THE INCOME ELASTICITY OF DEMAND

# FOR CASH BALANCES IN CANADIAN INDUSTRY: AN EMPIRICAL INVESTIGATION 

## by

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## ABSTRACT

This study examines the transactions demand for cash in several sectors of Canadian industry. In particular, the question of the existence of economies and diseconomies of scale in the holding of cash balances is investigated.

In the estimation procedure, sales were used as an approximation of transactions. Average cash balances were regressed on average annual sales figures for fourteen industrial groups in the years 1957. 1958 and 1960. Log. and ordinary linear formulations were used.

Regression coefficients indicated that in the majority of cases, elasticities of cash with respect to sales were approximately unity, as the Meltzer model of the demand for cash predicts. In three out of the fortytwo cases, elasticities were significantly (at the .05 level) less than unity, indicating that the Baumol-Tobin models might be relevant in some cases. In fiwe cases, elasticities significantly greater than unity were found.

## TABLE OF CONTENTS

PageCHAPTER I. INTRODUCTION
Objectives ..... 1.
The Relevance of the Study ..... 2.
Method ..... 8.
Limitations of the Study ..... 9.
CHAPTER II METHODOLOGY AND RESULTS
Source of Data ..... 12.
Manipulation of Data ..... 13.
Results ..... 15.
CHAPTER III EVALUATION OF RESULTS
Significance of the Regression Equations ..... 22.
Sales Elasticities ..... 23.
Effect of Changes in Economic Conditions ..... 31.
Comparison of Cross Section and Time series Results ..... 32.
CHAPTER IV CONCLUSIONS ..... 36.
Bibliography ..... 37.
Appendix I
A Description of the Derivation ..... 40
of Baumol's Cash Inventory Model.
IIST OF TABIES.

|  |  | Page |
| :---: | :---: | :---: |
| TABLE I. | Coefficients of Determination ( $r^{2}$ ) For Log. And Arithmetic Regressions. | 19. |
| TABLE II. | Elasticities Of Cash Balances With Respect To Sales For Selected Industrial Sectors For Three Taxation Years. | 20. |
| TABLE III. | Distribution of Elasticities By Taxation Year. | 21. |
| TABLE IV. | Data From All Firms Grouped By Income Class For 1960. | 26. |
| TABLE V. | Statistics Illustrating The Pattern Of Payments And Degree Of Capital Intensity of The Subclasses of The Wood And Paper Products Industrial Group - 1958. | 28. |
| TABLE VI. | Comparison Of Cross Section Elasticities With Time Series Cash/Sales Ratios. | 34. |

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## CHAPTER I.

## INTRODUCTION

## (A) OBJECTIVES

Recently, many empirical and theoretical papers have been published dealing with the holdings of cash and other liquid assets by business firms. ${ }^{1 .}$. It will be the purpose of this thesis to examine the manner in which the aggregate cash balances of Canadian business firms vary with income levels. Specifically, the question of the existence of economies of scale in the use of cash will be investigated. The study involves the determination of the income elasticity of the demand for cash balances for three distinct groupings of firms. The logs of average cash balances are regressed on the logs of average annual sales volumes for:
(1) A number of Canadian industries using average

1. See Richard T. Selden, "The Postwar Rise in the Velocity of Money: A Sectoral Analysis, Journal of Finance, Dec. ${ }^{\text {F }}$ 1961\% pp. 483-545; William J. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach;" Quarterly Journal of Economics November, 1952; pp. 545-556; James Tobing "The Interest Elasticity of Transactions Demand for Cash," Review of Economics and Statistics*, August, 1956 pp. 241-247; Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results; Journal of Political Economy, August, 1959; pp. 327-351; Allan H. Meltzer, "The Demand for Money: "A Cross-Section Study of Business Firms," Quarterly Journal of Economics, August, 1963, pp. $405-422$; R. C. Vogel and G. S. Maddala, "Cross Section Estimates of Liquid Asset Demand by Manufacturing Corporations" Journal of Finance, December 1967. pp. 557-575.
cash and sales figures for industrial subclasses as observations. (2)
(2) The manufacturing industry, which is broken down by asset size classes. Average cash and sales figures from each asset size class are used as observations.
(3) A sample of firms classified by asset groups and a sample of firms classified by income groups. Average cash and sales figures from each asset and income class respectively are used as observations.

The above regressions are performed using the ordinary linear formulation as well as the log form. The effect of changes in prevailing economic conditions on the income elasticity of cash is also investigated.
(B) THE RELEVANCE OF THE STUDY

The question of the existence of economies of scale in holding of cash is related to the broader question of the motives for holding cash balances. A widely accepted concept of the functions performed by money is based on the writings of J. M. Keynes,

Keynes' approach to the role of money differs from the classical analysis primarily because it focuses attention on the demand for money rather than on the supply
(2) e.g. The Elasticity for the Wood and Paper Products industry is determined by using average cash and sales figures from the subclasses of Plywood Mills, Sawmills, Furniture, Paper Bags and Boxes, Pulp and Paper Mills, Misc. Wood Products, and Misc. Paper Products as observations.
(3) J.M. Keynes - The General Theory of Employment, Interest, and Money - New York, Harcourt, Brace, 1936. by Keynes postulates three major motives why individuals and business firms want to hold money balances. These motives are: the transactions' motive, the precautionary motive, and the speculative motive. The focus of this thesis will be on the transactions motive which relates to the need to hold some quantity of money to carry on day-to-day economic dealings.

Almost all transactions in advanced economic systems involve an exchange of money, and since the cash receipts of individuals and business firms are not perfectly synchronized with transactions involving money outlays, it is necessary that some money be held in order to meet "daily" financial needs.

In the Keynesian system the actual amount of money held to satisfy the transactions motive $\left(L_{t}\right)$ is assumed to vary directly in proportion to income (Y); so, we can describe the relationship as: $L_{t}=a(Y)$.

The basis for this statement is that in a complex society the volume of economic transactions varies directly with income level. (5) Consequently, the absolute quantity of money balances needed to carry on these transactions also varies directly in proportion to the income level.

More recently, several economists, notably W. Baumol
(4) A statement of the classical position can be found in: W.C.Peterson Income, Employment, and Economic Growth - Norton and Co. New York, 1962.
(5) Ibid. P. 344
and J. Tobin have presented propositions which question the thesis that cash balances vary in direct proportion to income. Baumol (6) and Tobin (7) argue that if firms act in a rational manner, cash balances will tend to vary less than proportionally with income, implying that there are economies of scale in the use of cash. In essence, Baumol's contention is that the volume of cash held will vary in proportion to the volume of transactions and inversely with the costs related to holding cash. Similarly, Tobin contends that interest rates will influence the amount of cash held for transactions purposes. Both of these models, if associated variable costs are taken as zero collapse; into:

$$
\begin{equation*}
C=\sqrt{\frac{a Y}{2 r}} \tag{9}
\end{equation*}
$$

where $C=$ Cash balance
$a=$ fixed costs (pecuniary and non-pecuniary)
$Y=$ income
$r=$ interest rate
where "non-pecuniary" means costs which cannot be quantified.
The square root formula implies that if a firm acts to minimize the costs associated with the use of trans-
(6) Baumol; op. cit.
(7) Tobin, op. cit.
(8) A description of the derivation of Baumol's model is given in Appendix 1.
(9) Winiata, W. The Income Elasticity of the Demand for Money by Business Firms: An Empirical Test. (Unpublished), 1966.
-actions cash, its dollar balance will increase less than in proportion to income. In other words, if firms aet in accordance with the theory then the income elasticity of cash can be expected to be about 0.5 .

Another economist, M. Friedman, (10) has suggested that cash balances are "permanent-income" elastic and that in the long-run, real money balances increase more than proportionally to an increase in real permanent income. Friedman's empirical analysis shows that in the long run, the elasticity of money with respect to income was 1.8 over 20 business cycles from 1870 to 1954. An analysis of the behaviour of these variables within a cycle differed from the long-run trend in that the calculated elasticities were about 0.2. Interest rates were found to have little effect on the demand for money.

In a cross section study of American business firms A. H. Meltzer (11) proposes that the demand for cash 1 s a function of the firm's wealth (non-human assets). The author begins with the statement that the size of a firm's money balance is a function of a market rate of interest and net wealth. He then shows how sales are related to wealth by the internal rate of return on assets in the industry and a variable representing changes in the percentage of the firm's assets in use.
(10) Friedman, op. cit. P. 350
(11) Meltzer, op. cit. P 407

Meltzer performs a number of cross section cash on sales regressions and finds that the demand for money by firms is a function of sales to a first approximation, is linear in the logarithm and unit elastic. Meltzer qualifies his overall conclusions as being a "first approximation" since he does note that economies and diseconomies of scale may be found within individual industries.

In a recent paper, K. Brunner and A. H. Meltzer have re-examined the question of economies of scale in the use of cash in a critique of the Baumol and Tobin models. Brunner and Meltzer reject these models on empirical grounds, referring to the findings in Meltzer's (1963) cross-section study.

The authors conclude that:
".... the Baumol model provides little reason for abandoning the earlier conclusion that to a first approximation the quantity theory explains the observed cash holdings of business firms when the observations are from crosssections at a point of time."

They state further that economies and diseconomies of scale of the magnitude predicted by the Baumol model become apparent only when one is concerned with the behaviour of small firms or the distributions of cash (13) balances within industries.
(12) Brunner, K. and Meltzer, A. "Economies of Scale in Cash Balances Reconsidered." Quarterly Journal of Economics.
(13) Ibid. p. 427. The authors cite the findings of Meltzer's 1963 study as evidence for this statement.

With regard to Tobin's hypothesis, Brunner and Meltzer state that his principal concern is to show that interest rates will affect the amount of transactions balances held in the form of money. Tobin's analysis does not necessarily contradict the quantity theory of money because his analysis is not primarily concerned with the determinants of total money holdings but rather with computing the amount by which receipts must change before there is a change in the optimal number of transactions between cash and liquid securities. Tobin is not concerned with economies of scale in the use of cash for individual firms with the possible exception of those firms who are close to a point where a change in the number of transactions between bonds and cash is necessary. Brunner and Meltzer conclude that while the issue of economies of scale in the demand for money is primarily an empirical question it is possible to state theoretically that rational behaviour of the Baumol or Tobin type does not deny that the quantity theory of money explains the cross-section demand for money by firms to a first approximation.

It would appear that a conclusive theoretical basis for the existence of economies of scale in the demand for money is, as yet, undefined. It will not be the purpose of this exercise to pursue the theoretical argument and further, but rather to discover whether or not economies of scale existed at selected points in time.
(C) METHOD.

The analysis employed in this cross-section study is conceptually similar to that of Meltzer (1963). Data was obtained from Canada taxation statistics (14) which contains aggregate balance sheet and income statement data for all Canadian business firms submitting income tax returns. Cash and sales (or gross revenue) figures were obtained for firms in subdivisions of eleven industrial classes as well as for the total of all manufacturing firms grouped in ten asset classes, all Canadian companies grouped in asset classes, and all Canadian companies reporting a profit grouped in income classes. To investigate the effect of prevailing economic conditions on the demand for money; cash and sales figures were collected for three taxation years: 1957 and 1960 representing peaks in the business cycle and 1958 representing a trough year. (15) Averages of the cash and sales figures were calculated for each industrial subdivision and each asset and income group. The logar-
(14) Canada Dep't of National Revenue Taxation Statistics, Part II Tables 4-6.
(15) Turning points in the business cycle are as follows: April 1957 - peak April 1958 - trough January 1960 - peak March 1961 - trough.

1thms of these numbers were used in a linear regression to obtain the elasticity of cash with respect to sales. Cash on sales regression analyses were done in arithmetic as well as in log form. Sales was selected as the independent variable whenever possible for two reasons. Sales more closely reflects the scale of operation of most companies than does income. In the approximations we are not concerned with the increments to the value of the firm associated with net income but rather with the major source of cash inflows, Sales; Which more closely approximates transactions. Second; using sales facilitates a comparison with the results of other studies.
(D) LIMITATIONS OF THE STUDY.

Many of the results of this analysis are not strictly comparable to those obtained in other cross section studies such as those conducted by Meltzer; (1965); Frazer (1964), and Vogel and Maddala (1967). It is likely that the findings will differ because the classification of the Canadian taxation data used here is different from the American data. In Taxation Statistics aggregate financial figures for firms in industrial groups (Mining, Quarrying and Oilwells; Petroleum Products; etc.) are not organized into a range of asset classes. Industrial groups are subdivided as to various types of firms rather than various asset sizes. For example the group:of Wood and Paper Products is sub-
divided into seven types of firms: Planer Mills, Saw Mills, Furniture Manufacturing; Pulp; Paper Bags and Boxes, Miscellaneous Wood Products and Miscellaneous Paper Products. A breakdown of firms by asset size is available for only two groups: All firms submitting tax returns and all manufacturing firms. It is to be expected that because of the way in which the data is organized, regression equations obtained will not yield coefficients of determination as large as those of American studies since cash and sales figures are probably distributed more randomly in groups of firms of heterogeneous asset sizes.

In regard to cross section studies in general, several authors (notably Kuh; (1959). Grunfeld, (1961) and Vogel and Maddala, (1967) have recently suggested some limitations of a theoretical nature inherent in this type of analysis. In general; these authors state that one must be aware of two sources of error:
(1) errors introduced by the artificial grouping of dissimilar firms.
(2) errors introduced by aggregation over time; the problem in dealing with balance sheet figures is the selection of a moment in time which may not be representative. Balance sheet data are not derived from a single point in time because corporate year ends are distributed through the taxation year.

Another problem with the balance sheets of manu-
facturing companies is the practice of 'window dressing.'

Vogel and Maddala conclude their article with the warning that one must use caution in interpreting the results of cross section estimates and recommend that regression values obtained from this type of analysis be compared with time series values obtained from the same group of firms. While this latter point is well taken it is not possible to make this type of comparison when using aggregate tax statistics because the population of firms in each class changes from year to year. One must keep Vogel and Maddala's warning in mind, however, and recognize that the results of this study must be regarded as a first approximation of the income elasticities of cash balances.

## CHAPTER II.

> METHODOLOGY AND BESULTS.
(A) SOURCE OF DATA

The data for this thesis was obtained from Canada Department of National Revenue Taxation Statistics, Part Two, Tables 4 to 6 for the taxation years 1957, 1958 and 1960. This publication collects selected statistics from T2 Corporation Income Tax Returns filed by companies in Canada during a given taxation year. The tables mentioned above represent an aggregation of financial statement data from a sample of firms submitting tax returns; and are entitled:

Table 4 - Distribution of Fully Tabulated Companies by Industrial Class.

Table 5 - Distribution of Fully Tabulated Companies by Size of Total Assets.

Table 5A - Distribution of Fully Tabulated Manufacturing Companies by Size of Total Assets.

Table 6 - Distribution of Fully Tabulated Profit Companies by Income Class.

Tables 5 and 5 A are broken down into nine asset size classes while Table 6 is composed of ten income classes. (1) SAMPIE OF FIRMS: Canada Tax Statistics aggregates data from a sample of firms selected in the following manner: All corporations reporting with total assets of \$l million or more; or a current year profit of at least \$50 thousand are included. Certain unnamed industries considered subject to aberration are also sampled at a 100\% rate. The remaining corporations are stratified by
industrial classification and a $10 \%$ random sample selected in each industrial class. In the aggregate approximately $20 \%$ of corporation returns submitted are analyzed.
(2) PERIOD COVERED: Statistics are collected during the taxation year which embraces all company returns for fiscal periods ending between January 1 and December 31 of a given calendar year. Unless a company's fiscal year ends on December 31, the data submitted are likely to pertain to parts of two calendar years.
(3) SYSTEM OF CLASSIFICATION: The system by which firms are classified is based on the Standard Industrial Classification Manual issued by the Dominion Bureau of Statistics. Where a company's activities are diversified, it is normally grouped in the industry where it shows the greatest volume of sales.
(4) DEFINITIONS:

CASH: In Canada Taxation Statistics, cash is defined as cash on hand and in bank deposits after deducting outstanding cheques or bank overdrafts. It is assumed that "bank deposits" refers to both chequing and time deposits.

SALES: In general, this term includes only actual sales or revenue derived from operations. Capital profits, interbranch or interdepartment sales are excluded.
(B) MANIPULATION OF DATA.

Cash and sales figures were collected for each of the subdivisions of eleven industrial classes. To use the Transportation industrial class as an example, cash and sales totals were obtained for the following eight subdivisions:

Air Transport, Water Transport, Truck Transport, Bus Transport; Urban Transportation and Taxicabs; other Transportation, Pipe-lines; and Railways. Average cash and sales figures were then calculated for each of these eight transportation Subdivisions. (I) The same procedure was followed for subdivisions of the other ten industrial classes and each income or asset class in Tables 5,5A and 6.

The average figures obtained were then used to calculate a regression equation, using the least-squares method. Ordinary linear and linear in logarithms estimates of the relationship between the average cash and sales figures were computed. To use the Transportation group again as an example, average cash and sales figures for each of the eight subdivisions (Air; Water Transport, etc., were used as observations in deriving the regression equation. From the data grouped by size classes (the total Manufacturing Industry, all Firms grouped by Asset class and all Firms grouped by Income class) average cash and sales figures were taken from each class and used as observations. When these figures (cash and sales) were regressed in log form, the regression coefficient of the resulting regression equation is our measure of the elasticity of the demand for money with respect to sales for the Transportation industry as a whole. In logarithms the regression equation took the general form, $\log \mathrm{C}=\log \mathrm{a}+\mathrm{b}$ $\log S$ where $C=$ the average cash balance, $S=$ the average

1. Each subdivision is divided into two further categories; companies reporting a profit and companies reporting a loss. In this study; statistics for both profit and loss companies were added and an average was calculated using this total.
sales volume and $b=$ the elasticity of cash with respect to sales. Regression coefficients of approximate unity indicate that there are neither economies or diseconomies of scale in the use of cash in a given industrial class.

To test whether the elasticities obtained were significantly different from unity, a test of the hypothesis that $\mathrm{b}=1$ was performed using the statistic

$$
\begin{equation*}
T=\frac{b-1}{\sqrt{1-r^{2}}} \cdot \sqrt{N-2} \tag{2}
\end{equation*}
$$

Where $b$ is the elasticity in question, $r^{2}$ is the coefficient of determination of the regression equation and $N$ is the number of observations. This statistic has Student's distribution with N-2 degrees of freedom. (C) RESULTS

The findings of the cross section regressions are summarized in the tables at the end of the chapter. In general, the results shown describe how well the regression equations explain the relationship between the cash and sales variables and the nature of the elasticities of cash with respect to sales which were obtained.

TABLE I. COEFFICIENTS OF DETERMINATION ( $r^{2}$ ) FOR LOG. AND ARITHMETIC REGRESSIONS.

The coefficients of determination give an indication of how well the cross section estimations describe the relation(2) M. R. Spiegel. Schaum's outline of Statistics. Schaum Publishing Co. New York, 1961.
ship between the variables in that they indicate how much of the variation of observations from the regression line is explained by the equation. It must be argued that the arithmetic form of the approximation provides a better explanation, since $r^{2 \prime} s$ for this form were higher in 26 out of the 31 statistically significant equations. Coefficients of determination for the log equations ranged from 0.37 to 0.98 with a mean value of 0.85 .

Coefficients of determination obtained from the arithmetic equations ranged from 0.53 to 0.99 with a mean of 0.86 .

It was found that of the 42 regressions performed; seven of the log. and seven of the arithmetic equations had coefficients of correlation (r) which did not differ significantly from zero at the .05 significance level. The Service;; Food Products, and Finance and Investment categories account for most of the non-significant equations; although the coefficients for the Wholesale industry are fairly low as well.

Highest $r^{2} s$ were encountered in the data classified by size classes; the last three categories in Table I. TABLE II. - ELASTICITIES OF CASH BALANCES WITH RESPECT TO SALES FOR SELECTED INDUSTRIAL SECTORS FOR THREE TAXATION YEARS.

Elasticities of cash with respect to sales are equal to the regression coefficients (b) of the log. regression equations. The elasticities calculated were all positive, with a range of 0.54 to 1.51 over all industrial classes for all three years. Within some industrial classes, consider-
able variation was noted over the three years studied. The Mining and Quarrying class, for example, went from an elasticity of about 1.0 in 1957 to significantly greater than unity in 1958 and 1960. The overall mean elasticities. on the other hand, show little variation over the three years studied and in all cases are quite close to 1.0 (See Table III).

After applying the 't' test (2) to the regression coefficients, it was found that of the 34 statistically significant regression equations; 26 had elasticities which could be considered to be unity. Diseconomies of scale were found in three industrial classes; Construction (1957. 1958); Wood Products (1958); and Mining and Quarrying (1958, 1960). Economies of scale were found in two classes; Petroleum Products (1960); and Wholesale (1957:1958).

TABLE III. DISTRIBUTION OF ELASTICITIES BY TAXATION YEAR.

The purpose of this Table is to show the effect of changing economic conditions in the three years considered on the distribution and average size of the elasticities.

The range of elasticities for each year is as follows:

| 1957 | $0.76-1.38$ |
| :--- | :--- |
| 1958 | 0.54 |
| 1960 | 0.67 |

The only difference noted between the peak years of 1957
(2) The ' $t$ ' test is described in part (B) of this Chapter.
and 1960 and the trough year of 1958 was a wider dispersion of elasticities in the latter year.

## TABLE I.

COEFFICIENTS OF DETERMINATION ( $r^{2}$ ) FOR LOG. AND
ARITHMETIC REGRESSIONS.

1957
Log. Arith. Log. Arith. Log. Arith.
Finance and Invest-
ment
0.79
(0.53)
0.66
(0.24)
(0.07)
(0.002)

Service
Transportation
Construction
Steel
0.82
0.97
0.75
0.85
0.61
0.85

Petroleum Products
0.94
0.98
0.94
0.99
0.76
0.98

Wood Products
0.93
0.98
0.97
0.99
0.98
0.99

Food Products
(0.015) 0.08
(0.156)
0.41
0.15
(0.22)

Mining and -

| Quarrying | 0.67 | 0.58 | 0.86 | 0.64 | 0.85 | 0.68 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| nolesale | 0.37 | 0.66 | 0.43 | 0.62 | $(0.23)$ | $(0.28)$ |
| tail | 0.86 | 0.97 | 0.82 | 0.97 | 0.88 | 0.98 |
| tal Manufacturing | 0.98 | 0.99 | 0.98 | 0.98 | 0.98 | 0.99 |

All Firms (By Income Class)
0.99
0.98
0.98
0.99
0.98
0.99

All Firms -
(By Asset Class)
0.98
0.99
0.97
0.98
0.98
0.99

1. Coefficients in brackets are from equations whose coefficient of correlation did not differ significantly from zero at the .05 significance level.

## TABLE II.

ELASTICITIES OF CASH BALANCES WITH RESPECT TO INCOME FOR SELECTED INDUSTRIAL SECTORS FOR THREE TAXATION YEARS.

|  | 1957 | 1958 | 1960 |
| :--- | :---: | :---: | :---: |
| Finance and Investment | 1.32 | 1.23 | $(0.45)$ |
| Service | $(0.45)$ | $(0.45)$ | 0.94 |
| Transportation | 1.10 | 0.88 | 1.05 |
| Construction | $1.38 *$ | $1.45^{*}$ | $(1.25)$ |
| Steel | 0.77 | 0.83 | 0.92 |
| Petroleum Products | 0.83 | 0.85 | $0.67^{* *}$ |
| Wood Products | 1.24 | $1.27^{*}$ | 1.09 |
| Food Products | $(0.17)$ | $(0.47)$ | $(0.37)$ |
| Mining and Quarrying | 0.98 | $1.47 *$ | $1.51^{*}$ |
| Wholesale | $0.76 * *$ | $0.54 * *$ | $(0.53)$ |
| Retail | 0.83 | 0.81 | 0.88 |
| Total Manufacturing | 1.04 | 1.05 | 0.92 |
| All Firms (By income class) | 1.08 | 1.05 | 1.02 |
| All Firms (By asset class) | 1.09 | 1.06 | 1.04 |

1.     * Denotes elasticities significantly greater than unity at the .05 level.
2. ** Denotes elasticities significantly less than unity at the .05 level.
3. Elasticities in brackets are from regression equations whose coefficient of correlation did not differ significantly from zero at the .05 level.

## TABLE III.

## DISTRIBUTION OF ELASTICITIES <br> (b) BY TAXATION

 YEAR.|  | 1957 | 1958 | 1960 |
| :---: | :---: | :---: | :---: |
| 0.5-0.59 |  | 1 |  |
| 0.6-0.69 |  |  | 1 |
| 0.7-0.79 | 2 |  |  |
| $0.8-0.89$ | 2 | 4 | 1 |
| 0.9-0.99 | 1 |  | 3 |
| $1.0-1.09$ | 3 | 3 | 4 |
| $1.1-1.19$ | 1 |  |  |
| $1.2-1.29$ | 1 | 2 |  |
| $1.3-1.39$ | 2 |  |  |
| $1.4-1.49$ |  | 2 |  |
| $1.5-1.59$ |  |  | 1 |
| MEAN. | 1.03 | 1.04 | 1.00 |
| STANDARD DEV. | 0.2 | 0.33 | 0.2 |

## CHAPTER III

## EVALUATION OF THE RESULTS

(A) SIGNIFICANCE OF THE REGRESSION EQUATIONS.

One of the outstanding differences between the results of this study and those found in other studies is the relatively high number of regression equations which are not statistically significant, i.e., equations whose reoefficients do not differ significantly from zero at the .05 level. Almost $20 \%$ of the regression estimates were found to be non-significant, in comparison with other studies where virtually all of the regressions performed were claimed to be significant. This result can be explained by the fact that there is a basic difference in the manner in which the corporation data in Canada Tax Statistics and its American counterpart
(1) are organized.

As mentioned earlier (Chapter II, Parts A and B), the data for a given industrial class in Canada Taxation Statistics are grouped as to type of business while the data in American Statistics of Income are grouped by asset class. The former method results in some cases in a more random distribution of observations when average cash and sales figures are regressed, which in turn results in relatively lower $r$ and $r^{2}$ figures.

Further, the grouping of heterogeneous types of firms into one industrial class also results in a random distribution of observations. Many dissimilar firms are aggregated under the Food Products and Service groups in Canada Tax

[^0]Statistics and low $\mathbf{r}^{2}$ s result when regressions are performed using statistics from these categories. Industrial classes composed of more homogeneous types of firms, Wood and Paper Products for example, yielded more significant results. The high r - squares obtained from regressions performed on data classified by size class can be attributed to this method of classification. Further, the similarity in the results for all Firms grouped by Asset Class and all Firms grouped by Income Class can be attributed in part to the fact that these data are taken from approximately the same sample of firms. In comparing the $r^{2}$ figures of the $\log$ and the arithmetic equations for a given year, it can be seen that for most industries the arithmetic form gives a better approximation of the relationship between cash and sales. The relatively higher leviels of significance of the estimates of the ordinary linear equations indicate that hypotheses which suggest a roughly linear relationship between cash and sales should be given more attention than the square-root hypothesis. (B) SALES ELASTICITIES

In the significant regression equations, elasticities of cash with respect to sales ranged from 0.54 to 1.51. This range is considerably greater than those of other crosssection studies, particularly Meltzer's in which a range of 0.88 to 1.27 was reported and Vogel and Maddala's where a range of 0.929 to 1.077 was given. The relatively wide range of elasticities found here may again be explained by the nature of the data employed. Sales elasticities obtained
from the 'structured' (2) data in this study had a much narrower range ( $0.92-1.09$ ) than those obtained from the data classified by industrial types. It is most likely that the elasticities in other studies (and from certain data in this study) are very close to 1.0 because of the structured nature of the data employed. As Vogel and Maddala (1967) have observed.

```
* .... although regressions run with asset -
size observations for any industry or year
yield high R - Squares and a sales elasticity
for cash close to unity, it cannot be con-
cluded that liquid asset demand has been
explained or that there are no economies or
diseconomies of scale in money holding.
These results can be attributed to the
classification of the data and mis-
specification from excluded variables."
```

To examine the effect described by Vogel and Maddala, average cash and sales figures from each size class were plotted for several classes of data grouped by asset or income classes. Average cash and sales figures for all firms grouped by Income class for 1960 are given as anexample in Table IV. It was found in all cases that observations from the three largest size classes are given much more weight than observations from smaller size classes when average figures are plotted, and it was thought at first that high r - squares and unit elasticities were caused by this weighting. To test this hypothesis, regressions were performed for all data grouped by asset or income class with first the observations from the largest size class then the second largest class omitted,
(2) 'Structured' refers to data which are classified by asset or income size classes.
to see if this would result in different $r$ - squares or elasticities. It was found, however, that even with the two largest size class observations removed, in each case the $r$ - squares remained high and the elasticities were still close to unity as in the example below:

STATISTICS FROM ALL FIRMS GROUPED BY
ASSET CLASS - 1960

Statistics obtained when observations from all 9 size $0.98 \quad 1.04$ classes were regressed Statistics obtained when largest class was omitted 0.98 1.04

Statistics with two largest size classes omitted

Since the statistics obtained from regressions performed with the two largest size classes removed are not appreciably different than those obtained by regressing all size class observations the effect described by Vogel and Maddala can not be attributed to the greater weighting given to large size classes.

It is likely that high $r^{2}$ and unit elasticities are caused by another effect which results when data are grouped by size class. Table IV shows as an example the average cash, sales and asset figures for each size class for Firms grouped by income class, 1960. It can be seen that the percent change between size classes in the average figures are of roughly the same order of magnitude. Since the percent changes are roughly the same, unit elasticities will result if logs of the average figures are regressed.

TABLE IV.
DATA FROM ALL FIRMS GROUPED BY INCOME CLASS.

| 1960 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class <br> (\$ thousand) | Average | Cash | Average | Sales | Average | Assets |
| Under 5 | 0.004 | $(50.0)$ | 0.127 | (69.2) | 0.109 | (64.2) |
| 5-9.9 | 0.006 | (100) | 0.215 | (74.0) | 0.179 | 46.3 |
| 10-24.9 | 0.012 | (100) | 0.375 | (106.6) | 0.262 | (120.2) |
| 25-49.9 | 0.025 | (100.0) | 0.775 | (85.9) | 0.577 | (112.1) |
| 50-99.9 | 0.050 | $(112.0)$ | 1.441 | (97.9) | 1.224 | (113.4) |
| 100-249.9 | 0.106 | (126.4) | 2.852 | (83.2) | 2.613 | (86.5) |
| $250-499.9$ | 0.240 | $(58.7)$ | 5.277 | (85.0) | 4.874 | $(184.0)$ |
| 500-999.9 | 0.381 | $(163.7)$ | 9.671 | (149.2) | 13.845 | (146.7) |
| 1,000-4,999.9 | 1.005 | (266.0) | 24,107 | (472.0) | 34.157 | (432.5) |
| Over 5,000 | 3,679 |  | 137.91 |  | 181.90 |  |

Figures in brackets are percent change between statistics.

There does not appear to be any factor common to industrial classes which were shown to have economies of scale. While Sales elasticities significantly less than 1.0 were shown to exist in Construction, Wood and Paper Products and Mining, no common pattern is discernible in such statistics as average industry asset size, cash balances or sales volumes, shown in the figures below: (Figures are in Millions)

|  | Average <br> Assets | Average <br> Cash | Average <br> Sales. |
| :--- | :--- | :--- | :--- |
| Mining and Quarrying | 4.598 | 0.138 | 1.236 |
| Wood and PaperProducts. | 0.703 | 0.017 | 0.659 |
| Construction | 0.282 | 0.015 | 0.440 |

The same lack of an overall pattern is seen in the aggregate statistics of industries showing diseconomies of scale. A detailed examination of the pattern of payments, the capital intensity and the asset size of the various subclasses of firms in each industry might reveal some underlying relationships.

While only limited information is available from aggregate statistics, some similarities in capital intensity are found in the subclasses of the Wood and Paper Products industrial group shown in Table V. In 1958 this industry displayed diseconomies of scale in cash holdings.

Having examined the $r$ - squares and elasticities found in our investigation; it might be useful to compare them with the findings of two other cross section studies.
(1) THE NELTZER STUDY. The formulation of the regression estimates used in this thesis is essentially the same as Meltzer's since both are log. Cash $=\log a+b \log$ sales.

TABLE V.

STATISTICS ILLUSTRATING THE PATTERN OF PAYMENTS AND DEGREE OF CAPITAL INTENSITY OF THE SUBCLASSES OF THE WOOD AND PAPER PRODUCTS INDUSTRIAL GROUP. 1958

Cash as \% Receivables as \% Building Payables as \% Total Assets Total Assets and Equip- Total ment as \% Liabilities. Total Assets

| Plywood Mills | 4 | 16 | 40 | 11 |
| :---: | :---: | :---: | :---: | :---: |
| Sawmills | 1 | 6 | 48 | 4 |
| Furniture | 3 | 21 | 38 | 13 |
| Paper Bags and Boxes | 5 | 11 | 48 | 5 |
| Pulp and Paper Mills | 2 | 3 | 58 | 3 |
| Misc: Wood Products | 2 | 18 | 41 | 10 |
| Misc: Paper Products | 4 | 12 | 52 | 8 |

For his estimates Meltzer uses 126 individual regressions (14 industries in 9 different years) with asset class observations and finds that the demand for money is a function of sales, linear in logarithms and unit elastic. In each of the nine years and in 12 of the 14 industries the mean value of the elasticities was greater than 1.0. Elasticities of less than 1.0 were found in 26 cases largely concentrated in particular industries. Meltzer found some evidence that changes in business conditions raise or lower the estimates of the sales elasticities of cash balances. Years in which the business cycle reached a trough, 1938, 1946, and 1954, all had mean elasticities which were relatively high. Conversely, years in which business conditions reached a peak had relatively low mean elasticities.

The results of this study do not entirely agree with Meltzer's findings in spite of the fact that the average elasticities for all industries in the years examined were approximately unity (see Table III). The economies and diseconomies of scale found here are too significant to allow us to state, as Meltzer does, that the "unitary" hypothesis explains the demand for money.
(2) THE VOGEL AND MADDALA STUDY. These authors contend that both the Baumol - Tobin and the Meltzer models are essentially modifications of the central theory that marginal rates of return; though constrained by wealth, discounted by risk, and influenced by payments patterns and costs of financial transactions; are equated for various assets. Thus attempts
to distinguish unequivocally between the two models, even on theoretical grounds are somewhat beside the point. Further, the authors raise the empirical question of how the use of only the sales variable can allow statistical descrimination among the various models. (3)

In view of the above limitations of cross section analyses, Vogel and Maddala concentrate on answering specific empirical questions of economies of scale in cash balances;' substitution between money and government securities; and the influence of interest rates and other variables. Regression estimates were made for 14 asset size classes within 16 industries using the form:

$$
\log Y=a+\log \text { Sales }+ \text { Dummy variable }
$$

where $Y$ is alternately cash and government securities and Dummies are introduced alternately for asset size class and industrial class.

The sales elasticities of cash were found to be about umity ( 0.995 ) when industry dummies were used in the regression equation, about the same as those reported by Meltzer and obtained from 'structured' data in this study. When dummies were introduced which increased in size with increasing asset classes, the elasticities were substantially reduced to about 0.281. From this latter result the authors conclude that in some other studies the influence of asset size class differences have been incorrectly attributed to
(3) The sales variable is used as an approximation for both transactions and wealth in various studies.
the sales variable, and state further that a strong argument can be made for economies of scale in money demand.

The empirical results of this thesis are not directly comparable to those obtained by Vogel and Maddala because of the different formulations employed. It is possible to speculate, however, that if size class dummy variables had been used in the regressions performed using structured data; the resulting elasticities may have been considerably lower.
(C) EFFECT OF CHANGES IN ECONOMIC CONDITIONS.

Although interest rates are assumed to be constant in a cross section estimate; changing economic conditions between years can be expected to affect an industry's elasticities. One might expect that the size of the elasticities would vary inversely with interest rates since firms would be encouraged to shift funds from cash balances into short-term securities when yields are increasing; and to add to cash balances if yields decrease.

With regard to individual industries, the elasticities tended to vary considerably over the three sample years with no distinct pattern emerging in relation to peak and trough years. This result is probably due in part to the fact that although Taxation Statistics publishes data for a given calendar year, there is a dispersion of corporate year-ends. throughout the year; so that the effect of changes in prevailing interest rates is less noticeable. Further, the effect of interest rate changes on a firm's mix of cash and marketable securities varies with the size of the firm.

Firms of large asset size with large holdings of cash can be expected to react more noticeably to changes in the interest rates on marketable securities; while firms with small cash balances may not find it economical to invest in cash substitutes. The data for a given industrial class is an average of the cash and sales figures for firms of many different asset sizes, and changes in these figures caused by prevailing economic conditions are hidden by the averaging process.
(D) COMPARISON OF CROSS SECTION AND TIME SERIES RESULTS

It has been suggested by several authors (Grunfeld, 1961, and Kuh, 1959) that the findings of a cross section study should not be accepted at face value as an explanation of behaviour. Kuh suggests that biases from variables excluded from the approximation procedure will have strikingly different effects on time series and cross sections, and that as a result, cross section estimates cannot be taken as a valid approximation without a comparison with time series results from a uniform group of firms. While data found in Canada Taxation Statistics is taken from a sample of firms which changes from year to year, it is interesting to compare the time series performance of the cash/sales ratios with the cross section elasticities. This information is given for statistically significant equations in Table VI.

If the time series ratios are to support cross section elasticities which indicate economies of scale one would expect that if an industry's sales rise from one year to
the next; the cash/sales ratio should decline. If diseconomies exist; an increase in sales should be accompanied by an increase in the cash/sales figure. Third; if elasticities are approximately unity, the cash/sales ratio should remain unchanged.

Referring to Table VI, it can be seen that in six out of ten cases cross section results are generally consistent with the time series ratios. In two cases, Mining and Petroleum Products; time series ratios show a trend opposite to that suggested by the elasticities and in two other classes, Steel and Finance the comparison is inconclusive. One obVious contradiction, for example; is seen in the results for Mining in 1958 where an elasticity of 1.47 was found, indicating diseconomies. While sales increased from 1957 to 1958; the cash/sales ratio actually decreased indicating economies of scale. The conflicting results of the two estimation methods can be attributed to the biases mentioned previously which arise from the exclusion of dynamic variables such as interest rates. The main reason estimates differ is that time series typically reflect long run adjustments whereas cross sections reflect shorter run reactions so that dynamic specification errors that tend to bias time series results downwards are less observable in cross sections (Kuh, 1959).

The effect of changes in prevaliing economic conditions is more noticeable in the time series ratios than in the cross section results. The cash/sales ratio for all manu-

- 34 -

TABLE VI.

COMPARISON OF CROSS SECTION ELASTICITIES WITH TIME SERIES
CASH/SALES RATIOS.

|  |  | 1957 | 1958 | 1959 | 1960 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Finance and Investment | Elasticities | 1.32 | 1.23 |  | (0.45) |
|  | Cash/Sales | 0.292 | 0:247 | 0.234 | 0.29 |
|  | \% $\triangle$ Sales | +17.0 | +8.7 | +10.3 | -13.7 |
| Transportation | Elasticities | 1.10 | $0.88$ |  | 1.05 |
|  | Cash/Sales | 0.067 | $0.066$ | 0.053 | 0.056 |
|  | \% $\triangle$ Sales | +7.5 | 0 | +12.0 | +3.4 |
| Construction | Elasticities | 1.38 ${ }^{1}$ | 1.45 |  | (1.25) |
|  | Cash/Sales | 0.029 | 0.036 | 0.035 | 0.028 |
|  | \% $\triangle$ Sales | +15.3 | +5.3 | +5.5 | +4.3 |
| Steel | Elasticities | 0.77 | 0.83 |  | 0.92 |
|  | Cash/Sales | 0.029 | 0.043 | 0.032 | 0.035 |
|  | \% $\triangle$ Sales | +9.1 | -7.6 | +12.8 | -6.5 |
| Petroleum | Elasticities | 0.83 | 0.85 |  | $0.67{ }^{2}$ |
|  | Cash/Sales | 0.024 | 0.024 | 0.020 | 0.014 |
|  | \% $\triangle$ Sales | -33.0 | -4.1 | +15.0 | -6.0 |
| Wood and Paper | Elasticities | 1.24 | 1.27 |  | 1.09 |
|  | Cash/Sales | 0.018 | 0.025 | 0.020 | 0.024 |
|  | \% $\triangle$ Sales | -0.3 | -4.8 | +24.1 | -8.9 |
| Mining | Elasticities | 0.98 | 1.47 |  | 1.51 |
|  | Cash/Sales | 0.133 | 0.112 | 0.119 | 0.105 |
|  | \% $\triangle$ Sales | -1.7 | +1.7 | +7.4 | +3.6 |
| Wholesale | Elasticities | 0.76 2 | 0.54 |  | (0.53) |
|  | Cash/Sales | 0.016 | 0.016 | 0.016 | 0.015 |
|  | \% $\triangle$ Sales | +3.0 | 0 | +6.0 | +3.0 |
| Retail | Elasticities | 0.83 | 0.81 |  | 0.88 |
|  | Cash/Sales | 0.017 | 0.018 | 0.017 | 0.016 |
|  | \% $\triangle$ Sales | +9.0 | +5.0 | +9.0 | +1.0 |
| Total <br> Manufacturing | Elasticities | 1.04 | 1.05 |  | 0.92 |
|  | Cash/Sales | 0.028 | 0.034 | 0.027 | 0.025 |
|  | \% $\triangle$ Sales | -0.4 | -1.5 | +8.5 | +3.5 |

1. Denotes Diseconomy of Scale
2. Denotes Economy of Scale
3. Elasticities in brackets are from regression equations whose coefficients of correlation did not differ significantly from Zero at the . 05 level.
-facturing firms rose from . 028 to . 034 from 1957 to 1958. The increase in this ratio can be explained by a decline in the yields of marketable securities accompanying the decline in economic activity in the latel95? and 1958. This effect is more noticeable in the time series results since this form of estimation better reflects the influence of dynamic variables such as interest rates.

## CHAPTER IV.

CONCLUSIONS.

The results of this study have shown that, to a first approximation, there are economies and diseconomies of scale in the demand for cash in some sectors of Canadian industry. Although the overall average elasticities for all sectors in the years tested were approximately unity, cross section and time series results for individual industrial classes have shown that in several cases cash balances do not vary in proportion to sales.

Because of the wide range of the elasticities obtained, the results described here cannot be said to support either the Baumol - Tobin or the Meltzer hypothesis. Few economies of scale of the order predicted by the Baumol - Tobin hypothesis were found in the cross section estimates, although there is some evidence in the time series ratios that some elasticities may be over estimated or hidden by the cross section approximation. Because of the difficulties in assessing the extent of the bias inherent in the approximation methods, the main conclusion stated above must be qualified as a first approximation.

The existence of economies and diseconomies of scale suggests that Meltzer's 'unitary' hypothesis does not explain the demand for cash for the economy as a whole.

The results of the study do not disclose diseconomies of scale of the magnitude predicted by Friedman. Further, changes in economic conditions did not affect the overall average income elasticities of the demand for cash in the sectors studied.

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## APPENDIX I.

AN OUTLINE OF THE DERIVATION OF BAUMOL'S
CASH INVENTORY MODEL.
By keeping part of its assets in the form of cash a firm incurs costs similar to those associated with an inventory of any other asset, i.e. holding and ordering costs. Baumol applies the familar optimum lot size inventory model to minimize the costs involved in holding cash.

The focus of the author's analysis is on the transactions demand for cash dictated by rational behaviour; which means holding the cash balance that will do the job at minimum cost. Baumol first assumes a state in which transactions are perfectly foreseen and occur in a steady stream, so that in the course of a given period a firm will pay out $\$ T$ continuously. Cash is obtained either by borrowing or selling investments; in either case the interest cost (or interest opportunity cost) is \$1 per dollar per period. Cash is withdrawn in lots of $\$ C$ evenly spaced throughout the year and each time a withdrawal is made a fixed "broker's fee" of $\$ b$ must be paid ( $b$ includes all noninterest costs of borrowing or making a cash withdrawal). In this situation any value of $C \leq T$ will enable the firm to meet its payments equally well provided withdrawals are made often enough. Thus, the firm will make T/C witharawals over the course of the year at a total cost in "broker's fees" of $b T / C$. Since cash is spent in a steady stream between withdrawals; the average holdings
are \$C/2.
The annual interest cost of holding cash will then be $1 \mathrm{C} / 2$. Thus the total cost of using cash for the year will be
(1) $\quad \frac{b T}{C}+\frac{1 C}{2}$

To determine the optimum value of $C$ set the derivative of (1) with respect to $C$ equal to zero to obtain
or (2) $\quad C=\sqrt{\frac{2 b T}{i}}$
This model applies to two cases: Where cash is obtained from invested capital and where cash is obtained from borrowing in anticipation of future receipts. One other case is possible where receipts precede expenditures. Here, the firm has the option of withholding some or all of its receipts from investing and simply keeping the cash until it is needed. Baumol shows that as in the two previous cases the optimum cash balance after the initial cash holding is used up will vary with the square root of the volume of transactions. The optimum amount to be withheld from investment is shown to be

$$
R=C+T \quad\left[\frac{k+k_{d}}{1}\right]
$$

where $k_{W}$ and $k_{d}$ are constant terms associated with the "broker's fees" for withdrawing and investing cash respectively. The author states that even though $R$ varies more in proportion to $T$ than does $C$, the general nature of
the results is unaffected.
After outlining the model Baumol concedes that (2) is an oversimplification and that some of the underlying assumptions, namely that the firm's disbursements are constant over time, that there is one relevant interest rate and that the "broker's fee" is constant or varies linearly with the sum involved, may be invalid. He does propose, however, that (2) may be conceptually accurate with respect to the economy as a whole and that the demand for cash rises less than in proportion with the volume of transactions, so that there are, in effect, economies of large scale in the use of cash. Equation (2) also confirms that the rational transactions demand for cash will be $>0$.


[^0]:    (1) Internal Revenue Service, Statistics of Income: Corporation Income Tax Returns Washington, U. S. Treasury)

