Essays on New Open Economy Macroeconomics

by

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Abstract

This thesis consists of three essays on the issues related to endogenous currency of pricing in utility-based open economy models. The first essay endogenizes both the currency of liability denomination and the currency of export pricing to study twin dollarization in East Asian economies, a phenomenon where firms borrow in US dollars and also set export prices in US dollars. This essay shows that twin dollarization is an optimal strategy for all firms when exchange rate flexibility is limited, and it can reduce the welfare loss caused by a fixed exchange rate regime. The second essay examines the role of US dollar as an oil currency and its impact on the world dollar standard and the global economy in a two-country general equilibrium model with sticky prices. When the oil price is denominated in US dollars, US firms bear less exchange rate risk than foreign firms when facing oil price shocks. As a result of this asymmetry, all the firms have an incentive to set export prices in dollars, this will generate an endogenously determined dollar standard in international goods pricing. In such a case, the households in the US are better off than those in the rest of the world in term of welfare, though it is costly for the US to have a dollar standard. The third essay re-examines the issue of the degree of exchange rate flexibility in an open economy monetary policy game. It focuses on an environment where the currency of pricing is endogenous, and monetary authorities take into account the way in which firms make this choice. It is shown that there is a unique equilibrium to the monetary policy game, where all firms follow PCP, and Friedman's classic defense of flexible exchange rates is upheld. Meanwhile, an alternative method to sustain flexible exchange rates is also provided.

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Summary

This thesis consists of three essays on new open economy macroeconomics.

The first essay develops a small open economy general equilibrium model with nominal rigidities to study twin dollarization in East Asian economies, a phenomenon where firms borrow in US dollars and also set export prices in US dollars. The model endogenizes both the currency of liability denomination and the currency of export pricing. We show that the key factor that affects the firm's dollarization decision is exchange rate policy. Twin dollarization is an optimal strategy for all firms when exchange rate flexibility is limited, which implies that a fixed exchange rate regime will cause twin dollarization. Furthermore, we find that twin dollarization can dampen the welfare loss caused by the fixed exchange rate regime, as it helps to cushion the economy against the domestic nominal risk.

The second essay examines the role of US dollar as an oil currency and its impact on the world dollar standard and the global economy in a two-country general equilibrium model with sticky prices. When the oil price is denominated in US dollars, US firms bear less exchange rate risk than foreign firms when facing oil price shocks. As a result of this asymmetry, all firms in the world will set their export prices in US dollars, which implies a dollar standard in international goods pricing. We also find that, however, in the presence of active monetary policy, a dollar standard is not necessarily the equilibrium of firms' choices. The nature of the equilibrium will depend critically on the relative sizes of external shocks. Finally, we find that households in the US are always better off than those in the rest of the world in the situation of having a dollar standard. This is because the welfare gain, which comes from the cost advantage of US firms, exceeds the welfare cost of having a dollar standard, which is due to the lack of exchange rate pass-through.

Recent literature has emphasized that the degree to which exchange rate adjustment should be part of optimal monetary policy in an open economy depends critically on the way in which

export good prices are set. If export goods are priced in the producer's currency (PCP), then monetary policy should use exchange rate adjustment. But if export goods prices are set in the consumer's currency (LCP), then optimal monetary policy should keep the exchange rate constant. This paper examines the determination of optimal monetary policy in an environment where the currency of pricing is endogenous, and monetary authorities take into account the way in which firms make this choice. It is shown that there is a unique equilibrium to the monetary policy game, where all firms follow PCP, and Friedman's classic defense of flexible exchange rates is upheld. Furthermore, we show that there is a simple method to sustain flexible exchange rates by restricting monetary authorities to respond only to domestic economics conditions when monetary authorities cannot take account of the currency of pricing decision. In this case, restricting the form of monetary rule acts as an equilibrium selection mechanism.

Chapter 1

Twin Dollarization and Exchange Rate Policy

1.1 Introduction

This paper is motivated by a frequently observed phenomenon in East Asian economies. On the one hand, the vast majority of lending to these emerging markets is denominated in foreign currency, especially in US dollars. On the other hand, most export goods in these economies are priced in US dollars as well. For instance, in Thailand, approximately 76 billion dollars was recorded as raised by Thai firms on international debt markets between 1992 and mid-1997, while only 3.2 percent was denominated in Thai baht. Meanwhile, about 90 percent of Thai export goods were priced in US dollars.[1] This phenomenon represents a coexistence of liability dollarization and export pricing dollarization,[2] which we refer to as "twin dollarization".

While considerable attention has been paid to the macroeconomic implications of liability dollarization and of export pricing dollarization, there are few papers which relate them together and investigate a common cause. Intuitively, they should be related to each other. Firms have an incentive to match the currency of their sales revenue with the currency of their debt denomination so that they can use sales revenues to hedge their foreign debts against exchange rate risk. This could explain the coexistence, but it would still not reveal the cause of twin dollarization. In this paper, we try to analyze the "twin dollarization" phenomenon formally and answer the following questions: Why do firms want to borrow in dollars and set export prices in dollars at the same time? Furthermore, if twin dollarization can be rationalized as an optimal strategy for firms, what is the inducement? Finally, what are the welfare

[1] See Choi and Cook (2004) and Cook and Devereux (2004).

[2] Cook and Devereux (2004) use the term "dollar currency pricing" to describe the latter fact.

implications of twin dollarization?

To address these questions, we propose a small open economy stochastic general equilibrium model with nominal rigidities. The economy is subject to two types of external shocks, foreign demand shocks and world interest rate shocks. The key feature of the model is that both the currency of liability denomination and the currency of export pricing are endogenous. Our analysis concentrates on the behavior of export firms, which depend highly on the world market for both the sale of products and the supply of inputs.[3] The export firms are monopolistic competitive and supply differentiated goods to the rest of the world. Furthermore, they can set export prices either in domestic currency (peso) or in foreign currency (dollar). Whatever currency they choose, firms must set their export prices before the state of the world is realized, and the prices cannot be adjusted until the next period. Firms import intermediate goods and finance their purchases by borrowing from international lenders.[4] The loan is assumed to be contracted either in domestic currency or in foreign currency *ex ante* as well. Therefore, four feasible (pure) strategies are available for firms, and each of them represents a combination of the currency of liability denomination and the currency of export pricing.

Using this framework, we find that exchange rate policy is a key factor for firms' dollarization decisions. When exchange rate flexibility is low, twin dollarization is an optimal strategy for all firms. In other words, a fixed exchange rate will cause twin dollarization. We also find that, as exchange rate flexibility increases, the degree of dollarization decreases. That is, fewer firms will choose to borrow in dollars and set export prices in dollars. As a result, a floating exchange rate will lead all firms to borrow in the domestic currency and set export prices in the domestic currency. Under intermediate exchange rate regimes, the currency of export pricing and the currency of liability denomination can be different. In some cases, multiple equilibria may exist for firms' optimal currency strategies due to the presence of a strategic complementarity among firms.

The intuition behind these findings is straightforward. The optimal strategy for the export firm is the one that delivers the highest expected profit, which reflects the firm's consideration

[3]This is one of main features of export firms in the East Asian economies.

[4]Most Asian emerging market economies are export-oriented, and the export sectors are capital intensive. Thus, international lending to these countries is most often used by export firms.

of both the demand for its export goods and marginal cost. In a stochastic environment, exchange rate flexibility determines how the absorption of external shocks is divided into changes of the domestic interest rate and the exchange rate. When exchange rate flexibility is high, the exchange rate is volatile and the domestic interest rate is relatively stable. Thus, the export firm will choose to borrow in pesos to avoid exchange rate risk, as it implies a more stable marginal cost. Meanwhile, by setting prices in pesos, the firm can stabilize the demand for its export goods, as in this case the relative prices of export goods can be adjusted by exchange rates in the face of demand shocks. Therefore, the optimal strategy for all firms is to borrow in pesos and set export prices in pesos when exchange rates are flexible. In the case of low exchange rate flexibility, exchange rate volatility is low and the domestic interest rate volatility is high. Therefore, the export firm can stabilize the marginal cost and avoid the volatile domestic interest rate by borrowing in dollars. Regarding the currency of pricing, due to the lack of exchange rate flexibility, setting prices in pesos cannot help the firm to stabilize the demand for its export goods. Meanwhile, when the firm sets export prices in domestic currency, its demand is directly sensitive to exchange rate movement, which results in an increase in the firm's expected cost, ceteris paribus. So given a low exchange rate flexibility, the firm is more concerned with expected cost than revenue and should choose to set export prices in dollars. Therefore, twin dollarization is an optimal strategy for all firms under a fixed exchange rate regime.

For intermediate levels of exchange rate flexibility, the ranking of firms' currency strategies also depends on the decisions of other firms. Therefore, the interaction among firms will lead to mixed strategy or multiple equilibria.

Our findings suggest that twin dollarization is optimal for firms under a fixed exchange rate regime, but is it a beneficial arrangement for the whole economy in terms of welfare? Following Schmitt-Grohe and Uribe (2004), we use a perturbation method to calculate welfare, which is measured by the representative household's lifetime expected utility. In our model, the equilibrium under a flexible exchange rate regime implies a higher welfare than that generated by a fixed exchange rate regime, so there is always a welfare loss associated with a fixed exchange rate regime. Nevertheless, twin dollarization can deliver a higher welfare than other currency strategies under a fixed exchange rate regime. In this sense, it dampens the welfare

loss caused by the fixed exchange rate arrangement. The reason is simple. Under a fixed exchange rate regime, the exchange rate cannot insulate the economy from external shocks, so the economy is subject to both real shocks and nominal uncertainties. But with twin dollarization, some domestic nominal risk can be evaded and thus social welfare is improved.

Our paper provides a new angle to study the dollarization phenomenon in East Asian emerging market economies. We relate two aspects of dollarization and study their common cause, while most of the recent literature focuses solely on the macroeconomic implication of liability dollarization. For example, Calvo and Reinhart (2000) show that currency devaluation will lead to the deterioration of the balance sheets of firms with foreign currency debts, which causes real contraction in developing countries.[5] In this paper, we emphasize that it is the fixed exchange rate regime that causes twin dollarization, instead of Calvo and Reinhart (2002)'s suggestion that liability dollarization leads to the "fear of floating". In this sense, we show a new linkage between exchange rate policy and dollarization. We also show that there exists a welfare gain from twin dollarization under a fixed exchange rate regime. This is in contrast to the welfare implications of liability dollarization in most of the financial crisis literature, which typically emphasizes the welfare losses caused by foreign currency debt.

This paper is also closely related to two other lines of literature. With respect to endogenous currency of pricing, we follow the approach used by Devereux, Engel and Storgaard (2004). They endogenize the currency of export pricing and show that exporters wish to set prices in the currency of the country with a relatively stable monetary policy.[6] Our paper differs from theirs in two key dimensions. First, we focus on a small open economy and study how exchange rate flexibility affects firms' currency of export pricing. Second, we illustrate a linkage through which the currency of debt denomination can affect the firm's marginal cost and the currency choice of export pricing.

Another line of research that is related to this paper is the literature on endogenous liability

[5] Also see Aghion, Bachetta, and Bannerjee (2000, 2001), Calvo (2000), Cespedes et al. (2004), Cook (2000), and Krugman (2000)

[6] Engel (2005) discusses the parallels between the choice of invoicing when prices are sticky and the optimal degree of exchange rate pass-through when prices are flexible.

denomination.[7] Our work differs in several aspects. First, most papers in the literature argue that foreign currency debt exists because of market or institutional failure in emerging market economies, see for example, Jeanne (2000, 2003), and Caballero and Krishnamuthy (2003). Instead, we show that dollarization can be the result of firms' optimization behavior and that exchange rate policy is a key factor affecting firms' decisions. Second, our general equilibrium model setting has a natural advantage over the recent literature based on partial equilibrium or reduced form models, since it allows for welfare analysis.[8] Finally, we investigate the common cause of two aspects of dollarization, instead of analyzing liability dollarization in isolation.

This paper is organized as follows. Section 2 describes stylized facts of twin dollarization in East Asian economies. Section 3 presents the basic setup of the model. Section 4 solves the model and shows how the firms' dollarization decisions can be affected by exchange rate policy. Section 5 discusses the welfare implication. Section 6 concludes.

1.2 Stylized Facts

A frequently observed fact in East Asian economies is that firms borrow externally in foreign currency and set their export goods prices in foreign currency as well. Since the US dollar is the dominant foreign currency, we refer to this phenomenon as "twin dollarization". It reflects the fact that, in contrast to the practise of firms in developed countries, firms in emerging market economies seldom use their own currency in goods and financial market trading with the rest of the world.

Evidence of "twin dollarization" is given in Tables A.1 and A.2. Table A.1 reports the

[7]There is also a presumption in the literature that developing countries cannot borrow in local currency. Some call it the "original sin". This wing of the literature suggests that if somehow we could eliminate this "original sin" then currency and debt crises problems of the developing world would go away. Here we make a different point: there are well defined economic reasons why firms in developing countries may not want to borrow in local currency even if they could.

[8]Chammon and Hausman (2002) also argue that the central bank's preference may affect a firm's choice of liability denomination in a reduced form model.

use of US dollar in the currency of export pricing (invoicing currency) in selected countries.[9] For East Asian economies, we report data for Korea, Thailand, Malaysia, and Indonesia. The selected developed countries are the US, Germany, Japan, the UK, France, and Italy. The striking feature of Table A.1 is that East Asian economies seldom use their domestic currency as the currency of export pricing. In particular, for Korea and Thailand, almost all of their export goods are priced in foreign currency, mostly in US dollars. By contrast, the share of domestic currency invoicing is much higher in the developed countries. Even in Italy, about 40 percent of exports are priced in Lira.

Table A.2 presents the currency composition of external debt for the same set of countries. From Table A.2, we can see that the external debts of these East Asian economies are mainly denominated in foreign currency, while for developed countries, the share of domestic currency denominated external debt is much higher. Although the data in Table A.2 includes both public debt and private debt, statistics show that the share of private debts to total external debts is high in some East Asian economies. For example, data from the Japanese Ministry of Finance show that 81.3% of Thailand's external debts and 98.5% of Korea's external debts were private debts at the end of 1996. Therefore, it is reasonable to infer that most private debts raised by firms in East Asian economies are denominated in foreign currency. This argument is also well supported by other observations in Thailand and Korea: Of the approximately 76 billion dollars recorded as raised by Thai firms on international debt markets between 1992 and mid-1997 (according to the IFR Platinum database), about 3.2% was denominated in Thai baht. During the same period, less than 1% of the approximately 144 billion dollars recorded as raised by Korean firms was denominated in Korean won (Choi and Cook 2004). The detailed currency composition of external debt is not reported in Table A.2, but the dominant use of the US dollar in emerging market debt denomination is clear.

From Table A.1 and Table A.2, we can see that firms in East Asian emerging market economies tend to follow both export pricing dollarization and liability dollarization, which contrasts with borrowing and export pricing behavior in developed countries. Thus, the special

[9]Data for invoicing currency are not available for every country, so we will focus on the available data set in selected countries. Nevertheless, we believe these countries are representative for East Asian economies and developed economies

role of the US dollar in the global economy could not solely account for the twin dollarization phenomenon. Some other common characteristics in East Asian economies are needed as an explanation.

A possible explanation for this phenomenon is the exchange rate regimes in these economies. Before the 1997-98 Asian crisis, East Asian economies like Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, and Thailand all pegged their exchange rates to the US dollar. The exchange rate regimes ranged from a currency board hard peg in Hong Kong to a sliding or crawling peg in Indonesia. Although these pegs were often not openly admitted or were disguised as currency baskets, the common adherence to the dollar is easy to recognize (Table A.3). On the other hand, the selected developed countries adopted a flexible exchange rate regime. Table 3 also reports the pre-crisis exchange rate fluctuations of selected East Asian currencies and some major currencies against the US dollar. As shown in the first column of Table A.3, the volatilities of East Asian currencies were usually much lower than those of the major currencies.

Can the low exchange rate flexibility in East Asian Economies explain the twin dollarization phenomenon? How does exchange rate flexibility affect firms' debt-borrowing and price-setting behavior? In the next section, we construct a small open economy stochastic general equilibrium model with nominal rigidities to study the linkage between exchange rate regimes and twin dollarization.

1.3 Basic Model

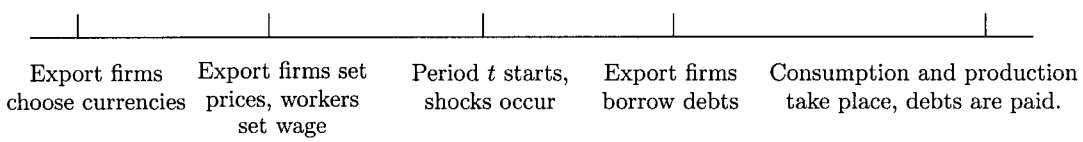
We consider a small open economy populated by two kinds of private agents: households and firms. The representative household owns firms and receives the average profit. There is a domestic currency we call peso and a foreign currency we call dollar. The household consumes both domestic non-traded goods and foreign goods. The domestic non-traded goods are homogenous and produced by a competitive non-traded goods sector.[10] The export firms

[10]This assumption implies that export goods are not sold to the domestic market, reflecting the fact that export firms are highly dependent on the world market. We can allow the domestic household to consume the domestically produced traded goods, and it just quantitatively alters the degree of dollarization given an exchange rate regime.

are monopolistic competitive, using local labor and imported intermediate goods to produce differentiated goods, which are sold to the rest of world. Each export firm sets its price to exploit its monopoly power, and chooses the currency in which to set export goods prices. Whatever currency is chosen, the firm must set its export price before the state of the world is realized. It is assumed that the export firm has to borrow externally to finance the purchases of imported intermediate goods. Finally, the currency of liability denomination must be chosen by the firm ex ante as well.[11]

The economy is subject to two kinds of external shocks: foreign demand shocks and world interest rate shocks. Monetary policy (or the exchange rate regime) is represented by a simple domestic interest rate targeting rule, which is used to determine how the absorption of external shocks is divided into changes of the domestic interest rate and the exchange rate. This rule reflects the central bank's preference.

Figure 1.1: Timing of Events



The timing of events is shown in Figure 1. Before the start of period t, export firms choose the currency in which to set their export good prices and the currency in which to contract their external debts. After that, they set prices to maximize expected discounted profits, based on the stochastic discount factors, and the anticipated market demand and marginal cost. Meanwhile, workers preset their wages.[12] After period t starts, external shocks occur, export firms will borrow to buy intermediate goods for production, households will supply labor, and choose their optimal consumption baskets and domestic bond holding, then consumption and

[11]The import of intermediate goods for firms' production reflects the dependence on foreign capital of this economy.

[12]There are two types of nominal rigidities in this model, the sticky price and the sticky wage. If prices are flexible, then the firm would be indifferent about the currency of export pricing, so the price stickiness is a key assumption. But the sticky wage assumption is just for the simplification of our analysis, and our results can still hold under a flexible wage.

production take place, and the exchange rate is determined. Finally, the debts are paid back to lenders at the end of period t.

The detailed structure of the economy is described below. Where appropriate, foreign currency (dollar) prices are indicated with an asterisk.

1.3.1 Households

The small open economy contains a unit interval [0,1] of households indexed by j. Each household supplies, in a monopolistic fashion, a distinctive variety of labor service. The expected utility of household j is given by

$$EU(j) = E_t \sum_{s=t}^{\infty} \beta^{s-t} [\frac{C_s^{1-\rho}(j)}{1-\rho} - \eta L_s(j)] \tag{1.3.1}$$

where C is a consumption index defined across domestic non-traded goods and foreign goods; E_t is the expectation operator conditional on information at time t; β is the discount factor; ρ is the inverse of the elasticity of intertemproal substitution; η is a scale parameter for the disutlity of labor supply. The consumption index C is defined as follows

$$C_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} C_{Nt}^{1-\alpha} C_{Ft}^{\alpha} \tag{1.3.2}$$

where C_N is the homogenous domestic non-traded goods produced by a competitive non-traded sector, C_F is the consumption of foreign goods, and α is the share of imported foreign goods in the total consumption expenditure of domestic households. The Cobb-Douglas form of equation (1.3.2) implies a unit elasticity of substitution between domestic goods and foreign goods in consumption.[13] The consumer price index for domestic households is derived as

$$P_t = P_{Nt}^{1-\alpha} P_{Ft}^{\alpha} \tag{1.3.3}$$

where P_N and P_F are the prices of domestic non-traded goods and imported foreign goods respectively.

Each household decides his consumption, domestic nominal bond purchase, and labor supply every period. The household derives income from wages, profits from export firms,[14] and

[13] This structure has been adopted by many papers in the open-economy literature. For instance, see Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2003), Corsetti and Pesenti (2001).

[14] As the non-traded sector is perfectly competitive, the profit from this sector is zero.

returns on domestic bond holding. Household j's budget constraint can therefore be written as:

$$P_tC_t(j) + B_{t+1}(j) = W_t(j)L_t(j) + (1+i_t)B_t(j) + \Pi_{Xt} \tag{1.3.4}$$

where $B_t(j)$ represents the household j's holding of domestic bonds, $W_t(j)$ is the nominal wage, and i_t is the domestic nominal interest rate between period $t-1$ and t. Π_X is the average profit from the export sector. It is assumed that households have no access to the international bond market, but export firms can finance their purchases of imported intermediate goods through the international capital market. This assumption implies that the standard uncovered interest rate parity condition(UIRP) does not hold in our model since there are no arbitrage opportunities for households.[15]

The aggregate labor service hired by an export or non-traded goods firm can be defined as

$$L_t = [\int_0^1 L_t(j)^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}} \tag{1.3.5}$$

Therefore, each household j faces a downward-sloping labor demand curve with elasticity $\theta > 1$,

$$L_t(j) = [\frac{W_t(j)}{W_t}]^{-\theta} L_t \tag{1.3.6}$$

where W is the production-based wage index and is given by $W_t = [\int_0^1 W_t(j)^{1-\theta}]^{\frac{1}{1-\theta}}$. We assume that the household sets nominal wages one period ahead and, *ex post*, supplies the amount of labor that firms demand at the predetermined nominal wage. Therefore, the optimal preset nominal wage is

$$W_t(j) = \frac{\theta}{\theta-1} \frac{E_{t-1}(\eta L_t(j))}{E_{t-1}(\frac{L_t(j)}{P_tC_t^{\rho}(j)})} \tag{1.3.7}$$

Without uncertainty, equation (1.3.7) would simply imply that the marginal utility of real wage equals a fixed markup over the marginal disutility of labor.

The household's optimal choice of consumption results in the standard Euler equation:

$$\frac{1}{1+i_{t+1}} = \beta E_t(\frac{P_tC_t^{\rho}(j)}{P_{t+1}C_{t+1}^{\rho}(j)}) \tag{1.3.8}$$

[15]This assumption captures the fact that households in emerging market economies can seldom smooth consumption by holding foreign assets. This imperfect financial market also implies that there might exist an interest rate differential between the domestic interest rate and the world interest rate even when exchange rates are fixed. For example, see Lahiri and Vegh (2003).

Finally, the demands for non-traded goods and foreign goods are given by:

$$C_{Nt}(j) = (1-\alpha)\frac{P_t C_t(j)}{P_{Nt}}, \qquad C_{Ft}(j) = \alpha\frac{P_t C_t(j)}{P_{Ft}} \tag{1.3.9}$$

In a symmetric equilibrium, $C_t(j) = C_t$, $W_t(j) = W_t$, and $L_t(j) = L_t$, $\forall j \in [0,1]$, so we can drop the subscript j.

1.3.2 Non-traded Goods and Imported Goods

We assume that the non-traded goods sector has a linear production technology:

$$Y_{Nt} = L_{Nt} \tag{1.3.10}$$

Since the sector is perfectly competitive, the price of non-traded goods equals the firm's marginal cost.

$$P_{Nt} = W_t \tag{1.3.11}$$

The foreign good has a constant price of one in terms of dollars in the world market. Thus, the price of imported foreign goods in terms of domestic currency (peso) is given by $P_{Ft} = S_t$, where S is the nominal exchange rate of dollar in terms of peso. Therefore, the domestic CPI price index is $P_t = W_t^{1-\alpha} S_t^{\alpha}$ and the degree of exchange rate pass-through into the domestic CPI equals α.

1.3.3 Export Firms

The export sector is monopolistic competitive and contains a unit interval [0,1] of firms indexed by i. Each firm i in this sector sells a differentiated good to the world market, and faces a downward-sloping demand function

$$Y_{xt}^d(P_{xt}^*(i)) = (\frac{P_{xt}^*(i)}{P_{xt}^*})^{-\lambda}\frac{X_t}{P_{x_t}^*} \tag{1.3.12}$$

where $P_{xt}^*(i)$ is the price that the foreign consumer pays for the export good i in period t, P_{xt}^* is the price index for all export goods sold in the world market. X_t is assumed to be a stochastic foreign demand shift term, following a log-normal distribution with mean $\bar{x}$ and variance σ_x^2 in period t. Without loss of generality, let $P_{xt}^*(i)$ and P_{xt}^* be denominated in dollars.[16] The

[16]Here, we just use $P_{xt}^*(i)$ to denote the price of export good i, whether the export goods prices is preset in pesos or dollars is endogenously determined and will be discussed in later section.

demand structure implies that foreign demand for the aggregate domestic export good is unit elastic, and each firm's own elasticity of demand is λ, where $\lambda > 1$.

In the export sector, the firm produces with both local labor and imported intermediate goods. The production technology for a typical export firm i is given by [17]

$$Y_{xt}(i) = AL_{xt}(i)^{1-\omega} I_t(i)^{\omega} \tag{1.3.13}$$

where $A = \frac{1}{\omega^{\omega}(1-\omega)^{1-\omega}}$ is a constant productivity term, and $I_t(i)$ represents the imported intermediate goods used in the production of good i. ω is the share of imported intermediate goods in the production of export goods. It also represents the capital dependence of this small open economy on the rest of the world. The aggregate output in the export sector is then defined as $Y_{xt} = [\int_0^1 Y_{xt}(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}}$.

Debt Contracts

We assume export firms must finance their purchases of imported intermediate goods through external borrowing. In the beginning of every period, the firm must borrow from the international capital market, represented here by a continuum of international lenders, and repay the principal and interest when the export goods are sold at the end of each period. Two types of debt contracts are provided by international lenders.[18] One is contracted in pesos, and the other is contracted in dollars. The export firm can borrow in either pesos or dollars, but the currency of debt contracting must be chosen ex ante. The existence of external shocks implies that the firm has to take some risk in financing, which is contingent on the debt contract. The amount of the loan (in terms of dollars) that firms need to borrow is equal to the import bill $q^* I_t$, where q^* and I_t are the dollar price and the quantity of imported intermediate goods respectively. Here, q^* is assumed to be exogenously given in the world market and to be constant over time.

[17] This production technology for emerging market economies has been used in previous literature. See, for example, Devereux and Poon (2004).

[18] The process of contract negotiation between international lenders and firms is not modelled, since our focus is to study how the firm chooses a debt contract, given the menu of contracts. We assume that the contracts are exogenously posted by the international lenders, and firms' choices in this small open economy have no effect on the design of contracts.

It is assumed that, if the firm borrows in pesos, the repayment is subject to the ex post gross domestic interest rate $1 + i_{t+1}$, where i_{t+1} is the domestic interest rate between period t and $t+1$.[19] In other worlds, one cannot write a peso contract in terms of a fixed domestic interest rate. This assumption captures the fact that borrowing through short-term local currency debt makes the firm vulnerable to shocks to the domestic interest rate at which the debt is rolled over.[20] With a peso contract, to import one unit of intermediate good, the firm will borrow $q^* S_{t-1}$ in pesos, where S_{t-1} is the exchange rate in period $t-1$. Therefore, in this case, the firm's total cost in terms of pesos to use one unit of imported intermediate good is given by

$$q_t^{peso} = q^*(1 + i_{t+1})S_{t-1} \tag{1.3.14}$$

If the firm borrows in dollars, the repayment is at the gross world interest rate $1 + r^*_{t+1}$,[21] where r^*_{t+1} is the world interest rate between period t and $t+1$, and it is assumed to follow a stochastic process given by

$$r^*_{t+1} = \bar{r^*} + \epsilon_t \tag{1.3.15}$$

where ϵ_t is an i.i.d. shock with mean zero and variance σ^2_ϵ in period t, and $\bar{r^*}$ is the steady state world interest rate. Therefore, with a dollar contract, the firm's total cost in terms of pesos to use one unit of imported intermediate good is given by

$$q_t^{dollar} = q^*(1 + r^*_{t+1})S_t \tag{1.3.16}$$

That is, when the export firm finances import purchases with dollar debt, it is vulnerable to both the exchange rate risk and the world interest rate shock.

[19] In general, we can assume that the gross return for a peso contract and a dollar contract are functions $h(i_{t+1}, r^*_{t+1}, S_t, S_{t-1})$ and $f(i_{t+1}, r^*_{t+1}, S_t, S_{t-1})$, respectively. As long as $h(.)$ is more sensitive to the domestic interest rate than $f(.)$, the results in our model will still hold.

[20] Mckinnon and Schnabl (2004) explain why firms in developing countries cannot borrow at a fixed interest rate. This is because these firms tend to be small, without well-developed accounting systems, and cannot issue bonds in their own name. Therefore, even firms with longer term projects must roll over short-term bank loans, or, at best, borrow at medium term with the variable interest rate tied to short rates.

[21] Since international lenders are risk-neutral, the expected return from each kind of contract should be equalized. Thus, $E_{t-1}[S_{t-1}(1+i_{t+1})\frac{1}{S_t}] = E_{t-1}[1+r^*_{t+1}] = 1+\bar{r^*}$. This condition can be used to determine the level of exchange rates or the dynamic path of exchange rates, and has no direct impact on the firms' decisions since the firms' dollarization decisions only depend on the second moments of aggregate variables.

Given the export firm's production technology, marginal cost is given by $MC_t = W_t^{1-\omega} q_t^{\omega}$, where q_t can be either q_t^{peso} or q_t^{dollar}. As q_t varies with the way the firm finances its debts, the marginal cost will also be affected. For simplicity, we normalize $q^* = 1$, so that the marginal cost under a peso contract and a dollar contract, respectively, can be derived as follows:

$$MC_t^{peso} = W_t^{1-\omega}[(1+i_{t+1})S_{t-1}]^{\omega}, \qquad MC_t^{dollar} = W_t^{1-\omega}[(1+r_{t+1}^*)S_t]^{\omega} \tag{1.3.17}$$

Since the nominal wage is predetermined, the change in the marginal cost depends completely on the change in the domestic interest rate, the exchange rate, or the world interest rate. As the firm has to choose the currency of debt denomination ex ante, each firm will choose the debt contract that delivers a higher expected profit. Nevertheless, the expected profit is also affected by the currency of export pricing. Therefore, when the export firm chooses the currency of debt contracting, the currency of export pricing should be considered as well.

The Currency of Export Pricing

Whatever currency it chooses, the firm must set its export price before the state of the world is realized. Given the debt contract that the firm chooses, that is, given the marginal cost, if the export firm sets its price in pesos, then the expected discounted profit in period $t-1$ is

$$E\Pi^{peso}(i) = E_{t-1}[d_t(P_{xt}^{peso}(i) - MC_t)(\frac{P_{xt}^{peso}(i)}{S_t P_{xt}^*})^{-\lambda}\frac{X_t}{P_{xt}^*}] \tag{1.3.18}$$

where d_t is the stochastic discount factor, which is equal to the marginal utility of consumption $\frac{1}{P_t C_t^p}$, as all export firms are assumed to be owned by domestic households. $P_{xt}^{peso}(i)$ is the price set by firm i in terms of pesos. P_{xt}^* is the export sector's price index, which is taken as given by all firms.[22] If the export firm sets its price in dollars, then the expected discounted profit in period $t-1$ is

$$E\Pi^{dollar}(i) = E_{t-1}[d_t(S_t P_{xt}^{*dollar}(i) - MC_t)(\frac{P_{xt}^{*dollar}(i)}{P_{xt}^*})^{-\lambda}\frac{X_t}{P_{xt}^*}] \tag{1.3.19}$$

where $P_{xt}^{*dollar}(i)$ is the price set by firm i in terms of dollars.

[22] Since not necessarily all firms will choose the same currency of export pricing, we will just use P_{xt}^* to denote the aggregate price index. How P_{xt}^* is determined will be given by equation (1.4.5) in Section 1.4.

The optimal price policies which maximize the expected profits in equations (1.3.18) and (1.3.19) can be derived as follows, respectively:

$$P_{xt}^{peso}(i) = \hat{\lambda}\frac{E_{t-1}(MC_t S_t^{\lambda} Z_t)}{E_{t-1}(S_t^{\lambda} Z_t)}, \qquad P_{xt}^{*dollar}(i) = \hat{\lambda}\frac{E_{t-1}(MC_t Z_t)}{E_{t-1}(S_t Z_t)} \tag{1.3.20}$$

where $\hat{\lambda} = \frac{\lambda}{\lambda-1}$ is the markup, and $Z_t = d_t P_{xt}^{*\lambda-1} X_t$ represents a market demand factor.

Using the above optimal price policies, we can compute the expected discounted profits as:

$$E\Pi^{peso} = \widetilde{\lambda}[E_{t-1}(S_t^{\lambda} Z_t)]^{\lambda}[E_{t-1}(S_t^{\lambda} Z_t MC_t)]^{1-\lambda} \tag{1.3.21}$$

$$E\Pi^{dollar} = \widetilde{\lambda}[E_{t-1}(S_t Z_t)]^{\lambda}[E_{t-1}(Z_t MC_t)]^{1-\lambda} \tag{1.3.22}$$

where $\widetilde{\lambda} = \frac{1}{\lambda-1}(\frac{\lambda}{\lambda-1})^{-\lambda}$. From now on, we will omit the time subscripts in the approximation of expected profit functions. Without explicit statement, all the expectation and variance operators are conditional on the information set in period $t-1$.

Strategies and Payoff

Since the firm has to choose both the currency of debt contracting and the currency of export pricing ex ante, the set of feasible pure strategies for each firm is given by

$$\Theta = [s_1, s_2, s_3, s_4] \equiv [(p,p), (p,d), (d,p), (d,d)] \tag{1.3.23}$$

For each strategy $s_i \in \Theta$, $i \in (1,2,3,4)$, the first letter represents the currency of export pricing, and the second represents the currency of debt contracting. The expected profit associated with the strategy s_i is defined as $E\Pi(s_i)$. Thus, the optimal strategy s_i for the firm can be determined by the following condition:

$$E\Pi(s_i) \geq E\Pi(s_{-i}), \qquad \forall s_i \in \Theta \tag{1.3.24}$$

To find the optimal strategy, following Devereux, Engel and Storgaard (2004), we simplify the expected profit as a function of aggregate variables by using a second order approximation approach. From now on, lower-case letters are the natural logs of their upper-case counterparts, that is, $x = \ln(X)$. Defining $\pi(s_i) = \ln E\Pi(s_i) - \Pi^*$ as the difference between the log of expected

profit and the common component in the log of expected profit for any strategy $s_i \in \Theta$,[23] we may rewrite the expected profit associated with each strategy as:[24]

$$\pi(p,p) = \frac{\lambda^2}{2}\sigma_s^2 + \frac{1-\lambda}{2}\sigma_{mcp}^2 + \lambda(1-\lambda)cov(mcp,s) + (1-\lambda)cov(mcp,z) \tag{1.3.25}$$

$$\pi(p,d) = \frac{\lambda^2}{2}\sigma_s^2 + \frac{1-\lambda}{2}\sigma_{mcd}^2 + \lambda(1-\lambda)cov(mcd,s) + (1-\lambda)cov(mcd,z) \tag{1.3.26}$$

$$\pi(d,p) = \frac{\lambda}{2}\sigma_s^2 + \frac{1-\lambda}{2}\sigma_{mcp}^2 + (1-\lambda)cov(mcp,z) \tag{1.3.27}$$

$$\pi(d,d) = \frac{\lambda}{2}\sigma_s^2 + \frac{1-\lambda}{2}\sigma_{mcd}^2 + (1-\lambda)cov(mcd,z) \tag{1.3.28}$$

The approximated expected profits are functions of conditional variance and covariance terms of the log exchange rate, marginal cost, and market demand. Since all terms are affected by monetary policy rules, we can express the above conditions as functions of the underlying monetary policy parameters. Nevertheless, before solving the model, we can derive the following Lemmas from equations (1.3.25)–(1.3.28).

Lemma 1 *Given a debt contract, the firm will set export price in pesos if and only if*

$$\frac{1}{2}\sigma_s^2 - cov(mc_k, s) > 0, \qquad k = p, d \tag{1.3.29}$$

The intuition behind this optimal condition for export pricing is as follows.[25] When the export firm sets export prices in domestic currency, exchange rate flexibility can stabilize the demand by adjusting its relative prices. Therefore, as shown by equation (1.3.18), the firm's expected profit is convex in the exchange rate. When the export firm sets prices in foreign currency, exchange rate changes will only have wealth effects on the firm's revenue, so the expected profit is linear in the exchange rate, as illustrated by equation (1.3.19). Hence, higher exchange rate volatility will encourage the firm to set prices in domestic currency. Meanwhile, when the firm sets export prices in domestic currency, its demand is directly sensitive to

[23] Here, Π^* is $\ln\sum + \frac{1}{2}\sigma_z^2 + \lambda cov(z,s)$, and $\ln\sum$ is the log of steady state profit.

[24] For the detailed derivation, see the Appendix.

[25] This optimal condition for export pricing was first derived by Devereux, Engle and Storgaard (2004).

exchange rate movements, which results in an extra covariance term between the marginal cost and the exchange rate. This will increase the firm's expected cost, ceteris paribus. Thus, these two effects have opposite impacts on the firm's choice of the export pricing currency, as shown by equation (1.3.29). Since, in our model, the covariance term between marginal cost and the exchange rate varies with the debt contract, the export pricing currency decisions also will depend on the choice of debt contract.

Lemma 2 *When the firm sets export prices in pesos, it will borrow in pesos if and only if*

$$\frac{1}{2}\sigma^2_{mcp} + \lambda cov(mcp, s) + cov(mcp, z) < \frac{1}{2}\sigma^2_{mcd} + \lambda cov(mcd, s) + cov(mcd, z) \tag{1.3.30}$$

When the firm sets export prices in dollars, it will borrow in pesos if and only if

$$\frac{1}{2}\sigma^2_{mcp} + cov(mcp, z) < \frac{1}{2}\sigma^2_{mcd} + cov(mcd, z) \tag{1.3.31}$$

Equations (1.3.30) and (1.3.31) show that it is optimal for the firm to choose the currency of the debt contract, which has a smaller marginal cost effect on the expected profit. When the firm sets export prices in pesos, the marginal cost effect of the debt contract, i.e., the terms associated with marginal cost in the expected profit function, is composed of three terms: the variance of marginal cost; the covariance of the marginal cost and the exchange rate; and the covariance of the marginal cost and the market demand. The first two terms are positive and reduce the expected profit as they both increase the expected cost, while the effect of the third term on the expected profit is ambiguous, as it varies with other firms' strategies. When the firm sets export prices in dollars, as the expected profit is linear in exchange rate, no covariance term exists between the marginal cost and the exchange rate. Therefore, the marginal cost effect will only involve the first and the third terms.

1.3.4 Monetary Policy

Following recent literature (Woodford 2003 and Clarida et al 2000) in abstracting from the details of the monetary mechanism, we simply assume that the monetary authority is committed to a domestic interest rate targeting rule

$$1 + i_{t+1} = (1 + \bar{r}^*)(\frac{S_t}{S_0})^\gamma, \qquad \gamma > 0 \tag{1.3.32}$$

where $\bar{r}^*$ is the steady state world interest rate. The parameter γ represents the degree of exchange rate flexibility or the coefficient of exchange rate intervention. As long as $\gamma > 0$, there is a determinate equilibrium value for the nominal exchange rate. γ is exogenously given and measures the preference of policy makers. The higher is γ, the closer the monetary rule approximates a pegged exchange rate regime, where the target for exchange rate peg is S_0. When γ approaches zero, it represents a flexible exchange rate regime.

1.3.5 Equilibrium

Given the stochastic processes of shocks (X, ϵ) and the exchange rate regime (γ), a symmetric equilibrium has the following properties: (a) Households preset wages and choose consumption and domestic bond holdings to maximize expected utility subject to their budget constraints; (b) The export firm chooses the optimal strategy s_i to maximize the expected profit; (c) Given the optimal currency strategies, firms set prices to maximize expected discounted profit; and (d) Labor, goods and bond markets clear. In equilibrium, we have

$$Y_{Nt} = C_{Nt} = (1-\alpha)\frac{P_t C_t}{P_{Nt}} \tag{1.3.33}$$

$$Y_{xt} = [\int_0^1 Y_{xt}(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}} = \frac{X_t}{P^*_{xt}} \tag{1.3.34}$$

$$L_{Nt} + \int_0^1 L_{xt}(i) di = L_t \tag{1.3.35}$$

$$B_t = 0 \tag{1.3.36}$$

The first two equations represent the goods market clearing conditions for non-traded goods and export goods, respectively. The third equation is the domestic labor market clearing condition, and the last one is the domestic bonds market clearing condition.[26]

[26] In a symmetric equilibrium, we will have $B_t(j) = 0, \forall j$.

1.4 Export Firms' Optimal Strategies

In this section, we analyze how monetary policy affects firms' decision. Since the firm's expected profit is a function of the second moments of endogenous aggregate variables (the exchange rate and other macroeconomics variables), to find the optimal strategies of firms, we have to solve the general equilibrium model to get the endogenous aggregate variables. Besides monetary policy, the determination of these variables also depends on the market structure - the distribution of firms who choose different optimal strategies. In other words, every export firms' payoff will be affected by other firms' optimal strategies. In equilibrium, the distribution of firms must be supported by each firm's optimal decision.

To solve for the equilibrium, we first assume that there exists a distribution of firms with different optimal strategies and derive the aggregate variables given the conjectured distribution. Then every firms' optimal strategy can be determined. Finally, to find out if the conjectured distribution is an equilibrium, we must check if these strategies support that distribution. We denote the distribution of firms as $\Omega = (\mu_1, \mu_2, \mu_2, \mu_4)$, where $\mu_i \in [0, 1]$ is the endogenously determined number of firms who choose the optimal strategy $s_i \in \Theta$, and $\Sigma_{i=1}^{4} \mu_i = 1$. For example, in an economy, if all firms set export prices and borrow in dollars, then the economy can be represented by $\Omega = (0, 0, 0, 1)$.

We solve the model by log-linearizing around a non-stochastic, symmetric steady state, which is described in the Appendix. The Appendix also gives the complete solution of the model, so we will just outline the important and intuitive steps of the solutions. Given the log-linearized system, the deviations of the exchange rate and other macroeconomic variables from their $t-1$ expectations are solved in terms of external shocks (X_t, ϵ_t). From now on, $\hat{k}_t = log(K_t) - log(\bar{K})$, where $\bar{K}$ is the non-stochastic steady state value of variables K_t, and $\tilde{k}_{t+j} = \hat{k}_{t+j} - E_{t-1}\tilde{k}_{t+j}$, $j \geq 0$.

1.4.1 Model Solution

In this subsection, we focus on deriving consumption, the exchange rate and other endogenous variables given the distribution of firms Ω.[27]

The output and net income of export sector Given the distribution of firms Ω, there are four types of firms in the export sector, and the demand for the goods produced by each type of firms is given by

$$Y_{xt}^{pp} = (\frac{P_{xt}^{pp}}{S_t P_{xt}^*})^{-\lambda} \frac{X_t}{P_{xt}^*} \tag{1.4.1}$$

$$Y_{xt}^{pd} = (\frac{P_{xt}^{pd}}{S_t P_{xt}^*})^{-\lambda} \frac{X_t}{P_{xt}^*} \tag{1.4.2}$$

$$Y_{xt}^{dp} = (\frac{P^{*dp}_{xt}}{P_{xt}^*})^{-\lambda} \frac{X_t}{P_{xt}^*} \tag{1.4.3}$$

$$Y_{xt}^{dd} = (\frac{P^{*dd}_{xt}}{P_{xt}^*})^{-\lambda} \frac{X_t}{P_{xt}^*} \tag{1.4.4}$$

where $P_{xt}^{s_i}$ and $Y_{xt}^{s_i}$ are the prices and demand (or output) for the firms who choose the optimal strategy s_i. The prices with an asterisk are in terms of dollars. The export sector's price index P_{xt}^* in terms of dollars is then given by

$$P_{xt}^* = [\mu_1(\frac{P_{xt}^{pp}}{S_t})^{1-\lambda} + \mu_2(\frac{P_{xt}^{pd}}{S_t})^{1-\lambda} + \mu_3(P^{*dp}_{xt})^{1-\lambda} + \mu_4(P^{*dd}_{xt})^{1-\lambda}]^{\frac{1}{1-\lambda}} \tag{1.4.5}$$

Log-linearizing the above equation, we have

$$\hat{p}_{xt}^* = \mu_1(\hat{p}_{xt}^{pp} - \hat{s}_t) + \mu_2(\hat{p}_{xt}^{pd} - \hat{s}_t) + \mu_3\hat{p}_{xt}^{*dp} + \mu_4\hat{p}_{xt}^{*dd} \tag{1.4.6}$$

As all prices $p_{xt}^{s_i}$ (or $p_{xt}^{*s_i}$) are preset in period $t-1$, $\hat{p}_{xt}^{s_i} - E_{t-1}\hat{p}_{xt}^{s_i} = 0$. Thus, $\tilde{p}_{xt}^* = \hat{p}_{xt}^* - E_{t-1}\hat{p}_{xt}^* = -(\mu_1 + \mu_2)\tilde{s}_t$. Using the market clearing condition $Y_{xt} = \frac{X_t}{P_{xt}^*}$, the deviation of (log) output of the export sector from its $t-1$ expectation is given by

$$\tilde{y}_{xt} = \tilde{x}_t - \tilde{p}_{xt}^* = \tilde{x}_t + (\mu_1 + \mu_2)\tilde{s}_t \tag{1.4.7}$$

[27] Note that Ω should be time variant, but we focus on a stationary equilibrium, so we drop the time subscript for Ω.

where $\mu_1 + \mu_2$ is the number of firms who set their export prices in pesos. Equation (1.4.7) shows that the more firms choose to set prices in pesos, the more stable export output will be. If all firms set export prices in dollars, that is, $\mu_1 + \mu_2 = 0$, then the exchange rate cannot adjust the terms of trade to stabilize output, so the export sector output will fully reflect the demand shock X_t.

In equilibrium, the household's expenditure is affected by the net income of the export sector, which is equal to wage income plus the profit from the export sector. For simplicity of presentation, we define the net income of export firm i as $G_{xt}(i)$, and it is given by

$$G_{xt}(i) = W_t L_{xt}(i) + \Pi_{xt}(i). \tag{1.4.8}$$

For firm i, $W_t L_{xt}(i) = (1-\omega) MC_t(i) Y_{xt}(i)$ and $\Pi_{xt}(i) = P_{xt}(i) Y_{xt}(i) - MC_t(i) Y_{xt}(i)$, thus we have

$$G_{xt}(i) = P_{xt}(i) Y_{xt}(i) - \omega MC_t(i) Y_{xt}(i) \tag{1.4.9}$$

Given the distribution of firms Ω, the total net income of the whole export sector can be rewritten as

$$G_{xt} = \sum_{i=1}^{4} \mu_i G_{xt}^{s_i} \tag{1.4.10}$$

where $G_{xt}^{s_i}$ is the net income of the firm who chooses the optimal strategy s_i. As shown in the Appendix, log-linearizing equation (1.4.10) yields the following relationship.

$$\tilde{g}_{xt} = b\tilde{s}_t + \tilde{x}_t - \xi \epsilon_t \tag{1.4.11}$$

where $b = ea - (a-1)[v + \gamma(1-v)]\omega + (1-e)$ and $\xi = (a-1)\omega v$. Note that $a = \frac{\lambda}{\lambda - (\lambda-1)\omega}$, the steady state ratio of output to net income in the export sector $(\frac{\bar{Y}_x}{\bar{G}_x})$. $e = \mu_3 + \mu_4$, is the number of firms who set export prices in dollars. $v = \mu_2 + \mu_4$, is the number of the firms who borrow in dollars. Therefore, (e, v) can be used to measure the degree of dollarization in the economy. For example, $(e = 1, v = 1)$ represents an arrangement of twin dollarization.

Intuitively, b measures the total effect of exchange rate changes on the net income of the export sector. There are three channels through which the exchange rate can affect G_{xt}. The first one is the wealth effect ea. For those firms who sell goods in dollars, an exchange rate depreciation will increase the nominal profit in terms of domestic currency. The second channel

is the cost effect, $(a-1)[v+\gamma(1-v)]\omega$. This is because an exchange rate depreciation causes an increase in the cost of intermediate goods import, which reduces net income. The third mechanism is the demand effect $(1-e)$.[28] For those firms who set price in peso, an exchange rate depreciation increases the demand for their goods and thus their net income. Therefore, adding these three effects together, we have the total effects of exchange rate changes on net income. Nevertheless, the sign of b is ambiguous and it depends on the exchange rate regime γ and the distribution of firms Ω in equilibrium.

From equation (1.4.11), we can also see that net income will be affected by the foreign demand shock X and the world interest rate shock ϵ directly. When more firms borrow in dollars, the world interest rate shock has a bigger effect on the net income of export sector. In particular, if all debts are borrowed in pesos, then the world interest rate has no impact on the economy.

The budget constraint In equilibrium, the household's budget constraint in period t is given by

$$P_tC_t = W_tL_t + \Pi_{xt} \tag{1.4.12}$$

The market clearing conditions for the non-traded goods and labor market implies $W_tL_{Nt} = (1-\alpha)P_tC_t$, and $L_t = L_{Nt} + L_{xt}$, so equation (1.4.12) can be rewritten as

$$\alpha P_tC_t = W_tL_{xt} + \Pi_{xt} = G_{xt} \tag{1.4.13}$$

It implies that the import expenditure of households is equal to the net income of the export sector. Log-linearizing and using the CPI price index $P_t = W_t^{1-\alpha}S_t^{\alpha}$ and equation (1.4.11), we have

$$\tilde{c}_t + (\alpha - b)\tilde{s}_t = \tilde{x}_t - \xi\epsilon_t \tag{1.4.14}$$

Equation (1.4.14) implies that external shocks to the economy will be absorbed by changes of current consumption and the current exchange rate. Since the household has no access to international bond market, the current account dynamics are completely shut down in this model. The term $\alpha - b$ represents the distribution of shock absorption between changes of current consumption and changes of exchange rate.

[28] We assume unitary income elasticity of foreign demand in the model, so we get $1-e$, otherwise, it should be multiplied by an elasticity term.

The Euler equation Combining the Euler equation (1.3.8) with the domestic interest rate targeting rule (1.3.32), we have

$$\frac{1}{(1+r^*)(\frac{S_t}{S_0})^\gamma} = \beta E_t[\frac{P_t C_t^\rho}{P_{t+1} C_{t+1}^\rho}] \tag{1.4.15}$$

Again, log-linearizing the above equation, and using the CPI price index and the preset wage equation (1.3.7), we have

$$\rho \tilde{c}_t + (\alpha+\gamma)\tilde{s}_t - E_t(\frac{\rho}{\alpha}\tilde{c}_{t+1} + \tilde{s}_{t+1}) = 0 \tag{1.4.16}$$

As shown by equation (1.4.14), the external shocks are fully absorbed by changes in current consumption and the current exchange rate. Therefore, with the assumption that the shocks X_t and ϵ_t are i.i.d, it is easy to show that $E_t(\frac{\rho}{\alpha}\tilde{c}_{t+1} + \tilde{s}_{t+1}) = 0$ (see the Appendix for details), which yields

$$\rho \tilde{c}_t + (\alpha+\gamma)\tilde{s}_t = 0 \tag{1.4.17}$$

This implies that the changes of the current exchange rate have two effects on the household's intertemporal consumption smoothing: depreciation of the exchange rate increases the imported goods price and thus the domestic CPI through the term $\alpha\tilde{s}_t$, which reduces current consumption; Meanwhile, it raises the domestic interest rate through the parameter γ, which also decreases current consumption. The magnitude of the total effect depends on the degree of openness α and exchange rate flexibility γ. Note that it is independent of the distribution of firms Ω.

The determination of exchange rate and consumption Putting equations (1.4.14) and (1.4.17) together, we may solve for $\tilde{c}_t$ and $\tilde{s}_t$,

$$\tilde{c}_t = \frac{\alpha+\gamma}{(\rho b+\gamma)-\alpha(\rho-1)}(\tilde{x}_t - \xi\epsilon_t), \qquad \tilde{s}_t = \frac{\rho}{\alpha(\rho-1)-(\rho b+\gamma)}(\tilde{x}_t - \xi\epsilon_t) \tag{1.4.18}$$

Using the domestic interest rate targeting rule (1.3.32), we can derive the deviation of the domestic interest rate from its $t-1$ expectation

$$1+\widetilde{i_{t+1}} = \frac{\rho\gamma}{\alpha(\rho-1)-(\rho b+\gamma)}(\tilde{x}_t - \xi\epsilon_t) \tag{1.4.19}$$

The above equations give the determination of consumption, exchange rate and domestic interest rate, which are all still conditional on Ω. Note that b and ξ are both functions of Ω.

Although the equilibrium values of b and ξ cannot be determined yet, from equations (1.4.18) and (1.4.19), we still can roughly find out how the aggregate variables respond to external shocks in two extreme cases. As $\gamma \to 0$, external shocks are mainly absorbed by the flexible exchange rate, thus consumption is less affected by external shocks. Meanwhile, the domestic interest rate is quite stable. As $\gamma \to \infty$, the exchange rate is almost fixed and cannot insulate consumption from external shocks. In this case, the domestic interest rate can absorb part of external shocks. But, without current account dynamics, households cannot spread the shocks over to future periods, therefore, current consumption becomes volatile.

Equations (1.4.18) and (1.4.19) show that consumption, the exchange rate and the domestic interest rate depend critically on exchange rate flexibility γ, but the effects of Ω (b, ξ) on these endogenous variables are relatively small, especially in the two extreme cases.

1.4.2 The Optimal Strategies

Given the above solution, we can calculate all conditional variance and covariance terms of the (log) exchange rate, consumption, marginal cost and other variables in period $t-1$ by using the property that $cov_{t-1}(x_t, y_t) = cov_{t-1}(\hat{x}_t, \hat{y}_t) = cov_{t-1}(\tilde{x}_t, \tilde{y}_t), \forall x_t, y_t$. Therefore, we can express firms' payoff functions as a simple function of second moments of external shocks $(\sigma_x^2, \sigma_\epsilon^2)$, and then find the optimal strategies. For simplicity, we drop the time subscript in the following analysis.

Using equations (1.3.17) and (1.3.32), we can express all conditional variance (covariance) terms associated with the (log) marginal cost (mcp, mcd) as follows,

$$\sigma_{mcp}^2 = \gamma^2\omega^2\sigma_s^2 \qquad cov(mcp, s) = \gamma\omega\sigma_s^2 \qquad cov(mcp, z) = \gamma\omega cov(s, z) \tag{1.4.20}$$

$$\sigma_{mcd}^2 = \omega^2(\sigma_s^2 + \sigma_\epsilon^2), \qquad cov(mcd, s) = \omega(\sigma_s^2 + \sigma_{s\epsilon}), \qquad cov(mcd, z) = \omega(cov(s, z) + cov(\epsilon, z)) \tag{1.4.21}$$

where exchange rate volatility σ_s^2 and the covariance between the exchange rate and the world interest rare shock $\sigma_{s\epsilon}$ can be derived as,

$$\sigma_s^2 = [\frac{\rho}{\alpha(\rho-1) - (\rho b + \gamma)}]^2(\sigma_x^2 + \xi^2\sigma_\epsilon^2) \qquad \sigma_{s\epsilon} = \frac{-\rho\xi}{\alpha(\rho-1) - (\rho b + \gamma)}\sigma_\epsilon^2 \tag{1.4.22}$$

Using the fact that $Z = \frac{1}{PC^{\rho}} P_x^{*\lambda-1} X$, we have $\tilde{z} = [\gamma - (\lambda - 1)(1 - e)]\tilde{s} + \tilde{x}$. Thus, the covariance terms $cov(s, z)$ and $cov(z, \epsilon)$ can be derived as

$$cov(s, z) = [\gamma - (\lambda - 1)(1 - e)]\sigma_s^2 + \frac{\rho}{\alpha(\rho - 1) - (\rho b + \gamma)}\sigma_x^2 \tag{1.4.23}$$

$$cov(z, \epsilon) = \frac{-\rho\xi[\gamma - (\lambda - 1)(1 - e)]}{\alpha(\rho - 1) - (\rho b + \gamma)}\sigma_\epsilon^2 \tag{1.4.24}$$

To further simplify the payoff functions, we may rewrite the second moments as

$$\sigma_{mcd}^2 = \omega^2 b_1 \sigma_s^2 \qquad cov(mcd, s) = \omega b_2 \sigma_s^2, \qquad cov(s, z) = \phi_1 \sigma_s^2, \qquad cov(s, \epsilon) = \phi_2 \sigma_s^2 \tag{1.4.25}$$

where $b_1 = 1 + \frac{\sigma_\epsilon^2}{\sigma_s^2}$, $b_2 = 1 + \frac{\sigma_{s\epsilon}}{\sigma_s^2}$, $\phi_1 = \frac{cov(s,z)}{\sigma_s^2}$ and $\phi_2 = \frac{cov(z,\epsilon)}{\sigma_s^2}$.

Substituting these variance and covariance terms into equations (1.3.25)–(1.3.28), we can rewrite the approximated payoff function for each strategy as a linear function of σ_s^2. Note that b_1, b_2, ϕ_1 and ϕ_2 are all functions of γ, b and ξ.

$$\pi(p, p) = \frac{1}{2}[\lambda^2 + \gamma^2\omega^2(1 - \lambda) + 2\gamma\omega(1 - \lambda)\lambda + 2(1 - \lambda)\gamma\omega\phi_1]\sigma_s^2 \tag{1.4.26}$$

$$\pi(p, d) = \frac{1}{2}[\lambda^2 + \omega^2 b_1(1 - \lambda) + 2\omega b_2(1 - \lambda)\lambda + 2(1 - \lambda)\omega(\phi_1 + \phi_2)]\sigma_s^2 \tag{1.4.27}$$

$$\pi(d, p) = \frac{1}{2}[\lambda + \gamma^2\omega^2(1 - \lambda) + 2(1 - \lambda)\gamma\omega\phi_1]\sigma_s^2 \tag{1.4.28}$$

$$\pi(d, d) = \frac{1}{2}[\lambda + \omega^2 b_1(1 - \lambda) + 2(1 - \lambda)\omega(\phi_1 + \phi_2)]\sigma_s^2 \tag{1.4.29}$$

From Lemmas 1 and 2 (equations 1.3.29-1.3.31), we may derive the following conditions.

$$\pi(p, p) > \pi(d, p), \qquad if \quad \gamma < \frac{1}{2\omega} \tag{1.4.30}$$

$$\pi(p, d) > \pi(d, d), \qquad if \quad b_2 < \frac{1}{2\omega} \tag{1.4.31}$$

$$\pi(p, p) > \pi(p, d), \qquad if \quad \omega(\gamma^2 - b_1) + 2\lambda(\gamma - b_2) + 2(\gamma\phi_1 - \phi_1 - \phi_2) < 0 \tag{1.4.32}$$

$$\pi(d,p) > \pi(d,d), \qquad if \quad \omega(\gamma^2 - b_1) + 2(\gamma\phi_1 - \phi_1 - \phi_2) < 0 \tag{1.4.33}$$

Therefore, we may establish the following proposition which describes the optimal strategies when γ approaches zero and γ approaches infinity. [29]

Proposition 1 *There always exist two critical values γ_L and γ_H ($\gamma_L < \gamma_H$), such that for any $\gamma \in (0, \gamma_L)$, the optimal strategy for each firm is (p,p), that is, all firms set export prices in pesos and borrow in pesos. The distribution of firms is $\Omega = [1,0,0,0]$. For any $\gamma \in (\gamma_H, \infty)$, the optimal strategy for each firm is (d,d), that is, all firms set export prices in dollars and borrow in dollars, and $\Omega = [0,0,0,1]$.*

The proof is given in the Appendix. Here, we focus on equations (1.3.29)-(1.3.31) to explain the intuition behind the proposition. First, we look at the case where exchange rate flexibility is low. From Lemma 1, we know that there are two effects which have opposite impacts on firms' choices of export pricing currency. When the export firm sets price in domestic currency (foreign currency), revenue is convex (linear) in the exchange rate. Hence, exchange rate volatility encourages the firm to set prices in its own currency (peso). Meanwhile, when the firm sets price in pesos, the positive covariance between marginal cost and the exchange rate will reduce firm's expected profit. As exchange rate flexibility decreases, regardless of which currency the firm chooses to contract its debts, the first effect will be dominated by the second, as γ gets bigger and σ_s^2 gets smaller.[30] As a result, the firm will always choose to set prices in dollars when the exchange rate is fixed.[31]

Why do firms borrow in dollars when exchange rate flexibility is low? The impact of exchange rate flexibility on the firm's choice of debt contract is mainly through the borrowing

[29] We assume $\omega < 0.5$ here since it is consistent with the calibration of ω for most emerging market economies.

[30] This is because $Cov(s, mcp) = \gamma\omega\sigma_s^2 > Cov(s, mcd) = \omega(\sigma_s^2 + \sigma_{s\epsilon}) > 0$, when $\gamma \to \infty$ and $\sigma_s^2 \to 0$.

[31] In our model, since we use a general interest rate rule, σ_s^2 is always positive, even in the fixed exchange rate regime where γ approaches infinity. To make our result hold when $\sigma_s^2 = 0$, we can change our model slightly according to Devereux, Shi and Xu (2004). Assume that the firm incurs a cost of adjusting prices ex-post and this cost arises only when the price facing consumers is adjusted. If the firm sets prices in its own currency, it faces a fixed nominal cost. Under such an environment, the optimal pricing condition can be rewritten as $\frac{1}{2}\sigma_s^2 - cov(md, s) > \delta_c$, where δ_c is a positive function of the menu cost. With this slight revision, our result holds even in the case $\sigma_s^2 = 0$.

cost channel. In our model, the borrowing cost of a peso contract is subject to ex post domestic interest rate risk, while the borrowing cost of a dollar contract depends on the world interest rate shock and exchange rate risk. As shown by equations (1.4.18) and (1.4.19), when exchange rate flexibility is low, the domestic interest rate will absorb the foreign demand shock and the world interest rate shock, and thus becomes quite volatile. Hence, borrowing in pesos implies a higher expected cost than borrowing in dollars, which is only subject to the world interest rate shock (exchange rate risk is almost zero). Therefore, firms tend to borrow in dollars as exchange rate flexibility decreases.

When exchange rate flexibility is high (γ is low), from equations (1.4.18)-(1.4.19), we know that the exchange rate absorb most foreign demand shocks and world interest rate shocks, so the domestic interest rate is stable and the exchange rate is volatile. Therefore, borrowing in pesos implies a stable borrowing cost for firms. As for the currency of price setting, with high exchange rate flexibility, the first term in equation (1.3.29) is always positive and the second term depends on the cost structure and the way in which firms borrow. If firms borrow in pesos, the covariance between marginal cost and the exchange rate is smaller than that under the dollar debt contract, which is subject to exchange rate risk. Hence, setting export prices in pesos gives firms a higher expected profit. Intuitively, this is because, under peso currency pricing, exchange rate changes can adjust the firm's relative prices, and then the firm can stabilize the demand for its export goods. Therefore, when exchange rate flexibility is high, firms will borrow in pesos and also set export prices in pesos.

Proposition 1 shows that exchange rate policy is the key factor affecting firms' dollarization decisions. From equations (1.4.30) - (1.4.33), however, we find that ω, the share of imported intermediates in the production of export goods, also can play a role in firms' dollarization decisions. If ω is small enough, then it requires a lower exchange rate flexibility (a bigger γ) to induce all firms to choose twin dollarization. If ω is big, however, then firms will choose to set prices in dollars and borrow in dollars even in the case where γ is small. In other words, the magnitude of ω will affect the impact of exchange rate flexibility on the degree of dollarization. The intuition is straightforward, a higher ω implies a higher share of borrowing cost in the total cost of production for export firms. As firms have to finance import purchases by external borrowing, when ω is bigger, the benefits of twin dollarization will be bigger. Thus, even a

small γ will cause export firms to choose twin dollarization. The dependence of export firms on the world market in the supply of inputs (measured by ω in this model) is also an important characteristic of East Asian economies that affects the degree of dollarization.

In the above discussion, we focus on the optimal strategies for firms in two extreme cases where $\gamma < \gamma_L$ and $\gamma > \gamma_H$. Now, we analyze the optimal strategies for firms in the intermediate case, where $\gamma_L < \gamma < \gamma_H$. As the optimal strategies for firms in intermediate cases cannot be solved analytically, we resort to a numerical solution.

1.4.3 Numerical Results

Calibration Our model has only a few parameters that need to be calibrated (Table A.4). The coefficient of risk aversion ρ, is set to 2 as is commonly assumed in the literature. The discount factor β is calibrated at 0.96, so that the steady state annual real interest rate is 4%. The elasticity of substitution across individual export goods λ is chosen to be 7, which implies a steady state markup of 16%. This is slightly higher than the common value of 10% (e.g., Basu and Fernald 1997) used in the literature for industrial economies. As pointed out by Cook and Devereux (2001), however, markups are usually higher in emerging markets.[32] Following Devereux and Poon (2004), we set the share of intermediate goods in production ω=0.4, which is consistent with the estimates for intermediate imports as a fraction of GDP in Braggion et al (2003).[33] α is set to equal 0.4, which implies that the share of non-traded goods in the consumer price index is set to 0.6. This is close to the evidence cited in Schmitt and Uribe (2000) for Mexico, and by Cook and Devereux (2001) for Malaysia and Thailand. With $\alpha = 0.4$ and $\omega = 0.4$, the total expenditure on imported goods (including the imported intermediate goods in the export sector) is about half of the GDP. Thus, the steady state debt to (GDP) ratio is about 17%.[34] The standard deviation of (log) foreign demand shock σ_x is set to 4%, so the standard deviation of log GDP is about 2%. Finally, the standard deviation

[32]Rotemberg and Woodford (1998) set $\lambda = 7.66$, Cook and Devereux (2001) choose $\lambda = 6$.

[33]In our model, ω also represents capital intensity in traded goods production.

[34]The value in Cook and Devereux (2004) is about 27.5%, but the debt in their model includes the debt of the non-traded goods sector.

of the world interest rate shock ϵ is set to 1%.[35]

Given the calibration of parameters, we can solve for the two critical values γ_L and γ_H. Table A.5 lists the changes of Ω, the distribution of firms when γ increases. Recall that μ_i is the number of firms who choose the strategy s_i. To highlight the extent of dollarization, we also report the degree of export pricing dollarization e and the degree of liability dollarization v separately in the last two rows of Table A.5. From Table A.5, we can see that, as exchange rate flexibility decreases (γ increases), both e and v increase, implying that the degree of dollarization increases with γ. The intuition is straightforward. As exchange rate flexibility decreases, the benefit of peso currency pricing and peso debt decreases, which induces more firms to set prices in dollars or to borrow in dollars. This finding also suggests that a policy that increases exchange rate flexibility may help to reduce the degree of dollarization in the economy, especially in some intermediate exchange rate regimes, as in these regimes the distribution of firms Ω is sensitive to exchange rate flexibility.[36]

We also find that multiple equilibria exist for some γ due to the strategic complementarity among firms. For instance, for $\gamma \in (13.368, 14.412)$, two equilibria are present.[37] The first is $\Omega = [0, 1, 0, 0]$, where all firms follow the strategy (p, d). The second is $\Omega = [0, 0, 0, 1]$, where all firms choose (d, d). The existence of multiple equilibria comes from the fact that one firm's dollarization decision will change the profit ranking of different strategies for all firms. Fox example, when $\gamma \in (13.368, 14.412)$, the difference between payoffs of strategies (p, d) and (d, d) is small, and it decreases with e, the degree of export pricing dollarization. When e increases, the sign of $\frac{1}{2}\sigma_s^2 - cov(mcd, s)$ will go from positive to negative. Therefore, a slight change of e will affect the relative ranking of $\pi(p, d)$ and $\pi(d, d)$. Intuitively, if one more firm set its price in dollars, it will increase the benefit of all firms who choose the dollar currency pricing, implying that the strategy (d, d) can deliver a higher payoff than strategy (p, d). As a result, all firms will follow the strategy (d, d). Obviously, in these intermediate exchange rate

[35] We set $\theta = 11$ and $\eta = 1$ in Table A.4, these two parameters have no impact on firms' dollarization decisions, but will be used for the welfare calculation.

[36] The strategy (d, p) is never an optimal strategy in the calibrated model.

[37] Another equilibrium is possible with $\Omega = [0, 1 - \mu_4(\gamma), 0, \mu_4(\gamma)]$, where $\mu_4(\gamma)$ is the number of firms choosing the strategy (d, d). In this equilibrium, the strategy (p, d) and (d, d) are indifferent for firms, and $\mu_4(\gamma)$ is a function of γ. Nevertheless, it cannot be a stable equilibrium due to the strategic complementarity among firms.

regimes, strategic complementarity has a big impact on the distribution of firms. Nevertheless, exchange rate flexibility is still the key factor for firms' dollarization decision.

1.4.4 Discussion

In the previous subsection, we have shown that, if exchange rate intervention (γ) is sufficiently large, then twin dollarization will be an optimal strategy for all firms. In other words, a fixed exchange rate will cause twin dollarization. This finding is consistent with the observation in East Asian emerging market economies, but is in contrast to the "fear of floating" literature, that argues that liability dollarization causes the fear of floating. In our model, it is the fixed exchange rate itself that leads to both liability and export pricing dollarization. In this sense, our finding builds a new linkage between fixed exchange rate regimes and dollarization.

Recently, fixed exchange rate regimes have been blamed for the over-borrowing and even financial turmoil in many East Asian economies. Fox example, Mckinnon and Pill (1998) argue that fixed exchange rates encourage excessively risky, unhedged external debts. Cook and Devereux (2001) explore the linkage between exchange rate regimes and capital inflow in a small open economy. They show that fixed exchange rates can lead to government subsidies to encourage international borrowing. Our result differ from their in that we focus on studying the impact of fixed exchange rate regimes on firms' currency choices of both debt contracting and export pricing.

Our research is also related to the literature on endogenizing liability denomination. That literature emphasizes that liability dollarization is caused by market or institutional failure in emerging market economies. For example, Jeanne (2000) argues that foreign currency debt arises because of commitment or signalling problems at the firm level. Jeanne (2003) also emphasizes the lack of monetary policy credibility. Caballero and Krishnamuthy (2003) attribute the presence of foreign currency debt to the lack of domestic financial development. By contrast, we show that dollarization can be an optimal arrangement for private agents, and is mainly affected by exchange rate policy. In addition, our utility based general equilibrium model setting has a natural advantage over the recent literature that relies on partial equilibrium or reduced form model, since it allows us to study the relevant welfare implication. Of course, the key contribution of our paper to the literature is that we investigate the common

cause of both types of dollarization, rather than liability dollarization alone.

Our findings imply that twin dollarization is optimal for firms under a fixed exchange rate regime, but is it a beneficial arrangement for the whole economy in terms of welfare? We will answer this question in the next section.

1.5 Welfare Implication

In this section, we discuss the welfare implications of firms' dollarization decisions for the economy. The welfare measurement we use here is the conditional expected lifetime utility of the representative household at time zero. Following Schmitt-Grohe and Uribe (2004), the expected lifetime utility is computed conditional on the initial state being the deterministic steady state, which is the same for all policy regimes and for the distribution of firms.[38] To measure the magnitude of welfare differential across regimes, we define ζ_k as the percentage change of deterministic steady state consumption that will give the same conditional expected utility EU under regime k, which is conditional on given γ and Ω. That is, ζ_k is given implicitly by:

$$\frac{\frac{1}{1-\rho}[(1+\zeta_k)\bar{C}]^{1-\rho} - \eta\bar{L}}{1-\beta} = EU_k \tag{1.5.1}$$

where a bar over a variable denotes the deterministic steady state of that variable. If $\zeta_k > 0 (< 0)$, the welfare under regime k is implied to be higher (lower) than that of the steady state case. Higher values of ζ_k correspond to higher welfare.

The welfare EU_k is computed by taking second order Taylor approximations of the structural equations around the deterministic steady state. The system of equations is solved using a perturbation method described in Schmitt-Grohe and Uribe (2004).[39] The values of structural parameters are those used in Section 4.

To find out whether or not twin dollarization is beneficial for the economy, we report the welfare effect of firms' endogenous dollarization decisions (Table A.6). We know from Proposition 1 that firms will choose to set prices in pesos and borrow in pesos under a flexible exchange

[38]This choice of initial state has the advantage of ensuring that the economy starts from the same initial point for all policy regimes considered.

[39]Matlab codes for welfare calculation are available to download at kang Shi' web site, http://grad.econ.ubc.ca/kangshi.

rate regime and choose twin dollarization under a fixed exchange rate regime. Therefore, we also focus on these two extreme exchange rate regimes, which are represented by $\gamma = 0.01$ and $\gamma = 900$, respectively.

From Table A.6, we can see that given any distribution of firms Ω, a flexible exchange rate regime is superior to a fixed exchange rate regime in terms of welfare. Therefore, if monetary policy is chosen ex ante endogenously to maximize the welfare of the economy, twin dollarization can never be an optimal strategy for firms in equilibrium. Under a fixed exchange rate regime, however, twin dollarization is not only optimal for firms, but also a beneficial outcome for the economy. We consider this economy in an initial equilibrium where the exchange rate is floating and all firms choose the strategy (p, p). In the case where the firm's pricing and borrowing behavior is given, if the government switches the exchange rate regime from flexible to fixed, then the welfare loss will be equivalent to about 0.214% steady state consumption. If, however, the firms' endogenous behavior is considered, then firms will choose to set export prices in dollars and borrow in dollars, and the welfare loss is about 0.091% steady state consumption. In other words, twin dollarization increases welfare, given that a fixed exchange rate is in place, and the gain in this case is about 0.123% steady state consumption.

This implies that endogenizing the currency choices of firms' export pricing and debt denomination will help to improve the welfare for the economy given an exchange rate regime. The welfare gain comes from the fact that firms can adjust their currency choices or dollarization decisions according to the changes in exchange rate regime. Table A.7 compares the mean and variance of consumption, labor and other variables for $\Omega = [1, 0, 0, 0]$ and $\Omega = [0, 0, 0, 1]$, under a fixed exchange rate regime. From Table A.7, we can see that twin dollarization delivers much higher expected consumption and lower consumption volatility than does the strategy (p, p) under a fixed exchange rate regime, which thus improves the economy's welfare. Intuitively, twin dollarization can help the economy to evade the volatile domestic interest rate. As a result, the mean and the volatility of marginal cost tend to be lower, which generates higher and more stable profit for export firms. Since export firms are assumed to be owned

by households, this implies higher expected consumption and low consumption volatility.[40] Therefore, twin dollarization is not only an optimal strategy for firms, but also delivers higher welfare for the whole economy.

Since ω is an important parameter for firms' dollarization decisions, we investigate the welfare consequences of changes in ω under a fixed exchange rate regime (Table A.8). We find that, ζ^{dd}, the welfare when all firms choose twin dollarization, does not greatly vary with ω. Meanwhile, ζ^{pp}, the welfare when all firms choose the strategy (p, p), sharply decreases with ω. Therefore, the welfare gain of twin dollarization will rise if ω increases. Since ω measures the share of imported intermediate goods in the production of export goods, the higher ω, the more debts firms need to borrow from abroad. Thus, twin dollarization will help to avoid welfare losses for economies that rely heavily on the international capital market.

Our results imply that twin dollarization can bring welfare gains to the economy in some environments. This is in contrast to the welfare implications of liability dollarization in most of the financial crisis literature, which typically emphasizes the welfare losses caused by foreign currency debt.

1.6 Conclusion

This paper studies twin dollarization - liability dollarization and export pricing dollarization in East Asian economies. We develop a small open economy general equilibrium model with nominal rigidities, where both the currency of export pricing and the currency of liability denomination are endogenous. Our main findings are that firms' dollarization decisions depend critically on exchange rate flexibility, and twin dollarization will be an optimal strategy for all firms if exchange rate flexibility is limited. Hence, our paper builds a new linkage between fixed exchange rate regimes and dollarization. Furthermore, we find that twin dollarization

[40] In the case with twin dollarization, the labor supply is also higher. Thus, higher consumption also can be attributed to the change of wage income, but, the increase of wage income cannot be the main source of consumption increase, since the magnitude of labor supply change is not big enough to account for the increase of consumption. Note that real wage is almost without any change in two cases. On the other hand, the higher labor supply leads to more disutility, which decreases welfare, but this effect is dominated by the increase of consumption.

can bring welfare gains to the economy under a fixed exchange rate regime. This contrasts with the welfare implications of liability dollarization in most of the financial crisis literature.

Our model can be extended in a number of ways. For example, it may be interesting to consider the optimal debt contract design in our model, so that we can explore the strategic interaction between firms' liability dollarization and export pricing dollarization. This interaction will be especially interesting if we allow the export output to be the collateral for external borrowing. And, once extended to incorporate the debt contract negotiation process, our model also can analyze the effect of default risk and country spread on firms' dollarization decisions and the welfare of the economy.

Chapter 2

Oil Currency and the Dollar Standard

2.1 Introduction

Ever since 1971, when the US broke the link between the dollar and gold that had been agreed to at the Bretton Wood Conference at the end of World War II, the dollar has been a global monetary instrument that the United States, and only the United States, can produce by fiat.[1] Although the United States has never imposed explicitly its dollar hegemony on other countries, the dollar has been used in the global economy as a reserve currency and a reference currency for international goods pricing and asset trading. McKinnon (2001, 2002) uses the term "dollar standard" to describe the dominance of the US dollar in the world economy.[2]

Recently, increased attention has been paid to the dominance of the US dollar and its impact on the global economy. For example, Eichengreen (2004) argues that the current international system is composed of a core and a periphery, where the US is the core and the rest of world is the periphery. As the center country, the US can continue running current account deficits because the periphery, Asian and Latin American emerging markets economies are happy to accumulate dollars and resist the appreciation of their currencies against the dollar. In a sense, world trade is a game in which the US produces dollars and the rest of the world produces goods that the dollar can buy. Different from the "core and periphery" literature, which focuses on the role of US dollar as an international reserve currency, Devereux, Shi, and Xu (2004) investigate one particular aspect of the multi-dimensional role of the dollar standard – a reference currency for international good pricing, which represents a situation where all firms set their export prices in US dollars.[3] They show that in such a setting, the US monetary

[1]This chapter is based on the joint work with Michael Devereux and Juanyi Xu.

[2]Also see McKinnon and Schnabl (2003).

[3]That is, for the US, the export prices are set in the producer's currency (PCP), while for the rest of the world, the export prices are set in the buyer's currency (LCP).

authority dominates in the equilibrium of international monetary policy game. Nevertheless, US residents are actually worse off than those in the rest of the world, due to the lack of an efficient expenditure-switching mechanism. In other words, it is costly for the US to have such a dollar standard.

The literature analyzes different aspects of the dollar standard and their implication for the world economy, but it leaves us with two intriguing questions: What causes a dollar standard? And what are the welfare consequences for the US and the rest of the world if the dollar standard is endogenously determined?

The emergence of an international currency may be attributed to a lot of factors, such as economic size, history, capital flows, and economic policies. Krugman (1984) notes that while economic size is likely to be an important factor, there is also a "snowballing" effect, whereby even if the countries are of similar size, if one currency becomes acceptable in exchange, then all countries will have an incentive to support this outcome. This suggests that the dollar standard may be caused by a historical accident as much as by current fundamentals. McKinnon (2002) takes a different view. He stresses the importance of the US monetary policy, arguing that the US dollar's role as an international currency is a result of low and stable US inflation rates in the post-WWII international system.

In this paper we want to argue that the dollar hegemony might be caused by a geopolitically constructed peculiarity that critical commodities, most notably oil, are denominated in dollars. Since oil has been the most important energy for production and life today, when the oil price is denominated in US dollars, it creates incentives for exporters in both the US and the rest of the world to set their goods prices in dollars, which in turn leads to the dominance of the dollar in the world goods market and financial market. Historical evidence is consistent with this explanation. For example, the history of oil being extensively used in both the United States and the world economy coincides with the growing path of the US dollar as an international currency.

To address our questions and explain our story, we will develop a two-country new open economy macroeconomic model, where the US dollar is assumed to be the oil currency. The key features in our model are: (a) the currency of export pricing is itself endogenous, so the emergence of a dollar standard will be the result of firms' optimal choices; (b) oil is introduced

as a required productive input and its world price is denominated in US dollars. We analyze two cases. In the benchmark case, both home and foreign countries are subject to monetary supply shocks and world oil price shocks. In such a setting, when the oil price is quoted in US dollars, the oil price shock will have an asymmetric impact on US and foreign firms, as US firms bear less exchange rate risk than foreign firms. This asymmetry will lead all firms in the world to set their export prices in US dollars, which implies a dollar standard in international goods pricing. In the extended case, besides the oil price shock, country-specific productivity shocks and active monetary policies are also introduced. We find that the optimal monetary policies can offset the asymmetry caused by the US dollar's role as an oil currency. Therefore, a dollar standard is not necessarily the equilibrium of firms' choices. Given different relative sizes of external shocks, the equilibrium can be quite different. If productivity shocks are relatively small, then a dollar standard can still be an equilibrium choice for firms. Moreover, contrary to Devereux, Shi, and Xu (2004), when there is a dollar standard in international goods pricing, we find that households in the US always are better off than those in the rest of the world. The net welfare difference between the US and the foreign country is composed of two components: the pass-through effect, which represents the cost of having a dollar standard; and the cost-advantage effect, which represents the benefit of having the US dollar as the oil currency. Our result suggests that the special role of the US dollar as an oil currency can help to build a dollar standard in international goods pricing.

This paper is closely related to the literature on endogenous currency of pricing. We follow the approach in Devereux, Engel, and Storgaard (2004). They endogenize the firms' currency of export pricing and show that the exporters wish to set their prices in the currency of the country with a relatively more stable monetary policy.[4] Our paper differs from theirs, as we focus on an asymmetric environment where one currency has a special role. To introduce the

[4] Corsetti and Pesenti (2002) analyze optimal monetary policies when the currency of pricing setting is itself endogenous. They show that there can be multiple equilibria. In one equilibrium, firms set prices using PCP, and an optimal monetary policy maintains a flexible exchange rate, while in another equilibrium, firms set prices using LCP, and the equilibrium is a fixed exchange rate.

effect of an oil price shock on the economy, we use the production approach,[5] That is, we assume that firms need oil to make their production plants work. This approach has been used recently by Leduc and Sill (2003). They develop a standard sticky-price, dynamic general equilibrium model, where monopolistically competitive firms use capital, labor, and oil to produce, to study the impact of oil price shocks on US output volatility.

This paper is organized as follows. Section 2 presents the benchmark model. Section 3 solves the model and shows that a dollar standard is the equilibrium of firms' choices. Section 4 extends the benchmark model to include the country-specific productivity shocks and active monetary polices. We find that there exist multiple equilibria of the game between firms and monetary authorities. Section 5 concludes.

2.2 Basic Model

The world economy consists of two countries, which will be referred to as the home country (the United States) and the foreign country. There is a continuum of home goods (and home population) and foreign goods (foreign population) of measures n and $1-n$ respectively. In each country, households maximize expected lifetime utility, taking prices and wages as given. Each firm is monopolistic competitive and use oil and labor to produce differential goods. In this section, we will focus on the baseline model where there only exist two shocks, the money supply shocks and the oil price shocks. Country-specific productivity shocks will be introduced later.

For simplicity, we abstract from any dynamics by considering a single period model with uncertainty.[6] The structure of events within the period is as follows; Before the period begins, households can trade in a full set of nominal state-contingent bonds. Then the firms choose

[5]The oil price shock also has a wealth effect on the economy. For example, Krugman (1985) develops a simple theoretical model to study the effect of an oil price increase on the US dollar. The model shows that the effect depends on a comparison between a direct balance of payment burden caused by oil price increase and a indirect balance of payments benefits of OPEC spending in investment.

[6]The assumption of a static model is certainly restrictive, however, it is a common assumption, either explicit (Obstfeld and Rogoff 2000, 2002), or implicit by shutting down the dynamics through the functional forms and asset structure (Corseetti and Pesenti 2001, Devereux and Engel 2003).

the currency in which they set their export goods prices, given the cross country risk-sharing rule, and taking into account the way in which they set prices, as well as the distribution of the stochastic monetary shocks and the oil price shocks. Following this, firms set prices in advance, contingent on the state-contingent discount factors, and the demand and marginal cost conditions that they anticipate to hold. After the realization of stochastic shocks, households work and choose their optimal consumption baskets, production and consumption take place, and the exchange rate is determined.

The detailed structure of the home country is described below. The foreign country has an identical structure. Where appropriate, foreign variables are indicated with asterisk.

2.2.1 Households

The representative household in home country maximizes the following expected utility: [7]

$$U = E(\frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{M}{P} - \eta L) \tag{2.2.1}$$

where $C = \frac{C_h^n C_f^{1-n}}{n^n (1-n)^{1-n}}$, $C_h = [\int_0^n n^{-\frac{1}{\lambda}} C_h(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}}$, and $\lambda > 1$. Here C is home aggregate consumption, composed of home goods and foreign goods with weights of n and $1-n$ respectively. Note that the elasticity of substitution between home and foreign goods is unit. C_h is the home sub-aggregate consumption of a continuum of home goods indexed by $[0, n]$, and λ is the elasticity of substitution between home individual goods. $\frac{M}{P}$ is the real money balance, and L is the costly labor effort. We assume that ρ, the inverse of inter-temporal elasticity parameter of substitution is greater than 1. η and χ are positive constant scale parameters. E is the expectation operator defined across all possible states of nature. The specification of (2.2.1) allows us to derive a closed form solution of the model. From the consumption structure, we may derive the consumption-based price index,

$$P = P_h^n P_f^{1-n} \tag{2.2.2}$$

[7] To give a closed form solution, our modelling approach is based on special assumptions, about both functional form and the stochastic environment, which are also used in Obstfeld and Rogoff (2000), Devereux and Engel (2003), and Devereux, Shi, and Xu (2004).

where P_h and P_f represent the prices for home goods and foreign goods in the home country, respectively. The consumption structure of foreign country is analogous, but with prices and quantities denoted by asterisks.

Home and foreign households can trade a full set of nominal state-contingent bonds, thus the budget constraint of the home household for a particular state of the world z can be written as:

$$P(z)C(z) + M(z) + \sum_{z \in Z} \xi(z)B(z) = W(z)L(z) + \Pi(z) + B(z) + M_0 + T(z) + \Pi^{oil}(z) \quad (2.2.3)$$

That is, the household obtains the wage income, with W being the nominal wage, gets the payoff of contingent securities (B) and receives the profit from household's ownership of home goods firms (Π) and the revenue from the share of world oil endowment Π^{oil}, the initial money balance M_0, and lump-sum transfers from the government (T). The household chooses how many state-contingent securities to purchase before the period begin, with ξ_z and B_z representing the price and holding, respectively, of a security paying off 1 unit of home currency in state $z \in Z$, where Z is the set of states. Then the household also chooses the holding of money, consumption and labor supply. We assume that government repays any seignorage revenue through a lump sum transfer, so that $M_0 - M(z) + T(z) = 0$. We will focus on the stochastic money supply in this section, active money supply rule will be discussed later.

The trade in state-contingent nominal assets across countries will lead to the following risk-sharing condition:

$$\frac{C^{-\rho}}{P} = \Gamma \frac{C^{*-\rho}}{SP^*}, \quad (2.2.4)$$

where S is the nominal exchange rate, and $P^* = P_h^{*n} P_f^{*1-n}$ is the foreign price level. Equation (2.2.4) implies that one dollar can get the same marginal utility of consumption across countries, or the ratio of marginal utilities of consumption is equal to the real exchange rate. Γ is the state-invariant weight,[8] from the appendix for Devereux and Engel (2003), we may show

$$\Gamma = \frac{EC^{(1-\rho)}}{EC^{*(1-\rho)}}. \quad (2.2.5)$$

[8] Γ represents the ratio of the Lagrange multiplier on the home household budget constraint to the Lagrange multiplier on the foreign households budget constraint.

In addition, the home household's optimization gives rise to the money demand function:

$$M = \chi P C^{\rho}, \tag{2.2.6}$$

and the implicit labor supply schedule:

$$W = \eta P C^{\rho}. \tag{2.2.7}$$

Therefore, the nominal wage is proportional to the money in circulation. Combining the money market equilibrium for the home and foreign countries with cross country risk-sharing condition (2.2.4), we can derive the exchange rate as:

$$S = \Gamma \frac{M}{M^*}. \tag{2.2.8}$$

2.2.2 Oil Market

We treat oil as a direct input in the production process suggested by Mork and Hall (1980) and Leduce and Sill (2003). It is assumed that the oil endowment is owned by a third party such as OPEC. Both home and foreign firms have to import oil from the OPEC and it is assumed that the oil price is determined by an exogenous shock and is quoted in the US dollar.[9] The supply of oil will be adjusted to equal to the total demand by both home and foreign firms, and the excess stock of oil endowment will be stored for future use. We also assume households in both home and foreign countries get some lump sum transfer of the revenue of oil endowment Π^{oil} from OPEC, which represents the net effect of oil wealth emphasized by Krugman (1985). [10] The oil price Q is quoted in home currency (the U.S. dollar) and follows a log normal distribution.

$$\ln Q = q, \qquad q \sim N(0, \sigma_q). \tag{2.2.9}$$

For the foreign firms, they face an oil price $\frac{Q}{S}$, in terms of foreign currency. Therefore, the oil price shock will have an asymmetric effect on home and foreign firms.

[9] For simplicity, we do not model the supply side of world oil market and the pricing behavior of OPEC explicitly in this paper. There is strong evidence that the oil price can reasonably be argued as exogenous during the period 1948-1972. See Hamilton 1983,1985.

[10] Krugman (1985) showed that the distribution of oil wealth and the expenditure of oil revenue may have an impact on the exchange rate and welfare. Nevertheless, in our model, the complete asset market assumption completely eliminates this effect.

2.2.3 Production

Each firm i in our economy has the following production technology.

$$Y(i) = AL(i)^{1-\alpha}O(i)^{\alpha} \tag{2.2.10}$$

where O represents the amount of oil used in the production, and $A = \frac{1}{\alpha^{\alpha}}\frac{1}{(1-\alpha)^{1-\alpha}}$ is a constant parameter. Since we assume the Dixit-Stiglitz consumption structure, each firm faces a downward-sloping individual demand curve and chooses its optimal price along the demand curve. The cost minimization problems for home and foreign firms give us marginal costs for home and foreign firms in term of their own currency, respectively.

$$MC = W^{1-\alpha}Q^{\alpha} \tag{2.2.11}$$

$$MC^{*} = W^{*1-\alpha}(\frac{Q}{S})^{\alpha} \tag{2.2.12}$$

Equations (2.2.11) and (2.2.12) imply that the oil price shock has a direct impact on the firm's production cost. Moreover,foreign firms may have a disadvantage in the cost of production, as they have to bear extra exchange rate risk when using oil, while the US firms do not need to bear this extra exchange rate risk.[11] We can show that this asymmetry in cost structure will affect the endogenous currency of pricing decision for both home and foreign firms later.

2.2.4 Monetary Shocks

In the baseline model, neither home or foreign monetary authorities are active. The money supply in each country just follows a log normal stochastic process.

$$M = \exp(u) \tag{2.2.13}$$

$$M^{*} = \exp(u^{*}) \tag{2.2.14}$$

[11] We assume that firms are not able to hedge against exchange rate risk in foreign exchange market. In our model, the main mechanism that generates a dollar standard comes from the firms' concern about extra exchange rate risk of using oil. In reality, some firms can hedge against the exchange rate risk, but this usually requires an extra management costs. In this sense, our assumption that firms cannot hedge exchange rate risk is not far away from the reality.

where the terms u and u^* represent uncontrollable disturbances to money supply. We assume that $u \sim N(0, \sigma_u^2)$, and $u^* \sim N(0, \sigma_{u^*}^2)$, and $\sigma_u^2 = \sigma_{u^*}^2$. Note that money supply shock and oil price shock are independent, i.e.,[12] Note that $cov(u, q) = cov(u^*, q) = 0$.

2.2.5 Endogenous Currency of Pricing

When a firm sells abroad, it can set its export prices in its own currency (PCP) or in buyer's currency (LCP). Whatever currency it chooses, it must set the price before the state of the world is known.[13] So the optimal currency of pricing decision depends on the difference between the expected profit under PCP and the expected profit under LCP. Following the approach used in Devereux, Engel, and Storgaard (2004), we can derive home and foreign firms' optimal conditions for currency of pricing decision.

For each firm i in the home country, who sells a differentiated good to the foreign market, the firm faces a CES demand curve,

$$X(P_{hf}^*(i)) = (\frac{P_{hf}^*(i)}{P_{hf}^*})^{-\lambda} \frac{P^*}{P_{hf}^*} C^* \tag{2.2.15}$$

where $P_{hf}^*(i)$ is the price the foreign consumer pays for home-produced goods i. P_{hf}^* is the price index for all home goods purchased by the foreign consumer, and P^* is the foreign country consumer price index. Without loss of generality, let $P_{hf}^*(i)$, P_{hf}^* and P^* be denominated in foreign currency.

If the home firm i sets its price in its own currency (the currency of producer, PCP), then the expected discounted profits is

$$E\Pi^{Pcp} = E[d(SP_{hf}^{Pcp}(i) - MC)(\frac{P_{hf}^{Pcp}(i)}{SP_{hf}^*})^{-\lambda} \frac{P^*}{P_{hf}^*} C^*] \tag{2.2.16}$$

[12]The mean of log money supply has no impact on the result of our model, for simplicity we assume a simple money rule in which the mean of log money supply equals zero.

[13]If the firms can freely reset their price when shocks are realized, firms would have no special preference on the currency in which they set their export goods prices, thus the currency of pricing decision for firms would not matter in the economy, and there is no chance for one currency to become a reference currency for international pricing. Also, if prices are flexible, the currency of pricing oil would not be a concern for firms.

where $d = \frac{1}{PC^p}$ is the marginal utility of home households, which is used as the stochastic discount factor for home firms. MC is the marginal cost of home firms.

If the home firm i sets its price in the foreign currency (the currency of buyer, LCP), then the expected discounted profit is

$$E\Pi^{Lcp} = E[d(P^{*Lcp}_{hf}(i) - MC)(\frac{P^{*Lcp}_{hf}(i)}{P^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^*] \tag{2.2.17}$$

The home firm will set its price in its own currency if the expected profit differential Ω is positive. Thus, it follows PCP whenever

$$\Omega = E\Pi^{Pcp} - E\Pi^{Lcp} > 0 \tag{2.2.18}$$

In Devereux, Engel, and Storgaard (2004), it is shown that the above optimal condition can be be approximated as below:

$$\frac{1}{2}\sigma_s^2 - cov(\ln(MC), s) > 0 \tag{2.2.19}$$

The equivalent condition for the foreign firm to follow PCP is given by

$$\frac{1}{2}\sigma_s^2 + cov(\ln(MC^*), s) > 0 \tag{2.2.20}$$

That is, if a firm chooses its export price optimally, then up to a second order approximation, its decision depends only on the variance of the exchange rate and the covariance of exchange rate with marginal cost, and is independent of the variance of market demand, the financial market structure and the prices of all other firms. In our model, due to the complete asset market assumption, exchange rate movement is fully determined by money supply shocks and will not be affected by the degree of exchange rate pass-through. Thus, all firms in one country will choose the same pricing strategy. In Devereux, Engel, and Storgaard (2004), if both countries have identical monetary stability, firms are indifferent between PCP and LCP. But this result will not hold in our model, as the oil price shock will generate an asymmetric effect on firms' marginal cost, which we discuss in detailed in the following section.

2.2.6 Equilibrium

Given the stochastic processes $\{u, u^*, q\}$, a symmetric equilibrium is a collection of allocations $\{C, C^*, L, L^*, M, M^*, B(z), B^*(z)$[14]. $\}$, and price system $\{P, P^*, W, W^*, P_h, P_f, P_h^*, P_f^*, S, \xi(z), \Gamma\}$ such that

- Given the price system, $\{C, C^*, L, L^*, M, M^*, B(z), B^*(z)\}$ solve the consumer's optimality problem, subject to his budget constraint.
- $\{P_h, P_f, P_h^*, P_f^*\}$ solve the individual firm's optimal pricing problem.
- The strategy of currency of pricing for each firm is optimal.
- The labor, goods, and money markets clear.

2.3 Model Solution

To derive the solution to the baseline model, firstly, we have to find the optimal currency of pricing decision for the firms. Then use the optimal pricing policies of firms to solve the endogenous variables contingent on the realizations of monetary shocks. Finally, we calculate the expected welfare for the home and foreign consumers.

2.3.1 The Currency of Pricing

From Appendix B, we can derive the expected profit differential for both home and foreign firms as follows:

$$\Omega = \alpha\sigma_u^2 > 0, \qquad \Omega^* = -\alpha\sigma_u^2 < 0 \tag{2.3.1}$$

This implies that all firms in the home country will choose PCP and all firms in the foreign country will choose LCP, and the home currency (US dollar) would be the reference currency for international goods pricing. Thus, we can state the following proposition.

[14] $B(z)$ and $B^*(z)$ are the holding of state-contingent nominal bond for home and foreign households in the state of z, and $\xi(z)$ is the price of bond in the state z

Proposition 2 *In the world where $\sigma_u^2 = \sigma_{u^*}^2$, all home and foreign firms will choose oil currency as the reference currency for international goods pricing. This represents a dollar standard in the world economy.*

Proposition 2 shows that there is an asymmetric equilibrium in our model. According to Devereux, Engel,and Storgarrd (2004), if a firm chooses its export price optimally, then its decision depends only on the trade-off between the variance of exchange rate and the covariance of the exchange rate with the marginal cost. If countries are identical in all aspects, then firms in home and foreign countries follow the same pricing policy. There exist three symmetric equilibria, and firms are indifferent between PCP and LCP. That is, the difference of expected profit between choosing PCP and choosing LCP is zero. However, in this paper, when the oil price is denominated in US dollars, for home firms, only the labor cost will be affected by changes of exchange rates. For foreign firms, both the labor cost and the oil cost would be affected by exchange rate changes. As (B.1.5) and (B.1.6) show, in absolute value, the covariance between home firms' marginal cost and exchange rates is less than the covariance between foreign firms' marginal cost and exchange rates. Thus, neither home or foreign firms will be indifferent to their currency of export pricing. The home firms will prefer PCP to LCP, while foreign firms would prefer to choosing LCP. This implies that all firms will wish to set their export prices in the US dollars. Since the economy is symmetric in all aspects expect that the oil price is denominated in US dollars, we may ascribe the emergence of US dollar as the reference currency for international goods pricing to the special role of US dollar as an oil currency.

For simplicity, we impose the condition that $\sigma_u^2 = \sigma_{u^*}^2$ in Proposition 2. However, we can show that all firms will want to set their export prices in the US dollar as long as $\frac{\sigma_u^2}{\sigma_{u^*}^2} < \frac{1}{1-2\alpha}$. This implies, a dollar standard can even exist in the case where monetary policy in the foreign country is slightly more stable than that in the US. The more the economies rely on oil, the more chance a dollar standard will exist.

2.3.2 Optimal Pricing Schedule

When the dollar is used as the reference currency for international goods pricing, we can derive the optimal pricing policy for home and foreign firms for goods sold in home and foreign

markets, respectively.

$$P_{hh} = \hat{\lambda}\frac{E[MCC^{1-\rho}]}{E[C^{1-\rho}]} \tag{2.3.2}$$

$$P_{hf} = \hat{\lambda}\frac{E[MCC^{*1-\rho}]}{E[C^{*1-\rho}]} \tag{2.3.3}$$

$$P^*_{ff} = \hat{\lambda}\frac{E[MC^*C^{*1-\rho}]}{E[C^{*1-\rho}]} \tag{2.3.4}$$

$$P_{fh} = \hat{\lambda}\frac{E[MC^*SC^{1-\rho}]}{E[C^{1-\rho}]} \tag{2.3.5}$$

where $\hat{\lambda}$ represents the markup, subscript hf represents the price of home goods sold in the foreign market. An asterisk over the price means the price is denominated in foreign currency. Given these prices, we can derive the price index for each country as follows:

$$P = P_{hh}^n P_{fh}^{1-n} \tag{2.3.6}$$

$$P^* = [\frac{P_{hf}}{S}]^n {P^*_{ff}}^{1-n} \tag{2.3.7}$$

This imply that the home CPI is completely predetermined and independent of external shocks, but there is positive exchange rate pass-through into the foreign CPI. This is an important channel through which a dollar standard may affect the global economy.

2.3.3 A Closed-form Solution

As the model is log-linear and the underlying monetary and oil price shock are log-normal, we may solve for the exact distribution of all endogenous variables in a closed-form. The solution allows a dichotomy between variables that are determined in advance of the realization of shocks (i.e. $P_{hh}, P_{hf}, P_{fh}, P^*_{ff}$), and Γ, and variables determined after the shocks have occurred (i.e. C, C^*, W, W^*, and S).

Equation (2.2.8) shows that exchange rate movements are fully determined by the realization of money supply shocks. Combining it with the market clearing conditions, we can get home and foreign consumption.

$$C = [\frac{1}{\chi}\frac{M}{P_{hh}^n P_{fh}^{1-n}}]^{\frac{1}{\rho}} \tag{2.3.8}$$

$$C^* = [\frac{1}{\chi}\frac{M^n M^{*(1-n)}}{P_{hf}^n P_{ff}^{*\,1-n}}]^{\frac{1}{\rho}}. \tag{2.3.9}$$

This implies that home country consumption is independent of the realization of the foreign country money supply. This follows directly from the fact that the home country CPI is predetermined, given that both home goods and imported foreign goods in the home market have prices preset in home currency. But with full exchange rate pass-through into imported goods prices, foreign country consumption is affected by the home country monetary shocks.

2.3.4 Welfare Comparison

We now compare the welfare of a representative household in the home country with that in the foreign country and check if the home household would gain when a dollar standard exists. It is assumed that the welfare of the household can be measured as [15]

$$E(\frac{C^{1-\rho}}{1-\rho} - \eta L) \tag{2.3.10}$$

As shown in Devereux and Engel (2003), the expected utility of the household in a stochastic environment is a function of variances and covariance terms of log consumption and log exchange rate. Thus, given the solution to the consumption and the exchange rate, we may rewrite the welfare in terms of the variance of monetary and oil shocks. From properties of the pricing setting equations in home and foreign countries, and the labor market clearing condition in the home country, in Appendix B, we can establish that

$$E(L) = (1-\alpha)\{\frac{n}{\hat{\lambda}\eta}E(C^{1-\rho}) + \frac{1-n}{\hat{\lambda}\eta}E(C^{*1-\rho})\Gamma\} \tag{2.3.11}$$

Combining (2.3.10) and (2.3.11), we may rewrite the expected utility of home household as

$$E(U) = \frac{\lambda - n(1-\alpha)(\lambda-1)(1-\rho)}{(1-\rho)\lambda}E(C^{1-\rho}) - \frac{(1-n)(1-\alpha)(\lambda-1)}{\lambda}\Gamma E(C^{*1-\rho}) \tag{2.3.12}$$

Since the log-normal distribution satisfies $EC^{1-\rho} = \exp\left\{(1-\rho)[E(c) + \frac{1-\rho}{2}\sigma_c^2]\right\}$, (2.3.12) ultimately depends only on the second moments of consumption and the exchange rate. To

[15] Obstfeld and Rogoff (1998, 2002) argue that the utility of real balance is small enough to be neglected.

derive these second moments, we rewrite the equations which give the closed-form solution to consumption and the exchange rate in log terms as:

$$s - E(s) = m - m^* \tag{2.3.13}$$

$$c - E(c) = \frac{1}{\rho}m \tag{2.3.14}$$

$$c^* - E(c^*) = \frac{1}{\rho}[nm + (1-n)m^*] \tag{2.3.15}$$

where small-case letters denote logarithms. The expected utility of foreign household can be derived in the same way.

To explain the model well, we focus on a special case where $\rho = 1$, then give the result for general case where $\rho > 1$. When $\rho = 1$, the expected labor supplies for both countries are a function of constant parameters,[16] $EL = EL^* = \frac{1-\alpha}{\eta\hat{\lambda}}$. Thus, the expected utility for the home country only depends on the mean of log consumption, the variance of log consumption has no impact on the expected utility of households.

$$EU = Ec - \frac{1-\alpha}{\hat{\lambda}} \tag{2.3.16}$$

From the money demand function $c = \ln\chi + m - p$ and the assumption $Em = 0$, we may find that, when $\rho = 1$, the expected utility for home country is fully determined by the expected log price level.

$$EU = -\ln\chi - \frac{1-\alpha}{\hat{\lambda}} - Ep. \tag{2.3.17}$$

The expected utility for foreign country can be derived in the same way, $EU^* = -\ln\chi - \frac{1-\alpha}{\hat{\lambda}} - Ep^*$. From Appendix B, we have

$$Ep = \ln\hat{\lambda}\eta^{1-\alpha} + \frac{(1-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 \tag{2.3.18}$$

$$Ep^* = \ln\hat{\lambda}\eta^{1-\alpha} + \frac{(1-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 + (1-n)\alpha\sigma_u^2 \tag{2.3.19}$$

[16] In the special case where $\rho = 1$, $\Gamma = 1$.

It is straightforward to show that $Ep < Ep^*$, and

$$EU - EU^* = (1-n)\alpha\sigma_u^2 > 0 \tag{2.3.20}$$

Equation (2.3.20) implies that, the home household has higher expected utility than the foreign household in the situation of having a dollar standard. This welfare difference between the US and the foreign country is obviously attributed to the special role of US as an oil currency, as the model is symmetric in all aspects except that the oil price is quoted in US dollars. This welfare gain increases with the share of oil cost in total production cost (α). Because these results can hold in the more general case when $\rho > 1$, we conclude in the following proposition.

Proposition 3 *A households in the home country has higher welfare than that in the foreign country in the situation of having a dollar standard in international goods pricing, i.e.* $EU > EU^*$.

Proof: See Appendix B.

When $\rho > 1$, the expected home utility will be a function of mean of log consumption and variance of log consumption. We can find that the welfare gain for US households decreases with risk-aversion coefficient ρ.

2.4 The Case with Active Monetary Policies

In this section, besides the oil price shock and the money supply shock, we introduce country-specific productivity shocks for each country. We also assume monetary authorities are active and can optimally respond to these shocks. Thus, we can analyze how the optimal monetary policies affect our finding in this section. Now, there exists a sequential game between firms and monetary authorities. The firms choose the currency in which they set their export prices before monetary authorities announce their monetary rules,[17] Then, given the currency of pricing, each monetary authority chooses its optimal monetary rules in an international non-coordinated game. Note that both firms and monetary authorities make decision before the state of the world is realized.

[17] If we change the timing of the action of firms and monetary authorities and let monetary authorities move first, then monetary authorities can use policy to indirectly choose the currency of pricing of firms, see Devereux, Shi, and Xu (2005).

In the economy with country-specific productivity shocks, the production technology for the representative firm in the home country is given by.

$$Y(i) = A(\theta L(i))^{1-\alpha} O(i)^{\alpha} \tag{2.4.1}$$

where θ is a country-specific shock in the home country, following a log normal distribution.

$$\theta = exp(z), \qquad z \sim N(0, \sigma_z^2) \tag{2.4.2}$$

The country-specific shock θ^* in the foreign country is analogously distributed, and $cov(z, z^*) = 0$ and $\sigma_z^2 = \sigma_{z^*}^2$. We assume that both monetary authorities can respond optimally to the oil price shock q, country-specific productivity shocks z and z^*. Also they commit to their money rules when shocks are realized as follows.

$$m = a_0 q + a_1 z + a_2 z^* + u \tag{2.4.3}$$

$$m^* = b_0 q + b_1 z + b_2 z^* + u^* \tag{2.4.4}$$

where {a, b} are policy parameters and will be discussed later. The term u and u^* are the disturbances to money supply, representing the financial innovation or implementation error for policy makers. Thus, the stability of monetary policy in each country not only depends on the response to real shocks, but is also affected by the shocks from nominal sectors.

The equilibrium in this extended case is a bit different from that in the baseline model. We have to take account of the interaction between monetary authorities and firms:

(a) Given firms' currency of pricing, monetary authorities in home and foreign countries choose the policy parameters {a, b} by solving the international monetary Nash game.

$$\max_a EU(a, b^n) \tag{2.4.5}$$

$$\max_b EU^*(a^n, b) \tag{2.4.6}$$

(b) The optimal policy parameters {a, b} from Equations (2.4.5) and (2.4.6) must support both home and foreign firms's pricing strategies.

To find the equilibrium, we express the expected profit differential of firms' currency of pricing decision as functions of policy parameters {a, b}. From Appendix B, we can establish

$$\begin{aligned}\Omega(a,b) &= \frac{1}{2}(a_0-b_0)[a_0-b_0-2((1-\alpha)a_0+\alpha)]\sigma_q^2+\frac{1}{2}(a_1-b_1)[a_1-b_1-2(1-\alpha)(a_1-1)]\sigma_z^2 \\ &+ \frac{1}{2}(a_2-b_2)[a_2-b_2-2(1-\alpha)a_2)]\sigma_{z^*}^2+\alpha\sigma_u^2 \end{aligned} \tag{2.4.7}$$

$$\begin{aligned}\Omega^*(a,b) &= \frac{1}{2}(a_0-b_0)[a_0-b_0+2(b_0-\alpha a_0+\alpha)]\sigma_q^2+\frac{1}{2}(a_1-b_1)[a_1-b_1+2(b_1-\alpha a_1)]\sigma_z^2 \\ &+ \frac{1}{2}(a_2-b_2)[a_2-b_2+2(b_2-\alpha a_2-(1-\alpha))]\sigma_{z^*}^2-\alpha\sigma_u^2 \end{aligned} \tag{2.4.8}$$

Thus, there exist four feasible pricing specification for home and foreign firms. They are {PCP, PCP}, {LCP, LCP}, {PCP,LCP}, and {LCP, PCP}. For simplicity, we focus on the case where $\rho = 1$, but the results also hold in the more general case where $\rho > 1$.[18] Table B.1 gives the optimal policy parameters for optimal monetary rules in these cases.

{PCP, PCP} If all firms in both home and foreign counties follow PCP, the optimal monetary policy requires that the home (foreign) authority fully respond to its own productivity shock but ignore the foreign (home) productivity shock. Since the oil price shock is a common shock to both countries, the optimal monetary policy requires both home and foreign monetary authorities to respond in the same way. To fully eliminate the effect of the oil price shock on firms' marginal cost, for one percent increase in the oil price, the nominal wage should decrease by $\frac{\alpha}{1-\alpha}$, so the monetary authorities should reduce the money supply by $\frac{\alpha}{1-\alpha}$ percent.

Now, we may check whether PCP is indeed an optimal pricing strategy for firms in home and foreign countries, if the monetary policy parameters are chosen as described above. From equations (2.4.7) and (2.4.8), we may establish:

$$\Omega(a,b) = \sigma_z^2 + \alpha\sigma_u^2 > 0 \tag{2.4.9}$$

$$\Omega^*(a,b) = (1-2\alpha)\sigma_z^2 - \alpha\sigma_u^2 \tag{2.4.10}$$

[18]The value of risk aversion parameter ρ only affects the welfare level and has no impact on the currency of pricing decisions for firms and the equilibrium of the game. Especially, in the symmetric pricing specification, the optimal monetary rules are independent of the parameter ρ.

It is straightforward to show that all home firms will wish to follow PCP, but the optimal pricing strategy for foreign firms depends on the relative size of productivity shocks to monetary supply shocks. So the case where all firms choose PCP would be an equilibrium if the following condition holds

$$\frac{\sigma_z^2}{\sigma_u^2} > \frac{\alpha}{1-2\alpha} \tag{2.4.11}$$

If there is no extra exchange rate risk ($\alpha = 0$) of using oil, all foreign firms will follow PCP. In the case with extra cost of using oil, when there is the productivity shock is relative large, the net benefit of choosing PCP can outweigh the extra exchange rate risk, PCP will still be an optimal strategy for foreign firms.

{LCP, LCP} If all firms in both home and foreign countries follow LCP, the optimal monetary policy requires the home (foreign) authority to adjust their money supplies to home and foreign shocks according to their weights in world output. This eliminates the effect of productivity shocks on the exchange rate. The policy response to the oil price shock is the same as that in the symmetric PCP case. Thus, the variance of optimal exchange rate in this case is completely affected by the disturbance to money supply. Again, from Equations (2.4.7) and (2.4.8), we establish:

$$\Omega(a,b) = \alpha\sigma_u^2 > 0 \tag{2.4.12}$$

$$\Omega^*(a,b) = -\alpha\sigma_u^2 < 0 \tag{2.4.13}$$

This implies LCP is not the optimal pricing strategy for home firms. So the case where all firms choose LCP is not an equilibrium when monetary authorities choose policies after the currency of pricing decision has been determined.

{PCP, LCP} If all home firms follow PCP and all foreign firms follow LCP, in this asymmetric case, the home authority responds to home and foreign productivity shock according to their weight in world output, but the foreign authority reacts less to the home productivity shock and more to its own productivity shock.[19] If the share of oil cost in production cost α

[19]The optimal monetary parameters listed in Table B.1 is for this case where $\rho = 1$, the optimal parameters for the more general case where $\rho > 1$ depends on ρ, but they give the same intuitions.

converges to zero, the foreign country would completely responds to its own shock. This is the exact case that Devereux, Shi, and Xu (2004) show. Now we check if the pricing strategy for home and foreign firms is optimal.

$$\Omega(a,b) = n(2-n)(1-\alpha)^2\sigma_z^2 + \alpha\sigma_u^2 > 0 \tag{2.4.14}$$

$$\Omega^*(a,b) = n^2(1-\alpha)^2\sigma_z^2 - \alpha\sigma_u^2 \tag{2.4.15}$$

We find that all home firms will wish to follow PCP, but the optimal pricing strategy for foreign firms also depends on the relative size of productivity shocks to monetary supply shocks. So the case where all home firms follow PCP and all foreign firms follow LCP would be an equilibrium if the following condition holds

$$\frac{\sigma_z^2}{\sigma_u^2} < \frac{\alpha}{n^2(1-\alpha)^2} \tag{2.4.16}$$

That is, when the relative size of productivity shock is less than some critical value, the extra exchange rate risk would be the main concern for foreign firms when they choose the currency of pricing. Now we conclude the three cases and state the following proposition.[20]

Proposition 4 *If $\frac{\sigma_z^2}{\sigma_u^2} \geq \frac{\alpha}{n^2(1-\alpha)^2}$, there is a unique equilibrium for the game between firms and monetary authorities that all the firms follow PCP; If $\frac{\alpha}{1-2\alpha} < \frac{\sigma_z^2}{\sigma_u^2} < \frac{\alpha}{n^2(1-\alpha)^2}$, there will be two equilibria: one is the case that all firms follow PCP, the other is the case that all home firms follow PCP and all foreign firms follow LCP. The latter case represents a dollar standard. If $\frac{\sigma_z^2}{\sigma_u^2} < \frac{\alpha}{1-2\alpha}$, there exists a unique equilibrium that all home firms follow PCP and all foreign firms follow LCP, that also represents a dollar standard.*

Proposition 4 implies that the equilibrium can be drastically different depending on the relative size of productivity shocks. If productivity shocks are relatively small, then a dollar standard can be an equilibrium. Intuitively, when there are big technology shocks, the variance of the optimal exchange rate increases. So this encourages both US firms and foreign firms to follow PCP, thus giving a flexible exchange rate regime. Given the size of shocks, an increase in α will increase the possibility of a dollar standard in international goods pricing. An increase in

[20] For any reasonable values of parameters, we always have $\frac{\alpha}{1-2\alpha} < \frac{\alpha}{n^2(1-\alpha)^2}$.

n is going to reduce the distance between two threshold values of $\frac{\sigma_z^2}{\sigma_u^2}$, which in turn lowers the possibility of multiple equilibria.

To analyze the welfare gain of the home household in the situation of having a dollar standard, we can derive the expected utility of the home household and the foreign household under a dollar standard, respectively. The detailed derivation is given in Appendix B.

$$EU = -\ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + n(1-n)(1-\alpha)^2\sigma_z^2 + \frac{(1-\alpha)^2}{2}\sigma_u^2 \tag{2.4.17}$$

$$EU^* = -\ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + n(1-n)^2(1-\alpha)^2\sigma_z^2 + \frac{n(1-\alpha)^2}{2}\sigma_u^2 + \frac{(1-n)(1+\alpha^2)}{2}\sigma_u^2 \tag{2.4.18}$$

This gives the difference between home and foreign households's expected utility:

$$EU - EU^* = (1-n)\alpha\sigma_u^2 - n^2(1-n)(1-\alpha)^2\sigma_z^2 \tag{2.4.19}$$

The welfare of households in both countries are only a function of the variance of productivity shocks and monetary shocks, as the optimal monetary policy fully eliminates the effect of oil price shock (common shock) on the welfare. The welfare differential between the US and the foreign country consists of two components: the first term is cost-advantage effect, which represents the benefit of having the US dollar as an oil currency; the second one is the pass-through effect, which represents the cost of having a dollar standard, as suggested by Devereux, Shi, and Xu (2004). From equation (2.4.19), we can state the following proposition.

Proposition 5 *In an environment with active monetary policies, the households in the U.S. are better off than those in the rest of the world when there is a dollar standard.*

The condition of have a dollar standard as a possible equilibrium is given by $\frac{\sigma_z^2}{\sigma_u^2} < \frac{\alpha}{n^2(1-\alpha)^2}$, which will ensure that $EU > EU^*$. That is, as long as there is a dollar standard, the cost-advantage effect always dominates the pass-through effect. Thus, there must exist a welfare gain for the US households under the dollar standard. Our result suggests that the US economy does gain from the role of the US dollar as an oil currency.[21]

[21] In the case where all firms choose PCP, US households are also better off than those in the rest of the world.

2.5 Conclusion

The paper analyzes the impact of the US dollar as an oil currency and its implication for the world dollar standard in international goods pricing in a two-country general equilibrium model with sticky prices. Our model gives a different insight about the dollar standard from the "core and periphery" literature. We relate the US dollar's role as an oil currency to the dollar standard in international goods pricing. We find that, when the currency of export pricing is itself endogenous and oil is required for production, all firms have an incentive to set their export prices in US dollars in the presence of oil price shocks, as the oil price is denominated in US dollars. But when there are active monetary policies, a dollar standard is not necessarily the equilibrium of firms' choices, since the optimal monetary policy can affect firms' currency choices of export pricing. Given different relative sizes of external shocks, the equilibrium can be quite different. If productivity shocks are relatively small, then a dollar standard is still an equilibrium. We also find that the households in the US always are better off than those in the rest of the world in the situation of having a dollar standard in international goods pricing.

For future research, we will endogenize the currency of oil pricing. Also, the complete asset market assumption can be relaxed and the effect of oil price shocks on the world wealth distribution can be explored.

Chapter 3

Flexible Exchange Rates and Endogenous Currency of Pricing

3.1 Introduction

The debate on fixed versus flexible exchange rates has been at the heart of international monetary economics for many years.[1] Friedman (1953) and later Mundell (1961) made the case for flexible exchange rates as efficient in responding to country specific shocks when domestic price levels cannot change quickly enough. Since country specific shocks in general require changes in relative prices (the terms of trade), it is more efficient to allow these changes to take place quickly via nominal exchange rate adjustment than a slower and potentially more costly process of nominal price adjustment.

Recent studies of monetary policy in utility-based open economy models have reached varying conclusions about the desirability of flexible exchange rates. Obstfeld and Rogoff (2000, 2002) develop models where an optimal monetary policy focuses mainly on domestic economic conditions, and an adjustable exchange rate is a key feature of the optimal response to shocks. On the other hand, Corsetti and Pesenti (2001, 2002), and Devereux and Engel (2003) reach quite different conclusions. In their analysis, monetary policy should respond to both domestic and foreign shocks, and in some cases, an optimal monetary policy should keep the exchange rate fixed, even in the presence of country specific productivity shocks.

The key difference between these two sets of results lies in the assumptions about price setting and the response of prices to the exchange rate. Obstfeld and Rogoff assume that export goods prices are set in the currency of the producer (PCP, or producer's currency pricing), so that an unanticipated exchange rate depreciation leads to a rise in the price of imported goods

[1]This chapter is based on the joint work with Michael Devereux and Juanyi Xu.

facing final consumers, causing a tilting of world demand towards the depreciating country's goods. This 'expenditure switching' is a key part of the optimal response to country specific shocks. Corsetti and Pesenti (2001, 2002) and Devereux and Engel (2003) on the other hand, point out that in many low inflation countries, nominal prices of final goods seem to be quite unresponsive to changes in the exchange rate. In light of this, they assume that export goods prices are set in the currency of the final consumer (LCP, or local currency pricing). When pricing is done in this way, there is no short run impact of a currency depreciation on the price of imported goods facing consumers. The expenditure switching mechanism cannot work. As a result, an optimal monetary policy response, even to country specific shocks, is consistent with a fixed exchange rate.

What determines the currency in which exporting firms will set their prices? In Corsetti and Pesenti (2002) and Devereux, Engel, and Stoorgard (2004), this decision is made endogenous. Firms are allowed to choose whether they set prices in domestic currency or foreign currency when selling abroad. Corsetti and Pesenti (2002) show that this can give rise to multiple equilibria - in one equilibrium, firms will set prices in producer's currency, and an optimal monetary policy maintains a flexible exchange rate, while in another equilibrium, firms set prices in local currency, and the equilibrium is a fixed exchange rate. Devereux Engel, and Storgaard (2004) examine the nature of the determination of the currency of pricing, and derive a simple second-order condition which governs this choice. They show that the stability of monetary policy is a critical determinant of the currency of pricing.

This paper re-examines the issue of the degree of exchange rate flexibility in an optimal monetary policy game and explores the methods to sustain flexible exchange rates, when the currency of export pricing is endogenous. The key innovation in the first part of this paper is to allow monetary authorities to *take into account* the way in which firms make their decisions about the currency in which they set prices. In order to sustain this equilibrium, monetary authorities must be able to commit to follow rules which are consistent with the currency of pricing decisions that firms follow.

The paper highlights one very sharp result. When monetary authorities take into account the currency of pricing decision in their choice of monetary policy rule, there is a unique equi-

librium in which all firms set prices in producer's currency.[2] In welfare terms, the equilibrium with PCP is better. Since it ensures that relative prices change efficiently, it sustains the outcome of the flexible price economy. In this equilibrium the exchange rate is flexible, and in essence supports the Friedman argument for flexible exchange rates.

Nevertheless, this may not be a realistic description of monetary policy, since product market structure is normally thought to be outside the purview of monetary policy. An alternative is to restrict the form of monetary rules itself. In the second part of this paper, we show that, if monetary authorities are precluded from reacting to foreign productivity shocks, then their optimal policies will rule out the equilibrium with LCP, and the unique equilibrium in this case is the one with PCP and flexible exchange rates. In a sense, restricting the form of the monetary policy rule acts as an equilibrium selection mechanism. The key insight is that there is a social benefit of exchange rate volatility that is not necessarily internalized in the policy makers' decision, when the currency of pricing is taken as given. By restricting monetary policy rules to focus only on domestic economic conditions, the policy makers are forced to follow rules that lead to volatile exchange rates. Given this exchange rate volatility, price setting rules adjust to ensure efficient exchange rate pass-through.

This organization of this paper is as follows. Section 2 lays out the model. Section 3 derives the equilibrium when monetary authorities can take into account the way in which the firms set export prices. Section 4 discusses the equilibrium when the money policy rule is restricted. Section 5 concludes.

3.2 Basic Model

The world economy consists of two countries, labelled as home and foreign. Each country is populated by a large number of atomistic households, a continuum of firms that choose the currency of pricing and set prices in advance, and a monetary authority which chooses optimal money rules to maximize the representative household's expected utility. Both countries

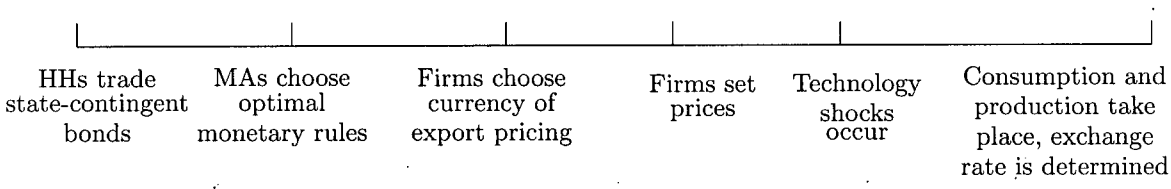
[2]When monetary policy is chosen conditional on the currency of pricing, then there are two Pareto ranked equilibria. Due to the problem of coordinating expectations of price setters and the actions of monetary authorities, the economy can be stuck in an equilibrium with low pass-through, fixed exchange rates, and social welfare lower than the flexible price equilibrium.

specialize in the production of a composite traded good.

3.2.1 The Timing of Events

All events take place within a single period. For simplicity, we abstract from any dynamics.[3] The structure of events within the period is described in Figure 3.1. At the beginning of the period, households can trade in a full set of nominal state-contingent bonds. Following this, the monetary authorities choose optimal monetary rules, given the cross country risk sharing condition, but taking into account the currency of pricing decisions of firms, the way in which firms set optimal prices, and the distribution of stochastic technology shocks. Firms then choose the currency in which they set their goods prices, and having chosen this, set prices in advance, based on their stochastic discount factors, and anticipated demand and marginal cost conditions. After the realization of technology shocks, households work and choose their optimal consumption baskets, production and consumption takes place, and the exchange rate is determined.

Figure 3.1: Timing of Events



The detailed structure of home country is described below. The foreign country has an identical structure. Where appropriate, foreign real variables and foreign currency prices are indicated with an asterisk.

3.2.2 Household

The representative household in the home country maximizes the following expected utility:

$$U = E(\frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{M}{P} - \eta L) \tag{3.2.1}$$

[3]Because we are considering a complete market environment, and there are no predetermined state variables, an infinite horizon model would have the same results as those below.

where

$$C = \frac{C_h^n C_f^{1-n}}{n^n(1-n)^{1-n}} \qquad C_h = [n^{-\frac{1}{\lambda}} \int_0^n C_h(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}} \tag{3.2.2}$$

Here C is aggregate consumption, C_h is the consumption sub-aggregate over a continuum of home goods indexed by $[0, n]$, $\frac{M}{P}$ is real money balances, and L is labor supply. We assume that the inverse of the intertemporal elasticity of substitution in consumption $\rho \geq 1$ and the elasticity of substitution between individual goods $\lambda > 1$. The foreign consumption sub-aggregate is analogously defined, but over a range of goods indexed by $[n, 1]$. η and χ are positive constant parameters. From the consumption structure, we may derive the CPI price index

$$P = P_h^n P_f^{1-n} \tag{3.2.3}$$

where P_h and P_f represent the price for home goods and foreign goods in home market, respectively.

Home and foreign households trade a full set of nominal state-contingent assets. Thus, the budget constraint of the representative home household for a particular state of the world z is written as:

$$P(z)C(z) + M(z) + \sum_{t \in Z} \xi(t)B(t) = W(z)L(z) + \Pi(z) + B(z) + M_0 + T(z) \tag{3.2.4}$$

That is, home households' income is derived from wage income (WL), payoff of state-contingent securities (B), the profits from home monopolistic firms which are assumed to be owned by home households (Π), the initial money balance M_0, and lump-sum transfers from the government (T). Before the state of the world is realized, households purchase state-contingent securities. Note that ξ_t and B_t represent the price and quantity, respectively, of a state-contingent bond paying off 1 unit of home currency in state $t \in Z$, where Z is the set of all states. After the state is realized, households choose their holding of money, consumption and labor supply. We assume that government repays any seignorage revenue through a lump-sum transfer, so that $M_0 - M(z) + T(z) = 0$. The specific money supply rules will be defined later.

In the appendix of Devereux and Engel (2003), it is shown that the trade in the state-contingent nominal assets across countries leads to the following optimal risk sharing arrangement:

$$\frac{C^{-\rho}}{P} = \Gamma \frac{C^{*-\rho}}{SP^*}, \tag{3.2.5}$$

where S is the nominal exchange rate, and $P^* = P_h^{*n} P_f^{*1-n}$ is the foreign price level. Equation (3.2.5) implies one dollar can get the same marginal utility of consumption across countries. The risk-sharing parameter Γ is state-invariant, but its level is determined by the outcome of state-contingent securities trade. As shown by the appendix of Devereux and Engel (2003), the risk-sharing parameter satisfies

$$\Gamma = \frac{EC^{(1-\rho)}}{EC^{*(1-\rho)}}. \tag{3.2.6}$$

In addition, the household's optimization problem gives rise to the money demand and labor supply conditions:

$$M = \chi PC^{\rho}, \tag{3.2.7}$$

$$W = \eta PC^{\rho}. \tag{3.2.8}$$

This means that the nominal wage is proportional to the money holding,

$$W = \frac{\eta}{\chi} M \tag{3.2.9}$$

Putting Equation (3.2.5), the home household's optimality condition and their foreign analogy together implies that the nominal exchange rate is determined by

$$S = \Gamma \frac{M}{M^*} \tag{3.2.10}$$

3.2.3 Production

It is assumed that firms are monopolistically competitive and each of them produces a differentiated good i by using a linear technology:

$$Y(i) = \theta L(i) \tag{3.2.11}$$

where θ is a home country-specific technology shock, following a log-normal distribution:

$$\theta = \exp(u), \qquad u \sim N(0, \sigma_u^2) \tag{3.2.12}$$

Therefore, the marginal cost for the home firm is then,

$$MC = \frac{W}{\theta} \tag{3.2.13}$$

As the individual goods are differentiated, each firm has monopolistic power characterized by the elasticity of substitution between individual goods λ. Prices of individual goods are assumed to be set before the state of the world is realized. It is also assumed that, due to high costs of arbitrage for consumers, each individual monopolist can price discriminate across countries and can set the price in term of the buyer's currency (local currency, LCP) or its own currency (producer currency, PCP).

The optimization problem of each firm is to choose the currency of pricing and then to maximize the discounted expected profits, taking the individual demand function as given. The endogenous currency of pricing will be discussed in detail in next subsection. The optimal pricing schedules given the choice of pricing currency are given as follows.

$$P_{hh} = \hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]} \tag{3.2.14}$$

$$P_{hf}^{*,PCP} = \frac{P_{hh}}{S} \tag{3.2.15}$$

$$P_{hf}^{*,LCP} = \hat{\lambda}\frac{E[\frac{WC^{*1-\rho}S^{-1}}{\theta}]}{E[C^{*1-\rho}]} \tag{3.2.16}$$

where $\hat{\lambda} = \frac{\lambda}{\lambda-1}$ is the markup, and P_{hh} represent the prices of home goods sold in home countries; $P_{hf}^{*,PCP}$ and $P_{hf}^{*,LCP}$ represents the prices of home goods sold in foreign country in term of producer currency (PCP) and buyer currency (LCP) respectively. The problem of the firms in the foreign country is entirely analogous and their prices schedule are given in Table C.1.

3.2.4 Endogenous Currency of Pricing

We now describe how firms choose the currency in which they will set prices for exporting abroad. We say the firm follows *Producer Currency Pricing*, or PCP, if it sets its export price in the domestic currency. If the firm chooses to set export price in the currency of the buyer, it follows *Local Currency Pricing*, or LCP. Whatever currency it chooses, it must set the price before the state of the world is known. Take a firm i in the home country selling a differentiated

good to a foreign market, it faces a downward sloping demand function

$$X(P^*_{hf}(i)) = (\frac{P^*_{hf}(i)}{P^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^* \tag{3.2.17}$$

$P^*_{hf}(i)$ is the price the foreign consumer pays for the home goods i. P^*_{hf} is the price index for all home goods purchased by the foreign consumer, and P^* is the foreign country consumer price index. Without loss of generality, let $P^*_{hf}(i)$, P^*_{hf} and P^* be denominated in foreign currency.

If a home firm sets its price in its own currency (producer currency, PCP), then the expected discounted profit is

$$E\Pi^{Pcp} = E[d(P^{Pcp}_{hf}(i) - MC)(\frac{P^{Pcp}_{hf}(i)}{SP^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^*] \tag{3.2.18}$$

where d is the stochastic discount factor. It equals to the marginal utility of consumption, as all home firms are assumed to be owned by home households. If the home firm set its price in the foreign currency (buyer's currency, LCP), then the expected discounted profit is

$$E\Pi^{Lcp} = E[d(SP^{*Lcp}_{hf}(i) - MC)(\frac{P^{*Lcp}_{hf}(i)}{P^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^*] \tag{3.2.19}$$

The home country firm will set its price in its own currency if the expected profit differential is positive. That is, it follows PCP if and only if

$$E\Pi^{Pcp} - E\Pi^{Lcp} > 0 \tag{3.2.20}$$

In Devereux, Engel, and Storgaard (2004), it is shown that Equation (3.2.20) can be approximated by the following inequality

$$\frac{1}{2}\sigma_s^2 - cov(\ln(MC), s) > 0 \tag{3.2.21}$$

From now on, small-case letters denote logarithms, that is, $x \equiv log(X)$. The equivalent condition for the foreign firm is

$$\frac{1}{2}\sigma_s^2 + cov(\ln(MC^*), s) > 0 \tag{3.2.22}$$

That is, if a firm chooses its currency of pricing optimally, then up to a second-order approximation, its decision depends only on the variance of exchange rate and the covariance of exchange rate with the marginal cost, and is independent of the variance of market demand and the prices of all the other firms. For simplicity, we define the left-hand side of 3.2.21 and 3.2.22 as Ω and Ω^* respectively.

3.2.5 Monetary Authorities

Following a number of recent papers, we assume the monetary authority in each country is concerned with the expected utility of consumption and the disutility of labor effort, but ignores the utility of real money balances.[4] Thus, the home country monetary authority chooses monetary policy to maximize

$$E(\frac{C^{1-\rho}}{1-\rho} - \eta L) \tag{3.2.23}$$

Each monetary authority can commit to an optimal rule. The model and the shocks are log-linear, so without any loss of generality, we can write the form of the monetary rules (for both home and foreign authorities) in the log-linear representation given by

$$m = a_1 u + a_2 u^* \tag{3.2.24}$$

$$m^* = b_1 u + b_2 u^* \tag{3.2.25}$$

These rules are *unrestricted* in that they allow the monetary authority to respond freely to both home and foreign shocks. We will also discuss restricted rules in the later section. The policy feedback rule parameters [a,b] are determined by the following international monetary Nash game

$$\max_a EU(a, b^n) \tag{3.2.26}$$

$$\max_b EU^*(a^n, b) \tag{3.2.27}$$

Given the money rules, the log of exchange rate can be determined by

$$s = \ln \Gamma + (a_1 - b_1)u + (a_2 - b_2)u^* \tag{3.2.28}$$

Note that, since the monetary authorities act after the trade in state-contingent assets, they take the the risk-sharing parameter Γ as given. [5]

[4]See for instance, Obstfeld and Rogoff (2002), Corsetti and Pesenti (2001), and Devereux and Engel (2003).

[5]In Devereux and Engel (2003), it is shown that allowing monetary authorities to choose rules before asset trade makes no difference to the results. Note, because our model is symmetric, $\Gamma = 1$ in equilibrium in any cases.

3.3 A Unique Equilibrium with Flexible Exchange Rates

The key innovation of the model described above is to allow monetary authorities to take account of the way in which firms make their decisions about the currency in which they set prices. The equilibrium for this model can defined as follows: Given the stochastic process $\{\theta, \theta^*\}$, a symmetric equilibrium is a collection of allocations $\{C, C^*, L, L^*, M, M^*, B(z)\}$, price system $\{P, P^*, W, W^*, P_h, P_f, P_h^*, P_f^*, S, \Gamma\}$, and monetary authorities policy {a, b}, such that

- Given the price system, $\{C, C^*, L, L^*, M, M^*, B(z)\}$ solves the household's optimality problem, subject to his budget constraint.
- $\{P_h, P_f, P_h^*, P_f^*\}$ solves the individual firm's optimal pricing problem.
- The firm's currency of pricing decision is optimal given optimal monetary rules.
- {a, b} solves the international monetary Nash game and maximizes the representative household's expected utility.
- The labor, goods, and money markets clear.

3.3.1 Monetary Rules chosen after the Pricing Decision Is Made

We first construct an equilibrium conditional on the optimal monetary policy feedback parameters. The properties of the economy will differ depending on whether the home firm, as well as the foreign firm, follows PCP as opposed to LCP. Because in equilibrium all firms within a country will follow the same pricing strategy, we impose this at the beginning, without any loss of generality. We will construct the equilibrium in the following manner. First, we compute the mean and variance of (log) consumption (for both countries), as a function of the policy rule parameters, in the case where all firms in both countries follow PCP, where all firms in both countries follow LCP, or where home firms follow PCP and foreign firms follow LCP (and vice versa). For each case, we may compute expected utility of home and foreign agents. We then briefly characterize the outcome to an optimal monetary policy game, where monetary authorities take the currency of pricing decisions as given. Finally, we define the game in

which governments choose policy rules taking into account that their choices will influence the currency of pricing of firms, and show that a flexible exchange rate is a unique equilibrium of this game.

Case 1. All Firms Follows PCP

When all firms follow PCP, we may compute the mean and variance of (log) consumption, as functions of the underlying feedback parameters

$$Ec = -\frac{1}{\rho}\ln(\hat{\lambda\eta}) - \frac{2-\rho}{2}\sigma_c^2 - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} + n(1-n)\frac{\sigma_{su} - \sigma_{su^*}}{\rho} \tag{3.3.1}$$

$$\sigma_c^2 = \frac{(na_1 + (1-n)b_1)^2\sigma_u^2 + (na_2 + (1-n)b_2)^2\sigma_{u^*}^2}{\rho^2} \tag{3.3.2}$$

where the other variance and covariance terms in Equation (3.3.1) except σ_u^2 and $\sigma_{u^*}^2$ are also functions of policy parameters, as shown in the appendix Devereux and Engel (2003). Then, as outlined in their appendix, with all firms following PCP, expected utility for the home country may be written as:

$$EU = \Theta\exp(1-\rho)[-\frac{1}{2}\sigma_c^2 - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} + n(1-n)\frac{\sigma_{su} - \sigma_{su^*}}{\rho}] \tag{3.3.3}$$

The expected utility of foreign country is analogous.

Then, with all firms following PCP, a Nash equilibrium to the game in which each monetary authority chooses its feedback parameters to maximize the expected utility (but taking the currency of pricing decision as given) is defined as

$$\max_a EU(a, b^n) \tag{3.3.4}$$

s.t. *Home firms choose PCP*

Foreign firms choose PCP

$$\max_b EU^*(a^n, b) \tag{3.3.5}$$

s.t. *Home firms choose PCP*

Foreign firms choose PCP

Solving for the above equilibrium, the optimal policy rules for this game are

$$a = [1, 0], \qquad b = [0, 1] \tag{3.3.6}$$

The intuition behind this equilibrium is straightforward (see Devereux and Engel 2003). If each monetary authority focuses its monetary policy only on the domestic productivity shock, then this allows for output, consumption, and the terms of trade to respond to the shocks exactly as they would in a flexible price equilibrium. For terms of trade adjustment, it is necessary that the exchange rate responds to productivity shocks, and in order to facilitate this, monetary policies must respond to shocks in different ways across countries.

Finally, we may now check whether PCP is indeed an optimal pricing strategy for firms in the home and foreign countries, if monetary policy is chosen as described in 3.3.6. As shown in Appendix C.1.2, using conditions 3.2.21 and 3.2.22, we may establish that

$$\Omega = \Omega^* = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{3.3.7}$$

where Ω and Ω^* are expected profit differential, as on the left hand side of Equations (3.2.21) and (3.2.22). Hence, all firms will wish to follow PCP, and PCP is an equilibrium when monetary authorities choose policies *after* the currency of pricing decision has been determined, and therefore choose policies as in Equation (3.3.6).

Given the optimal policy parameters, we may compute the expected utility in this equilibrium, identical across countries, as

$$EU = EU^* = \Theta \exp\left\{(\frac{1-\rho}{\rho})^2(\frac{n^2\sigma_u^2 + (1-n)^2\sigma_{u^*}^2}{2})\right\} \tag{3.3.8}$$

where $\Theta = \frac{\lambda-(1-\rho)(\lambda-1)}{(1-\rho)\lambda}(\lambda\eta)^{\frac{\rho-1}{\rho}} < 0$ is a constant function of parameters.

Case 2. All Firms Follow LCP

If all firms in both countries follow LCP, then the mean and variance of log consumption for the home country can be written as

$$Ec = -\frac{1}{\rho}\ln(\hat{\lambda\eta}) - \frac{2-\rho}{2}\sigma_c^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} \tag{3.3.9}$$

$$\sigma_c^2 = \frac{a_1^2\sigma_u^2 + a_2^2\sigma_{u^*}^2}{\rho^2} \tag{3.3.10}$$

where, again, the covariance terms σ_{cu} and σ_{cu^*} are also functions of policy parameters $\{a, b\}$, as shown in appendix of Devereux and Engel (2003). In this case, home consumption variance depends only on home monetary policy rules, because there is no pass-through of exchange rate changes into the domestic price level.

The appendix then shows that expected utility may be represented as:

$$EU = EC^{1-\rho}[\frac{\lambda - n(1-\rho)(\lambda-1)}{\lambda(1-\rho)}] - \frac{(1-n)(\lambda-1)}{\lambda}EC^{*1-\rho} \tag{3.3.11}$$

$$EU^* = EC^{*1-\rho}[\frac{\lambda - (1-n)(1-\rho)(\lambda-1)}{\lambda(1-\rho)}] - \frac{n(\lambda-1)}{\lambda}EC^{1-\rho} \tag{3.3.12}$$

The expected utility is a combination of two separate function of consumption variance and the covariance between consumption and productivity shocks.

$$EC^{1-\rho} = \Upsilon \exp(1-\rho)[-\frac{1}{2}\sigma_c^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho}] \tag{3.3.13}$$

$$EC^{*1-\rho} = \Upsilon \exp(1-\rho)[-\frac{1}{2}\sigma_{c^*}^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{c^*u} + (1-n)\sigma_{c^*u^*})}{\rho}] \tag{3.3.14}$$

where $\Upsilon = (\lambda\eta)^{\frac{\rho-1}{\rho}}$.

Thus, the monetary rules that represent an equilibrium of the Nash game in which monetary authorities choose monetary policies taking as given the LCP currency of pricing in both countries, can be derived as

$$a = [n, 1-n], \qquad b = [n, 1-n] \tag{3.3.15}$$

In this case, monetary authorities follow identical policies in the two countries. The intuition behind this (see Devereux and Engel 2003), is that the exchange rate does not play any allocational role, given zero pass-through in each country. An optimal monetary rule therefore does not try to attain the terms of trade adjustment that would occur in a flexible price economy, and so the equilibrium has zero exchange rate variance.

Again, from Appendix C.1.2, we may use Equations (3.2.21) and (3.2.22) to establish that

$$\Omega = \Omega^* = 0 \tag{3.3.16}$$

Hence, weakly, LCP is also an equilibrium when monetary authorities choose policies *after* the currency of pricing decision has been determined. Since both LCP and PCP constitute equilibria where policy is chosen taking the pricing decision as given, there are multiple equilibria in this case. [6]

Expected utility in this equilibrium can then be computed as

$$EU = EU^* = \Theta \exp(1-\rho)\{\frac{[n^2 - \rho n]\sigma_u^2 + [(1-n)^2 - \rho(1-n)]\sigma_{u^*}^2}{2\rho^2}\} \tag{3.3.17}$$

Case 3. Home Firms Follow PCP, Foreign Firms Follow LCP

Now let us look at the asymmetric case. If the home firm follows PCP, and the foreign firm LCP, there will be zero pass-through of exchange rate changes into the home economy, but full pass-through into the foreign economy. As shown in Devereux, Shi, and Xu (2004), we may compute the mean and variance of (log) consumption in the home and foreign economy, respectively, as

$$Ec = -\frac{1}{\rho}\ln[\hat{\lambda}\eta\Gamma^{1-n}] - \frac{2-\rho}{2}\sigma_c^2 - \frac{n\sigma_u^2 + (1-n)\sigma_{u^*}^2}{2\rho} + \frac{n\sigma_{cu} + (1-n)\sigma_{cu^*}}{\rho} \tag{3.3.18}$$

$$\begin{aligned} E(c^*) &= -\frac{1}{\rho}\ln(\Gamma^{-n}\hat{\lambda}\eta) - \frac{2-\rho}{2}\sigma_{c^*}^2 - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{n\sigma_u^2 + (1-n)\sigma_{u^*}^2}{2\rho} \\ &\quad + \frac{n\sigma_{c^*u} + (1-n)\sigma_{c^*u^*}}{\rho} + \frac{n(1-n)(\sigma_{su} - \sigma_{su^*})}{\rho} \end{aligned} \tag{3.3.19}$$

$$\sigma_c^2 = \frac{1}{\rho^2}[a_1^2\sigma_u^2 + a_2^2\sigma_{u^*}^2] \tag{3.3.20}$$

$$\sigma_{c^*}^2 = \frac{1}{\rho^2}[(na_1 + (1-n)b_1)^2\sigma_u^2 + (na_2 + (1-n)b_2)^2\sigma_{u^*}^2] \tag{3.3.21}$$

where the variance and covariance terms other than σ_u^2 and $\sigma_{u^*}^2$ are all functions of policy parameters.

[6] This was first pointed out, although in quite a different modelling framework, by Corsetti and Pesenti (2001).

Therefore, we also can derive expected utility for the home and foreign agent in the following way.

$$E(U) = \frac{\lambda - n(\lambda-1)(1-\rho)}{(1-\rho)\lambda} E(C^{1-\rho}) - \frac{(1-n)(\lambda-1)}{\lambda} \Gamma E(C^{*1-\rho}) \tag{3.3.22}$$

$$EU^* = \frac{\lambda - (1-n)(\lambda-1)(1-\rho)}{(1-\rho)\lambda} E(C^{*1-\rho}) - \frac{n(\lambda-1)}{\lambda} \Gamma^{-1} E(C^{1-\rho}) \tag{3.3.23}$$

Following Devereux, Shi and Xu (2004), we may then obtain the equilibrium of the Nash game in which the home and foreign monetary authorities choose policy rules, taking the {PCP, LCP} pricing configuration as given. The rules for the general case where $\rho > 1$ are quite complicated, but the rules for the case where $\rho = 1$ is quite straightforward.

$$a = [n, 1-n], \qquad b = [0, 1] \tag{3.3.24}$$

In this case, the home authority responds approximately equally to home and foreign shocks, while the foreign authority puts more weight on its domestic shock.

Now we may ask if in fact the {PCP, LCP} configuration is an equilibrium for the economy. The answer is no. From Appendix C.1.2, we may show that

$$\Omega > 0, \qquad \Omega^* > 0 \tag{3.3.25}$$

In each country, firms would wish to set prices in their own currency. Hence in fact, the {PCP, LCP} configuration is *not* an equilibrium, if monetary policy rules are chosen taking the currency of pricing as given.

3.3.2 Monetary Rules Chosen before the Pricing Decision Is Made

Now we focus on the game where each monetary authority chooses its policy before the currency of pricing decision is made. In general, the maximization problem is quite complex, because the monetary authorities' choice of monetary rule will partly determine the pricing policies of firms in both countries, and hence will lead to a switching across pass-through outcomes that makes the decision discontinuous. But we can circumvent these difficulties by defining a simpler game, in which the choice set of the monetary authorities in each country is simply binary. Although this seems excessively restrictive, in fact it is not really, since we show that

this game supports the constrained Pareto optimum of the world economy, which is the flexible price allocation. [7] The game that we focus on allows the monetary authority of each country to choose either the Nash equilibrium monetary rules of the ex-post {PCP, PCP} game defined above, or the Nash equilibrium rules of the ex-post {LCP, LCP} game defined above.

The binary game is then defined in the matrix in the Figure below.

Figure 3.2: Binary Game

		Foreign Monetary Authority	
		$b = [0, 1]$	$b = [n, 1-n]$
Home Monetary Authority	$a = [1, 0]$	(PCP, PCP)	(PCP, PCP)
	$a = [n, 1-n]$	(PCP, PCP)	(LCP, LCP)

Proposition 6 *The {PCP, PCP} configuration, and flexible exchange rates represent the unique equilibrium of the monetary policy game when monetary authorities take account of the currency of pricing.*

The payoffs of this binary game are given in Appendix C.2. Once we know them, then the proof is straightforward. Say that Foreign is following $b = [n, 1-n]$. Then if home follows $a = [1, 0]$, it generates a switching to all PCP, and then $a = [1, 0]$ is clearly better for home. If foreign is following $b = [0, 1]$, then the optimal policy for home is to follow $a = [1, 0]$.

Corollary 1 *The equilibrium of the ex-ante monetary policy game supports the full flexible price equilibrium.*

The logic of the proposition is two-fold. First, it is clear that the outcome under {PCP, PCP} is preferable to both countries, since it supports the full flexible price equilibrium, whereas the outcome under {LCP, LCP} prevents exchange rate pass-through, does not allow optimal

[7] We have not explored the possibility of equilibria in the game where monetary policy rules are unrestricted, and policy makers take account of the currency of pricing decision, but clearly in welfare terms, these equilibria would be worse than the unique equilibrium of the game defined here.

terms of trade adjustment, and hence gives lower expected utility. But the key feature of the result is that by committing to follow the optimal monetary rules that are an equilibrium of the ex-post {PCP, PCP} game (case 1 above), the monetary authorities will actually ensure that {PCP, PCP} constitutes the equilibrium price configuration. Hence, by taking into account the way in which firms choose the currency of pricing, the monetary authorities support producer currency pricing and attain the flexible price outcome in the world economy. As a by product, it implies that optimal monetary policy in an environment where the currency of pricing is endogenous restores the Friedman case for flexible exchange rates, and satisfies the central results of the optimal currency area literature.

3.4 Sustaining the Flexible Price Equilibrium with Restricted Monetary Policy Rules

When monetary policy is chosen conditional on the currency of pricing, and monetary policy rules are unrestricted, then there are two Pareto ranked equilibria. Due to the problem of coordinating expectations of price setters and the actions of monetary authorities, the economy can be stuck in an equilibrium with low pass-through, fixed exchange rates, and social welfare lower than the flexible price equilibrium. One way to eliminate this bad equilibrium is to require monetary authorities to take account of the currency of pricing decision. We have show this possibility in our model where the monetary authorities move 'first' in the game. If monetary policy rules were set *before* the firms make their currency of pricing decision as the sequence of actions described in Figure 3.1, then there is a *unique* equilibrium to the policy game, characterized by PCP, and flexible exchange rates, which coincides with the flexible price world equilibrium.

However, this equilibrium requires that monetary policy take account of the structural determination of pricing. To the extent that our model captures a pricing decision that may evolve more slowly over time than we actually allow for in the simple timing structure here, it may be unrealistic to assume that monetary policy can have the degree of commitment and far-sightedness necessary to sustain such a rule. Real world monetary policy making does not usually consider product market structure to be within its influence. For these reasons, it may

be that the timing sequence of Figure 3.1 is not the most relevant way to describe monetary policy making.

As an alternative, we consider placing restrictions on the monetary policy rule itself in order to select the PCP pricing equilibrium of the monetary policy game. Assume that monetary authorities choose monetary policy after firms' currency of pricing decision, but they are restricted to follow rules that focus only on domestic productivity shocks. What will be the outcome in this case? In principle it would seem undesirable to restrict monetary authorities from taking into account all shocks. However, in the presence of coordination externalities in price-setting, this conclusion may no longer hold. Hence we define a *restricted* monetary policy rule, as follows:

$$m = au, \quad m^* = bu^* \tag{3.4.1}$$

Under these rules, monetary policy can respond only to domestic shocks. The policy feedback rule parameter vector {a,b} is determined by the same international monetary Nash game.

We present the analysis as follows. First, assume a given currency of pricing. Then derive the outcome of an international monetary Nash game, where monetary authorities take the currency of pricing as given. Finally, use the solution of the monetary game to check whether or not the given currency of pricing configuration is consistent with the optimal strategies of home and foreign firms.

3.4.1 Case 1: All Firms Follow PCP

When all firms follow PCP, Appendix C.3 shows that the equilibrium of a Nash game between monetary authorities using restricted monetary rules monetary rules is given as:

$$a = 1, \qquad b = 1 \tag{3.4.2}$$

This is in fact equivalent to (3.3.6). Since, in the case of unrestricted monetary policy, in a Nash equilibrium with PCP, monetary authorities choose to respond to the domestic shock alone, then the restriction on the form of monetary rules is irrelevant.

We can use the same logic as above to show that the monetary rules given by (3.4.2) are consistent with PCP pricing behavior on the part of firms. Given (3.4.2), Appendix C.3 shows

that $\Omega = \Omega^* = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0$, so both home and foreign firms will choose PCP. Hence {PCP, PCP} is an equilibrium of the game with restricted monetary rules.

3.4.2 Case 2: All Firms Follow LCP

What happens if all firms in both countries follow LCP? Appendix C.3 shows that in this case, the restricted optimal monetary rules are

$$a = n, \qquad b = 1 - n \tag{3.4.3}$$

In this instance, the optimal monetary rules differ from the unrestricted rules under LCP. Monetary authorities offset their own national shock, in proportion to their country weight. But as described in (3.3.15) above, they would strictly prefer to offset the foreign shock as well, since in the absence of exchange rate pass-through, they would wish to use monetary policy to replicate the flexible price response of consumption.

What does this monetary rule imply for the currency of pricing decision? Note that unlike the economy with LCP and unrestricted setting of monetary policy, where the exchange rate is fixed in equilibrium, under the restricted money rule, there is inevitably fluctuation in exchange rate, since by design monetary authorities cannot respond to shocks in the same way. But the presence of exchange rate volatility will encourage firms to follow PCP pricing rules. Again, using (3.2.21), we may establish that $\Omega = \frac{n(2-n)}{2}\sigma_u^2 + \frac{(1-n)^2}{2}\sigma_{u^*}^2 > 0$ and $\Omega^* = \frac{n^2}{2}\sigma_u^2 + \frac{(1-n)(1+n)}{2}\sigma_{u^*}^2 > 0$. This implies that all firms in both countries will choose PCP. Thus {LCP, LCP} is not an equilibrium when monetary authorities can only use the restricted money rules.

From the analysis of the above two cases,[8] we may state the following proposition.

Proposition 7 *The {PCP, PCP} configuration, and flexible exchange rates represent the unique equilibrium of the international monetary Nash game, where the currency of pricing is endogenous, and monetary policy rules are restricted as in (3.4.1).*

[8]We have not allowed for the possibility of asymmetric pricing equilibria, where one country follows LCP and another country follows PCP. From Appendix C.1.2 however, it is shown that such an asymmetric outcome cannot be an equilibrium. The reason is that if one country follows a PCP rule, then the monetary policy game will imply sufficient exchange rate flexibility that firms in *both* countries will choose PCP. Hence the asymmetric outcome is ruled out for the same reasons as in Case 2.

Corollary 2 *The full flexible price equilibrium can be sustained by the restricted monetary rules.*[9]

The key effect of the restrictions on monetary rules is that they ensure exchange rate adjustment. Without any restrictions, there is an equilibrium where the monetary authorities find it optimal to eliminate exchange rate adjustment. Given LCP pricing decisions, exchange rate volatility hinders cross country consumption risk-sharing, while at the same time, has no benefit in terms of relative price adjustment (because there is no pass-through to domestic prices). By ensuring that monetary policy cannot deliver absolute exchange rate stability, the restricted rules cause all firms to follow PCP pricing decisions, leading to full exchange rate pass-through. The ensuing optimal monetary policy then brings *both* efficient international consumption risk-sharing, and efficient relative price adjustment.

3.5 Conclusion

This paper re-examines the issues of the degree of exchange rate flexibility in an optimal monetary policy game, when the currency of export pricing is endogenous. We show that if monetary policy can take account of the currency of pricing decision, then there is a unique equilibrium to the monetary policy game, where all firms follow PCP, and Friedman's classic defense of flexible exchange rates is upheld. Nevertheless, we also realized that this may not be a realistic description of monetary policy, since product market structure is normally thought to be outside the purview of monetary policy. An alternative is to restrict the form of monetary rules itself. If monetary authorities are precluded from reacting to foreign productivity shocks, then their optimal policies will rule out the equilibrium with LCP. In this sense, restricting the form of the monetary policy rule acts as an equilibrium selection mechanism, which sustains flexible exchange rates.

[9] It can be shown that to sustain the full flexible price equilibrium, we only need one monetary authority to use the restricted monetary rules. In other words, as long as one country promises to restrict itself and only respond to domestic shocks, Friedman's argument for flexible exchange rates can be upheld.

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Appendix A

Appendices of Chapter 1

A.1 Approximation of Expected Profit Functions

As shown in the first paper, the profits under domestic currency pricing are given by

$$E\Pi^{peso} = \widetilde{\lambda}[E(S^{\lambda}Z)]^{\lambda}[E(S^{\lambda}ZMC)]^{1-\lambda} \tag{A.1.1}$$

where $\widetilde{\lambda} = \frac{1}{\lambda-1}(\frac{\lambda}{\lambda-1})^{-\lambda}$, $Z = dP_x^{*\lambda-1}X$, MC depends on the choice of debt contract. This expression may be rewritten as:

$$\widetilde{\lambda}[E\exp(\ln Z)\exp(\lambda\ln S)]^{\lambda}[E\exp(\ln Z)\exp(\lambda\ln S)\exp(\ln MC)]^{1-\lambda} \tag{A.1.2}$$

Now using the second order approximation, we have

$$\begin{aligned} E\exp(\ln Z)\exp(\lambda\ln S) &\approx \exp(E\ln Z)\exp(\lambda E\ln S) \\ &\times\{1+\frac{1}{2}var(\ln Z)+\frac{\lambda^2}{2}var(\ln S)+\lambda cov(\ln Z,\ln S)\} \end{aligned} \tag{A.1.3}$$

Using the same approximation for the expression $E\exp(\ln Z)\exp(\lambda\ln S)\exp(\ln MC)$, we get an approximation for $E\Pi^{peso}$ as follows:

$$\begin{aligned} &\sum[1+\frac{1}{2}var(\ln Z)+\frac{\lambda^2}{2}var(\ln S)+\lambda cov(\ln Z,\ln S)]^{\lambda} \\ &\times[1+\frac{1}{2}var(\ln Z)+\frac{\lambda^2}{2}var(\ln S)+\frac{1}{2}var(\ln MC)+\lambda cov(\ln Z,\ln S) \\ &+cov(\ln Z,\ln MC)+\lambda cov(\ln S,\ln MC)]^{1-\lambda}, \end{aligned} \tag{A.1.4}$$

where $\sum = \tilde{\lambda}\exp[E(\ln Z)]\exp[\lambda E(\ln S)]\exp[(1-\lambda)E(\ln MC)]$. Taking logs, we get expected discounted profits equal to [1]

$$\begin{aligned} E\Pi^{peso} &\approx \ln\sum+[\frac{1}{2}var(\ln Z)+\frac{\lambda^2}{2}var(\ln S)+\frac{1-\lambda}{2}var(\ln MC)]+ \\ &\{\lambda cov(\ln Z,\ln S)+\lambda(1-\lambda)cov(\ln MC,\ln S)+(1-\lambda)cov(\ln Z,\ln MC)\} \end{aligned} \tag{A.1.5}$$

where $\ln\sum$ is the steady state profit. Therefore, using the same approximation, we can rewrite $E\Pi^{dollar} = \widetilde{\lambda}[E(SZ)]^{\lambda}[E(ZMC)]^{1-\lambda}$ as:

$$\begin{aligned} E\Pi^{dollar} &\approx \ln\sum+[\frac{1}{2}var(\ln Z)+\frac{\lambda}{2}var(\ln S)+\frac{1-\lambda}{2}var(\ln MC)]+ \\ &\{\lambda cov(\ln Z,\ln S)+(1-\lambda)cov(\ln Z,\ln MC)\} \end{aligned} \tag{A.1.6}$$

[1] Here, we use the approximation $\ln(1+x)\approx x$.

A.2 The Steady State

The model in the steady state can be described as the following system. There are 13 steady state variables $\{P,\ W,\ S,\ r, MC,\ C, L,\ L_x,\ L_N,\ P_x,\ Y_x,\ Z,\ \Pi_x\ \}$.

$$PC = WL_N + WL_X + \Pi_x \tag{A.2.1}$$

$$W = \hat{\theta}\eta PC^{\rho} \tag{A.2.2}$$

$$L = L_N + L_x \tag{A.2.3}$$

$$WL_N = (1-\alpha)PC \tag{A.2.4}$$

$$P = W^{1-\alpha}(S)^{\alpha\epsilon} \tag{A.2.5}$$

$$Y_x = \frac{\bar{X}}{P_x} \tag{A.2.6}$$

$$MC = W^{1-\omega}[S(1+r)]^{\omega} \tag{A.2.7}$$

$$WL_x = (1-\omega)Y_x MC \tag{A.2.8}$$

$$P_{xt} = \hat{\lambda} MC \tag{A.2.9}$$

$$Z = P^{-1}C^{-\rho}P_x^{\lambda-1}X \tag{A.2.10}$$

$$\Pi_x = P_x Y_x - MCY_x \tag{A.2.11}$$

$$1 + r = \frac{1}{\beta} \tag{A.2.12}$$

$$S = 1 \tag{A.2.13}$$

Given the foreign demand $\bar{X}$ and the structure parameters β, α, ω, λ, ρ and η, the steady state variables can be analytical solved. Note that, we use $\bar{K}$ to denote the steady state value of variable K_t in the model.

A.3 Model Solution

Given the distribution of firms $\Omega = (\mu_1, \mu_2, \mu_3, \mu_4)$, we can express the export sector's price index P_x^* in terms of dollar as below:[2]

$$P_x^{*1-\lambda} = \mu_1(\frac{P_x^{pp}}{S})^{1-\lambda} + \mu_2(\frac{P_x^{pd}}{S})^{1-\lambda} + \mu_3 P_x^{*dp\,1-\lambda} + \mu_4 P_x^{*dd\,1-\lambda} \tag{A.3.1}$$

Log-linearizing the export sector's price index, we have

$$\hat{p}_x^* = \mu_1(\hat{p}_x^{pp} - \hat{s}) + \mu_2(\hat{p}_x^{pd} - \hat{s}) + \mu_3\hat{p}_x^{*dp} + \mu_4\hat{p}_x^{*dd} \tag{A.3.2}$$

As all prices p_x^{pp}, p_x^{pd}, p_x^{*dp} and p_x^{*dd} are preset in period $t-1$, it implies that $\hat{p}_x^{s_i} - E_{t-1}\hat{p}_x^{s_i} = 0$, thus

$$\tilde{p_x^*} = \hat{p}_x^* - E_{t-1}\hat{p}_x^* = -(\mu_1 + \mu_2)\tilde{s} \tag{A.3.3}$$

where $\mu_1 + \mu_2$ is the number of firms who set their export prices in pesos. Using the market clearing condition $Y_x = \frac{X}{P_x^*}$, the deviation of the log export output from its $t-1$ expectation is given by

$$\tilde{y_x} = \tilde{x} - \tilde{p_x^*} = \tilde{x} + (\mu_1 + \mu_2)\tilde{s} \tag{A.3.4}$$

The log forms of individual demand for the firms who choose the strategy s_i are given by, respectively,

$$y_x^{pp} = -\lambda[p_x^{pp} - s - p_x^*] + x - p_x^* \tag{A.3.5}$$

$$y_x^{pd} = -\lambda[p_x^{pd} - s - p_x^*] + x - p_x^* \tag{A.3.6}$$

$$y_x^{dp} = -\lambda[p_x^{*dp} - p_x^*] + x - p_x^* \tag{A.3.7}$$

$$y_x^{dd} = -\lambda[p_x^{*dd} - p_x^*] + x - p_x^* \tag{A.3.8}$$

The net income for an export firm who chooses the strategy (p, p) is given by

$$G_x^{pp} = (P_x^{pp} - \omega MC_p)Y_x^{pp} = [P_x^{pp} - \omega W^{1-\omega}((1 + i_{t+1})S_{t-1})^{\omega}]Y_x^{pp} \tag{A.3.9}$$

Note that, in the steady state $\bar{MC} = (1+\bar{r})^{\omega}\bar{W}^{1-\omega}$, $\bar{\Pi} = \frac{1}{\lambda-1}(1+\bar{r})^{\omega}\bar{W}^{1-\omega}\bar{Y}_x$, $\bar{P}_x = \frac{\lambda}{\lambda-1}(1+\bar{r})^{\omega}\bar{W}^{1-\omega}$, and $\bar{G} = (\frac{\lambda}{\lambda-1} - \omega)(1+\bar{r})^{\omega}\bar{W}^{1-\omega}\bar{Y}_x$. Taking the log-linearization, we have

$$\hat{g}^{pp} = a\hat{p}_x^{pp} - (a-1)[(1-\omega)\hat{w} + \omega\gamma\hat{s} + \omega\hat{s_{t-1}}] + \hat{y}_x^{pp} \tag{A.3.10}$$

where $a = \frac{\lambda}{\lambda-(\lambda-1)\omega} > 1$. After eliminate the predetermined terms, we have

$$\tilde{g}^{pp} = -(a-1)\omega\gamma\tilde{s} + \tilde{y}_x^{pp} \tag{A.3.11}$$

[2]For simplicity, we drop the subscript t for all variables.

Following the same technique, we can have the following derivatives of the net income for the export firm who chooses the other strategy,

$$G_x^{pd} = (P_x^{pd} - \omega MC_d)Y_x^{pd} = [P_x^{pd} - \omega W^{1-\omega}((1 + r_{t+1}^*)S_t)^\omega]Y_x^{pd} \tag{A.3.12}$$

$$\hat{g}^{pd} = a\hat{p}_x^{pd} - (a-1)[(1-\omega)\hat{w} + \omega(s+\epsilon)] + \hat{y}_x^{pd} \tag{A.3.13}$$

$$\tilde{g}^{pd} = -(a-1)\omega(\tilde{s} + \epsilon) + \tilde{y}_x^{pd} \tag{A.3.14}$$

$$G_x^{dp} = (SP_x^{*dp} - \omega MC_p)Y_x^{dp} = [SP_x^{*dp} - \omega W^{1-\omega}((1 + i_{t+1})S_{t-1})^\omega]Y_x^{dp} \tag{A.3.15}$$

$$\hat{g}^{dp} = a(\hat{s} + \hat{p}_x^{*dp}) - (a-1)[(1-\omega)\hat{w} + \omega\gamma\hat{s} + \omega\hat{s_{t-1}}] + \hat{y}_x^{dp} \tag{A.3.16}$$

$$\tilde{g}^{dp} = a\tilde{s} - (a-1)\omega\gamma\tilde{s} + \tilde{y}_x^{dp} \tag{A.3.17}$$

$$G_x^{dd} = (SP_x^{*dd} - \omega MC_d)Y_x^{dd} = [SP_x^{*dd} - \omega W^{1-\omega}((1 + r_{t+1}^*)S_t)^\omega]Y_x^{dd} \tag{A.3.18}$$

$$\hat{g}^{dd} = a(\hat{s} + \hat{p}_x^{*dd}) - (a-1)[(1-\omega)\hat{w} + \omega(s+\epsilon)]] + \hat{y}_x^{dd} \tag{A.3.19}$$

$$\tilde{g}^{dd} = a\tilde{s} - (a-1)\omega(\tilde{s} + \epsilon) + \tilde{y}_x^{dd} \tag{A.3.20}$$

As the total net export income is given by $G_x = \mu_1 G_x^{pp} + \mu_2 G_x^{pd} + \mu_3 G_x^{dp} + \mu_4 G_x^{dd}$, we can express

$$\tilde{g_x} = \mu_1\tilde{g}^{pp} + \mu_2\tilde{g}^{pd} + \mu_3\tilde{g}^{dp} + \mu_4\tilde{g}^{dd} \tag{A.3.21}$$

Using the fact that $\tilde{y}_x = \mu_1\tilde{y}_x^{pp} + \mu_2\tilde{y}_x^{pd} + \mu_3\tilde{y}_x^{dp} + \mu_4\tilde{y}_x^{dd} = \tilde{x} + (\mu_1 + \mu_2)\tilde{s}$, we can have

$$\tilde{g_x} = b\tilde{s} + \tilde{x} - \xi\epsilon \tag{A.3.22}$$

where $b = ea - (a-1)[v + \gamma(1-v)]\omega + (1-e)$ and $\xi = (a-1)\omega v$. Note that $e = \mu_3 + \mu_4$ and $v = \mu_2 + \mu_4$.

A.4 The Euler Equation

Given $\tilde{c}_t + (\alpha - b)\tilde{s}_t = \tilde{x}_t - \xi\epsilon_t$, we conjecture that the solution to consumption and the exchange rate is only determined by current shocks and can be given by,

$$\tilde{c}_t = a_1 f_t, \qquad \tilde{s}_t = a_2 f_t \tag{A.4.1}$$

where $f_t = \tilde{x}_t - \xi\epsilon_t$, a_1 and a_2 are function of parameters and will be determined later. Since all shocks are i.i.d., and by the definition of $\tilde{x_{t+j}} = \hat{x_{t+j}} - E_{t-1}(\hat{x_{t+j}})$, $j \geq 0$, we have $E_t\tilde{x_{t+1}} = 0$. This implies that $E_t f_{t+1} = 0$. Therefore, we have

$$E_t(\frac{\rho}{\alpha}\tilde{c}_{t+1} + \tilde{s}_{t+1}) = (\frac{\rho}{\alpha}a_1 + a_2)E_t f_{t+1} = 0 \tag{A.4.2}$$

From equations 1.4.16, it yields

$$\rho\tilde{c}_t + (\alpha + \gamma)\tilde{s}_t = 0 \tag{A.4.3}$$

Combined with $\tilde{c}_t + (\alpha - b)\tilde{s}_t = \tilde{x}_t - \xi\epsilon_t$, we can solve for a_1 and a_2 and verify our conjecture. Therefore, we prove that $E_t(\frac{\rho}{\alpha}\tilde{c}_{t+1} + \tilde{s}_{t+1}) = 0$.

A.5 The Proof of Proposition 1

As $\gamma \to 0$, we conjecture that $\Omega = [1, 0, 0, 0]$, and then check if the optimal strategy for each firm support this distribution. Given Ω, we have $e = 0$ and $v = 0$, thus $b_1 > 1$, $b_2 = 1$, $\phi_1 = \lambda$ and $\phi_2 = 0$. From conditions (1.4.30), (1.4.31) and (1.4.32), we have $\pi(p,p) > \pi(d,p)$, $\pi(d,p) > \pi(d,d)$ and $\pi(p,p) > \pi(p,d)$. This implies, given Ω, (p,p) will be the optimal strategy for all firms and no firm would deviate from the distribution $\Omega = [1,0,0,0]$. So $\Omega = [1,0,0,0]$ is an equilibrium distribution of firms when $\gamma \to 0$. Obviously, this equilibrium may hold for any $\gamma < \gamma_L$, where γ_L is the maximum value of γ which can satisfy conditions (1.4.30), (1.4.31) and condition that $\pi(p,p) > \pi(d,d)$.

As $\gamma \to \infty$, we conjecture that the distribution of firms $\Omega = [0,0,0,1]$, that is, $e = 1$ and $v = 1$. As b_2 increases with γ, then from conditions (1.4.30) and (1.4.31), we can show $\pi(p,p) < \pi(d,p)$, $\pi(p,d) < \pi(d,d)$. As γ is big enough, approximately, $b_1 \approx \frac{\gamma^2}{\rho^2}\frac{\sigma_\epsilon^2}{\sigma_x^2+\xi^2\sigma_\epsilon^2}$, $\phi_1 \approx \gamma - \frac{\gamma}{\rho}\frac{\sigma_x^2}{\sigma_x^2+\xi^2\sigma_\epsilon^2}$, $\phi_2 \approx \frac{\xi\gamma^2}{\rho}\frac{\sigma_\epsilon^2}{\sigma_x^2+\xi^2\sigma_\epsilon^2}$, where $\xi = (a-1)\omega$, substituting them in the inequality 1.4.33, we can have

$$\omega\gamma^2 - \omega\frac{\gamma^2}{\rho^2}\frac{\sigma_\epsilon^2}{\sigma_x^2 + \xi^2\sigma_\epsilon^2} + 2(\gamma - 1)(\gamma - \frac{\gamma}{\rho}\frac{\sigma_x^2}{\sigma_x^2 + \xi^2\sigma_\epsilon^2}) - 2\frac{\xi\gamma^2}{\rho}\frac{\sigma_\epsilon^2}{\sigma_x^2 + \xi^2\sigma_\epsilon^2} > 0 \tag{A.5.4}$$

It implies that $\pi(d,d) > \pi(d,p)$.[3] Therefore, (d,d) would be the optimal strategy for all firms, and this is consistent with our conjecture $\Omega = [0,0,0,1]$. This result will hold not only in the case where $\gamma \to \infty$, but also in any case where $\gamma > \gamma_H$, where γ_H is the minimum value of γ which violate conditions (1.4.31) and (1.4.33) and satisfy the condition $\pi(p,p) < \pi(d,d)$.

[3]This inequality will hold for most reasonable values of parameters ρ and ω and sizes of shocks.

Table A.1: The Currency Denomination of Export Invoicing of Selected Countries (%)[a]

Countries	Observation year	US dollar	Own currency
Korea	1995	88.1	< 2.2*
Thailand	1996	91.7	1.3
Malaysia	1996	66	< 20*
Indonesia	1995	90	–
United States	1996	98.0	98.0
Germany	1994	9.8	76.4
Japan	1995	52.7	35.7
United Kingdom	1992	22.0	62.0
France	1995	18.6	51.7
Italy	1994	23.0	40.0

[a] Source: The data for developed countries are from Tavlas (1995). The data for Korea and Thailand are from Cook and Devereux (2004) and Mckinnon and Schnabl (2003). Malaysian data are from Coldberg and Till (2005). Indonesia data are from World Bank. The data with asterisk are calculated by the author using data from above resources.

Table A.2: The Currency Denomination of External Debt of Selected Countries (%)[b]

Countries	Observation year	share of foreign currency debt
Korea	1995	98.5
Thailand	1995	98.8
Malaysia	1995	93.3
Indonesia	1995	98.7
United States	1993-1998	30
Germany	1993-1998	69
Japan	1993-1998	64
United Kingdom	1993-1998	56
France	1993-1998	59
Italy	1993-1998	86

[b] Source: The data for selected emerging market economies are from Goldstein and Tuner (2004). The data for selected developed countries are from Eichengreen, Hausmann and Panizza (2003).

Table A.3: Standard Deviation of Daily Exchange Rate Fluctuations against the Dollar (%)[c]

Countries	Pre-crisis	Pre-crisis Ex regime
Hong Kong Dollar	0.02	CBA
Indonesian Rupiah	0.17	CP
Korea Won	0.22	CP
Malaysian Ringgit	0.25	Limited flexibility w.r.t. US dollar
Philippine Peso	0.37	De facto peg to US dollar (or fixed)
Singapore Dollar	0.20	De facto moving band to US dollar
Thai Baht	0.21	De facto peg to US dollar (or fixed)
Japan Yen	0.67	IF
Deutsche Mark	0.60	IF
Swiss Franc	0.69	IF

[c] Source: The data of pre-crisis exchange rate flexibility are from Mckinnon and Schnabl (2003); the classification of exchange rate regimes are from Reinhart and Rogoff (2002). CBA = Currency board; CP = Crawling peg; IF = Independently floating.

Table A.4: Calibration Parameters

Parameters	value	Parameters	value	Parameters	value
ρ	2	β	0.96	α	0.4
λ	7	ω	0.4	σ_x	4%
σ_ϵ	1%	θ	11	η	1

Table A.5: The Impact of γ on the Distribution of Firms Ω [d]

γ	(0, 1.004)	(1.004,1.025)	(1.025,13.368)	(13.368,14.412)	(14.412,∞)
(pp)μ_1	1	$1-\mu_2(\gamma)$	0	0(0)	0
(pd)μ_2	0	$\mu_2(\gamma)$	1	1(0)	0
(dp)μ_3	0	0	0	0(0)	0
(dd)μ_4	0	0	0	0(1)	1
$e=\mu_3+\mu_4$	0	0	0	0(1)	1
$v=\mu_2+\mu_4$	0	$\mu_2(\gamma)$	1	1	1

[d] $\mu_2(\gamma)$ is the number of firms which choose the strategy s_2=(p,d) where $\gamma \in (1.004, 1025)$, and it is also a increasing function of γ, which satisfies with $\mu_2^s(1.004)=0$ and $\mu_2^s(1.025)=1$.

Table A.6: Welfare Comparison

ζ	$\gamma=0.01$	$\gamma=900$
(p,p) $\Omega=(1,0,0,0)$	-0.009%	-0.223%
(d,d) $\Omega=(0,0,0,1)$	-0.012%	-0.10%

Table A.7: Variables Comparison [e]
($\gamma = 900$)

	$\Omega = [1,0,0,0]$ (p,p)	$\Omega = [0,0,0,1]$ (d,d)
$\widetilde{C}$	-0.2715	-0.0004
$\sigma^2_{\widetilde{C}}$	0.0047	0.0016
$\widetilde{L}$	-0.0035	-0.0009
$\sigma^2_{\widetilde{L}}$	9.16×10^{-4}	9.2×10^{-4}
$\widetilde{MC}$	0.0036	0.0018
$\sigma^2_{\widetilde{MC}}$	0.0026	3.4×10^{-5}
$\widetilde{P}$	0.0048	0.0018
$\widetilde{W}$	0.0105	0.0039

[e] $\widetilde{X} = EX_t - \bar{X}$, where $\bar{X}$ is the steady state variable of X.

Table A.8: Welfare Comparison with Increasing ω [f]
($\gamma = 900$)

ζ	$\omega = 0.36$	$\omega = 0.4$	$\omega = 0.45$	$\omega = 0.48$
ζ^{pp}	-0.16%	-0.223%	-0.46%	-0.85%
ζ^{dd}	-0.0997%	-0.10%	-0.101%	-0.102%
$\zeta^{dd} - \zeta^{pp}$	0.061%	0.091%	0.359%	0.748%

[f] Note that, ζ^{pp} and ζ^{dd} represent the measure of welfare for the two extreme cases, $\Omega = [1,0,0,0]$ and $\Omega = [0,0,0,1]$, respectively. $\zeta^{dd} - \zeta^{pp}$ can be defined as the welfare gain of twin dollarization under a fixed exchange rate regime.

Appendix B

Appendices of Chapter 2

B.1 Proof for Proposition 2

Firstly, we rewrite the equations (2.2.8), (2.2.11), and (2.2.12) in log term.

$$s = \log \Gamma + u - u^* \tag{B.1.1}$$

$$mc = (1-\alpha)\log\frac{\eta}{\chi} + (1-\alpha)u + \alpha q \tag{B.1.2}$$

$$mc^* = (1-\alpha)\log\frac{\eta}{\chi} + (1-\alpha)u^* + \alpha(q-s) \tag{B.1.3}$$

where small-case letters denote logarithms. Using the fact that $cov(q,u) = cov(q,u^*) = cov(u,u^*) = 0$. Equations (B.1.1)-(B.1.3) give us:

$$\frac{1}{2}\sigma_s^2 = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) \tag{B.1.4}$$

$$cov(mc,s) = (1-\alpha)\sigma_u^2 \tag{B.1.5}$$

$$cov(mc^*,s) = -(1+\alpha)\sigma_{u^*}^2 + \alpha(\sigma_u^2 + \sigma_{u^*}^2) \tag{B.1.6}$$

So from Equation (2.2.19) and (2.2.20), we have the expected profit differentials Ω and Ω^*

$$\Omega = \frac{1}{2}\sigma_s - cov(mc,s) = (\alpha - \frac{1}{2})\sigma_u^2 + \frac{1}{2}\sigma_{u^*}^2 \tag{B.1.7}$$

$$\Omega^* = \frac{1}{2}\sigma_s + cov(mc^*,s) = (\frac{1}{2} - \alpha)\sigma_u^2 - \frac{1}{2}\sigma_{u^*}^2 \tag{B.1.8}$$

If $\sigma_u^2 = \sigma_{u^*}^2$, then we have

$$\Omega = \frac{1}{2}\sigma_s - cov(mc,s) = \alpha\sigma_u^2 > 0 \tag{B.1.9}$$

$$\Omega^* = \frac{1}{2}\sigma_s + cov(mc^*,s) = \alpha\sigma_u^2 < 0 \tag{B.1.10}$$

If $\sigma_u^2 \neq \sigma_{u^*}^2$, the condition to hold the above results need

$$\frac{\sigma_u^2}{\sigma_{u^*}^2} < \frac{1}{1-2\alpha} \tag{B.1.11}$$

QED.

B.2 The Labor Market Clearing Condition

Given the production technology for firms, we can write the labor market clearing condition in home country as

$$L = (1-\alpha)\frac{MC}{W}Y \tag{B.2.1}$$

and

$$Y = n\frac{PC}{P_{hh}} + (1-n)\frac{P^*C^*}{\frac{P_{hf}}{S}} \tag{B.2.2}$$

Using the implicit labor supply condition $W = \eta PC^{\rho}$ and the pricing equation for home goods in home market and foreign market,

$$P_{hh} = \hat{\lambda}\frac{E[MCC^{1-\rho}]}{E[C^{1-\rho}]} \tag{B.2.3}$$

$$P_{hf} = \hat{\lambda}\frac{E[MCC^{*1-\rho}]}{E[C^{*1-\rho}]} \tag{B.2.4}$$

we can rewrite the labor market clearing condition as

$$L = (1-\alpha)n\frac{MC}{\eta PC^{\rho}}\frac{PCE[C^{1-\rho}]}{\hat{\lambda}E[MCC^{1-\rho}]} + (1-\alpha)(1-n)\frac{MC}{\eta PC^{\rho}}\frac{SP^*C^*E[C^{*1-\rho}]}{\hat{\lambda}E[MCC^{*1-\rho}]} \tag{B.2.5}$$

Using the risk-sharing condition that $\Gamma PC^{\rho} = SP^*C^{*\rho}$ and taking the expectation, we may have

$$EL = (1-\alpha)\{\frac{n}{\hat{\lambda}\eta}E[C^{1-\rho}] + \frac{1-n}{\hat{\lambda}\eta}E[C^{*1-\rho}]\Gamma\} \tag{B.2.6}$$

B.3 Expected Utility for the Special Case $\rho = 1$

From the price index and pricing equation, we have

$$P = {P_{hh}}^n {P_{fh}}^{1-n} = \hat{\lambda}E[MC]^n E[MC^*S]^{1-n} \tag{B.3.1}$$

Since

$$MC = W^{1-\alpha}Q^{\alpha} = (\frac{\eta}{\chi})^{1-\alpha}M^{1-\alpha}Q^{\alpha} \tag{B.3.2}$$

$$MC^* = W^{*1-\alpha}(\frac{Q}{S})^{\alpha} = (\frac{\eta}{\chi})^{1-\alpha}M^{*1-\alpha}(\frac{Q}{S})^{\alpha} \tag{B.3.3}$$

Thus,

$$P = \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha}E[M^{1-\alpha}Q^{\alpha}]^n E[M^{*1-\alpha}Q^{\alpha}S^{1-\alpha}]^{1-n} = \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha}E[M^{1-\alpha}Q^{\alpha}] \tag{B.3.4}$$

Using the log normal property of these shocks and taking log, we have

$$Ep = p = \ln\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{(1-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 \tag{B.3.5}$$

The price index in foreign country is given by

$$P^* = [\frac{P_{hf}}{S}]^n (P_{ff})^{1-n} \tag{B.3.6}$$

We denote $P^* = \frac{Z}{S^n}$, where

$$Z = P_{hf}^n (P_{ff})^{1-n} = \hat{\lambda} E[MC]^n E[MC^*]^{1-n} \tag{B.3.7}$$

We can rewrite it as

$$Z = \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} E[M^{1-\alpha} Q^\alpha]^n E[M^{*1-\alpha} Q^\alpha S^{-\alpha}]^{1-n} \tag{B.3.8}$$

Thus

$$\ln Z = \ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{(1-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 + (1-n)\alpha\sigma_u^2 \tag{B.3.9}$$

Since $p^* = \ln Z - n \ln S$, we have

$$Ep^* = \ln Z = \ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{(1-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 + (1-n)\alpha\sigma_u^2 \tag{B.3.10}$$

So that, we have

$$Ep < Ep^*, \qquad EU > EU^* \tag{B.3.11}$$

QED.

B.4 Proof for Proposition 3

When $\rho > 1$, using $\Gamma = \frac{EC^{(1-\rho)}}{EC^{*(1-\rho)}}$, we can simplify EU and EU^* as :

$$EU = \frac{\lambda - (\lambda-1)(1-\rho)}{(1-\rho)\lambda} EC^{(1-\rho)} \tag{B.4.1}$$

$$EU^* = \frac{\lambda - (\lambda-1)(1-\rho)}{(1-\rho)\lambda} EC^{*(1-\rho)} \tag{B.4.2}$$

Since $\frac{\lambda-(\lambda-1)(1-\rho)}{(1-\rho)\lambda}$ is negative, to prove $EU > EU^*$ is equivalent to proving $\Gamma < 1$ or $EC^{(1-\rho)} < EC^{*(1-\rho)}$. Thus, we only need to show the following condition holds

$$Ec + \frac{1-\rho}{2}\sigma_c^2 > Ec^* + \frac{1-\rho}{2}\sigma_{c^*}^2 \tag{B.4.3}$$

From the Equation (2.3.14) and (2.3.15), we have

$$\sigma_c^2 = \frac{1}{\rho^2}\sigma_u^2 \tag{B.4.4}$$

$$\sigma_{c^*}^2 = \frac{1-2n(1-n)}{\rho^2}\sigma_u^2 \tag{B.4.5}$$

From home and foreign money demand functions, we have

$$Ec - Ec^* = \frac{1}{\rho}(Ep^* - Ep) \tag{B.4.6}$$

From the pricing equation and price index, we have

$$P = \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha}\frac{E[M^{1-\alpha}Q^{\alpha}C^{1-\rho}]^n E[M^{*1-\alpha}Q^{\alpha}S^{1-\alpha}C^{1-\rho}]^{1-n}}{E[C^{1-\rho}]} \tag{B.4.7}$$

$$P^* = S^{-n}\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha}\frac{E[M^{1-\alpha}Q^{\alpha}C^{*1-\rho}]^n E[M^{*1-\alpha}Q^{\alpha}S^{-\alpha}C^{*1-\rho}]^{1-n}}{E[C^{*1-\rho}]} \tag{B.4.8}$$

Using the log normal distribution property of underline shocks, we can derive

$$Ep = \ln\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{(\frac{1}{\rho}-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 - \frac{(\frac{1}{\rho}-1)^2}{2}\sigma_u^2 \tag{B.4.9}$$

$$Ep^* = \ln\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{(\frac{1}{\rho}-\alpha)^2}{2}\sigma_u^2 + \frac{\alpha^2}{2}\sigma_q^2 - \frac{(\frac{1}{\rho}-1)^2}{2}\sigma_u^2 + [\frac{2n(1-n)(\rho-1)+(1-n)\alpha}{\rho}]\sigma_u^2 \tag{B.4.10}$$

Thus,

$$Ec - Ec^* = \frac{1}{\rho^2}[2n(1-n)(\rho-1)+(1-n)\alpha]\sigma_u^2 \tag{B.4.11}$$

and

$$\frac{1-\rho}{2}(\sigma_{c^*}^2 - \sigma_c^2) = \frac{1}{\rho^2}(\rho-1)n(1-n)\sigma_u^2 \tag{B.4.12}$$

$$EU - EU^* = \frac{n(1-n)(\rho-1)+(1-n)\alpha}{\rho}\sigma_u^2 \tag{B.4.13}$$

Thus, Equation (B.4.3) holds. QED

B.5 The Case with Productivity Shocks and Active Monetary Authorities

Given the production technology with country-specific shock, the marginal cost for home and foreign firm in log terms are, respectively,

$$mc = (1-\alpha)\ln\frac{\eta}{\chi} + (1-\alpha)(m-z) + \alpha q \tag{B.5.1}$$

$$mc^* = (1-\alpha)\ln\frac{\eta}{\chi} + (1-\alpha)(m^*-z^*) + \alpha(q-s) \tag{B.5.2}$$

We express the log marginal cost and log exchange rate as a function of policy parameters {a, b}:

$$s = \ln\Gamma + (a_0-b_0)q + (a_1-b_1)z + (a_2-b_2)z^* + u - u^* \tag{B.5.3}$$

$$mc = (1-\alpha)\ln\frac{\eta}{\chi} + (1-\alpha)(a_0 q + (a_1 - 1)z + a_2 z^* + u) + \alpha q \tag{B.5.4}$$

$$mc^* = (1-\alpha)\ln\frac{\eta}{\chi} + (b_0 - \alpha a_0 + \alpha)q + [b_1 - \alpha a_1]z + [b_2 - \alpha a_2 - (1-\alpha)]z^* + u^* - \alpha u \tag{B.5.5}$$

This gives the following variance terms.

$$\frac{1}{2}\sigma_s^2 = \frac{1}{2}[(a_0 - b_0)^2\sigma_q^2 + (a_1 - b_1)^2\sigma_z^2 + (a_2 - b_2)^2\sigma_{z^*}^2 + \sigma_u^2 + \sigma_{u^*}^2] \tag{B.5.6}$$

$$cov(mc, s) = (a_0 - b_0)[(1-\alpha)a_0 + \alpha]\sigma_q^2 + (1-\alpha)(a_1 - b_1)(a_1 - 1)\sigma_z^2 + (1-\alpha)(a_2 - b_2)a_2\sigma_{z^*}^2 + (1-\alpha)\sigma_u^2 \tag{B.5.7}$$

$$cov(mc^*, s) = (b_0 - \alpha a_0 + \alpha)(a_0 - b_0)\sigma_q^2 + (a_1 - b_1)(b_1 - \alpha a_1)\sigma_z^2 + (a_2 - b_2)(b_2 - \alpha a_2 - (1-\alpha))\sigma_{z^*}^2 - (1+\alpha)\sigma_u^2 \tag{B.5.8}$$

Thus, the expected profit differentials between PCP and LCP for home and foreign firms are given by:

$$\begin{aligned}\Omega(a,b) &= \frac{1}{2}(a_0 - b_0)[a_0 - b_0 - 2((1-\alpha)a_0 + \alpha)]\sigma_q^2 + \frac{1}{2}(a_1 - b_1)[a_1 - b_1 - 2(1-\alpha)(a_1 - 1)]\sigma_z^2 \\ &+ \frac{1}{2}(a_2 - b_2)[a_2 - b_2 - 2(1-\alpha)a_2)]\sigma_{z^*}^2 + \alpha\sigma_u^2 \end{aligned} \tag{B.5.9}$$

$$\begin{aligned}\Omega^*(a,b) &= \frac{1}{2}(a_0 - b_0)[a_0 - b_0 + 2(b_0 - \alpha a_0 + \alpha)]\sigma_q^2 + \frac{1}{2}(a_1 - b_1)[a_1 - b_1 + 2(b_1 - \alpha a_1)]\sigma_z^2 \\ &+ \frac{1}{2}(a_2 - b_2)[a_2 - b_2 + 2(b_2 - \alpha a_2 - (1-\alpha))]\sigma_{z^*}^2 - \alpha\sigma_u^2 \end{aligned} \tag{B.5.10}$$

Since the solution to the symmetric case PCP and LCP are the same as Devereux and Engel (2003), we focus on the asymmetric case (PCP,LCP). When $\rho = 1$, EU is equivalent to $-Ep$ and EU^* is equivalent to $-Ep^*$. Thus, we can express the expected utilities for home households and foreign households as a function of monetary policy feedback parameters {a, b}. [1]

$$\begin{aligned}EU = \quad & -\{\ln\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{n}{2}[((1-\alpha)a_0 + \alpha)^2\sigma_q^2 + (1-\alpha)^2(a_1 - 1)^2\sigma_z^2 \\ & +(1-\alpha)^2 a_2^2\sigma_{z^*}^2 + (1-\alpha)^2\sigma_u^2] + \frac{1-n}{2}[((1-\alpha)a_0 + \alpha)^2\sigma_q^2 \\ & +(1-\alpha)^2 a_1^2\sigma_z^2 + (1-\alpha)^2(a_2 - 1)^2\sigma_{z^*}^2 + (1-\alpha)^2\sigma_u^2]\} \end{aligned} \tag{B.5.11}$$

$$\begin{aligned}EU^* = \quad & -\{\ln\hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + \frac{n}{2}[((1-\alpha)a_0 + \alpha)^2\sigma_q^2 + (1-\alpha)^2(a_1 - 1)^2\sigma_z^2 \\ & +(1-\alpha)^2 a_2^2\sigma_{z^*}^2 + (1-\alpha)^2\sigma_u^2] + \frac{1-n}{2}[(-\alpha a_0 + b_0 + \alpha)^2\sigma_q^2 \\ & +(-\alpha a_1 + b_1)^2 a_1^2\sigma_z^2 + (-\alpha a_2 + b_2 - (1-\alpha))^2\sigma_{z^*}^2 + \sigma_u^2 + \alpha^2\sigma_u^2]\} \end{aligned} \tag{B.5.12}$$

[1] Note that, we have assumed $\sigma_z^2 = \sigma_{z^*}^2$.

The Nash solution to the international monetary game can be derived as

$$a_0 = -\frac{\alpha}{1-\alpha}, \qquad a_1 = n, \qquad a_2 = 1-n \tag{B.5.13}$$

$$b_0 = -\frac{\alpha}{1-\alpha}, \qquad b_1 = n\alpha, \qquad a_2 = 1-n\alpha \tag{B.5.14}$$

It is very straightforward that the monetary solutions support (PCP,LCP). We may derive the expected utilities for home and foreign households as:

$$EU = -\{\ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + n(1-n)(1-\alpha)^2\sigma_z^2 + \frac{(1-\alpha)^2}{2}\sigma_u^2\} \tag{B.5.15}$$

$$EU^* = -\{\ln \hat{\lambda}(\frac{\eta}{\chi})^{1-\alpha} + n(1-n)^2(1-\alpha)^2\sigma_z^2 + \frac{n(1-\alpha)^2}{2}\sigma_u^2 + \frac{(1-n)(1+\alpha^2)}{2}\sigma_u^2\} \tag{B.5.16}$$

This gives the welfare gain for home households:

$$EU - EU^* = -n^2(1-n)(1-\alpha)^2\sigma_z^2 + (1-n)\alpha\sigma_u^2 \tag{B.5.17}$$

Table B.1: The Optimal Monetary Parameters in Three Cases

Parameters	(PCP,LCP)	(LCP,LCP)	(PCP,LCP)
a_0	$-\frac{\alpha}{1-\alpha}$	$-\frac{\alpha}{1-\alpha}$	$-\frac{\alpha}{1-\alpha}$
a_1	1	n	n
a_2	0	1-n	1-n
b_0	$-\frac{\alpha}{1-\alpha}$	$-\frac{\alpha}{1-\alpha}$	$-\frac{\alpha}{1-\alpha}$
b_1	0	n	$n\alpha$
b_2	1	1-n	$1-n\alpha$

Appendix C

Appendices of Chapter 3

C.1 Firms

C.1.1 Preset Prices

Listed in Table C.1.

C.1.2 Ω and Ω^*

Here we express the expected profit differential between PCP and LCP for home firms and foreign firms Ω and Ω^* as functions of the underling monetary policy parameters respectively. If the monetary authorities follow the unrestricted money rules, we have

$$\sigma_s^2 = (a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_{u^*}^2. \tag{C.1.1}$$

$$cov(mc, s) = (a_1 - b_1)(a_1 - 1)\sigma_u^2 + (a_2 - b_2)a_2\sigma_{u^*}^2. \tag{C.1.2}$$

$$cov(mc^*, s) = (a_1 - b_1)b_1\sigma_u^2 + (a_2 - b_2)(b_2 - 1)\sigma_{u^*}^2. \tag{C.1.3}$$

Thus,

$$\Omega = \frac{1}{2}(a_1 - b_1)(2 - a_1 - b_1)\sigma_u^2 + \frac{1}{2}(a_2 - b_2)(-a_2 - b_2)\sigma_{u^*}^2. \tag{C.1.4}$$

$$\Omega^* = \frac{1}{2}(a_1 - b_1)(a_1 + b_1)\sigma_u^2 + \frac{1}{2}(a_2 - b_2)(a_2 + b_2 - 2)\sigma_{u^*}^2. \tag{C.1.5}$$

Therefore, under (PCP, PCP) specification, the solution to international monetary game is $a = [1, 0]$ and $b = [0, 1]$, we have

$$\Omega[(1,0),(0,1)] = \Omega^*[(1,0),(0,1)] = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{C.1.6}$$

Under (LCP, LCP) specification, the solution to international monetary game is $a = [n, 1-n]$ and $b = [n, 1-n]$, we have

$$\Omega[(n,1-n),(n,1-n)] = \Omega^*[(n,1-n),(n,1-n)] = 0 \tag{C.1.7}$$

Under (PCP, LCP) specification, the solution to international monetary game is a little complicated, see Devereux, Shi and Xu (2004) for more detail. The optimal monetary policy rules in this case have the following properties:

$$n \le a_1 < 1, \quad 0 < a_2 \le (1-n), \quad b_1 \le 0, \quad b_2 \ge 1 \tag{C.1.8}$$

$$a_1 + a_2 = 1, \qquad b_1 + b_2 = 1 \tag{C.1.9}$$

which give us

$$\Omega[a,b] = \frac{1}{2}(a_1 - b_1)(2 - a_1 - b_1)(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{C.1.10}$$

$$\Omega^*[a,b] = \frac{1}{2}(a_1 - b_1)(a_1 + b_1)(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{C.1.11}$$

This is because $a_1 - b_1 > 0$ and $2 - a_1 - b_1 > 0$ and $a_1 + b_1 > 0$. But, if monetary authorities follow restricted money rules, we then have

$$\Omega = \frac{1}{2}a(2-a)\sigma_u^2 + \frac{1}{2}b^2\sigma_{u^*}^2 \tag{C.1.12}$$

$$\Omega^* = \frac{1}{2}a^2\sigma_u^2 + \frac{1}{2}b(2-b)\sigma_{u^*}^2 \tag{C.1.13}$$

C.2 The Proof of Binary Game

The binary game where each monetary authority chooses its policy before the currency of pricing decision is made is defined in the matrix of chapter 3. From this binary game, it can be shown that the {PCP, PCP} configuration with the monetary rules (a=[1,0], b=[0,1]) and flexible exchange rates represent the unique equilibrium of the monetary policy game when monetary authorities take account of the currency of pricing. The proof will be straightforward if we can show the following payoff inequalities

$$EU^{PCP}_{[1,0],[0,1]} > EU^{LCP}_{[n,1-n],[n,1-n]} \tag{C.2.1}$$

$$EU^{PCP}_{[1,0],[n,1-n]} > EU^{LCP}_{[n,1-n],[n,1-n]} \tag{C.2.2}$$

C.2.1 follows directly from the comparison between 3.3.8 and 3.3.17. To show C.2.2, suppose that the foreign monetary authority follows $b = [n, 1-n]$, then if the home country chooses $a = [1,0]$ and given the {a=([1,0], b=[n,1-n]} configuration, we have

$$\Omega > 0, \qquad \Omega^* > 0 \tag{C.2.3}$$

So, all the firms will follow PCP. So under this situation, the expected utility for home and foreign country are given by

$$EU^{pcp}_{[1,0],[n,1-n]} = \Theta \exp(1-\rho)\frac{A_1\sigma_u^2 + A_2\sigma_{u^*}^2}{2\rho^2} \tag{C.2.4}$$

where $A_1 = -n^2(1-n)^2 - n(1-n)^3\rho - n\rho + n^2 + 2n\rho(1-n)^2$ and $A_2 = -(1-n)^4 - n(1-n)^3\rho - (1-n)\rho + 2(1-n)^3 + 2n(1-n)^2\rho$.

Nevertheless, if the home monetary authority choose $a = [n, 1-n]$ instead of $a = [1,0]$, then the home and foreign monetary rules are the optimal rules associated with the equilibrium of {LCP, LCP}, so the expected utility for this case is

$$EU^{Lcp}_{[n,1-n],[n,1-n]} = \Theta \exp\{(1-\rho)\frac{(n^2 - \rho n)\sigma_u^2 + ((1-n)^2 - (1-n)\rho)\sigma_{u^*}^2}{2\rho^2}\} \tag{C.2.5}$$

We can show

$$A_1 - (n^2 - n\rho) = n(1-n)^2(\rho + \rho n - n) > 0 \tag{C.2.6}$$

$$A_2 - [(1-n)^2 - (1-n)\rho] = n(1-n)^2(\rho + \rho n - n) > 0 \tag{C.2.7}$$

Therefore, C.2.2 holds.

Thus, say that foreign is following $b = [n, 1-n]$. Then if home follows $a = [1, 0]$, it generates a switching to all PCP, and then $a = [1, 0]$ is clearly better for home since C.2.2 holds. Similarly, if foreign is following $b = [0, 1]$, then the optimal policy for home is to follow $a = [1, 0]$. So the {PCP, PCP} configuration with the monetary rules (a=[1,0], b=[0,1]) and flexible exchange rates represent the unique equilibrium of the ex-ante monetary policy game, which also supports the full flexible price equilibrium.

C.3 Equilibrium under Restricted Monetary Policy Rules

We now use the same approach to find the equilibrium under restricted monetary policy rules. The restriction of the monetary policy rules will affect the level of the expected utility by changing the variance and covariance terms.

C.3.1 Case 1 {PCP,PCP}

First, we may derive the exchange rate and consumption in log terms:

$$s - E(s) = m - m^* \tag{C.3.1}$$

$$c - Ec = \frac{nm + (1-n)m^*}{\rho} \tag{C.3.2}$$

Under the restricted monetary policy rules, from C.3.1 and C.3.2, we can derive the following variance and covariance terms.

$$\sigma_s^2 = a^2\sigma_u^2 + b^2\sigma_{u^*}^2, \qquad \sigma_c^2 = \frac{n^2a^2\sigma_u^2 + (1-n)^2b^2\sigma_{u^*}^2}{\rho^2}, \tag{C.3.3}$$

$$\sigma_{cu} = \frac{na\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = \frac{(1-n)b\sigma_{u^*}^2}{\rho} \tag{C.3.4}$$

$$\sigma_{su} = a\sigma_u^2, \qquad \sigma_{su^*} = -b\sigma_{u^*}^2 \tag{C.3.5}$$

Substituting the above terms into 3.3.3, and solving the international monetary game, we have

$$a = 1, \qquad b = 1 \tag{C.3.6}$$

Then using C.1.12 and C.1.13, we find

$$\Omega = \Omega^* = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{C.3.7}$$

Thus, all firms choosing PCP is an equilibrium and the optimal monetary policy associated with this equilibrium ensures the flexible exchange rate where $s - E(s) = u - u^*$. The expected utility associated with this equilibrium is exactly the same as in 3.3.8.

C.3.2 Case 2 {LCP,LCP}

Under LCP, we may get the home and foreign countries log consumption:

$$c - Ec = \frac{m}{\rho} \qquad c^* - Ec^* = \frac{m^*}{\rho} \tag{C.3.8}$$

Under the restricted monetary policy rule, from C.3.8, we can derive the following variance and covariance terms.

$$\sigma_c^2 = \frac{a^2\sigma_u^2}{\rho^2}, \qquad \sigma_{c^*}^2 = \frac{b^2\sigma_{u^*}^2}{\rho^2}, \tag{C.3.9}$$

$$\sigma_{cu} = \frac{a\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = 0 \tag{C.3.10}$$

$$\sigma_{c^*u} = 0, \qquad \sigma_{c^*u^*} = \frac{b\sigma_{u^*}^2}{\rho} \tag{C.3.11}$$

Using a similar approach as in above sections, we can derive

$$a = n, \qquad b = 1 - n \tag{C.3.12}$$

This gives us

$$\Omega = \frac{n(2-n)}{2}\sigma_u^2 + \frac{(1-n)^2}{2}\sigma_{u^*}^2 > 0, \qquad \Omega^* = \frac{n^2}{2}\sigma_u^2 + \frac{1-n^2}{2}\sigma_{u^*}^2 > 0 \tag{C.3.13}$$

This implies, all the firms will follow PCP. Thus {LCP, LCP} is not an equilibrium when monetary authorities can only use the restricted money rules.

C.3.3 Case 3 {PCP, LCP}

Under this asymmetric pricing specification, the exchange rate and consumption in log terms are:

$$s - Es = m - m^* \tag{C.3.14}$$

$$c - Ec = \frac{m}{\rho} \qquad c^* - Ec^* = \frac{1}{\rho}[nm + (1-n)m^*] \tag{C.3.15}$$

Under restricted monetary policy rules, we may solve for the variances and covariances terms.

$$\sigma_s^2 = a^2\sigma_u^2 + b^2\sigma_{u^*}^2 \tag{C.3.16}$$

$$\sigma_c^2 = \frac{a^2\sigma_u^2}{\rho^2} \tag{C.3.17}$$

$$\sigma_{c^*}^2 = \frac{1}{\rho^2}[n^2a^2\sigma_u^2 + (1-n)^2b^2\sigma_{u^*}^2] \tag{C.3.18}$$

$$\sigma_{cu}^2 = \frac{1}{\rho}a\sigma_u^2, \qquad \sigma_{cu^*}^2 = 0 \tag{C.3.19}$$

$$\sigma_{c^*u}^2 = \frac{na}{\rho}\sigma_u^2, \qquad \sigma_{c^*u^*}^2 = \frac{(1-n)b}{\rho}\sigma_{u^*}^2 \tag{C.3.20}$$

$$\sigma_{su} = a\sigma_u^2, \qquad \sigma_{su^*} = -b\sigma_{u^*}^2. \tag{C.3.21}$$

Using the same methodology, we can derive the following solution

$$a = n\frac{\hat{\lambda} - (1-\rho)[n + (1-n)(\rho - \rho n + n)]}{\hat{\lambda} - n(1-\rho)[1 + (1-n)(\rho - \rho n + n)]}, \qquad b = 1 \tag{C.3.22}$$

It can be shown that $n \leq a < 1$. [1] It gives us

$$\Omega = \frac{a(2-a)}{2}\sigma_u^2 + \frac{1}{2}\sigma_{u^*}^2 > 0, \qquad \Omega^* = \frac{a^2}{2}\sigma_u^2 + \frac{1}{2}\sigma_{u^*}^2 > 0 \tag{C.3.23}$$

In each country, firms would wish to set prices in their own currency. Hence, the {PCP, LCP} configuration is *not* an equilibrium if restricted monetary policy rules are chosen by the monetary authorities, taking the currency of pricing as given.

[1] In the special case with $\rho = 1$, we have $a = n$.

Table C.1: Three Optimal Pricing Policies

Price	{PCP, PCP}	{LCP, LCP}	{PCP, LCP}
P_{hh}	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$
P^*_{hf}	$\frac{P_{hh}}{S}$	$\hat{\lambda}\frac{E[\frac{WC^{*1-\rho}S^{-1}}{\theta}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{S^{-1}E[\frac{WC^{*1-\rho}}{\theta}]}{E[C^{*1-\rho}]}$
P^*_{ff}	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$
P_{fh}	SP^*_{ff}	$\hat{\lambda}\frac{E[\frac{SW^*C^{1-\rho}}{\theta^*}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{SW^*C^{1-\rho}}{\theta^*}]}{E[C^{1-\rho}]}$