Imperfect Information and Macroeconomics

by

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Abstract

This dissertation addresses the role that imperfect information about economic fundamentals plays in shaping the equilibrium outcomes of the economy. The three chapters each look at different issues.

The first chapter examines the optimal amount of "noise" that a policy authority should inject into its public announcements. Private sector agents may individually incur a cost of acquiring private information about the state of the economy and this private information may be incorporated into their actions. The policy authority attempts to infer the state of the economy from a noisy measure of aggregate private sector actions and makes a public announcement to inform the private sector of this inference. The policy authority faces a trade-off between informing the private sector of the state of the economy, and gathering information about the state of the economy. It is shown that there may exist conditions under which a policy authority can increase the welfare of the agents in the economy by making policy statements that do not perfectly reveal the policy authority's information.

The second chapter examines the ability of equilibrium job search and matching models to produce highly volatile unemployment rates when subjected to small fluctuations in labour productivity. A model is presented in which there is asymmetric information between firms and workers concerning their output as well as the ability for workers to search on-the-job for new jobs. This model is able to amplify the effects of labour productivity well beyond that observed in the standard equilibrium job search models.

The third chapter examines the ability of standard monetary business cycle models to yield immediate increases in aggregate consumption and investment in response to the arrival of news concerning future increases in productivity. Such responses are not possible in the standard real business cycle models. It is found that the combination of nominal price and wage rigidities and interest rate rules that target expected inflation can result in immediate increases in consumption and investment upon the arrival of news indicating future productivity increases.

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Dedication

For Mom and Dad

Chapter 1

Introduction

This dissertation addresses the role that imperfect information about economic fundamentals plays in shaping the equilibrium outcomes of the economy. It is generally accepted that information imperfections can result in interesting macroeconomic outcomes. In the following chapters I illustrate three separate macroeconomic models and show how informational issues can either result in novel macroeconomic problems or be used to resolve problems in models that typically have abstracted from informational issues.

The first chapter examines the information gathering and dissemination role of the monetary policy authority. The enormous attention paid by the private sector to monetary policy announcements gives rise to the question of whether monetary policy announcements can transmit quality information about important economic conditions to the private sector. A chief difficulty faced by central banks is to obtain an accurate measure of underlying macroeconomic conditions through observations of private sector economic behaviour and to simultaneously signal these conditions to imperfectly informed members of the private sector. This chapter examines the role of transparency in a benevolent monetary authority's policies. Private sector agents may individually incur a cost of acquiring private information about the state of the economy and this private information may be incorporated into their actions. The policy authority attempts to infer the state of the economy from a noisy measure of aggregate private sector actions and makes a public announcement to inform the private sector of this inference. The policy authority faces a trade-off between informing the private sector of the state of the economy, and gathering information about the state of the economy. When the policy announcement is of very high quality, private sector agents have an incentive not to gather private information and to base their actions solely on information contained in the policy announcement. However, this makes the observed actions of the private sector less informative to the policy authority. The monetary authority should be vague about what it believes to be the state of the economy when doing so pushes more private sector agents into gathering information thereby making the policy signal more informative to those for whom it is relatively more costly to gather private information.

The second chapter examines the role that private information held by firms concerning output production plays in the determination of equilibrium wages and how such wage setting institutions determine the effects of productivity changes on the dynamics of the unemployment rate. An equilibrium model of job search and unemployment is presented that can produce large fluctuations in unemployment while allowing for wage dynamics in response to plausible productivity shocks. Informational asymmetries between firms and their workers create an upper bound to wage contracts offered by firms in equilibrium. At the same time, the threat of unobservable on-the-job search by the worker links wages to market conditions. In times of high productivity, wages do not increase to absorb all the gains of productivity resulting in an incentive for firms to create job vacancies. The result is that small productivity gains can cause large reductions in the unemployment rate. It has been shown that the workhorse model of equilibrium unemployment theory, the Mortensen-Pissarides model, is unable to generate realistic unemployment and vacancy movements in response to small productivity shocks. This is of importance given that recent observations have emphasized the role of fluctuations in the job finding rate and downplayed the role of variation in the job separation rate as driving forces behind unemployment and wage dynamics.

The third chapter examines role of monetary policy interest rate rules in determining equilibrium outcomes of consumption and investment in a standard monetary business cycle model. Shocks to optimism about future events have been shown to account for almost 50% of business cycle fluctuations and any serious business cycle model should be able to account for such observations. The aspects of monetary policy that allow for the possibility of booms in consumption and investment, driven by optimism, is studied in a standard one-sector model; a framework that has been shown to have difficulties generating such expectationally driven booms in consumption and investment. The third chapter clarifies the role of that monetary policy plays in allowing for the possibility of consumption and investment booms driven by optimism about future events. It has been shown that standard one-sector RBC models are unable to generate immediate increases in both consumption and investment in response to optimistic beliefs about future events when current productivity remains unchanged. This chapter shows that standard monetary business cycle models with nominal wage rigidities are able to generate increases in both consumption and investment at impact in response to anticipated technological changes when current productivity remains unchanged. The key component to the model used in this paper that gives rise to both a rise in consumption and investment in the face of such anticipated future gains to TFP is an interest rate rule that results in a lower expected real rate of return to savings as compared to the economy's "natural rate" counterpart. Conditional on such a rule, nominal wage rigidities allow for large investment increases following an increase in expected TFP gains when equilibrium prices result in lower real wages and nominal price rigidities that result in variable real marginal costs can also allow for investment increases. Use of standard Taylor rules do not result in the aforementioned dynamics.

Chapter 2

Information Acquisition, Dissemination and Transparency of Monetary Policy

> ... the problem of what is the best way of utilizing knowledge initially dispersed among all the people is at least one of the main problems of economic policy ...- *F. A. Hayek*

Over the last decade as more central banks have adopted inflation targeting policies, there has been an increased emphasis on the role of transparency in the conduct of monetary policy. In an effort to increase transparency[1] under inflation targeting policy regimes, countries such as Canada, New Zealand, the United Kingdom and Sweden announce explicit inflation targets, issue publications of inflation forecasts, and may even publish the ongoings of the policy committee's meetings. The amount of attention paid to these publications certainly suggests that there could be an important information dissemination role played by policy announcements.[2] Improper maintenance of this information channel could result in private sector agents making costly decisions based on inaccurate beliefs about the state of the economy.

A complication that arises in the conduct of policy is that this information channel works in both directions. The policy authority's instrument setting may transmit information to members of the private sector that influences their actions. On the flip-side, actions taken by agents in the private sector provide data that may inform the policy authority about the state of the economy thereby affecting the setting of the policy instrument. Thus there is a feedback from the action of the policy authority to the data that it observes, from which it draws inference concerning the state of the economy. The information gathering and dissemination role of the policy

[1]The common (theoretical) measure of transparency is the distance between what the private sector thinks the policy authority's information set is and what the policy authority's information set actually is.

[2]See Romer and Romer (2000) for evidence that there may be informational content in policy instrument settings.

authority begets the question of whether increased transparency is optimal when the informational content of actions is endogenous.

This paper highlights a potential danger of transparent policy when the policy authority must gather information from observing the actions of private sector agents.[3] Agents populating the economy may incur a cost to acquire private information concerning underlying supply and demand conditions.[4] As information acquisition costs may vary across private agents there may be asymmetric knowledge concerning underlying macroeconomic conditions. The policy authority is confronted with the task of gathering information about the unobservable fundamentals underlying the economy through its inference based on the observed actions of private sector agents. At the same time, the policy authority is charged with making a policy statement to better inform the private sector about the unobservable fundamentals. As the policy statement is publicly observed, when it is very informative, private sector agents have an incentive not to acquire private information. A trade-off emerges between increasing the accuracy of the beliefs amongst private sector agents and decreasing the quality of the information contained in private sector actions observed by the policy authority.

The main result of this paper is that the policy authority should be vague about its beliefs concerning macroeconomic conditions when doing so causes a sizable fraction of private sector agents to gather information about economic conditions thereby enabling the policy authority to make more informative policy statements. By increasing the noise in policy statements the authority increases the risk to agents who choose their actions based solely on inference of macroeconomic conditions from the policy statement. Thus a greater incentive for private sector agents to acquire private information about macroeconomic conditions is provided. This information is worked into the actions of informed private agents, hence increasing the information quality of data that the policy authority observes. With higher quality data the policy authority is better equipped to inform agents of the private sector through its statements. Therefore, in highlighting a new avenue by which policy transparency affects the economy, this paper exposes another margin on which policy authorities must exercise care when making policy decisions.

A model is constructed in which firms must set prices for their output and a policy authority attempts to influence the prices set by firms so that the prices best reflect underlying macroeconomic conditions. The profits of firms are determined in part by how their prices reflect underlying cost and demand conditions but there is a strategic complementarity component in that payoffs are also affected by the deviation of their price from the prices set by firms selling substitutable goods. All firms in the economy would like a clear measure of underlying macroeconomic conditions, however, it is costly for firms to acquire such information. Firms differ in their cost of information

[3]For a seminal paper concerning information aggregation by a price system see Grossman and Stiglitz (1980). Vives (1997) discusses the speed of information aggregation and corresponding welfare over time. Bernanke and Woodford (1997) present a model of inflation forecast targeting with a flavour similar to the argument used in this paper.

[4]In the model that follows, for purposes of exposition, the environment is pushed to the extreme in that some agents may choose to be perfectly informed while others are imperfectly informed.

acquisition. By choosing their price optimally given their information, prices may reflect the private information of firms. Unfortunately, agents are hamstrung by their inability to directly communicate with one another and cannot observe prices set by other firms in the economy. It is assumed that a benevolent policy authority exists that has access to a noisy measure of aggregate activity, that is, the policy authority is assumed to be able to observe a measure of the aggregate prices set by firms. This is the policy authority's sole source of information about underlying macroeconomic conditions. It is this information that the policy authority would like to pass on to the agents in the economy through its policy statement, which in this paper is a "policy" signal.

A problem arises that if the policy authority is too clear about its information, then the agents in the economy pay more attention to the policy signal in choosing their action and thereby have little incentive to acquire private information. If this is the case, then the aggregate actions of the agents mostly reflect information the policy authority incorporates into its signal and not what they know through their private signals. Ironically, by trying to be more informative, the policy authority is able to gather less information thereby making the policy signal less informative. This is the major control problem faced by the authority in this paper.

When there is a sizable fraction of agents with relatively similar information acquisition costs and there is a sufficient degree of strategic complementarity in the payoff function of the private sector agents then the monetary authority will choose to make statements that do not perfectly reveal its beliefs to the private sector. In other words, non-transparent policy will be optimal. This result arises because all firms choose their prices partly to be as close to those of their competitors as possible. This element of the payoff function results in informed firms choosing prices that may not necessarily be those optimal given supply and demand conditions but rather to conform to prices chosen by firms that are not perfectly informed about macroeconomic conditions. Uninformed firms learn about economic conditions solely from policy announcements so if policy announcements are more informative then both the informed firms and the uninformed firms are better off. Cases exist where increasing the level of noise in policy statements will cause a large fraction of firms to pay their costs to become informed. In these cases, the information obtained by informed firms is worked into their prices allowing the data observed by the policy authority to be more informative about the underlying macroeconomic conditions. This allows for more informative policy statements to those for whom it is excessively costly to directly acquire information about the underlying economic conditions. It is precisely when this spillover effect dominates the harm of more noise in the policy statement that the policy authority should choose to be non-transparent.

The major contribution of this paper to the monetary policy literature is that it considers the effect that policy announcements have on an economy where both the policy authority and the private sector are asymmetrically, imperfectly informed.[5]

[5]Much of the previous work on transparency stresses the incentive of the policy authority to be vague in order to prevent the private sector from learning about the objectives of the policy authority which is private information of the policy authority. The policy authority may choose not

In doing so it is recognized that the policy authority's source of information are endogenous variables and the informational content of these variables depend on policy actions. Svensson and Woodford (2003, 2004), and Aoki (2003) address the problems that a monetary policy authority faces in controlling a forward-looking dynamic economy when either the policy authority and the private sector are equally ignorant about the state of the world or when the private sector is more knowledgable about the state of the world than is the policy authority. Amato and Shin (2004) address the problem when information is dispersed amongst agents in the economy but the policy authority is perfectly informed.

Recently, a paper by Morris and Shin (2002) studying the social value of public information has shown that increasing the quality of publicly observed policy announcements issued by a benevolent policy authority may be harmful to the economy. By emphasizing the interaction between imperfect information about unobservable payoff relevant fundamentals and strategic complementarities in agents' actions, their paper shows that small errors in commonly observed exogenous policy signals can be amplified to the point where the economy would be better off having agents ignore the policy signal and act only on their imperfect private information. This paper differs in that the quality of the policy statement is endogenous and depends on the behaviour of the policy authority. As a result it is shown that the reason in favour of less transparency is quite different. It is optimal to reduce transparency in order for the policy authority to gather more information thereby making it possible for policy statements to be more informative. This makes clear the consequences of recognizing the fact that the policy authority's source of information is an endogenous variable and that the informational quality of the authority's data depends heavily on the actions of the policy authority.

The following section presents the environment to be studied. In Section 2.2 the equilibrium is defined. Section 2.3 contrasts the mechanism highlighted in this paper from those commonly discussed in the monetary policy transparency literature. Section 2.4 presents an extension concerning discretionary policy under which the policy authority does not possess a commitment technology. An alternative set-up is also considered in which the main results can be obtained in a model with cost externalities. The model with cost externalities clarifies the role of increasing returns to information acquisition used in this paper. Section 2.5 concludes.

2.1 The Model

The private sector agents in this model are firms who are setting the price of their good. Ex ante, firms are unaware of the underlying macroeconomic conditions

to be transparent about its objective function in order to be able to sacrifice inflation for output when desired. In other words, the policy authority does not want to be forthright in order to take advantage of its informational advantage. For some examples see Stein (1989), Cukierman and Meltzer (1986), and Faust and Svensson (2002). Geraats (2002) provides a comprehensive literature review. The practices of central banks following inflation targeting policies appear to make such arguments dated.

that are relevant to their profit maximizing price. If these supply and demand conditions were known perfectly, then all firms would know exactly what price to charge.[6] These market conditions can encompass anything from labour market conditions to the development of new cost reducing technologies. In the model, these conditions are captured by an unobserved random variable. It is assumed that these unobserved conditions affect all firms alike. Each firm faces a cost of acquiring information about the underlying macroeconomic conditions. These costs can be thought of as a research cost or effort expended to research market conditions. If a given firm incurs this cost then that firm learns perfectly about the macroeconomic conditions.[7] It is assumed that firms can learn about the macroeconomic conditions because they have access to information about its workers, firm specific productivity, market competition, market demands, etc. There also exists a policy authority who would like prices to best reflect the unobservable economic fundamentals. The policy authority can send a policy signal concerning its beliefs about the realization of the unobservable, a signal that is common knowledge. However, the policy authority gathers all its information from a noisy observation of private sector actions.

The timing is as follows : the policy authority decides on the quality of its policy signal, firms decide whether to become informed, nature chooses the shocks to the economy at which point informed firms learn the realization of macroeconomic conditions, and finally the prices chosen by firms and the policy signal are simultaneously determined. It is assumed that agents know all the parameters governing the economy when they make their decisions. An elaboration on details is given below.

2.1.1 The Agents in the Private Sector

There exists a continuum of firms indexed on the unit interval. Firm i chooses the its price[8], $p_i \in \Re$, and also decides whether to incur an idiosyncratic cost, c_i, in order to obtain a signal about the realization of payoff relevant macroeconomic conditions. The underlying macroeconomic conditions will be denoted by the random variable $\tilde{\theta}$. Following convention, random variables are denoted with a tilde, while the same variables without a tilde denote the realization of the random variable. Costs of information acquisition are drawn from a distribution $G(c)$ that is continuously differentiable with density $g(c)$. The information cost can be thought of as research costs incurred to learn about macroeconomic conditions. Let z_i denote firm i's decision of becoming informed about $\tilde{\theta}$.

$$z_i = \begin{cases} 1 & \text{if informed} \\ 0 & \text{if uninformed} \end{cases}$$

[6] One can also think of the problem of the firms in a Lucas Island world where firms want to know macroeconomic conditions because it helps them distinguish between the component of the demand for their good that is common across all firms versus idiosyncratic to their specific firm. The underlying mechanism of this paper should carry through.

[7] Alternatively, a firm can receive a noisy signal about the underlying macroeconomic conditions if it incurs the cost of becoming informed. By allowing firms to receive a noiseless signal, the exposition is simplified immensely.

[8] It may be useful to think of the firm setting the logarithm of its price as prices may be negative.

The payoff function for firm i is given by

$$u_i(p,\theta) = -(1-r)(p_i - \theta)^2 - r(L_i - \overline{L}) - z_i c_i$$

where r is a constant such that $r \in (0,1)$, $\underset{\sim}{p}$ is the profile of prices chosen by all firms, θ is an unobserved random variable with $\tilde{\theta} \sim N(0, \sigma_\theta^2)$,

$$L_i \equiv \int_0^1 (p_j - p_i)^2 dj$$

and

$$\overline{L} \equiv \int_0^1 L_j dj.$$

Clearly, aside from the information acquisition cost, the loss function has 2 components:

1. The first component is a standard quadratic loss in the distance between the underlying state variable θ and firm i's action p_i.

2. The second component is the strategic complementarity term. The loss L_i is increasing in the average distance between i's action and the action profile of the entire population. The effect of the quality of firm i's forecast, as measured by L_i, on total loss u_i depends on how good firm i's forecast of θ is relative to the measure of the average quality of forecasts across the population, $\overline{L}$.

There is an externality in which an individual agent tries to second guess the decisions of other individuals in the economy. The parameter r puts the weight on this externality. This externality is socially inefficient in that it is of a zero-sum nature. In the game of second guessing, the winners gain at the expense of the losers.

To put the variable $\tilde{\theta}$ in context, it is useful to think of θ as representing the price level that would prevail if everyone in the economy could observe the values of all the fundamental shocks buffeting the economy with common knowledge. These include all the unobserved market conditions referred to earlier. In this model, all the agents want to know what this price level should be but individual firms do not have direct observations of the fundamental shocks unless an information acquisition cost is incurred. In this setting the demand for their good will depend on their relative price. However, if all firms set prices too high (given fundamental shocks) then it may be better for a given firm to choose prices to closer reflect what it believes the fundamental shocks warrant.

Social welfare, defined as the average of individual payoffs is

$$\begin{aligned} W &= \int_0^1 u_i(p,\theta) di \\ &= -(1-r)\int_0^1 (p_i - \theta)^2 di - \int_0^1 z_i c_i di \end{aligned}$$

Thus a social planner who cares only about welfare seeks to keep p_i close to θ, for all i.

Before choosing its price, each firm must decide whether to incur the cost of becoming informed about θ. If a firm decides to become informed it will learn the realized macroeconomic conditions. Otherwise, firms will only observe a signal issued by the policy authority concerning macroeconomic conditions. When firms decide whether or not to become informed about macroeconomic conditions, the quality of the policy signal, y, which is described later, is common knowledge. Firm i will become informed if it expects a higher payoff when informed than if it remains uninformed. That is, $z_i = 1$ if

$$E\left\{E[-(1-r)(p_i-\theta)^2 - r(\tilde{L}_i - \bar{\tilde{L}}) - c_i|\theta, y]\right\} \geq E\left\{E[-(1-r)(p_i-\tilde{\theta})^2 - r(\tilde{L}_i - \bar{\tilde{L}})|y]\right\} \tag{2.1.1}$$

and $z_i = 0$ otherwise.

After making its information acquisition decision, each firm chooses the price of its good, p_i, to solve the problem

$$\max_{p_i} E_i[u_i(p_i, \theta)] = E_i\left[-(1-r)(p_i-\theta)^2 - r(L_i - \overline{L})\right] \tag{2.1.2}$$

for which the first order necessary condition is:

$$p_i = (1-r)E_i(\theta) + rE_i(P) \tag{2.1.3}$$

where

$$P = \int_0^1 p_j dj \tag{2.1.4}$$

is the average action across all agents. Here $E_i(\cdot)$ is the conditional expectations operator and accounts for whether firm i is informed about θ. When choosing their prices, all firms observe the realization of a policy signal, y, to be discussed below. Therefore when choosing its price, firm i's information set includes θ and y if it is informed, and when firm i is uninformed, it only observes y when choosing its price. Thus uninformed firms have an incentive to extract all information about θ contained in y. Note that agent i treats $\overline{L}$ as a constant as his action is trivial given the continuum of agents. Thus agent i puts positive weight on θ and the actions of everyone else. If θ is common knowledge the equilibrium entails $p_i = \theta$ for all i so social welfare is maximized in equilibrium. Therefore when perfect information obtains there is no conflict between individual rational actions and the socially optimal actions.

Denote by $p_i(\mathcal{I}_i)$ the decision by firm i as a function of his information set $\mathcal{I}_i$. $\mathcal{I}_i$, which contains the pair $(z_i\theta, y)$, captures all the information available to i at the time of the decision.[9] Notice that this set-up implies that the actions of the other firms, p_{-i}, is not observable by firm i.

[9]For notational convenience the parameters governing the economy and the general structure of the economy are omitted from $\mathcal{I}_i$.

2.1.2 The Monetary Policy Authority

Reiterating the timing of the model; the policy authority decides on the quality of its policy signal, firms decide whether to become informed, nature chooses a vector of shocks at which point informed firms learn the realization of macroeconomic conditions, and finally the prices chosen by firms and the policy signal are simultaneously determined. As the policy authority chooses the quality of its signal before the state of the world is revealed to the informed firms it maximizes the unconditional expected social welfare function

$$\begin{aligned} E\left[E_M(\tilde{W})\right] &= E\left\{E\left[\int_0^1 u(\tilde{p}_i, \tilde{z}_i)di|\mathcal{I}_M\right]\right\} \\ &= -E\left[(1-r)\int_0^1 (\tilde{p}_i - \tilde{\theta})^2 di + \int_0^1 \tilde{z}_i\tilde{c}_i di\right]. \end{aligned} \tag{2.1.5}$$

The policy authority observes a noisy measure of the aggregate action of the agents and sends a policy signal that is seen by all agents and is common knowledge. In other words, the policy authority's objective is to maximize the expected social welfare by informing the private sector of its expectations of θ conditional on observing its noisy measure of the aggregate price level, P. Formally, the policy authority observes A where

$$A = P + \eta \qquad \tilde{\eta} \sim N(0, \sigma_\eta^2). \tag{2.1.6}$$

In this model, the policy authority's set of instruments is restricted to a linear report of its conditional expectations of θ. This restriction is motivated by the policies of inflation targeting central banks. The idea is that central banks give statements reflecting what it believes are the inflationary pressures buffeting the economy. Denoting the policy instrument by y,

$$y = E[\tilde{\theta}|A] + u \qquad \tilde{u} \sim N(0, \sigma_u^2). \tag{2.1.7}$$

The policy authority chooses σ_u^2 to maximize expected social welfare given that all the agents understand the policy authority's problem. With common knowledge of the structure of the economy, all agents can solve the policy authority's problem and are able to deduce the policy authority's optimal choice for σ_u^2. Similarly, all firms understand the problem that each firm faces and can determine the fraction of firms, x, that will become informed for any given σ_u^2. This determines the optimal weights that the firms place on any announcement, y, by the policy authority. To be explicit the policy authority's problem is

$$\max_{\sigma_u^2} E\left[-(1-r)\int_0^1 (\tilde{p}_i - \tilde{\theta})^2 di - \int_0^1 \tilde{z}_i\tilde{c}_i di\right] \tag{2.1.8}$$

subject to equations (2.1.1)-(2.1.3) and (2.1.6)-(2.1.7).

2.2 A Rational Expectations Equilibrium

Definition 1 *A **rational expectations equilibrium** (REE) is a joint distribution of $\tilde{\theta}, \tilde{\eta}, \tilde{u}, \tilde{z}_i, \tilde{x}, \tilde{p}_i$ and $\tilde{y}$ such that*

1. *for informed firms,*

$$p_i \in \arg\max_{\hat{p}_i \in \mathcal{R}} \left\{ E\left[-(1-r)(\hat{p}_i - \theta)^2 - r(\tilde{L}_i - \tilde{\bar{L}}) | \theta, y(\theta, \tilde{\eta}, \tilde{u}) \right] \right\}$$

2. *for uninformed firms p_i is optimal given an observed policy signal y, that is*

$$p_i \in \arg\max_{\hat{p}_i \in \mathcal{R}} \left\{ E\left[-(1-r)(\hat{p}_i - \tilde{\theta})^2 - r(\tilde{L}_i - \tilde{\bar{L}}) | y(\tilde{\theta}, \tilde{\eta}, \tilde{u}) \right] \right\}$$

3. *z_i is optimal given c_i in that $z_i = 1$ if*

$$E\left\{ E[-(1-r)(\tilde{p}_i - \tilde{\theta})^2 - r(\tilde{L}_i - \tilde{\bar{L}}) - c_i | \tilde{\theta}, \tilde{y}] \right\} \geq E\left\{ E[-(1-r)(\tilde{p}_i - \tilde{\theta})^2 - r(\tilde{L}_i - \tilde{\bar{L}}) | \tilde{y}] \right\}$$

4. *σ_u^2 is optimal in that*

$$\sigma_u^2 \in \arg\max_{\hat{\sigma}_u^2 \in \mathcal{R}_+} -E\left\{ (1-r) \int_0^1 (\tilde{p}_i - \tilde{\theta})^2 di + \int_0^1 \tilde{z}_i \tilde{c}_i di \right\}$$

In words, a rational expectations equilibrium is a joint distribution for the variables $\tilde{\theta}, \tilde{\eta}, \tilde{u}, \tilde{z}_i, \tilde{x}, \tilde{p}_i$ and $\tilde{y}$ such that, the policy authority chooses σ_u^2 optimally and the firms make their information acquisition decisions optimally, knowing the structure of the economy which encompasses the values of σ_θ^2, and σ_η^2 (the volatility in the fundamental θ and the measurement error in the policy authority's observable), and then, conditional on the realizations of $\tilde{\theta}$, $\tilde{\eta}$, and $\tilde{u}$,

1. given expectations conditional on observing the pair (θ, y), informed firms choose p_I optimally,[10]

2. given expectations conditional on observing y, uninformed firms choose p_U optimally,

3. given expectations conditional on observing A, the policy authority chooses a policy signal, y, that is generated by its choice of σ_u^2, and its conditional expectations,

4. expectations for each firm i are formed such that given private signals and the equilibrium policy signal function, the actions chosen by agents result in the expectations generated by the observed private and policy signals,

5. the expectations of the policy authority are formed such that the policy signal function (ie. the policy rule), $y(A, u)$, generates equilibrium beliefs for agents which result in actions that, when aggregated, result in the A which generates the policy signal as specified by the policy rule.

[10]For notational convenience, variables subscripted with I and U denote variables corresponding to informed and uninformed firms respectively.

2.2.1 Equilibrium

In order to solve for the rational expectations equilibrium it is necessary to work backwards.[11] Suppose that a fraction of firms, x, have become informed and learned the value of θ in the first stage. Then these x firms will choose their price such that $p_i = p_I$. The remaining $1 - x$ uninformed firms choose their prices optimally given that they observe the policy signal y so that $p_i = p_U$. From the firm's optimal policy rule it is known that

$$p_i = (1-r)E_i(\theta) + rE_i(P).$$

Substituting for P and writing $\overline{E}(\theta)$ for the average expectations of θ across agents we have

$$p_i = (1-r)E_i(\theta) + (1-r)rE_i\left[\overline{E}(\theta)\right] + (1-r)r^2E_i\left[\overline{E}^2(\theta)\right] + ...$$

which upon continued iteration yields

$$p_i = (1-r)\sum_{k=0}^{\infty} r^k E_i\left[\overline{E}^k(\theta)\right]. \tag{2.2.9}$$

Given the linear-normal structure of the economy it is conjectured that $y(\theta, \eta, u)$ and $A(\theta, \eta, u)$ are normally distributed random variables. If so, then Bayesian beliefs for the uninformed firms are given as

$$E_U[\tilde{\theta}|y] = b_U y.$$

Noting that for the informed firm that $E_i(\theta) = \theta$ substitution of beliefs into equation (A.1.1) and forward iteration yields the price function

$$p_I = \frac{(1-r)}{(1-rx)}\theta + \frac{r(1-x)}{1-rx}b_U y \tag{2.2.10}$$

$$p_U = b_U y \tag{2.2.11}$$

Aggregating across firms

$$A = \frac{(1-r)x}{1-rx}\theta + \frac{(1-x)}{1-rx}b_U y + \eta$$

and so the monetary authority's beliefs are given by

$$E_M(\tilde{\theta}) = f_A A_p.$$

where

$$A_p = A - f_U u.$$

[11]Proofs and details about the equilibrium solution are relegated to the Appendix.

The term $f_U u$ is the linear least squares projection of A on u and the term b_U is the linear least squares projection coefficient of y on θ. The coefficients governing the belief functions can be expressed as

$$b_U = \frac{cov(y,\theta)}{var(y)} \quad , \quad f_A = \frac{cov(A_p,\theta)}{var(A_p)} \quad , \quad f_U = \frac{cov(A,u)}{var(A)} \tag{2.2.12}$$

Substitution of the authority's expectations into its instrument rule and then solving the resulting expression for the policy instrument gives an expression for y in terms of the exogenous variables:

$$y = \frac{f_A(1-r)x}{1-rx-(1-x)f_A b_U}\theta + \frac{f_A(1-rx)}{1-rx-(1-x)f_A b_U}\eta - \frac{(f_A f_U - 1)(1-rx)}{1-rx-(1-x)f_A b_U}u \tag{2.2.13}$$

Using this expression for the policy signal allows the policy observable to be expressed as

$$A = \frac{(1-r)x}{1-rx-(1-x)f_A b_U}\theta + \frac{(1-rx)}{1-rx-(1-x)f_A b_U}\eta - \frac{(1-x)(f_A f_U - 1)b_U}{1-rx-(1-x)f_A b_U}u \tag{2.2.14}$$

From (2.2.13) and (2.2.14) it is clear that the policy signal and the policy authority's observable are both functions of the realization of θ, the measurement error in the policy authority's observable, η, and the noise that the policy authority injects into its policy signal, u. The weights on these variables will depend on the coefficients governing the belief functions of the agents in the economy. These coefficients are complicated functions of the structural parameters of the economy, including the level of noise chosen by the policy authority, σ_u^2. Note that in equilibrium the expressions for y and A are linear functions of θ, the measurement error, η and the policy noise u. This allows for the use of linear least squares projection formulas to calculate the beliefs of the agents.

It is clear from (2.2.13) that the measurement error in A is worked into the policy signal as the policy authority is unable to disentangle the measurement error from the realization of θ. This prevents a fully revealing equilibrium from being obtained. From (2.2.14) it can be seen that the noise injected by the policy authority works its way into the policy authority's observable. As the policy authority knows the realization of u, it can clean this noise out of A when forming its expectations concerning θ.

Now consider the first stage during which the firms decide whether or not to become informed and the policy authority sets the quality of its signal, σ_u^2. Firm i understands that if it pays its cost c_i then it will choose a price p_I. Otherwise it will pick a price p_U. Thus firm i will only become informed if

$$E[u_i^I(\tilde{p};\tilde{\theta},\tilde{\eta},\tilde{u})|\sigma_u^2] \geq E[u_i^U(\tilde{p};\tilde{\theta},\tilde{\eta},\tilde{u})|\sigma_u^2] \tag{2.2.15}$$

or

$$c_i \leq (1-r)\left[1 - \frac{r^2(1-x)^2}{(1-rx)^2} - \frac{(1-r)^2 r}{(1-rx)^2}(1-2x)\right] E[(b_U\tilde{y}-\tilde{\theta})^2|\sigma_u^2] \tag{2.2.16}$$

Finally expected social welfare is given as

$$E(\tilde{W}) = -(1-r)\left\{x\left[\frac{r(1-x)}{1-rx}\right]^2 + 1 - x\right\} E(b_U\tilde{y}-\tilde{\theta})^2 - \int_0^{c^*} c_i g(c_i)dc_i \quad (2.2.17)$$

where c^* denotes the highest cost of all informed firms.

It is instructive to notice that, everything else constant, uninformed firms pay more attention to the policy signal as the fraction of informed firms increases; that is $\frac{\partial b_u}{\partial x} > 0$. Thus, there is always a free-rider effect in that uninformed firms prefer not to incur the cost of becoming informed if the policy signal is more precise. The strategic complementarity component in payoffs adds an extra coordination effect in that if costs are such that a large fraction of agents become informed then the remaining agents have an increased incentive to become informed; a force working against the free rider effect. Furthermore it can be shown that the direct effects of increased policy noise is to worsen the welfare of the uninformed, $-\frac{\partial E[(b_U\tilde{y}-\tilde{\theta})^2]}{\partial \sigma_u^2} \leq 0$, and that the direct effect of increasing the fraction of informed agents on the welfare of the uninformed is positive, $-\frac{\partial E[(b_U\tilde{y}-\tilde{\theta})^2]}{\partial x} \geq 0$.

Lemma 1 *Let the density $g(c_i)$ be such that it is continuous and non-zero on $\Re_+$. Let c^* be the highest cost type of all informed firms. For every level of policy noise, σ_u^2, there exists at least one cut-off cost, c^*, such that the firm with this cost is indifferent between becoming informed or remaining uninformed.*

Proposition 1 *There exists an equilibrium.*

Differentiating the social welfare function with respect to σ_u^2 we obtain

$$\begin{aligned}\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} &= -(1-r)\left\{\left[\left(\frac{r(1-x)}{1-rx}\right)^2 - 2x\left(\frac{r(1-x)}{1-rx}\right)\frac{r(1-r)}{(1-rx)^2} - 1\right]\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial \sigma_u^2}E(b_u\tilde{y}-\tilde{\theta})^2\right.\\ &\quad \left.+\left[x\left(\frac{r(1-x)}{1-rx}\right)^2 + 1 - x\right]\left[\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial \sigma_u^2}\right]\right\}\\ &\quad -c^*g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} \qquad (2.2.18)\end{aligned}$$

The first line on the right-hand side equals the benefits accrued to the fraction of firms that are the marginally informed. If there were no strategic complementarity in payoffs then these firms would choose $p_i = \theta$ so that $E(p_i - \theta)^2 = 0$ and their gain from becoming informed would be $E(b_U y - \theta)^2$. In other words by becoming informed these firms would rid of any uncertainty. The change to total welfare from this gain would then be $E(b_U y - \theta)^2$ times the change in the fraction of firms becoming informed, $\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial \sigma_u^2}$. However, with the presence of strategic complementarity in the payoff function, the informed do not choose their prices to be equal to θ. The informed firms place a weight $\frac{r(1-x)}{(1-rx)}$ on the price that the uninformed firms will choose and with a fraction x of firms being informed, the change in expected welfare coming from the deviation of the informed firms' price from θ is given by the first two terms on the first line of equation (2.2.18).

The second line on the right-hand side describes the gains from increasing policy noise that arise from the accuracy of setting prices conditional on only the policy signal. There are direct costs of increasing policy noise which is given by the term $\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial\sigma_u^2}$. On the other hand, there are indirect benefits in that as more firms are pushed into becoming informed, the policy signal becomes more informative - a benefit captured by the term $\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial\sigma_u^2}$.

Lemma 2 *In a stable equilibrium, as policy noise increases, the cut-off cost of the marginal informed firm cannot decrease,* $\frac{\partial c^*}{\partial\sigma_u^2} \geq 0$. *In an unstable equilibrium, as policy noise increases, the cut-off cost of the marginal informed firm decreases,* $\frac{\partial c^*}{\partial\sigma_u^2} <$ 0.

Thus in the equilibria of interest increases in policy noise push firms towards becoming informed.

As shown in the appendix, by making use of the indifference condition for the marginal informed firm, the change in expected social welfare can be rewritten as the sum of two components.

$$\begin{aligned}\frac{\partial E(\tilde{W})}{\partial\sigma_u^2} &= (1-r)\left\{\left[\frac{r(1-r)^2(1-2x)}{(1-rx)^2}+\frac{2xr^2(1-r)(1-x)}{(1-rx)^3}\right]E(b_u\tilde{y}-\tilde{\theta})^2\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial\sigma_u^2}\right\}\\ &\quad -(1-r)\left\{\left[x\left(\frac{r(1-x)}{1-rx}\right)^2+1-x\right]\left[\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial\sigma_u^2}+\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial\sigma_u^2}\right]\right\}\end{aligned}$$

The first component describes the "strategic complementarity externality". Informed firms choose their prices as a weighted sum of the realization of macroeconomic conditions, θ, and the average price chosen by all other firms. When firms make their information acquisition decision, they consider their own benefits arising from the strategic complementarity component to payoffs; if there is a large fraction of informed firms (more than one-half) then becoming informed allows price to be closer to the average price set by other firms. In the social welfare function, when summed across all firms the strategic complementarity component to payoffs nets to zero. The policy authority wants firms to choose their prices to best reflect macroeconomic conditions. However, the policy authority is also concerned about minimizing the costs of information acquisition. When policy noise is increased firms compare the benefits of becoming informed to the costs, which includes the changes in benefits arising from the strategic complementarity to payoffs. Thus in considering the effects of policy noise on total costs of information acquisition, the policy authority must account for the effects of policy noise on the payoffs arising from the strategic complementarity. The complementarity thus sneaks back into the welfare evaluation via the costs of information acquisition.

Individual firms do not consider the effect that becoming informed has on the prices set by all the other firms. In making the information acquisition decision, each individual firm considers itself too small to have any impact on aggregate variables.

However, when a positive measure of firms with identical costs become informed, the uninformed are made worse off on average because their guess concerning the prices set by all other firms is now less accurate as the uninformed do not know what price the informed agents are setting with certainty. In contrast with more informed firms, informed firms gain in the complementarity component to payoffs as their price is closer to the prices of more firms. The overall effect of this on aggregate welfare is ambiguous and depends on the fraction of informed firms. Furthermore, the informed firms are better off because their prices are now closer to θ which has a positive effect on welfare.

The second component is the "informational spillover" effect. This term captures the informativeness of the policy signal. When this spillover effect is positive it means that increasing policy noise results in a more informative policy signal. There are two forces at work here. First, as policy noise increases the policy signal is less informative because the firms now have a harder time deciphering the component of the policy signal that is related to the policy authority's beliefs and the component that is random noise. Second, this increases the value of being informed and so more firms are willing to incur the cost to become informed. With a larger fraction of informed firms, the policy authority's observable is more informative, allowing the policy authority to better extract the truth about macroeconomic conditions, θ, from its data, A. With better knowledge of θ the policy authority's beliefs are refined and are passed along to the private sector via its statement y. It is when a large fraction of firms are pushed into acquiring information that this informational spillover dominates the direct costs of increased policy noise. As can be seen in the following proposition, it is the interaction between the complementarity parameter, r, and the distribution of costs that generates this informational spillover.

Proposition 2 *In stable equilibria there is a positive informational spillover effect when*

$$(1-r)\left[\frac{2r(1-r)[(1-r)-r^2(1-x)]}{(1-rx)^3}\right] E(b_u\tilde{y}-\tilde{\theta})^2 g(c^*) > 1.$$

To understand the role played by the complementarity, consider the case where $r = 0$. In this case the indifference condition for the marginal informed firm is

$$c^* = E[(b_u\tilde{y}-\tilde{\theta})^2|\sigma_u^2].$$

If the informational spillover effect is positive then the uninformed firms are better off meaning that the expected deviation of an uninformed firm's price from that best reflecting macroeconomic conditions is lower, $\frac{dE(b_u\tilde{y}-\tilde{\theta})^2}{d\sigma_u^2} < 0$. The net benefits of becoming informed are eroded and uninformed firms are willing to pay less for information about θ. Thus the cut-off c^* must fall. However, if the cut-off falls then there must be less informed firms and as $\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial\sigma_u^2} > 0$ and $\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x} < 0$ it is impossible for the spillover effect to be positive. The only way that the spillover effect can be positive is if *more* firms become informed to offset the direct harm caused by increased policy noise.

Now consider the indifference equation for the marginal informed firm when $r \neq 0$.

$$-\left[\frac{r^2(1-x)^2}{(1-rx)^2} + \frac{(1-r)^2 r}{(1-rx)^2}(1-2x)\right] E[(b_U\tilde{y} - \tilde{\theta})^2|\sigma_u^2] - c^* = -E[(b_U\tilde{y} - \tilde{\theta})^2|\sigma_u^2]$$

The first term on the left-hand side accounts for the penalty that informed firms incur for not setting their price equal to macroeconomic conditions, θ. The second term accounts for the difference in the complementarity payoff component between the pricing as an informed firm versus pricing as an uninformed firm. If increased policy noise is to result in sufficiently large informational spillovers then an uninformed firm's price should reflect economic conditions better on average. This means that the right-hand side increases providing more incentive for the marginal firm to remain uninformed. For this spillover to occur a large number of firms must become informed requiring a higher threshold c^*. As an equilibrium outcome, this can only occur if the first two terms on the left-hand side increase significantly to make-up for the increased threshold such that the marginal firm will choose to become informed. In other words there must be an increased incentive to become informed. The strategic complementarity in payoffs provides a mechanism for such incentives to arise. When a large fraction of firms are informed then a firm's benefit from becoming informed increases (i) because the optimal price it sets will be closer to θ and (ii) as more firms are informed, the payoff arising from the strategic complementarity is higher if price is set equal to the price that the large fraction of informed firms, x, have set their prices to.

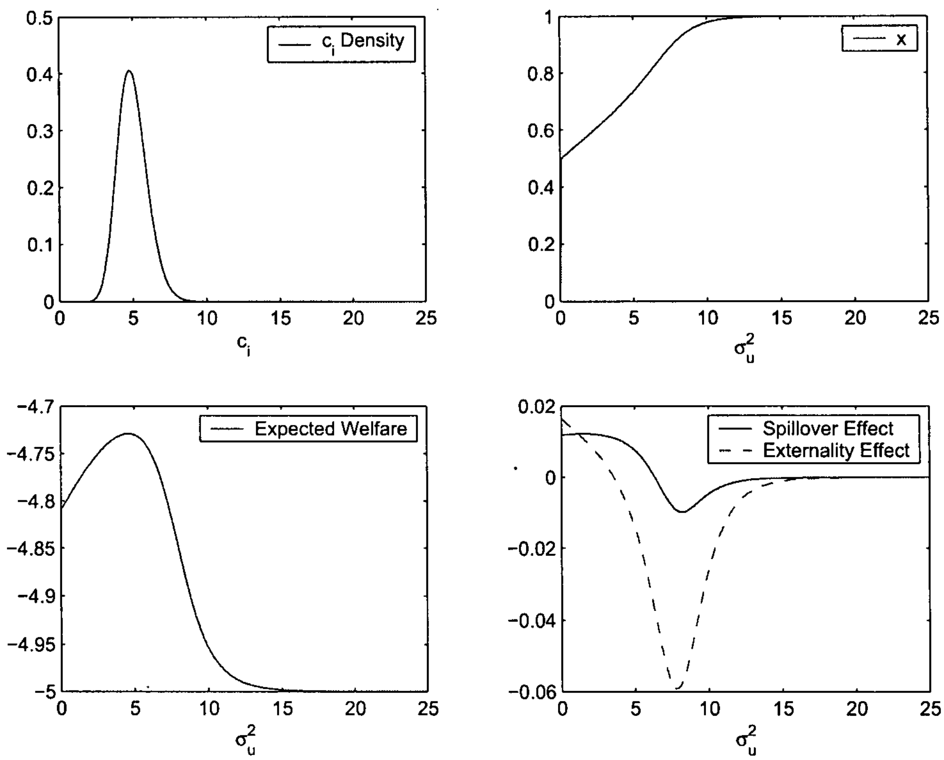
Remark 1 *A sufficient condition for an equilibrium to arise where perfect transparency is not optimal is as follows : when policy noise is zero, the positive informational spillover condition obtains, and* $(1-r)(1-rx) \geq 2x[1-r-r(1-rx)]$ *so that the strategic complementarity externality is positive.*

Figures 2.1 and 2.2 illustrate the effects of changing the distribution of costs. In the numerical example below the costs follow a gamma distribution with parameters α and β. As the density of costs becomes more dispersed, it becomes more difficult to obtain a positive informational spillover effect because small increases in policy noise do not generate a large flow of firms into the informed state. When the positive spillover effect is not obtained and the externality effect is relatively small, it is optimal for the policy authority to be perfectly transparent ($\sigma_u^2 = 0$). In contrast, figure 2.1 presents an example where perfect transparency is not optimal. Rather, the informational spillover effect is large enough so that the welfare can be improved by giving some firms the incentive to acquire information

2.3 Discussion

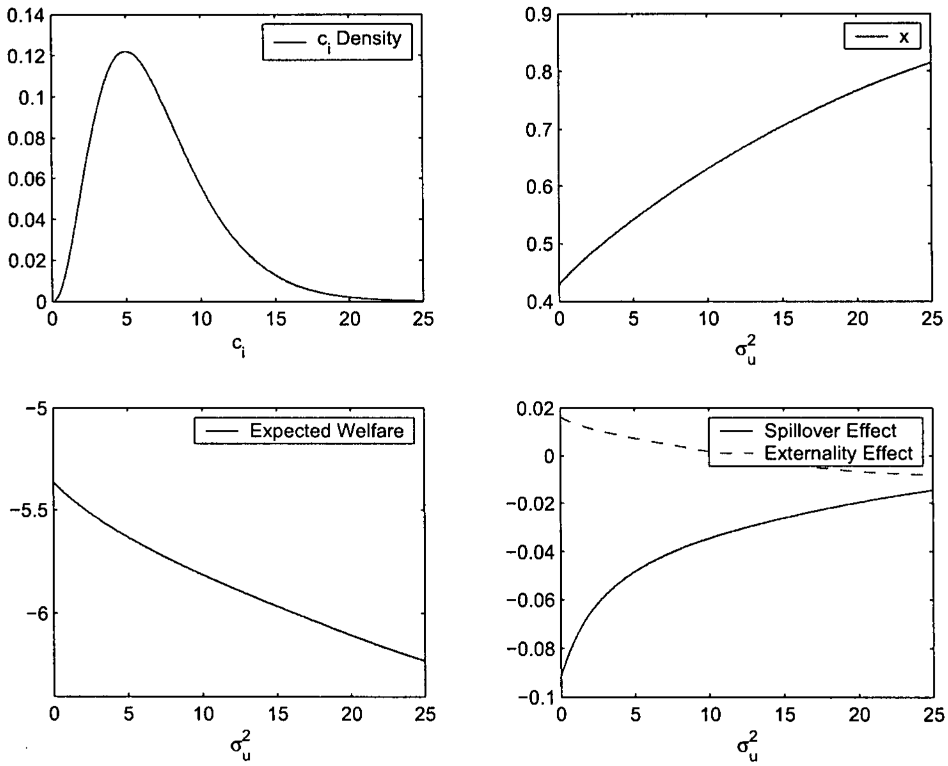
Recently, the paper of Morris and Shin (2002) discussed how public information is a "double-edged" instrument for a policy authority. When there is a strategic complementarity component to payoffs they argue that agents may overweight public

Figure 2.1: The Effects of Transparency on Welfare



Note: Costs are drawn from a gamma distribution. The parameters of the model are $\sigma_\theta^2 = 20$, $\sigma_\eta^2 = 2$, $r = 0.35, \alpha = 25, \beta = 0.2$.

Figure 2.2: The Effects of Transparency on Welfare with Dispersed Costs



Note: Costs are drawn from a gamma distribution. The parameters of the model are $\sigma_\theta^2 = 20$, $\sigma_\eta^2 = 2$, $r = 0.35, \alpha = 3.5, \beta = 2$.

information resulting in a magnification of mistakes in policy statements. Even worse, if private information were costly to obtain, agents may not obtain informative private information at all, instead relying on public information of inferior quality. They further argue that central banks must be careful to guard against the potential damage created by noise in their statements.

By endogenizing the source of noise in the public signal, this paper characterizes a different reason for the "double-edged" nature of the policy instrument. In the Morris and Shin model, the policy authority receives an exogenous noisy signal about the state of macroeconomic conditions. A policy authority can choose to add additional noise to its exogenous signal when making policy statements. Private sector agents are endowed with a noisy private signal in addition to observing the policy signal. Welfare can be lowered by decreasing the amount of policy noise in the public signal in situations where the noise in private signals is relatively small. Notice that in the Morris and Shin model, the informational quality of the public signal is *increased* when its noise is reduced. However, as the information quality of the public signal is considerably worse than that of the private signals, agents are better off weighing the two signals according to their signal-to-noise ratios. Unfortunately, agents overweight the public signal because it acts as a coordinating device in the "beauty contest" component of the economy.[12]

In the present model, when there is a large mass of agents with low idiosyncratic costs of information acquisition, lowering the endogenous noise chosen by the policy authority results in lower welfare as agents place more weight on the policy signal. This results in a large fraction of firms foregoing the opportunity to acquire information at a low cost. Interestingly, by doing so, the informational quality of the policy signal is *decreased*, thus reducing its service to the private sector. This is the key to the results of this paper. It is the fact that lowering the amount of noise in the policy signal results in a worsening of the quality of the information in the policy signal that makes agents worse off.

It is also interesting to note that the information spillover effect adds a complementary channel to the strategic complementarity channel emphasized in Morris and Shin (2002). This additional channel allows the degree of complementarity in the payoff function, r, to be reduced while still allowing for optimal policy noise to be strictly positive. By modelling the costly acquisition of information the model sidesteps the recent criticism of the original Morris and Shin model that without large degrees of complementarities in the payoff function, the Morris and Shin model is really an argument for perfect transparency.[13]

Another closely related paper is that of Bernanke and Woodford (1997). In their paper they discuss the difficulties of inflation forecast targeting. The idea in their paper is that the policy authority wishes to minimize the variance of one-period ahead inflation. All the information concerning underlying inflationary pressures is known by the private sector and the monetary authority has to infer this information

[12] Amato, Morris, and Shin (2003) directly implement the mechanism of Morris and Shin (2002) in a monetary policy setting.

[13] See Svensson (2005) for such a critique.

from the private sector's inflation forecast. It is shown that it is impossible for the authority to eliminate all sources of variation in inflation from the underlying inflationary pressures because attempting to do so would cause the private sector forecasts to be void of any information about the underlying inflationary pressures. Hence the impossibility of the monetary authority to learn about these pressures in order to defend against them. In other words, the monetary authority will offset any useful information incorporated into the inflation forecasts so forecasters will not incorporate this information into their forecasts.

One remedy that they present is for central banks to infer information about unobservable inflationary pressures from variables other than the variables that they target in their objective function. So long as the private sector works information about inflationary pressures into other variables that are not targeted by the monetary authority, and that are observable by the monetary authority, then the authority has the ability to draw inference about the inflationary pressures without losing the ability to target inflation.

The implications of the results in this paper suggest that so long as private sector agents have imperfect information about the underlying inflationary pressures, any statement by the monetary authority revealing information about inflationary pressures can result in a reduction of information quality in the monetary authority's observable. This means that even if the policy authority extracts information from non-targeted variables the quality of information in the non-targeted variables is diluted the more informative is the policy signal. Of course this is contingent on the actions of the private sector agents being functions of their beliefs about the unobserved underlying inflationary pressures. The key difference from the mechanism in Bernanke and Woodford's paper is that agents have a choice of drawing inference from a policy signal of endogenous quality or a private signal of known high quality. As the quality of the policy noise is reduced there can be a shift in the fraction of agents drawing inference from private signals into drawing inference from the policy signal. The result is the endogenous quality of the authority's observable data is reduced, thereby reducing the endogenous quality of the policy signal.

2.4 Extensions

2.4.1 Policy Under Discretion

In this section the policy authority is allowed to alter its choice of policy noise after the firms make their information acquisition decisions. A time-inconsistency problem is present in that after firms choose to become informed, the policy authority can then choose to re-optimize its level of policy noise. As the fraction of informed agents is now fixed, by reducing policy noise, the policy authority can now increase the expected payoff to uninformed agents at no cost.

Consider the case where the policy authority maximizes social welfare taking the fraction of informed agents as given. In this case, taking the derivative with respect to σ_u^2 yields

$$\begin{aligned}\frac{\partial E(W)}{\partial \sigma_u^2} &= -\left\{x\left[\frac{r(1-x)}{1-rx}\right]+1-x\right\}\frac{\partial E[(b_U y-\theta)^2]}{\partial \sigma_u^2} \\ &\leq 0\end{aligned}$$

The key to notice here is that once the fraction of informed is fixed, increasing policy noise can only reduce social welfare as it makes the policy signal less informative to the uninformed, the only agents who make use of the policy signal. By backward induction, as firms understand the policy authority's incentives, they will make their information acquisition decision under the knowledge that when prices are chosen there will be no policy noise.

Remark 2 *Under discretionary policy, the zero policy noise equilibrium will be obtained.*

An implication of this result is that the policy authority can never do better than the zero policy noise case, even though adding policy noise may increase social welfare. The benefits of large informational spillovers are impossible to obtain because the policy authority cannot commit to reducing the informativeness of its policy signal in order to induce marginal firms into becoming informed.

2.4.2 A Model with Cost Externalities

In this section the model is extended such that firms no longer have a strategic complementarity in payoffs but rather face a cost externality in the cost of acquiring information. While the strategic complementarity used in the previous model arises naturally in models of monopolistic competition, a large complementarity is required to obtain the results of interest. It is shown below that the existence of externalities in the cost of acquiring information can also produce the result that perfect transparency may not be optimal policy. While not explicitly derived in this paper, the existence of cost externalities may reduce the amount of strategic complementarity required to bring forth case against perfect transparency due to positive information spillover effects. The derivation of the result is similar to the case with strategic complementarities in payoffs but is much simpler.

There exists a continuum of firms indexed on the unit interval. Firm i chooses the log of its price, $p_i \in \Re$, and also decides whether to incur a cost, $c(c_i, x)$, in order to obtain a signal about the realization of payoff relevant macroeconomic conditions. The cost of becoming informed is a function of a firm specific cost, c_i, and the fraction of firms that choose to become informed, x. Below, cases where $c(c_i, x)$ is a decreasing function of x are considered. Cases when $c(c_i, x)$ is a decreasing function of the number of informed firms can be thought of as examples where as more firms become informed, firms learn about where information about macroeconomic conditions can be found. Informally, complementarity in information acquisition costs can arise as a result of network externalities. For example, as other firms gather information, a given firm may be alerted to news or events of interest, reducing the cost of effort required in searching for the relevant information. Nevertheless, it is not costless for uninformed firms to learn about the state of the economy and effort and costs still must be incurred to obtain accurate readings of the true underlying macroeconomic conditions. Let z_i denote firm i's decision of becoming informed about $\tilde{\theta}$.

$$z_i = \begin{cases} 1 & \text{if informed} \\ 0 & \text{if uninformed} \end{cases}$$

The payoff function for firm i is given by

$$u_i(p_i, z_i) = -(p_i - \theta)^2 - z_i c(c_i, x)$$

where θ is an unobserved random variable with $\tilde{\theta} \sim N(0, \sigma_\theta^2)$ and $\tilde{c}_i \sim g(c_i)$. It should be noted that firm i knows its cost of becoming informed when making any decisions.

Social welfare, defined as the sum of individual payoffs, is

$$\begin{aligned} W &= \int_0^1 u_i(p_i, z_i) di \\ &= -\int_0^1 (p_i - \theta)^2 di - \int_0^1 z_i c(c_i, x) di \end{aligned}$$

Thus a social planner who cares only about welfare seeks to keep p_i close to θ, $\forall i$ while minimizing the aggregate costs across firms of becoming informed.

Before choosing its price, each firm must decide whether to incur the cost of becoming informed about θ. If a firm decides to become informed it will learn the realized macroeconomic conditions. Otherwise, firms will only observe a signal issued by the policy authority concerning macroeconomic conditions. When firms decide whether or not to become informed about macroeconomic conditions, the quality of the policy signal, which is described later, is common knowledge. Firm i will become informed if it expects a higher payoff when informed than if it remains uninformed. That is, $z_i = 1$ if

$$E\left\{E[(p_i - \theta)^2 - c(c_i, x)|\theta, y]\right\} \geq E\left\{E[(p_i - \tilde{\theta})^2|y]\right\} \tag{2.4.19}$$

and $z_i = 0$ otherwise.

After making its information acquisition decision, each firm chooses the price of its good, p_i, to solve the problem

$$\max_{p_i} E_i\left[-(p_i - \theta)^2\right] \tag{2.4.20}$$

for which the optimality condition is:

$$p_i = E_i(\theta) \tag{2.4.21}$$

where $E_i(\cdot)$ is the conditional expectations operator and accounts for whether firm i is informed about θ. When choosing their prices, all firms observe the realization of a policy signal, y, as above. Therefore when choosing its price, firm i's information set includes θ and y if it is informed, and when firm i is uninformed, it only observes y when choosing its price. Again, uninformed firms have an incentive to extract all information about θ contained in y. All else remains as in the model with strategic complementarities in the payoff function.

2.4.3 Equilibrium

As before a rational expectations equilibrium is constructed. In equilibrium there is a cut-off cost c^* which denotes the maximum idiosyncratic cost-type that will become informed. Firms with idiosyncratic costs c^* are indifferent between acquiring information about macroeconomic conditions or remaining uninformed and inferring the realization of θ from the policy signal y.

For simplicity the cost function is assumed to be additively separable with an idiosyncratic cost and a common cost of becoming informed. Specifically, let the cost function take the form $c(c_i, x) = v(x) + c_i$ where $v'(x) < 0$ and $v''(x) > 0$. In this case the indifference condition for the marginal informed firm becomes

$$E[c^* + v(x)|\sigma_u^2] = E[(b_u y - \tilde{\theta})^2|\sigma_u^2] \tag{2.4.22}$$

and the social welfare function is now

$$E(\tilde{W}) = -E\left[(1-x)(b_U\tilde{y} - \tilde{\theta})^2 + \int_0^{c^*} c_i g(c_i) + v(x) dc_i\right]$$

To understand the policy authority's trade-off differentiate the social welfare function with respect to the level of policy noise.

$$\begin{aligned}
\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} &= E[(b_U\tilde{y} - \tilde{\theta})^2]g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} - [c^* + v(x)]\, g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} - xv'(x)g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} \\
& \quad -(1-x)\left[\frac{\partial E[(b_u\tilde{y} - \tilde{\theta})^2]}{\partial \sigma_u^2} + \frac{\partial E[(b_u\tilde{y} - \tilde{\theta})^2]}{\partial x}\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial \sigma_u^2}\right]
\end{aligned}$$

The first term is the direct gain to the social welfare of pushing a fraction of uninformed firms into becoming informed. These firms now obtain the certain payoff from knowing the realization of economic conditions. The second term represents the costs that these marginal informed firms incur of acquiring information about economic conditions. From the indifference condition, the first two terms offset each other as the marginal firms are indifferent between being informed and uninformed. The third term represents the reduction in information acquisition costs accrued to all informed firms from increasing the fraction of informed firms. Finally, the last term represents both the decrease in welfare of the uninformed from additional policy noise and the increase in welfare of the uninformed from having a more informative policy signal.

Imposing the indifference conditions for the marginal informed firms[14]

$$\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} = -(1-x)\frac{\partial E[(b_U\tilde{y}-\tilde{\theta})^2]}{\partial \sigma_u^2}\left\{\frac{1+v'(x)g(c^*)}{1-\left[\frac{\partial E[(b_u\tilde{y}-\tilde{\theta})^2]}{\partial x}-v'(x)\right]g(c^*)}\right\} -xv'(x)g(c^*)\frac{\partial c^*}{\partial \sigma_u^2}. \quad (2.4.23)$$

In order to construct an equilibrium where the policy authority chooses to be non-transparent, it is necessary for at least the lowest cost firms to become informed when there is no policy noise. The reason for this is that the policy authority can never push lowest cost firms into investing. If no firms are investing then there is no information in the policy signal and so adding policy noise to the policy statement does not make the uninformed any worse off as the uninformed firms understand that there is already no information contained in the policy signal.[15] In such a case expected welfare is $E(\tilde{W}) = -\sigma_\theta^2$.

It is easily seen that when $1+v'(x)g(c^*) < 0$ and $1+v'(x)g(c^*) < E[(b_u\tilde{y}-\tilde{\theta})^2]g(c^*)$ then the cost of the marginal informed firm is increasing with policy noise and social welfare is increasing in policy noise. Cases where social welfare is increasing due solely to decreasing costs are not of interest here so it is of interest to construct an equilibrium where total costs are always increasing as more firms become informed. In such an equilibrium, as more firms become informed the rising idiosyncratic costs of becoming informed dominate the decreasing common cost. Looking at the first order condition of the policy authority, when the above conditions are met, social welfare in increasing mainly because the information transmitted to the uninformed via the policy signal is increasing as policy noise increases. Eventually, as the cost of becoming informed belonging to the marginal firm becomes too large, increasing policy noise does not push enough firms to become informed and the uninformed are worse off with added policy noise.

To ensure that a unique cut-off cost exists for any given level of policy noise, it is enough that the lowest cost firm always becomes informed, $1+v'(x)g(c^*) < 0$ when there is no policy noise, and for any level of policy noise a single-crossing property is satisfied.

Definition 2 *The pair of functions $v(x)$ and $g(c_i)$ satisfies* the single crossing property *if for any level of policy noise there is a cut-off cost, c^*, such that for all $c_i < c^*$, $c_i + v(x) < E[(b_U\tilde{y}-\tilde{\theta})^2]$ and otherwise, $c_i + v(x) \geq E[(b_U\tilde{y}-\tilde{\theta})^2]$.*

Proposition 3 *Consider any function $v(x)$ such that $v'(x) < 0$, $v''(x) > 0$ and any density function $g(c_i)$ with $g'(c_i) < 0$ and $g''(c_i) > 0$. Let the single-crossing property be satisfied. If $v(x)$ and $g(c_i)$ are such that with no policy noise, $1+v'(x)g(c^*) < 0$*

[14] In a unique equilibrium it is common knowledge that a given level of policy noise is associated with a particular fraction of informed agents.

[15] This feature is absent if the policy authority is endowed with an exogenous noisy signal concerning the realization of θ.

at the cut-off cost c^*, then there exists an equilibrium in which perfect transparency is not optimal.

Clearly, the distribution of costs and the cost externalities must work together to produce an equilibrium in which the policy authority chooses to add policy noise to its signal. It must be the case that increasing policy noise pushes a large fraction of firms to become informed and only as there is a cost externality is it that a large enough fraction of firms become informed. The cost externality allows costs to fall so that while the benefits to the uninformed increases the costs to become informed also falls. Total costs across all informed firms rise as a large fraction of firms become informed so the policy authority faces a trade-off between increasing the information transmitted to the uninformed via the policy signal and increasing the cost incurred by the informed.

Figures 2.3 and 2.4 display the case where $v(x) = \frac{\bar{c}}{e^x}$ and $g(c_i) = \frac{1}{\beta}e^{-\frac{c_i}{\beta}}$ so that costs follow the exponential distribution. As costs become widely dispersed across firms, increasing policy noise entices a smaller fraction of firms to become informed and the common cost component does not fall enough to attract marginal firms to become informed. Without a large enough fraction of firms becoming informed when policy noise increases, the increased informativeness of the policy signal to the uninformed is not enough to dominate the increase noise in the policy signal to justify the increase ins policy noise. Thus even though there exists a cost externality, when costs are too dispersed across firms, it is optimal for the policy authority to be transparent.

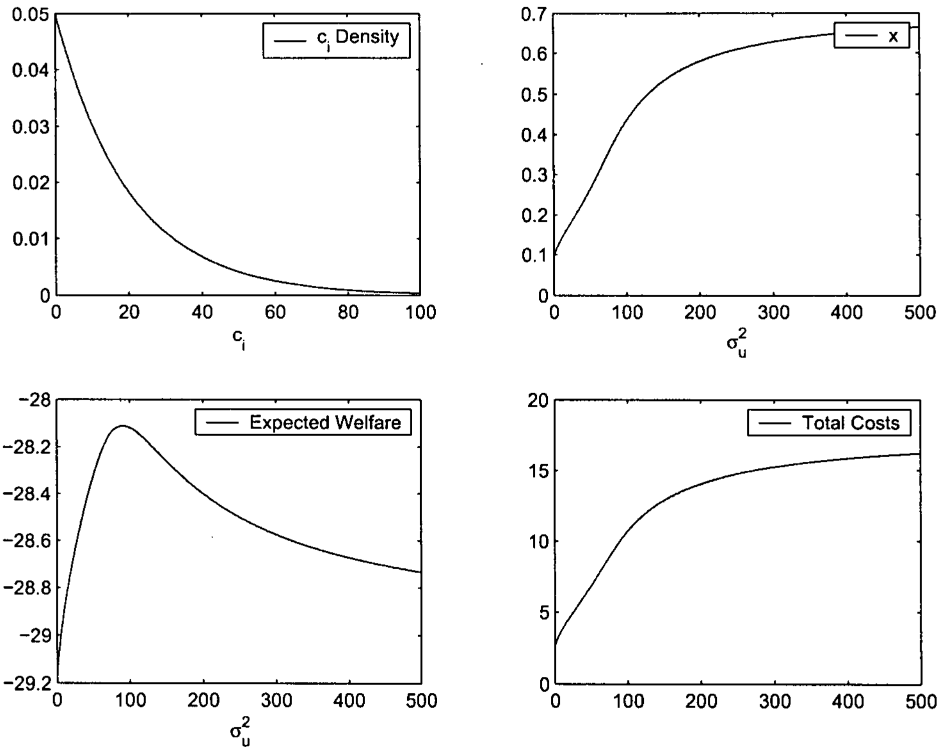
2.4.4 Fixed Costs with No Learning Externalities

Now consider the extreme case where $c(c_i, x) = c_i$. This property of the cost function elucidates the role of cost externalities in obtaining the result of proposition 3. In this case the cost of becoming informed faced by firm i is not affected by the number of firms that are informed. Thus each firm faces some cost of learning about underlying macroeconomic conditions and this cost is independent of the activities of other firms.

Corollary 1 *When $c(c_i, x) = c_i$ and the density function $g(c_i)$ is continuous and nonzero, there exists a unique equilibrium in which the policy authority always chooses to be transparent ($\sigma_u^2 = 0$).*

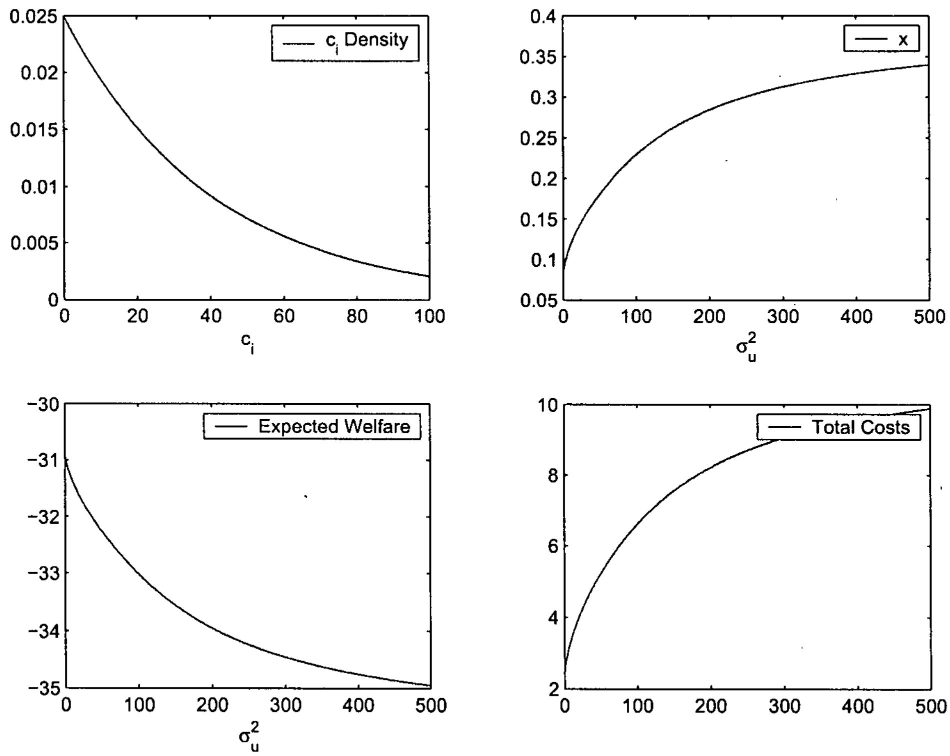
Notice that the only way for the policy authority to increase the informativeness of its signal is to push more firms to become informed. Then as the informed firms choose their prices, the policy observable, A, is more informative about the macroeconomic conditions, θ. The policy authority only has one instrument at its disposal, σ_u^2, so in order to increase the informativeness of its observable, the policy authority must increase the amount of noise it injects into the policy signal. Doing so increases the benefits of becoming informed pushing the marginal firms into becoming informed.

Figure 2.3: The Effects of Transparency on Welfare with Cost Externalities



Note: Costs are drawn from an exponential distribution. The parameters of the model are $\sigma_\theta^2 = 40$, $\sigma_\eta^2 = 1$, $\beta = 20$, $\bar{c} = 30$.

Figure 2.4: The Effects of Transparency on Welfare with Cost Externalities and Dispersed Costs



Note: Costs are drawn from an exponential distribution. The parameters of the model are $\sigma_\theta^2 = 40$, $\sigma_\eta^2 = 1$, $\beta = 40$, $\bar{c} = 30$.

Looking at the derivative of the social welfare function with respect to the policy instrument,

$$\begin{aligned}\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} &= E[(b_U\tilde{y}-\tilde{\theta})^2]g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} - c^*g(c^*)\frac{\partial c^*}{\partial \sigma_u^2} \\ &\quad -(1-x)\left[\frac{\partial E[(b_u\tilde{y}-\tilde{\theta})^2]}{\partial \sigma_u^2} + \frac{\partial E[(b_u\tilde{y}-\tilde{\theta})^2]}{\partial x}\frac{\partial x}{\partial c^*}\frac{\partial c^*}{\partial \sigma_u^2}\right].\end{aligned}$$

The first term is the benefit to the firms that become newly informed and the second term is the cost to these firms. By the indifference condition at the margin, in equilibrium it must be that

$$c^* = E[(b_U\tilde{y}-\tilde{\theta})^2] \tag{2.4.24}$$

so that the marginal firm is indifferent between becoming informed and not becoming informed. This implies that the first and last terms of the policy authority's first order condition net to zero and rewriting

$$\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} = -(1-x)\left[\frac{\partial E[(b_u\tilde{y}-\tilde{\theta})^2]}{\partial \sigma_u^2} + \frac{\partial E[(b_u\tilde{y}-\tilde{\theta})^2]}{\partial x}g(c^*)\frac{\partial c^*}{\partial \sigma_u^2}\right] \tag{2.4.25}$$

The first term in equation (2.4.25) represents the decrease in welfare that increasing noise in the policy signal causes. This arises because the fraction of uninformed firms now have more noise in the only signal that they receive about the macroeconomic conditions. The last term represents the increase in the informativeness of the policy signal to the uninformed agents due to the increase in the fraction of informed firms. As more firms become informed the policy authority's observable, A, is then more informative making the policy signal more informative. When the first term dominates the second term, social welfare decreases with more policy noise. This is the policy authority's trade-off.

Given the trade-off faced by the policy authority, it must be that in the case without learning externalities, welfare in decreasing in the amount of policy noise. Taking the total differential of the marginal firm's indifference condition yields

$$\partial c^* = \frac{\partial E[(b_U\tilde{y}-\tilde{\theta})^2]}{\partial \sigma_u^2}\partial \sigma_u^2 + \frac{\partial E[(b_U\tilde{y}-\tilde{\theta})^2]}{\partial x}g(c^*)\partial c^*. \tag{2.4.26}$$

It is shown in the Appendix that the first term on the right-hand side is positive so that the direct effect of increased noise in the policy instrument is to decrease the welfare of the uninformed by making their guess about θ worse. The second term on the right-hand side is the indirect effect of increasing policy noise. Doing so increases the fraction of informed making the policy signal more informative and making the guess of the uninformed firms better. For social welfare to increase with policy noise, the indirect effect must dominate the direct effect. If this is to occur then it must be that the right-hand side is negative but this cannot be if c^* is to increase, resulting in the requisite rise in x. It is clear that as policy noise increases the cost of the marginal informed firm is never decreasing.

Intuitively, if the benefits of increasing policy noise is so high, then the marginal firm would prefer to remain uninformed. However, if the marginal firm remains uninformed, then the policy signal cannot possibly become more informative with more policy noise because it can only become more informative when the marginal firm decides to become informed. It is impossible for increased policy noise to increase the informativeness of the policy signal when there are no learning externalities and transparent policy is optimal.

Corollary 2 *As policy noise is increased, the less informative is the policy signal.*

This must be the case as social welfare decreases with policy noise. Although the covariance between the policy signal and the macroeconomic conditions increases as policy noise increases, the variance of the policy signal increases relatively more. If this were not the case then the value of being uninformed would increase relative to being informed and fewer firms would become informed unless the cost of acquiring information falls. This is ruled out as the cost of becoming informed is fixed.

Summarizing, when the cost of acquiring information is fixed for all firms, firms pay less attention as the policy noise increases choosing to become informed. This increases the fraction of informed firms raising the attention that the policy authority pays to the prices set by firms. This increases the amount of information regarding macroeconomic conditions in the policy signal. However the amount of noise injected into the signal swamps the increase in informational content provided by the signal, thereby reducing social welfare.

2.5 Conclusion

This paper has presented an argument for the determination of the optimal level of transparency for a central bank. The model isolated a particular informational channel through which policy has effects. The appealing feature of the model is that the effect of policy only comes through announcements from the central bank that are common knowledge to the private sector. In understanding how agents form their beliefs about unobservable fundamentals and how these beliefs are mapped into the actions chosen by the agent, the policy authority chooses the optimal transparency in its announcements in order to affect the beliefs of private sector agents. In doing so, the policy authority must trade-off increasing the quality of the information in its announcement with the amount of private information that agents build into their actions. This is important as the monetary authority has no private information and must rely on private sector agents incorporating their private information into their actions in order to form policy announcements that contain useful informational content. The results do not consider the effects of instruments such as the interest rate or the money supply. While isolating a particular channel of communication, this transmission mechanism seems germane given the recent literature on inflation targeting policies.

An interesting implication of this paper is that more transparency does not necessarily result in lower inflation. If the bulk of private sector agents have easy access to high quality information then the policy authority may choose not to be transparent and low inflation may be achieved. If the costs of acquiring information are widely dispersed across private sector agents then the policy authority may choose to be very transparent leading to lower inflation than if it chooses to be vague. However, in this model, the private sector as a whole has all the information about the underlying inflationary forces. If the quality of this information is good then lack of transparency may lead to lower inflation outcomes. Thus while the mechanism emphasized in this paper is not the only channel impinging on the transparency decision of the central banks (for example, hedging against mistakes that may result in loss of credibility) it may provide something to ponder about when testing the transparency-inflation outcomes empirically.

Chapter 3

Unemployment and Wage Dynamics in a Competitive Search Model with Private Information

Until recently, it was believed that the fundamental driving force behind cyclical unemployment was variations in the job separation rate. Recessions were periods in which an economy experienced a sudden burst of job losses and slowly recovered whereas booms were characterized by relatively little job losses. However, recent empirical observations by Hall (2005b), Shimer (2005b), and Nagypál (2004) have shown that, at least for the two most recent recessions, job finding rates were very volatile over the business cycle and job separation rates were nearly constant.

The empirical findings emphasizing the importance of job finding rates has renewed interest in the theory of unemployment and particularly, in whether the workhorse Mortensen-Pissaridies model of equilibrium unemployment theory (see Pissarides (2001)) can deliver realistic unemployment dynamics given plausible productivity variation. The influential work of Shimer (2005a) showed that within the anonymous search framework, the typical Nash Bargaining institution for wage determination prohibited small productivity shocks from producing large variations in the unemployment rate. Specifically, in a properly calibrated version of the model, he shows that unemployment is half as volatile as labour productivity and wages are as volatile as labour productivity. Related work by Hall (2005a) and Shimer (2004) posited that the addition of sticky wages could deliver realistic unemployment dynamics. In obtaining these results these models posit permanently sticky wages resulting in unrealistic wage dynamics. This paper presents an equilibrium model of job search and unemployment that can produce large fluctuations in unemployment while allowing for wage dynamics in response to small variations in exogenous labour productivity.

A conjecture is that introducing informational asymmetries between workers and firms can result in wage rigidities that amplify the effects stemming from fluctuations in labour productivity. However, this is a particularly daunting task as the introduction of firm-specific informational asymmetries produces distributional issues that are difficult to track in an equilibrium framework. Thus as a first step, this paper begins addressing such informational issues in a relatively parsimonious manner by introducing private information to the firm. It is shown quantitatively, that indeed such informational margins show promise as a propagation mechanism generating 10 times the volatility in unemployment rates relative to the standard job search model.

The main obstacle that the standard equilibrium search models face is that firms must have a small profit margin absent any shocks to the economy. Furthermore, wages must not rise proportionally with productivity gains in order to leave a large amount of the increased productivity to profits for the firms. In such a situation, the gains from a small and persistent increase in labour productivity translates into a large percentage gain in profits for the firm if it can capture the full increase in the match surplus. If such gains are transitory, then firms have an incentive to open vacancies in attempt to hire a worker and immediately take advantage of the temporary productivity gains. However, wage data also suggests that wages are moderately procyclical so a model of unemployment must also address this observation on wage variation as a central issue in macroeconomics is the relation between prices and allocations. Finally, to retain the spirit of unemployment as a costly state, a reasonable model should have a large difference between wages and unemployment benefits.

In order to achieve the desired unemployment dynamics the model presented in this paper includes three ingredients. First, all workers can search on-the-job at a minimal cost and this search cannot be detected by employers. It is assumed that search is directed in that firms post wage contracts and workers apply to attractive wage contract postings in the job market. The threat of on-the-job search ties the wages paid by employers to the prevailing market wage. Second, firms have private information in that workers are unable to perfectly observe the level of output produced by the their own worker-firm productive unit. This puts an upper bound on any credible wage contract that firms can offer to workers. Third, firms face a feasibility constraint such that workers cannot be paid an amount that exceeds output. This eliminates any signing bonuses that workers can be paid when first hired by the firm. As firms observe output that workers do not, the firms can always claim that their output does not permit them to pay the worker a wage above the lowest possible level of output. Thus any contract that specifies a wage above this threshold is not a credible wage offer. When productivity is such that this threshold is binding the firm can reap most or all of the gains to productivity. This result must arise in equilibrium as the threshold must be binding for firms hiring workers as all wages are tied to market wage contracts through on-the-job search. The feasibility constraint adds a condition such that firms cannot front-load payments to workers to the beginning of the worker-firm relationship forcing wages payments to be spread out over the duration of the job match. In equilibrium, market competition for labour services push wages to the upper bound on wages.

This paper contributes to the recent literature concerning the role of wage rigidities in the propagation of labour productivity fluctuations in equilibrium job search models. Although sticky wages have shown promise in generating reasonable unemployment fluctuations, the work by Hall (2005a) points out that sticky wages have a negligible contribution if wages are not set in a manner that leave firms with little of the match surplus. In his paper, such high wages are obtained by a "social norm" and without an equilibrium theory of social norms, there is not a theory of equilibrium unemployment. Hagedorn and Manovskii (2005) show that the standard model can generate realistic dynamics if the flow benefits of unemployment are near equal to the level of output produced by a firm-worker pair and the worker has nearly no bargaining power in negotiating period wages. This argument, while technically correct, is not in the spirit of unemployment as workers are shown to choose work over unemployment for an increase in surplus of about one percent[1]. Another paper that examines the properties of private information held by firms is by Kennan (2005). Private information plays a similar role in his paper although the wage setting mechanism differs from that in this paper.

Section 3.1 provides some detail on the wage and unemployment dynamics that the paper aims to address. An equilibrium model of unemployment and wage determination is then presented in section 3.2 followed in section 3.3 by an examination of the quantitative properties of the model. Section 3.4 provides a discussion of the model. Section 3.5 provides some concluding remarks.

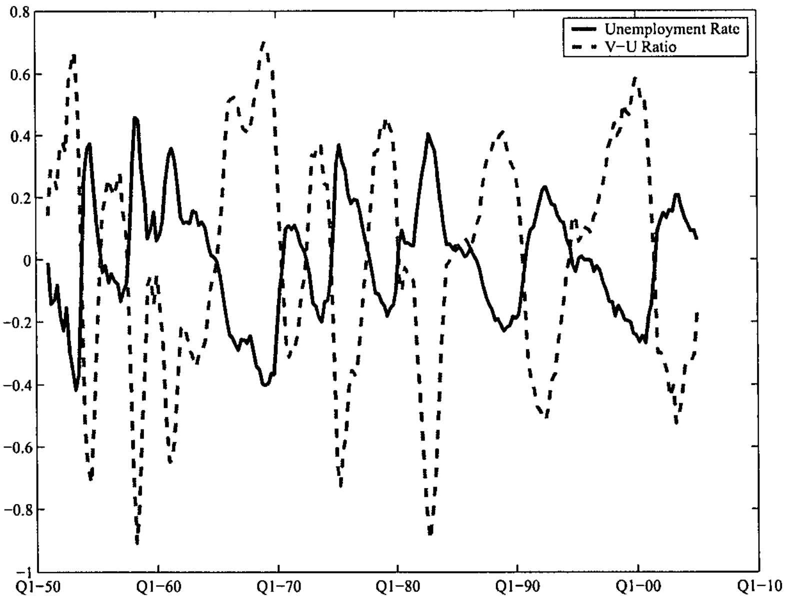
3.1 Empirical Observations

This section examines the time series behaviour of the unmemployment rate, u, and the average real wage, w. Figure 3.1 plots the cyclical behaviour of unemployment rates and the vacancy-unemployment ratio. The figure makes clear that the unemployment rate is very volatile with maximum deviations of more than 40% from its trend. Similarly the vacancy-unemployment ratio is also very volatile and consistently moves in the opposite direction as cyclical fluctuations of the unemployment rate. The levels in the plot for the vacancy-unemployment ratio must not be taken too seriously as the vacancy data is an index of job advertisements constructed by measuring the amount of job advertisements in newspapers. Thus the Conference Board Index of Help-Wanted Advertising Index is not a direct measure of the number of job vacancies. Beginning in December of 2000, the Job Openings and Labor Turnover Survey (JOLTS) provides direct measures of job vacancies but the sample size is too small to draw any definitive facts about the behaviour of job vacancies. However, to provide an idea about how well the Help-Wanted Advertising Index captures the cyclical behaviour of job vacancies, the JOLTS data on job vacancies is regressed on a constant and the Help-Wanted Advertising Index. Figure 3.2 provides a plot of the actual JOLTS data and the predicted value of the vacancy series using only a constant and the Help-Wanted Advertising Index as predictors.

To examine the cyclical behaviour of real wages, Figure 3.3 presents the detrended behaviour of two measures of average real hourly wages. Traditionally real wages are measured by the average real wage in the manufacturing sector. However, average real wages captured in total private industry may capture more of the response of

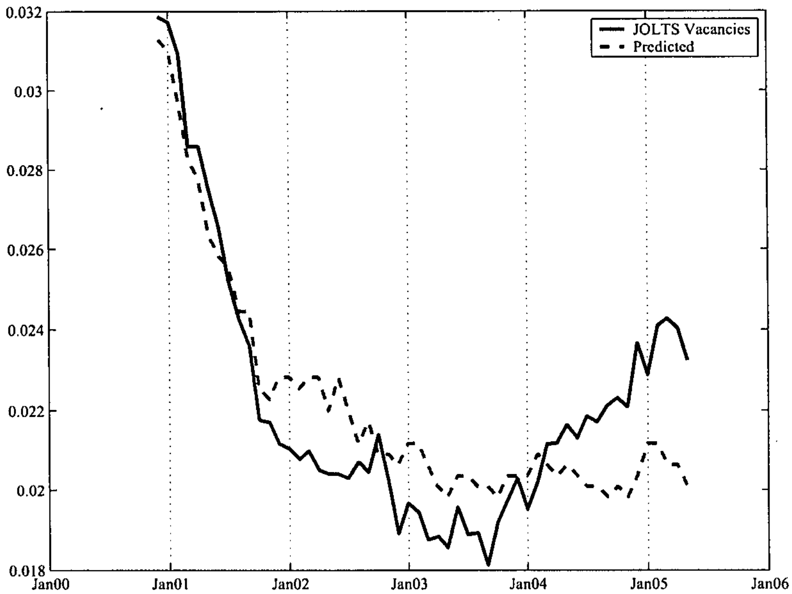
[1]See Mortensen and Nagypal (2005) for a full derivation of this argument.

Figure 3.1: Cyclical Fluctuations of Quarterly U.S. Unemployment Rates and Vacancy-Unemployment Ratios



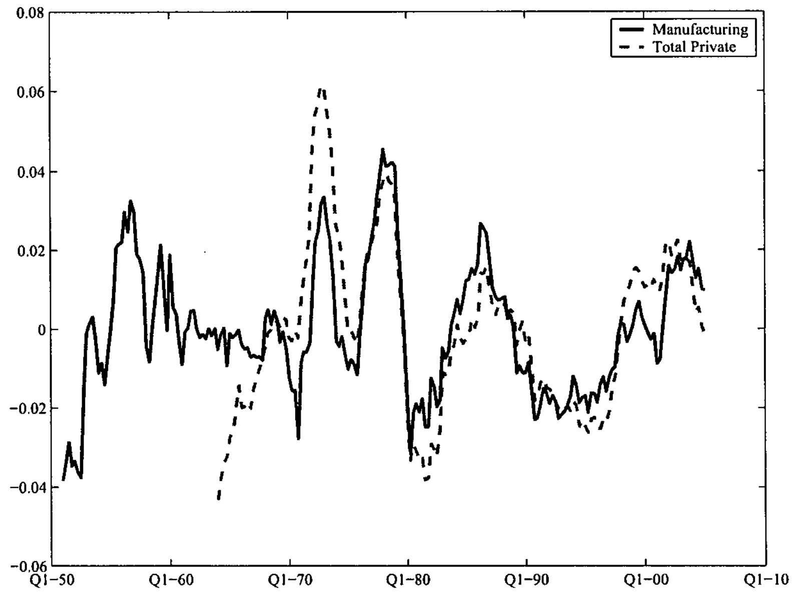
Notes: Data are quarterly averages of monthly data. The quarterly averages are detrended using an HP-filter with a smoothing parameter of 10^5 to make the results comparable with Shimer (2005a). The unemployment rate data is obtained from the BLS website and the vacancy series used is the Conference Board Help-Wanted Advertising Index (with normalization 1987 = 100) obtained from the Federal Reserve Bank of St. Louis database.

Figure 3.2: JOLTS Vacancy Data vs. Predicted Vacancy Data



Notes: Regression : $V_t^{JOLTS} = c + \beta V_t^{CB_{Index}} + e_t$. Adjusted $R^2 = 0.6684$.

Figure 3.3: Cyclical Fluctuations of Quarterly U.S. Average Real Wage Measures



Notes: Data are quarterly averages of monthly data except for the average real hourly compensation index (1992=100) which is quarterly data. The data are detrended using an HP-filter with a smoothing parameter of 10^5. The wage data are all obtained from the BLS website.

wages to aggregate shocks. The measure of real wages from private industry only runs from 1964 forwards so for the rest of the paper the cyclical behaviour of the average real hourly wages from the manufacturing sector is used for the measure of cyclical real wage behaviour. The property to be noted here is that the maximum deviation of the real wages from their trends is about five percent for the manufacturing wage and six percent for the private sector wage. It is clear that the manufacturing wage tracks the private sector wage over time. Real wages are much less volatile than unemployment rates, an observation that has been at the heart of business cycle research for decades.

Table 3.1 provides a compact summary of the cyclical behaviour of wages, unemployement and the vacancy-unemployment ratio. It can easily seen from the second row of the table that unemployment and vacancies are very persistent processes. The observation that unemployment is strongly negatively correlated with vacancies is consistent with the existence of a Beveridge curve. It is also seen that the unemployment rate exhibits a negative correlation with the average real wage.

The negative correlation between the unemployment rate and wages can be gleaned from figure 3.4 which plots the unemployment rate against the average real hourly compensation.

Table 3.1: Summary Statistics, Quarterly U.S. Data : 1951Q1 - 2005Q1

	u	w	$\frac{v}{u}$	z
Standard Deviation	0.1885	0.0167	0.3793	0.0162
Quarterly Autocorrelation	0.9383	0.9282	0.9462	0.8658
Correlation Matrix				
u	1.0000	-0.0781	-0.9739	-0.1924
w	-	1.0000	0.1351	0.3437
$\frac{v}{u}$	-	-	1.0000	0.1645
z	-	-	-	1.0000

Figure 3.4: Cyclical Fluctuations of Quarterly U.S. Labour Productivity, Unemployment Rate and Average Real Hourly Wage

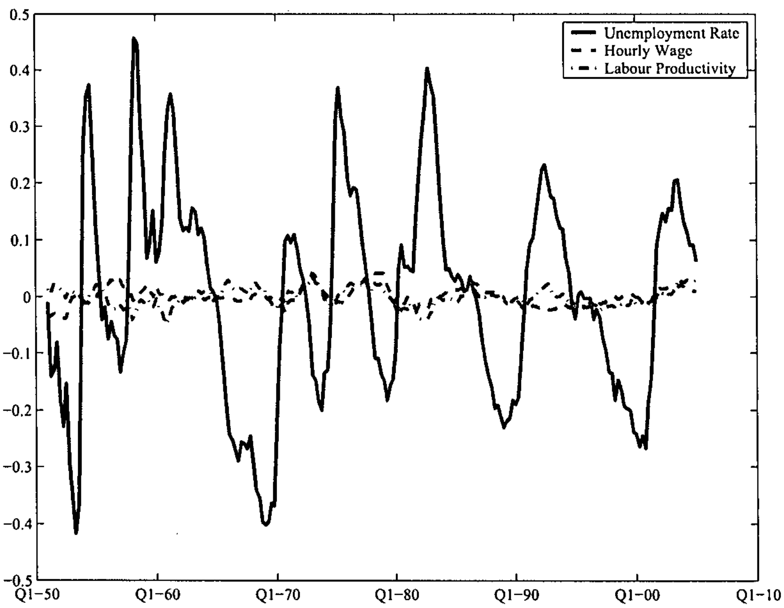


Table 3.2: Dynamic Correlations Between Wages and Unemployment Rates

$Corr(w_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
U.S. Data	-0.1105	-0.1333	-0.1365	-0.1000	-0.0038	0.0966	0.1922

To give a final observation on the relationship between the detrended quarterly data on unemployment rate and wages, I present the dynamic correlations between wages and the unemployment rate. The clear pattern is that unemployment is negatively correlated with wages contemporaneously and up to three lags but is positively correlated with wages up to three leads.

3.2 The Model

There exists a continuum of risk-neutral workers with unit mass and a continuum of risk-neutral firms with infinite mass. At the beginning of period t, there is a measure, u_t, of unemployed workers. The remaining workers are all employed. At the beginning of each period the level of aggregate productivity is revealed to the economy. Firms currently employing workers then make wage contract offers to their workers. Firms are constrained such that wages must be paid out of current output and thus cannot exceed output. Once workers receive their wage contract offers, they choose whether to quit, search while on-the-job, or stay and produce without searching for a new job. Workers must incur a small cost to search on the job. Next job searchers and firms with vacant jobs enter a matching process. This matching process is modelled as a competitive search model in which firms post wage contracts. Once the matching process is finished, firm-worker pairs produce output. The output of a productive unit is a function of the aggregate productivity level and a match specific productivity shock. Firms observe output and then pay their workers. All period output must be consumed as it is non-storable.

3.2.1 Production

The output of a production unit is given by

$$y_{it} = z_t + \epsilon_{it}^{z_t}.$$

There are n possible states for aggregate productivity, $z_t \in \{z_1, z_2, ..., z_n\}$ with $z_{n'} < z_{n''}$ for all $n' < n''$. The process governing aggregate productivity is given by an n-state Markov chain. The subscript i is a firm specific index signifying that each firm draws a firm-specific shock ϵ_{it}^{z} each period. These firm-specific shocks, revealed only during production, are independent across time and firms. The superscript indicates that for different levels of aggregate productivity, z_t, there is a different

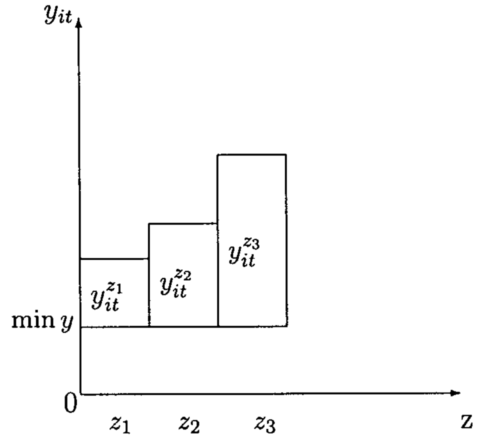


Figure 3.5: Example of Output Levels for Production Units

process governing the random variable ϵ_{it}^{z}. It is assumed that for every state of aggregate productivity the firm-specific shocks can take on m values $\epsilon_{it}^{z} \in \{\epsilon_1^z, \epsilon_2^z, ..., \epsilon_m^z\}$ with each firm-specific shock taking place with some non-zero probability. The firm specific shocks have mean zero, $E_t(\epsilon_{it}^{z}) = 0$. An important property of the firm-specific shocks is that the lowest value of the firm-specific shock for every state of aggregate productivity is such that the minimum output produced by a production unit in a given state, $\underline{y}^{z_{n'}}$, equals the lowest amount of that can be produced when aggregate productivity is in its lowest state $\underline{y}^{z_1} = z_1 + \epsilon_1^{z_1}$. Thus it is such that $\underline{y}^{z_n} = \underline{y}^{z_{n'}} = \underline{y}^{z_1}$ for all $n', n'' \in \{1, 2, ..., n\}$. Figure 3.5 provides an example of the production structure.

3.2.2 Workers

Entering a period, any given worker is in one of two states : unemployed or employed. At the beginning of the period, a measure δ of existing worker-firm pairs are separated exogenously. Workers re-enter the unemployment pool when separated. When unemployed, a worker searches for a job. The job finding probability for an unemployed worker is JFU_t. If a worker is unable to find work in a given period then that worker enjoys a period utility of b which can be thought of either as unemployment benefits, or the output of leisure and home production.

Let W_t^U, and W_t^E be the value of being unemployed and employed at the beginning of date t, respectively. The value of being unemployed at the beginning of period t can then be given by

$$W_t^U = JFU_t \left\{ w_t^M + \beta E_t \left[W_{t+1}^E \right] \right\} + (1 - JFU_t) \left(b + \beta E_t \left[W_{t+1}^U \right] \right) \qquad (3.2.1)$$

where w_t^M is the wage that a market contract offers a newly hired worker.

An employed worker can choose to quit a job, search on-the-job at a small utility cost s, or stay on the job and not search. If a worker chooses to quit a job prior to job search then the value of employment in this case is W_t^U Define $W_t^{E,OJS}$ as the value of being employed at the beginning of date t and choosing to search on-the-job. Then the value of being employed at the beginning of period t is given by

$$W_t^E = (1-\delta)\max\left\{W_t^U, W_t^{E,OJS}, w_t^E + \beta E_t W_{t+1}^E\right\} + \delta W_t^U \tag{3.2.2}$$

with

$$W_t^{E,OJS} = JFU_t\left[w_t^M + \beta E_t W_{t+1}^E\right] + (1-JFU_t)\left[w_t^C + \beta E_t W_{t+1}^E\right] - s \tag{3.2.3}$$

where JFU_t is the job finding rate for a worker in period t and w_t^E is the wage that the incumbent employer offers in a contract before the period t matching market opens.

3.2.3 Firms

If the worker-firm pair is not separated at the beginning period t, then the firm offers the worker a wage contract which specifies a wage that the worker will receive after production.[2] The firm must decide if it will offer its worker a wage that guarantees that the worker will not search on-the-job, or if it will offer the worker a wage that will ensure that the worker will incur the small cost of job search. If the worker searches on the job, she will leave for a new match if a better offer is obtained in the matching market. The firm must always offer its worker a wage contract that ensures that the worker will not prefer unemployment.

One key feature of this economy is that the exact amount of output produced by a firm is private information known only to the firm. That a production unit has produced some output is commonly observable but only the firm knows the exact amount of the output. Furthermore, a firm is constrained to pay its worker out of its current output. As this output is unobservable to anyone but the firm, the firm can always claim that it did not produce enough to pay its worker if the contracted wage is higher than the minimum output given the current aggregate state. Given the assumptions on the production structure, this minimum output is $\underline{y}^{z_1}$. As firm profits are decreasing in wages no wage contract can credibly offer a worker a wage that exceeds $\underline{y}^{z_1}$. Any wage contract offering a wage that exceeds the respective wage threshold is effectively offering a wage that equals the threshold.

First consider a firm matched with a worker prior to the exogenous destruction of jobs in period t. Let J_{it}^S denote the value of a given firm i that employs a worker if it offers a contract that causes a worker to search on-the-job. In contrast, let J_{it}^C denote the value of a given firm i that employs a worker if it offers a contract that causes a worker not to search on-the-job. Then the value of a firm matched with a worker at the beginning of period t is

$$J_{it}^E = (1-\delta)\max[J_{it}^S, J_{it}^C]$$

[2]For tractability, I restrict firms to offering only current period wages instead of long-term state-contingent wage contracts. The effects of more complicated contracts remain an area for future research.

where

$$
\begin{aligned}
J_{it}^S &= \max_{w_t^S} E_t \left\{ (1 - JFU_t) \left[y_{it} - w_t^S + \beta J_{it+1}^E \right] \right\} \\
\text{s.t.} & \\
& W_t^{E,OJS} \geq W_t^U \\
& w_t^S + \beta E_t \left[W_{t+1}^E \right] \geq b + \beta E_t W_{t+1}^U \\
& w_t^S \leq \underline{y}^{z_1}
\end{aligned}
\tag{3.2.4}
$$

The first constraint specifies that a worker must prefer searching on-the-job to quitting. The second constraint specifies that even if a worker does not find a better job, the worker is better off staying with the firm rather quitting. Finally, any wage offered in a contract must be credible. As profits are decreasing in w_t^S, if the worker is to search then the firm should set the wage offered such that the participation constraint binds. The result is that

$$
E_t[(1 - JFU_t)w_t^S] = E_t \left\{ (1 - JFU_t) \left[b - \beta \left(W_{t+1}^E - W_{t+1}^U \right) \right] \right\} + s
$$

as long as $w_t^S < \underline{y}^{z_1}$. Similarly, if a firm with a worker wishes its worker not to search it must offer a wage that solves the following problem

$$
\begin{aligned}
J_{it}^C &= \max_{w_t^C} E_t \left\{ y_{it} - w_t^C + \beta J_{it+1}^E \right\} \\
\text{s.t.} & \\
& w_t^C + \beta E_t[W_{t+1}^E] \geq W_t^{E,OJS} \\
& w_t^C + \beta E_t \left[W_{t+1}^E \right] \geq b + \beta E_t W_{t+1}^U \\
& w_t^C + \beta E_t \left[W_{t+1}^E \right] \geq W_t^U \\
& w_t^C \leq \underline{y}^{z_1}
\end{aligned}
$$

yielding the optimal wage policy[3]

$$
E_t(JFU_t w_t^C) = E_t(JFU_t w_t^M) - s \tag{3.2.5}
$$

if $w_t^C < \underline{y}$ and $w_t^C = \underline{y}$ otherwise.

3.2.4 The Matching Market

The competitive search job market here uses the coordination frictions as set forth by Burdett, Shi, and Wright (2001), Peters (2000), Moen (1997), Stevens (2004) and Burdett and Coles (2003). In the matching market firms incur a cost, c, to post a job vacancy. Firms solve their recruitment problem by offering contracts to entice

[3]For the problem to be interesting it is assumed that the cost of on-the-job search, s, is small enough that should a firm not offer an incentive compatible wage contract, the worker will always choose to search (on-the-job) for a new job. Furthermore, for small enough search costs it will not be the case that the participation constraint is binding while the incentive compatibility constraint holds.

workers to their firms. All job searchers and firms in a given labour market are identical. Firms can only post one vacancy and workers can only apply to one vacancy in any given period. A firm chooses a wage, w_t^d, which they offer to potential job applicants. Workers observe the menu of posted wages and then choose a probability with which to apply to any given posted wage. If a firm only receives one job applicant then that applicant is hired. As workers cannot coordinate their job search decisions, there is a probability that more than one worker will apply to the same firm. In this case, firms randomly select one of the applicants with equal probability and the unsuccessful job applicants remain unemployed. It is this coordination friction that gives rise to involuntary unemployment in the job search process. Due to the inability to coordinate some firms will receive no job applicants and some will receive multiple job applicants so firms that post a job vacancy are not guaranteed a match.

In solving for an equilibrium I impose the restriction all workers in the labour market use the same job application strategies conditional on the menu of posted wages. This seems like a sensible restriction when there is a large number of job searchers as there is no reason to believe that all the workers can get together and coordinate their application strategies.[4] In the first stage of the wage posting process, firms rationally anticipate that for any wage contract that it posts, w_t^d it will on average attract a corresponding queue of job applicants q_t^d from which it will select one applicant. In the second stage of the wage posting process, job applicants observe the menu of posted contracts and choose the optimal probability with which it should apply to each wage posting. In choosing its optimal application probabilities, the worker should not expect to gain from readjusting its application probabilities. This implies that application probabilities are selected so that the expected value of applying to any given wage posting is equal to the expected value of applying to any other wage posting.

A firm with a vacancy chooses its wage, w_t^d to solve

$$\max_{w_t^d}(1 - e^{-q_t^d})E_t\left\{y_{it} - w_t^d + \beta J_{t+1}^E\right\}$$

with the constraint that it must offer a worker the expected return to search offered by the labour market, R_t,

$$\frac{1 - e^{-q_t^d}}{q_t^d}\left\{w_t^d + \beta E_t W_{t+1}^E\right\} + \left(1 - \frac{1 - e^{-q_t^d}}{q_t^d}\right)(b + \beta E_t W_{t+1}^U) = R_t \tag{3.2.6}$$

and the credible wage constraint $w_t^d \leq \underline{y}_{it}$. The probability that the firm receives at least one job application is $(1 - e^{-q_t^d})$ and the probability that a worker is hired by

[4]See Burdett, Shi, and Wright (2001), Peters (2000) and Shi (2001) for good discussions about the properties of equilibria in directed search models. In these papers it is shown that the limiting properties of a wage posting game with a finite number of players is well approximated by the competitive wage posting model as the number of players on both sides of the market approach infinity. Thus, in this paper, the competitive equilibrium is used as an approximation of the limit to the finite player game where the relationships between payoff functions and equilibrium strategies as well as the matching function and equilibrium strategies are well specified.

a given firm is $\frac{(1-e^{-q_t^d})}{q_t^d}$.[5] In the competitive search labour market firms can increase the probability with which it receives job applications by increasing its posted wage. By choosing the posted wage contract the firm is essentially choosing the probability with which workers apply so to solve the problem, the firm can instead choose its optimal queue, q_t^d, and let the wage be determined such that the worker is offered exactly the market return, R_t. This market return will be the value of employment to a newly hired worker. Offering a wage contract that offers less than the expected return offered by the market to workers mean that no worker will apply to the posted contract. Then the optimal policy for the firm is given by

$$e^{-q_t^d} E_t \left\{ y_{it} + \beta \left[W_{t+1}^E + J_{t+1}^E \right] \right\} + (1 - e^{-q_t^d})(b + \beta E_t W_{t+1}^U) - \lambda \frac{\partial w_t^d}{\partial q_t^d} = R_t \quad (3.2.7)$$

where λ is the multiplier on the credible wage constraint. When the credible wage constraint is not binding then the optimal policy rule determines the optimal queue length and the wage is chosen so that the worker is offered exactly the market return. When the credible wage constraint is binding then the optimal queue length is chosen so that the worker obtains the market return.

In a symmetric equilibrium, all firms in a given market offer workers the same job finding probability and all workers in a market apply to all firms with equal probability. Thus in equilibrium the average queue faced by firms are $q_t^d = q_t = \frac{u_t + \delta e_t}{v_t}$. Then the job finding rate of a job searcher is $JFU_t = \frac{1-e^{-q_t}}{q_t}$ and all firms post a market wage, w_t^M.

Firms that enter the matching market must pay a fixed cost, c, to post a vacancy. The value of a vacancy is given by

$$V_t = -c + (1 - e^{q_t}) E_t \left\{ y_{it} - w_t^M + \beta \left[J_{t+1}^E \right] \right\} + e^{q_t} \beta E_t V_{t+1}.$$

When determining whether to post a vacancy, firms can deduce the wage contract that they will post if a vacancy is posted. As there are no barriers to entry, there are zero expected profits to posting a vacancy in equilibrium so $V_t^N = V_t^S = 0$ for all t.

3.2.5 Equilibrium

The flow equation accounting for the flow of workers into and out of unemployment is

$$u_{t+1} = (1 - JFU_t)u_t + (1 - JFU_t)\delta e_t.$$

with employment at the beginning of period t being $e_t = 1 - u_t$.

[5]Here I follow the logic used in the finite player game and then push the number of players on both sides of the market towards infinity. To see this let the probability that a worker applies to firm i be π_i. Then if there are n workers, the probability that no worker applies to firm i is $(1-\pi_i)^n$. This may be rewritten as $(1 - \frac{\pi_i n}{n})^n$. Holding $\frac{\pi_i n}{n}$ constant and pushing n to infinity $lim_{n \to \infty}(1 - \frac{\pi_i n}{n})^n = e^{-\pi_i n}$. In a symmetric equilibrium all firms offer the same contracts and workers apply to all firms with equal probability, $\pi_i = \frac{1}{v}$, where v is the measure of vacancies. Thus the probability that a firm obtains at least one application is $(1 - e^{-q})$ where q is the ratio of job applicants to job vacancies. By similar logic the probability that a worker is hired by a given firm is $\frac{1-e^{-q}}{q}$.

Definition 3 *An equilibrium is a sequence of exogenous productivity levels* $\{z_t\}_{t=0}^{\infty}$*, allocations* $\{u_t, v_t\}_{t=0}^{\infty}$ *and wages* $\{w_t^M, w_t^C\}_{t=0}^{\infty}$ *such that*

1. *firms make optimal wage contract offers,*
2. *workers make optimal job search decisions, and*
3. *there are zero expected profits from opening vacancies.*

3.3 Quantitative Results

In this section I present results illustrating the quantitative properties of the model in equilibrium where all firms offer incentive compatible contracts so that there is no on-the-job search. As a benchmark I also present a model in which there are no firm-specific shocks. In the benchmark model, period output is known to all agents when wage contracts are offered. The only constraint on wage contracts is that specified wages not exceed output.

3.3.1 Parameterization

Most of the parameters are set to retain comparability to the work of Shimer (2005a) who illustrates the shortcomings of the standard Mortensen-Pissarides model. Many of the parameterizations are standard in the business cycle literature. Table 4.1 describes the values chosen for the parameters of the model. The subjective discount factor is chosen to yield a quarterly discount rate of one percent. In the simulation of the model the productivity process is approximated by a 15-state Markov chain where the median state is normalized to one. The method of Tauchen and Hussey is used to discretize the productivity process. The choice for the states of labour productivity and the transition probability returns a productivity process that has an average standard deviation of 0.016 and an average autocorrelation of 0.87, matching the observed properties of the sample data. The exogenous separation rate is chosen to conform with the monthly separation rates calculated by Shimer (2005b). The approximate monthly separation rate is adopted to match recent findings of a monthly separation rate of 0.034.

The value of period unemployment utility is the same as that used in Shimer (2005a). One interpretation is that this represents unemployment benefits so that it is near the upper end of the replacement ratio. The cost of opening a vacancy, c, is chosen so that the average simulated unemployment rate is 5.7% which is equal the the average unemployment rate over the period of 1951Q1 through 2005Q1. An assumption is made that it is relatively costless to send out a job application each quarter so the utility cost of on-the-job search is arbitrarily set so that this cost is negligible. This conforms loosely with findings by Fallick and Fleischman (2004) who show that new results from the Current Population Survey indicate that the majority of workers who move from one job to another find jobs do not actively seek new jobs.

Table 3.3: Calibration Values

Parameter	Value	Description
β	0.99	Subjective discount factor
z	1	Steady state level of aggregate productivity
δ	0.1	Exogenous job separation rate
b	0.4	Period utility received by an unemployed worker
s	0.0001	Utility cost of on-the-job search
c	0.385	Cost of posting a job contract
ϵ	0.012	Determines the highest credible wage offer

Finally, I had to make the choices for the credible wage bounds represented by $z_1 - \epsilon$. The choice of ϵ is made so that the average job finding probability is 0.62 per quarter. There is a wide band of observations concerning the monthly job finding rate for the unemployed. An often cited job finding probability is based on Blanchard and Diamond (1990) which places the average monthly job finding rate at 0.24. More recently, some unemployment accounting by Shimer (2005b) places the monthly rate at approximately 0.4 while data from Hall (2005b) suggests the average monthly job finding rate of the unemployed to be 0.276. The parameterization chosen in the paper yields a average quarterly job finding rate of 0.62 which corresponds to a monthly job finding rate that is close to 0.27 assuming that workers cannot find a job and lose a job within the same quarter.

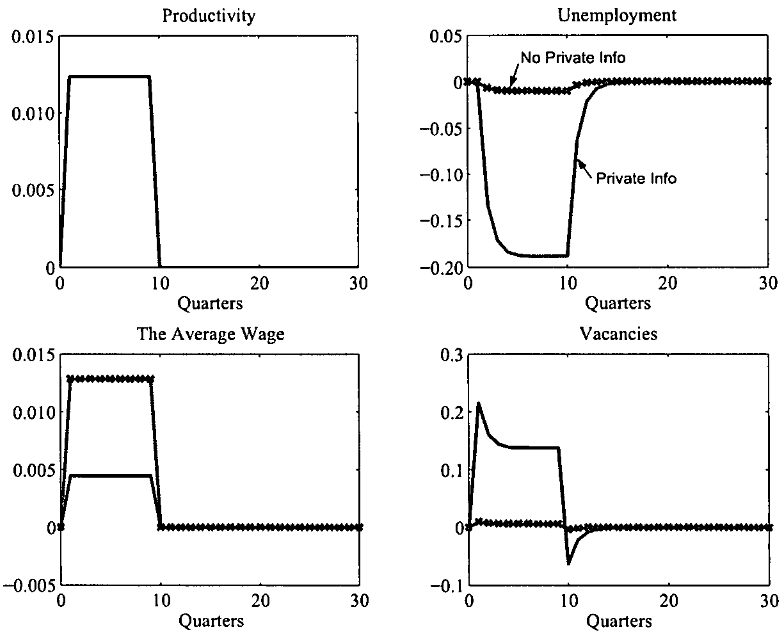
The parameters remain the same for the benchmark model with the exception that the cost of posting a vacancy, c, is adjusted so that the mean unemployment rate is 5.7%.

3.3.2 Dynamic Responses

Simulation of the model is complicated by the existence of an occasionally binding constraint in the wage functions. In order to provide the numerical simulations of the model I use a minimum weighted residuals method to approximate the equilibrium decision rules for vacancies in the matching market, the decision rules for the market wages and also the value functions for the workers. Each decision rule is approximated for each possible level of productivity by a chebychev polynomial in the level of the unemployment rate. For each decision rule, at each level of productivity, a grid of unemployment rates is used as approximation nodes and the weighted distance between the approximated value of the variable given by the decision rule and the exact value of the variable given by the decision rule in equilibrium is minimized using the polynomials themselves as weights.

In order to give an understanding of the dynamics of the model figure 3.6 displays the dynamic effects of unemployment, wages and vacancies to a small increase in labour productivity while figure 3.7 displays the dynamic effects for the same variables to a small decrease in labour productivity. Clearly the upper bound on wages

Figure 3.6: Simulated time series. Model responses to a small increase in labour productivity.

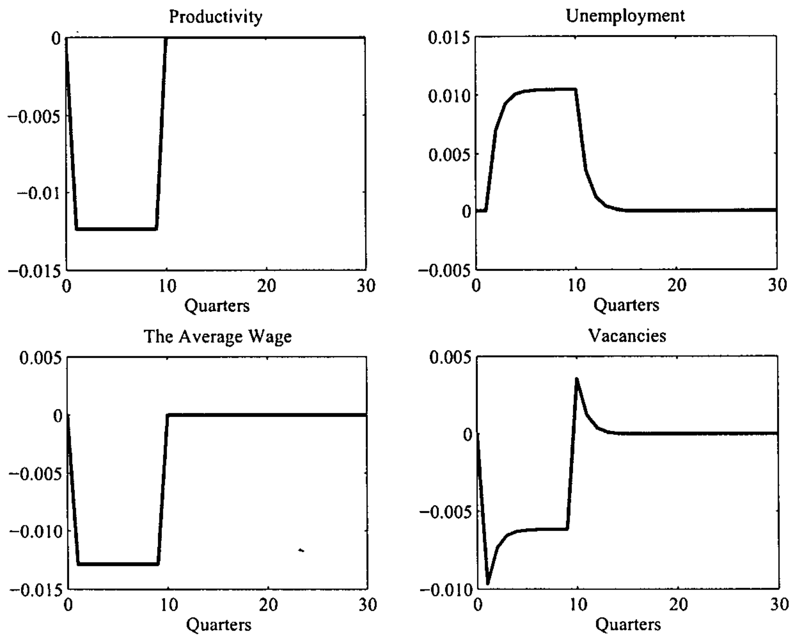


Notes: All plots display the percentage deviations of the variables from their values in the stochastic steady state.

creates an asymmetric response to productivity shocks. In response to positive gains in labour productivity, the wage ceiling allows the firm to capture a large fraction of the increase in productivity. This results in a large increase in job vacancies pulling the unemployment rate downwards. When the wage bound is in effect, for a given level of output z_t the ex-post profits of hiring a worker cannot decrease by having wages increase. Thus ex-post profits are higher. When steady state profits from employing a worker are small then a small increase in such profits results in a large percentage increase in these profits. For the zero expected profits of opening a vacancy to hold, firms must have a much lower probability of hiring a worker. This requires that the vacancy-unemployment ratio increase. If the elasticity of the job-filling probability is low, a large increase in vacancies is required to lower the job-filling probability. With a large increase in vacancies, a large measure of workers transit out of unemployment. Thus the result that a small productivity shock results in a large change in unemployment is obtained.

The responses are contrasted with a model in which there are no firm-specific shocks. In this "no private information model" the only constraint on wage contracts becomes that firms cannot offer wages that exceed expected output. In this case the level of productivity is known when wage contracts are offered and fluctuations in

Figure 3.7: Simulated time series. Model responses to a small decrease in labour productivity.



Notes: All plots display the percentage deviations of the variables from their values in the stochastic steady state. The model with no private information exhibits the same impulse responses.

labour productivity are largely absorbed by wages. As a result, the percentage increase in profits is small given a small productivity increase and there is not much incentive for firms to open job vacancies. This explains the lack of response in the unemployment rate.

In contrast to a positive shock, when productivity decreases, there is no lower bound on wages so wages fall absorbing the fall in productivity. With wages falling, vacancies do not react to decreased ex-post profits and thus there is not much change in the unemployment rate.

3.3.3 Volatility

In this section I show that while the model still requires much work to match the observed behaviour of the U.S. economy, the model adds considerable volatility to the unemployment rate relative to the benchmark model, which behaves much like the standard Mortensen-Pissarides model. Table 3.4 presents the simulated volatilities of model along with some unconditional moments. All the variables with the exception of the unemployment rate are not as volatile as in the observed U.S.

Table 3.4: Simulated Statistics, Quarterly Data

	u	w	$\frac{v}{u}$	z
Standard Deviation	0.1594	0.0109	0.1577	0.0156
	(0.0358)	(0.0020)	(0.0333)	(0.0020)
Quarterly Autocorrelation	0.8902	0.8302	0.8070	0.8635
	(0.0387)	(0.0496)	(0.0675)	(0.0350)
Correlation Matrix				
u	1.0000	-0.5229	-0.7980	-0.7312
		(0.0422)	(0.0687)	(0.0633)
w	-	1.0000	0.5506	0.9030
			(0.0342)	(0.0428)
$\frac{v}{u}$	-	-	1.0000	0.8446
				(0.0546)
z	-	-	-	1.0000

Notes: To construct the data the model is simulated 1000 times. The logarithm of the simulated data are differenced from the logarithm of their respective steady state values. The mean of the relevant statistics are presented in the table with their standard deviations in parenthesis.

data. The v-u ratio in particular is much less volatile in the model relative to the data. Wages and the v-u ratio also lack the persistence relative their empirical counterparts but it is known that the standard business cycle models lack persistence without adding mechanisms directly targeting increased persistence.[6] It can also be seen that relative to the observed U.S. data, the unemployment rate is much more negatively correlated to the wage in the model and also much more negatively correlated to labour productivity. Meanwhile wages are far more positively correlated to the v-u ratio and labour productivity than in the data while the v-u ratio is almost perfectly correlated with labour productivity in the model whereas in the data it is moderately positively correlated to labour productivity.

In comparing the dynamic correlations generated by the model, given in table 3.5, with the sample data it can be seen that while negative correlation between unemployment and lagged wages dies out as the lag in wages decrease it does not have the sign turnaround that the sample data exhibits.

However, the proper measure of success in this paper is to compare the model with private information to its counterpart that does not have private information. The model without private information exhibits the properties of the standard Mortensen-Pissarides model in that volatility of unemployment is negligible and the volatility in the v-u ratio is very small. Table 3.7 displays simulation results of an economy where

[6]Fujita (2003) addresses the issue of persistence of vacancies in the context of a job search framework.

Table 3.5: Simulated Dynamic Correlations Between Wages and Unemployment Rates

$Corr(w_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
Simulated Data	-0.5140	-0.5527	-0.5659	-0.5228	-0.4728	-0.4222	-0.3735
	(0.0449)	(0.0386)	(0.0371)	(0.0424)	(0.0540)	(0.0684)	(0.0830)

there are no firm-specific shocks and thus no wage ceiling. The point to note here is that the volatility of unemployment in the private information model is more than 10 times that in the model where firms are stripped of their private information. Coming hand-in-hand with this result is that the vacancy-unemployment ratio is much more volatile in the private information model. Wages in the benchmark model virtually track aggregate productivity so that all the gains from productivity are absorbed by wages leaving firms with no incentives to open vacancies. On account of the wage bound, the model with private information displays less than half the volatility in wages relative to the benchmark model.

3.4 Discussion

The model presented in this paper achieves the three goals of producing large swings in unemployment in response to small productivity fluctuations while allowing for some flexibility in wages and allowing for unemployment to be a costly state for workers. A recent paper by Hagedorn and Manovskii (2005) has shown that the standard Mortensen-Pissarides model can yield plausible unemployment dynamics if the benefits that unemployed workers enjoy when unemployed are upwards of 90% of the output that would be produced in a match if the worker were employed. In addition, workers are to have near zero bargaining power in wage negotiations which are determined through Nash Bargaining. In response to this observation, Mortensen and Nagypal (2005) has argued that the gain to employment to a worker would be close to 1% of the value of unemployment. He continues to argue that this is an unrealistic view of the costs of unemployment.

It is interesting to notice that the argument of Hagedorn and Manovskii is close to that made in Hall (2005a) who produces a model in which the bargaining institution between workers and firms allows for wages to be chosen from a large range of wages in equilibrium. Hall imposes social norms to select an equilibrium in which wages are high. However, there is no theory of social norms accompanying his argument which effectively leaves wages to be a free parameter. In my paper, I go the opposite route and impose a set of shocks that leaves an upper bound to wages, not a lower bound. It turns out that in equilibrium, market competition for workers push the wage up to this upper bound on wages.

Table 3.6: Simulated Statistics for No Private Information Model, Quarterly Data

	u	w	$\frac{v}{u}$	z
Standard Deviation	0.0125	0.0163	0.0122	0.0157
	(0.0018)	(0.0021)	(0.0016)	(0.0020)
Quarterly Autocorrelation	0.9304	0.8657	0.8657	0.8657
	(0.0192)	(0.0347)	(0.0347)	(0.0347)
Correlation Matrix				
u	1.0000	-0.8511	-0.8511	-0.8512
		(0.0384)	(0.0384)	(0.0384)
w	-	1.0000	1.0000	1.0000
			(0.0000)	(0.0000)
$\frac{v}{u}$	-	-	1.0000	1.0000
				(0.0000)
z	-	-	-	1.0000

Notes: To construct the data the model is simulated 1000 times. The logarithm of the simulated data are differenced from the logarithm of their respective steady state values. The mean of the relevant statistics are presented in the table with their standard deviations in parenthesis.

Table 3.7: Simulated Dynamic Correlations Between Wages and Unemployment Rates for the No Private Information Model

$Corr(w_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
Simulated Data	-0.8367	-0.9352	-0.9842	-0.8509	-0.7346	-0.6330	-0.5444
	(0.0438)	(0.0180)	(0.0038)	(0.0388)	(0.0682)	(0.0916)	(0.1096)

One aspect of my model that may require further examination is that in equilibrium the vacancy-unemployment ratio during the job search process is near 1. This is important as the job matching function that comes for the directed search modelling has no parameters so that the measure of matches is determined entirely by the vacancy-unemployment ratio. Recent data from the JOLTS database has shown that, at least over the last business cycle, the average vacancy-unemployment ratio has been close to $\frac{1}{2}$. This would result in workers facing a much tighter labour market and perhaps pushing the equilibrium wages away from the upper bound on credible wage offers. On the other hand, decreasing the vacancy-unemployment ratio would lower the elasticity of the job filling probability so that if wages fall above the credible wage ceiling, then a small decrease in productivity would result in an even larger decrease in job vacancies than observed under the current calibration. In the calibration process nothing was chosen to match the level of the vacancy-unemployment ratio, partially because of the lack of data on the levels of vacancies across business cycles and partially because the model is so tightly parameterized that achieving a vacancy-unemployment ratio of $\frac{1}{2}$ in the median state would result in an unrealistically high rate of unemployment. Amending this property of the model, conditional on more data on the level of vacancies, is left to future work.

3.5 Conclusion

This paper presented a model in which private information held by firms concerning period output of its productive unit that is not observable by its worker can add some rigidity in wages. The result is a drastic increase in the volatility of unemployment rates; a result that eludes the standard Mortensen-Pissarides model of equilibrium unemployment. The main mechanism in the model that allows the volatility of unemployment to be high while keeping the volatility of wages low is that the private information held by firms keeps an upper bound on wages and market competition for workers drive the wages up to this wage bound. In this case wages do not absorb transitory increases in productivity allowing for a large percentage increase in firm profits. This increase in profits then results in a large increase in job vacancies pulling workers out of the unemployment pool. It is of interest to note that in the model, wages are high due to market forces and not because the reservation wage of workers is high. Unemployment is a costly state for workers.

As the model can create more realistic dynamics in unemployment and wages than the standard model, this paper shows that models in which informational issues between workers and firms may be a promising line of research in understanding the joint business cycle dynamics of unemployment and wages.

Chapter 4

Nominal Rigidities, Interest Rate Rules and the Effects of Anticipated Technological Change

"As we can see, in retrospect at least, the year 2000 was one of re-evaluation, particularly for high-tech investment. Though the evidence is strong that high-tech investments have greatly enhanced productivity in the economy, by 2000 many managers had apparently become concerned that the long-term profit potential of their investments in computers and communications equipment was smaller than they had expected... and sometimes the productivity enhancements were less than anticipated." - Federal Reserve Governor Ben S. Bernanke[1]

The recent downturn in business investment of the early 2000's has brought great attention to the role of expectations as a driving force of business cycles. Economic pundits point to the explosion in the information technology sector of the late nineties and its subsequent bust as an important observation that unfulfilled optimism can result in episodes of large welfare costs as resources are slowly reallocated across the economy. The difficulties in ascertaining whether economic optimism will be validated is a major concern of central banks that are charged with providing the appropriate stimulus to control economic fluctuations. When the economy plunges into recessions that are deemed a result of unfulfilled optimism, questions naturally arise as to whether monetary policy design contributed to the economic downturn.

Empirically, Beaudry and Portier (2004b) show that shocks to expectations regarding TFP that are unaccompanied by current gains account for about 50% of business cycle fluctuations. Furthermore, a chief characteristic of such business cycles is that both consumption and investment rise when the economy is hit with a shock to expectations about future productivity gains. In an exposition of expectationally driven business cycles Beaudry and Portier (2004a) show that the standard one-sector real business cycle model is unable to generate such simultaneous increases

[1] Quote taken from Governor Ben S. Bernanke's speech "Will Business Investment Bounce Back?" delivered April 24, 2003.

in investment and consumption in response to such anticipated increases in productivity. Given the empirical importance of expectationally driven business cycles, an important question is whether the standard preference and technology structure of business cycle models render such models incapable of matching these empirical observations. This paper shows how a model with nominal rigidities can overcome the problems of the standard one-sector model. It is shown that a certain class of interest rate rules that target inflation (current or expected), with little weight on the output gap, can result in booms in investment and consumption given shocks to expected TFP gains that are not accompanied by current TFP gains. The objective of this paper is to push the boundaries of the monetary business cycle literature in order to increase the understanding of the effects of commonly discussed interest rate rules in generating equilibrium responses to expectational shocks. Given that the standard one-sector monetary business cycle model has become the benchmark model within which various monetary policy regimes are evaluated it is crucial to understand whether this framework can help explain the empirical importance of shocks to optimism.

The standard RBC model as documented in King, Plosser, and Rebelo (1988) has difficulties in generating increases in consumption and investment when the economy anticipates future increases in total factor productivity with current productivity unchanged. Intuitively, if the marginal product of capital is expected to rise in the future it seems natural to expect the economy to increase its stock of capital inputs so that when the technological gains are realized, the economy has the capital inputs in place to take advantage of the increase in its marginal product. However this does not occur in the standard RBC model. The reason is that when households expect lifetime income to rise there is an incentive to increase current consumption due to consumption smoothing motives. As current productivity is unchanged and the capital stock fixed, the only way to increase output sufficiently to provide for increase consumption and investment is for labour hours to rise. However, when leisure is a normal good, as is typically assumed in the literature, then households also prefer to increase current leisure. This gives rise to an increase in the marginal rate of substitution between leisure and consumption, increasing the real wage with a resulting reduction in equilibrium labour. Given the reduction in labour, output falls and so consumption is substituted for investment; investment *falls* and consumption rises.

The key components to the model used in this paper that gives rise to both a rise in consumption and investment in the face of anticipated future gains to TFP are (i) an interest rate rule that results in a lower expected real rate of return to savings as compared to the economy's "natural rate" counterpart,[2] and (ii) either nominal price rigidities that result in variable real marginal costs or nominal wage rigidities that result lower real wages which encourage labour to be drawn into production.

As the main difficulties in the RBC model seems to be the link between wealth and labour supply this paper uses nominal wage stickiness (or predetermined wages)

[2]The natural rate counterpart economy is the flexible price economy with a passive nominal interest rate policy. This concept is discussed in detail in Woodford (2003).

to push the household off its labour supply schedule. In such a specification the household sets a wage at which it is willing to meet the labour demand from firms. Then as long as demand for goods is large enough that firms demand more labour, the household is forced to increase labour, pushing the production productivity frontier outwards. Alternatively, sticky prices can be used to force firms to accommodate the increase in demand for final good output at prevailing prices. This results in *variable* real marginal costs that permit inequality between movements in factor input prices and their marginal products. The task then becomes to provide an environment where anticipated gains in future productivity translates into increased demand for investment and consumption. To this end nominal interest rate policies are examined to determine the specifications that induce households to increase capital savings in response to anticipated increases in the future marginal product of capital. By examining a model with imperfections, this paper differs from the work of Beaudry and Portier (2004c) in that their work focuses mainly on the ability of frictionless, competitive economies to generate the desired comovements between investment and consumption conditional on anticipated technological change.

The following section displays the economic model. Sections 4.2 and 4.3 provide the response of the model to shocks to anticipated productivity gains and discusses the mechanisms at work. Section 4.4 concludes.

4.1 A Fully-Specified Model with Nominal Rigidities

The remainder of the paper examines a model with nominal rigidities under various interest rate rules that have been discussed recently in the monetary policy literature.[3]

4.1.1 The Final Good Firms

In period t, a final consumption good is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods indexed by $i \in [0,1]$, using the technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$. The final good firm takes the price of its output, P_t and the price of its inputs, $P_t(i)$, for all i as given. The profit maximizing demand for intermediate goods is then

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t.$$

[3]Examples of business cycle models with nominal rigidities in both output and factor markets include the papers of Christiano, Eichenbaum, and Evans (2001), Smets and Wouters (2003), and Kim (2000).

Integrating across the demand for intermediate goods and making use of the production function of the final goods firm yields a relationship between the price of final goods and the price of intermediate goods

$$P_t = \left[\int_0^1 P_t(i)^{\epsilon-1} di\right]^{\frac{1}{1-\epsilon}}.$$

4.1.2 The Intermediate Goods Firms

Intermediate good firm i produces its output using the production technology

$$Y_t(i) = A_t \tilde{K}_t(i)^{\alpha}(z^t N_t(i))^{1-\alpha}$$

where A_t is a productivity variable, $\tilde{K}_t$ is capital services, N_t is labour services and z^t is labour-augmenting technological progress. The productivity variable follows the first-order autoregressive process

$$\ln(A_{t+1}) = (1-\rho_A)\ln(A) + \rho_A A_t + \Psi_t s_{At+1-n} + \nu_{At+1}$$

where $A > 0$, $\rho_A \in (-1,1)$ and ν_{At} is normally distributed with zero mean and variance σ_A^2. The variable s_{At} is a forecastable component of technology. The value of this innovation to technology is revealed to the economy n periods before it is potentially realized. In this specification, Ψ_t is an indicator variable that takes the value one with probability p and zero otherwise. The economy receives a signal each period that productivity gains may occur n periods into the future. Unfortunately, the signals may be imperfect which can be rationalized as in the herd behaviour literature. The signals, s_{At}, are normally distributed with mean zero and variance σ_S^2.[4]

Intermediate good firms take the nominal prices of labour services, W_t, and capital services, R_{Kt}, as given. As the intermediate goods are imperfect substitutes for each other, each intermediate good firm has monopolistic power and sets a price, $P_t(i)$, for its good at which it is willing to meet all demand. It is assumed that prices are set following a standard Calvo-pricing process. Specifically, in each period, a given intermediate good firm faces a probability, $1-\theta_p$ of being able to reoptimize the price of its good. The ability to reoptimize across time is independent across time and firms. Firms that cannot reoptimize their price simply index to lagged inflation so that

$$P_t(i) = \pi_{t-1} P_{t-1}(i)$$

where $\pi_t = P_t/P_{t-1}$.

Let $\tilde{P}_t(i)$ denote the price that firm i chooses if it can reoptimize its price in period t. In periods where firm i is able to reoptimize its price it will choose $\tilde{P}_t(i)$ to

[4]The models in the literature concerning anticipated technological change usually model news concerning *likely* future productivity gains. To highlight the mechanisms at work, I will assume that all news correctly reports future productivity gains.

maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \theta_p^j \lambda_{t,t+j} \left[\frac{\tilde{P}_t(i)}{P_t} X_{tj} - \frac{MC_t(i)}{P_t} \right] Y_t(i)$$

where

$$X_{tj} = \begin{cases} \prod_{k=0}^{j} \pi_{t+k-1} & \text{for } j > 0 \\ 1 & \text{for j} = 0 \end{cases}$$

and $\lambda_{t,t+j}$ is the value of an additional unit of period $t+j$ real wealth to the household discounted back to period t. Given the demand for its output in any period, firm i then chooses the amount of capital services and labour services that minimizes the cost of meeting that demand.

The optimality conditions for firm i are

$$\begin{aligned} \frac{R_t^K}{P_t} &= \frac{MC_t(i)}{P_t} \alpha \tilde{K}_t(i)^{\alpha-1} (z^t N_t)^{\alpha} \\ \frac{W_t}{P_t} &= \frac{MC_t(i)}{P_t} (1-\alpha) \tilde{K}_t(i)^{\alpha} (z^t N_t)^{\alpha-1} \end{aligned}$$

and

$$E_t \sum_{j=0}^{\infty} \beta^j \theta_p^j \lambda_{t,t+j} \left[(\epsilon - 1) \frac{\tilde{P}_t(i)}{P_{t+j}} X_{tj} Y_{t+j}(i) - \epsilon \frac{MC_{t+j}}{P_{t+j}} Y_{t+j}(i) \right] = 0$$

for the firms that reoptimize their price in period t.

4.1.3 Households

As before there is a continuum of households of unit mass indexed by $h \in [0, 1]$. These monopolistically competitive household provide differentiated labour services to the production sector; that is, firms regard the labour services of each household as imperfect substitutes of the services provided by other households. The labour services of the households are aggregated by a representative "employment agency" producing a labour service bundle, N_t, that is rented to the final goods firms at the aggregate nominal wage, W_t. The employment agency has access to the technology

$$N_t = \left[\int_0^1 N_t(h)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}$$

and maximizes its profits by choosing labour inputs, $N_t(h)$, for all $h \in [0, 1]$ so that it solves

$$\max_{N_t(h)} \frac{W_t}{P_t} \left[\int_0^1 N_t(h)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} - \frac{W_t(h)}{P_t} N_t$$

taking the nominal wages $W_t(h)$ and W_t as given. The demand for household h's labour services is then

$$N_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\phi} N_t$$

Households make a number of decisions in each period. It makes a consumption decision and chooses how to allocate its savings between money, one-period (non-state-contingent) nominal bonds issued by the government, and capital. It also purchases securities whose payoffs are contingent on whether they can reoptimize its wage decision as well as paying its taxes. These assets are traded amongst households and allow households to perfectly insure themselves against wage reoptimization shocks. This follows arguments made in Erceg, Henderson, and Levin (2000) which show that existence of state contingent securities ensures that, in equilibrium, households are homogenous with respect to their consumption and asset holdings. Given this, the notation will treat households homogeneously with respect to consumption and assets but differently with respect to their wages and hours worked. Finally, when possible, households reoptimize their nominal wage.

Preferences are assumed to be of the Hansen-Rogerson variety so that the household maximizes the expected discounted payoff[5]

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\log(c_{t+j}) - \eta N_{t+j}(h)\right].$$

Households face a cash-in-advance constraint so that any purchases of consumption must be paid for in cash. However, it is assumed that the asset market is open at all times so that households can trade excess cash for bonds at any point in time. Specifically, the household faces the following constraints

$$\begin{aligned}
\frac{M_t}{P_t} + \frac{B_t}{P_t} &= \frac{M_t^d}{P_t} - c_t - I_t \\
K_t &= (1 - \delta(u_t))K_{t-1} + I_t \\
c_t + I_t &\leq \frac{M_t^d}{P_t} \\
\frac{M_t^d}{P_t} &= R_{t-1}\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{W_t(h)}{P_t}N_t(h) + \frac{R_t^K}{P_t}u_t K_{t-1} + \frac{D_t}{P_t} - \frac{T_t}{P_t} + \frac{A_{t,h}}{P_t}
\end{aligned}$$

where M_t is money balances, B_t is one-period nominal bond holdings, c_t is consumption, I_t is investment, R_t is the gross interest paid by the government on nominal bonds, u_t is the rate of utilization of capital, K_t is capital holdings, T_t is nominal taxes, $A_{t,h}$ is household h's net cash inflow from partaking in the state-contingent securities market, and D_t is dividends paid to the household by firms.

Notice that capital services, $\tilde{K}_t$ is related to the capital stock by

$$\tilde{K}_t = u_t K_{t-1}.$$

It is assumed that the household chooses the rate of utilization of capital in each period t. It is also assumed that the depreciation function is given by

$$\delta(u_t) = \psi \frac{u_t^{\omega} - \chi}{\omega}$$

[5]Beaudry and Portier (2004c) give conditions under which Hansen-Rogerson preferences can generate the desired conditional correlations between consumption and investment.

Finally, a Calvo-style wage setting environment is assumed in which households face a probability $1-\theta_w$ that they will be able to reoptimize their nominal wage in any period. If the household is unable to reoptimize its wage then it sets $W_t(h) = \pi_{t-1}W_{t-1}(h)$. There are two timing assumptions that will be followed in this paper with respect to the timing of wage setting. Under the first assumption, the households that are able to reoptimize for period t must set their new wages prior to the realization of period t shocks. In other words they reoptimize based on end of period $t-1$ information sets. Under the second assumption, the households that are able to reoptimize for period t do so after observing period t shocks. Households meet the employment agency's demand for their labour services at their given nominal wage. The optimal policy for households that are able to reoptimize its nominal wage is given by

$$E_t\left\{\sum_{j=0}^{\infty}\beta^j\theta_W^j\left[\phi\eta+(1-\phi)\frac{1}{c_{t+j}}\left(\frac{W_t(h)}{P_{t+j}}\right)X_{tj}\right]N_{t+j}(h)\right\}=0$$

and the optimal savings decisions using capital and nominal bonds are

$$\begin{aligned}\frac{1}{c_t} &= \beta E_t\left[\left(1+\frac{R_{Kt+1}}{P_{t+1}}-\delta(u_{t+1})\right)\frac{1}{c_{t+1}}\right]\\ \frac{1}{c_t} &= \beta E_t\left[\frac{R_tP_t}{P_{t+1}}\frac{1}{c_{t+1}}\right]\end{aligned}$$

respectively.

4.1.4 The Government

The government issues nominal assets, collects taxes and incurs government expenditures in a manner that satisfies a flow constraint in every period, taking prices P_t as given. Formally, the government's flow constraint is given by

$$\frac{M_t}{P_t}+\frac{B_t}{P_t}=G_t-\frac{T_t}{P_t}+\frac{M_{t-1}}{P_t}+\frac{R_{t-1}B_{t-1}}{P_t}.$$

For simplicity let $G_t = G$ be constant. Taxes are collected to satisfy the constraint given monetary policy. Monetary policy will be characterized by interest rate rules of the form

$$\hat{R}_t=\gamma_1E_t\hat{\pi}_{t+k}+\gamma_2\hat{\pi}_t+\gamma_3\hat{Y}_t+\gamma_4(\hat{Y}_t-\hat{Y}_t^f)+\rho_R\hat{R}_{t-1}$$

where $\hat{x}_t$ denotes the deviation of x_t from its steady state value. The value of output in the flexible price and wage counterpart to this economy is given by Y_t^f so that the output gap is given by $(\hat{Y}_t-\hat{Y}_t^f)$. Various values of the coefficients governing the interest rate rule will be subsequently examined.

Table 4.1: Calibration Values for Model with Nominal Rigidities

Parameter	Value	Description
β	0.99	Household's Subjective Discount Factor
η	2.75	Preference Parameter on Disutility of Labour
ω	1.5	Parameter Governing Elasticity of the Depreciation Rate
ψ	0.0425	Multiplicative Constant in Depreciation Rate Function
χ	-0.1058	Constant in Depreciation Function
ϕ	21	Elasticity of Substitution Between Labour Inputs in Labour Aggregator
θ_w	0.64	Fraction of Households Unable to Reoptimize Nominal Wage
ϵ	6	Elasticity of Substitution Between Intermediate Goods in Final Goods Production
θ_p	0.6	Fraction of Firms Unable to Reoptimize Price
α	0.25	Capital's Share in Output
ρ_A	0.9	AR(1) Coefficient in TFP Process
σ_A	0.0056	Standard Deviation of TFP Shock
σ_S	0.0056	Standard Deviation of Forecastable TFP Shock
p	0.65	Probability of Receiving a Correct Signal
γ_z	1.0046	Gross Trend Growth in Labour Augmenting TFP
π	1.0058	Steady State Inflation

4.2 Calibration

The length of a period is meant to be one quarter. Much of the calibration follows Christiano, Eichenbaum, and Evans (2001), with some exceptions. At the chosen values consumption's share of output is 0.67 and investment's share of output is 0.16. Labour hours is 0.32 of the time endowment which is normalized to unity. Steady state gross inflation is chosen to be 1.0058 which matches the mean gross rate of inflation from 1987 : $Q1$ through to 2004 : $Q2$. The share of capital in production, $\alpha = 0.25$, is taken from Ireland (2004) in which a standard RBC model is estimated using Kalman Filter techniques. The value for the quality of the TFP signals is a new parameter that has little precedence. This value is chosen to be low and should be estimated within the context of such a model in the future. Given the variations in information sets available to wage and price setters to be considered in the simulations, the log-linearized model is simulated using the method of moments proposed by Christiano (2001).

4.3 Dynamics Under Taylor Rules

Simulation results of the model under various versions of the interest rate rule proposed by Taylor (1993) are presented in this section. The motivation for examination of interest rate rules is that recent research has emphasized the use by central banks of nominal interest rates as the chief policy instrument. Furthermore, a major motivation of this paper is to establish whether positive increases in both consumption and investment can occur after the realization of optimism concerning future productivity increases within the context of standard monetary business cycle models with nominal rigidities.

In order to highlight the role that different nominal rigidities play in determining the effects of shocks to expectations, the model is simulated for TFP shocks with no persistence and that are realized one period after the signal is received. It is assumed that the signal is correctly realized. Furthermore, the nominal interest rate rule is first set to be a function of only expected inflation or current inflation. A systematic presentation of impulse responses to shocks of expectations concerning TFP follows. First the wage and price rigidities are shutdown one at a time to examine their importance in driving the results. Then importance of preset versus stickiness of wages and prices are examined.

4.3.1 The Role of Nominal Rigidities When $\hat{R}_t = 1.5E_t\hat{\pi}_{t+1}$

First the importance of the flexibility of wages and prices are examined. Clearly from the investment panel in Figure 4.1 both predetermined prices and predetermined wages play an important role in generating increases in investment following a shock to expected TFP gains. Consider first the case with predetermined wages that are set before the signal is realized and near-flexible prices ($\theta_p = 0.01$) that are set after receiving the signal shock. In this case there is an enormous jump in investment on impact. One of the chief reasons for this is that the real wage falls in response to the shock as shown in Figure 4.2. When the signal is received, the agents believe that next period marginal costs will fall. This means that inflation expectations are lowered as monopolistically competitive firms set their prices as a mark-up on marginal costs. The interest rate rule is set such that the expected real rate of return to bonds are lowered which leads to a substitution towards savings through capital goods. This increases the demand for output. In order for output to increase, labour and capital utilization must increase. While the nominal wage does not adjust when labour is brought into production, the rental rate on capital services increases increasing the nominal marginal costs of production. Hence the increase in the price of output charged by monopolistically competitive firms. Moreover, as prices rise, the real wage decreases resulting in firms drawing labour inputs into production. The rise in prices exacerbates the reduction in the expected real return to nominal assets. Hence, the interest rate rule accommodates the rise in investment by lowering the real rate of return on bonds and encouraging substitution into capital savings.

Next to isolate the effects of predetermined prices, the probability for being unable to readjust nominal wages is set to $\theta_w = 0.01$ and wages are set after the signal shock.

As before, the interest rate rule results in expected inflation to fall on impact, though the fall is not near the magnitude as in the case of predetermined wages and near price flexibility. As before, along the equilibrium path the expected real rate of capital falls requiring a build-up of capital in the impact period. In contrast to the predetermined wage with near-flexible price case, here the real wage increases. However, the increase in real wages is accompanied by a rise in marginal costs. As firms are forced to meet output demand at their posted prices, and the demand for investment and consumption goods rises, more labour must be drawn into production lowering the marginal product of labour. With near-flexible wages, household demand higher wages for labour services which helps fulfill the equilibrium increase in demand. In this case, the marginal cost wedge plays a major role in allowing output to rise. With flexible prices and flexible wages this marginal cost wedge does not allow the flexibility in the marginal product of labour to result in a rise of both consumption and investment in equilibrium.

Even more important though is the role played by the interest rate rule. Here the nominal rate policy requires a large drop in expected inflation along the equilibrium path. As households substitute capital savings for bonds savings the real rate of capital drops and yields an increase in investment. As the nominal rate falls so significantly, arbitrage incentives provide a strong mechanism for increasing investment. It is key to understand that with the present interest rate policy that the expected real rate of return is given by

$$\begin{aligned} E_t\hat{r}_{t+1} &= 1.5E_t\hat{\pi}_{t+1} - E_t\hat{\pi}_{t+1} \\ &= 0.5E_t\hat{\pi}_{t+1} \end{aligned}$$

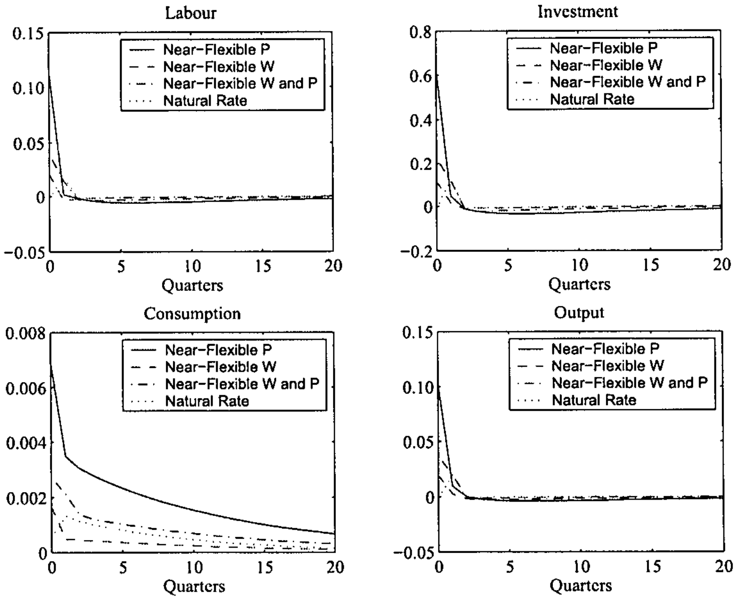
so with marginal costs expected to fall, inflation should also fall. This means that the deviation of the real rate of return from its steady state value should be negative helping give rise to the increase in investment. To isolate the interest rate effects Figures 4.1 - 4.3 also show the effects when both prices and wages are set after the signal shocks are received. In these plots $\theta_p = \theta_w = 0.01$ so that wages and prices are almost flexible. Finally the interest rate rule is set to

$$\hat{R}_t = 1.5E_t\hat{\pi}_{t+1} + 0.2\hat{R}_{t-1}$$

with the interest rate inertia term added to avoid indeterminacy. It is interesting to note that with just enough rigidity to provide determinacy, such an interest rate rule can provide for a large increase in investment following a signal shock.[6] In this case, the real wage falls (though by a small magnitude) on impact and marginal costs rise substantially allowing for a decrease in the marginal product of labour as workers are brought into production to provide for enough output to let both investment and consumption rise. This illustrates the importance of an interest rate rule that causes substitution away from nominal assets into capital in the face of expectation driven demand.

[6]The case with almost flexible prices and wages is not perfectly comparable to the other cases as the interest rate rule is different.

Figure 4.1: Dynamic Responses : Varying the Flexibility of Price and Wage Setting (Panel 1)



Note : All impulse responses present the percentage deviation of the respective variable from its steady state value.

Also displayed is the outcome in which the policy authority sets the nominal interest rate in order to accommodate the natural rate of interest.[7] Under such policy, the real rate of return is taken as given (as is the sequence of allocations determined in a flexible price model) and the nominal rate is set so as to leave the real rate of return unaffected. The natural rate of interest is then a passive nominal interest rate policy which provides a useful benchmark against which to compare the effects of all other interest rate rules. It is useful to note the flexible price and wage economy where the interest rate accommodates the natural rate of interest, the plot shows that the economy cannot produce increases in investment in response to a TFP signal. Under perfectly flexible prices and wages, real marginal costs are constant. This prevents decreases in the marginal product of labour to be accommodated by increases in marginal costs. The result is that equilibrium labour falls as does output and consumption is substituted for investment.

[7]The concept of the natural rate of interest is discussed in detail in Woodford (2003).

Figure 4.2: Dynamic Responses : Varying the Flexibility of Price and Wage Setting (Panel 2)

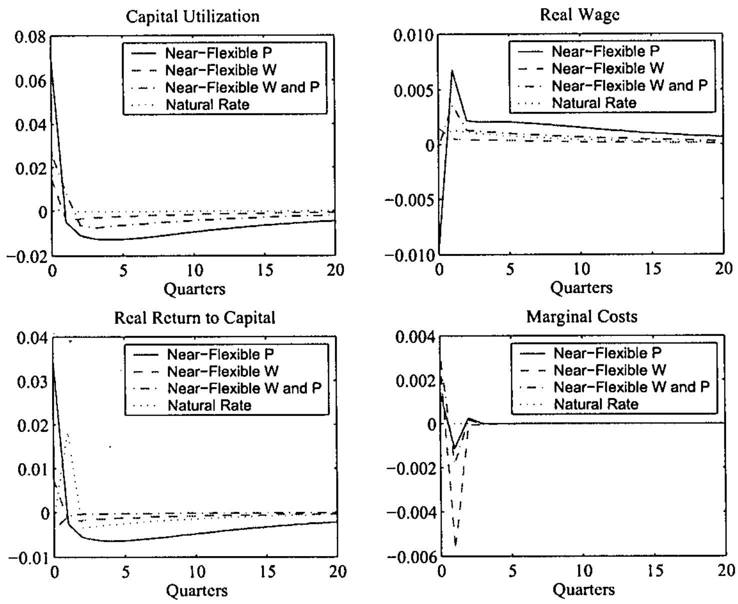


Figure 4.3: Dynamic Responses : Varying the Flexibility of Price and Wage Setting (Panel 3)

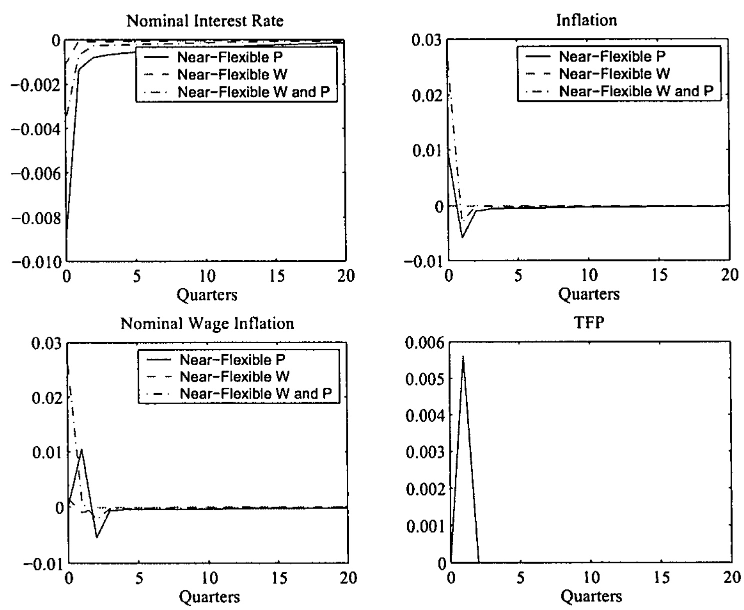
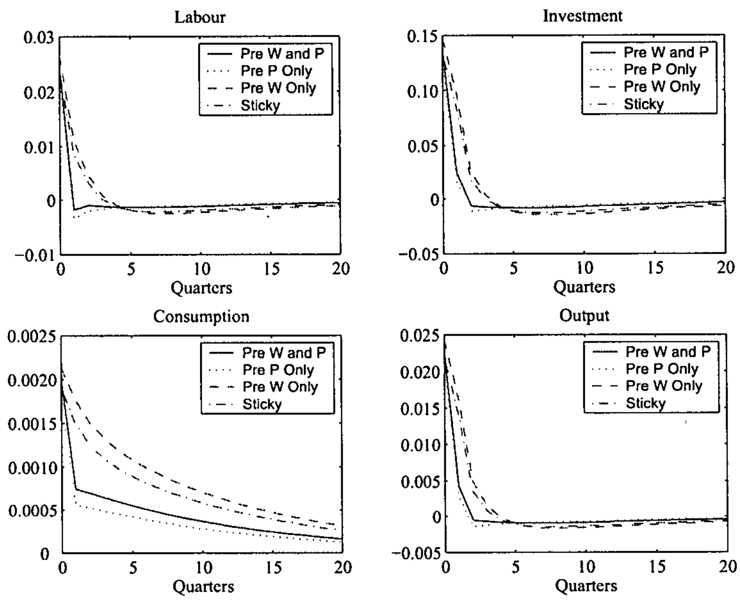


Figure 4.4: Dynamics Responses : Varying the Timing of Wage and Price Setting (Panel 1)



4.3.2 The Role of Timing of Expectational Shocks for $\hat{R}_t = 1.5E_t\hat{\pi}_{t+1}$

In discussing the importance of timing of the expectational shock, it is found that investment increases largest when only wages are predetermined. The effects are still strong when both wages and prices are predetermined and when only prices are predetermined. Finally, an increase in investment is still obtained when prices and wages are sticky but not predetermined. These cases are displayed from figures 4.4 - 4.6.

All the cases are mostly similar in outcomes. The behaviour of the economy in these cases suggest that the role of marginal costs is important. As in the predetermined price and almost flexible wages case shown above, real wages rise on impact. In order for output to increase with a rising real wage, the economy relies heavily on rising marginal costs to provide this flexibility as capital services do not rise significantly enough to provide this output itself.

Summarizing the results so far, it appears that the important factors contributing to the rise in investment and consumption following a shock to expectations about future TFP are

1. the interest rate rule, by forcing a low real rate of return on nominal assets, provides strong incentives for households to substitute away from nominal assets towards capital goods when making their optimal savings decision,

Figure 4.5: Dynamics Responses : Varying the Timing of Wage and Price Setting (Panel 2)

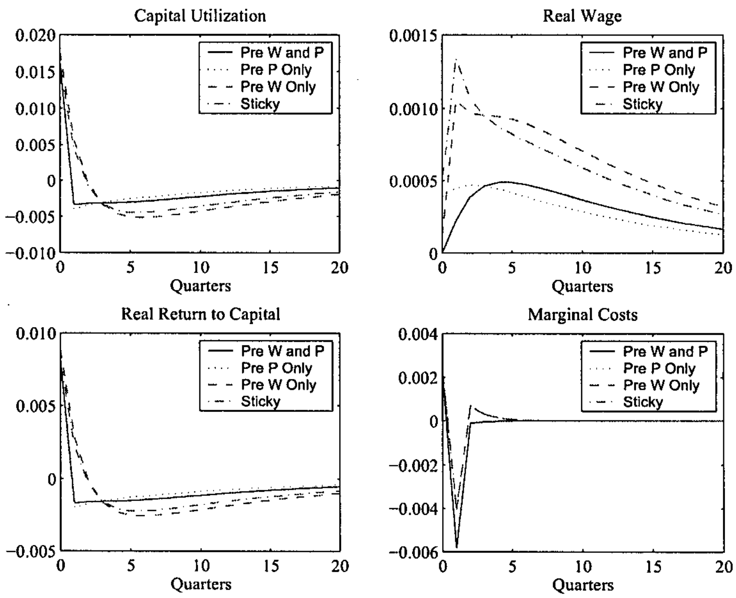
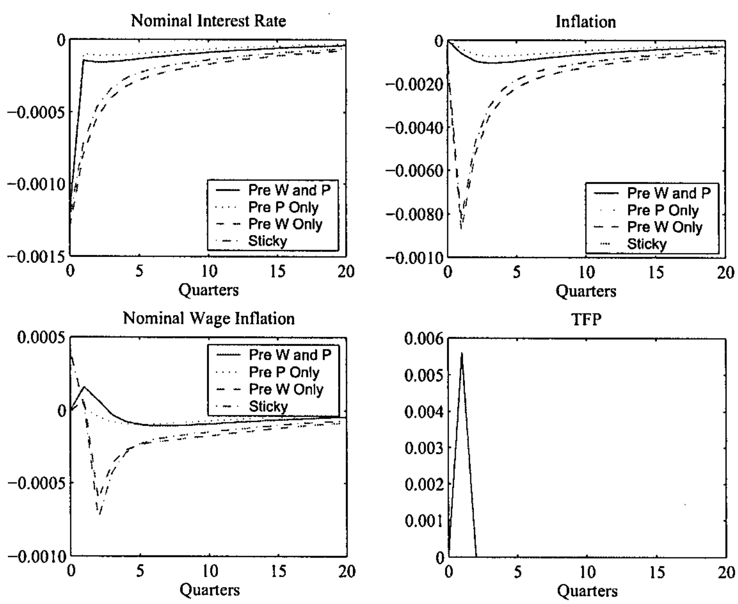


Figure 4.6: Dynamics Responses : Varying the Timing of Wage and Price Setting (Panel 3)



2. there is either some nominal rigidity in prices so that real marginal costs movements accommodate decreases in the marginal product of labour or nominal wage rigidities are present to allow a fall real wages on impact. The possibility of a large decrease in the real wage allows for a large decrease in the marginal product of labour as workers are drawn into production.

The results of this section are robust to the inclusion of lags of the nominal rate of interest so that

$$\hat{R}_t = 1.5E_t\hat{\pi}_{t+1} + 0.6\hat{R}_{t-1}$$

or to using current inflation as opposed to expected inflation so that

$$\hat{R}_t = 1.5\hat{\pi}_t + 0.6\hat{R}_{t-1}.$$

These two alternative rules provide significant rises in investment in response to a positive shock to anticipated TFP but the effects are not as drastic as seen in the benchmark rule. Also, when the shocks to TFP are persistent, the effects remain qualitatively the same as above with the magnitudes of the resulting responses being reduced but still large.

4.3.3 Effects in the Presence of a Generalized Taylor Rule

In this section it is shown that the use of the type of Taylor Rules generally used in the literature does not allow for an increase in investment to accompany the increase in consumption that occurs when the economy anticipates an increase in future productivity.

To show this the interest rate rule is set to be

$$\hat{R}_t = 1.5E_t\hat{\pi}_t + 0.5(\hat{Y}_t - \hat{Y}_t^f) + 0.6\hat{R}_{t-1}.$$

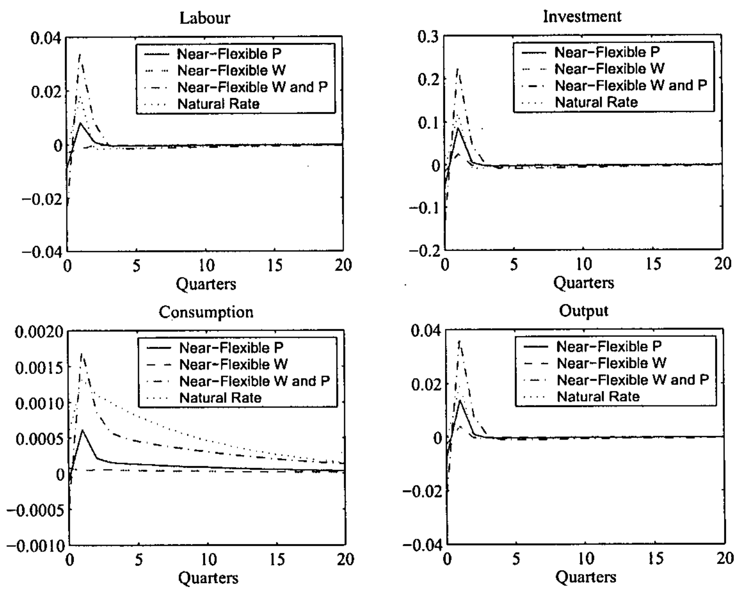
where, as discussed earlier, $\hat{Y}_t^f$ is the natural rate of output. The term $\hat{Y}_t - \hat{Y}_t^f$ is referred to as the output gap.

It is clear from Figures 4.7 - 4.9 that in all specifications illustrated in the previous section, whether prices or wages are predetermined or not, investment falls on impact of a positive shock to the expectations of future TFP.

From the previous section it is seen that in the absence of an output gap term, the economy reacts to positive anticipated TFP gains by increasing its stock of capital in order to take advantage of the expected increase in the marginal product of capital inputs. Once the output gap is added to the rule, the nominal interest rate is expected to increase if output is to deviate from its natural rate counterpart. However, as previously shown, in the flexible price model with the nominal rate of interest defined by the natural rate of interest, investment does not increase on impact of expected TFP gains.

Consider the economy with nominal rigidities. After a positive signal arrives indicating future TFP gains, the household considers increasing its savings via capital in response to expected increases in the marginal product of capital. On account

Figure 4.7: Dynamic Responses with a Generalized Taylor Rule (Panel 1)



of consumption smoothing motives, it also wishes to consume more as its lifetime income is expected to rise. However, if both consumption and investment are to increase then output must increase giving rise to a positive output gap. Given the interest rate rule, although marginal costs are expected to fall in the future, the size of the coefficient on the output gap will result in a rise to the nominal interest rate. This rise in the nominal rate is sufficient enough so that the rise in the expected real rate of returns from holding bonds increases. Arbitrage then prevents the expected real rate of return from falling low enough that there is sufficient substitution by households away from bonds and into capital so that investment rises.[8]

Summarizing, for the sizes of coefficients on the output gap that reflect those estimated in the empirical literature on Taylor rules[9] the nominal rate is expected to rise if investment and consumption are to boom on impact. This rise would precipitate a substitution of capital for nominal assets and prevents a rise of investment on impact in the equilibrium outcome.

Before concluding this section, interest rate rules that replace the output gap with output are considered. If it is the natural rate of output that is preventing investment from rising on impact then the question arises as to whether a rule that specifies the nominal rate as a function of output can result in a rise of investment on impact. Specifically, consider rules given by

$$\hat{R}_t = 1.5E_t\hat{\pi}_t + \rho_y\hat{Y}_t + \rho_r\hat{R}_{t-1}.$$

[8]The timing of shocks is not important enough to reverse these results and are not shown here.
[9]For examples see Clarida, Gali, and Gertler (2000) and Orphanides (2001).

Figure 4.8: Dynamic Responses with a Generalized Taylor Rule (Panel 2)

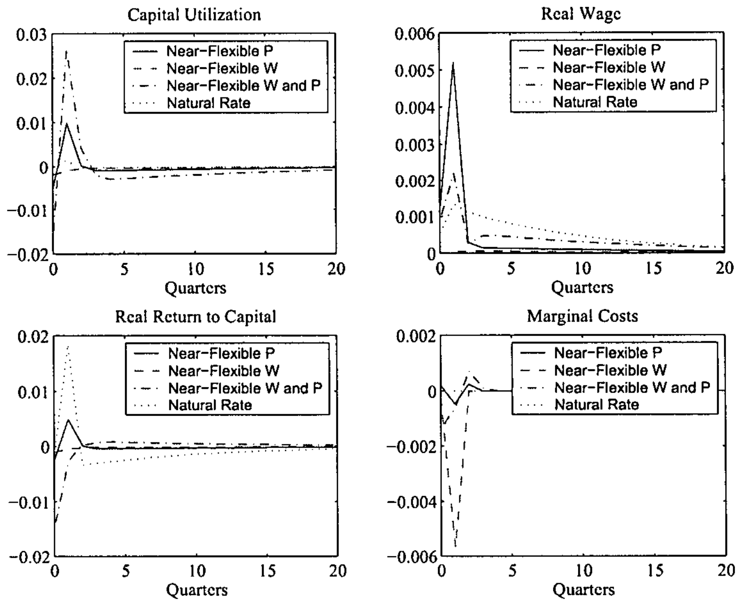


Figure 4.9: Dynamic Responses with a Generalized Taylor Rule (Panel 3)

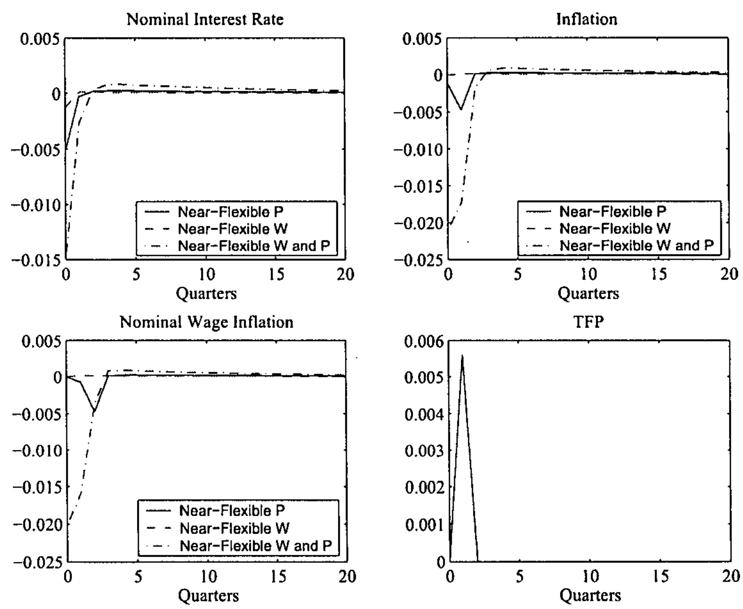
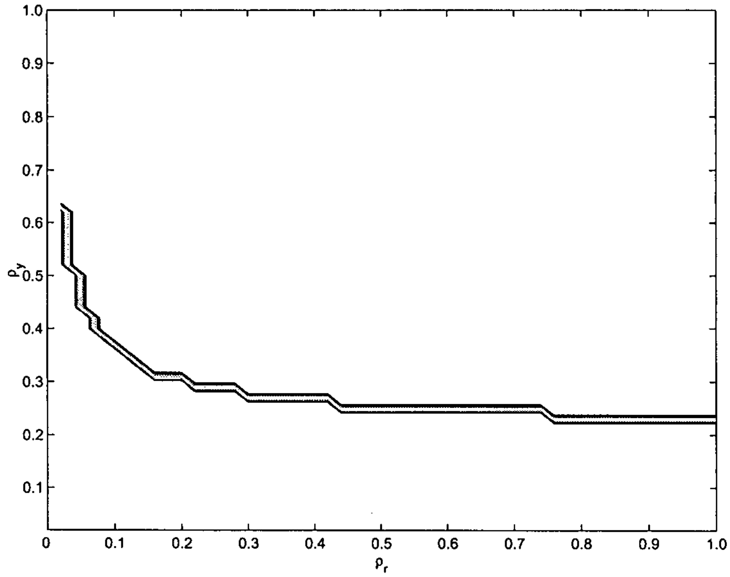


Figure 4.10: The Generalized Taylor Rule Frontier



Note : In this figure the interest rate rule takes the form $\hat{R}_t = 1.5E_t\hat{\pi}_t + \rho_y\hat{Y}_t + \rho_r\hat{R}_{t-1}$. In the region below the frontier, the economy exhibits an immediate rise in both consumption and investment upon the arrival of news concerning future gains in productivity.

The model is simulated for various combinations of ρ_y and ρ_r to find combinations under which investment rises on impact.[10] The area below the line in Figure 4.10 shows the combinations of ρ_y and ρ_r that allow for investment to rise on impact. Without a large coefficient on output the nominal rate allows for the expected real rate of return on nominal assets to be low enough that arbitrage between capital and nominal assets generates a large rise in demand for investment.

It is necessary to note that the model has been simulated with the realization of expectations occurring a period after the signal of future TFP gains are received. As the time between periods when the signals are received and when the signals are revealed to be correct (or incorrect) increases the inflation targeted in the interest rate rule must adjust commensurately in order for investment to boom on impact of the signal. This is because the nominal interest rate must cause the expected real rate of return on nominal assets to fall on impact. More flexibility may be gained if the information structure is altered so that signals are currently received about TFP gains in several future periods.

[10]In this exercise prices and wages are sticky but not predetermined.

4.4 Conclusion

This paper has shown that the possibility for investment to rise after an innovation to anticipated TFP, even with the presence of nominal rigidities, depends heavily on the specification of monetary policy. Particularly, given the underlying structure of the model with no nominal rigidities, when measures of the output gap are included in the nominal interest rate rule, the features that prevent the standard RBC model from producing the desired conditional correlations in output and investment also prevent such conditional correlations from arising in the economy with nominal rigidities. Work such as that of Clarida, Gali, and Gertler (2000) raise questions about the weight placed on the output gap or the expected output gap in various specifications of interest rate rules during the Volcker-Greenspan years. Some specifications result in insignificant estimates on this weight while other specifications yield large and significant estimates. If measures of the output gap are deemed a central feature of monetary policy rules then the implications of this paper seem to be that current economic structures used in macroeconomics may need revision.

Chapter 5

Concluding Remarks

This dissertation examines the role of information imperfections and asymmetries in shaping macroeconomic equilibrium outcomes. Chapter 1, describes a new channel through which monetary policy can affect the equilibrium outcomes of an economy. An environment is constructed in which private sector agents can incur a cost to learn about the state of the economy but can also try to infer about the state of the economy through the announcements of the central bank at no cost. The central bank learns about the state of the economy through its observations of private sector actions and conveys its knowledge through its announcements to the private sector. The policy authority faces a trade-off between informing the private sector of the state of the economy, and gathering information about the state of the economy. In some cases it can be shown that the policy authority may find it optimal to garble its messages to the private sector in order to maximize the aggregate welfare of the agents in the economy. This garbling provides incentives for private sector agents to incur costs to learn about the state of the economy thereby allowing the central bank to learn about the state of the economy through its observations of private sector actions. At the same time, this makes its policy announcements more informative to the private sector.

Chapters 2 shows that informational asymmetries between workers and firms can be a promising mechanism in the propagation of productivity shocks in job search and matching models. It has been shown that the standard models of job search and unemployment have been unable to generate volatile unemployment rates in response to plausible variations in labour productivity. This has called into question the quantitative relevance of this class of models as a useful tool for which to study the aggregate allocation of labour resources. In Chapter 2, it is shown that an equilibrium job search model incorporating informational asymmetries can be used to produce wage rigidities which along with on-the-job search yields an economy exhibiting highly volatile unemployment rates when subjected to productivity shocks.

The final chapter of the dissertation examines whether a monetary business cycle model can result in increases in both consumption and investment on impact of news about future gains in productivity. It is now known that a standard real business cycle model is incapable of producing such comovements of aggregate consumption and investment in response to new of future gains in productivity. Given that empirically, such news accounts for almost 50% of the variation in consumption and investment, any reasonable equilibrium macroeconomic model should be able generate such behaviour in aggregate consumption and investment. In Chapter 3, a standard monetary business cycle model is augmented with anticipated productivity gains to examine whether the interaction between nominal rigidities and monetary policy can generate both increases in consumption and investment. It is found that combining nominal rigidities and interest rate rules that do not place much weight on output gaps can result in consumption and investment to rise on the arrival of news concerning future productivity gains.

Appendix A

Addendum to Chapter 1

A.1 Solving for the Rational Expectations Equilibrium

In order to solve for the rational expectations equilibrium it is necessary to work backwards. Suppose that a fraction of firms, x, have become informed and learned the value of θ in the first stage. Then these x firms will choose their price such that $p_i = p_I$. The remaining $1 - x$ uninformed firms choose their prices optimally given that they observe the policy signal y so that $p_i = p_U$. From the firm's optimal policy rule it is known that

$$p_i = (1-r)E_i(\theta) + rE_i(P).$$

Substituting for P and writing $\overline{E}(\theta)$ for the average expectations of θ across agents we have

$$p_i = (1-r)E_i(\theta) + (1-r)rE_i\left[\overline{E}(\theta)\right] + (1-r)r^2E_i\left[\overline{E}^2(\theta)\right] + ...$$

which upon continued iteration yields

$$p_i = (1-r)\sum_{k=0}^{\infty} r^k E_i\left[\overline{E}^k(\theta)\right]. \tag{A.1.1}$$

Given the linear-normal structure of the economy it is conjectured that $y(\theta, \eta, u)$ and $A(\theta, \eta, u)$ are normally distributed random variables. If so, then Bayesian beliefs for the uninformed firms and the monetary authority are given as

$$E_U(\theta) = b_U y.$$

$$E_M(\theta) = f_A A_p.$$

respectively, where

$$A_p = A - f_U u.$$

The term $f_U u$ is the linear least squares projection of A on u and the term b_U are the linear least squares projection coefficient of y on θ. The coefficients governing the belief functions can be expressed as

$$b_U = \frac{cov(y,\theta)}{var(y)} \quad , \quad f_A = \frac{cov(A_p,\theta)}{var(A_p)} \quad , \quad f_U = \frac{cov(A,u)}{var(A)} \tag{A.1.2}$$

Noting that for the informed firm that $E_i(\theta) = \theta$ substitution of beliefs into equation (A.1.1) and forward iteration yields the price function

$$p_I = \frac{(1-r)}{(1-rx)}\theta + \frac{r(1-x)}{1-rx}b_U y \tag{A.1.3}$$

$$p_U = b_U y \tag{A.1.4}$$

for the informed and uninformed firms, respectively. Aggregating across firms

$$\begin{aligned} A &= \int_0^1 p_i di + \eta \\ &= \frac{(1-r)x}{1-rx}\theta + \frac{1}{1-rx}(1-x)b_U y + \eta \end{aligned}$$

Define the variable

$$A_P = A - f_u u$$

which is the data from which the policy authority learns about the realization of θ. The component $f_u u$ accounts for the fact that the policy authority knows the realization of u. Thus the policy authority can clean this noise out of its data, A when forming its expectations about θ. Then

$$E_M(\theta) = f_A A - f_A f_U u$$

Given that $y = E_M(\theta) + u$ it is easily verified that

$$y = \frac{f_A(1-r)x}{1-rx-(1-x)f_A b_U}\theta + \frac{f_A(1-rx)}{1-rx-(1-x)f_A b_U}\eta - \frac{(f_A f_U - 1)(1-rx)}{1-rx-(1-x)f_A b_U}u \tag{A.1.5}$$

$$A = \frac{(1-r)x}{1-rx-(1-x)f_A b_U}\theta + \frac{(1-rx)}{1-rx-(1-x)f_A b_U}\eta - \frac{(1-x)(f_A f_U - 1)b_U}{1-rx-(1-x)f_A b_U}u \tag{A.1.6}$$

which shows that in equilibrium the $\tilde{y}$ and $\tilde{A}$ are normally distributed allowing for the conjectured belief coefficients. The belief coefficients are found to be

$$b_U = \frac{(1-r)^2 x^2 \sigma_\theta^4}{(1-r)^2 x^2 \sigma_\theta^4 + [(1-r)^2 x^2 \sigma_\theta^2 + (1-rx)^2 \sigma_\eta^2]\sigma_u^2} \tag{A.1.7}$$

and

$$f_A = \frac{(1-r)x(1-rx)\sigma_\theta^2}{(1-r)x[(1-r)x + (1-x)b_U]\sigma_\theta^2 + (1-rx)^2\sigma_\eta^2} \tag{A.1.8}$$

$$f_U = \frac{(1-x)b_U}{(1-rx)} \tag{A.1.9}$$

Now it is possible to specify the payoff functions of the informed and uninformed firms. The payoffs to an informed firm is

$$u_i^I(p;\theta,\eta,u) = -(1-r)\left(\frac{r(1-x)}{1-rx}\right)^2 (b_U y - \theta)^2 - r(1-2x)(1-x)\left(\frac{1-r}{1-rx}\right)^2 (b_U y - \theta)^2 - c_i \tag{A.1.10}$$

and the payoff to an uninformed firm is

$$u_i^U(p;\theta,\eta,u) = -(1-r)(b_U y-\theta)^2 - rx(2x-1)\left(\frac{1-r}{1-rx}\right)^2 (b_U y-\theta)^2 \quad \text{(A.1.11)}$$

It can easily be shown that

$$E(b_u\tilde{y}-\tilde{\theta})^2 = \frac{\{(1-r)^2x^2(1-rx)^2\sigma_\theta^4\sigma_\eta^2 + [(1-r)^2x^2\sigma_\theta^2 + (1-rx)^2\sigma_\eta^2]^2\sigma_u^2\}\,\sigma_\theta^2}{(1-r)^2x^2\sigma_\theta^4[(1-r)^2x^2\sigma_\theta^2 + (1-rx)^2\sigma_\eta^2] + [(1-r)^2x^2\sigma_\theta^2 + (1-rx)^2\sigma_\eta^2]^2\sigma_u^2}.$$

Stepping back to the information acquisition decision, letting c^* denote the information cost for the marginal informed firm, c^* then must satisfy

$$c^* = (1-r)\left[1 - \frac{r^2(1-x)^2}{(1-rx)^2} - \frac{(1-r)^2 r}{(1-rx)^2}(1-2x)\right] E[(b_U\tilde{y}-\tilde{\theta})^2|\sigma_u^2] \quad \text{(A.1.12)}$$

Notice that in equilibrium the fraction of informed agents, conditional on σ_u^2 is common knowledge. Finally, expected social welfare is given by

$$E(\tilde{W}) = -(1-r)\left\{x\left[\frac{r(1-x)}{1-rx}\right]^2 + 1 - x\right\} E(b_U\tilde{y}-\tilde{\theta})^2 - \int_0^{c^*} c_i g(c_i)dc_i \quad \text{(A.1.13)}$$

which the policy authority maximizes by choice of σ_u^2.

A.2 Proofs

A.2.1 Proof of Lemma 1

Consider the case of the marginal investing firm when $x = 0$. For the firm to be indifferent about acquiring information or not it must be that

$$\begin{aligned} c_i &= (1-r)^2(1+r^2)\sigma_\theta^2 \\ &\geq 0 \end{aligned}$$

Given the assumptions on $g(c_i)$ this means that the marginal firm will choose to become informed when $x = 0$ and so $c^* > 0$. From the indifference conditions

$$c^* = (1-r)\left[1 - \frac{r^2(1-x)^2}{(1-rx)^2} - \frac{(1-r)^2 r}{(1-rx)^2}(1-2x)\right] E[(b_U\tilde{y}-\tilde{\theta})^2|\sigma_u^2]$$

where the function on the right-hand side is continuous and starts above zero. Looking at the marginal firm and taking the derivative of the indifference equation with respect to c^*

$$\partial c^* = (1-r)g(c^*)\left[\frac{\partial \Xi}{\partial x}E(b_u\tilde{y}-\tilde{\theta})^2 + \Xi\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}\right]\partial c^*$$

where

$$\Xi = \frac{(1-r)[1+r^2(1-2x)]}{(1-rx)^2} > 0.$$

Evaluating the right-hand side at $\lim_{c^* \to \infty}$

$$\lim_{c^* \to \infty} RHS = g(c^*) \cdot Constant = 0.$$

As the slope of the left-hand side is constant and equal to one there is at least one crossing point. □

A.2.2 Proof of Proposition 1

This is direct given the result of lemma 1 and continuity of the expected social welfare in x. □

A.2.3 Proof of Lemma 2

Consider the indifference equation for the marginally informed firm

$$c^* = (1-r)\left[1 - \frac{r^2(1-x)^2}{(1-rx)^2} - \frac{(1-r)^2 r}{(1-rx)^2}(1-2x)\right] E[(b_U\tilde{y} - \tilde{\theta})^2 | \sigma_u^2].$$

The left-hand side plots the explicit cost of becoming informed. The right-hand side plots the net benefits of being informed. Notice that the direct cost has a slope of one as the cut-off cost is varied. Taking the derivative of the benefits with respect to policy noise, while holding the cut-off cost constant, the change in net benefits is given by

$$(1-r)\left[1 - \frac{r^2(1-x)^2}{(1-rx)^2} - \frac{r(1-r)^2}{(1-rx)^2}(1-2x)\right] \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2}$$

which is positive. Thus as policy noise increases, holding the fraction of informed firms constant, the guess of any firm concerning θ is worse on average. This yields an incentive to become informed.

Now consider the change in the net benefits as the cut-off cost is varied, holding the level of policy noise constant. The derivative of the net benefits with respect to the cut-off cost yields

$$\begin{aligned}\frac{\partial \text{Net Benefits}}{\partial c^*} &= (1-r)\left\{\left[\frac{1 - r(1-x) + r^2(1-2x)}{(1-rx)^3}\right] E(b_u\tilde{y} - \tilde{\theta})^2 + \right. \\ &\left. \left[\frac{(1-r)[1+r^2(1-2x)]}{(1-rx)^2}\right] \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial x}\right\} g(c^*)\end{aligned}$$

The first term on the right-hand side is the change in benefits attributed to the strategic complementarity component to payoffs. As more firms are informed, the uninformed firms receive less expected benefits from forecasting the prices set by other firms because they don't know what price the informed firms will set as informed prices are dependent on the realization of θ. In contrast, informed firms are better off because they set prices as a function of the realization of θ and the prices set by uninformed firms. The lower the fraction of uninformed firms, the closer is the price of informed firms to θ. In other words, on average, the strategic complementarity in payoffs pulls the optimal price of informed firms further away from θ when a larger fraction of firms are uninformed. Thus the larger the fraction of uninformed firms, the more incentive the strategic complementarity gives firms to remain uninformed. It can then be easily shown that

$$\frac{\partial c^*}{\partial \sigma_u^2} = \frac{\overbrace{(1-r)\Xi\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial \sigma_u^2}}^{(+)}}{1-(1-r)\left\{\underbrace{\left[\frac{1-r(1-x)+r^2(1-2x)}{(1-rx)^3}\right]E(b_u\tilde{y}-\tilde{\theta})^2}_{(+)}+\underbrace{\Xi\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}}_{(-)}\right\}g(c^*)} \tag{A.2.14}$$

For the cut-off to decrease as policy noise increases it is necessary for the second term in the denominator to be overwhelmingly large. That is, the increase in the fraction of uninformed firms from a fall in the cut-off cost must be so large that the gains from guessing the same price as all uninformed firms makes up for the expected deviation of the price from θ. Notice for this case to hold the net benefits from an increase in c^* must exceed unity. Thus the case in which $\frac{\partial c^*}{\partial \sigma_u^2} < 0$ holds at the set of unstable equilibria. □

A.2.4 Proof of Proposition 2

Totally differentiate the indifference equation for the marginal informed firm to obtain $\frac{\partial c^*}{\partial \sigma_u^2}$. Substitute into equation (2.2.18) to obtain

$$\begin{aligned}\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} &= -(1-r)\left\{\left[\frac{r(1-r)[(1-r)(1-rx)-2x(1-r^2x)]}{(1-rx)^3}\right]g(c^*)E(b_u\tilde{y}-\tilde{\theta})^2\frac{\partial c^*}{\partial \sigma_u^2}\right\}\\ &\quad -(1-r)\left[x\left(\frac{r(1-x)}{1-rx}\right)^2+1-x\right]\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial \sigma_u^2}\\ &\quad \cdot\left\{\frac{1-(1-r)\left[\frac{2r(1-r)[(1-r)-r^2(1-x)]}{(1-rx)^3}\right]E(b_u\tilde{y}-\tilde{\theta})^2 g(c^*)}{1-(1-r)\left[\frac{2r(1-r)[(1-r)-r^2(1-x)]}{(1-rx)^3}E(b_u\tilde{y}-\tilde{\theta})^2+\Xi\frac{\partial E(b_u\tilde{y}-\tilde{\theta})^2}{\partial x}\right]g(c^*)}\right\}.\end{aligned}$$

The result is direct. □

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