A METHOD FOR THE COMPUTER ANALYSIS OF MUSIC UTILIZING THE IBM 360 DIGITAL COMPUTER

by

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We accept this thesis as conforming to the required standard.

THE UNIVERSITY OF BRITISH COLUMBIA

APRIL, 1971
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Department of MUSIC

The University of British Columbia
Vancouver 8, Canada

Date April 29, 1971
ABSTRACT

A METHOD FOR THE COMPUTER ANALYSIS OF MUSIC
UTILIZING THE IBM 360 DIGITAL COMPUTER

by

EDWIN WAYNE CARR B.Mus.

A method for the computer analysis of certain melodic and rhythmic elements, such as intervals, range, and frequency of occurrence of elements, of selected musical works is presented and results are obtained. The method also utilizes new techniques of musical analysis which would otherwise be too lengthy or cumbersome to be practical, i.e., information content, redundancy, and autocorrelation. The analytical results are discussed and both the method and the results are evaluated with a view to alteration and expansion leading to a more comprehensive analysis of music.

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Thesis Advisor

THE UNIVERSITY OF BRITISH COLUMBIA

MARCH, 1971
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The author wishes to acknowledge Mr. Wayne Fung for his assistance in the programming of the method, and Professor Cortland Hultberg for his assistance in the entire study.
INTRODUCTION

The purpose of this paper is to present a method for the computer analysis of selected musical works. The research method used is concerned only with certain melodic and rhythmic elements of a work, such as intervals, range, and frequency of occurrence of these elements. The method also allows for the exploration of new techniques of musical analysis which would otherwise be too cumbersome or lengthy to be practical, i.e., entropy, redundancy, and autocorrelation. The analytical results are discussed in terms of single and comparative analyses, and in terms of trends which appear in a comparative historical sampling of musical works. In addition, both the method and the results are assessed with a view to possible corrections and expansions that could ultimately lead to a more comprehensive computer analysis of music.

¹A detailed description of these aspects of information theory and their musical applications is included in Appendix 5.
METHOD OF ANALYSIS

The method of analysis described in this paper uses as input data the pitches and rhythms of each instrumental or vocal line of a musical work represented by numerical equivalents. This input data is subjected to a series of analytical operations which are now briefly described:

1. Sections
Each line of a work is subdivided according to musical phrasing, allowing various sections of the work to be effectively compared. The number of sections in each musical line and the number of the last note of each section is printed out.

2. Melodic and Rhythmic Range
The melodic range is determined by subtracting the value of the lowest note from the highest in each line, while the rhythmic range is found by subtracting the smallest from the largest duration value in each line.

3. Melodic Intervals
Two orders of intervals are taken into consideration. Intervals of the first order occur between adjacent notes and are found by subtracting the numerical value of one note from the

\(^2\)See Appendix 3
\(^3\)See Appendix 2
\(^4\)Ibid.
value of the note preceding it. Intervals of the second order are found in the same manner between every other note. The numerical representation of the intervals is:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>unison</td>
<td>0</td>
</tr>
<tr>
<td>minor second</td>
<td>1</td>
</tr>
<tr>
<td>major second</td>
<td>2</td>
</tr>
<tr>
<td>minor third</td>
<td>3</td>
</tr>
<tr>
<td>major third</td>
<td>4</td>
</tr>
<tr>
<td>perfect fourth</td>
<td>5</td>
</tr>
<tr>
<td>augmented fourth</td>
<td>6</td>
</tr>
<tr>
<td>perfect fifth</td>
<td>7</td>
</tr>
<tr>
<td>minor sixth</td>
<td>8</td>
</tr>
<tr>
<td>major sixth</td>
<td>9</td>
</tr>
<tr>
<td>minor seventh</td>
<td>10</td>
</tr>
<tr>
<td>major seventh</td>
<td>11</td>
</tr>
<tr>
<td>octave</td>
<td>12</td>
</tr>
</tbody>
</table>

etc.

No additional enharmonic spellings of intervals were found in the selected works.

4. Frequency of Occurrence

The frequency of occurrence of pitch is found by counting the number of times each different pitch is found in each line, and expressing the result as a part of 1.0. Frequency of occurrence is also expressed as a computer holograph, i.e., a visual representation (graph) of the results obtained. The frequency of occurrence of the first and second-order intervals, and of duration values is found in the same way.

5. Autocorrelation

The application of the autocorrelation function was an attempt to obtain a graphic representation of melodic and rhythmic repetitions in each line of the selected work, and thereby determine the formal structure of the work. However, 

5Ibid.
since the autocorrelation function was not applied to the entire work the results obtained have little meaning.

6. Information Content Per Line

Information content is a measure of randomness, such that a completely ordered musical work would have no information and a completely random work would have a maximum amount of information. The information content is found for each different pitch, duration, and interval in each line of the selected work, and is expressed in bits per symbol.

7. Information Content Per Section

The information content per section is the amount of information conveyed by each section of a work and is found by summing the information content of the data analyzed for that section.

8. Information Content of the Work and Maximum Information

The information content of the work is amount of information conveyed by each musical line in the entire work. The maximum information is the maximum amount of information that it was possible for the work to convey.

9. Redundancy

The redundancy is a percentage figure which shows how much of the work is repetitious.

The method is unique in the following ways: the use of FORTRAN (a prevalent programming language), the application of autocorrelation to musical works, the application of information

6See Appendix 5 for more detailed description.
theory to the selected works with a view to historical trends, and the results obtained.
ANALYSIS RESULTS

Nine musical works,\(^7\) chosen as a historical sampling, were analyzed and three analytical viewpoints are discussed:

1. the analysis of a single work: W. A. Mozart's "Ave Verum Corpus", K. 618, Waldheim-Eberle Edition, Wien, 1954, chosen as the central work in the historical sample,


3. a discussion of the indications of historical trends which become apparent when nine examples from musical history are analyzed.

\(^7\)See Appendix 4
1. The analysis of Mozart's "Ave Verum Corpus", K. 618.

The results of the computer analysis found the range of the work to be two octaves and a minor sixth, while the ranges of the individual voices were: soprano, a major tenth; alto, a minor seventh; tenor, an octave; and bass, a major tenth, the ranges being representative of choral part-writing of this and earlier periods.

The frequency of occurrence calculations for the three most frequent pitches in each line resulted in the following:

<table>
<thead>
<tr>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>A .219</td>
<td>E .373</td>
<td>A .284</td>
<td>A .279</td>
</tr>
<tr>
<td>G .180</td>
<td>D .230</td>
<td>G# .215</td>
<td>D .158</td>
</tr>
<tr>
<td>F# .133</td>
<td>G# .131</td>
<td>D .159</td>
<td>G .085</td>
</tr>
</tbody>
</table>

Therefore, A seems to be the most frequent pitch in the work, accounting for 21.9% of all pitches in the soprano line, 28.4% in the tenor, and 27.9% in the bass. In the alto line E is very prominent, accounting for 37.3% of all pitch occurrences. The frequency of occurrence of pitch A can be seen to reinforce the given tonality of D major since it is the fifth of the tonic triad and the root of the dominant, these two triads accounting for the bulk of the harmonic content. The pitch E also reinforces the tonality since it is the fifth of the dominant triad and the root of the supertonic.

The frequency of occurrence of the three most frequent melodic intervals was:

\(^8\)See Appendix 4.
The unison was the most frequent melodic interval in all the lines, indicating the use of many repeated notes. The unison was also used significantly more often in the tenor and alto than in the soprano and bass, i.e., in the alto and tenor lines the unison occurred approximately 46% of the time, while in the soprano and bass the unison averaged approximately 36% of the intervals. Another interesting result was the alteration of the frequencies of occurrence of major second and minor second between the lines, i.e., the major second was the more frequent interval of the two in the soprano and tenor, while the minor second was the more frequent in the alto and bass. Such similarities between the soprano and tenor, and between the alto and bass, are also found in other aspects of this analysis.

The second-order interval calculations were found to be essentially the same as the first-order, so there was no need to consider them. As in most choral music of this period the alto and tenor were melodically subordinate, with the soprano and bass having fewer repeated pitches.

The most frequently used rests in all lines were the whole and half rests:

<table>
<thead>
<tr>
<th></th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole</td>
<td>.888</td>
<td>.666</td>
<td>.692</td>
<td>.642</td>
</tr>
<tr>
<td>Half</td>
<td>.111</td>
<td>.333</td>
<td>.307</td>
<td>.357</td>
</tr>
</tbody>
</table>

It should be noted that the frequency of occurrence of the
whole rest generally decreases from the soprano through to the bass while the half rest generally increases. The quarter-note was the most frequent note value with significant occurrences of the whole-note, dotted half-note, half-note, and eighth-note:

<table>
<thead>
<tr>
<th></th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter</td>
<td>.641</td>
<td>.510</td>
<td>.574</td>
<td>.530</td>
</tr>
<tr>
<td>Half</td>
<td>.160</td>
<td>.239</td>
<td>.241</td>
<td>.289</td>
</tr>
<tr>
<td>Dotted-Half</td>
<td>.056</td>
<td>.108</td>
<td>.114</td>
<td>.120</td>
</tr>
</tbody>
</table>

The quarter-note was most prominent in the soprano with 64.1% of all occurrences and 57.4% in the tenor. The half and dotted half-notes increased in frequency of occurrence from soprano to bass. The frequency of occurrence of rests and notes combined shows quarter, half, and whole-note durations to be the most frequent:

<table>
<thead>
<tr>
<th></th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter</td>
<td>.596</td>
<td>.456</td>
<td>.505</td>
<td>.458</td>
</tr>
<tr>
<td>Half</td>
<td>.157</td>
<td>.252</td>
<td>.252</td>
<td>.302</td>
</tr>
<tr>
<td>Whole</td>
<td>.122</td>
<td>.116</td>
<td>.121</td>
<td>.135</td>
</tr>
</tbody>
</table>

Therefore, in all voices the quarter-note value, for both notes and rests, accounted for approximately 50% of all occurrences and the half-note for another 25%, i.e., Mozart's rhythmic activity was generally constant from voice to voice. Again, the second-order results were essentially the same and were not considered here. It should be noted that in all frequency of occurrence calculations the holograph restates all results in graphic form.

Since the information content calculations for pitches, ⁹

⁹See Appendix 2.
intervals, and rhythms were based on their frequency of occurrence, the results were merely a restatement of the frequency of occurrence results, i.e., the most frequent elements had the highest information content. However, the results did reveal that throughout the range of each voice the most information was found in notes in the center of the range with a gradual decrease in information on each side of this central area. The information content of the intervals showed a gradual decrease in the amount of information content as the intervals became larger.

The information content per section was found to be:

**Pitches**

<table>
<thead>
<tr>
<th>Section</th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm. 1-23</td>
<td>2.365</td>
<td>2.008</td>
<td>.953</td>
<td>1.722</td>
</tr>
<tr>
<td>2 mm. 24-44</td>
<td>2.355</td>
<td>2.052</td>
<td>1.739</td>
<td>1.486</td>
</tr>
<tr>
<td>3 mm. 45-67</td>
<td>2.712</td>
<td>1.533</td>
<td>2.915</td>
<td>3.046</td>
</tr>
<tr>
<td>4 mm. 68-88</td>
<td>2.321</td>
<td>2.353</td>
<td>2.413</td>
<td>2.744</td>
</tr>
<tr>
<td>5 mm. 89-105</td>
<td>3.127</td>
<td>2.074</td>
<td>2.625</td>
<td>2.749</td>
</tr>
</tbody>
</table>

**Intervals**

<table>
<thead>
<tr>
<th>Section</th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm. 1-23</td>
<td>1.782</td>
<td>1.180</td>
<td>.565</td>
<td>1.340</td>
</tr>
<tr>
<td>2 mm. 24-44</td>
<td>1.712</td>
<td>1.327</td>
<td>1.345</td>
<td>1.486</td>
</tr>
<tr>
<td>3 mm. 45-67</td>
<td>1.404</td>
<td>1.069</td>
<td>1.587</td>
<td>1.717</td>
</tr>
<tr>
<td>4 mm. 68-88</td>
<td>1.379</td>
<td>1.307</td>
<td>1.192</td>
<td>1.826</td>
</tr>
<tr>
<td>5 mm. 89-105</td>
<td>1.765</td>
<td>1.214</td>
<td>1.342</td>
<td>1.441</td>
</tr>
</tbody>
</table>

**Rhythm**

<table>
<thead>
<tr>
<th>Section</th>
<th>Soprano</th>
<th>Alto</th>
<th>Tenor</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm. 1-23</td>
<td>.974</td>
<td>1.162</td>
<td>1.650</td>
<td>1.668</td>
</tr>
<tr>
<td>2 mm. 24-44</td>
<td>1.280</td>
<td>1.728</td>
<td>1.673</td>
<td>1.445</td>
</tr>
<tr>
<td>3 mm. 45-67</td>
<td>1.511</td>
<td>1.727</td>
<td>1.067</td>
<td>1.284</td>
</tr>
<tr>
<td>4 mm. 68-88</td>
<td>1.549</td>
<td>1.549</td>
<td>1.271</td>
<td>1.233</td>
</tr>
<tr>
<td>5 mm. 89-105</td>
<td>1.954</td>
<td>1.913</td>
<td>1.774</td>
<td>1.665</td>
</tr>
</tbody>
</table>
Each section of the piece has one dominant voice in terms of the highest information content for that section:

<table>
<thead>
<tr>
<th>Section</th>
<th>Pitches</th>
<th>Intervals</th>
<th>Rhythm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Soprano</td>
<td>Soprano</td>
<td>Bass</td>
</tr>
<tr>
<td>2</td>
<td>Soprano</td>
<td>Soprano</td>
<td>Alto</td>
</tr>
<tr>
<td>3</td>
<td>Tenor</td>
<td>Bass</td>
<td>Alto</td>
</tr>
<tr>
<td>4</td>
<td>Bass</td>
<td>Bass</td>
<td>Alto &amp; Soprano</td>
</tr>
<tr>
<td>5</td>
<td>Soprano</td>
<td>Soprano</td>
<td>Soprano</td>
</tr>
</tbody>
</table>

These results indicate the kind of part-writing used in the work, i.e., in section one melodic information was mainly conveyed by the soprano and rhythmic information by the bass, in section two rhythmic activity shifted to the alto while the soprano maintained the melody, in section three the tenor and bass took over melodic dominance and the alto continued with rhythmic dominance, in section four the bass dominated all melodic information and the soprano joined the alto in rhythmic dominance, and in section five the soprano dominated both melodically and rhythmically. In addition, each voice has a section with the highest information content for each voice:

<table>
<thead>
<tr>
<th>Voice</th>
<th>Pitches Section</th>
<th>Intervals Section</th>
<th>Rhythms Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soprano</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Alto</td>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Tenor</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Bass</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Most noticeably, these results show that in the last section of the piece the soprano, alto, and tenor all have their highest amount of rhythmic information as well as the soprano having its highest amount of pitch information.

Calculations for the information content of the work, the
maximum information possible, and the redundancy produced the following results:

Pitches

<table>
<thead>
<tr>
<th>Voice</th>
<th>INF</th>
<th>MAX</th>
<th>RED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soprano</td>
<td>.323</td>
<td>.671</td>
<td>51%</td>
</tr>
<tr>
<td>Alto</td>
<td>.249</td>
<td>.650</td>
<td>61%</td>
</tr>
<tr>
<td>Tenor</td>
<td>.267</td>
<td>.645</td>
<td>58%</td>
</tr>
<tr>
<td>Bass</td>
<td>.354</td>
<td>.635</td>
<td>44%</td>
</tr>
</tbody>
</table>

Intervals

<table>
<thead>
<tr>
<th>Voice</th>
<th>INF</th>
<th>MAX</th>
<th>RED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soprano</td>
<td>.231</td>
<td>.670</td>
<td>65%</td>
</tr>
<tr>
<td>Alto</td>
<td>.189</td>
<td>.649</td>
<td>70%</td>
</tr>
<tr>
<td>Tenor</td>
<td>.208</td>
<td>.644</td>
<td>67%</td>
</tr>
<tr>
<td>Bass</td>
<td>.235</td>
<td>.633</td>
<td>62%</td>
</tr>
</tbody>
</table>

Rhythm

<table>
<thead>
<tr>
<th>Voice</th>
<th>INF</th>
<th>MAX</th>
<th>RED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soprano</td>
<td>.178</td>
<td>.683</td>
<td>73%</td>
</tr>
<tr>
<td>Alto</td>
<td>.205</td>
<td>.668</td>
<td>69%</td>
</tr>
<tr>
<td>Tenor</td>
<td>.183</td>
<td>.662</td>
<td>72%</td>
</tr>
<tr>
<td>Bass</td>
<td>.176</td>
<td>.658</td>
<td>73%</td>
</tr>
</tbody>
</table>

(INF is the information content of the work, MAX is the maximum possible information, and RED is the redundancy).

The maximum information of the pitches tells us how much information the pitches in each voice could contain in this work, i.e., it was possible for the soprano pitches to have .671 bits of information content (and .670 in intervals, .683 in rhythm). However, the soprano pitches contained only .323 bits of a possible .671 and was therefore 51% redundant in pitches. The information content of all voices was higher in their pitches than in intervals or rhythm. The bass pitches had the highest information content, .354 bits, as well as low maximum information content, resulting in the lowest redundancy of any voice, 44%.

In all voices the maximum possible information of the intervals was almost the same as the pitches, and since the
intervals were originally calculated from the pitches, the possible information of each should be the same. The information content of the intervals was lower than the pitches, resulting in redundancy values approximately 10% higher, i.e., the soprano pitches were 51% redundant while the soprano intervals were 65%. The rhythm in all voices had low information content, high maximum information, and therefore higher redundancy than the pitches or intervals. Rhythm in the soprano and bass had the highest redundancy of the work, 73%, as well as the lowest information content, .178 bits in the soprano and .176 in the bass. The rhythm of the soprano also had the highest maximum information in the work, .683 bits. Therefore, in all voices generally one-half of all musical information considered was melodically conveyed, one-third conveyed by intervals, and one-sixth rhythmically conveyed.

The holographs\textsuperscript{10} of pitches, intervals, and rhythm simply restate in another form the results which have been already discussed and will therefore not be considered in this discussion.

In terms of the fluctuation of information content, when a fuller use is made of the musical vocabulary the resulting work becomes more complex and less predictable, while with less of the musical vocabulary in use the opposite is true. More generally, the fluctuation of the elements creating a musical structure can be judged from the basis of any musical \textsuperscript{10}Ibid.
style and result in meaningful quantitative measures, i.e., information content. In addition, information theory analysis, as used in this paper, expresses a viewpoint independent of historical content and without qualitative connotations.

Since the comparative analysis of two works is often a practise in traditional analysis, the Bach and Webern examples are compared in terms of the results of an information theory analysis. Both examples are vocal works with three sections and similar three octave ranges. In this discussion results are considered in terms of all voices, but the specific examples of results are only given for the soprano. Also, only three results are considered in the examples. This is to allow a clear and concise discussion of the more significant results.

The three most frequent pitches in each work were:

Bach ...... B .190, E .119, C .097
Webern ... D .108, B\textsubscript{b} .102, B .095

Since B is much more frequent than any other pitch in the Bach it is the tonic or dominant of the key, but in the Webern the pitches are very similar in frequency of occurrence, indicating a less prominent tonal center, although several analysts have concluded that G serves a tonic function in the work and it is interesting that D, the dominant of G, should be the most frequent pitch.

The three most frequently occurring intervals in each were:

Bach ...... Major Second .316, Minor Second .218, Unison .114
Webern ... Minor Second .424, Major Second .205, Minor Third .102

\textsuperscript{11}See Appendix 4.
As indicated, both works are dominated by major and minor seconds although Bach concentrated on smaller intervals and Webern tended toward larger ones. In both examples the most frequently used rhythmic value was the eighth-note, although Webern generally tended to use smaller values:

<table>
<thead>
<tr>
<th></th>
<th>Bach</th>
<th>Webern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eighth</td>
<td>.614</td>
<td>.650</td>
</tr>
<tr>
<td>Quarter</td>
<td>.266</td>
<td>.205</td>
</tr>
<tr>
<td>Dotted Quarter</td>
<td>.032</td>
<td>.136</td>
</tr>
</tbody>
</table>

It should be noted that the unit beat differs in these two works (Bach: quarter-note, Webern: eighth-note), and must be taken into consideration. In their use of rests Bach used only eighth and quarter rests, while Webern used sixteenth, eighth, and quarter rests:

<table>
<thead>
<tr>
<th></th>
<th>Bach</th>
<th>Webern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter</td>
<td>.529</td>
<td>.411</td>
</tr>
<tr>
<td>Eighth</td>
<td>.470</td>
<td>.352</td>
</tr>
<tr>
<td>Quarter</td>
<td>.032</td>
<td>.176</td>
</tr>
</tbody>
</table>

The information content results were:

<table>
<thead>
<tr>
<th></th>
<th>Bach</th>
<th>Webern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitches</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>.455</td>
<td>D</td>
</tr>
<tr>
<td>E</td>
<td>.366</td>
<td>B♭</td>
</tr>
<tr>
<td>C</td>
<td>.328</td>
<td>B</td>
</tr>
</tbody>
</table>

| Intervals  |      |        |
| Maj. Second | .525 | Min. Second | .524 |
| Min. Second | .479 | Maj. Second | .469 |
| Unison     | .358 | Min. Third  | .337  |

| Rhythm     |      |        |
| Quarter    | .523 | Sixteenth | .480  |
| Eighth     | .447 | Eighth    | .426  |
| Dotted Quarter | .142 | Quarter   | .397  |
The information content of the pitches in each work indicates that Webern employed a more equiprobable choice of pitches since the pitches had a more even information content than those in the Bach. Both works had very similar information content for the dominant interval of a second. Concerning the information content of rhythm, eighth and quarter-notes dominate the Bach, but in the Webern the sixteenth, eighth, and quarter-notes are dominant at a level of information content similar to the Bach.

The results of the information per section calculations were again similar in both pieces. Concerning the information content of their pitches, both have a center section with the lowest information content, but in the Bach the first section has the highest while in the Webern the last is highest:

<table>
<thead>
<tr>
<th>Section</th>
<th>Bach</th>
<th>Webern</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitches</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 mm.1-54</td>
<td>3.54</td>
<td>3.66</td>
<td>1 mm.1-43</td>
</tr>
<tr>
<td>2 mm.55-91</td>
<td>3.29</td>
<td>3.53</td>
<td>2 mm.44-95</td>
</tr>
<tr>
<td>3 mm.92-184</td>
<td>3.43</td>
<td>3.80</td>
<td>3 mm.96-147</td>
</tr>
<tr>
<td>Intervals</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 mm.1-54</td>
<td>2.66</td>
<td>2.11</td>
<td>1 mm.1-43</td>
</tr>
<tr>
<td>2 mm.55-91</td>
<td>2.73</td>
<td>2.25</td>
<td>2 mm.44-95</td>
</tr>
<tr>
<td>3 mm.92-184</td>
<td>2.18</td>
<td>2.42</td>
<td>3 mm.96-147</td>
</tr>
<tr>
<td>Rhythm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 mm.1-54</td>
<td>1.31</td>
<td>.70</td>
<td>1 mm.1-43</td>
</tr>
<tr>
<td>2 mm.55-91</td>
<td>1.39</td>
<td>1.28</td>
<td>2 mm.44-95</td>
</tr>
<tr>
<td>3 mm.92-184</td>
<td>1.58</td>
<td>.89</td>
<td>3 mm.96-147</td>
</tr>
</tbody>
</table>

The information content of the intervals in the Webern shows a gradual increase as the work progresses, while in the Bach the opposite is true. In the rhythm information content the Bach
shows a gradual increase, but the Webern begins very low, increases, and then decreases to a similar low section of information content. Again in both works, the amount of information in different parameters is used to balance or strengthen one another; when one parameter, such as melody or rhythm, is in the foreground in terms of information being conveyed, another is in the background and vice versa.

The results of the calculations for the information content of the works, the maximum possible information, and the redundancy were:

<table>
<thead>
<tr>
<th></th>
<th>Bach</th>
<th>Webern</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pitches</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information Content</td>
<td>.396</td>
<td>.360</td>
</tr>
<tr>
<td>Maximum Information</td>
<td>.719</td>
<td>.752</td>
</tr>
<tr>
<td>Redundancy</td>
<td>44.9%</td>
<td>52.0%</td>
</tr>
<tr>
<td><strong>Intervals</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information Content</td>
<td>.260</td>
<td>.287</td>
</tr>
<tr>
<td>Maximum Information</td>
<td>.718</td>
<td>.751</td>
</tr>
<tr>
<td>Redundancy</td>
<td>63.7%</td>
<td>61.7%</td>
</tr>
<tr>
<td><strong>Rhythm</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information Content</td>
<td>.140</td>
<td>.153</td>
</tr>
<tr>
<td>Maximum Information</td>
<td>.735</td>
<td>.776</td>
</tr>
<tr>
<td>Redundancy</td>
<td>80.8%</td>
<td>80.2%</td>
</tr>
</tbody>
</table>

In both works information content was highest in the pitches, with less information in the intervals, and the least in the rhythm. There was more information content in the pitches of the Bach than in the Webern, but in terms of intervals and rhythm the Webern had more information content; the maximum possible information results were higher in the Webern in all parameters although the difference was a small one. In fact,
all parameters were very similar in maximum possible information. The redundancy percentages for both were very similar in intervals and rhythm, however, the Bach was somewhat less redundant, especially in pitches. Lastly, in both works rhythm was the most redundant, the intervals less redundant, and the pitches the least redundant.
3. Historical Trends

The nine selected works were:


Calculations were made for ranges and rhythmic values in each of the selected works, however, since several works were transcriptions and since both instrumental and vocal works were included, no valid comparisons could be made regarding their ranges or their rhythms.

Even though autocorrelation results were generally found to be insignificant, some interesting trends were found in the autocorrelations of the beginning of each work. In all examples the autocorrelation graph of the pitches gradually decreased to a point between .760 for Chopin to .916 for Beethoven, that is, in all works the relative dependence of one note on the next is quite high. The Beethoven autocorrelation fell the least of any of the examples indicating the highest degree of dependence between pitches, even though the range of the results covered only 15.6% of the possible range. The autocorrelation graph of the intervals was more erratic but always began with a drop to a point between .252 in the Schubert and .668 in the Bach, indicating that intervals have low to medium influence over the intervals which follow them. The rhythmic values also began their autocorrelation graph with a drop, though not as severe as the intervals, to a point between .400 in the Schubert and .729 in the Machaut, indicating that rhythmic values are somewhat more predictable.

The information of the piece, maximum information possible, and redundancy results for all parameters were:
The pitch information content tends to increase gradually as the example becomes more recent, with the highest information in pitches and corresponding decrease in redundancy found in the Beethoven. The Schubert work shows a noticeable increase in redundancy and decrease in information, contrary to the trends found in chronological comparison. Maximum information content shows no particular trends. However, it is interesting that the Perotinus and Webern examples should share such similar maximum information results. The Beethoven had the highest maximum information while the Chopin had the lowest. The intervals also have a gradual increase in information content and a corresponding decrease in redundancy as the examples become more recent, although the amount of increase is approximately half that of the increase in pitch information content. The intervals' maximum information again shows no particular trend and the first and last examples are again very close in their maximum possible information. The rhythmic values
gradually tend toward higher information content except for noticeable increases in the Victoria example and a marked decrease in the Webern. The Webern example again reveals a reversal in trends. As found in the pitch calculations the maximum information content of the rhythm shows no trend, with the lowest value in the Chopin and the highest in the Beethoven. The redundancy percentages gradually drop in magnitude, but revert to a higher value in the Webern example. The Machaut was found to be the most rhythmically redundant, followed by the Bach, Webern, and Perotinus, while the Chopin and Schubert were found to be the least redundant.

There is therefore a general increase in the information content of pitches, intervals, and rhythm as the selected works become more recent. There is also a corresponding decrease in redundancy, but the maximum possible information for the selected works showed no trends but varied from work to work.

The information results per section showed that in most examples the last section had the highest information content in terms of pitches. There was also a general tendency for all sections to contain more information as the example became more recent. Beethoven, Webern, and Bach had the highest information content in all sections while the Machaut and Victoria had the lowest pitch information. The information content of the pitches was found to be the highest of all parameters, indicating that in all the works the composers placed great stress on the pitches they chose. The information content of the intervals
was found to be the highest in the Bach and Webern, and lowest in the Perotinus and Machaut.

The following graphs are based on the information content per section for various parameters in two works (arbitrarily chosen), showing how different parameters are given more or less emphasis from section to section in the amount of information conveyed.

Victoria: "O Vos Omnes," motet

Information Content

<table>
<thead>
<tr>
<th>Section</th>
<th>Pitch</th>
<th>Rhythm</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mozart: "Ave Verum Corpus," K. 618

Information Content

<table>
<thead>
<tr>
<th>Section</th>
<th>Pitch</th>
<th>Rhythm</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the Victoria work the information content for the rhythm remains quite similar for section one through four but shows a marked increase at section five to balance the information content of the pitches and intervals which remains similar for sections four and five. Also, the rhythmic information decreases in section six as the information of the
pitches and intervals increases.

In the Mozart example the interval information content tends to decrease as the information of the pitches and rhythm increases. All parameters increase in information in the last section, indicating a climax. The decrease in information of pitches in section four would tend to place more emphasis on the pitches as their information increased in section five for the climax.

The information content of rhythmic values in all nine examples revealed overall lower values than the pitch and interval values, except in the Schubert and Chopin where parameter importance in terms of information conveyed would be ordered pitch, rhythm, and interval. These two examples also had the highest rhythmic information content while the Webern had the lowest.
EVALUATION

The method of analysis used in this study could be improved by a more direct handling of the input data. Possibly the best approach to the handling of input data would be to have the piece of music read directly into the computer, via the appropriate interface, from tape or disc recordings. Necessarily, the method of analysis would have to be modified to accept the music as it sounds rather than as it is notated, and obtain the same results. One obvious advantage to this method is that pieces without traditional scores could be computer analyzed. There is also the possibility that a more accurate analysis of any work could result since the actual input data to the computer would be similar to a listener's perception of a work. The analysis results and their presentation could be modified to be more concise and thorough in the following ways:

1. simultaneously print out all voices of a work for each parameter (pitch, interval, or rhythm) in a graph,
2. add harmony as another parameter of the analysis,
3. print in a table the highest and lowest elements and their range for all parameters and all voices,
4. print the frequency of occurrence, autocorrelation, information measure, information measure per section, and maximum information in one graph for all voices,
5. take great care in the selection of works for the
analysis of historical trends, i.e., the works should be written for the same performance media and should tend to be "typical" of the composer, especially if only several of his works are represented,

6. take the autocorrelation of all points of a work rather than only the first few,

7. present redundancy results as a table of percentages including the redundancy of all parameters for each voice as well as cumulative results,

8. include information rate per second to reflect the tempo, texture, and harmonic density of the music,

9. include a table of the average rates of information transmission per second for each section of a work,

10. delete second-order calculations since they merely repeat the results of the first-order,

11. give all graphs the same axis divisions,

12. include with the frequency of occurrence calculations an additional set of calculations which would assume octave equivalence,

13. restate various results in the form of comparative graphs to make results as concise and usable as possible,

14. include provision for the simultaneous computer comparison of several different musical works,

15. include in each graph the average of the results,

16. analyze larger pieces, especially orchestral works, by the consideration of parameters rather than single lines,
17. include calculations to analyze timbre, dynamics, and orchestration, and
18. compare the information content of chords and one aspect of Hindemith's\textsuperscript{12} compositional theory. Hindemith assigned relative values to chords according to their relative harmonic tension, such that motion from a high to a lower value chord will cause a decrease in tension, with the reverse also being true. As a result, harmonic fluctuation becomes a change of tension, making tension approximately analogous to information trends. For example, when a chord has a low tension value in a harmonic sequence it will also have been a relatively certain choice in terms of the listener's expectations, and will therefore have low information. On the other hand, chords with high information content would have high tension values.

An aesthetic problem arises when a computer analysis of music is attempted since it is difficult to attach literal meanings to music even though an aim of analysis is to determine the significance of a musical composition as well as to understand the mental and technical processes involved in creating and responding to a musical composition. Meyer\textsuperscript{13} states that musical meaning depends upon learned responses to musical stimuli and he proposed an "affect theory of music" based on the concept that emotion is generated when a tendency:


to respond is inhibited, and musical meaning is a product of expectation. Tischler¹⁴ felt that an aesthetic appreciation of music must be based on a familiarity with the medium and its technical possibilities. Tischler defines two types of relationships which characterize musical aesthetics:

1. internal ... these change with the medium and in music consist of rhythm, melody, harmony, counterpoint, tone color, expression, and form or contour,

2. external ... these are true of all the arts and consist of gesture, program, ethics, technical mastery, psychological drives of the artist, function, relevant historical and sociological data, and performance.¹⁵

He proposed that the greater the number of relationships a work of art reveals, the greater aesthetic significance we must attach to it. This may or may not be true, but his definition of internal and external relations is helpful in determining what can be readily extracted from a musical score and performance by the computer, and what must be read into a score to determine its general significance.

Several possible future musical applications of the computer have become evident as a result of this analysis. However, the investigation of specific aspects of musical works rather than general concepts seems a better application of the computer for the present time in seeking more precise descriptions of musical concepts. One such application is information theory analysis of music of the type presented in this paper, which bases its analysis on the premise that most musical works

¹⁵Ibid.
reflect a balance between the extremes of order and disorder, and that stylistic differences can depend to a large degree on fluctuations between these two extremes. Included in this application could be comparative stylistic analyses which might be of interest to musicologists in determining characteristics of particular historical styles.

Another application might be the analysis of musical sounds themselves as sound waves. Acoustical applications are also possible, such as spectrum analyses of musical compositions to study orchestration, dissonance, density, and texture as well as performance. A comparative study of instrumental timbres would also be possible using spectrum analysis.
CONCLUSION

The method presented in this paper was applied to various musical parameters, and significant results regarding information content and redundancy were obtained, in addition to determining intervals, ranges, and frequency of occurrence of elements in the selected works. Frequency of occurrence results were made possible by the use of the digital computer to perform operations on musical parameters which would be too time consuming to be considered by any other method.

This computer analysis of music is unique in several ways. The method uses FORTRAN, a prevalent programming language available at most computer installations. The autocorrelation function was applied to the selected works and was found to be a useful analytical tool for determining forms and contours. Also, the selected works had not previously been subjected to an information theory analysis. It was found that the musical analyst can increase his understanding of historical trends in music as well as individual works by adding information content and redundancy calculations to his analytical method.
APPENDIX 1
APPENDIX 2

These appendixes, the computer program, printouts, and input cards, have not been included here because of their size but are available from the Library, Music Department, University of B.C.
APPENDIX 3

The numerical representation of musical pitch was based on the range of the normal piano keyboard and assumed enharmonic equivalency, for example:

\[ \text{\textcopyright} \]

Rhythmic representation was as follows:

\[ \text{\textcopyright} \]

rests were indicated as negative durations.
APPENDIX 4:


Information theory is a statistical theory of communication. This theory, originated in 1948 by C. E. Shannon, is an outgrowth of applied probability theory with extensive applications to communications systems.

The fundamental problem of communication is to reproduce at one point a message transmitted from another point. Whether the message has meaning or not is irrelevant; what is significant is that the actual message is one selected from a large number of possibilities, and the communication system is designed to operate for each possible message.

Three levels of communication determine a message: the technical level to transmit symbols accurately, the semantic level to make symbols convey the desired meaning, and the effectiveness level to determine how the received message affects conduct in the desired way. The message and network of a communications system may be of any nature, but the entire system has certain fundamental criteria:

1. any communication system can be considered to be independent of the human receiver and sender,
2. the message is a sequence of signals or some physical representation of signals,
3. there is no concern for the meaning of the information,

but the number of symbols and how fast they are transferred is of great concern.

Since information theory grew out of probability theory, many aspects of probability theory are carried over. Probability can be defined as an event's relative frequency of occurrence, and the information content of a system reflects the probability of occurrence of events.

If the system has no defined properties or restrictions, the choice is random and the information content is at a maximum, but when a restriction is applied the information content is reduced. A completely random message consists of a sequence of symbols chosen independently from those available with equiprobability; any conceivable message might be generated.

Entropy can be thought of as a measure of missing information, so that when an outcome is certain, entropy will disappear. Entropy can also be a measure of randomness, i.e., the tendency of a system to become more and more unorganized as entropy increases, so that when the entropy is said to be low there is little choice within the system. When the entropy of an information source has been calculated, it can be compared to the maximum value the entropy could have to give the relative entropy. For example, a .8 relative entropy means the source is 80% free in its choice of symbols from a given set to form a message. Conversely, redundancy is the fraction of the message determined by rules governing the use of the elements in question; therefore, the relative entropy is a measure of the free results
while the redundancy is a measure of the governed results.

Shannon developed the basic equation for information content computations derived from probabilities \( p(i) \) where \( i \), which is equal to \( 1, 2, \ldots, N \), is associated with each symbol in an alphabet of \( N \) symbols and the \( p(i) \) are not equal in value. Therefore, information content \( (H_{N}) \) is:

\[
H_{N} = \sum_{i=1}^{N} p(i) \log_{2} p(i) \text{ bits/symbol.}^{17}
\]

Because the magnitude of the information content in a message depends not only on the size of the alphabet, but also on the probability distribution of symbols drawn from the alphabet, the information content in messages is analogous to the entropy. Actually, entropy and information are often interchangeable.

The formula for information rate becomes:

\[
H_{T} = -M \sum_{i=1}^{N} p(i) \log_{2} p(i) \text{ bits / second,}
\]

and the total information conveyed becomes:

\[
H' = -MT \sum_{i=1}^{N} p(i) \log_{2} p(i) \text{ bits.}
\]

The scale of information goes from maximum information at total disorder to zero information at total order. Conversely, if redundancy is defined as \( R = \frac{H_{\text{max}} - H}{H_{\text{max}}} \times 100\% \) where \( H_{\text{max}} = \frac{\log_{2} N \text{ bits/symbol, or as } R = 1 - \frac{H_{N}}{\log_{2} N} \times 100\%, \text{ then the predictability goes from 100\% for total order to 0\% for total disorder.}}{100\%} \)

It should be noted that Shannon's measure of information

content through his $H_N$ formula cannot be considered as a measure of meaning because of the quantitative nature of the results. Also, the $H_N$ formula applies to ensembles of events or messages whereas meaning applies to a single message. In fact, the theory relates to what one could say, rather than what one does say. Information is a measure of one's freedom of choice when one selects a sequence of symbols to form a message.

A. A. Moles has worked out a general approach to relating music and information theory. He relates ambiguity, form, and entropy variances, and thereby shows the importance of sequential choice processes in building up musical structures.

MEMORY

1. Instantaneous Memory
2. Dated Memory
3. Undated Memorization

PERCEPTION

1. Semantic Mode
2. Aesthetic Mode

The semantic mode is a system of symbols which can be coded, i.e., translated into another language (a score for example), and the aesthetic mode is any element of sensory appeal. The aesthetic and semantic modes are related by acoustical quantities which are symbols on a given scale of duration and information rate which can be computed. It should be noted that the average person can grasp 10 to 20 bits of information per second with a maximum capacity of perhaps 100 bits per second; therefore,


Ibid.
perception is usually a selection of definite symbols from the whole message. In terms of perception the timbre, density, and amount of repetition should also be considered. Memory is simply divided in terms of span. More specifically, music can be seen as a discrete system with the most diffuse music being a successive note selection which is completely random, and which forms the basis from which to fashion more characteristic structures. In most cases the main methods considered for restricting the note selection are combinatorial principles and statistical methods with the computation of transition probabilities of information theory. Other methods of restriction are possible, but these concepts receive the main usage in relating information theory and music.

To apply information theory analytically to music certain aspects and ideas of information theory are directly transferred or assumed to be directly transferable. Two general types of assumptions are made:

1. mathematical assumptions of the musical system being applicable to the analytic calculation of information theory:
   a. no element not already known to be in a sequence can occur; i.e., unused elements have zero probability,
   b. a sufficiently large sample from an infinite sequence has the same statistical structure as the infinite sequence,
   c. the statistical structure of a sequence is in-

---

dependent of the observer's starting point within the message,
d. there is the same order of patterning throughout the sample,
e. infinite memory is available for storing music as numbers.

2. aesthetic assumptions of the suitability of applying information theory to music.
Aesthetic assumptions can be reduced to one basic question, i.e., do the statistical probabilities correspond to the listener's expectations. However, for this assumption to be entirely true the listener would necessarily completely store the music like a computer; the listener's experience is much more like a constantly changing probability system affected by such things as past experience, averaging, attitude, comprehension, etc.

W. Meyer-Eppler's\textsuperscript{21} approach to the information theory analysis of music is based on the premise that music is made up of definable discrete elements which can be described, although the sum of these elements may not necessarily result in music. Information theory serves as a criterion of form by the observation of the frequency of occurrence of elements. For example, $p_i$ is the frequency of occurrence of the element "i" in the works to be analyzed and therefore, first-order information entropy is the statistical distribution of elements.

with no reference to mutual relations. In calculating second-order frequencies $p_1$ becomes $p(1,j)$ suggesting pairs of elements. The second and higher orders begin to deal with Markov chains and the frequency of transfer from one element to another, making considerable mathematical calculations necessary.

Professor W. Fuchs\textsuperscript{22} of Germany has applied a statistical method to the analysis of a representative group of works from the Renaissance period through the modern period. His method is based on the computation of frequency distributions and related parameters such as mean values, variances, higher orders, skewness (the state of asymmetry or symmetry as shown by the frequency distribution), and kurtosis (the state of peaked or flat graphic representations of a statistical distribution). Also, his method has an entropy basis, i.e., information values as obtained from Shannon's formula. Professor Fuchs applied these methods to the analysis of note and interval counts, and when he plotted the various parameters against the historical periods he found definite trends in, for example, pitch distribution which seemed to correlate with the historical development of tonality. Typical results show that serial compositions indicate a reversal in the trend to more uniform distribution.

J. G. Brawley applied information theory to the analysis of rhythm, being concerned "not so much with the intent of describing individual styles as with investigating the possibilities\textsuperscript{22}Described in J. V. Cohen, "Information Theory and Music," Behaviour Science, Vol. 7, 1962.
of a method for using information theory as a tool in describing styles. He makes two preliminary assumptions: rhythm in music is a discrete system of communication and, style in music is a stochastic process with the structure of a stationary Markov chain. Brawley's rhythmic analysis described the elements of this discrete system such that

"... in perception of occurrences in time, these occurrences are grouped into patterns," and "whether these groupings are regular or not, we shall adopt this grouping of any number of unaccented beats with a single accented beat as one of the basic principles of this study. More specifically, this will be the basis for determining the length of the primary elementary symbols of our discrete system, i.e., the length of a single rhythmic unit, or rhythm."24

Tempo was restricted within the range of one to two beats per second.

Initially Brawley analyzed Bach's Two-Part Invention #14 and listed each rhythmic pattern, its frequency of occurrence, and its relative frequency. Both parts of the invention were considered but preference was given to the upper part. The eight rhythmic patterns that were found had an average information content of 2.6 bits and a redundancy of 13.1%. Because of the limited nature of this application no generalizations were made until other samples had been analyzed, with emphasis being given to redundancy calculations for comparison:

23Ibid., p. 153.
24Ibid., p. 153.
<table>
<thead>
<tr>
<th>Composition</th>
<th>Redundancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hec dies (Perotinus, 12c.)</td>
<td>25.8%</td>
</tr>
<tr>
<td>Acun Motet (P. de Cruice)</td>
<td>15.0%</td>
</tr>
<tr>
<td>Two ballads of M. dePenusio</td>
<td>8.9%</td>
</tr>
<tr>
<td>Schoenberg, 4th String Quartet</td>
<td>6.4%</td>
</tr>
<tr>
<td>Schoenberg, Verklärte Nacht</td>
<td>18.8%</td>
</tr>
<tr>
<td>Schubert, Three songs</td>
<td>5.7%</td>
</tr>
<tr>
<td>Mendelssohn, Three songs</td>
<td>19.6%</td>
</tr>
<tr>
<td>Brahms, Three songs</td>
<td>16.3%</td>
</tr>
</tbody>
</table>

Finally Brawley analyzed a large closed body of material, the minuets of Mozart's string quartets. Each measure was considered a rhythmic group and the redundancy of these was found to be 19.4%.

Dr. J. E. Youngblood\(^2\) attempted "to explore the usefulness of information theory as a method of identifying and defining musical styles." "Musical style may be considered a probability system which must be stationary."\(^2\) To begin this study Youngblood selected a number of Romantic songs in major keys: eight songs from Schubert's "Die Schone Mullerin," six arias from Mendelssohn's "St. Paul," and six songs from Schumann's "Frauenliebe und Leben." After finding an approximation of the probability of each of the twelve notes of the scale for each composer he calculated first-order information content and redundancy. Secondly, he found the frequency of occurrence of all pairs of tones and calculated the second-order frequency as well as the information gained by one note from another,

\(^2\)Ibid., p. 151.

\(^2\)Ibid., p. 151.
and the redundancy of this order:

\[
\begin{array}{ccc}
\%R_1 & \%R_2 & \%R_{2c} \\
Schubert & 12.5 & 20.4 & 35.6 \\
Mendelssohn & 14.9 & 24.0 & 43.5 \\
Schumann & 14.7 & 22.4 & 37.3 \\
Cumulative & 13.4 & 20.5 & 29.2 \\
\end{array}
\]

Some conclusions could be reached from these figures, especially if a harmonic analysis were also considered; for example, Mendelssohn's use of chromaticism was less frequent than Schubert's and Schubert is less redundant than Mendelssohn.

To compare this data on Romantic music to that of another style, Youngblood analyzed the Gloria, Sanctus, and Agnus Dei from the First Mass for Solemn Feasts (Liber Usualis pp. 19-22) and the Kyrie from the mass Orbis Factor (Liber Usualis p. 46). Using seven and twelve note systems the results were:

\[
\begin{array}{ccc}
\%R_1 & \%R_2 & \%R_{2c} \\
Seven Note & 3.2 & 23.9 & 28.8 \\
Twelve Note & 23.9 & 41.7 & 44.0 \\
\end{array}
\]

Gregorian chant as a twelve-note system is therefore slightly more redundant than any of the Romantic composers and much more redundant than the three of them combined. Although these results are very interesting as they stand, Youngblood concluded that to obtain precise analytical results analysis of at least twice the number analyzed here, or another type of analysis, would be necessary.

27Ibid., p. 151.
L. A. Hiller and C. Bean used information theory to study the structural details of musical compositions. Four sonata expositions were analyzed and compared using elementary information theory analysis:

1. Mozart ... Sonata in C Major, K. 545, 1788
2. Beethoven ... Sonata in E Minor, Op. 90, 1814
4. Hindemith ... Sonata in G Minor, 1936

The analysis was limited to the study of pitch distributions in the expositions of the first movements of each sonata. The expositions were subdivided at phrase endings or other natural division points so that changes in information content could be seen as each exposition progressed. Two basic element sets were used, a twelve-note set of the chromatic scale assuming octave equivalence and a twenty-one-note set to include differences in enharmonic content. Two counts were made for each phrase (both using the twelve and twenty-one-note sets), count A based on the number of note attacks only and count B which included durations of notes where a sixteenth note is equal to one unit, with smaller durations disregarded.

Eight calculations were made for each exposition: note occurrences, the probabilities of note occurrences, the information content, the total information content of the symbols in each of the subsections, the average information value

of the symbols in each exposition, redundancy, rates of information transmission, and the average rate of note transmission for each subsection. Generally, in counts A and B, there is little sharing of information content or redundancy values. However, chronologically there is an increase in average information and a decrease in redundancy. For example, a comparison of the extreme high and low values in the pitch calculations shows section V of the Mozart to have an information content of 2.19 and a redundancy of 50.1%, and section IV of the Bérg to have an information content of 3.97 and a redundancy of 9.5%. Differences between count A and count B are a result of a general tendency of durational values to cause lower information content.

This information theory analysis of four sonata expositions is based on zero and first-order stochastic processes. A zero-order approximation to some message is that order generated by the random choice process with equiprobable choices, while a first-order approximation is based on a probability distribution derived from the frequency of occurrence of symbols and does not allow for the possible ways each choice may affect all subsequent choices. The problem seems to be how one might consider the long-range structure of music, i.e., overall form in individual compositions as well as the historical continuity or lack of continuity of all music.

Conditional probabilities have been used in the analysis
of Webern's Symphonie, Op. 21. L. A. Hiller and R. Fuller carried out a comparison of two analyses of the first movement of Webern's Symphonie: structural analysis and information theory analysis. Their purpose was to show any complementary aspects which the two methods might have and to intensively analyze the movement using information theory, showing the procedures used, the types of results obtained, and restrictions, such as sample size, which affect the final results. The Webern Symphonie was chosen because it is a "classic" example of highly ordered serial music, and therefore the structural organization would be directly reflected in information content measurements. The first movement was chosen because certain recent tendencies occur (such as a rigidly controlled pitch structure), and the three main sections of the movement are all long enough for valid statistical analysis.

The purpose of the structural analysis was to reconstruct precompositional procedures presumably used by Webern himself, i.e., affected by the composer's logic and not by any independent evaluation of his plan. Seven aspects were considered: tone row, sectional plan, canonic structure, vertical structure, transpositions of the row, instrumentation, and rhythm.

Four parameters of analysis by information theory were considered: pitch, four types of intervallic relations bet-

ween pitches, rhythm, and pitch and attack intervals combined. Redundancy plots were found for each section and information content plots were found for each section for one, two, and three element groups; one element determined by another, a two element group determined by one element, and one element determined by a two element group. The pitches in each section were analyzed according to their own symbol alphabet. For example, the exposition section was analyzed according to a 13-pitch set, the development according to a 27-pitch set, and the recapitulation according to a 14-pitch set.

The analysis of interval structure was subdivided into four types and rhythm was analyzed according to the "attack interval" which was the time span between note attacks measured in eighth notes. The analysis was concluded with the calculation of the information rates in bits per second for each section, and the total information in bits per second.

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