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DISCRETE HEDGING IN INSURANCE RISK MANAGEMENT

by

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ABSTRACT

Based upon the Black-Scholes option pricing model, Schwartz developed an equilibrium pricing definition of the equity-linked life insurance contract with an asset value guarantee. Under the conditions of this contract, the beneficiary may elect to receive the value of a reference portfolio of securities or a minimum guaranteed amount, whichever is greater. In this sense, the contract is synonymous to investment in a mutual fund and a term insurance policy.

The guarantee provision, however, gives rise to nondiversifiable risk. In the event of a general market collapse, the company becomes simultaneously liable for the guarantee on all mature contracts. Such an eventuality could prove to be disasterous. The equilibrium model proposes a hedging strategy which eliminates the probability of this type of a loss. At any point in time, the benefits of the contract may be viewed as the present value of the guarantee plus the value of a call option on the reference portfolio. Conversely, it can also be stated as the present value of the reference portfolio plus the value of the put option. Since the call option is sold short, a fully hedged position may be established by the appropriate investment in the reference portfolio. Maintenance of this position implies that no gains

or losses will occur.

In practise, however, continuous hedging is impossible because of transaction costs. Adoption of a policy of periodic revision to re-establish the hedged position will result in gains and losses. Exposure to risk, therefore, is not eliminated.

This dissertation deals with the problem of risk exposure resulting from a discrete revision strategy. Through the employment of simulation techniques, the impact of various revision strategies and transaction cost levels on the losses is examined. As a basis of comparison, a naive strategy was also developed. That is, under the naive strategy, the company buys the market portfolio with the premium and holds it until maturity. The case under consideration is that of the single premium contract with a known maturity date.

The results of the analysis establish the dominance of the periodic revision strategy over the naive. Furthermore, for lower transaction cost levels, the dispersion of losses is reduced as the number of revisions is increased. Although the simulation model does not provide an optimal solution, it does provide the framework for the establishment of a management strategy which is consistent with the firm's perception of acceptable risk. It is hoped that the proposed strategy will find acceptance not only by the insurance industry but also in the areas of mutual and pension fund management.

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Introduction

With the emergence of equity linked life insurance contracts in Canada and the United Kingdom as a viable alternative to straight insurance, a number of important issues arise with respect to the value and the risks associated with this type of a contract. It can be argued with some justification that this type of a contract is not really a life insurance instrument, but a security or asset in the case of the single premium contract, or an investment schedule in the case of the periodic premium contract. This controversy has only recently been resolved in the United States, with the Securities Exchange Commission establishing the procedures for the registration of the sale of these instruments. To January 1976, only the Equitable Life Assurance Society has registered with the Securities Exchange Commission and published its intention to sell the product in a limited market.

In its simplest form, the equity linked life insurance contract may be viewed as an instrument with a fixed terminal asset value, plus a premium, the magnitude of which depends on the performance of a reference portfolio of securities. The insurance component is the fixed terminal asset value, or a guarantee by the company that in the event of a considerable stock market decline, the beneficiary will still be able to elect to receive a guaranteed amount, which is determined at the creation of the contract. Obviously, the lower bound of the premium is zero. Put another way, for the investor, the benefits of the contract is either the guaranteed amount, or the value of the reference portfolio of securities, whichever is greater. The reference portfolio may be any portfolio of securities or mutual fund, or in the extreme, a portfolio consisting of one stock. Under equilibrium conditions, only the variance rate of the reference portfolio is of importance, not the return, for reasons which become evident in the discussion in Chapter 2. There is no requirement that the company invest all the premiums in this portfolio, and in fact it has been shown that it would be suboptimal to do so. (11) Briefly, the investor purchases the present value of the guaranteed amount plus the right to exercise his option with respect to the reference portfolio at the termination of the contract, should its value exceed the guaranteed amount.

The company, on the other hand, is faced with a different set of problems. The most obvious is the pricing of the equity linked life insurance contract. The second is the management of the funds invested in the reference portfolio. Thirdly, since a portion of the premium is invested in securities, the company must develop a strategy to minimize the probability of bankruptcy or disaster in the event of a market collapse. Portfolio theory suggests that most of the unsystematic risk associated with a security may be diversified away by selecting a large enough portfolio, or that it may be completely eliminated by buying the market portfolio. From the point of view of the insurance company, however, neither of these alternatives seem satisfactory because of the guaranteed amount. In the event of a market collapse, the beneficiary of the contract will obviously elect to receive the guaranteed amount, as he would in all cases where this amount exceeded the value of the reference portfolio. It should be noted, however, that disaster occurs only if the value of the reference portfolio plus the amount invested in the risk free asset is less than the guaranteed amount. The objective function of the company, therefore, may be viewed as the development of a strategy to minimize the probability of this type of bankruptcy.

The theoretical framework for such a strategy has been proposed by E. Schwartz in his doctoral dissertation. In a very general sense, he shows that the valuation of the equity linked life insurance contract is closely related to the option pricing problem. This interpretation gives rise to a hedging strategy. Theoretically, if a fully hedged position is maintained by continuous adjustment and the appropriate price is charged for the contract, then no gains or losses can occur. At a practical level, this strategy is not feasible because of transaction costs. A discrete hedging policy may, however, prove to be attractive, in the sense that the probability of the disaster losses discussed previously, may be reduced.

The objective of this thesis is to examine the hypothesis that the probability of disaster losses may be reduced by adopting a discrete hedging policy in the management of equity linked life insurance contracts.

Although numerous alternatives exist, the analysis will focus entirely on the single period, single premium contract. In order to test the hypothesis, computer simulation techniques were adopted.

The development of the analysis conforms to the following outline. Chapter 1 provides a brief discussion of the evolution of equity linked life insurance contracts. A review of some of the literature pertinent to this topic is also presented in this section. Chapter 2 focuses mainly on the Black-Scholes option valuation formula and the relevant sections of the Schwartz dissertation, which in effect forms the foundations of this analysis. Chapter 3 contains a discussion of simulation and the simulation model employed. Chapter 4 summarizes the findings of the thesis and provides recommendations for further analysis. Chapter 5 is a restatement of the problem and a brief conclusion. An appendix has been provided for the relevant statistics and the simulation program.

Chapter 1

1.1 Background

Interestingly enough, the first life insurance contracts marketed by companies, were infact term insurance policies, as opposed to whole life instruments. Although the reason for this phenomenon is somewhat unclear, it has been argued that a general misunderstanding of the theory of insurance or the infant state of the theory caused companies to take this position. Briefly, term insurance, as the name implies, provides coverage for a specified period. In this sense, term insurance is concerned primarily with the probability of accidental death and natural death within the specified period of time, not with the eventual certainty. As such, it may be viewed as a bet between the company and the insured, the company taking the position that the individual will survive the contract period, the latter that he will not.

The whole life contract, on the other hand, recognizes the eventuality of death as a certainty. The company assumes the liability for the face value of the contract at its creation, and therefore must build up a reserve during the life of the contract in order to meet its obligation at its termination.

The premiums associated with these contracts vary considerably. Term insurance premiums tend to be lower than whole life, reflecting the fact that the company may not be liable for the face value of the contract, and the reality that aged individuals are excluded from this type of an instrument.

With the evolution of nonforfeiture clauses in the contracts and the inclusion of loan provisions, the whole life policy became a very versatile instrument from a marketing point of view. The value of the loan option is self evident, the nonforfeiture clause can be interpreted as the surrender value of the contract if it is terminated before maturity, and of course the insurance aspect needs no explanation. Although intuitively appealing as a package, serious doubts have been raised concerning the real value of these benefits, given the investment required in the policy. Furthermore, the long run average rate of return on whole life contracts has been about 3% to 5%, suggesting that any saving or investment motive which may have been a part of the decision making criteria, was ill founded.

The third type of policy, the endowment contract, offers some interesting features to the insured. In the most general sense it may be viewed as a savings plan with a guarantee attached to it. This type of insurance attempts to incorporate certain features of both the term and whole life contracts. The policy generally provides for the payment of the face value to the beneficiary in the event of premature death of the insured, (ie. within the specified time of the contract). If the insured is alive at the termination of the contract, the insured sum is still paid by the company. In essence then, it is simply an insured savings plan.

The rapid increase in inflation rates since 1945 and the sharp fluctuations have caused policy holders and for that matter insurance companies to review the terminal benefits that current policies purport to yield. As mentioned previously, a 3 - 5% return on a whole life policy is anything but satisfactory, given that inflation rates have been as high as 13% per year. Those on fixed incomes, or individuals cashing in endowment type policies have incurred considerable losses due to the spiraling inflation rates. It can be argued that insurance companies should have done very well, repaying uninflated dollars with inflated ones, but the returns declared by companies does not seem to reflect this. It remains to be seen what impact current efforts to reduce the rate of inflation will have on the contracts signed in the last five or six years.

1.2 Risk

Although the risk of death is often aluded to by insurance salesmen as a justification for the premiums charged, in a theoretical sense this arguement is inconsistent. Basically, the theory of insurance is synonymous to portfolio theory. The pooling of independent risk implies that the unsystematic portion, or the risk unique to a particular case may be diversified away. Consequently, only the risk inherent in the industry remains. The key to the argument is the independence factor. If this criterion is observed, then the number of deaths per period may be calculated with considerable accuracy, assuming that there is a sufficienty large enough sample. The only thing that remains, therefore, is the matching of the liquidation of interest earning assets and claims against the company, as they become due. If the theory did not hold, then there could be no justification for the liability exemption clauses such as war, natural disasters, act of God, etc., included in all policies. In essence, these phenomena violate the independence assumption.

As the above argument implies, under the conditions of the policies discussed thus far, the insurance company assumes all the risk. Primarily because of the rapid increase in inflation and the general dissatisfaction of policy holders with the subsequent decline in the value of the policy, insurance companies began to market an equity linked instrument first in the Netherlands in the early fifties, then later in Britain and Canada, and now in the United States. The primary difference between this type of policy and the conventional instrument is that a portion of the risk is borne by the insured. As discussed previously, the benefits of the contract may be viewed as the value of a reference portfolio of securities, or a guaranteed amount, whichever is greater. The theory behind this approach is that the individual is willing to assume a part of the risk because of the belief that securities provide a hedge against inflation. Since the risk structure is different from the conventional approach, the valuation of this instrument and the determination of the premium structure presents some unique problems. In a narrow sense, the policy is similar to buying a mutual fund and term insurance as a complement, from the point of view of the insured. It should be noted that this type of instrument provides the departure point from generally accepted insurance theory. The risk associated with the asset value guarantee can not be diversified away. A general market decline will result in a catastrophe as the guarantee will be exercised under all maturing contracts.

Although a considerable number of papers have been written on the subject of equity linked life insurance policies, most of them pursue what may be defined as a naive approach. That is, most of them focus on the problem of establishing adequate reserves, without considering all the parameters of the problem. In the traditional sense, this meant average or mean reserve requirements. If companies enjoyed the same degree of experience with the equity linked products as they do with whole life contracts, then such a strategy may well be acceptable to a certain extent. This, however, is not the case.

1.3 Literature Overview

Squires, (16) in a 1974 paper correctly identifies the inadequacies of existing models attempting to explain stock market behavior. Unfortunately, his assumptions about the market are also open to criticism, which tend to negate his conclusions. In particular, it will be shown that in the case of single premium policies, it becomes irrelevant when the policy is effected. His assertion is that if the policy is effected when the market is at its peak, then the company is subjected to substantial risks during the subsequent trough. This assertion clearly ignores the random walk or efficient market hypothesis. If the random walk is an accurate description of price behavior, of which there is considerable evidence, then a "market peak" can only be identified in retrospect. Put another way, if the random walk hypothesis holds, then it is impossible to determine if today's stock price is at its "peak" because in order to accomplish that, tomorrow's price must be known. This is not possible because of the condition:

$$P (X_t > 0) = P (X_t < 0) = .5$$

that is, the probability of a positive price change equals the probability of a negative price change (ie. 0.5). If such is the case, then the best estimate of tomorrow's price must be today's price. This can be derived by algebra. In effect then, since tomorrow's price is not known for certain, troughs and peaks cannot be identified. The major point to note however, is that his approach completely ignores the implications of the hedging strategy.

The major contributions to the understanding of the nature of the equity linked life insurance contract were simulation models developed by Turner (18), DiPaolo (6), and Kahn (8). Their work may be viewed as the point of departure from the conventional interpretation of the valuation problem. Generalizing, their approach may be interpreted as aggressive, whereas the traditional position is defensive. As Turner (17) states, in reference to a paper by Sidney Benjamin, " the stated approach to valuation, that is, to the establishment of additional reserves for asset value guarantees, is to determine on each valuation date any reserves, which, in the opinion of the actuary, would be required considering the nature of the guarantees provided and the financial situation at that time". But clearly, the most important element of the decision making criteria is lacking in this definition, that is, the expected performance of the reference portfolio, or in a more general sense, the expected performance of the market. Since the basic theory of the equity linked product is that the investor is willing to assume a portion of the risk in order to attain a higher yield, this must be the least desirable alternative from both points of view. The reserve requirement, therefore, must be a function of the expected rate of return on the market, and the probability of attaining an equity position greater than the amount of the guarantee. As pointed out in the various critiques of the aforementioned papers, there is a general reluctance on the part of actuaries to accept the current theory of capital asset pricing. This may account for the hesitation observed in viewing the instrument as an option pricing problem, as opposed to a reserve problem.

Turner (17) basically views the problem in three stages. First, he recognizes that some conclusions must be made concerning the nature of the probability density function of security returns. Second, he focuses on the evaluation of the net risk premium of an asset value guarantee at the end of the contract period, given the equity linked instrument. Thirdly, he analyzes the sensitivity of the net risk premium to changes in underlying parameters, such as investment period, charges against the return on equity, taxes, and decrements in mortality and withdrawals. The overall implications of his analysis is the presentation of a framework which forces actuaries into viewing the problem of equity linked contracts with an asset value guarantee in a more quantitative or analytic setting than the traditional approach.

DiPaolo relies on Monte Carlo techniques to generate or simulate security trends, which are then utilized to evaluate the adequacy of the investment risk premium charged, for an equity based endowment policy. His basic assumption is that a risk premium is deemed to be adequate if the probability of the risk fund being in a state of ruin is small after the last contract matures. Ruin, in this sense, occurs if the risk fund incurs losses after the termination of the last contract. As is the case with most simulation models, his does not provide an optimal solution rather a distribution of the various outcomes. Nevertheless the model does recognize the fact that the rate of return on the securities is an inseparable factor in the management of the funds involved.

Kahn's approach to the problem is similar to that of Turner and DiPaolo. He also utilizes simulation and analytic techniques to generate a market trend or return and uses the results to project various insurance alternatives. In general, his findings show the extreme sensitivity of earnings of a variable life insurance company with respect to investment performance. He also shows that the cost of a minimum death benefit guarantee varies widely with investment performance.

In general, the significance of these analyses lies in the interpretation or definition of the problem. As stated previously, the departure from the traditional defensive position that the overriding factor is the establishment of reserves, must be viewed as a breakthrough. All three authors explicitly recognize that the critical issue is the performance of the investment portfolio. Under their assumption, therefore, the probability of ruin is a function of the probability of a sustained market decline or a general collapse. The following chapter will show that in a theoretical sense it is possible to view the problem in such a framework that the probability of ruin may be completely eliminated through a process of hedging. In order to accomplish this, a comprehensive overview of the Black-Scholes option pricing model will be presented, followed by a discussion of the major issues of the Schwartz dissertation. This will provide the necessary background for the simulation model employed in this analysis.

Chapter 2

The Equilibrium Model

2.1 The Pricing of an Option

In the most general sense, an option may be defined as the right to buy or sell an asset, subject to certain conditions, within a specified period of time. The price paid for the option is generally referred to as the striking or exercise price. The last day on which it can be exercised is the maturity or expiration date. There are basically two types of options; the European, and the American. An American option may be exercised any time up to and including the maturity date, whereas the European can only be exercised on the specified future date.

The simplest kind of option is the right to buy a single share of common stock, or call option. Its counterpart, the put option, is the right to sell one share of common to another party. It can be readily seen that a number of combinations of the two basic option types are possible, depending on the objectives of the individual. For the purposes of this portion of the analysis, however, the focus will be on the call option.

Clearly, a relationship must exist between the value of the option and the price of the underlying security. It can be expected that the higher the price of the stock, the greater should be the value of the option. If the stock price is considerably greater than the exercise price, then, the option will probably be exercised. More formally, at this point, the value of the option will be approximately equal to the price of the stock minus the price of a pure discount bond that matures on the same date as the option, and has a face value equal to the striking price of the option. That is:

$$VO_t = PE_t - B(e^{-rt^*}) \quad (2\text{-}1)$$

Where VO_t is the value of the option at time t; PE_t the price of the security at time t; and the expression $B(e^{-rt^*})$ the price of the discount bond. Since the probability of exercising the option becomes very high as the maturity date approaches (ie. as per the above assumption), the process may be viewed as a deferred purchase plan. The value of the discount bond at the creation of the option represents the amount which must be invested in the risk free asset in order to insure that sufficient funds are available to exercise the option at maturity. In essence, this is the same as a deferred plan.

On the other hand, if the stock price, PE is considerably less than the striking price, the option will probably expire, so that its value should be near zero. Furthermore, if the expiration date is far off, then the price of the discount bond will be low, implying that the option value will be approximately the same as the stock price. If the expiration date is near, then the option value should approximately equal the difference between the stock price and the exercise price, or zero if the stock price is less than the striking price. Normally, it can be expected that the value of the option should decline, if there is no change in the stock price. Within this framework it can be expected that the option is more volatile than the stock. That is, for a given percent change in the price of the stock, a larger percent change will occur in the value of the option, given that maturity is held constant. It should be noted, however, that the relative volatility of the option is not constant as it depends on stock price and maturity.

2.2 The Black-Scholes Valuation Formula

The origins of the Black-Scholes valuation formula may be found in the works of Sprenkle (1961), Ayres (1963), Samuelson (1965), etc. These earlier works dealt primarily with the valuation of warrants, but for all intents and purposes, the theory is equally applicable to other options. The major problem with these earlier formulations is the fact that some of the parameters were left undefined. The key assumption that they do utilize, however, is a conclusion of the work of Thorpe and Kassouf (1967). They note that the valuation of the warrant hinges upon the ratio of stock options to shares needed to create a hedged position by going short in one security and long in the other. As Black and Scholes point out, what they failed to recognize was the fact that the expected rate of return on such a position must be equal to the risk free rate of return. Given this equilibrium condition, they proceed to develop their theoretical valuation formula. Before proceeding to their model, however, a review of their assumptions is in order. Generally, ideal market conditions are assumed for the stock and the option. More specifically:

a) The short term interest rate is assumed to be known and constant through time. This may be relaxed under certain conditions.

b) Stock prices follow a random walk in continuous time with a variance rate proportional to the square of the stock price. That is, the distribution of stock prices is log normal and the variance rate of the return on the stock is constant.

c) There are no dividends or any other distributions.

d) The option is European. (ie. exercisable at maturity only).

e) No transactions costs in buying or selling the stock or the option. (This will be relaxed in the subsequent analysis).

f) It is possible to borrow at the short term rate and buy any portion of a security.

g) No penalties for short selling.

It can be readily seen that under these assumptions, the value of the option depends only on the stock price and time, and on parameters which are taken as known constants. Under such circumstances it is possible to form a hedged position by shorting the option and taking a long position in the stock such that the value of the option will not depend on the stock price but on time and the constants. That is, more formally the value of the option may be expressed as:

(2-2) $$w\ (x,\ t)$$

or as a function of the stock price x and time t. To form the hedged position, the number of options that must be sold against one stock may be written as:

(2-3) $$1/w_1\ (x,\ t)$$

The subscript denotes the partial with respect to the first argument. To show that the value of the hedged position does not depend on stock price, note that $w_1\ (x,t)$ is the ratio of the change in the option value to the change in the stock price. That is, if x changes by Δx, the option price will change by $w_1\ (x,t)\ \Delta x$, so that the value of $1/w_1$ (x,t) options will change by Δx. Therefore a change in the value of the long position in x will be approximately offset by the change in the value of the short position in $1/w_1$ options. If continuity is assumed, then it can be shown that the approximations become exact and the return on the hedged position is completely independent of the changes in the value of the stock. That is, the return on the hedged position becomes certain.

It is important to note that the argument is consistent with existing portfolio and market theory. Under the random walk and constant variance rate assumptions the covariance between the returns on the equity and the stock will be zero. The same argument applies to the market portfolio concept. Consequently, under a continuous adjustment policy, the risk in the hedged position is zero. Even if continuous adjustment does not occur, it is expected that the risk will be small. The criticial factor, however, is that it may be completely diversified away by holding a portfolio of hedged positions. These generalizations have definite implications for the rest of this analysis.

The value of the equity in the position, given one share long and $1/w_1$ options short is defined as:

$$Ve = x - w/w_1 \qquad (2\text{-}4)$$

and the change in Ve over a short interval t as:

(2-5) $$\Delta Ve = \Delta x - \Delta w / w_1$$

Under the assumption of continuous adjustment, Δw can be expanded through stochastic calculus and be shown that the value of the equity becomes:

(2-6) $$-(\tfrac{1}{2} w_{11} \; v^2 x^2 + w_2) \Delta t / w_1$$

where the subscripts refer to partial derivatives and v^2 is the variance rate of return on the stock. Furthermore, since the return on the equity is known for certain to be $r \Delta t$, the change in the equity can be expressed as:

(2-7) $$(x - w/w_1) \, r \Delta t$$

Equating (2-6) and (2-7), dropping t and rearranging gives the following differential equation for the value of the option:

(2-8) $$w_2 = rw - rxw_1 - \tfrac{1}{2} (v^2 x^2 w_{11})$$

Assuming t* to be the maturity date and c the exercise price and the following boundary conditions,

(2-9) $$w(x, t^*) = x - c \quad \text{for } x \geq c$$

and $$w(x, t^*) = 0 \quad \text{for } x < c$$

and solving (2-8) subject to (2-9) results in the option valuation formula. (2). The formula may be stated as:

(2-10) $$w(x,t) = x\,N(d_1) - c e^{r(t-t^*)}\,N(d_2)$$

where

(2-11) $$d_1 = \frac{\ln(x/c) + (r + \tfrac{1}{2} v^2)(t^* - t)}{v(t^*-t)^{\frac{1}{2}}}$$

$$d_2 = \frac{\ln(x/c) + (r - \tfrac{1}{2} v^2)(t^* - t)}{v(t^*-t)^{\frac{1}{2}}}$$

and N(d) is the cumulative normal density function given by:

(2-12) $$N(d) = 1/(2\pi)^{\frac{1}{2}} \int_{-\infty}^{d} e^{-\frac{1}{2}(x)^2}\,dx$$

Taking the partial derivative of equation (2-10) with respect to the first argument and simplifying, results in the following definition:

(2-13) $$w_1(x,t) = N(d_1)$$

which is of particular importance to this analysis, as the expression defines the ratio of stock to options in the hedged position.

Black and Scholes continue by pointing out that as can be seen from expression (2-10), the value of the option is not a function of the expected return on the stock. This, however, does not negate the proposition that the expected return on the option is a function of the expected return on the stock. In effect, then, the formulation confirms that the price of the option is independent of the investors' utility functions. As presented in a previous argument, from equations (2-10) and (2-13) it can be shown that the volatility of the option is always greater than the volatility of the stock. That is, since the ratio

$$xw_1 / w \qquad (2\text{-}14)$$

is always greater than one, it implies that the relative volatilities will maintain a relationship such that the volatility of the option will always exceed the volatility of the security.

Black and Scholes also show that equation (2-8) may be derived by using the capital asset pricing model, but for the purposes of this analysis, the previous development will suffice. It is worthwhile to note the observed empirical test results reported by Black and Scholes when they compared the theoretical valuation prediction to actual call-option data. They report that the observed values tend to deviate from the predicted values in a systematic manner.

The purchasers of call options tend to overpay, but the writers of the option receive approximately what the valuation formula predicts. According to Black and Scholes, the difference must be the transactions costs. They also note that the observed differences tend to be greater for options on low risk stocks than high risk securities. The transaction costs,however, remove the potential profit opportunities implied.

Since its derivation, the option valuation formula has received a considerable amount of attention from academics. As stated in the assumptions, Black and Scholes restricted the analysis to non-dividend paying securities. Merton (10) extends the model and shows that this assumption may be relaxed to stocks paying continuous dividends. He also shows that if there are no dividends, then it will never pay to exercise an American option before the maturity date, implying that the valuation formula is equally valid for the latter option. With respect to sensitivity, he notes that the value of the option will increase continuously to the extent that the maturity t, the risk free rate r, or the variance rate v^2 increases. The upper bound, he concludes, must be the stock price.

Briefly, the valuation formula must be considered as one of the most important breakthroughs in finance. It not only offers a simplified solution to a rather complex problem, but also causes a re-examination of the existing financial theory. For example, in the area of corporate finance it can be shown rather easily that the stock holders, in effect, have an option on the assets of the firm assuming bonds to be outstanding. Furthermore, each bond interest payment may be viewed as an option contract. In general, it can be shown that the total number of options in this context is equal to n + 1, where n equals the number of interest payments to be made. Clearly, the arguments may be extended to convertible instruments, etc., but these become rather complicated.

Having developed the option valuation formula to this point, it becomes necessary to integrate it into the equity linked life insurance framework. In order to achieve this, the next section will focus on the Schwartz dissertation referred to in the previous chapter.

2.3 The Schwartz Dissertation

This dissertation offers one of the most important challenges to the life insurance industry. In essence, it calls upon the industry to completely re-evaluate its position with respect to the management of equity linked life insurance contracts. In this sense it also challenges regulatory bodies to review their interpretation of the concept of risk.

The dissertation may be divided into two distinct areas. One, the previously cited problems concerning the valuation of certain options and two, the interpretation of the equity linked life insurance instrument in the option pricing framework. In the first part, the author develops numerical methods to evaluate options on securities paying discrete dividends. He proceeds to show that under certain conditions, it pays to exercise American options prior to the maturity date (ie. the security pays discrete dividends). Briefly, he shows that a "critical stock price" can be determined, above which it pays to exercise the option.

The second part of the dissertation deals with equity linked instruments with an asset value guarantee, in the option pricing framework. Within this framework, the optimal investment strategy of the firm is developed. Along the same lines, partial differential equations are given for the option components of the constant, continuous premium contract, the periodic premium contract and the single premium contract. For the purposes of this analysis, the single premium contract is the relevant focus of discussion.

2.4 The Single Premium Case

Restating earlier definitions, an equity linked life insurance contract with an asset value guarantee, is an instrument providing the benefit of either the value of a reference portfolio of securities, or a guaranteed amount, whichever is greater. A call option permits the holder to purchase an asset at a given price, whereas the put option is the right to sell an asset for a certain or predetermined amount. Within this framework, the contract may be described as follows. If, upon maturity, the value of the reference portfolio exceeds the guaranteed amount, the beneficiary may exercise a call option on the reference portfolio. In this sense, the exercise price is the guaranteed amount. The value of the option must therefore be the difference between the guarantee and the portfolio value. The call option, obviously will not be exercised if the value of the guarantee exceeds the value of the portfolio.

Put another way, the value of the put option plus the value of the reference portfolio must equal the value of the guarantee plus the value of the call option. Either definition may be viewed as the benefits of the contract.

It should be noted that the previously stated assumptions of the Black-Scholes model, in particular, that the investment is made in non-dividend paying securities, still apply to the above argument. Also, given that the investment in the reference portfolio is made at the time the contract is purchased, Schwartz shows that the value of the option at any point in time corresponds exactly to the Black-Scholes call option formulation. That is, the partial differential equation may be expressed as:

$$\tfrac{1}{2} v^2 \; x^2 \; w_{11} + rxw_1 - rw - w_2 = 0 \qquad (2\text{-}15)$$

with the boundary conditions:

(2-16) $$w\ (x,t) = \max\ (\ 0,\ x-g\ )$$

Therefore, at any time τ , the value of the option may be stated as:

(2-17) $$w(x(\tau), t-\tau, g(t)) = x(\tau)\ N(d_1) - g(t)e^{-r(t-\tau)}N(d_2)$$

(2-18) where $$d_1 = (\ln(x(\tau)/g(t) + (r + \tfrac{1}{2}v^2)\ (t-\tau)/v\ (t-\tau)^{\frac{1}{2}}$$

(2-19) $$d_2 = d_1 - v\ (t-\tau)^{\frac{1}{2}}$$

(2-20) $$N(d) = 1/(2\pi)^{\frac{1}{2}}\int_{-\infty}^{d} e^{-\frac{1}{2}(x)^2}\,dx$$

and N(d) is the cumulative normal density function, as before. Although the asset value guarantee, g, has been substituted for the exercise price, c, these equations are identical to equations (2-10) to (2-12), respectively. The basic conclusions that the value of the option at any time t, can be expressed in terms of the current price of the reference portfolio, the variance rate of return on the portfolio, the time to maturity and the rate of interest, still holds. Only the variance rate is unobservable, but this can be estimated from historical data.

Schwartz goes on to point out that if equation (2-17) did not hold in the sense that the calculated value was not the market equilibrium value of the option, then arbitrage profits would exist.

Within this framework, the total value of the contract to the insured, at the time the instrument is created, can be expressed as the present value of the guaranteed amount plus the value of the call option:

$$PV_0\ (b(t)) = g(t)\ e^{-rt} + w(x(0),\ t,\ g(t)) \tag{2-21}$$

where $PV_0\ (b(t))$ is the present value of the benefits. Similarly, the present value of the benefits may be expressed in terms of the present value of the reference portfolio and the value of the put option:

$$PV_0\ (b(t)) = PV_0\ (x(t)) + p(x(0),\ t,\ g(t)) \tag{2-22}$$

Since the value of the call option is given by equation (2-17), the value of the put may be calculated from (2-21) and (2-22). That is:

$$p(x(0),\ t,\ g(t)) = w(x(0),\ t,\ g(t)) + g(t)e^{-rt} - PV_0(x(t)) \tag{2-23}$$

Under equilibrium conditions, and staying within the stated assumptions of the previous section, the present value of the benefits may be viewed as the premium charged for the contract. It is important to note that within this framework, the value of the call option must be determined before the premium can be established. Since all the parameters of the valuation formula, (2-17), are known, or can be estimated, the procedure becomes mechanical. It should be recognized, however, that from the point of view of the company, the most relevant calculation is the determination of the value of the put option. In a strict sense, this amount represents the charge for the guarantee, or the amount which the company receives for assuming the investment risk.

It was indicated in the discussion of the Black-Scholes option pricing model that a hedged position may be formed, such that no gains or losses will be encountered as long as a continuous revision policy is observed. This hedged position was formed by going long in the security and short in the call option. The same logic is applicable to the insurance case in question. Since the company has sold a call option on the reference portfolio short, in order to eliminate the associated risk, it must take a long position in the portfolio. In order to maintain the hedged position, it must revise the long position continuously. That is from equation (2-14) it can be shown that the position in the portfolio is equal to

$$x w_1 (x,t) \qquad (2\text{-}24)$$

where x is the value of the reference portfolio and w_1 (x,t), as before, the partial derivative of the option value with respect to the first arguement. This may also be interpreted as the proportion which acutally must be invested in the reference portfolio. From this it follows that not all the funds have to be actually invested in the portfolio. This can be shown by the following arguments. Firstly, the value of the option must always be less than or equal to the value of the portfolio, (ie. $w \leq x$). Secondly, it must exceed or be equal to the difference between the portfolio value and the exercise price (ie. $w \geq x-e$). Thirdly, it is assumed that w(x,t) is concave upward. Given these boundary conditions, it follows that the partial derivative with respect to x is increasing always within the specified range. In fact the range of the derivative must be:

$$(2\text{-}25) \qquad 0 \leq w_1 (x,t) \leq 1$$

It will equal zero if (x-e) is less than or equal to zero. On the other hand, w_1 (x,t) can only equal one if x becomes infinite (ie. $w_1(\infty,t)=1$).
From (2-25) it follows that:

$$(2\text{-}26) \qquad x\, w_1 (x,t) \leq x$$

2.5 Summary

Briefly, in this section, it was necessary to follow the Schwartz dissertation very closely, as it provides the theoretical framework for the subsequent simulation problem. In essence, it is this framework that provided the initiative to build a simulation model and test the hypothesis, that given transactions costs, a hedging strategy of this nature will still reduce disaster losses. Because of transactions costs, a continuous model is not practical. Because of this liability, certain assumptions must be made concerning the distribution of returns on the market or portfolio, about the size of the transactions costs per revision and the revision schedule itself. The next chapter of the analysis deals with these assumptions, the foundations of simulation and the subsequent model developed for the hypothesis test. It also considers the results of the various alternatives tested.

Chapter 3

Development of the Simulation Model

3.1 Basic Concepts of Simulation

In the most general sense, simulation may best be described as the process of designing, building, validation, analysis and operation of a formal model designed to represent only those features of the system under study which are believed to be significant in view of the objectives behind the investigation. In other words, it is the synthesis and analysis of a system with the functioning of the real system being represented. This does not, however, mean that the system being modeled must exist. For example, certain physical phenomena simply take too long for the analyst to observe, whereas a simulation program can reduce the time factor such that it becomes very simple to study the problem. Simulation is not intended to provide optimal solutions to the problem, rather it permits the analyst to employ an algorithm, the parameters of which may be altered by the analyst so that a range of solutions may be generated. In this sense, the model is predictive, given that certain assumptions about the relevant parameters have been made.

In the case of the equity linked life insurance contract, given the complexities of the problem and the potential costs involved, clearly, a considerable amount of research and analysis must be performed before a company should undertake such a proposal. Since the system discussed in the previous chapter does not exist in the real world, and therefore is unobservable, the analyst must build a model of the theoretical framework and simulate real world externalities which can affect the model in order to determine the sensitivity of the model to these externalities. This implies that it is not enough to quantify the model parameters, but also the relevant externalities, if possible; if not, then certain assumptions must be made about them. These externalities are the topics of discussion in the next section.

3.2 The Relevance of Portfolio Composition

In the previous chapter, under the discussion of the theoretical model, constant reference was being made to the reference portfolio of securities. The actual composition of the portfolio was not considered at all however. The obvious question therefore, is the relevance of the composition of the portfolio to the model being developed. Portfolio theory suggests that the investor should simply buy the market portfolio and revise it only to maintain the ratios. That is, given the amount of wealth available for investment it should be distributed among the securities in such a way that the ratio of the amount invested in each security to the total wealth is the same as the ratio of the value of the securities of each company to the total value of the market portfolio. If the ratios in the market portfolio change resulting from the reinvestment of dividends, new issues, etc., then the investment portfolio should be altered so that the ratios remain the same.

If the policy of continuous revision could be pursued, (ie. no transactions costs), then the composition of the reference portfolio is irrelevant since the amount invested in the portfolio depends on the current value of the portfolio, not on an implicit risk return relationship which has been established. Looking at it from another angle, since the objective under the continuous revision strategy is to be fully hedged at all times, it becomes irrelevant what the value of the portfolio is, or what the return on the portfolio has been. This must be true, because as it was stated before, if a fully hedged position is maintained, then the insurance company will not experience losses or gains as it has not assumed any risk. In this sense, only the variance of the reference portfolio is of importance because the value of the call or put option depends on the variance rate.

Since in a practical situation, only a discrete or periodic revision policy can be pursued, because of transactions costs, the company will be exposed to some risk. In this sense, the composition of the portfolio does become relevant, as does the return on the portfolio. The extent to which this is true depends on the number of revisions that the company can undertake during the life of the contract. That is, assuming that the expiration date is known for certain, the company is subject to portfolio related risk from the last revision point to the expiration date, given that a market collapse has not occurred in the previous periods.

For the purposes of this analysis, it is assumed that the insurance company forms the reference portfolio by buying the market portfolio, as represented by the Toronto Stock Exchange (ie. TSE). Given this assumption, it is only logical that the variance rate to be employed by the model be the observed historical rate of the TSE. It should be noted, however, that the amount to be invested in the reference portfolio is still governed by the differential equations discussed in the previous chapter. It is assumed that the cost of maintaining the market portfolio is exogenous to the model under consideration.

3.3 The Return on the Portfolio

Within the simulation framework, it becomes necessary to generate a rate of return on the reference portfolio. The simplest way to achieve this objective is to employ a random number generator, which is capable of generating from various distributions. Given the Black-Scholes assumption that security prices are lognormally distributed, with a constant variance rate, it is assumed that the returns on the market portfolio are lognormally distributed, with the same variance restriction. This is the position taken with respect to the simulation model.

Since most random number generators do not generate directly from a lognormal distribution, but from a standard normal, transformations become necessary. The simplest reconciliation can be presented by reviewing the two-parameter distribution's defintion, as given by Aitchison and Brown (1). Assuming an essentially positive variate $X(0 < x < \infty)$ such that:

$$Y = \ln X \qquad (3\text{-}1)$$

is normally distributed with mean u and variance v^2, then it can be said that X is lognormally distributed, and write X is $\Lambda(u, v^2)$ and correspondingly Y is $N(u, v^2)$. The distribution of X is completely specified by the two parameters u and v^2. Obviously, this is the simplest natural definition. It is evident, however, that X cannot assume zero values as the transformation Y = ln X is not defined for X = 0. Since X and Y have the relationship Y=lnX, the distributions of X and Y are related by:

$$\Lambda(x) = N(\ln x) \qquad (x > 0) \qquad (3\text{-}2)$$

$$\Lambda(x) = 0 \qquad (x \leq 0) \qquad (3\text{-}3)$$

(3-4) and $d\Lambda(x) = (1/xv(2\pi)^{\frac{1}{2}}) \exp(-(1/2v^2)(\ln x - u)^2)\ dx \qquad (x > 0)$

which describes the frequency curve with a single mode at:

(3-5) $$x = e^{u - v^2}$$

The mean may be defined as:

(3-6) $$\bar{x} = e^{u + \frac{1}{2}v^2}$$

and the variance:

(3-7) $$v^2 = e^{2u + v^2}(e^{v^2} - 1)$$

(3-8) $$= \bar{x}^2 \; n^2$$

where n is the coefficient of variation of the distribution. The median is simply e^u. It should be noted that the two-parameter lognormal distribution does possess reproductive properties, which is the justification for the assumption that the returns on securities is distributed lognormally. That is, if the return R on security j, at time t, is given by:

(3-9) $$R_{j_t} = (P_{j_t} - P_{j_{t-1}}) / P_{j_{t-1}}$$

where P is the price of the security, and has a $\Lambda(u, \hat{v}^2)$, it follows that R has a Λ distribution from the corollary that:

$$\ln X_1 - \ln X_2 = \ln(X_1/X_2) \qquad (3\text{-}10)$$

implying that the lognormal distribution will have divisible reproductive properties.

In order to generate values from a lognormal distribution with a known mean SM and a standard deviation SX, only the following transformation is necessary:

$$\hat{v} = (\ln(1.0 + XS^2/XM^2))^{\frac{1}{2}} \qquad (3\text{-}11)$$

and

$$\bar{x} = \ln(XM) - \tfrac{1}{2}(\hat{v}^2) \qquad (3\text{-}12)$$

Since the generator selects a value s from S which is N(0,1), the following evaluation occurs:

$$x = \exp(\bar{x} + \hat{v} \cdot s) \qquad (3\text{-}13)$$

In the simulation program, the mean return was specified as 8% and the variance rate on the TSE as .01846.

It is recognized that in reality, a computer random number generator is at best a pseudo-random number generator. The bias which this may introduce, is probably so minimal that it is not worthwhile to pursue the effect by performing randomness tests.

3.4 The Simulation Program

Given that transactions costs are to be included in the analysis, it is no longer valid from a practical point of view to assume that continuous adjustment of the ratio of the long position in the reference portfolio to the short in the call option on this portfolio is possible. Since, as stated previously, the objective of this analysis is to examine the potential losses which may be incurred by the company under a discrete revision policy, it is necessary to determine or define the types of losses which may occur.

Reviewing the previous arguments, it becomes evident that two types of potential losses may occur. The first type may be viewed as simply additional costs of conducting business, arising from the fact that transactions costs are now relevant. In this sense the word loss is a misnomer as the company will simply charge the policy holder for these additional costs; but for the sake of simplicity, the aforementioned terminology will be adhered to. The magnitude of these losses over the life of the contract will be related to the size of the imbalances in the hedged position at the time of revision and the number of revisions planned for, during the contract period. This point will be pursued further in another section.

The second type of loss may be defined as a "disaster loss". Although it is expected that the occurence of this types of loss should be infrequent, nevertheless, from the point of view of the company, these are very important. This type of loss can occur from a general collapse in the market. Looking at it from another angle, if the value of the reference portfolio, plus the amount invested in the risk free asset is less than the guaranteed amount, the company must either have the ability to borrow, or face bankruptcy. It should be noted that the criteria of the reference portfolio being greater than the guaranteed amount is overstating the requirement by the amount invested in the risk free asset. As stated previously, it is not necessary, and as a matter of fact, suboptimal to invest all of the premium in the reference portfolio. Consequently, the excess can be invested in a risk free asset, assuming no additional costs besides the transactions requirements.

The initial parameters of the simulation program were set arbitrarily. More specifically:

Market return	8%
Variance	0.01846

Risk free rate	6%
Contract period	10 years
Number of revisions	10
Transactions costs	1%
Guaranteed amount	$100.00
Initial value, Ref. Port.	$100.00
Number of simulations	500

The risk free rate, contract period, guaranteed amount and initial investment in the reference portfolio were held constant for all the simulations. The number of simulations were varied from 500 to 2,000 in order to establish the stability of the results. In general, it was found that about a one one-hundredth cent change occurred in the mean losses if the number of simulations were expanded from 500 to 2,000. About the same magnitude change occurred in the standard deviation of these losses. Consequently, 500 simulations were adopted for all the runs, as the changes described were deemed to be insignificant.

Initially, the transactions costs were permitted to vary from 1% to 2.5% by increments of 0.5%. This, in effect,results in a cost of 2 to 5% to get in and out of the market, which is fairly representative of reality. These costs may be divided in the following manner:

a) The cost of buying the initial portfolio.

b) Revision costs.

c) The cost of selling the portfolio.

In this sense, it is assumed that at the end of the contract, the reference portfolio must be liquidated. It should also be noted that transactions costs only apply to the reference portfolio, not to the investment in the risk free asset. The latter is assumed to be the equivalent of a savings account, which typically does not incur transactions costs.

As stated previously, the mean return on the portfolio was assumed to be 8% with a variance of 0.01846. This was also held constant for the runs. Various arguments may be made about the appropriateness of the assumed mean return, but it should be noted that the cirtical assumption is the variance rate.

The revision parameter was permitted to vary across the simulations. Initially, annual revisions were adopted as the policy of the insurance company, but in subsequent simulations it was changed to six month, four month and three month intervals. In essence the simulation paramenters may be defined as the transaction cost range and the revision policy range, as per the predefined increments.

In order to facilitate an easier understanding of the computer program provided in Appendix (A), some of the notation of the previous chapters will be altered to conform to that of the program. Since some of the mechanics of the program are not relevant to this discussion, or may be summarized in one equation, a dictionary of the variables and functions has been provided at the end of the program in order to clarify the logic.

The main portion of the program may be viewed in four stages:

a) The initial period. This is synonymous to the creation of the contract and the subsequent valuation of the benefits.

b) The revision policy. This portion deals with the revision of the hedged position, given the hypothetical market return for the period, as described by the firm's policy.

c) Termination of the contract. This section determines the firm's performance with respect to the contract under consideration. In effect, it establishes the firm's liability to the policy's beneficiary.

d) Overall performance evaluation. The final section may be viewed as an evaluation of the performance of a large portfolio of contracts, managed under the same criteria (ie. with respect to revision and transactions costs).

Given the assumption that at the time of the initiation of the contract, the value of the reference portfolio X, is equal to the guaranteed amount, from equations (2-17) to (2-20), the value of the option may be determined as follows:

(3-14) since $\ln(X(0)/g(t) = 0$

from $\ln(100/100) = 0$

(3-15) d_1 becomes $d_1 = ((r + v^2)\ (t))/v(t)^{\frac{1}{2}}$

(3-16) and $d_2 = d_1 - v(t)^{\frac{1}{2}}$

so that the value of the option at time zero becomes:

(3-17) $OPT(0) = X(0)\ N(d_1) - g(t)e^{-rt}\ N(d_2)$

where the previous variable definitions apply. Having determined the value of the option, the firm's total liability may be written as:

(3-18) $L(0) = g(t)\ e^{-rt} + OPT(0)$

and the actual amount invested in the reference portfolio x, as:

(3-19) $x\ (0) = X(0) \cdot N(d_1)$

Since the initial wealth position must equal the initial liability of the firm, after the investment in the reference portfolio is made, the wealth position becomes:

$$W(0) = L(0) - (x(0) \cdot TR) \tag{3-20}$$

where TR is the incurred transactions cost, as a percentage. The amount invested in the risk free asset becomes:

$$RF(0) = W(0) - x(0) \tag{3-21}$$

These equations summarize the company's position at the creation of the contract. It should be noted that since it is assumed that the initial value of the reference portfolio equals the guaranteed amount, the initial investment in the reference portfolio will be identical for all the simulations. The initial wealth will depend on the size of the transactions costs in the current calculations, since L(0) and x(0) are constant in equation (3-20). Similarly, the amount invested in the risk free asset depends on the value of W(0) in equation (3-21).

The second stage of the program evaluates the above relationships at discrete points in time, as defined by the revision policy under consideration. At each revision point a rate of return is generated, so that the value of the reference portfolio at time t is:

$$X(t) = X(t\text{-}1) \cdot Z\ (\Delta t) \quad (3\text{-}22)$$

where Z is the simulated return on the portfolio. That is, if Z is greater than one, the result is a profit or increase in the value of the portfolio. If it is less than one, a loss or reduction in the value is implied. Since the value of the reference portfolio has changed, the hedged position has also changed, therefore the investment in the portfolio must be altered. From equations (3-14) to (3-17) the value of the option may be calculated for time t, and from (3-19) the hedged position may be re-established. The new wealth position at time t can be defined as:

$$W(t) = (W(t\text{-}1) - x(t\text{-}1)) \cdot e^{r\,\Delta\,t} + x(t\text{-}1) \cdot Z\ (\Delta\ t) \\ -((x(t) - x(t\text{-}1)))\quad TR \quad (3\text{-}23)$$

The firm's liability at time t is determined from equation (3-18).

The third section of the program determines the final financial position of the firm with respect to the contract. The final wealth position is determined by equation (3-23), but it should be noted that the terminal transactions costs are considerably larger than for the revisions, as the whole portfolio is liquidated. To clarify the point, the final wealth position may be simplified to:

$$W(t^*) = (W(t^*-1) - x(t^*-1) \quad e^{r \Delta t} + x(t^*-1) \cdot Z) - ((x(t^*-1) \cdot Z) \cdot TR) \qquad (3\text{-}24)$$

where t* is the maturity date. The value of the option at maturity is:

$$OPT(t^*) = X(t^*) - g \qquad (3\text{-}25)$$

for $X(t^*) > g$, or:

$$OPT(t^*) = 0 \qquad (3\text{-}26)$$

for $X(t^*) < g$. The company's liability, therefore, equals:

$$L(t^*) = g + OPT(t^*) \qquad (3\text{-}27)$$

The liability may also be expressed as:

$$L(t^*) = X(t^*) \qquad (3\text{-}28)$$

for $X(t^*) > g$, or

$$L(t^*) = g \qquad (3\text{-}29)$$

for $X(t^*) < g$. If the value of the reference portfolio is greater than the guarantee, then the profit to the firm is:

(3-30) $$PR(t^*) = W(t^*) - X(t^*)$$

otherwise it is:

(3-31) $$PR(t^*) = W(t^*) - g$$

Since no provisions were made for transactions costs at the creation of the contract, the profit represents the amount the company must charge the insured at the creation of the contract, over and above the other costs, in order to break even. In this sense, the profit is negative. It should be noted, however, that in very unique cases the profit may in fact be positive. This situation can arise if the value of the reference portfolio is less than the guarantee, but the wealth position is greater. This is possible since a portion of the premium is invested in the risk free asset, so that in certain instances, the following conditions may result:

(3-32) $$X(t^*) < g$$

but

(3-33) $$X(t^*) + RF(t^*) > g$$

where RF(t*) represents the terminal value of the investment in the risk free asset, so that W(t*) is actually greater than the guaranteed amount. Since the outcome described by equation (3-32) has occurred, the beneficiary would naturally elect to receive the guaranteed amount, leaving the positive profit given by (3-31) to the company. The scarcity of this phenomenon is evidenced by the fact that it only occurred under the condition of annual revision and one percent transaction cost criteria.

Appendix B provides examples of the initial, intermediate and terminal values derived from the previous equations, for various revision strategies and transactions costs. The summary page of the appendix provides a description of the variables listed and a reconciliation of those variables to the equations of this section.

To this point, the arguments presented have dealt with only the single simulation or contract. The rest of the program focuses on the accumulation and processing of the relevant information over the whole simulation. For each simulation the program stores the terminal value of the reference portfolio, the loss (profit) associated with it, and the classification of the loss into transaction cost derived, or disaster loss, as per the criterion discussed previously. That is, if the loss occurs because the value of the reference portfolio is less than the guaranteed amount, resulting in the beneficiary exercising the guarantee, the loss is deemed to be disaster loss. Otherwise, the loss is assumed to be the result of the transactions costs incurred because of the revision policy under consideration. Upon the completion of the simulation under the existing parameter values, the program calculates the mean and standard deviation of the losses and of the value of the reference portfolio and summarizes the statistics in the form of tables. (Refer to Appendix D). Tables are also produced for the mean and standard deviation of the disaster losses. (Appendix E). In order to analyse the magnitude of ordinary or transaction cost losses, the program also produces tables of the actual values, mean and standard deviation of the largest five percent and ten percent losses incurred, respectively. These tables may be found in Appendix F.

The final section of the program summarizes the above statistics across all the simulations. The tables generated represent the mean and standard deviation of the above described loss categories under the various combinations of transaction cost levels and revision policies. These may be referred to in Appendix E.

With slight modification to the program, the naive strategy was also tested. This alternative may simply be viewed as the formation of the reference portfolio at the creation of the contract, with the assumption that no revisions will take place during its life. Under this policy the only additional costs incurred are the initial and terminal transactions costs. The results of this run are presented in Appendix C.

Chapter 4

Analysis of Results

The analysis of the results proceeds from an in depth examination of the intermediate relationships to the evaluation of the simulation output. Because of the volume of output provided by the simulation, only the most relevant statistics have been accumulated in tabular form in the appendices. As a result, an examination of each of the appendices is presented.

4.1 Intermediate Results

Appendix B provides the results of intermediate calculations for one percent transactions costs and annual, six and four month revisions, respectively. It should be noted that the initial investment in the reference portfolio, X-J, the value of the reference portfolio, RPORT, the initial wealth WLTH, the value of the option, OPVAL,and the liability to the firm, LIAB, is the same for all simulations. RAND-Z is the return on the reference portfolio in any period under consideration.

The first two examples of Table 1 clearly indicate that the beneficiary will not exercise the guarantee, but elect to receive the terminal value of the reference portfolio, since in both cases this exceeds the value of the guarantee. In the third example, however, the guarantee is exercised, as the value of the reference portfolio is less than the guarantee. This, in effect, is an example of the disaster losses discussed in the previous chapters. This table also shows the impact of the decline in the value of the reference portfolio with respect to the actual investment in the portfolio. The fluctuations in the returns and subsequent losses on the reference portfolio result in the value of the option becoming zero, and the investment in the portfolio being reduced considerably. On the other hand, when positive returns are experienced, as given in tables two and three, the investment in the reference portfolio is almost equal to the value of the portfolio. That is, the quantity $X(t) \cdot N(d_1)$ is almost equal to one, and infact becomes one in some of the cases. Since in these cases the investment in the reference portfolio is greater than the actual wealth position of the company, small amounts of borrowing must occur in order to arrive at the terminal conditions of the particular simulation. It is assumed that this borrowing is accomplished at zero cost for simplicity. The increase in the losses, (ie. the negative profit values), can be attributed to the increase in transaction costs resulting from the additional revisions undertaken.

It is important to recognize that doubling or tripling the number of revisions does not necessarily result in doubling or tripling the losses, respectively. Infact, the losses incurred, as given in

Table 3, are less than those indicated in Table 2, which result from ten less revisions. While the comparison of the results of two simulations is inadequate to draw concrete conclusions from, nevertheless, the implication is present that the larger the number or revisions, the less the company is exposed to abnormal gains or losses. Furthermore, it follows that the more often the portfolio is revised, the smaller the required change in the investment in the reference portfolio, in order to re-establish the hedged position. The overall implication is that if these assertions can be substantiated, then the firm's exposure to abnormal losses may be reduced by following the hedging strategy, which in effect is the hypothesis of the analysis.

The intermediate results also substantiate the argument that the volatility of the option is always greater than the volatility of the security or reference portfolio. For example, an 11% change in the value of the reference portfolio, in Table 1, results in a 15.6% change in the value of the option. Similarly, a 14% decline in the portfolio value causes a 40% decline in the value of the option. It should be noted that the relative volatility of the option is dependent not only on the value of the reference portfolio, but also on the maturity period.

4.2 The Naive Strategy

Under the naive strategy it is assumed that the company simply invests the total premium in the reference portfolio and does not pursue a revision strategy, but holds the portfolio until the termination of the contract (Appendix C). The initial investment in the portfolio is assumed to be $100.00 which is only 21 cents less than the calculated premium under the revision strategy option. Since only initial and terminal transaction costs can be incurred under this strategy, they are omitted from the calculations for the sake of simplicity. The number of simulations were set at 500, as for the revision strategy option. The number of periods is synonymous to the revision period concept. That is, for example, 10 periods implies the calculation of the return on the portfolio annually, 20 periods, semi-annually, etc.

Table 2 summarizes the disaster losses the company may incur by following this strategy. The number of losses refers to the number of times the guarantee is exercised over 500 simulations. The percentage simply re-expresses the number of losses in terms of the number of simulations. The average loss is the mean of the disaster losses under the particular strategy, the standard deviation, the deviation from that mean.

The outcomes under the various options of the naive strategy suggest that the process is random. That is, it is irrelevant how many periods are considered over the number of simulations. The average losses and the deviations do not display any trend. It is interesting to note, however, the magnitude of the deviations in relation to the magnitude of the mean losses. This can be attributed partly to the limited number of observations under consideration. When the individual loss magnitudes are considered, they appear to be random. The mean and the standard deviation of the losses become critical, as the naive strategy statistics are used as the benchmark to compare the revision strategy against.

The first Table summarizes the losses in terms of the total number of simulations per strategy performed, and the corresponding mean values of the reference portfolio. Since under the naive strategy the company cannot make a profit, the upper bound is set at zero. That is, since there is no investment in a risk free asset, the possibility of total wealth exceeding the guaranteed amount when the value of the reference portfolio is less than the guarantee cannot occur. As stated previously, this outcome can only occur if a hedging policy is adopted.

As expected, the average loss per 500 simulations is very close to zero. Infact the range over the periods considered is from -0.33 to -0.77 dollars. The large standard deviations and the fact that zero is a boundary condition (ie. no profits possible), imply that the distribution is very highly skewed to the left. In effect, the standard deviation provides a good indication of the magnitude of the losses incurred, especially when compared to the average.

The average values of the reference portfolio over the number of periods is also within expectations. The range of the averages is from \$236.67 to \$255.03 which can be translated into compounding \$100.00 investment at a continuous rate of from 8.5% to about 9.5%. This is reasonable as the mean return parameter on the reference portfolio was set at 8%. One standard deviation from the mean translates into a range of about 4% to 13%, which is acceptable in terms of the variance rate assumed.

In general, it may be concluded that although the number of disaster losses compared to the number of simulations is not very high, (ie. from 2 to 4.2%), given the average size and deviation of the losses, the strategy cannot be considered as very effective.

The rest... of the analysis is concerned with comparing the revision strategy to the naive procedure, and showing that the former is dominant over the latter.

4.3 The Revision Strategy: Overall Losses

Appendix D summarizes the overall losses incurred by the company under the various revision strategies. The average losses represent disaster losses as well as the transaction costs associated with revisions. The tables present summaries of transactions costs ranging from 1% to 2.5%, as well as the case in which no transactions costs are incurred.

This case is similar to the naive strategy discussed in the previous section, except that the investment in the reference portfolio is revised at the end of each period.

As expected, as the transaction costs are increased the average loss also increases, holding the number of revisions constant. The standard deviations associated with these means do not, however, increase at a proportional rate. That is, doubling the transaction cost does not result in doubling the standard deviation. For example, doubling the rate from 1% to 2% increases the average loss from -6.98 to -13.66 in the case of annual revisions, but the standard deviation only changes from 3.34 to 5.24 respectively.

If transaction costs are held constant, a comparison of the average losses over the various revision strategies reveals that the averages tend to increase at a decreasing rate, as the revision period is increased ten periods at a time. This tends to hold for all the cases except the special case of zero transactions cost. In this case, the average loss is very close to zero. Furthermore, comparing the latter case to the naive strategy with zero transactions cost, results in the conclusion that in all cases the revision strategy is dominant over the naive, in terms of average losses. If positive transaction costs are considered, then this comparison cannot be made.

The standard deviations of the average losses under the various strategies also provide valuable information. Holding transaction costs constant and increasing the number of revisions, results in an initial decline in the standard deviation, but as the number of revisions reaches forty to fifty per contract period, it begins to rise. For the lower transaction cost levels, after eighty revisions the deviation is about the same or lower than for ten revisions, but for the higher levels, it exceeds the deviation after ten revisions. If the transaction costs are eliminated (ie. set at 0%), then it becomes clear that the more often the portfolio is revised, the lower the standard deviation of the losses. This is entirely consistent with expectations. Comparison of these deviations to those derived under the naive strategy leads to the conclusion that the revision strategy is dominant. The deviations are not only declining, but in magnitude they are considerably smaller than the ones derived under the naive plan. As discussed previously, under the naive plan the deviations do not display any trend, but suggest that they are random.

The average value of the reference portfolio, as given by the various tables, are within expectations. The dollar values translate into a 7.5 to 9.5% annual return over ten years on an initial investment of about a hundred dollars. This is reasonable, given the original assumptions about the mean return and variance on the market portfolio. It should be noted that the average value and deviations of the reference portfolio are the same for the naive strategy and the zero and one percent revision strategies because the same sequence of random numbers were generated in these evaluations. This fact tends to reinforce the conclusion that in terms of total losses the revision strategy is superior to that of the naive plan, since the same percent losses are generated. The difference in the results must then be attributed to the effects of following the hedging strategy. The average values for the reference portfolios of the other strategies are different because a different sequence of random numbers are generated due to the changes in the revision periods. The results are still acceptable, however, in terms of the original parameters.

4.4 The Revision Strategy: Disaster Losses

As defined previously, disaster losses occur if the guarantee is exercised by the beneficiary because of a general market decline and the subsequent poor performance of the reference portfolio in relation to the guarantee. Appendix E summarizes these losses over the various transaction costs and revision strategies.

As noted with the overall losses, the average disaster loss incurred under the various revision strategies, (holding transaction costs constant), tend to increase at a decreasing rate. The fact that the average losses do increase as the transaction costs are increased is within expectations, and does not conflict with the above conclusion. It is not possible however, to isolate the incremental losses attributable to the increase in transaction costs, (ie. holding revision periods constant), within the existing simulation program. In order to accomplish this, a considerable amount of reprograming is necessary.

If zero transaction costs are considered, (Table 5), then both gains and losses appear under the average loss category, but they tend to be close to zero in magnitude. The gains can be attributed to the investment in the risk free asset. That is, although the beneficiary elects to receive the guaranteed amount because the value of the reference portfolio is less than the guarantee, the total wealth position of the company is not. The difference between the value of the reference portfolio and the wealth position is the amount invested in the risk free asset.

Comparison of these results to those derived under the naive strategy reveals that for transaction costs of 0% to 1½% the revision strategy is dominant over the naive, in terms of average losses. The results tend to be inconclusive for transaction costs of 2 and 2½%, in terms of average losses. It may be concluded, however, that if the number of revisions are kept to ten per contract period, then even at these transaction cost levels the revision strategy is superior to the naive.

Analysis of the standard deviations of the average losses reveals the actual impact of the revision strategy. At all levels of transaction costs the deviations tend to improve as the number of revisions is increased. The improvement may be viewed as a decline in the magnitude of the deviation as the number of revisions are increased, or in terms of the ratio of the mean loss to the standard deviation associated with the loss. That is, although the standard deviation increases in some cases as the number of revisions are increased, the relationship of the average loss to the deviation should be considered instead of the absolute magnitude of the deviation.

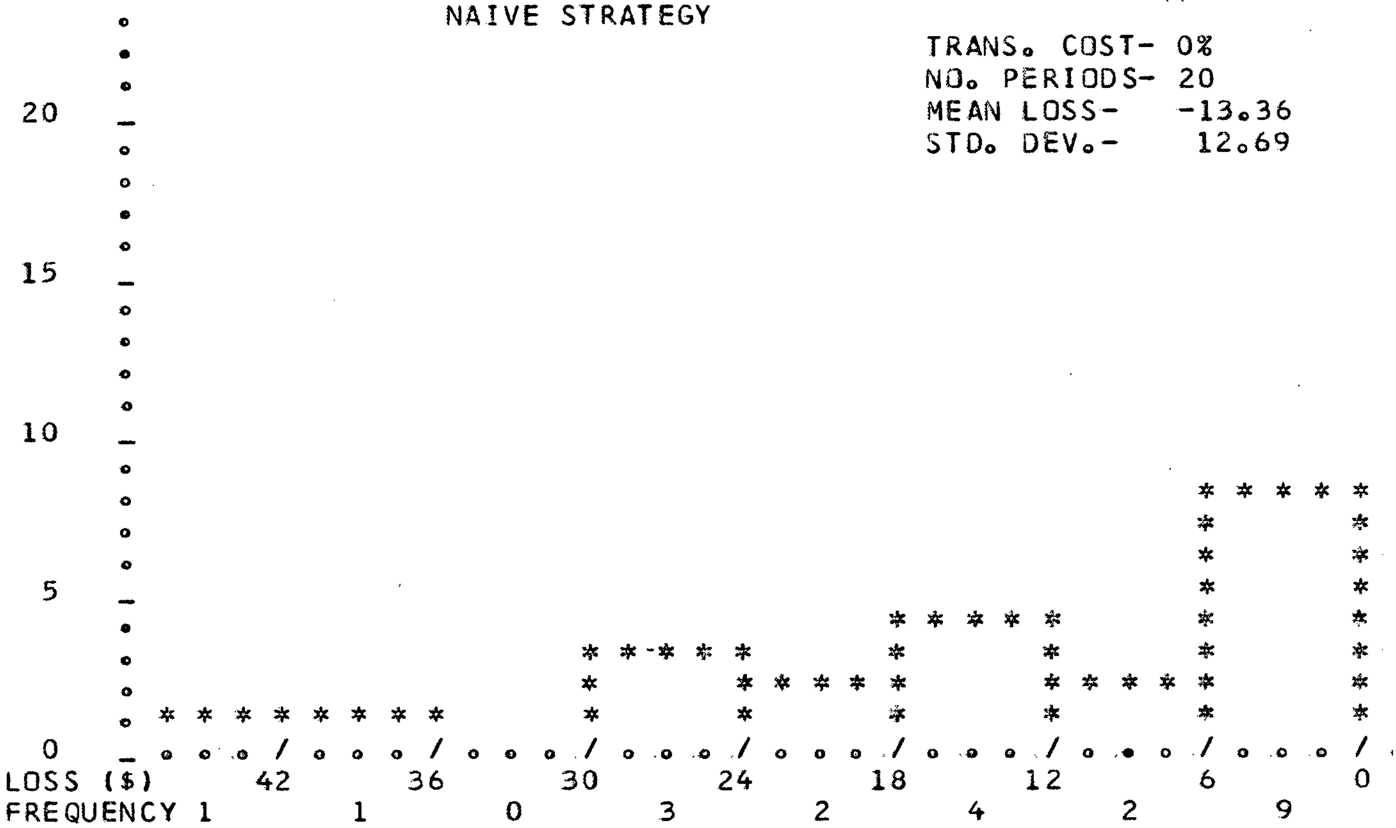
With respect to the naive strategy, in all cases the revision policy is dominant, in terms of the derived deviations. The differences are especially significant at the lower transaction cost levels. For example, at 1% transaction costs and 10 revisions the deviation is 5.55, at 80 revisions it is 2.00. Under the naive strategy, however, the respective deviations are 10.95 and 11.11. For the case of no transaction cost, the decline is even more dramatic, ranging from 5.50 to 1.47, respectively. Even at the highest cost level considered, (ie. 2½%), the respective deviations are 6.19 and 3.47 which are still considerably below the naive results.

Generalizing, the implications of the average losses and the respective standard deviations are that by increasing the number of revisions, the company reduces the disperison of the losses, which can be viewed as a reduction or risk. The increase in the average losses as the number of revisions is increased, on the other hand, can be interpreted as the cost of reducing the risk. The measure of risk may also be interpreted as the magnitude of losses incurred by the company, but the major weakness of considering absolutes is the random component. That is, if the largest loss under each strategy is considered as an indicator of the risk associated with that strategy, then the random component must be isolated in order to determine the effectiveness of the revision strategy.

The following histograms summarize the disaster losses experienced under the naive strategy and the revision strategy for the 20 period alternative. This is synonymous to revising the portfolio every six months, or in the case of the naive strategy, generating a return on the portfolio semi-annually. It is evident from the histograms that not only are the mean and standard deviations reduced, but also the absolute magnitude of the losses. In absolute terms, the reduction in the largest loss is $29.22 (ie. from 42.33 to 13.11). Analysis of the behavior of the largest loss is not pursued, however, because of the changes in the return generating sequence resulting from increasing the number of revisions. Doubling the number of revisions results in doubling the number of random returns generated. This, in turn can result in the generation of an extremely large loss. Because of this possibility, the more conservative approach of examining the means and standard deviations of the losses is adopted.

NAIVE STRATEGY

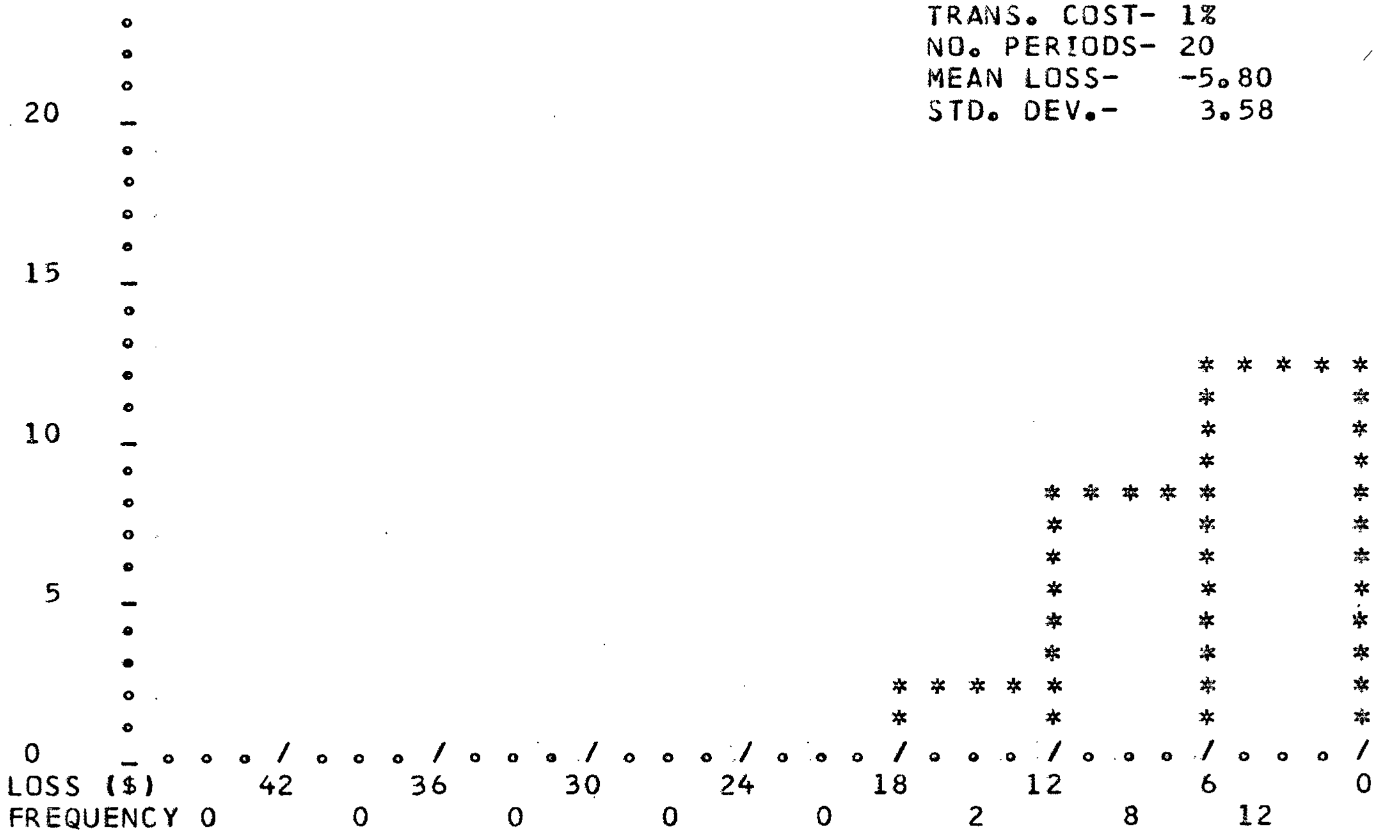
TRANS. COST- 0%
NO. PERIODS- 20
MEAN LOSS- -13.36
STD. DEV.- 12.69



REVISION STRATEGY

DISASTER LOSSES

TRANS. COST- 1%
NO. PERIODS- 20
MEAN LOSS- -5.80
STD. DEV.- 3.58



4.5 The Revision Strategy: Largest Losses

The final set of tables, presented in Appendix F, summarize the largest 5 and 10% losses over the 500 simulations per revision strategy and the various levels of transaction costs. The average losses represent the means of the 25 and 50 largest losses incurred under each revision strategy, respectively. In essence it is an analysis of the tail end of the total loss distribution.

As in the previous tables, the average losses increase with the number of revisions, but the deviations from the mean do not increase linearly. As the transaction costs increase from 1 to 2½% the deviations do increase more rapidly as the number as the number of revisions is increased. Even at 2½%, the deviations only double if the number of revisions is increased from 10 to 80. For the case of zero transaction cost, the average loss and corresponding standard deviation declines as the number of revisions is increased. In essence the implications of these tables is that if the company is able to reduce the transaction cost level (ie. the percent cost), considerable savings will result, along with a reduction in the risk associated with the losses.

These results may be examined from another point of view. If the company adopts a policy of charging the investor for the average of the 25 or 50 largest losses, per contract, a considerable decrease occurs in the risk associated with the portfolio of contracts. The following table summarizes the minimum and maximum number of losses (in percentage terms) which occur over the various transaction cost levels, given the above strategy.

Percent Losses

Trans. Cost	5% Min. Loss	5% Max. Loss	10% Min. Loss	10% Max. Loss
1%	1.6%	2.6%	3.0%	4.0%
1½%	1.2%	2.4%	3.0%	4.6%
2%	1.4%	2.2%	2.8%	4.0%
2½%	1.4%	2.2%	3.2%	4.2%

That is, for example, if the company incurs transaction costs of 1%, under the policy of charging the average of the 25 largest losses (ie. 5%), irregardless of which revision strategy it pursues, the minimum number of contracts on which it will incur a loss will be 1.6% or 8, the maximum, 2.6% or 13. Clearly, for a specific revision strategy, the percentage can vary, but it will not exceed these limits for a given transaction cost level. It should also be noted that the limits are relatively constant for the various transaction cost levels (although the average losses increase as the transaction cost increases).

If zero transaction costs are assumed, the trends in the results are very similar to those obtained in the analysis of the disaster losses. As the number of revisions is increased, the average loss declines and the standard deviation of the losses becomes smaller. In effect, the distribution of the losses at the five and ten percent level becomes tight as the number of revisions are increased. This result is consistent with the hypothesis of the analysis.

4.6 Summary

In this section an attempt has been made to analyse the losses which occur when a hedging strategy is adopted by the company. As a basis of comparison, the results of the naive or buy and hold strategy have also been presented. Although the losses have been considered from a number of different points of view, from the results it is evident that the hedging strategy is dominant over the naive. The average losses may be viewed as the cost of reducing the standard deviation or risk associated with the revision strategy under consideration. With the naive strategy, however, there is no attempt made to reduce or eliminate risk. Consequently, the additional risk associated with the guarantee provision is also ignored.

Comparison of the various revision strategies over the different transaction cost levels indicates that the increases in the average losses and their respective standard deviations may be attributed to the change in the transaction cost. Although a direct linear relationship cannot be concluded from the evidence supplied by the program, the changes are consistent enough to support the above conclusion. From the results it is evident that it is in the interest of the company to minimize the transaction cost per share, to the extent that this is possible. Although the point was not pursued in the analysis, savings resulting from a reduction in the transaction cost per share provide the opportunity to increase the number of revisions and consequently reduce the risk of losses. This is not a fact arising from the results of the analysis, but a hypothesis based on the observed trends. This aspect could be considered in subsequent analyses.

In the case of lower transaction costs per share, (ie. 0 to 1½%), the evidence supports the hypothesis that by increasing the number of revisions, the deviation or risk of loss is reduced. If the transaction costs are in the range of 2 to 2½% per share traded, the trend is towards a decline in the deviations as the revisions are increased, but the fluctuations do cause concern. The fluctuations tend to occur when the number of revisions is increased to about 60 to 70 per period. Up to these levels, and after, the deviation declines with the increases in the revisions. Analysis of the actual magnitude of losses generated during these simulations reveals that extreme losses were created by the random number generating process. There is also the possibility that at higher transaction cost levels a significant underinvestment or overinvestment in the referenence portfolio may be occuring. This may be the result of the fact that in the theoretical model no consideration is given to the potential impact of transaction costs on the required investment in the reference portfolio.

The special case of zero transaction costs clearly shows the impact of the revision strategy on the losses and the deviations when compared to the naive strategy. As discussed previously, losses can only occur under these conditions if the guarantee is exercised. From the evidence presented it is clear that if the number of revisions becomes very large, the average loss and the standard deviation should approach zero, resulting in no gains or losses for the company.

Chapter 5

Conclusions

The Schwartz dissertation proposed that an equity linked life insurance contract with an asset value guarantee may be explained in terms of the Black-Scholes option valuation framework. Within this framework a model was developed which proved that under conditions of equilibrium no gains or losses can accrue to the insurance company. That is, by maintaining a fully hedged position between the investment in the reference portfolio and the call option on the reference portfolio continuously, the probability of a loss or gain becomes zero. Since under equilibrium conditions transaction costs are ignored, direct application of the model to a practical situation is not possible. Furthermore, if transaction costs are included in the model, a continuous hedging strategy is not possible because such a strategy implies infinite transaction costs. Therefore if a discrete revision strategy is adopted, the company will be subject to losses and gains. In light of this, it becomes necessary to develop a procedure to analyse the nature of the losses and to determine if the discrete strategy is superior to some benchmark, such as a naive buy and hold strategy. Superiority in this context results from obtaining a significant reduction in the risk of loss by adopting the proposed strategy.

More specifically, the objective of this dissertation was to prove that by adopting a discrete revision strategy, an insurance company can reduce the dispersion of losses which can arise as a result of a general market decline. As a basis of comparison, it was assumed that the market portfolio would be bought and held. Put another way, if management rejects the option pricing interpretation of the problem, then it is not unreasonable to assume buying the market portfolio as a viable alternative. In addition, an attempt has been made to examine the implications of variable transaction costs.

Briefly, it was shown that the hedging concept is valid within this framework as the company takes a long position in the reference portfolio and a short position in the call option on the portfolio. According to the Black-Scholes formulation, in order to form the hedged position $1/w_1$ options must be sold short against each share held. With respect to the insurance contract, since one option on the reference portfolio is sold short, to form the hedged position, only the determination of the necessary investment in the portfolio is required. This was given by the expression:

$$x(t) = X(t) \cdot N(d_1) \qquad (5\text{-}1)$$

The simulation model generated a rate of return on the portfolio at specified times, which caused the hedged position established in the previous period to be no longer valid. In order to re-establish the hedged position, the company had to either sell a portion of the portfolio or buy additional shares, depending on whether a positive or negative return was generated. In this way, the company incurred not only initial and terminal transaction costs, but also the costs of revising the portfolio.

Analysis of the intermediate calculations indicated that if positive returns were generated on the reference portfolio, the percent investment in the portfolio approached 100. On the other hand, in the case of losses on the portfolio, the proportion invested declined considerably. This is entirely within expectations. Put another way, if the value of the reference portfolio is increasing over time, then the company should be almost fully invested in the portfolio. If it is losing on the portfolio, however, it stands to reason that the investment should be reduced. It should be noted that the difference between the amount invested in the reference portfolio and the total wealth is always invested in a risk free asset. This principle accounts for the fact that even though the beneficiary exercises the guarantee, the company may still show a profit. That is, the terminal value of the reference portfolio may be less than the guaranteed amount, resulting in the guarantee being exercised, but the sum of the portfolio value plus the amount invested in the risk free asset can exceed the guaranteed amount. Under such circumstances the company will show a profit in spite of the fact that the value of the reference portfolio did not exceed the guaranteed amount. During the course of the simulations, this situation only occurred for low transaction cost levels.

In the course of the analysis, two types of losses were examined. The first may be defined as transaction cost derived losses. These are simply the result of the initial and terminal transaction costs, as well as the costs of revising the portfolio. In other words, these are the costs of adopting the hedging strategy. The terminal value of the loss per simulation is the amount which the insured must be charged for, in order to ensure that the company breaks even at the termination of the contract. The second type of loss was defined as a disaster loss. This type of loss occurs when the beneficiary exercises the guarantee, because the value of the reference portfolio is less than this amount. As mentioned above, the company experiences a loss under these conditions only if the value of the portfolio plus the amount invested in the risk free asset does not exceed the value of the guarantee. This type of a loss will occur only if there is a general decline in the market. In essence, the minimization of the dispersion of this type of a loss is the objective function of the hedging strategy.

Within the framework of these definitions, the losses were examined from a number of different points of view. Firstly, consideration was given to the average loss and its dispersion over the 500 simulations per strategy. Secondly, the disaster losses were isolated and analysed in terms of the mean and standard deviation, per case. Thirdly, the largest 5 and 10% of the losses per case were examined, in terms of the means and deviations. Lastly, a special case of zero transaction costs were simulated for each case. The criterion for the acceptance of the hedging strategy over the naive was established in terms of the behavior of the deviation of the losses over the various alternatives. That is, the deviations of the losses had to be not only less than those given under the naive strategy, but they should also decline as the number of revisions was increased.

The results of the analysis indicate that the hedging strategy is dominant over the naive. While the average losses increase as the number of revisions is increased, the dispersion of the losses decreases. It is expected that the average loss should increase as the number of revisions is increased, because of the additional transaction costs. The smaller deviations imply that the distribution of the losses is considerably tighter as the revisions are increased. In this sense, the risk associated with the strategy is reduced. That is, the average loss may be viewed as the cost of reducing the risk to the level indicated by the standard deviation. Put another way, the additional loss incurred by increasing the number of revisions is the cost of reducing the risk by the amount indicated by the incremental change in the standard deviation.

From the results of the analysis, a number of management strategies may be developed. For example, in the last chapter consideration was given to the strategy of charging the insured the average of the largest 5% losses over and above the value of the option and the present value of the guarantee. It should be noted that this proposal is simply an alternative. No effort has been made in the course of this analysis to examine the marketability of the instrument, given these additional costs. Furthermore, the degree of risk which a company may assume depends on the risk aversion of management, not on an optimal solution which one may expect. The constraints, in this sense are exogenous to the outlined procedure. Utility functions have been ignored in the course of the analysis because at any point in time, the required investment in the reference portfolio depends not on its expected value, but the current value. The expected return on the instrument, does however, depend on the expected performance of the reference portfolio. Furthermore, the level of acceptable losses also depend on utility functions.

In this analysis, an attempt has been made to reinforce the contention that the option pricing interpretation of the equity linked life insurance contract with an asset value guarantee is the correct interpretation of the problem. Furthermore, it has been shown that the adoption of the hedging strategy yields superior results to those given by the conventional buy and hold option. The problem of mortality has been excluded from the analysis, primarily to simplify the results. It is recommended that in subsequent analyses of the problem, this variable be included to determine its impact on the results. Because of the exclusion of the mortality problem, this analysis may be viewed also as an investment in a mutual fund and a term insurance policy on that investment.

This analysis has provided a viable alternative to the management of risk within the existing framework. It remains to be seen whether the necessary legal conditions will be brought about in order to provide the flexibility required to adopt the proposed strategy. Before such changes can be expected, authorities will have to re-examine the existing legislation governing the management of risk.

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Appendix A

Simulation Program

This appendix contains the simulation program which generates the supportive statistics for the analysis. The tables of the following appendices are edited from the output of this program. It should be noted that if CONT=2 then the results of intermediate calculations are also produced as output (ie. as per Appendix B), otherwise, if CONT=1, only the terminal values are provided as output. Similarly, if NAIV=2, then the simple or naive buy and hold policy is evaluated (ie. no revisions), otherwise, if NAIV=1, the general model with appropriate revisions is evaluated and written. When evaluating the naive strategy, the following changes must be made:

1) Initialize:

```
TR=0.0
DO 1500 JM=1,1
DO 1400 NREV= 10, 10, 10
```

2) Add before the comment card 'BEGIN':

```
IF (NAIV. EQ. 2) GO TO 1
```

3) Add at the bottom of page one of the program:

```
1  X(1) = 100.00
   OPVAL (1) = 0.0
   WLTH(1) = 100.00
   RPORT(1) = 100.00
```

Initial and terminal transactions costs are calculated externally because considerable changes must be made in the program in order to accomplish this internally.

SIMULATION PROGRAM

```
      DIMENSION RPORT(99),WLTH(99),OPVAL(99),X(99),RAN(99),XLAB(99),
     1CSUM(500),AVE(500),PROF(500),RPORT1(500),AVEPRT(500),VARPFT(500),
     2VARPRT(500),DUM(500),DUM1(500),XLOSS(500)
      INTEGER BARRAY(10),FMT/'F7.3'/
      INTEGER CARRAY(10),FMA/'F7.3'/
C     INITIALIZE VARIABLES
C     * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CONT=1.
      NAIV=1
      NSIM=500
      KS=NSIM-40.
      TR=0.01
      DO 1500 JM=1,4
      DO 1400 NREV=10,80,10
      NPER=NREV+1
      NPER1=NPER-1
      GAR=100.
      RFRE=.06*(10./NREV)
      VAR=0.01846*(10./NREV)
      SUM=0.
      TOT=0.
      TRBL=0.0
      NO=0.
C      BEGIN
C     CALCULATION OF INITIAL PERIOD (T=NPER-1)
C     * * * * * * * * * * * * * * * * * * * * * * * * * * *
      S=0.0
      STD=SQRT(VAR)
      FM=.08*(10./NREV)
      XX=100.
      RAN(1)=0.0
      T=NREV
      RPORT(1)=100.0
      XL=ALOG(XX/100.)
      DD1=XL+(RFRE+.5*VAR)*T
      DD2=XL+(RFRE-.5*VAR)*T
      DB=SQRT(VAR*(2*T))
      D1=DD1/DB*(-1)
      D2=DD2/DB*(-1)
      A=EXP(-RFRE*T)
      OPVAL(1)=((XX*ERFC(D1))-(100*A*ERFC(D2)))*.5
      XLAB(1)=100*EXP(-RFRE*T)+OPVAL(1)
      WLTH(1)=100.0*EXP(-RFRE*T)+OPVAL(1)
      X(1)=RPORT(1)*ERFC(D1)*.5
      IRF=WLTH(1)-X(1)
      WLTH(1)=WLTH(1)-(X(1)*TR)
```

```
C      CALCULATION OF SECOND STAGE (T=NPER-2 TO 1)
C      * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       DO 1000 K=1,NSIM
       L=NPER
       DO 100 J=2,NPER1
       T=NREV
       Z=RANDL(S,FM,STD)
       RAN(J)=Z
       JJ=J-1
       RPORT(J)=RPORT(JJ)*Z
       WLTH(J)=((WLTH(JJ)-X(JJ))*EXP(RFRE))+X(JJ)*Z
C      REVISION
C      * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       T=T-JJ
       IF(NAIV.EQ.2) GO TO 1234
       XC=ALOG(RPORT(J)/100)
       A=EXP(-RFRE*T)
       D1=(XC+((RFRE+.5*VAR)*T))/SQRT(VAR*(2*T))*(-1)
       D2=(XC+((RFRE-.5*VAR)*T))/SQRT(VAR*(2*T))*(-1)
       OPVAL(J)=((RPORT(J)*ERFC(D1))-(100*A*ERFC(D2)))*.5
       X(J)=RPORT(J)*(ERFC(D1))*.5
       GO TO 3456
 1234  X(J)=X(JJ)*Z
       OPVAL(J)=RPORT(J)-100.
       IF(OPVAL(J).LT.0.) OPVAL(J)=0.0
 3456  TRCST=TR*ABS(X(J)-X(JJ))
       WLTH(J)=WLTH(J)-TRCST
C      CALCULATE LIABILITY
       XLAB(J)=100*EXP(-RFRE*T)+OPVAL(J)
  100  CONTINUE
C      CALCULATION OF 3RD STAGE (T=0)
C      * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       Z=RANDL(S,FM,STD)
       RAN(L)=Z
       LL=L-1
       WLTH(L)=((WLTH(LL)-X(LL))*EXP(RFRE))+X(LL)*Z
       WLTH(L)=WLTH(L)-((X(LL)*Z)*TR)
       RPORT(L)=RPORT(LL)*Z
       OPVAL(L)=RPORT(L)-100.
       IF(OPVAL(L).LT.0.) OPVAL(L)=0.0
       X(L)=0.0
       XLAB(L)=100.+OPVAL(L)
       IF(CONT.EQ.1.) GO TO 97
       IF(K.GT.4) GO TO 97
       WRITE(6,91)
   91  FORMAT(6X,'INTERMEDIATE CALCULATIONS',//6X,'X-J',6X,'RPORT',
      16X,'WLTH',6X,'RAND-Z',4X,'OPVAL',5X,'LIAB')
       DO 96 J=1,NPER
       WRITE(6,95)X(J),RPORT(J),WLTH(J),RAN(J),OPVAL(J),XLAB(J)
   95  FORMAT(1X,6F10.2)
   96  CONTINUE
   97  CONTINUE
```

```
C      PROFIT AND AVE. PROFIT CALCULATIONS.
C      * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       IF(RPORT(NPER).LE.100) GO TO 110
       PROF(K)=WLTH(NPER)-RPORT(NPER)
       GO TO 990
 110   PROF(K)=WLTH(NPER)-100.
       TRBL=TRBL+1.
       NO=NO+1
       XLOSS(NO)=PROF(K)
 990   SUM=SUM+PROF(K)
       CSUM(K)=SUM
       AVE(K)=CSUM(K)/K
       IF(CONT.EQ.1.) GO TO 993
       IF(K.GT.4) GO TO 993
       WRITE(6,991)PROF(K)
 991   FORMAT('0',5X,'VALUE OF PROFIT',F10.4)
       WRITE(6,992)
 992   FORMAT('0',4X,14('*  *'))
 993   RPORT1(K)=RPORT(L)
       TOT=TOT+RPORT1(K)
       AVEPRT(K)=TOT/K
       Q=0.
       XY=0.
       DO 995 JJ=1,K
       IF(K.EQ.1) GO TO 996
       Q=Q+((RPORT1(JJ)-AVEPRT(K))**2)/(K-1)
 995   XY=XY+((PROF(JJ)-AVE(K))**2)/(K-1)
 996   VARPRT(K)=SQRT(Q)
       VARPFT(K)=SQRT(XY)
 1000  CONTINUE
       IF(CONT.EQ.2) GO TO 1400
       WRITE(6,1549)
 1549  FORMAT('1')
       WRITE(6,1300)
       WRITE(6,5300)
 1300  FORMAT(7X,'PROFIT',5X,'AVE',7X,'STD',5X,'RPORT1',5X,'AVE',7X
      2,'STD')
 5300  FORMAT(7X,'------',5X,'---',7X,'---',5X,'------',5X,'---',7X
      2,'---')
       DO 1010 KK=KS,NSIM
       WRITE(6,1011)PROF(KK),AVE(KK),VARPFT(KK),RPORT1(KK),AVEPRT(KK),VAR
      1PRT(KK)
 1011  FORMAT(2X,6F10.2)
 1010  CONTINUE
       TRBLP=(TRBL/NSIM)*100.
       WRITE(6,1012)TR,NREV,NPER,RFRE,VAR,FM
 1012  FORMAT('0','TRANS. COST',8X,F10.6,6X,'NO. OF REVISIONS',8X,I10,/,'
      1NO. OF PERIODS',5X,I10,6X,'RISK FREE RATE',10X,F10.6,/,'VARIANCE',
      211X,F10.6,6X,'MEAN',20X,F10.6)
       WRITE(6,1100)TRBL,TRBLP
 1100  FORMAT('0','DISASTER',11X,F10.0,6X,'PERCENT',17X,F10.6)
```

```
C      PLOT ROUTINE
C      * * * * * * * * * * * * * * * * * * * * * * *
       DO 1301 IJ=1,NSIM
       DUM1(IJ)=RPORT1(IJ)
 1301  DUM(IJ)=PROF(IJ)
       CALL SSORT(DUM,NSIM,3)
       CALL SSORT(DUM1,NSIM,3)
       N=DUM(1)-1
       M=DUM1(1)-1
       XMIN=N
       YMIN=M
       DX=2.0
       DY=40.
       NX=NSIM
       NY=NSIM
       CALL HISTGM (XMIN,DX,10,BARRAY,NX,PROF,6.5,FMT,7)
       WRITE(6,1310) BARRAY
 1310  FORMAT(/' THE DISTRIBUTION IS'/7X,10I6)
       CALL HISTGM (YMIN,DY,10,CARRAY,NY,RPORT1,6.5,FMA,7)
       WRITE(6,1312) CARRAY
 1312  FORMAT(/'THE DISTRIBUTION IS'/7X,10I6)
C      * * * * * * * * * * * * * * * * * * * * * * * * *
C      ADDITIONS TO MAIN. -DISASTER LEVEL CALCULATIONS
       SAM=0.0
       NE=NO
       I1=0
       IF(NO.EQ.0) GO TO 1360
       DO 1340 J=1,NO
       I1=I1+1
 1340  SAM=SAM+XLOSS(J)
       SAMMN=SAM/I1
       SVAR=0.0
       I2=0
       DO 1345 I=1,NE
       I2=I2+1
 1345  SVAR=SVAR+((SAMMN-XLOSS(I))**2)
       IF(NO.EQ.1) I2=2
       SVAR=SQRT(SVAR/(I2-1.))
       WRITE(6,1346)
 1346  FORMAT('1','  DISASTER LOSSES.')
       WRITE(6,1550)
 1550  FORMAT('0')
       WRITE(6,1350)(XLOSS(K),K=1,I2)
 1350  FORMAT(5F10.5)
       WRITE(6,1355)SAMMN,SVAR
 1355  FORMAT('0','  MEAN LOSS',10X,F10.2,2X,//'   STANDARD DEVIATION'
      1,1X,F10.2)
```

```
      GO TO 1365
 1360 WRITE(6,1361)
 1361 FORMAT('0','NO DISASTER LOSSES')
1365  WRITE(6,1550)
      WRITE(6,1370)(DUM(J),J=1,25)
 1370 FORMAT('0',2X,'LARGEST LOSSES -UP TO 5% OF TOTAL',//,(5F8.2))
      AD1=0.0
      DO 1371 K=1,25
 1371 AD1=AD1+DUM(K)
      ADM1=AD1/25.
      V1=0.0
      DO 1372 K=1,25
 1372 V1=V1+((DUM(K)-ADM1)**2)
      VSD1=SQRT(V1/24.)
      WRITE(6,1373)ADM1,VSD1
 1373 FORMAT('0',2X,'MEAN LOSS',10X,F10.2,//,2X,' STANDARD DEVIATION',1X,
     1F10.2)
      AD2=AD1
      V2=V1
      DO 1375 J=26,50
 1375 AD2=AD2+DUM(J)
      ADM2=AD2/50.
      DO 1376 J=26,50
 1376 V2=V2+((DUM(J)-ADM2)**2)
      VSD2=SQRT(V2/49.)
      WRITE(6,1550)
      WRITE(6,1377)(DUM(J),J=1,50)
 1377 FORMAT('0',2X,'LARGEST LOSSES -UP TO 10% OF TOTAL',//,(5F8.2))
      WRITE(6,1373)ADM2,VSD2
      WRITE(6,1311)
 1311 FORMAT('1')
 1400 CONTINUE
      TR=TR+0.005
 1500 CONTINUE
      STOP
      END
```

*** SUBPROGRAM DICTIONARY ***

NAME	ATTRIBUTES	REFERENCES						
ABS		72						
ALOG		34	62					
ERFC		41	44	66	67			
EXP		40	42	43	57	63	75	82
HISTGM		165	168					
RANDL		53	79					
SQRT		28	37	64	65	126	127	187
		210	221					
SSORT		155	156					
<EXIT>		231						

*** VARIABLE DICTIONARY ***

NAME	ATTRIBUTES	REFERENCES						
A		40*	41	63*	66			
ADM1		206*	209	211				
ADM2		218*	220	225				
AD1		203*	205*	206	214			
AD2		214*	217*	218				
AVE	(D)	1	110*	125	139			
AVEPRT	(D)	1	119*	124	139			
BARRAY	I*4 (D)	4	165	166				
CARRAY	I*4 (D)	5	168	169				
CONT		8*	89	111	129			
CSUM	(D)	1	109*	110				
DB		37*	38	39				
DD1		35*	38					
DD2		36*	39					
DUM	(D)	1	154*	155	157	201	205	209
		217	220	223				
DUM1	(D)	1	153*	156	158			
DX		161*	165					
DY		162*	168					
D1		38*	41	44	54*	56	67	
D2		39*	41	65*	66			
FM		29*	53	79	144			
FMA	I*4	5	168					
FMT	I*4	4	165					
GAR		17*						
I		183*	185					
IJ		152*	153	154				
IRF		45*						
I1		175*	178*	180				
I2		182*	184*	186*	187	192		
J		51*	54	55	56	57	62	66
		67	69	70	71	72	73	75
		94*	95	177*	179	201	216*	217
		219*	220	223				
JJ		55*	56	57	60	69	72	122*

		124	125					
JM		13*						
K		49*	90	102	104	107	108	109
		110	112	113	117	118	119	122
		123	124	125	126	127	192	204*
		205	208*	209				
KK		138*	139					
KS		11*	138					
L		50*	80	81	82	83	84	85
		86	87	88	117			
LL		81*	82	83	84			
M		158*	160					
N		157*	159					
NAIV		9*	61					
NE		174*	183					
NO		23*	106*	107	174	176	177	186
NPER		15*	16	50	94	101	102	104
		144						
NPER1		16*	51					
NREV		14*	15	18	19	29	32	52
		144						
NSIM		10*	11	49	138	143	152	155
		156	163	164				
NX		163*	165					
NY		164*	168					
OPVAL	(D)	1	41*	42	43	66*	70*	71*
		75	85*	86*	88	95		
PROF	(D)	1	102*	104*	107	108	113	125
		139	154	165				
Q		120*	124*	126				
RAN	(D)	1	31*	54*	80*	95		
RFRE		18*	35	36	40	42	43	57
		63	64	65	75	82	144	
RPORT	(D)	1	33*	44	56*	62	66	67
		70	84*	85	95	101	102	117
RPORT1	(D)	1	117*	118	124	139	153	168
S		27*	53	79				
SAM		173*	179*	180				
SAMMN		180*	185	194				
STD		28*	53	79				
SUM		20*	108*	109				
SVAR		181*	185*	187*	194			
T		32*	35	36	37	40	42	43
		52*	60*	63	64	65	75	
TOT		21*	118*	119				
TR		12*	46	72	83	144	229*	
TRBL		22*	105*	143	148			
TRBLP		143*	148					
TRCST		72*	73					
VAR		19*	28	35	36	37	64	65
		144						
VARPFT	(D)	1	127*	139				
VARPRT	(D)	1	126*	139				
VSD1		210*	211					
VSD2		221*	225					
V1		207*	209*	210	215			
V2		215*	220*	221				

WLTH	(D)	1	43*	45	46*	57*	73*	82*
		83*	95	102	104			
X	(D)	1	44*	45	46	57	67*	69*
		72	82	83	87*	95		
XC		62*	64	65				
XL		34*	35	36				
XLAB	(D)	1	42*	75*	88*	95		
XLOSS	(D)	1	107*	179	185	192		
XMIN		159*	165					
XX		30*	34	41				
XY		121*	125*	127				
YMIN		160*	168					
Z		53*	54	56	57	69	79*	80
		82	83	84				

*** STATEMENT LABEL DICTIONARY ***

LABEL	DEF'N	REFERENCES	
91	92	91	
95	96	95	
96	97	94	
97	98	89	90
100	76	51	
110	104	101	
990	108	103	
991	114	113	
992	116	115	
993	117	111	112
995	125	122	
996	126	123	
1000	128	49	
1010	142	138	
1011	141	139	
1012	145	144	
1100	149	148	
1234	69	61	
1300	134	132	
1301	154	152	
1310	167	166	
1311	227	226	
1312	170	169	
1340	179	177	
1345	185	183	
1346	189	188	
1350	193	192	
1355	195	194	
1360	198	176	
1361	199	198	
1365	200	197	
1370	202	201	
1371	205	204	
1372	209	208	
1373	212	211	225
1375	217	216	

1376	220	219		
1377	224	223		
1400	228	14	129	
1500	230	13		
1549	131	130		
1550	191	190	200	222
3456	72	68		
5300	136	133		

*** LOGICAL I/O UNITS DICTIONARY ***

UNIT	REFERENCES						
6	91	95	113	115	130	132	133
	139	144	148	166	169	188	190
	192	194	198	200	201	211	222
	223	225	226				

Appendix B

Summary Statistics: Intermediate Calculations

The following tables provide examples of the more important intermediate values calculated under various revision strategies. For all the tables a one percent transaction cost is assumed.

Revision Policy:

Table 1 - Annual revision

Table 2 - Every six months

Table 3 - Every four months

Label Definitions:

X-J: The actual amount invested in the reference portfolio at time t, as given by equation (3-19).

RPORT: The value of the reference portfolio at time t, as given by (3-22).

WLTH: The wealth position of the company at time t, as given by (3-23).

RAND-Z: The simulated return on the reference portfolio for the period under consideration.

OPVAL: The value of the option at time t, as defined by the generalized version of (3-17).

LIAB: The liability of the company at time t, as given by the generalized version of (3-18), or by the conditions described in equations (3-27) to (3-29).

The value of profit given at the conclusion of each simulation is the amount the company must charge the investor in order to break even, if the value is negative. The derivation of the profit is given by equations (3-30) and (3-31).

It should be noted that the initial values of the variables are identical for all the simulations. The terminal value of the investment in the reference portfolio (ie. X*J) is always zero because of the assumption that the portfolio must always be liquidated at the termination of the contract. Finally, the option value, OPVAL, must always be greater than or equal to zero.

TABLE_1

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
106.87	110.99	110.84	1.11	53.50	111.78
105.42	110.36	110.46	0.99	49.39	111.27
125.50	128.16	127.57	1.16	62.86	128.56
140.82	142.32	141.41	1.11	72.74	142.51
178.97	179.17	177.53	1.26	105.10	179.19
192.65	192.71	190.83	1.08	114.05	192.71
178.13	178.21	176.07	0.92	94.68	178.21
167.76	167.81	165.45	0.94	79.12	167.81
150.04	150.08	147.41	0.89	55.90	150.08
0.0	154.52	150.14	1.03	54.52	154.52

VALUE OF PROFIT -4.3823

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
123.64	125.92	124.80	1.26	68.03	126.31
142.41	143.65	142.08	1.14	81.95	143.82
137.88	139.48	137.89	0.97	74.00	139.71
130.75	133.09	131.50	0.95	63.64	133.40
129.16	131.83	130.29	0.99	58.08	132.16
143.73	144.97	143.09	1.10	66.43	145.09
139.40	140.77	138.85	0.97	57.36	140.89
182.65	182.66	179.86	1.30	93.96	182.66
154.75	154.76	151.52	0.85	60.59	154.77
0.0	160.77	155.71	1.04	60.77	160.77

VALUE OF PROFIT -5.0571

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
86.97	94.69	95.46	0.95	38.13	96.40
108.72	113.13	112.70	1.19	52.05	113.92
86.69	96.79	97.02	0.86	33.12	98.82
112.44	117.36	115.82	1.21	48.34	118.11
88.68	100.91	100.03	0.86	28.93	103.01
67.10	90.44	91.32	0.90	16.20	94.86
92.32	106.01	104.12	1.17	24.27	107.80
75.27	99.51	99.01	0.94	13.86	102.55
81.23	103.77	103.65	1.04	11.46	105.64
0.0	91.97	95.08	0.89	0.0	100.00

VALUE OF PROFIT -4.9210

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

TABLE 2

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
88.54	95.44	96.00	0.95	40.42	96.97
66.58	79.75	81.46	0.84	24.96	83.23
50.79	70.21	73.79	0.88	16.08	76.13
65.53	80.73	81.95	1.15	22.75	84.63
87.24	96.56	95.08	1.20	34.72	98.48
86.95	96.97	95.69	1.00	33.28	98.99
83.38	95.17	94.30	0.98	29.83	97.54
96.03	104.71	102.87	1.10	36.46	106.22
108.79	114.75	112.15	1.10	43.76	115.65
89.38	101.37	99.38	0.88	29.33	103.41
115.40	120.47	116.27	1.19	44.80	121.14
122.25	126.11	121.62	1.05	47.89	126.56
103.18	112.16	107.89	0.89	32.25	113.31
113.36	119.43	114.62	1.06	36.55	120.08
123.61	127.08	121.82	1.06	41.32	127.39
148.43	148.82	142.67	1.17	60.16	148.85
154.13	154.22	147.82	1.04	62.83	154.23
149.82	149.85	143.22	0.97	55.68	149.86
133.62	133.67	126.67	0.89	36.63	133.67
0.0	139.22	130.62	1.04	39.22	139.22

VALUE OF PROFIT -8.6014

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
109.74	113.25	112.77	1.13	57.36	113.91
123.55	125.84	124.93	1.11	67.96	126.23
128.07	130.12	129.12	1.03	70.40	130.45
143.67	144.84	143.49	1.11	83.13	145.01
136.70	138.31	136.94	0.95	74.78	138.55
151.70	152.58	150.90	1.10	86.99	152.70
161.80	162.36	160.50	1.06	94.73	162.43
172.95	173.27	171.22	1.07	103.54	173.31
200.87	200.94	198.50	1.16	129.06	200.95
181.58	181.76	179.06	0.90	107.69	181.77
156.58	157.22	154.21	0.86	80.94	157.28
151.57	152.34	149.24	0.97	73.75	152.42
161.48	161.83	158.51	1.06	80.80	161.86
179.00	179.07	175.45	1.11	95.55	179.08
184.76	184.78	180.99	1.03	98.71	184.78
210.27	210.27	206.11	1.14	121.58	210.27
224.40	224.40	219.96	1.07	133.01	224.40
260.66	260.66	255.73	1.16	166.49	260.66
259.05	259.05	253.95	0.99	162.01	259.05
0.0	236.37	228.75	0.91	136.37	236.37

VALUE OF PROFIT -7.6193

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

TABLE_3

INTERMEDIATE CALCULATIONS

X-J	RPORT	WLTH	RAND-Z	OPVAL	LIAB
94.64	100.00	100.21	0.0	46.28	101.16
105.88	109.78	109.47	1.10	54.56	110.55
100.65	105.49	105.35	0.96	49.35	106.47
102.61	107.36	107.21	1.02	50.02	108.30
91.33	98.42	98.65	0.92	40.47	99.92
98.91	104.77	104.61	1.06	45.26	105.92
92.59	100.05	100.21	0.95	39.68	101.56
103.53	109.03	108.56	1.09	46.91	110.04
117.22	120.75	119.66	1.11	56.92	121.33
117.70	121.31	120.24	1.00	56.17	121.88
106.55	112.18	111.32	0.92	46.09	113.12
116.14	120.26	118.99	1.07	52.50	120.89
129.13	131.63	129.91	1.09	62.21	131.98
148.14	149.21	146.97	1.13	78.15	149.33
160.54	161.11	158.64	1.08	88.55	161.17
161.58	162.09	159.58	1.01	88.06	162.15
168.05	168.39	165.75	1.04	92.84	168.42
175.39	175.59	172.82	1.04	98.50	175.61
168.54	168.82	165.93	0.96	90.18	168.84
169.34	169.56	166.61	1.00	89.33	169.58
148.07	148.91	145.72	0.88	67.11	148.98
152.90	153.43	150.11	1.03	69.94	153.47
139.65	140.88	137.42	0.92	55.76	140.97
140.62	141.62	138.10	1.01	54.75	141.69
135.13	136.45	132.86	0.96	47.85	136.54
150.14	150.36	146.44	1.10	59.88	150.37
160.97	161.00	156.88	1.07	68.68	161.00
169.02	169.02	164.74	1.05	74.85	169.02
166.56	166.56	162.17	0.99	70.48	166.56
150.32	150.32	145.67	0.90	52.30	150.32
0.0	152.41	146.15	1.01	52.41	152.41

VALUE OF PROFIT -6.2643

* ** ** ** ** ** ** ** ** ** ** ** ** ** *

Appendix C

Summary Statistics: Naive Strategy

This appendix provides the summary statistics for the naive strategy. This strategy simply assumes that the initial premium of $100.00 is invested in a portfolio of securities and held for the duration of the contract. No portfolio revision occurs during this period.

The label definitions of the previous appendices apply to the tables exhibited. The number of periods is synonymous to the revision periods of the previous tables.

TABLE 1

TRANSACTIONS COST 0%.

NAIVE STRATEGY

NUMBER PERIODS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-0.33	2.64	255.03	115.98
20	-0.59	3.78	242.45	115.00
30	-0.35	2.98	242.58	108.42
40	-0.40	2.78	236.37	103.15
50	-0.57	3.67	246.58	115.07
60	-0.58	3.95	245.90	113.61
70	-0.77	4.79	240.38	113.66
80	-0.44	3.08	246.09	113.28

TABLE 2

TRANSACTIONS COST 0%.

DISASTER LOSSES; NAIVE STRATEGY

NUMBER PERIODS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	15	3.00	-11.15	10.95
20	22	4.40	-13.36	12.69
30	10	2.00	-17.43	12.69
40	13	2.60	-15.38	8.52
50	16	3.20	-17.74	11.15
60	14	2.80	-20.66	12.35
70	21	4.20	-18.41	15.20
80	16	3.20	-13.65	11.11

Appendix D

Summary Statistics: Overall Losses

Label Definitions

AVERAGE LOSS:

The mean loss incurred by the company over five hundred simulations. This is the average amount the company must charge the insured in order to break even under the particular revision strategy.

AVERAGE PORTFOLIO:

The mean value of the reference portfolio over five hundred simulations, given the particular revision strategy.

Table 5 is the special case of zero transaction costs. It clarifies the effect of increasing the number of revisions by eliminating the accumulating effect of the transaction costs.

TABLE 1

TRANSACTIONS COST 1%.

NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-6.98	3.34	255.03	115.98
20	-8.03	2.81	242.45	115.00
30	-8.84	2.68	242.58	108.42
40	-9.05	2.71	236.37	103.15
50	-9.96	2.86	246.58	115.07
60	-10.62	2.73	245.90	113.61
70	-10.84	3.00	240.38	113.66
80	-11.41	2.97	246.09	113.28

TABLE 2

TRANSACTIONS COST 1.5%

NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-10.44	4.00	250.16	109.50
20	-11.66	3.67	240.87	107.05
30	-13.05	3.82	243.04	113.01
40	-13.81	3.52	237.12	101.81
50	-14.81	3.78	245.05	104.95
60	-15.63	4.15	245.48	113.91
70	-16.66	3.95	247.31	109.37
80	-17.26	4.02	246.39	104.39

TABLE 3

TRANSACTIONS COST 2%.

NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-13.66	5.24	245.43	115.24
20	-15.29	5.32	232.92	113.95
30	-17.49	4.73	247.00	110.13
40	-18.86	4.96	245.90	110.58
50	-20.02	4.92	246.88	105.77
60	-20.86	4.95	244.20	106.81
70	-22.03	5.11	218.08	106.23
80	-23.02	5.66	250.38	115.76

TABLE 4

TRANSACTIONS COST 2.5%

NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-16.88	6.01	240.07	110.74
20	-19.33	5.62	238.75	104.19
30	-21.57	5.69	240.41	106.14
40	-23.26	6.37	240.69	115.24
50	-25.34	6.36	256.88	114.91
60	-25.52	5.59	233.24	99.44
70	-26.90	6.39	237.85	107.96
80	-28.49	6.55	240.84	108.64

TABLE 5

TRANSACTIONS COST 0%.

NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE PORTFOLIO	STANDARD DEVIATION
10	-0.05	1.89	255.03	115.98
20	-0.21	1.51	242.45	115.00
30	-0.17	1.14	242.58	108.42
40	0.08	1.01	236.37	103.15
50	0.00	1.02	246.58	115.07
60	-0.10	0.83	245.90	113.61
70	0.02	0.88	240.38	113.66
80	0.02	0.80	246.09	113.28

Appendix E

Summary Statistics: Disaster Losses

The following tables summarize the losses which the company incurs if the guarantee is exercised.

Label Definitions

NUMBER LOSS: The number of times the guarantee was exercised over 500 simulations, per revision strategy.

PERCENT LOSS: The number of losses as a percent of the total number of simulations.

AVERAGE LOSS: This is the average dollar loss incurred by the company as a result of the guarantee being exercised.

Table 5 is the special case of no transaction costs.

TABLE 1

TRANSACTIONS COST 1%.

DISASTER LOSSES

NUMBER REVISIONS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	15	3.00	-4.11	5.55
20	22	4.40	-5.80	3.58
30	10	2.00	-7.38	2.82
40	13	2.60	-6.70	2.91
50	16	3.20	-6.47	2.60
60	14	2.80	-7.77	2.18
70	21	4.20	-7.23	2.94
80	16	3.20	-7.55	2.00

TABLE 2

TRANSACTIONS COST 1.5%

DISASTER LOSSES

NUMBER REVISIONS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	12	2.40	-8.63	6.64
20	20	4.00	-7.88	3.76
30	19	3.80	-8.68	2.91
40	19	3.80	-10.44	2.41
50	13	2.60	-10.96	3.80
60	13	2.60	-9.95	2.93
70	22	4.40	-12.54	3.08
80	13	2.60	-12.87	3.43

TABLE 3

TRANSACTIONS COST 2%.

DISASTER LOSSES

NUMBER REVISIONS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	21	4.20	-9.03	4.47
20	23	4.60	-10.94	3.89
30	11	2.20	-11.63	2.70
40	15	3.00	-12.14	2.10
50	8	1.60	-14.55	2.29
60	9	1.80	-15.06	3.37
70	17	3.40	-15.92	4.11
80	16	3.20	-15.48	2.34

TABLE 4

TRANSACTIONS COST 2.5%

DISASTER LOSSES

NUMBER REVISIONS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	18	3.60	-10.08	6.19
20	15	3.00	-11.91	3.31
30	9	1.80	-14.47	3.28
40	14	2.80	-15.09	2.83
50	10	2.00	-15.69	2.52
60	16	3.20	-18.95	4.54
70	19	3.80	-19.36	4.79
80	17	3.40	-21.76	3.47

TABLE 5

TRANSACTIONS COST 0%.

DISASTER LOSSES

NUMBER REVISIONS	NUMBER LOSS	PERCENT LOSS	AVERAGE LOSS	STANDARD DEVIATION
10	15	3.00	0.16	5.50
20	22	4.40	-0.62	3.31
30	10	2.00	-1.65	2.16
40	13	2.60	-0.88	2.46
50	16	3.20	0.00	2.26
60	14	2.80	-0.82	1.68
70	21	4.20	-0.11	2.66
80	16	3.20	0.16	1.47

Appendix F

Summary Statistics: Largest Losses

These tables summarize the magnitude of losses the company incurs given the revision policy and the transaction costs. Table 5 is again the special case of no transaction cost.

Label Definitions:

AVERAGE LOSS: FIVE PERCENT

The mean of the 25 largest losses occuring over 500 simulations, under each revision strategy.

AVERAGE LOSS: TEN PERCENT

The same as above, except the largest 50 losses are considered.

It should be noted that these losses include exercising the guarantee, as well as the losses created by transaction costs.

TABLE 1

TRANSACTIONS COST 1%.

LARGEST LOSSES

	FIVE PERCENT		TEN PERCENT	
NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE LOSS	STANDARD DEVIATION
10	-14.95	2.01	-13.41	1.83
20	-14.53	1.79	-13.35	1.53
30	-15.60	2.04	-14.26	1.73
40	-15.67	2.46	-14.34	1.98
50	-16.83	1.98	-15.52	1.69
60	-17.80	1.89	-16.23	1.77
70	-19.06	2.92	-17.07	2.51
80	-19.16	2.93	-17.43	2.41

TABLE 2

TRANSACTIONS COST 1.5%.

LARGEST LOSSES

	FIVE PERCENT		TEN PERCENT	
NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE LOSS	STANDARD DEVIATION
10	-20.18	2.14	-18.33	2.04
20	-20.47	2.08	-18.82	1.91
30	-23.20	2.95	-21.10	2.60
40	-22.66	2.19	-21.00	2.00
50	-24.74	2.82	-22.71	2.48
60	-27.04	4.46	-24.40	3.69
70	-27.13	3.54	-24.94	2.98
80	-27.86	4.22	-25.59	3.40

TABLE 3

TRANSACTIONS COST 2%.

LARGEST LOSSES

	FIVE PERCENT		TEN PERCENT	
NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE LOSS	STANDARD DEVIATION
10	-27.00	3.93	-24.19	3.46
20	-28.83	9.29	-25.33	6.97
30	-30.32	5.86	-27.16	4.70
40	-31.25	3.67	-28.93	3.08
50	-33.75	3.61	-30.73	3.41
60	-34.17	2.99	-31.70	2.80
70	-36.09	4.67	-33.09	3.97
80	-39.26	5.41	-35.58	4.69

TABLE 4

TRANSACTIONS COST 2.5%.

LARGEST LOSSES

	FIVE PERCENT		TEN PERCENT	
NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE LOSS	STANDARD DEVIATION
10	-32.25	3.06	-29.07	3.18
20	-34.36	3.77	-31.29	3.50
30	-37.94	5.46	-33.94	4.85
40	-41.48	8.19	-36.82	6.70
50	-43.06	6.26	-38.85	5.36
60	-39.95	4.06	-37.09	3.59
70	-43.74	4.92	-40.53	4.20
80	-47.20	7.54	-43.05	6.09

TABLE 5

TRANSACTIONS COST 0%.

LARGEST LOSSES

	FIVE PERCENT		TEN PERCENT	
NUMBER REVISIONS	AVERAGE LOSS	STANDARD DEVIATION	AVERAGE LOSS	STANDARD DEVIATION
10	-4.99	2.33	-3.68	1.89
20	-4.49	1.86	-3.27	1.58
30	-3.34	1.38	-2.51	1.14
40	-2.38	1.17	-1.71	0.95
50	-2.62	1.03	-1.84	0.92
60	-2.28	0.73	-1.77	0.64
70	-2.15	0.81	-1.58	0.71
80	-1.69	0.67	-1.29	0.56