NUCLEAR ORIENTATION OF Bi^{206} IN NICKEL

by

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ABSTRACT

A study of the Pb^{206} decay was performed by Nuclear Orientation of Bi^{206} nuclei in a Nickel host at very low temperatures. The hyperfine field of $\text{Bi}^{206}/\text{Ni}$ was determined to be 400^{+34} kilogauss, with only 65 % of the Bismuth nuclei occupying 'good' lattice sites. Mixing ratios were determined for several transitions in the Pb^{206} decay. Attenuation coefficients due to reorientation effects in the isomeric state at 2200 keV. excitation in Pb^{206} were measured.

A discussion of the fractional occupation of 'good' lattice sites by the Bismuth nuclei is given.

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CHAPTER I

THEORY OF NUCLEAR ORIENTATION

1.1 Theoretical Review

Nuclear Orientation refers to the production of an ensemble of nuclei whose spins are ordered along some preferred axis in space. The relative populations of the magnetic substates of such a system are not equal, that is $a(M) \neq (2I + 1)^{-1}$, where $a(M)$ is the relative population of the substate M for a nucleus of spin I . If there exists a distinction between up and down along this preferred axis (i.e. $a(M) \neq a(-M)$ for at least one value of M), then the ensemble of nuclei is referred to as polarized. If the nuclei are oriented but there exists an up-down symmetry (i.e. $a(M) = a(-M)$ for all M), then the ensemble is said to be aligned. It is often useful to note whether a specific orientation process produces polarization or alignment of the spin ensemble. These aspects of nuclear orientation will be dealt with in greater detail in the discussion of experimental techniques.

In a system of oriented nuclei, the angular distribution of the radiation from the decaying nucleus is dependent on the quantum numbers describing the states involved in the decay process. Consider a nucleus of spin I_0 , with the population of the magnetic substates M_0 given by $a(M_0)$, decaying to an intermediate state I_1 by emission of radiation with angular momentum L_0 . If this state promptly decays to a final state of spin I_2 by a transition involving radiation of angular momentum L_1 , the angular dependence of this radiation is given by

$$W(\theta) = \sum_{m,n} a(M_0) |C(I_1, L_0, I_0; M_0 - m, n)|^2 |C(I_2, L_1, I_1; M_0 - m - n, n)|^2 F_{LM}(\theta) \quad (1.1)$$

where C is the Clebsch-Gordan coefficient and

$$F_{LM}(\theta) \propto \sum_{\mu=-1}^{+1} |C(L_1, 1, L; M - \mu, \mu)|^2 |Y_{L, M - \mu}(\theta)|^2 \quad (1.2).$$

Here M refers to the z component of the radiation L .

FIGURE 1.1 Typical decay scheme for a nucleus of spin I_0 decaying to final state spin I_2 .

The many sums involved in (1.1) can be reduced to a simplified form using Racah coefficients and Legendre Polynomials, yielding

$$W(\theta) = \sum_{\nu \text{ even}} B_{\nu} U_{\nu} F_{\nu} P_{\nu}(\cos\theta) \quad (1.3).$$

Here ν , restricted to even values for parity-conserving transitions, is

carried to the maximum index value of $2I_0$, $2I_1$, or $2L_1$, whichever is the smallest. In practice, the sum is usually terminated at $v = 4$.

The B_v coefficients describe the initial state and involve only parameters describing this state. Thus

$$B_v = (2v + 1)^{\frac{1}{2}} \sum_{M_0} C(I_0, v, I_0; M_0, 0) a(M_0) \quad (1.4).$$

In this work static nuclear orientation is employed, in which case the populations of the nuclear magnetic substates are given by the Boltzman distribution (this shall be discussed in detail later). Therefore

$$a(M_0) = \frac{\exp(\beta M_0)}{\sum_{M_0} \exp(\beta M_0)} = \frac{\sinh \beta(I + \frac{1}{2})}{\sinh(\beta/2)} \quad (1.5).$$

and

$$\beta = \frac{\mu H}{I k_B T} \quad (1.6).$$

The term β involves the hyperfine interaction and contains the temperature dependence of the observed radiation. At high temperatures where $\beta \approx 0$, the intensity of the observed radiation is normalized such that $W(\theta) = 1$, an isotropic distribution. The ratio of the yield at low temperatures, cold counts, to the yield at high temperatures, warm counts, is defined as the anisotropy.

The U_{ν} coefficients describe reorientation effects caused by the unobserved transition(s) to the intermediate state. It involves only the spins of the states involved and the angular momentum connecting them. For the decay sequence considered here,

$$U_{\nu} = (-1)^{I_1 + I_0 - L_0 + \nu} [(2I_0 + 1)(2I_1 + 1)]^{1/2} W(I_0 I_1, I_1, I_1; L_0 \nu) \quad (1.7)$$

where W is a Racah coefficient.

In the case of several unobserved transitions, the U_{ν} factor is the product of factors $U_{\nu}^{(k)}$, where $U_{\nu}^{(k)}$ is the reorientation factor for each of the preceding transitions. For example, the cascade

$$I_0 \xrightarrow{L_0} I_1 \xrightarrow{L_1} I_2 \xrightarrow{L_2} \dots \xrightarrow{L_{m-1}} I_m \quad (1.8)$$

would have an effective U_{ν} coefficient given by

$$U_{\nu} = U_{\nu}^{(0)}(L_0) \cdot U_{\nu}^{(1)}(L_1) \dots U_{\nu}^{(m)}(L_{m-1}) \quad (1.9)$$

If the transition is of one of the mixed multipolarity, involving radiation with angular momentum L and L' , the U_{ν} coefficients must be replaced by the incoherent average of the combination. The mixing ratio is defined by the parameter δ , such that

$$\delta = \frac{\langle I_1 || L' || I_2 \rangle}{\langle I_1 || L || I_2 \rangle} = \frac{\text{amplitude of radiation } L'}{\text{amplitude of radiation } L} \quad (1.10)$$

Then

$$U_V = \frac{U_V(L) + \delta^2 U_V(L')}{1 + \delta^2} \quad (1.11)$$

In most cases, one can restrict the mixtures to the two highest multipoles, $L = \Delta J$ and $L' = \Delta J + 1$.

If there exists an appreciable interaction between the intermediate state of the nucleus and any extranuclear fields, an attenuation of U_V coefficients may be observed. The angular distribution of the radiation L_1 observed in the cascade

$$I_0 \xrightarrow{L_0} I_1^* \xrightarrow{L_1} I_2 \quad (1.12)$$

is altered when the intermediate state I_1^* is appreciably perturbed. Such perturbations can be caused, for example, by the interaction of the magnetic dipole moment μ of the nucleus with a magnetic field H , or by the electric quadrupole moment Q interacting with an electric field gradient $\partial^2 V / \partial z^2$. The classical interpretation describes this in terms of the Larmor precession of the nuclear spin about the symmetry axis. If the lifetime of the intermediate state is long enough, the orientation of the spin axis changes before the emission of the observed radiation L_1 . Quantum mechanically this corresponds to transitions among the magnetic substates of the intermediate state I_1^* . The observed radiation then occurs from a state with an altered population distribution. Depending upon the type of interaction, this can result in an attenuation of the anisotropy of the observed radiation. Mathematically the effect can be

expressed in terms of attenuation coefficients, A_ν , as follows:

$$U_\nu(L_1) = A_\nu U_\nu(L_1) \quad (1.13)$$

For most states with lifetimes less than 10^{-10} seconds, no attenuation is observed and we have $A_\nu = 1$. However, there are cases where the intermediate state lifetime is long enough to cause appreciable perturbation of the intermediate state, with the result that $A_\nu < 1$. A detailed discussion of this effect will be given in Chapter III.

The observed gamma-ray transition is described by the F_ν coefficients, and involves only the angular momentum of the observed radiation and the spins of the states which it connects. Its formula, in terms of the Clebsch-Gordan and Racah coefficients, is

$$F_\nu = (-1)^{I_1 - I_2 + 1} (2L_1 + 1)(2I_1 + 1)^{1/2} C(L_1 L_1 \nu; 1 - 1) W(I_1 L_1 I_1 L_1; I_2 \nu) \quad (1.14)$$

where C is the Clebsch-Gordan coefficient and W is the Racah coefficient.

In the case of a transition of mixed multipolarity, the F_ν coefficient for a transition of dominant multipole L and lesser multipole L' is given by the coherent average

$$F_\nu = \frac{F_\nu(I_1 I_2 LL) + 2\delta F_\nu(I_1 I_2 LL'^1) + \delta^2 F_\nu(I_1 I_2 L'^1 L'^1)}{1 + \delta^2} \quad (1.15)$$

using the sign convention of Biedenharm and Rose (1953). Here the mixing ratio δ is the same as defined in (1.10).

The $P_\nu(\cos\theta)$ are Legendre Polynomials, containing the angular

dependence of the observed radiation at an angle θ relative to the orientation axis. In practice, a relatively large solid angle is subtended by the detectors which monitor the radiation. As a result the detectors do not look at a unique value of θ . In the case of cylindrical detectors it is possible to calculate the correction to the angular distribution exactly. Rose (1953) has shown that for cylindrical detectors, the form of the distribution remains unchanged, with the exception of an attenuation factor multiplying the Legendre Polynomial terms.

A detailed discussion of the derivation of these attenuation factors is given by Rose (1953) and Yates (1964).

The solid angle correction factors give the corrections to the anisotropies which must be made because of the finite solid angle subtended by the detector. In the analysis of the data, no energy dependence of this factor was considered. It was felt that since the cold counts were being normalized to warm counts of the same energy, that any corrections for the energy of the peak would be negligible. In particular, the attenuation factors are given by

$$g_v^u = \frac{1}{1 - \cos\alpha} \int_0^\alpha P_v(\cos\beta) \sin\beta \, d\beta$$

(1.16)

where the angles α and β are defined in FIGURE 1.2.

FIGURE 1.2 Definition of the angles α and β used in the determination of the solid angle correction factor.

Here α refers to the half-angle subtended by the detector face. The result is a new term which appears in the anisotropy expression given by (1.3) as

$$P_V^1(\cos\theta) = g_V P_V(\cos\theta) \quad (1.17).$$

The B_V coefficients are calculated explicitly using the expression given in (1.4). Other methods exist for describing the initial state population (see DeGroot, Tolhoek, and Huiskamp (1965)).

The U_V and F_V coefficients can be evaluated from tables of Yamazaki (1967) or using tables of Clebsch-Gordan and Racah coefficients. Ferentz and Rosenzweig (1955) have tabulated the F coefficients up to $I = 12$. A Fortran computer program is included in Appendix D for the calculation of the U_2 and U_4 coefficients.

1.2 Methods of Orienting Nuclei

There are several methods available for obtaining nuclear orientation at low temperatures. All of the methods reviewed in this work make use of the interaction of the nuclear magnetic dipole moment or the electric quadrupole moment with magnetic and electric fields respectively. Abragam and Pryce (1951) express the interaction by a Hamiltonian of the form

$$\begin{aligned}
 H = & g_N \mu_N H \cdot I + g_{\parallel} \mu_B H_z S_z + g_{\perp} \mu_B (H_x S_x + H_y S_y) \\
 & + A S_z I_z + B (S_x I_x + S_y I_y) \\
 & + D \left[S_z^2 - \frac{1}{3} S(S+1) \right] + P \left[I_z^2 - \frac{1}{3} I(I+1) \right]
 \end{aligned}
 \tag{1.18}$$

where

$$g_N = \mu/I$$

μ_N = magnetic moment in units of nuclear magnetons

H = external applied magnetic field

S = effective nuclear spin quantum number

A & B = hyperfine structure coupling constants

D & P = crystalline electric field splitting parameters

$g_{\parallel}$ & $g_{\perp}$ = spectroscopic splitting factors

μ_B = Bohr magneton

Only the 'Brute Force' method and method of orientation in ferromagnets

shall be discussed in this section. Further methods of orienting nuclei are discussed by Jeffries (1963) and Frauentfelder and Steffen (1965).

1) Brute Force Method

This method involves the application of a large magnetic field to a system of free nuclei, causing Zeeman splitting of the level I into its $2I + 1$ magnetic substates. The energy levels are given by

$$E_m = \frac{\mu H m}{I} \quad (1.19)$$

and the relative populations of these levels are given by the Boltzman distribution

$$a_m = \frac{\exp(-\beta m)}{\sum_m \exp(-\beta m)} \quad (1.20)$$

In this case, polarization of nuclear spins results since $a(m) \neq a(-m)$.

Appreciable polarization of the spin ensemble is achieved when β is the order of unity. In case of typical nuclei, such as Co^{60} with $\mu = 3.56 \mu_n$ and $I = 5$, using 100 kilogauss as the limit of current magnet technology, temperatures of the order of $10^{-2} \text{ } ^\circ\text{K}$ are required.

Dabbs et.al. (1955) first used this technique, followed by Stolovy (1960), to polarize In^{115} . Using the polarization parameter f_N of Rose (1949)

$$f_N = \frac{1}{I} \frac{\sum_i m_i \exp(-\beta m_i)}{\sum_i \exp(-\beta m_i)} \quad (1.21)$$

which reduces to (for $\mu H/kT \ll 1$)

$$f_N = 1/3 \frac{1 + 1}{I} \frac{\mu H}{kT} \quad (1.22)$$

they were able to obtain polarizations for 2.1% and 3.0% respectively.

ii) Ferromagnets

In ferromagnetic materials, the unpaired electron spins are aligned spontaneously along the easy axis of magnetization of the crystal domain. Upon application of an external magnetic field, the domains whose magnetization is more or less parallel to the external field increase in size, while the remainder decrease. The magnetization of the domains remain parallel to the direction of spontaneous magnetization during this process, which requires fields the order of a few hundred gauss. Application of even larger fields, the order of a few kilogauss, cause saturation of the domain magnetization so as to polarize them parallel to the direction of the external field.

The orientation of impurity nuclei in ferromagnets occurs via the Fermi contact interaction between the 4s conduction electrons and the nucleus. These 4s electrons are polarized by the 3d electrons localized on the ferromagnetic atoms, and maintain this polarization. Impurity nuclei in the ferromagnetic host see a large hyperfine magnetic field produced by interaction with the polarized conduction electrons. This results in Zeeman splitting of the nuclear magnetic substates and subsequent polarization of the impurity nuclei at low temperatures.

Marshall (1958) considers the orientation of nuclei in ferromagnets in detail. The effective magnetic field at the impurity nuclei may be written as

$$H_{\text{eff}} = H_1 + H_c + H_a$$

(1.23)

If we neglect the Lorentz field, the local field H_1 is given by

$$H_1 = H_e - DM$$

(1.24)

where H_e is the external field and DM is the demagnetizing field. This latter term is dependent upon the specimen shape.

The contact term H_c results from the contact interaction from the 4s conduction electrons which are spin polarized by the 3d electrons. A detailed discussion of this contribution to the effective field is given by Shirley and Westenbarger (1965).

The H_a term arises from the interaction of the nucleus with the electrons of the same atom. The relevant terms in the spin Hamiltonian describing this interaction are

$$H = AS_z I_z + B(S_x I_x + S_y I_y)$$

(1.25)

from (1.18). For the case of ferromagnetic hosts, the electron spins are aligned by the exchange interaction, hence S_x and S_y can be replaced by zero. The result is

$$H_a = -AS_z I_z = \frac{-AS_z I_z}{\mu}$$

(1.26)

The result of this interaction is that the nuclei within a domain are polarized at sufficiently low temperatures.

In the typical multidomain specimen, an external applied field is necessary to align the individual domain magnetization along the preferred axis. Grace et.al. (1955) were the first to observe this effect when they oriented ^{60}Co nuclei in a Co lattice.

Orientation of non-magnetic impurity nuclei is a result of the conduction electron polarization term H_c . Samoilov et.al. (1960) obtained orientation of In^{114} , Sb^{122} , and Au^{198} in an iron host.

Campbell (1969) has calculated the hyperfine field in a wide range of impurities for several hosts.

The advantage to this method is that a great many impurities are soluble to some extent in ferromagnetic hosts. A review article by Shirley (1971) gives a partial compilation of hyperfine fields in iron, cobalt, and nickel hosts.

1.3 Experimental Techniques

In the experiments described here, the radiation emitted from a system of oriented nuclei is measured as a function of temperature and of angle relative to the orientation axis (which is taken as the direction of the applied external field). The temperature dependence of the anisotropy is contained in the B_ν terms of Equation (1.3). By studying the temperature dependence of $W(\theta)$, the hyperfine interaction μH_{eff} may be obtained. Once the hyperfine interaction has been determined, and hence the B_ν terms established, the $U_\nu F_\nu$ terms can be extracted from the data. The U_ν and F_ν coefficients are exactly determined by the spin assignments to the states involved and the multipolarity of the transitions between these states. This allows assignments to be made for the decay scheme of the radiation being studied. The importance of such a method for studying nuclear decay was first proposed by Halban (1937). It should be mentioned that the F_ν coefficients in (1.3) are dependent upon the mixing ratio δ , as shown in (1.15). The quadratic form allows not only the magnitude of δ to be determined, but often the sign as well. One of the two possible values of δ can be selected on the basis of available internal conversion data.

Other experimental methods which determine these quantities are:

i) Angular Correlation

This method is quite similar to that of nuclear orientation. For example, in the case of a gamma-gamma cascade, detection of the first emitted gamma-ray establishes a quantization axis relative to which the succeeding transition is measured as a function of the angle. For an $I_0 \longrightarrow I_i \longrightarrow I_f$ transition, the angular correlation function is given by

$$W(\cos\theta) = \sum_{k \text{ even}} F_k^{(1)} F_k^{(2)} P_k(\cos\theta) \quad (1.27)$$

Here the $F_k^{(1)}$ and $F_k^{(2)}$ terms are the F_k coefficients of Ferentz and Rosenzweig (1955) for gamma-rays 1 and 2 respectively. The $P_k(\cos\theta)$ are the Legendre polynomials. It has the advantage that it can be performed at room temperature. The decay scheme is depicted schematically in FIGURE 1.3.

A more complete review of this method is given by Fraunfelder and Steffen (1965).

ii) Perturbed Angular Correlation

If one of the intermediate states in a gamma cascades is a long lived one, the interaction between internal, or external, electromagnetic fields and the nuclear moments cause a perturbation of the angular correlation. For example, in the case of an electric quadrupole interaction, an attenuation of the angular correction is observed, of the form

$$W(\theta) = \sum_k F_k^{(1)} F_k^{(2)} G_{kk} P_k(\cos\theta) \quad (1.28)$$

where the G_{kk} term is the attenuation factor.

The original application of this technique was in the measurement of nuclear g-factors, first successfully applied by Aeppli et.al. (1951). It is also extensively used in hyperfine studies where the hyperfine interaction may be derived from the precession of the nuclear spin. A detailed discussion of this technique is given in Karlsson et.al. (1964), Fraunfelder and Steffen (1965), and Steffen (1955).

FIGURE 1.43 Experimental arrangement for detecting angular correlations of successive gamma radiations.

iii) Internal Conversion

When a nucleus decays from an excited state, it may do so by emission of electromagnetic radiation, or it may transfer the energy to an orbital electron resulting in its ejection from the atom. This electron ejection process is referred to as internal conversion. The ratio of the number of conversion electrons per unit time to the number of photons per unit time is defined as the internal conversion coefficient α . These coefficients are determined by the amount of energy involved in the transition, the angular momentum change, and the parity change. A detailed treatment is given by Rose (1960) and (1965).

iv) Mossbauer Effect

Recoil-free absorption of nuclear resonance radiation was first discovered by Mossbauer (1958). This resonance radiation makes possible the study of the hyperfine structure in the excited states of many nuclei. The detection of the resonance radiation arises from the change in the resonance effect as a function of the relative velocity between the source and absorber.

The degeneracy of the excited levels involved in the resonant transition can be removed, say by the hyperfine interaction, and the magnetic substates in the absorber split. As long as the hyperfine splitting of the resonance line is greater than the level half-width, there will be minimal absorption of the resonance radiation (for the case considered in FIGURE 1.4).

In FIGURE 1.4 (a) i, there exists six different energies for the transition between the $3/2^-$ excited state and the $1/2^-$ ground states in the absorber. The mono-energetic transition in the absorber does not match

FIGURE 1.4 (a) Ground and excited states of iron and Fe₂O₃
 i) the 14.4 keV gamma decay of Fe⁵⁷ and
 ii) the hyperfine splitting of the two levels. The splitting of these levels in FeO is shown for ii) magnetic hyperfine splitting and iii) quadrupole splitting. The chemical shift Δ is also shown.
 (b) The absorption spectrum for Fe₂O₃. Kistner and Sunyar (1960).

any of these. As a result, there is minimal resonance absorption of the source radiation.

By moving the source relative to the absorber, the Doppler shift of the source line can be brought into resonance with any one of the transitions in the absorber. It is possible to 'scan' a relatively larger energy spectrum about the resonance line, about 10^{-4} ev range, and observe all of the transitions between the magnetic substates of the two spin levels in the absorber nuclei. The results of Kistner and Sunyar (1960) on the Fe^{57} , 14.4 keV nuclear gamma resonance are shown in FIGURE 1.5 (b) for Fe_2O_3 absorber.

Another interesting application of the Mossbauer effect is to alloy systems in which the impurity nuclei occupy non-equivalent sites. Bauminger et.al. (1961) used recoil free 14.4 keV gamma radiation to study internal fields acting on iron nuclei in Iron Garnet. The absorption spectrum for Yttrium Iron Garnet is shown in FIGURE 1.5.

In FIGURE 1.6, the stronger internal field is assigned to the components of the doublets. Superposition of the doublets in lines 3 and 4 causes broadening of the absorption line.

Similar results have been found by de Waard et.al. (1971) for various atoms implanted in metals. In some cases (such as Cs^{133}Fe) they have found evidence high, intermediate, and low field sites.

The theory of the Mossbauer effect is given by Frauenfelder (1963) and Mossbauer (1965). An excellent description of the experimental technique used in deriving the nuclear information from Mossbauer spectra is given by Kistner (1966).

FIGURE 1.5 Mossbauer Spectrum for Yttrium Iron Garnet showing the effect of non-equivalent lattice sites on the Iron hyperfine splitting.
Bauminger et.al, (1961)

v) NMR

Nuclear magnetic resonance involves the absorption of resonant electromagnetic radiation between magnetic substates which are split by the hyperfine interaction. Impurity nuclei in ferromagnetic hosts have been studied, but the impurity concentration must be larger than 0.1 atomic percent for a measurable effect. There are several cases in which the impurity concentration is so low that the NMR technique is no longer feasible.

The theory of the magnetic resonance technique is given, for example, by Abragam (1961) and Schumacher (1970).

vi) NMR/ON

This technique employs NMR techniques on an oriented system of nuclei and the detection of decay radiation from them. The nuclei are polarized in a ferromagnetic sample by standard nuclear orientation techniques and irradiated with an externally applied R.F. field. Resonant absorption of the electromagnetic radiation causes transitions between the magnetic substates of the level I . This tends to equalize the populations of the m substates and destroy the anisotropy of the decay radiation (resonant destruction of nuclear orientation). Removal of the external R.F. field allows relaxation of the nuclear spins back to population distribution characteristic of the lattice temperature.

The energy difference between the $2I + 1$ substates is given by

$$\Delta E = |E_{m+1} - E_m| = \frac{\mu H}{I} \quad (1.29).$$

The resonant frequency ν_o is then

$$\nu_o = \frac{\Delta E}{h} = \frac{\mu H}{hI} \quad (1.30)$$

where h is Planck's constant.

FIGURE 1.6 The effect of R.F. radiation of frequency ν_o on the population distribution of magnetic substates in a system of polarized nuclei.

Advantages to this method are:

- a) very high accuracy is obtained in these resonance experiments $\sim 0.05\%$. This limit results mainly from the inhomogeneous broadening of resonance line, as mentioned by Stone (1971); and
- b) very low impurity concentrations, ~ 0.01 atomic percent may be studied. This makes NMR/ON very attractive for impurities which are not very soluble in ferromagnetic hosts.

The resonant destruction of nuclear orientation was first observed by Matthias and Holliday (1966) for Co^{60} in an Fe host. Resonance absorption curves are shown in FIGURE 1.7.

FIGURE 1.7. Resonant absorption curve for Co^{60}/Fe
[Matthias and Holliday (1966)]

Detailed discussions of NMR/ON are given by Shirley (1968), and Stone (1971).

It is interesting to compare angular correlation, Mossbauer effect, and Nuclear Orientation as methods of studying the nucleus. All three of these methods are quite different in various aspects.

A general description of these techniques is given in Table 1.1, where the method of determining various nuclear properties is given.

Each of the three particular techniques has its own merits, depending upon the system being investigated. In some cases it is possible to compare results of a particular decay which were obtained by three entirely different methods.

TABLE 1.1

A summary of various experimental techniques.

	LOW TEMPERATURE MOSSBAUER EFFECT	ANGULAR CORRELATIONS	NUCLEAR ORIENTATION
Preparation	Polarized source	preceding radiation	Polarized parent nuclei
Detection	resonant absorption of radiation by absorber	coincidence rate of succeeding radiation	count rate of radiation as a function of θ and/or T.
Nuclear Properties			
Spin of Level	number of absorption lines in spectrum	fitted values of F_γ coefficients to $W(\theta)$	values of $U_\gamma F_\gamma$ for observed radiation
Angular momentum of radiation	intensity of individual components in the spectrum	through F_γ coefficient	through $U_\gamma F_\gamma$ coefficient
δ	angular distance of component radiations and their intensity	through F_γ	through F_γ and/or U_γ
Hyperfine field	splitting of absorption lines	values of attenuation coefficient as a function of delay between coincidence of γ -radiation correlation.	form of $W(\theta)$ vs. T. Appear in the $B_\gamma(T)$ terms.

1.4 Present Measurements

The present nuclear orientation studies of Bi^{206} in an iron host lattice were undertaken for several reasons, involving combined aspects of both nuclear and solid state interest.

A great deal of theoretical and experimental work has been done on nuclei in the region of the doubly closed shells of Pb^{208} . Nuclei differing from the Pb^{208} nucleus by a few nucleons constitute one of the few regions that can be described fairly accurately by conventional shell-model theory. The predicted energy levels of these nuclei are not very sensitive to the nuclear state wave functions. However, the mixing ratios of transitions between states are quite sensitive. Measurements of these mixing ratios can provide theorists with an accurate method for determining the wave functions of the states in these isotopes.

From a nuclear structure standpoint, a great deal is known about Pb^{206} , the daughter product in the decay of Bi^{206} . Very little data, however, is available on E2/M1 mixing ratios for several transitions and, for reasons mentioned above, it was felt that measurements of these would be of interest. In addition, the existence of an isomeric state at an excitation energy of 2200 keV, in Pb^{206} offered the possibility of studying reorientation effects.

The well-known level structure of Pb^{206} is of great advantage when attention is focussed on aspects of interest to solid state physics. Several studies of the hyperfine fields acting at Bismuth impurity sites in ferromagnetic hosts have yielded large discrepancies. This is illustrated in Table 1.2, where values of the hyperfine fields obtained by

TABLE 1.2

Hyperfine fields (kGauss) at Bismuth impurity nuclei.

Group	nuclide	Host		
		Nickel	Cobalt	Iron
Balabanov & Delyagin (1968)	Bi	255	—	935
Zawislak & Cook (1969)	Bi ²¹¹	160	—	—
Bowman & Zawislak (1969) ¹⁾	Bi ²⁰⁷	115	430	660
Bacon <u>et. al.</u> (1972) ²⁾	Bi ²⁰⁴	325	—	1180
Kaplan <u>et. al.</u> (1972) ²⁾	Bi ²⁰⁶	390	—	800-1000
Daly (1973) ²⁾	Bi ²⁰⁷	—	400 - 600	—

1) Perturbed Angular Correlation. 2) Nuclear Orientation.

various experimental groups are shown. In this respect, it was felt that, in view of the low solubility of Bismuth in ferromagnetic host lattices, the use of Bi^{206} as a probe would be particularly advantageous; the short half-life of the isotope (6.24 days) allows high activities to be realized at small concentration levels, while low temperature problems from internal heating of the source are minimized because of the electron capture decay. The possibility that sample inhomogeneities might be responsible for such large discrepancies provided a further aegis for these studies, in that nuclear orientation measurements might give information on the distribution of Bi^{206} nuclei within the sample.

CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURES

2.1 Methods for Producing Low Temperatures

Several methods exist for obtaining the very low temperature required to polarize a system of nuclei. The two most common methods are dilution refrigeration and adiabatic demagnetization. A description of both methods is given to show the merits of each system.

i) Dilution Refrigeration

This method produces refrigeration by forcing He^3 to dissolve from a concentrated He^3 phase into a saturated solution of He^3 in superfluid He^4 . The adiabatic dilution of He^3 causes cooling in the mixing chamber. A schematic representation of a dilution refrigerator is given in FIGURE 2.1.

In the apparatus of FIGURE 2.1, He^3 gas enters the vacuum jacket and condenses in a condenser coil immersed in pumped He^4 at a temperature of about 0.9°K . The He^3 then goes through the throttling capillary and a series of heat exchangers into the top of the mixing chamber. The lighter He^3 diffuses across the phase boundary created by the $\text{He}^3 + \text{He}^4$ mixture, causing cooling in the mixing chamber. The diluted He^3 continues to the still, in which the temperature is kept at about 0.6°K . This heating allows the He^3 circulation rate to be controlled, due to the vapour-pressure dependence on the temperature. Typical He^3 extraction rates are the order of $10^{-5} - 10^{-6}$ moles/second. The extraction rate also influences the temperature of the mixing chamber. Temperatures as low as $4.5\text{ m}^\circ\text{K}$ have been attained by Wheatley et. al. (1968).

Commercially available units are capable of cooling rates of the order of 750 ergs/sec. at 0.1°K . For nuclear orientation experiments, this allows higher activity samples to be used. This would improve the counting

FIGURE 2.1 Elements of a typical dilution refrigerator.

statistics and also allow weak transitions to be studied. One of the major problems in adiabatic demagnetization is the heating of the sample by charged particle emission.

The fact that the large cooling rates can be realized for the dilution refrigerator, plus the fact that it can be operated on an almost continuous basis, make it a very desirable apparatus for low temperature physics. For these reasons, the dilution refrigerator is rapidly replacing older techniques for producing very low temperatures.

The technique of dilution refrigeration is discussed, for example, by Wheatley (1970).

ii) Adiabatic Demagnetization

The process of adiabatic demagnetization is probably the most widely used technique for obtaining temperatures below 1°K . It was first suggested by Debye (1926) that the adiabatic demagnetization of a paramagnetic substance should produce low temperatures. This was proven experimentally by Giauque and MacDougall (1933) who obtained a temperature of 0.25°K using Gadolinium Sulfate.

For a paramagnetic substance at temperatures the order of 1°K , the entropy of the substance is due mainly to the ionic spins. It is frequently the case in solids that they produce the only significant contributions to the net magnetic moment, due to quenching of the orbital angular momentum by the crystal fields. These fields produce a very slight splitting of the $2S + 1$ degenerate energy levels, the splitting being much less than $k_B T$. The population of these levels is therefore very nearly equal. If a large magnetic field H is applied to the system of paramagnetic ions in an isothermal manner, the levels are widely split ($\Delta E \gtrsim k_B T$). This results in the lowest energy levels being more heavily populated, or a net

decrease in the entropy of the system. The thermodynamics of this system can be represented by the equation

$$TdS = C_H dT + T \left(\frac{\partial M}{\partial T} \right)_H dH \quad (2.1)$$

where S is the system entropy, C_H is the heat capacity at constant field H , and M is the magnetization. For an isothermal magnetization, $(\partial M / \partial T)_H < 0$, and $dT = 0$. Therefore

$$dS = \left(\frac{\partial M}{\partial T} \right)_H dH \quad (2.2)$$

and there is a net decrease in entropy of the system. If the paramagnetic ions are removed from all external sources and the system of ions is adiabatically demagnetized,

$$dS = \frac{dQ}{T} = 0 \quad (2.3)$$

since $dQ = 0$. In this case, Equation (2.1) can be simplified to

$$C_H dT = - T \left(\frac{\partial M}{\partial T} \right)_H dH \quad (2.4),$$

Since $dH < 0$, then $dT < 0$ and the temperature of the system will decrease from the initial temperature.

The final temperature obtained for a paramagnetic substance is given by Zemansky (1968) to be

$$\left(\frac{T_i}{T_f} \right)^2 \cong 1 + \frac{C_c}{A} H_i^2 \quad (2.5)$$

for the case where the final field after demagnetization is very weak ($H_f \rightarrow 0$).

The constant term C_c/A can be derived from the data of Zemansky (1968), and is given in Table 2.1 for various salts.

Table 2.1 Data for paramagnet salts

SALT	$C_c \frac{\text{cm}^3 - \text{deg}}{\text{gr.} - \text{Ion}}$	$A/R \text{ deg}^2$	gram-ionic mass
$\text{Cr}_2(\text{SO}_4)_3 \cdot \text{K}_2\text{SO}_4 \cdot 24\text{H}_2\text{O}$	1.84	0.018	499
$\text{Fe}_2(\text{SO}_4)_3 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 24\text{H}_2\text{O}$	4.39	0.013	482
$2\text{Ce}(\text{NO}_3)_3 \cdot 3\text{Mg}(\text{NO}_3)_2 \cdot 24\text{H}_2\text{O}$	0.317	6.1×10^{-6}	765

Here C_c is the Curie constant, A is a constant term developed from Schottky's Equation, and $R = N_A k_B$. The term gram-ion refers to the mass of a crystal containing exactly N_A (Avogadro's number) magnetic ions.

The magnetic heat capacity per gram-ion of a paramagnetic salt at temperatures above the Schottky maximum are given by

$$\frac{C_H}{R} = \frac{A/R}{T^2} \quad (2.6).$$

From the data of Table 2.1, it can be seen that the lowest temperatures will be obtained if Cerium Magnesium Nitrate is used, the constant C_c/A of eqn (2.5) being almost 100 times larger than that of Iron Ammonium Sulphate. However, the magnetic specific heat of the Iron Ammonium Sulphate is about 2000 times greater than that of Cerium Magnesium Nitrate.

A detailed description of the theory involved is given by Zemansky (1968) and White (1968).

2.2 Experimental Apparatus

The low temperature apparatus used in these experiments is described in detail by Kieser (1974). A brief description of this apparatus is given here.

The apparatus used in these experiments is shown in FIGURE 2.2. Temperatures as low as $10\text{ m}^{\circ}\text{K}$ are obtained by the adiabatic demagnetization of a Chrome potassium alum salt pill in the inner jacket of the cryostat assembly. The cryostat has stages of cooling to provide temperatures the order of 1.2°K necessary for the demagnetization of the salt pills.

i) Cryostat

The initial stage of cooling in the cryostat is provided by liquid nitrogen at 77°K in a glass dewar surrounding the Helium dewar. This minimizes helium boil-off during the experiments, thus reducing the number of helium transfers that must be made. A level indicator was placed in the nitrogen dewar and controlled a liquid nitrogen transfer system to fill the dewar automatically.

The helium dewar was an evacuated, silvered, pyrex vessel which was supported from the top plate of the apparatus stand by a four wire support system. In order to reduce the heat flow down the pumping lines and electrical leads, they were passed through a liquid nitrogen pot mounted on the top plate of the apparatus. This reduces the amount of boil-off in the helium dewar. Several radiation traps were soldered to the pumping lines to ensure as much heat as possible is absorbed by the evaporating helium gas. The helium level was monitored visually through unsilvered strips along the length of both the nitrogen and helium dewars.

A system of brass jackets is used to house the salt pill assembly. The outer jacket contains the helium pot and the inner jacket. Helium gas

FIGURE 2.2 Details of low temperature cryostat.

is admitted to the outer jacket to precool the system. After the thermal equilibrium temperature of 4.2°K is reached, the outer jacket is evacuated to a pressure of 0.001 micron. The helium pot is then thermally isolated from the inner dewar. A large capacity pump reduces the pressure in the helium pot to approximately 600 microns and a resultant temperature of 1.2°K . Helium exchange gas in the inner jacket at a pressure of 10 microns thermally couples the salt pill assembly to the helium pot. The temperature of the guardpill is monitored by a $420\ \Omega$ carbon resistor. When thermal equilibrium is reached in the pill assembly, the inner jacket is evacuated to a pressure of approximately 10^{-6} microns.

The details of the salt pill assembly are shown in FIGURE 2.3. A Chrome potassium alum pill constitutes the mainpill. This pill is suspended by nylon supports from the helium pot. The mainpill has a ferric ammonium alum guardpill to isolate it further from the helium pot. The temperature of the guardpill is approximately $50\ \text{m}^{\circ}\text{K}$ after a typical demagnetization. An auxilliary guardpill of manganese ammonium sulphate was attached to the lower end of the heat shield, which is thermally isolated from the mainpill. This helps to reduce heat conduction to the mainpill produced by eddy current heating of the heat shield during demagnetization. Thermal contact between the mainpill and the sample is provided by a copper heat link.

A system of three superconducting solenoids surround the jacket assembly. Two of these are used for demagnetization of the salt pill assembly. A Ventron Magnion 40kG. superconducting solenoid completely surrounds the mainpill assembly, while a smaller 22kG. superconducting solenoid surrounded the guardpill. The polarizing solenoid had a Helmholtz configuration and was located at the end of the copper heat link.

FIGURE 2,3 Details of the inner jacket assembly.

The conversion factor for this solenoid was 320 Gauss per Ampere and a maximum current of 40 A. could be reached without quenching.

During magnetization, the pressure in the inner jacket is maintained at approximately 10 microns to facilitate the removal of the heat of magnetization produced in the pills. The current in the solenoids was increased at 5 A./min. up to 25 A., and 1.5 A./min. to the maximum current desired. Once the magnets were in the persistent mode, the current was lowered to zero at the rate of 5 A./min. The polarizing solenoid was energized in the same fashion. When the salt pill reached thermal equilibrium with the helium pot, the inner jacket exchange gas was evacuated. The current to the magnets was increased to the original magnetization value in about 5 minutes to reduce Joule heating in the leads. When this value was reached, the 22 KG. and 50 KG. magnets were put in the normal mode and the current reduced at 5 A./min. to reduce eddy current of the copper heatshields. The demagnetization of the salt pill assembly produced temperatures of about 10 m°K. Test runs on the apparatus indicated that temperatures below 12 m°K could be obtained for up to 50 hrs. (Kieser 1974). Typical system performance during a normal experiment are given in FIGURE 2.4. It is quite apparent from FIGURE 2.4 (a) that there was a large helium boil-off when there were large currents present in the leads to the solenoids. FIGURE 2.4 (b) shows the cooling of the guardpill to 1.2 °K, which corresponds to a guardpill resistor value of 750 ohms. The system was allowed to stabilize for slightly longer than one hour before demagnetizing the pills.

FIGURE 2.4 Typical system performance.

2.3 Co^{60} Gamma-Ray Thermometry

The anisotropy of gamma radiation from polarized radioactive nuclei for thermometry was first used by Stone and Turrell (1962). In the case of Co^{60} nuclei in an iron lattice, all parameters necessary for the description of the gamma-ray anisotropy are known, other than the temperature. Each B_γ is a single valued function of the nuclear spin temperature T . Measurements of the gamma-ray anisotropy gives these B_γ values, and hence the value of T .

The decay scheme of Co^{60} is given in FIGURE 2.5. Angular momentum considerations of the Co^{60} decay reveal that both the 1173 keV and 1332 keV gamma-rays exhibit identical anisotropies. This allows an averaging of the combined gamma-ray counts to obtain better statistical accuracy. The parameters describing the anisotropy of the Co^{60} decay are given in Table 2.2.

Table 2.2 Factors Describing the Co^{60}/Fe Anisotropy.

Ground State Spin	5^+
Magnetic Moment	$3.754 \mu_N$
Hyperfine Field	287.7 kG
Radiation	E2
$U_2 F_2$	-0.42056
$U_4 F_4$	-0.24281

The gamma-ray anisotropy for Co^{60}/Fe was calculated by the computer routine ANGDIST. Corrections to the hyperfine field at the Co^{60} nuclei were made for the polarizing field. A correction was also made for the finite solid angle subtended by the detectors. The temperatures of the spectra were obtained by correlating the corrected anisotropy with the temperature. ANGDIST is described in Appendix B with a sample output.

FIGURE 2.5 The decay scheme of Co^{60} .

The use of nuclear orientation thermometers is considered in some detail by Berglund et.al.(1972).

2.4 Gamma-Ray Detection System

The gamma radiation of the Co^{60} decay and the Bi^{206} decay was detected by a lithium drifted germanium detector positioned along the orientation axis ($\theta = 0^\circ$). It was necessary to use this type of detector rather than high efficiency sodium iodide detectors because of the higher resolution available.

The Ge(Li) detector used for the $\text{Bi}^{206}/\text{Ni}$ experiment had a relative photopeak efficiency of 13.8%. The peak-to-Compton ratio was 20 to 1, and the resolution of the 1332 keV Co^{60} peak was 3.2 keV. For the $\text{Bi}^{206}/\text{Co}$ experiment, a detector with a relative photopeak efficiency* of 3.9% was used. The peak-to-Compton ratio was 9 to 1, and the resolution of the 1332 keV Co^{60} peak was 5.0 keV. Here, the term resolution means the full width at half maximum. Sodium Iodide detectors were not used because of the much poorer resolution of these detectors compared to the Ge(Li) type. Typical resolution for NaI(Tl) are 60 keV at the 1332 keV Co^{60} peak. The Pb^{206} spectrum for a 2" x 2" Na(I) Tl detector is shown in FIGURE 2.6, which demonstrates the inability of the detector to resolve several of the transitions.

The block diagram of the electronic processing and analyzing system is shown in FIGURE 2.7. When a gamma-ray interacts with the Ge(Li) crystal, a current pulse is produced which is then converted to a voltage pulse by the preamplifier. This pulse has a short rise time of approximately 10^{-7} seconds and a fall time of about 10^{-5} seconds. The preamplifier output is

* The relative photopeak efficiency of a Ge(Li) detector is measured relative to that of a 3" x 3" NaI detector for the 1332 keV peak of Co^{60} source at 25 cm from the detector.

FIGURE 2.6

The decay spectrum of Pb²⁰⁶ for a 2" x 2" NaI(Tl) detector. In this spectrum, the two groups of closely spaced peaks cannot be resolved into the individual transitions which are indicated on the diagram.

FIGURE 2.7 Block diagram of electronic systems.

further amplified and shaped by the main amplifier. The unipolar output signal from the main amplifier was processed by a baseline restorer before analysis by the multi-channel analyzer. This processing reduces the degradation of the lower energy side of photopeaks, and the poorer resolution which arises from this.

The output from the linear amplifier and baseline restorer was presented to a multi-channel analyzer (MCA) to record the spectrum. A second output from the main amplifier was analyzed by a series of single channel analyzers (SCA's) and scalers. The SCA windows were set on particular peaks in the decay spectrum being observed. These were useful during the experiment to determine the approximate temperature of the sample and size of effect. An accurate determination of these parameters cannot be made by the scaler outputs because they include not only the photopeak counts, but the background counts as well.

The full spectrum was recorded in a Northern Scientific NS900, 1024 channel analyzer. In this unit, a pulse height analysis was performed on the linear output of the amplifiers and the data stored in the memory. The MCA could be programmed to count for a preset time, generally 1000 to 4000 seconds, and then the memory contents could be put onto punched paper tape and printed rolls. After the memory contents had been obtained, the memory could be erased and a new count begun. The printed paper output allowed a reliable determination of pill temperature to be made from the spectrum. This indicated if the demagnetization was a successful one.

2.5 Analysis Procedure

The punched paper tape output of the MCA was converted to magnetic tape for analysis on an IBM 370/184 computer. Separation of the photopeak counts was performed by the computer program SPECTAN II, described in Appendix A. This program made corrections for gain shifts in the amplifiers and decay of the source. The warm counts were determined by the program and the cold counts were normalized to these.

Further analysis of the SPECTAN II output was performed by the computer program NOFIT, described in Appendix C1. Using the anisotropy of the peak of interest and the temperature as determined by the Co⁶⁰/Fe thermometer, several nuclear orientation parameters could be determined.

Chapter III

Nuclear Orientation of Bi^{206} in Ni.3.1 Introduction

As mentioned earlier in Chapter I, nuclear orientation experiments can yield information on both decay scheme parameters of nuclei, of relevance to shell model predictions, and on the electro-magnetic environment in which the decay occurs. These and other specific interests provided a stimulus for the present nuclear orientation studies of Bi^{206} in hosts of nickel and cobalt.

The level scheme of Pb^{206} has been studied extensively from the decay of the parent nucleus Bi^{206} . This decay scheme has been investigated by several groups (Alburger and Pryce 1954, Allan 1971, Kanbe et.al. 1972, and Manthurithil et.al. 1972). The spins, parities, and energies of all levels are well established from these experiments (except for the spin of the 3225.53 keV level). In addition, internal conversion measurements have placed upper limits on E2 admixtures to the predominantly M1 transitions. True and Ford (1958) have shown that there will be small E2 admixtures due to weak surface coupling to the neutron hole states. The nuclear orientation measurements yield two solutions, together with the signs, for this admixture. In most cases, the internal conversion measurements can be used to select one of the two solutions. The internal conversion measurements of Manthurithil et.al. (1972) have been used in conjunction with the present data.

The spin and magnetic moment of the Bi^{206} ground state has been determined by Lindgren and Johansson (1959), using the atomic beam

magnetic resonance method. This leaves the hyperfine field as the only unknown in the description of the hyperfine interaction.

In the previous nuclear orientation of Bi^{206} in nickel by Kaplan et.al. (1973), attenuation of the 516 keV transition from the 2200 keV isomeric state in Pb^{206} was observed. A similar attenuation was observed in the subsequent decays through the 803 keV and 881 keV transitions. This effect could not be attributed to uncertainties in the decay scheme or higher order multipole admixtures to these transitions. Kaplan et.al. (1973) and Johnston et.al. (1974) have attributed this attenuation to a reorientation of the nuclear spin during the lifetime of the 120 μsec . isomeric state. The very small magnetic moment of 0.150 nuclear magnetons (Vary and Ginocchio, 1971) for the isomeric state gives rise to Zeeman splitting a factor of 10^{-2} smaller than typical values of the magnetic splitting. They investigated the effect of an off-axis quadrupole interaction not coaxial with the dipole interaction. Attenuation factors A_v were calculated in terms of electric quadrupole and magnetic dipole contributions to the Hamiltonian. The A_v coefficients of their calculations are shown in FIGURE 3.1, where the interaction parameter X is given as

$$X = \frac{3eQ}{4I(2I-1) \gamma H_{\text{eff}}} \frac{\partial^2 V}{\partial^2} \quad (3.1)$$

It is interesting to note that quadrupole effects from the crystal arise only if the symmetry of the field is lower than cubic. The crystal structure of Nickel is cubic and should not contribute to the quadrupole splitting of the 2200 keV isomeric state in Pb^{206} . Johnston et.al. (1974) used this fact to suggest a non-substitutional, unique lattice site in these hosts for Bismuth impurities. This will be discussed in detail later.

FIGURE 3.1 Variation of the A_v parameters as a function of the quadrupole interaction parameter X .

In order to resolve some of the questions about the Bi^{206} impurities in ferromagnetic hosts, a systematic study of the hyperfine interactions in Nickel was carried out. Mixing ratios for several transitions in the daughter Pb^{206} decay were measured, and attenuation coefficients due to reorientation in the 2200 keV. isomeric state were determined. Apart from the desirability of resolving the discrepancies in the values for the hyperfine fields, a question of central importance devolves on the criterion for a good sample. High solubility of the radioactive label in the host material may not necessarily produce a sample with a high fraction of the impurity nuclei at lattice sites. It would be of great interest if the analysis of such nuclear orientation measurements could yield information on this question. A more detailed consideration of this point is considered following the presentation of the results at the end of this chapter.

A study of the Pb^{206} decay in Iron and Cobalt was undertaken, but the data from the experiments was not of sufficient quality to be analyzed.

3.2 Bi^{206} Decay

The decay scheme of Pb^{206} is shown in FIGURE 3.2, following Manthurithil et.al. (1972). The primary decay of 6^+ Bi^{206} ground state proceeds by electron capture to the 3279 keV. and 3403 keV. levels of Pb^{206} , both of which are 5^- levels. All other secondary E.C. decays were ignored in the analysis.

The 1719 keV. transition from the 3403 keV. 5^- level to the 1684 keV. 4^+ level proceeds by emission of a pure E1 gamma-ray. This transition was used to determine the hyperfine field and fraction of nuclei at lattice sites, since all other parameters necessary to the calculations were known. The 516 keV. transition from the 2200 keV. 7^- isomeric level to the 1684 keV. 4^+ level has been determined to be a pure E3 transition. The attenuation resulting from reorientation in the 2200 keV. isomeric state was calculated by comparing the experimental anisotropies of the 516 keV. (E3), 803 keV. (E2), and 881 keV. (E2) radiations with those expected with no reorientation. Small E2 admixtures for several other gamma transitions were calculated by determining the F_2 coefficients of the particular transitions of interest.

FIGURE 3.2 The decay scheme of Pb²⁰⁶, showing only those transitions which were analyzed.

3.3 The Samples

The Bi²⁰⁶ activity was obtained from Amersham/Searle Corp. in the form of carrier free ²⁰⁶BiCl₃ in HCl solution. Since BiCl₃ boils at 400 °C, it was not possible to prepare the sample by the regular method of evaporating the chloride onto metal foils and furnacing at 700 - 900 °C.

The Bi ions are more electronegative than Ni, Co, or Fe. This being the case, the Bi ions were allowed to plate spontaneously onto the surface of about 0.25 grams of Ni, Co, and Fe powders. The reactions were

The ferromagnetic host were very high purity powders. Details are shown in Table 3.1.

Table 3.1

host	form	purity	source
Fe	1-5 micron powder	99.9 %	Atlantic Equip. Eng. ⁺
Ni	nickel sponge	99.99%	Johnson and Matthey ⁺
Co	coarse powder	99.9 %	UBC Metallurgy Dept. ⁺

⁺ SEE References in Appendix

The use of the powder specimen maximized the surface area available for spontaneous plating.

About 150 microcuries of activity was added to each of the host powders.

in a sterile Pyrex[®] vial and they were allowed to stand for 5 minutes. These samples were then rinsed with a 0.1 M. HCl solution and again with distilled water. The activity remaining in the powders was 71% for Fe, 46% for Co, and 35% for Ni.

The powders were individually sealed in quartz vials, 0.9 cm. in diameter and approximately 5 cm. in length. They were sealed in a hydrogen atmosphere (at 13 cm. pressure (Hg) which acted as a reducing agent. Quartz was used in these cases because it softens at a temperature of about 1500 °C, which is near the melting point of the host metals.

The Bi was melted with the host metals by heating them in an R.F. induction furnace. Temperatures in excess of the melting point of each host metal were obtained. In order to heat the vial to these temperatures, the vial was placed in a graphite block of dimensions 2.5 x 2.5 x 5.0 cm.. The R.F. eddy current heating enabled temperatures up to 1550 °C to be obtained. Temperatures were determined using an optical pyrometer.

At 1500 °C, the pressure in the vial rose to about 78 cm. pressure (Hg), causing a slight over-pressure in the vial. This over-pressure caused expansion of the vial, which was at its softening point. The walls of the vial came in contact with the graphite surface upon expansion, ensuring an even temperature distribution in the vial.

A graphite block was maintained at a temperature in excess of the host melting point for several minutes and then it was rapidly quenched in a water bath to a temperature less than 100 °C. The resulting pellets were all hemispherical in shape. The measured activities were 25 μ C for Fe, 15 μ C for Co, and 13 μ C for Ni. A diagram of the quartz vial and

graphite block is shown in FIGURE 3.3.

The surface of each pellet was lightly sanded and etched in 0.1 M. HCl solution. No change in the pellet activity could be detected. Bismuth concentration in these hosts were determined to be $\sim 10^{-8}$ atomic percent.

In order to ensure that an applied polarizing field would saturate the magnetic domains, the magnetization curve for each pellet was obtained. This was necessary because of the relatively large demagnetizing factor (~ 3) arising from the sample geometry. The magnetization curve for each sample was obtained by measuring the self-inductance of a small coil wound around the specimen as a function of applied magnetic field. Alignment of the specimen relative to the direction of the applied field was as close as possible to the geometry to be used in the actual experiment. The resultant curves are shown in FIGURE 3.5.

The results of this plot showed that the polarizing field had to be in excess of 7 kilogauss for Fe and Co, and in excess of 4 kilogauss for Ni.

Graphite Block

Graphite Chips

Graphite Powder

All dimensions in cm.

FIGURE 3.3

The quartz vial and graphite block dimensions as used in the sample preparation.

FIGURE 3.4

Block diagram of the apparatus used to determine the saturation magnetization of the samples.

FIGURE 3.5

Data obtained from the circuit shown in FIGURE 3.4. Here ΔL refers to the change in inductance of the sample coil,

3.4 The Bi²⁰⁶/Ni Experiment

3.4.1. Procedure:

The Bi²⁰⁶/Ni sample was soldered to the end of the copper heat link from the paramagnetic salt pill. The activity of the pellet at this time was 16 μ Curies. A Co⁶⁰/Fe foil of activity ~ 5 μ Curies was soldered next to the sample pellet. Due to problems with the equipment, only one Ge(Li) detector could be used for these experiments. This was placed in the axial direction. A typical spectrum taken at $\sim 1^\circ\text{K}$ is shown in FIGURE (3.6). The polarizing field used in this run was 4.7 kilogauss, which ensured saturation of the nickel host pellet. The geometry of the Co⁶⁰/Fe foil was arranged so that the plane of the foil was parallel to the polarizing field. This resulted in a minimal demagnetizing factor. Tests by Daly (1973) found that saturation of the Fe foil occurs at 1.5 kilogauss in this geometry.

The anisotropy of the observed gamma rays were analyzed as a function of sample temperature over the range 12 to 53 m^oK. Temperatures above the 12 m^oK obtained by the initial demagnetization were produced by introducing small persistent currents (up to 1.3A) in the main magnets. Seventeen different temperature points were obtained in this manner. Every time the temperature of the pill was changed by this method, a period of 10 minutes was allowed for the sample to come into thermal equilibrium with the pill before counting was resumed.

Spectra were accumulated for 1000 sec. of analyzer live time. Temperatures during the experiment were determined by calculating anisotropies from the teletype output of the MCA contents at the end of every spectrum. The system for gamma-ray detection is discussed in--

FIGURE 3.6 $^{206}\text{Bi}/^{60}\text{Co}$ warm spectra with Ge(Li) detector.

section 2.4 .

The resolution of the detection system was determined prior to the actual runs using a 5 μ Curie Co^{60} source. The system resolution was 4.00 ± 0.08 keV at 1.332 MeV. The solid angle subtended by the detector in the axial position during the experiment was calculated to be 0.224 ± 0.015 radians.

3.4.2. Analysis of Bi²⁰⁶/Ni Data

The flow chart in FIGURE 3.7 summarizes the analysis procedure for the Bi/Ni measurements. The anisotropies of the observed gamma-rays were extracted from the spectra using the computer routine Spectan II, which is described further in Appendix A. Data for the Co⁶⁰ decay were compared to the temperature-anisotropy table for Co⁶⁰/Fe and the temperature of the sample during the run was determined.

In order to determine whether non-substitutional Bi²⁰⁶ nuclei played a significant role, two models were tried in fitting the data for the hyperfine field determination.

In the first case, a simple δ -function profile was adopted, in which it was assumed that a fraction f of the Bi²⁰⁶ nuclei were located at lattice sites where they experienced a unique hyperfine field H ; the remaining nuclei were then assumed to experience an average zero field. The effect in the anisotropies of the observed Bi²⁰⁶ decays was expressed as

$$[W(\theta) - 1]_{\text{obs.}} = f [W(\theta) - 1]_{\text{Th.}} \quad (3.3).$$

This model was incorporated into the nuclear orientation fitting programme NOFIT, described in Appendix C, in which the experimental data were fitted by the method of least squares to Equation(3.3). This programme also determined the best fit to the value of the hyperfine field. In this manner, the fraction f of the Bi²⁰⁶ nuclei feeling a unique hyperfine field H could be determined, providing there were no further unknown parameters entering into Equation (3.3).

In the second case, a model similar to that used in the first case was used. This model assumed that a fraction f of the Bi²⁰⁶ nuclei felt

a distribution of hyperfine fields, while the remainder felt an average zero field. In this case, the $W(\theta)$ of Equation (3.3) would become

$$W(\theta) = \sum_{\nu} U_{\nu} F_{\nu} P_{\nu}(\cos\theta) \sum_{H_i} B_{\nu}(H_i) dP_{H_i} \quad (3.4)$$

instead of that given in Equation (1.3). The distribution function was taken to be a Gaussian of the form

$$dP_{H_i} = \frac{1}{\sigma\sqrt{2\pi}} e^{\left[-\frac{(H_i - H_0)^2}{2\sigma^2} \right]} dH_i \quad (3.5),$$

where H_0 is the centroid of the Gaussian distribution and σ is the standard deviation of the distribution. The details of the analysis procedure are given in the description of the program HYPFIT in Appendix E.

The Pb^{206} decay contains several transitions of pure multipolarity, mainly E1 in character. Several of these transitions occur between the higher energy states and remain free of any possible perturbation from the 2200 keV. isomeric state. The strongest of these is the 1719 keV transition for which all parameters in the decay scheme are known. It is 32.2 % as intense as the 803 keV. ground state transition, but the lower efficiency of the Ge(Li) detector at this higher energy alter this to an observed intensity of 20% relative to that of the 803 keV. ground state transition.

The only feed to the 3402.8 keV. level is by the $6^+ \rightarrow 5^+$ electron capture transition from the Bi^{206} ground state. It has a log ft value of 6.25, consistent with a first forbidden, non-unique transition. This allows the assignment of $\Delta J_{EC} = 1$ to this transition. The 3402.8 keV. 5^-

FIGURE 3.7 Flow chart for the analysis of the Pb^{206} spectra.

level decays through the 1719 keV $E1$ transition to the 1684.1 keV 4^+ level. The theoretical values for the $U_\nu F_\nu$ parameters of this decay route are,

$$\begin{aligned} U_2 &= 0.95803 & F_2 &= 0.86006 \\ U_4 &= 0.00000 & F_4 &= 0.00000 . \end{aligned}$$

The magnetic moment for the Bi^{206} ground has been determined by Lindgren and Johansson (1959) to be $4.56 \pm 0.05 \mu_N$ by the atomic beam resonance method.

Using these parameters, the anisotropy data for the 1719 keV γ -ray were fitted by the method of least squares to Equation (3.3), using only the parameters f and H as variables (see NOFIT, Appendix C). The data was also analyzed by the method of least squares using Equation (3.4) (see HYPFIT, Appendix F). In this case, the parameters to be determined by the fit were the fraction of nuclei feeling the distribution of hyperfine fields, the centroid of the distribution, and the standard deviation of the distribution.

3.4.3. Results:

The data obtained from the experiment was analyzed by NOFIT and HYPFIT. No problems were encountered when the data was analyzed using the model given by Equation (3.3) in NOFIT. However, the data was not good enough to allow an analysis for the model of Equation (3.4) in HYPFIT. In order to analyze the data with HYPFIT, it was necessary to use trial values of the distribution centroid H_0 and standard deviation σ while allowing f to vary. This was done, and the results of both models are presented in Table 3.2.

Table 3.2 Fitted Values of the fraction f , Hyperfine field H , and Standard Deviation for the two models used in the analysis.

Case	f	H_0 kilogauss	σ kilogauss	χ^2	Degrees of Freedom	Confidence Level
1 a)	0.654 ± 0.043	400 ± 34	NA	15	14	0.378
b)	1.000	237 ± 17	NA	79	15	1×10^{-6}
2	0.662 ± 0.005	405	30	14.8	15	0.391

It was felt that the results of the fit using the distribution of hyperfine fields did not give conclusive evidence for such a model for the following reasons:

- i) the difference between the best fit anisotropies for NOFIT and HYPFIT ranged from -0.08% to $+0.14\%$ while the experimental error in the data ranged from $\pm 1.3\%$ to $\pm 1.5\%$.
- ii) the data obtained was not accurate enough to allow the parameters H and σ to be fitted by the least squares method.

For these reasons, only the model of Equation (3.3) was used in the subsequent analysis. However, the results of the analysis of HYPFIT are shown in Table 3.2 for comparison with NOFIT. (Case 1(a)). The results indicate that the data is not very sensitive to the width of the distribution function of Equation (3.3). If a distinction were to be made between the two models, data with an experimental error of about 0.1 % would be required for nuclear orientation experiments.

The results of the fitting routine for the 1719 keV. transition are indicated in Table 3.2 and FIGURE 3.8. These show that, within the limitations of the simple model, only 65.4 ± 4.3 % of the Bi^{206} nuclei in this sample were located at lattice sites. The hyperfine field obtained of 400 ± 34 kilogauss is in excellent agreement with the results of Kaplan et.al. (1973), who reported a value of 390 ± 15 kilogauss.

For comparison, the data were also fitted under the assumption that all nuclei were at lattice sites (i.e. $f = 1.0$). In this case a much poorer fit was obtained, as evidenced by the fivefold increase in the χ^2 , yielding a value of 237 ± 17 kG. for the hyperfine field.

Moreover, when applied to Kaplan's data, the model was consistent, yielding values of $f = 96.8 \pm 2.3$ % and $H = 391 \pm 10$ kG. Reasons for the higher lattice site occupancy achieved in their sample are presented in the discussion at the end of this chapter.

A hyperfine field of 400 kilogauss and a fraction of 65.4 % were used in the subsequent analysis of the remaining Pb^{206} gamma-rays. Using these parameter values in the program NOFIT, data for the remainder of the observed gamma decays were analyzed. The parameters allowed to vary in the analysis were the U_{22} and U_{44} values. Results of the analysis on the gamma-rays are given in Table 3.4.

Table 3.3 Results of the analysis of the Bi²⁰⁶ data

E_{γ} keV	L/L^1	$U_2 F_2$	$U_4 F_4$	χ^2	Deg. Freedom	$P(\chi^2)$
* 343	M1/E2	0.270 ± 0.051	0.202 ± 0.169	9.1	14	0.825
497	M1/E2	-0.300 ± 0.010	0.000	10.8	15	0.767
* 516	E3	-0.432 ± 0.026	-0.125 ± 0.079	13.9	14	0.457
* 537	M1/E2	-0.472 ± 0.096	-0.035 ± 0.024	14.3	14	0.428
620	M1/E2	0.462 ± 0.029	0.000	8.4	15	0.907
* 803	E2	-0.372 ± 0.016	-0.092 ± 0.050	21	14	0.102
* 881	E2	-0.368 ± 0.015	-0.127 ± 0.046	9.8	14	0.777
895	M1/E2	0.233 ± 0.035	0.000	43	15	0.00016
1019	M1/E2	0.280 ± 0.036	-0.033 ± 0.136	17.5	14	0.231
1099	E1	0.131 ± 0.013	0.000	9.6	15	0.844
1595	E1	0.286 ± 0.020	0.000	7.5	15	0.942

* Transitions involving isomeric state at 2200 keV.

Table 3.4 Details of the Pb^{206} Decay (from Manthurithil et.al. (1972))

Initial level keV.	π J	Final level keV.	π J	E_γ keV.	Multipole	$I_\gamma\%$ relative to GS. trans.	Branching Ratio From Initial State
3402	5^-	1684	4^+	1719	E1	32.2	0.6902
		1998	4^+	1405	E1	1.45	0.0311
		2384	6^-	1019	M1	7.68	0.1646
		2782	5^-	620	M1	5.32	0.1140
3279	5^-	1684	4^+	1595	E1	5.07	0.1286
		2384	6^-	895	M1	15.8	0.4015
		2782	5^-	497	M1	15.5	0.3926
		3016	5^-	262	M1	3.05	0.0774
3244	4^-	1341	3^+	1903	E1	0.353	1.000
3016	5^-	2384	6^-	632	M1	4.52	0.9488
		2782	5^-	234	M1	0.244	0.0512
2782	5^-	1684	4^+	1099	E1	13.65	0.5569
		2384	6^-	398	M1	10.86	0.4431
2647	3^-	803	2^+	1844	E1	0.575	1.000
2384	6^-	2200	7^-	184	M1	16.00	1.000
2200	7^-	1684	4^+	516	E3	41.2	0.9989
		1998	4^+	202	E3	0.004	0.0011
1998	4^+	1340	3^+	657	M1	1.930	0.9883
		1684	4^+	314	M1	0.363	0.0117
1684	4^+	803	2^+	881	E2	66.90	0.7384
		1341	3^+	343	M1	23.70	0.2616
1341	3^+	803	2^+	537

— $H = 400 \pm 34$ kG.

$f = 0.654 \pm 0.043$

$\chi^2 = 15.0$

- - - $f = 1.00$

$H = 237 \pm 17$ kG.

$\chi^2 = 79$

1.719 MeV.

FIGURE 3.8

Data for the 1719 keV. gamma-ray with best fits for the two cases discussed.

The U_ν coefficients were calculated using the data on the Bi^{206} decay of Manthurithil et.al. (1972). In these calculations, only transitions with an intensity of greater than 1% of the 803 keV. ground state transition were considered. The branching ratios, level spins and parities, and multipolarities of the transitions obtained from their paper are shown in Table 3.4. For convenience, individual U_ν coefficients used in the calculations are given in Appendix D, along with a sample calculation of the type required for this analysis:

In calculating the U_ν coefficients for the transition of interest, it was found that the multipole mixing in the preceding transitions could be neglected. Since the mixing ratio appears as δ^2 , the following approximation was made.

$$\frac{U_\nu(I_i, I_f, L_1) + \delta^2 U_\nu(I_i, I_f, L_1 + 1)}{1 + \delta^2} \cong U_\nu(I_i, I_f, L_1) \quad .(3.6)$$

Tabulated values of the U_ν coefficients for the transitions of interest are given in Table 3.5. For transitions which involve contributions from the 2200 keV. isomeric state, the attenuation factors A_ν appear. (Equation (1.13)).

In the analysis of the gamma-ray anisotropies, it was necessary to consider effects from Compton scattering in the detector and escape of any gamma-rays caused by pair production. The energy of the Compton edges are given by

$$E_c = \frac{E_\gamma}{1 + \frac{M_0 c^2}{2E_\gamma}} \quad (3.7)$$

Table 3.5 Table of U_γ Coefficients*

E_γ keV.	U_2	U_4
343	$0.50595 + 0.47798 A_2$	$0.53518 \pm 0.27955 A_4$
497	0.95803	0.86006
516	$0.83344 A_2$	$0.61004 A_4$
537	$0.46032 + 0.28491 A_2$	$0.30469 + 0.12048 A_4$
620	0.95803	0.86006
803	$0.37978 + 0.35808 A_2$	$0.10813 + 0.10195 A_4$
881	$0.50695 + 0.47798 A_2$	$0.53518 + 0.27955 A_4$
895	0.95803	0.86006
1.019	0.95803	0.86006
1.099	0.85554	0.55856
1.595	0.95803	0.86006
1.720	0.95803	0.86006

* Based on the data of Manthurithil et.al. (1972) shown in Table 3.4

and the energy of the single escape peak is

$$E_{SE} = E_{\gamma} - MoC^2 \quad (3.8).$$

Here E_{γ} is the energy of the gamma-ray and MoC^2 is the rest energy of the electron. A list of these energies is given in Table 3.6. These peaks are indicated on Pb^{206}/Co^{60} decay spectrum in FIGURE 3.9. The energies for the single escape peaks are only indicated for $E_{\gamma} > 1$ MeV, since the probability for such an occurrence below this energy zero.

The mixing ratios were determined by solving the quadratic expression for δ , which is of the form

$$\delta = \frac{2F_V(I_i I_f LL+1) \pm \sqrt{4(F_V(I_i I_f LL+1))^2 - 4(F_V(I_i I_f L+1L+1) - F_V')(F_V(I_i I_f LL) - F_V')}}{2(F_V(I_i I_f L + 1 L + 1) - F_V')} \quad (3.9)$$

Using the internal conversion measurements, it was possible to select one of the two roots for the transition of interest. The U_V coefficients of Table 3.6 were used to determine the experimental F_V' coefficients, which appear in Equation (3.9). Results are given in Table (3.9 a) for several mixing ratio determinations.

Since the 516 keV., 803 keV., and 881 keV. gamma-rays are pure multipole transitions, in these cases the F_V coefficients are known exactly. These are labelled $F_{V_{th}}$ in FIGURE 3.7. Thus the fitted values of $U_V F_V$ obtained from the data of Table 3.4 were used to derive the U_V coefficients. In the case of transitions involving the 2200 keV. isomeric state, however, the quantity of interest is $U_V' = A_V U_V$, as described in Equation (1.13).

Table 3.6

Energies of Compton edges and single escape peak associated with gamma-rays of the Pb^{206} and Co^{60} delays.

E_γ in keV.	I_γ in % rel. to 803 keV.	E_C in keV.	E_{SE} in keV.
1879	2.03	1585	1368
1719	32.2	1495	1208
1595	5.07	1375	1084
1405	1.45	1185	896
1332	N/A	1118	821
1173	N/A	965	662
1099	13.7	892	588
1019	7.68	818	508
895	15.8	696	-
881	66.9	685	-
803	100.0	610	-
620	5.32	439	-
537	30.8	364	-
516	41.2	345	-
497	15.5	330	-
343			

$$E_C = \frac{E_0}{1 + \frac{mc^2}{2E_0}}$$

$$E_{SE} = E_0 - 0.511 \text{ MeV.}$$

I_γ = intensity of
transition
relative to
g.s. trans.

FIGURE 3.9 Decay scheme of Pb^{206} showing the location of the Compton edges of Table 3.6 .

Comparison of these values with the coefficients of Table 3.6 then yielded the A_V coefficients for the 2200 keV isomeric state. These results are given in Table 3.9 b.

The A_V coefficients from this analysis were used to determine the U_V coefficients for analysis of the 343 keV. and 537 keV. transitions. Experimentally determined F_V coefficients were then used to determine the mixing ratios using Equation (3.8). These results are given in Table 3.7 c.

i) 1595 keV. Transition

This is a pure E1 transition from the 3279 keV. levels. Analysis of the Pb^{206} decay spectrum in FIGURE 3.6 shows the Compton edge of the 1879 keV. transition situated at 1585 keV. The latter is a weak transition and its Compton edge produces a very small effect on the background calculation. The experiment U_2F_2 coefficient was 0.278 ± 0.020 , which is in excellent agreement with the theoretical value of 0.282.

ii) 497 keV. Transition

This is a mixed E2/M1 transition from the 3279 keV. level. The E2/M1 mixing ratio was determined to -0.194 ± 0.021 , which is in disagreement with the value derived by Kaplan et.al. (1973) of -0.09. Spectra obtained in their experiment show a very large Compton edge at 610 keV., arising from the 803 keV. gamma-ray. This could have an adverse effect on the background calculations, and subsequent effects on the experimental values of F_V . The use of detectors with a high peak to Compton ratio were used in these experiments to eliminate such effects. A sample comparison of this is shown in FIGURE 3.7, and the effect on the background radiation is shown schematically for the method used in this analysis.

Table 3.7

(a) Pure Multipole Transitions

E_γ MeV.	δ	a	%E2/M1 b	c
0.497	-0.194 ± 0.021	3.61 ± 0.76	0.81 ± 0.20	< 2.3
0.621	-0.082 ± 0.010	0.64 ± 0.16	-	< 6.3
0.895	0.047 ± 0.025	0.25 ± 0.01	0.09 ± 0.01	< 13
1.019	0.055 ± 0.020	0.30 ± 0.01	0.03 ± 0.01	< 22

(b) Pure Transitions Involving Isomeric State

E_γ MeV.	A_2	A_4
0.516	0.758 ± 0.045	-1.17 ± 0.74
0.803	0.677 ± 0.077	-0.213 ± 0.459
0.881	0.658 ± 0.070	-0.43 ± 0.54

Average Attenuation: $A_2 = 0.718 \pm 0.034$ $A_4 = -0.46 \pm 0.32$

(b) Mixed Transitions Involving Isomeric State

E_γ	δ	a	%E2/M1 b	c
0.343	0.002 ± 0.020	0.001 ± 0.043	0.01 ± 0.01	< 0.8
0.537	-0.211 ± 0.080	4.41 ± 3.36	0.00 ± 0.02	< 2.9

a) This work

b) Kaplan et.al.c) Manthurithil et.al.

The detector used in this analysis produced Pb^{206} spectra as shown in FIGURE 3.9 and 3.10 (a) with a peak to Compton ratio of 20:1. A smaller detector with a peak to Compton ratio of 9:1 was used to obtain the spectrum in FIGURE 3.10 (b). Comparison of the two spectra in FIGURE 3.7 show that the lower peak-to-Compton ratio can result in an over-estimate of the background radiation for the 497 keV., 516 keV., and 537 keV gamma-rays. The effect on the 516 keV. and 537 keV. gamma-rays will be relatively unaffected by this since they are quite intense transitions. The 497 keV. transition, however, is much weaker and, as a result of this effect, exhibits an apparently lower anisotropy at lower temperatures. The 803 keV. transition has a lower anisotropy at lower temperatures as well, and will show the same effect in its Compton edge at 610 keV. This could result in an underestimate of the U_2F_2 coefficients, which would result in a higher E2 admixture being determined for this transition.

In the analysis of the 497 keV. data, the U_4F_4 coefficient was set equal to 0.000, since internal conversion data set a maximum value of 0.00784.

iii) 621 keV. Transition

This is a mixed E2/M1 transition from the 3402 keV. level. Internal conversion measurements set an upper limit of the U_4F_4 coefficient of 0.00195. For this reason, the U_4F_4 coefficient was set equal to zero in the analysis. The E2/M1 mixing ratio was determined to be -0.082 ± 0.010 , or a 0.64 ± 0.16 % E2 admixture to the M1 transition. This result is in agreement with the internal conversion measurement.

iv) 895 keV. Transition

This is a mixed transition from the 3279 keV. level, populated by the

FIGURE 3.10

Spectra for the 497 keV , 516 keV , and 537 keV transitions showing the effect of the Compton edge from the 803 keV transition on the background calculations. The spectra are for detectors with peak to Compton ratios of 9:1 and 20:1.

123 keV transition from the 3402 keV level and the β decay from the Bi^{206} ground state to the 3279 keV level of Pb^{206} . Internal conversion measurements set an upper limit to the U_4F_4 coefficient of 0.00354, and for this reason it was set equal to 0.000 for the analysis. The mixing ratio was determined to be $+0.047 \pm 0.025$.

Analysis of FIGURE 3.8 and Table 3.7 shows that the Compton edge of the 1.099 keV transition at 892 keV interferes with the 895 keV peak. The 881 keV peak is also very close to the 895 keV peak. These two factors introduce considerable uncertainty into the determination of the mixing ratio for this transition. The reason for this arises from the large value of the $F_2(I_i I_f L + 2L)$ term in the expression for the F_2 coefficients. This is shown in Equation (3.10).

$$F_2 = \frac{0.16984 + 2\delta \times 0.77831 + \delta^2 \times 0.26689}{1 + \delta^2} \quad (3.10)$$

The effect of the 1.099 keV Compton edge would be an underestimate of the U_2F_2 coefficient and a smaller mixing ratio determination. It was felt that this effect introduced considerable error in the analysis of the transition.

v) 1019 keV. Transition

This is a mixed E2/M1 transition from the 3402 keV level. Internal conversion measurements set an upper limit on the U_4F_4 coefficients of 0.00334, and for this reason it was set equal to 0.000. The E2/M1 mixing ratio was determined to be $+0.055 \pm 0.020$. There were no effects from Compton edges of other transitions in this analysis. This value is in disagreement with the value -0.18 determined by Kaplan et.al. (1973).

vi) 516 keV Transition

This is a pure E3 transition from the 2200 keV isomeric state. The attenuation coefficient A_ν for this transition was determined for $\nu = 2, 4$. The values were $A_2 = 0.758 \pm 0.045$ and $A_4 = -1.17 \pm 0.74$. The upper limit of the spectrum used to determine the background was 563 keV, hence the Compton edge of the 803 keV transition at 610 keV would have little, if any, effect on this result.

vii) 803 keV Transition

This is a pure E2 transition from the 803 keV level to the ground state of Pb^{206} . The attenuation coefficients were determined to be $A_2 = 0.677 \pm 0.077$ and $A_4 = -0.213 \pm 0.459$. There were no effects to interfere with the analysis of this transition. The analysis in this case is not complicated by interfering peaks.

viii) 881 keV Transition

This is a pure E2 transition from the 1684 keV level. The attenuation coefficients were determined to be $A_2 = 0.658 \pm 0.070$ and $A_4 = -0.43 \pm 0.54$. The effect of the 1099 keV Compton edge at 892 keV and the proximity of the 895 keV peak were felt to be negligible in the analysis.

ix) 343 keV Transition

This is a mixed E2/M1 transition from the 1684 keV level, which is populated by several transitions from higher levels. Included in these is the 516 keV transition from the 2200 keV isomeric state. This requires the average attenuation, as determined by the 516 keV, 803 keV, and 881 keV transitions, to determine the mixing. Using the average A_2 coefficient of 0.718 ± 0.034 , a mixing ratio of $+0.002 \pm 0.020$ was

calculated. The coefficients used in the determination of the F_2 coefficient are

$$F_{2\text{EXP}}^1 = \frac{0.31339 - 2\delta_{\text{EXP}} \times 0.94017 + \delta_{\text{EXP}}^2 \times 0.04477}{1 + \delta_{\text{EXP}}^2} \quad (3.11)$$

The mixing ratio is extremely sensitive to the value determined for F_2^1 due to the large $F_2(I_1, I_1 L L + 1)$ coefficient. This allows only an upper limit of 0.044 % for the E2/M1 admixture to this transition.

The Compton edge of the 516 keV transition occurs at 345 keV, but it was not felt that this had an effect on the analysis.

x) 537 keV Transition

This is a mixed E2/M1 transition from the 1340 keV level which is populated by the 657 keV and 343 keV transitions. Both of these transitions involve attenuation in the 2200 keV isomeric state. Using the average A_2 value of 0.718 ± 0.034 , the mixing ratio was determined to be $\delta = -0.211 \pm 0.080$, or a 4.41 ± 3.36 % E2/M2 admixture. This result is in agreement with internal conversion measurements on the 537 keV transition. The $U_4 F_4$ coefficient was not used in the determination of δ due to the large uncertainty, it could only be stated the $\delta < \pm 0.06$.

3.5 Discussion

The interpretation of the data for the 1719 keV gamma-ray in terms of the full field-no field model gives better agreement with the data than a model which assumes all nuclei feel a unique hyperfine interaction. The $\text{Bi}^{206}/\text{Ni}$ experiment gives a hyperfine field value which is in excellent agreement with the data of Kaplan et.al. (1973). For this case, the data indicates that $65.4 \pm 4.3\%$ of the Bi^{206} nuclei feel a hyperfine field of 400 ± 34 kiloGauss, with the remaining nuclei feeling zero field. Failure to account for this zero field fraction results in a hyperfine field value of 237 ± 17 kiloGauss but with a much poorer fit to the data, as indicated in Table 3.2 and FIGURE 3.7.

The distribution of impurity nuclei in host materials has been studied by channelling techniques. For example, Feldman et.al. (1968) used this technique to study the lattice location of Bi^{206} implanted into iron single crystals. Their results indicated that not all Bismuth nuclei assumed substitutional lattice sites, but that some of these nuclei assumed interstitial sites. An annealing temperature dependence of the Bismuth nuclei at substitutional sites was also found. Their data is shown in FIGURE 3.11.

FIGURE 3.11 Fraction of Bismuth nuclei at substitutional lattice sites for various annealing temperature (Feldman et.al. (1968)).

The reason for different lattice locations of implanted nuclei has been attributed to radiation damage of the lattice, and the temperature dependence of the substitutional fraction to differences in annealing processes (de Waard (1975)). Although the samples used in these experiments were not prepared by implantation, the method of sample preparation will certainly cause lattice damage. This will allow some comparisons between results.

Kaplan et.al. (1972) found they could only obtain good occupancy of substitutional lattice sites by rapid quenching of the alloy from 1500 °C to room temperature. In their iron samples, which were allowed to cool slowly, they could not obtain very good fits to the data if they assumed all nuclei felt a unique hyperfine interaction. They found it necessary to introduce a variable population factor. This effect could arise from Bismuth nuclei at non-substitutional lattice sites which experience a hyperfine field different from that at substitutional sites. An effect of this type has been displayed in $\text{Xe}^{133}/\text{Fe}$ system (de Waard et.al. (1968)) where high, intermediate and low field sites are observed of 273 kiloGauss, 132 kiloGauss, and <30 kiloGauss respectively.

A strong correlation between the variation of the average hyperfine interaction and the backscattering ratio in channelling experiments as a function of the annealing temperature has been found for Yb and Fe. This effect, which has been attributed to the annealing of defects, is shown in FIGURE 3.12 (Bernas 1975).

τ %
(Backscatter Ratio)

H
HF
1.74 MegaGauss

FIGURE 3.12 Annealing behaviour for Yb in Fe
(Bernas, 1975)

A similar explanation may explain the $\text{Bi}^{206}/\text{Fe}$ experiments of Kaplan et.al. (1973) when they allowed their samples to cool slowly.

By rapid quenching of binary alloys, one cannot always expect to have impurity atoms occupy substitutional lattice sites. The reason for the insolubility of many elements in the 3-d ferromagnetic hosts can be traced in part to the Hume-Rothery rules for alloys. These rules place limits on the difference in atomic radii which allow the formation of good alloys. The general rule (Hume-Rothery, (1936)) is that when the atomic diameters differ by $\pm 15\%$, the size factor is unfavourable and the substitutional solid solution is restricted. The atomic diameters of Bi, Fe, Co, and Ni are given in TABLE 3.8 .

Table 3.8 Atomic diameters for Bi, Fe, Co, and Ni.

Element	Goldschmidt Atomic ⁺ Diameter in $\text{\AA}$	Size Factor Difference Relative to Bi
Bi	3.64	-
Fe	2.47	47%
Co	2.50	45%
Ni	2.487	46%

⁺ from Hume-Rothery (1936).

Dr. R. Butters of the Metallurgy Department of the University of British Columbia suggests that the rapid quenching of the binary system might form a substitutional solid solution of Bi²⁰⁶ in Ni. Any subsequent annealing, or even slower cooling, would cause the Bismuth atoms to anneal out of the substitutional sites to take up interstitial ones, in agreement with predictions of the Hume-Rothery rules. This could probably explain the results of Feldman et.al. (1968) as shown in FIGURE 3.11 and the Bi/Fe experiments of Kaplan et.al. (1973) when the samples were allowed to cool slowly. Such effects will cause the results to be strongly dependent on the thermal history of the sample in question.

The samples used in these experiments were purposely not annealed. This was done to minimize the formation of interstitial impurity lattice sites. One of the problems with this method is the formation of a very fine grain structure in the sample, with a large number of grain

boundaries owing to the relatively low activation energy of interstitial solutes (Brick et.al. (1965)). Bismuth nuclei at these sites will effectively feel a zero field. In other experiments where the samples were annealed, the grain boundary occupation was reduced by recrystallization and grain growth effects. For example, the results of Bowman et.al. (1969) on Bi^{206} in Fe whose results are shown in FIGURE 3.13.

FIGURE 3.13 Hyperfine field for $\text{Bi}^{206}/\text{Fe}$ as a function of the annealing time at 800 °C. (Bowman et.al. (1969)).

Their results clearly demonstrate the recrystallization effect. The annealing, however, also promotes the formation of defect clusters which can interact with the impurity nuclei (Bernas 1975) in a complicated manner. The sample preparation in their work differs drastically from that used in the present experiments, and it must be mentioned that their

results also show the largest discrepancy with previous results (Table 1.2). In their analysis, they fail to account for a fraction of the nuclei feeling a unique field and the remainder zero field, or some field different from that at the substitutional sites.

The data of Kaplan et.al. (1973) for $\text{Bi}^{206}/\text{Ni}$ was consistent with 97-100% of the Bismuth nuclei feeling a unique hyperfine field of 390 kiloGauss (the 97% figure was obtained by fitting Kaplan's data with NOFIT, Appendix C). This result is in excellent agreement with that obtained in this work of 400 ± 34 kiloGauss. The lower fractional occupation evidenced in the present case may possibly be explained by the soldering of the sample to the cold finger of the chrome alum pill. Some difficulty was encountered in the soldering which required the sample to be kept at the melting point of solder ($\sim 270^\circ\text{C}$) for a longer time than would usually be required. This may have produced some annealing effects in the sample. The fact that such good agreement can be obtained only by the introduction of a variable population factor offers a good explanation of the discrepancies between previous results in the Bismuth in Nickel system. Failure to account for this population factor results in a hyperfine field value of 237 kiloGauss with a much poorer fit to the data.

The mixing ratios for the E2/M1 transitions obtained from the $\text{Bi}^{206}/\text{Ni}$ experiment are in good agreement with the internal conversion measurements of Manthurithil et.al. (1972), with the exception of the 497 keV transition and the 1099 keV transition. Both of these can be explained by the effect of the Compton edge arising from higher energy transitions. As was explained in the analysis of the data, the Compton edge from the 803 keV transition interferes with the calculation of the background for

the 497 keV transition. The Compton edge from the 1.333 MeV Co^{60} decay occurs at 1118 keV, in very close proximity to the 1099 keV transition. To complicate this even further, the Co^{60} gamma-rays exhibit a decrease in yield anisotropy in the axial direction, with decreasing temperature, while the 1099 keV gamma-ray exhibits an opposite effect. This will result in a much lower U_2F_2 coefficient being determined experimentally, as is shown in Table 3.9. The possible reason for the discrepancy between this data and that of Kaplan et.al. (1973) is discussed in the analysis, but is basically due to the extreme sensitivity of the F_V coefficient to the mixing ratio δ in most cases.

The results do, however, agree with the theoretical predictions of small E2 admixtures to the M1 transitions.

The attenuation coefficients arising from the intermediate state reorientation in the 2200 keV isomeric state of the Pb^{206} daughter nucleus for a Nickel host are in good agreement with the results of Kaplan et.al. (1973). The negative value for the A_4 coefficient is consistent with the parameter X in Equation 3.1, representing the quadrupole interaction, having a negative value. The value of this parameter was determined to be -0.09 ± 0.02 for the Nickel host.

3.6 Conclusions

The results of the present experiments indicate that great care should be exercised in the evaluation of the data from the point of view of sample quality. Metallurgical problems in the alloying of these systems, and possibly other unknown problems, are sufficient to cause doubt about the precise lattice locations of the impurity nuclei. This system should be studied to see if there is any vacancy clustering or point defects around the impurity lattice locations. Resistivity measurements should also be made to study the annealing behaviour as a function of temperature. If any effects arising from lattice defects were to occur, they would make themselves apparent in such measurements (Mantl 1975, de Waard 1975).

The analysis has shown, however, that to some extent, information on sample quality can be extracted from the Nuclear Orientation data, even with a very simple model. These experiments do give a reasonable explanation for the lower hyperfine fields of Bi nuclei in ferromagnets reported previously. The failure to account for non-substitutional impurity nuclei, and consequently a non-unique value for the hyperfine interaction, can lead to an erroneously small value for the latter. While a simple δ function model affords a reasonable fit to the data in the present measurements, it is clear that varying degrees of complexity will be incurred depending on the history of the sample.

These remarks also apply to samples prepared by ion implantation. The results indicate that nuclear orientation measurements are sensitive to such defects and suggest that, wherever indicated, specimens should be checked by other techniques.

Appendix A Spectrum Analysis Program SPECTAN II

Spectan II is a program to remove the photopeak from the background contributions and correct the spectrum for any gain shifts which might have occurred in the amplifiers. It determined the counts in the warm and cold spectra, hence calculating the anisotropy from these data. Spectan II is a modified version of a program developed by P.W. Daly for analysis of nuclear orientation spectra.

The background radiation of radioactive sources exhibits an approximately exponential intensity as a function of energy in a Ge(Li) detector. In the vicinity of a photopeak, this background is essentially linear. A typical Ge(Li) spectrum is shown in FIGURE A1. The linear background was described by the Equation (A.1).

$$Y = mN + B \quad (A.1).$$

The counts in AREA 1 and AREA 2 were used to calculate the equation of the background running under the photopeak. This equation was used to determine the background radiation in the window. To obtain the number of counts in the photopeak, the total background in the window was subtracted from the total counts in the window.

Further details of this program are given by Daly (1973). The effects of the Compton edges are discussed in the analysis of the data.

$$\text{Window Counts} = \sum_{10}^{25} Y(N)$$

$$\text{Background Counts} = \sum_{10}^{25} (mN+B)$$

$$\text{Counts in Peak} = \sum_{10}^{25} Y(N) + \sum_{10}^{25} (mN+B)$$

FIGURE A1

Sample calculation of type performed in SPECTAN II.

MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)			MAIN	10-28-75	18135140	PAGE P001
SPECTRA -						
C	C	SPECTRA, VERSION 2, JUNE 11, 1974, P.R.H. MC CONNELL				1,000
C	C	SPECTRA GIVEN FROM I/O UNIT 2				2,000
C	C	FIRST DATA CARD GIVES FILE NUMBER (I3)				3,000
C	C	NEXT IS A RUN CARD GIVING NNAME= NUMBER OF WARM SPECTRA (I3)				4,000
C	C	NSIRT= FIRST CHANNEL REPEATED				5,000
C	C	NSTOP= LAST CHANNEL FOR EACH				6,000
C	C	JAI= WIDTH OF BG) PEAK (MAX=5)				7,000
C	C	JGI= GAP OF BG) 5(213,412)				8,000
C	C	JG2= GAP OF BG)				9,000
C	C	JG2= WIDTH OF BG)				10,000
C	C	CAYE= NUMBER OF DECAYING PEAKS (I1)				11,000
C	C	MOVE= MAXIMUM CENTROID SHIFTS (I2)				12,000
C	C	IF CAY.GT.0, MUST GIVE CAY CARDS EACH WITH PEAK NUMBER AND				13,000
C	C	HALFLIFE(HRS)				14,000
C	C	FORMAT(I3,F4.0)				15,000
C	C	UP TO (MOVE) CENTROID CORRECTIONS TO THE WINDOWS ARE PERFORMED.				16,000
C	C	THE FIRST SPECTRUM'S CENTROIDS ARE TAKEN AS THE STANDARDS.				17,000
C	C	BACKGROUND IS CALCULATED AS A STRAIGHT LINE (YXX+H) UNDER THE				18,000
C	C	PEAKS USING A GAP OF JGI & JG2 CHANNELS AND AVERAGING OVER JAI				19,000
C	C	A JG2 CHANNELS TO DETERMINE EBN. OF THE BG				20,000
C	C	THERE FOLLOW SPECTRUM CARDS IDEF= FIRST SPECTRUM				21,000
C	C	IDEF= LAST SPECTRUM; DEF=IDF				22,000
C	C	IDE= RELATIVE COUNT TIME, DEF=1				23,000
C	C	NSP= EVERY NSP-TH SPECTRUM IS DONE				24,000
C	C	DEF=2				25,000
C	C	JBL= SHIFT IN BASELINE)				26,000
C	C	JWE= SHIFT IN WIDTH) .X5 (IUI3)				27,000
C	C	FORMAT(I4I3)				28,000
C	C	THE WARM SPECS MUST ALL COME FIRST				29,000
C	C	I/O UNIT 8 RECEIVES ID,I,NSUM(I),CAY,NCOR(I),EREG(I)				30,000
C	C	FORMAT(I3I7,F9.5,I7,E12.6)				31,000
C	C	I/O UNIT 9 RECEIVES ID,I,ERR,C,ERRC FROM SPECTAN OUTPUT FOR				32,000
C	C	ANALYSIS BY AVSPEC WHICH AVERAGES THE ANISOTROPIES OVER A				33,000
C	C	SPECIFIED				34,000
C	C	NUMBER OF SPECTRA AND PRINTS THE AVERAGES AND THEIR ERRORS.,				35,000
C	C	FORMAT(2I4,F6.4)				36,000
C	C	THE RUN IS ENDED BY A BLANK CARD, AND ANOTHER RUN BEGINS				37,000
C	C	A SENDFILE FINISHES EVERYTHING				38,000
C	C	IN THE DESCRIPTION OF SPECTRA ON TAPE, DE(2) SHOULD CONTAIN THE				39,000
C	C	TIME				40,000
C	C	*****				41,000
C	C	*****				42,000
C	C	*****				43,000
C	C	*****				44,000
C	C	*****				45,000
C	C	*****				46,000
C	C	*****				47,000
C	C	*****				48,000
C	C	*****				49,000
C	C	*****				50,000

```

0001  DIMENSION I(19),DE(17),NSIRT(5),NSTOP(5),MSTRT(5),MSTOP(5),NSU
      2M(5),NCOR(5),ATCOR(5),NXB(5),JBL(5),JWB(5),ERTB(5),ERB(5),SIG(5
      3),SIGC(5),CCW(5),J1(5),J2(5),NTOT(5),MON(12),NOVE(5),TAU(5),
      4JAI(5),JG2(5),JGI(5),JG2(5)
0002  LOGICAL NURM,I1,DECA(5)
0003  INTEGER Y(1024),DIE(5)
0004  DATA JSL,JMW,IU=0,MON/JAN,1,FEB,1,MAR,1,APR,1,MAY,1,JUN,1,
      5JUL,1,AUG,1,SEP,1,OCT,1,NOV,1,DEC,1/
0005  CALL DATEI(01E)
0006  MTHEOTE(1)

```

0007		NPAGE=1	51,000
0008		READ(5,101) MFILE	52,000
0009		CALL SKIP(MFILE,0,2)	53,000
0010		READ(2) MFILE,TITLE	54,000
0011		WRITE(6,102) MFILE,(TITLE(N),N=1,19)	55,000
	C		56,000
	C	START OF RUN	57,000
	C		58,000
0012	1	DO 2 I=1,5	59,000
0013		TAU(I)=100000.	60,000
0014		DECAY(I)=.FALSE.	61,000
0015		ERSG(I)=0.	62,000
0016		ERIRG(I)=0.	63,000
0017		SIG(I)=0.	64,000
0018		SISC(I)=0.	65,000
0019		NTOT(I)=0	66,000
0020	2	NTCON(I)=0	67,000
0021		INIT=0	68,000
0022		ITI=.TRUE.	69,000
0023		ISAR=1	70,000
0024		HEAD(5,10),END=52) NARM,(NSTRT(N),NSTOP(N),JA1(N),JG1(N),JG2(N), 2JAZ(N),N=1,5),KAY,MOVE	71,000
			72,000
0025		NORM=.FALSE.	73,000
0026		IF(NARM,GE.1) NORM=.TRUE.	74,000
0027		NPA=5	75,000
0028		DO 3 I=1,5	76,000
0029		II=6-I	77,000
0030		IF(NSTOP(II),NE.0) GO TO 4	78,000
0031	3	NPA=NPA-1	79,000
0032	4	WRITE(6,116) (TITLE(N),N=1,3),MON(MTH),DTE(2),DTE(3),NPAGE,(TITLE(2N),N=4,9)	80,000
			81,000
0033		NPAGE=NPAGE+1	82,000
0034		IF(KAY,EG.0) GO TO 201	83,000
0035		DO 202 I=1,KAY	84,000
0036		READ(5,117) N,X	85,000
0037		TAU(N)=X/0.693147	86,000
0038	202	DECAY(N)=.TRUE.	87,000
0039	201	WRITE(6,103)	88,000
0040		DO 17 I=1,NPK	89,000
0041	17	WRITE(6,104) I,JA1(I),JG1(I),JG2(I),JA2(I),TAU(I)	90,000
0042		WRITE(6,107) MOVE	91,000
0043		LINE=7+NPK	92,000
0044		IF(.NOT.NORM) GO TO 200	93,000
0045		WRITE(6,108)	94,000
0046		LINE=LINE+2	95,000
	C		96,000
	C	START OF SPECTRUM	97,000
	C		98,000
0047	200	READ(5,106,END=52) IDF,IDL,INT,NSP,(JBL(N),JWW(N),N=1,5)	99,000
0048		IF(IDF,EG.0) GO TO 1	100,000
0049		IF(IDL,EG.0) IDL=IDF	101,000
0050		IF(NSP,EG.0) NSP=2	102,000
0051		IF(INT,EG.0) INT=1	103,000
0052		DO 33 IDJ=IDF,IDL,NSP	104,000
0053		DO 39 I=1,NPK	105,000

MICHIGAN TERMINAL SYSTEM FORTKRN G(41336)			MAIN	10-28-75	18135140	PAGE P003
0054			MSIRT(I)=MSIRT(I)+JBL(I)+JA1(I)+JG1(I)		106,000	
0055	39		MSTOP(I)=MSTOP(I)+JBL(I)+JWH(I)-JA2(I)-JG2(I)		107,000	
0056			IEND=0		108,000	
0057	18		READ(2,END=19) ID,DE,Y		109,000	
0058			IF((ID-IDD) 18,20,18		110,000	
0059	19		CALL SKIP(-2,0,2)		111,000	
0060			CALL SKIP(1,1,2)		112,000	
0061			IF(IEND.NE.0) GO TO 51		113,000	
0062			IEND=1		114,000	
0063			GO TO 18		115,000	
0064	20		DO 60 I=1,NPK		116,000	
0065	44		MOVE(I)=0		117,000	
0066			IF(MOVE.EQ.0) GO TO 35		118,000	
	C				119,000	
	C		CENTROID CORRECTIONS		120,000	
	C				121,000	
0067			DO 36 I=1,NPK		122,000	
0068			DO 38 II=1,MUVE		123,000	
0069			N1=MSIRT(I)		124,000	
0070			N2=MSTOP(I)		125,000	
0071			N3=0		126,000	
0072			N4=0		127,000	
0073			DO 37 N=N1,N2		128,000	
0074			N3=N3+N*Y(N)		129,000	
0075	37		N4=N4+Y(N)		130,000	
0076			NCENT=N3/N4		131,000	
0077			LEFT=(10*N3)/N4-10*NCENT		132,000	
0078			IF(LEFT.GE.5) NCENT=NCENT+1		133,000	
0079			NCENT=NCENT-MSIRT(I)		134,000	
0080			IF(IBAR.EN.1) NXBR(I)=NCENT		135,000	
0081			IF(NXBR(I).EQ.NCENT)GO TO 36		136,000	
0082			MOVE(I)=MOVE(I)+1		137,000	
0083			MSIRT(I)=MSIRT(I)+NCENT-NXBR(I)		138,000	
0084	38		MSTOP(I)=MSTOP(I)+NCENT-NXBR(I)		139,000	
0085	36		CONTINUE		140,000	
0086			IBAR=0		141,000	
0087	35		NWARN=NWARN+1		142,000	
	C				143,000	
	C		INTENSITY AND BACKGROUND CALCULATIONS		144,000	
	C				145,000	
0088			DO 61 I=1,NPK		146,000	
0089			N1=MSIRT(I)		147,000	
0090			N2=MSTOP(I)		148,000	
0091			N3=0		149,000	
0092			DO 62 N=N1,N2		150,000	
0093	62		N3=N3+Y(N)		151,000	
0094	61		NSUM(I)=N3		152,000	
0095			DO 60 I=1,NPK		153,000	
0096			N1=MSIRT(I)		154,000	
0097			N2=MSTOP(I)		155,000	
0098			NN1=N1-JA1(I)-JG1(I)		156,000	
0099			NN2=N1-JG1(I)-1		157,000	
0100			NN3=N2+JG2(I)+1		158,000	
0101			NN4=N2+JA2(I)+JG2(I)		159,000	
0102			LIM=NN2-NN1+1		160,000	

MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)			MAIN	10-28-75	18:35:40	PAGE P004
0103		KS=0				161,000
0104		NSUM1=0				162,000
0105		NSUM2=0				163,000
0106		DO 500 I2=NN1,NN2				164,000
0107	500	NSUM1=NSUM1+Y(I2)				165,000
0108		DO 501 I3=NN3,NN4				166,000
0109	501	NSUM2=NSUM2+Y(I3)				167,000
0110		ASUM1=NSUM1				168,000
0111		ASUM2=NSUM2				169,000
0112		ALIM=ALIM				170,000
0113		AJAI=AJAI(I)				171,000
0114		AJAZ=AJAZ(I)				172,000
0115		AJG1=AJG1(I)				173,000
0116		AJG2=AJG2(I)				174,000
0117		AKN1=AJAI+AJAZ				175,000
0118		I1=ASUM1/AJAI				176,000
0119		I2=ASUM2/AJAZ				177,000
0120		WISE=Y2-Y1				178,000
0121		MUNE=2-N1+1				179,000
0122		RUNE=RUN+(AJAI+AJAZ)/2,0*AJG1+AJG2				180,000
0123		SLOPE=RISE/RUN				181,000
0124		B1=AJAI/2.0-0.5				182,000
0125		X1=NI-1				183,000
0126		A2=A1-B1				184,000
0127		AINTER=Y1-SLOPE*X2				185,000
0128		YSUM=0				186,000
0129		DO 502 I4=NI,N2				187,000
0130		POINT=I4				188,000
0131	502	YSUM=YSUM+SLOPE*POINT+INTER				189,000
0132		MYSUM=YSUM				190,000
0133		KS=NSUM1-NSUM2				191,000
0134		IBGERS				192,000
0135		SBG=ALIM/AKNT				193,000
0136		ERBG(I)=NSUM(I)+IBG*SBG*SBG				194,000
0137		NCOR(I)=NSUM(I)-MYSUM				195,000
0138		CAY=1				196,000
0139		IF(.NOT.DECAY(I)) GO TO 66				197,000
0140		I=TIME(DE(2))/TAU(I)				198,000
0141		CAY=EXP(I)				199,000
0142		X=NCOR(I)				200,000
0143		NCOR(I)=X*CAY				201,000
0144		ERBG(I)=ERBG(I)+CAY*CAY				202,000
0145	C	WRITE(118) ID,I,NSUM(I),CAY,NCOR(I),ERBG(I)				203,000
0146		OUTPUT TO I/O UNIT 8				204,000
0147		IF(NARM,GE,0) GO TO 29				205,000
0148		C IF GOLD				206,000
0149		C				207,000
0150		C				208,000
0151		C				209,000
0152		IF(Y1) GO TO 42				210,000
0153	43	IF(LINE+APK,GE,60) GO TO 90				211,000
0154	90	WRITE(6,109) ID,DE(2),INT				212,000
0155		DO 31 I=1,NPK				213,000
0156		Z1=NSUM(I)				214,000
0157		Z2=NCOR(I)				215,000

MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)			MAIN	10-28-75	18135140	PAGE 0005
0154	IF(NORM) GO TO 40					216,000
0155	M=1.					217,000
0156	WC=1.					218,000
0157	GO TO 41					219,000
0158	40 M=Z1/(C*(1)+INT)					220,000
0159	WC=Z2/(C*(1)+INT)					221,000
0160	41 ERRC=NSORT((1,Z1)*SIG(I))					222,000
0161	ERRC=NSORT((1,Z2)*SIG(I))					223,000
0162	WRITE(9,119)ID,I,ERRC					224,000
0163	31 WRITE(9,110) I,NSUM(I),ERR,NCOR(I),MSRT(I),MS10					225,000
0164	2P(I),NOVE(I)					226,000
0165	LINE=LINE+NPK+1					227,000
0166	GO TO 53					228,000
0167	42 TIME=FALSE					229,000
0168	IF(LINE+NPK,GE,58) GO TO 90					230,000
0169	WRITE(9,113)					231,000
0170	LINE=LINE+2					232,000
	GO TO 43					233,000
	C					234,000
	C					235,000
	C					236,000
0171	29 DO 25 I=1,NPK					237,000
0172	ERRG(I)=ERRG(I)+ERRG(I)					238,000
0173	NTOT(I)=NTOT(I)+NSUM(I)					239,000
0174	25 NCOR(I)=NCOR(I)+NCOR(I)					240,000
0175	INIT=INIT+INT					241,000
0176	LB=1					242,000
0177	IF(LINE+NPK,GE,60) GO TO 90					243,000
0178	WRITE(9,109) ID,DE(2),INT					244,000
0179	DO 26 I=1,NPK					245,000
0180	26 WRITE(9,110) I,NSUM(I),NCOR(I),MSRT(I),MSTOP(I),NOVE(I)					246,000
0181	LINE=LINE+NPK+1					247,000
0182	IF(NORM,GT,0) GO TO 53					248,000
	C					249,000
	C					250,000
	C					251,000
0183	LB=2					252,000
0184	IF(LINE+NPK,GE,60) GO TO 90					253,000
0185	WRITE(9,111) INT					254,000
0186	DO 27 I=1,NPK					255,000
0187	27 WRITE(9,110) I,NTOT(I),NCOR(I),ERRG(I),ERRG(I)					256,000
0188	LINE=LINE+NPK+1					257,000
0189	LB=3					258,000
0190	IF(LINE+NPK,GE,60) GO TO 90					259,000
0191	WRITE(9,112)					260,000
0192	DO 28 I=1,NPK					261,000
0193	28 NTOT(I)					262,000
0194	22=NCOR(I)					263,000
0195	WRITE(9,113)					264,000
0196	CC(I)=Z1/X					265,000
0197	CC(I)=Z2/X					266,000
0198	SIG(I)=1/Z1					267,000
0199	SIG(I)=ERRG(I)/(Z2*Z2)					268,000
0200	NSUM(I)=C*(I)					269,000
0201	NCOR(I)=C*(I)					270,000

MICROSCAN TERMINAL SYSTEM FORTRAN G(41336)			MAIN	10-28-75	18:35:40	PAGE 0006
0202		VART=ERTBU(I)/(X*X)			271,000	
0203	28	WRITE(6,110) I,NSUM(I),NCOR(I),VART			272,000	
0204		LINE=LINE+NPK+1			273,000	
0205		GO TO 55			274,000	
0206	51	NNARM=NNARM+1			275,000	
0207		LB=5			276,000	
0208		IF(LINE,GE,58) GO TO 90			277,000	
0209	95	WRITE(6,105) IOD			278,000	
0210		LINE=LINE+3			279,000	
0211		GO TO 53			280,000	
0212	90	WRITE(6,116) (TITLE(N),N=1,3),MON(MTH),DTE(2),DTE(3),NPAGE,(TITLE(			281,000	
		2N),N=4,9)			282,000	
0213		IF(NNARM,GE,0) WRITE(6,108)			283,000	
0214		IF(NNARM,LT,0) WRITE(6,113)			284,000	
0215		LINE=4			285,000	
0216		NNPAGE=NPAGE+1			286,000	
0217		GO TO (91,92,93,94,95),LB			287,000	
0218	53	CONTINUE			288,000	
0219		GO TO 200			289,000	
0220	52	REWIND 2			290,000	
0221		STOP 1			291,000	
0222	101	FORMAT(13,5(213,412),11,12)			292,000	
0223	102	FORMAT(7H0FILE #14/19A4)			293,000	
0224	103	FORMAT(34H0 LINEAR BG CALCULATIONS USING 1/7X, 44MNPX AREA1 GAP			294,000	
		21 GAP2 AREA2 LIFETIME(HRS))			295,000	
0225	104	FORMAT(1H 7X,11,5X,4(12,4X),2X,F7.2)			296,000	
0226	105	FORMAT(47H *****/21H YOU			297,000	
		2HAVE ASKED FOR #14,22H BUT IT DOES NOT EXIST/47H *****			298,000	
		3*****)			299,000	
0227	106	FORMAT(14I5)			300,000	
0228	107	FORMAT(46H0 MAXIMUM NUMBER OF CENTROID CORRECTIONS 1914)			301,000	
0229	108	FORMAT(7SHOWARMS: ID TIME RCT PEAK COUNTB COUNTA VARIANCE			302,000	
		2 WINDOW NCC)			303,000	
0230	109	FORMAT(14 ,5X,14,1X,44,14)			304,000	
0231	110	FORMAT(14 ,20X,12,5X,217,1X,E12,6,4X,1H(13,1H,13,1H)2X,I3)			305,000	
0232	111	FORMAT(14 ,7X,7HTOTALS ,14)			306,000	
0233	112	FORMAT(14 ,8X,10HMEANS 1)			307,000	
0234	113	FORMAT(102HOCOLS: ID TIME RCT PEAK COUNTB NORMB ERROR CO			308,000	
		2UNTA NORMA ERROR VARIANCE WINDOW NCC)			309,000	
0235	114	FORMAT(14 ,20X,12,2(5X,17,2F6,3),1X,E12,6,4X,1H(13,1H,13,1H)2X,I3)			310,000	
0236	116	FORMAT(1H1,3A4,25H: SPECTAN ANALYSIS ON A4,12,4H, 19,12,29X,4HPAG			311,000	
		2E13/16X,6A4)			312,000	
0237	117	FORMAT(13,F4,0)			313,000	
0238	118	FORMAT(317,F0,6,17,E12,6)			314,000	
0239	119	FORMAT(214,4F6,4)			315,000	
0240		END			316,000	
OPTIONS IN EFFECT IO,ESCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP						
OPTIONS IN EFFECT NAME = MAIN , LINECNT = 57						
STATISTICS SOURCE STATEMENTS = 240,PROGRAM SIZE = 12104						
STATISTICS NO DIAGNOSTICS GENERATED						
NO ERRORS IN MAIN						

Appendix B The Angular Distribution Program ANGDIST

The program was developed by P. W. Daly to give the anisotropy of gamma radiation over a range of temperatures for any of the nuclei used in nuclear orientation experiments. Corrections are made to the anisotropy for the solid angle subtended by the detectors, allowing the thermometer temperature to be determined by looking up the anisotropy in the output table.

*COMPILE			
C	ANGOST	*	5
C		*	6
C	ENTER SPIN, HYPERFINE FIELD IN KGauss, MAGNETIC MOMENT IN N.M., TITLE	*	7
C	FORMAT: I4,1,F7.2,F7.4,I5A4	*	8
C		*	9
C	ENTER UNOBSERVED TRANSITION: NFE0, FINAL SPIN, L1, DELTA SQUARED, L2	*	10
C	FORMAT: I1,2F4.1,F5.2,F4.1	*	11
C	IF THERE IS MORE THAN ONE UNOBSERVED, THEY MUST BE ENTERED IN ORDER	*	12
C	END-OF-FILE STOPS PROGRAM	*	13
C		*	14
C	ENTER OBSERVED GAMMA: NFE1, FINAL SPIN, L1, DELTA, L2	*	15
C	FORMAT: I1,2F4.1,F5.2,F4.1	*	16
C	L2 DEFAULTS 10/L1+1	*	17
C		*	18
C	ENTER SOLID ANGLE DATA: HALF ANGLES IN RADIANS AT DETECTOR, A(0), A(90)	*	19
C	FORMAT: 2F7.4	*	20
C		*	21
C	ENTER TEMPERATURE LIMITS: IC=0 FOR BETA	*	22
C	=1 FOR TEMP IN MK	*	23
C	ST= INITIAL VALUE (OF BETA OR T)	*	24
C	SP= FINAL VALUE	*	25
C	NUM= NUMBER OF VALUES	*	26
C	FORMAT: I1,2F7.3,I3	*	27
C	WHEN FINISHED, IT RETURNS TO READ MORE TEMP LIMITS	*	28
C	END-OF-FILE RETURNS TO READ MORE UNOBSERVED PARAMETERS	*	29
C		*	30
C	NOTE THAT TWO END3-OF-FILE NECESSARY TO STOP PROGRAM	*	31
C	*****		32
C			33
1	DIMENSION C2(10), C4(10), FIR(6), A(15)		34
2	COMMON/FACULC/F(60)		35
3	LOGICAL YF1		36
4	READ(5,100) S,4,D,(A(N),N=1,15)		37
C			38
C	CLESSH-GORDAN COEFFICIENTS		39
C			40
5	SS=2.*S		41
6	SR=ABS(SS)-AINT(SS)		42
7	IF(SR.EQ.0.) GO TO 24		43
8	WRITE(6,111) S		44
9	STOP 10		45
10	24 Q21=S*(S+1.)		46
11	Q22=SQRT((2.*S+3.)*(S+1.)*S*(2.*S-1.)/5.)	ANGD0004	47
12	Q41=3.*(S+2.)*(S+1.)*S*(S-1.)	ANGD0005	48
13	Q42=5.*(6.*S*S+6.*S-5.)	ANGD0006	49
14	Q43=2.*SQRT((2.*S+5.)*(S+2.)*(2.*S+3.)*(S+1.)*S*(2.*S-1.)*(S-1.)*(2.*S-3.)/9.)	ANGD0007	50
		ANGD0008	51
15	N1=S+1.1	ANGD0009	52
16	N2=S+0.6	ANGD0010	53
17	N3=N1-N2	ANGD0011	54
18	DO 10 A=1,N1	ANGD0012	55
19	SK=K	ANGD0013	56
20	S4=S-SK+1.	ANGD0014	57
21	S42=S4*S4		58
22	C2(K)=0.		59
23	C4(K)=0.		60
24	IF(S.EQ.1.) C2(K)=(3.*S42-Q21)/Q22		61
25	WRITE(6,201) C2(K), C4(K)		62
26	201 FORMAT(' ',2F10.7)		63

27	10	IF(S,GE,2.) C4(K)=(Q41-Q42*SM2+35, *SM2*SM2)/Q43	64
28		SETL=0,0365846*H*D/S	65
	C		ANGD00017
	C	U COEFFICIENTS	66
	C		67
	C		68
29	15	U2=1.	69
30		U4=1.	70
31		NUNOB=1	71
32		S2=S	72
33		YES=TRUE.	73
34	18	S1=S2	74
35		READ(5,102,END=16) NF,S2,EL1,DELTA,EL2	75
36		IF(YES) WRITE(6,101) (ATN),N=1,15),S,H,D,BETE	76
37		YES,FALSE.	77
38		IF(EL2,1,0,1,AND,NF,NE,0) EL2=EL1+1.	78
39		I1=2, *S1	79
40		I2=2, *S2	80
41		IF(NF,NE,0) GO TO 21	81
42		L2, *EL1	82
43		DO 19 I=1,2	83
44		IF(I,GI,1) L=2, *EL2	84
45		DO 20 NJ=4,8,4	85
46		N=1,NH/2-2	86
47		N=SI*J(I,1),11,NJ,I2,I2,L)	87
48		F1(N)=N*SQRT((2, *S1+1, *)*(2, *S2+1, *))	88
49	20	IF(MOD((I1+I2+1)/2,2),NE,0) FIR(N)=FIR(N)	89
50	19	CONTINUE	90
51		UT2=(FIR(1)+DELTA*FIR(2))/(1, *DELTA)	91
52		UT4=(FIR(3)+DELTA*FIR(4))/(1, *DELTA)	92
53		*RITE(6,103) NUNOB,S1,S2,EL1,EL2,DELTA,UT2,UT4	93
54		NUNOB=NUNOB+1	94
55		U2=U2*UT2	95
56		U4=U4*UT4	96
57		GO TO 18	97
	C		98
	C	F COEFFICIENTS	99
	C		100
58	21	WRITE(6,104) U2,U4	101
59		L1=2, *EL1	102
60		L2=EL1	103
61		DO 22 I=1,3	104
62		IF(I,GI,1) L2=2, *EL2	105
63		IF(I,GI,2) L1=EL2	106
64		DO 23 NJ=4,8,4	107
65		N=1+3*(NJ/4-1)	108
66		C=H*REJ(I,1,L2,NJ,I2,=2,0)	109
67		*SIAJ(I,1,1,NJ,L2,L1,I2)	110
68		X=(I1+1)*L2+1)*(I1+1)*(NJ+1)	111
69		FIR(N)=C**SQRT(X)	112
70	23	IF(MOD((I1+I2)/2,2),EQ,0) FIR(N)=FIR(N)	113
71	22	CONTINUE	114
72		DELTA=DELTA*DELTA	115
73		F2=(FIR(1)+2, *DELTA*FIR(2)+DELTA*FIR(3))/(1, *DELTA)	116
74		F4=(FIR(4)+2, *DELTA*FIR(5)+DELTA*FIR(6))/(1, *DELTA)	117
75		WRITE(6,105) S1,S2,EL1,EL2,DELTA,F2,F4	118
76		U2=U2*F2	119
77		U4=U4*F4	120
	C		121
	C	SOLID ANGLE FACTORS	122
	C		123

78		READ(5,106) A0,A90			124
79		X0=COS(A0)	ANGD0019		125
80		X90=COS(A90)	ANGD0020		126
81		G20=.5*X0*(1.+X0)	ANGD0021		127
82		G40=G20*(7.*X0*X0-3.)/4.	ANGD0022		128
83		G290=.5*X90*(1.+X90)	ANGD0023		129
84		G490=G290*(7.*X90*X90-3.)/4.	ANGD0024		130
85		WRITE(6,107) A0,A90,G20,G40,G290,G490			131
	C				132
	C	BETA VALUES READ			133
	C				134
86	14	READ(5,110,END=15) IC,ST,SP,NUM			135
87		WRITE(6,108)			136
88		IF(IC,EQ.0) GO TO 11	ANGD0029		137
89		ST=BETE/ST	ANGD0030		138
90		SP=BETE/SP	ANGD0031		139
91	11	RUNS=NUM-1	ANGD0032		140
92		DEL=(SP-ST)/RUNS	ANGD0033		141
93		T=0.			142
94		B=100.			143
95		B2=C2(1)			144
96		B4=C4(1)			145
97		E2=B2*U2			146
98		E4=B4*U4			147
99		W0=1.+E2+E4			148
100		W90=1.-.5*E2+.375*E4			149
101		WC0=1.+G20*E2+G40*E4			150
102		WC90=1.-.5*G290*E2+.375*G490*E4			151
103		WRITE(6,109) B,T,B2,B4,W0,W90,WC0,WC90			152
104		B=S7	ANGD0034		153
105		DO 12 I=1,NUM	ANGD0034		154
	C				155
	C	X(0),X(90) CALCULATION			156
	C				157
106		Z= SINH(B*(S+.5))/SINH(B*.5)	ANGD0035		158
107		B2=0.	ANGD0036		159
108		B4=0.	ANGD0037		160
109		DO 15 K=1,N2	ANGD0038		161
110		SK=K	ANGD0039		162
111		SM=S-SK+1.	ANGD0040		163
112		X=2.*COSH(B*SM)	ANGD0041		164
113		B2=B2+C2(K)*X	ANGD0042		165
114	13	B4=B4+C4(K)*X	ANGD0043		166
115		IF(N0,EQ.0) GO TO 17	ANGD0044		167
116		B2=B2+C2(N1)	ANGD0045		168
117		B4=B4+C4(N1)	ANGD0046		169
118	17	B2=B2/Z	ANGD0047		170
119		B4=B4/Z	ANGD0048		171
120		T=BETE/B	ANGD0049		172
121		E2=U2*B2			173
122		E4=U4*B4			174
123		W0=1.+E2+E4	ANGD0052		175
124		W90=1.-.5*E2+.375*E4	ANGD0053		176
125		WC0=1.+G20*E2+G40*E4	ANGD0054		177
126		WC90=1.-.5*G290*E2+.375*G490*E4	ANGD0055		178
127		WRITE(6,109) B,T,B2,B4,W0,W90,WC0,WC90			179
128	12	B=B+DEL	ANGD0058		180
129		GO TO 14	ANGD0059		181
130	16	STOP	ANGD0060		182
131	100	FORMAT(F4.1,F7.2,F7.4,15A4)			183

132	101	FORMAT(1H,30X,15A4//11X,'PARENT PARAMETERS:/21X,'SPIN='F4,1/21X, 2'FIELD='F7,2,' KGAUSS/21X,'MAG MOMENT='F7,4,' NUCLEAR MAGNETONS/ 321X,'BETA TIMES TEMP (IN MK)='F6,2)	184 185 186
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR N='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR T='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR J='F6	
133	102	FORMAT(11,2F4,1,F5,2,F4,1)	187
134	103	FORMAT(1H0,10X,'UNOBSERVED TRANSITION'/21X,'INITIAL SPIN='F4,1/2 21X,'FINAL SPIN ='F4,1/21X,'RADIATION L ='F4,1,' PLUS'F4,1,' (DELT 3A SQUARED='F5,2,')/16X,'THEN U2='F8,5,4X,'U4='F8,5)	188 189 190
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR ON'I2	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR N='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR U='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR U5='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F5	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 2='F8	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 4='F8	
135	104	FORMAT(1H0,14X,'TOTAL U2='F8,5,4X,'U4='F8,5)	191
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 14X'I	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 2='F8	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 4='F8	
136	105	FORMAT(1H0,10X,'OBSERVED GAMMA TRANSITION'/21X,'INITIAL SPIN='F4,1 2/21X,'FINAL SPIN ='F4,1/21X,'MULTIPOLE ='F4,1,' PLUS'F4,1,' (D 3ELTA='F5,2,')/16X,'THEN F2='F8,5,4X,'F4='F8,5)	192 193 194
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR N='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR U='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR U5='F4	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR A='F5	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 2='F8	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 4='F8	
137	106	FORMAT(2F7,4)	195
138	107	FORMAT(1H0,10X,'SOLID ANGLE CORRECTIONS'/21X,'ALPHA(0)='F7,4,'RAD 21ANS'/21X,'ALPHA(90)='F7,4,'RADIANS'/16X,'THEN G2(0)='F7,4,4X,'G4 3(0)='F7,4/21X,'G2(90)='F7,4,4X,'G4(90)='F7,4)	196 197 198
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR J='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR D='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR J='F7	
***WARNING**		EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR J='F7	
139	108	FORMAT(1H0,7X,4MBETA,3X,4HTEMP,6X,2HB2,7X,2HB4,8X,4HW(0),5X,5HW(90 2),6X,4HW(0),5X,5HW(90)/15X,4H(MK),25X,13H(UNCORRECTED),3X,24H(SOLI 3D ANGLE CORRECTION))	199 200 201
140	109	FORMAT(1H,5X,F0,3,2X,F6,2,2X,2(F7,4,2X),2X,2(F8,5,1X),2X,2(F8,5,1 2X))	202 203
141	110	FORMAT(11,2F7,3,13)	204
142	111	FORMAT(1H0,F4,1,30H IS UNACCEPTIBLE VALUE OF SPIN)	205
143		END	206
144		FUNCTION THREEJ(JJ1,JJ2,JJ3,MM1,MM2,MM3)	TREE0002 207
		C-----4ARGUMENTS MUST HAVE THEIR DOUBLE VALUES. FACILITIES SEE FUNCTION F,TREE0003	208
		C-----IGNOR 3-J-SYMBOL. EDMONDS (3,6,10) AND (3,7,5)	TREE0004 209
145		COMMON/FANULC/F(60)	TREE0005 210
146		DATA NN/2/	TREE0006 211
147		IF(NN,GT,1) GOTO 4	TREE0007 212
148	1	J1=JJ1	TREE0008 213
149		J2=JJ2	TREE0009 214

150	J3=J1+J2	TREE0010	215
151	M1=MM1	TREE0011	216
152	M2=MM2	TREE0012	217
153	M3=MM3	TREE0013	218
154	THREEJ=0	TREE0014	219
155	I1=J1+J2+J3	TREE0015	220
156	N=11+4	TREE0016	221
157	IF(N.GT.60) WRITE(6,6) N	TREE0017	222
158	6 FORMAT(16,26H IS OUT OF RANGE OF FAK(N) /1H0119(1H*))	TREE0018	223
***WARNING** EXPECTING COMMA BETWEEN FORMAT ITEMS NEAR 1H011			
159	IF(MOD(J1+J2+J3,2),NE,0) RETURN	TREE0019	224
160	IF((J1+J2-J3.LT.0).OR.(J3-1ABS(J1-J2).LT.0).OR.(M1+M2+M3.NE.0))	TREE0020	225
	1 RETURN	TREE0021	226
161	IF((M1.EQ.0).AND.(M2.EQ.0)) GO TO 3	TREE0022	227
162	N1=J1+J2-J3+1	TREE0023	228
163	N2=J1-M1+1	TREE0024	229
164	N3=J1-J2+J3+2	TREE0025	230
165	N4=J2-M2+1	TREE0026	231
166	N5=J2+J3-J1+2	TREE0027	232
167	N6=J3-M3+1	TREE0028	233
168	N7=J1+M1+2	TREE0029	234
169	N8=J3+M3+1	TREE0030	235
170	N9=J2+M2+2	TREE0031	236
171	Z=SQRT(0.125*F(N)*F(N1)*F(N2)*F(N3)*F(N4)*F(N5)*F(N6)*F(N7)*F(N8)*	TREE0032	237
	1 F(N9))	TREE0033	238
172	KMIN=MAX0((J3-J2-M1),0)+1	TREE0034	239
173	KMAX=MIN0((J1-M1),(J3+M3))+1	TREE0035	240
174	DO 2 K=KMIN,KMAX,2	TREE0036	241
175	N1=J1+M1+K	TREE0037	242
176	N2=J1-M1-K+3	TREE0038	243
177	N3=J2+J3-M1-K+2	TREE0039	244
178	N4=J3+M3-K+3	TREE0040	245
179	N5=J2-J3+M1+K+1	TREE0041	246
180	2 THREEJ=F(K+1)*F(N1)*F(N2)*F(N3)*F(N4)*F(N5) - THREEJ	TREE0042	247
181	THREEJ=THREEJ*Z	TREE0043	248
182	IF(MOD(((KMAX-1+M3-M1+J2)/2),2),NE,0) THREEJ=-THREEJ	TREE0044	249
183	RETURN	TREE0045	250
184	3 IF(MOD(I1,4),NE,0) RETURN	TREE0046	251
185	I2=I1/2	TREE0047	252
186	N1=I1-2*J1+1	TREE0048	253
187	N2=I1-2*J2+1	TREE0049	254
188	N3=I1-2*J3+1	TREE0050	255
189	N4=I2-J1+2	TREE0051	256
190	N5=I2-J2+2	TREE0052	257
191	N6=I2-J3+2	TREE0053	258
192	THREEJ=SQRT(.125*F(N)*F(N1)*F(N2)*F(N3))*F(I2+1)*F(N4)*F(N5)*F(N6)	TREE0054	259
193	IF(MOD(I2,4),NE,0) THREEJ=-THREEJ	TREE0055	260
194	RETURN	TREE0056	261
195	4 X=1.	TREE0057	262
196	Y=.125	TREE0058	263
197	DO 5 L=1,50,2	TREE0059	264
198	F(L)=X	TREE0060	265
199	F(L+1)=1./X	TREE0061	266
200	X=X*Y	TREE0062	267
201	5 Y=Y+.125	TREE0063	268
202	NN=0	TREE0064	269
203	GO TO 1	TREE0065	270
204	END	TREE0066	271
C-----V.22 FUNCTION SIXJ		SIXJ0001	272
C WIGNER SIX-J-SYMBOL, EDMONDS (6,3,7).		SIXJ0002	273

C-----ARGUMENTS MUST HAVE THEIR DOUBLE VALUES, FACULTIES SEE FUNCTION F SIXJ0003			274
C		SIXJ0004	275
205	FUNCTION SIXJ(JJ1,JJ2,JJ3,LL1,LL2,LL3)	SIXJ0005	276
206	COMMON/FAKULC/F(60)	SIXJ0006	277
207	LOGICAL TRL,MU	SIXJ0007	278
208	TRL(K,L,M)=(K,GT,L+M),OR,(K,LT,IABS(L-M))	SIXJ0008	279
209	MO(K,L,M)=MOD(K-IABS(L-M),2),NE.0	SIXJ0009	280
210	DATA NR/2/	SIXJ0010	281
211	IF(VN,GT,1) GOTO 4	SIXJ0011	282
212	1 J1=JJ1	SIXJ0012	283
213	J2=JJ2	SIXJ0013	284
214	J3=JJ3	SIXJ0014	285
215	L1=LL1	SIXJ0015	286
216	L2=LL2	SIXJ0016	287
217	L3=LL3	SIXJ0017	288
218	SIXJ=0.	SIXJ0018	289
219	IF(TRL(J1,J2,J3),OR,TRL(J1,L2,L3),OR,TRL(L1,J2,L3),OR,TRL(L1,L2,J3	SIXJ0019	290
220	1) RETURN	SIXJ0020	291
221	IF(MO(J1,J2,J3),OR,MO(J1,L2,L3),OR,MO(L1,L2,J3)) RETURN	SIXJ0021	292
222	I1=J1+J2+J3-2	SIXJ0022	293
223	I2=J1+L2+L3-2	SIXJ0023	294
224	I3=L1+J2+L3-2	SIXJ0024	295
225	I4=L1+L2+J3-2	SIXJ0025	296
226	N1=J1+J2+L1+L2+2	SIXJ0026	297
227	N2=J2+J3+L2+L3+2	SIXJ0027	298
228	N3=J3+J1+L3+L1+2	SIXJ0028	299
229	MINZ=MAX(I1,I2,I3,I4)+2	SIXJ0029	300
230	MAXZ=MIN(N1,N2,N3)-2	SIXJ0030	301
231	IF(MAXZ+2,GT,60) GOTO 12	SIXJ0031	302
232	NA1=J1+J2+J3+4	SIXJ0032	303
233	NA2=J1+J2+J3+1	SIXJ0033	304
234	NA3=J1-L2+J3+1	SIXJ0034	305
235	NA4=J2-J1+J3+1	SIXJ0035	306
236	NB1=J1+L2+L3+4	SIXJ0036	307
237	NB2=J1+L2-L3+1	SIXJ0037	308
238	NB3=J1-L2+L3+1	SIXJ0038	309
239	NB4=-J1+L2+L3+1	SIXJ0039	310
240	NC1=L1+L2+J3+4	SIXJ0040	311
241	NC2=L1+L2-J3+1	SIXJ0041	312
242	NC3=L1-L2+J3+1	SIXJ0042	313
243	NC4=-L1+L2+J3+1	SIXJ0043	314
244	ND1=L1+J2+L3+4	SIXJ0044	315
245	ND2=L1+J2-L3+1	SIXJ0045	316
246	ND3=L1-J2+L3+1	SIXJ0046	317
247	ND4=-L1+J2+L3+1	SIXJ0047	318
248	U=0,125*(-1.)**MOD(MINZ/2,2)*SQRT(F(NA1)*F(NA2)*F(NA3)*F(NA4)	SIXJ0048	319
249	1 *F(NB1)*F(NB2)*F(NB3)*F(NB4)*F(NC1)*F(NC2)*F(NC3)*F(NC4) *F(ND1)	SIXJ0049	320
250	2 *F(ND2)*F(ND3)*F(ND4))	SIXJ0050	321
251	DO 11 N=MINZ,MAXZ,2	SIXJ0051	322
252	NA1=N-11	SIXJ0052	323
253	NA2=N-12	SIXJ0053	324
254	NA3=N-13	SIXJ0054	325
255	NA4=N-14	SIXJ0055	326
256	NB1=N-1	SIXJ0056	327
257	NB2=N-2	SIXJ0057	328
258	NB3=N-3	SIXJ0058	329
259	SIXJ=SIXJ+U*(V+3)*F(NA1)*F(NA2)*F(NA3)*F(NA4)*F(NB1)*F(NB2)*F(NB3	SIXJ0059	330
260	1)	SIXJ0060	331
261	11 U=-U	SIXJ0061	332

* CO-60/FE, H=287.70KG, *

PARENT PARAMETERS:

SPIN= 5.0
FIELD= 287.70 KGAUSS
MAG MOMENT= 3.7540 NUCLEAR MAGNETONS
BETA TIMES TEMP (IN MK)= 7.90

UNOBSERVED TRANSITION 1

INITIAL SPIN= 5.0
FINAL SPIN = 4.0
RADIATION L = 1.0 PLUS 0.0 (DELTA SQUARED= 0.00)
THEN U2= 0.93937 U4= 0.79772

TOTAL U2= 0.93937 U4= 0.79772

OBSERVED GAMMA TRANSITION

INITIAL SPIN= 4.0
FINAL SPIN = 2.0
MULTIPOLE = 2.0 PLUS 3.0 (DELTA= 0.00)
THEN F2=-0.44770 F4=-0.30438

SOLID ANGLE CORRECTIONS

ALPHA(0) = 0.0550RADIANS
ALPHA(90) = 0.0550RADIANS
THEN G2(0) = 0.9977 G4(0) = 0.9925
G2(90) = 0.9977 G4(90) = 0.9925

BETA	TEMP	B2	B4	W(0)	W(90)	W(0)	W(90)
(MK)				(UNCORRECTED)	(SOLID ANGLE CORRECTION)		
*****	0.00	1.6984	1.1767	0.00001	1.25000	0.00378	1.25000
0.790	10.00	1.0077	0.2412	0.51765	1.18993	0.51405	1.18961
0.782	10.10	0.9998	0.2362	0.52217	1.18874	0.52356	1.18843
0.774	10.20	0.9919	0.2311	0.52674	1.18753	0.52811	1.18722
0.767	10.31	0.9839	0.2260	0.53134	1.18630	0.53270	1.18599
0.759	10.42	0.9757	0.2210	0.53599	1.18505	0.53732	1.18474
0.751	10.53	0.9675	0.2160	0.54067	1.18377	0.54199	1.18346
0.743	10.64	0.9591	0.2110	0.54540	1.18247	0.54670	1.18215
0.735	10.75	0.9506	0.2061	0.55017	1.18114	0.55145	1.18082
0.727	10.87	0.9421	0.2011	0.55497	1.17978	0.55624	1.17947
0.719	10.99	0.9334	0.1962	0.55982	1.17840	0.56107	1.17809
0.711	11.11	0.9246	0.1913	0.56471	1.17699	0.56594	1.17668
0.703	11.24	0.9156	0.1865	0.56963	1.17556	0.57085	1.17525
0.695	11.36	0.9066	0.1817	0.57460	1.17410	0.57580	1.17379
0.688	11.49	0.8975	0.1769	0.57951	1.17261	0.58079	1.17230
0.680	11.63	0.8882	0.1722	0.58466	1.17109	0.58583	1.17079
0.672	11.76	0.8788	0.1674	0.58976	1.16955	0.59090	1.16924
0.664	11.90	0.8693	0.1628	0.59489	1.16797	0.59601	1.16767
0.656	12.05	0.8597	0.1581	0.60006	1.16637	0.60117	1.16607
0.648	12.20	0.8499	0.1535	0.60528	1.16474	0.60637	1.16444
0.640	12.35	0.8401	0.1490	0.61053	1.16308	0.61160	1.16278
0.632	12.50	0.8301	0.1445	0.61583	1.16139	0.61688	1.16109
0.624	12.66	0.8200	0.1400	0.62116	1.15967	0.62220	1.15938
0.616	12.82	0.8097	0.1356	0.62654	1.15792	0.62756	1.15763
0.608	12.99	0.7994	0.1312	0.63195	1.15614	0.63296	1.15585
0.601	13.16	0.7889	0.1269	0.63741	1.15433	0.63839	1.15404
0.593	13.33	0.7783	0.1227	0.64290	1.15248	0.64387	1.15220
0.585	13.51	0.7676	0.1185	0.64843	1.15061	0.64938	1.15033
0.577	13.70	0.7567	0.1143	0.65401	1.14871	0.65494	1.14843

0.569	13.89	0.7457	0.1102	0.65962	1.14677	0.66053	1.14649
0.561	14.08	0.7306	0.1062	0.65526	1.14480	0.66616	1.14453
0.553	14.29	0.7234	0.1022	0.67094	1.14281	0.67182	1.14253
0.545	14.49	0.7121	0.0983	0.67666	1.14078	0.67752	1.14050
0.537	14.71	0.7006	0.0945	0.68242	1.13872	0.68326	1.13845
0.529	14.93	0.6890	0.0907	0.68820	1.13662	0.68903	1.13636
0.522	15.15	0.6773	0.0870	0.69403	1.13450	0.69483	1.13424
0.514	15.38	0.6655	0.0834	0.69986	1.13235	0.70067	1.13209
0.506	15.63	0.6536	0.0798	0.70576	1.13016	0.70653	1.12990
0.498	15.87	0.6415	0.0763	0.71168	1.12795	0.71243	1.12769
0.490	16.13	0.6294	0.0729	0.71762	1.12570	0.71836	1.12545
0.482	16.39	0.6171	0.0696	0.72359	1.12343	0.72431	1.12318
0.474	16.67	0.6047	0.0663	0.72959	1.12112	0.73029	1.12088
0.466	16.95	0.5922	0.0631	0.73561	1.11879	0.73629	1.11855
0.458	17.24	0.5797	0.0600	0.74165	1.11643	0.74232	1.11619
0.450	17.54	0.5670	0.0569	0.74772	1.11404	0.74836	1.11381
0.443	17.86	0.5542	0.0540	0.75380	1.11153	0.75443	1.11140
0.435	18.18	0.5414	0.0511	0.75990	1.10919	0.76051	1.10896
0.427	18.52	0.5285	0.0483	0.76602	1.10672	0.76661	1.10650
0.419	18.87	0.5155	0.0456	0.77214	1.10423	0.77272	1.10402
0.411	19.23	0.5024	0.0430	0.77828	1.10172	0.77884	1.10151
0.403	19.61	0.4892	0.0405	0.78442	1.09919	0.78496	1.09898
0.395	20.00	0.4760	0.0380	0.79057	1.09663	0.79109	1.09643
0.387	20.41	0.4628	0.0357	0.79672	1.09406	0.79722	1.09387
0.379	20.83	0.4495	0.0334	0.80286	1.09147	0.80335	1.09128
0.371	21.28	0.4361	0.0312	0.80900	1.08887	0.80948	1.08868
0.363	21.74	0.4228	0.0291	0.81514	1.08625	0.81559	1.08607
0.356	22.22	0.4094	0.0271	0.82126	1.08362	0.82170	1.08344
0.348	22.73	0.3960	0.0251	0.82736	1.08098	0.82779	1.08080
0.340	23.26	0.3826	0.0233	0.83345	1.07833	0.83386	1.07816
0.332	23.81	0.3692	0.0215	0.83951	1.07567	0.83990	1.07551
0.324	24.39	0.3558	0.0199	0.84554	1.07301	0.84592	1.07285
0.316	25.00	0.3425	0.0183	0.85154	1.07035	0.85190	1.07020
0.308	25.64	0.3292	0.0167	0.85751	1.06769	0.85785	1.06754
0.300	26.32	0.3159	0.0153	0.86343	1.06503	0.86376	1.06489
0.292	27.03	0.3027	0.0140	0.86930	1.06236	0.86962	1.06223
0.284	27.78	0.2895	0.0127	0.87512	1.05974	0.87542	1.05961
0.277	28.57	0.2760	0.0115	0.88089	1.05711	0.88117	1.05699
0.269	29.41	0.2637	0.0104	0.88659	1.05450	0.88686	1.05438
0.261	30.30	0.2509	0.0093	0.89222	1.05191	0.89247	1.05180
0.253	31.25	0.2382	0.0084	0.89777	1.04934	0.89802	1.04923
0.245	32.26	0.2257	0.0075	0.90325	1.04679	0.90348	1.04669
0.237	33.33	0.2134	0.0066	0.90864	1.04427	0.90886	1.04418
0.229	34.48	0.2013	0.0058	0.91394	1.04179	0.91414	1.04169
0.221	35.71	0.1893	0.0051	0.91914	1.03934	0.91933	1.03925
0.213	37.04	0.1776	0.0045	0.92423	1.03693	0.92441	1.03685
0.205	38.46	0.1661	0.0039	0.92921	1.03456	0.92938	1.03449
0.198	40.00	0.1546	0.0034	0.93408	1.03224	0.93423	1.03217
0.190	41.67	0.1438	0.0029	0.93862	1.02997	0.93886	1.02991
0.182	43.48	0.1331	0.0025	0.94342	1.02776	0.94356	1.02770
0.174	45.46	0.1227	0.0021	0.94790	1.02561	0.94802	1.02555
0.166	47.62	0.1126	0.0018	0.95222	1.02351	0.95233	1.02346
0.158	50.00	0.1028	0.0015	0.95640	1.02149	0.95650	1.02144
0.150	52.61	0.0934	0.0012	0.96042	1.01954	0.96051	1.01949
0.142	55.56	0.0844	0.0010	0.96428	1.01765	0.96436	1.01761
0.134	58.63	0.0757	0.0008	0.96797	1.01585	0.96804	1.01581
0.126	62.50	0.0675	0.0006	0.97146	1.01413	0.97155	1.01410
0.119	67.07	0.0595	0.0005	0.97482	1.01249	0.97497	1.01246
0.111	71.45	0.0522	0.0004	0.97796	1.01094	0.97801	1.01091
0.103	76.93	0.0452	0.0003	0.98092	1.00948	0.98096	1.00946

0.095	83.34	0.0387	0.0002	0.98368	1.00812	0.98372	1.00810
0.087	90.91	0.0326	0.0001	0.98624	1.00585	0.98627	1.00683
0.079	100.00	0.0271	0.0001	0.98859	1.00568	0.98861	1.00567

CORE USAGE OBJECT CODE= 15248 BYTES,ARRAY AREA= 420 BYTES,TOTAL AREA AVAILABLE= 102400 BYTES

DIAGNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 30, NUMBER OF EXTENSIONS= 0

COMPILE TIME= 0.36 SEC,EXECUTION TIME= 0.18 SEC, WATFIV - JUL 1973 V1L4 18:49:14 TUESDAY 28 OCT 75

SSTOP
EXECUTION TERMINATED

SSIG

Appendix C The Nuclear Orientation Fitting Program NOFIT

The Program:

A program was developed which would fit, by the least squares method, various Nuclear Orientation parameters to the experimental data points. The parameters which were to be fit to the data were;

f the fraction of nuclei feeling a unique hyperfine field. The remainder feel zero field.

H the unique hyperfine field

$\left. \begin{array}{l} U_2 F_2 \\ U_4 F_4 \end{array} \right\}$ the $U_\nu F_\nu$ coefficients

The expression for the anisotropy was written as a Taylor Series Expansion of the form

$$F(T, H + \Delta H, \theta = 0^\circ) = f \left[F(T, H, \theta = 0) + \frac{\partial F(T, H, \theta = 0)}{\partial H} \Delta H \right] \quad (C.1)$$

where

$$F(T, H, \theta = 0) = A_2 U_2 F_2 B_2 + A_4 U_4 F_4 B_4 \quad (C.2)$$

and

$$\frac{\partial F}{\partial H} (T, H, \theta = 0) = A_2 U_2 F_2 \frac{\partial B_2}{\partial H} + A_4 U_4 F_4 \frac{\partial B_4}{\partial H} \quad (C.3)$$

In Equations (C.2) and (C.3), the $U_2 F_2$ and $U_4 F_4$ coefficients are the

initial estimates. Their best fit values are found by varying the A_2 and A_4 multiplicative constants. Hence the best fit values would be,

$$(U_2 F_2)^{fit} = A_2 U_2 F_2 \qquad (U_4 F_4)^{fit} = A_4 U_4 F_4$$

(C.4)

A Restricted Least Squares Fitting method developed by Froese et.al. (1974) was used to fit the parameter values to the experimental data points. The routine allowed the parameters to be varied individually, or any combination of them to be varied simultaneously. A Listing of the NOFIT program is given in Table C1.

```

C ***** 1,000
C ***** 1,000
C ***** NOFIT ***** 2,000
C ***** 3,000
C ***** 3,000
C A FITTING PROGRAM TO FIT THE HYPERFINE FIELD AND THE FRACTION 4,000
C OF NUCLEI FEELING THE HYPERFINE FIELD TO A SET OF EXPERIMENTAL 5,000
C DATA POINTS. 6,000
C 7,000
C FUNCTION IS EXPANDED AS A TAYLOR SERIES OF THE FORM , 8,000
C 9,000
C  $F(X,H+\Delta H)=P(1)*(F(X,H)+(D F(X,H)/D H)*P(2))$  10,000
C 11,000
C WHERE P(1) IS THE FRACTION OF NUCLEI FEELING THE HYPERFINE FIELD 12,000
C AND P(2) IS AN EST. OF  $\Delta H$ , P(3) AND P(4) ARE COEFFICIENTS 13,000
C WHICH APPEAR AS- 14,000
C 15,000
C  $W(\theta)=1+P(3)A2B2P2+P(4)A4B4P4$  16,000
C 17,000
C THEY ARE MEANT TO SOLVE FOR THE ATTENUATION COEFFICIENTS OF ANY 18,000
C OBSERVED TRANSITION. 19,000
C THE SUBROUTINE "AUX" 20,000
C X IS THE TEMPERATURE IN MILLI DEGREES KELVIN AND H IS THE FIELD 21,000
C IN KILOGAUSS. POLARIZING MUST BE ADDED TO H WHEN FITTING AN 22,000
C UNKNOWN HYPERFINE FIELD. 23,000
C ***** 24,000
C ***** 24,000
C 25,000
C ENTER: SPIN, MAGNETIC MOMENT IN NUCLEAR MAGNETONS, TITLE 26,000
C FORMAT : F4,1,F7,4,15A4 27,000
C 28,000
C ENTER : U2,U4,F2,F4 29,000
C FORMAT: 4F10,7 30,000
C 31,000
C ENTER : SOLID ANGLE DATA ; HALF ANGLES IN RADIANS AT THE DETECTORS 32,000
C FORMAT : 2F7,4 33,000
C 34,000
C ENTER: NUMBER OF DATA POINTS, NUMBER OF PARAMETERS , NUMBER OF 35,000
C ITERATIONS, EPSILON, AXIS 36,000
C FORMAT : 3I2,2F10,5 37,000
C AXIS = 0.0 SOLVED FOR AXIAL ANISOTROPY DATA POINTS 38,000
C =90.0 SOLVED FOR EQUATORIAL ANISOTROPY DATA POINTS 39,000
C 40,000
C ENTER : MODE INDICATES TYPE OF WEIGHTING TO BE GIVEN TO THE DATA 41,000
C POINTS DURING THE FITTING ROUTINE. 42,000
C FORMAT : I2 43,000
C MODE=-1 , STATISTICAL WEIGHTING  $W(I)=1./SIGMA(I)$  44,000
C MODE= 0 , EQUAL WEIGHTING  $W(I)=1.0$  45,000
C MODE=+1 , INSTRUMENTAL WEIGHTING  $W(I)=1./SIGMA(I)**2$  46,000
C  $W(I)=W(I)/WMEAN$   $WMEAN=W(1)+W(2)+-----+W(N)$  47,000
C 48,000
C ENTER : DATA POINTS ; TEMP(MILLIK),ANISOTROPY,ERROR IN ANIS 49,000
C FORMAT : 3F10,5 50,000
C 51,000
C ENTER: HYPERFINE FIELD ESTIMATE, ESTIMATE OF FRACTION OF NUCLEI 52,000

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C      FEELING THE HYPERFINE FIELD ,INITIAL ESTIMATE OF DELTA H          53,000
C      ESTIMATES OF THE ATTENUATION COEFFICIENTS G2 AND G4 RESPECTIVELY  54,000
C      FORMAT : F7,2,5F10,5      55,000
C      P(1) IS THE FRACTION OF NUCLEI FEELING THE HYPERFINE FIELD      56,000
C      DELTA H IS PARAMETER P(2)      57,000
C      P(3) IS THE ATTENUATION COEFFICIENT G2      58,000
C      P(4) IS THE ATTENUATION COEFFICIENT G4      59,000
C      ENTER : MD      60,000
C      FORMAT : 5I1      61,000
C      MD=0 P(1) IS HELD AT ITS LAST GIVEN VALUE      62,000
C      MD=1 P(1) IS ALLOWED TO VARY      63,000
C      ---SUPPLY AS MANY DIFFERENT MD CARDS AS YOU WISH      64,000
C      EXAMPLE : P(1) HELD CONSTANT, VARY P(2), THEN MD CARD WOULD BE  65,000
C      01      66,000
C      01      67,000
C      01      68,000
C      TO ANALYZE DATA FOR MORE THAN 1 TRANSITION , PUT SEND BEFORE THE  69,000
C      DATA OF THE NEXT GROUP OF DATA TO BE ANALYZED, TO STOP PROGRAM,  70,000
C      THREE END-OF-FILES ARE NECESSARY      71,000
C      *****      72,000
C      *****      72,000
0001      EXTERNAL AUX      73,000
0002      DIMENSION X(50), Y(50), YF(50), SIGMAY(50), W(50), P(5), E1(5), E2(5  74,000
          2), FIR(6), A(15), MD(5)      75,000
0003      COMMON A20, A40, A290, A490, S, DIPMOM, H, AXIS, C2(10), C4(10)      76,000
0004      COMMON/FAKULC/F(60)      77,000
0005      LOGICAL YET      78,000
0006      READ(5,100,END=1000) S, DIPMOM, (A(N), N=1,15)      79,000
C      80,000
C      CALCULATE THE CLEBSH= GORDAN COEFFICIENTS      81,000
C      82,000
0007      SS=2,*S      83,000
0008      SR=ABS(SS)-AINT(SS)      84,000
0009      IF(SR,EQ,0.) GO TO 24      85,000
0010      WRITE(6,111) S      86,000
0011      STOP 10      87,000
0012      24 Q21=S*(S+1.)      88,000
0013      Q22=SQRT((2,*S+3.)*(S+1.)*S*(2,*S-1.)/5.)      89,000
0014      Q41=3.*(S+2.)*(S+1.)*S*(S-1.)      90,000
0015      Q42=5.*(S+3.)*S*(S+1.)*S*(S-1.)      91,000
0016      Q43=2.*SQRT((2,*S+5.)*(S+2.)*(2,*S+3.)*(S+1.)*S*(2,*S-1.)*(S-1.)*  92,000
          22,*S-3.)/9.)      93,000
0017      N1=S+1.1      94,000
0018      N2=S+0.6      95,000
0019      N0=N1-N2      96,000
0020      DO 10 K=1,N1      97,000
0021      SK=K      98,000
0022      SM=S-SK+1.      99,000
0023      SM2=SM*SM      100,000
0024      C2(K)=0.      101,000
0025      C4(K)=0.      102,000
0026      IF(S,GE,1.) C2(K)=(3.*SM2-Q21)/Q22      103,000
0027      10 IF(S,GE,2.) C4(K)=(Q41-Q42*SM2+35.*SM2*SM2)/Q43      104,000
C      105,000
0028      READ(5,102,END=1000) U2, U4, F2, F4      106,000

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MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)			MAIN	08-25-75	15:28:12	PAGE P003
0029	102	FORMAT(4F10,7)			107,000	
0030		WRITE(6,103)U2,U4,F2,F4			108,000	
0031	103	FORMAT('01,10X,'U COEFFICIENTS:1/21X,'U2=1,F10,7,5X,'U4=1,F10,7//1			109,000	
		20X,'F COEFFICIENTS:1/21X,'F2=1,F10,7,5X,'F4=1,F10,7)			110,000	
	C	CALCULATE THE SOLID ANGLE CORRECTION FACTORS			112,000	
	C				113,000	
0032		READ(5,106) A0,A90			114,000	
0033		X0=COS(A0)			115,000	
0034		X90=COS(A90)			116,000	
0035		G20=.5*X0*(1.+X0)			117,000	
0036		G40=G20*(7.*X0*X0-3.)/4.			118,000	
0037		G290=.5*X90*(1.+X90)			119,000	
0038		G490=G290*(7.*X90*X90-3.)/4.			120,000	
0039		WRITE(6,107) A0,A90,G20,G40,G290,G490			121,000	
0040		A20=U2*F2*G20			122,000	
0041		A40=U4*F4*G40			123,000	
0042		A290=U2*F2*G290			124,000	
0043		A490=U4*F4*G490			125,000	
	C				126,000	
0044		READ(5,200) N,M,NI,EP,AXIS			127,000	
0045		WRITE(6,202) N,M,NI,EP			128,000	
	C	AXIS=0, FOR AXIAL DETECTOR			129,000	
	C	=90, EQUATORIAL DET.			130,000	
0046		IF(AXIS,EQ,0.) GO TO 204			131,000	
0047		WRITE(6,203)			132,000	
0048		GO TO 206			133,000	
0049	204	WRITE(6,205)			134,000	
0050	206	IF(N,LE,50,OR,M,LE,5) GO TO 208			135,000	
0051		WRITE(6,207)			136,000	
0052		GO TO 1000			137,000	
0053	208	NPTS=N			138,000	
0054		MPARAM=M			139,000	
	C				140,000	
	C	GIVE THE DATA THE APPROPRIATE TYPE OF WEIGHTING			141,000	
	C				142,000	
0055		READ(5,209) MODE			143,000	
	C	MODE SPECIFIES THE TYPE OF WEIGHTING TO BE USED.			144,000	
0056		WZ=1.			145,000	
	C	IF THE WEIGHTS ARE TO BE SUPPLIED BY THE USER, WZ=1.			146,000	
0057		SUM=0.0			147,000	
0058		DO 215 I=1,NPTS			148,000	
0059		READ(5,210) X(I), Y(I), SIGMAY(I)			149,000	
0060		Y(I)=Y(I)-1.			150,000	
	C	THIS IS DUE TO THE FACT THAT THE FUNCTION BEING FITTED IS OF THE			151,000	
	C	FORM (W-1)OBS=F*(W-1)			152,000	
0061		IF(SIGMAY(I),NE,0.) GO TO 212			153,000	
	C	IF SIGMA(I)=0.0, SET THE WEIGHT=1.0 TO AVOID THE FACTOR 1/SIGMA			154,000	
	C	GOING TO INFINITY.			155,000	
0062		WRITE(6,211) I			156,000	
0063		GO TO 214			157,000	
0064	212	IF(MODE) 213,214,215			158,000	
	C	STATISTICAL WEIGHTING GIVEN TO Y(I)			159,000	
0065	213	W(I)=1./SIGMAY(I)			160,000	
0066		GO TO 216			161,000	
	C	EQUAL WEIGHTING GIVEN TO Y(I)			162,000	

0067	214	W(I)=1,	163,000
0068		GO TO 216	164,000
	C	INSTRUMENTAL WEIGHTING GIVEN TO Y(I)	165,000
0069	215	W(I)=1./SIGMAY(I)**2	166,000
0070	216	SUM=SUM+W(I)	167,000
0071		WMEAN=SUM/FLOAT(NPTS)	168,000
0072		DO-217-I=1,NPTS	169,000
0073	217	W(I)=W(I)/WMEAN	170,000
	C		171,000
	C	READ IN THE INITIAL ESTIMATES OF FRACTION OF NUCLEI AT LATTICE	172,000
	C	SITES AND HYPERFIN FIELD BEING FELT BY THE NUCLEI, THE DELTA H IN	173,000
	C	THE TAYLOR SERIES IS P(2) AND THE FRACTION OF NUCLEI IS P(1)	174,000
	C	P(3) AND P(4) ARE THE ATTENUATION COEFF. G2 AND G4 RESPECTIVELY	175,000
0074		READ(5,218) H, (P(1), I=1,MPARAM)	176,000
0075		WRITE(6,251)H, (P(1), I=1,MPARAM)	177,000
	C		178,000
	C	READ IN THE CARDS ALLOWING THE P(I)'S TO VARY	179,000
	C		180,000
0076	219	READ(5,220,END=999) MD	181,000
0077		MOCHCK=MD(2)	182,000
0078		CALL DPRLOF(X,Y,YF,W,E1,E2,P,Z,N,M,NI,ND,EP,AUX,MD)	183,000
	C	INCREMENT H BY P(2) ONLY WHEN YOU ARE ALLOWING P(2) TO VARY	184,000
0079		IF(MOCHCK.EQ.1) H=H+P(2)	185,000
0080		WRITE(6,221)	186,000
0081		WRITE(6,222)	187,000
0082		DO 223 K=1,MPARAM	188,000
0083		WRITE(6,224) K,P(K),E1(K),E2(K)	189,000
0084	223	CONTINUE	190,000
0085		IF(ND) 901,901,902	191,000
0086	901	WRITE(6,905)	192,000
0087		GO TO 229	193,000
0088	902	WRITE(6,906)	194,000
0089	229	GO TO 219	195,000
	C	READ ANOTHER SET OF MD VALUES AND VARY THE PARAMETERS AGAIN	196,000
	C		197,000
	C	PRINT OUT THE FINAL RESULTS OF THE FITTING	198,000
	C		199,000
0090	999	WRITE(6,1001)	200,000
0091		WRITE(6,230) P(1)	201,000
0092		WRITE(6,225) H	202,000
0093		WRITE(6,250) P(3),E2(3),P(4),E2(4)	203,000
0094		WRITE(6,226)	204,000
0095		DO 228 K=1,NPTS	205,000
0096		Y(K)=Y(K)+1,	206,000
0097		YF(K)=YF(K)+1,	207,000
	C	THIS IS TO CORRECT FOR THE FACT THAT THE POINTS BEING FITTED WERE	208,000
	C	ACTUALLY Y(I)=1.0 AND NOT Y(I)	209,000
0098		DIFF=Y(K)-YF(K)	210,000
0099		WRITE(6,227) K,X(K),Y(K),SIGMAY(K),W(K),YF(K),DIFF,MODE	211,000
0100	228	CONTINUE	212,000
	C		213,000
	C	CALCULATE STATISTICAL DATA	214,000
	C		215,000
0101		NFREE=NPTS-MPARAM-1	216,000
			217,000

		2FIELD =',F8,2/10X,'FRACTION OF NUCLEI FEELING H =',F8,5/10X,'ATTEN	265,000
		3UATION COEFFICIENTS ;G2=',F8,5,5X,'G4=',F8,5/10X,'P(S)=',F8,5)	266,000
0140	315	FORMAT('01','STATISTICAL ANALYSIS OF RESULTS ;1/10X,'DEGREES OF FRE	267,000
		2EDUM=',F10,5/10X,'WEIGHTED AVERAGE OF VARIANCES =',F10,5/10X,'CHI	268,000
		3SQUARED=',F10,5/10X,'REDUCED CHI-SQUARED=',F10,5/10X,'VARIANCE OF	269,000
		4FIT =',F10,5)	270,000
0141	325	FORMAT('01','*****ERROR*** DEGREES OF FREEDOM = 1,13,1 CHECK FOR	271,000
		2 FAULT BEFORE CONTINUING !!)	272,000
0142	905	FORMAT('01','***** EQUATIONS WERE NOT SOLVED SUCCESSFULLY *****)	273,000
0143	906	FORMAT('01','***** EQUATIONS WERE SOLVED SUCCESSFULLY *****)	274,000
0144	910	FORMAT('11')	275,000
0145	1001	FORMAT('11','----- END OF FITTING ROUTINE, -----',//THE RESULTS OF	276,000
		2 THE FITTING ARE ,1.)	277,000
0146		END	278,000
		OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP	
		OPTIONS IN EFFECT NAME = MAIN , LINECNT = 57	
		STATISTICS SOURCE STATEMENTS = 146, PROGRAM SIZE = 6712	
		STATISTICS 008 DIAGNOSTICS GENERATED, HIGHEST SEVERITY CODE IS 0	
		8 WARNING ERRORS IN MAIN	

MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)				AUX	08-25-75	15:28:15	PAGE 0001
0001	0001	FUNCTION AUXIP,D,X,L)				279.000	
0002	0002	DIMENSION P(S), D(S)				280.000	
0003	0003	COMMON A20,A40,A290,S,DIPMOM,H,AXIS,C2(10),C4(10)				281.000	
0004	0004	N1=S+1,N2				282.000	
0005	0005	N2=S+0.6				283.000	
0006	0006	N2=N1-N2				284.000	
0007	0007	H=0.0365846*H+DIPMOM/(S*AX)				285.000	
0008	0008	Z=SINH(B*(S+.5))/SINH(B*.5)				286.000	
0009	0009	H2=0.0				287.000	
0010	0010	H=0.0				288.000	
0011	0011	DO 13 K=1,N2				289.000	
0012	0012	SK=K				290.000	
0013	0013	S=S-SK+1				291.000	
0014	0014	XX=2.*COSH(B*SM)				292.000	
0015	0015	B2=B2+C2(K)*XX				293.000	
0016	0016	B=84+C4(K)*XX				294.000	
0017	0017	IF(N0.EQ.0) GO TO 17				295.000	
0018	0018	B2=B2+C2(N1)				296.000	
0019	0019	B=84+C4(N1)				297.000	
0020	0020	B2=B2/Z				298.000	
0021	0021	B=B4/Z				299.000	
0022	0022	DDB=0.0365846*DIPMOM/(S*AX)				300.000	
0023	0023	DDB=(.5*SINH(B*(S+.5))*COSH(B*.5)+(9+.5)*SINH(B*.5)*COSH(B*(S+.5) 2)/SINH(B*(S+.5))*2)				301.000	
0024	0024	DERIV=DDB+DBDH				302.000	
0025	0025	D2=0.				303.000	
0026	0026	D4=0.				304.000	
0027	0027	DO 25 N=1,N2				305.000	
0028	0028	SK=K				306.000	
0029	0029	S=S-SK+1				307.000	
0030	0030	XX=2.*COSH(B*SM)				308.000	
0031	0031	YY=2.*SM*SINH(B*SM)				309.000	
0032	0032	YY=YY*DBDH				310.000	
0033	0033	D2=D2+(DERIV*XX+C2(K))*XX+C2(K)/Z				311.000	
0034	0034	D4=D4+(DERIV*XX+C4(K))*XX+C4(K)/Z				312.000	
0035	0035	IF(N0.EQ.0) GO TO 27				313.000	
0036	0036	D2=D2+DERIV*C2(N1)				314.000	
0037	0037	D4=D4+DERIV*C4(N1)				315.000	
0038	0038	IF(AXIS.NE.0.) GO TO 28				316.000	
0039	0039	E=43)+A20+82+P(4)+A40+B4				317.000	
0040	0040	DERIV=P(3)*A20+D2+P(4)*A40+D4				318.000	
0041	0041	D(1)=E+DERIV*P(2)				319.000	
0042	0042	D(2)=P(1)*DERIV				320.000	
0043	0043	D(3)=P(1)*A20+82+P(1)*P(2)+A20+D2				321.000	
0044	0044	D(4)=P(1)*A40+84+P(1)*P(2)+A40+D4				322.000	
0045	0045	AUX=P(1)*D(1)				323.000	
0046	0046	GO TO 30				324.000	
0047	0047	E=-0.5*P(3)*A290+82+0.375*P(4)*A490+B4				325.000	
0048	0048	DERIV=-0.5*P(3)*A290+D2+0.375*P(4)*A490+D4				326.000	
0049	0049	D(1)=E+DERIV*P(2)				327.000	
0050	0050	D(2)=P(1)*DERIV				328.000	
0051	0051	D(3)=P(1)*A290+82+P(3)*A290+D2				329.000	
0052	0052	D(4)=P(1)*A490+84+P(2)*A490+D4				330.000	
0053	0053	AUX=P(1)*D(1)				331.000	
0054	0054	RETURN				332.000	
						333.000	

Testing NOFIT:

In order to test NOFIT, the parameters f , H , U_2F_2 and U_4F_4 were least squares fitted to theoretical and pseudo-experimental data for Co^{60} . The pseudo experimental points were generated for standard deviations in the anisotropy of 0.010 and 0.020. The pseudo-experimental points had a random-normal distribution about the theoretical data. A list of the theoretical and pseudo-experimental data points for ten temperature values is given in Table C1. These values are for all nuclei ($f = 1.00$) feeling a hyperfine field of 287.7 kiloGauss.

Table C1

Theoretical and Pseudo-Experimental Data Points for Co^{60}

Temperature milli °K	Theoretical	Anisotropy * ($\theta = 0$)	
		Experimental	
		$\sigma = 0.010$	$\sigma = 0.020$
10.00	0.51905	0.513	0.506
11.63	0.58583	0.591	0.596
12.05	0.60117	0.603	0.605
14.08	0.66616	0.677	0.688
15.38	0.69483	0.695	0.694
20.00	0.79109	0.779	0.768
25.00	0.85190	0.842	0.831
29.41	0.88686	0.873	0.859
34.48	0.91414	0.910	0.907
50.00	0.95650	0.968	0.979

* These points are generated for a detector which subtends 0.055 radians, therefore the anisotropies are those which can be seen by the detector.

1) Best Fit to Theoretical Data

- a) The data of Table C1 was used to fit the values of f and H . The $U_V F_V$ values were kept fixed at the theoretical values of $U_2 F_2 = -0.42056$ and $U_4 F_4 = -0.24281$. Only f and H were allowed to vary. The order of variation was f , H , then f and H simultaneously. The results were

$$f = 0.9931 \pm 0.0119$$

$$H = 289.71 \pm 3.06 \text{ kiloGauss}$$

In this run it was necessary to give all points equal weighting, since no error was associated with the data. The results are plotted in FIGURE C1 (a).

- b) The data of Table C1 was used to determine the best fit values of $U_2 F_2$ and $U_4 F_4$. The parameters f and H were fixed at the values 1.000 and 287.7 kiloGauss respectively. Only $U_2 F_2$ and $U_4 F_4$ were allowed to vary. The order of variation was $U_2 F_2$, $U_4 F_4$, then $U_2 F_2$ and $U_4 F_4$ simultaneously. The best fit to the data was

$$U_2 F_2 = -0.42056 \times (1.00640 \pm 0.00713)$$

$$U_4 F_4 = -0.24281 \times (0.94561 \pm 0.06794)$$

All data points were given equal weighting for reasons given above. The results are plotted in FIGURE 3.1 (b).

FIGURE C1

Results of the fitting routine NOFIT on theoretical Co^{60} data for a) variation of f and H only, and b) variation of U_2F_2 and U_4F_4 only.

2) Best Fit to Pseudo-Experimental Data for Std. Dev. = 0.010

- a) The pseudo-experimental data was used to obtain the best fit values of f and H . The U_2F_2 and U_4F_4 values were -0.42056 and -0.24281 respectively. The order of parameter variation was the same as that in C.1 (a).

Best fit values to the data were

$$f = 0.9345 \pm 0.0582$$

$$H = 306.57 \pm 16.96 \text{ kiloGauss}$$

$$\chi^2 = 6.15$$

The results are plotted in FIGURE C2(a).

- b) The pseudo-experimental data was used to obtain the best fit values of U_2F_2 and U_4F_4 . The parameters f and H were fixed at 1.000 and 287.7 kiloGauss respectively. Parameter variation order was the same as in C1 (b). The best fit values to the data were,

$$U_2F_2 = -0.42056 \times (1.03726 \pm 0.03660)$$

$$U_4F_4 = -0.24281 \times (0.66264 \pm 0.34862)$$

$$\chi^2 = 7.00$$

The results are shown in FIGURE C2 (b).

FIGURE C2 Results of NOFIT on Co^{50} pseudo-experimental data for a) variation of f and H only, b) variation of U_{22}^F and U_{44}^F only.

3) Best Fit to Pseudo-Experimental Data for Std. Dev. = 0.020

- a) These pseudo-experimental data pts. were generated with a larger standard deviation ($\sigma = 0.020$) than those of section C2. The fitting procedures were identical to Section C2 (a). The best fit to the data was obtained with

$$f = 0.88001 \pm 0.11529$$

$$H = 324.6 \pm 38.1 \text{ kiloGauss}$$

$$\chi^2 = 6.65$$

The results are shown in FIGURE C3 (a).

- b) The pseudo experimental data was used to obtain the best fit values of U_2F_2 and U_4F_4 . The fitting procedure was identical to that of Section C2 (b). Best fit values were obtained with

$$U_2F_2 = (-0.42056)(1.06714 \pm 0.07319)$$

$$U_4F_4 = (-0.24281)(0.39755 \pm 0.69713)$$

$$\chi^2 = 7.00$$

The results are shown in FIGURE C3 (b).

FIGURE C3 Results of NOFIT on Co⁶⁰ pseudo-experimental data with a standard deviation of $\sigma = 0.20$.

Table C2 Tabulation of the results when f and H were varied

Temp. mili °K	W(θ) Theory		W(O) _{EXP.} $\sigma = 0.010$		W(O) _{EXP.} $\sigma = 0.020$	
	Data	Best Fit	Data	Best Fit	Data	Best Fit
10.00	0.51905	0.51879	0.513	0.523	0.506	0.526
11.63	0.58583	0.58530	0.591	0.586	0.596	0.586
12.05	0.60117	0.60056	0.603	0.601	0.605	0.600
14.08	0.66616	0.66514	0.677	0.663	0.688	0.661
15.38	0.69483	0.69958	0.695	0.697	0.694	0.693
20.00	0.79109	0.79008	0.779	0.786	0.768	0.781
25.00	0.85190	0.85102	0.842	0.845	0.831	0.842
29.41	0.88686	0.88609	0.873	0.882	0.859	0.878
34.48	0.91414	0.91351	0.910	0.910	0.907	0.907
50.00	0.95650	0.95615	0.968	0.979	0.979	0.952

One of the problems in the testing of the program NOFIT was the generation of the pseudo-experimental data points. If they were truly random, the sum of the differences between the Theoretical and pseudo-experimental anisotropies would be very close to zero. In the case of the data points generated for these tests, the sum of the differences was +0.045 for a standard deviation of 0.020. These points exhibited smaller anisotropies than the theoretical values they were generated from. For the lower temperatures these points exhibited a larger anisotropy than the theoretical data.

The effect of these pseudo-experimental points will be to generate a hyperfine field which is larger than that of the theoretical value and a fraction f which is lower than the figure of 100% used in the calculations. This effect is demonstrated in FIGURE C4.

FIGURE C4 Exaggerated effect of f and H on the fit to the data.

This effect explains the results of least squares fitting of f and H to the psuedo-experimental data. In both cases a fraction $f < 1$ and hyperfine field $H > 287.7$ kiloGauss was obtained. The effect is quite apparent in the comparison of FIGURE C2 and C3 with FIGURE C4.

Appendix D Calculation of the U_ν Coefficients

The U_ν coefficient can be written in terms of the Wigner 6J symbols as

$$U_\nu = [(2I_1 + 1)(2I_2 + 1)]^{\frac{1}{2}} (-1)^{I_1 + I_2 - L} \begin{bmatrix} I_1 & I_1 & \nu \\ I_2 & I_2 & L \end{bmatrix}$$

where the bracketed term is the SIX-J symbol. Here I_1 is the initial spin of the unobserved transition, I_2 is the final spin, and L is the multipolarity of the unobserved transition.

A listing and the output of the values used in these experiments follows.

This program is a modified version of one developed by P.W. Daly (1973) to give the angular distribution for gamma radiation from oriented nuclei.

SCOMPIL

C*****
C THIS PROGRAMME CALCULATES THE U COEFFICIENTS DESCRIBING UNOBSERVED
C PRECEDING NUCLEAR DECAYS.

C ENTER INITIAL SPIN, FINAL SPIN, L1, L2, WHERE L1 & L2 ARE THE
C MULTIPOLARITIES OF THE DECAY, (L2 DEFAULTS TO L1 + 1)

C FORMAT: 4F4,1

C*****

```

1  DIMENSION FIR(6)
2  COMMON/FAKULC/F(60)
3  WRITE(6,1)
4  1  FORMAT('1',5X,'THE UK COEFFICIENTS APPEAR AS UK(L1,LF,L1), WHERE L1
    2/15X,'L1 INITL SPIN 1/15X,'LF=FINAL SPIN1/15X,'L1=MULTIPOLARITY OF
    3THE TRANSITION')
5  2  READ(5,3,END=1000)S1,S2,EL1,EL2
6  3  FORMAT(4F4,1)
7  IF(EL2.LT.0.1)EL2=EL1+1.
8  I1=2.*S1
9  I2=2.*S2
10 L=2.*EL1
11 DO 4 I=1,2
12 IF(I.GT.1)L=2.*EL2
13 DO 5 NU=4,8,4
14 N=I+NU/2-2
15 WW=SIXJ(I1,I1,NU,I2,I2,L)
16 FIR(N)=WW*SQRT((2.*S1+1.)*(2.*S2+1.))
17 5  IF(MOD((I1+I2+L)/2,2).NE.0) FIR(N)=-FIR(N)
18 4  CONTINUE
19 WRITE(6,6)S1,S2,S1,S2,EL1,FIR(1),S1,S2,EL1,FIR(3),S1,S2,EL2,FIR(2)
20 6  FORMAT('10',F4,1,1X,F4,1,2(1X,'U2('1,F4,1,1,1,F4,1,1,1,F4,1,1)=1,F10
    2,7,2X,'U4('1,F4,1,1,1,F4,1,1,1,F4,1,1)=1,F10,7))
21 GO TO 2
22 1000 WRITE(6,7)
23 7  FORMAT('11')
24 STOP
25 END

```

C-----V,22 FUNCTION SIXJ

C WIGNER SIX-J SYMBOL, EDMONDS (6.3.7).

C-----ARGUMENTS MUST HAVE THEIR DOUBLE VALUES, FACULTIES SEE FUNCTION F

C

```

26 FUNCTION SIXJ(JJ1,JJ2,JJ3,LL1,LL2,LL3)
27 COMMON/FAKULC/F(60)
28 LOGICAL TRL,M0
29 TRL(X,L,M)=(K.GT.L+M).OR.(K.LT.IABS(L-M))
30 M0(K,L,M)=MOD(K-IABS(L-M),2).NE.0
31 DATA NN/2/
32 IF(NN.GT.1) GOTO 4.
33 1 J1=JJ1
34 J2=JJ2
35 J3=JJ3
36 L1=LL1
37 L2=LL2
38 L3=LL3
39 SIXJ=0.
40 IF(TRL(J1,J2,J3).OR.TRL(J1,L2,L3).OR.TRL(L1,J2,L3).OR.TRL(L1,L2,J3)

```

SIXJ0001

SIXJ0002

SIXJ0003

SIXJ0004

SIXJ0005

SIXJ0006

SIXJ0007

SIXJ0008

SIXJ0009

SIXJ0010

SIXJ0011

SIXJ0012

SIXJ0013

SIXJ0014

SIXJ0015

SIXJ0016

SIXJ0017

SIXJ0018

SIXJ0019

41	1) RETURN	SIXJ0020	64
42	IF(MO(J1,J2,J3).OR,MO(J1,L2,L3).OR,MO(L1,L2,J3)) RETURN	SIXJ0021	65
43	I1=J1+J2+J3-2	SIXJ0022	66
44	I2=J1+L2+L3-2	SIXJ0023	67
45	I3=L1+J2+L3-2	SIXJ0024	68
46	I4=L1+L2+J3-2	SIXJ0025	69
47	N1=J1+J2+L1+L2+2	SIXJ0026	70
48	N2=J2+J3+L2+L3+2	SIXJ0027	71
49	N3=J3+J1+L3+L1+2	SIXJ0028	72
50	MINZ=MAXO(I1,I2,I3,I4)+2	SIXJ0029	73
51	MAXZ=MINO(N1,N2,N3)-2	SIXJ0030	74
52	IF(MAXZ+2-G1*60) GOTO-12	SIXJ0031	75
53	NA1=J1+J2+J3+4	SIXJ0032	76
54	NA2=J1+J2+J3+1	SIXJ0033	77
55	NA3=J1+J2+J3+1	SIXJ0034	78
56	NA4=J2-J1+J3+1	SIXJ0035	79
57	NB1= J1+L2+L3+4	SIXJ0036	80
58	NB2= J1+L2+L3+1	SIXJ0037	81
59	NB3= J1+L2+L3+1	SIXJ0038	82
60	NB4= J1+L2+L3+1	SIXJ0039	83
61	NC1= L1+L2+J3+4	SIXJ0040	84
62	NC2= L1+L2+J3+1	SIXJ0041	85
63	NC3= L1+L2+J3+1	SIXJ0042	86
64	NC4= L1+L2+J3+1	SIXJ0043	87
65	ND1= L1+J2+L3+4	SIXJ0044	88
66	ND2= L1+J2+L3+1	SIXJ0045	89
67	ND3= L1+J2+L3+1	SIXJ0046	90
68	ND4= L1+J2+L3+1	SIXJ0047	91
69	U=0.125*(-1)**MOD(MINZ/2,2)*SQRT(F(NA1)*F(NA2)*F(NA3)*F(NA4)	SIXJ0048	92
70	1*F(NB1)*F(NB2)*F(NB3)*F(NB4)*F(NC1)*F(NC2)*F(NC3)*F(NC4)-F(ND1)	SIXJ0049	93
71	2 *F(ND2)*F(ND3)*F(ND4))	SIXJ0050	94
72	DO 11 N=MINZ,MAXZ,2	SIXJ0051	95
73	NA1=N-11	SIXJ0052	96
74	NA2=N-12	SIXJ0053	97
75	NA3=N-13	SIXJ0054	98
76	NA4=N-14	SIXJ0055	99
77	NB1=N1-N	SIXJ0056	100
78	NB2=N2-N	SIXJ0057	101
79	NB3=N3-N	SIXJ0058	102
80	SIXJ=SIXJ+U*(N+3)*F(NA1)*F(NA2)*F(NA3)*F(NA4)*F(NB1)*F(NB2)*F(NB3)	SIXJ0059	103
81	1)	SIXJ0060	104
82	11-U=U	SIXJ0061	105
83	RETURN	SIXJ0062	106
84	4 X=1.	SIXJ0063	107
85	Y=1.25	SIXJ0064	108
86	DO 5 L=1,59,2	SIXJ0065	109
87	F(L)=X	SIXJ0066	110
88	F(L+1)=1./X	SIXJ0067	111
89	X=X*Y	SIXJ0068	112
90	5 Y=Y*1.25	SIXJ0069	113
91	NN=0	SIXJ0070	114
92	GOTO 1	SIXJ0071	115
	12 WRITE(6,6) MAXZ	SIXJ0072	116
	5-FORHAT(-16,26H-15-OUT-OF-RANGE-OF-PAK(N) /IHO119(IH*))	SIXJ0073	117
	RETURN	SIXJ0074	118
	END	SIXJ0075	119
	SDATA		

THE UK COEFFICIENTS APPEAR AS UK(LI,LF,L1), WHERE
 LI=INITIAL SPIN
 LF=FINAL SPIN
 L1=MULTIPOLARITY OF THE TRANSITION

7.0	5.0	U2(7.0, 5.0, 2.0)= 0.9285535	U4(7.0, 5.0, 2.0)= 0.7718214	U2(7.0, 5.0, 3.0)= 0.7196288	U4(7.0, 5.0, 3.0)= 0.1929555
7.0	4.0	U2(7.0, 4.0, 3.0)= 0.8722591	U4(7.0, 4.0, 3.0)= 0.6157001	U2(7.0, 4.0, 4.0)= 0.5451620	U4(7.0, 4.0, 4.0)= -0.1539254
7.0	3.0	U2(7.0, 3.0, 4.0)= 0.7891173	U4(7.0, 3.0, 4.0)= 0.4195294	U2(7.0, 3.0, 5.0)= 0.2959194	U4(7.0, 3.0, 5.0)= -0.4544905
6.0	7.0	U2(6.0, 7.0, 1.0)= 0.9692257	U4(6.0, 7.0, 1.0)= 0.8974037	U2(6.0, 7.0, 2.0)= 0.8480727	U4(6.0, 7.0, 2.0)= 0.5234646
6.0	5.0	U2(6.0, 5.0, 1.0)= 0.9580334	U4(6.0, 5.0, 1.0)= 0.8600592	U2(6.0, 5.0, 2.0)= 0.7937989	U4(6.0, 5.0, 2.0)= 0.3685972
6.0	4.0	U2(6.0, 4.0, 2.0)= 0.8999511	U4(6.0, 4.0, 2.0)= 0.6660888	U2(6.0, 4.0, 3.0)= 0.6106812	U4(6.0, 4.0, 3.0)= -0.0490061
5.0	7.0	U2(5.0, 7.0, 2.0)= 0.9285535	U4(5.0, 7.0, 2.0)= 0.7718214	U2(5.0, 7.0, 3.0)= 0.7196288	U4(5.0, 7.0, 3.0)= 0.1929552
5.0	6.0	U2(5.0, 6.0, 1.0)= 0.9580334	U4(5.0, 6.0, 1.0)= 0.8600585	U2(5.0, 6.0, 2.0)= 0.7937996	U4(5.0, 6.0, 2.0)= 0.3685969
5.0	5.0	U2(5.0, 5.0, 1.0)= 0.8999985	U4(5.0, 5.0, 1.0)= 0.6666643	U2(5.0, 5.0, 2.0)= 0.7102544	U4(5.0, 5.0, 2.0)= 0.1536454
5.0	4.0	U2(5.0, 4.0, 1.0)= 0.9393724	U4(5.0, 4.0, 1.0)= 0.7977214	U2(5.0, 4.0, 2.0)= 0.7045276	U4(5.0, 4.0, 2.0)= 0.1324539
5.0	3.0	U2(5.0, 3.0, 2.0)= 0.8498341	U4(5.0, 3.0, 2.0)= 0.5435554	U2(5.0, 3.0, 3.0)= 0.4249168	U4(5.0, 3.0, 3.0)= -0.3623703
4.0	4.0	U2(4.0, 4.0, 1.0)= 0.8499979	U4(4.0, 4.0, 1.0)= 0.4999982	U2(4.0, 4.0, 2.0)= 0.5733747	U4(4.0, 4.0, 2.0)= -0.1493499
4.0	3.0	U2(4.0, 3.0, 1.0)= 0.9046819	U4(4.0, 3.0, 1.0)= 0.6813838	U2(4.0, 3.0, 2.0)= 0.5428065	U4(4.0, 3.0, 2.0)= -0.2271280
4.0	2.0	U2(4.0, 2.0, 2.0)= 0.7491475	U4(4.0, 2.0, 2.0)= 0.2847196	U2(4.0, 2.0, 3.0)= 0.0749144	U4(4.0, 2.0, 3.0)= -0.5694390
3.0	2.0	U2(3.0, 2.0, 1.0)= 0.8280776	U4(3.0, 2.0, 1.0)= 0.4178545	U2(3.0, 2.0, 2.0)= 0.2070192	U4(3.0, 2.0, 2.0)= -0.6267818
2.0	0.0	U2(2.0, 0.0, 2.0)= 0.0000000	U4(2.0, 0.0, 2.0)= 0.0000000	U2(2.0, 0.0, 3.0)= 0.0000000	U4(2.0, 0.0, 3.0)= 0.0000000

Appendix E Hyperfine Distribution Fitting Program HYPFIT

This program fits, by the method of least squares, experimental data to a Gaussian distribution of hyperfine fields. The anisotropy for such a distribution is given in Equations (3.4) and (3.5). A variable fraction of nuclei which felt this distribution of hyperfine fields was also included. This allowed the distribution centroid, distribution width, and fraction of nuclei feeling the distribution to be determined from the data.

A listing of this program is given.

```

C***** HYPERFINE FIELD DISTRIBUTION, *****
C
C THIS ROUTINE FITS, BY LEAST SQUARES, A GAUSSIAN HYPERFINE FIELD
C DISTRIBUTION TO NUCLEAR ORIENTATION DATA ( W(THETA), TEMP. ).
C A FRACTION P(1) OF THE NUCLEI FEEL A GAUSSIAN FIELD DISTRIBUTION
C CENTERED ABOUT A MEAN FIELD P(2) OF WIDTH P(3), THE REMAINING
C NUCLEI ARE ASSUMED TO FEEL ZERO FIELD,
C
C JOB CARDS IN ORDER ARE:
C
C ENTER TEMP(MILLI DEGREES KELVIN), W(0), ERROR IN W(0)
C FORMAT: 3F6,3
C ENTER INITIAL ESTIMATES OF THE PARAMETERS:
C P(1)= FRACTION OF NUCLEI FEELING HYPERFINE DISTRIBUTION,
C P(2)= MEAN VALUE OF THE GAUSSIAN FIELD DISTRIBUTION IN
C KILOGAUSS,
C P(3)= FIELD WIDTH IN KILOGAUSS,
C FORMAT: 3F10,5
C ENTER MAXIMUM NUMBER OF ITERATIONS, EPSILON (DELTA P/P)
C FORMAT: I3,F5,3
C ENTER MD VALUES, A / HOLDS THE PARAMETER FIXED AND THE U ALLOWS THE
C PARAMETER TO VARY,
C FORMAT: 3I1
C SEND TERMINATES THE PROGRAMME, TWO REQUIRED,
C
C WRITTEN BY P. R. H. MC CONNELL, JULY 23, 1975
C
0001 EXTERNAL AUX
0002 DIMENSION X(17),Y(17),YF(17),DELTAY(17),WT(17),E1(3),E2(3),P(3),MD
2(3)
0003 DATA N,M,WZ,A2/17,3,1,0,0,27935/
0004 DO 10 I=1,17
0005 READ(5,101)X(I),Y(I),DELTAY(I)
0006 101 FORMAT(3F6,3)
0007 Y(I)=(Y(I)-1,0)/A2
0008 WT(I)=1,0/DELTAY(I)**2
0009 10 CONTINUE
0010 READ(5,102)(P(I),I=1,3)
0011 102 FORMAT(3F10,5)
0012 READ(5,103)NI,EPS
0013 103 FORMAT(I3,F5,3)
0014 WRITE(6,104)(P(I),I=1,3),NI,EPS
0015 104 FORMAT('1', 'INITIAL PARAMETER VALUES', // '5X', 'FRACTION OF NUCLEI=', F
26,4/5X, 'MEAN FIELD=', F6,2, ' KILOGAUSS', /5X, 'FIELD WIDTH=', F6,2, ' KI
3LOGAUSS' // 'NUMBER OF ITERATIONS ALLOWED (MAX)=', I3, 'EPSILON=', F6,4
4)
0016 11 READ(5,105,END=14)(MD(I),I=1,3)
0017 105 FORMAT(3I1)
0018 CALL DPLQF(X,Y,YF,WT,E1,E2,P,WZ,N,M,NI,ND,EPS,AUX,MD)
0019 WRITE(6,106)
0020 106 FORMAT('01',8X,'P(1)',16X,'E1(1)',15X,'E2(1)')
0021 DO 12 I=1,3
0022 12 WRITE(6,107)P(I),E1(I),E2(I)
0023 107 FORMAT('1',3(5X,G15,5))
0024 IF(ND)13,13,11

```


0025	13	WRITE(6,108)	55,000
0026	108	FORMAT('0','***WARNING** EQUATIONS IN THE PROGRAMME DPRLQF WERE	56,000
		2NOT SOLVED SUCCESSFULLY, /14X, 'PROGRAMME WILL CONTINUE USING THE N	57,000
		3EW PARAMETER VARIATIONS MD,1)	58,000
0027		GO TO 11	59,000
	C		60,000
	C	PRINT OUT THE RESULTS OF THE LEAST-SQUARES FIT,	61,000
	C		62,000
0028	14	WRITE(6,109)	63,000
0029	109	FORMAT('1','THE HYPERFINE FIELD DISTRIBUTION RESULTS FOR B1=206/M1	64,000
		2 USING THE 1719 KEV DATA,')	65,000
0030		WRITE(6,110)	66,000
0031	110	FORMAT('1',///'TEMP MILLI K',2X,'N EXP',4X,'N FIT',3X,'NEXP=NFIT')	67,000
0032		CHISQ=0.0	68,000
0033		DO 15 I=1,17	69,000
0034		YF(I)=YF(I)*A2+1.0	70,000
0035		Y(I)=Y(I)*A2+1.0	71,000
0036		DIFF=Y(I)-YF(I)	72,000
0037		WRITE(6,111)*X(I),Y(I),YF(I),DIFF	73,000
0038	111	FORMAT('0',3X,F5,2,5X,F6,3,3X,F6,3,4X,F6,3)	74,000
0039		CHISQ=CHISQ+DIFF**2/DELTAY(I)**2	75,000
0040	15	CONTINUE	76,000
0041		A=PCHI(CHISQ,13,0)	77,000
	C ***	CHECK FOR INTEGER OR FP 13 IN CHISQ,	78,000
0042		WRITE(6,112)*CHISQ,A	79,000
0043	112	FORMAT('0','STATISTICS,///'DEGREES OF FREEDOM=13/'CHI-SQUARED = '	80,000
		2,F6,2/'CHI-SQUARED PROBABILITY='E13,0)	81,000
0044		WRITE(6,113)*(P(I),I=1,3)	82,000
0045	113	FORMAT('0','PARAMETER VALUES,///'FRACTION F='F7,5/'MEAN FIELD='F	83,000
		27,3,' KILOGAUSS,/'FIELD WIDTH OF GAUSSIAN DISTRIBUTION='F7,3,' KI	84,000
		3LOGAUSS')	85,000
0046		STOP	86,000
0047		END	87,000
OPTIONS IN EFFECT IO,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP			
OPTIONS IN EFFECT NAME = MAIN , LINECNT = 57			
STATISTICS SOURCE STATEMENTS = 47, PROGRAM SIZE = 2454			
STATISTICS NO-DIAGNOSTICS-GENERATED			
NO ERRORS IN MAIN			

MICHIGAN TERMINAL SYSTEM FORTRAN G(41356)			AUX	08-08-75	15124100	PAGE P001
0001	FUNCTION AUX(P,D,X,L)					86.000
0002	IMPLICIT REAL*8(A-H,O-Z)					89.000
0003	REAL*4 P(3),D(3),AUX,X					90.000
	C ALL CALCULATIONS WITHIN THIS SUBPROGRAMME ARE IN DOUBLE PRECISION					91.000
	C					92.000
0004	COMMON/8L7C(13)					93.000
0005	IDELTA=10					94.000
0006	DELTAH=10.0D0					95.000
0007	BTWO=0.0D0					96.000
0008	DONE=0.0D0					97.000
0009	DTWO=0.0D0					98.000
0010	DTHREE=0.0D0					99.000
0011	DIPNOM=4.56D0					100.000
0012	SPIN=6.0D0					101.000
0013	DBOH=0.0365846*DIPNOM/(SPIN**3)					102.000
	C THE FACTOR IDBOH IS THE DERIVATIVE OF BETA W-R-T= H					103.000
	C					104.000
	C					105.000
	C					106.000
	C SUM OVER THE FIELD INCREMENTS HI.					107.000
	C					108.000
0014	DO 10 I=300,500,IDELETA					109.000
0015	HI=DFLOAT(I)					110.000
0016	BETA=DSINH(HI)					111.000
0017	PHI=DSORT(5,D0/6,28318)*DEXP(-0.5*(HI-P(2))*2/P(3)**2)					112.000
0018	SHI=DSINH(0.5*BETA)/DSINH(6.5*BETA)					113.000
0019	BSUM=0.0					114.000
0020	DISUM=0.0					115.000
0021	D2ISUM=0.0					116.000
0022	D2SUM=0.0					117.000
0023	D3ISUM=0.0					118.000
0024	D3SUM=0.0					119.000
	C SUM OVER THE MAGNETIC SUBSTATES, M=-6 TO M=6.					120.000
	C					121.000
0025	DO 20 J=1,13					122.000
0026	M=J-7					123.000
0027	U=(0.5*DCOSH(0.5*BETA)/DSINH(6.5*BETA))*(M*SHI)*(6.5*SHI*DCOSH(6.5					124.000
	1*BETA)/DSINH(6.5*BETA))					125.000
0028	CHI=C(J)*DEXP(BETA*M)					126.000
0029	B2=SHI*CHI/P(3)					127.000
0030	D1=B2					128.000
0031	D21=((P(2)-HI)/P(3))*3*SHI*CHI					129.000
0032	D22=((1.0/P(3))*CHI*U*DBDH					130.000
0033	D31=((1.0/P(3))*2)*B2					131.000
0034	D32=((HI-P(2))*2/P(3))*3*B2					132.000
0035	BSUM=BSUM+B2					133.000
0036	DISUM=DISUM+D1					134.000
0037	D2ISUM=D2ISUM+D21					135.000
0038	D2SUM=D2SUM+D22					136.000
0039	D3ISUM=D3ISUM+D31					137.000
0040	D3SUM=D3SUM+D32					138.000
0041	CONTINUE					139.000
0042	BTWO=BTWO+BSUM*PHI					140.000
	20					141.000
	C					142.000

0043	DONE=DONE+DISUM*PHI	143,000
0044	DTWO=DTWO+(D21SUM+D22SUM)*PHI	144,000
0045	DTHREE=DTHREE+(D31SUM+D32SUM)*PHI	145,000
0046	10 CONTINUE	146,000
0047	AUX=P(1)*BTWO*DELTAH	147,000
0048	D(1)=DUNE*DELTAH	148,000
0049	D(2)=P(1)*DTWO*DELTAH	149,000
0050	D(3)=P(1)*DTHREE*DELTAH	150,000
0051	RETURN	151,000
0052	END	152,000

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP

OPTIONS IN EFFECT NAME = AUX , LINECNT = 57

STATISTICS SOURCE STATEMENTS = 52, PROGRAM SIZE = 1846

STATISTICS NO DIAGNOSTICS GENERATED

NO ERRORS IN AUX

MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)

BLK DATA

08-08-75

15:24:02

PAGE P001

0001
0002
0003

BLOCK DATA
COMMON/BL/C

REAL*8 C(13)/0.7928241,0.3964120,0.0720749,-0.1801874,-0.3603744,-
20.4684858,-0.5045245,-0.4684858,-0.3603744,-0.1801874,0.0720749,0,
33964120,0.7928241/

153,000
154,000
155,000
156,000
157,000
158,000

0004

END

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP

OPTIONS IN EFFECT NAME = BLK DATA, LINECNT = 57

STATISTICS NO DIAGNOSTICS GENERATED

NO ERRORS IN BLK DATA

NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.
EXECUTION TERMINATED

\$RUN HYPFIT
EXECUTION BEGINS

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