THE DEVELOPMENT OF NUMERACY: A LONGITUDINAL STUDY
OF CHILDREN FROM FIRST THROUGH FOURTH GRADE

by

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Abstract

The present study investigated the development of numeracy in children from first to fourth grade and was based on a model from research in literacy (Juel, 1988). The present study sought to answer the following questions: Do the same children struggle with numeracy year after year? What skills do children with numeracy difficulties lack? What underlying factors support numeracy achievement? What are the early indicators of numeracy difficulties? The results indicated that the majority of fourth grade children with numeracy difficulties experienced similar difficulties in previous years, indicating a stability of functioning in this area. Furthermore, working memory and mathematical background knowledge were found to have a robust relationship with numeracy achievement and numeracy difficulties, whereas visual-spatial ability, short-term memory, processing speed, and phonological processing did not demonstrate a reliable relationship with numeracy. Together, these results indicate that working memory and math background knowledge are involved in numeracy development in children from first to fourth grade and that across this time period, children with numeracy difficulties struggle with tasks that require complex numerical processing. Educational implications are discussed.
# Table of Contents

Abstract ............................................................................................................... ii

Table of Contents ............................................................................................ iii

List of Tables ..................................................................................................... iv

List of Figures .................................................................................................. v

Acknowledgements .......................................................................................... vi

Dedication .......................................................................................................... vii

Introduction ....................................................................................................... 1

The Simple View of Numeracy ........................................................................ 2
Underlying Processes Implicated in Numeracy Development ....................... 5
Present Study ..................................................................................................... 12

Method ............................................................................................................. 12

Participants ...................................................................................................... 12
Materials .......................................................................................................... 13
Procedure ......................................................................................................... 17

Results ............................................................................................................. 18

Do the same children struggle with numeracy year after year? ................. 18
What skills do children with numeracy difficulties lack? ......................... 21
What underlying factors support numeracy achievement? ....................... 23
What are the early indicators of numeracy difficulties? ........................... 28

Discussion ....................................................................................................... 29

Educational Implications ............................................................................... 35
Limitations ....................................................................................................... 38

References ....................................................................................................... 53
List of Tables

Table 1 Grade 4 Numeracy Outcomes Based on First to Third Grade Classifications ..... 41
Table 2 Descriptive Characteristics of Children with Numeracy Difficulties per Grade ..... 42
Table 3 Means, Standard Deviations, and Effect Sizes across Measures and Years.......... 43
Table 4 Correlations among Variables Involved in First and Second Grade Analyses....... 45
Table 5 Correlations among Variables Involved in Third and Fourth Grade Analyses ...... 46
Table 6 Multiple Regression of First to Fourth Grade Calculation Skills ......................... 47
Table 7 Multiple Regression of First to Fourth Grade Problem Solving .......................... 48
Table 8 Relative Risk of Developing Numeracy Difficulties Based on Grade 1 Measures . 49
List of Figures

Figure 1 Task Requirements of WJ-III Block Rotation Test .................................................... 50
Figure 2 Percentage of Numeracy Difficulties per Grade .............................................................. 51
Figure 3 Numeracy Achievement of Children with Numeracy Difficulties .............................. 52
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Dedication

To Stan Chung - the journey began with the Power Sentence
Introduction

This study examined the development of numeracy in a cohort of children from first through fourth grade with the specific goal of answering the following questions: Do the same children struggle with numeracy year after year? If so, what skills do children with numeracy difficulties lack? What underlying factors support numeracy achievement? What are the early indicators of numeracy difficulties? This study was modelled after a study that investigated the development of literacy in children from first through fourth grade (Juel, 1988). Juel’s study was important in demonstrating the persistent nature of reading difficulties in children identified as at-risk for reading failure in first grade, as well as identifying some of the factors that contributed to poor literacy acquisition. In the same way, the present study examined the longitudinal nature of numeracy difficulties in order to identify some of the factors that contributed to the numeracy difficulties faced by children in their early school years.

Most definitions of learning disabilities (LD) include numeracy as a potential source of difficulty for individuals with LD (e.g., Learning Disabilities Association of Canada, 2002; National Joint Committee On Learning Disabilities, 1998). Despite this, there is little understanding about numeracy difficulties. In fact, compared to the research on reading difficulties, research on numeracy difficulties is scant and researchers are still trying to determine how best to measure and define numeracy difficulties, whether there are core deficits involved in numeracy difficulties, and how to reliably identify children at-risk for numeracy difficulties in the early stages of formal math instruction.

Given the well-developed research programs and successes in the area of reading, it has been suggested that modelling reading research can lead to advances in our knowledge of
numeracy development and difficulties (Chiappe, 2005; Kulak, 1993). The present study took such an approach by applying a well-known reading paradigm - the Simple View of reading (Juel, 1988; Juel, Griffith, & Gough, 1986) to the study of numeracy. The Simple View of reading conceptualizes reading as consisting of only two components: decoding and comprehension. Although there is recognition in the Simple View paradigm that both decoding and comprehension are complex and may be separated into many subcomponents, only decoding and comprehension seem necessary to characterize reading. From such a simple framework, it is possible to study the underlying factors associated with literacy development at the word and text levels. In fact, a great deal has been learned about reading acquisition by studying reading globally at the word and text level. Such research has led to significant improvements in the ability to prevent reading difficulties and to provide effective interventions for those children who struggle to acquire age-appropriate literacy skills (e.g., Snow, Burns, & Griffin, 1998).

*The Simple View of Numeracy*

In a similar way to Juel’s (1988) study, the present study took a ‘simple view’ of numeracy to study children’s development in this area. In order to propose a simple view of numeracy, it was necessary to conceptualize mathematical cognition as representing a primary lower-level skill and a primary higher-order skill that together make up the critical components of numeracy. Although such critical components have yet to be identified in the numeracy literature, for the purposes of the present study, these two components were computational arithmetic (lower-level skill) and mathematical problem solving (higher-order skill). Thus, the present study was guided by the assumption that being numerate hinges upon well-developed calculation and mathematical problem solving skills; in other words, those
with numeracy difficulties must be either poor calculators or poor mathematical problem solvers.

There is much evidence that individuals with low numeracy achievement have difficulties with computational arithmetic (e.g., Geary 1990; Geary, Bow-Thomas, & Yao, 1992; Geary, Brown, & Samaranayake, 1991; Geary, Hoard, & Hamson, 1999; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000; Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, & Hanich, 2002; Jordan & Montani, 1997). For example, Hanich et al. found that second grade children with numeracy difficulties performed worse than typically achieving children on tests of oral and written computations. In a follow-up study, children with low math achievement continued to have difficulties with written computations in third grade (Jordan et al., 2003).

There is also evidence that mathematical problem solving is a significant source of difficulty for children with low numeracy achievement (e.g., Fuchs & Fuchs, 2002; Fuchs, Fuchs, & Prentice, 2004; Hanich et al., 2001; Jordan et al., 2002, 2003; Jordan & Hanich, 2000; Jordan & Montani, 1997). For example, Fuchs and Fuchs found that fourth grade children with low numeracy had significant difficulty with simple story problems (i.e., one-step problems, sums or minuends of nine or less), complex story problems (i.e., multi-step problems containing irrelevant information), and real-world mathematical problems (e.g., students are required to solve problems involving tabular and graphical information). Jordan and Hanich found that compared to typically achieving peers, second grade children with low numeracy had significant difficulty solving mathematical story problems including change problems (e.g., Maggie has 5 candies. Then Joanna gave her 2 more candies. How many candies does Maggie have now?), combine problems (e.g., Julie has 3 candies. Matt has 4
candies. How many candies do they have altogether?), compare problems (e.g., Kevin has 8 candies. Veronica has 3 candies. How many more candies does Kevin have than Veronica?), and equalize problems (e.g., Walter has 2 candies. Mary has 9 candies. How many candies does Walter need to have as many as Mary?).

Given the evidence that supports that children with low numeracy achievement have difficulties with calculation and mathematical problem solving, these aspects of mathematical cognition appear to be an appropriate starting point from which to examine the underlying sources of difficulty in children with numeracy difficulties using a simple view paradigm. It is important to note that although representing numeracy as calculation and problem solving may appear overly simplistic especially considering the diversity of mathematical domains (Geary, 2004), calculation and problem solving skills are typically considered foundational numeracy skills that support the acquisition of more complex and sophisticated mathematical concepts. Furthermore, it is worth recognizing that reading research has not been hindered by characterizing reading as word recognition and reading comprehension. Thus, a simple view of numeracy was adopted in the current study in order to determine whether such a paradigm was useful in understanding the underlying sources of difficulty in children with numeracy difficulties.

Before a discussion of the underlying processes implicated in numeracy, it is important to briefly address the subtype research that is sometimes conducted in the area of numeracy difficulties so that the present study can be placed appropriately. Some researchers studying numeracy make a distinction between those with specific numeracy difficulties (MD-only) and those with both numeracy and reading difficulties (MD/RD) and there is evidence that these groups present with somewhat distinct profiles (e.g., Badian, 1999; Fuchs...
& Fuchs, 2002; Geary et al., 1999; Rourke, 1993; Rourke & Conway, 1997; Shafrir & Siegel, 1994; Siegel & Ryan, 1989). However, MD-only and MD/RD groups appear more similar to each other than to typically achieving children (e.g., Landerl, Bevan, & Butterworth, 2004). For instance, children with numeracy difficulties – with or without RD – commit more procedural errors than typically achieving peers (Geary et al.), do not have the same number-fact knowledge as their typically achieving peers (Hanich et al., 2001; Jordan & Hanich, 2000), perform story problems more poorly than their peers (Hanich et al.; Jordan & Hanich), and the groups do not differ in their rate of mathematical skill acquisition (Jordan et al., 2003). As the purpose of the present study was to understand those factors that contributed to numeracy development rather than understanding subtypes, consistent with other researchers in this area (e.g., Mazzocco & Meyers, 2003; Mazzocco & Thompson, 2005; Swanson & Beebe-Frankenberger, 2004), the primary focus was on children for whom numeracy was a significant concern, with or without concurrent reading difficulties.

Underlying Processes Implicated in Numeracy Development

Although a core deficit underlying numeracy difficulties has yet to be identified, there are some candidate systems that warrant continued investigation. These include working memory, visual-spatial abilities, processing speed, mathematics background knowledge, and phonological processing. However, there are three significant limitations in the body of research that investigates the underlying sources of numeracy difficulties that make it difficult to interpret and evaluate findings: 1) most of the studies are cross-sectional and thus do not provide a longitudinal perspective of the role of these candidate systems in numeracy development; 2) little research has investigated all of the candidate systems in one study, making it difficult to compare the overall contribution of each candidate system to
numeracy development; and 3) studies in this area are not consistent in their definition of numeracy difficulties, making it difficult to draw conclusions across studies (i.e., it is difficult to evaluate whether the lack of significant findings have to do with the way numeracy difficulty was defined or if the underlying process actually is not involved in numeracy). The present study addressed some of these limitations by providing a longitudinal investigation of the major underlying processes implicated in numeracy development according to a simple view paradigm. By systematically studying numeracy from an explicit framework, it may be possible to identify those factors that are related to numeracy difficulties and under what conditions.

*Working Memory.* Working memory is conceptualized as a limited capacity central-executive system responsible for allocating cognitive resources between the processing and storage demands created by the flow of information in immediate awareness (Baddeley & Hitch, 1994). The central executive system interacts with two passive storage systems often referred to as short-term memory in the literature: the speech-based phonological loop, which is responsible for the temporary storage of verbal information; and the visual sketchpad, which is responsible for the temporary storage of visual-spatial information. The working memory system is implicated in numeracy because of the complexity of the skills involved in solving mathematical problems and the need for a system to coordinate the simultaneous processes occurring during numeracy tasks. Working memory is associated with both calculation and problem solving (e.g., Swanson & Beebe-Frankenberger, 2004).

There is considerable evidence that numeracy difficulties are associated with reduced working memory performance, particularly the central executive component of the working memory system (e.g., Geary, 1990, 1993; Geary et al., 1992, 1999; Geary, Hamson, &
Hoard, 2000; Passolunghi & Siegel, 2001; Siegel & Ryan, 1989; Swanson & Beebe-Frankenerberger, 2004). However, given the limitations of research in this area, particularly the lack of longitudinal studies, it is unclear the extent to which central executive functioning is involved in numeracy development or that working memory deficits are a core deficit of numeracy difficulties. Furthermore, although working memory has been identified as an important component of an early numeracy screening battery (Gersten, Jordan, & Flojo, 2005), it is not known to what extent working memory difficulties predict later numeracy difficulties.

The role of short-term memory processes in numeracy difficulties is also inconclusive. For instance, some research has found that verbal short-term memory is significantly associated with growth in children’s calculation skills from second to third grade (Hecht, Torgesen, Wagner, & Rashotte, 2001), whereas other research has found that verbal short-term processes are uniquely involved in math problem solving but not math calculation (Swanson & Beebe-Frankenerberger, 2004). In a study with fourth grade children, Passolunghi and Siegel (2001) found that children with problem solving difficulties did not show impairments in short-term memory for linguistic information, but did for numerical information, suggesting a specific short-term memory deficit for numerical information. Swanson and Beebe-Frankenerberger found that compared to typically achieving children, first through third grade children with problem solving difficulties had smaller short-term memory spans for both numerical and linguistic information. In contrast, others have found no differences on short-term memory span between children with numeracy difficulties and typical achievers (e.g., Bull & Johnston, 1997; Geary et al., 1999, 2000; Landerl et al., 2004). Thus, more research - particularly longitudinal research - is needed to examine whether
short-term memory processes are involved in numeracy and if deficits in short-term memory processes characterize children with numeracy difficulties over time.

Visual-Spatial Abilities. Visual-spatial abilities refer to the cognitive ability to generate and manipulate visual-spatial information. Visual-spatial processes are thought to underlie numeracy because of the nonverbal nature of mathematical concepts and processes (Friedman, 1995) and because there is evidence that both visual-spatial and quantitative information are similarly stored and accessed in the brain (Dehaene, 1997). Thus, in relation to numeracy, visual-spatial deficits may manifest as difficulties with discriminating between numbers and arithmetic signs, spatially organizing calculations, interpreting spatially represented numerical information (e.g., place value, maps, charts), and “visualizing” mathematical problems (Friedman; Geary, 1993).

The evidence that visual-spatial deficits are involved in numeracy difficulties is mixed. For instance, the research by Rourke and colleagues (e.g., Harnadek & Rourke, 1994; Rourke, 1993; Rourke & Finlayson, 1978; Rourke & Conway, 1997) demonstrates that children with both computational arithmetic and reading difficulties (MD/RD) tend to have intact visual-spatial abilities whereas children with specific calculation difficulties (MD-only) tend to show visual-spatial deficits, suggesting that visual-spatial deficits may be specific to children with MD-only. Other researchers have found that visual-spatial deficits are not specific to MD-only groups and instead have found that individuals with MD/RD may also show visual-spatial deficits (e.g., Shafrir & Siegel, 1994; Share, Moffitt, & Silva, 1988). Yet others have found limited evidence of visual-spatial deficits in numeracy difficulties: Geary et al. (2000) found that children with problem solving difficulties did not show visual-spatial deficits; and Jordan, Levine, and Huttenlocher (1995) found that children
with visual-spatial deficits did not differ from children with typical visual-spatial abilities on measures of calculation or problem solving. Thus, more research is needed to determine the relationship between numeracy development and visual-spatial abilities, including whether visual-spatial deficits characterize children with numeracy difficulties and the ability of visual-spatial processes to predict numeracy difficulties.

**Processing Speed.** In his synthesis of the literature, Geary (1993) suggested that cognitive processing speed may represent a unique source of difficulty for children with low numeracy as these children tend to perform mathematical problems more slowly than typically achieving children. In relation to numeracy difficulties, slow processing speed is thought to interfere with the ability to store and retrieve numerical information from long-term memory. There has been little research that has examined the relationship between direct measures of processing speed and numeracy. Instead, studies in this area tend to use indirect measures of processing speed such as time taken to solve arithmetic problems, or counting speed (e.g., Geary, 1990; Geary & Brown, 1991). It is not clear, however, that such slowness is related to underlying processing speed as children with numeracy difficulties have also been found to use less efficient problem-solving strategies.

In the few studies using direct measures of processing speed, children with numeracy difficulties have been found to perform significantly slower than typically achieving children (Bull & Johnston, 1997; Swanson & Beebe-Frankenberger, 2004), suggesting that slow processing speed may in fact underlie numeracy difficulties. Processing speed - also referred to as rapid retrieval from long-term memory - has been found to explain unique variance in both calculation skills (e.g., Swanson & Beebe-Frankenberger) and mathematical problem solving (e.g., Bull & Johnston; Swanson & Frankenberger). Furthermore, Hecht et al. (2001)
found that processing speed was significantly associated with growth in children’s
calculation skills from second to third grade. Given the limited work done in this area,
however, the role of processing speed in numeracy is far from complete, particularly
knowledge surrounding how processing speed is related to numeracy skills over time.

Mathematics Background Knowledge. Just as general background knowledge is
important for reading, so too is math background knowledge important for the acquisition of
numeracy skills (Carpenter & Moser, 1984; Gersten et al., 2005; Ginsburg, 1997; Russell &
Ginsburg, 1984). Mathematics background knowledge includes skills such as counting (e.g.,
oral counting, counting objects, counting knowledge), number identification, quantity
discrimination (which of two numbers is larger), and quantitative vocabulary (e.g., largest,
smallest, first, last). Mathematics background knowledge in children develops spontaneously
through interactions with the environment and these early math skills allow children to
transcend the concrete world of objects and enter the abstract world of higher mathematical
thinking and analysis (Dehaene, 1997; Ginsburg). Mastery of early math concepts is thought
to be critical to the development of formal math (Clarke & Shinn, 2004; Gersten & Chard,
1999; Gersten et al.; Jordan, 1995; Russell & Ginsburg).

There is evidence that children with numeracy difficulties have deficits in their early
math concepts. For instance, children with numeracy difficulties have been found to be
delayed in their knowledge and understanding of counting procedures (Geary, 1990; Geary et
counting strategies (Geary; Geary et al., 1991, 1992; Hanich et al.), to execute counting
strategies poorly (Geary; Geary et al., 1991; Hanich et al.), and to be unfamiliar with
numerical representations (Geary et al., 1999). In addition, early math skills such as number
identification and quantity discrimination measured at the beginning of first grade have been shown to be correlated both with calculation and problem solving at the end of first grade (Clarke & Shinn, 2004). Given the limitations noted above, however, more longitudinal research is needed to determine the relationship between numeracy and mathematics background knowledge, including whether deficits in math background knowledge are a defining feature of numeracy difficulties and whether math background knowledge predicts numeracy outcomes beyond first grade.

**Phonological Processing.** Many children who have numeracy difficulties also have reading difficulties (e.g., Badian, 1983; Siegel & Ryan, 1989). Thus, it has been suggested that phonological processes - which are strongly related to reading development - may also be involved in numeracy (Geary, 1993). In addition, phonological processes are implicated in numeracy achievement because of the common finding that children often rely on speech-based phonological systems, such as counting (verbally or mentally) or naming numbers aloud when performing complex calculations, to complete mathematical problems (Bull & Johnston, 1997; Dehaene, 1997; Geary). Thus, phonological processing deficits may interfere with the processing of quantitative information that occurs within the phonological system.

Although there is evidence supporting the role of phonological processes in numeracy, as with processing speed, few studies directly measure phonological abilities, relying instead on indirect measures such as articulation speed as a proxy for phonological abilities (e.g., Geary et al., 2000). In one study using direct measures, Hecht et al. (2001) found that phonological awareness was related to growth in calculation skills from second to fifth grade, even after controlling for prior math achievement, general verbal ability, and reading ability. The authors concluded that the same phonological abilities that influence
growth in reading also appear to influence growth in computational arithmetic. Thus,
phonological processes may influence the development of both reading and math skills.
More research is needed to investigate the relationship between phonological processes and
numeracy, including whether phonological deficits are a defining feature of numeracy
difficulties and whether early phonological processes predict numeracy difficulties.

Present Study

The primary purpose of the study was to take a simple view of numeracy in order to
investigate those factors related to numeracy difficulties in children from first through fourth
grade. Given the longitudinal nature of the study, a secondary purpose was to examine the
early indicators of numeracy difficulties. This study was based on a study conducted in the
area of literacy (Juel, 1988) and followed similar research questions: 1) Do the same children
struggle with numeracy year after year? 2) What skills do children with numeracy difficulties
lack? 3) What underlying factors support numeracy achievement? 4) What are the early
indicators of numeracy difficulties?

Method

Participants

The participants of this study included children who were part of a longitudinal study
of numeracy development that began in their first grade year. All participants were enrolled
in five participating schools within the diverse school district of North Vancouver, Canada.
The schools were located primarily in working class neighbourhoods characterized by high
concentrations of rental housing and high mobility rates (43% - 67% mobility rate; City of
North Vancouver, 2004). As a result, the study began with 164 children and ended with 85
children who remained in the study for all four years, 97 children who remained in the study for three of four years, and 101 children who were only assessed in grade 1 and grade 4.

The present study included data from the 97 children (53 girls and 44 boys) who participated in the study in at least three of four years from grade one to grade four. The children were 64.9% majority culture, 16.5% First Nations, 9.3% Middle Eastern, 5.2% Asian, and 4.1% other. Although demographic variables were not of primary interest in the study, it was important to determine whether demographic characteristics were necessary to consider in further analyses. There were no differences on grade 1 measures between the longitudinal sample and the children who were lost due to attrition. In addition, there were no gender or group background differences on the measures used in the study. Given the lack of such differences, these variables were not considered further.

Materials

Computational arithmetic. Computational arithmetic was assessed with the Calculation test of the Woodcock Johnson - III Tests of Achievement (WJ-III): Research Edition (Woodcock, McGrew, & Mather, 2001). In this standardized task children completed a series of written calculation problems of increasing difficulty (e.g., \(1 + 2 = \_\_\)). Questions were presented either in horizontal or vertical format and were not read to the children. This test was administered in all grades. The reliabilities for the Calculation test are reported in the range of .80 - .96 for the ages of the children in the study; however, as the WJ-III Research Editions are shortened versions of the published tests, the standard error of measurement for these tests is somewhat larger than the published versions, although well within an acceptable range for research purposes (K. McGrew, personal communication, May 2002). Age level percentile scores are reported.
Mathematical problem solving. Mathematical problem solving was assessed with the Applied Problems test of the WJ-III: Research Edition (Woodcock et al., 2001). In this standardized test children were required to analyze and solve practical problems in mathematics (e.g., Lindsay had $6. She bought a kite for $3.35 and a pencil for $1.05. How much money did she have left?). Questions were read to the children to eliminate the confound of reading ability. This test was administered in all grades. The reliabilities for Applied Problems are reported in the range of .88 - .93 for the ages of the children in the study. Age level percentile scores are reported.

Number-fact fluency. Automaticity of number facts was assessed through the Math Fluency test of the WJ-III: Form B (Woodcock et al., 2001). In this standardized test children were presented with two pages of simple single-digit addition, subtraction, and multiplication facts and were given 3 minutes to complete as many problems as possible. Children were instructed to complete the questions they knew automatically and to skip questions that they did not know. This test was administered in grades 2, 3, and 4 and was included in the battery as number fact knowledge has been identified as a defining feature of numeracy difficulties (Geary, 1993). The reliabilities are reported in the range of .66 - .81 for the ages of the children in the study. Age level percentile scores are reported.

Underlying Cognitive Processes

Working memory. Working memory capacity was assessed through an experimental counting span task designed by Case, Kurland, and Goldberg, 1982, as cited in Sigel & Ryan, 1989. This task involved counting the yellow dots from a field of blue and yellow dots arranged in an irregular pattern on an index card. The child was asked to count aloud the yellow dots on each card in the set and at the end of the set to recall each number counted.
The child was presented with three sets of cards at each span length (i.e., 2, 3, 4, and 5). The number of yellow dots on each card varied from one to nine. Working memory capacity was defined as the largest number span correctly recalled on two of three sets. For example, if the child correctly recalled two of three sets at span length 4 but failed all sets at span length 5, working memory capacity was 4. If a child passed two of three sets at span length 4 and only one of three sets at span length 5, working memory capacity was marked as 4.5. This test was administered in all grades. The reliability for the working memory numbers test was in the range of .71 - .74.

Short-term memory capacity was assessed with the Forward Digit Span portion of the Digit Span test of the *Wechsler Intelligence Scale for Children - Third Edition* (WISC-3; Wechsler, 1991). In this task children listened to a series of digits (increasing in length) and were required to repeat the numbers in the same order that the digits were presented. The child was presented with two trials at each span length and short-term memory span was defined as the largest number span correctly recalled on both trials. The digit span test was administered in all grades. The reliability for forward digit span is reported in the range of .79 - .84 for the ages of the children in the study.

**Visual-spatial abilities.** Visual-spatial abilities were assessed with the Block Rotation test of the *WJ-III: Diagnostic Supplement* (Woodcock et al., 2001). This standardized task measured the child’s ability to mentally manipulate three-dimensional objects. In this task the child was presented with a stimulus object and then chose which two of five items represented the stimulus that had been rotated in three dimensional space (see Figure 1). This test was administered in all grades. The reliability for Block Rotation is reported in the range of .77 - .84 for the ages of the children in the study. Age level percentile scores are reported.
**Processing speed.** Processing speed was assessed with a rapid number naming task that is based on other rapid naming tests (Denckla & Rudel, 1974). In this task, children were required to name individual numbers (1 - 9) presented in random order on a 5 x 5 array. The primary measure of interest was the time taken in seconds to name all the numbers on the page. The rapid naming task was administered in all grades.

**Mathematics Background Knowledge.** Early math skills were assessed with the Quantitative Concepts test of the WJ-III: Form A (Woodcock et al., 2001). This test consists of two components: concepts and number series. In the concepts component, students demonstrated basic numeracy skills (e.g., counting, number identification, identification of concepts such as “first” and “last”) and identified math terms and formulae (e.g., children were shown an addition symbol and are asked what the sign meant). In Number Series, students were required to determine the next number in a series (e.g., 6 7 8 __). Number series can be considered a measure of numerical reasoning. These tests were administered in grades 1 through 4. The reliability for the Quantitative Concepts test is reported in the range of .86 - .92 for the ages of the children in the study. For the purpose of analyses, concepts and number series were considered separately and thus raw scores are reported.

**Phonological Processing.** Phonological processing was assessed with the Word Attack test of the WJ-III: Form B (Woodcock et al., 2001), which is a standardized measure of phonological decoding that requires children to pronounce pseudowords that conform to English spelling rules (e.g., flib, bungic). Children were required to read as many words as possible from a list of pseudowords of increasing difficulty. This test was administered in all grades. The reliability is reported in the range of .89 - .94 for the ages of the children in the study. Age level percentile scores are reported.
Measures of literacy. Reading ability was measured using the Letter-Word Identification (LWID) test of the WJ-III: Research Edition (Woodcock et al., 2001). This task required children to identify and pronounce isolated letters (e.g., g, r) and words of increasing difficulty (e.g., cat, palm). Each child was required to identify as many items as possible from the list. This task was administered in grades 1 through 4. The reliability is reported in the range of .94 - .98 for the ages of the children in the study. Age level percentile scores are reported.

The Analogy portion of the WJ-III Form B: Reading Vocabulary test (Woodcock et al., 2001) was administered in order to examine whether verbal reasoning processes were involved in numeracy. In this test, children were shown three words of an analogy and were required to provide the fourth word to complete the analogy (e.g., pencil is to lead as pen is to ...). To eliminate any confounds of reading ability, the words were read by the experimenter. This test was administered in grades 2, 3, and 4. The reliability is reported in the range of .88 - .89 for the ages of the children in the study. Raw scores are reported.

Procedure

Children were assessed in the spring (i.e., April/May) of each year for all grades except third grade, in which children were assessed in the winter (i.e., January/February). Trained research assistants conducted the assessments in the schools. In first grade, children were assessed individually in a quiet room for all tasks; testing time was approximately 30 minutes per child. In grades 2 through 4, children were individually assessed for all tasks except for WJ-III Calculation and WJ-III Math Fluency, which were administered in small groups in the children’s classrooms. Testing time was approximately 30 minutes per child for individually administered tasks and 15 minutes for the small group testing.
Results

Do the same children struggle with numeracy year after year?

In order to determine whether the same children struggled with numeracy year after year, children were classified as either typically achieving or as having numeracy difficulties. Typically achieving was initially defined as performance at or above the 40th percentile on both *WJ-III* Calculation and *WJ-III* Applied Problems. Children were initially classified as having numeracy difficulties in they performed at or below the 25th percentile on either *WJ-III* Calculation or *WJ-III* Applied Problems. Using these criteria, 12, 10, 8, and 7 children in grades 1, 2, 3, and 4, respectively, could not be classified. However, given the nature of the research question, it was important to classify all children. Upon closer examination of those children who could not be classified, approximately 80% of those who performed between the 25th and 33rd percentile ended up having low numeracy achievement in grade 4 (i.e., percentile scores at or below the 30th percentile), with approximately 75% having scores below the 25th percentile in grade 4. In comparison, children who performed between the 34th and 39th percentile tended to become typical achievers in later years.

Thus, the initial classification criteria were modified slightly to account for those few children who could not be classified but who in general seemed to demonstrate performance more consistent with either typical achievers or children with numeracy difficulties. Effectively, this modification meant that although the majority of typically achieving children performed at or above the 40th percentile on both Calculation and Problem Solving, there were a few children with percentile scores between the 34th to the 39th percentile who were included in the typically achieving group. Similarly, the majority of children in the numeracy difficulties group demonstrated performance at or below the 25th percentile on at
least one measure of numeracy\(^2\), although the numeracy difficulties group also included a few children whose performance between the 26th and 33rd percentile on either Calculation or Applied Problems was sufficient to be classified into the numeracy difficulties group.

It is important to note that although this modification effectively resulted in using a cut-score of the 33rd percentile, which is a higher cut-score than is typically used to identify children with learning difficulties (Lyon et al., 2001; Siegel, 1999), such cut-scores are commonly used in numeracy research given the nature of standardized numeracy achievement tests. In particular, such tests tend to include only a few items of each skill (e.g., single-digit addition; multi-digit addition; single-digit subtraction; multi-digit subtraction). Thus, a cut-off of the 25th percentile is likely to overestimate the competencies of children with numeracy difficulties (Geary, 2004; Geary et al., 2000; Hanich et al., 2001).

In total, 61, 58, 56, and 58 children were classified as typically achieving in grades 1, 2, 3, and 4, respectively, and 36, 33, 40, and 39 children were identified with numeracy difficulties in grades 1, 2, 3, and 4, respectively. Table 1 displays the grade 4 outcomes of typically achieving children and children with numeracy difficulties in each grade. As shown in Table 1, 72%, 78%, and 84% of the typically achieving children in grades 1, 2, and 3, respectively, remained typically achieving in grade 4; 28%, 22%, and 16% of the typically achieving children in grades 1, 2, and 3, respectively, developed numeracy difficulties by grade 4. Thus, typically achieving children were more likely to remain typically achieving than to develop numeracy difficulties from the period from first to fourth grade, although

\(^2\) In preliminary analyses, no differences were found among children with specific calculation difficulties, children with specific problem solving difficulties, or children with both calculation and problem solving difficulties. In addition, classifying children on the basis of their performance on either problem solving or calculation resulted in a classification scheme that was more sensitive in detecting those children who demonstrated low numeracy achievement in later years. For these reasons, it was deemed appropriate to include in the numeracy difficulties group those children who demonstrated below average performance on either problem solving or calculation, although it is worth noting that approximately 50% of the numeracy difficulties group demonstrated difficulties on both calculation and problem solving.
there certainly existed typically achieving children who developed numeracy difficulties by fourth grade.

In comparison, Table 1 shows that 61%, 67%, and 73% of the children with numeracy difficulties in grades 1, 2, and 3, respectively, continued to struggle with numeracy in grade 4; 39%, 33%, and 28% of the children with numeracy difficulties in grades 1, 2, and 3, respectively, no longer met criteria for numeracy difficulties in grade 4. Thus, most of the children with numeracy difficulties from grade 1 to grade 3 continued to struggle with numeracy in fourth grade. Table 2 displays the characteristics of the children per grade meeting criteria for numeracy difficulties.

Table 2 shows that of those children whose numeracy difficulties lasted until grade 4, the majority struggled either with specific calculation difficulties or with both calculation and problem solving; few children struggled only with problem solving. This finding is depicted more clearly in Figure 2. Furthermore, Table 2 shows that at least 90% of children with persistent numeracy difficulties also had math fluency difficulties, and between 30-40% also had reading difficulties (defined as performance below the 25th percentile on WJ-III LWID). Finally, Table 2 shows that although the majority of children with numeracy difficulties in each grade demonstrated numeracy difficulties in previous years, there existed a small portion of children in each grade who did not show evidence of numeracy difficulties in earlier grades.

In sum, these results show that most typically achieving children remained typically achieving from first through fourth grade and that most children with numeracy difficulties continued to struggle with numeracy. In fact, by fourth grade, 84.6% of children with numeracy difficulties experienced numeracy difficulties at some point before grade 4.
Furthermore, numeracy difficulties tended to be accompanied by math fluency difficulties and to a lesser extent reading difficulties. In the early years numeracy difficulties tended to reflect problem solving difficulties and in later years, children with numeracy difficulties tended to struggle with both calculation and problem solving, although a large portion of children struggled only with calculation procedures. Finally, there existed in each grade a small portion of children who did not show any numeracy difficulties in previous grades.

What skills do children with numeracy difficulties lack?

In order to determine the skills children with numeracy difficulties lacked, those children who remained typically achieving throughout grade 1 to grade 4 (n = 37) were compared to the children with numeracy difficulties in each grade whose difficulties lasted until grade 4 (n = 22, 22, 29, and 33 in grades 1, 2, 3, and 4, respectively). The dependent variables for the analyses of variance (ANOVAs) were percentile scores for the Calculation, Applied Problems, Math Fluency, Block Rotation, Word Attack, and LWID tests, and raw scores for the remaining tests. Percentile ranks were selected whenever possible due to more desirable distributional properties of percentile scores compared to raw scores and because percentile scores are more informative than raw scores as indicators of level of performance (Zimmerman & Zumbo, 2005). Furthermore, in many situations, statistical power is gained by using percentile scores in place of raw scores (Zimmerman & Zumbo). In order to minimize the chance of a Type I error when making multiple comparisons, the Bonferroni correction factor was used resulting in a corrected alpha of \( p = .005 \) in grade 1, and \( p = .004 \) in grades 2, 3, and 4. Table 3 summarizes the mean scores and the corresponding effect sizes across tasks and grades for typical achievers and children with numeracy difficulties.
Table 3 shows that in each grade children with numeracy difficulties performed significantly lower than typically achieving children on calculation, problem solving, and math fluency. In addition, the numeracy difficulties group performed significantly lower on word recognition in each grade, although the mean scores were in the average range. This indicates that although the numeracy difficulties group had significantly lower reading scores than typical achievers, as a group they were not characterized by word reading deficits. Children with numeracy difficulties had significantly lower verbal reasoning scores only in grade 2.

In terms of the underlying cognitive processes, children with numeracy difficulties in each grade had smaller working memory spans for numbers, knew fewer quantitative concepts, had lower numerical reasoning skills, and performed significantly lower than typical achievers on phonological processing (though still in the average range). Children with numeracy difficulties showed evidence of a smaller digit span only in grades 3, and 4, lower visual-spatial abilities only in grade 1, and slower rapid number naming only in grade 4. Otherwise, digit span, visual-spatial abilities, and rapid number retrieval did not differentiate between children with and without numeracy difficulties.

Given the significant differences on LWID between the groups in each grade, and given that some of the underlying processes have a well-established relationship with reading (i.e., short-term memory, rapid retrieval, and phonological processing), it was of interest to examine whether group differences existed on the underlying processes after statistically controlling for reading ability. Thus, the above comparisons were reanalyzed with a series of univariate analysis of covariance (ANCOVAs) using LWID as a covariate (Bonferroni corrected alpha of \( p = .004 \) in grade 1, and \( p = .005 \) in grades 2 through 4).

22
Table 3 shows that with reading ability statistically controlled children with numeracy difficulties continued to perform significantly lower than typically achieving children on calculation, problem solving, and fluency. The group difference on grade 2 verbal analogy was no longer significant with reading ability statistically controlled.

With respect to the underlying cognitive processes, children with numeracy difficulties in each grade continued to have smaller working memory spans for numbers, and controlling for reading ability did not diminish the group differences on numerical reasoning or math concepts (except for concepts in grade 1). In contrast, controlling for reading ability completely eliminated the group differences on phonological skills, rapid number naming, and forward digit span. Controlling for reading ability did not change the relationship between numeracy difficulties and block rotation: visual spatial abilities differentiated children with and without numeracy difficulties only in grade 1.

Figure 3 graphically depicts an interesting finding that emerged when analyzing group differences. Specifically, Figure 3 shows that in the early years, children with numeracy difficulties had more difficulty with mathematical problem solving than calculation procedures whereas in later grades, children with numeracy difficulties performed better on mathematical problem solving than on isolated skills such as calculation and math fluency. This is an interesting finding as it suggests that children with numeracy difficulties have better calculation skills than would be suggested by their performance on discrete tasks.

What underlying factors support numeracy achievement?

Calculation Skills. In order to examine the underlying factors involved in calculation skills, four regression models were fit: model 1 examined the relationship between first grade
calculation and first grade underlying processes; model 2 examined the relationship between second grade calculation and second grade processes; model 3 investigated the relationship between third grade calculation and third grade processes, and model 4 examined the relationship between fourth grade calculation and fourth grade processes. For each model, variables were entered simultaneously such that the beta values reflected unique variance. The explanatory variables were: working memory, forward digit span, block rotation, rapid number naming, concepts, number series, and word attack. The correlations among the variables used in the first and second grade regression analyses are presented in Table 4 and the correlations among the variables used in the third and fourth grade regression analyses are presented in Table 5. Note that as shown in Tables 4 and 5, calculation was significantly correlated with all the underlying processes at each time point except for forward digit span in grade 1 and 3, block rotation in grades 1 and 4, and word attack in grade 4. In preliminary regression analyses, these variables that were not significantly correlated with calculation acted to distort the relationship between calculation and various explanatory variables that in fact shared univariate relationships with calculation. Thus, these variables were removed from multiple regression analyses in the grade for which there was not a significant univariate relationship. The results of the calculation skills regression analyses are presented in Table 6.

All four regression models were significantly different from zero: Grade 1 model, $R^2 = .339$, $F(5, 91) = 9.32, p<.001$; Grade 2 model, $R^2 = .432$, $F(7, 62) = 6.73, p<.001$; Grade 3 model, $R^2 = .514$, $F(6, 89) = 15.71, p<.001$; Grade 4 model, $R^2 = .199$, $F(5, 91) = 4.53$, $p=.001$. In grade 1, working memory was the only variable that contributed uniquely to first grade calculation (5%). The first grade explanatory variables in combination contributed .289
in shared variance. In second grade, the explanatory variables contributed .312 in shared variance and grade 2 concepts and number series contributed 5% and 7% unique variance respectively. The third grade explanatory variables contributed .374 in shared variance, with block rotation, concepts, and number series contributing 5%, 3%, and 6% unique variance respectively. In the grade 4 model, number series was the only variable to contribute unique variance (5%), with the explanatory variables contributing 14.9% in shared variance.

Each model was further analyzed to determine the relative importance of the variables in the model. Variable importance was determined using the Pratt Index (Thomas, Hughes & Zumbo, 1998), which partitions R-squared in such a way that the contribution of each variable to the overall variance accounted for by the model can be examined. To calculate the Pratt index, the beta-weight is multiplied by the simple correlation and this value is divided by the model R-squared. For instance, the Pratt Index for grade 1 working memory is calculated by multiplying .24 (beta-weight) by .42 (simple correlation) and dividing by .339 (R-squared), resulting in a Pratt Index of .297; this value is interpreted to mean that 29.7% of the explained variance in the grade 1 model ($R^2 = .339$) is attributed to working memory. A variable is interpreted as "important" to the model if the Pratt Index value is greater than $1/(2\times \text{number of explanatory variables})$, according to the criteria set by Thomas (1992). The Pratt Index values for the calculation skills regression models are also presented in Table 6.

Based on the Pratt Index, working memory, number series, word attack, rapid number naming, and concepts made, in relative order, important contributions to the grade 1 model. In the grade 2 model, number series, concepts, working memory, word attack, and block rotation made, in relative order, important contributions, whereas forward digit span and
rapid number naming were found to be "unimportant" to the model. The variables in the grade 3 model that made important contributions were, in relative order, number series, concepts, block rotation, and working memory; rapid number naming and word attack were found to be unimportant to the model. In the grade 4 model, the variables that made important contributions were, in relative order, number series, working memory, concepts, rapid number naming, and forward digit span.

Mathematical Problem Solving Skills. As above, four regression models were fit to determine the factors that contributed to mathematical problem solving: model 1 examined the relationship between first grade problem solving and first grade processes; model 2 examined the relationship between second grade problem solving and second grade processes; model 3 investigated the relationship between third grade problem solving and third grade processes, and model 4 examined the relationship between fourth grade problem solving and fourth grade processes. The explanatory variables were: working memory, forward digit span, block rotation, rapid number naming, concepts, number series, and word attack. As shown in Tables 4 and 5, applied problems was significantly correlated with all of the underlying processes except for block rotation in grade 2. For the reasons stated above, block rotation was not included in the grade 2 regression model. Table 7 presents the results of the mathematical problem solving regression analyses, including the Pratt Index.

All four regression models were significantly different from zero: Grade 1 model, $R^2 = .614$, $F(7, 89) = 20.23, p<.001$; Grade 2 model, $R^2 = .571$, $F(6, 77) = 17.09, p<.001$; Grade 3 model, $R^2 = .532$, $F(7, 88) = 14.29, p<.001$; Grade 4 model, $R^2 = .541$, $F(7, 89) = 15.00, p<.001$. In the grade 1 model, working memory, quantitative concepts, number series, and word attack each contributed uniquely to first grade problem solving (5%, 5%, 3%, and 4%,
respectively). Together, the first grade explanatory variables contributed .444 in shared variance. In second grade, working memory, concepts, and number series contributed unique variance to second grade problem solving (3%, 14%, 6%, respectively), with the explanatory variables in combination contributing .341 in shared variance. The third grade explanatory variables contributed .372 in shared variance, with concepts and number series contributing 4% and 12% unique variance respectively. In the grade 4 model, working memory, concepts, and number series contributed unique variance (4%, 3%, and 13% respectively), with the explanatory variables contributing .341 in shared variance.

Based on the Pratt Index, of the variables in the grade 1 model, concepts, working memory, number series, and word attack each made, in relative order, important contributions; forward digit span, block rotation, and rapid number naming did not make important contributions to the model. In the grade 2 model, concepts, number series, and working memory made, in relative order, important contributions, whereas forward digit span, rapid number naming, and word attack were found to be “unimportant” to the model. The variables in the grade 3 model that made important contributions were, in relative order, number series, concepts, and forward digit span; working memory, block rotation, rapid number naming, and word attack were found to be unimportant to the model. In the grade 4 model, the variables that made important contributions were, in relative order, number series, concepts, working memory, rapid number naming, and block rotation; forward digit span was found to be unimportant to the model.

In summary, there were three findings that emerged from the regression analyses. First, the underlying processes accounted for more variance in the problem solving than in the calculation models. Second, of the underlying processes considered, working memory,
concepts, and number series consistently made unique and important contributions to both
calculation and problem solving, whereas the relationship between the remaining underlying
processes and numeracy outcomes was unreliable and often "unimportant." Finally, in
general, number series shared the strongest relationship with both calculation and problem
solving; concepts and working memory played relatively equal roles in explaining the
variability in calculation and problem solving.

What are the early indicators of numeracy difficulties?

In order to investigate the relationship between first grade skills and fourth grade
numeracy outcomes, the relative risk statistic was computed. Relative risk is a measure of the
strength of the association between a variable and a particular outcome and is useful in
identifying those factors that indicate that certain individuals are at greater risk than others of
developing an outcome (Christie, Gordon, & Heller, 1997; Unwin, Carr, Leeson, & Pless-
Mulloli, 1997) - in this case numeracy difficulties. For the current study, the relative risk
ratio provided an indicator of the risk of developing numeracy difficulties if a child
experienced difficulties in first grade on the underlying processes associated with numeracy
difficulties. Children's performance on each first grade measure (i.e., working memory,
forward digit span, block rotation, rapid number naming, quantitative concepts, number
series, and word attack) was classified as average or below average based on the entire
sample (n = 97): below average was defined as performance in the bottom 25th percentile of
the sample; average was defined as scores above the 25th percentile. The rate of below
average grade 1 skills in fourth grade typical achievers (n = 58) and fourth grade children
with numeracy difficulties (n = 39) was compared. Table 8 displays the results of the relative
risk analyses. Table 8 shows that below average scores on grade 1 working memory span,
quantitative concepts, and number series were all significant risk factors for grade 4 numeracy difficulties.

Discussion

The purpose of the present study was to investigate those factors associated with numeracy development in typically achieving children and children with numeracy difficulties when numeracy difficulties was defined on the basis of calculation and problem solving skills. Similar to Juel’s (1988) study, the current study was designed to determine whether the same children struggled with numeracy over time, the skills children with numeracy difficulties lacked, the cognitive factors that supported numeracy achievement, and the early indicators of numeracy difficulties. In general, the findings from the current study indicated that by fourth grade, the majority of children with numeracy difficulties had also experienced numeracy difficulties in previous grades and that of the underlying cognitive processes measured, working memory, numerical reasoning, and mathematical background knowledge were consistently implicated in numeracy difficulties and numeracy achievement.

Juel (1988) found that poor first-grade readers almost invariably remained poor readers in grade 4. In the present study, the likelihood of having persistent numeracy difficulties beginning in grade 1 was not as strong as in Juel’s study, although by grade 4, 85% of the children with numeracy difficulties experienced numeracy difficulties in previous years. Thus, for the children in the current study numeracy difficulties did not necessarily emerge as early as first grade but once a child experienced numeracy difficulties, these difficulties tended to be persistent at least through grade 4. Furthermore, numeracy difficulties almost invariably included difficulties with math fluency. This is consistent with previous literature that suggests that difficulties with math facts are a defining feature of
numeracy difficulties (e.g., Geary, 1993; Jordan, 1995) and extends this research to include a longitudinal sample of children followed from first through fourth grade. That children with numeracy difficulties also had difficulties with number fact retrieval supports a definition of numeracy difficulties that is based on calculation skills and mathematical problem solving.

Another interesting finding that emerged from examining whether the same children struggled with numeracy difficulties was that at each grade there existed between 14-32% of children with numeracy difficulties who had not experienced such difficulties in previous years. In the reading literature, recent attention has been paid to those children with late-emerging reading difficulties, that is, those children who experience reading difficulties for the first time in grade 4 or grade 5 (Leach, Scarborough, & Rescorla, 2003). The current study indicates that in the area of numeracy difficulties, late-developing difficulties may not be restricted to the middle elementary years but instead occur at each grade, including the early elementary years. This finding suggests that it cannot be assumed that a child who demonstrates typical numeracy skills in one year will necessarily remain typical in the skill area in subsequent years. Instead, for a small portion of children, the numeracy skills carried over from earlier years may not be sufficient to meet the heavier numeracy demands in later years. This is an area that should be explored in future research.

In terms of the roles of the underlying cognitive factors involved in numeracy, the present study is unique in simultaneously investigating the major underlying processes implicated in numeracy development in the same children over time. The present study found that at each grade, children with numeracy difficulties were characterized by smaller working memory capacity, and weaknesses in numerical reasoning and mathematical background knowledge, even after controlling for reading ability. Children with numeracy difficulties
had lower phonological skills than typical achievers, although these differences were no longer significant after reading ability was controlled. It is important to note that children with and without numeracy difficulties did not differ significantly on the verbal analogy task, which suggests that any group differences were not due to differences in verbal reasoning processes but instead likely reflected differences in numerical processing.

The finding that children with numeracy difficulties had smaller working memory capacity than typical achievers is consistent with previous research (e.g., Geary, 1990; Geary et al., 1992, 1999, 2000; Siegel & Ryan, 1989; Swanson & Beebe-Frankenberger, 2004), and provides convincing evidence from a longitudinal sample that working memory deficits are involved in numeracy difficulties. This was also supported in the regression analyses demonstrating that working memory consistently contributed unique and important variance to calculation and problem solving, as well as the finding that children with low working memory in grade 1 were significantly more likely to have numeracy difficulties in grade 4 than children with average working memory capacity in grade 1. Thus, with respect to the relationship between working memory and numeracy difficulties, the present study provides clear evidence that deficits in central executive functioning are a defining feature of numeracy difficulties and that the central executive system is involved in numeracy achievement.

Given that the working memory measure used in the current study tapped only numerical working memory, the present study is unable to provide conclusive evidence that a general working memory system as opposed to a numerical working memory system accounts for the relationship between working memory and numeracy. Other research has found that latent working memory variables that cut across numerical, verbal, and spatial
working memory contribute uniquely to both calculation and problem solving (e.g., Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). More studies are needed to confirm whether a number-specific or general working memory system explains the relationship between numeracy and working memory.

Mathematical background knowledge - including numerical reasoning and knowledge of quantitative concepts - was also found to differentiate between children with and without numeracy difficulties, and the measures of mathematical background knowledge consistently explained unique and important variance in both calculation and problem solving. Furthermore, children who had below average math background knowledge in grade 1 were significantly more likely to have numeracy difficulties in fourth grade than children with average skills in first grade. The importance of early math experiences to formal math achievement has been discussed in the literature (e.g., Gersten & Chard, 1999; Gersten et al., 2005; Jordan, 1995). The current study extends this notion to suggest that weaknesses in math background knowledge characterize children with numeracy difficulties across the time period from first to fourth grade. More importantly, exposure to formal math instruction did not appear to eliminate the difficulties children with numeracy difficulties had with identifying math terms and concepts, or with reasoning with numbers. Again, it is important to emphasize that children with numeracy difficulties were not characterized by verbal reasoning deficits. Thus, these findings suggest that it is quantitative representation and reasoning in particular that children with numeracy difficulties struggled with, as opposed to general reasoning abilities, and that such difficulties are not necessarily overcome with exposure to formal math instruction. In terms of early identification, it appears that math background knowledge is an important skill area to assess in order to identify children with
math difficulties. Future research is needed to determine whether remediating such deficits would have an impact on a child’s numeracy achievement.

In the current study, short-term memory, visual-spatial ability, and rapid number retrieval did not reliably differentiate between children with and without numeracy difficulties, and these variables rarely made important contributions to either calculation or problem solving regression models. Phonological processing was a discriminating variable, although children with numeracy difficulties had mean phonological skills in the average range, indicating that as a group, children with numeracy difficulties were not characterized by phonological deficits. Furthermore, differences on phonological skills disappeared when reading ability was statistically controlled, and phonological skills rarely made important contributions to regression models. The results of the relative risk analyses determined that below average phonological abilities, short-term memory, visual-spatial ability, and rapid number retrieval were not significant risk factors for numeracy difficulties in fourth grade. Together, these findings suggest that these underlying processes are not defining features of numeracy difficulties, which is inconsistent with studies that have found relationships with these variables and numeracy difficulties (e.g., Rourke, 1993; Rourke & Finlayson, 1978; Siegel & Passolunghi, 2001; Swanson & Beebe-Frankenberger, 2004), but consistent with the literature that has suggested that these processes are not related to numeracy difficulties (e.g., Bull & Johnston, 1997; Geary et al., 1999, 2000; Jordan et al., 1995; Jordan et al., 2003; Mazzocco & Thompson, 2005).

One possibility to explain the inconsistencies in the literature is that relationships between these processes and numeracy may reflect an indirect relationship with reading, given that there is overlap between children with reading difficulties and numeracy
difficulties, and given that variables such as short-term memory, rapid retrieval, and phonological processing have well-established relationships with reading (e.g., Torgesen, Wagner, Rashotte, Burgess, & Hecht, 1997). For instance, some studies have found that children with numeracy difficulties have smaller short-term memory spans than typically achieving children (e.g., Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004), whereas other studies have not found such differences (e.g., Bull & Johnston, 1997; Geary et al., 1999, 2000). It is of note that studies that find group differences on measures of short-term memory do not tend to take word reading ability into account. In other words, children with numeracy difficulties do not appear to have short-term memory deficits when reading ability is taken into account, either statistically or by comparing typically achieving children to children with MD-only (e.g., Bull & Johnston). This notion is supported by the findings in the present study in that controlling for reading ability eliminated the few group differences on short-term memory, rapid number naming, and phonological abilities. In any case, the findings from the current study suggest that numeracy is not associated with short-term numerical memory, rapid number retrieval, visual-spatial ability, or phonological awareness, independent of reading. However, more studies are needed to verify this finding. In particular, it will be important for future studies to assess these cognitive processes with tasks that include stimuli other than numerical stimuli in order to determine how number-specific versus general cognitive systems are related to numeracy.

It is interesting that in the present study numeracy difficulties were not associated with forward digit span or rapid number naming, given that both tasks involved numerical stimuli. Landerl et al. (2004) hypothesized that numeracy difficulties reflect a deficit in the “representation or processing of specifically numerical information” (p. 121). That digit span
and rapid number naming in the present study were not reliably involved in understanding numeracy difficulties suggests that children with numeracy difficulties do not uniformly show deficits on tasks that contain numerical information; in other words, numbers in and of themselves do not necessarily cause problems for children with numeracy difficulties. Rather, numeracy difficulties may reflect deficits with more complex numerical processing, as in the working memory and math background tasks which involved working with or manipulating numbers and quantitative information in some way, in contrast to numerical tasks that required passively interacting with numbers, such as digit span and rapid number naming tasks. In a related study, Mazzocco and Thompson (2005) also found that not all numeric tasks necessarily increased the prediction of numeracy difficulties. Future research is needed to determine the extent to which numerical abilities - basic and complex - are impaired in children with numeracy difficulties.

**Educational Implications**

A number of important educational implications emerged from the current study. First, the finding that there existed children in each grade who did not show numeracy difficulties in earlier years suggests that early numeracy screening, although important, may not identify all children who develop numeracy difficulties. Thus, the present findings indicate that there is a need to continually assess children’s numeracy skills in order to identify any difficulties as they emerge so that timely interventions can be provided. However, it is possible that the measures used in the current study were not sensitive enough to detect all numeracy difficulties in first grade so that what appeared as late-emerging numeracy difficulties in fact reflects measurement error. The existence of later-emerging numeracy difficulties should be confirmed by future studies. In addition, studies are needed
to examine whether those children whose numeracy difficulties begin in first grade (or earlier) differ in any way from those whose difficulties develop after first grade as this may have implications for early identification, intervention, and instruction.

Another important finding that has educational implications is that in the current study, mathematical problem solving appeared to operate somewhat independently from discrete skills such as calculation and math facts. That is, for children with and without numeracy difficulties, the mean score on mathematical problem solving tended to be higher than the mean score on measures of computational arithmetic and math fact fluency, particularly in the later grades. This finding is interesting and quite opposite to the finding in reading that lower-level discrete skills (e.g., word recognition, word reading fluency) tend to be better developed than applied skills (e.g., reading comprehension). In fact, in reading, it is very rare to find children with word reading difficulties who demonstrate average reading comprehension abilities. In contrast, the results of the present study show that it is not unusual to find children with difficulties in discrete skills (i.e., calculation) but who nonetheless demonstrate average applied skills (i.e., problem solving). Instead, it is much more unusual to find children with specific problem solving difficulties. Furthermore, there did not appear to be any differences between children with specific calculation difficulties, children with specific problem solving difficulties, and children with both calculation and problem solving difficulties, although the present study was not able to definitively answer this question given the sample size. In any case, such results demonstrate that math is fundamentally different from reading so that although reading research may be useful in guiding numeracy research, what has been learned about how to best teach reading may have limited utility in informing math education. More research in this area is warranted.
That applied math in the present study was better developed than discrete math skills is consistent with the notion that formal math as taught in schools does not capitalize on children’s inherent math abilities (e.g., Carraher, Carraher, & Schliemann, 1985; Nunes, Schliemann, & Carraher, 1993). More specifically, Nunes et al. have suggested that the way discrete math skills are taught in school does not empower children to become mathematically literate, as demonstrated by findings such as working-class Brazilian children who have difficulties with school algorithms (e.g., calculation procedures, number-facts) despite being able to demonstrate complex math skills during the course of daily street vending transactions.

There is growing recognition that traditional math instruction that emphasizes rote learning of discrete skills to the exclusion of developing children’s underlying awareness of how and why these skills apply to contexts other than math class does not lead to mathematically competent children or adults (e.g., National Council of Teachers of Mathematics, 1989). The current study supports the notion that children lack a conceptual understanding of how to execute discrete skills, given that many children - including those with numeracy difficulties - were able to perform calculations in context but not in isolation. Deficiencies in the execution of calculation procedures represents a significant concern given the innumerable numeracy demands placed on citizens in the 21st century; at some point, the numeracy demands created by certain tasks outweigh the inherent limits imposed by the architecture of the brain and it becomes necessary to resort to pencil-and-paper calculations (Dehaene, 1997). It is the role of the school to provide children with a solid foundation of how to execute such calculations appropriately and accurately. Studies are needed to examine what specifically about calculation procedures proves challenging for all
children, but especially children with numeracy difficulties. In addition, more research is needed to determine the most appropriate instructional context that supports the development of calculation skills commensurate with problem solving skills.

Limitations

The results from the present study demonstrated that of the various cognitive candidate systems considered, working memory, numerical reasoning, and math background knowledge had the most robust relationship with numeracy achievement and numeracy difficulties in children from first through fourth grade. It is important to note though that the regression models in the current study accounted for between 19% and 61% of the variance in numeracy outcomes, indicating that factors beyond cognitive processes are necessary to study in order to fully understand numeracy in children. Other researchers have noted that comprehensive models of academic achievement need to consider the role of cognitive, economic, social, motivational, and instructional factors (Francis, Fletcher, Stuebing, Lyon, Shaywitz, & Shaywitz, 2005). In particular, there is a growing awareness of the need for researchers studying learning difficulties to link achievement data to specific behavioural criteria directly related to classroom performance, as well as regularly collect data on response to instruction and intervention (e.g., Francis et al.; Fuchs & Fuchs, 2001; Landerl et al., 2004). That such data were not collected is a limitation of the current study and the inclusion of such variables would likely have contributed to a more comprehensive understanding of children's numeracy abilities. More studies are needed in this area to determine how cognitive factors interact with instructional factors, as well as how to provide effective instruction to prevent or reduce numeracy difficulties.
It is also important to note that although the present study was longitudinal and offered important developmental insights, the design was correlational. In other words, as no attempt was made to manipulate the underlying processes in children with numeracy difficulties across development, the present study is limited in the extent to which it can be concluded that the underlying processes measured in the current study - particularly working memory, numerical reasoning, and knowledge of quantitative concepts - are causes or consequences of numeracy difficulties. Training studies are necessary to determine whether interventions aimed at addressing such weaknesses correspond to gains in numeracy achievement. Such evidence would provide important information concerning the nature of the relationship between numeracy and underlying cognitive processes.

Although the relationship between numeracy difficulties and reading difficulties (RD) was not a specific focus of the current study, it is worth noting that the present results may not be typical with respect to the overall number of children identified with RD. Specifically, 10, 8, 15, and 16 children in grades 1, 2, 3, and 4, respectively, were identified with word reading difficulties. Of these children with RD, most tended to have numeracy difficulties: only 1, 0, 2, and 4 children were identified with RD in the typically achieving group in grades 1, 2, 3, and 4, respectively. The overall number of children with RD is much smaller than would be expected in the general population given our selection criteria. The present study was conducted within a school district committed to providing early phonological instruction to children beginning in kindergarten. Thus, it is possible that the number of children with RD in the current study is an underestimate of what would be expected in a district that does not provide such balanced literacy training. In addition, it is unknown whether such early literacy exposure had any effect on the development of numeracy skills in
these children and consequently, whether the present results are typical. Replication of the present study with larger samples of children is necessary.

Finally, it is important to note that the findings from the current study may be limited with respect to how numeracy was defined and measured. The present study examined the development of numeracy in children according to a simple view of numeracy - that is, a view that numeracy difficulties reflected difficulties with either calculation or problem solving. However, Mazzocco and Myers (2003) found that those children with low achievement on one measure of numeracy did not necessarily perform in the below average range on other measures that emphasized different aspects of numeracy achievement. In fact, Mazzocco and Myers found that over half of the children in their study met criteria for numeracy difficulties on at least one standardized numeracy measure at some point during primary school. Thus, the results of the current study may not generalize to other studies that do not use the same or similar types of classification measures. This is an important topic to pursue in order to determine the best way to identify numeracy difficulties and the corresponding deficits associated with low numeracy achievement. As noted at the outset of the current report, there is much variability in the literature with respect to how numeracy difficulties are defined. That the majority of children with numeracy difficulties in the present study also showed difficulties with number fact retrieval supports a definition of numeracy difficulties that is based on calculation skills and mathematical problem solving.
Table 1

*Grade 4 Numeracy Outcomes Based on First to Third Grade Classifications*

<table>
<thead>
<tr>
<th>Grade 4 Outcome</th>
<th>Number of Typically Achieving Children</th>
<th>Number of Children with Numeracy Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 1 Typically Achieving (n = 61)</td>
<td>44 (72%)</td>
<td>17 (28%)</td>
</tr>
<tr>
<td>Grade 2 Typically Achieving (n = 58)</td>
<td>45 (78%)</td>
<td>13 (22%)</td>
</tr>
<tr>
<td>Grade 3 Typically Achieving (n = 56)</td>
<td>47 (84%)</td>
<td>9 (16%)</td>
</tr>
<tr>
<td>Grade 1 Numeracy Difficulties (n = 36)</td>
<td>14 (39%)</td>
<td>22 (61%)</td>
</tr>
<tr>
<td>Grade 2 Numeracy Difficulties (n = 33)</td>
<td>11 (33%)</td>
<td>22 (67%)</td>
</tr>
<tr>
<td>Grade 3 Numeracy Difficulties (n = 40)</td>
<td>11 (28%)</td>
<td>29 (73%)</td>
</tr>
</tbody>
</table>
Table 2

Descriptive Characteristics of Children with Numeracy Difficulties per Grade

<table>
<thead>
<tr>
<th></th>
<th>Grade 1</th>
<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number with numeracy difficulties</td>
<td>36</td>
<td>33</td>
<td>40</td>
<td>39</td>
</tr>
<tr>
<td>Number whose difficulties lasted to grade 4</td>
<td>22</td>
<td>22</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>• Percent with calculation difficulties</td>
<td>14 (n = 3)</td>
<td>23 (n = 5)</td>
<td>38 (n =11)</td>
<td>42 (n= 14)</td>
</tr>
<tr>
<td>• Percent with problem solving difficulties</td>
<td>55 (n= 12)</td>
<td>18 (n = 4)</td>
<td>10 (n = 3)</td>
<td>12 (n = 4)</td>
</tr>
<tr>
<td>• Percent with both numeracy difficulties</td>
<td>32 (n = 7)</td>
<td>59 (n= 13)</td>
<td>52 (n= 15)</td>
<td>45 (n= 15)</td>
</tr>
<tr>
<td>• Percent with math fluency difficulties</td>
<td>-</td>
<td>91 (n= 20)</td>
<td>90 (n= 26)</td>
<td>91 (n= 30)</td>
</tr>
<tr>
<td>• Percent with reading difficulties</td>
<td>32 (n = 7)</td>
<td>32 (n = 7)</td>
<td>38 (n= 11)</td>
<td>33 (n= 11)</td>
</tr>
<tr>
<td>Number with previous numeracy difficulties</td>
<td>-</td>
<td>15</td>
<td>25</td>
<td>33</td>
</tr>
<tr>
<td>Number with no difficulties in previous years</td>
<td>-</td>
<td>7</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
Table 3

Means, Standard Deviations, and Effect Sizes across Measures and Years

<table>
<thead>
<tr>
<th>Measure</th>
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<th></th>
<th>Grade 2</th>
<th></th>
<th>Grade 3</th>
<th></th>
<th>Grade 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD M (SD)</td>
<td>NA M (SD)</td>
<td>(\eta^2)</td>
<td>(\eta^2)</td>
<td>MD M (SD)</td>
<td>NA M (SD)</td>
<td>(\eta^2)</td>
<td>(\eta^2)</td>
</tr>
<tr>
<td>Age</td>
<td>6.86 (0.30)</td>
<td>6.95 (0.26)</td>
<td>ns</td>
<td>ns</td>
<td>7.71 (0.37)</td>
<td>7.90 ns</td>
<td>ns</td>
<td>8.44 (0.22)</td>
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<tr>
<td>Calculation a</td>
<td>41.05 (22.30)</td>
<td>78.38 (16.00)</td>
<td>.49</td>
<td>.25</td>
<td>27.50 (18.17)</td>
<td>79.38 (18.22)</td>
<td>.60</td>
<td>.27</td>
</tr>
<tr>
<td>Applied Problems a</td>
<td>24.50 (17.53)</td>
<td>76.22 (21.00)</td>
<td>.62</td>
<td>.42</td>
<td>26.44 (19.89)</td>
<td>82.19 (16.36)</td>
<td>.69</td>
<td>.39</td>
</tr>
<tr>
<td>Math Fluency a</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>21.67 (12.38)</td>
<td>69.58 (18.82)</td>
<td>.60</td>
<td>.43</td>
</tr>
<tr>
<td>WM Span b</td>
<td>2.07 (0.79)</td>
<td>3.00 (0.91)</td>
<td>.22</td>
<td>.12</td>
<td>2.53 (0.59)</td>
<td>3.28 (0.82)</td>
<td>.18</td>
<td>.10</td>
</tr>
<tr>
<td>Forward Span c</td>
<td>4.41 (1.25)</td>
<td>4.92 (1.00)</td>
<td>ns</td>
<td>ns</td>
<td>4.59 (1.27)</td>
<td>5.46 (1.06)</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Block Rotation a</td>
<td>41.91 (27.79)</td>
<td>67.70 (29.00)</td>
<td>.17</td>
<td>.13</td>
<td>39.50 (34.59)</td>
<td>58.84 (32.58)</td>
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<td>ns</td>
</tr>
<tr>
<td>Number Naming d</td>
<td>17.91 (5.94)</td>
<td>14.68 (3.33)</td>
<td>ns</td>
<td>ns</td>
<td>14.56 (5.66)</td>
<td>11.86 (2.87)</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Concepts</td>
<td>12.23</td>
<td>14.78</td>
<td>.33</td>
<td>ns</td>
<td>13.94</td>
<td>17.08</td>
<td>.51</td>
<td>.16</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>-------</td>
<td>-----</td>
<td>----</td>
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<td>-------</td>
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</tr>
<tr>
<td>(2.00)</td>
<td>(1.65)</td>
<td>(1.57)</td>
<td>(1.38)</td>
<td>(1.68)</td>
<td>(1.62)</td>
<td>(2.21)</td>
<td>(1.29)</td>
<td></td>
</tr>
<tr>
<td>9 - 17</td>
<td>11 - 18</td>
<td>12 - 17</td>
<td>14 - 19</td>
<td>11 - 21</td>
<td>13 - 20</td>
<td>11 - 21</td>
<td>15 - 22</td>
<td></td>
</tr>
<tr>
<td>Number Series</td>
<td>7.50</td>
<td>9.81</td>
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<td>.14</td>
<td>8.69</td>
<td>11.62</td>
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<td>.12</td>
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<tr>
<td>(1.50)</td>
<td>(1.94)</td>
<td>(1.35)</td>
<td>(1.80)</td>
<td>(1.80)</td>
<td>(1.64)</td>
<td>(1.87)</td>
<td>(1.63)</td>
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<tr>
<td>5 - 10</td>
<td>5 - 13</td>
<td>7 - 11</td>
<td>6 - 15</td>
<td>6 - 13</td>
<td>9 - 16</td>
<td>7 - 18</td>
<td>11 - 18</td>
<td></td>
</tr>
<tr>
<td>Word Attack</td>
<td>62.45</td>
<td>87.54</td>
<td>.33</td>
<td>ns</td>
<td>50.44</td>
<td>80.03</td>
<td>.38</td>
<td>ns</td>
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<tr>
<td>(22.76)</td>
<td>(13.35)</td>
<td>(25.27)</td>
<td>(13.36)</td>
<td>(23.34)</td>
<td>(16.00)</td>
<td>(22.02)</td>
<td>(15.70)</td>
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<tr>
<td>LWID</td>
<td>52.14</td>
<td>86.38</td>
<td>.40</td>
<td>-</td>
<td>37.44</td>
<td>81.19</td>
<td>.59</td>
<td>ns</td>
</tr>
<tr>
<td>9 - 95</td>
<td>39 - 99</td>
<td>3 - 78</td>
<td>51 - 99</td>
<td>3 - 97</td>
<td>27 - 96</td>
<td>5 - 99</td>
<td>6 - 97</td>
<td></td>
</tr>
<tr>
<td>Verbal Analogy</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.50</td>
<td>6.24</td>
<td>.17</td>
<td>ns</td>
</tr>
<tr>
<td>(2.25)</td>
<td>(1.62)</td>
<td>(2.28)</td>
<td>(1.38)</td>
<td>(1.66)</td>
<td>(1.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 - 7</td>
<td>3 - 10</td>
<td>0 - 8</td>
<td>4 - 9</td>
<td>4 - 9</td>
<td>3 - 13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Dashes indicate the measure was not administered. Analyses based on n = 37 in the normally achieving group and n = 22, 22, 29, and 33 in the numeracy difficulty group in grade 1, 2, 3, and 4, respectively. Significant differences are significant at least at p = .002. Partial eta squared (\( \eta^2 \)) was used as the measure of effect size: .01 (small effect), .059 (medium effect), and .138 (large effect). Two \( \eta^2 \) values are reported: the first is the effect size for the ANOVA analyses and the second value reflects the effect size for the ANCOVA analyses. MD = numeracy difficulties group; NA = normally achieving group; WM = working memory; LWID = Letter-Word Identification. Results presented for Calculation, Applied Problems, Math Fluency, Block Rotation, Word Attack, and Letter-Word Identification are in percentile scores. Scores for working memory span and forward span are span scores. Scores for all other measures are presented in raw scores.

These measures were taken from the \textit{WJ-III} (Woodcock et al., 2001). \textsuperscript{b} Working Memory Span was measured with working memory for numbers (Siegel & Ryan, 1989). \textsuperscript{c} Forward Span was measured with the Forward Digit Span portion of the Digit Span subtest of the \textit{WISC-3} (Wechsler, 1991). \textsuperscript{d} Rapid Naming was measured with a rapid number naming task. Lower scores on rapid naming tasks indicate better performance.
Table 4

Correlations among Variables Involved in First and Second Grade Analyses

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Calculation</td>
<td>—</td>
<td>.64***</td>
<td>.30**</td>
<td>.29**</td>
<td>.25*</td>
<td>-.26*</td>
<td>.49***</td>
<td>.47***</td>
<td>.40***</td>
</tr>
<tr>
<td>2. Applied Problems</td>
<td>.48***</td>
<td>—</td>
<td>.39***</td>
<td>.31**</td>
<td>.12</td>
<td>-.30**</td>
<td>.66***</td>
<td>.53***</td>
<td>.42***</td>
</tr>
<tr>
<td>3. Working Memory Span</td>
<td>.42***</td>
<td>.53***</td>
<td>—</td>
<td>.23*</td>
<td>.01</td>
<td>-.22*</td>
<td>.19*</td>
<td>.29**</td>
<td>.27**</td>
</tr>
<tr>
<td>4. Forward Digit Span</td>
<td>.06</td>
<td>.23*</td>
<td>.25**</td>
<td>—</td>
<td>.09</td>
<td>-.04</td>
<td>.31**</td>
<td>.08</td>
<td>.44***</td>
</tr>
<tr>
<td>5. Block Rotation</td>
<td>.12</td>
<td>.40***</td>
<td>.35***</td>
<td>.31**</td>
<td>—</td>
<td>-.21*</td>
<td>.13</td>
<td>.11</td>
<td>.15</td>
</tr>
<tr>
<td>6. Rapid Number Naming</td>
<td>-.37**</td>
<td>-.37**</td>
<td>-.25**</td>
<td>.00</td>
<td>-.21*</td>
<td>—</td>
<td>-.30**</td>
<td>-.15</td>
<td>-.28**</td>
</tr>
<tr>
<td>7. Concepts</td>
<td>.42***</td>
<td>.65***</td>
<td>.36***</td>
<td>.25**</td>
<td>.31**</td>
<td>-.35**</td>
<td>—</td>
<td>.41***</td>
<td>.47***</td>
</tr>
<tr>
<td>8. Number Series</td>
<td>.43***</td>
<td>.59***</td>
<td>.37***</td>
<td>.04</td>
<td>.40***</td>
<td>-.33**</td>
<td>.56**</td>
<td>—</td>
<td>.34**</td>
</tr>
<tr>
<td>9. Word Attack</td>
<td>.37***</td>
<td>.50***</td>
<td>.20*</td>
<td>.27**</td>
<td>.12</td>
<td>-.36**</td>
<td>.44**</td>
<td>.29**</td>
<td>—</td>
</tr>
</tbody>
</table>

*Note.* The left of the diagonal shows the correlations among the first grade variables and the right of the diagonal displays the correlations among the second grade variables.

*p < .05. **p < .01. ***p < .001.
Table 5

Correlations among Variables Involved in Third and Fourth Grade Analyses

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>1. Calculation</td>
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<td>.37***</td>
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</tr>
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<td></td>
<td>.49***</td>
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<td>-.37***</td>
<td>.54***</td>
<td>.53***</td>
<td>.24**</td>
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<tr>
<td>3. Working Memory Span</td>
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<td>.39***</td>
<td></td>
<td>.32**</td>
<td>.33***</td>
<td>-.37***</td>
<td>.46***</td>
<td>.14</td>
<td>.29**</td>
</tr>
<tr>
<td>4. Forward Digit Span</td>
<td>.19</td>
<td>.37***</td>
<td>.29**</td>
<td></td>
<td>.29**</td>
<td>-.26*</td>
<td>.34***</td>
<td>.09</td>
<td>.37***</td>
</tr>
<tr>
<td>5. Block Rotation</td>
<td>.40***</td>
<td>.26**</td>
<td>.09</td>
<td>.02</td>
<td></td>
<td>-.13</td>
<td>.34***</td>
<td>.19*</td>
<td>.16</td>
</tr>
<tr>
<td>6. Rapid Number Naming</td>
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<td>-.22*</td>
<td>-.42**</td>
<td>-.09</td>
<td>-.28**</td>
<td></td>
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<td>-.18*</td>
<td>-.33***</td>
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<td>7. Concepts</td>
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<td>.57***</td>
<td>.38***</td>
<td>.35***</td>
<td>.27**</td>
<td>-.24**</td>
<td></td>
<td>.33**</td>
<td>.41***</td>
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<td>8. Number Series</td>
<td>.58***</td>
<td>.63***</td>
<td>.53***</td>
<td>.23*</td>
<td>.19*</td>
<td>-.37***</td>
<td>.46***</td>
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<td>.23*</td>
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<tr>
<td>9. Word Attack</td>
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<td>.39***</td>
<td>.29**</td>
<td>.31**</td>
<td>.06</td>
<td>-.25**</td>
<td>.51***</td>
<td>.36***</td>
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</tbody>
</table>

Note. The left of the diagonal shows the correlations among the third grade variables and the right of the diagonal displays the correlations among the fourth grade variables.
*p < .05. **p < .01. ***p < .001.
Table 6

**Multiple Regression of First to Fourth Grade Calculation Skills**

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<tr>
<th>Explanatory Variables</th>
<th>Grade 1</th>
<th></th>
<th>Grade 2</th>
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<th></th>
<th>Grade 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$ (SE)</td>
<td>$\beta$</td>
<td>Pratt Index</td>
<td>$B$ (SE)</td>
<td>$\beta$</td>
<td>Pratt Index</td>
<td>$B$ (SE)</td>
<td>$\beta$</td>
</tr>
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<td>.24</td>
<td>.297</td>
<td>0.59 (.37)</td>
<td>.16</td>
<td>.111</td>
<td>0.41 (.29)</td>
<td>.12</td>
</tr>
<tr>
<td>Forward Digit Span *</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.22 (.30)</td>
<td>.08</td>
<td>.054</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Block Rotation b</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.01 (.01)</td>
<td>.14</td>
<td>.081</td>
<td>0.02 (.01)**</td>
<td>.25</td>
</tr>
<tr>
<td>Rapid Naming</td>
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<td>-.15</td>
<td>.164</td>
<td>-0.07 (.09)</td>
<td>-.08</td>
<td>.048</td>
<td>-0.05 (.09)</td>
<td>-.05</td>
</tr>
<tr>
<td>Concepts b</td>
<td>0.11 (.12)</td>
<td>.11</td>
<td>.136</td>
<td>0.43 (.18)*</td>
<td>.27</td>
<td>.306</td>
<td>0.31 (.14)*</td>
<td>.22</td>
</tr>
<tr>
<td>Number Series b</td>
<td>0.23 (.13)</td>
<td>.18</td>
<td>.228</td>
<td>0.43 (.16)**</td>
<td>.29</td>
<td>.316</td>
<td>0.47 (.14)**</td>
<td>.31</td>
</tr>
<tr>
<td>Word Attack b</td>
<td>0.06 (.04)</td>
<td>.16</td>
<td>.175</td>
<td>0.05 (.06)</td>
<td>.09</td>
<td>.083</td>
<td>0.06 (.04)</td>
<td>.12</td>
</tr>
<tr>
<td>Model $R^2$</td>
<td>.339***</td>
<td></td>
<td></td>
<td>.432***</td>
<td></td>
<td></td>
<td>.514***</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Dashes indicate the variable was removed from the final model. A Pratt Index score less than .10, .07, .08, and .10 in grades 1, 2, 3, and 4, respectively, indicates that the variable is "unimportant" to the model, as determined by the criteria set by Thomas (1992). Forward Digit Span was measured with the Forward Digit Span portion of the Digit Span subtest of the WISC-3 (Wechsler, 1991). These variables were measured using tests from the WJ-III (Woodcock et al., 2001).

*p < .05. **p < .01. ***p < .001
### Table 7

**Multiple Regression of First to Fourth Grade Problem Solving**

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Grade 1</th>
<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$ (SE)</td>
<td>$\beta$ Pratt Index</td>
<td>$B$ (SE)</td>
<td>$\beta$ Pratt Index</td>
</tr>
<tr>
<td>Working Memory</td>
<td>1.31 (.39)**</td>
<td>.25 .216</td>
<td>0.94 (.43)*</td>
<td>.18 .123</td>
</tr>
<tr>
<td>Forward Digit Span a</td>
<td>-0.06 (.33)</td>
<td>-.01 .004</td>
<td>0.42 (.33)</td>
<td>.11 .060</td>
</tr>
<tr>
<td>Block Rotation b</td>
<td>0.02 (.01)</td>
<td>.10 .065</td>
<td>- -</td>
<td>0.02 (.01)</td>
</tr>
<tr>
<td>Rapid Naming</td>
<td>-0.02 (.08)</td>
<td>-.02 .012</td>
<td>-0.09 (.10)</td>
<td>-.08 .042</td>
</tr>
<tr>
<td>Concepts b</td>
<td>0.62 (.18)**</td>
<td>.30 .318</td>
<td>1.06 (.21)**</td>
<td>.46 .532</td>
</tr>
<tr>
<td>Number Series b</td>
<td>0.53 (.22)*</td>
<td>.21 .202</td>
<td>0.59 (.19)**</td>
<td>.27 .251</td>
</tr>
<tr>
<td>Word Attack b</td>
<td>0.18 (.06)**</td>
<td>.23 .187</td>
<td>-0.08 (.07)</td>
<td>-.01 .007</td>
</tr>
</tbody>
</table>

**Model $R^2$**

- Grade 1: .614***
- Grade 2: .571***
- Grade 3: .532***
- Grade 4: .541***

*Note.* Dashes indicate the variable was removed from the final model. A Pratt Index score less than .07, .08, .07, and .07 in grades 1, 2, 3, and 4, respectively, indicates that the variable is "unimportant" to the model.

*a* Forward Digit Span was measured with the Forward Digit Span portion of the Digit Span subtest of the *WISC-3* (Wechsler, 1991).

*b* These variables were measured using tests from the *WJ-III* (Woodcock et al., 2001).

*p < .05.  **p < .01.  ***p < .001*
Table 8

*Relative Risk of Developing Numeracy Difficulties Based on Grade 1 Measures*

<table>
<thead>
<tr>
<th>Grade 1 Measure (below average score)</th>
<th>Proportion of Children with Low Grade 1</th>
<th>Relative Risk</th>
<th>Skill</th>
<th>Grade 4</th>
<th>Grade 4</th>
<th>Ratio (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Numeracy Group</td>
<td>Typical Group</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working Memory (Span &lt; 2.0)</td>
<td>.487</td>
<td>.259</td>
<td>1.88</td>
<td>(1.10 - 3.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward Digit Span (Span &lt; 4.0) a</td>
<td>.436</td>
<td>.310</td>
<td>1.40</td>
<td>(0.83 - 2.37)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Block Rotation (Percentile &lt; 25) b</td>
<td>.333</td>
<td>.224</td>
<td>1.49</td>
<td>(0.77 - 2.86)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Naming (Seconds &gt; 19)</td>
<td>.385</td>
<td>.379</td>
<td>1.01</td>
<td>(0.61 - 1.70)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concepts (Raw score &lt; 12) b</td>
<td>.615</td>
<td>.328</td>
<td>1.88</td>
<td>(1.20 - 2.93)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Series (Raw Score &lt; 7) b</td>
<td>.564</td>
<td>.293</td>
<td>1.92</td>
<td>(1.18 - 3.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Word Attack (Percentile &lt; 25) b</td>
<td>.436</td>
<td>.328</td>
<td>1.33</td>
<td>(0.80 - 2.22)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Analyses based on n = 39 in the grade 4 low numeracy group and n = 58 in the grade 4 typical group. Relative risk ratios that do not include 1.00 in the confidence interval are significant at the .05 level.

a Forward Digit Span was measured with the Forward Digit Span portion of the Digit Span subtest of the *WISC-3* (Wechsler, 1991). b These measures were taken from the *WJ-III* (Woodcock et al., 2001).
Figure 1. An example of the task requirements of the *WJ-III* Block Rotation test (Woodcock et al., 2001). Children were required to select two of five items that represented the stimulus object (boxed items) rotated in three dimensional space.
Figure 2. A breakdown of the percentage of numeracy difficulties per grade that reflect either calculation-only difficulties, problem solving-only difficulties, or difficulties with both calculation and problem solving. This Figure demonstrates that in the early years, numeracy difficulties reflected difficulties primarily with mathematical problem solving and in later years, numeracy difficulties reflected either specific calculation difficulties or difficulties with both calculation and problem solving.
Figure 3. Numeracy achievement of children with numeracy difficulties. This Figure demonstrates that in the early years, children with numeracy difficulties had more difficulty with mathematical problem solving than calculation procedure. In later grades, children with numeracy difficulties performed better on mathematical problem solving than on isolated skills such as calculation procedures and math fluency.
References


growth of word-reading skills in second- to fifth-grade children. Scientific Studies of Reading, 1, 161-185.


