Optimal Survey Design for Skewness Measurements in Weak Gravitational Lensing

by

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Abstract

The properties and structure of the Universe we live in has been the subject of many studies since human kind started to watch the sky. Today, with rapid advancements in technology, cosmology has become a science where theories are rigorously tested against the evidence of observation. Many attempts have been taken in order to find ways to explain the behavior and nature of the Universe as a whole. Already we know that only a small fraction of our Universe is made up of the baryonic matter and the rest comprises an unknown “dark matter” component and an unknown “dark energy” component that drives the accelerating expansion of the Universe. The exact fraction of each of these components drives the fate of the Universe. More importantly, in order to understand the nature of these components, we need to perform very accurate measurements before the theoretical models can be ruled out. Ultimately cosmologists believe that understanding dark matter and dark energy will help to understand some of the most fundamental questions in modern physics.

We can infer the existence of dark matter by observing its gravitational effect on the distant galaxies that we see. Computational simulations confirmed by observations, suggest that dark matter collects into long filament structures, clusters and sheets. The best method to detect the dark matter is to investigate its gravitational effects on the light of distant galaxies traveling in the Universe along the line of sight, since this effect is totally independent of the nature of dark matter. Due to the strong gravitational field of such massive body, the light rays are deflected and typically travel through a zigzag path rather than a straight line. The resulting images of the distant galaxies are magnified, and distorted. By studying these sheared and distorted images we can reconstruct the distribution of the mass which caused it. Gravitational Lensing is the only method which probes the total mass distribution of the Universe including the dark matter directly. Weak Gravitational Lensing deals with small image distortions. In order to get more accurate results deeper and wider surveys are needed.

The detection of weak gravitational lensing by large-scale structure enables us to set constraints on the cosmological parameters such as amplitude of the matter power spectrum and the matter density parameter. Measuring third-order statistics such as the skewness allows us to break the degeneracies that exist between these two cosmological parameters when they are determined from two-point statistics. In this thesis, we ask what type of survey should be performed for an optimal constraint of the third-order statistics of weak gravitational lensing. We use an extensive set of cosmological lensing simulations and determine the signal-to-noise ratio of different third-order statistics for ground and space-based surveys of various depths. We conclude with the prospects of measuring the skewness with existing and forthcoming surveys.
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5.14 The comparison between survey size. The solid lines show the signal to noise ratio for the survey area of 46 deg$^2$ and the dotted line shows the same for survey area of 184 deg$^2$ which is roughly the size of CFHTLS wide field. For these plots, the statistical noise as well as cosmic variance are taken into the account for ground based survey with limiting magnitude of 24. (a), (c) and (e) are measured with tophat smoothing filter while (b), (d) and (f) are measured aperture filter.
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1. Introduction

Cosmology is the study of the Universe as a whole, its content, origin, evolution and fate. According to the standard model of cosmology and confirmed by observations, the content of the Universe includes baryonic matter, dark matter and dark energy. Approximately 4% of the Universe content is the baryonic matter; 21% is dark matter and the rest 75% is made of dark energy which is responsible for the accelerated expansion of the Universe [41]. The properties of the standard cosmological model are expressed in terms of various cosmological parameters. Constraining these parameters and at the end obtaining their most probable values will describe the fundamental physics governing the Universe.

It is assumed that matter in the Universe started as small Gaussian fluctuations, and as the Universe evolves they grow to form the large scale structures. These small density fluctuations are well described by the matter power spectrum, of which the amplitude normalization is not yet well constrained. Whenever we study how, and where and when the structures form, we are dealing with the matter density fluctuations and its power spectrum. Whenever we are dealing with the matter power spectrum, inevitably we have to deal with its normalization. Since the matter density fluctuations appear in every probe of the Universe, and as a result the normalization of its power spectrum enters into all cosmological studies as a degenerate parameter.

The physical properties of the Universe are different for different values of the matter density fluctuations and its normalization. For example to understand the structure formation we need to know the matter density, since a Universe with higher matter density grows clusters at later times in its history compared to a case where the matter density is lower. Or we can learn about the nature of dark matter, how it clusters and evolves. Larger values of the normalization imply higher levels of clumpiness in the structure. Finally by tightly constraining the normalization we get tighter constraints on the matter density which will push constraints on the cosmological constant, and as a result the nature of dark energy which is one of the most interesting and least known components of our Universe.

In cosmology, we can only constraint the cosmological parameters by observations. Fortunately different observations probe different physics and quantities. Each of them results in some degenerate constraints on some of the cosmological parameters. By combining the surveys we can have tighter constraints or even break the degeneracy between them. Figure 1.1(a) shows the constraints between the matter density parameter $\Omega_m$ and the normalization of the amplitude of the power spectrum $\sigma_8$ when the WMAP and weak lensing results are combined. The WMAP and weak lensing each provide different degenerate constraints on matter density and matter power spectrum normalization. When combined together, a tighter constraint is generated. Figure 1.1(b) shows the dark energy density and matter density probability contours from WMAP, supernovae and large scale structure studies. Again combining different probes, helps to develop a tighter constraint. Supernovae probes provide constraints on the dark energy and matter content in the Universe as well. Figure 1.2 shows the constraints provided by first year WMAP, 2dfGRS large scale structure study and supernovae. The large scale structure constraints on $\Omega_m$ are almost independent from the dark energy equation of state.
Figure 1.1: (a) The cosmological constraints from Cosmic Microwave Background (WMAP3) and weak lensing (from CFHTLS) for the power law ΛCDM model. [41]. (b) 95% constraints in the (Ω_m,Ω_Λ) plane [44].

Figure 1.2: Constraints on Dark energy properties. The marginalized maximum likelihood surface for the first year WMAP (CMB probe), and WMAP + 2dFGRS (Large scale structure probe), overlapped with the Supernovae constraints. [42].

parameter, but it again is degenerate with σ_8. So tightly constraining the value of σ_8, provides a non-degenerate Ω_m measurement which then combined with supernovae and CMB, generated a tighter constraint on the dark energy equation of state parameter which is very favorable. Al-
though the normalization of the matter density power spectrum is not a fundamental physics parameter, in addition to the shape of the power spectrum, it describes the matter density fluctuations. The matter distribution in the Universe is clumpier when $\sigma_8$ is larger, and it is less clumpy and more smoothly distributed when $\sigma_8$ is smaller. Various methods on different data sets are used in order to find strong constraints on the amplitude of the power spectrum normalization. Large scale structures studies, cosmic microwave background anisotropies and weak gravitational lensing separately and combined were used for this matter. The results so far have not been entirely consistent with each other. Table 1.1 shows some of the results for the normalization labeled as $\sigma_8$ by several groups. Mapping the matter distribution in the Universe has always been a challenge in cosmology.

One of the methods used is weak gravitational lensing. Mass inhomogeneities in the Universe causes light rays to deflect. Due to the gravitational lensing effect by large scale structure, the image of distant background galaxies becomes distorted. This effect is called the cosmic shear. Figure 1.3 shows the light propagation throughout the large scale structure of the Universe. As seen in the left hand side of the figure this path is not a straight line, but the light bundles are bent when passing by massive structures. On the right hand figure 1.3 the resulting distorted image from background galaxies is shown.

![Figure 1.3](image1.png)

Figure 1.3: (Left) The large scale structure of the Universe effect the propagation of light rays coming from the background galaxies. (Right) The bent light coming through the large scale structure causes distorted images of the background galaxies. The 2 dimensional projections are observed by the observer. From S.Colombi (IAP).

Weak gravitational lensing is superior to the other methods measuring $\sigma_8$ because it is sensitive to the total mass (irrespective to its nature) and requires a modest amount of modeling to be understood. The advantage of lensing in particular, is its independence regarding the state and nature of matter, so it probes the total matter regardless of its dynamical state.

Cosmic shear is a tool for measuring the mass distribution and the cosmological parameters in the Universe. It is a measure of the alignments of the lensed galaxies over large angular distanced on the sky. By measuring the amplitude of the cosmic shear as a function of scale, one can directly calculate the projected mass power spectrum, convolved with a selection function which is only dependent on the cosmological parameters and the redshift distribution of the sources and lenses.

In this thesis, we explain how weak lensing can provide better measurements for the matter
Analysis

Clusters:
Voevodkin and Vikhlinin 2004
Bahcall and Bode 2003, z < 0.2
Bahcall and Bode 2003, z > 0.5
Pierpaoli et al. 2002
Allen et al. 2003
Schuecker et al. 2002
Viana et al. 2002
Seljak 2002
Reiprich and Bohringer 2002
Borgani et al. 2001
Pierpaoli et al. 2001

Measurements

\[ \sigma_8 = 0.60 + 0.28 \Omega_m^{0.5} \pm 0.04 \]
\[ \sigma_8(\Omega_m/0.3)^{0.60} = 0.68 \pm 0.06 \]
\[ \sigma_8(\Omega_m/0.3)^{0.14} = 0.92 \pm 0.09 \]
\[ \sigma_8 = 0.77_{-0.04}^{+0.05} \]
\[ \sigma_8(\Omega_m/0.3)^{0.28} = 0.69 \pm 0.04 \]
\[ \sigma_8 = 0.771_{-0.031}^{+0.039} \]
\[ \sigma_8 = 0.78_{-0.03}^{+0.15} \text{ (for } \Omega_m=0.35) \]
\[ \sigma_8(\Omega_m/0.3)^{0.44} = 0.77 \pm 0.07 \]
\[ \sigma_8 = 0.96_{-0.12}^{+0.15} \]
\[ \sigma_8 = 0.66 \pm 0.06 \]
\[ \sigma_8(\Omega_m/0.3)^{0.60} = 1.02_{-0.076}^{+0.070} \]

Weak Lensing
Hoekstra et al. 2005
Heymans et al. 2005
van Waerbeke et al. 2005
Heymans et al. 2003
Jarvis et al. 2002
Brown et al. 2002
Hoekstra et al. 2002
Refregier et al. 2002
Bacon et al. 2002
van Waerbeke et al. 2002
Hamana et al. 2002

\[ \sigma_8 = 0.85 \pm 0.054 \]
\[ \sigma_8 = 0.68 \pm 0.13 \]
\[ \sigma_8 = 0.83 \pm 0.07 \]
\[ \sigma_8(\Omega_m/0.3)^{0.6} = 0.67 \pm 0.10 \]
\[ \sigma_8(\Omega_m/0.3)^{0.57} = 0.71_{-0.08}^{+0.06} \]
\[ \sigma_8(\Omega_m/0.3)^{0.56} = 0.74 \pm 0.09 \]
\[ \sigma_8(\Omega_m/0.3)^{0.52} = 0.86_{-0.07}^{+0.05} \]
\[ \sigma_8(\Omega_m/0.3)^{0.44} = 0.94_{-0.24}^{+0.24} \]
\[ \sigma_8(\Omega_m/0.3)^{0.68} = 0.97 \pm 0.13 \]
\[ \sigma_8(\Omega_m/0.3)^{0.24_{-0.18}^{+0.18}} = 0.94_{-0.12}^{+0.14} \]
\[ \sigma_8(\Omega_m/0.3)^{-0.37} = 0.78_{-0.27}^{+0.27} \]

Table 1.1: The recent constraints in the (\(\Omega_m, \sigma_8\)) plane.

density parameter, independent of the normalizations of the amplitude of the power spectrum. Then we suggest an optimal survey design for such measurements, and then the prediction on CFHTLS data.
2. Weak Gravitational Lensing by Large Scale Structure

Figure 2.1: The effect of weak lensing by large scale structure [32].

2.1 Cosmic Shear and Convergence

Weak gravitational lensing provides us with a direct probe for both baryonic and non-baryonic matter distribution in the Universe. This method is independent of the dynamical state of the matter which makes it a unique tool for mapping the matter in the Universe and measuring the cosmological parameters. Although it has been discovered in 1987 [40] only, we now know that weak lensing is very common and happens all over the sky. Each single photon along its path, would be deflected by the inhomogeneous matter distribution in the Universe. The angular size, the apparent position, the shape and the apparent magnitude of the lensed distant object are modified by any inhomogeneous matter distribution along the path of the photons emitted by the source. However since we do not know the exact position of the source objects, it is not normally possible to detect the position change. Only the distortion is detectable and in some cases the magnitude changes as well. The distortion is small, and not detectable on individual galaxy images, so statistical measurements should be applied. The reason is that the weak lensing signal has an amplitude of 1%, which is very small compared to the intrinsic dispersion of ellipticity distribution of the galaxies. To detect such small signal, large survey areas are needed in order to collect the signal statistically. In this thesis, we assume that the systematics including the PSF distortion (the telescope point spread function) and atmospheric smearing (for ground based surveys) are properly controlled. Since the lensing effect is of order of 1% on the shear field, the systematics should be very well understood, because they could create
Figure 2.2: The effect of shear and convergence in the image of a circular object. The convergence causes magnification and the shear stretches it.

A spurious alignment of distant galaxies that could be mistaken as a lensing signal. Note that on small scales, non linear structures dominate which makes the lensing signal stronger.

The deflected light causes the shape of distant galaxies to be amplified and modified. The magnification effect provides us with a correlation between density of the foreground lenses and the apparent luminosity of the distant sources, known as magnification bias. Figure 2.2 shows the rough effect of shear and magnification on a circular source. The amplification bias is also not included here, although it is potentially a powerful probe of dark matter distribution. A recent measurements by Scranton et al. (2006) [37] shows that magnification can now be detected.

In the following, I briefly establish the connection between the shear and the projected mass distribution needed in this work. The Poisson's equation described the Newtonian gravitational potential:

\[ \nabla^2 \Phi = -4\pi \rho \]  

where \( \rho \) is the total matter density. The projected gravitational potential of a lens, however is:

\[ \varphi(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(D_L \vec{\theta}, z) dz \]  

Figure 2.3 shows how the angular diameter distances, \( D_S, D_L, D_{LS} \) and the position angle \( \theta \) are defined.

Distortion produces a correlation between ellipticity distributions of lensed galaxies, which is known as cosmic shear. The shapes of the galaxies are characterized by the surface brightness second moments:

\[ M_{ij} = \frac{\int I(\theta) \theta_i \theta_j d^2 \theta}{\int I(\theta) d^2 \theta} \]  

If the intrinsic galaxy ellipticity is \( e \), due to gravitational lensing the observed ellipticities would be \( e + \delta \), where \( \delta \) is called the gravitational distortion. The relation between the shear and ellipticity is given by:

\[ \delta = 2\gamma \frac{(1 - \kappa)}{(1 - \kappa)^2 + |\gamma|^2} = \left( \delta_1 : \frac{M_{11} - M_{22}}{Tr(M)} ; \delta_2 : \frac{2M_{12}}{Tr(M)} \right) \]  

\( \kappa \) is the projected mass known as the gravitational convergence and \( \gamma \) is the shear, which both depend on the second derivatives of the projected gravitational potential \( \varphi \):
Figure 2.3: The basic thin lens setting. $S$ is the background source, $S_1$ and $S_2$ are the observed imaged of the source due to light deflection by the lens $L$, between the observer $O$ and the source $S$. The distance between the observer and the source is $D_S$, the distance between the observer and the lens is $D_L$, and the distance between lens and source is $D_{LS}$. $\alpha$ is the deflection angle, $\alpha$ is the reduced deflection angle, $\beta$ is the observed angular position of the source and $\theta$ is the source original location. In the small angle limit, $\alpha$ and $\alpha$ are equal.
Figure 2.4 shows the geometry of shear and ellipticity. A positive shear component $\gamma_1$ corresponds to an elongation along the $x$ axis. Shear component $\gamma_1$ when negative, corresponds to a compression along $x$ axis, while positive or negative values for shear component $\gamma_2$ is correspondent to elongation or compression along the $x=y$ axis. The ellipticity components, $\delta_1$ and $\delta_2$ are defined the same way as the shear. Note that the ellipticity of a circular object is set to be zero.

Weak lensing occurs when $\kappa \ll 1$, $|\gamma| \ll 1$ and $\bar{\delta} \approx 2\bar{\gamma}$, so the observed ellipticity can be written as:

$$\bar{\varepsilon}_{\text{obs}} = \bar{\varepsilon} + \bar{\delta} = \bar{\varepsilon} + 2\bar{\gamma}$$

We can quantitatively reconstruct the surface mass distribution of a cluster lens using Kaiser & Squires [25] method: We know that the convergence $\kappa(\theta)$ and the two components of the shear $\gamma_1$ and $\gamma_2$ are linear combinations of the second derivative of the effective lensing potential $\varphi(\theta)$. The method connects the two with a mathematical relation. When $\gamma_1$ and $\gamma_2$ are estimated by measuring the distortions on the back ground galaxy images, using this

\[\kappa(\theta) = \frac{1}{2} (\varphi_{,11} + \varphi_{,22}) \quad (2.5)\]
\[\gamma_1(\theta) = \frac{1}{2} (\varphi_{,11} - \varphi_{,22}) \quad (2.6)\]
\[\gamma_2(\theta) = \varphi_{,12} \quad (2.7)\]

We use the following notation to then simplify the partial derivations of $\varphi$ so that:

$$\frac{\partial^2 \varphi}{\partial \theta_i \partial \theta_j} = \varphi_{,ij} \quad (2.8)$$
mathematical relation one can determine the convergence $\kappa$.

First step is to Fourier transform the expressions for the convergence and shear:

$$\hat{\kappa}(\vec{k}) = -\frac{1}{2}(k_1^2 + k_2^2)\hat{\varphi}(\vec{k})$$  \hspace{1cm} (2.10)

$$\hat{\gamma}_1(\vec{k}) = -\frac{1}{2}(k_1^2 - k_2^2)\hat{\varphi}(\vec{k})$$  \hspace{1cm} (2.11)

$$\hat{\gamma}_2(\vec{k}) = -k_1^2 k_2^2 \hat{\varphi}(\vec{k})$$  \hspace{1cm} (2.12)

where $k$ is the two dimensional wave vector conjugate to $\theta$. The linear relation between the transformed components $\hat{\kappa}$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$ can be written as:

$$\begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} = k^{-2} \begin{pmatrix} \left((k_1^2 - k_2^2) \right) \\ 2k_1 k_2 \end{pmatrix} \hat{\kappa}$$  \hspace{1cm} (2.13)

$$\hat{\kappa} = k^{-2} \left[(k_1^2 - k_2^2), (2k_1 k_2)\right] \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix}$$  \hspace{1cm} (2.14)

By solving for $\hat{\kappa}$, we can inverse Fourier transform and get an estimate for the convergence value $\kappa$. In particular, an observed shear map can be turned into a convergence (projected mass) map.

The deflected light is observed at an angular position $\vec{\beta}$ on the sky, while its original location is $\vec{\theta}$. See figure 2.3 for the illustration. The two positions (the intrinsic and the observed) are related to each other by:

$$A = \frac{\partial \beta_i}{\partial \theta_i} = \begin{pmatrix} 1 - \kappa - \gamma_1 & \gamma_2 \\ \gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$  \hspace{1cm} (2.15)

$A$ is called the magnification matrix.

Figure 2.5 shows the shear field overlapped on a reconstructed projected mass map. One can clearly see the high correlation between the shear and mass clumps.

The cosmic shear signal depends on cosmological matter density $\Omega_m$, the mass power spectrum normalizations $\sigma_8$ and the source redshift $z_s$ primarily. As seen in figure 2.6 the mass power spectrum starts to behave non-linearly at scales, $\log(k/h^{-1}\text{Mpc})$ around $-0.4$ and larger ($k/h^{-1}\text{Mpc}$ around 0.4 and larger). On the other hand the weak lensing sensitivity peaks at about 10 arc minutes on the sky, which is correspondent to structure size of $k/h\text{Mpc}^{-1}$ around 1, since that is the scale of the galaxy clusters. This scale however, falls in the non linear regime of the power spectrum (the $k/h^{-1}\text{Mpc}$ larger than 0.4). This means that the scales probes by weak lensing are within the non linear part of the power spectrum. We can obtain constraints on these parameters by comparing the lensing signal with the non-linear predictions. The non-linear scales correspond to galaxy clusters and measurements of their abundance yields a robust measure of the power near this scale for a given matter density $\Omega_m$. The surveys are not yet big enough to probe the linear regime, however the power in non linear regime is computable numerically.
2.2 The Limitation of the Observed Mass Power Spectrum

As predicted by inflation, the primordial density fluctuations are assumed to be Gaussian, and so can be completely characterized by their power spectrum. Figure 2.7 shows the density fluctuations as a function of scale using different probes. As seen, gravitational lensing, cosmic microwave background, cluster abundance, galaxy clustering and intergalactic gas clumping probe different scales, and together they show the density fluctuations for all scales.

Historically, cosmological constraints from cosmic shear came first from 2-point statistics, because they are computationally affordable compared to higher-order statistics and the survey size was a limitation for higher-order statistics. The variance (as the 2-point statistics) of shear was measured and suggested a cosmological origin for the signal for various reasons. First, there was consistency in measurements done by several groups with different telescopes, cameras, depths and filters. Second, by using different lensing statistics, power spectrum or correlation function the results were consistent. The third reason lies on the fact that after correcting for the B modes (induced by intrinsic ellipticities) from lensed stars, the signal contained only E modes (produced by gravity induced distortion), which is as expected. Lastly it satisfied the expectation of a lower lensing signal at lower redshift due to less lensing effect. All the above reasons confirm the existence of a real signal in the observations and make 2-point statistics have
a very important role in weak lensing studies. So many lensing groups measured this variance on several data sets, including the corrections for systematics. When it comes to constraints on cosmological parameters though, 2-point statistics offers nothing but a degeneracy between \( \Omega_m \) and \( \sigma_8 \) which causes a significant discrimination between the different cosmological models. Depending on the value of \( \sigma_8 \) the shear variance measurements can be interpreted differently. For low signal to noise ratio data, measuring variance and power spectra is best to be done with two point correlation function.

One can show that the measured shear power spectrum is identical to the convergence (projected mass) power spectrum \( P_\kappa(k) \). \( P_\kappa(k) \) is the projected power spectrum and is related to the 3-dimensional power spectrum \( P_{3D} \) by the following [36]:

\[
P_\kappa(k) = \frac{9}{4} \Omega_b^2 \int_0^{w_H} \frac{dw}{\alpha^2(w)} P_{3D} \left( \frac{k}{f_K(w)}; w \right) \left[ \int_w^{w_H} dw' n(w') \frac{f_K(w') - f_K(w)}{f_K(w')} \right]^2
\]  

(2.16)

where \( w_H \) is the horizon distance, \( k \) is the 2-dimensional wave vector perpendicular to the line of sight, \( f_K(w) \) is the comoving angular diameter distance out to a distance \( w \), \( n(w(z)) \) is the redshift distribution of the sources and \( a(w) \) is the scale factor.

The power spectrum is characterized by its shape and normalization \( \sigma_8 \), which is by definition the fluctuation r.m.s. in a tophat sphere of radius \( 8 h^{-1}\text{Mpc} \). As shown in table 1.1, different methods used to measure \( \sigma_8 \) do not totally agree. The shape of the power spectrum has been confirmed for the linear regime, and is still unknown by \( \sim 5\% \) for the non linear regime [39].

Figure 2.6: The convergence power spectrum from equation 2.16. The linear (solid) and non-linear (dashed) power spectrum for \( \Omega_m = 0.3, \Omega_\Lambda = 0.7, z_{\text{source}} = 0.8 \) and \( h = 0.5 \) normalized to the cluster abundance of \( \sigma_8 = 0.9 \). The non linear part of the power spectrum increases for larger \( k \) scales, [4].

Normalization by Cosmic Microwave Background (CMB) anisotropies: ([2]) The fluctua-
tions in the temperature of the microwave background can be translated into the amplitude of the power spectrum. These measurements are done on large angular scales, at $z = 0$, so such method is valid for large physical scales (small $k$) only. The other contamination raises from the fact the CMB fluctuations measure the amplitude of both scalar and tensor perturbation modes, where the density fluctuations resulting in growth is only originating from scale modes. $\sigma_8$ is degenerate with $\tau$ (the reionization optical depth) and $\Omega_m$. See figure 1.1

Normalization by the local variance of the galaxy counts: ([3], [13], [59] and [24]) This method is based on the idea that galaxies are unbiased tracers of the underlying dark matter fluctuations. So if we can measure the local variance of galaxy counts within a fixed volume as well as setting an expression for the bias, we can fix the normalization of the density power spectrum. This fixed volume has been decided to be a sphere with radius of $8h^{-1}$Mpc, being the origin of the naming of the normalization as $\sigma_8$. The main problem is the exact expression we choose for the bias.

Normalization by the local abundance of galaxy clusters: ([60]) Assuming that galaxy clusters form as a result of dark matter density perturbations, we can use the spatial cluster number density to determine the amplitude of the power spectrum. The point in this method is that the cluster normalization can only determine the amplitude of the power spectrum at scales of order of $10h^{-1}$Mpc, which is the typical dark matter fluctuation scale for the galaxy clusters to collapse. When dealing with gravitational lensing by large scale structure, the scale sensitivity is around $k_0^{-1} \sim 12(\Omega_0 h^2)$Mpc, which makes the galaxy cluster normalization method favorable. The main problem with this method however, arises from the mass calibration which uses X-ray mass-temperature relation, which is poorly known.

On one hand, we know that due to theoretical and observational limitations, $\sigma_8$ is not known better than few percent [39] and [28]. On the other hand the shape of the power spectrum which is determined by the exact matter content in the Universe (baryons, neutrinos, dark matter and any other massive component), is not known better than few percent either [41]. For the shape of power spectrum we use a generic shape parameter $\Gamma = \Omega_m h$ from (BBKS) [3]. For $\sigma_8$ however, there is a degeneracy with $\Omega_m$. Our goal here is to explore the use of skewness to break this degeneracy and first measure the $\Omega_m$ independent of $\sigma_8$ and shape, and then by combining the result with other measurements, measure $\sigma_8$ more accurately.

The motivation of this thesis is to develop an optimal survey design, with which the skewness of the convergence can be measured accurately, and provide us with measuring $\Omega_m$ independent of $\sigma_8$. When constraining $\Omega_m$ independently, we can break the degeneracy that currently exists between $\Omega_m$ and $\sigma_8$ and obtain a tighter constraint on normalization of matter power spectrum and as a result tighter constraints on dark matter and dark energy densities.
Figure 2.7: The new SDSS results (black dots) are the most accurate measurements to date of how the density of the Universe fluctuates from place to place on scales of millions of light years. The larger the scales we average over, the more uniform the Universe appears [43].
3. Skewness

3.1 2-point and 3-point Statistics

The cosmic shear signal depends on the cosmological mean density \( \Omega_m \), the normalization of the amplitude of the mass power spectrum \( \sigma_8 \), the source redshift \( z_s \), and the shape of the power spectrum \( \Gamma \). As pointed out in §2.2 \( \Gamma \) combines the effect of matter content of all types, and the physical state (i.e. cooling, pressure and gravity) and is not a cosmological parameter. As a result any statistical measurements will provide some constraints on these parameters. Figure 3.1 shows the joint constraint on matter density and \( \sigma_8 \), when the lensing signal was compared to the non-linear predictions. The survey size here is not big enough (8.5 deg\(^2\)) to probe the linear scales accurately.

As explained in §2.2 measuring 2-point statistics is much easier since it only requires the two point correlation function measurements which are computationally cheap. However, we need higher order statistics in order to break this degeneracy, since the study of power spectrum is not sufficient to obtain knowledge about non linear features.

Skewness cannot be measured on the shear data directly, and mass reconstruction (using KSB method, explained in §2.1) is needed. Mass reconstructed \( \kappa \) maps are generated by Fourier transforms, and in Fourier space 3-point calculations are much faster and more feasible compared to shear maps. Mass reconstruction however, is very sensitive to the geometry of the survey, since the projected mass \( \kappa \) reconstruction is a non linear process. Map making is a local process and needs to be done with very high accuracy to result in cosmic shear measurements. One way to avoid mass reconstruction for surveys without complicated geometry is to measure the third moments with an aperture mass filter.

Measuring the third moment of the shear field is unfortunately not trivial, since all odd moments of the components of the vector field result in zero. Bernardeau et al (2002) [8] used three point correlation function, and Pen et al (2002) [30] computed the aperture mass 3 point function from an integral of shear 3 point function to measure the third moment of the shear field. Both methods were used on VIRMOS-DESCART data, due to high noise level, no cosmological constraints could be extracted from the results.

The theory behind skewness measurements was developed by Bernardeau, van Waerbeke and Mellier (1997) [10]. The efficiency of such measurements were studied by van Waerbeke et al. (1999) [49] and the first simulations were generate by Jain, Seljak and White (2000) [22] and White and Hu (2000) [58], which confirmed the feasibility of detecting the skewness signal. The initial detection has been reported by Bernardeau van Waerbeke and Mellier (2002) [8]. Later the skewness was measured by Pen et al (2003) [31], on VIRMOS DESCART survey which had a 5 \( \sigma \) signal, but couldn’t provide much constraint on \( \Omega_m \), because the data was limited by sample variance and analysis techniques. Pen et al (2003) [31], used the VIRMOS-DESCART data (VIRMOS-DESCART data contains four uncorrelated patches, each 4 deg\(^2\)) to measure the first dark matter skewness.

The results from this study shows \( \kappa^2 = (5.32 \pm 0.62 \pm 0.98) \times 10^{-5} \), \( \kappa^3 = (1.06 \pm 0.06) \times 10^{-6} \) and as a result \( S_3 = 375_{-124}^{+342} \). By calibrating to mock catalogs from N-body simulations, Pen
Figure 3.1: The constraints on the matter density $\Omega_m$ and $\sigma_8$ from 2-point statistics studies for flat cosmology. The confidence intervals are 68\%, 95\% and 99.9\%. There is degeneracy between the $\Omega_m$ and $\sigma_8$ [52].

et al (2003) [31] found $\Omega_0 < 0.5$ at 90\% and $\Omega_0 < 0.25$ at 80\% confidence. Looking at smaller scales will provide better constraints since at these scales the sample variance is smaller, but this requires higher resolution simulations and faster three-point analysis. We need a detailed analysis to forecast $S_3$ measurements as well as an optimal way to perform such a detailed analysis.

3.2 Skewness from Second Order Perturbation

At large scales (larger than 10 $h^{-1}$Mpc) the amplitude of the r.m.s. fluctuation observed in the cosmic fields are below unity which means that these cosmic fields are still undergoing formation. Matter density fluctuations in the Universe start Gaussian, and as the Universe evolves and structures form, the overdensities grow, while the underdensities can not get smaller. This means that the evolving fluctuations do not remain Gaussian, leading to higher-order statistics in the distribution which can be observed by using the second order perturbation theory. See Peebles (1980) [29].

At scales below the transition we expect the skewness to become significant. When using the second order statistics, only the power spectrum is probed. However non-linear structure evolution causes the non-Gaussian features in the density field, which can only be probed with the higher order statistics.

One of the main motivations to use higher-order statistics is to break the degeneracy between $\Omega_m$ and $\sigma_8$ which the second order statistics measurements contain. We can write the second order perturbation theory for the growth of the density contrast for a density fields with scales on which the inhomogeneities are weakly non-linear. Thus we have: $\delta = \delta^{(1)} + \delta^{(2)} + \cdots$ where $\delta^{(1)}$ is the density contrast obtained from the linear perturbation theory and $\delta^{(2)}$ is the second order term which is quadratic in the linear density field, $\delta^{(2)} \propto (\delta^{(1)})^2$. The linear density fields $\delta$ is $\propto \sigma_8$ and the projected mass density $\kappa \propto \Omega_m \sigma_8$. In the linear regime, $\langle \kappa^2 \rangle \propto \Omega_m^2 \sigma_8^2$ where $\langle \kappa^2 \rangle$ shall denote any second order estimator. As the result, the lowest order contribution to
Figure 3.2: (Left panel) Top-hat, (Right Panel) compensated statistics for the $\Lambda$CDM model, from [50].
The third order statistics is:

$$\langle \kappa^3 \rangle \propto (\delta^{(1)})^2 \delta^{(2)} \propto \Omega_m^3 \sigma_8^4$$  \hspace{1cm} (3.1)

and we know that the term $$(\delta^{(1)})^3$$ has no contribution since we assumed the linear density field is Gaussian. Putting it all together we get:

$$S_3 = \frac{\langle \kappa^3 \rangle}{(\langle \kappa^2 \rangle)^2} \propto \frac{\Omega_m^2 \sigma_8^4}{(\Omega_m^2 \sigma_8^2)^2} \propto \frac{1}{\Omega_m}$$  \hspace{1cm} (3.2)

which is independent of the $\sigma_8$. In this thesis $S_3$ is referred to as Skewness from now on.

3.3 $S_3$ dependence on $z_s$, $\Omega_m$ and $\sigma_8$

The expression for $S_3$ comes from second-order perturbation theory as stated before. This quantity is independent of the normalization of the amplitude of the matter power spectrum which was shown previously. See [10] for detailed calculations. However in order to work with skewness one should notice that it has dependence on source redshift distribution as well as the matter density parameter. Using skewness to break the degeneracies between $\Omega_m$ and $\sigma_8$ would be only possible if we understand how does it depend on $\Omega_m$ and $z_s$. Bernardeau, van Waerbeke and Mellier (1997) [10] have studied the dependence of skewness on redshift as well as on $\Omega_m$. The variance of the smoothed convergence in an Einstein-de Sitter Universe is dependent on source redshift as:

$$\langle \kappa^2(\theta_0) \rangle \propto z_s^{1.5}$$  \hspace{1cm} (3.3)

This is assuming that all the sources all at the redshift $z_s$ and the shape of the power spectrum is power law and that the source redshift is 1.

The dependence of the variance of the smoothed convergence on the matter density param-
eter for $z_s = 1$ is:

$$\langle \kappa^2(\theta_0) \rangle \propto \Omega_m^{1.6} \quad (3.4)$$

They found that the dependence of skewness $S_3$ will then follow as: On redshift:

$$S_3(z_s) \approx -42z_s^{-1.35} \quad (3.5)$$

which has a noticeable dependence of source redshift.

On matter density parameter:

$$S_3(\Omega_0) \approx -42\Omega_m^{-0.8} \quad (3.6)$$

for source redshift $z_s \approx 1$ and less dependent on $\Omega_m$ for higher redshifts. It is as well shown that the dependence of $S_3$ on cosmological constant $\Lambda$ is very weak. So in total the skewness follows as:

$$S_3 \equiv \frac{\langle \kappa^3 \rangle}{\langle \kappa^2 \rangle^2} \propto \Omega_m^{-0.8}z_s^{-1.35} \quad (3.7)$$

There are two sources of noise which should be dealt with properly. One is the intrinsic ellipticity which adds to the measured shear, and the other one is the galaxy number density, which can be calculated from the limiting magnitude of the survey.
4. Skewness Measurement and Optimal Survey Design

4.1 Motivation to Optimize the Survey Design

In previous chapters I explained why skewness measurement is a great method to break the degeneracy which exists between $\Omega_m$ and $\sigma_8$. Several measurements have been done so far on VIRMOS-DESCART data; however, the low signal level is not sufficient to constraint the cosmological parameters. It is important to have a proper survey design with which the skewness measurements result in best signal to noise ratio.

The main motivation for this thesis is to determine what the optimal survey design is for such purpose considering the depth and area of the survey with a limited observation time and to compare the results to some of the present surveys such as CFHTLS wide and deep.

In a previous work done by Zhang et al. (2003) [61], they found that high order statistics are better measured with the compensated Gaussian filter at 2.5 arc minutes as a generic optimal smoothing scale. They did not take the galaxy redshift distribution into account and the depth and size of the survey was fixed. Figure 4.1 shows the measurement for different smoothing filters. The goal in [61] was to optimize the smoothing filter and smoothing scale.

Here however, the optimization was done differently. In this thesis we looked at different survey areas and depths as well and implementing the difference between a ground based survey and a space based survey. In addition to an optimal filtering scheme, we obtain an optimal survey design for high order statistics. Our results are realistic, because we populated the simulations with a galaxy shape noise determined by the galaxy number density determined from the Hubble deep field data. This chapter explains the methods and steps taken in this thesis.
Figure 4.1: Skewness measurements with four different smoothing filters as a function of smoothing scale, for different models. The top line (black) corresponds to $\Omega_m = 0.2$, the second line from top (red line) $\Omega_m = 0.3$ the third line from the top (green line) $\Omega_m = 0.4$, and the bottom line (blue line) corresponds to $\Omega_m = 1$. The smooth curves are the best fit to skewness for each model, [61].
4.2 Ray-Tracing Simulations

Ray-tracing simulations are widely used in weak lensing studies. They can be used to test the fundamental assumptions adopted in making analytical predictions, or to explore the higher-order statistics, or to examine the systematic effects, all to work out an optimal survey strategy. We used \( \kappa \) map ray-tracing simulations generated by Takashi Hamana [15], with 60 different lines of sight each containing 62 redshift slices from \( z = 0.05 \) to \( z = 3.04 \). These are generated from one set of N-body data by randomization. As result, the different lines of sight are not independent on large scales but since we are using the very large size (46.6 degree\(^2\)) they can be considered approximately independent on scales smaller than 1 degree. The simulations are done with Normal FORTRAN binary data of 1024\( \times \)1024 grids (DEC-alpha endian). The grid spacing is 0.41 arc minute, thus the total area is about 7\( \times \)7 deg\(^2\). For technical details see [15].

The cosmological model used in these simulations are \( \Lambda_{CDM} \) with \( \Omega_m = 0.3 \), \( \Omega_A = 0.7 \) and \( \sigma_8 = 0.9 \).

Figure 4.2: A sample of ray tracing simulations done by Takashi Hamana, which were used for this work. (left): Halo distribution in the N-body simulation. The large symbols are of \( M > 3 \times 10^{14} \), small symbols, \( 3^{14} > M > 8 \times 10^{13} \) and dots \( 8 \times 10^{13} > M > 1 \times 10^{13} \). (right): weak lensing mass map sample from the ray-tracing simulations for a \( \Lambda \)CDM model with \( \sigma_8 = 0.9 \). The \( \kappa \) values are shown on the color bar.

Figure 4.2 shows a sample \( \kappa \) map generated by Takashi Hamana. Here we used the 60 different lines of sight and each contained 62 redshift slices. The first step is to populate each slice with the corresponding galaxy number density and then stacking them together. Figure 4.3 shows the stacking. We stacked the noisy images which again resulted in a 1024 pixels \( \times \)
1024 pixels $\kappa$ map, and then measured the 2-point ($\langle \kappa^2 \rangle$) and 3-point ($\langle \kappa^3 \rangle$ and $S_3$) statistics with tophat and aperture mass smoothing filter. Figure 4.3 illustrates the stacking of different redshift slices. For this particular illustration the redshifts are 0.04, 0.11, 0.19, 0.35, 0.47, 0.87, 1.87 and 3.02. After adding ground based noise corresponding to limiting magnitude 25 and stacking the 60 slices, the resulting $\kappa$ map is ready for measurements. The PDF (probability density function) of this result is shown in figure 4.4. Next section explains the properties of different smoothing filters and the reason they are used.
Figure 4.3: The stacking of the 62 redshift slices of each line of sight into one final map. The slices here are of redshift 0.04, 0.11, 0.19, 0.35, 0.47, 0.87, 1.87 and 3.02.
Figure 4.4: The probability density function of $\kappa$ for the stacking 60 $\kappa$ maps of figure 4.3 for $\Lambda$CDM model, with $\Omega_m = 0.3$ and $\sigma_8 = 0.9$. The smoothing filter used is tophat at 8 arc minute scale.
4.3 Smoothing Filters

Smoothing has a very important role when dealing with noisy maps. By smoothing one finds an optimum smoothing scale on which the signal to noise ratio is maximum. This is a huge advantage since one can suppress the noise and signal enough so that the signal becomes more obvious. The filters normally used on $\kappa$ maps are the tophat filter, the Gaussian filter, the aperture filter and the compensated Gaussian filter. We use the Fourier transform of the filter on $\kappa$ maps, however the aperture mass and compensated Gaussian filters can be used on shear maps as well. Different filters have different spectral properties. A tophat and Gaussian filters are normalized to have a sum of unity in the two dimensional real space windows. The aperture however, defined in such way to have zero mean and the compensated Gaussian has zero area with a normalized peak of unity in Fourier space. The aperture filter is very much like a wavelet filter, which allows us to perform a Fourier analysis localized in wavelength.

Not filtering the maps corresponds to choosing a filter of scale infinity in the k space, which is equivalent to smoothing with scale zero in real space. This causes an infinite variance map which contains no useful information. Essentially, filtering avoids this problem. This makes the study to be only sensitive to very small physical scales in real space which is obviously not what is desirable by a weak lensing study.

Here in figure 4.5 we show the shape of all these filters in real space and then the power spectrum density of them in $k$ space. The power spectral density of a filter is the square of the two dimensional Fourier transform of it. As seen in the figure 4.5, the right column shows why aperture and compensated Gaussian filters are band pass and the tophat and Gaussian filters are low pass filters. Here we see the mathematical form of the filters in real space. This mathematical form was used to generate figure 4.5. The physical scale $\theta_f$ is chosen to be 10 for all the plots.

Tophat filter:

$$W(|\theta|) = \frac{1}{2\pi \theta_f^2} \text{ if } |\theta| < \sqrt{2\theta_f^2} \text{ and zero elsewhere}$$ (4.1)

Gaussian filter:

$$W(|\theta|) = \frac{1}{2\pi \theta_f^2} e^{-\frac{\theta^2}{2\theta_f^2}}$$ (4.2)

Compensated Gaussian filter:

$$W(\theta) = \frac{1}{2\pi \theta_f^2} \left(1 - \frac{2\theta^2}{\pi \theta_f^2}\right) e^{-\frac{\theta^2}{2\theta_f^2}}$$ (4.3)

Aperture filter:

$$W(\theta) = \frac{9}{\pi} \left(\frac{1}{\theta_f}\right)^2 \left(1 - \left(\frac{\theta}{\theta_f}\right)^2\right)^2 \left(\frac{1}{3} - \left(\frac{\theta}{\theta_f}\right)^2\right)$$ (4.4)
Figure 4.5: The smoothing filters in real space (left column) and the power spectral density corresponding to each filter (right column). The physical scale of $\theta_f$ is chosen to be 10 to generate these plots. (a) and (b): Tophat filter, (c) and (d): Gaussian filter, (e) and (f): Aperture filter, and (g) and (h): Compensated Gaussian filter.

As seen in the figure 4.5, an aperture filter and a tophat filter with the same physical size $\theta_f$ have different maximum $k$ values in the Fourier space. This ratio is around 1 to 5, which makes an aperture filter 5 times more sensitive to the smaller scales than a tophat filter. In the
results presented in this thesis, when aperture filter is used, we can only trust smoothing scales above 2, since with aperture we are probing scales around $2/5 = 0.4$ which is the resolution of our simulations and beyond which we can not interpret much.

4.3.1 Filter Cross Correlation Matrix

It is important to know how correlated different scales are when using different filters. If a filter shows strong correlation at for all scales, measurement can be done for one scale only, since additional scales do not add extra information. While if the filter is uncorrelated at different scales, one may want to do the measurements at various scales, since each will provide new information to the study. In figure 4.6 we calculate the correlation coefficient of the tophat and aperture filter. The elements of the matrices correspond to scales of 0.4, 0.8, 1.2, 1.6, 1.8, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 8.0, 12.0, 16.0, 20.0 and 40.0 arc minutes. These are measured on noisy $\kappa$ maps of limiting magnitude 24 for typical ground based survey. The diagonal elements are the self correlation and are marked as white corresponding to 1. As seen for all $\langle \kappa^2 \rangle$, $\langle \kappa^3 \rangle$ and $S_3$, the tophat filter shows much higher correlation between scales, while for the aperture filter, the off diagonal terms are much below 1. This shows that the using the aperture filter, measurements on different scales are less correlated with each other, which means they contain more information. While in case of tophat filter, the strong correlation between the scales causes the measurements on different scales have less new information.
Figure 4.6: The cross correlation coefficient matrix for the tophat (left column) and aperture (right column) filters. Each matrix element corresponds to one of the scales, 0.4, 0.8, 1.2, 1.6, 1.8, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 8.0, 12.0, 16.0, 20.0 and 40.0 arc minutes. The diagonal elements show the self correlations which are all equal to 1 coded with white color. (a) and (b) are the cross correlations for \( \langle \kappa^2 \rangle \), (c) and (d) for \( \langle \kappa^3 \rangle \) and (e) and (f) for S3 measurements of noisy ground based survey with limiting magnitude of 24.
4.4 Noise properties

4.4.1 Galaxy Shape Noise

To mimic the noise from the intrinsic ellipticity distribution of the galaxies, and shape measurement errors, we assume a width in the observed galaxy ellipticities of $\sigma_e = 0.44$ for ground-based observations and $\sigma_e = 0.36$ for space-based observations. We assume that $\sigma_e$ is constant as a function of galaxy redshift.

The noise due to the randomly oriented intrinsic ellipticities of source galaxies is modeled as Gaussian random fields with variance:

$$\sigma_{noise}^2 = \frac{\sigma_e^2}{2 \pi \theta_f^2 n(z)}$$

where the $\sigma_e$ is the r.m.s. amplitude of the intrinsic ellipticity distribution, $\theta_f$ is the Gaussian smoothing scale and $n(z)$ is the number density of source galaxies which in this work we used the $n(z)$ as determined by van Waerbeke [48].

The pure noise was generated over 1000 runs to reduce the sample variance of the gaussian noise by 0.001. The noise maps include the corresponding $n(z)$ depending on the limiting magnitude, and $\sigma_e$ for both ground and space based case, appearing as $\sigma_{noise}$. The noise maps were then smoothed with different smoothing filters with the same scale that the $\kappa$ maps were smoothed. To obtain the noisy $\kappa$ maps results, we subtracted the pure Gaussian noise variance from the noisy measured $\kappa^2$. Figure 4.7 (a) is the pure Gaussian noise, (b) is the pure $\kappa$ maps and (c) shows the noisy $\kappa$ map, when the pure noise map is added to the pure signal map. In next section we explain how we determined the proper $n(z)$ used.

![Figure 4.7](image)

Figure 4.7: (a) pure Gaussian noise of $\sigma_e = 0.44$ for typical ground based survey, (b) noise free signal from simulations with 5 galaxy/arcmin$^2$ corresponding to limiting magnitude of 24, and (c) the signal plus noise. These are 46 deg$^2$ maps smoothed with tophat filter with smoothing scale of 4 arc minutes and mean redshift of 0.17.

4.4.2 Cosmic Variance

The cosmic variance is the result of looking at a fraction of the sky rather than studying the sky as a whole. The smaller the patch of sky observed the large the cosmic variance becomes. By looking at different sky samples one can reduce the cosmic variance. To minimize the cosmic variance effect we used all 60 lines of sight we had. It is important to know how much contribution to the total noise is from the cosmic variance in order to determine the most
efficient ways to deal with systematics and assessing the noise properties. This effect is studied by Semboloni et al (2006) [38] and will discuss our results later in §5.2. The error bars in the “without noise” plots are representing the cosmic variance. Stacking the ray-tracing simulations of different redshift slices was done with and without adding an extra galaxy shape noise. The latter case resulted in a variance in measurements due to the cosmic variance, while the error bars in the former case take into the account both cosmic variance and the galaxy shape noise.

4.4.3 Redshift Distribution

To mimic different types of observations, we populate these simulations with a realistic redshift distribution of galaxies that is dependent on the limiting magnitude of the survey we want to simulate, with a number density that is also dependent on the type of observation; space-based or ground-based. The difference between the two cases come from the fact that the telescope PSF distortion is smaller in space, so more objects are resolved compared to the ground based case. We need to be able to resolve the galaxies in order to measure the shear. To model the galaxy redshift distribution $n(z)$ of different magnitude limited surveys, we use the method described in [16] and [55], modeling $n(z, \text{mag})$ as:

$$n(z, \text{mag}) \propto z^{2.2} \exp \left[ - \left( \frac{z}{z_0(\text{mag})} \right)^{1.0} \right] \text{(4.6)}$$

which corresponds to the best-fitting function to the HDF (Hubble deep field) photometric redshift distribution from [12]. We calculate the $z_m$ for each magnitude bin from:

$$z_m = -3.132 + 0.164 V_{\text{mag}} \text{ (4.7)}$$

where $V_{\text{mag}}$ is the F606 AB magnitude from [16]. Using these parameters $z_0 = z_m/2.87$ where the median redshift $z_m$ is taken from the redshift-magnitude relation of [16]. The relation is:

$$\phi(z) = \frac{\sum_{i=1}^{M} N(i)n(z, m(i))}{\sum_{i=1}^{M} N(i)} \text{(4.8)}$$

where $i = 1, \cdots, M$ are the magnitude bins, with mean magnitude $m(i)$, and each bin contains $N(i)$ galaxies. The $n(z, m(i))$ is found by equation 4.6 with $z_0 = z_m/2.87$.

We estimate the number density of resolved galaxies as a function of limiting magnitude from the galaxy number counts in the HST ultra-deep field. We basically determined the $n(z)$ for ground based from the space based data. In order to do that we assumed that the galaxy number density is the same functionality of limiting magnitude for $n(z)$ in space and ground. Then for a given limiting magnitude, more galaxies are resolved from space compared to the ground, since they are too small to be resolved from ground, which could be interpreted as deeper median redshift. Having the galaxy number density, and $z_m$ from equation 4.7 we calculate the $n(z, \text{mag})$.

We define a source to be adequately resolved for lensing studies if the object’s half light radius is greater than the resolution limit set by the atmospheric seeing (ground, 0.7 arc sec), or pixel scale (space, 0.1 arc sec). Figure 4.8 shows the galaxy number density for each limiting magnitude bin for a typical ground based survey, while figure 4.9 shows the space based result. In figure 4.10 we see the comparison between space and ground galaxy number density. The space based survey has a significantly higher number density for all limiting magnitudes. The

$^1$HST UDF: www.stsci.edu/hst/udf
galaxy number density, mean redshift and the redshift separation for each limiting magnitude is shown in table 4.1 and 4.2 for ground and space case respectively. The numbers come from the UDF which was selected to have no bright galaxies in them. As a result these numbers contain cosmic variance in them and this is why no bright galaxies seem to be taken into the account.

Figure 4.8: The cumulative galaxy number density (resolved galaxies) for different limiting magnitudes for a typical ground based survey.

Figure 4.9: The cumulative galaxy number density (resolved galaxies) for different limiting magnitudes for a typical space based survey.
Figure 4.10: The cumulative galaxy number density (resolved galaxies) for space and ground based surveys is compared here. The space based survey has significantly higher galaxy number density for the same limiting magnitudes.

<table>
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<tr>
<th>Limiting Magnitude</th>
<th>ngal/arcmin²</th>
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<th>$(z^2)^{1/2}$</th>
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Table 4.1: The cumulative galaxy number density, mean redshift and r.m.s. redshift for different limiting magnitudes in ground based survey. The magnitudes are AB F606 magnitudes.
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<tr>
<th>Limiting Magnitude</th>
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<th>$\langle z^2 \rangle^{1/2}$</th>
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<tr>
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<td>497.00</td>
<td>1.47</td>
<td>1.63</td>
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Table 4.2: The cumulative galaxy number density, mean redshift and r.m.s. redshift for different limiting magnitudes in space based survey. The magnitudes are AB F606 magnitudes.
4.5 Ground vs. Space

As discussed in the previous section, deeper surveys have higher $n(z)$ and space based ones have a higher overall $n(z)$ for each limiting magnitude bin. The reason is that smaller galaxies are resolved from space which adds to the $n(z)$. This is the reason why the galaxy shape noise is lower for the space based surveys and we see its effect as an overall higher signal to noise ratio for all the results in this thesis. Figure 4.11 shows the simulated maps for typical ground based and space based surveys of different depth. As seen in the picture the deeper we go for a ground based observation the signal would seem stronger and at some point it would be comparable to a space based observation with less depth. Part (a) is the pure $\kappa$ with no added noise, part (b) is the ground based noisy signal with limiting magnitude of 26, part (c) shows the space based noisy signal with the same limiting magnitude, 26, part (d) is a deeper ground based noisy signal, with limiting magnitude of 27, and part (e) shows an even deeper ground based noisy signal with limiting magnitude of 28.

Figure 4.11: The effect of space and ground noise level added to a noise free image. (a) Pure signal from simulations at limiting magnitude 26. (b) ground based noisy signal at limiting magnitude 26. (c) space based noisy signal at limiting magnitude 26. (d) ground based noisy signal at limiting magnitude 27. (e) ground based noisy signal at limiting magnitude 28.
Here in figure 4.12 we show the different values of $n(z)$ as a function of redshift for different limiting magnitudes. This plot compared the galaxy number density of different limiting magnitudes as a function of redshift. The point curves indicate the space based and the dashed line curves show the ground based results. The ground based results are shown with different types of lines, and the space based results are shown by different types of points. One can compare the galaxy number densities between space and ground for the same limiting magnitude at different redshifts.

![Figure 4.12: The galaxy number density for each redshift slice used in the galaxy shape noise calculations. Note that the point curve represents the space case and the lines are the ground case.](image)

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Figure 4.12: The galaxy number density for each redshift slice used in the galaxy shape noise calculations. Note that the point curve represents the space case and the lines are the ground case.
4.6 Simulating CFHTLS

As discussed in the earlier chapters, skewness was measured on VIRMOS-DESCART data which was not wide enough to provide satisfactory constraints. This however will not continue to be the case since CFHTLS (Canada France Hawaii Legacy Survey) wide is now almost ready for measurements. In this thesis we predict the results from CFHTLS like survey for 2-point and 3-point statistics.

4.6.1 CFHTLS specifications

CFHTLS wide is providing us with the widest area to the weak lensing study, and is a great survey to try to measure the skewness signal. Canada and France have dedicated around 50% of the CFHT telescope time for the large project, the Canada-France-Hawaii Telescope Legacy Survey (CFHTLS). More than 450 nights over 5 years will be devoted to the survey using the wide field imager MegaPrime equipped with MegaCam, a 36 CCDs 1 degree x 1 degree field of view camera. This project has started in 2003 and has released the first pixel data on August 2006.

The CFHTLS wide fields covers 170 square degrees in three patches of 49 to 72 square degrees through the whole filter set (u*, g', r', i', z') down to i'=24.5, which allow the study of the large scale structures and matter distribution in the universe through weak lensing and galaxy distribution, as well as the study of clusters of galaxies through morphology and photometric properties of galaxies. The forth wide field was added later to make up for bad weather of previous winters.

Figure 4.13 shows the W1, W2, W3 and W4 targets on the sky. The camera used is the MegaPrime which is the newest wide-field imaging facility at CFHT (the official first light took place in January 2003), and represents a major upgrade of the telescope upper-end as well as the largest astronomical CCD mosaic ever built.

The wide-field imager, consists of 40 2048 x 4612 pixel CCDs (a total of 340 mega pixels), covering a full 1 degree x 1 degree field-of-view with a resolution of 0.187 arc second per pixel to properly sample the 0.7 arc second median seeing offered by the CFHT at Mauna Kea.

4.6.2 Simulation of CFHTLS wide

For previous parts of this study we uses 60 different lines of sight, with area of 46 deg², 1024x1024 pixels, and the resolution of each is 0.4 arc minutes. CFHTLS wide however is covering almost 4 times the size of one simulation we used. By combining each four lines of sight together we simulated an area almost the same as CFHTLS wide. We used the 60 lines of sight to make now 15 CFHTLS wide lines of sight as figure 4.14 shows the combined maps. In the next chapters, we show the promising results obtained for CFHTLS skewness predictions.
Figure 4.13: CFHTLS deep and wide targets. From CFHTLS website
Figure 4.14: By putting each four 46 deg$^2$ simulations beside each other we simulated the CFHTLS wide area.
5. Results

5.1 Results: Ground vs. Space

In this section we compared the difference in signal level between a typical ground based and space based survey. We used the tophat filter for this study. The area of the surveys are 46 deg$^2$ and the limiting magnitude is 24. From table 4.1 we see that the galaxy number density for the ground based case at this limiting magnitude is 5 galaxies/arcmin$^2$ while for the space case it is 8 galaxies/arcmin$^2$.

In figure 5.1 I show the mean redshift as a function of limiting magnitude for both ground based and space based surveys. As seen the mean redshift for space seems to be lower than that of the ground below limiting magnitude 25, and above 25 in stays larger than the ground case. The main difference between ground and space comes from the number of galaxies, which is larger at brighter magnitudes. This is because we see smaller objects and objects with lower surface brightness from space which are not resolved from ground. This reduces the shot noise for space based surveys.

Here we compared the signal and signal to noise ratio for 2-point and 3-point statistics measurements for typical ground and space based surveys. The survey area is 24 deg$^2$ and the tophat filter is used for smoothing the maps. Figure 5.2 shows the results. The solid lines represent ground based results, and dashed lines represent space based results. The error bars contain cosmic variance and added statistical noise.

As seen in 5.2(a) and 5.2(c) the signal for $\langle \kappa^2 \rangle$ and $\langle \kappa^3 \rangle$ is slightly higher for ground compared to space, and for $S_3$ the ground signal is slightly lower than the space based one. However when signal to noise ratio is considered, the space based results for all $\langle \kappa^2 \rangle$, $\langle \kappa^3 \rangle$ and $S_3$ show significant improvement by going to space. This gain peaks between 2 to 10 arc minutes as in 5.2(b), 5.2(d) and 5.2(f).

![Figure 5.1: The comparison between mean redshift as a function of limiting magnitude for space and ground based surveys.](image)
Figure 5.2: The solid lines represent ground based results, and dashed lines represent space based results. The limiting magnitude is 24 and the survey area in both cases is 46.6 deg². The smoothing filter used is tophat filter. (a): $\langle \kappa^2 \rangle$, (b): signal to noise ratio of $\langle \kappa^2 \rangle$, (c): $\langle \kappa^3 \rangle$, (d): signal to noise ratio of $\langle \kappa^3 \rangle$, (e): $S_3$, (f): signal to noise ratio of $S_3$. These plots contain both cosmic variance and statistical noise.
5.2 Result: The Effect of Depth

In this section we studied the effect of depth on the signal to noise ratio of \( \langle \kappa^2 \rangle \), \( \langle \kappa^3 \rangle \) and \( S_3 \). The goal was to see how the signal and signal to noise ratio would be affected by increasing the depth of the survey with fixed size. The higher the limiting magnitude is the dimmer the objects are and as a result longer exposure is needed to observe them. We stacked the 46 deg\(^2\) noise free and ground based noisy simulations smoothed with a tophat filter and measured the 2 and 3-point statistics. Figure 5.3 shows the comparison of the error contribution from cosmic variance and statistical noise. The statistical noise has a significant contribution at smaller scales and decreases for larger scales, while cosmic variance has the exact opposite effect. So naturally without adding the galaxy shape noise, the signal to noise ratio drops at larger scales, affected by cosmic variance.

The plots in 5.4 show the signal and the signal to noise ratio for three different limiting magnitudes as a function of smoothing scale when the only noise source is the cosmic variance. The solid lines have limiting magnitude of 24, the dashed lines 25.5 and the dotted ones 27. The signal is higher for \( \langle \kappa^2 \rangle \) and \( \langle \kappa^3 \rangle \) in deeper surveys as in figure 5.4(a) and 5.4(c), although the skewness \( S_3 \) shows larger signal for shallower survey 5.4(e). When signal to noise ratio is considered, however, at smaller space deeper surveys tend to have higher signal to noise for \( \langle \kappa^2 \rangle \) and \( \langle \kappa^3 \rangle \) as in 5.4(b) and 5.4(d), while as seen in 5.4(f) the signal to noise ratio of the noise free skewness is completely independent of the depth of the survey. This is not surprising as was shown in §3.3 in equation 3.7, the deeper surveys tend to reduce the skewness signal. So by going deeper the skewness signal will drop due to increase of the redshift, but on the other hand more objects are observed which decreases the noise. These two effect cancel each other out and as figure 5.4(f) the signal to noise ratio remain the same. This is not true any more if the statistical noise is added to the maps. Then the galaxy number density for these limiting magnitudes would be 5, 16 and 32 galaxies/arcmin\(^2\) respectively and as seen in figure 5.5 deeper surveys improve the skewness measurements especially at smaller scales.

A factor of 3 improvements in signal to noise ratio is obtained by increasing the limiting magnitude by one for skewness measurement, which encourages deeper surveys. The disadvantage however is the time it takes to obtain a deeper survey and makes this result unrealistic. It is interesting to note that this however is not due to higher signal but to less noise, since more objects are observed, so the statistical noise drops. This is why the gain is more obvious at small scales on which the statistical noise is more dominating than the cosmic variance.
Figure 5.3: The cosmic variance and statistical noise contribution to the total noise. Here the survey area is 46 deg$^2$, limiting magnitude is 24 and the smoothing filter is tophat. The cosmic variance (solid line) increases for larger scales, while the statistical noise (dotted line) behaves the opposite. (a): N/S for $\langle \kappa^2 \rangle$, (b) N/S for $\langle \kappa^3 \rangle$ and (c) for $S_3$. 

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Figure 5.4: The comparison between different survey depths for a ground based survey of size 46 deg$^2$ with tophat smoothing filter. The solid line represents limiting magnitude 24, (5 gal/arcmin$^2$), the dashed line shows the limiting magnitude 25.5 (16 gal/arcmin$^2$) and the dotted line shows the limiting magnitude 27 (32 gal/arcmin$^2$). Error bars here only contain cosmic variance.
Figure 5.5: The comparison between different survey depths for ground based survey of size 46 deg\(^2\) with tophat smoothing filter. The solid line represents limiting magnitude 24, (5 gal/arcmin\(^2\)), the dashed line shows the limiting magnitude 25.5 (16 gal/arcmin\(^2\)) and the dotted line shows the limiting magnitude 27 (32 gal/arcmin\(^2\)). The noise contains both cosmic variance and statistical noise.
5.2.1 Prospective Application:

When an optimal skewness measurements is achieved, the goal as mentioned in the introduction, is to constraint the $\sigma_8$ and $\Omega_m$. To do so, 90 different $(\Omega_m, \sigma_8)$ theoretical models were calculated. The values for $\sigma_8$, are 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, and 1.2 and the $\Omega_m$ values are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0. The models are ΛCDM, so $\Omega_\Lambda = 1 - \Omega_m$.

The shape of the matter power spectrum is set to $h \times \Omega_m$ (according to standard CDM). The likelihood contours $\Omega_m$ and $\sigma_8$ were calculated for the skewness measurements by:

$$\chi^2(\Omega_m, \sigma_8) = \sum_{i=1}^{15} (d_i - m)^T C^{-1} (d_i - m)$$

$$C = m^T m$$

where $m$ is the fiducial model vector and $C = \langle (m)^T (m) \rangle$ is the covariance matrix (15 by 15) and $d_i$s are the 90 different theoretical models.

$$\mathcal{L} = \frac{1}{(2\pi)^n |C|^{1/2}} e^{[(d_i - m)^T C^{-1} (d_i - m)]}$$

Figure 5.6 shows the 68 %, 95 % and 99.9 % probability contours, when skewness measurements are used. It is very promising to note that the result show that the contours show independence of $\sigma_8$ values, as was predicted theoretically.

![Figure 5.6: The likelihood function of $\Omega_m$ vs. $\sigma_8$. The white central part shows the 1$\sigma$ (68%) probability contour, the gray part corresponds to 2$\sigma$ (95%) and the black part indicates the 3$\sigma$ (99%) contour.](image-url)
Skewness combined with \( \langle \kappa^2 \rangle \) and \( \langle \kappa^3 \rangle \) measurements will provide an even tighter likelihood constraint. This is however, only valid if these measurements are uncorrelated. We calculated the cross correlation coefficient of these statistics. The cross correlation coefficient is calculated as:

\[
 r_{xy} = \frac{\langle xy \rangle}{\langle xx \rangle^{1/2} \langle yy \rangle^{1/2}} \tag{5.4}
\]

Here \( x \) and \( y \) are either \( \kappa^2, \kappa^3 \) and \( S_3 \). Figure 5.7 shows the results in matrix form. The row and columns correspond to the measurements at 15 different scales. In 5.7(a) we see the cross correlation between \( \kappa^2 \) and \( \kappa^3 \), which its highest value is 0.6 marked as white squares. In 5.7(b) we see an overall lower correlation between \( \kappa^2 \) and \( S_3 \), and finally in 5.7(c), we see \( \kappa^3 \) and \( S_3 \) are partially correlated. These levels of correlation let us step further and measure the joint likelihood between 2-point and 2-point statistics. In figure 5.8 we present the joint \( \langle \kappa^3 \rangle, S_3 \) probability contours in \( \Omega_m - \sigma_8 \) plane.

Figure 5.7: The cross correlation coefficient for all combinations of \( \kappa^2 \) and \( \kappa^3 \) and \( S_3 \). (a) shows the cross correlation between \( \kappa^2 \) and \( \kappa^3 \), (b) shows the cross correlation between \( \kappa^2 \) and \( S_3 \), and (c) shows the cross correlation between \( \kappa^3 \) and \( S_3 \).

This result is very encouraging to measure skewness on CFHTLS wide data which is going to be the first priority of future work. We showed that skewness measurements are very useful for \( (\Omega_m - \sigma_8) \) degeneracy breaking purposes.
Figure 5.8: The $\langle \kappa^3 \rangle$ and $S_3$ joint likelihood. The white central part shows the $1\sigma$ (68%) probability contour, the gray part corresponds to $2\sigma$ (95%) and the black part indicates the $3\sigma$ (99%) contour.
5.3 Result: The Effect of Filters

In this part of the study we compare the effect of tophat and aperture smoothing filter on the signal to noise ratio of all statistics. First it is interesting too see that in the noise free scenario, the signal level when the mass map is smoothed with aperture filter is much higher than when smoothed with tophat filter as shown in figure 5.9.

In figure 5.10 we see the comparison between the signal to noise ratio for noise free and noisy study. The survey area and limiting magnitude are the same in all cases. Although aperture filter has higher signal to noise ratio for a cosmic variance limited survey (see figure 5.10(a), 5.10(c), and 5.10(e)), when the realistic conditions are applied, tophat filter shows an overall higher signal to noise ratio.

On one hand, as discussed in the previous chapter, the tophat filter shows higher correlation between the scales, while for aperture this is not the case. On the other hand from the results presented in this section, we see that in the realistic noisy conditions, the tophat filter has higher signal to noise ratio for all statistics. Therefore to answer the question that which filter is appropriate for skewness measures, we calculated the reduced $\chi^2$ for each statistics for tophat and aperture filter:

$$\chi^2_{\text{reduced}} = m_i^T C^{-1} m_i \equiv \left( \frac{S}{N} \right)^2$$

(5.5)

where $m_i$ are the measurements on different scale and $C^{-1}$ is the inverse of the covariance matrix and we know that the square root of the reduced $\chi^2$ is equivalent to the signal to noise ratio. The results are summarized in the table 5.1 below. Tophat filter provides extra $2\sigma$ measurement for $\langle \kappa^2 \rangle$, $1.5\sigma$ for $\langle \kappa^3 \rangle$ and most importantly more than $3\sigma$ measurements for the skewness.

<table>
<thead>
<tr>
<th></th>
<th>S/N $\langle \kappa^2 \rangle$</th>
<th>S/N $\langle \kappa^3 \rangle$</th>
<th>S/N $S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tophat</td>
<td>15.33</td>
<td>5.24</td>
<td>7.17</td>
</tr>
<tr>
<td>Aperture</td>
<td>13.15</td>
<td>3.95</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Table 5.1: The signal to noise ratio for tophat and aperture filter calculated from the reduced $\chi^2$. 
Figure 5.9: The skewness signal, when only cosmic Variance is taken into account.
Figure 5.10: The comparison between the signal to noise ratio of tophat and aperture smoothing filters. The solid lines show the tophat filter results and the dashed lines show the aperture filter results. The error bars in the right column plots contain only the cosmic variance, while in the left panel plots, they contain both cosmic variance and statistical noise. (a) and (b) : the signal to noise ratio for : $\langle \kappa^2 \rangle$, (c) and (d) show the signal to noise ratio for $\langle \kappa^2 \rangle$ and (e) and (f) show the signal to noise ratio for $S_3$ as a function of scale for both filters.
5.4 Result: Deep and Narrow vs. Wide and Shallow

Now we want to compare a deep and narrow survey to a wide and shallow one, so that the observing time for is equal. The two chosen areas are $46\,\text{deg}^2$ (as the wide field) and $8\,\text{deg}^2$ (as the narrow field), first with limiting magnitude 25 (wide) and the second with limiting magnitude 26 (deep). The galaxy shape noise was added to all the simulations and the maps were smoothed by tophat filter with different smoothing scales.

Since the limiting magnitude are different, the galaxy number densities would not be the same for the two. After adding the corresponding galaxy shape noise to each case, we measured the statistics. As seen from figure 5.11(a) and 5.11(b), the $\langle \kappa^2 \rangle$ and $\langle \kappa^3 \rangle$ signal is higher for the deep and narrow survey compared to wide and shallow one. This is while the skewness shows a higher signal for the wide and shallow case.

More interestingly is the signal to noise ratio. We see in figure 5.11(b), 5.11(d) and 5.11(f) that the signal to noise ratio for all statistics is higher when the wide and shallow survey is used. When the survey is wide and shallow the cosmic variance decrease, with the price of loosing the numerous faint objects and as a result the statistical noise will increase. For the deep and narrow survey it is the opposite. So far our results suggested that for the same observing time, wide and shallow is better than deep and narrow to measure the statistics.

We need to however, asses the other extreme case as well, when for the same observing time the survey is very wide and very shallow. The advantage of such survey is the low cosmic variance, but on the other side, due to its shallowness, very few objects will be observed that increases the statistical noise. We performed such study for the survey size 700 deg$^2$ and limiting magnitude 23 so that the observing time stays the same as previous parts. As seen in figure 5.12 the signal to noise ratio for skewness measurement for the very wide survey drops. This implies that there is an optimal survey area for which the skewness measurement is most effective.
Figure 5.11: Deep and narrow, vs. wide and shallow ground based surveys: The solid lines show the deep and narrow survey with area of 8 deg\(^2\) and limiting magnitude of 26 (corresponding to 11 gal/arcmin\(^2\)) and the dotted lines show the wide and shallow survey of 46 deg\(^2\) and limiting magnitude of 25 (corresponding to 21 gal/arcmin\(^2\)). The smoothing filter is tophat and the depth and area is calculated for the same observing time (around 115 hours). The error bars contain both cosmic variance and statistical noise. (a) and (b) show the signal and signal to noise ratio of \(\langle \kappa^2 \rangle\), (c) and (d) show the signal and signal to noise ratio of \(\langle \kappa^3 \rangle\), and (e) and (f) show the signal and signal to noise ratio of \(S_3\).
Figure 5.12: The comparison between signal to noise ratio of noisy skewness measurements of the wide and deep and very wide for the same observing time.
5.5 CFHTLS Predictions

The last section’s results suggested that skewness measurements are very promising for wide surveys, and in the previous chapter we stated the method to simulate the CFHTLS wide survey. We performed measurements on the simulated CFHTLS wide to predict the outcomes. The limiting magnitude used was 24 for ground based survey smoothed by tophat filter and the realistic noise was added. The original simulation area is 46 deg\(^2\) and the simulated CFHTLS is four times larger, around 184 deg\(^2\). The 184 deg\(^2\) corresponds to the complete CFHTLS size and 46 deg\(^2\) is the current size. Here is figure 5.13 we show the skewness signal level for both surveys.

Now we measure the signal to noise ratio for both of the survey sizes, using tophat and aperture smoothing filter. In right column of figure 5.14 we see the tophat results and in the left column the aperture results. The solid line in each panel represents the survey area of 46 deg\(^2\), and the dotted line show the simulated CFHTLS area of around 184 deg\(^2\). It is interesting to note that the signal to noise ratio for tophat filter peaks around smaller scales compared to that of the aperture filter, which was explained in §5.3 previously. In general however, the point is to appreciate the significant gain for skewness measurements using a survey like CFHTLS wide. The signal is the same for both areas and what changed is the error bars which decrease.

![Figure 5.13: The comparison between survey size. The solid line shows the noisy $S_3$ signal for the survey area of 46 deg\(^2\) and the dotted line shows the same for survey area of 184 deg\(^2\) which is roughly the size of CFHTLS wide field.](image-url)
for the larger survey since the cosmic variance effect becomes smaller. Both cases have the same
depth, i.e. limiting magnitudes are equal and as a result the galaxy number density is the same.
This means that the statistical noise in both case are the same but the cosmic variance noise
drops for the lager one by factor of $\sqrt{4}$, since the large survey is 4 times large. Interestingly
the signal level does not change much for different scales.
Figure 5.14: The comparison between survey size. The solid lines show the signal to noise ratio for the survey area of 46 deg$^2$ and the dotted line shows the same for survey area of 184 deg$^2$ which is roughly the size of CFHTLS wide field. For these plots, the statistical noise as well as cosmic variance are taken into the account for ground based survey with limiting magnitude of 24. (a), (c) and (e) are measured with tophat smoothing filter while (b), (d) and (f) are measured aperture filter.
6. Conclusion and Future Work

Measuring the higher-order statistics has been theoretically suggested to be the means to break the degeneracy between the matter density parameter and normalization of the matter power spectrum. In this thesis the goal was to assess how the realistic skewness signal can be affected by different realistic survey specifications. We used 60 projected mass ($\kappa$) lines of sight ray-tracing simulations with 62 redshift slices. Then to perform the realistic study, we populated the projected mass maps with redshift and limiting magnitude dependent galaxy number densities, and measured the 2-point and 3-point statistics. The cosmic variance due to the size of the surveys, dominates the large scales, and the statistical noise, dominates at small scales, due to the lack of objects. Both effects were realistically taken into the account in our study. The number counts were calibrated on deep space based survey (UDF+HDF). We showed that, as expected, the space based surveys have higher signal to noise ratio, mainly because the telescope PSF is smaller and more objects are observed. Deeper surveys with the same area (same cosmic variance contribution), have higher signal to noise ratio as well, due to higher signal and due to less noise, since more objects are observed. When comparing the smoothing filters, we showed that the tophat filter is highly correlated on different scales, while aperture filter shows small correlation. On the other hand for the realistic noisy conditions the tophat filter has higher signal to noise. To answer the question of which filter measures skewness the best, we calculated the reduced $\chi^2$ and the conclusion was that tophat filter measured the skewness with $3\sigma$ more than aperture filter. Then for a fixed observing time, we compared a deep and narrow survey to a wide and shallow one, and we concluded that wide and shallow does a better job by a factor of a few. When it comes to very wide and very shallow, our results show that the signal to noise drops down again, indicating that there is an optimal "wide" size corresponding to the chosen limiting magnitude of the survey. At last, using the simulations, we made predictions of the statistics measurements on CFHLS wide, which resulted in very promising outcome. Finally using the theoretical models and our measured results, we calculated the probability contours from skewness measurements and illustrated the degeneracy breaking that was claimed theoretically. Finally assessing the cross correlation between the different statistics, we calculated the joint likelihood of $\langle \kappa^3 \rangle$ and $S_3$. These results are extremely encouraging the next step of a larger project that is to perform the skewness measurements on CFHTLS real data.

There are some limitations in this work as well. First, the source clustering is not taken into the account, which in fact will affect the results. The redshift estimates are not very accurate since they were obtained by photometric redshifts, rather than spectroscopy. One of the noise sources is the intrinsic alignments of the galaxies, which were not included in this work.

The first step in future work would be to establish and use a way to get around these limitations. For the next step of this project, mass reconstruction will be done on CFHTLS data, and finally measure the skewness on this wide survey, in order to find an independent measure of $\Omega_m$. During the course of this project, I worked on another study for independent $\Omega_m$ measurement as well, which is still in progress and would be completed soon. The idea behind that study is to measure the $\Omega_m$ using the minimum convergence ($\kappa$) value, which would
be determined from the emptiest line of sight. The minimum $\kappa$ value is directly proportional to the matter density parameter and is independent of $\sigma_8$, which makes it very interesting as another means of degeneracy breaking.
Bibliography


