APPLICATION OF MODERN CONTROL TECHNIQUES TO POWER SYSTEMS

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in the Department of

Electrical Engineering

We accept this thesis as conforming to the

required standard

Research Supervisor...

Members of Committee...

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THE UNIVERSITY OF BRITISH COLUMBIA

October, 1971

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ABSTRACT

A power system may be subjected to different types of disturbances. The control strategy to be taken in order to preserve system stability depends on the severity of the disturbance.

For very severe disturbances, power system stability can be improved by sudden changes in the electric power network such as the insertion of braking resistors, generator dropping or load shedding. A unified treatment of optimum switching is presented by considering the switching instants to be elements of a generalized control vector. Dynamic optimization is then applied to determine optimum switching instants.

Less severe disturbances can be overcome by employing governor and/or voltage regulator controls. The governor control problem for a large signal model of interconnected power plants is investigated via the multi-level concept. A two-level controller for interconnected power plants is discussed. Each plant has a first-level local optimal or suboptimal controller. The second level of control is an intervention control performed by a central co-ordinator. If a sudden system disturbance causes the system angular acceleration to exceed preset tolerances, a priority interrupt to the central co-ordinator initiates intervention control. Angular velocity deviations of all plants are transmitted to the co-ordinator. This data is used to generate coefficient data for each plant. On receiving its coefficient data, each plant generates a local second-level intervention control which augments first-level local control.

The Load-Frequency Control problem, due to minor or routine disturbances caused by load changes, is investigated. Since the incremental power demand in a power system is not always known a priori, direct application of the

optimum linear-state regulator to Load-Frequency Control is not possible. Furthermore, Load-Frequency Control generally requires the use of an integral-type control operation to meet the system operating specifications. This requirement is introduced into the formulation of the optimum Load-Frequency Control problem presented in this thesis.

Two methods are suggested for demand identification. The first method makes use of differential approximation. The second method makes use of a Luenberger observer to identify unmeasured states. The optimum control is a linear function of measured states, identified unmeasured states, and the identified incremental power demand.

A method is given for solving, suboptimally, the problem of optimum-load frequency sampled-data control with either unknown deterministic power demand or randomly varying system disturbances. It is shown how to modify an optimum continuous control to obtain optimum control in the case of discrete-data transmission and unknown deterministic demand.

The case of random power demand and random disturbances is treated by introducing an adaptive observer. A three stage systematic design procedure is given. The effectiveness of Load-Frequency Control using an adaptive observer is illustrated by an example.

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ACKNOWLEDGEMENT

I am indebted to my Supervisor, Dr. E.V. Bohn, for his guidance and assistance throughout the period of this research.

Thanks are due to Dr. Y.N. Yu for his interest and his valuable comments on the draft. Reading the Draft by Dr. M.S. Davies and Dr. B.J. Kabriel is duely appreciated. I am grateful to my colleagues Terry Curran and Emile Struyk for proof reading the manuscript.

I also wish to thank Miss Linda Morris for her patience in typing the manuscript.

The financial support by the National Research Council of Canada, grant 67-3134 (1968-1971), and the Scholarship by B.C. Telephone Company (1970) are appreciated.

Finally, in lieu of the time I might have otherwise spent with them, I would like to thank my wife Nadia and my son Ehab for their patience and understanding.

1. INTRODUCTION

Interconnection between adjoining power systems is an inevitable development in the utility industry because it offers the mutual benefits of inherent economy, reliability of operation and improved stability. The prospect of transferring large blocks of power over long transmission distances between neighbouring systems in different time zones, to assist each in turn through its respective peak load period is very attractive and a great encouragement to large scale interconnection. In its realization, many new problems and difficulties are confronting the power utilities. With ever increasing demands for electric power and the desire to improve still further the quality of service, utilities must meet the challenge of seeking improved methods of regulating power generation.

1.1 System Decomposition

An interconnected power system is a complex system whose primary goal is to furnish electrical energy as required by customers and as long as required. Requisite to this objective is a quality of service characterized by stable electrical frequency and voltage and by continuity in time.

A general solution to the problem of controlling the whole system such that all the operating objectives are satisfied at all times is impractical. Decomposition of the system "space-wise" and "time-wise" is necessary to solve such a complicated problem.

1.1.1 Spatial Decomposition

We can take the whole power system and decompose it into subsystems or areas. The need for subdivision into areas will depend on: (1) The complexity and computational difficulty associated with a single control model.

(2) The geographical location of generating sources and the associated load allocation and political boundaries. (3) Overall consideration of reliability.

Decomposition is generally guided by the specific nature of the network. For control reliability it is usually required to have each area as self-sufficient as possible in generating capacity and in interconnection support. In a power system consisting of strong areas connected by weak ties the following policies are usually recommended:

a) each area has the responsibility of seeing that its internal disturbances do not disrupt the interties or impair service in other areas;

b) both terminal areas are responsible for counteracting disturbances on the interties between those areas.

1.1.2 Sequential Decomposition

Even for each area, single control policy would be inadequate for all operating states. Characterization of power system operation by operating states has been investigated in [1], according to "conditions for operation". It is,in effect, decomposing the total operating problem into a set of sequential problems, corresponding essentially to "before, during and after" a severe system disturbance.

According to [1], the power system can be considered in one of the following states;

(a) Preventive operating state (normal operation):

In this state the power system is being operated so that the demands of all customers are satisfied at standard frequency and voltage. The control problem in this state is to continue indefinitely the satisfaction of customer demand without interruption and with minimum cost.

(b) Emergency operating state:

Aside from the causes that brings the system to that state, an emergency operating state comes about when the rating of some components are exceeded, or when the voltage cannot be maintained at a safe minimum, or when the system frequency starts to decrease toward a value at which motors will stall, or when the electrical system is in the process of loosing synchronism. The control objective in this state is to relieve the system distress and forestall further degradation while satisfying a specified customer demand. Economic considerations become secondary in this state.

(c) Restorative state:

This is the state where the power system will be in partial load operation, when some customer loads has been lost. Usually this is the aftermath of an emergency. The control objective in the restorative state is the safe transition from partial to 100 percent satisfaction of all customer demand in minimum time.

It is hard to define general boundaries between the different states of operation. This is because of the different design and nature for different parts of the system. To clarify this point,consider for example two areas of the same capacity. Two equal disturbances at both areas may cause instability to one area while the other may be able to survive the disturbance. The first area would be in the emergency state and emergency control action should be taken according to the nature of disturbance. For example,braking resistors may be necessary to decelerate the accelerating generators. For the second area, because of the design of its generators, stability could be maintained by using existing controls such as governors and/or voltage regulators. There is a limit, however, on the amount of disturbance after which the second area must take emergency control measures.

This thesis is concerned with the problem of control in the emergency and preventive state. In the next Section, we are going to examine some control aspects related to these states from the energy balance and dynamic behaviour viewpoint, along with the current work and thesis layout.

1.2 Background and Thesis Layout

During steady state operation of power system there is equilibrium between the mechanical power input of generators and the sum of losses and electrical power output. Non-equilibrium can occur as a result of either a change in mechanical input or a change in electrical output. However, from a practical viewpoint, it is obvious that changes in electrical output can occur almost instantaneously because of change in the network, whereas the mechanical input cannot change nearly so fast. The difference between input and output power is known as the accelerating or decelerating power. The immediate sign of non-equilibrium is system acceleration (the accelerating or decelerating power divided by the inertia constant).

Depending on the severity of the disturbance and consequently the acceleration, a power system can be in any of the operating states mentioned before. The controls for each state depend on the severity of disturbance. The following points briefly outline the contents of this thesis.

1. For a very severe disturbance caused by change in the network, a discontinuous form of control is required to prevent excessive system upset. This control problem has been investigated[2,3,4,5,6,7], but no unified technique has been given to determine the optimum switching times. In Chapter 2, the problem of optimum network switching is investigated[8]. A unified treatment of optimum switching is presented by considering the switching instants to be elements of a generalized control vector. Dynamic optimization is then applied to determine optimum switching instants.

2. Moderate disturbances can be overcome by employing: (a) faster control of prime movers[9]; (b) high speed excitation system with supplementary signals for providing strong damping of swings. The above continuous type of control action can be augmented by discontinuous control action.

The solution to the linear regulator problem with a quadratic cost index is well known. Its practical application to the control of power systems, however, poses severe problems. These are the modelling of high order non-linear systems by a linear system, the computation, and transmission of large quantities of data between different plants. Some form of suboptimal control is therefore essential. Presently known suboptimal controllers, such as specific optimum controllers, reduce somewhat the severity of the problem. However, it is difficult to account for system interaction and non-linearities in such controllers without requiring continuous communication of large amounts of data between the plants. Significant improvement in design technique and system response over the conventional methods has been achieved by the application of the linear regulator problem to power systems with some degrees of sub-optimality[10,11,12].

In Chapter 3, the possibility of implementing a simple control for a large signal model of interconnected power plants is examined via the concept of multi-level control. A two-level control is proposed. The first level consists of independent linear subsystems, which have local feedback controllers. The second-level controller co-ordinates the sub-systems by an intervention open-loop control[13].

3. Minor or routine disturbances causes small deviations from the fixed references and are corrected by governor and/or voltage regulator controls. The main problem in this state is Load-Frequency Control (LFC) problem*. This is the problem of regulating the power output of electric generators within a prescribed area such that each area satisfies its own demand.

For improved dynamic performance, the linear regulator solution was adopted by Elgerd[14]. In reference 14, the state deviations are expressed in terms of the final states, the states the system is supposed to reach after a certain demand is applied. The final states cannot be known unless the demand is known a priori which is not the situation in practice. A feasible optimum control must identify the unknown demand.

In Chapter 4, two methods are suggested for identifying the demand. The first method uses differential approximation and is suitable for slowly changing demands. In the second method the system states are augmented by a demand equation, and a minimal order Luenberger observer is utilized. In the second method, it is assumed that the tie-line and frequency deviations

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* See Appendix I for detailed definition.

are the only measurements available. A modified form of the regulator problem solution[15] is considered, which gives, in conjunction with the demand identifier, a feasible suboptimum control for the LFC problem.

4. A practical situation which must be considered is that the tie-line deviation may not be instantaneously available for utilization in the controller. In addition, for practical systems, both plant and measurement devices are disturbed by noise. In the last part of the thesis, Chapter 5, these two points are investigated. The sampled-data regulator[16], which is suitable for continuous systems that have a communication link in the feedback loop, is considered. Because of the noise present in the system, the system states are estimated by a suboptimal filter. The filter does not require detailed a priori knowledge of noise statistics. The filter is essentially an adaptive observer and is based on adaptively changing a scalar gain. Updating the gain is based on minimizing an instantaneous cost index. The cost index realizes a trade off between a deterministic performance index and an estimation error. The proposed scheme is simple and does not require excessive computer memory or computation time.

2. OPTIMUM NETWORK SWITCHING IN POWER SYSTEMS

2.1 Introduction

Power system stability can be improved by discontinuous changes in the electric power network[2]. The actions to be taken in order to bring the system to equilibrium after a severe disturbance depend on the nature of the disturbance, brief or prolonged. Sometimes there are different actions to choose from. The choice of action not only depends on the type of disturbance, but also on economical and practical considerations.

Consider, for example, a brief disturbance, lasting typically a fraction of a second, such as a short circuit cleared in normal time. Such a fault near a generating plant accelerates the generators. This disturbance can be counteracted by a short application of a shunt braking resistor located at the generating plant[3].

Other disturbances, for example the loss of a large load, produce a sustained non-equilibrium condition. The control action should likewise be sustained, and in this event it is logical to disconnect an amount of generating capacity equal to the lost load.

In the latter case, prolonged application of a braking resistor would be effective, but it would be uneconomical to provide resistors of the required rating. In the former case, dropping of generation would be inappropriate unless it could be restored to service rapidly and with accurate timing.

Kimbark has discussed the possibilities of improving power system stability by control of discontinuous changes in the electrical network[2]. In his paper he does not present a systematic method for evaluating the switching times. A practical implementation has been reported by the British

Columbia Hydro and Power Authority[3]. A braking resistor is switched on at time of isolating the faulted line and switched off at a later time. This time was determined by digital simulation and numerical experimentation.

Transient control by using network parameters and by series capacitor switching has recently received considerable attention[4,5,6,7]. The methods discussed in references 4 and 5 are based on derivation of switching functions from energy flow considerations. These methods do not appear suitable for the case of third or higher order machine models or for multi-machine systems. References 6 and 7 treat series capacitor switching by optimal control theory. The cost index used is minimum time. No systematic iterative numerical method is given for obtaining the solution. A trial-and-error approach using an analog computer with subsequent digital computer refinement is used. Furthermore, the case of general network switching with cost indices other than minimum time is not discussed.

It is the purpose of this Chapter to present a systematic method for evaluating optimum switching times. The method is general and is applicable to any of the possibilities discussed by Kimbark. In determining the critical switching time, the method appears to offer both computational as well as practical advantages over stability approaches based on the construction of Liapunov functions[20,21].

2.2 Problem Formulation for Optimum Switching Times

A steepest descent method for solving a combined continuous and bang-bang optimum control problem has been proposed by Vachino[22]. This is based on considering the switching instants for the bang-bang control as a parametric control and augmenting it with a continuous control to form a generalized control vector. A simplified version of this method seems to be applicable to the problem of optimum switching times in power systems, as formulated by Kimbark. This is a consequence of the structural form of the differential equations describing power system dynamics as well as the relatively small and known number of switching instants.

The differential equations describing the state of the system have the form

$$\dot{x} = f(x, \alpha_k) \tag{2.1}$$

where x is an n state vector and α_k is a system parameter vector which is constant for $t_{k-1} \leq \tau < t_k$ and which can change at any of N switching instants $t_1, t_2, \ldots t_N$. The problem to be considered is to choose the switching instants so that a cost index

$$\phi = \phi(x_f), \quad x_f \triangleq x(t_f) \tag{2.2}$$

is minimized at a final time $t_f = t_{N+1}$ defined by a given stopping condition. The initial state, $x(t_o) \triangleq x_o$, is considered known.

In formulating a solution to the optimization problem it is convenient to consider the set of differential equations given by (2.1) as one differential equation of the form

$$\dot{x} = F(x, \alpha, v), \quad x(t_o) = x_o \tag{2.3}$$

where

$$F(x, \alpha, v) \triangleq \sum_{k=1}^{N+1} f(x, \alpha_k)[h(t-t_{k-1}) - h(t-t_k)] \tag{2.4}$$

In (2.4), h(t) is the unit step function, α is a composite parameter vector formed from the α_k and

$$v' = (t_1, t_2, \ldots t_N) \tag{2.5}$$

is considered a generalized control vector (prime denotes transposition). The optimization problem defined by (2.2), (2.3), (2.4) and (2.5) can be formulated as an optimal control problem[23]. The simplest numerical method of solution of optimal control problems is steepest descent in control function space. However, the parametric form of the control vector (2.5) and the discontinuities in (2.4) make it necessary to modify this method.

The required modification is obtained by considering the incremental change in state δx resulting from an incremental change δv in the control vector (see (2.5)). Linearization of (2.3) yields:

$$\delta\dot{x} = F_x \delta x + F_v \delta v, \quad \delta x_o = 0 \tag{2.6}$$

where

$$F_x \triangleq \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial F_n}{\partial x_1} & \cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \tag{2.7}$$

To apply a steepest descent method, the incremental cost $\delta\phi$ must be expressed in terms of the incremental control δv. The desired expression is

$$\delta\phi = \phi_x' \delta x_f = \phi_v' \delta v \tag{2.8}$$

where

$$\begin{aligned} \phi_x' &\triangleq \left[\frac{\partial \phi}{\partial x_1}, \ldots \frac{\partial \phi}{\partial x_n}\right] \\ \phi_v' &\triangleq \left[\frac{\partial \phi}{\partial t_1}, \ldots \frac{\partial \phi}{\partial t_N}\right] \end{aligned} \tag{2.9}$$

In (2.8) ϕ_x is the gradient of ϕ with respect to x, and ϕ_v is the gradient of ϕ with respect to v. In order to find ϕ_v an alternative expression must be found for $\delta\phi$. This expression is obtained by use of the identity (see (2.6)),

$$\frac{d}{dt}(p'\delta x) = \delta x'[\dot{p}+F_x'p] + p'F_v\delta v \tag{2.10}$$

where p is the costate vector which is defined by equating the coefficient of δx in (2.10) to zero:

$$\dot{p} = -F_x'p \tag{2.11}$$

Integrating (2.10) with the initial condition $\delta x_o = 0$ and choosing

$$p_f = -\phi_x(x_f) \tag{2.12}$$

for the terminal condition, it is seen that

$$\delta\phi = \phi_x'\delta x_f = -p_f'\delta x_f = -\int_{t_o}^{t_f} H_v'\delta v\, dt \tag{2.13}$$

where $H \triangleq p'F$ is the Hamiltonian. Since δv is constant it follows by comparison of (2.8) and (2.13) that

$$\phi_v = -\int_{t_o}^{t_f} H_v dt \tag{2.14}$$

The evaluation of H_v is straightforward. From (2.4) it is seen that

$$\frac{\partial H}{\partial t_k} = p'\frac{\partial F}{\partial t_k} = p'[f(x, \alpha_k)-f(x, \alpha_{k+1})]\Delta(t-t_k) \tag{2.15}$$

since

$$\frac{\partial}{\partial t_k} h(t-t_k) = -\Delta(t-t_k) \tag{2.16}$$

where $\Delta(t)$ is the unit impulse function. Substituting (2.15) into (2.14) yields

$$\phi_v = -\begin{bmatrix} p'(t_1)[f_1(t_1) - f_2(t_1)] \\ \cdot \\ \cdot \\ \cdot \\ p'(t_N)[f_N(t_N) - f_{N+1}(t_N)] \end{bmatrix} \tag{2.17}$$

where, for notational convenience, $f_k(t_k) \triangleq f(x(t_k), \alpha_k)$, $f_{k+1}(t_k) \triangleq f(x(t_k), \alpha_{k+1})$. It follows from (2.8) that the steepest descent adjustment is given by:

$$\delta v = -\ell\phi_v \qquad (2.18)$$

where $\ell > 0$ is a step size.

2.3 Algorithm

A systematic method for evaluating v is given by the following algorithm.

1. Starting with a nominal control $\bar{v}$ and a given initial state x_o integrate (2.3) forward in time from t_o until the stopping condition is satisfied, which defines t_f.
2. Integrate (2.11) backward in time using (2.12) to initialize the costate vector.
3. Use (2.18) to update the control ($v = \bar{v} + \delta v$).
4. Terminate the computation when $||\phi_v||$ is sufficiently small.

2.4 Applications

To illustrate its effectiveness, the general technique developed in the previous section is applied to the following problems:

I. To find the critical switching time for:

(a) One machine infinite bus system (second order model; constant voltage behind transient reactance).

(b) One machine infinite bus system (third order model; taking account of field flux linkage changes).

(c) Multi-machine system (second order model for each machine).

II. To find the optimum time of switching off a braking resistor for (a), (b) and (c).

2.4.1 Critical Switching Time

The system equations for the multi-machine case are given by:[20]

$$\dot{\delta}_i = \omega_i$$

$$\dot{\omega}_i = \frac{1}{M_i}[P_{mi}-D_i\omega_i-P_{ei}],\ i=1,2,\ \ldots,\ N_m \tag{2.19}$$

where N_m is the number of machines. M_i, P_{mi} and D_i are the inertia constant, the mechanical input power and the damping coefficient for the ith machine, respectively.

For a single machine the electrical output power P_{ei} and the damping coefficient D_i in (2.19) are defined by,

$$P_e = C_1\left(\frac{\psi_F}{\tau_o}\right)^2 + (C_2\cos\delta + C_3\sin\delta)\left(\frac{\psi_F}{\tau_o}\right)$$
$$+ C_4\sin 2\delta + C_5\sin^2\delta + C_6\cos^2\delta. \tag{2.20}$$

$$D = \frac{C_{11}+C_{12}}{2} + \frac{C_{11}-C_{12}}{2}\cos 2\delta \tag{2.21}$$

where ψ_F = a flux linkage proportional to field flux linkage, and τ_o is the direct-axis transient open circuit time constant.

In the case of a third order model the rate of change of ψ_F is given by:

$$\dot{\psi}_F = C_7+C_8\sin\delta+C_9\cos\delta+C_{10}\psi_F \tag{2.22}$$

where:

$$C_7 \triangleq -C_{10}\psi_F(0)-C_9\cos\delta(0)-C_8\sin\delta(0) \tag{2.23}$$

In the case of a second order model, ψ_F in (2.20) is replaced by $\psi_F = \psi_F(0)$ = constant. The parameters C_1, C_2, ..., C_{12} are the elements of the parameter vector α(see (2.4)) and their values depend on machine parameters and network impedances (see Appendix II).

In the multi-machine case the quantities in (2.19) are given by:[20]

$$P_{ei} \triangleq E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{N_m} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \tag{2.24}$$

$$D_i = d_i = \text{constant} \quad i = 1, 2, \ldots, N_m \tag{2.25}$$

where: $E_i \angle\delta_i$ = internal phasor voltage of the i<u>th</u> machine, $G_{ij}+jB_{ij}$ = short circuit transfer admittance between the i<u>th</u> and the j<u>th</u> machine, and G_{ii} is the load conductance at the i<u>th</u> machine bus (Gij is neglected in (2.24)).

The rotor angle of each machine is fixed with respect to the electrical phase angle of the voltage behind the transient reactance. The angles are measured with respect to a common axis rotating at synchronous speed (the infinite bus in the one machine case). In the multi-machine case the parameters G_{ii} and B_{ij} are the elements of the α vector.

Figure (2.1) illustrates situations (a) and (b). A salient pole synchronous generator is connected to an infinite bus by two transmission lines. The machine supplies a complex power P + jQ at a terminal voltage V_t. The infinite bus has a fixed voltage V_o. A sudden three phase symmetrical short circuit to ground is considered to occur at position (x) at t = 0. The faulted line section between A and B is disconnected after 8 cycles at time t_o. The faulted line is reconnected after m cycles with the fault cleared.

Figure (2.2) illustrates the power angle diagram for the three stages: (a) fault on, (b) faulted line disconnected, (c) line restored with fault cleared. The critical switching time (in cycles) is the maximum number of cycles, m, for which the machine stays in synchronism with the infinite bus. An equivalent definition can be given by the equal area criterion ($A_1 = A_2$) which defines the critical switching time t_c at $\delta = \delta(t_c)$.

It is evident from Fig. (2.2), that the switching time equals the critical switching time, $t_s = t_c$, when the conditions

$$\begin{aligned} \omega(t_f) &= 0 \\ P_m(t_f)-P_e(t_f) &= 0 \end{aligned} \tag{2.26}$$

are satisfied, where t_f is the time of the first swing. A possible cost index is t_f. The switching time becomes critical when t_f is a maximum subject to the conditions (2.26). However, the terminal conditions on the costate are then unknown and a more involved iterative procedure is required. This complication can be avoided by choosing a penalty function cost index based on (2.26) as a target set. That is, a cost index of the form

$$\phi = 0.5[W_1\omega^2(t_f)+W_2(P_m(t_f)-P_e(t_f))^2] \tag{2.27}$$

can be chosen, where W_1 and W_2 are positive weighting factors. The choice of (2.27) allows the simple algorithm given in the previous Section to be used. The stopping condition, which defines t_f, is taken to be the instant of time when one of the following conditions is satisfied;

$$\begin{aligned} \omega(t_f) &< 0 \quad , \\ P_m(t_f)-P_e(t_f) &> 0 \quad . \end{aligned} \tag{2.28}$$

Problems I(a) and (b) are defined by (2.19)-(2.23); the value of α for various stages are shown in Table (2.1). The pre- and post-fault value is α_2, during the fault it is α_F and with the faulted line disconnected it

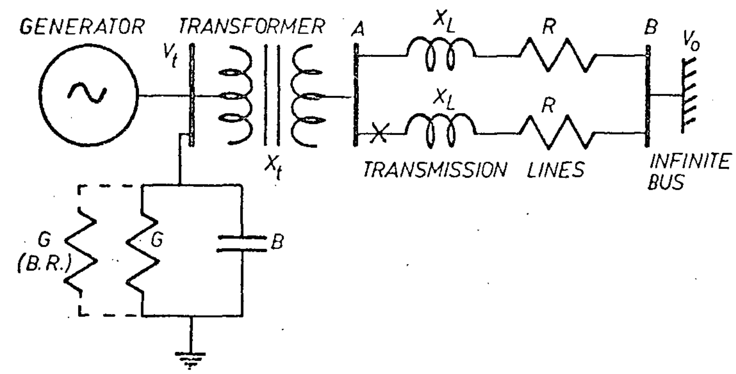

Fig. (2.1) One machine infinite bus system

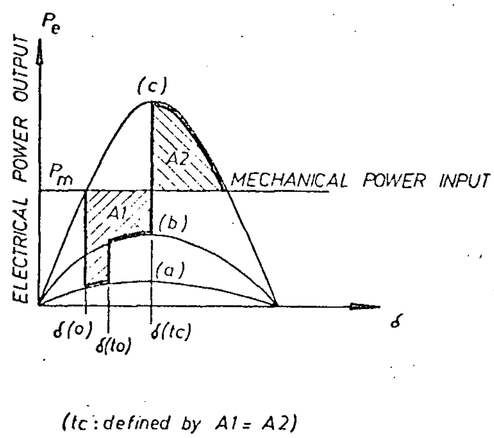

Fig. (2.2) Power angle diagram

Table (2.1)

Prob I (a)(b)	$\alpha 1$	$\alpha 2$			αF
Prob II (a)(b)		$\alpha 3$	$\alpha 1$	$\alpha 2$	
C_1	0.226	0.244	0.358	0.345	$.38\times10^{-3}$
C_2	-.258	-.298	-.412	-.424	0
C_3	0.519	0.907	0.528	0.921	0
C_4	-.046	0.113	0.059	0.124	0
C_5	-.028	-.043	-.045	-.061	0
C_6	0.034	0.056	0.057	0.082	0
C_7	1.268	1.268	1.268	1.268	1.268
C_8	0.138	0.134	0.216	0.188	0
C_9	0.396	0.678	0.428	0.704	0
C_{10}	-.156	-.187	-.159	-.190	-.397
C_{11}/M_1	0.272	0.556	.388	0.626	0
C_{12}/M_1	0.036	-.090	.056	0.106	0

Table (2.2)

(Prob. I(a))

ITERATION	t_s SECONDS	$\omega(t_f)$	P_A^*	ϕ	t_f
1	0.1167	-.29	-.38	23	0.35
2	0.200	-.18	-.25	9.5	0.48
3	0.2167	-.06	-.21	4.8	0.52
4	0.233	-.09	-.15	3.4	0.58
5	0.250	-.03	-.08	.69	0.71

$^*P_A \triangleq P_m - P_e(t_F)$

Table (2.3)

(Prob. I(b))

ITERATION	t_s	$\omega(t_f)$	P_A	ϕ	t_f
1	.1167	-.087	-.27	8.2	0.40
2	.133	-.045	-.24	5.9	0.43
3	.15	-.07	-.19	4.3	0.48
4	.1667	-.02	-.14	1.9	0.55

is α_1. Equation (2.19) is initialized at $t = 0$ with $M = .0212$ (pu power sec^2), $P_m = .735$ (pu), $\psi_F = 9.48$ (pu), $\delta(0) = .9414$ (rad), $\omega(0) = 0$(rad/sec) and $\alpha = \alpha_F$. (The subscript '1' is omitted for the one machine case).

Problem 1(a) is solved by integrating (2.19) with ψ_F = constant $= \psi_F(0)$ from $t = 0$ to $t = t_o$. This yields $x_1(t_o) = \delta(t_o) = 1.183$ (rad), $x_2(t_o) = \dot{\omega}(t_o) = 4.14$ (rad/sec) (a fourth-order Runge Kutta Subroutine is used with a step size of 1 cycle = .01667 sec.). In (2.27) the choice $W_1 = W_2 = 200$ is made. The results for the proposed method are given in Table (2.2). Note the steady decrease in the cost index. A critical switching time of 15 cycles is found after five iterations.

Problem I(b) is solved by using (2.22) with the same values at $t = 0$, defined in I(a). Integration during the fault stage yields $x_1(t_o) = 1.183$ rad, $x_2(t_o) = 4.14$ rad/sec., and $x_3(t_o) = \psi_F(t_o) = 9.195$ pu. The results are shown in Table (2.3). The critical switching time is 10 cycles.

Problem I(c) is the multi-machine problem. When a fault occurs, the machine having the greater ratio of initial accelerating power to momentum constant would be expected to accelerate faster than the other machines. In reference 20, the critical switching time is defined with respect to the fastest machine and it has been shown that this definition is a useful one. Consequently, it is possible to use (2.27) for the multi-machine problem, choosing the variables to be those of the fastest machine.

The fault is taken to be a sudden three phase symmetric short circuit to ground at any one of the tie lines between two machines. The fault is assumed permanent. The circuit breakers open t_s seconds from fault occurence to isolate the faulted section. A four-machine system with data taken from reference 20 is considered. The relevant data are shown in Tables (2.4), (2.5) and (2.6). The one line diagram of the system is shown in Fig. (2.3) and the reduced system in Fig. (2.4).

The fastest machine in this case is machine number 3. The results obtained from the proposed method are shown in Table (2.7). The critical switching time is 155 radians (time unit used, τ, in radians for comparison with reference 20, $\tau = 2\pi ft.$). This is in agreement with the numerical result using a Liapunov function given in reference 20.

Since the methods are completely different, a direct comparison of the two approaches is not possible. The advantage of the proposed approach is that it is not affected by the order of the model. Governor and voltage regulator effects can be included, if desired. The approach based on constructing a Liapunov function becomes impractical for system models greater than second order.

2.4.2 Optimum Switching Time of a Braking Resistor

The braking resistor is connected to the generator bus at the instant t_s at which the circuit breaker opens. The resistor is disconnected after a period t_b. The size of the resistor is usually determined by economic considerations and the power demand under normal operating conditions[2,3]. For a given resistor and cost index there is an optimum time of switching t_b.

For the multi-machine system, the resistor is applied to the fastest machine. A suitable cost index for defining an optimal t_b is given by:

$$J = 0.5 \int_{t_o}^{t_f} (W_1 \delta^2 + W_2 \omega^2) dt = \int_{t_o}^{t_f} f_{n+1} dt \tag{2.29}$$

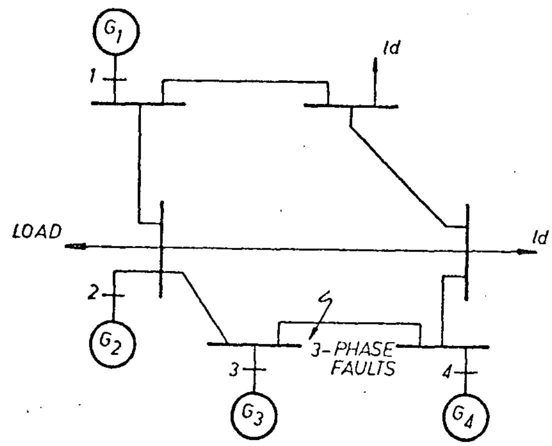

Fig. (2.3) One line diagram of a four machine system

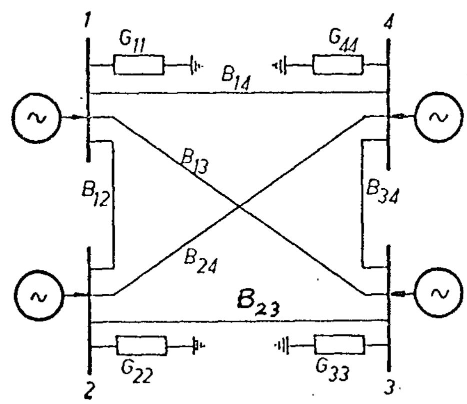

Fig. (2.4) Reduced system of Fig. (2.3)

Table (2.4)

(DATA AND INITIAL CONDITION)

GEN.	MVA CAPACITY	M_{pu}	D_{pu}	E_{pu}	δ (to) rad	$P_{m\,pu}$
1	100	75350	1.0	1.0004	.0013	.332
2	15	1130	12.0	1.0410	.1030	.100
3	40	2260	2.5	1.1900	.1970	.300
4	30	1508	6.0	1.0710	.0772	.200

Table (2.5)

(DURING FAULT ADMITTANCES)

B_{ij}:

i \ j	1	2	3	4
1	-3.582	0.546	0.0	0.303
2	0.546	-.871	0.0	0.062
3	0.0	0.0	-2.0	0.0
4	0.303	0.062	0.0	-1.216

G_{ii}:

i	G_{ii}
1	1.456
2	0.027
3	0.0
4	0.22

Table (2.6)

(AFTER FAULT ADMITTANCES)

B_{ij}:

i \ j	1	2	3	4
1	-2.310	.664	.656	.751
2	.664	.880	.121	.062
3	.656	.121	-.868	.062
4	.751	.062	.062	-.984

G_{ii}:

i	G_{ii}
1	.864
2	.029
3	.104
4	.225

Table (2.7)

(Prob I(c))

ITER.	t_s (rad)	100 ω	P_A	ϕ
1	130	-.12	-.84	70.8
2	135	-.046	-.81	64.8
3	140	-.11	-.76	58.0
4	145	-.12	-.68	47.0
5	155	-.045	-.39	15.4

The cost index (2.29) is a measure of the mean square frequency and rotor angle deviation. Augmenting (2.1) by $\dot{x}_{n+1} = f_{n+1}$, $x_{n+1}(t_o) = 0$, it is seen that the cost index is $\phi = J = x_{n+1}(t_f)$. The costate p has to be augmented by p_{n+1} and the Hamiltonian becomes $H = p'F + p_{n+1}f_{n+1}$. It follows from (2.11) and (2.12) that

$$\begin{aligned} \dot{p} &= -H_x = -\sum_{k=1}^{N} f'_{kx}p[h(t-t_{k-1})-h(t-t_k)] + f_{(n+1)x},\quad p(t_f) = 0 \\ \dot{p}_{n+1} &= \frac{-\partial H}{\partial x_{n+1}} = 0 \qquad p_{n+1}(t_f) = \frac{\partial \phi}{\partial x_{n+1}} = -1 \end{aligned} \tag{2.30}$$

The algorithm remains otherwise unchanged. In (2.29) the choice $W_1 = 100$, $W_2 = 10$ is made.

Consider problem II(a). The braking resistor has a value of 5.55 (pu), equal to the local load resistance (dotted lines in Fig. (2.1)). The braking resistor is applied at $t = t_o$ and the circuit breaker is assumed to reclose after 12 cycles. The values of the α vector for various stages are given in Table (2.1). The results obtained by the algorithm for $t_f = 90$ cycles (from t_s) are shown in Table (2.8). The swing curves for the cases (a) no braking resistor, (b) a braking resistor applied for the optimum time interval of 18 cycles, are shown in Fig. (2.5).

In problem II(b) the circuit breaker is assumed to reclose after 9 cycles. The results are given in Table (2.9). The optimum time interval is 18 cycles.

In problem II(c) the braking resistor has a value of 0.2 (pu) and is applied to machine no. 3 at the instant t_s when the circuit breaker opens (155 radians from t_o). The results obtained from the proposed algorithm

Table (2.8)

(Prob II(a))

ITER.	t_b(sec)	ϕ
1	.0667	248.7
2	.0883	247.9
3	.1	247.6
ϕ (no B.R.) = 373.3		

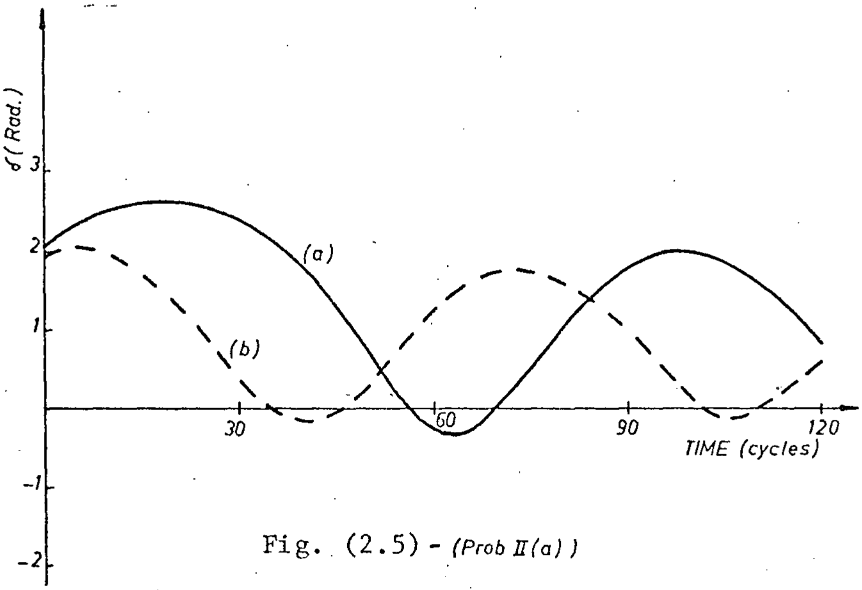

Fig. (2.5) - (Prob II(a))

Table (2.9)

(Prob II(b))

ITER.	t_b(sec)	ϕ
1	.0667	233.7
2	.0833	231.5
3	.1	229.8
4	.1167	228.6
5	.133	227.9
6	.15	227.7
ϕ (no B.R.) = 354.1		

are given in Table (2.10). The optimum time interval is 80 radians. The swing curves for the cases (a) no braking resistor, (b) braking resistor switched off after 200 radians, are shown in Fig. (2.6).

Note that the use of braking resistor damps out the first swing during the first half second (30 cycles Fig. (2.5)). It is possible to reapply the braking resistor and disconnect it again for further damping in the subsequent swings. This, however would be uneconomical since a braking resistor of higher rating would be required.

The subsequent swings can be damped out by using governor and/or voltage regulator controls. In the previous examples, those controls were totally neglected, and therefore there is no damping effect after the first swing. It is the purpose of next Chapter to investigate the governor control for large signal model of interconnected power plants.

Table (2.10)

(Prob. II (c))

ITER	t_b (rad)	$10^{-2}\phi$
1	30	462
2	50	430
3	80	427
$10^{-2}\phi$(no B.R.)= 643.2		

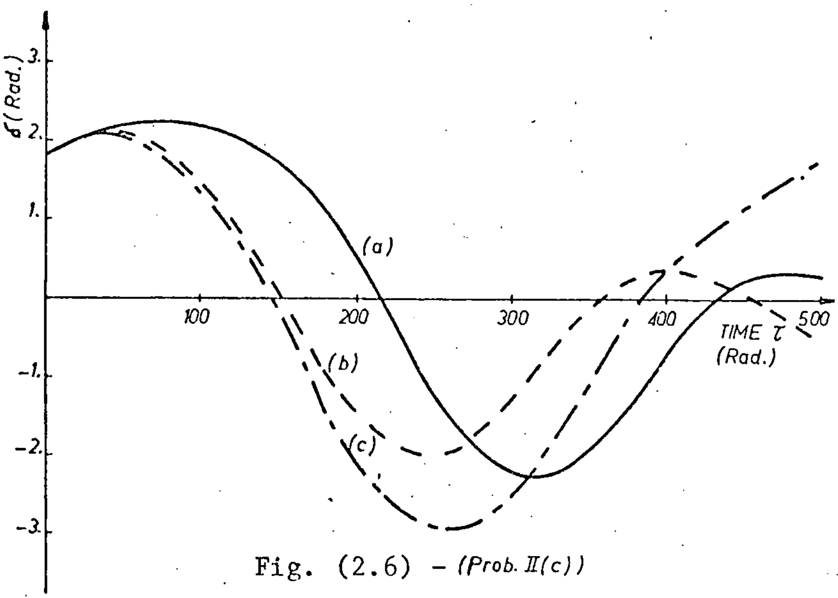

Fig. (2.6) – (Prob. II(c))

3. TWO-LEVEL CONTROL OF INTERCONNECTED POWER PLANTS

3.1 Introduction

Severe disturbances, caused by sudden changes in the electrical network, can be counteracted by discontinuous control as explained in Chapter 2. However, for optimum system performance, discontinuous controls should be augmented by continuous or modulated controls namely (a) voltage regulator control and (b) governor control.

By representing the dynamics of a power system by a linear model and choosing a quadratic cost index, the above control problems (a) and (b) can be formulated as the well known infinite-time linear state regulator problem of optimal control theory[24]. The state regulator control problem objective is to control the system so that the system states are kept small in some sense. The solution of this problem leads to an optimal controller which is a linear function of the states of the system.

Application of the solution of the linear state regulator problem to the optimum control of machine excitation in a one-machine infinite bus system is given in reference 11. The optimum control was derived from a linear low-order model and was tested on a non-linear high order model representation for one machine and multimachine systems. Even though this control was found effective in damping oscillations, no attempt was made to find the optimal control for a large signal model of interconnected machines.

The problem of governor control of the prime mover for a one-machine infinite bus system can be treated in a similar manner[10]. The regulator problem solution has also been suggested for the Load-Frequency Control problem[14], and was applied to two interconnected similar power plants*.

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* Interconnected areas were considered in reference 14. Here, it is assumed that each area has only one plant.

Because of the coupling in the model representing the plants, the optimum control for each plant is a linear combination of its own states and the states of the other plants. In reference 14, for the example used, the gain associated with the states of the other plant was very small and was neglected. That is, each plant uses only local state information and consequently is controlled in a suboptimal fashion. This suboptimal solution may be adequate for small-signal model as in the example used, but it cannot be adopted as a general policy. This fact is shown in the present study of two typical interconnected steam and hydro plants[25,26]. It was found that a local suboptimal control based on a linearized system model can result in system instability. Consequently, coupling between plants and system non-linearities cannot always be neglected and a good suboptimum control requires that each plant have information available about the states of other plants. Implementation of such control would be expensive due to the high cost of continuous communication required between the plants.

The purpose of this Chapter is to examine the possibility of implementing a simpler control for a large-signal model of interconnected power plants. It will be assumed that the generator voltages are held constant by the voltage regulator controls. Excitation control, however, can be considered without much difficulty.

For large deviations, the dynamics of the system can be represented by a set of first order non-linear differential equations. By inspecting the system dynamics it is found to belong to a class of non-linear systems for which a multi-level suboptimal control can be developed. The concept of multi-level control[27] is attractive since it appears applicable to many complex systems such as power systems.

A two-level govenor control of a power system is proposed. The first level consists of independent linear subsystems (plants), which have local feedback controllers. The second-level controller co-ordinates the subsystems by an intervention open-loop control. The intervention control is used to compensate for a decrease in system performance due to neglecting interaction and non-linearities at the local level.

For ease in real-time on-line implementation, a suboptimal solution to the optimum intervention control is determined as a function of initial conditions and real time. The scheme is simple, fast and requires a comparatively small amount of computer memory.

3.2 Large Signal Model for Interconnected Power Plants

A block diagram representation for a steam and a hydro plant [14,25,26] is shown in Fig. (3.1) and Fig. (3.2) respectively. The state variables and controls for the ith plant are defined as follows: (there are 4 states for the steam plant and 5 states for the hydro plant)

$x_{1i} \triangleq \Delta\delta_i$ = voltage angle deviation in radians

$x_{2i} \triangleq \Delta\omega_i$ = angular frequency deviation in rad/sec.

$x_{3i} \triangleq \Delta P_{gi}$ = deviation in mechanical power = deviation in generated power, in pu power (assuming that the time constant of the generator is negligible in comparison to governor and turbine time constants, also the two plants are assumed of equal capacity).

$x_{4i} \triangleq \Delta X_{gi}$ deviation in governor position in pu power (steam plant)

$x_{4i} \triangleq \Delta g_i$ deviation in gate position in pu power (hydro plant)

$x_{5i} \triangleq \Delta X_{gi}$ deviation in governor position in pu power (hydro plant)

$u_i \triangleq \Delta P_{ci}$ speed changer position in pu power

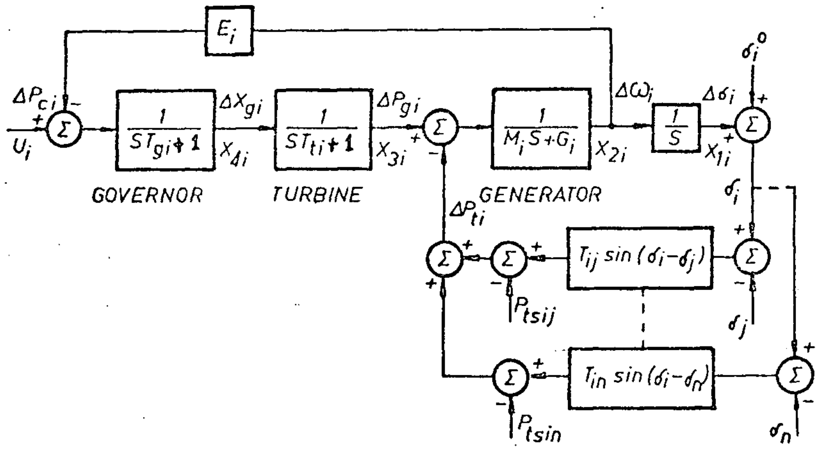

Fig. (3.1) Steam plant block diagram

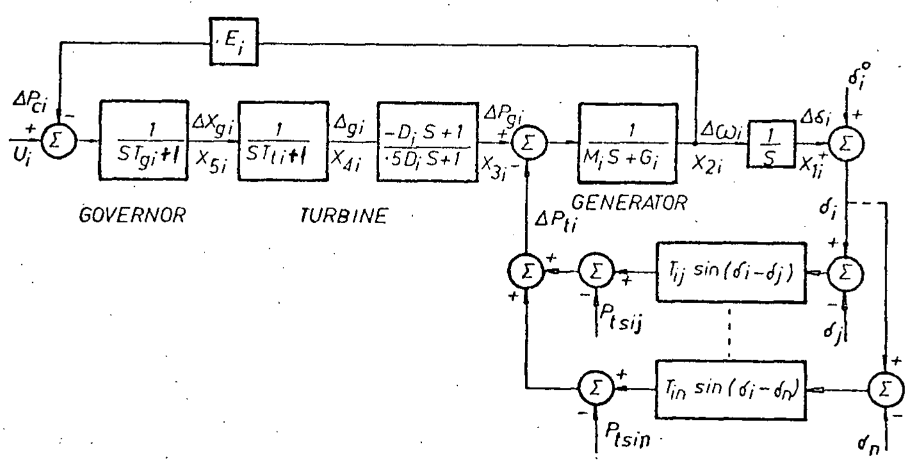

Fig. (3.2) Hydro plant block diagram

ΔP_{ti} = deviation in tie-line flows in pu power

P_{tsij} = scheduled tie-line power between the ith and jth plant.

The dynamics of the hydro plant are different from that of the steam plant because of water inertia. In the hydro plant, when the gates open, the turbine torque tends to decrease momentarily and then increases. The extra block in Fig. (3.2) represents this situation.

The tie-line flows are given by

$$P_{ti} = \sum_{\substack{j \\ j\neq i}} P_{tij} \tag{3.1}$$

where

$$P_{tij} = |v_i| \, |v_j||Y_{ij}|\sin(\delta_i-\delta_j) \triangleq T_{ij} \sin \delta_{ij} \tag{3.2}$$

where

$$T_{ij} \triangleq |v_i||v_j||Y_{ij}| \tag{3.3}$$

is assumed constant, where

$$\delta_{ij} \triangleq (\delta_i-\delta_j) \tag{3.4}$$

and where $v_i = |v_i| \, e^{j\delta_i}$ is the voltage at the ith plant bus bar in pu.

Y_{ij} is the line admittance between the ith and the jth plant (line resistance is neglected). The expression for the tie line deviation ΔP_{ti} is derived by considering a deviation $\Delta\delta_{ij}$ from the nominal δ^o_{ij}:

$$P^o_{ti} + \Delta P_{ti} = \sum_{\substack{j \\ j\neq i}} T_{ij} \sin(\delta^o_{ij}+\Delta\delta_{ij}) \tag{3.5}$$

since

$$P^o_{ti} = \sum_{\substack{j \\ j\neq i}} P_{tsij} = \sum_{\substack{j \\ j\neq i}} T_{ij} \sin \delta^o_{ij} \; . \tag{3.6}$$

It follows that

$$\Delta P_{ti} = \sum_{\substack{j \\ j\neq i}} T_{ij}[\sin\delta^{o}_{ij}(\cos\Delta\delta_{ij}-1)+\cos\delta^{o}_{ij}\sin\Delta\delta_{ij}] \tag{3.7}$$

Equation (3.7) can be decomposed into a linear local term which is a function of $\Delta\delta_i$ only, and non-linear coupling terms:

$$\Delta P_{ti} = [\sum_{\substack{j \\ j\neq i}} T^{o}_{ijc}]\Delta\delta_i + \sum_{\substack{j \\ j\neq i}} [T^{o}_{ijs}(\cos\Delta\delta_{ij}-1) + T^{o}_{ijc}(\sin\Delta\delta_{ij} - \Delta\delta_i)] \tag{3.8}$$

where

$$T^{o}_{ijc} \triangleq T_{ij} \cos \delta^{o}_{ij},$$

$$T^{o}_{ijs} \triangleq T_{ij} \sin \delta^{o}_{ij}, \text{ and}$$

$$\Delta\delta_{ij} \triangleq \Delta\delta_i - \Delta\delta_j.$$

No unique method exists for decomposing the system into linear subsystems for which a feedback control is used and then accounting for subsystem interaction by an open-loop intervention control. For the interconnected plants discussed here, the decomposition (3.8) proved successful. This decomposition results in the local voltage angle deviation being used in a local feedback control.

3.3 System Dynamics In State Variable Form

The dynamics of each plant can be represented in the form:

$$\dot{X}_i = A_i X_i + B_i U_i + F_i(X_1, X_2, \ldots X_N) \tag{3.9}$$

where

X_i is a vector of state variables of dimension n_i (for the model used, n_i=4 and 5 for steam and hydro plants, respectively).

U_i is a vector of control variables of dimension m_i (m_i=1 for both steam and hydro plants).

F_i is a vector function of dimension n_i, which contains the coupling and non-linear terms.

A_i, B_i are time invariant matrices of appropriate dimensions.

The matrices A_i, B_i and the vector F_i are defined for each steam and hydro plant as follows:

Steam Plant

$$A_i \triangleq \begin{bmatrix} 0 & 1 & 0 & 0 \\ [\sum_{\substack{j \\ j\neq i}} T^o_{ijc}]/M_i & -G_i/M_i & 1/M_i & 0 \\ 0 & 0 & -1/T_{ti} & 1/T_{ti} \\ 0 & -E_i/T_{gi} & 0 & -1/T_{gi} \end{bmatrix}$$

$$B'_i \triangleq [0 \quad 0 \quad 0 \quad 1/T_{gi}]$$

$$F_i \triangleq \begin{bmatrix} 0 \\ -\frac{1}{M_i} \sum_{\substack{j \\ j\neq i}} [T^o_{ijs}(\cos\Delta\delta_{ij}-1)+T^o_{ijc}(\sin\Delta\delta_{ij}-\Delta\delta_i)] \\ 0 \\ 0 \end{bmatrix}$$

Hydro Plant

$$A_i \triangleq \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ [\sum\limits_{\substack{j \\ j\neq i}} T^o_{ijc}]/M_i & -G_i/M_i & 1/M_i & 0 & 0 \\ 0 & 0 & -2/D_i & (2/D_i+2/T_{ti}) & -2/T_{ti} \\ 0 & 0 & 0 & -1/T_{ti} & 1/T_{ti} \\ 0 & -E_i/T_{gi} & 0 & 0 & -1/T_{gi} \end{bmatrix}$$

$$B_i' \triangleq [0 \quad 0 \quad 0 \quad 0 \quad 1/T_{gi}]$$

$$F_i \triangleq \begin{bmatrix} 0 \\ -\frac{1}{M_i} \sum\limits_{\substack{j \\ j\neq i}} [T^o_{ijs}(\cos\Delta\delta_{ij}-1)+T^o_{ijc}(\sin\Delta\delta_{ij}-\Delta\delta_i) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For N interconnected plants the dynamics of the system can be written as a composite state vector equation of the form

$$\dot{X} = AX + BU + F(X), \; X(t_o) = X_o \tag{3.10}$$

where the matrices A and B are known block diagonal time invariant matrices so that, for example

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ & A_2 & & \\ \vdots & & \ddots & \\ 0 & & & A_N \end{bmatrix} \tag{3.11}$$

The composite state vector X and the composite control vector U are composed of the state vectors X_i and the control vector U_i, i=1, 2,...,N, respectively so that

$$X' = (X_1', X_2', \ldots, X_N') \text{ and } U' = (U_1', U_2', \ldots, U_N').$$

(the prime is used to denote transpose of a vector or matrix). Similarly $F' = (F_1', F_2', \ldots, F_N')$.

It should be pointed out here that the control vector could be augmented to include excitation control and in this case we would have a two-dimensional control vector for each plant. The system dynamics can still be formulated in the general form (3.10). Excitation control has been shown to give damping effect on system oscillation[12]. However, to clearly illustrate the proposed control, only governor control is considered.

The control problem is the following. It is required to find a control vector U such that the deviations in the states resulting from a system disturbance is minimized without excessive control effort. The problem can be formulated by introducing a cost index

$$J = \frac{1}{2} \int_{t_o}^{t_f} (X'QX + U'RU)\,dt \tag{3.12}$$

where Q and R are block diagonal weighting matrices (Q is a positive or semipositive definite matrix, R is a positive definite matrix), and choosing U so that J is a minimum. The state X is subject to the dynamical constraint (3.10).

3.4 Two-Level Structure of the Control Problem

A two-level structure is chosen. It is assumed that the structure specifies a feedback control of the form

$$U_i = -C_i X_i \tag{3.13}$$

for the local controllers, and that the second-level controller co-ordinates the subsystems (plants) by an intervention control V. The resultant composite control is

$$U = -CX + V \tag{3.14}$$

where C is a composite block diagonal matrix of the form (3.11) composed from the C_i, i=1, 2, ... N.

The first level of the control is obtained by neglecting the coupling function, F(X), in (3.10). The problem then reduces to the well known linear control problem with quadratic cost index[24]. The optimum control can be obtained as a feedback control given by

$$U = -R^{-1}B'KX = -CX \tag{3.15}$$

where K is the solution of the matrix Ricatti equation

$$\dot{K} = -KA - A'K + KSK - Q, \quad K(t_f) = 0 \tag{3.16}$$

where

$$S \triangleq BR^{-1}B' \tag{3.17}$$

Since A, B, Q and R are block diagonal matrices, the solution for K is a block diagonal matrix (from (3.16) and (3.17)). The solution for each block K_i is obtained by solving

$$\begin{aligned} \dot{K}_i &= -K_iA_i - A_i'K_i + K_iS_iK_i - Q_i\,, \quad K_i(t_f) = 0, \\ S_i &\triangleq B_iR_i^{-1}B_i' \qquad\qquad i = 1,2,\ldots N \end{aligned} \tag{3.18}$$

For A, B, Qand R time invariant, let K_s be the steady state solution of (3.16):

$$0 = -K_sA - A'K_s + K_sSK_s - Q. \tag{3.19}$$

An easily implemented control results if

$$C = R^{-1}B'K_s \tag{3.20}$$

is used as a suboptimal local gain matrix. The choice (3.20) is optimum if $t_f = \infty$ in (3.12) and it often gives an excellent suboptimal control.

There is an increase in the cost index associated with the subsystem interaction and system non-linearities which have been neglected in the suboptimal choice (3.20). To account for these effects a second-level control of the form

$$V = R^{-1}B'h \tag{3.21}$$

is introduced. To determine the optimum h, the original problem must be reformulated. Substituting (3.14),(3.20), and (3.21) into (3.10) gives

$$\dot{X} = (A-SK_s)X + F + Sh, \quad X(0) = X_o \tag{3.22}$$

$$U = R^{-1}B'(-K_sX+h) \tag{3.23}$$

The problem is to choose the intervention control h so that the cost index (3.12) is a minimum subject to the dynamical constraint (3.22). The Hamiltonian for this problem is

$$H = p'[(A-SK_s)X + F + Sh] - \frac{1}{2}X'QX \tag{3.24}$$

$$- \frac{1}{2}(-K_sX+h)'S(-K_sX+h) \tag{3.24}$$

Applying the necessary conditions of optimal control theory yields

$$\dot{p} = -H_x = -(A-SK_s+F_x)'p + QX-K_sS(-K_sX+h), \quad p(t_f) = 0 \tag{3.25}$$

$$0 = H_h = S(p+K_sX-h) \tag{3.26}$$

where

$$F_x \triangleq \begin{bmatrix} \frac{\partial F_1}{\partial X_1} \cdots \frac{\partial F_1}{\partial X_N} \\ \vdots \\ \frac{\partial F_N}{\partial X_1} \cdots \frac{\partial F_N}{\partial X_N} \end{bmatrix} \qquad H_h \triangleq \begin{bmatrix} \frac{\partial H}{\partial h_1} \\ \vdots \\ \frac{\partial H}{\partial h_N} \end{bmatrix} \tag{3.27}$$

Equations (3.22), (3.25) and (3.26) define a two-point boundary value problem whose solution gives the optimum intervention control h. Since the equations are non-linear, iterative methods are required to obtain the solution. Consequently, since on-line implementation is desired, optimal control is not feasible and a good suboptimal control policy must be determined.

If $h = \bar{h} = 0$, then local control only is applied. Let $\bar{X}$, $\bar{p}$ be the solution of (3.22) and (3.25) for this control. A fundamental result of optimal control theory is that an incremental control δh results in an incremental cost index, given by

$$\delta J = - \int_{t_o}^{t_f} \bar{H}_h' \, \delta h \, dt \tag{3.28}$$

where

$$\bar{H}_h = S(\bar{p} + K_s \bar{X}) \, . \tag{3.29}$$

The intervention control is to be chosen so that system performance is improved. That is, so that δJ, the incremental cost index, is decreased. Since S is positive definite, it is seen from (3.28) and (3.29) that

$$\delta h = \ell(\bar{p} + K_s \bar{X}), \tag{3.30}$$

where $\ell > 0$ is a step size parameter, accomplishes this objective. In control theory terminology (3.30) is a steepest descent increment in function space.

For computational reasons it is convenient to derive an equation for

$$q \triangleq \bar{p} + K_s \bar{X} \, . \tag{3.31}$$

With the aid of (3.19), (3.22), (3.25) and (3.31) it is seen that

$$\dot{q} = -(A-SK_s + \bar{F}_x)'q+\bar{F}_x'K_s\bar{X}+K_s\bar{F} \ , \ q(t_f) = K_s\bar{X}(t_f) \tag{3.32}$$

(the terms with an overbar are evaluated for the nominal $\bar{X}$, which is the solution of (3.22) for $h = \bar{h} = 0$).

The algorithm for finding h is simple one. Equation (3.22) is integrated in the forward direction, taking $h = 0$. Equation (3.32) is then integrated in the backward direction to find q. The intervention control is then $h = \bar{h} + \delta h = \ell q$. The optimum value ℓ_o of ℓ which minimizes J can be found by a simple direct search procedure. The details of this algorithm is given in Section (3.6.2). However, on-line implementation of this algorithm for computing h(t) is not practical. The next section considers this problem.

3.5 On-Line Control Implementation

The intervention control h(t) is a function of time and initial conditions of all the subsystems (see (3.22) and (3.32)). Suppose that this function is known. To generate h(t), initial conditions must be transmitted to the second level. After generating h(t), the second-level coordinator continuously transmits the components of h(t) back to the subsystems.

A more feasible way of implementing this control is to transmit a minimum amount of information between the local controllers and the central co-ordinator. In order for this to be accomplished, h(t) must be approximated in the form

$$h_i(t) = \tilde{h}_i = g_i(\alpha_i(X_o),t) \tag{3.33}$$

where g_i is a non-linear function of time with unknown coefficients. Each coefficient is non-linear function of initial conditions. The coefficients are chosen such that $||h_i(t) - \tilde{h}_i||^2$ is minimum. A suitable algebraic form for g_i and α_i is given in Section (3.6.3) and Appendix III, respectively.

On-line implementation would be as follows. Each subsystem is locally controlled by a feedback control $U_i = -C_i X_i$. At the time $t = t_o$ of a system disturbance the initial conditions of each subsystem are transmitted to the central co-ordinator which generates the coefficients for (3.33). The intervention control $h_i(t)$ is generated locally by a function generator after receiving the coefficients from the central co-ordinator (see Fig. (3.3)).

In implementing (3.33), preliminary off-line computations are required to determine the unknown coefficients which are stored by the central co-ordinator. On-line control requires relatively few multiplications and additions at the central co-ordinator level (after receiving X_o from the subsystems) to generate the coefficients.

As will be shown later, each α_i is a vector. For the system under investigation a dimension of four for each vector α_i was found adequate for generating the $\tilde{h}_i$'s.

3.6 Off-Line Control Design

Implementing (3.23) and (3.33), the control design follows three steps: a) design of the feedback control $u_i = -C_i X$, b) design of the intervention control $h(t)$, and c) design of the approximate intervention control $\tilde{h}$. The design details for each of the three steps follow.

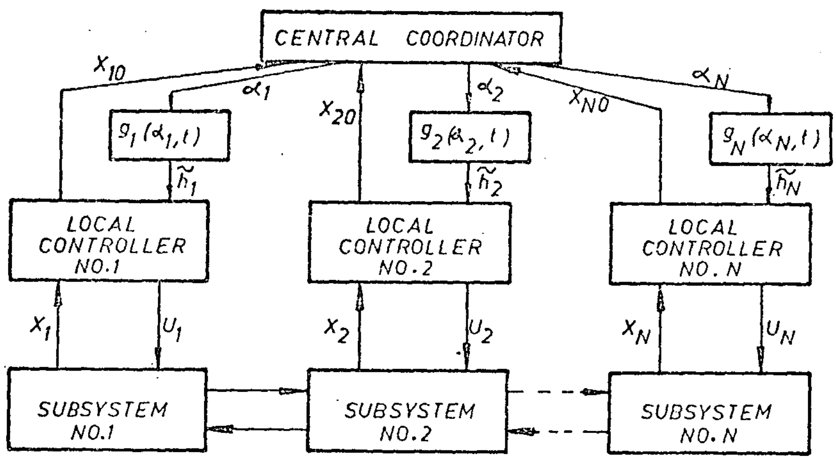

Fig. (3.3) On-Line control implementation

3.6.1 Design of the Feedback Control ($U_i = -C_i X_i$)

Synthesizing this control requires solutions of (3.19) to obtain K_{si}. Since coupling is neglected at this level, the solution reduces to solving (3.19) for each plant separately. One method of solving (3.19) is by the Newton-Raphson technique[28]. For fast convergence, the initial guess for K_{si} is obtained by integrating (3.18) for a short period (e.g. 10 seconds). C_i is given by $C_i = R_i^{-1} B_i' K_{si}$.

3.6.2 Design of the Intervention Control h(t)

The open loop intervention control h(t) depends on the initial conditions. Given X_o, the optimum value $h^*(t)$ is obtained as follows:

(1) Integrate (3.22) forward from $t=t_o$ to $t=t_f$ with $h=\bar{h}=0$. The trajectory obtained is the nominal trajectory $\bar{X}(t)$. Evaluate the final condition of (3.32), $q(t_f) = K_s \bar{X}(t_f)$.

(2) Integrate (3.32) backward from $t=t_f$ to $t=t_o$ (with $X(t)=\bar{X}(t)$) and store q(t).

(3) Choose a step size $\ell > 0$, and with $h(t) = \ell q(t)$ evaluate the cost index J.

(4) Find the optimum step $\ell = \ell_o$ at which J is minimum. (This is easily done by incrementing ℓ).

(5) The optimum intervention control is chosen to be $h^* = \ell_o q(t)$.

Note that only some components of h(t) (and consequently q(t)) are to be stored. For one control component in each plant, for example, only one component of h(t) would be required to be stored since the other components are multipled by zeros (see (3.17) for S and (3.22) for Sh). Therefore, for the case of two interconnected power plants having the model of Section (3.2), only two components for $h_i(t)$ $(i = 1,2)$ are required.

3.6.3 Design of the Approximate Intervention Control $\tilde{h}$

As pointed out before, the previous algorithm for h(t) cannot be implemented for on-line control of the power system under investigation. However, by approximating h(t) by a suitable function of known algebraic form, on-line implementation is feasible. Suitable functions are polynomials or splines.

By plotting the intervention controls as a function of time for each set of initial conditions, k, it was noticed that a cubic polynomial in t could be fitted to each $h_i^k(t)$ (the superscript k denotes that $h_i(t)$ is evaluated for the set of initial conditions k). Consider the case of approximating one intervention control, for example $h_1(t)$ of the first plant. The same procedure is followed for the intervention controls of the other plants.

(1) Let $h_1^k(t)$ be approximated by a cubic polynomial in t:

$$h_1^k(t) = \sum_{m=1}^{4} \alpha_m^k t^{m-1} \tag{3.34}$$

A curve-fitting routine based on a least-squares approach can be employed to find the coefficients α_m^k. This is done for different sets of initial conditions k=1,2,...M.

(2) In general, the coefficients α_m^k are functions of the initial conditions. Consider the <u>mth</u> coefficient α_m. Let α_m^k be approximated by

$$\tilde{\alpha}_m^k = \alpha_m(X_o^k) \tag{3.35}$$

where $\alpha_m(X_o^k)$ is a specified function of X_o^k with unknown coefficients. It is required to find the coefficients in $\alpha_m(X_o)$ such that

$$||\alpha_m^k - \tilde{\alpha}_m^k||^2, \; m=1,2,3,4 \;, \; k=1,2,\ldots M \tag{3.36}$$

is minimum for all k. The choice of the function (3.35) is arbitrary. A choice of (3.35) suitable for the power system studied is given in Appendix III.

For on-line implementation the coefficients (3.35) are stored by the central co-ordinator. On receiving the initial conditions from each subsystem, the central co-ordinator computes four parameters α_1, α_2, α_3 and α_4 for each plant and transmits them to the different subsystems. Each subsystem then generates its own intervention control $\tilde{h}_i$.

3.7 Application

An example of an interconnected steam and hydro plant [25] is considered. Let the subscripts 1 and 2 denote the steam and hydro plants, respectively.

The first sign of impending trouble in a power system disturbed by loss of load is acceleration. The speed deviation (time integral of acceleration) appears later, and the angular change (integral of speed) still later (see Fig. (3.9)). Because of the relatively long time constants associated with the governor and turbine, their outputs do not change instantaneously and can be assumed constant during the very short period following a disturbance. Consequently, detection of angular acceleration is the most promising way of quickly initiating control action[2].

In this application, the disturbance is assumed to be a speed deviation from nominal at $t=t_o$, all other states are assumed zero. The cost index is chosen as

$$J = \frac{1}{2}\int_0^{20} (\Delta\delta_1^2 + \omega_1^2 + \Delta\delta_2^2 + \Delta\omega_2^2 + U_1^2 + U_2^2)\,dt$$

That is

$$Q_1 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{bmatrix}, \quad R_1=1,\ R_2=1.$$

The choice $t_f = 20$ seconds is made since the system settling time is around 30 seconds.

The different parameters for each plant represented by Fig. (3.1) and Fig. (3.2) are as follows[25].

$M_1 = .04$, $G_1=.01$, $T_{t1} = 0.5$, $T_{g1} = 0.5$, $E_1 = .03$

$M_2 = .03$, $G_2 = .008$, $D_2 = 0.5$, $T_{t2} = 0.5$, $T_{g2} = 1.2$, $E_2 = .013$

$T_{12} = 0.05$, $\delta_{12}^o = -1$

The local feedback controls for each plant is obtained as explained in Section (3.6.1) and are given by:

$U_1 = -0.336\ \Delta\delta_1 - .607\ \ \Delta\omega_1 - .416\ \Delta P_{g1} - 1.6\ \Delta X_{g1}$

$U_2 = .515\Delta\delta_2 - 1.16\Delta\omega_2 - 11.5\ \Delta P_{g2} - 41.6\Delta g_2 - 9.22\ \Delta X_{g2}$

For the design of the intervention control, 9 sets of speed deviations are considered. Three different sets for each of the following cases are taken:

(a) Disturbance at plant (1) only.
(b) Disturbance at plant (2) only.
(c) Disturbance at both plants simultaneously.

Following the design procedure of Section (3.6.3), it was noticed that

$h^*(t)$ is almost zero after 10 seconds. Consequently, the fitting routine for (3.39) was given data up to 10 seconds only. Therefore, the generation of h at the local level is done for 10 seconds after which intervention control is removed. The coefficients to be stored at the central co-ordinator which are used for generating h (see Appendix III) are given in Table (3.1).

For the 9 sets of initial conditions, a comparison is made between the cost function J for the cases (a) using local control only (J_a); (b) using local plus intervention control $h^*(t)$, (J_b); and (c) using local plus intervention control $\tilde{h}$, (J_c). The results are given in Table (3.2).

To show the effectiveness of on-line generation of $\tilde{h}(t)$, a different set ($\omega_{1o}=5$, $\omega_{2o}=0$) was tested (Test, Table (3.1)). With local control only the system was unstable. By introducing the intervention control $\tilde{h}$ the system was stabilized. Figure (3.5) shows these results.

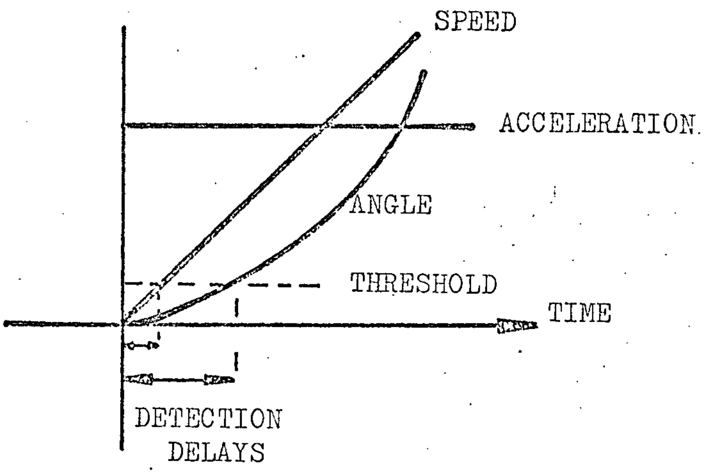

Fig. (3.4) Angle and angular speed as a function of time for a step change of angular acceleration

Table (3.1)

SET NO.	INITIAL CONDITIONS ω_{10}, ω_{20}	J_a	J_b	J_c
1	1, 0	.446	.360	0.364
2	2, 0	1.957	1.470	1.487
3	3, 0	6.233	3.427	3.611
4	0, −1.33	1.625	1.435	1.453
5	0, −2.667	6.191	5.610	5.619
6	0, −4	13.95	12.738	12.785
7	1, −1.33	1.5	1.45	1.459
8	2, −2.667	5.9	5.690	5.720
9	3, −4	13.73	13.3	13.35
TEST	5, 0	10.4×10^4		14.84

Table (3.2)

	m	β_{m0}	β_{m1}	β_{m2}	β_{m3}	β_{m4}	β_{m5}
STEAM PLANT	1	-.996	-42.9	-46.0	-2.78	-3.7	-2.9
	2	-2.70	2.28	8.82	7.89	0.99	4.79
	3	0.55	0.77	-0.68	-1.99	-1.50	-1.08
	4	-0.03	-0.07	-0.02	1.23	0.01	.07
HYDRO PLANT	1	18.0	-91.7	6.88	-3.19	-1.91	-16.6
	2	-11.8	40.6	-29.8	-5.77	-0.88	2.64
	3	2.26	-7.22	6.87	1.91	0.28	0.11
	4	-0.134	-0.418	-0.42	-0.136	-0.02	-.022

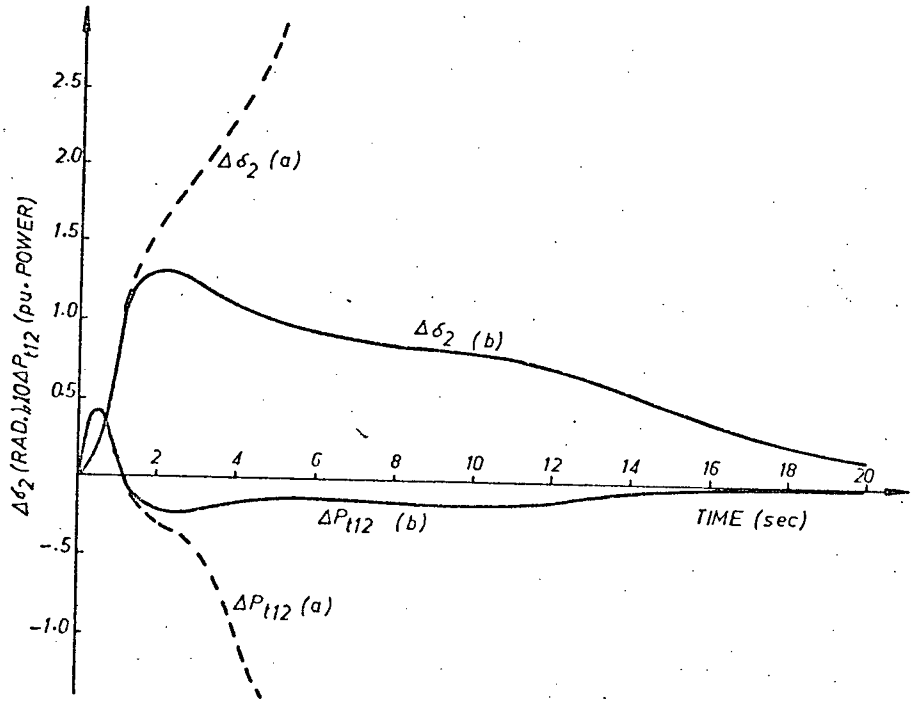

Fig. (3.5) Angular and tie-line deviations for $\omega_{1o} = 5$, $\omega_{2o} = 0$ with (a) local control only, (b) local plus intervention controls.

4. OPTIMUM LOAD-FREQUENCY CONTINUOUS CONTROL WITH UNKNOWN DETERMINISTIC POWER DEMAND

4.1 Introduction

Power system disturbances caused by load-fluctuations result in changes in tie-line real power and system frequency, necessitating some form of load-frequency control. The form of Load-Frequency Control (LFC) presently in use is based on an error signal which is a linear combination of the net interchange and frequency errors. A simple integral-type control action drives the error signals to zero.

Modern optimal control theory has led to the development of design techniques which can result in significant improvement in the control of high-order systems. The applications of these techniques to improve LFC is currently receiving increasing attention[14,17,25,32]. Elgerd and Fosha[14] applied the solution of the state-regulator problem to the LFC problem. This approach, however, requires knowledge of the new steady-state operating point. Consequently, the control is not a feasible optimum control, since the information required for its implementation is not available.

Feasible optimum load-frequency control requires the identification of the incremental power demand in order to optimally compensate for load-frequency deviations. This fact was recognized in reference 17 where a modified Kalman filter was introduced to perform the identification. The approach suggested in 17 has, however, several shortcomings. The assumption is made that the incremental tie-line power ΔP_t is a known (through measurements) function of time. Actually, ΔP_t depends on the system state and this must be accounted for in the optimal control formulation of the LFC problem.

A rather serious shortcoming in 17 is the manner in which the Kalman filter is used.

The Kalman filter output gives an estimate $\hat{x}$ of a state x. Normally, it is the filter output $\hat{x}$ which is used for implementing control action. In 17 however, the filter input $\dot{\hat{x}}$, is used instead of the filter output. Invariably, the filter is realized in a discrete form on a digital computer. The numerical generation of $\dot{\hat{x}}$ from sampled data could introduce highly undesirable noise problems. The unconventional requirement for $\dot{\hat{x}}$ seems to arise out of the manner in which integral control action is introduced in the problem formulation.

In the method presented here, $\hat{x}$ is used to implement control action so that the Kalman filter can be used in its conventional form. However, there is a practical difficulty associated with implementing a Kalman filter. Detailed statistical data about plant and measurement noise is required and such data is generally not available for a power system. Consequently, instead of a Kalman filter, two alternative methods are suggested for demand identification. The first method is extremely simple and identifies incremental demand by the differential approximation technique. The second method is based on using a Luenberger-type observer to perform the identification of the demand and to estimate the unmeasured states. The advantage of these alternative methods is that they do not require data about noise statistics.

4.2 Problem Formulation

A typical model of two interconnected power areas is shown in Fig. (4.1)[14,17,25,26,32]. The controlling station in the first and second area is taken to be a steam-plant and a hydro-plant, respectively.

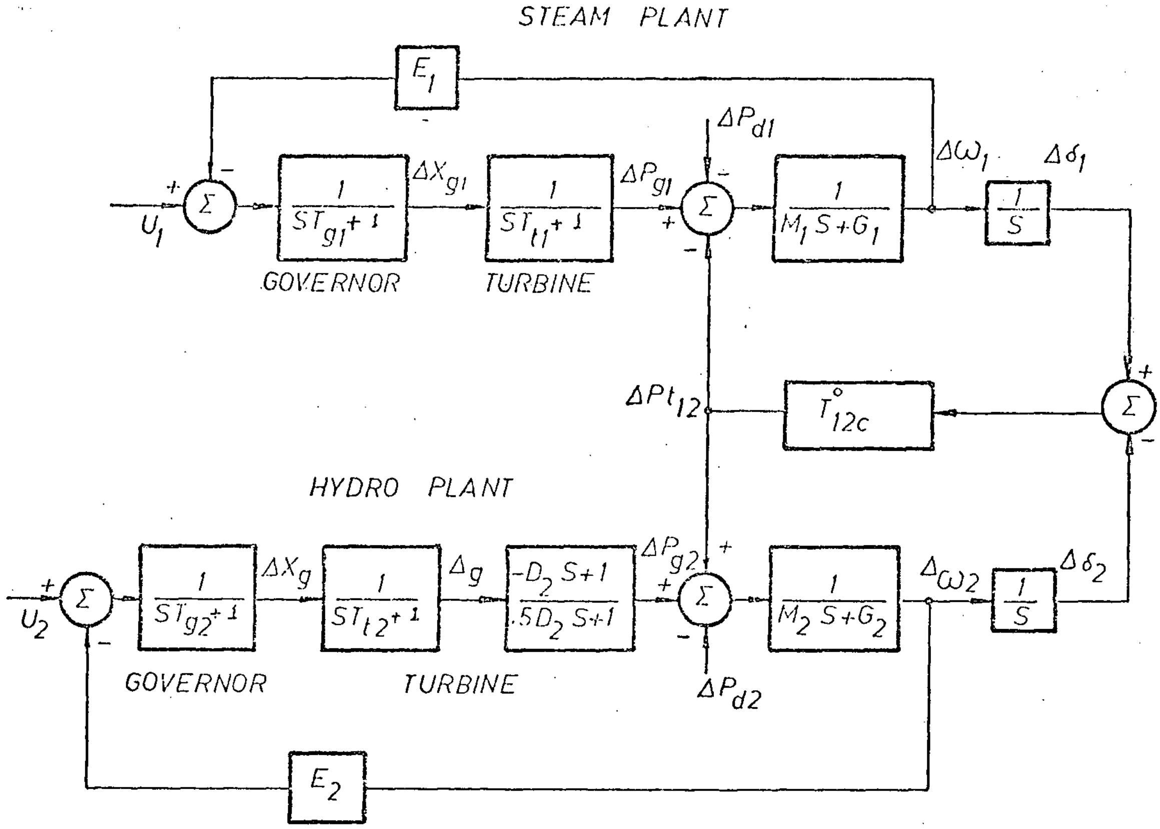

Fig. (4.1) Block diagram of two interconnected steam and hydro areas.

In state variable form the i-th controlling plant dynamics in an N-interconnected system has the form

$$\dot{x}_{pi} = A_i x_{pi} + B_i u_i + \Gamma_i \Delta P_{di} + \Omega_{ij}(\omega_j) \qquad j \neq i, j = 1, 2, \ldots, N \tag{4.1}$$

where for a steam-plant the state vector is (prime denotes transposition)

$$x_p' = [\Delta P_t \quad \Delta\omega \quad \Delta P_g \quad \Delta X_g] \tag{4.2}$$

and for hydro-plant the state vector is

$$x_p' \triangleq [\Delta P_t \quad \Delta\omega \quad \Delta P_g \quad \Delta g \quad \Delta X_g] \tag{4.3}$$

The matrices in (4.1), for a steam-controlling plant are given by: (The matrices for a hydro plant have a similar structure).

$$A_i \triangleq \begin{bmatrix} 0 & \sum\limits_{\substack{j \\ j\neq i}} T^o_{ijc} & 0 & 0 \\ -1/M_i & -G_i/M_i & 1/M_i & 0 \\ 0 & 0 & -1/T_{ti} & 1/T_{ti} \\ 0 & -E_i/T_{gi} & 0 & -1/T_{gi} \end{bmatrix} \tag{4.4}$$

$$B_i' \triangleq [0 \quad 0 \quad 0 \quad 1/T_{gi}] \tag{4.5}$$

$$\Gamma_i' \triangleq [0 \quad -1/M_i \quad 0 \quad 0] \tag{4.6}$$

$$\Omega_{ij}' \triangleq [-T^o_{ijc} \quad 0 \quad 0 \quad 0] \tag{4.7}$$

where the terms in (4.2)-(4.7) are as defined in Section (3.2).

To avoid unnecessary complications in notation, the coupling terms between the areas are set equal to zero in the initial problem formulation and the subscript i is dropped (non-zero coupling is considered in the example). Equation (4.1) then takes the form

$$\dot{x}_p = A\,x_p + Bu + \Gamma\Delta P_d \tag{4.8}$$

To obtain a feasible control, the LFC problem must be considered to be composed of two separate problems: (1) Problem of identifying the unknown power demand ΔP_d. (2) Problem of optimally controlling the dynamic response so that the generation becomes equal to the demand at the specified frequency.

Consider the second of the above problems and assume for the moment that ΔP_d is a known constant. The terminal conditions to be satisfied are*

$$\Delta P_t(\infty) = 0,\ \Delta\omega(\infty) = 0,\ \Delta P_g(\infty) - \Delta P_d = 0 \text{ and } \Delta X_g(\infty) - \Delta P_d = 0 \tag{4.9}$$

To formulate an optimal control problem, a change of variables is introduced:

$$\tilde{x} = x_p - \rho\Delta P_d \tag{4.10}$$

where

$$\rho' \triangleq (0 \quad 0 \quad 1 \quad 1). \tag{4.11}$$

Substituting (4.10) into (4.8) (and setting $\Delta\dot{P}_d$ to zero) yields;

$$\dot{\tilde{x}} = A\tilde{x} + Bu + (A\,\rho + \Gamma)\Delta P_d \tag{4.12}$$

The terminal condition (4.9) requires that

$$\dot{\tilde{x}}(\infty) = 0 = \tilde{x}(\infty) \tag{4.13}$$

It is seen from (4.11) and the definition of the system matrices that

$$A\,\rho + \Gamma = -B \tag{4.14}$$

An essential characteristic of the load-frequency control is the requirement for an integral-type operation on the error signal. To introduce this control requirement into the formulation of an optimal control problem necessitates augmenting (4.12) by

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* See Appendix I for LFC criteria

$$\tilde{x}_{n+1}(t) \triangleq u(t) - \Delta P_d \tag{4.15}$$

so that

$$\dot{x}_{n+1} = \tilde{u} \tag{4.16}$$

where

$$\tilde{u} \triangleq \dot{u} \tag{4.17}$$

The augmented system is therefore (see (4.14))

$$\dot{\tilde{X}} = \tilde{A}\tilde{X} + \tilde{B}\,\tilde{u} \tag{4.18}$$

where

$$\tilde{X} \triangleq \begin{bmatrix} \tilde{x} \\ \tilde{x}_{n+1} \end{bmatrix}, \quad \tilde{A} \triangleq \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \text{ and } \tilde{B} \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4.19}$$

The cost index is taken to have the quadratic form

$$J = \frac{1}{2}\int_0^\infty (\tilde{X}'Q\tilde{X} + \tilde{u}'R\tilde{u})dt \tag{4.20}$$

where Q and R are positive definite matrices.

The optimal control for the problem defined by (4.18) and (4.20) is given by

$$\tilde{u} = c'\tilde{X} \tag{4.21}$$

where

$$c' = [s \quad s_{n+1}] \tag{4.22}$$

is a constant vector which can be found by solving a steady-state Riccati equation (see Section 3.6.1).

In terms of the original state variables the control is given by (see (4.10), (4.15) and (4.19))

$$\dot{u} = s'x_p + s_{n+1}u - (s'\rho + s_{n+1})\Delta P_d \tag{4.23}$$

where u(0) is arbitrarily taken to be zero. The control (4.23) is similar to the conventional proportional plus integral control which is presently used for load-frequency control. It is seen, however, that the control (4.23) is feasible only if ΔP_d can be identified. The next Section discusses the demand identifier.

4.3 Demand Identifier - Differential Approximation

To implement the control given by (4.23) requires the identification of the parameter ΔP_d. A simple identifier for ΔP_d can be constructed by using the method of differential approximation[23].

Let $T_o = t_k - t_{k-1}$ be a fixed identification period which starts at time t_{k-1} and terminates at time t_k. The sequence $(t_o, t_1, t_2, \ldots)$ defines a set of identification intervals. In actual power systems, ΔP_d is constant or slowly varying. Consequently, a reasonable approximation for ΔP_d in (4.23) is to use the demand identified during the previous period. That is, by taking

$$\Delta P_d = \Delta P_d(k-1) \tag{4.24}$$

for the interval $t_k < \tau \leq t_{k+1}$, where $\Delta P_d(k-1)$ is determined by integrating the power equilibrium equation (see Fig. (4.1)).

$$\Delta P_g - \Delta P_d = M\Delta\dot{\omega} + G\Delta\omega + \Delta P_t \tag{4.25}$$

over the previous interval $t_{k-1} < \tau \leq t_k$. This yields

$$\Delta P_d(k-1) = \frac{1}{T_o}\Big[-M(\Delta\omega(t_k) - \Delta\omega(t_{k-1})) + \int_{t_{k-1}}^{t_k} (-G\Delta\omega(t)+\Delta P_g(t)-\Delta P_t(t))dt\Big] \tag{4.26}$$

The quantities on the right-hand side of (4.26) are determined by measurements on the system.

The estimate given by (4.26) could be improved by averaging over several identification intervals. The type of averaging and the number of intervals used would depend on the kind of load disturbance.

The structure of the composite plant and controller is illustrated in Fig. (4.2). In load-frequency control there is the problem of the controller following rapidly changing random-load disturbances. This is inefficient and contributes to unncessary wear in the controller mechanism. Ross[19] treated this problem and suggested the use of an Error Adaptive Control Computer (EACC). The EACC monitors the error signals and computes the probability that load-frequency control action is required. Control action is initiated only when the computed probability exceeds a preset threshold. As indicated in Fig. (4.2), an EACC can be used to augment proposed load-frequency control given by (4.23) and (4.26).

4.4 Applications

The proposed load-frequency controller is tested on two interconnected steam and hydro-plants (Fig. (4.1)). The parameter values used are as given in Section (3.7)

Due to the coupling between the plants, the optimum feedback control is a function of all the states. The complexity of such a controller makes it essential to investigate various forms of suboptimum controllers. Two different suboptimum controllers are considered, and they are compared to the optimum control.

The optimum feedback control has the form

$$\tilde{u}_0' \triangleq [\tilde{u}_{01} \quad \tilde{u}_{02}], \tag{4.27}$$

$$\tilde{u}_{01} = s_{011}'\tilde{X}_1 + s_{012}'\tilde{X}_2, \tag{4.28}$$

$$\tilde{u}_{02} = s_{021}'\tilde{X}_1 + s_{022}'\tilde{X}_2, \tag{4.29}$$

where $\tilde{X}_1$ and $\tilde{X}_2$ are the state vectors (in the form (4.19)) for the first and second plant, respectively. The gain vectors s_{0jk} (j, k=1, 2) are solutions of a steady-state matrix Riccati equation[24].

By neglecting the coupling between the plants, an optimum control

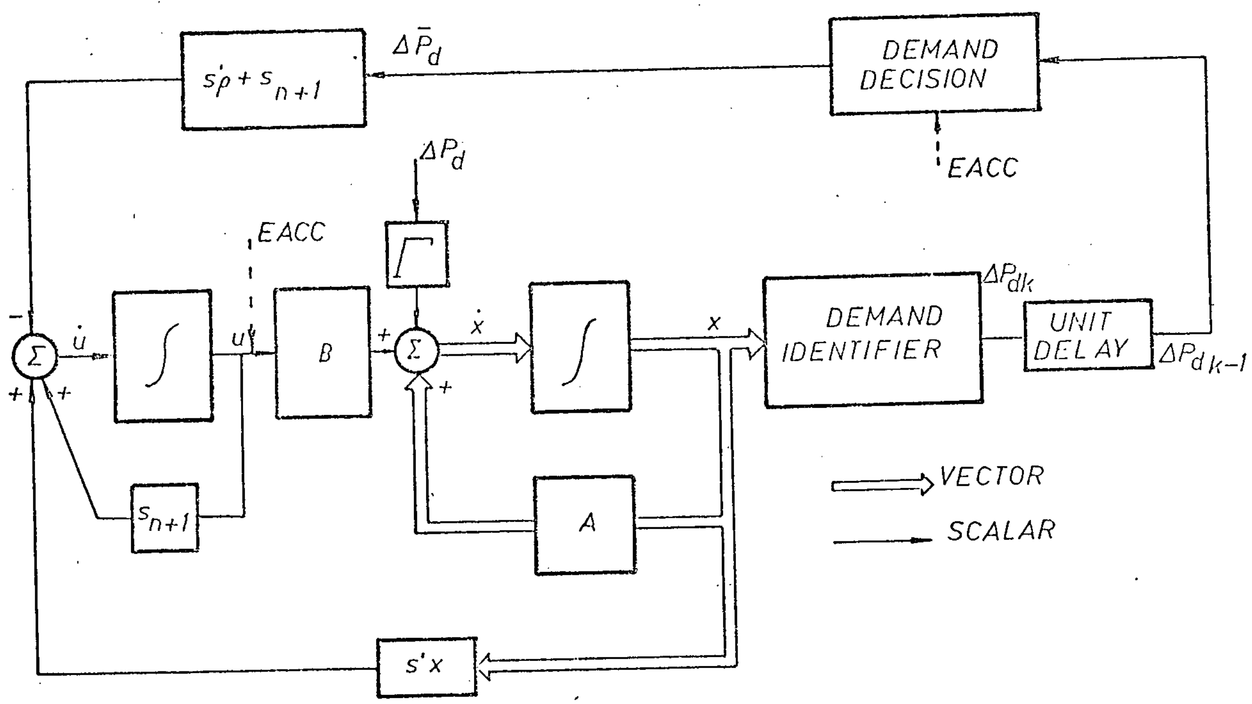


Fig. (4.2) Block diagram of a power plant with a load-frequency controller of the form

$$\tilde{u}_r' \triangleq [\tilde{u}_{r1} \quad \tilde{u}_{r2}], \tag{4.30}$$

$$\tilde{u}_{r1} = s_{r11}'\tilde{X}_1, \tag{4.31}$$

$$\tilde{u}_{r2} = s_{r22}'\tilde{X}_2, \tag{4.32}$$

is obtained for each plant. The control (4.30) is suboptimum for the coupled system.

By neglecting the coupling terms in the optimum control (4.27), a suboptimum control of the form

$$\tilde{u}_s' \triangleq [\tilde{u}_{s1} \quad \tilde{u}_{s2}], \tag{4.33}$$

$$\tilde{u}_{s1} = s_{011}'\tilde{X}_1 \tag{4.34}$$

$$\tilde{u}_{s2} = s_{022}'\tilde{X}_2 \tag{4.35}$$

is obtained. The type of suboptimal control given by (4.34) and (4.35) is discussed in reference 33.

<u>Example (4.1)</u> The Q and R matrices in (4.20) are chosen to be the unit matrices and the assumed demands are taken to be slowly time-varying and given by

$$\Delta P_{d1} = \begin{cases} 0.1 \sin(\pi t/20) & 0 < t \leq 10 \\ 0.1 & t > 10 \end{cases} \tag{4.36}$$

$$\Delta P_{d2} = 0.0$$

The identification interval T_o and the final time t_f are chosen to be 0.5 and 30 seconds, respectively (at $t_f = 30$ the system has essentially reached the steady-state). The initial conditions on the controllers are arbitrarily set equal to zero.

Table (4.1) gives the numerical values of the gain vectors (in terms of the original states (see (4.23)) and the performance cost J.

Table (4.1)

	STATE FED BACK	GAINS FOR THE CONTROLS					
		$\dot{u}_o$		$\dot{u}_r$		$\dot{u}_s$	
		$\dot{u}_{o1}$	$\dot{u}_{o2}$	$\dot{u}_{r1}$	$\dot{u}_{r2}$	$\dot{u}_{s1}$	$\dot{u}_{s2}$
STEAM PLANT	ΔP_{t12}	7.5		15.62		7.5	
	$\Delta \omega$	-.62	-.48	-.5		-.62	
	ΔP_{g1}	-8.	-3.3	-7.2		-8.	
	ΔX_{g1}	-5.7	-1.4	-5.4		-5.7	
	u_1	-4.76	-.47	-4.66		-4.76	
	ΔP_{d1}	18.46	5.17	17.26		18.46	
HYDRO PLANT	ΔP_{t21}		21.		27.5		21.
	$\Delta \omega_2$	-.22	.19		.22		.19
	ΔP_{g2}	-1.71	-.09		-.09		-.09
	Δg_2	-5.7	-10.7		-11.3		-10.7
	ΔX_{g2}	-2.02	-12.9		-13.4		-12.9
	u_2	-.47	-4.63		-4.7		-4.63
	ΔP_{d2}	9.9	28.32		29.49		28.32
	J	0.53		0.78		0.65	

Example (4.2) Table (4.1) shows that the suboptimal control $\tilde{u}_s$ gives a lower cost than $\tilde{u}_r$. Consequently, $\tilde{u}_s$ is used in this Example which illustrates the effect of T_o on identification and control. The following demands are assumed:

$$\Delta P_{d1} = \begin{cases} 0.15 \sin (\pi t/20) & 0 < t \leq 10 \\ 0.15 & t > 10 \end{cases}$$
$$\Delta P_{d2} = \begin{cases} -0.1 \sin (\pi t/20) & 0 < t \leq 10 \\ -0.1 & t > 10 \end{cases} \qquad (4.37)$$

The system responses for (a) $T_o = 0.5$, and (b) $T_o = 1$, are shown in Fig. (4.3). It is evident from Fig. (4.3) that the load-frequency control (4.34) and (4.35), with power demand identification given by (4.24) and (4.26) results in a satisfactory system response.

Example (4.3). In this example the control $\tilde{u}_s$ is compared with a conventional load-frequency controller given in transfer function form by[25].

$$u = \left(\frac{a}{s}\right)\left(\frac{1}{bs+1}\right)(c\Delta\omega + \Delta P_t) \qquad (4.38)$$

The optimum parameter values for the controller, as given in reference 25, are:

a) Steam plant; $a_1 = .09$, $b_1 = 0.3$, $c_1 = .02$

b) Hydro plant; $a_2 = .4$, $b_2 = 0.3$, $c_2 = .02$

Since power demand is not identified, the optimization is performed after averaging over a specified set of power demand profiles. Consequently, for a given power demand, (4.38) is suboptimum.

Figure (4.4) illustrates the comparison of the system responses for: (a) the conventional control u given by (4.38), and (b) the proposed control $\tilde{u}_s$. The demands were chosen to be[25],

$$\Delta P_{d1} = -.005 \qquad\qquad \Delta P_{d2} = .005$$

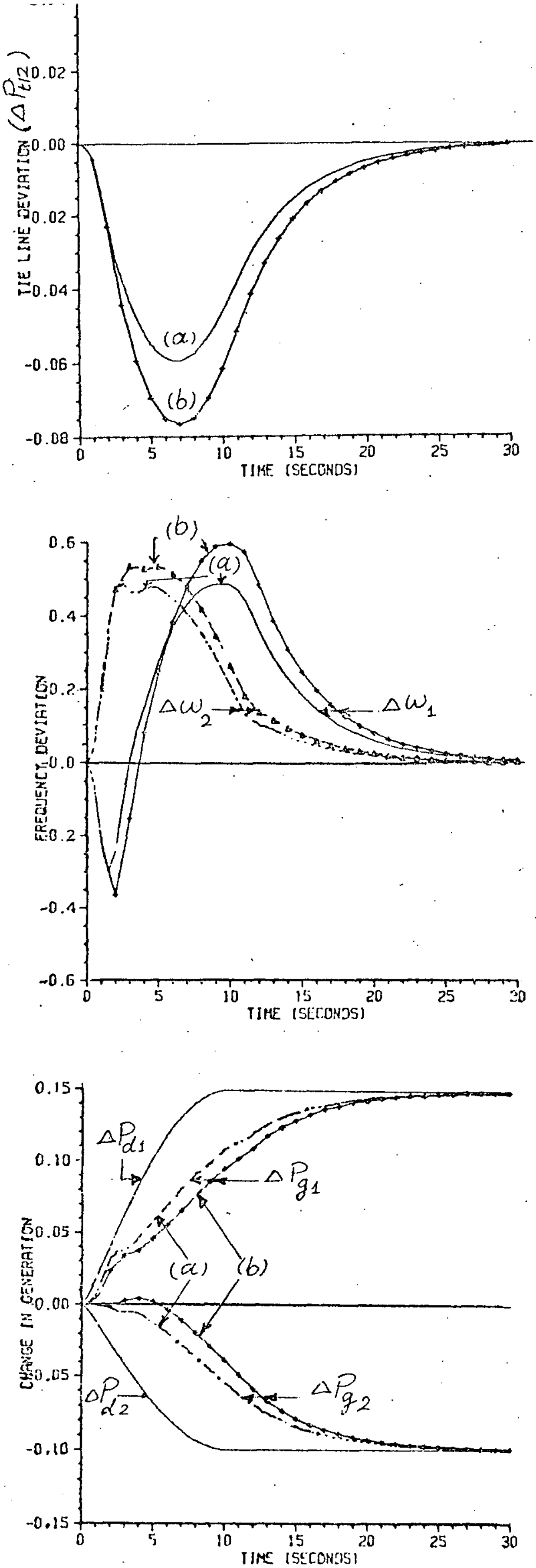


Fig. (4.3) System response for (a) $T_o = 0.5$, (b) $T_o = 1$.

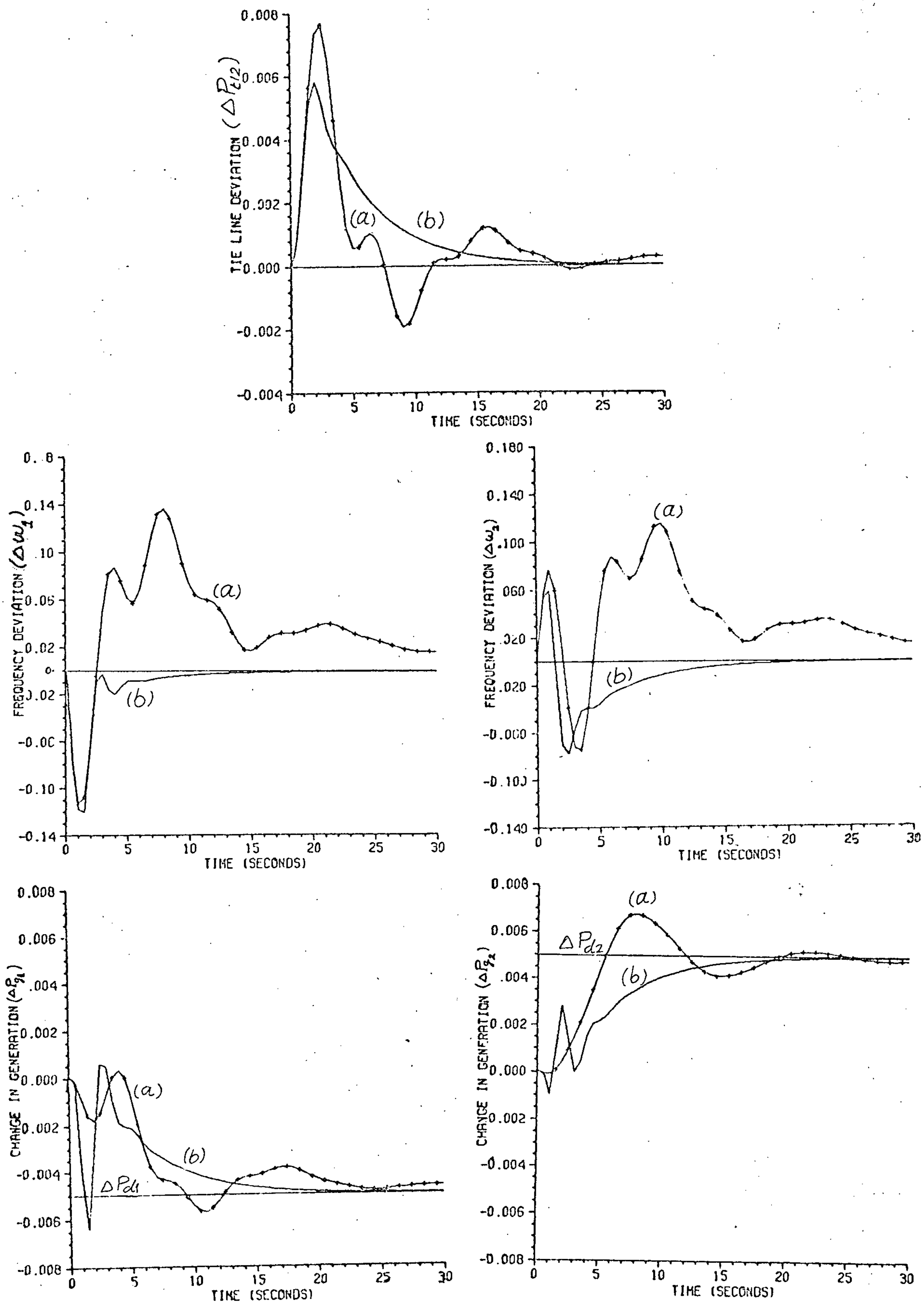


Fig. (4.4) Response Comparison (a) Conventional and (b) proposed controls

It is evident from Fig. (4.4) that the identification of the power demand and the use of the load-frequency control $\tilde{u}_s$ results in a significant improvement in system response.

4.5 Demand Identifier - Luenberger Observer

Tie-line power and frequency deviations (the first two states in the model given by (4.1)) are the only measurements required in presently used load-frequency controllers. The optimum control given by (4.23) requires that all the states be measured and that ΔP_d be identified. For an observable system, measurements of some of the states can be used to reconstruct the complete state by use of a Luenberger observer[34].

Consider a system model and a measurement system of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ z &= Hx \end{aligned} \tag{4.39}$$

where x is an n-state vector and z is an m measurement vector (in general m<n). It is assumed that (4.39) is observable. Luenberger has shown that a class of (n-m) dimensional observers can be structured from (4.39). The observer outputs give an asymptotically correct estimate of the unmeasured states. In theory, arbitrarily small settling time of the observer can be achieved.

To illustrate the use of observers to identify unmeasured power system states and a constant ΔP_d, a single steam plant is considered. The system state equations are augmented by $\Delta\dot{P}_d = 0$. This yields

$$\dot{x} = A_a x + B_a u \tag{4.40}$$

where $x' = [x_p \quad \Delta P_d]$, and where (see (4.4)).

$$A_a \triangleq \left[\begin{array}{cc|ccc} 0 & T^{o}_{12c} & 0 & 0 & 0 \\ -1/M_1 & -G_1/M_1 & -1/M_1 & 0 & -1/M_1 \\ \hline 0 & 0 & -1/T_{t1} & 1/T_{t1} & 0 \\ 0 & -E_1/T_{g1} & 0 & -1/T_{g1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \quad (4.41)$$

$$B'_a \triangleq [0 \quad 0 \mid 0 \; \frac{1}{T_{g1}} \; 0] = [0 \quad 0 \mid B_2] \quad (4.42)$$

Assuming that only the first two states are measured, the measurement matrix in (4.39) takes the form

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (4.43)$$

The partitioning indicated in (4.41) and (4.42) is used to decompose (4.40) into the form

$$\dot{\xi}_1 = A_{11}\, \xi_1 + A_{12}\, \xi_2 \quad (4.44)$$

$$\dot{\xi}_2 = A_{21}\, \xi_1 + A_{22}\, \xi_2 + B_2\, u \quad (4.45)$$

where ξ_1 is the measured m-vector and ξ_2 is an n-m vector which is to be reconstructed by an observer.

Consider the observer defined by[36]

$$\dot{\zeta} = N_{11}\, \zeta + N_{12}\, \xi_1 + B_2\, u \quad (4.46)$$

$$\hat{\xi}_2 = \zeta - M_{11}\, \xi_1, \quad (4.47)$$

where

$$\begin{aligned} N_{11} &\triangleq M_{11}A_{12} + A_{22} \\ N_{12} &\triangleq M_{11}(A_{11} - A_{12}M_{11}) + (A_{21} - A_{22}M_{11}). \end{aligned} \quad (4.48)$$

In (4.48) M_{11} is an arbitrary (n-m)xm matrix. Let $\sigma \triangleq \xi_2 - \hat{\xi}_2$ be the error between the unmeasured vector ξ_2 and the observer output $\hat{\xi}_2$, as given by (4.46) and (4.47). It is seen that

$$\dot{\sigma} = (A_{22} + M_{11}A_{12})\sigma \tag{4.49}$$

Consequently, if M_{11} can be chosen so that (4.49) is asymptotically stable, it follows that

$$\hat{\xi}_2(t) \rightarrow \xi_2(t) \qquad \text{as} \quad t \rightarrow \infty.$$

To determine such a matrix, consider the auxiliary system[35].

$$\dot{\mu} = (A_{22} + M_{11}A_{12})'\mu \tag{4.50}$$

and let

$$\dot{V} = -\mu' C\mu \tag{4.51}$$

where C is an arbitrary positive definite constant matrix. It follows from (4.50) and (4.51) that

$$V = \mu' K\mu \tag{4.52}$$

where

$$(A_{22} + M_{11}A_{12})K + K(A_{22} + M_{11}A_{12})' + C = 0 \tag{4.53}$$

By taking

$$M_{11} = -\frac{1}{2} KA_{12}'S \tag{4.54}$$

where S is an arbitrary positive definite matrix, (4.53) takes the form

$$A_{22}K + KA_{22}' - KA_{12}'SA_{12}K + C = 0 \tag{4.55}$$

Equation (4.55) is the algebraic matrix Riccati equation which can be solved for a positive definite symmetric matrix. With this choice of M_{11} it is seen from (4.51) and (4.52) that the auxiliary system (4.50) is asymptotically stable. Since (4.49) and (4.50) have the same eigenvalues, it follows that (4.49) is asymptotically stable.

Figure (4.5) illustrates a block diagram of optimum load-frequency control with partial measurements of the state and an observer to reconstruct the remaining state components. Notice that the demand ΔP_d is included in $\hat{\xi}_2$ which is reconstructed by the observer from (4.46) and (4.47).

Example (4.4) The data for the single steam-plant considered in this example is the same as used in Example (4.1). Only the first two states (tie-line and frequency deviations) are measured. Digital simulation indicated that

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}, \qquad C = 1000 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.56}$$

was a reasonable choice for this example. The observer matrices resulting from this choice are

$$N_{11} = \begin{bmatrix} -4.97 & 2 & 2.97 \\ -1.33 & -2 & 1.33 \\ .79 & 0 & -7.9 \end{bmatrix}, \quad N_{12} = \begin{bmatrix} 2.97 & -1.4 \\ 1.33 & -.73 \\ -7.9 & 3.36 \end{bmatrix}, \tag{4.57}$$

$$M_{11} = \begin{bmatrix} 0 & -.12 \\ 0 & -.05 \\ 0 & 0.32 \end{bmatrix} \tag{4.57}$$

A constant demand of $\Delta P_d = 0.1$ is assumed. Figure (4.6) illustrates the system responses for Load-Frequency Control using: (a) measurement of all the states assuming ΔP_d is known; (b) measurement of two of the state components and observer reconstruction of remaining components. The slight difference in responses indicates that the observer reconstructs the unmeasured states with adequate (for control purposes) accuracy.

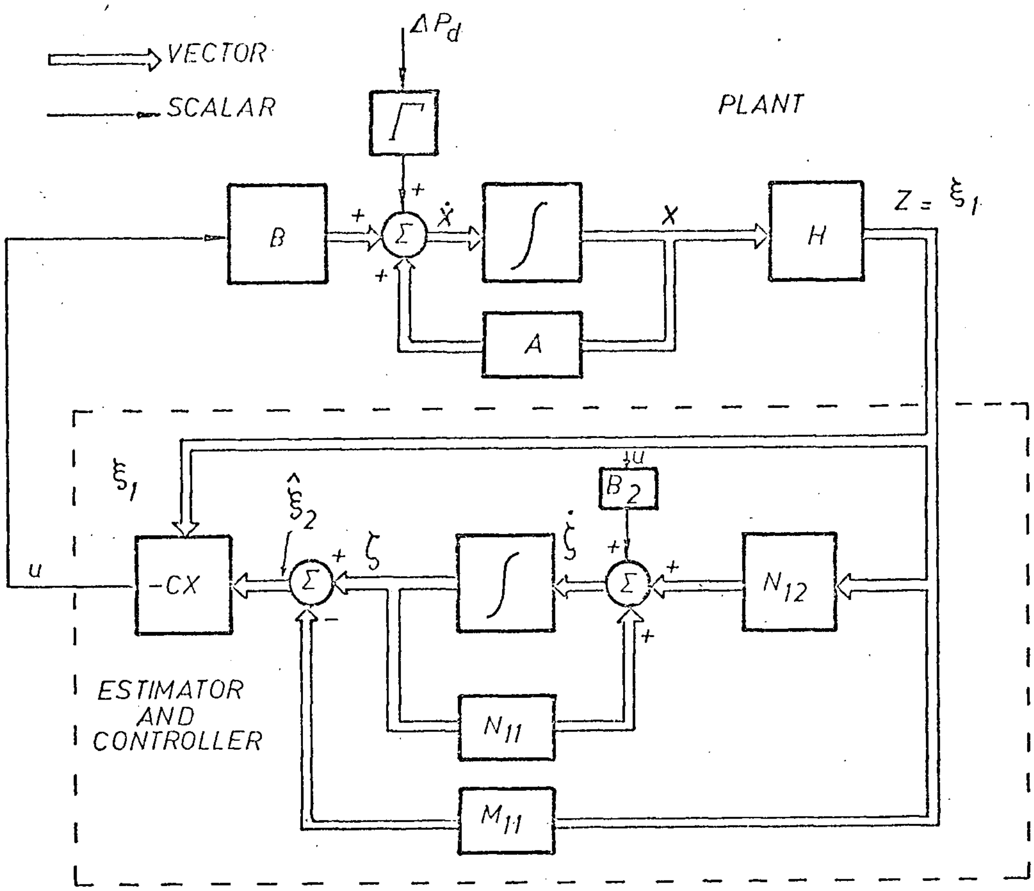
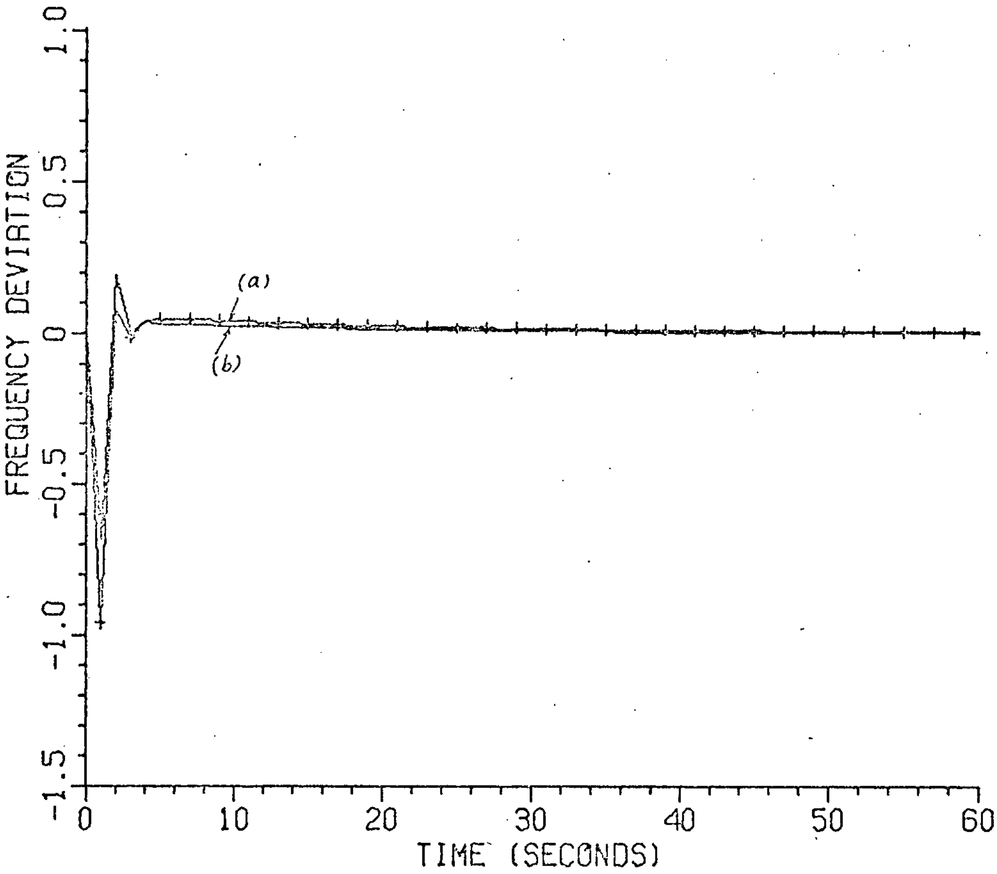


Fig. (4.5) Block diagram of optimum load-frequency control with partial measurements



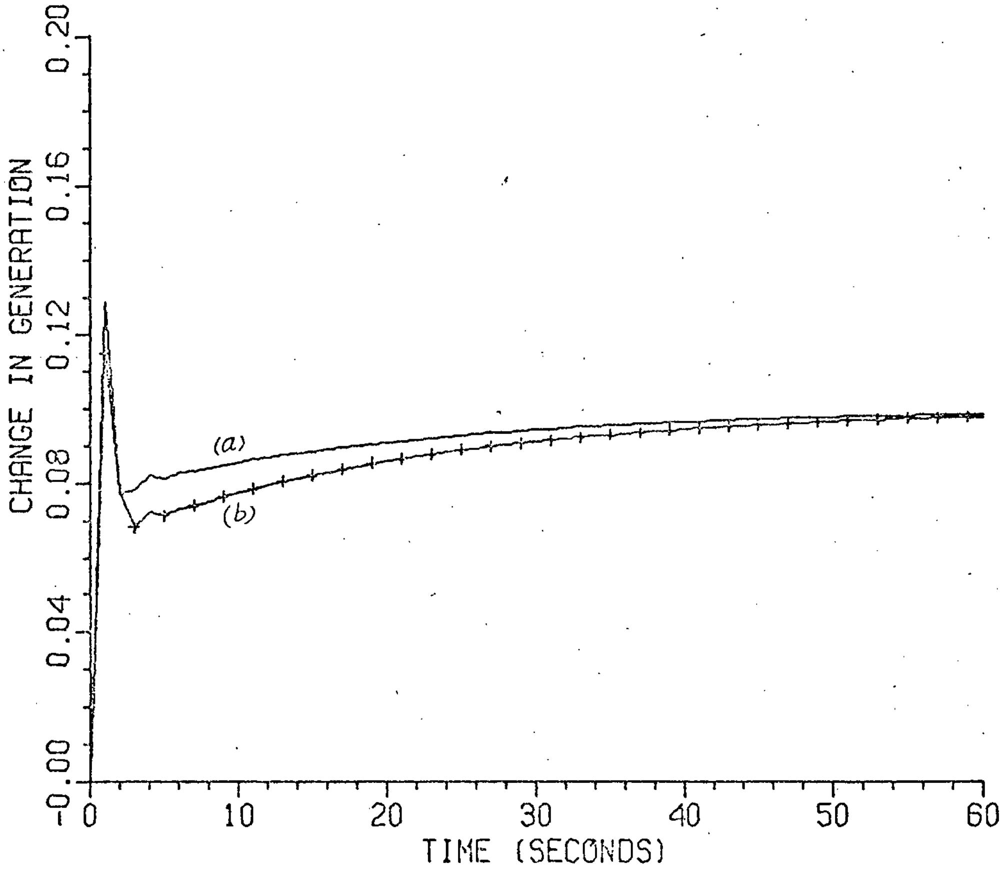


Fig. (4.6) System response - Example (4.4)

5. OPTIMUM LOAD-FREQUENCY SAMPLED-DATA CONTROL WITH RANDOMLY VARYING SYSTEM DISTURBANCES

5.1 Introduction

A power system area generally has interconnections which are physically remote from the controlling station or dispatching center. Feedback regulatory control of the system requires the measurement of tie-line flows at interconnections and the transmission of measured data over data links to the controlling plant or dispatching center. It is essential, therefore, to investigate the effect of sampling time on a control strategy based entirely on continuous signals[17]. In reference 17 the effect of tie-line measurement delay is tested on the continuous system. No attempt, however, is made to take this effect into account in the optimal control design.

This Chapter deals, essentially, with the sampled-data or discrete-time version of the controller discussed in Chapter 4. There is, however, another important aspect of power system control that is considered. Many load disturbances are random in nature and measurements of the system state are often perturbed by noise. The problem of optimal control in the presence of plant and measurement noise is known as the stochastic optimal control problem[31]. The problem has been solved for the case of a quadratic cost index. The resulting controller consists of a cascade combination of a Kalman filter with the standard optimum controller for a linear deterministic system. The detailed statistical data about plant and measurement noise required to implement the Kalman filter is generally not available in a power system. A suboptimal stochastic controller is investigated which does not require extensive statistical data for its implementation. The small number of parameters in the controller makes on-line tuning feasible.

5.2 Optimal Sampled-Data Regulator

The problem formulation for continuous optimal load-frequency control is given by (see Chapter 4, (4.18) and (4.20)).

$$\dot{\tilde{X}} = \tilde{A}\,\tilde{X} + \tilde{B}\,\tilde{u} \tag{5.1}$$

$$\tilde{J} = \frac{1}{2}\int_{t_o}^{\infty} (\tilde{X}'\,Q\,\tilde{X} + \tilde{u}'\,R\,\tilde{u})dt \tag{5.2}$$

The introduction of a data-link in the regulatory loop of a power system results in a sampled-data system and a continuous optimal control is then no longer realizable.

Consider a set of sampling instants (t_o, t_1, t_2, ...) and let $T = t_k - t_{k-1}$ be a constant sampling interval. In a sampled-data system, the control is constrained to be constant between sampling instants:

$$\tilde{u}(\tau) = \tilde{v}_k\ ,\quad t_k \leqq \tau < t_{k+1} \tag{5.3}$$

The formulation of an optimal control problem with a sampled-data control of the form (5.3) requires that (5.1) and (5.2) be transformed into a discrete-time equivalent set of equations. Since A and B are time-invariant, the solution of (5.1) for the interval $t_k \leqq \tau < t_{k+1}$ is given by

$$\begin{aligned}\tilde{X}_{k+1} \triangleq \tilde{X}(t_{k+1}) &= \tilde{\Phi}(t_{k+1}, t_k)\tilde{X}_k + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)\tilde{B}\tilde{v}_k\, d\tau \\ &= \tilde{\tilde{\Phi}}\,\tilde{X}_k + \tilde{D}\tilde{v}_k\end{aligned} \tag{5.4}$$

where $\tilde{\Phi} \triangleq \tilde{\Phi}(T,0)$ is the state-transition matrix of (5.1):

$$\dot{\tilde{\Phi}} = \tilde{A}\,\tilde{\Phi},\quad \tilde{\Phi}(0,0) = I \tag{5.5}$$

(I is the unit matrix), and where

$$\tilde{D} \triangleq \int_0^T \tilde{\Phi}(T,t)\tilde{B}\, dt. \tag{5.6}$$

The cost index (5.2) can be expressed as the sum of N integrals, each integral being evaluated over a sampling interval. Using (5.4) it is seen that

$$J = \frac{1}{2} \sum_{k=0}^{\infty} (\tilde{X}_k'\hat{Q}\tilde{X}_k + 2\tilde{X}_k'M_k\tilde{v}_k + \tilde{v}_k'\hat{R}_k\tilde{v}_k) \tag{5.7}$$

where

$$\hat{Q} \triangleq \int_0^T \tilde{\Phi}'(T,t)\tilde{Q}\,\tilde{\Phi}(T,t)dt \tag{5.8}$$

$$M \triangleq \int_0^T \tilde{\Phi}'(T,t)\tilde{Q}\tilde{D}(T,t)dt \tag{5.9}$$

$$\hat{R} \triangleq \int_0^T [R + \tilde{D}'(T,t)\tilde{Q}\tilde{D}(T,t)]dt \tag{5.10}$$

The optimum feedback control for the discrete-time problem given by (5.4) and (5.7) is[16]

$$\tilde{v}_k^* = -C\,\tilde{X}_k \tag{5.11}$$

where

$$C = \hat{R}^{-1}M' + (\hat{R} + \tilde{D}'\tilde{K}\tilde{D})^{-1}\tilde{D}\tilde{K}\Theta \tag{5.12}$$

The nxm constant matrix $\tilde{K}$ is the steady-state solution of the matrix-Riccati difference equation

$$\tilde{K}_k = \Theta'[\tilde{K}_{k+1} - \tilde{K}_{k+1}\tilde{D}(\hat{R} + \tilde{D}'\tilde{K}_{k+1}\tilde{D})^{-1}\tilde{D}'\tilde{K}_{k+1}]\Theta + \Lambda \tag{5.13}$$

where $\tilde{K}_\infty = 0$ is the boundary condition and where

$$\Theta \triangleq \tilde{\Phi} - \tilde{D}\,\hat{R}^{-1}\,M' \tag{5.14}$$

$$\Lambda \triangleq \hat{Q} - M\,\hat{R}^{-1}\,M' \tag{5.15}$$

Expressing (5.11) in terms of the original system states (see Chapter 4, (4.8), (4.23)) it is seen that the following equations describe the optimum sampled-data load-frequency control:

$$\dot{x}_p(\tau) = A\, x_p(\tau) + B\, u(\tau) + \Gamma\, \Delta P_d \qquad \tau \varepsilon [t_k, t_{k+1}] \tag{5.16}$$

$$u(\tau) = u_k + t\, \tilde{v}_k \qquad t \varepsilon [0,T] \tag{5.17}$$

$$u_{k+1} = u_k + T\, \tilde{v}_k \tag{5.18}$$

$$\tilde{v}_{k+1} \triangleq s'\, x_{pk} + s_{n+1}\, u_k - (s'\rho + s_{n+1})\Delta P_d \tag{5.19}$$

$$c' \triangleq (s, s_{n+1}) \tag{5.20}$$

It is seen from (5.11), (5.12) and (5.17) that the optimum control for a sampled-data system depends parametrically on the sampling time T. It is shown in reference 16 that, as $T \to 0$, the continuous optimum control is the limiting case of the sampled-data control.

Example (5.1) Consider the single steam plant discussed in Chapter 4, Example (4.4). The following three control policies are considered for a demand $\Delta P_d = 0.1$:

(a) A continuous control u_1 which uses continuous state information (see Chapter 4, Table (4.1)).

(b) A sampled-data control u_2 which uses state information at a sampling rate of 1 second (T=1).

(c) Control u_3, same as (b) above with T=2.

The feedback coefficients (5.20) for the above controls are:

$$C_1' = [15.62 \quad -.5 \quad -7.2 \quad -5.4 \quad -4.66] \tag{5.21}$$

$$C_2' = [2.04 \quad 0.03 \quad -0.18 \quad -0.48 \quad -1.58] \tag{5.22}$$

$$C_3' = [.172 \quad 0.02 \quad 0.17 \quad 0.11 \quad -0.51] \tag{5.23}$$

The cost indicies for the three above cases are:

Control	u_1	u_2	u_3
Cost Index	0.754	1.465	2.319

The cost index has practical usefulness only if the associated control strategy results in acceptable system responses. These are shown in Fig. (5.1). It is seen from Fig. (5.1) that u_1 gives the fastest response. However, there is no significant deterioration in dynamic performance by using a sampling rate of one second.

5.3 Stochastic Optimum and Suboptimum Control

The stochastic optimal control problem for a discrete-time linear system with linear measurements is defined by

$$x_{k+1} = \Phi_k x_k + D_k u_k + w_k \tag{5.24}$$

$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1} \tag{5.25}$$

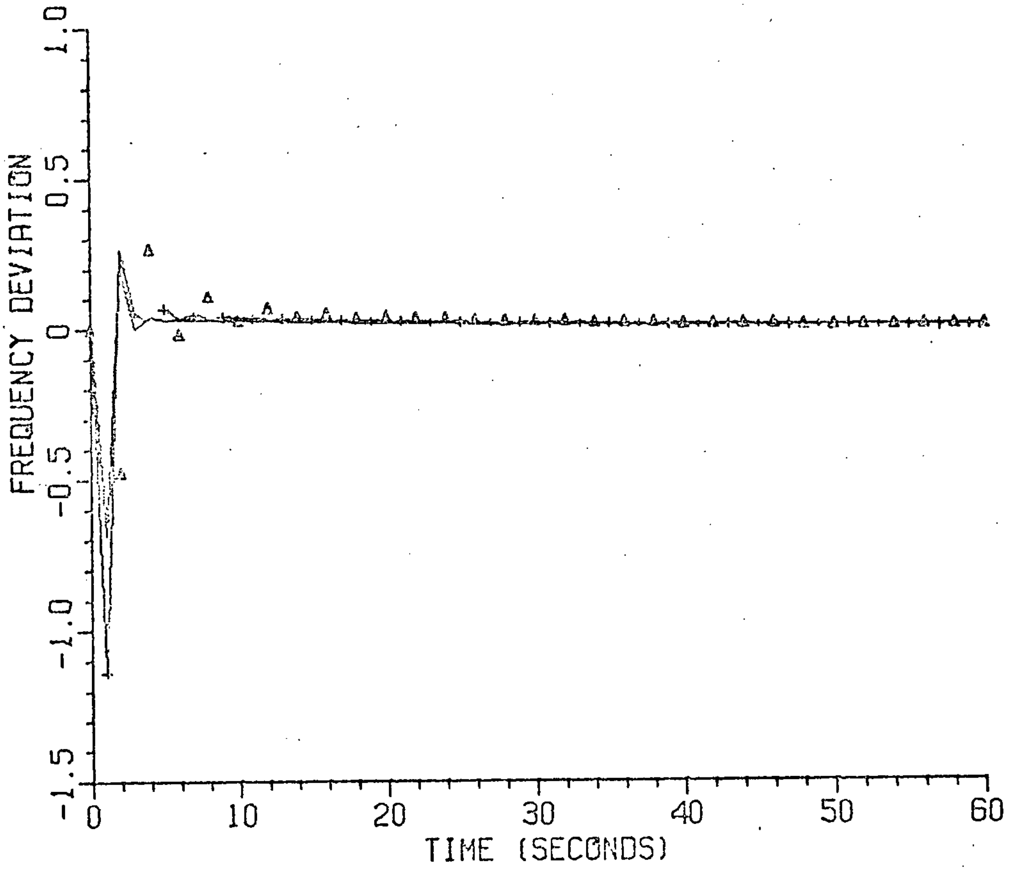
$$J_k = \frac{1}{2} \sum_{j=1}^{N} x_j' \bar{Q}_j x_j + u_j \bar{R}_j u_j \tag{5.26}$$

where w_k is an n-dimensional plant noise vector, z_k is an m-dimensional measurement vector and v_k is an m-dimensional measurement noise vector. In (5.26), $\bar{Q}_j$ is a positive semi-definite weighting matrix sequence and $\bar{R}_j$ is a positive definite weighting matrix.

The problem is to determine a realizable control sequence which minimizes $E(J_o)$, the expected value of J_o, given the measurement sequence and the following statistical data ($E(\cdot)$ is the expectation operator):

$$\begin{gathered} E(x_o) = \hat{x}_o \ , \quad E(x_o x_o') = P_o \ , \quad E(w_k) = E(v_k) = 0 \\ E(w_j w_k') = 0, \quad E(v_j v_k') = 0 \ , \quad E(w_j v_k') = 0 \quad (j \neq k) \end{gathered} \tag{5.27}$$

$$E(w_j w_j') = Q_j \qquad E(v_j v_j') = R_j$$



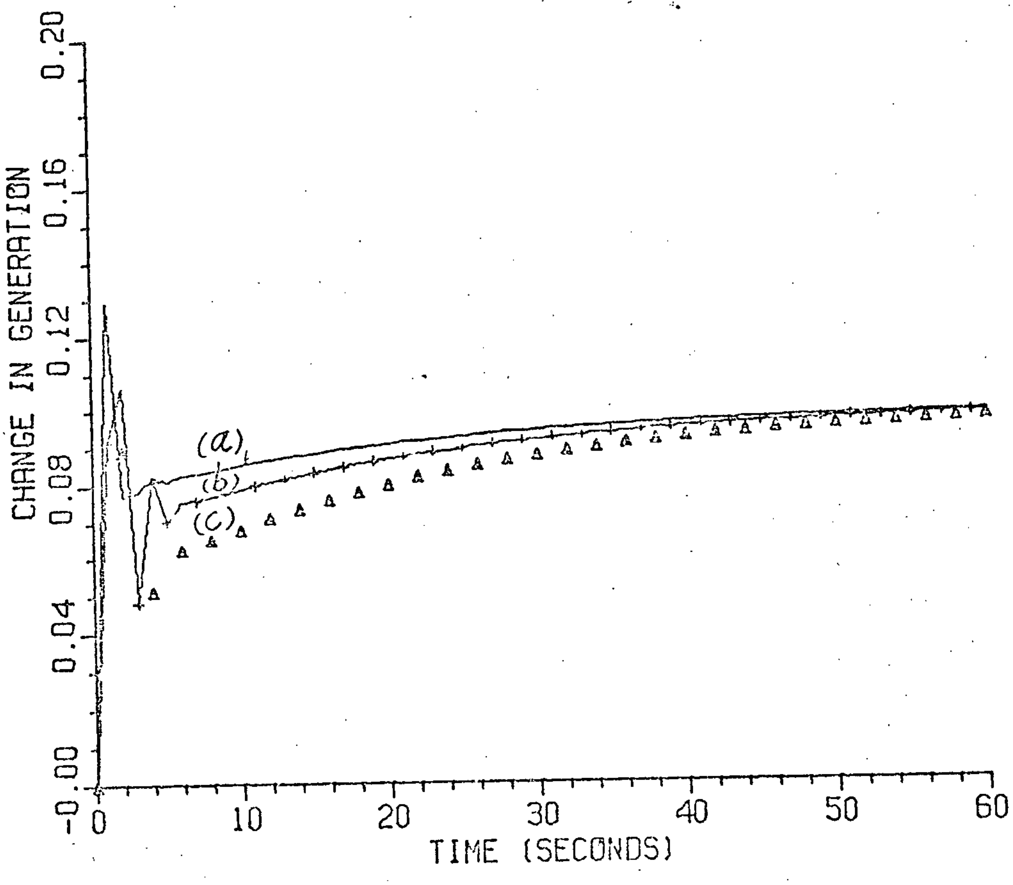


Fig. (5.1) Response comparison for (a) continuous control, (b) sampled-data control T = 1 and (c) sampled data control T = 2.

If the statistics are Gaussian, it is known that the optimal control sequence is given by

$$u_k = -C_k \hat{x}_k \quad , \quad (k = 0, 1, \ldots N-1) \tag{5.28}$$

where $\hat{x}_k$ is the conditional mean of x_k and where C_k is the control gain for the optimum deterministic (noise-free) control[31]. The estimate $\hat{x}_k$ can be generated recursively on-line by use of the Kalman filter:

$$\hat{x}_k = (\Phi_{k-1} - D_{k-1} C_{k-1}) \hat{x}_{k-1} + K_k y_k \tag{5.29}$$

$$y_k \triangleq z_k - H_k (\Phi_{k-1} - D_{k-1} C_{k-1}) \hat{x}_{k-1} \tag{5.30}$$

The vector y_k is the difference between the measurement vector and the predicted measurement vector, and K_k is the optimum filter gain. It is seen from (5.24) - (5.30) that the error, $e_k = x_k - \hat{x}_k$, and the error covariance, P_k, are given respectively by

$$e_k = (I - K_k H_k)(\Phi_{k-1} e_{k-1} + w_{k-1}) - K_k v_k \tag{5.31}$$

$$P_k \triangleq E(e_k e_k') = (I - K_k H_k)(\Phi_{k-1} P_{k-1} \Phi_{k-1}' + Q_{k-1})(I - K_k H_k)' + K_k R_k K_k' \tag{5.32}$$

The optimum filter gain, K_k, minimizes $Tr(P_k)$, the trace of the error covariance matrix.

From a mathematical point of view the solution is surprisingly simple. The control gain matrix in (5.28) can be determined by solving the noise-free optimal control problem and $\hat{x}_k$ is determined by solving, separately, an optimal filter problem. From a practical point of view, however, the on-line implementation of (5.28) raises severe problems, particularly in power system applications. Even in the noise-free case, due to the complexity of the optimum controller, optimum control of a power system is both impractical and uneconomical.

It is essential to investigate suboptimum controllers and to simplify the system model. Consequently, the noise sequences in (5.24) and (5.25) may in part arise from modelling errors, thus invalidating the Gaussian white noise assumption used in deriving (5.28). Furthermore, data in the form (5.27) is generally not available for a power system. It is reasonable, however, to retain the control structure defined by (5.28), (5.29) and (5.30) in a suboptimum stochastic controller. This follows from the fact that, for arbitrary K_k, (5.29) is an observer for the system (5.24). The usefulness of an observer has been shown in Chapter 4. The observer (5.29) gives improved estimates if the observer gain K_k is chosen to minimize an estimation error cost index (such as $Tr(P_k)$).

Let L be a constant observer matrix gain which results in a stable observer and let

$$K_k = g_k L \tag{5.33}$$

where g_k is a scalar gain. A class 0_g of stable observers is defined by the stability limits,

$$g_m < g_k < g_M \tag{5.34}$$

on the scalar gain. All subsequent observers are considered to belong to 0_g. An optimum gain could be defined in 0_g by associating a cost index with P_k (see (5.32)). Consider the choice

$$Tr(\bar{\bar{Q}}_k P_k) = \alpha_k\, g_k^2 - 2\beta_k\, g_k + \gamma_k \tag{5.35}$$

where

$$\begin{aligned} \beta_k &\triangleq Tr[\bar{\bar{Q}}_k L H_k (\Phi_{k-1} P_{k-1} \Phi'_{k-1} + Q_{k-1})] \\ \alpha_k &\triangleq Tr(\bar{\bar{Q}}_k L (H_k (\Phi_{k-1} P_{k-1} \Phi'_{k-1} + Q_{k-1}) H'_k + R_k) L') \end{aligned} \tag{5.36}$$

and where $\bar{\bar{Q}}_k$ is a positive semi-definite weighting matrix. The optimum gain which minimizes the cost index (5.35) is given by

$$g_k^* = \beta_k/\alpha_k. \tag{5.37}$$

Filtering and minimization of estimation error do not, however, represent the complete problem, which is to determine a suboptimal control. Furthermore, even though there are only two statistical parameters in (5.37), it is desirable to reduce further the need for statistical data. The dependency of the optimum stochastic control (5.28) on the data (5.27) arises out of the global minimization of the cost index $E(J_o)$. A suboptimal stochastic control can be determined by a local (stage-wise) minimization of an instantaneous cost $\bar{J}_k$ associated with a control decision at stage k. The cost $\bar{J}_k$ must be related in some meaningful manner to (5.26).

In the noise-free case, the system dynamics are

$$\bar{\bar{x}}_{j+1} = \Phi_j \bar{\bar{x}}_j + D_j \bar{\bar{u}}_j \quad , \bar{\bar{x}}_k = \hat{x}_k \, , \quad j \geq k \tag{5.38}$$

the optimal control sequence is given by $\bar{\bar{u}}_j = -C_j \bar{\bar{x}}_j$, and (5.26) can be expressed in the closed form

$$\hat{J}_k = \frac{1}{2} \hat{x}_k' V_k \hat{x}_k \tag{5.39}$$

where V_k is the solution of a discrete matrix-Riccati equation[31]. Let $\hat{x}_k$ be defined by (5.27) where $\hat{x}_{k-1}$ is known from the previous stage and let

$$\bar{\bar{e}}_j \triangleq x_j - \bar{\bar{x}}_j \tag{5.40}$$

A simple estimate of the effect of the error $e_k = \bar{\bar{e}}_k = x_k - \hat{x}_k$ at stage k on the cost J_k can be obtained by taking the predicted value of future noise at any stage $j > k$ to be equal to the mean value, which is zero, as given by (5.27). Consequently

$$\begin{aligned} x_{j+1} &= (\Phi_j - D_j C_j) x_j = \Psi(j+1, k) x_k \\ \bar{\bar{e}}_{j+1} &= (\Phi_j - D_j C_j) \bar{\bar{e}}_j = \Psi(j+1, k) e_k \, , \quad (j \geq k) \end{aligned} \tag{5.41}$$

are predicted future values of x_k and e_k, respectively. In (5.41), $\Psi(j,k)$ is the system state transition matrix. Introducing (5.40) and (5.41) into (5.26) and making use of (5.39) yields

$$J_k = \hat{J}_k + e_k' \bar{\bar{Q}}_k \hat{x}_k + \frac{1}{2} e_k' \bar{\bar{Q}}_k e_k \tag{5.42}$$

where

$$\bar{\bar{Q}}_k \triangleq \sum_{j=k}^{N} \Psi'(j,k)(\bar{Q}_j + C_j'\bar{R}_j C_j)\Psi(j,k) \tag{5.43}$$

The expectation of (5.42) is

$$E(J_k) = E(\hat{J}_k) + Tr(\bar{\bar{Q}}_k E(\hat{x}_k e_k')) + \frac{1}{2} Tr(\bar{\bar{Q}}_k P_k) \tag{5.44}$$

Equation (5.44) represents a decomposition of an estimate of the average cost into a deterministic control cost (first term) and a cost of estimation error (third term). The second term couples the two costs. It is seen from (5.35) and (5.37) that $g_k = g_k^*$ minimizes the third term. Consider the effect that this choice has on the second term. Since $\hat{x}_{k-1}$ is assumed given, and $E(x_{k-1}) = \hat{x}_{k-1}$, it follows that

$$E(\hat{x}_{k-1}\, e_{k-1}') = 0 \tag{5.45}$$

It can be shown from (5.29), (5.31) and (5.45) that (see (5.35), (5.36)).

$$T_r(\bar{\bar{Q}}_k E(\hat{x}_k e_k')) = \beta_k\, g_k = \alpha_k\, g_k^2 \tag{5.46}$$

From (5.37) it is seen that (5.46) vanishes when $g_k = g_k^*$, consequently, if

$$\bar{J}_k \triangleq \hat{J}_k + \frac{1}{2}(g_k^2 - 2g_k^* g_k + a_k)\alpha_k \tag{5.47}$$

where $a_k \triangleq \gamma_k/\alpha_k$, it follows that

$$E(J_k) = E(\bar{J}_k) \tag{5.48}$$

when $g_k = g_k^*$. Equation (5.48) justifies considering the control defined by (5.28), (5.29), (5.33) and (5.37) as a suboptimum stochastic control.

The cost index (5.47) has two terms. The first term is the cost of deterministic control and the second term is a cost associated with estimation error. Both terms depend on g_k and consequently there is a trade-off between the two costs, with α_k representing the trade-off or weighting factor. This suggests the possibility of choosing g_k adaptively within the observer class 0_g so that there is a decrease in $\bar{J}_k$. To formulate such an adaptive control strategy requires that the parameters in (5.47) be estimated, as far as this possible, from the available measurements.

From (5.30) and (5.36) it is seen that

$$\begin{aligned} \alpha_k &= Tr(\bar{\bar{Q}}_k \; L \; E \; (y_k y_k') L') \\ &= E(y_k' \; L' \; \bar{\bar{Q}}_k \; L \; y_k) \end{aligned} \tag{5.49}$$

The random variable in (5.49) is always positive and vanishes only when $y_k = 0$. Since α_k enters (5.47) as a weighting factor, it is reasonable to replace (5.49) by an instantaneous estimate

$$\alpha_k \cong y_k' \; W_k \; y_k \tag{5.50}$$

where W_k is a positive definite weighting matrix. Consequently, (5.47) can be replaced by the instantaneous cost index

$$\bar{J}_k = \frac{1}{2} \hat{x}_k' \; V_k \; \hat{x}_k + \frac{1}{2}(g_k^2 - 2\bar{g} \; g_k + a_k) y_k' \; W_k \; y_k \tag{5.51}$$

where $\bar{g}$ is a threshold level determined by off-line computer simulation (or by on-line tuning).

To prevent erratic gain changes due to the estimation (5.50), a step size constraint

$$(g_k - g_{k-1})^2 = \delta\ell(g_{k-1})^2 \tag{5.52}$$

where $\delta\ell$ is fixed, is imposed. The optimum adaptive gain is defined to be the gain that minimizes (5.51) subject to the constraints (5.52) and (5.34). If (5.34) is satisfied, the optimum gain is determined by the method of steepest descent,which yields

$$g_k = g_{k-1}[1 - \delta \ell \operatorname{sgn}(G_{k-1})] \tag{5.53}$$

$$G_{k-1} \triangleq \left[\frac{\partial \bar{J}_k}{\partial g_k}\right]_{g_j = g_{k-1}} = y_k'\left(V_k \bar{x}_k + (g_{k-1} - \bar{g}) W_k y_k\right) \tag{5.54}$$

$$\bar{x}_k \triangleq (\Phi_{k-1}' - D_{k-1} C_{k-1}) \hat{x}_{k-1} + g_{k-1} L y_k \tag{5.55}$$

If (5.53) violates (5.34), then g_k is replaced by the appropriate upper or lower bound.

The adaptive nature of g_k can be seen from (5.51). If estimation error becomes excessive, (5.50) increases and more weight is given to choosing g_k to minimize estimation error. As estimation error increases, it is desirable to place more weight on the use of measurements. This weighting is done optimally if (5.37) is used. The threshold can be set so that g_k^* approaches $\bar{g}_k$ as estimation error increases. On the other hand, if the estimation error is acceptable, then more weight is given to choosing g_k to minimize the cost of control. This means that, as long as the estimate is acceptable, a small control effort should be used.

The choice of W_k in (5.54) is governed largely by computational convenience. A simple and reasonable choice is $W_k = wV_k$, where w is a positive number. Another simple possibility is to choose w so that

$$G_{k-1} = y_k' V_k(\bar{x}_k + b \operatorname{sgn}(g_{k-1} - \bar{g}) y_k) \tag{5.56}$$

where b is a positive number. The advantage of the adaptive approach is that explicit evaluation of statistical data is not required. The controller is "tuned" by off-line computer simulations. The small number of tuning parameters (two) makes on-line tuning feasible.

5.4 Application - Single Steam Plant

The model of a single steam plant given in Example (5.1), is used to evaluate the load-frequency control capabilities of the proposed adaptive controller. The augmented state model has the form (5.24). It is assumed that frequency and tie-line deviations are the only measurements available so that the measurement matrix in (5.25) has the form

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{5.57}$$

The discrete control sequence represented by u_k in (5.24) must be replaced by $U_k' = (u_k, v_k)'$, where (see (5.17) - (5.19))

$$u_k = u_{k-1} + T\,\tilde{v}_{k-1} \tag{5.58}$$

$$\tilde{v}_k = s'\,x_{pk} + s_{n+1}\,u_k - (s'\,\rho + s_{n+1})\Delta\bar{P}_d. \tag{5.59}$$

The Φ and D matrices for the augmented model are given by (see Chapter 4, (4.40), and (5.4))

$$\dot{\Phi} = A_a\,\Phi \qquad \Phi(0,0) = I, \tag{5.60}$$

$$D \triangleq [D_1 \quad D_2] \tag{5.61}$$

where

$$D_1 = \int_0^T \Phi(T,t)\;B_a\;\mathrm{ft} \tag{5.62}$$

$$D_2 = \int_0^T \Phi(T,t)\;B_a\cdot t\cdot dt \tag{5.63}$$

For simulation purposes, the noise vectors w_k and v_k are taken in the form

$$w_k = \alpha_w\,I_{wk}\,x_{k+1}\quad,\qquad v_k = \alpha_v\,I_{vk}\,z_k \tag{5.64}$$

where (see (4.40)), $x_k' = [\, x_{pk} \quad \Delta P_{dk} \,]$. I_{wk} and I_{vk} are diagonal matrices whose elements are pseudo-random numbers with a uniform distribution between -1 and +1. The scalars α_w and α_v are used to set noise level.

It should be noticed that the noise (5.64) is state-dependent. This occurs, for example, when the system parameters undergo random disturbances. The control (5.28), with $\hat{x}_k$ given by the Kalman filter, whose gain is based on the Gaussian statistics (5.27), is suboptimum for the type of noise given by (5.64). Tuning of the time-varying matrix gain to improve system performance is impractical. The proposed suboptimum adaptive control, however, is easily tuned to a variety of noise statistics, including those defined by (5.64).

The design of the adaptive control proceeds in three stages. The first stage is to determine the control gain matrix in (5.28) for the deterministic system. This has been done in Section (5.2) (Example (5.1)).

The second stage is the design of a deterministic observer with a constant gain L (see (5.33)). The design details are given in Appendix IV where the result

$$L' = \begin{bmatrix} .663 & .005 & .091 & -.128 & .364 \\ .007 & .935 & -.006 & -.014 & -.053 \end{bmatrix} \tag{5.65}$$

is obtained. The system response for an incremental power demand $\Delta P_d = 0.1$ using a suboptimum controller with the observer gain (5.65) is shown in Fig. (5.2). It is seen that the response meets the specified conditions in that, as $t \to +\infty$, the frequency deviation and incremental generation approach zero and 0.1, respectively. In the absence of system noise, it is seen that the control (5.28), where $\hat{x}_k$ is the deterministic observer output, gives acceptable dynamic performance.

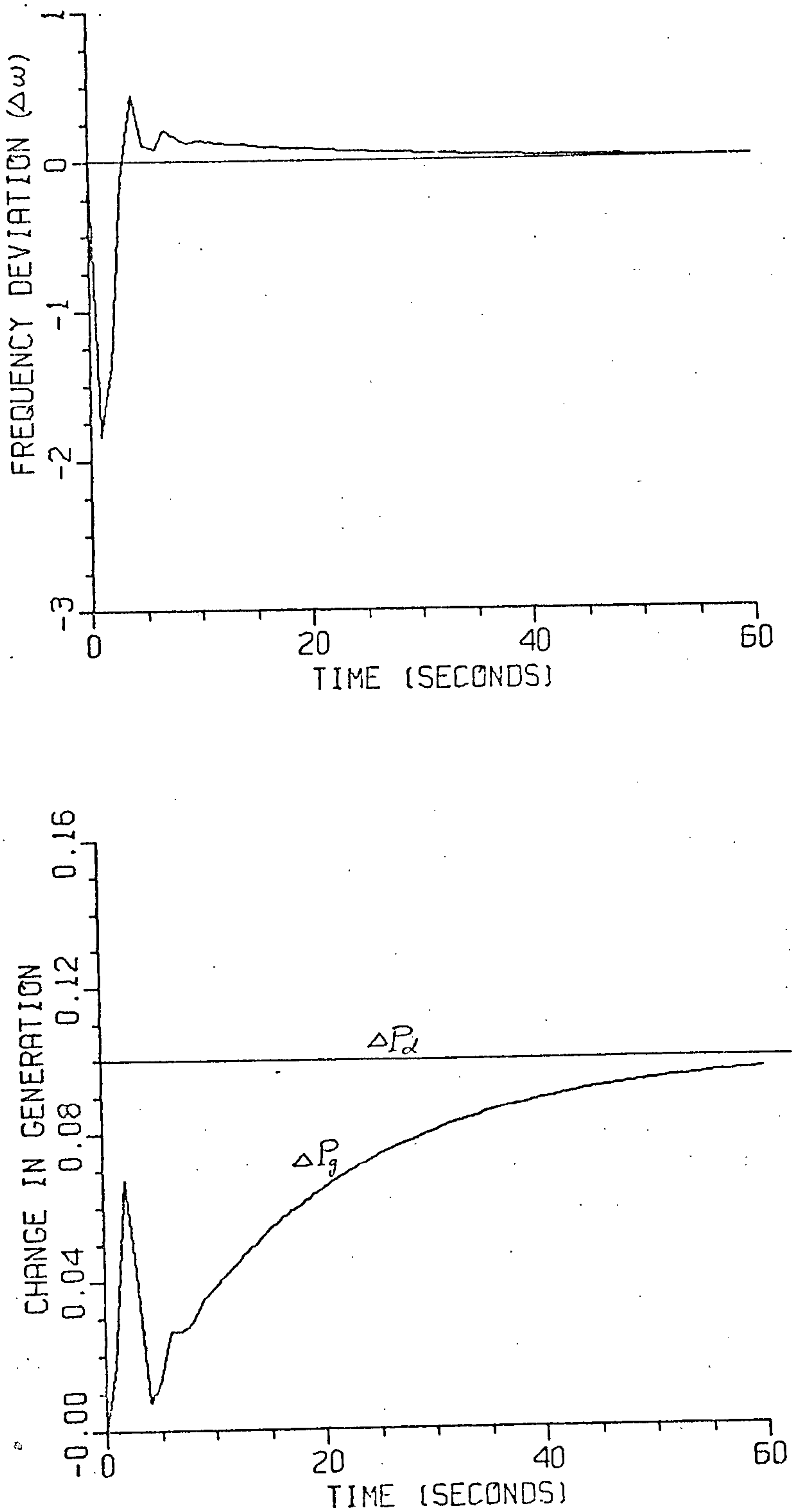


Fig. (5.2) Steam plant response. Frequency and tie-line measurements sampled with T = 1.

The third stage is the design of the adaptive controller which is based on changing the scalar gain g_k according to the strategy given by (5.53) and (5.56). The stability limits (5.34) for the system under consideration are easily shown to be $g_m = 0$ and $g_M = 2$, respectively. The step-size parameter $\delta\ell$ is set equal to 0.2 (see (5.52)). This choice is a commonly used compromise between using a small step to meet linearity and numerical stability requirements and using a large step to reduce the number of steps. The only parameters which require detailed investigation are b and $\bar{g}$. For the system considered the choice b = 0.2 appeared reasonable after some preliminary simulation studies. To investigate the effect of different values for the threshold level $\bar{g}$, the cost

$$J = \sum_{k=0}^{60} (\tilde{X}_k' \tilde{X}_k + u_k^2) \tag{5.66}$$

is investigated and averaged over ten runs. The initial frequency deviation is taken to be $\Delta\omega(0) = 2$ and all other initial states are set equal to zero. Figure (5.3) illustrates the average cost as a function of $\bar{g}$ for different noise levels. From Fig. (5.3) the best average value is taken to be $\bar{g} = 0.5$.

The adaptive controller is now "tuned" and its effect on system performance with different noise levels can be evaluated. Figure (5.4) illustrates the results for: (a) The deterministic observer ($g_k = 1$). (b) The adaptive observer. It is seen that the adaptive observer results in a lower cost. The three values chosen for α_w correspond to random changes in the elements of Φ and D of 10%, 20% and 30%, respectively. Heavy measurement noise ($\tilde{\alpha}_v = 1$) could be considered to arise when there are faulty measurements or faulty data transmission.

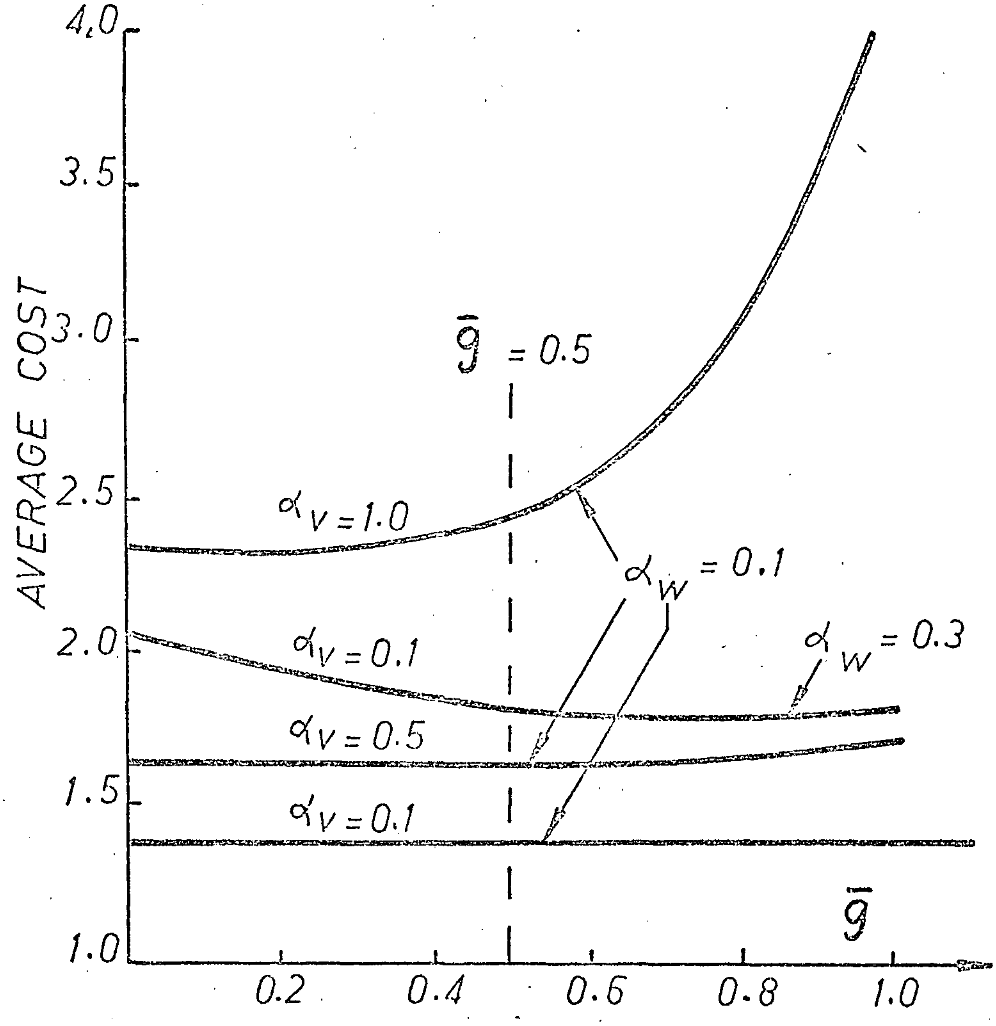


Fig. (5.3) Average cost as a function of threshold level $\bar{g}$

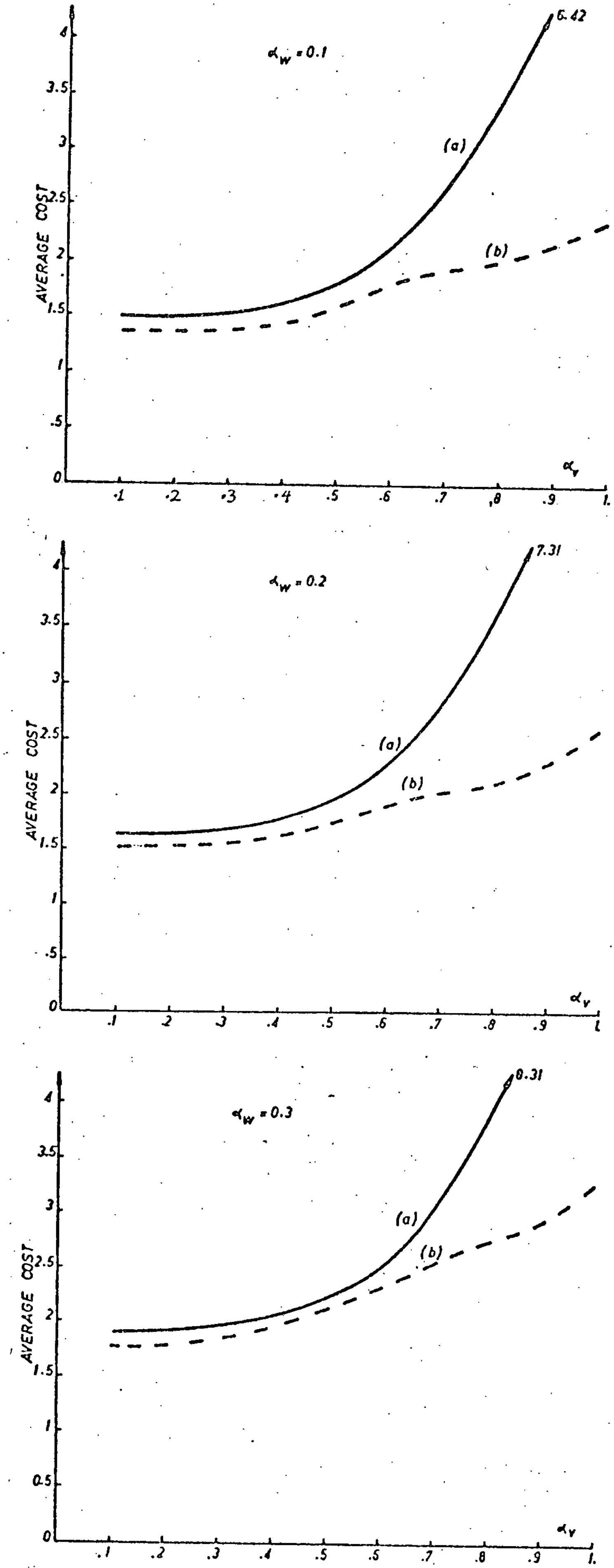


Fig. (5.4) Effect of adaptive gain (a) deterministic observer
(b) adaptive observer.

The adaptive observer is a filter whose output is an estimate of the state of the system. It is of interest to evaluate the filtering (or tracking) capbilities of the adaptive observer in the presence of system noise. For the evaluation, an incremental power demand of $\Delta P_d = 0.1$ is assumed and the noise levels are chosen to be $\alpha_v = 0.2$ and $\alpha_w = 0.1$. Figure (5.5) illustrates the system frequency $\Delta\omega$, the estimated frequency $\Delta\hat{\omega}$, the estimated demand $\Delta\hat{P}_d$, and the control signal u. It is seen from Fig. (5.5) that the filter output gives a good estimate of the average behaviour of the states.

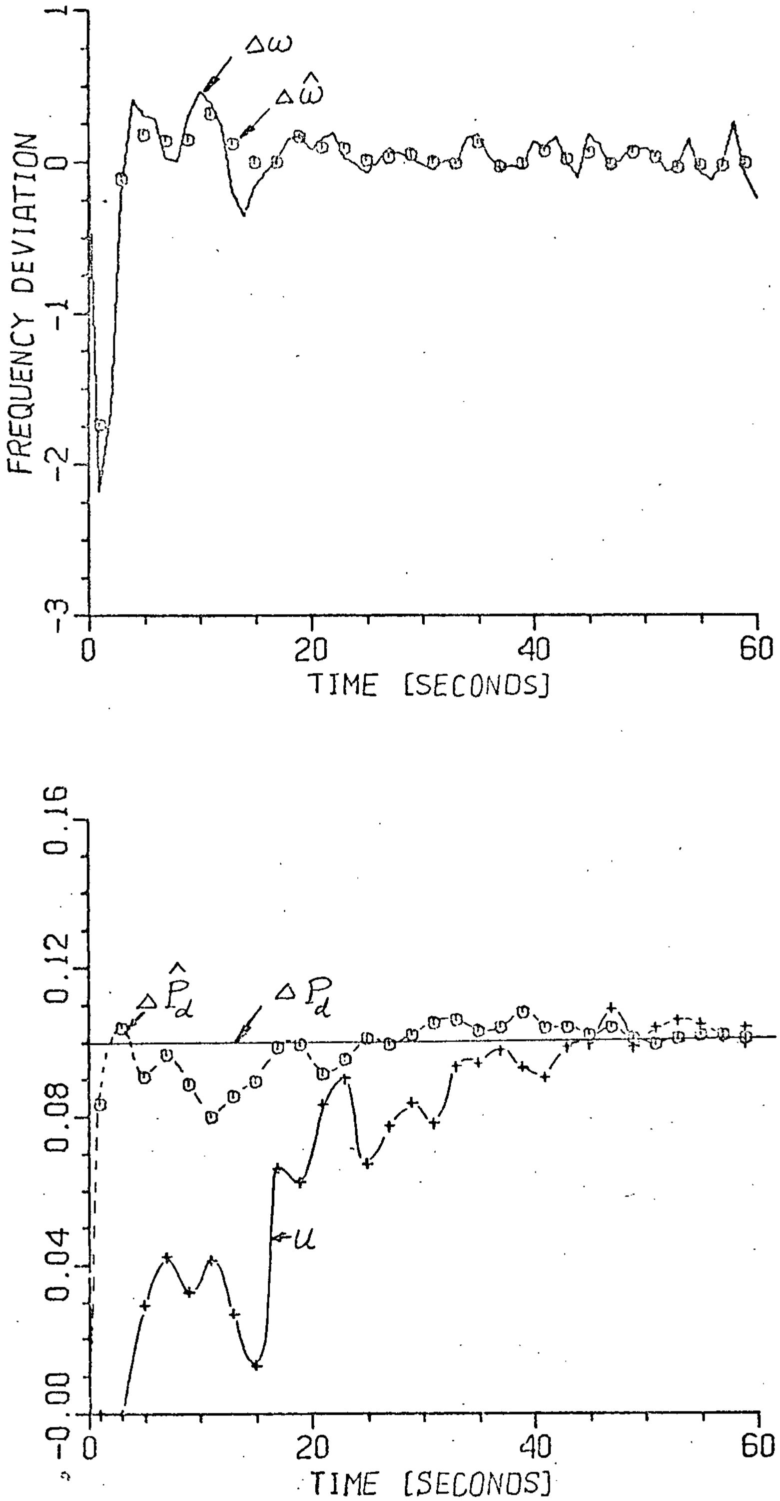


Fig. (5.5) Tracking capability of the adaptive observer.

6. CONCLUSIONS

For very severe disturbances in a power system, an algorithm is presented, in Chapter 2, for the evaluation of optimum switching instants for parameter changes in the network so as to improve system stability. The method appears to offer practical as well as computational advantages over the Liapunov function approach, in finding the critical switching time.

On-line implementation seems possible. Preliminary off-line computation could obtain the relationships between optimum switching instants and initial system states. Efficient numerical curve-fitting methods are becoming available which would make it possible to store these relationships in parametric form which require a limited computer memory. This would eliminate the necessity for fast on-line solution of sets of differential equations.

For less severe disturbances, digital simulation results show that system non-linearities and plant interaction must be accounted for. Because of the large number of state variables, optimal control of an interconnected power system is not feasible and some form of suboptimal control is essential. A suboptimum local control based on a linearized model which neglects plant interaction is physically feasible but can result in system instability.

By introducing the concept of two-level control a satisfactory suboptimal feasible control is obtainable. The local control is augmented by a second-level intervention control. A feasible on-line method for generating this control is given. Off-line computations are used to determine the intervention control in a parametric form as a function of time and initial states. When a system disturbance occurs, a second-level coordinator can generate these parameters on-line and transmit them back to the subsystems which generate the local intervention control signals.

Because most of the computations are done off-line, other controls such as exciter voltage control, and associated system non-linearities can be accounted for without much difficulty. The multi-level control scheme with high-level intervention control in the parameterized form suggested here appears to be a promising feasible approach to the control of interconnected power systems.

The intervention control is an open-loop control which augments the closed-loop local controllers. The composite control results in improved system performance. Intervention control would only be applied if the system disturbances are significant.

The load-frequency control problem, due to routine small disturbances experienced in everyday operation of power systems, is discussed in Chapter 4. Because incremental power demand is not known a priori, the problem of optimal load-frequency control cannot be solved by direct application of the optimal linear-state regulator control. A feasible optimal control is obtainable by a state variable transformation and by identification of the incremental power demand. Two methods have been shown suitable for power demand identification. One method is based on differential approximation and is very simple. Improved identification accuracy can be achieved by the second method which uses a Luenberger observer. A further advantage of the second method is that it can cope with the situation where not all the states are measured. The observer is driven by measurements of some of the states and its output is an estimate of the unmeasured states and the incremental power demand.

In Chapter 5, a suboptimum solution to the problem of sampled data optimum load-frequency control with unknown deterministic incremental power demand is given. Trade-off between system response and sampling rate can be easily studied. The case of random system disturbance is considered. The optimum stochastic controller is excessively complex to be used in controlling a power system. Furthermore, the statistical data and accurate models required to achieve optimum performance are generally not available. It is essential, therefore, to study suboptimum controllers. A three stage procedure is given for the design of a suboptimum stochastic controller. The first stage consists in determining the control gain for a deterministic optimum control, the second stage consists in the design of a class of deterministic observers, the third and final stage consists in adaptively choosing a scalar observer gain so as to minimize an instantaneous cost index. An example is used to illustrate the design procedure. Comparative studies of system performance for different parameter values show the effectiveness of the design procedure and the ease with which "tuning" can be accomplished.

The proposed load-frequency control discussed in Chapters 4 and 5 is compatible with an EACC-type control. The load-frequency controller is activated only after the EACC has decided that control action is required. This prevents the load-frequency controller from attempting to correct for rapidly changing load fluctuation. An EACC controller could be programmed to make decisions concerning the type of control to be used. One mode of load-frequency control would be for unknown but deterministic disturbances. The controller for this mode could be of the form discussed in Chapter 4 (or in a sampled-data form as given in Section (5.2)). A second mode of control would occur if the disturbances are random. The change from one mode of control to another is basically very simple. It amounts to setting $g_k = 1$ in the case of deterministic disturbances and making g_k adaptive in the case of random disturbances.

APPENDIX I

(Definitions)[18,19]

1. The term area[18] identifies that part of an interconnected power system which is to absorb its own load changes. It may be a single company, responding to its own load changes; it may be part of a company operating to respond to load changes that occur in only a given part of the company's network; it may be a whole group of companies pooled together to absorb the load changes that occur anywhere within their collective boundaries.

2. A single area interconnected system is one in which load changes are absorbed by the system as a whole, regardless of where on the system they occur. No one part of the system is expected to adjust its own generation to absorb its own load changes. Load changes that occur in any part of the system may be absorbed elsewhere within the system, in accordance with allocation practices prevailing at that particular time. Tie-line power flows are, therefore, neither scheduled nor controlled.

3. A multiple area interconnected system is one that consists of a number of operating areas, each of which is expected to adjust its own generation to absorb its own load changes. Tie-line power flows between areas are scheduled and maintained.

During our study of the Load-Frequency Control problem (as defined later), the term "plant" shall refer to the controlling generating station in the area. Two common types of plants are considered, namely, steam and hydro plants.

4. Megavar-voltage (Q-V) control problem is the problem of controlling the reactive power in the system. In this problem, the main concern is the voltage level at the different buses throughout the system. Due to the relatively fast action of voltage regulators, it is common practice to assume that the bus voltages are maintained at their nominal values. This assumption is adopted in this thesis.

5. Megawatt-Frequency (P-f) control problem is the problem of controlling the real power. Load-Frequency Control (LFC) is an alternate term for this particular control job. The following definition of the LFC problem is accepted by the IEEE (AIEE 94, Proposed Definitions, December 1962):

"Load-Frequency Control is the regulation of the power output of electric generators within a prescribed area in response to changes in system frequency, tie-line loading, or the relation of these to each other, so as to maintain the scheduled system frequency and/or the established interchanges with other areas within predetermined limits."

6. LFC criteria: Some of the criteria of LFC as generally defined are given below. Neglecting the contraints on measurements, control, and system dynamics, then the ideal "Static" control criteria may be stated as follows[19]:

Minimize (a), (b) or (c) where

(a) Area Control Error (ACE) = tie-line deviation + frequency bias x frequency deviation

$$[ACE \triangleq \Delta P_t + B\Delta f \rightarrow 0], \qquad (I.1)$$

(b) Inadvertent Interchange (II) = Integral of the line deviation

$$[II \triangleq \int \Delta P_t \, dt \rightarrow 0], \qquad (I.2)$$

(c) Time Deviation (TD) = $\frac{1}{60}$ x Integral of frequency deviation

$$[TD \triangleq \frac{1}{60} \int \Delta f \, dt \rightarrow 0], \tag{I.3}$$

and also mimimize

(d) Area Supplementary Control (ASC) $\triangleq$ function of ACE, II, TD.

$$[ASC \triangleq f(ACE,\ II,\ TD) \rightarrow \text{minimum}] \tag{I.4}$$

APPENDIX II

The transmission system and local impedance of Fig. (2.1) can be reduced by Thevenin's theorem to a series impedance $r_e + j\, x_e$ and an equivalent infinite bus voltage V_o'. The electrical power, damping coefficient and flux linkage equations can be reduced to the form given by (2.20), (2.21) and (2.22) respectively[29].

The coefficients are defined by the following relations:

$C_1 \triangleq B_1$

$C_2 \triangleq B_2 \cos\beta + B_3 \sin\gamma$

$C_3 \triangleq B_4 \cos\gamma - B_2 \sin\beta$

$C_4 \triangleq 0.5\, B_4 \cos(\gamma-\beta)$

$C_5 \triangleq -B_4 \cos\gamma \sin\beta$

$C_6 \triangleq B_4 \sin\gamma \cos\beta$

$C_7 \triangleq$ constant (defined by (2.23))

$C_8 \triangleq A_6 \cos\gamma$

$C_9 \triangleq A_6 \sin\gamma$

$C_{10} \triangleq -A_7/\tau_o$

$C_{11} \triangleq D_1$

$C_{12} \triangleq D_2$

where:

$B_1 \triangleq A_2(A_5 + A_4/x_q)$

$B_2 \triangleq -A_1 V_o'(A_5 + A_4/x_q)$

$B_3 \triangleq A_2 A_3 V_o'(x_d' - x_q)/x_d' x_q$

$B_4 \triangleq A_1 A_3 V_o'^2 (x_q - x_d')/x_d' x_q$

$D_1 \triangleq V_o'(x_q - x_q'')\tau_{qo}''/(x_q + x_e)^2$

$D_2 \triangleq V_o'^2 (x_d' - x_d'')\tau_{do}''/(x_d' + x_e)^2$

$\beta \triangleq \text{Arctan}\,[(x_d' + x_e)/r_e]$

$\gamma \triangleq \text{Arctan}\,[(x_q + x_e)/r_e]$

where:

$A_1 \triangleq (\sqrt{r_e^2 + (x_d' + x_e)^2})/\Delta_1 x_d'$

$A_2 \triangleq r_e/\Delta_1 x_d'$

$A_3 \triangleq (\sqrt{r_e^2 + (x_q x_e)^2})/\Delta_1 x_q$

$A_4 \triangleq (r_e + x_e(x_q + x_e))/\Delta_1 x_q x_d'$

$A_5 \triangleq (x_q + x_e)/\Delta_1 x_d' x_q$

$A_6 \triangleq (x_d - x_d') A_3 V_o'/x_d'$

$A_7 \triangleq x_d(1 - A_4)/x_d' + A_4$

$\Delta_1 \triangleq r_e^2 + (x_q + x_e)(x_d' + x_e)/x_d' x_q$

In the above definitions:

x_d, x_q = synchronous reactance in d and q axes, respectively.

x'_d, x'_q = transient reactance in d and q axes, respectively.

x''_d, x''_q = substransient reactance in d and q axes, respectively.

τ''_{do}, τ''_{qo} = subtransient open-circuit time constant in d and q axes, respectively.

The values (in pu) of the different parameters are:

$x_d = 1.0$, $x_q = 0.6$, $x'_d = .27$, $x''_d = 0.22$, $x''_q = 0.29$, $\tau_o = 9.0$,

$\tau''_{do} = .04$, $\tau''_{qo} = .07$, $x_t = .013$, $R = .15$, $x_L = .7488$, $B = .067$,

$G = .18$, $V_t = 1.05$, $P+JQ = .753 + J.03$.

APPENDIX III

As explained in Section (3.7), the initial conditions for each plant are taken to be zero with the exception of the frequency deviation. Consequently $\alpha_m(X_o)$, for each of the two plants considered, is assumed in the form

$$\alpha_m(X_o)=\beta_{mo}+\beta_{m1}\Delta\omega_{1o}+\beta_{m2}\Delta\omega_{2o}+\beta_{m3}\Delta\omega_{1o}^2+\beta_{m4}\Delta\omega_{2o}^2+\beta_{m5}\Delta\omega_{1o}\Delta\omega_{2o} \tag{III.1}$$

where the coefficients β_{mi}, $i=0, 1, \ldots 5$ are chosen so that (3.36) is minimum. The algorithm for finding β_{mi} for one plant is as follows

(1) Find the optimum $h^*(t)$ for M different sets of initial conditions (Section (3.6.2)).

(2) Find the coefficients α_m^k, $m=1,2,3,4$ for each set of initial conditions $k=1,2, \ldots M$.

(3) For $m=1$ and $k=1, \ldots M$ form

$$\zeta_m = W\Gamma_m \tag{III.2}$$

where

$$\zeta_m = \begin{bmatrix} \alpha_m^1 \\ \alpha_m^2 \\ \vdots \\ \alpha_m^M \end{bmatrix}, \qquad \Gamma_m \triangleq \begin{bmatrix} \beta_{mo} \\ \beta_{m1} \\ \vdots \\ \beta_{m5} \end{bmatrix}, \quad \text{and}$$

$$W \triangleq \begin{bmatrix} [1 & \Delta\omega_{1o} & \Delta\omega_{20} & \Delta\omega_{1o}^2 & \Delta\omega_{2o}^2 & \Delta\omega_{1o}\cdot\Delta\omega_{20}]^1 \\ \vdots \\ [1 & \Delta\omega_{1o} & \Delta\omega_{2o} & \Delta\omega_{1o}^2 & \Delta\omega_{2o}^2 & \Delta\omega_{1o}\cdot\Delta\omega_{2o}]^M \end{bmatrix}$$

The only unknown in (III.2) is Γ_m. For $M > 6$, Γ_m is given by[30]

$$\Gamma_m = (W'W)^{-1} W' \zeta_m \qquad \text{(III.3)}$$

(4) Repeat (3) for m=2,3,4.

APPENDIX IV

The optimum filter gain can be determined by solving the equations[23]

$$K_{k+1} = P_{k+1} H' R_{k+1}^{-1} \tag{IV.1}$$

$$P_{k+1} = N_{k+1} - N_{k+1} H'(H N_{k+1} H' + R_{k+1}) H N_{k+1} \tag{IV.2}$$

$$N_{k+1} = \Phi P_k \Phi' + Q_k \tag{IV.3}$$

recursively. The initial covariance matrix, P_o, and the matrix sequences Q_k, R_k ($k = 0, 1, \ldots$) are assumed known. In general, however, the data P_o, Q_k, R_k is not available. However, equations (IV.1), (IV.2) and (IV.3) still prove useful in determining a constant observer matrix gain L (see (5.33)). Simulation studies or system operating experience usually allow some initial guess to be made for the unknown parameters. A reasonable choice is to take constant positive-definite diagonal matrices P_o, Q_o and R_o. Based on this choice, (IV.2) and (IV.3) can be recursively solved for the steady-state convariance matrix P_∞. Equation (IV.1) then yields the steady-state gain matrix and a reasonable choice for L is to take

$$L = K_\infty H' R_o^{-1} \tag{IV.4}$$

For the example given in Section (5.4), the choice

$$R_o = b_1 I \quad , \quad Q_o = b_2 I \quad , \quad P_o = I \tag{IV.5}$$

where $b_1 = b_2 = 10^{-4}$, is made. Evaluation of (IV.4) yields (5.65).

An alternative approach is to apply a discrete version of the method presented in reference 35. This alternative approach has been used in Section (4.5) to determine an observer gain. Equation (IV.4) is to be preferred if a limited amount of statistical data is available.

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