SELECTING RESEARCH AND DEVELOPMENT PROJECTS

by

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B.Sc., McGill University, 1971

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We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

April 1974
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ABSTRACT

Previously proposed algorithms for selecting research and development projects are reviewed. A new method for analysis of project selection decisions is developed. This method includes such features as multiple criteria, funding ranges, and multiple solutions. Methods of handling multiple criteria are discussed. An interactive project selection algorithm is developed and implemented. Three characteristics of the algorithm improve its usefulness and acceptability to decision makers: 1) the algorithm is based on a relatively realistic model of the decision situations, 2) it requires direct participation on the part of the decision maker and allows some user control of evaluation criteria, and 3) it's data requirements are modest.
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Chapter 1

A SURVEY OF PROJECT SELECTION ALGORITHMS

The problem of selecting research and development projects is an important and difficult one. The future competitive position of many companies depends on the effectiveness of their research and development program. The decisions that are made now will determine whether the company will remain in business, and if so, what business it will be in. Research and development programs are equally important to governments in their attempt to control, direct, and stimulate change. Since many billions of dollars are spent each year on research and development there is justifiable concern that this money should be invested wisely.

Much of the difficulty in selecting research and development projects arises from the uncertainties surrounding the selection decision. The outcome of each project (success or failure)\(^1\) may be uncertain due to technological considerations,

\(^1\)It is often difficult to classify the outcome of a project as success or failure. However, for planning purposes a project may be considered successful if it achieved the goal it set out to, and unsuccessful otherwise.
and the value of each project, even if successful, is uncertain due to unforeseeable future needs and conditions. Since more projects are available than can be undertaken, due to budget and other resource constraints, the organization must select the ones which they consider to be of most value. However, the goals toward which the organization should be striving are often unclear, and the contribution of research and development to the achievement of these goals even more vague. This makes the selection problem difficult. "If basic data and clear-cut measures of the effectiveness of applied research were available there would appear to be little difficulty establishing an objective basis for selecting projects" [64].

Any attempt to establish a selection procedure is based, implicitly or explicitly, on a model of the decision process. Ideally this model would include all the significant features of the problem. However, since any model is only an abstraction of the real-world situation, it necessarily incorporates many assumptions and simplifications. This is especially true when dealing with a large number of uncertainties. However, despite the approximations inherent in the model and the uncertainty of the data, the selection procedure may still be valuable for "... decisions have to be made, usually on inadequate data,

——

*Most project selection algorithms assume there is a large set of available projects from which to choose. Whitman and Landau [82] and Gee [34] dispute this assumption, especially in the chemical industry, but in general it seems to be a reasonable one.*
and they will be made intuitively on the data, whatever its quality. Anything that can be done to quantify the bases for these decisions and to demonstrate the logical consequences of the assumptions is a step in the right direction" [7].

Much of the value resulting from the development of a selection procedure would come from the model-development phase itself. The development of a model would force the decision maker to specify, in detail, his goals and any assumptions or implicit constraints he imposes. Any irrationalities would then be clearly illustrated. An analysis of current practices would help to eliminate inconsistencies between the decision maker's personal objectives and the overall organizational objectives. Current "rules of thumb" could be tested for their rationality and effectiveness. Long range planning and discussion of goals would be stimulated. Thus, an analysis of the decision problem would result in a better understanding of the many uncertainties involved and their effect on the results of the decision process. The selection procedure that would result from the development of the model would hopefully facilitate a more consistent treatment of decision problems.

Many models and selection procedures have been suggested in the literature. They may be broadly divided into qualitative and quantitative approaches. The qualitative methods generally consist of a check list of desirable properties. Projects are rated with respect to each checklist criterion as
simply "favorable" or "unfavorable" or on a numeric scale. To be selected, a project's ratings must follow an "acceptable" pattern, or must meet certain specified minimum levels with respect to each criterion. Such methods are imprecise for they depend heavily upon the ability of the decision maker to rate objectively and consistently. Furthermore, they provide no indication of the relative value of the acceptable projects and hence furnish little information or guidance concerning the appropriate funding level for each project. The selection procedure may also be difficult to define and apply. What constitutes an "acceptable" pattern of ratings? What should the minimum acceptable level of a criterion be? Is it reasonable to reject a project that does not meet the minimum level in one area but far exceeds it in another? Clearly these questions are important. Answering them is the first step in the development of a more precise, and sophisticated decision method.

The quantitative models that have been suggested in the literature may be classified into eight general categories: scoring models, linear, non-linear, zero-one, and utility models, profitability indices, risk analysis and decision trees.

1.1 SCORING MODELS

The scoring models are the least sophisticated of the quantitative methods. They use the same type of ratings as the qualitative methods to determine a numerical project score.
The Mottley-Newton method [64] rates each project on a three point scale with respect to five criteria. The ratings are then multiplied to produce the project score. Garguilo et al. [32] and Hertz and Carlson [42] suggest a method whereby each project is rated as "favorable," "unfavorable," or "no opinion" with respect to the criteria which are divided into three classes; economic, technical, and commercial factors. The number of each type of response in each class is counted and a score for that class calculated. The scores for the three classes are multiplied together to form the overall project score.

The scoring models have many of the defects of the qualitative methods. They depend heavily on the decision maker's rating ability, and provide only an ordinal ranking of projects. Since the ratings are on an arbitrary scale, it is not possible to know how much better one project is than another. It may not even be possible to know if the best project is "good." (They may all be of little value to the organization.) Scoring models do however have several desirable features. Since much reliance must be placed on the decision maker, very little data on the project is needed. Thus, the method is most useful for decisions concerning pure research projects and projects in their early stages of development where little concrete information is available on their costs and benefits. The project scores can also be used to help diagnose a project's weak points. They can illustrate those areas where the project
could be improved. Furthermore, scoring models can be constructed
to include non-economic criteria that are difficult to quantify
for use in more sophisticated models.

1.2 LINEAR MODELS

    Linear models often use a type of scoring system as
well. The methods suggested by Pound [68] and Dean and Nishry
[21] use a weighted sum of project ratings as the objective
function. Pound determines the appropriate weights by interview­
ing the decision maker. Dean and Nishry suggest obtaining them
from statistical analyses of past decisions. Both methods
provide only an ordinal ranking of projects. Nutt [66] has
developed a linear model for selecting military projects. The
"effectiveness" of each different project at six discrete fund­
ing levels is calculated by considering various military needs
and goals. A linear program is solved and the results indicate
the level at which each project should be funded, and the man­
power that will be required. Asher [1] suggests maximizing
expected profit in an L.P. model which allocates a non-homogeneous
work force to projects.

    The linear models generally provide more information
about funding levels and the relative values of the various
projects. However they require more data to do this; estimates
of profit, or effectiveness, and probabilities of success are
required.
1.3 NON-LINEAR MODELS

Many of the non-linear models view the selection procedure as a sequential problem. At the beginning of each planning period old projects are reviewed, and new projects are evaluated. The research and development program for the next period is selected from this collection of old and new projects. This view of the problem leads naturally to a dynamic programming formulation. Hess [44] suggests a method for maximizing the expected discounted net profit. A discount factor must be specified and estimates are required of the total expected discounted gross profit accruing from each project if it is successful in the $n^{th}$ period, as well as the probability it will be successful in the $n^{th}$ period, for all periods in the planning horizon. The probability of success is assumed to be an exponential function of the current funding level and past funding levels or current funding level alone. The result of the procedure is an optimal funding level for each project in each planning period.

Bobis et al. [7, 9] have suggested modifications to Hess's method. They developed a distribution of the cost of completing a project (success or failure) by requiring estimates of the least, most likely, and greatest expected completion cost. The probability of success in any year is then the probability of completion at the current expenditure level multiplied by the probability of technical, legal, engineering and
commercial success. The optimal allocation of funds is determined from estimates of sales, costs, prices, time required for commercialization and probability of success. Since the method is so dependent on the data, they suggest replacing point estimates by distributions and simulating to obtain a more accurate value.

Souder [73] and Rosen [69] have attempted to simplify Hess's method by allowing each project to be funded at certain discrete values only, and by assuming that the probability of success is a function of the current funding level alone. They incorporate the additional constraint that there is a "minimum and maximum amount that can be spent on each project over its research and development life" [69].

These methods are based on a more realistic view of the decision process. However their onerous data requirements make them difficult to use except on commercial projects in an advanced state of development. The objective function of these models is to maximize profit. No other possible goals are considered. Dean and Hauser [20], however, have suggested dynamic programming methods for use in a military context which optimize several different criteria.

1.4 ZERO-ONE MODELS

The zero-one models such as those devised by Minkes and Samuels [61], Freeman [31], and Dean and Nishry [21] are all very similar. They all propose maximizing an index of value
subject to budget and resource constraints. Minkes and Samuels suggest the possibility of maximizing expected present value but impose an additional constraint that the total risk involved in the program (weighted sum of the variances of project return) is less than a specified amount. Freeman uses an index of value which is not necessarily profit oriented and which must be developed by the organization concerned. He provides for three discrete levels of funding to be considered. Dean and Nishry develop two models; one which maximizes the present value of future profits and another using a scoring-type approach which maximizes some non-economic measure of value.

The zero-one models are very similar in approach to the linear ones, and require about the same amount of data. The ones which restrict the possible funding level to one value are more appropriate for projects with a fairly well determined cost.

1.5 Utility Models

The utility models suggested by Cramer and Smith [16] and Green [40] are an attempt to explicitly handle risk considerations. They take into account the fact that it is more important to minimize loss than to maximize gain. Cramer and Smith attempt to reduce the value of a project to its certainty equivalent by estimating a coefficient of risk aversion and a coefficient of
diversification from the utility function obtained from the decision maker. Projects are then selected or rejected on the basis of their certainty equivalent. Green also obtains a utility function from the decision maker and uses it to aid in the decision process.

1.6 PROFITABILITY INDEX MODELS

The profitability index models, Hirsch and Fisher [45], Olsen [67], Bobis and Atkinson [8], and Disman [23] are all based on the same idea; the ratio of some measure of the value of the project to some measure of the cost is used to indicate the project's desirability. The differences lie in the measures of value and cost used. Disman's method is considered useful [23]. He suggests calculating the maximum expenditure justified (MEJ) which is the present value at some acceptable rate of return of the income generated by the project. This is multiplied by the probability of technical and commercial success and divided by the total estimated research and development costs. Several other more specialized indices have been devised [36].

Profitability index methods are similar to scoring models in that they provide a single numeric measure of the desirability of each project, and an ordinal ranking of projects. Whereas scoring methods are most suitable for pure research projects and those projects in their early stages of development
which have only imprecise data available, profitability index models are most suitable for commercial projects, and projects near completion, where accurate estimates can be made of costs and benefits.

1.7 RISK ANALYSIS

Risk analysis is a simulation technique for determining the probability distribution of return on a project [41, 59, 80]. Distribution functions must be specified for each factor which affects return. Malloy [59] suggests using a beta distribution so that only estimates of the lowest, most likely, and highest possible values are required to define the distributions. The project's development is simulated by choosing a value for each factor according to its distribution and combining these values in the appropriate way. Bobis et al. [7, 9] have suggested using a similar technique to determine more reliable estimates of the factors required in their non-linear model.

1.8 DECISION TREES

Many of the decision tree models use risk analysis as a solution technique. A decision tree is a graphical representation of the expected stages of project development. Each future decision point and chance outcome point is represented by
a node in the tree. Hespos and Strassman [43] suggest using risk analysis on each possible path through the tree (if there are not too many), or eliminating some paths by dominance, and analyzing the rest in more detail. Lockett and Freeman [55] and Lockett and Gear [56] suggest sampling at each chance outcome point, and reducing the problem to a deterministic linear program. This procedure is repeated many times, resulting in a set of feasible programs which are optimal for one particular state of the world. The final program is selected by examining this set for projects which are always selected or never selected and for other significance patterns. An integer programming method of solving the decision tree problem directly (without simulation) is given by Gear and Lockett [33]. It becomes unsolveable however, when there are many chance outcome points.

1.9 LIMITATIONS OF THE MODELS

Very few of the models and selection procedures that have been suggested in the literature have actually been implemented. Baker and Pound [5] suggest two reasons for this: lack of testing and computational experience, and lack of realism in many of the models.

The most clearly unrealistic feature of many of the models is the objective function. The goal of profit maximization is the only one considered in most cases. In a study of
the utility and acceptability of project selection models, Souder found that this need not necessarily be the only, nor even the primary goal. In his study, "none of the seven administrators interviewed indicated a strong proclivity to pursue the maximization of expected values. Several administrators indicated a definite rejection of such objectives. All the administrators viewed several non-monetary goals as paramount considerations. Some administrators indicated that various intrinsic properties of the portfolios themselves could be more important considerations than the short term profitability statistics" [78]. The scoring-type models and some of the linear models consider other types of goals. However the method of combining them into an objective function is generally quite arbitrary and thus their relative importance in the model is not the same as their relative importance in the eyes of the decision maker. What is needed then, is a technique to combine any of the possible goals into an objective function in accordance with the decision maker's priorities. "The assumption that there exists an optimal solution or a set of optimal solutions to a problem involving multiple criteria implies the existence of some preference ordering defined over the set of feasible values of the criteria" [24]. The problem is then reduced to one of finding this "preference ordering," a subject which is discussed in the next chapter.
Another unrealistic feature of many of the models concerns the assumption made about funding levels. The zero-one models assume funding is possible at only one level. Many of the linear and non-linear models assume funding is possible at any level. A more realistic approach would be to allow a project to be funded in a range or not at all. Freeman [31] considers this idea but does not implement it. He suggests that there is a "critical cost level" below which the value and probability of success of a project is very small, and a "satiation point" above which additional funding creates little additional value and increases the probability of success by an insignificant amount. These upper and lower bounds could also be dictated by organizational policy. There may be an upper limit on the amount that may be risked on any one project and a lower limit determined by the least amount that is "reasonable" to invest in a project.

The probability of success of a project is directly related to its funding level. Most of the more sophisticated models require a subjective estimate of the probability of success of each project at a given level. This estimate is used to develop the relationship between funding and probability of success. A study by Souder [74] on the validity of subjective probability of success estimates, suggests that they are generally valid and reliable although they may not always be accurately communicated due to ulterior motives on the part of the decision maker and organizational pressures.
The project selection model which will be developed in the following chapters will attempt to describe the selection process more realistically by including multiple criteria, funding ranges, and probability of success estimates.
RATIONAL DECISION MAKING WITH MULTIPLE CRITERIA

Rational decision making implies a goal-directed choice among alternative courses of action. Goal-directed choice among alternatives requires both a knowledge of the correspondence between actions and outcomes, and a subjective preference ordering among outcomes. This chapter focuses on the latter, the development of methods for evaluating outcomes in terms of goals and ordering them on the basis of their subjective value.

Outcomes can be described by a set of variables or attributes, which reflect the dimensions through which the outcome contributes to or detracts from the ultimate goals or objectives of the decision maker. For example, in choosing a house, the alternatives can be presented in terms of such attributes as space, price, convenience of location, condition and facilities, etc., which describe each house completely with respect to the decision maker's goals, and thus define criteria for evaluation. These criteria may correspond directly to the goals, or they may be simply indicators which are related to the goals. For another example consider a government agency
that wishes to fund a number of projects. Two of its goals might be to increase employment and to maintain Canadian sovereignty. One of the attributes used in this case for project evaluation corresponds directly to the first goal, i.e. the number of jobs created. The second goal can be measured in several ways, for example, by (1) the resulting percentage increase in Canadian ownership of firms, and (2) the increase in the amount of local raw materials processed in Canada. These last two criteria are merely indicators which are related to the second goal.

Multiple criteria decision problems have a well-defined solution if there exists a preference ordering defined over the feasible values of the criteria which is complete and transitive. Completeness means that all alternatives can be compared and the one with the greatest relative value can be found. This may be a difficult requirement in practice. The decision maker may be able to choose between two alternatives which give him $500 or $800, but may find it more difficult to choose between alternatives which give him $500 or a trip to Hawaii. In the first case he need only compare the levels of the relevant attribute (money). In the second case he must relate the different attributes (money, trip) to an underlying goal (possibly prestige) in order to determine which one has the greater relative value. In the context of the project selection problem the alternatives (programs of projects) will
generally differ along the level of the relevant attributes rather than by having different attributes altogether. Therefore they will tend to be easier to compare.

Transitivity means that if alternative B is preferred to alternative A (the relative value of B is greater than the relative value of A), and alternative C is preferred to alternative B, then alternative C is preferred to alternative A. This is a reasonable assumption when all alternatives are readily comparable, i.e. completeness holds.

In many situations all that is required of the decision maker is that he rank his alternatives, by considering each criterion and the relationships between them. However, in the case of a project selection decision, the number of alternatives is large. An alternative in this case is a portfolio of R&D projects with a specified funding level for each. Each project under consideration is not simply one possible component of a research and development program. It is a representative of a set of possible components, distinguished by their funding levels. Each funding level implies a different probability of success for the project, and thus results in a different amount of the attributes of that project. Therefore there is an infinite number of possible research and development programs, and it is impossible to solve the selection problem by simply ranking the alternatives. What is required is an explicit preferences ordering that may be used to find an optimal portfolio of projects.
There are many methods of determining preference orderings described in the literature. These may be classified into sequential elimination methods, spatial proximity methods, mathematical programming methods, and utility function methods (directly assessed preference techniques and inferred preference techniques). Table 1 summarizes the classification and lists the methods which fall into each category.

2.1 SEQUENTIAL ELIMINATION METHODS

Sequential elimination methods order the alternatives by comparing them either to each other or to a standard. If alternatives are compared to each other, those which provide less of all criteria than another can be eliminated. This dominance technique is used by Terry [79] as an initial filter in selecting new product areas. Generally very few alternatives can be eliminated this way. Other methods must be used to determine the ordering of the remaining alternatives.

Another method of ordering alternatives by comparing them to each other is lexicography. The alternatives are ranked on the basis of their rating on the most important criterion. If any of the alternatives prove to be equal in value, then the rating on the next most important criterion is considered.

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1This classification is essentially the one proposed by MacCrimmon [58].
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<th>CLASSIFICATION</th>
<th>METHOD</th>
<th>CHARACTERISTICS</th>
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<tr>
<td>A. Sequential Elimination Methods</td>
<td>Dominance</td>
<td>alternatives are compared to each other; those alternatives with lower levels of all criteria may be eliminated.</td>
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<tr>
<td></td>
<td>Lexicography</td>
<td>alternatives are ordered on the basis of their rating on the most important criterion</td>
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<td></td>
<td>Elimination by Aspects</td>
<td>most discriminatory criterion used to order alternatives</td>
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<td></td>
<td>Conjunctive</td>
<td>alternatives are compared to a standard; eliminate all whose worst criterion level does not meet the standard; non-compensatory</td>
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<tr>
<td></td>
<td>Disjunctive</td>
<td>accept alternatives if any criterion level exceeds the standard, non-compensatory</td>
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<tr>
<td>B. Spatial Proximity Methods</td>
<td>Indifference Map</td>
<td>a set of indifference or tradeoff curves are obtained for each pair of criteria; alternatives can then be positioned with respect to these curves, and or ordering obtained</td>
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<td>Multi-dimensional Scaling</td>
<td>alternatives are located in a multi-dimensional space so that their value is inversely proportional to their distance from the ideal point</td>
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<td>C. Mathematical Programming Methods</td>
<td>Goal Programming</td>
<td>an alternative is generated which minimizes the weighted sum of deviations from a set of goals; uses L.P.</td>
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<td>Interactive Tradeoff Technique</td>
<td>better alternatives are iteratively generated by requiring the user to specify tradeoffs between criteria at the current point and using them to find the best direction to move</td>
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<td>D. Utility Function Methods</td>
<td>Direct Rating Method</td>
<td>decision maker rates criterion levels on an arbitrary scale criterion levels ranked generally using pairwise comparisons</td>
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<td>1. Directly Assessed Preference</td>
<td>Ranking Methods</td>
<td>one variant, decision maker asked for probability p(z) which makes a gamble made up of two criterion levels x and y occurring with probabilities p(z) and 1 - p(z) respectively, as valuable to him as level z</td>
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<td>Techniques</td>
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<td>Ordered Metric Methods</td>
<td>differences between utility adjacent criterion levels are ranked, and utilities assigned accordingly</td>
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<td>Successive Comparison Methods</td>
<td>inequalities between sums of utilities are obtained and used to assign utilities to criterion levels</td>
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<td>Tradeoff Methods</td>
<td>e.g. single tradeoff method derives the utility function for one criterion from a tradeoff or indifference curve between it and another criterion and the utility function of the other criterion</td>
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<td>Linear Forms</td>
<td>$U = \sum_{k=1}^{n} a_k x_k$ or $U = \sum_{k=1}^{n} a_k^2 x_k + \left[ \left( a_k - a_k^2 \right) \min \left( x_k, B_k \right) \right]$</td>
</tr>
<tr>
<td></td>
<td>Conjunctive Form</td>
<td>$U = \prod_{k=1}^{n} x_k^{a_k}$</td>
</tr>
<tr>
<td></td>
<td>Disjunctive Form</td>
<td>$U = \prod_{k=1}^{n} \frac{1}{\left( a_k x_k - d_k \right)^{a_k}}$</td>
</tr>
<tr>
<td></td>
<td>Logarithmic Form</td>
<td>$U = \sum_{k=1}^{n} a_k \log x_k$</td>
</tr>
<tr>
<td></td>
<td>Exponential Form</td>
<td>$U = \prod_{k=1}^{n} e^{a_k x_k}$</td>
</tr>
<tr>
<td></td>
<td>Combination Form</td>
<td>$U = \prod_{k=1}^{n} \left[ \frac{x_k}{a_k} \right]^{a_k}$</td>
</tr>
</tbody>
</table>

coefficients estimated by applying linear regression to past decisions or simulated decision situations presented to the decision maker.
Elimination by aspects is similar to lexicography except that instead of choosing the most important criterion as the basis for comparison, the one with the most discriminatory power (greatest range, fewest ties, etc.) is chosen. These two methods of handling multiple criteria seem to have been developed because they are convenient rather than because they model any conscious or logical strategy for finding the best alternative.

There are two methods of comparing alternatives to a standard. One is to examine each alternative for its "worst" criterion level, and accept or reject it on the basis of whether or not this worst level meets the standard. This conjunctive strategy results in a set of acceptable alternatives which exceed the standard with respect to each criterion. If a government agency was concerned with keeping its fund granting program free of public criticism, it might adopt such a decision procedure to prevent an obviously poor performance with respect to any one criterion.

The other method is a disjunctive strategy. In this case the best criterion level of each alternative is found, and if it exceeds the standard, then the alternative is acceptable. This strategy results in a set of alternatives which exceed the standard with respect to at least one criterion. A research centre that wishes to develop an excellent reputation might opt to use such a strategy to recruit scientists on the basis of their strongest field of endeavor.
In each case the problem of ordering the accepted alternatives remains, and must be resolved using one of the other methods. Because of the restrictive logic used in these methods, they are applicable in a limited number of cases only. The resulting acceptable set of alternatives is very different with the two methods. Dawes [17] discusses the characteristics of groups of people selected for various positions by the two methods.

The sequential elimination methods are non-compensatory. A high level of achievement with respect to one criterion cannot compensate for a deficiency in another area. In many situations this assumption is too restrictive. However there are cases where these techniques have been effective. Kleinmuntz [52] uses a sequence of elimination methods of this type to model a clinical psychologist judging test profiles (MMPI) as being "normal" or "abnormal." Smith and Greenlaw [72] use a similar technique to model a psychologist selecting applicants for jobs. In both studies the subjects were asked to describe the techniques they were using as they made a set of decisions. Their descriptions and decisions were examined to derive a set of sequential rules which could be applied to other problems.

2.2 SPATIAL PROXIMITY METHODS

Spatial proximity methods are ordering techniques
which rely more heavily on a geometric representation. The indifference map technique obtains indifference curves for each pair of criteria. An indifference curve is a set of points, each having two co-ordinates, which represent the levels of the two criteria. All combinations of the two criteria which are seen by the decision maker as having the same relative value are located on the same indifference curve. MacCrimmon and Toda [57] describe an efficient method of constructing indifference curves. One possible combination of the two criteria, say \((a,b)\), is chosen as a reference point. Since it is assumed that a higher level of any criterion is always preferred to a lower one, all points "north east" of the reference point are more valuable than the reference point, and can be thought of as the "accept region." All points "south west" of the reference point are less valuable, and can be thought of as the "reject" region. These areas may be blocked off and excluded from further consideration as in Figure 2.1. A circle is drawn around \((a,b,)\). The decision maker is then asked to compare in turn, the mid-point of each of the areas which fall in the open region to the reference point. If the mid-point is accepted (rejected) more points on the arc in the direction of the reject (accept) region are compared to the reference point until one of them
Figure 2.1
Obtaining an Indifference Curve by the MacCrimmon-Toda Method
is rejected (accepted). Each time a point is compared to the reference point, another area may be blocked off as belonging to either the accept or reject region. This process is continued until the open area is narrow enough for an indifference curve to be drawn through it. All points on the indifference curve would have approximately the same subjective value as the reference point \((a,b)\). Other indifference curves for the same two criteria, corresponding to different value levels are derived by choosing a different reference point. The same technique is applied to all other pairs of criteria to derive a complete indifference map. The alternatives can then be positioned in this multi-dimensional space with respect to the indifference curves and a complete ordering among them obtained.

Multi-dimensional scaling is another proximity ordering technique. An ideal alternative is assumed to exist. The value of any other alternative is assumed to be inversely proportional to the "distance" of that alternative from the ideal point. Klahr [51] describes a method of locating the real and ideal alternatives in a multi-dimensional space. He first requires the decision maker to judge the similarity of a set of alternatives. The alternatives are then positioned, consistent with the nearness assumption. This can always be done in an \(N-1\) dimensional space, where \(N\) is the number of alternatives. The next step is to iteratively reduce the dimensionality of the space until
it is less than or equal to the number of criteria, since one would expect to need no more than one dimension for each criterion that is significant in the decision making process. The dimension finally chosen is the one that minimizes stress (a measure of departure from perfect fit). Klahr applies this technique to the problem of graduate admission decisions. The method however is computationally difficult and has not been particularly successful.

2.3 MATHEMATICAL PROGRAMMING METHODS

There are two mathematical programming methods: goal programming and an interactive tradeoff technique. Goal programming requires the decision maker to specify desired or acceptable levels of a set of goals. A goal may correspond to one of the criteria or may be combinations of the criteria. The amount of dissatisfaction accompanying any alternative which does not satisfy the goals is determined by obtaining weighting coefficients for deviations in each direction from each goal. The weighted sum of these deviations is used as the objective function in a standard LP minimization procedure. The resulting optimal solution is the criterion levels of the best alternative. Lee and Clayton [54] apply the technique to the scheduling of an academic department.
The interactive tradeoff technique assumes that a global objective function exists but does not require it to be defined. At any feasible alternative the decision maker's tradeoffs among the criteria in the neighbourhood of the alternative are determined. The decision maker is asked to choose between alternatives of the form \((f_1, \ldots, f_r)\) and \((f_1 + \Delta f_1, f_2, \ldots, f_i + \Delta f_i, \ldots, f_r)\) where \(f_i\) are the criterion levels. The \(\Delta f_1\) and \(\Delta f_i\) are varied until the decision maker is indifferent between the two alternatives. (This technique is similar to the method used in deriving indifference curves.) Then the tradeoff between criterion \(i\) and criterion \(1\), \(w_i = -\Delta f_1/\Delta f_i\) is used in approximation to the gradient of the global utility function at the current point. This approximation is used in the objective function of a mathematical programming algorithm which determines the best direction to move, i.e. the direction in which better points than the current one lie. A set of points at various distances from the current point in that direction is then generated and the decision maker is asked to choose the best one. If this new alternative is "good enough" the procedure terminates, otherwise it is repeated. Geoffrion, Dyer, and Feinberg [25, 35] have developed this technique and applied it to the scheduling of an academic department.

These techniques construct the best alternative rather than choosing it from a set of explicitly pre-defined alternatives. They can only be used in situations such as a project
selection problem where the criteria take on continuous values. Applying these techniques to the project selection problem results in the optimal combination of criterion levels.

2.4 UTILITY FUNCTION METHODS

The last method of dealing with multiple criteria to be considered here is the use of utility functions. In the literature the term "utility function" has a specific meaning which depends on the axiomatic system being followed. However, here it will simply mean a mapping which assigns to an alternative a real number (utility) which indicates the relative worth of the alternative. This mapping has the two properties mentioned previously; it is complete and it is transitive.

There are two main classes of methods used in deriving utility functions; directly assessed preferences and inferred preferences. Fishburn [29] describes 24 methods of directly assessing preferences. The basic ideas behind these methods are discussed below.

2.4.1 Directly Assessed Preference Techniques

All of the methods assume that the utility of an alternative is the sum of the utilities of the individual criterion levels, i.e.
\[ U \text{ (alternative)} = u_1(x_1) + u_2(x_2) + \ldots + u_n(x_n) \]

where the \( x_i \) are the levels of the criteria achieved by the alternative and \( u_i \) is the utility function for the \( i^{th} \) criterion. Fishburn [28,29,30] and v. Winterfeldt and Fischer [83] discuss the conditions under which this additivity assumption holds.

The directly assessed preference methods find utility functions \( u_i \) for each of the criteria. The simplest method is the direct rating method. The decision maker is asked to assign a utility to each criterion level by rating it on an arbitrary scale. The other methods ask the decision maker questions on his preferences, then attempt to assign utilities consistent with his responses.

Ranking methods ask for an ordering of the feasible criterion levels. This generally can be reduced to a series of pairwise comparisons. For example, to find the most preferred criterion level of \( n \) feasible ones, the decision maker may choose a candidate, then compare this level with each other feasible level in turn, in order to verify the hypothesis that it is more preferred. Utilities are then assigned to each feasible criterion level in accordance with this ranking. One way would be to assign the most preferred level a utility of \( n \), the next preferred \( n-1 \), etc.

Gamble methods require the decision maker to choose between gambles made up of various criterion levels which may occur with various probabilities. One such method uses the least and most desirable level of a criterion, say \( X_i \) and \( Y_i \).
respectively. For a set of other criterion levels \( (z_i) \), the decision maker is asked to estimate the probability \( p(z_i) \) for which \( z_i \) has the same value to him as a gamble between \( x_i \) and \( y_i \), where \( y_i \) occurs with probability \( p(z_i) \) and \( x_i \) occurs with probability \( 1 - p(z_i) \). The utility of criterion level \( z_i \) is then \( p(z_i)u(y_i) + [1 - p(z_i)] u(x_i) \). The utility of levels \( x_i \) and \( y_i \) can be arbitrarily set (for example to 1 and 100) and the utility of \( z_i \) determined.

Ordered metric methods require a ranking of the differences between criterion levels which are adjacent in utility. An initial ranking of the criterion levels is necessary. One example, the direct ordered metric method, requires the decision maker to rank the utility differences between four adjacent criterion levels. Suppose \( u(a) < u(b) < u(c) < u(d) \) and \( 0 < [u(d) - u(c)] < [u(c) - u(b)] < [u(b) - u(a)] \). Numerical assignments consistent with this metric ranking may be made. One possible way of assigning numbers is to set the utility of the least desired level to 0 and set \( u(y) - u(x) \) to \( k \) when this difference is the \( k^{th} \) one in the ranking. For the example above this implies

\[
\begin{align*}
  u(a) &= 0, \\
  u(b) - u(a) &= 3, & u(b) &= 3, \\
  u(c) - u(b) &= 2, & u(c) &= 5, \\
  u(d) - u(c) &= 1, & u(d) &= 6.
\end{align*}
\]
Successive comparison methods work with groups of utility adjacent criterion levels, and attempt to assign utilities to them by examining inequalities between sums of utilities. For example, suppose again \( u(a) < u(b) < u(c) < u(d) \). Compare \( u(d) \) with \( u(b) + u(c) \). If \( u(d) < u(b) + u(c) \), compare \( u(c) \) with \( u(a) + u(b) \). If \( u(d) > u(b) + u(c) \) compare \( u(d) \) with \( u(a) + u(b) + u(c) \). Continue in this manner until a complete set of inequalities is obtained. Numerical utilities may now be assigned to each criterion level consistent with these inequalities. Many variants of this method are possible.

There are several useful types of trade-off methods. The single trade-off method uses a set of indifference curves between two criteria, and the previously derived utility function for one of the criteria in order to derive the utility function for the other. If \( (t_i, s_i) \) is on the same indifference curve as \( (t_2, s_2) \) then \( u(t_i) + u(s_i) = u(t_2) + u(s_2) \). If \( u(t_i) \) is known, a set of such equations can be solved for \( u(s_i) \) by setting \( u(s_i) \) for some \( i \), to an arbitrary value. One possibility would be to set \( u(s_{\min}) \) to 0, where \( s_{\min} \) is the least preferred s level.

Directly assessed preference techniques are effective in determining utility functions, however the conditions under which the additivity assumption holds are fairly restrictive and may not be satisfied in all project selection decision situations.
2.4.2 Inferred Preference Techniques

Inferred preference methods attempt to deduce the decision maker's utility function from choices he makes. These choices may be actual past decisions in similar situations or they may be decisions made on simulated problems presented to him. Some hypothesis must now be made about the form of the utility function. If it is assumed to be linear or "quasi-linear"\(^2\) then the coefficients are estimated using standard linear regression techniques. If the utility function is thought to be non-linear and involve interactions among the attributes, then analysis of variance is used to determine the utility function.

Inferred preference techniques have been particularly successful in predicting decisions. Huber, Sahney and Ford [49] have used the technique to model professionals judging the quality of hospital wards. Huber, Daneshgar and Ford [48] have applied the techniques to job selection decisions. Both studies used several types of functional forms and found that the linear one had the greatest predictive power. Einhorn [27] used a linear, and two "quasi-linear" functions to model job selection decisions and graduate school admission decisions. He found that the conjunctive form was most predictive in the

\(^2\)A quasi-linear function is any function which can be made linear by a simple transformation. For example, the multiplicative form \(U = \pi X_i\) is quasi-linear since \(\log U = \sum \log X_i\) is linear.
job selection case, and the linear function was most applicable to the admission decision problem. The conjunctive form models the strategy which rejects alternatives unless they exceed some standard with respect to all criteria. If even one criterion level is less than the standard then the alternative has a low utility (i.e. it is a non-compensatory strategy). He argues that this is a reasonable strategy in situations where choosing the wrong alternative would be very costly (such as job selection). In such cases, that type of conservative, non-compensatory strategy would minimize the chances of costly mistakes.

Goldberg [39] used the same functions plus two more "quasi-linear" functions to model psychologists judging (MMPI) test profiles. He concluded that the linear function best predicted decisions. Hoffman and Wiggins [47] studied the same problem using a linear function, a quadratic function, and a sign model (a linear combination of scores and functions of scores). There was some evidence that interactions among criteria affected decisions (configurality), but even so the linear function predicted decisions well.

Several authors have suggested using inferred preference techniques to develop a utility function which would then be used as a decision making tool to help eliminate costly inconsistencies. Bowman [10] and Kunreuther [53] have used the techniques in several managerial decision making situations, ranging from inventory policy and production scheduling decisions,
to equipment replacement policy decisions. They claim that mistakes due to a manager's inconsistency are more costly than mistakes caused by his misconception of the problem. Their experience has indicated that most managers have a good understanding of the problems that face them, and from similar past situations are aware of what factors and indicators are most important. In several cases they have been able to show that this type of analysis of past decisions has led to a decision-making procedure which attained better results than decisions made by following a policy developed from a more analytical study of the situation.

Yntema and Torgerson [84] have suggested a computer-aided decision system which would reduce the amount of time spent on routine decisions as well as help eliminate inconsistencies. They use an analysis of variance technique to derive the decision maker's utility function which is then used as the decision rule by the computer. They found that "main effects" (the linear portion of the function) provided an excellent approximation to the utility function if utility was monotonically increasing in all criteria, i.e. a higher criterion level is always preferred to a lower one.

In all of the studies mentioned the decisions predicted with a linear utility function correlated highly with actual decisions. This is not meant to suggest that the human decision process is necessarily linear. However a linear model has been
shown to be an excellent "paramorphic representation" [18] of decision makers in many situations. That is, while the decision process may not be linear, the general linear model is powerful enough to reproduce most decisions with very little error. Goldberg [39] has found that

\[ \ldots \text{for a number of different judgment tasks and across a considerable range of judges, the simple linear model appeared to characterize quite adequately the judgmental processes involved - in spite of the reports of the judges that they were using cues in a highly configural manner.} \ldots \]

Consequently if one's purpose is to reproduce the responses of most judges, then a simple linear model will normally permit the reproduction of 90-100% of their reliable judgment variance, probably in most - if not all - clinical judgment tasks.

Since our purpose in deriving a utility function to help analyze the project selection problem is to reproduce decisions, it would seem appropriate to use an inferred preference technique. From the studies cited one would expect a linear function to predict decisions well. However, to allow more flexibility to model people and situations where it is not the most appropriate form, one might wish to consider some of the non-linear functions as well. The functions which are included in the Interactive Utility Assessment Procedure (IUAP) developed are the linear, conjunctive, disjunctive, logarithmic and exponential functions, and a combination of the conjunctive and disjunctive functions.
The linear function is of the following form:

\[ U = \sum_{k=1}^{n} a_k X_k, \]

where \( U \) is the utility of an alternative, \( X_k \) is the level of criterion \( k \) achieved by the alternative, and \( a_k \) is the coefficient found by the linear regression procedure. In many cases it seems to be true that while utility is monotonically increasing with respect to each criterion, at some point the incremental utility of a unit increase in criterion level decreases. In other words the utility function exhibits decreasing returns. The simplest method of modelling the decreasing returns characteristic is with a piecewise linear concave function. In that case the utility of a unit of criterion \( k \), up to the point \( B_k \) is \( a_k^1 \). When the criterion level is greater than \( B_k \), each additional unit is worth \( a_k^2 \) rather than \( a_k^1 \), and \( a_k^2 < a_k^1 \). Thus the utility function would be as follows:

\[ U = \sum_{k=1}^{n} a_k X_k + \left( a_k^1 - a_k^2 \right) \min\left( x_k, B_k \right). \]

The linear forms are compensatory in that a high level of one criterion can make up for a low level of another.

The conjunctive utility function used by Goldberg [39] was originally proposed by Einhorn [27]:
where the $a_k$ are coefficients obtained by applying linear regression to the equation

$$\log U = \sum_{k=1}^{n} a_k \log X_k.$$ 

In one dimension the function is a parabola (see Figure 2.2(a)). A low level of any criterion would reduce the utility of the alternative regardless of how high the other criterion levels might be.

The disjunctive function, also originally proposed by Einhorn, models the strategy which chooses alternatives on the basis of their "best" criterion level. The general form is

$$U = \prod_{k=1}^{n} \left( \frac{1}{d_k - X_k} \right)^{a_k},$$

where $d_k$ is the maximum possible value of criterion $k$. The coefficients $a_k$ are determined by applying linear regression to

$$\log U = -\sum_{k=1}^{n} a_k \log \left( d_k - X_k \right).$$
Figure 2.2
Conjunctive Utility Functions
In one dimension the function is a hyperbola (see Figure 2.3 (a)). The closer a criterion level is to its maximum value $d_k$, the larger the fraction $1/(d_k-X_k)$ and the larger the utility assigned to the alternative. When the criterion level is high the function provides a great deal of discriminatory power (i.e. the derivative is large). This corresponds to choosing between alternatives on the basis of their best criterion level. The disjunctive form is also non-compensatory. If an alternative's best criterion level is only average, then similar average levels of other criteria will not make it more attractive than an alternative which has one very high score and the rest very low.

The next two models, the logarithmic and the exponential were suggested by Goldberg [39] to determine whether a logarithmic transformation of the criterion levels (logarithmic), or the value judgements (exponential) would provide a better fit to the observed utility function. The logarithmic form is

$$U = \sum_{k=1}^{n} a_k \log X_k,$$

and the exponential form is

$$U = \prod_{k=1}^{n} e^{a_k X_k},$$

or

$$\log U = \sum_{k=1}^{n} a_k X_k.$$

Another non-linear function to consider is a combination of the conjunctive and disjunctive forms:
Figure 2.3
Disjunctive Utility Functions
\[ U = \prod_{k=1}^{n} \left( \frac{X_k}{d_k - X_k} \right), \]

or

\[ \log U = \sum_{k=1}^{n} a_k \left( \log X_k - \log (d_k - X_k) \right), \]

where \( d_k \) is the maximum possible value of criterion \( k \) and the \( a_k \) are coefficients found by applying linear regression to the second form. Alternatives which have a high level with respect to one criteria are given a high utility since \( X_k \) and \( 1/d_k - X_k \) would both be large. However, none of the other criteria can be too low or the utility would be reduced.

In the Interactive Utility Assessment Procedure the coefficients for all of the forms are obtained by applying a linear regression technique to the results of a set of decisions. The decision maker is first asked to specify the lowest and highest possible values of each of the criteria he wishes to consider. This information is used to randomly generate alternatives (set of criterion levels). The decision maker is then interactively asked to rate the alternatives on an arbitrary scale from 1-100. One is the utility of an alternative which has the lowest possible values of all criteria, and one hundred is the utility of an alternative with the highest possible values of all criteria. These ratings reflect the decision maker's
conception of the difference in value between such sets of criterion levels. They have no intrinsic significance, and are meaningful only in relation to one another. The decision maker could just as well have used a 100-300 scale to rate the alternatives.

The alternatives are rated one at a time as they are generated. The decision maker is then presented with the entire set of alternatives and may adjust his ratings. A consistency check is applied to the set of ratings. Any alternative which dominates another (has higher levels on all criteria), and yet has a lower rating, is pointed out, and the decision maker is asked to re-evaluate it. Once the ratings are internally consistent i.e. satisfy the transitivity assumption, and the decision maker is satisfied with them, they are used as the dependent variable in a linear regression procedure, and the coefficients for the various types of utility functions are determined.

Slovic and Lichtenstein [71] discuss the problems that can arise when the alternatives used in the decision situations posed to derive the utility function have randomly generated criteria levels. If the assigned criteria levels are unrealistic or outside the range the decision maker is accustomed to handling, then he may not be able to accurately rate the alternatives. However, the technique used in the IUAP program, assigns random values between the maximum and minimum possible values of each criterion. In that way each value is realistic and the problem does not arise.
Another problem occurs when the decision criteria are not independent. In that case randomly generated criteria levels could well violate expected relations between criteria. For example, if two of the criteria were the amount of energy consumed and the increase in the pollution level, one would expect a high level of one criterion to be associated with a high level of the other. The random generation technique used here would not guarantee that. In situations where the expected relationship is not observed, the decision maker essentially disregards one of the conflicting criteria. In cases where other criteria support the hypothesis that one of the criterion levels is wrong, this is a reasonable procedure. However at other times it leads to inconsistency and unrelability. One method of obviating the problem would be to employ a rejection technique. The generated alternatives could be examined before being presented to the decision maker, and rejected if the criterion levels violate the expected relationships.

Another cause of inconsistency is the availability of a large number of criteria. Einhorn [27] has shown that as the number of criteria increases, consistency decreases. One possible explanation of this phenomenon is that when the number of criteria to be considered exceeds the decision maker's information processing capacity, he chooses only a subset of them on which to base his judgment. This subset, however, is not the same each time. This hypothesis is strengthened by the fact that the number of criteria that are statistically
significant in decisions is generally smaller than the decision maker would estimate. He may remember using them all at one time or another, but the more important ones dominate. Slovic and Lichtenstein feel that the most important criterion generally accounts for 40% of a decision maker's predictable variance, and the three most important account for 80%. The number of criteria that may be used without causing serious inconsistencies appears to depend a great deal on the problem being considered and the relationship between the criteria. Whereas only the most important 4 or 5 are generally statistically significant, 10 are allowed in the IUAP program and could probably be considered before the decision maker would be faced with an "information overload" which is assumed to lead to inconsistency.

Of the many methods for handling multiple criteria considered, the inferred preference technique for deriving utility functions used by the IUAP program seems to be the most appropriate in analyzing the project selection problem. Most of the sequential elimination methods reduce the set of possible best alternatives, but the remaining ones must still be ordered using some other method. The restrictive logic of several of the methods and their non-compensatory nature make them inappropriate in many cases. They are heuristic methods of handling multiple-criteria decision situations which are not based on a logical strategy. The spatial proximity methods are computationally more complex than the inferred preference technique. They are also more difficult for the decision maker to under-
stand and use. Therefore, from a practical point of view, the inferred preference technique is more suitable. The mathematical programming methods are applicable to the project selection problem since it has continuous criterion levels. They have been successful in solving several real-world problems and might prove effective in solving the project selection problem as well. They are more difficult to implement than the inferred preference technique however, and have not been used for that reason. The directly assessed preference techniques for deriving utility functions are based on the assumption that the utility functions for each criteria are additive. The conditions under which this additivity assumption holds may not be met in many project selection decision situations and thus these methods are not always appropriate. Inferred preference techniques such as IUAP have been shown to be very successful in modeling decision makers and predicting their preferences. Since this modeling capability is the major goal in developing a project selection algorithm, this technique seems to be the appropriate one to use.
Chapter 3

A NEW PROJECT SELECTION ALGORITHM

In any rational decision process it is necessary to evaluate outcomes in terms of goals. In the last chapter it was suggested that the most appropriate evaluation method for the project selection problem is to derive a measure of the utility of each alternative as a function of its criterion levels. The most efficient and practical method of deriving a utility function in this case is the inferred preference technique. This technique derives a utility function by asking the decision maker to reveal his preferences through choice. There are several forms this utility function may take but many studies suggest that a linear form often predicts decisions best. Therefore, the linear form, and a modification of it, the piecewise linear form, were selected for the new decision algorithm.

3.1 DERIVING A LINEAR UTILITY FUNCTION

The Interactive Utility Assessment Procedure (IUAP) for identifying a utility function is used in the project
selection algorithm\(^1\) to derive the linear form. The decision maker is first asked by a conversational subroutine to describe his decision problem. How many projects are being considered? How many criteria are relevant? What is the suggested or requested level of funding for each project, and what is the maximum and minimum funding level possible? What level of each criterion would each project achieve (assuming it is successful and funded at the requested level)? This information is used to determine the maximum and minimum value of each criterion, \(X_k^{\text{max}}\) and \(X_k^{\text{min}}\). Then, sets of criterion levels between these bounds are randomly generated. These sets of criterion levels can be thought of as "pseudo-projects." Since each project can be funded at an infinite number of levels between its upper and lower funding bounds, a combination of projects funded with various amounts might result in a set of criterion levels with the same values as the pseudo-project. Each pseudo-project is therefore representative of a possible research and development program (portfolio of projects). At this point in the algorithm, IUAP is used to interactively illicit preference ratings and determine regression coefficients for the linear form.

The next step in the procedure is to determine how well the linear model predicts the decision maker's ratings. Three different coefficients of the correlation between the

\(^1\)Figure 3.1 is a flow chart of the algorithm.
Describe the Projects

Double the number of pseudo-projects to be generated at each stage.

Have you already derived a utility function?

Is your utility function linear?

Generate pseudo-projects, rate them.

Apply regression to find coefficients, calculate correlations.

Do you wish to find a better fitting utility function?

Is any correlation greater than 0.9?

Have you already attempted to derive the piecewise linear form?

Have you found and replaced the worst point \( \max(3, 4m/5) \) times?

Find the worst point.

Is this worst point near any other?

Is this the first time the worst point has been found?

Generate a new pseudo-project in the neighborhood of the worst point and replace the worst point with it.

Figure 3.1 Flowchart of the Algorithm
Are all of the criterion levels of the worst point in the middle of the range of possible values?

Use its criterion levels as the breakpoints. Generate new pseudo-projects, rate them, find the coefficients of the piecewise linear form by regression, calculate the correlations.

Is the piecewise linear utility function concave?

Are any of the correlations for the piecewise linear form better than the corresponding ones for the linear form?

Is the piecewise linear function accurate enough?

Find the next worst point, i.e. the point with the next largest difference between actual and predicted scores.

Are the points been tested?

Formulate and solve with the piecewise linear form. STOP.
actual and predicted ratings are calculated by the program; the Pearson or product moment correlation, the Spearman or rank correlation, and a distance-error correlation. Since these correlations are calculated on the same data used to derive the regression coefficients one would expect them to be high. At this point a new set of pseudo-projects could be generated and rated. The correlations between the actual and predicted ratings of this set would be a better indicator of the goodness of fit of the linear model. However, another set of ratings for this purpose seems to place an unnecessary burden on the decision maker. A correlation level of 0.9 was arbitrarily chosen as a standard. If any of the correlations are greater than 0.9 then the linear model will probably fit well enough to be useful. However, the decision maker is given the option to continue the process in the hope of finding a better fitting utility function. If none of the correlations are greater than 0.9, or if the decision maker exercises his option to continue, then a piecewise linear utility function is derived in an attempt to improve the correspondence between choices and predictions.

3.2 DERIVING A PIECEWISE LINEAR UTILITY FUNCTION

If the decision maker's utility function is best approximated by a piecewise linear function, we would expect the
configuration of pseudo-projects to resemble one of the cases depicted in Figure 3.2. The pseudo-project with the most poorly predicted score is found by computing the absolute value of the difference between the predicted score and the actual score for all pseudo-projects, and identifying the largest such difference $d$. To eliminate the possibility that this deviation is simply an error on the part of the decision maker, he is asked to rate another pseudo-project in the same neighborhood as the worst point. The criterion levels of this new point are set at randomly generated values between the two adjacent criterion levels. This new pseudo-project replaces the most poorly fitted one and the linear regression procedure is reapplied using the new set of ratings as dependent variables. Once again the pseudo-project with the most poorly predicted score is found. This cycle is repeated until one of three

\[2\text{The adjacent criterion levels are found by examining each pseudo-project to find the greatest criterion level less than the corresponding one for the worst point, and the smallest criterion level greater than the corresponding one for the worst point, for all criteria. Another method of generating a new pseudo-project in the neighbourhood of the worst point would be to consider a circle of radius } r \text{ about the point. Criterion level } k \text{ of the new point would then be generated at random from the interval } (X_k^{WP} - r, X_k^{WP} + r), \text{ where } X_k^{WP} \text{ is the criterion level of the worst point.}\]
things happens. 1.) The last worst point found is near one of the previous worst points. In that case an area has been identified where the linear form does not predict well, and one might suspect that a piecewise linear function would be more appropriate. 2.) The correlation between the actual and predicted ratings improves enough that the decision maker decides the linear form is adequate. 3.) The iteration counter \( k \) becomes greater than \( \max(3, \frac{4m}{5}) \), where \( m \) is the number of criteria being considered. In this case, after a "reasonable" number of iterations, no area has been found where the linear form consistently predicts ratings poorly. One may conclude therefore, that the decision maker's underlying utility function is not piecewise linear and that a piecewise linear representation of it would not likely predict ratings better than the linear form.

If case 1.) occurs then the next step in the algorithm is to find the breakpoints \( B_k \) of the piecewise linear form

\[
U = \sum_{k=1}^{m} \left[ a_k^2 x_k + (a_k^1 - a_k^2)\min(x_k, B_k) \right].
\]

The breakpoints are where the phenomenon of decreasing returns

\[3^\text{The new worst point is near the last worst point if all of its criterion levels fall in the interval } \left( x_k^\min - \frac{x_k^\max - x_k^\min}{N}, x_k^\max + \frac{x_k^\max - x_k^\min}{N} \right) \text{, where } x_k \text{ is the } k^{th} \text{ criterion level of the old worst point and } N \text{ is the total number of pseudo-projects generated.} \]
occurs, i.e. where the marginal value of a unit increase in the criterion level decreases. For criterion levels less than the breakpoint, the added value of one unit of criterion $k$ is $a_k^1$. When the criterion level is greater than $B_k$, the added value of one unit is $a_k^2$ and $a_k^2 < a_k^1$.

One possible method of obtaining the breakpoints is to ask the decision maker for them. He may be aware of a discontinuity in his utility function with respect to one or more criteria, and may be able to indicate where it occurs. In most instances however, this will not be the case. The computerized procedure finds the breakpoints by examining the last worst point found. If all of its criterion levels are in the middle half of the range of possible values of that criterion, i.e. between $X_k^{.25}$ and $X_k^{.75}$ in Figure 3.2, then case (a) in Figure 3.2 is applicable. The criterion levels of the point are then taken as the breakpoints $B_k$ for each criterion. If all of the criterion levels of this worst point are not in the middle half of the range of possible values of the criterion, then case (b) or (c) of Figure 3.2 may apply. In this case the criterion levels of this point cannot be used as the breakpoints. Some other poorly fitted pseudo-project with criterion levels closer to the middle of the range must be found. The pseudo-projects are searching in decreasing order of $d$ (distance between actual and predicted ratings) until one satisfying the "middle of the range" rule is found (in which case its criterion levels are used as the breakpoints), or until half of the
pseudo-projects have been examined. If the most poorly predicted half of the pseudo-projects does not contain a point in the middle of the range, then the distribution of sample points does not resemble case (b) or (c) of Figure 3.2, and it is unlikely that a piecewise
linear form would be a more accurate representation of the decision maker's actual utility function, than the linear form.

Once the breakpoints of the piecewise linear form have been determined, the next step is to find the coefficients $a_k^1$ and $a_k^2$, $k=1,...,m$. This is done by applying regression to

$$U = \sum_{k=1}^{m} a_k^1 X_k^1 + a_k^2 X_k^2,$$

where $X_k^1 = \min (X_k, B_k)$ and

$$X_k^2 = \max (X_k - B_k, 0).$$

Since twice as many coefficients as before are being calculated, another 4m pseudo-projects are generated and interactively rated before the regression technique is applied.

The next step in the procedure is to determine which of the two forms (the linear or the piecewise linear) fits best. The correlation coefficients between the actual ratings given the original set of pseudo-projects by the decision maker, and the ratings predicted by the piecewise linear form are calculated. These correlation coefficients are compared with the corresponding ones for the linear form. The linear form has an unfair advantage in this comparison since its correlation coefficients are calculated on the same data as was used to derive its regression coefficients. Here again another set of pseudo-projects could be generated and used
to test the two models more fairly. But the imposition of a third set of ratings was found to be unacceptable to many decision makers. If any of the correlation coefficients for the piecewise linear form are greater than the corresponding ones for the linear form, despite the bias in favour of the linear form, then the piecewise linear form would seem to fit best. If all of the coefficients for the linear form are greater than the ones for the piecewise linear form, then the linear form may or may not provide a better fit. However, for lack of another method of measuring the fit of the two forms, the linear one would be chosen in that case. If the decision maker is still not satisfied with the fit of the chosen utility function the whole procedure could be repeated. This time more pseudo-projects would be generated at each stage in order to find a better fitting utility function.

The question of how many pseudo-projects to generate at each stage is an important one. The more ratings available the better the fit of the utility function. However, if too many ratings are required, the decision maker will become tired and/or annoyed with a resulting decline in accuracy. There must be an optimum number of pseudo-projects to generate which would minimize these two types of problems. Clearly this number should be related in some way to the number of criteria used. As the number of criteria (and, in a sense, degrees of freedom in the utility function) increase, more points will be needed for a good fit. The number of pseudo-projects to generate was set at $4m$, when $m$ is the number of
criteria. If 5 criteria were considered relevant, which is more than the number most studies found to be statistically significant in influencing ratings, then 20 pseudo-projects would be generated. Twenty was found to be a reasonably large number of ratings with which most decision makers can cope. The number of criteria is certainly not limited to 5, however. If the decision maker wishes to use more, then he must rate a correspondingly larger number of pseudo-projects.

When the piecewise linear form is being derived, another set of 4m pseudo-projects is generated. The question arises as to whether the decision maker can rate this new set of pseudo-projects so that the ratings are consistent with the ones for the previous set. Since the type of decision maker this procedure is intended to model is one who is familiar enough with his problem and his preferences so that his viewpoint would not change as a consequence of the exercise, he is considered to be capable of consistency. As an aide to achieving consistency the previous set of pseudo-projects and ratings are presented to him as reference points.

A third question which arises is why $a^2_k$ should be less than $a^1_k$. The regression procedure certainly does not guarantee this. If in fact it turns out that $a^2_k$ is greater than $a^1_k$, then the decision maker's responses do not satisfy the assumption of decreasing returns with respect to criterion k (see Figure 3.3). Since our solution procedure depends on concavity, and it would seem unlikely that a decision maker's
utility function should exhibit increasing returns when dealing with real problems if this situation occurs, then the piecewise linear form is discarded and the linear form chosen.

Another method of dealing with the problem would be to re-apply the regression procedure to a new form which is piecewise linear with respect to only those criteria which satisfy the decreasing returns assumption, i.e.

\[ U = \sum_{k \in D} a_k x_k^1 + \sum_{k \in D} a_k x_k^2, \]

where \( D \) is the set of those criteria which exhibit decreasing returns.

3.3 FORMULATING THE OBJECTIVE FUNCTION

Once the appropriate utility function has been chosen, it can be combined with the decision maker's knowledge of the criterion levels of each of the projects under consideration, to construct an objective function for use in a mathematical programming algorithm. Consider first the linear form. The total utility of any portfolio of projects is

\[ U = \sum_{k=1}^{m} a_k \left( \sum_{j=1}^{n} \overline{x}_{kj} \right), \]

where \( \overline{x}_{kj} \) is the level of criterion \( k \) that project \( j \) actually achieves. \( \overline{x}_{kj} \) varies with the amount of funds allocated to
project \( j \) and cannot be known exactly until after the project is completed. However, the decision maker has some subjective estimate of the probability that project \( j \) will be successful when funded at various levels. The expected value of \( \bar{X}_{kj} \) is then \( X_{kj} P_j(x_j) \), where \( X_{kj} \) is the level of criterion \( k \) that the decision maker specifies in his description of the project, and \( P_j(x_j) \) is the probability that project \( j \) will succeed when funded at \( x_j \). The expected utility of any research and development program is then

\[
E(U) = \sum_{k=1}^{m} \alpha_k \left( \sum_{j=1}^{n} X_{kj} P_j(x_j) \right).
\]

Souder [77,78] has studied the effect on resulting decisions of three different forms of the \( P_j(x_j) \) function. The three forms he chose were a linear function, a piecewise linear function and a non-linear function (either exponential or S-shaped). These functions were derived by fitting curves through points specified by the decision maker (see Figure 3.4). He found that the linear function resulted in decisions which maximized profit, but that the piecewise linear function was most often preferred because it had other desirable properties. "The piecewise model was more frequently preferred than the other two forms, largely because of its ability to select compromise portfolios. The 'compromise' portfolios were those
All three forms are derived by asking the decision maker for the probability project \( j \) will succeed if funded at the various levels.

Figure 3.4
Probability of Success Functions
yielding acceptable anticipated profits while still maintain-
ing minimum funding on imminent failure outcome projects and
providing a balance of intrinsic portfolio attributes" [78].
The piecewise linear function is more suitable for multiple
criterion situations because of this compromise characteristic.

In that phase of the procedure where the decision
maker is asked to describe the projects he is considering, he
must specify the requested or suggested level of funding \( f_j \),
and the upper and lower funding bounds \( u_j \) and \( l_j \) respectively,
for each project \( j \). These bounds may be set by organizational
policy, or they may be simply limits which seem reasonable to
the decision maker. The upper bound might be the point where
the decision maker feels increased funding would not signifi-
cantly increase the probability of success. The lower bound
might be such that if the project were to be funded at an
amount below it, there would be virtually no possibility of
success. In addition to specifying these funding levels, the
decision maker must estimate the probability of success of each
project when funded at its upper and lower bounds. The prob-
ability of success of project \( j \) when funded at level \( x_j \), \( (x_j =
0, \text{ or } l_j \leq x_j \leq u_j) \) is then

\[
P_j(x_j) = \alpha_j z_j + (\beta_j - \alpha_j) y_j
\]
where \( P_j(x_j) \) is the probability project \( j \) will succeed when it is funded at \( x_j \), \( \beta_j \) is the probability it will succeed when funded at \( u_j \), \( z_j \) is a 0 or 1 variable which indicates whether or not project \( j \) is funded at all (i.e. \( x_j \) is at least \( x_j \)), and \( y_j \) is a variable between 0 and 1 which is the amount of funding above \( x_j \) allocated to project \( j \). Using this notation,

\[
x_j = x_j z_j + (u_j - x_j) y_j,
\]

and

\[
E(U) = \sum_{k=1}^{m} a_k \left[ \sum_{j=1}^{n} X_{kj} (\alpha_j z_j + (\beta_j - \alpha_j) y_j) \right].
\]

This is the expression which will be used as the objective function in the mathematical programming algorithm.

### 3.4 Formulating the Constraints

The next step is to determine the constraints within which the decision maker must work. These are typically constraints on resources such as money and manpower. The decision maker must specify the amount of each resource \( i \) used by each project \( j \) when funded at its requested funding level \( f_j \), denoted by \( S_{ij} \), and the total amount of each resource available, \( R_i \). The resource constraints may then be formulated as
\[ \sum_{j=1}^{n} x_j \leq B, \quad \text{or} \quad \sum_{j=1}^{n} \left( k_j z_j + (u_j - k_j) y_j \right) \leq B \quad \text{and} \]
\[ \sum_{j=1}^{n} \frac{S_{ij}}{f_j} x_j \leq R_i, \quad \text{or} \quad \sum_{j=1}^{n} \frac{S_{ij}}{f_j} \left( k_j z_j + (u_j - k_j) y_j \right) \leq R_i \quad \forall i, \]

where \( \frac{S_{ij}}{f_j} \) is the amount of resource \( i \) required per dollar invested in project \( j \) and \( B \) is the total budget.

There is an assumption implicit in this formulation of the resource constraints. The amount of resource \( i \) required by project \( j \) is assumed to vary linearly with the amount of funding \( x_j \). This is a fairly reasonable assumption for most types of resources when \( x_j \) falls in the range \([k_j, u_j]\). This range is where changes in funding result in significant changes in the probability of success and thus in the amount of each attribute produced and each resource consumed. Outside of this range changes in funding may not be reflected by changes in resource requirements. It would seem reasonable that the consumption of resources such as energy and manpower, which are essentially continuous (can be purchased in any amounts) would vary linearly with funding in this range. However there are some types of resources which are zero-one in nature. For example, if a piece of elaborate equipment is necessary (such as a computer or a cyclotron), three-quarters of the machine would not be particularly useful. In cases like this
then, the assumption is not valid. Most resources considered in project selection problems however, are of the continuous type.

Other types of constraints are possible as well. There may be relationships among the projects such that if one project or group of projects is funded, then others may not be. For example, a government agency may be concerned that no more than \( N \) projects from each region of the country are funded. In that case the necessary set of constraints are

\[
\sum_{j \in J_{\ell}} z_j \leq N \quad \forall \ell,
\]

where \( J_{\ell} \) is a set which indexes all projects submitted from region \( \ell \). Another possible relationship among project occurs when one project depends on results from another. In that case the dependent project A should not be funded unless the independent project B is also funded. The constraint \( z_A \leq z_B \) will insure that project A is not funded (\( z_A = 0 \)) if project B is not funded (\( z_B = 0 \)). Other types of relationships may be envisaged and can be readily included in the constraint set.

Another useful type of constraint would be to specify minimum levels of some or all criteria which must be achieved by the portfolio of projects. These constraints would have the following form:
where \( M_k \) is the minimum acceptable level of criterion \( k \).

A government agency might wish to define such a set of minimum standards in order to ensure approval of all their funding programs.

### 3.5 THE FORMULATION WITH A LINEAR UTILITY FUNCTION

The complete formulation of the project selection problem with a linear utility function is then as follows:

\[
\begin{align*}
\max \quad & \sum_{k=1}^{m} a_k \sum_{j=1}^{n} \left( \alpha_j z_j + (\beta_j - \alpha_j) y_j \right) x_{kj} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \left( \ell_j z_j + (u_j - \ell_j) y_j \right) \leq B & (1) \\
& \sum_{j=1}^{n} \frac{S_{ij}}{r_j} \left( \ell_j z_j + (u_j - \ell_j) y_j \right) \leq R_i & \forall i & (2) \\
& y_j \leq z_j & \forall j & (3)^4
\end{align*}
\]

4Constraints (3) insure that project \( j \) is funded at least at its lower bound if it is funded at all.
The project selection problem may now be solved using a mixed integer branch and bound algorithm.

3.6 THE FORMULATION WITH A PIECEWISE LINEAR UTILITY FUNCTION

If the piecewise linear utility function is chosen as the one which predicts the decision maker's ratings best, then the objective function in this formulation must be changed. In that case, the total utility of any portfolio of projects is

\[ U = \sum_{k=1}^{m} \left[ a_k^2 \sum_{j=1}^{n} \bar{X}_{kj} + (a_k^1 - a_k^2)\min\left\{ \sum_{j=1}^{n} \bar{X}_{kj}, B_k \right\} \right] \]

where \( \bar{X}_{kj} \) is the level of criterion \( k \) actually achieved by project \( j \). Again this may be translated into expected utility using the two probability of success estimates provided by the decision maker.

\[ E(U) = \sum_{k=1}^{m} \left[ a_k^2 \sum_{j=1}^{n} X_{kj} P_j(x_j) + (a_k^1 - a_k^2)\min\left\{ \sum_{j=1}^{n} X_{kj} P_j(x_j), B_k \right\} \right] \]

\[ = \sum_{k=1}^{m} \left[ a_k^2 \sum_{j=1}^{n} X_{kj} \left\{ \alpha_j z_j + (\beta_j - \alpha_j) y_j \right\} + (a_k^1 - a_k^2)\min\left\{ \sum_{j=1}^{n} X_{kj} \left( \alpha_j z_j + (\beta_j - \alpha_j) y_j \right), B_k \right\} \right] \]
In order to convert this objective function into a useful form, two new sets of variables are required: $t_k$ and $s_k$. $t_k$ is the total expected level of criterion $k$ and $s_k$ is the amount by which this level exceeds the breakpoint $B_k$, if any. Now the problem may be formulated as follows:

$$\max \sum_{k=1}^{m} a_k^1 t_k - \left[a_k^1 - a_k^2\right] s_k$$

s.t. $$t_k = \sum_j \left[\alpha_j z_j + (\beta_j - \alpha_j) y_j\right] x_{jk} \quad \forall k \quad (1)$$

$$s_k \geq \sum_j \left[\alpha_j z_j + (\beta_j - \alpha_j) y_j\right] x_{jk} - B_k \quad \forall k \quad (2)$$

$$\sum_{j=1}^{n} \left[\kappa_j z_j + (u_j - \kappa_j) y_j\right] \leq B \quad (3)$$
\[
\sum_{j=1}^{n} \frac{S_{ij}}{f_j} \left( \ell_j z_j + (u_j - \ell_j) y_j \right) \leq R_i \quad \forall i \quad (4)
\]

\[y_j \leq z_j \quad \forall j \quad (5)\]

other constraints \quad (6)

\[z_j = 0, 1 \text{ and } 0 \leq y_j \leq 1 \quad \forall j \quad (7)\]

\[t_k, s_k \geq 0 \quad \forall k \quad (8)\]

Constraints (3)-(7) are the same as the constraints in the previous formulation. Constraints (1) set \(t_k\) to the expected level of criterion \(k\) and constraints (2) set \(s_k\) to the difference between the level of criterion \(k\) and the breakpoint \(B_k\). If this difference is negative, then \(s_k = 0\) by (8). \(s_k\) will never be assigned a value greater than the excess of criterion level \(k\) over \(B_k\) because the coefficient of \(s_k\) in the objective function is negative (by concavity \(a_k^1 > a_k^2\)). Therefore, optimality will force \(s_k\) to be as small a non-negative value as possible.
This formulation of the project selection problem can also be solved by a branch and bound mixed integer algorithm. The result of applying the algorithm is the funding level of each project for the five best solutions to the problem. The decision maker is not simply presented with the optimal solution for two reasons. One is that the data he provides and the objective function that is derived contain errors and inaccuracies. There is no guarantee then that the "optimal" solution is really optimal. It should have no more significance than another solution whose value is very close. These ideas will be investigated more thoroughly in the next chapter.

The second reason is that there may be other considerations such as political ones which could not be incorporated into the utility function or constraint set. If the decision maker is presented with a range of solutions with near "optimal" values, then he may choose between them on the basis of such considerations without seriously affecting performance.

The project selection problem, when viewed as a multiple objective decision problem, can be solved by the interactive procedure described above. The decision maker is asked to rate sets of pseudo-projects in order to derive a measure of the utility of alternatives as a function of their criterion levels. This utility function is combined with probability of success estimates provided by the decision maker, to produce an expression for the expected utility of any research and
development program which is used as the objective function in a mathematical programming algorithm. When this objective function is combined with constraints formulated with the data provided by the decision maker, the problem can be solved with a branch and bound mixed integer algorithm. The usefulness of the resulting solutions and some of the ways the procedure may be extended and improved are discussed in the next chapter.
Chapter 4

THE UTILITY AND ACCEPTABILITY OF THE ALGORITHM

Most project selection algorithms proposed to date have received little acceptance from those involved in the funding of research and development projects. Several authors have attempted to explain this phenomenon. Baker and Pound [5] suggest three reasons for it: (1) the unrealistic features and assumptions implicit in many of the models, (2) the onerous data requirements of most of the algorithms, and (3) the lack of comprehensive testing and reported computational experience with real problems.

4.1 FEATURES OF THE ALGORITHM

The algorithm suggested here is based on a more realistic model of the project selection decision situation. In particular profit maximization is not assumed to be the only objective. For many organizations such as government fund granting agencies, this objective is irrelevant. In other cases the organization is interested in profit but not to the exclusion of other goal
dimensions. Often once a reasonable return on investment is ensured, other criteria are triggered. The algorithm suggested here considers all criteria, in the manner and to the extent they are reflected in the decision maker's judgements. This is in contrast to other multi-criteria project selection algorithms, such as scoring type models, which specify pre-determined methods of combining goal dimensions.

Most other project selection algorithms assume the cost of a project is fixed. The project is either funded at that level or it is not funded at all. However, the cost of any project is only an estimate of the amount required to ensure a reasonable probability of success. Like any estimate it is subject to errors. If the project were to be funded at an amount less than its "cost," then it would be less likely to succeed, but there would still be some probability of success. Conversely, if the project were funded above its "cost," then it would be more likely to succeed. This is true only within a certain range, however. For any project, there exists a lower funding bound such that if it is allocated any less, there is virtually no probability of success, and an upper funding bound such that if it is allocated any more, there is no significant increase in the probability it will succeed. The proposed algorithm allows the amount of funding to fall anywhere between these lower and upper bounds. The relationship between the funding level and the probability of success of the project is
assumed to be a piecewise linear function as described in the last chapter.

These two features, multiple-criteria and funding in an interval, make this model a more accurate representation of the project selection decision situation and improve the usefulness of the algorithm for real decision situations.

4.2 DATA REQUIREMENTS

The data required by the algorithm is not as extensive as that required by most other algorithms. The decision maker must be able to describe each project. He must be able to specify the levels of all relevant criteria that each project achieves, and the amount of each scarce resource each project requires. This data is the minimum required for rational selection. Often project descriptions or proposals contain all the data necessary. However, the required probability of success estimates may not be as readily available and often must be provided by the project evaluator. A study by Souder [74] of the validity of subjective probability of success estimates indicates that most experienced project evaluators can assess the probability of success of any project surprisingly well, though in some cases this assessment is not accurately communicated for political reasons.
4.3 THE USEFULNESS OF THE ALGORITHM

While the algorithm was not tested in real problem situations, it has some attractive features. The algorithm is general enough to be useful in any project selection decision situation. Any type of criteria may be used and enough data would generally be available on even somewhat less-structured pure research projects. It is probably least useful however, for development-type projects whose only goal is monetary. Several of the other algorithms such as the dynamic programming ones, are better suited to such cases.

Since the problem is a combinational one (involves integer constrained variables), the value of the optimal solution is not a continuous function of the value of the right-hand-side (resource availabilities). It is thus possible that a slight increase in the available amount of any resource would result in a large increase in the value of the optimal solution, and a very different funding pattern. To test the sensitivity of the solution to changes in the right-hand-side, the algorithm may be re-run without re-deriving the utility function.

Souder has discussed the "utility and acceptability" of project selection algorithms. He suggests (and is echoed by Beattie and Reader [6]) that a serious shortcoming of the

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\[1\] A sample problem and its solution are described in the Appendix.
algorithms is that only one solution, the supposedly optimal solution, is provided. This defect was overcome in the proposed algorithm by providing the five best solutions (if five feasible ones exist). Since the data the decision maker provides is not exact, and the utility function derived is only an approximate measure of the subjective value of any portfolio of projects, no solution can be considered optimal. Several solutions may have almost the same value, and given the accuracy of the data, none of them can be said to be the best. There are often considerations which cannot be included in the utility function nor in the problem's constraints. If the decision maker has a set of solutions to choose from, he can discriminate among them on the basis of these other considerations. In that way the algorithm is useful as a tool for analyzing the decision situation. It simplifies the decision by reducing the number of possible choices. However, it is not intended to replace the decision maker. His particular skills and insights are still used. He is in fact an integral part of the algorithm.

Rather than replacing the decision maker, the algorithm attempts to formalize some of his thought processes. The utility function which is derived is a formalization of preference choices made by the decision maker. It can be applied to other decision situations and would result in consistent responses. The decision maker cannot be consistent. He is affected by extraneous factors he can neither control nor co-
pensate for. Shepard [70] has discussed this phenomenon and terms it "subjective non-optimality." It occurs when a decision maker chooses an alternative he believes to be best, and indeed, it may be best with respect to his current state of mind and the stimuli he is exposed to. However, at a later time, detached from these stimuli he recognizes that the choice was not optimal with respect to his true preferences. The door-to-door encyclopedia salesman for example, and other "pressure" salesmen capitalize on this foible of human nature. Often, a "paramorphic representation" of a decision maker, by alleviating strong situational stimuli can out-perform the decision maker by reflecting his true preferences. Bowman [10] and Kunreuther [53] have suggested that inconsistency is the major cause of costly decision errors. They have used decision models based on past decisions made by the decision maker to improve his performance, and have had excellent results in a variety of areas.

Because of the decision maker's direct involvement in the algorithm, and the control of criteria and evaluation methods allowed, it is hoped that this algorithm will be more acceptable as a decision making tool than previously suggested ones. It has another use as well, however, the derived utility function can be used for diagnostic purposes, showing the decision maker how he actually makes choices, i.e. which criteria he considers most important. Any discrepancies between these criteria and the criteria he believes he uses can be pointed out. Any
illogical features can thus be eliminated from his utility function. His utility function can be compared with those of other decision makers in the organization. Any differences can be discussed, and possibly an organizational objective function can be agreed upon. A better utility function found by such analysis can be used to help analyze future decision problems more consistently. The optimization section of the algorithm can be used alone, if a utility function was previously estimated.

4.4 POSSIBLE EXTENSIONS AND IMPROVEMENTS TO THE ALGORITHM

Several extensions and improvements to the algorithm are possible. The piecewise linear function can be extended to include more than two pieces in the hope of finding a better fitting utility function. If N pieces are desired N-1 poorly fitted points are required as breakpoints. The current algorithm may be easily modified to find these break points and the coefficients $a_k^i$ for each piece $i$. More pseudo-project ratings would be required to fit a line accurately in each of the new pieces. The piece-wise linear formulation may be readily extended to include N pieces by introducing $s_k^i$, $i = 1, \ldots, N-1$. $s_k^i$ is the amount by which the total level of criterion $k$, $t_k$ exceeds the $i$th breakpoints. The complete formulation is as follows:
\[
\begin{align*}
\max & \quad \sum_{k=1}^{m} \left( a_k t_k - \sum_{\ell=1}^{N-1} \left( a_k^\ell - a_k^{\ell+1} \right) s_k^\ell \right) \\
\text{s.t.} & \quad t_k = \sum_{j=1}^{n} \left( \alpha_j z_j + (\beta_j - \alpha_j) y_j \right) x_{kj} \quad \forall k \\
& \quad s_k^\ell \geq \sum_{j=1}^{n} \left( \alpha_j z_j + (\beta_j - \alpha_j) y_j \right) x_{kj} - b_k^\ell \quad \forall k, \ell
\end{align*}
\]

\[
\sum_{j=1}^{n} \left( \ell_j z_j + (u_j - \ell_j) y_j \right) \frac{S_{jk}^\ell}{x_j} \leq R_{kj} \quad \forall k, \ell
\]

\[
y_j \leq z_j \quad \forall j
\]

\[
\text{other constraints}
\]

\[
z_j = 0, 1 \quad \text{and} \quad 0 \leq y_j \leq 1
\]

\[
t_k^\ell, s_k^\ell \geq 0 \quad \forall k, \ell
\]
where $B_k^f$ is the upper bound on criterion level $k$ for the $f^{th}$ piece. If $B_k^f \leq t_k \leq B_k^{f+1}$, then $s_k^f \geq 0$ for $f = 1, \ldots, f$ and optimality forces $s_k^f = 0$ for all $f > f$, since $(a_k^f - a_k^{f+1}) \leq 0$ by the concavity of the piecewise linear form. The value of level $t_k$ of the $k^{th}$ criterion is then $a_k^1 \cdot t_k$ minus $(a_k^1 - a_k^2)$ for each unit $t_k$ exceeds $B_k^1$, minus $(a_k^2 - a_k^3)$ for each unit $t_k$ exceeds $B_k^2$, etc. The amount of $t_k$ that falls into the $f^{th}$ piece is thus valued at $a_k^f$. This formulation can be readily solved by a branch and bound mixed integer algorithm as before.

Another possible extension would be to approximate some of the five non-linear utility function forms with a piecewise linear function. There may be cases where one of these forms would provide a better fit to the decision maker's utility function than the linear form or a directly derived piecewise linear form. The conjunctive form, $U = \prod_{k=1}^{m} X_k^a$, is concave if the $a_k$ are greater than 1, and the logarithmic form, $U = \sum_{k=1}^{m} a_k \log X_k$, is concave if the $a_k$ are positive. These forms could be accurately approximated by a concave piecewise linearization. The piecewise linear formulation given above would then be applicable.

These extensions would make the algorithm more flexible; able to model more decision makers and decision situations. In the great majority of cases however, the linear form or the piecewise linear form with only two pieces will be as accurate a representation of the decision maker's utility function as the quality of the data justifies.
The decision model developed in the last chapter is applicable to projects where the benefit levels are related to the funding level through the probability of success term. Such projects either succeed, in which case all the benefits are received, or fail, in which case none of the benefits are achieved. Many projects however, do not have this success-failure structure. The benefit levels in such cases are directly related to funding, with no probability of success considerations necessary. To model such situations, \( P_j(\lambda_j) \) or \( \alpha_j \) could be interpreted as the proportion of the benefit level \( X_{kj} \) which would be achieved if the project were to be funded at \( \lambda_j \) rather than the requested funding level \( f_j \). Similarly \( P_j(u_j) \) or \( \beta_j \) would be the proportion of \( X_{kj} \) achieved if project \( j \) were to be funded at \( u_j \). \( P_j(x_j) \) would then be linear between \( u_j \) and \( \lambda_j \).

\[
P_j(x_j) = \alpha_j z_j + (\beta_j - \alpha_j) y_j,
\]

where \( x_j = \lambda_j z_j + (\beta_j - \alpha_j) y_j \). Often projects are judged on sets of criteria which fall into both categories. For example, consider a project with the aim of developing a new optical instrument. If two of the criteria used to evaluate it are export sales, and jobs created, then the former falls into the success-failure category, while the latter depends directly on funding level and is essentially independent of
the outcome of the project. These situations may be modelled by considering \( P_j(x_j) \) to be a probability in the one case and a proportion in the other.

Other types of projects that the algorithm may be modified to handle are those which extend across more than one planning period. For the linear utility function the formulation would be as follows:

\[
\max \sum_{i=1}^{P} \sum_{k=1}^{m} a_k \sum_{j=1}^{n} \left[ \alpha_{ij} z_{ij} + (\beta_{ij} - \alpha_{ij}) \right] x_{ikj}
\]

s.t. \[
\sum_{j=1}^{n} \left[ z_{ij} z_{ij} + (u_{ij} - z_{ij}) y_{ij} \right] < B_i \quad \forall i \quad (1)
\]

\[
\sum_{j=1}^{n} \frac{S_{ij} x_{ij}}{f_{ij}} \left[ z_{ij} z_{ij} + (u_{ij} - z_{ij}) y_{ij} \right] < R_{i\lambda} \quad \forall i, \lambda \quad (2)
\]

\[
y_{ij} < z_{ij} \quad \forall i, j \quad (3)
\]

other constraints \( (4) \)

\[
z_{ij} = 0 \text{ or } 1, 0 < y_{ij} < 1 \quad \forall i, j \quad (5)
\]

where the index \( i = 1, \ldots, P \) refers to the planning period, and all the variables are as previously defined.

The project selection decision model presented here
attempts to be a more realistic representation of the real decision situation. Because of this greater realism and the participation required of the decision maker it is hoped that it will be a more acceptable and useful tool for analyzing project selection decision situations than previously developed algorithms.
BIBLIOGRAPHY


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APPENDIX - A SAMPLE PROBLEM

Consider a government fund-granting agency which has a number of projects under consideration. In order to use the project selection algorithm as a decision tool in analyzing the situation, three types of inputs to the program are required.

The first input required is information about goals and constraints. The decision maker, in consultation with the policy analysis staff of the agency must decide upon the goals which should be achieved by the particular program, and the indicators that best reflect achievement with respect to these goals. Often goals may be cast as constraints. For example, one goal might be the encouragement of economic growth in depressed regions. This goal could be represented by a requirement that a minimal proportion of the funds be spent in such regions. Other goals such as decreasing unemployment, increasing the GNP, and improving the quality of life could be included directly in the objective function.

The resources required by the projects must then be appraised in terms of total resources available. Clearly capital, or funds to be allocated, is considered to be a scarce resource. Other examples of scarce resources may be various types of skilled labour, energy, or specific types of equipment. Resource availabilities may impose constraints on subsets of projects or on all of them.
The second type of input necessary for the program is information from the project evaluator. The evaluator is responsible for estimating the probability of success of each project. Consequently, he must evaluate the competence and capabilities of the applicants, the merits of the proposal in terms of technological requirements, and finally, the compatibility of the applicants' capabilities and the proposed project. On the basis of these considerations, he should be able to indicate the lower funding bound or critical point below which the project would have little probability of success and the upper funding bound or satiation point above which an increase in funding would not be reflected in a significant increase in the probability of success of the project. Probability of success estimates given that the project is funded at the two bounds are required.

The applicant must provide the third phase of input to the program. He must specify the level of funding desired for the project, and describe the project in terms of resources required, benefits produced, and other costs incurred. A proposed schedule is also necessary so that the decision maker can discount the benefits to facilitate comparisons between projects with different timing patterns.

"Other costs" may include social costs and externalities such as an increase in pollution level.
In the example chosen, the government agency has the following three goals; 1) increasing employment 2) improving the balance of payments, and 3) improving the quality of life. Achievement with respect to the first goal can be measured directly by the number of jobs created by each project. Two indicators may be used to measure achievement with respect to the second goal; 1) export sales, and 2) added value. Quality of life according to this agency, concerns stabilization rather than improvement, and consequently the pollution produced by each project is considered to be an indicator related to the third goal. The agency has five projects under consideration with the following characteristics.

<table>
<thead>
<tr>
<th>Resource Required</th>
<th>Project 1</th>
<th>Projects 2</th>
<th>Project 3</th>
<th>Project 4</th>
<th>Project 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>capital (million $)</td>
<td>1</td>
<td>5</td>
<td>15</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>skilled labour 1</td>
<td>300</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>150</td>
</tr>
<tr>
<td>skilled labour 2</td>
<td>-</td>
<td>-</td>
<td>100</td>
<td>50</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Benefits Produced</th>
<th>Project 1</th>
<th>Projects 2</th>
<th>Project 3</th>
<th>Project 4</th>
<th>Project 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>jobs created</td>
<td>500</td>
<td>100</td>
<td>150</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>export sales ($10,000)</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>value added ($10,000)</td>
<td>5</td>
<td>10</td>
<td>30</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>pollution created</td>
<td>-1</td>
<td>-1</td>
<td>-10</td>
<td>-1</td>
<td>-13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Funding (in million $)</th>
<th>lower bound</th>
<th>Project 1</th>
<th>Projects 2</th>
<th>Project 3</th>
<th>Project 4</th>
<th>Project 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>requested level</td>
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<td>2</td>
<td>10</td>
<td>1</td>
<td>5</td>
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</tr>
<tr>
<td>upper bound</td>
<td>2</td>
<td>15</td>
<td>20</td>
<td>4</td>
<td>15</td>
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</table>

<table>
<thead>
<tr>
<th>at lower bound</th>
<th>Project 1</th>
<th>Projects 2</th>
<th>Project 3</th>
<th>Project 4</th>
<th>Project 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.8</td>
<td>.5</td>
<td>.7</td>
<td>.8</td>
<td>.7</td>
</tr>
<tr>
<td>at upper bound</td>
<td>1.0</td>
<td>.85</td>
<td>.98</td>
<td>1.0</td>
<td>.95</td>
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</tbody>
</table>

TABLE II
SAMPLE PROBLEM DATA

The following pages are the output from the interactive project selection algorithm applied to this problem.
WELCOME TO 'MODEM', A MULTIPLE OBJECTIVE DECISION MAKING PROGRAM. MODEM MAY HELP YOU DECIDE WHICH OF A NUMBER OF PROJECTS TO SUPPORT AND TO WHAT EXTENT SUPPORT SHOULD BE GIVEN.

BACKUP: ENTERING 'BACK' WILL CAUSE REVERSION TO A PREVIOUS QUESTION. TERMINATION: ENTERING 'END', 'HALT' OR 'STOP' WILL STOP MODEM IMMEDIATELY.

HOW MANY PROJECTS ARE THERE? (MAXIMUM IS 10.)
5
WHAT IS THE NAME OF PROJECT 1? one
WHAT IS THE NAME OF PROJECT 2? two
WHAT IS THE NAME OF PROJECT 3? three
WHAT IS THE NAME OF PROJECT 4? four
WHAT IS THE NAME OF PROJECT 5? five

WHAT IS THE TOTAL NUMBER OF BENEFITS (OBJECTIVES, PAYOFFS, OUTPUTS) FOR ALL OF THESE PROJECTS? (MAXIMUM OF 10.)
4
WHAT IS THE NAME OF BENEFIT 1? jobs
WHAT IS THE NAME OF BENEFIT 2? export sales
WHAT IS THE NAME OF BENEFIT 3? value added
WHAT IS THE NAME OF BENEFIT 4? pollution

TIME TO FILL IN THE PROJECT-BENEFIT MATRIX.

WHAT IS THE JOBS PAYOFF FOR ONE? 500
WHAT IS THE EXPORT SALES PAYOFF FOR ONE? 2
WHAT IS THE VALUE ADDED PAYOFF FOR ONE? 5
WHAT IS THE POLLUTION PAYOFF FOR ONE? -1
WHAT IS THE JOBS PAYOFF FOR TWO? 100
WHAT IS THE EXPORT SALES PAYOFF FOR TWO? 3
WHAT IS THE VALUE ADDED PAYOFF FOR TWO? 10
WHAT IS THE POLLUTION PAYOFF FOR TWO? -5
WHAT IS THE JOBS PAYOFF FOR THREE? 150
WHAT IS THE EXPORT SALES PAYOFF FOR THREE? 5
WHAT IS THE VALUE ADDED PAYOFF FOR THREE? 30
WHAT IS THE POLLUTION PAYOFF FOR THREE? -10
WHAT IS THE JOBS PAYOFF FOR FOUR? 300
WHAT IS THE EXPORT SALES PAYOFF FOR FOUR? 2
WHAT IS THE VALUE ADDED PAYOFF FOR FOUR? 5
WHAT IS THE POLLUTION PAYOFF FOR FOUR? -1
WHAT IS THE JOBS PAYOFF FOR FIVE? 200
WHAT IS THE EXPORT SALES PAYOFF FOR FIVE? 6
WHAT IS THE VALUE ADDED PAYOFF FOR FIVE? 10
WHAT IS THE POLLUTION PAYOFF FOR FIVE? -13
WHAT IS THE INITIAL REQUESTED LEVEL OF FUNDING FOR ONE? 1
WHAT IS THE MINIMUM ACCEPTABLE SUPPORT FOR ONE? 1
WHAT IS THE PROBABILITY OF SUCCESS OF ONE AT THIS FUNDING?
ANSWER SHOULD BE IN INTEGRAL PERCENT -- A NUMBER BETWEEN 0 AND 100.
80
WHAT IS THE MAXIMUM ACCEPTABLE SUPPORT FOR ONE? 2
WHAT IS THE PROBABILITY OF SUCCESS OF ONE AT THIS FUNDING? 100
WHAT IS THE INITIAL REQUESTED LEVEL OF FUNDING FOR TWO? 5
WHAT IS THE MINIMUM ACCEPTABLE SUPPORT FOR TWO? 2
WHAT IS THE PROBABILITY OF SUCCESS OF TWO AT THIS FUNDING? 50
WHAT IS THE MAXIMUM ACCEPTABLE SUPPORT FOR TWO? 15
WHAT IS THE PROBABILITY OF SUCCESS OF TWO AT THIS FUNDING? 85
WHAT IS THE INITIAL REQUESTED LEVEL OF FUNDING FOR THREE? 15
WHAT IS THE MINIMUM ACCEPTABLE SUPPORT FOR THREE? 10
WHAT IS THE PROBABILITY OF SUCCESS OF THREE AT THIS FUNDING? 70
WHAT IS THE MAXIMUM ACCEPTABLE SUPPORT FOR THREE? 20
WHAT IS THE PROBABILITY OF SUCCESS OF THREE AT THIS FUNDING? 98
WHAT IS THE INITIAL REQUESTED LEVEL OF FUNDING FOR FOUR? 2
WHAT IS THE MINIMUM ACCEPTABLE SUPPORT FOR FOUR? 1
WHAT IS THE PROBABILITY OF SUCCESS OF FOUR AT THIS FUNDING? 80
WHAT IS THE MAXIMUM ACCEPTABLE SUPPORT FOR FOUR? 4
WHAT IS THE PROBABILITY OF SUCCESS OF FOUR AT THIS FUNDING? 100
**What is the initial requested level of funding for five?** 10

**What is the minimum acceptable support for five?** 5

**What is the probability of success of five at this funding?** 70

**What is the maximum acceptable support for five?** 15

**What is the probability of success of five at this funding?** 95

**How many distinct resources are there for the 5 projects (maximum of 10.)?** 3

**What is the name of resource 1?** capital

**What is the name of resource 2?** labor 1

**What is the name of resource 3?** labor 2

**Please fill in the project-resources matrix.**

<table>
<thead>
<tr>
<th></th>
<th>Project One</th>
<th>Project Two</th>
<th>Project Three</th>
<th>Project Four</th>
<th>Project Five</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>1</td>
<td>5</td>
<td>15</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>Labor 1</td>
<td>300</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>Labor 2</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**What is the total maximum available for capital?** 20

**What is the total maximum available for labor 1?** 450

**What is the total maximum available for labor 2?** 120
Would you like to derive a utility function? Yes

The following are pseudo-project profiles with benefits ordered the way you presented them. Please score each pseudo-project from 1 (worst possible) to 100 (best possible) corresponding to a benefit profile of 40 1 2 -19.

<table>
<thead>
<tr>
<th>Pseudo-Project</th>
<th>Score?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>59</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>13</td>
<td>65</td>
</tr>
<tr>
<td>14</td>
<td>30</td>
</tr>
</tbody>
</table>
PSEUDO-PROJECT 15:
948 2 39 -10
SCORE? 65
PSEUDO-PROJECT 16:
816 8 6 -8
SCORE? 66

FOLLOWING IS AN ORDERED DISPLAY OF THE PSEUDO-PROJECTS:

<table>
<thead>
<tr>
<th>PROJ #</th>
<th>SCORE</th>
<th>BENEFITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>895</td>
</tr>
<tr>
<td>2</td>
<td>72</td>
<td>808</td>
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<td>8</td>
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<td>264</td>
</tr>
<tr>
<td>16</td>
<td>30</td>
<td>140</td>
</tr>
</tbody>
</table>

YOU MAY NOW CHANGE SCORES.
WHICH PSEUDO-PROJECT'S SCORE DO YOU WISH TO CHANGE?
A ZERO MEANS THAT NO MORE CHANGES ARE DESIRED.

WHAT IS 4'S NEW SCORE? 64

FOLLOWING IS AN ORDERED DISPLAY OF THE PSEUDO-PROJECTS:

<table>
<thead>
<tr>
<th>PROJ #</th>
<th>SCORE</th>
<th>BENEFITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>895</td>
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<tr>
<td>16</td>
<td>30</td>
<td>140</td>
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</tbody>
</table>

YOU MAY NOW CHANGE SCORES.
WHICH PSEUDO-PROJECT'S SCORE DO YOU WISH TO CHANGE?
A ZERO MEANS THAT NO MORE CHANGES ARE DESIRED.
FINAL RANKING OF PSEUDO-PROJECTS:

<table>
<thead>
<tr>
<th>PROJ #</th>
<th>SCORE</th>
<th>BENEFITS</th>
</tr>
</thead>
<tbody>
<tr>
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<td>76</td>
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COEFS: 4.6E-02 4.9E 00 6.5E-01 1.4E 00

PSEUDO-PROJECT ACTUAL PREDICTED

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<td>30</td>
<td>18.78</td>
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</table>

SCORING BY MISTAKES*DISTANCE METHOD:
SCORE = 4

R-SQUARE = 383.8

CORRELATION BY PEARSON'S METHOD:
R = 0.96

SPEARMAN CORRELATIONS:
R-SUB-S = 0.94

DISTANCE*ERROR METHOD:
TAU = 0.82
**Lexicographic Ordering of Benefits:**

<table>
<thead>
<tr>
<th>Benefit</th>
<th>Number of Order Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>3</td>
<td>7</td>
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<tr>
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</tbody>
</table>

**Is This Utility Function Accurate Enough?** No

**Worst Point is 16**

**Another Pseudo-Project to Score:**

| 136      | 5   | 9   | -9 |

**Score?** 36

**Following is an Ordered Display of the Pseudo-Projects:**

<table>
<thead>
<tr>
<th>Proj #</th>
<th>Score</th>
<th>Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>895 3</td>
</tr>
<tr>
<td>2</td>
<td>72</td>
<td>808 7</td>
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<td>309 7</td>
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<td>948 2</td>
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<td>64</td>
<td>816 8</td>
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<td>60</td>
<td>146 7</td>
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<tr>
<td>10</td>
<td>57</td>
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<td>53 8</td>
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<tr>
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<td>42</td>
<td>422 4</td>
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<tr>
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<td>39</td>
<td>716 2</td>
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<td>36</td>
<td>136 5</td>
</tr>
<tr>
<td>16</td>
<td>35</td>
<td>264 6</td>
</tr>
</tbody>
</table>

**You May Now Change Scores.**

**Which Pseudo-Project’s Score Do You Wish to Change?**

A Zero Means That No More Changes Are Desired.

**Final Ranking of Pseudo-Projects:**

<table>
<thead>
<tr>
<th>Proj #</th>
<th>Score</th>
<th>Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>895 3</td>
</tr>
<tr>
<td>2</td>
<td>72</td>
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<td>42</td>
<td>422 4</td>
</tr>
<tr>
<td>14</td>
<td>39</td>
<td>716 2</td>
</tr>
</tbody>
</table>
COEFFS: 4.5E-02 5.0E 00 6.4E-01 1.4E 00

PSEUDO-PROJECT  ACTUAL  PREDICTED
1       76       76.22
2       72       70.65
3       67       65.44
4       65       70.99
5       65       70.64
6       65       63.66
7       64       69.12
8       60       60.25
9       59       51.23
10      57       52.22
11      51       50.39
12      51       54.68
13      42       41.11
14      39       31.34
15      36       23.86
16      35       38.08

SCORING BY MISTAKES*DISTANCE METHOD:
SCORE = 7

R-SQUARE = 413.5

CORRELATION BY PEARSON'S METHOD:
R = 0.95

SPEARMAN CORRELATIONS:
R-SUB-S = 0.94

DISTANCE*ERROR METHOD:
TAU = 0.82

LEXICOGRAPHIC ORDERING OF BENEFITS:
BENEFIT  NUMBER OF ORDER ERRORS
1       7
2       5
3       8
4       7

IS THIS UTILITY FUNCTION ACCURATE ENOUGH?  no

WORST POINT IS 15

THE FOLLOWING ARE PSEUDO-PROJECT PROFILES
WITH BENEFITS ORDERED THE WAY YOU PRESENTED THEM.
PLEASE SCORE EACH PSEUDO-PROJECT FROM 1 (WORST POSSIBLE)
CORRESPONDING TO A BENEFIT PROFILE OF
40 1 2 -19
TO 100 (BEST POSSIBLE) CORRESPONDING TO A BENEFIT PROFILE OF
1000 9 40 0
BENEFITS:
1 2 3 4
PSEUDO-PROJECT 1:
895 3 39 -3
SCORE= 76
PSEUDO-PROJECT 2:
808 7 10 -5
SCORE= 72
PSEUDO-PROJECT 3:
309 7 35 -4
SCORE= 67
PSEUDO-PROJECT 4:
873 7 26 -14
SCORE= 65
PSEUDO-PROJECT 5:
988 5 35 -15
SCORE= 65
PSEUDO-PROJECT 6:
948 2 39 -10
SCORE= 65
PSEUDO-PROJECT 7:
816 8 6 -8
SCORE= 64
PSEUDO-PROJECT 8:
146 7 34 -2
SCORE= 60
PSEUDO-PROJECT 9:
507 4 22 -4
SCORE= 59
PSEUDO-PROJECT 10:
851 4 17 -12
SCORE= 57
PSEUDO-PROJECT 11:
53 8 13 0
SCORE= 51
PSEUDO-PROJECT 12:
644 7 19 -15
SCORE= 51
PSEUDO-PROJECT 13:
422 4 39 -16
SCORE= 42
PSEUDO-PROJECT 14:
716 2 5 -10
SCORE= 39
PSEUDO-PROJECT 15:
136 5 9 -9
SCORE= 36
PSEUDO-PROJECT 16:
264 6 30 -16
SCORE= 35
PSEUDO-PROJECT 1:
860 7 27 -12
SCORE= 70
FOLLOWING IS AN ORDERED DISPLAY OF THE PSEUDO-PROJECTS:

<table>
<thead>
<tr>
<th>Proj #</th>
<th>Score</th>
<th>Benefits</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>955</td>
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<tr>
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<table>
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</table>

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<th>Proj #</th>
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<tbody>
<tr>
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<tr>
<td>21</td>
<td>60</td>
<td>877</td>
</tr>
</tbody>
</table>

FOLLOWING IS AN ORDERED DISPLAY OF THE PSEUDO-PROJECTS:
YOU MAY NOW CHANGE SCORES.
WHICH PSEUDO-PROJECT'S SCORE DO YOU WISH TO CHANGE?
A ZERO MEANS THAT NO MORE CHANGES ARE DESIRED.

AN ORDINAL DOMINANCE CONFLICT IN SCORING HAS BEEN FOUND BETWEEN 2 AND 3.
THESE TWO ARE INDICATED BY ASTERISKS IN THE FOLLOWING PRESENTATION.
PLEASE CHANGE ONE OR BOTH SCORES SO THAT A CONFLICT WILL NOT OCCUR.

<table>
<thead>
<tr>
<th>PROJ #</th>
<th>SCORE</th>
<th>BENEFITS</th>
</tr>
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<tbody>
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<td>* 2</td>
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</table>

WHICH PSEUDO-PROJECT'S SCORE DO YOU WISH TO CHANGE
A ZERO MEANS THAT NO MORE CHANGES ARE DESIRED.

WHAT IS 2'S NEW SCORE? 47

FOLLOWING IS AN ORDERED DISPLAY OF THE PSEUDO-PROJECTS:

<table>
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<th>PROJ #</th>
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<tbody>
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<td>31</td>
<td>760</td>
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</tbody>
</table>
YOU MAY NOW CHANGE SCORES.

WHICH PSEUDO-PROJECT'S SCORE DO YOU WISH TO CHANGE?
A ZERO MEANS THAT-NO MORE CHANGES ARE DESIRED.

FINAL RANKING OF PSEUDO-PROJECTS:

<table>
<thead>
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<th>PROJ #</th>
<th>SCORE</th>
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<tbody>
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BENEFIT 1, COEFFS: 0.0519567, 0.0300714, BREAKPOINT: 507.0000000
BENEFIT 2, COEFFS: 4.6440487, 3.3582411, BREAKPOINT: 4.0000000
BENEFIT 3, COEFFS: 0.6867285, 0.5346403, BREAKPOINT: 22.0000000
BENEFIT 4, COEFFS: -0.8256531, 2.1860657, BREAKPOINT: -4.0000000

PSEUDO-PROJECT ACTUAL PREDICTED

| 1 | 76 | 78.62 |
| 2 | 72 | 72.03 |
| 3 | 67 | 70.07 |
| 4 | 65 | 64.69 |
| 5 | 65 | 64.06 |
| 6 | 65 | 63.27 |
| 7 | 64 | 66.32 |
| 8 | 60 | 59.41 |
| 9 | 59 | 63.33 |
| 10| 57 | 52.75 |
| 11| 51 | 55.74 |
| 12| 51 | 51.42 |
| 13| 42 | 41.77 |
| 14| 39 | 35.53 |
| 15| 36 | 27.55 |
| 16| 35 | 35.46 |

LINEAR R-SQUARE = 413.5222 ; PIECEWISE LINEAR = 168.978

CORRELATION BY PEARSON'S METHOD:
R = 0.98

SPEARMAN CORRELATIONS:
R-SUB-S = 0.96
DISTANCE*ERROR METHOD:
   TAU = 0.88

LEXICOGRAPHIC ORDERING OF BENEFITS:

<table>
<thead>
<tr>
<th>BENEFIT</th>
<th>NUMBER OF ORDER ERRORS</th>
</tr>
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<tr>
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</tbody>
</table>

SOLUTION 1

VALUE=80.4617:

SUPPORT ONE WITH RESOURCES AS FOLLOWS:
   1.366666 OF CAPITAL
   409.9995 OF LABOR 1
   0. OF LABOR 2

SUPPORT TWO WITH RESOURCES AS FOLLOWS:
   1.999999 OF CAPITAL
   40. OF LABOR 1
   0. OF LABOR 2

SUPPORT THREE WITH RESOURCES AS FOLLOWS:
   14.25 OF CAPITAL
   0. OF LABOR 1
   95. OF LABOR 2

SUPPORT FOUR WITH RESOURCES AS FOLLOWS:
   1. OF CAPITAL
   0. OF LABOR 1
   25. OF LABOR 2

DO NOT SUPPORT FIVE.

SOLUTION 2

VALUE=80.6269:

SUPPORT ONE WITH RESOURCES AS FOLLOWS:
   1.116665 OF CAPITAL
   334.9993 OF LABOR 1
   0. OF LABOR 2

SUPPORT TWO WITH RESOURCES AS FOLLOWS:
   1.999999 OF CAPITAL
   40. OF LABOR 1
   0. OF LABOR 2
SUPPORT THREE WITH RESOURCES AS FOLLOWS:
9.999999 OF CAPITAL
0. OF LABOR 1
66.6667 OF LABOR 2

SUPPORT FOUR WITH RESOURCES AS FOLLOWS:
1.883332 OF CAPITAL
0. OF LABOR 1
47.0833 OF LABOR 2

SUPPORT FIVE WITH RESOURCES AS FOLLOWS:
5. OF CAPITAL
75. OF LABOR 1
0. OF LABOR 2

SOLUTION 3

VALUE=78.0611:

SUPPORT ONE WITH RESOURCES AS FOLLOWS:
1.249999 OF CAPITAL
374.9995 OF LABOR 1
0. OF LABOR 2

DO NOT SUPPORT TWO.

SUPPORT THREE WITH RESOURCES AS FOLLOWS:
12.2045 OF CAPITAL
0. OF LABOR 1
81.3636 OF LABOR 2

SUPPORT FOUR WITH RESOURCES AS FOLLOWS:
1.545455 OF CAPITAL
0. OF LABOR 1
38.6364 OF LABOR 2

SUPPORT FIVE WITH RESOURCES AS FOLLOWS:
5. OF CAPITAL
75. OF LABOR 1
0. OF LABOR 2

SOLUTION 4

VALUE=77.3058:

SUPPORT ONE WITH RESOURCES AS FOLLOWS:
1.499999 OF CAPITAL
449.9995 OF LABOR 1
0. OF LABOR 2
DO NOT SUPPORT TWO.

SUPPORT THREE WITH RESOURCES AS FOLLOWS:
14.25 OF CAPITAL
0. OF LABOR 1
95. OF LABOR 2

SUPPORT FOUR WITH RESOURCES AS FOLLOWS:
1. OF CAPITAL
0. OF LABOR 1
25. OF LABOR 2

DO NOT SUPPORT FIVE.

SOLUTION 5

VALUE=71.9458:

SUPPORT ONE WITH RESOURCES AS FOLLOWS:
1.116665 OF CAPITAL
334.9993 OF LABOR 1
0. OF LABOR 2

SUPPORT TWO WITH RESOURCES AS FOLLOWS:
1.999999 OF CAPITAL
40. OF LABOR 1
0. OF LABOR 2

DO NOT SUPPORT THREE.

SUPPORT FOUR WITH RESOURCES AS FOLLOWS:
3.999998 OF CAPITAL
0. OF LABOR 1
99.9999 OF LABOR 2

SUPPORT FIVE WITH RESOURCES AS FOLLOWS:
5. OF CAPITAL
75. OF LABOR 1
0. OF LABOR 2

DO YOU WISH TO BEGIN ANOTHER RUN (B) OR END (E)? e
MODEM TERMINATING NORMALLY...
The five solutions provided by the algorithm have similar subjective values but suggest very different funding patterns. At this point the decision maker would be required to choose between them on the basis of any non-quantifiable information or criteria he may possess. Sensitivity runs with different amounts of resources available would point out any important discontinuities in the value of the objective and facilitate the final decision.