DEVELOPMENT OF A FORECASTING MODEL FOR DEPOSITS OF CREDIT UNIONS

by

CRISTOBAL TADEUSZ DOBRZANSKI
B.A., Sir George Williams University, 1972
M.A., University of British Columbia, 1973

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Abstract

The purpose of this thesis is to develop a forecasting model to predict demand deposits and term deposits of credit unions. It begins with a survey of the literature on demand functions for liquid assets. Both single equation models and simultaneous equation systems are summarized. The hypothesis for a likely structural model of credit union financial behaviour is also presented. However, a structural model cannot be estimated because there are no published data on credit unions' interest rates and there is a limited number of observations for the dependent variable.

The forecasting technique that is being developed in this thesis is an application of time series analysis. The basic idea behind this approach is to express the time series of demand deposits and of term deposits as a weighted sum of the past values of deposits. The weights in the sum are determined so as to achieve the greatest predictive power by minimizing the mean square error of the forecasts. The data are quarterly time series for demand deposits and for term deposits for each of three credit unions in the Vancouver Region in British Columbia from the second quarter of 1962 to the fourth quarter of 1974. The data are printed in the Appendix.

The strength of the mixed autoregressive moving average process (ARIMA) as a forecasting tool for financial intermediaries such as a credit union is evaluated by using a large sample of monthly data of personal demand deposits and personal term deposits of Canadian chartered banks. The best models for each credit union's demand deposits and term deposits are matched against the naive model of a random walk process. They are compared with respect to their minimum mean square error of prediction for the four quarters of 1974.

For both the three credit unions and the chartered banks, in all cases the best ARIMA model outperformed all other candidates.

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I. Introduction

Financial intermediaries are firms that are primarily engaged in borrowing funds (savings) from households and businesses and in lending funds (loans) to other households and businesses. In the process of carrying out these transactions they face the likelihood of withdrawals of savings and the risk of default on loans. The old approach to this liquidity problem was to balance the expected turnover in liabilities with the maturity of assets. The percentages of total funds held in short-term loans, consumer loans and mortgages would then be similar to the percentages of liabilities in savings deposits, term deposits and capital funds respectively. This method neither maximizes the return on invested funds nor takes advantage of diversification in the savings portfolio.

To benefit from both factors the institution must engage in a dynamic financial management process similar to the one illustrated in Figure I:1 (Cramer and Miller (1973)). Using statistical information on interest rates, demand for loans, and demand for savings deposits and share capital, the decision maker would apply an optimization technique to decide on the best mix of loans to issue and on the least cost combination of savings to attract. The decisions to commit funds today for one, five, or ten years hence are based on forecasts of interest rates, loan demand, and deposit levels. To forecast each of these three factors for a particular financial intermediary involves a sizeable study of time series, models and techniques. In this thesis we will focus our attention on the development of a forecasting technique to predict the demand for deposits for credit unions.

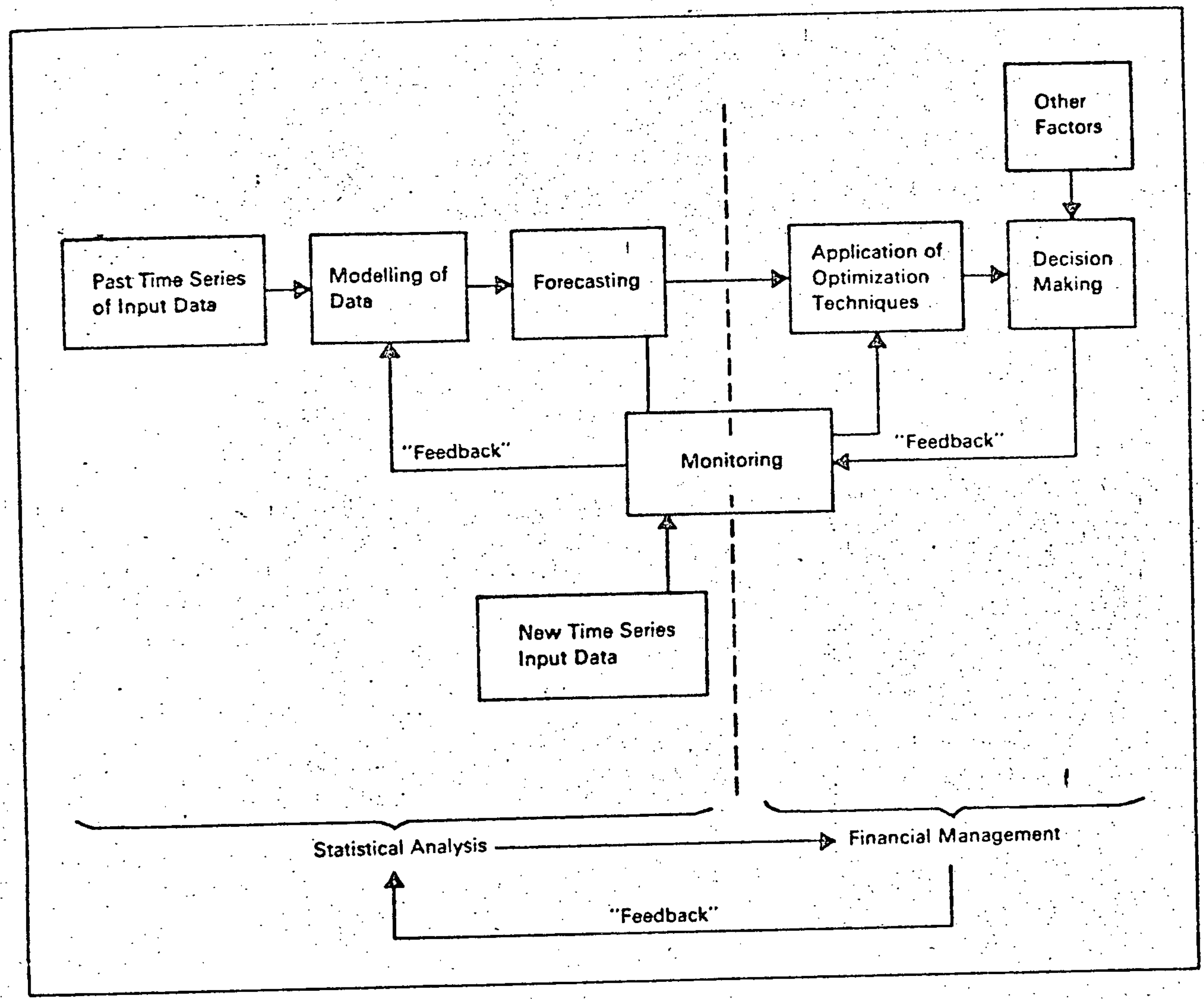
A credit union is a cooperative institution that provides financial services similar to those of other financial intermediaries such as chartered banks and trust companies. It operates as an autonomous unit that has few branch operations and deals only with its members. The borrowers and the lenders are the shareholders and owners of the assets. The credit union faces the same liquidity problem of financial intermediation and an optimal allocation of the credit union's resources is made through the same dynamic financial management process (Figure I.1). Although the essential difference between a credit union and other financial institutions is the former's cooperative philosophy, they all have the forecasting problem of estimating future levels of both demand and term deposits.

The traditional approach to the development of a forecasting model for deposits is to use economic theory of demand for liquid assets in order to formulate their demand equations. This is called a structural equation because it uses predetermined variables representing price of deposits, incomes of consumers, tastes and preferences of consumers, and prices of substitutable liquid assets. Whereas this demand equation models the behaviour of households, the single equation approach does not hold for financial institutions. The latter has control over the price of deposits causing the structural equation to have a current endogenous variable and the demand for deposits can no longer be estimated by a single equation.

The structural model must be expanded into two or more equations in order to capture the two-step decision making process of a financial intermediary (a credit union). In the first step, the credit union sets the interest rates on demand and term deposits. In the second step, there is a stochastic

Figure I 1

Dynamic Financial Management Process



movement in the level of deposits in response to the new interest rates. This adjustment in the liabilities will create a feedback to the decisions taken in the first stage. The interest rates may have to change again depending on competitive market conditions, consumer preferences, or because an unfavourable portfolio structure warrants it (i.e. unfavourable liquidity position whereby interest payments are rising faster than interest revenues). In other words the interrelations among the interest rates of the credit union and the level of deposits must be expressed as a system of simultaneous equations.

The strength of either the single equation model or the simultaneous equation system can only be evaluated empirically. To test the hypothesised models, one must have a sufficient number of observations and time series for all variables. However, the only data available at the time of this study are quarterly series on deposits from 1962 to 1974. There are no published quarterly data on credit unions' interest rates and there are no proxies for interest rates paid on credit unions' demand and term deposits for the 1962-1966 period. Therefore we cannot meaningfully test the structural approach because of missing data and limited number of degrees of freedom.

Another approach is to use time series analysis to formulate a forecasting model for credit unions' deposits. The method is data oriented because it incorporates economic information through subjective decisions made in modelling the time series of deposits. We assume that there exists a basic underlying pattern for the series of demand deposits and term deposits. This pattern is expressed as a weighted sum of past values of these variables where the weights in the sum are determined so as to achieve the greatest predictive power (i.e. minimize the forecasting error).

The analysis involves three stages: (1) identify the series as a stationary autoregressive process, a moving average process or a mixed autoregressive moving average process; (ii) estimate the parameters in the model just identified and verify if it is adequate; and (iii) forecast future values for the series of deposits. The data are quarterly time series for demand deposits and for term deposits for each of three credit unions in the Vancouver Region of British Columbia from the second quarter of 1962 to the fourth quarter of 1974. The best models for each credit union's demand deposits and term deposits are matched against the naive model of a random walk process. They are all compared with respect to the minimum mean square error of prediction for the four quarters of 1974.

To obtain "a priori" identification of credit union's series, but more important to evaluate the strength of the time series method used, we also examined a larger sample of monthly data for personal demand deposits and personal term deposits held in chartered banks in Canada (1967:9 - 1974:11, 87 observations).

The thesis begins with a survey of the literature on demand functions for liquid assets. The hypothesis for a likely structural model of credit union financial behaviour is also presented. Time series analysis is developed theoretically in Chapter III using the notation of Box and Jenkins (1970) for a mixed autoregressive integrated moving average process. In Chapter IV the data for demand deposits and term deposits of the three credit unions and for personal deposits of chartered banks are discussed and the models are estimated and evaluated. Some concluding remarks are presented in Chapter V. Finally the Appendix:Data lists the natural logarithms and the raw data for the deposits of both credit unions and chartered banks.

II. Survey of the Literature

There are various representations of the demand for demand deposits and term deposits and they depend upon the assumptions made about economic behaviour of individuals (or institutions) and the level of aggregation in the data. The literature is grouped into (a) single equation models, and (b) simultaneous equation systems; and analysed with respect to the economic theory, applicability of structural equations to financial intermediation in Canada, and problems with the data and estimation.

A. Single Equation Models

Classical demand theory states that the demand for a good or a service is determined by: its own price, consumers' incomes, consumers' tastes and preferences, and prices of substitute goods or services. Feige (1964) uses this hypothesis to estimate the demand for "demand deposits, a non-pecuniary flow of services that provide the owner with liquidity, salability, safety and convenience. Since the value of the stream of services cannot be observed the value of the stock is used as a proxy. The assumption is made that there exists a fixed relationship between the stock and the flow of services rendered by a given stock of demand deposits and hence the demand for non-precuniary services is equivalent to the demand for demand deposits.

Its own price (R_{dd})is the sum of the nominal interest rate (zero) and the positive service charges. R_{dd} is negative and is defined as total service charges divided by the average balance of demand deposits. Consumers' incomes are a weighted average of past and present values of personal income, where weights are those developed by Friedman (1957) to represent permanent personal income (Y_p). Tastes are assumed to be given and to remain constant over time but preferences are proxied by the per capita number of offices of financial institutions (#/Pop) to measure convenience (time-space utility provided by location). Finally, the prices of substitutes are the actual interest rates paid on: commercial bank time deposits (R_{td}); savings and loan association shares (R_s) and on mutual savings bank deposits (R_m). The actual rate is defined as total interest paid divided by the average size of assets and it represents the true opportunity cost faced by the holder of wealth.

Using a linear form, the demand function for demand deposits is estimated by using ordinary least squares. The expected signs for R_{dd}, Y_p, A, and #/Pop are positive, while the expected signs for the coefficients of substitues are negative. The data is a pooling of cross-section and time series observations from 1949 to 1959 (U.S.). In the best equation (2.1) Feige found R_{dd}, Y_p and R_{td} to be significant and to have the expected sign where $\frac{DD}{Pop}$, the dependent variable, is per capita commercial bank demand deposits (Feige, 1964, p. 24).

$$(2.1)\quad \frac{DD}{Pop} = \underset{(48)}{535R_{dd}} + \underset{(.080)}{.365Y_p} - \underset{(13)}{35R_{td}} + \underset{(13)}{53R_s} + \underset{(15)}{25R_m} + \text{regional dummies} \qquad R^2 = .98$$

$$(2.2)\quad \frac{TD}{Pop} = -\underset{(87)}{101R_{dd}} + \underset{(.037)}{.122Y_p} + \underset{(10)}{76R_{td}} - \underset{(10)}{44R_s} - \underset{(11)}{82R_m} + \text{regional dummies} \qquad R^2 = .94$$

For per capita commercial bank time deposits $\frac{(TD)}{Pop}$, R_{dd} is now a price of a substitute (expected sign negative) and R_{td} is the own price (expected sign positive). In this equation (2.2) all coefficients are statistically significant and have the right sign.

In a more recent study, Boyd (1973) uses the same theory but makes an explicit assumption about imperfect competition: that there exists product differentiation among deposits of various financial intermediaries because of minimum balances and minimum terms to earn interest. In his study of savings and loan associations, advertising is now introduced along with the classical determinants of demand. The functional form assumes that each variable affects per capita demand deposits as an exponential growth (decay) and that "given a change in the desired level of deposits, (individual) savers will quickly adjust their account balance to the new equilibrium" (Boyd, 1973, p. 746). The results for the cross-section sample for January 1969, using semi-annual data, are presented below in equations (2.3) and (2.4) where DD/Pop, demand deposits per capita; R_{dd} average DD interest rate; Y/Pop, 12 month average of per capita personal disposable income; A/Pop, per capita promotional expenses; $\frac{\#offa}{\#offc}$, ratio of number of associations' branches to number of competitors' branches; R_{td}, average rate on term deposits; R_b, average competitor's savings rate (banks); and TD/Pop, term deposits per capita.

$$(2.3)\ \ln\frac{DD}{Pop} = 5.8 + \underset{(1.24)}{3.50}\ln R_{dd} + \underset{(.34)}{.51}\ln\frac{Y}{Pop} - \underset{(3.73)}{3.33}\ln R_{td} + \underset{(.10)}{.57}\ln\frac{A}{Pop} + \underset{(.24)}{.55}\ln\frac{\#offa}{\#offc} - \underset{(1.66)}{2.22}\ln R_b + \text{regional dummies} \qquad R^2 = .60$$

$$(2.4)\ \ln\frac{TD}{Pop} = -15.93 - \underset{(3.20)}{4.92}\ln R_{dd} + \underset{(7.00)}{17.78}\ln R_{td} + \underset{(.17)}{.69}\ln\frac{A}{Pop} + \text{regional dummies} \qquad R^2 = .67$$

In the demand deposit equation all the coefficients have the expected sign and only own price (R_{dd}), tastes and preferences ($\frac{\#offa}{\#offc}$), and advertising ($\frac{A}{Pop}$) are significant determinants. In the cross-section equation for TD/Pop R_{td} and A/Pop are statistically significant. The other variables were used but they never entered significantly in (2.4) and their estimates were not published, unfortunately, because Boyd (1973, p. 741) admits that R_b was of the wrong sign and significant (probably due to misspecification bias).

Finally, Boyd tests his hypothesis that consumers' instantaneously adjust their deposits to changes in deposit rates. If transaction costs, imperfect information, etc., invalidate that assumption, the regression model is misspecified and empirical estimates may be biased." (Boyd, 1973, p. 746). Five year averages are calculated for the independent variables. They are then added as additional variables into the original equation for the reason is that if demand for deposits adjusts partially over time then the averages should be significant because they represent the values of the independent variables over time. The results for demand deposits and term deposits are not impressive as only the new average of advertising per capita proves to be significant and some variables have the wrong sign (probably due to multicollinearity caused by his specification). Thus Boyd concludes that depositors respond fully to the changes in the economic environment that take place within the six month interval of his data (semi-annual observation points).

In a paper by Motley (1970), he assumes that households are unable or unwilling to adjust asset holdings instantaneously to desired long-run levels. The desired asset level is a function of expected income (Y*), rates of return on all assets (vector R), and the implicit rentals on durable goods (μ).

(2.5) $$TD^* = f(Y^*, R_{td}, R_{dd}, R_s, \ldots, \mu)$$

The form of the function is linear in the logarithms and the demand for assets (at constant prices) is homogeneous of degree zero in general price level and unit elastic with respect to population

(2.6) $$\log TD^* = \alpha_o + \alpha_1 \log Y^* + \sum_{j=1}^{n} \alpha_{2j} \log R_j$$

"A constant proportion of any relative divergence between actual and desired stock of (term deposits) is corrected in each period (and) may be approximated by" (Motley, 1970, p. 236)

$$\text{2.7} \qquad \frac{TD_t}{TD_{t-1}} = \left[\frac{TD_t^*}{TD_{t-1}}\right]^{\lambda_i} \qquad \lambda > 0$$

where λ, the desired rate of adjustment, depends upon the change in stock of both term deposits and all other assets in the portfolio, ratio of current to expected income and some non-quantifiable liquidity preference and expectations parameter. For example more lucrative interest rates on term deposits will increase the demand and level of TD^*. This will draw funds away from securities that are substitutes and it will affect the latter's market equilibrium (their market interest rates and their quantities held). The readjusting in the portfolio is depicted by the following adjustment process:

$$\text{(2.8)} \qquad \log TD_t - \log TD_{t-1} = \lambda(\log TD_t^* - \log TD_{t-1}) + \sum_{j=1}^{n} \lambda_j (\log S_{jt}^* - \log S_{jt-1}) + \mu_i(\log Y_t - \log Y_t^*)$$

Substituting for desired levels of assets (TD^*, S_i^*) in (2.8), we obtain n seemingly unrelated equations.

$$\text{(2.9)} \qquad \log S_t = A + B\log Y_t^* + \Gamma \log R_t - (I - \Lambda) \log S_{t-1} + M \log (Y_t - Y_t^*)$$

where A, B, and M are n-vectors and Γ, Λ are nxm matrices and in particular the equation for term deposits is:

$$\text{(2.10)} \quad \log TD_t = \alpha_0 + \sum_j^n \lambda_j \alpha_1^j \log Y_t^* + \sum_j^n \lambda_j \sum_k^n \alpha_2^{jk} \log R_{kt} + (1-\lambda)\log TD_{t-1} - \sum_{j \neq 1}^{p} \lambda_{ij} \log S_{jt-1} + \mu(\log Y - \log Y^*)$$

The coefficients are estimated using non-linear techniques with only four other assets (i.e., n = 4). They are: money (M), savings deposits (TD), debt (D), and real assets (RA). Expected income Y^*) is defined as a geometrically weighted average of personal disposable income where the weights are those used by Friedman (1957). The significant determinants of savings deposits in (2-11) are: transitory income $(Y-Y^*)_t$, and lagged holdings (TD_{t-1}). These results illustrate the partial adjustment process but not the reallocation of funds in the portfolio, for quarterly U.S. data between 1953 and 1965.

$$(2.11)\quad \log TD_t = 5.72 + \underset{(.30)}{.14} \log Y_t^* + \underset{(.04)}{.03} \log R_{td,t} - \underset{(.13)}{.36} \log TD_{t-1} - \underset{(.04)}{.00} \log M_{t-1} - \underset{(.13)}{.23} \log D_{t-1} - \underset{(.40)}{.14} \log RA_{t-1} + \underset{(.08)}{.28} \log (Y-Y^*)_t$$

Batra (1973) uses the same formulation as Motley. The assets are interdependent inasfar as they compete with one another in the financial portfolio. Changes in the portfolio at any point in time are also affected by the capital gains (losses) incurred. Adjustments to the desired levels of assets are made in some proportion in a given quarter.

$$(2.12)\quad TD_t - TD_{t-1} = \lambda(TD_t^* - (TD_t + G_t)) + \sum_j^n \lambda_j [S_{jt}^* - (S_{jt-1} + G_{jt})]$$

where G_{jt} is the capital gains on j^{th} financial asset. The demand for the desired stock of term deposits is a function of expected income (Y^*), expected capital gains (G^*), past preferences and habits (S_{pt-1}), its own price (R_{td}) and those of substitutes $(R_{dd}, R_s, \ldots)$. Assuming a linear function

$$(2.13)\quad TD_t^* = \alpha_0 + \alpha_1 R_{td} + \alpha_2 Y^* + \alpha_3 G^* + \alpha_4 S_{pt-1} + \sum_i^n \alpha_{5i} R_i$$

Expected income is defined as a linear function of current income. Expected capital gains are assumed to be a linear function of current capital gains. Capital gains on asset i are defined as:

$$G_{it} = \frac{R_{it}S_{it-1}}{C.P.I._t} - \frac{R_{it-1}S_{it-1}}{C.P.I._{t-1}}$$

where C.P.I., implicit price deflator of personal consumption expenditure. By simplifying $\sum_j^n \lambda_j \, [S^*_{jt} - (S_{jt-1} + G_{jt})]$ to $\theta_i D_k$ and substituting for TD^* in (2.12) we get (2.14):

$$(2.14)\quad TD_t - TD_{t-1} = \delta_0 + \delta_1 Y^*_t + \delta_2 G_t + \delta_3 S_{pt-1} + \delta_4 R_{td} + \delta_5 R_o + \delta_6 TD_{t-1} + \delta_7 D_k$$

$$(2.15)\quad \Delta TD_t = -7861 + \underset{(2100)}{6922}\, R_{td} + \underset{(.03)}{.098}Y^* - \underset{(.15)}{.49}\, TD_{t-1} + \underset{(2300)}{8564}S_{pt-1} - \underset{(.01)}{.03}D_k \quad R^2 = .92$$

The empirical results are given after elimination of all nonsignificant variables. Data sources for the 1947-1969 time series are not listed but capital gains (G_t) and the price of substitutes (R_o) did not prove to be significant. Since Batra does not state what assets are included in D_k and does not explain what services are measured by S_{pt-1}, it is difficult to conclude that Motley's hypothesis of interdependence is statistically important for savings deposits.

B. Simultaneous Equation Systems

In the previous section, the single equation approach assumed that the variables on the right hand side of the model are predetermined-exogenous, or lagged endogenous- and hence they are all independent of the error term and ordinary least squares can give consistent estimates. This assumption is true for the behaviour of individuals but it cannot be made for financial institutions, especially at the macroeconomic level because interest rates on deposits are decision variables in the management of financial intermediaries, the own price becomes a current endogenous variable and single equation least squares will no longer result in consistent estimates for the coefficients. The econometrics of the situation requires that the determinants of own deposit rates be specified and the equations be estimated simultaneously.

Cohan (1973) applies a recursive system to determine the interest rates on certificates of deposit issued to (a) corporations and (b) households, and the level of these certificates acquired by (a) and (b). The desired deposit rate on certificates issued to corporations (R_{cd}) is influenced by: (1) anticipated strength in loan demand, (ii) yields on loans, (iii) yields on competitive assets (i.e. Treasury Bills R_{tb}); and constrained by (iv) ceiling rate restrictions (R_q). A partial adjustment process is assumed to explain movements in R_{cd}.

(2.16.1) $$\Delta R_{cd,t} = \lambda(R^*_{cd,t} - R_{cd,t-1})$$

(2.16.2) $$R^*_{cdt} = R_q - \gamma(R_q/R_{tb})$$

where R_q/R_{tb} approximates a cost mark-up factor for financial intermediation ($\lim R_q/R_{tb} \rightarrow 0 => R^* = R_q$). Assuming that $0 < \lambda < 1$ substitute for R^*_{cd} in (2.16.1) and the following equation is estimated using quarterly data (1961-1967) for U.S. commercial banks.

(2.17) $$R_{cd,t} = 2.88 + \underset{(.05)}{.83}R_q - \underset{(.20)}{2.64}\frac{R_q}{R_{tb}} + \underset{(.05)}{.15}R_{cd,t-1} \qquad R^2 = .99$$

(2.18) $$R_p = -4.34 \underset{(.17)}{\pm .89}R_{sd} + \underset{(.17)}{.83}R_s + \underset{(.06)}{.15}R_{cd} + \underset{(.06)}{.20}R_b \qquad R^2 = .98$$

The supply price for personal certificates of deposits (R_p) is determined by the same factors as R_{cd} as well as returns on competitive liquid assets. R_{cd} is a proxy for the above factors affecting desired deposit rate and the price of substitutes are: R_{sd}, savings and loan savings deposit rate; R_s, bank's savings deposit rate; R_b, short term bank lending rate. Whereas the author assumed a partial adjustment process for R_{cd}, R_p is assumed to adjust fully once R_{cd} is set and competitors' prices are known. The linear functional forms are estimated by two stage least squares and all the coefficients proved to be significantly different from zero. The structure of equation (2.17) is not applicable to the Canadian financial system because of the absence of a legal ceiling rate.

The demand function for certificates of deposit (CD) is based upon the traditional demand theory. The dependent variable is defined as the ratio of CD to liquid assets (LA) held by corporations and individuals. Liquid assets consist of corporate and individual holdings of demand deposits and currency, savings and time deposits at commercial banks and at savings institutions, short term treasury securities and commercial and finance company paper, and short term U.S. government securities. To avoid the high correlation among interest rates the spread between own price and a substitute is used. The estimated demand curve is linear and assumes instantaneous adjustment to exogenous factors.

$$(2.19) \quad \frac{CD}{LA} = -7.63 + \underset{(.22)}{.42}(R_{cd} - R_{tb}) + \underset{(.24)}{.46}(R_p - R_{sd}) + \underset{(.44)}{7.61}\, Y_p - \underset{(.02)}{.03}(\Delta Y\text{-}k) + \text{seasonals}$$

$$R^2 = .99$$

where Y_p, wealth measured weights adopted from Friedman's permanent income theory $Y_p = .139 \sum_{1}^{11} (.9)^i \, GNP_{-i}$, and $(\Delta Y\text{-}k)$ is the change in GNP less the average quarterly growth in GNP. This variable is intended to measure the effects of transitory income. "This type of income is likely to be held in temporary money balances rather than being shifted into an interest yielding liquid asset. [It] is expected to be inversely related to the CD's" [Cohan (1973), p. 107]. The spread between interest rates (α_1, α_2) are of marginal statistical significance and only Y_p proves to be significant.

Cohan's three equation model is a block recursive system: two equation supply block (estimated by 2SLS) and a unique demand equation (estimated by OLS). The two deposit rates are set interdependently and then they are part of the final determinants for CD's. "[The] institutional arrangements in this market are such that the determination of supply and demand may be considered sequential rather than simultaneous in nature". (Cohan, (1973), p. 109).

DeLeeuw's paper (1965) was the first simultaneous analysis of the monetary sector. It is a model of financial behaviour in the many financial markets in the U.S. At this level of aggregation the market interest rates and the quantities of liquid assets held are interdependent. Of the nineteen equations that make-up the complete model only three equations will be discussed below. They are: demand deposit holdings; time deposit holdings, and interest rate on time deposits. The model is based upon four assumptions: (i) There exists a "desired" relationship between portfolio composition and interest rates. The consumer maximizes net worth and will choose those combinations of assets that will give him the highest risk-return utility. (ii) At any period there is a partial adjustment to the "desired" portfolio. Adjustments are not immediate because lags in information, decision making and plan execution. (iii) There are short-run constraints that limit behaviour by both consumers and financial intermediaries. These refer to total savings, current income, liquidity considerations and reserve requirements. (iv) The final assumption states that all relationships are homogeneous of degree one in all dollar magnitudes.

The change in the level demanded of asset x is a function of its stock in the previous period, rates of return (its own and those of substitutes) (R_x, R_1, R_2, ..., R_n) and current and lagged short run constraints. ($f(x)_t$, $f(x)_{t-1}$)

$$(2.20)\quad \frac{\Delta(x)_t}{Y_{t-1}} = \alpha_0 + \alpha_1 \frac{x_{t-1}}{Y_{t-1}} + \alpha_2 R_x + \alpha_3 R_1 + \ldots + \alpha_j \frac{f(x)_t}{Y_{t-1}} + \alpha_{j+1} \frac{f(x)_{t-1}}{Y_{t-1}}$$

The changes in the quantities demanded are expressed as a percentage of total demand in the sector. The latter is measured by the proxy $Y_{t-1} = 0.114 \sum_{i=0}^{19} (0.9)^i \cdot GNP_{-i}$. It is lagged one period to facilitate simulation. Any measurement error arising from Y_{t-1} instead of Y_t is assumed to be negligible. The interest rates are nominal rates expressed as percentages. The constraints are particular to the demand equation.

In DeLeeuw's condensed model (1969), the change in demand deposits is determined by its previous stock (DD_{t-1}), average yield on U.S. securities maturing or callable in ten years or more (R_{gbl}), yield in commercial bank time deposits (R_{td}), personal disposable income (Y_d), and business gross investment in plant and equipment (I_b) plus private nonbusiness, nonresidential construction (I_c). The latter two variables serve as a proxy for the expected return on capital goods. Current and lagged values of disposable income represent sources of funds to households. One is struck by the absence of "own" price in the above hypothesis. This is because DeLeeuw assumes that the demand deposits have their interest rates fixed at zero and his model does not deal with service charges.

The results of 2SLS using U.S. quarterly data for the 1948-1962 time period are presented in equation (2.21) and the coefficients of DD_{-1}, R_{gbl}, Y_d, and (I_b+I_c) have the right sign and statistical significance.

$$(2.21)\quad \frac{\Delta DD}{Y_{t-1}} = -.003 - \underset{(.04)}{.11}\frac{DD_{t-1}}{Y_{t-1}} - \underset{(.002)}{.005}R_{gbl} - \underset{(.002)}{.002}R_{td} + \underset{(.03)}{\frac{.07Y_d}{Y_{t-1}}}$$

$$+ \underset{(.02)}{.03}\frac{Y_{d,t-1}}{Y_{t-2}} - \underset{(.06)}{.20}\frac{(I_b+I_c)}{Y_{t-1}}$$

$$(2.22)\quad \frac{\Delta TD}{Y_{t-1}} = -.002 - \underset{(.03)}{.12}\ \frac{TD_{t-1}}{Y_{t-1}} - \underset{(.000)}{.003}\ R_{tb} + \underset{(.001)}{.006}\ R_{td} + \underset{(.008)}{.02}\ \frac{Y_{d,t-1}}{Y_{t-2}}$$

For changes in term deposits the functional form is the same as that of ΔDD and the determinants are last period stock of term deposits (TD_{t-1}), the three month treasury bill rate (R_{tb}) the average yield on bank term deposits (R_{td}), and disposable income (Y_d). All estimates in (2.22) are significant.

The change in the interest rate on term deposits is a result of adjusting the financial intermediaries' present portfolios toward their desired portfolios. The quarterly change in R_{td} is assumed to depend on the desired rate and last quarter's actual rate. The desired rate depends upon U.S. security yields maturing or callable in ten years or more (R_{gbl}), on ceiling restrictions ($R_{gbl} - R_q + 1$), and on the ratio of loans to total deposits ($TL/(DD+TD)$)

$$(2.23.1)\qquad \Delta R_{td,t} = \lambda\ (R^*_{td,t} - R_{td,t-1})$$

$$(2.23.2)\qquad R^*_{td,t} = f(R_{gbl},\ R_{gbl} - R_q + 1,\ \frac{TL}{DD + TD})$$

The following equation (2.24) approximates this behaviour. All the coefficients proved to statistically significant by 2SLS. Again the absence of legal ceilings in Canada on R_{td} limits any direct application of (2.24) to our context.

$$(2.24)\quad \Delta R_{td} = -1.26 - .39\ R_{td,t-1} + .14R_{gbl} + .33R_q + 1.02\ \frac{TL}{(DD + TD)}_{-2}$$

A simultaneous model at the microlevel is the Dhrymes and Taubman (1969) study of the savings and loan associations in the U.S. Each firm will set deposit and loan interest rates to alter the demand for their assets and liabilities such that profits are maximized in each time period. But the actual changes in deposits and shares may fall short of the expected levels and this may force the financial manager to further readjust interest rates. The changes in interest rates and in deposits and loans are interdependent and are determined in each time period simultaneously. Their underlying theory is the partial adjustment process in a competitive framework Variables such as advertising per capita and number of S & L offices per capita reflect consumers' tastes and preferences rather than product differentiation.

The S & L have only one type of deposit and the desired level of term deposits is a function of own interest rate, permanent per capita income (Y/Pop), number of S & L offices per capita (#/Pop), and prices of substitutes (treasury bill rate R_{tb} and a regional rate R_e). For a district, the estimates linear in natural logs using quarterly data (1958-65) show logged stock, R_{td}, and convenience to be significant. However, the presence of TD_{t-1} biases the Durbin-Watson statistic and the coefficients estimated may be inconsistent. Coefficient on income is significantly of the wrong sign.

$$(2.25)\quad \ln \frac{TD}{Y} = .37 + \underset{(.06)}{.20} \ln R_{td} - \underset{(.01)}{.06} \ln \frac{Y}{Pop} + \underset{(.006)}{.04} \ln \frac{\#}{Pop} - \underset{(.014)}{.02} \ln R_{tb}$$

$$- \underset{(.14)}{.15} \ln R_e + \underset{(.005)}{.93} \ln \left(\frac{TD}{Y}\right)_{t-1} + \text{seasonals} \qquad R^2 = .95$$

$$(2.26)\quad R_{td} = .21 + \underset{(.01)}{.94} R_{td,t-1} + \underset{(.007)}{.02} R_{tb} + \underset{(.02)}{.01} \left(\frac{\Delta M}{TD_{t-1}}\right) + \text{regional dummies} \qquad R^2 = .91$$

The adjustment in the interest rate on term deposits is also expected to follow a partial adjustment process to the optimal level. It is determined by competitive rate (three month treasury bill rate R_{tb}) and the demand for own mortgages (ΔM). The latter variable illustrates that an excess demand for assets will put pressure on S & L firms to attract additional savings that they can channel into higher yielding loans, however it did not prove to be significant in the 1960-66 period.

C. Structural Model

The thrust of the literature and research surveyed is to model a simultaneous structural system of equations for the behaviour of financial intermediaries. The motivation for this approach rests with the fact that the firm has a number of policy variables at its discretion and the manager wants to know the various responses to those parameters (i.e. elasticities of demand with respect to interest rates, advertising, and exogenous interest rates, incomes or wealth). A hypothesis for such a model for demand deposits and term deposits, and the respective interest rates for a credit union is developed below. The discussion is divided into two parts: (i) Consumer Behaviour; and (ii) Financial Behaviour of a Credit Union. As usual the viability of this approach depends on the availability of a large number of observations.

(i) Consumer Behaviour

The consumer is expected to maximize his net worth in a world where there exists limited information and a time lag in realizations. In particular the consumer is expected to maximize risk-return utility for his portfolio of securities. We will consider his demand for only two such securities: demand deposits and term deposits. The demand for demand deposits stems from the desire to hold liquid assets. These assets provide the individual with liquidity, salability, safety, convenience and chequing facilities. It is these services that the individual purchases when he acquires demand deposits. The stock of demand deposits will serve as a proxy for the amount of services purchased.

On the other hand, the consumer purchases a term deposit in order to receive a positive rate of return. Optimizing over time, the consumer attempts to reconcile his earned income stream with his desired consumption pattern in every period. He may have surplus funds to invest in financial securities if his expected income is greater than his desired consumption or if the return from purchasing financial instruments is greater than the return from consumption of durable and non-durable goods. The stock of term deposits is the measure for quantity purchased.

Limited information implies that the consumer does not know all the opportunities available to him and their costs or profits, so his behaviour does not attain the optimum solution in every time period. There is a time lag in executing decisions due to delays in communications and due to uncertain expectations about the future (i.e. lagged response in deposit accounts in answer to changes in interest rates; lagged updating of expected income). Thus households are unable or unwilling to adjust asset holdings to long-run desired levels instantaneously.

Tastes and preferences are influenced by advertising and convenient location. There exist many substitutes among durable goods and financial instruments. The prices of substitutes are also expected to explain the size and nature of (dis)equilibrium in the respective markets of these assets in the portfolio. We adopt the assumption that "the demand for assets (at constant prices) is homogeneous of degree zero in general price level and unit-elastic with respect to population." (Motley, 1970, p. 236). Consumers' income is an exogenous variable in the model. Permanent income is estimated by adapting the weights developed by Friedman (1957) in his consumption study. Transitory income is expected to have a significant effect on both types of purchases of liquid assets.

Therefore the demand for a liquid asset is hypothesized to be an adjustment between this period's desired stock and last period's actual stock. The desired stock is determined by its own price by the size of the consumers' budget (permanent income and transitory income), by per capita advertising expenditures and number of offices per capita, and by prices of substitutes.

(ii) Financial Behaviour of a Credit Union

The credit union sets interest rates on demand and term deposits to alter their respective levels such that the surplus of revenues minus costs is maximized in each time period. The decision maker has a desired rate but because of a lag in decision-making or an unfavourable liquidity position he is unable to reach the desired rate instantaneously. The desired rate on demand deposits is determined by: service charges (S_e); competitors' rates (R_{dd}); credit union's demand for demand deposits (DD); the mortgage rate (R_m), liquidity constraint); and the desired rate on credit union term deposits (joint decision making on the two rates). The desired rate on term deposits is determined by: competitors' rates; credit union's demand for term deposits; the mortgage rate; the desired rate on credit union demand deposits; and the demand for term deposits. To maintain the desired distribution of low cost funds, we postulate a constant relationship between desired demand deposit rate and desired term deposit rate. To keep attracting funds into nonchequable savings rather than into higher cost term deposits an adaptive expectations equation is used. (2.27.1).

(2.27.1) $$R^*_{dd} - R^*_{dd-1} = \gamma\ (R_{td} - R^*_{dd-1})$$

(2.27.2) $$R^*_{dd}\ (1-(1-\gamma)L) = R^*_{dd}(1-\delta L) = \gamma R_{td}$$ where L, lag operator

(2.27.3) $$R^*_{dd} = \frac{\gamma}{1-\delta L}\ R_{td}$$

Partial adjustment model for term deposit rates (R_{td})

(2.28.1) $\Delta R_{td} = \lambda_1(R^*_{td} - R_{td-1})$

(2.28.2) $R^*_{td} = f_1(R^B_{td}, R_m, R^*_{dd}, TD)$

Substituting for R^*_{td} in (2.28.1) and using the result of (2.27.3)

(2.28.3) $\Delta R_{td} = \lambda_1 (R^B_{td} + R_m + \frac{\gamma}{1-\delta L} R_{td} + TD - R_{td-1})$

which simplifies to (2.28.4)

(2.28.4) $R_{td} = \alpha_1 R^B_{td} - \alpha_2 R^B_{td-1} + \alpha_3 R_m - \alpha_4 R_{m-1} + \alpha_5 TD - \alpha_6 TD_{-1}$

$+ \alpha_7 R_{td-1} + \alpha_8 R_{td-2}$

Partial adjustment model for demand deposit rate (R_{dd})

(2.29.1) $\Delta R_{dd} = \lambda_2 (R^*_{dd} - R_{dd-1})$

(2.29.2) $R^*_{dd} = f_2 (S_e, R^B_{dd}, R_m, R^*_{td}, DD)$

Substituting the result of (2.27.3) in (2.29.1) and bring R_{dd-1} to the right hand side

(2.29.3) $R_{dd} = \frac{\lambda_2\gamma}{1-\delta L} R_{td} - (1-\lambda_2) R_{dd-1}$

(2.29.4) $R_{dd} = \lambda_2\gamma R_{td} + (\lambda_2-\gamma) R_{dd-1} + (1-\gamma)(1-\lambda_2) R_{dd-2}$

From consumer behaviour the demand demand deposits (DD) and term deposits (TD) can be written with (2.28.4) and (2.29.4) to form the complete structural model of four simultaneous equations.

(2.30) $DD = \alpha_{11}R_{dd} + \alpha_{12}R_{td} + \alpha_{13}R^B_{dd} + \alpha_{14} \frac{A}{Pop} + \alpha_{15} \frac{\#off}{Pop} + \alpha_{16}Y_p$

$+ \alpha_{17}(Y-Y_p) + \alpha_{18}DD_{-1}$.

(2.31) $TD = \alpha_{21}R_{dd} + \alpha_{22}R_{td} + \alpha_{23}R^{B}_{td} + \alpha_{24}\frac{A}{Pop} + \alpha_{25}\frac{\#off}{Pop}$

$+ \alpha_{26}Y_p + \alpha_{27}(Y-Y_p) + \alpha_{28}TD_{-1}$

(2.32) $R_{dd} = + \alpha_{31}R_{td} + \alpha_{32}R_{dd-1} + \alpha_{33}R_{dd-2}$

(2.33) $R_{td} = \alpha_{41}R_m + \alpha_{42}R_{m-1} + \alpha_{43}R_{td-1} + \alpha_{44}R_{td-2} + \alpha_{45}R^{B}_{td} + \alpha_{46}R^{B}_{td-1}$

$+ \alpha_{47}TD - \alpha_{48}TD_{-1}$

All the equations are over identified but they meet the rank condition (necessary and sufficient condition for identification). This condition states that the structural equation is identified if and only if the rank of the matrix formed by the excluded variables is equal to the number of equations less one.

Unfortunately for the development of this hypothesis, data constraints are severe. There is no published quarterly information on credit union interest rates and hence the critical parameters of the model are unobservable.

III Theoretical Development of Time Series Analysis

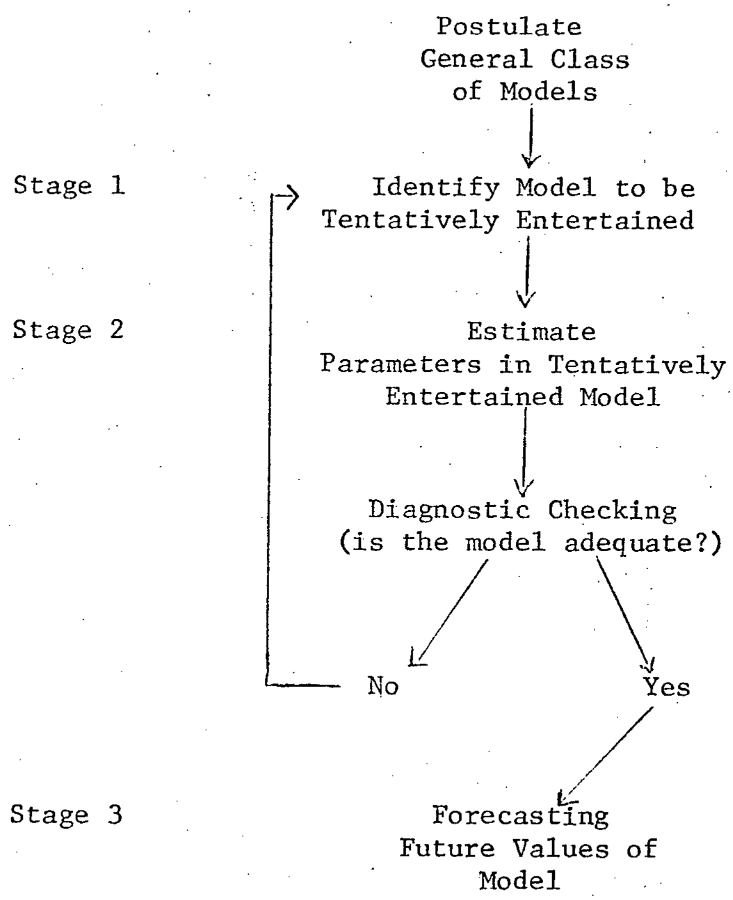
This method of forecasting is data oriented as it incorporates economic information through subjective decisions made in model specification. We assume that there exists a basic underlying pattern for the series that is represented by historical data and this pattern can be expressed as a weighted sum of past values of the data. The weights in the sum are determined so as to achieve the greatest possible predictive power. This analysis involves three stages and n iterations on these stages, (illustrated below in Figure III; Box and Jenkins (1970, p. 19).

Our concern is to fit a stationary model for the series of demand deposits and the series of term deposits of credit unions that will be used to forecast their respective values. We optimize the pattern of a set of data by minimizing its forecasting error. The components of the time series model are: (i) autoregressive process where there exists an association among values of the same variable but at different time periods (serial or seasonal); (ii) moving average process where there exists some mutual correspondence among successive values of residuals (trends or seasonal); and (iii) a mixture of the above mentioned processes. Each is presented below.

A. General Class of Models

Stationary process. A time series z_t is considered to be stationary if it has an equilibrium point about a constant mean and if the variances of the observations are the same (i.e. $E(z_t) = E(z_{t+n})$ and $E(z_t^2) - E(z_t)^2 = E(z_{t+n}^2) - E(z_{t+n})^2$). A nonstationary process has no natural mean and it is assumed that some suitable difference equation will re-

Figure III.1


Postulate General Class of Models
Stage 1
Identify Model to be Tentatively Entertained
Stage 2
Estimate Parameters in Tentatively Entertained Model
Diagnostic Checking (is the model adequate?)
No
Yes
Stage 3
Forecasting Future Values of Model

present the process as being stationary. We introduce ∇ as the backward difference operator which can be written in terms of B where $B^k z_t = z_{t-k}$

(3.1.1) $\nabla z_t = z_t - z_{t-1} = (1 - B)\, z_t$

(3.1.2) $\nabla^d z_t = z_t - dz_{t-1} + 1/2\, d(d - 1)\, z_{t-2} + \ldots + (-1)^d z_{t-d}$

Autoregressive process. Consider a time series z with observations from 1 to T. Assume that it is stationary and can be written as

(3.2.1) $z_t = \phi_1\, z_{t-1} + \phi_2\, z_{t-2} + \ldots + \phi_p\, z_{t-p} + a_t$

where z_t, is a random observation at period t, ϕ_t is an adjustable weight, and a_t is a series of random shocks ("white noise"). Introducing the autoregressive operator $\phi(B)$ we now write

(3.2.2) $(1 - \phi_1 B^1 - \phi_2 B^2 - \ldots - \phi_p B^p)\, z_t = \phi(B) z_t = a_t$

Moving average process. The time series z is stationary and can be written as (3.3.2) where $\theta(B)$ is moving average operator

(3.3.1) $z_t = a_t - \theta_1\, a_{t-1} - \theta_2\, a_{t-2} - \ldots - \theta_q\, a_{t-q}$

(3.3.2) $z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots \theta_q B^q)\, a_t = \theta(B)\, a_t$

Mixed autoregressive-moving average (ARMA) model is $\phi(B) z_t = \theta(B)\, a_t$ where ϕ (B) and θ (B) are polynomials of degree p and q respectively. This process is referred to as an ARMA (p,q) process (assumed to be stationary). (Box-Jenkins (1970), p. 74).

(3.4.1) $z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + a_t - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}$

A complete model is called the autoregressive integrated moving average process with a pth autoregressive scheme, a dth stationary difference, and a qth moving average: ARIMA (p,d,q)

(3.4.2) $$\phi(B)\nabla^d z_t = \Theta(B)\, a_t$$

An example of a (1.1.1) process is:

(3.5.1) $$(1 - \phi_1 B)\, \nabla z_t = (1 - \theta_1 B)\, a_t$$

(3.5.2) $$\nabla z_t - \phi_1 \nabla z_{t-1} = a_t - \theta_1 a_{t-1}$$

An example of a (0, 2, 2) process is:

(3.6.1) $$\nabla^2 z_t = (1 - \theta_1 B - \theta_2 B^2)\, a_t$$

(3.6.2) $$z_t - 2z_{t-1} + z_{t-2} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

Seasonal ARIMA process. Assume that z_t is a monthly series and that we intend to link current behaviour for month t with behaviour for the month in the previous year t-12 and so on for each of the twelve months. The series can be written as a stationary process by differencing it D times. The seasonal autoregressive process of level P is represented by the polynomial (3.7.1) and the seasonal moving average process of level Q is represented by the polynomial (3.7.2), where the seasonal length is denoted by s = 12 (in our example of a monthly series).

(3.7.1) $$\Phi(B_{12}) = 1 - \Phi_1 B_{12} - \Phi_2 B_{12}^2 - \dots - \Phi_P B_{12}^P$$

(3.7.2) $$\Theta(B_{12}) = 1 - \Theta_1 B_{12} - \Theta_2 B_{12}^2 - \dots \Theta_Q B_{12}^Q$$

It is assumed that the parameters Φ and Θ contained in this monthly model is approximately the same for each month and that the errors α_t's are random. The seasonal ARIMA can thus be written as (P,D,Q)s model specified in (3.9.3)

(3.7.3) $$\Phi_p(B_s)\, \nabla_s^D z_t = \Theta_Q(B_s)\alpha_t$$

Multiplicative model. In our seasonal series discussed above, we relax the assumption about the errors and now allow α_t, α_{t-1}, α_{t-2}, ... to be correlated, represented by (3.8.1)

(3.8.1) $$\phi(B)\nabla\alpha_t = \theta(B)\, a_t$$

where a_t is white noise. Substitute (3.8.1) in (3.7.3) and premultiplying the latter by $\phi(B)$ yields (3.8.2) - the (p,d,q) X (P,D,Q)s model.

(3.8.2) $$\phi(B)\,\Phi_P(B_s)\,\nabla^d\nabla_s^D z_t = \theta(B)\,\Theta_Q(B_s)\,a_t$$

This type of specification differs from the traditional approach to treat seasonality as an additive component in a time series. As Nelson (1973) points out such methods as dummy variables assume deterministic seasonality whereas it is more plausible to conceive of the pattern and intensity of seasonal variations as undergoing change over time. We consider instead a particular class of linear stochastic processes that display seasonal behaviour as the basis for models of seasonal time series (Nelson (1973) p. 169).

An example of a $(0,1,1) \times (0,1,1)_{12}$ model is presented below for a monthly series. To link the monthly z_t's one year apart we write (3.9.1) and to link the correlated α_t's one month apart we write (3.9.2). The multiplicative model is presented by (3.9.3).

(3.9.1) $$\nabla_{12} z_t = (1 - \Theta B_{12})\,\alpha_t$$

(3.9.2) $$\nabla\alpha_t = (1 - \theta B)\,a_t$$

(3.9.3) $$\nabla\nabla_{12} z_t = (1 - \theta B)(1 - \Theta B_{12})\,a_t = a_t - \theta a_{t-1} - \Theta a_{t-12} + \theta\,\Theta\,a_{t-13}$$

The moving average operator is now of order $q + sQ = 13$ and we have thirteen adjustable coefficients (i.e. twelve monthly contributions and one yearly contribution). We observe that the seasonal behaviour is picked up by the weighted error terms on the right hand side of equation (3.9.3)

B. Identification of a Model

The individual time series of deposits will be identified as an ARIMA (p,d,q) or multiplicative ARIMA (p,d,q) X (P,D,Q)s. The identification is broken up into three stages: (i) to identify the degree of differencing to obtain a stationary series expressed as a transform of the original series z_t; (ii) to identify the resultant stationary series as an ARMA process, and (iii) to identify the absence or presence of seasonality in the ARIMA.

If the theoretical autocorrelation function defined below in (3.10) does not die out fairly rapidly for the raw data, then one may consider ∇z_t, or some higher difference. "It is assumed that the degree of differencing d, necessary to achieve stationarity, has been reached when the autocorrelation function ρ_k of $\nabla^d z_t$ dies out fairly quickly. In practise d is normally either 0, 1 or 2 ...." (Box-Jenkins (1970), p. 175).

The autocorrelation for $z_t = \phi_1 z_{t-k} + a_t$ at lag k is

(3.10) $$\rho_k = \frac{E[(z_t - \mu_t)(z_{t-k} - \mu_t)}{\sigma_z^2}$$

The stationary AR process of order k $\phi(B) z_t = a_t$, premultiplied by z_{t-k} can be expressed as (3.11.1), and taking expectations and dividing through by σ_z^2 we get an equation (3.11.2) that presents the kth autocorrelation ρ_k as a weighted sum of ϕ's and k-1 autocorrelations.

(3.11.1) $z_{t-k}z_t = \phi_1 z_{t-k}z_{t-1} + \phi_2 z_{t-k}z_{t-2} + \ldots + \phi_k z_{t-k}z_{t-k}$

(3.11.2) $\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + \ldots + \phi_k$

To estimate the autocorrelation function, replace the ρ's by the estimated autocorrelations $\hat{\rho}_k$s defined by (3.10). We can write k linear equations for the ρ_k's in terms of ϕ's and ρ's. They are given by the Yule-Walker equations (3.12) where the kth equation is simply (3.11.2). (Box and Jenkins (1970), p. 54-55)

(3.12)
$$\rho_1 = \phi_1 + \phi_2\rho_1 + \ldots + \phi_k\rho_{k-1}$$
$$\rho_2 = \phi_1\rho_1 + \phi_2 + \ldots + \phi_k\rho_{k-2}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + \ldots + \phi_k$$

We have assumed that the AR process is of order k and hence can be expressed in terms of k non-zero autocorrelation parameters. To be sure of the length of the AR polynomial we examine the partial autocorrelation parameters in equations (3.12) - the kth ϕ in the kth equation of (3.12) is called ϕ_{kk}, partial autocorrelation. These ϕ_{jj} coefficients are found by solving the Yule Walker equations for j = 1,2, ..., k. Should the ϕ_{k+1} in the K+1 equation of an AR(k+1) be zero ($\phi_{k+1,k+1} = 0$), then we conclude that ρ_k has at cut-off point at k+1 and the process is of length k. This is our identification rule for an AR(ρ) process: if the autocorrelation function tails off and the partial autocorrelations have a cut-off point after lag ρ then ρ is the expected order of the autoregressive process.

For the MA process of order q $z_t = \Theta(B)\, a_t$, its variance and autocovariances are given by (3.13.1) and (3.13.2) respectively. Given that

$E(a_t a_{t+k}) = 0 \; k \neq 0$ is obvious that the autocovariance γ_k is zero for all k greater than the length of the polynomial q.

(3.13.1) $$\gamma_0 = E\,[(a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q})]^2 = \sigma_a^2\,(1 + \theta_1^2 + \dots + \theta_q^2)$$

(3.13.2) $$\gamma_1 = E\,[(a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q})(a_{t-k} - \theta_1 a_{t-k-1} - \dots - \theta_q a_{t-k-q})] = \sigma_a^2(-\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q) \qquad k < q$$

Using our definition of autocorrelation (3.10) we define the autocorrelation function for the MA(q) process by (3.14)

(3.14) $$\rho_k = \begin{cases} \dfrac{-\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q}{1 + \theta^2 + \dots + \theta_q^2} & k = 1,2,\dots q \\ 0 & k > q \end{cases}$$

This result yields an obvious identification rule for the MA(q) process for if the autocorrelation has a cut-off point after lag q and the partial autocorrelation function tails off, then q is the expected order of the moving average process.

A mixed process ARMA (p,q) is suggested if both the autocorrelation function and the partial autoregressive function tail off. The autocorrelation function has an irregular pattern at lags 1 through q, then it tails off according to its functional values. Conversely the partial autocorrelation function is dominated by an irregular pattern at lags p-q and onward.

The presence of any seasonal influence in the data can be observed by searching for peaks in the autocorrelation function and in the partial autocorrelation function that appear at regular intervals. The length of the seasonal is suggested by the definition of the data (i.e., monthly, quarterly, etc). and the levels of (P,D,Q) will be those in which the spikes at t-s lag die out quickly for both functions.

C. Estimation of Parameters and Diagnostic Checking

Once we have identified the series as an ARIMA (p,d,q) X (P,D,Q)s, the next step is to estimate the p autoregressive parameters, P seasonal autoregressive parameters, q moving average parameters and Q seasonal moving average parameters. The criterion for efficient estimation is to minimize the squared dfference between the actual value of z_t and its estimated value $\hat{z}_t$. We proceed to maximize the likelihood function of the joint normal distribution of $p(\nabla^d \nabla^D z_t / \phi, \Phi, \theta, \Theta \sigma_a^2)$ in the multiplicative model. Thus we try out all combinations of vectors ϕ , Φ , θ, Θ and σ^2 such that they maximize the likelihood of these parameters by minimizing the sum of squares in (3.15).

(3.15) $\exp \; [-\frac{1}{2\sigma^2} \Sigma \, [a_t / \phi, \; \Phi, \theta, \; \Theta, z]^2)$

To begin the estimation procedure we must give start-up values to the parameters in the four polynomials and to get the algorithm started we must calculate the a_{t-q}'s which specify the moving average part of the model. The values are estimated by using the given parameters by back forecasting on z_t (or its differenced level) (see Box and Jenkins, 1970, p. 212-220). Assume that the model is an ARMA (0,0,1), initialize the value of Θ and express the model in terms of the forward shift operator F where $Fe_t = e_{t+1}$

(3.16) $z_t = (1-\Theta B)\, a_t$ and $z_t = (1-\Theta F) e_t \qquad |\Theta|<1$

Set $E[e_{T-1}\ \Theta, z_t] = 0$ and solve for z_o by back forecasting.

$$
(3.17)\quad
\begin{aligned}
E[e_T \mid \Theta, z_t] &= E[z_T] + \Theta E[e_{T+1} \mid \Theta, z_t] = z_T \\
E[e_{T-1} \mid \Theta, z_t] &= E[z_{T-1}] + \Theta E[e_T \mid \Theta, z_t] = z_{T-1} + \Theta z_T \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
E[e_o \mid \Theta, z_t] &= E[z_o] + \Theta E[e_1 \mid \Theta, z_t] = 0
\end{aligned}
$$

This last equation in (3.17) gives us an initial value for $E[z_o] = -\Theta E[e_1 \mid \Theta, z_t]$ from which we can start our forward forecasting for a's conditional on Θ and z_t. Recall that $E[a_{-1} \mid \Theta, z_t] = 0$ since it is distributed independently of z_t.

$$
(3.18)\quad
\begin{aligned}
E[a_o \mid \Theta\ z_t] &= E[z_o] + \Theta E[a_1 \mid \Theta, z_t] = E[z_o] \\
E[a_1 \mid \Theta\Theta\ z_t] &= E[z_1] + \Theta E[a_o \mid \Theta, z_t] = E[z_1] + \Theta E[z_o] \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
E[a_T \mid \Theta, z_t] &= E[z_T] + \Theta E[[a_{T-1} \mid \Theta, z_t] = E[z_T] + \Theta E[z_{T-1}]
\end{aligned}
$$

Our inputs are now complete and we can start the iterations for the non-linear estimation of the parameters that will minimize the sum of squares function. The algorithm used in the computer program is Marquandt's iterative procedure which is a compromise between the methods of Gauss-Newton and steepest descent (Nelson (1974), p. 8).

The estimated coefficients are then examined for their significance levels and the model is diagnosed with respect to the estimated residuals. The former is done by testing the hypothesis that any parameter is different from zero (t-test). In particular the hypothesis that $\phi = 1$ is a test for non-stationarity and should we accept the null hypothesis we would take first differences of the data and test if $\phi = 1$ in $\phi \nabla z_{t-1}$, etc. If the constant term of the raw data is significantly non-zero then we also conclude that there is a drift in the series and that its mean is not independent of time. The presence of seasonality can be ascribed to the significance of either autoregressive or moving average coefficients of order P and Q respectively.

The estimation programs also yield two model characteristics. The first is the hypothesis that the population represented by the model and the population illustrated by the data come from the same population. This non-parametric test is the Kolmogorov-Smirnov Statistic which gives confidence bands for the differences between the distribution function of both populations. The second is the hypothesis that we have reduced the model to white noise. This is a chi-square test known as the Box-Pierce Statistic and if the sum of the first k sample autocorrelations of the errors is less than the chi-square value with (k-p-q-P-Q) degrees of freedom, then the residuals are said to be random.

D. Forecasting

The concern for efficiency in estimation and significance of coefficients stems from the desire to have efficient forecasts for the series z_{t+1} where l is some period into the future. Efficiency is defined here in the same way as in estimation: minimize the mean square error. When the model is identified and parameter estimates are obtained, the algorithm once again generates the disturbance terms (a_t's) and we can easily forecast next period's value for z_{t+1} conditional on this period's autoregressive parameters and moving average terms. It should be noted that if the raw-data has been transformed to natural logarithms then the forecasts are log-normally distributed and the antilogs must be adjusted based on the log-normal distribution (Nelson (1973), p. 161-163).

E. Transfer Function

Throughout this chapter we have shown that a particular times series can be represented by an ARIMA process but it may also be sensitive to some external shocks. If our series z_t is dependent on the current exogenous variable X_t or its past values X_{t-1}'s then equation (3.19) is the transfer function for z_t (i.e. a structural equation).

(3.19) $$z_t = \alpha_1 X_t + \alpha_2 X_{t-1} + \dots + \alpha_n X_{t-n} + a_t = \alpha(B) X_t + a_t$$

In our context of time series for demand deposits and term deposits, the economic theory reviewed by Chapter II suggest likely candidates for the X's. In particular for term deposits the most likely independent variable is the interest rate on term deposits. The empirical part of this study will examine the significance of this variable and the equation specified in (3.19), will be compared with the ARIMA models for its power to predict future values of time deposits. The former is measured by the t-test and the latter is measured by the mean square error.

IV Data and Empirical Results

A. Data

A credit union is a financial intermediary that is an autonomous entity. It has its own operating charter, its own Board of Directors and management, and its own borrowing and lending policies. The common bond of association may vary from employees in one firm to a community of a million people. The most homogeneous credit unions were to be considered in order to compare the estimated ARIMA models. The credit unions that had the longest history of providing both demand deposits and term deposits to their members would provide the largest number of observations to evaluate forecasts. The intersection of these two criteria resulted in the choice of three of the largest credit unions in the Vancouver Metropolitan Region in British Columbia and they had the following characteristics: (i) they face the same external market; (ii) they have the same common bond of community association; (iii) they are multi-branch operations; and (iv) they each have over thirty-million dollars in assets, and both demand deposits and term deposits are offered to their members.

The two accounts examined are total demand deposits and total term deposits. The credit unions primarily deal with non-corporate bodies and economic theory suggests two different behaviours by consumers to the two types of deposits; (i) demand deposits are purchased by individuals for liquidity and convenience in carrying out transactions; and (ii) term deposits are purchased as an investment of savings in low risk securities. The data are gathered quarterly and date back to the second quarter of 1962. From 1962:2 to 1974:4 we have 52 observations for demand deposits and no less than 36 observations for term deposits (some credit unions did not start to offer these services until 1966). The three credit unions examined below are referred to as C.U.1, C.U.2 and C.U.3 and they are ordered with respect to their asset size (This data series are listed in Appendix:Data).

B. Model for Chartered Banks' Deposits

To obtain "a priori" identification of the credit unions' series, but more important to evaluate the strength of the time series method to be used, we first look at the behaviour of demand deposits and term deposits for a similar financial intermediary for which we have a larger number of data points. The time series are the total of personal demand deposits and of personal term deposits held in chartered banks in Canada as published monthly in the Bank of Canada Statistical Review (1967:9 - 1974:11, 87 observations).

The Box and Jenkins technique suggests a monthly seasonal model for demand deposits. The simplest model whose coefficients proved to be statistically significant is (0,1,0) x (0,1,1) 12 multiplicative model where the data has been transformed to natural logarithms. This model indicates that the series is stationary after first differences have been taken and that there is a significant seasonal moving average term that determines the level of demand deposits in banks. Indeed this accurately depicts the seasonal troughs of June and December when the absolute levels of these deposits decrease with respect to the balance held in the previous month. The Kolmogrov-Smirnov Statistic and the Box-Pierce Statistic both suggest that the fitted model adequately represents the data (equation 4.1)

$$(4.1) \quad \nabla \ln DD_t - \nabla \ln DD_{t-12} = \underset{(.001)}{.008} + \underset{(.012)}{.128}\, a_{t-12} \qquad B\&P = 23 \quad n-k = 22$$

The transformed series of term deposits was identified to be (1,1,1) model (data transformed to $\log_e$) and the diagnostic checks suggest that the residuals are white noise (Equation 4.2).

$$(4.2) \quad \nabla \ln TD_t = \underset{(.06)}{.87}\nabla \ln TD_{t-1} - \underset{(.12)}{.34}\, a_{t-1} \qquad B\&P = 13 \quad n-k = 22$$

The transfer function was also tested for term deposits (TD), where the exogenous variables were interest rate on 90 day bank term deposits (R) and time (T). Both coefficients had the right sign and were significant statistically (although the standard errors are underestimated due to high serial autocorrelation, it is unlikely that they are not significant). For the equation with R and TD lagged one period, the monthly model again revealed the expected signs and significance, however, the Durbin-Watson Statistic indicates autocorrelation among the residuals.

$$(4.3.1) \quad \ln TD = \underset{(.07)}{7.05} + \underset{(.01)}{.07}R + \underset{(.001)}{.02}T \qquad D.W. = .07 \quad R^2 = .93$$

$$(4.3.2) \quad \ln TD = \underset{(.04)}{.20} + \underset{(.002)}{.01}R + \underset{(.005)}{.97} \ln TD_{-1} \qquad D.W. = .54 \quad R^2 = .99$$

Thus our digression on banks has confirmed our "a priori" behaviours of the two series in relation to consumers' choices and preferences, it has suggested an order of differencing for the model and the significance of transfer functions, and the time series analysis has been shown to fit the data.

C. Demand for Credit Unions' Demand Deposits

Upon inspection of the quarterly series from 1962-1974 for each of the three credit unions it was evident that there is an exponential growth trend in the data. The best model of demand deposits of C.U. 1 and C.U.3 (transformed into natural logarithms) was the second difference first order moving average model. For C.U.2 there were two models, not quite comparable, whose coefficients were significantly different from zero: (i) seasonal model (1,1,1) x (0,1,1)4; and (ii) second difference model (0,2,1). The study of banks' personal demand deposits showed a significant seasonal trough at the end of the sixth and twelfth months (identical to our second and fourth quarter observations), and we expect a similar seasonal pattern for demand deposits of credit unions. However, if one is using quarterly data as we are it is not obvious that the observations suggest a seasonal pattern. As Figure IV.1 shows, the dotted line is the expected monthly level with June and December being significantly lower than May and November (respective month prior). Whereas the monthly data suggest that $DD_{12} < DD_{11}$ and $DD_6 < DD_5$, they do not suggest that DD_{12} (4th quarter) $<$ DD_9 (3rd quarter) or that DD_6 (2nd quarter $<$ DD_3 (1st quarter), so that our quarterly data does not pick up the seasonal demand for credit union demand deposits--except in the case of C.U.2.

The best equations and their parameters are listed below in Table 4.1. We are satisfied that the unexplained variance in these models is white noise.

FIGURE IV.1

EXPECTED MONTHLY LEVELS

OF DEMAND DEPOSITS

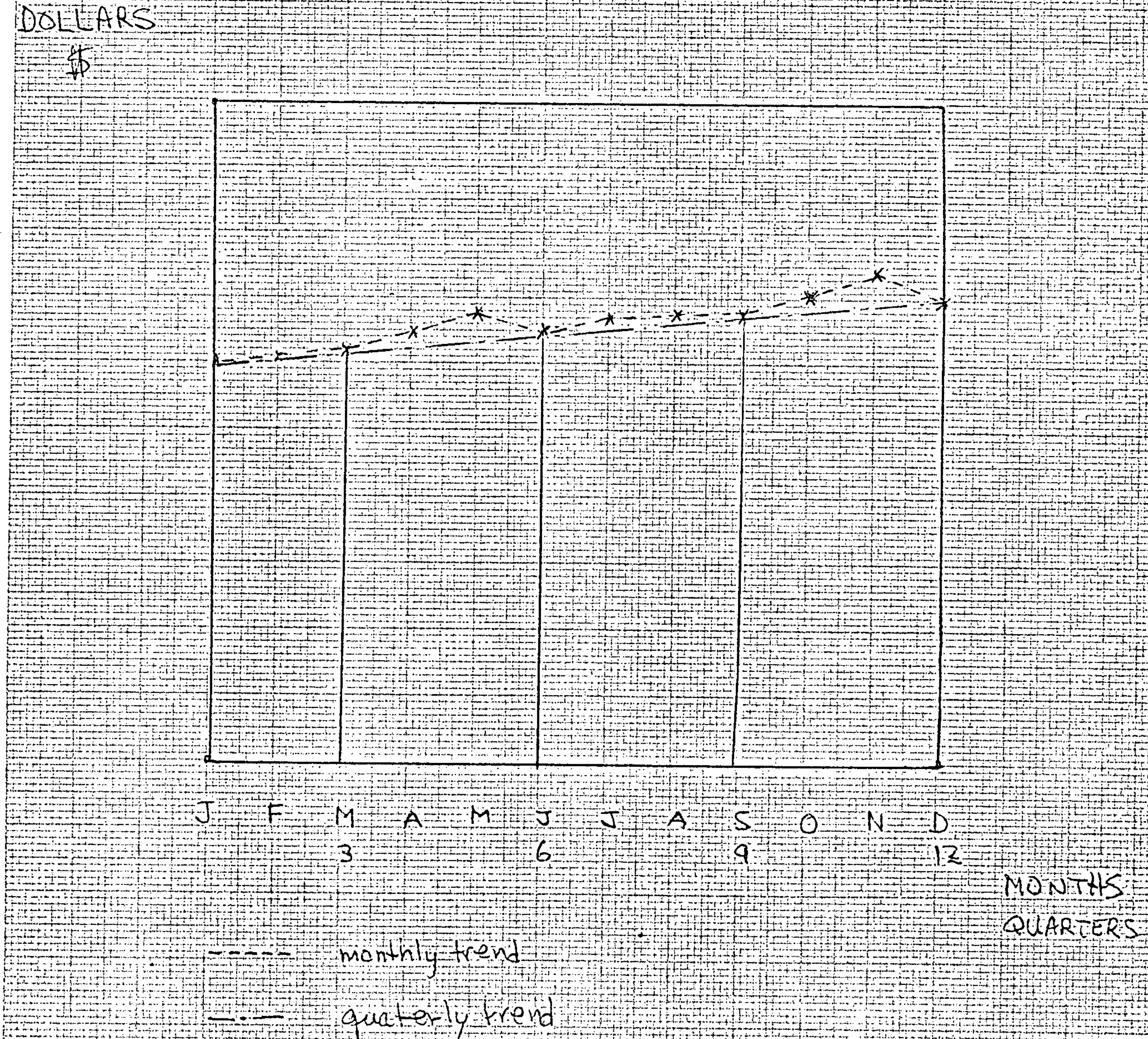


TABLE 4.1

ARIMA MODELS FOR DEMAND DEPOSITS OF CREDIT UNIONS

(0,2,1)	$\nabla ln_t DD_t - \nabla lnDD_{t-1} = a_t + \hat{\theta} a_{t-1}$			
C.U.1	$\hat{\theta} = .81$	$\sigma = .09$	B&P = 22	n-k = 23
C.U.2	$\hat{\theta} = .88$	$\sigma = .07$	B&P = 17	n-k = 23
C.U.3	$\hat{\theta} = .84$	$\sigma = .09$	B&P = 11	n-k = 23
(1,1,1)x(0,1,1)4	$\nabla lnDD_t - \nabla lnDD_{t-4} = \hat{\phi}\nabla lnDD_{t-1} + a_t - \hat{\theta} a_{t-1} - \hat{\Theta} a_{t-4} + \hat{\theta}\hat{\Theta} a_{t-5}$			
C.U.2.	$\hat{\phi} = -.77$ $\hat{\theta} = .46$ $\hat{\Theta} = -1.10$	$\sigma = .05$ $\sigma = .14$ $\sigma = .05$	B&P = 26	n-k = 21

D. Demand for Credit Unions' Term Deposits

The series are transformed to natural logarithms and the first difference first order autoregressive scheme is the best model with the smallest number of coefficients for C.U.1 and C.U.2 (The $\hat{\phi}$'s are significantly different from zero and one). For C.U.3 (1,1,1) model is significant and the residuals of this model are less correlated than those in (1,1,0) model for the time series. Since the autoregressive parameter in the former model is .92 - significantly different from zero but lieing just within the 95% confidence interval (one-tail test) - the second difference model was fitted. The (0,2,1) specification was significant and the evaluation of the three models for C.U.3 is postponed until the later section on prediction. The estimated ARIMA equations are listed below in Table 4.2

TABLE 4.2

ARIMA MODELS FOR TERM DEPOSITS OF CREDIT UNIONS

(1,1,0)	$\nabla \ln TD_t = \phi \nabla \ln TD_{t-1} + a_t$			
C.U.1	$\hat{\phi} = .62$	$\sigma = .06$	B&P = 28	n-k = 23
C.U.2	$\hat{\phi} = .32$	$\sigma = .09$	B&P = 16	n-k = 23
C.U.3	$\hat{\phi} = .64$	$\sigma = .12$	B&P = 23	n-k = 23
(1,1,1)	$\nabla \ln TD_t = \phi \nabla \ln TD_{t-1} + a_t + \theta a_{t-1}$			
C.U.3	$\hat{\phi} = .92$	$\sigma = .04$	B&P = 18	n-k = 22
	$\hat{\theta} = .70$	$\sigma = .13$		
(·,2,1)	$\nabla \ln TD_t - \nabla \ln TD_{t-1} = a_t + \theta a_{t-1}$			
C.U.3	$\hat{\theta} = .64$	$\sigma = .14$	B&P = 18	n-k = 23

The transfer function for demand for credit unions' term deposits was also fitted. Recall that in the example for chartered banks the explanatory variables were the interest rate on 90 day term deposits (R) and time (T). The appropriate interest rate for the time series of credit unions is the rate paid by credit unions on a comparable security. However, as stated in Chapter II the interest rates paid by credit unions are not published on a quarterly basis hence the rate paid by chartered banks is used as a proxy. Ordinary least squares was used to estimate the significance of R and T. For each of the three credit unions the former variable was not significantly different from zero (see Table 4.3). Again it is interesting to speculate as to why this result is so different from that of the chartered banks. Either there has been a structural change in promotion and preference of term deposits in the 1962-1966 and 1967-1974 periods or that in the quarterly data the changes in R may be too discrete to be positively correlated with the new level of term deposits.

TABLE 4.3

O.L.S. RESULTS FOR TRANSFER FUNCTION OF TERM DEPOSITS OF CREDIT UNIONS

	$\ln TD_t = c$	$+ \hat{\alpha}_1 R$	$+ \hat{\alpha}_2 T$		
C.U.1	12.49 (.16)	-.01 (.04)	.12 (.004)	D.W. = .29	R^2 = .97
C.U.2	11.18 (.18)	-.02 (.04)	.13 (.005)	D.W. = .59	R^2 = .97
C.U.3	8.53 (.20)	-.03 (.03)	.18 (.005)	D.W. = .26	R^2 = .98

E. Forecast Evaluation 1974:1-1974:4

The best ARIMA model for the demand for demand deposits of credit unions is evaluated against a naive random walk model ($z_t = \theta z_{t-1} + a_t$)

and against any other models that proved to be significant in the estimation stage. Table 4.4 presents the results and the ARIMA outperform the random walk. The (0,2,1) model is the best specification for forecasting credit unions' demand deposits.

TABLE 4.4

PREDICTION ERRORS, DEMAND DEPOSITS OF CREDIT UNIONS
1974:1 - 1974:4 PREDICTIONS

	Average Absolute Error	Root Mean Square Error
C.U.1		
(0,2,1)	0.29	0.30
(1,0,0)	0.33	0.39
C.U.2		
(0,2,1)	0.07	0.09
(1,1,1)x(0,1,1)4	0.09	0.11
(1,0,0)	0.12	0.20
C.U.3		
(0,2,1)	0.15	0.21
(1,0,0)	0.51	0.80

note: the root-mean square-error is $(\Sigma a^2/n)^{\frac{1}{2}}$, where a's are the errors, the summation is over all observations, and n is the number of observations

Similarly the various models of term deposits series are evaluated with respect to the minimum mean square error criteria for the quarterly forecasts in 1974. In all cases the ARIMA models outperformed the transfer functions and the random walk equation (Table 4.5) illustrates that the best models for demand of credit unions' TD are the (1,1,0) model for

C.U.1 and C.U.2 and (1,1,1) model for C.U.3. Figure IV.2 shows the plot of the calculated and actual values of demand deposits and term deposits for the forecast interval. In almost all the cases our forecasts were too optimistic, overstating the actual balances of deposits held by C.U.1, C.U.2 or C.U.3.

TABLE 4.5

PREDICTION ERRORS, TERM DEPOSITS OF CREDIT UNIONS
1974:1 - 1974:4 PREDICTIONS

	Average Absolute Error	Root Mean Square Error
C.U.1		
(1,1,0)	0.23	0.23
(1,0,0)	0.32	0.35
f(R,T)	0.26	0.27
C.U.2		
(1,1,0)	0.09	0.11
(1,0,0)	0.38	0.44
f(R,T)	0.61	0.65
C.U.3		
(1,1,1)	0.11	0.11
(1,1,0)	0.12	0.18
(0,2,1)	0.28	0.30
(1,0,0)	0.37	0.41
f(R,T)	0.67	0.69

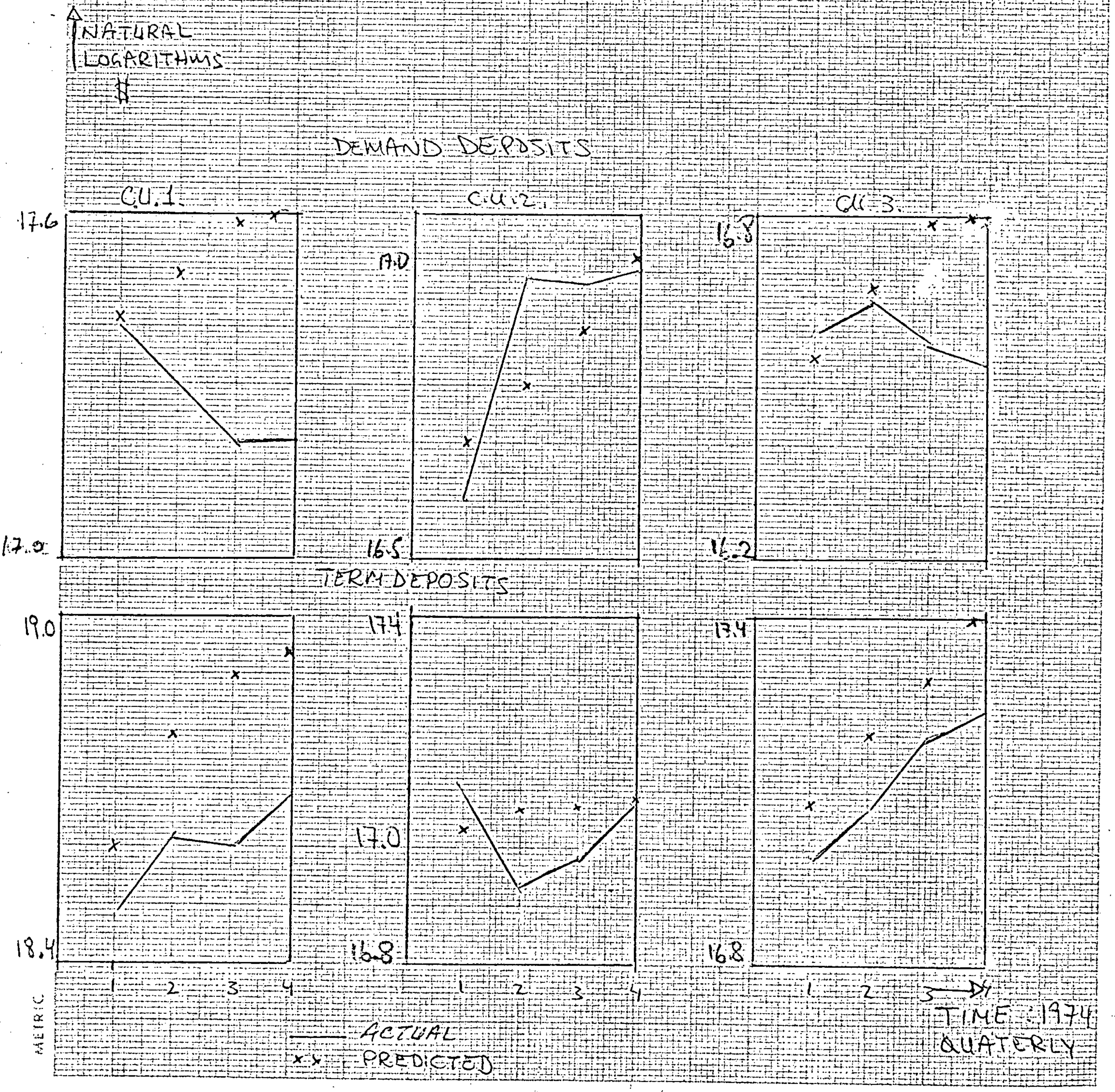
note: see note in table 4.4; f(R,T) is the transfer function where R is the interest rate and T is time as defined above

FIGURE IV.2

PLOT OF ACTUAL AND PREDICTED VALUES FOR CREDIT UNIONS'

DEMAND DEPOSITS AND TERM DEPOSITS

1974.1 — 1974.4



V. Concluding Remarks

We have found that the ARIMA models furnish the best forecasts for demand deposits and term deposits of credit unions and for personal demand deposits and term deposits of chartered banks. For demand deposits, the (0,2,1) model best describes the demand for credit unions' deposits and the (0,1,0) x (0,1,1) 12 process is the one we identified for chartered banks. For term deposits, C.U. 1 and C.U. 2 data follow a (1,1,0) process while the (1,1,1) model is the best formulation for C.U. 3 and chartered banks. In all the cases we are satisfied that the unexplained variation in the series of demand deposits or term deposits is white noise.

The predictions for 1974 proved to be too optimistic. This suggests that constant feedback must be maintained in order to update the forecasts and to monitor the turning points in the series. It is likely that the values of the parameters may change as these ARIMA models are fitted to new data points. Future research should try to use monthly data because there is a likely seasonal pattern that is not being picked up by the quarterly data. This will also give more observations to the time series and strengthen the model identification and estimation.

There is another problem with having used the quarterly series for the years between 1962 and 1974. This period is by no means a homogeneous one for financial intermediation in Canada or for credit unions in British Columbia. The market structure was quite different prior to 1967 at which time the Bank Act was changed and chartered banks increased their activities in the consumer market. There has been a marked shift in the growth rate of credit unions' assets in British Columbia since 1970 and perhaps the underlying pattern of the data is not the same as for the 1962-69 period. If this shift should be significant then our 1962-74 ARIMA models may have introduced an archaic pattern into the model and into the forecasts. As the number of observations will increase with time it will be possible to test the homogeneity of the time series for credit union deposits.

Thus our thesis has successfully modelled the time series for demand deposits and term deposits of a credit union. The financial manager in a credit union can generate the forecasts for deposits using our ARIMA models and with forecasts of interest rates and of loan demand he can implement them in an optimization technique.

BIBLIOGRAPHY

Batra, H. (1973). "Dynamic Interdependence in Demand for Savings Deposits", *Journal of Finance*, May, 1973, vol XXVIII, No. 2, p. 507-514.

Box, G. and G. Jenkins (1970). *Time Series Analysis: forecasting and control*, Holden Day, San Francisco, 1971.

Boyd, J. (1973). "Some Recent Developments in the Savings and Loan Deposit Markets", *Journal of Money, Credit and Banking*, August, 1973, Vol. v, No. 3, p. 733-750.

Cohan, S. (1973). "The Determinants of Supply and Demand for Certificates of Deposit", *Journal of Money, Credit and Banking*, February, 1973, Vol. v, No. 1, p. 100-112.

Cramer, R. and R. Miller (1973). "Development of a Deposit Forecasting Procedure for Use in Bank Financial Management", *Journal of Bank Research*, Summer, 1973, p. 122-138.

DeLeeuw, F. (1965). "A Model of Financial Behavior", in *Brookings Quarterly Econometric Model of the United States*, eds. J.B. Duesenberry et al., Rand-McNally, Chicago, 1965, p. 465-532.

DeLeeuw, F. (1969). "A Condensed Model of Financial Behavior," in *The Brookings Model. Some Further Results*, eds. J.B. Duesenberry et al., Rand-McNally, Chicago, 1969, p. 270-316

Dhrymes, P. and P. Taubman (1969). "An Empirical Analysis of the Savings and Loan Industry" in *Study of the Savings and Loan Industry*, ed. I. Friend, Federal Home Loan Bank Board, Washington, D.C., 1969, p. 67-182.

Feige, E. (1964). *The Demand for Liquid Assets: A Temporal Cross-Section Analysis*, Prentice-Hall, New Jersey, 1964.

Feige, E. (1974), "Alternative Temporal Cross-Section Specifications of the Demand for Demand Deposits", *Journal of Finance*, June, 1974, vol. XXIX, no. 3, p. 923-940.

Friedman, M. (1957). *A Theory of the Consumption Function*, National Bureau of Economic Research, Princeton, 1957.

Motley, B. (1970). "Household Demand for Assets: A Model of Short-Run Adjustments", *Review of Economics and Statistics*, August, 1970, Vol. XII, No. 3, p. 236-241.

Nelson, C. (1973). *Applied Time Series Analysis for Managerial Forecasting*, Holden-Day, San Francisco, 1973.

APPENDIX : DATA

DEMAND DEPOSITS C.U.1

283728.	288410.	277601.	290200.
328629.	397636.	483302.	579923.
670283.	664819.	700115.	859983.
1145867.	1286335.	1168532.	1315540.
1570730.	1361218.	1561367.	1623500.
1976460.	2667036.	2467194.	3069123.
3451364.	3778215.	4433987.	4638152.
5338262.	6508993.	6116369.	6064195.
6573045.	7154165.	8641735.	9423604.
13240078.	16313127.	17623152.	19440848.
21584896.	24559728.	27228336.	32179760.
38420704.	37876336.	32794640.	35345552.
32716080.	30331968.	29406736.	

DEMAND DEPOSITS LN C.U.1

12.556	12.572	12.534	12.578
12.703	12.893	13.088	13.271
13.415	13.407	13.459	13.665
13.952	14.067	13.971	14.090
14.267	14.124	14.261	14.300
14.497	14.796	14.719	14.937
15.054	15.145	15.305	15.350
15.490	15.689	15.626	15.618
15.698	15.783	15.972	16.059
16.399	16.607	16.685	16.783
16.887	17.017	17.120	17.287
17.464	17.450	17.306	17.381
17.303	17.228	17.197	

TERM DEPOSITS C.U.1

1.	85500.	231000.	390500.
507000.	616500.	695000.	800000.
863500.	949723.	1130535.	1278997.
1591329.	1727357.	2198997.	2412211.
2568230.	3113829.	3496095.	3946960.
4101678.	4466207.	5006250.	5546293.
5940851.	6321382.	6540961.	7158364.
7795252.	8087713.	8395023.	8454867.
9291387.	10108465.	13068507.	15679535.
19320080.	21975056.	29106528.	32807952.
35523872.	40226720.	49773856.	58248880.
66961040.	76191184.	104511584.	107059568.
121612864.	120154240.	131267616.	

TERM DEPOSITS LN C.U.1

0.0	11.356	12.350	12.875
13.136	13.332	13.452	13.592
13.669	13.764	13.938	14.062
14.280	14.362	14.604	14.696
14.759	14.951	15.067	15.188
15.227	15.312	15.426	15.529
15.597	15.659	15.694	15.784
15.869	15.906	15.943	15.950
16.045	16.129	16.386	16.568
16.777	16.905	17.186	17.306
17.386	17.510	17.723	17.880
18.020	18.149	18.465	18.489
18.616	18.604	18.693	

DEMAND DEPOSITS C.U.2

42566.	41730.	49070.	48420.
81171.	94573.	112242.	125489.
113742.	125952.	147022.	167439.
210250.	271566.	263801.	444177.
493942.	378563.	626290.	632440.
782152.	882650.	1054217.	1272136.
1465475.	1468787.	1783469.	1930989.
2200238.	2257850.	2234457.	2313589.
2615548.	2710551.	2880538.	3182165.
4333078.	4990218.	6097890.	6852933.
7941170.	9464295.	9860277.	11098798.
14047351.	16122663.	15874823.	16234764.
23868592.	22500016.	25584384.	

DEMAND DEPOSITS LN C.U.2

10.659	10.639	10.801	10.788
11.304	11.457	11.628	11.740
11.642	11.744	11.898	12.028
12.256	12.512	12.483	13.004
13.110	12.844	13.348	13.357
13.570	13.691	13.868	14.056
14.198	14.200	14.394	14.474
14.604	14.630	14.620	14.654
14.777	14.813	14.873	14.973
15.282	15.423	15.623	15.740
15.888	16.063	16.104	16.222
16.458	16.596	16.580	16.603
16.988	16.929	17.057	

TERM DEPOSITS C.U.2

1.	1.	1.	1.
1.	1.	34800.	135900.
204200.	250900.	316375.	375375.
411908.	447175.	501977.	361300.
449200.	846244.	729636.	834899.
991779.	1214559.	1395449.	1493096.
1796670.	2141400.	2566803.	2808933.
3575329.	3928884.	4180953.	4638880.
5071935.	5619708.	6515980.	7376612.
8505891.	9221807.	9992905.	10870671.
11891481.	12879057.	14714702.	16288378.
17563008.	18394128.	23261632.	28583872.
22601120.	23471504.	26045296.	

TERM DEPOSITS LN C.U.2

0.0	0.0	0.0	0.0
0.0	0.0	10.457	11.820
12.227	12.433	12.665	12.836
12.929	13.011	13.126	12.797
13.015	13.649	13.500	13.635
13.807	14.010	14.149	14.216
14.401	14.577	14.758	14.848
15.090	15.184	15.246	15.350
15.439	15.542	15.690	15.814
15.956	16.037	16.117	16.202
16.291	16.371	16.504	16.606
16.681	16.728	16.962	17.168
16.934	16.971	17.075	

DEMAND DEPOSITS C.U.3

30211.	29922.	35076.	36119.
47415.	40146.	54531.	58095.
54238.	50398.	61514.	67321.
70655.	102755.	85070.	83590.
104508.	129995.	145659.	193789.
247533.	286000.	300776.	373105.
470602.	486069.	588993.	719561.
890417.	948713.	969893.	1210128.
1489363.	1501195.	2063821.	2392084.
3401836.	3854162.	4818345.	5859625.
7983876.	8676195.	10504561.	12132317.
15228363.	15218569.	13645977.	15597718.
17014160.	16019679.	15431547.	

DEMAND DEPOSITS LN C.U.3

10.316	10.306	10.465	10.495
10.767	10.600	10.907	10.970
10.901	10.828	11.027	11.117
11.166	11.540	11.351	11.334
11.557	11.775	11.889	12.175
12.419	12.564	12.614	12.830
13.062	13.094	13.286	13.486
13.699	13.763	13.785	14.006
14.214	14.222	14.540	14.688
15.040	15.165	15.388	15.584
15.893	15.976	16.167	16.311
16.539	16.538	16.429	16.563
16.650	16.589	16.552	

TERM DEPOSITS C.U.3

1.	1.	1.	1.
1.	1.	1.	1.
1.	1.	1.	1.
1.	1.	1.	1.
59000.	97000.	113500.	132500.
176500.	186000.	205500.	363500.
419500.	569700.	858700.	1013400.
1470231.	1607531.	1696131.	1835406.
2150876.	2425786.	2932926.	3343537.
4428624.	5866774.	7293652.	8168730.
8880908.	9554125.	11617550.	12667943.
14149311.	17151568.	22396832.	23459472.
26597312.	29519456.	30429056.	

TERM DEPOSITS LN C.U.3

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
10.985	11.482	11.640	11.794
12.081	12.134	12.233	12.804
12.947	13.253	13.663	13.829
14.201	14.290	14.344	14.423
14.581	14.702	14.892	15.023
15.304	15.585	15.803	15.916
15.999	16.072	16.268	16.355
16.465	16.658	16.924	16.971
17.096	17.201	17.231	

DEMAND DEPOSITS X10**6 CANADIAN BANKS

1040.	1083.	1174.	1261.	1326.	1400.
1506.	1640.	1853.	2099.	2293.	2408.
2450.	2487.	2502.	2539.	2634.	2772.
2875.	2950.	3048.	3140.	3243.	3389.
3508.	3570.	3579.	3594.	3636.	3711.
3781.	3873.	4005.	4104.	4202.	4306.
4391.	4428.	4465.	4481.	4551.	4648.
4706.	4602.	4442.	4328.	4235.	4198.
4182.	4207.	4150.	4127.	4234.	4324.
4416.	4493.	4595.	4697.	4788.	4922.
5058.	5130.	5114.	5191.	5349.	5544.
5675.	5789.	5989.	6273.	6537.	6796.
7034.	7384.	8117.	8579.	8987.	9457.
9785.	10000.	10504.	11170.	11751.	12360.
12739.	13038.	12490.			

DEMAND DEPOSITS LN X10**6 CANADIAN BANKS

6.947	6.987	7.068	7.140	7.190	7.244
7.317	7.402	7.525	7.649	7.738	7.787
7.804	7.819	7.825	7.840	7.876	7.927
7.964	7.990	8.022	8.052	8.084	8.128
8.163	8.180	8.183	8.187	8.199	8.219
8.238	8.262	8.295	8.320	8.343	8.368
8.387	8.396	8.404	8.408	8.423	8.444
8.457	8.434	8.399	8.373	8.351	8.342
8.339	8.345	8.331	8.325	8.351	8.372
8.393	8.410	8.433	8.455	8.474	8.501
8.529	8.543	8.540	8.555	8.585	8.620
8.644	8.664	8.698	8.744	8.785	8.824
8.859	8.907	9.002	9.057	9.104	9.155
9.189	9.210	9.260	9.321	9.372	9.422
9.452	9.476	9.433			

TERM DEPOSITS X10**6 CANADIAN BANKS

10443.	10535.	10532.	10367.	10461.	10539.
10587.	10694.	10767.	10702.	10768.	10200.
10865.	11031.	11038.	10979.	11021.	11077.
11136.	11281.	11302.	11296.	11357.	11394.
11426.	11516.	11473.	11297.	11355.	11463.
11543.	11696.	11871.	11742.	11716.	11888.
12002.	12125.	12101.	11987.	12106.	12239.
12367.	12782.	12945.	13156.	13418.	13654.
13879.	14075.	13655.	13406.	13625.	13834.
14161.	14301.	14408.	14379.	14531.	14625.
14791.	14962.	14797.	14540.	14781.	14932.
15016.	15214.	15323.	15381.	15589.	15742.
15829.	15966.	15834.	15699.	15867.	16029.
16204.	16601.	16940.	16860.	17043.	17167.
17343.	17625.	17562.			

TERM DEPOSITS LN X10**6 CANADIAN BANKS

9.254	9.262	9.262	9.246	9.255	9.263
9.267	9.277	9.284	9.278	9.284	9.230
9.293	9.308	9.309	9.304	9.308	9.313
9.318	9.331	9.333	9.332	9.338	9.341
9.344	9.351	9.348	9.332	9.337	9.347
9.354	9.367	9.382	9.371	9.369	9.383
9.393	9.403	9.401	9.392	9.401	9.412
9.423	9.456	9.468	9.485	9.504	9.522
9.538	9.552	9.522	9.503	9.520	9.535
9.558	9.568	9.576	9.574	9.584	9.590
9.602	9.613	9.602	9.585	9.601	9.611
9.617	9.630	9.637	9.641	9.654	9.664
9.670	9.678	9.670	9.661	9.672	9.682
9.693	9.717	9.737	9.733	9.743	9.751
9.761	9.777	9.773			