Substituting into the viscous creep relation:

$$
C_\sigma^n e^{\frac{-Q}{RT}} = \left( \frac{W}{A_a} \right)^{1/\nu} \left( -\frac{1}{\nu} \right) \sigma \frac{d\sigma}{dt}
$$

Separating the variables and integrating with respect to time:

$$
\sigma \left( n+\frac{1}{\nu} \right) = \left( \frac{n+\frac{1}{\nu}}{\nu} \right) k c e^{\frac{-Q}{RT}} \left( \frac{W}{A_a} \right)^{1/\nu} \frac{1}{t} + C_1
$$

The stress $\sigma$ is identical to the pressure obtained in the static friction relation.

$$
\mu_s = \frac{s_1}{F}
$$

Substituting:

$$
\mu_s = s_1 \left\{ \left( \frac{n+\frac{1}{\nu}}{\nu} \right) k c e^{\frac{-Q}{RT}} \left( \frac{W}{A_a} \right)^{1/\nu} \frac{1}{t} \right\} \frac{1}{n+\frac{1}{\nu}} + \frac{1}{n+\frac{1}{\nu}} + C_2
$$

Noting the fact that at zero time of stick the static coefficient of friction is equivalent to the kinetic coefficient, hence:

$$
\mu_s - \mu_k = K_1 e^{\frac{-Q}{RT}} \left( \frac{1}{n+\frac{1}{\nu}} \right) t \frac{1}{n+\frac{1}{\nu}}
$$

The above equation relates the static coefficient of friction to the time.
of stationary contact of the asperities and includes a self-diffusion mechanism to give temperature dependence along with basic viscous creep.
CHAPTER III

III.1 APPARATUS

In the interests of having compact equipment and to facilitate the study of the variation in friction with length of run-in the apparatus was designed with a rotating disk as the moving surface. A wide range of system parameters such as surface velocity, normal load, spring constants and atmospheric conditions were general requirements. Hence the apparatus was designed with three main sections; a variable speed turntable, a cantilever beam to provide both normal load and spring force, and a deflection measuring device to determine the friction force. (Fig. 2)

The rotating disk was supported by ball bearings and was driven by a D.C. servo-feedback motor through a series of spur reduction gears and a bevel gear. This motor had speed regulation better than 5% from no-load to full-load conditions and the response was nearly instantaneous so
that the stick-slip vibration would not affect adversely the surface velocity. The turntable speed could be varied infinitely from 0.005 rpm to 5 rpm, thus providing a typical running track velocity of 0.001 in/sec to 1 in/sec. The turntable speed was monitored by means of studs mounted on the disk which tripped a micro-switch thus providing a continuous trace on a chart recorder.

The cantilever beam had a detachable rigid specimen holder mounted at one end and was clamped solidly at the other end. The specimen holder overhung the turntable so that the radius of curvature for the specimen vibration was in the same sense as the running track on the rotating disk. The specimens had a flat surface either \(\frac{3}{4}\) in. or \(\frac{3}{8}\) in. in diameter and were mounted in a drilled ball bearing which rode in a conical retaining cup in the specimen holder. This type of joint allowed the specimens to make excellent contact with the disk at all times. The length of the cantilever beam was adjustable and provision for interchangeability of beams was made so that the spring constant of the system was variable. The beam pivoted on two low friction bearings and load was applied by means of weights which imposed a moment on the system. This loading arrangement kept the vibrating mass constant even though the normal load was changed. The loading arrangement, although appearing awkward, kept the apparatus compact thus satisfying one of the initial requirements. Also, a removable cover could be provided which would allow for atmospheric control. Beams were manufactured to give spring constants from 4 lb/in to 42 lb/in force per unit deflection measured at the specimen. The normal load was variable from 0 to 5 lb. By shifting the entire cantilever-load mechanism various running tracks on the disk could be obtained.
Fig. 2—Diagram of Apparatus
Of the many devices available to measure the deflection of the beam in order to determine the friction force, a modified Metrisite displacement transducer provided the greatest flexibility and accuracy of measurement. In order to reduce the moving mass and eliminate friction in the transducer so as not to affect the motion of the beam, a teflon slider was manufactured to replace the original steel one. The Metrisite transducer, of the moving E-coil design, provided a useable range of $\frac{1}{8}$ in. beam deflection and resolved displacements accurately to 0.0001 in.

III.2 INSTRUMENTATION

The Metrisite displacement transducer was mounted on a slider so that the unit could be mechanically zeroed in the electrical null position before a test was begun. It was found that with all beams the output from the transducer was proportional to the displacement of the specimen. The details of the calibration of the unit are contained in Appendix I.

The transducer was excited by a Daytronic differential transformer indicator (Model 300 BF) which also analyzed the output and provided a visual reading. The analyzed signal from the Daytronic was fed to an oscilloscope for quick analysis of the results and to an Edin modulator amplifier (Model 8108 A) which provided excitation for a Brush dual channel oscillograph. One channel of the oscillograph displayed a displacement-time graph of the specimen motion while the other channel monitored the turntable velocity.
III.3 SPECIMENS

Initially steel-on-steel experiments were run in order to determine the effect of surface contamination and wear on the friction-time of stick curves as well as to study the effect of lubricants and controlled atmosphere. Hence the first friction-couple tried was a mild steel (C 1020) specimen running on a mild steel disk which had been ground and lapped.

It was immediately obvious that this combination yielded very inconsistent friction properties probably because of the non-homogenity of the metals. Next a disk of Atlas Nutherm hardened and annealed to $R_c 53.5$ and ground and lapped was tried, yet it also yielded inconsistent friction results. It was thought that if the friction couple were reversed, better results would be obtained so a specimen of Keewatin steel hardened to $R_c 55$ was run on a fully annealed disk of C 1020 ($R_k 77$). The resultant friction properties were much better and this system was used.

Since it appeared that the friction-couple was very important and since the experiment was designed to determine fundamental properties of the static coefficient of friction it was decided to try specimens made from pure metals. By using metallic elements the factor of alloying and material hardness became less important. Also, for comparison purposes, results reported by other workers in the field invariably dealt with the pure metals. Hence high purity samples of iron, gold, silver, nickel, lead, indium, aluminium, copper, zinc, and cadmium were fabricated into specimens for test purposes. Where applicable the metals obtained were fully annealed.
III.4 TEST PROCEDURE

A standard test procedure, determined after trying various methods to obtain the most consistent friction results, was followed for all specimens. Experiments with steel-on-steel showed that for best results the turntable should be ground flat and lapped on a special lapping plate with medicinal petroleum oil and alum powder as a cutting agent. The resultant finish was 10-12 μin. RMS and had no directional finish. The specimens were also ground and lapped. The surfaces were cleaned with trichlorethylene and then ethyl alcohol and were immediately run after the finishing process. At the completion of every run the disk was lapped to remove wear debris and cleaned before the next run. In this way it was possible to ensure that the surfaces were uniform between experimental runs.

Even with the precautions taken to ensure uniformity of surfaces it was found that the values obtained for static friction during the stick-slip phenomenon around the disk varied by too much to be useful in predicting a $\mu_s - t_s$ curve. The first method tried was to average the values for static friction around the disk at one constant velocity, then remove the disk and refinish it, then average the static friction values for a second velocity and so on. It was not possible to run the disk more than one revolution without refishing because the wear track influenced the friction values to a great extent. However this method gave very poor results for the $\mu_s - t_s$ curve primarily because in refishing the disk the system parameters were changed enough to make a comparison of runs inconclusive. After a trial and error procedure it was found that if a very small section of the running track were used to obtain the $\mu_s - t_s$ curve
the results were reasonable. Hence the procedure was to take two or three
stick-slip cycles at one velocity, then change the velocity and take three
more stick-slip cycles and so on until a full range of velocities had been
covered so that the results obtained included times of stick from in the
order of fifty seconds to pure sliding. This method gave a $\mu_s - t_s$ curve
obtained from less than one-sixth of a turntable revolution and allowed
three or four different curves to be taken for one revolution of the turn-
table thus eliminating running more than once over the wear track. The
value for the kinetic coefficient of friction was taken at the lowest turn-
table velocity when the specimen was sliding without vibration.

To study the effect of run-in on the static friction values,
$\mu_s - t_s$ curves were obtained from the first, second and tenth consecutive
revolution of the turntable. It was found that after ten revolutions very
little change in friction values was apparent. For the steel and copper
specimens it was decided to try various runs using ethyl alcohol as a pro-
tective film for the running track to inhibit oxidation of the surfaces and
wear debris. The effect of ethyl alcohol as an active chemical or hydro-
dynamic lubricant was thought to be negligible.

The effect of different loads and spring constants was studied for
the copper and steel specimens. For all other specimens the standard
system was a spring of 42.2 lb/in stiffness and a load of 5.05 lb., except
indium where the frictional values were so high that the load had to be
reduced to 3.55 lb. to ensure linearity of results. The standard universal
joint that provided intimate specimen contact with the disk proved useless
with lead, gold and indium because these metals deformed so readily that
flat edges resulted during the experimental runs. Thus a special rigid
specimen holder was made to ensure that these specimens would remain
perpendicular to the running surface.

All tests were conducted at 80°F room temperature. The magnitude of structural damping was checked by means of free vibration of the beam. The damping was found to be of a combined coulomb and viscous nature and was negligible. Typical values of the damping coefficients were 0.005 lb·sec/in for the 7.4 lb/in beam and 0.01 lb·sec/in for the 42.2 lb/in beam. The equivalent vibrating weight was determined to be 0.24 lb. All $\mu_s - t_s$ curves presented in the results are typical of a series of experimental runs and in most cases only one curve is given for each particular system in order to preserve clarity of the data.
CHAPTER IV

IV.1 RESULTS

Steel specimen

The first step in determining the friction parameters was to run the steel specimens for as many consecutive turntable revolutions as was found necessary to determine a stable value for the static coefficient of friction with a fully run-in wear track. For a specimen of annealed C 1020 the velocity was 0.03 in/sec and the turntable finish was 13μin. RMS at the start of run-in. After ten revolutions the static friction value had stabilized and the wear track was 15–18μin. across the track. (Fig. 8) For the hard specimen of Atlas Keewatin, also run at 0.03 in/sec, the static coefficient of friction stabilized after three revolutions. (Fig. 9) The friction–velocity curve in the stick-slip region for the hardened steel specimen on the fully annealed turntable was also considered. The kinetic
friction coefficient was calculated from Cameron's formula:

\[ \mu_k = \frac{K}{2W} S_{\text{min}} \frac{\mu_s}{2} \]

Pure sliding resulted at velocities greater than 0.12 in/sec with a spring of 42.2 lb/in and load of 5.05 lb. (Fig. 10)

The static friction values for the soft specimen were so erratic when run on the annealed turntable that any information obtained concerning the time of stick was meaningless. However the \( \mu_s - t_s \) curves for the hardened specimen were uniform and could be duplicated relatively easily. The effect of load and spring constant was determined by running tests at 42.2 lb/in, 5.05 lb and 7.4 lb/in, 3.55 lb. (Fig. 11) The effect of run-in was determined by taking one set of results at one revolution and another after ten revolutions of the disk. (Fig. 12)

The influence of ethyl alcohol on the static coefficient of friction was demonstrated for the hardened specimen by experimental runs with spring constants and loads of 42.2 lb/in, 5.05 lb and 7.4 lb/in, 3.55 lb respectively. The effects of run-in were also determined. (Figs. 13 & 14)

The wear track for the steel specimens in the lubricated and unlubricated conditions, even after ten revolutions, was barely noticeable. The average roughness across the wear track had increased by not more than 10\( \mu \text{in. RMS} \) after run-in for both the lubricated and unlubricated cases.

**Copper specimen**

The \( \mu_s - t_s \) relation for the copper specimen was obtained for a spring-load system of 7.4 lb/in and 3.55 lb. The effect of ethyl alcohol as a lubricant and the effect of run-in of the surfaces was demonstrated. (Figs. 15 & 16) For a spring-load system of 42.2 lb/in and 5.05 lb it was impossible to obtain stable stick-slip vibration since after a very few
cycles the stick-slip deteriorated into pure sliding even for surface velocities of 0.0005 in/sec. The kinetic coefficient of friction was determined to be velocity independent for all configurations. A wear track was evident and consisted primarily of copper debris on the running surface.

Iron specimen

An iron specimen, Ferrovac E, was run for two springload configurations, 7.4 lb/in, 2.1 lb and 42.2 lb/in, 5.05 lb. The corresponding $\mu_s - t_s$ curves were obtained for each springload combination including one and ten revolutions run-in. (Figs. 17 & 18) The wear track consisted of black iron oxide. The kinetic coefficient of friction was velocity independent.

Cadmium specimen

For a spring-load configuration of 42.2 lb/in and 5.05 lb the $\mu_s - t_s$ curves were obtained for one and ten revolution of run-in. (Fig.19) A wear track of cadmium particles was evident after full run-in. The kinetic coefficient of friction appeared to be velocity independent. For a given surface velocity the magnitude of the static coefficient of friction decreased slightly during stick-slip until a stable value was reached after about five cycles.

Lead specimen

A rigid specimen holder was necessary to prevent excessive deformation of the edges of the lead specimen during the test. The lead was so soft that any pressure on an edge caused immediate plastic deformation thus changing the orientation of the specimen with respect to the running track. For a spring-load system of 42.2 lb/in and 5.05 lb the $\mu_s - t_s$ curves were obtained for one and ten revolutions of run-in. (Fig. 20) Similar to the cadmium results the values of the static coefficient decreased with an
increasing number of cycles of stick-slip at constant velocity. However, to reach a stable limit took usually ten to twenty cycles and in some cases the stick-slip phenomena died out completely and pure sliding resulted. The kinetic coefficient of friction also appeared very velocity sensitive; in fact, the friction value rose progressively with sliding speed until a limiting velocity of 1 in/sec was reached by the apparatus. It was impossible to obtain an accurate $\mu_k - V$ curve because the kinetic friction value changed erratically depending on the position on the turntable. A very heavy wear track was composed of pure lead.

**Aluminium specimen**

The aluminium specimen was unusual in that slip-stick vibration could not be induced for a spring of 7.4 lb/in and loads of 2.1 lb or 3.55 lb or for a spring of 42.4 lb/in and a load of 5.05 lb. The specimen slid smoothly for over ten revolutions even though a very prominent wear track of white aluminium oxide deposited itself on the turntable. The kinetic coefficient of friction averaged 0.30 around the disk and did not change with run-in. Even manually induced vibration at the slowest surface velocity did not cause any stick-slip vibration indicating that the specimen was in a stable configuration while sliding.

**Zinc specimen**

For a spring-load combination of 42.2 lb/in and 5.05 lb the $\mu_s - t_s$ curves were determined for the zinc specimen. (Fig. 21) No noticeable difference in the curves was obvious even after ten revolutions of the turntable and the kinetic coefficient of friction remained fairly constant through a wide velocity range. A slight amount of zinc debris was evident on the wear track after run-in.
Nickel specimen

The $\mu_s - t_s$ curves for nickel were determined after one and ten turntable revolutions for a spring-load system of 42.2 lb/in and 5.05 lb. (Fig. 22) There was no noticeable wear track after run-in and the specimen itself showed very little wear. The kinetic coefficient of friction remained constant.

Silver specimen

It was difficult to determine static friction values for the silver specimen because it exhibited an unusual phenomenon which might be termed rapid creep. During the stick regime the specimen displacement-time curve should have had a constant slope (the surface velocity) but for silver the slope changed rapidly in a positive direction just before slip commenced. (Fig. 3)

![Graph of Slider Displacement Versus Time](image)

Fig. 3
Graph of Slider Displacement Versus Time

Taking the static coefficient at the point where sliding commenced, the $\mu_s - t_s$ curves for one and ten revolutions of the turn-table were obtained for a spring-load combination of 42.2 lb/in and 5.05 lb. (Fig. 23) A heavy wear track was evident.
Gold specimen

Similar to the lead specimen a rigid specimen holder was necessary for the gold specimen in order to prevent excessive deformation of the corners. The $\mu_s - t_s$ curves were obtained for one and ten revolutions of the turntable with a spring-load system of 42.2 lb/in and 5.05 lb. (Fig. 24) A very prominent wear track resulted after run-in.

Indium specimen

The rigid specimen holder was also used for the indium specimen because indium deformed even more readily than lead. For a spring-load combination of 42.2 lb/in and 3.55 lb the static and dynamic friction coefficients were extremely high, in the order of unity. (Fig. 25) At slow surface velocities, around 0.001 in/sec, indium exhibited phenomenal creep. The resultant displacement-time curve resembled silver (Fig. 3) except that the displacement would reach some stable value at low velocities and stick-slip would disappear. By increasing the surface velocity, stable stick-slip could be induced until a sufficient velocity was reached at which pure sliding commenced. Similar to lead, at some constant velocity, the amplitude of stick-slip would diminish until a stable value was obtained. The wear track, after ten revolutions of the turn-table, consisted of a very heavy layer of indium deposited during run-in. The values of static friction did not change by any large amount during run-in.

Theory predicted that the relationship between the static coefficient of friction and the time of stationary contact would have the form at constant temperature:

$$\mu_s - \mu_k = K_2 t_s^\beta$$

where $\beta = \frac{1}{n+1}/\nu$ and $K_2$ are some constants dependent on the rate effects of
plastic deformation of the asperities in physical contact. The general
trend of the experimental $\mu_s - t_s$ curves agreed with the shape predicted by
theory. The $\mu_s - t_s$ curves for the various metals tested showed that for
any particular friction couple the curves were of the same basic shape but
were generally displaced vertically on the $\mu_s$ axis from one another. Hence
by taking the difference $\mu_s - \mu_k$, the curves collapsed onto a single plot.

Replotting the $\mu_s - t_s$ curves on log-log axes showed that the ex-
perimental results did agree with those predicted by theory. On a log
$(\mu_s - \mu_k) - \log(t_s)$ plot the slope of the linear line was $\beta$ and the value of
the constant $K_2$ was determined at unity time of stick where $K_2 = \mu_s - \mu_k$.
(Figs. 26 - 38)

The equations from the log-log graphs for the various metals tested
are summarized in Table 1.

IV.2 DISCUSSION OF RESULTS

An examination of the $\mu_s - t_s$ curves for the hardened steel
specimen showed that the effect of the spring-load system and run-in was
slight. Similarly zinc showed little or no change with run-in. The dynamic
coefficient of friction decreased with run-in for the copper, cadmium, lead
and silver specimens whereas it increased for the zinc, nickel and gold
specimens which indicated, as all of these specimens left a wear track,
that the final phase of run-in was the specimen sliding on pure specimen
wear debris. The resulting friction-couple after run-in was much different
from the original specimen running on the annealed steel disk. It can be
seen that the steel, cadmium, lead and zinc specimens gave a curve which,
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<td>$\mu_s - \mu_k = 0.36t_{0.35}$</td>
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<td>Cadmium One to Ten Rev. Run-in, Dry</td>
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<td>Fig. 33</td>
<td>Indium Ten Rev. Run-in, Dry</td>
<td>$\mu_s - \mu_k = 0.153t_{0.081}$</td>
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Note: In all tests the running surface was annealed C1020

**TABLE 1**
disregarding the dynamic coefficient of friction, was independent of run-in. This indicated that if one of these pure metals were run together as a friction-couple the resultant friction curves would be similar to those that were obtained when the metal was run on annealed steel. Iron was the only material which showed a dependence on the normal load although an explanation for this effect is not immediately available.

The effect of ethyl alcohol as a lubricant was dubious and its inclusion in the test program was to determine the effect of removing the atmosphere from the wear track in order to prevent oxidation of the surface. Thus the effect of running in an oxygen-free environment could be determined. For the steel specimen there was no different in the $\mu_s - t_s$ curves with run-in and the ultimate values of friction closely resembled the results obtained for dry friction. However, the copper specimen showed a marked decrease in kinetic friction with run-in for the lubricated condition yet the initial static friction values were much higher in the lubricated state than in the unlubricated state. Oxidation of the steel was probably unaffected by the ethyl alcohol because sufficient oxygen was trapped in the fluid to effect full oxidation of the wear debris. Hence there was no noticeable difference in the $\mu_s - t_s$ curves for the lubricated and unlubricated conditions. However, for the copper specimen, the initial oxygen requirements for full oxidation of the wear particles and asperities in contact could not be obtained from suspended air in the lubricant and, as a result, the first turntable revolution showed a marked increase in friction values over the unlubricated condition because metals in contact without an oxide surface film tend to cold weld. As the number of revolutions increased, gradually the wear debris became oxidized and the static friction coefficients fell to a value equal to full run-in for the unlubricated case.
Aluminium was the only material tested which would not exhibit the stick-slip phenomenon. Because of a very coherent and strong oxide layer which surrounded the specimen the asperities in contact with the steel disk were probably oxide particles which would not exhibit plastic deformation in the same manner as the other specimens. Hence there was little time dependence of the hardness of the aluminium and it would not stick-slip under the conditions imposed by the test. Further tests showed that the static coefficient of friction was not measurably larger than the kinetic coefficient and this criterion immediately eliminated the possibility of stick-slip occurring.

The velocity dependence of lead and indium played an important role in determining the static friction values from the stick-slip phenomenon. During the slip portion of stick-slip the velocity of the specimen was much higher than the disk velocity thus providing a resisting force greater than would be encountered during normal sliding. This higher resisting force is similar to non-linear damping and would tend to decrease the amplitude of vibration at any given surface velocity. Thus, for velocity sensitive materials, the $\mu_s - t_s$ curves probably would have a different shape from that which would be obtained from velocity independent materials.

The erratic results obtained from the lead specimen could be attributed to the fact that lead consists of fairly large crystals of random orientation which, as wear proceeded, would present different crystallographic faces with different friction properties. Hence the static and kinetic coefficients of friction for lead could not be predicted with great accuracy.

The rapid creep effect evident in the silver and indium specimens could only be expected from very ductile materials. However, the gold and
lead specimens did not exhibit this phenomenon thus eliminating ductility as a factor. Exactly why there should be a sudden relaxation is shear strength for silver and indium is not known. Perhaps the materials reached the point at which sliding should have commenced yet adhesion between the asperities was so great that plastic flow commenced in the bulk material until deformation broke the surface bonds. This mechanism resembles large scale junction growth.

In previous work several authors obtained experimental results which agreed fairly well with the values for the $\mu_s - t_s$ curves presented in this thesis. (Fig. 39) Dokos (3) obtained for a soft steel specimen running under high load on a soft steel disk:

$$\mu_s - \mu_k = 0.092t_s^{0.24}$$

Similarly in a study of the time dependence of static friction Spurr (6) gave results for indium on glass,

$$\mu_s - \mu_k = 0.049t_s^{0.31}$$

and for zinc on glass:

$$\mu_s - \mu_k = 0.012t_s^{0.51}$$

However Spurr determined the static coefficients of friction for indium by loading the specimen for a predetermined length of time, then removing the load and applying a tangential force. Probably the values should have been much higher.

From data presented by Pomey et al, (8) for hardness tests of steel at high temperatures:

$$\sigma = 54.7t_s^{-0.16} \text{ (Kips/in}^2)$$

where $\sigma$ is the yield strength of the metal. Combining this with the
relation:

$$\mu_s - \mu_k = \frac{s_i}{\sigma}$$

from the theory and taking an interface shear stress value of approximately 5 Kips/in².

$$\mu_s - \mu_k = 0.1t_s^{0.16}$$

This compares reasonably well with results obtained experimentally in this thesis:

$$\mu_s - \mu_k = 0.030t_s^{0.34}$$

Similar data from Bowden and Tabor (5) taken from indentation hardness tests on indium (Fig. 4) yielded the relation.

$$\sigma = 0.83t_s^{-0.095} \text{ (Kg/mm}^2\text{)}$$

Assuming an interface shear stress of approximately 0.11 Kg/mm² then:

$$\mu_s - \mu_k = 0.1t_s^{0.1}$$

Fig. 4 - Variation in yield pressure with loading time for indium
The experimental results for indium gave:

$$\mu_s - \mu_k = 0.153 t_s^{0.061}$$

Data by Williamson (14) taken from indentation hardness tests on work-hardened gold yielded the relation, (Fig. 5)

$$\sigma = 83.7 t_s^{-0.074} \text{ (Kg/mm}^2\text{)}$$

Assuming an interface shear strength of approximately 10 Kg/mm$^2$ for the work-hardened material then:

$$\mu_s - \mu_k = 0.12 t_s^{0.074}$$

The experimental results for gold in the fully run-in condition yielded:

$$\mu_s - \mu_k = 0.112 t_s^{0.086}$$

Fig. 5 - Variation in yield pressure with loading time for work-hardened gold.
Therefore the results obtained by independent authors in this field of research tend to support the theory that the static coefficient of friction varies with the time of stick in the manner:

\[ \mu_s - \mu_k = K_2 t_s^\beta \]

The effect of ambient temperature on the \( \mu_s - t_s \) curves as predicted by theory can be determined from the formula,

\[ \mu_s - \mu_k = K_1 e^{\frac{-Q}{RT}} t_s^\beta \]

Rearranging:

\[ \ln \left( \frac{\mu_s - \mu_k}{K_1 t_s^\beta} \right) = \frac{Q}{RT} \]

Taking two temperatures \( T_2 > T_1 \) and subtracting gives:

\[ \frac{Q}{R} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right] = \ln \left( \frac{\mu_{s_2} - \mu_k}{\mu_{s_1} - \mu_k} \right) \]

Evaluating:

\[ \mu_{s_2} - \mu_k = (\mu_{s_1} - \mu_k) e^{\frac{Q}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} \]

To determine the magnitude of this variation, typical values for indium were used.

\[ Q = 16 \text{ kcal/mole} \]
\[ R = 2 \text{ cal/deg mole} \]
\[ \beta = 0.061 \]

Assuming \( T_1 \) is the ambient temperature (20°C) where

\[ \mu_{s_1} = 1.00 \]
\[ \mu_k = 0.80 \]
\[ T_2 = 100^\circ C \]
Hence:

\[ \mu_{s_2} = 0.80 + 0.20 \frac{0.061(16,000)}{2} \left( \frac{1}{293} - \frac{1}{393} \right) \]

\[ \mu_{s_2} = 1.086 \]

The change is approximately 9% for a 27% change in absolute temperature.
CHAPTER V

V.1 CONCLUSIONS

The experimental evidence verified the theoretical prediction that the variation of the static coefficient of friction with time of stationary contact followed the general relation:

$$\mu_s - \mu_k = K_2 t^\beta_s$$

where in usual practice for metals both constants $K$ and $\beta$ were less than unity. The experimental verification was obtained using the stick-slip phenomenon in which the time dependence of static friction plays an important role. For the various metals tested it was noted that except for the very ductile metals such as gold and indium the values of the constants $K$ and $\beta$ were remarkably similar when the running surface was annealed steel. In fact little error would result if, for unknown friction characteristics, a general curve were used:
The effect of run-in for the pure metals was that eventually the running track became coated with wear debris and the friction-couple changed from a pure metal run on annealed steel to a pure metal running on a pure metal. The $\mu_s - t_s$ curve changed accordingly depending on the metal's friction parameters. Generally, however, the static friction coefficients decreased with run-in.

Since ethyl alcohol failed to decrease the stick-slip phenomenon by any appreciable amount its effect as a lubricant was considered to be negligible. Also it did not seem to have a pronounced effect on the values of static friction as run-in progressed indicating that oxide formation continues regardless of atmospheric inhibitors although perhaps in a modified form. To prevent oxidation would probably require the elimination of the surrounding atmosphere.

The static coefficient of friction generally appeared to be independent of load and shearing force but showed a strong dependence on the rate of applied shear force which agreed with the theoretical prediction of junction area growth as a time dependent function. This effect was also manifest in the rapid creep phenomenon in shear exhibited by some of the more ductile and cohesive metals. The anomaly in the tests was the aluminium-steel friction-couple which did not show any form of relaxation oscillation.

V.2 RECOMMENDATIONS FOR FUTURE WORK

Further research should be carried out in three main areas. First,
a check on the effects of humidity could be attempted. The plexiglass cover constructed for the apparatus would provide an easy method of controlling the humidity. At the same time the temperature could be varied to check the theoretical predictions for the change in the static coefficient of friction values by using an electrically heated turntable. Secondly, some attempt should be made to determine the effects of surface finish and active chemical lubricants. From the experimental results in this thesis it seems that the surface finish directly affects the static friction values. Thirdly, there seems to be experimental verification that the determining factor in the variation of the static coefficient of friction is not necessarily the time of stationary contact but rather the rate of applied shear stress. It is suggested that further experimental work be conducted wherein the shear stress rate is correlated to the static friction values.
APPENDIX I

Calibration of the Beam, Transducer and Oscillograph

With the beam in its equilibrium position and the slider on the displacement transducer attached to the beam, the null position of the transducer was obtained by moving the transducer relative to the beam. After locking the transducer in the null position a micrometer barrel was attached rigidly to the apparatus with the spindle just touching the specimen holder. The micrometer was given 0.005 in. increments up to 0.10 in., increasing and decreasing, and the gain on the Daytronic indicator unit was adjusted until full scale deflection was obtained. Linearity of the beams was within 1% and hysteresis error was insignificant. Attenuation of the indicator unit provided full scale deflection from 0.01 in. to 0.25 in. specimen displacement using the initial calibration. An internal calibration signal provided a reference for future resettings.

With the transducer calibrated for displacement the stiffness of the
cantilever beam was determined by applying a known force to the specimen holder. A light cord was attached at one end to the specimen holder and, after running over a low friction pulley, at the other end to a weight-pan. The cord was carefully aligned to pull the specimen holder at right angles to the beam. By loading the weight-pan and determining the specimen displacement from the indicator unit the stiffness of the beam was obtained. The stiffness was found to be linear within 2% with the maximum error occurring at maximum displacement.

To determine the normal load the specimen holder was placed on a strain gauge ring which was analyzed by a Baldwin strain gauge analyzer and weights were applied to the weightpan. The load was obtained for the four weights used in the experiment. A quick check by considering the moment arms of the system about the frictionless bearings verified that the normal loads were correct. For the one beam length used in the experiment the normal loads were 0.70 lb, 2.1 lb, 3.55 lb and 5.05 lb. A moveable balance weight allowed the system to be statically balanced for the two beams used in the experiment without affecting the load calibration.

The oscillograph was calibrated by adjusting the gain on the modulator amplifier so that full scale deflection on the displacement unit corresponded to full scale deflection on the chart paper of the oscillograph.
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Fig. 6 - General View of Apparatus and Instrumentation
Fig. 8 - Change of Static Coefficient with Run-in-Annealed C1020 Steel Specimen on Annealed Steel Disk.

Fig. 9 - Change of Static Coefficient with Run-in - Hardened Atlas Nutherm Steel Specimen on Annealed Steel Disk.
Fig. 10 - Kinetic Coefficient as Calculated During Stick-Slip vs. Sliding Velocity for Atlas Nutherm.
Fig. 11 - Atlas Nutherm Specimen Showing Effect of Spring-Load System - One Rev. Run-in - Annealed Steel Disk.
Fig. 12 - Atlas Nutherm Specimen Showing Effect of Ten Revolutions Run-In
- Annealed Steel Disk.
Fig. 13 - Atlas Nutherm Specimen with Ethyl Alcohol Lubricant - One Rev. Run-In
- Annealed Steel Disk.
Fig. 14 - Atlas Nutherm Specimen with Ethyl Alcohol Lubricant - Ten Rev. Run-In
- Annealed Steel Disk.
Fig. 15 - Copper Specimen Showing Effect of Run-In
Annealed Steel Disk.
Fig. 16 - Copper Specimen Showing Effect of Ethyl Alcohol as Lubricant
- Annealed Steel Disk.
Fig. 17 - Iron Specimen Showing Effect of Run-In For $\frac{K}{W} = 7.4/2.1$
- Annealed Steel Disk.

Legend: ○ - One Rev. Run-In
△ - Ten Rev. Run-In
Legend:  
○ - One Rev. Run-In  
△ - Ten Rev. Run-In

Fig. 18 - Iron Specimen Showing Effect of Run-In for $K/W = \frac{42.2}{5.08}$  
Annealed Steel Disk.
Fig. 19 - Cadmium Specimen Showing Effect of Run-In $- \frac{K/W}{5.05} = \frac{42.2}{5.05}$
- Annealed Steel Disk.
Fig. 20 - Lead Specimen Showing Effect of Run-In for $\frac{K}{W} = \frac{42.2}{5.05}$
- Annealed Steel Disk.
Fig. 21 - Zinc Specimen for $\frac{K}{W} = \frac{42.2}{5.05}$ (One to Ten Revolutions)
- Annealed Steel Disk.
Fig. 22 - Nickel Specimen Showing Effect of Run-In for $\frac{K}{W} = \frac{42.2}{5.05}$
- Annealed Steel Disk.
Fig. 23 - Silver Specimen Showing Effect of Run-In for \( \frac{K}{W} = \frac{42.2}{5.05} \)
- Annealed Steel Disk.

Legend:
- \( \bigcirc \) - One Rev. Run-In
- \( \triangle \) - Ten Rev. Run-In
Fig. 24 - Gold Specimen Showing Effect of Run-In for $\frac{K}{W} = \frac{42.2}{5.05}$
- Annealed Steel Disk.
Fig. 25 - Indium Specimen for $\frac{K}{W} = \frac{42.2}{3.55}$ and Ten Rev. Run-In
- Annealed Steel Disk.
Fig. 26 - Atlas Nutherm Specimen on Annealed Steel Disk

\[ \log(\mu_s - \mu_k) \]

\[ \mu_s - \mu_k = 0.030t_s^{0.337} \]
Fig. 27 - Atlas Nutherm Lubricated Specimen on Annealed Steel Disk.
Fig. 28 - Annealed Copper Specimen on Annealed Steel Disk.
Fig. 29 - Annealed Copper Lubricated Specimen on Annealed Steel Disk.
Fig. 30 - Iron ($K/W = 7.4/2.1$) Specimen on Annealed Steel Disk.

- $\Delta$ = Ten Rev. Run-In
- $\bigcirc$ = One Rev. Run-In

$\mu_s - \mu_k = 0.043 + 0.163$

$\mu_s - \mu_k = 0.352 + 0.352$

$\log(t_s)$

$t_s$
Fig. 31 - Iron \((K/W = \frac{42.2}{5.05})\) Specimen on Annealed Steel Disk.
Fig. 32 - Cadmium Specimen on Annealed Steel Disk.
Fig. 33 - Lead Specimen on Annealed Steel Disk.
One to Ten Rev.

\[ \log(\mu_s - \mu_k) \]

\[ \mu_s - \mu_k = 0.065 t_s^{0.171} \]

Fig. 34 - Zinc Specimen on Annealed Steel Disk.
Fig. 35 - Nickel Specimen On Annealed Steel Disk.

\[ \log(\mu_s - \mu_k) = 0.025 t_s^{0.358} \]

\[ \log(\mu_s - \mu_k) = 0.011 t_s^{0.655} \]
\[ \log(M, g - \Delta k) \]

- One Rev. Run-In
- Ten Rev. Run-In

\[ \mu_s - \mu_k = 0.042t_s^{0.251} \]
\[ \mu_s - \mu_k = 0.039t_s^{0.171} \]

Fig. 36 - Silver Specimen On Annealed Steel Disk.
Fig. 37 - Gold Specimen On Annealed Steel Disk.
Fig. 38 - Indium Specimen on Annealed Steel Disk.

\[ \mu_s - \mu_k = 0.153 t^{0.0614} \]
Fig. 39 - Comparison Data.

- Dokos - soft steel on soft steel
  \[ \log (\mu_s - \mu_k) = 0.092t_s^{0.244} \]

- Spurr - indium on glass
  \[ \log (\mu_s - \mu_k) = 0.049t_s^{0.31} \]

- Spurr - zinc on glass
  \[ \log (\mu_s - \mu_k) = 0.012t_s^{0.505} \]