TRIADIC TRANSFORMATION AND HARMONIC COHERENCE
IN THE MUSIC OF GAVIN BRYARS

by

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Abstract

Recent developments in music theory have offered new ways of analyzing and interpreting music that uses major and minor triads differently than in traditional dominant-tonic tonality. Riemannian theory, developed and adapted from the dualist theories of Hugo Riemann (1849-1919), is perhaps the most noteworthy example. However, there is still a broad class of triadic compositions that this theory does not satisfactorily describe. In a 2002 article, Julian Hook proposes a family of “Uniform Triadic Transformations” (UTTs) that encompasses Riemannian transformations, along with a variety of other triadic transformations. The pitch classes in a triad other than the root are not explicit in his representation, making his model distinct from neo-Riemannian and other triadic models that focus on triads’ voice-leading and common-tone relationships. In this thesis, I demonstrate the applicability of Hook’s model to the analysis of chord progressions in musical passages from two works by the prominent contemporary British composer Gavin Bryars. Specifically, I show how a special simply-transitive subgroup of UTTs can offer understanding and insight into some of Bryars’s compositional practices. I then suggest further extensions of Hook’s theory, which were not explicit in his article, and apply them to a third work by Bryars.
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Chapter 1: An Introduction to “Uniform Triadic Transformations”

Recent developments in music theory have offered new ways of analyzing and interpreting music that uses major and minor triads differently than in traditional dominant-tonic tonality. Neo-Riemannian theory, developed and adapted from the dualist theories of Hugo Riemann (1849-1919), is perhaps the most noteworthy example. It incorporates some types of triadic successions, especially those involving changes of mode and root-change by third, into a coherent formal system that can be useful for analyzing a variety of musical genres.¹ Neo-Riemannian concepts have also been adapted to treat non-triadic music, specifically that of the early 20th-century.²

However, there is still a broad class of triadic compositions that this theory does not satisfactorily describe. Consider, for instance, Example 1, taken from Rehearsal A of the Second String Quartet of the prominent contemporary British composer Gavin Bryars (b. 1943). The example shows the cello arpeggiating a series of triads that change every two measures; the roots and qualities of the triads are labeled below the score as appropriate.³ (The other three instruments often support the triadic chord tones in the

³ Note that in all examples, “M” refers to a major triad, and “m” refers to a minor triad.
cello, but sometimes add other pitch classes that can be understood as passing or neighboring tones.)

Example 1: Excerpt taken from Bryars's Second String Quartet (1990), mm. 21-32.
Example 1 (cont.): Excerpt taken from Bryars's Second String Quartet (1990), mm. 33-42.

Properties of this passage suggest the possibility of a neo-Riemannian analysis: the cello chords are consistently triadic; the progressions are not traditionally tonal; and some changes, like C major to C minor, which involves a Parallel transformation, are neo-Riemannian. However, such an analysis, when considering the entire passage, is inconsistent, and does not provide a satisfactory account of the compositional process taking place. For example, the first change of this passage, from E minor to C minor, is not one of the basic operations in Richard Cohn’s system (P, L, or R), but a composite. We could understand the change more directly as a Terzschritt in the Schritt/Wechsel-group formulation of Riemannian theory, but that and other Schritt operations do not appear in the rest of the passage. Following this change is the succession that raised our expectations: a short Parallel-chain, which transforms C minor to C major and back again. But the rest of the passage contains transformations for which the standard neo-Riemannian operations provide a rather indirect account. For example, the change from E minor to Ab major, and from B minor to Eb major, each occurring twice in the passage (mm. 29-36 and mm. 37-44 respectively), must be expressed as a product of three basic Riemannian-transformations: PLP or LPL (see Example 2). These two chord pairs are related by perfect fifth, which could raise the question as to whether or not we should regard this transposition as an integral, basic operation. Furthermore, there is no evidence (possibly except for the alteration of C major to C minor) that the transformation from one triad to another is also its own inverse transformation (indicated in the example by the lower, right-facing arrows), as must be the case with P, L, and R.


5 This is the “hexatonic pole” relation treated in Cohn, “Maximally Smooth Cycles,” 19.
Example 2: A neo-Riemannian analysis of mm. 29-44 from Bryars’s Second String Quartet

A theoretical approach to such analytical issues is a 2002 article by Julian Hook. He proposes a transformational theory within a single, simple algebraic structure that encompasses all the neo-Riemannian transformations mentioned above, along with a variety of other triadic transformations. He defines a “uniform triadic transformation” (henceforth UTT) as an operation that acts on the collection of major and minor triads. The pitch classes (pcs) in a triad other than the root are defined in his representation, but are not explicitly manipulated, unlike neo-Riemannian and other triadic models that focus on voice-leading and common-tone relationships. Each UTT affects all major triads the same way and all minor triads the same way (but possibly in a different way than major triads), as explained in Example 3. It is expressed in the form $<+, m, n>$ or $<-, m, n>$: the $+$ or $-$ indicates whether the operation preserves or reverses the mode of the triad, respectively; the integers $m$ and $n$ indicate the pc-interval of transposition (modulo 12) of the root if the triad is major or minor, respectively. UTTs are effectively defined on roots

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and on the modal quality of the triad built over the root; the third and fifth of the triad are implicitly determined by these two factors. Example 4 provides some examples of UTTs.\(^8\)

**Example 3:** Julian Hook's theory of UTTs

Signifies whether the UTT preserves or reverses the mode of the triad to which it is applied.

\[
\text{UTT} = <+, m, n> \text{ or } <-, m, n>
\]

Signifies the interval (mod 12) by which the UTT transposes the root of a major triad to which it is applied.

Signifies the interval (mod 12) by which the UTT transposes the root of a minor triad to which it is applied.

**Example 4:** Some examples of UTTs

\(<-, 3, 4>\) transforms E\(^b\)M to G\(^b\)m and G\(^b\)m to B\(^b\)M
\(<+, 1, 6>\) transforms B\(^b\)M to C\(^b\)m and C\(^b\)m to F\(^b\)m
\(<-, 4, 3>\) transforms A\(^b\)M to C\(^b\)m and C\(^b\)m to E\(^b\)M
\(<+, 7, 7>\) transforms C\(^b\)M to G\(^b\)m and C\(^b\)m to Gm

There are 288 UTTs in total, the result of the product 2 x 12 x 12 of the number of possible values for each of the three UTT arguments: 2 for the mode-preserving or mode-reversing argument, and 12 for each of the transpositional integers, \(m\) and \(n\). They form a closed group: every UTT has an inverse that is a UTT, and the combination of any two

---

\(^8\) UTTs of the form <+, \(m, m\)> are equivalent to transposition by \(m\), as shown in the final example of Example 4.
UTTs results in another UTT. Example 5 demonstrates how to combine UTTs. It uses left-to-right orthography (I will apply this orthography in the rest of this paper), that is, the leftmost UTT is applied first, then the rightmost UTT.

Example 5: The combination of two UTTs, using left-to-right orthography

i. \(<+, m, n> <-, p, q> = <-, m+p, n+q>
ii. \(<-, m, n> <+ , p, q> = <-, m+q, n+p>
iii. \(<+, m, n> <+ , p, q> = <+, m+p, n+q>
iv. \(<-, m, n> <-, p, q> = <+, m+q, n+p>

Example 5 shows that, in general, UTTs do not commute. For example, \(<+, 4, 7> <-, 6, 5> = <-, 10, 0>\), but \(<-, 6, 5> <+ , 4, 7> = <-, 1, 9>\); or \(<-, 4, 5> <-, 1, 6> = <+ , 10, 6>\), but \(<-, 1, 6> <+ , 4, 5> = <+ , 6, 10>\). However, any two mode-preserving UTTs will always commute; a mode-preserving and a mode-reversing UTT will commute if and only if the mode-preserving UTT is equivalent to some transposition (for example \(<+, 4, 4> = T_4\)); two mode-reversing UTTs, \(<-, m, n>\) and \(<-, p, q>\), will commute if and only if \(n-m = q-p\) (for example \(<-, 3, 4>\) and \(<-, 7, 8>\), where \(4 - 3 = 8 - 7\)).

The group of UTTs contains many subgroups described throughout the article, including the neo-Riemannian subgroup, which is comprised of UTTs of the form \(<+, m, -m>\) and \(<-, n, -n>\); these correspond respectively to the 12 Schritt (mode-preserving) and 12 Wechsel (mode-reversing) operations in neo-Riemannian theory. The Wechsels
include the Parallel (P), Leittonwechsel (L), and Relative (R) operations. Example 6 shows UTT equivalents of these and demonstrates how they transform triads.

Example 6: P, L, and R and their UTT equivalents, plus resulting transformations

\[
P = <-, 0, 0> \quad L = <-, 4, 8> \quad R = <-, 9, 3>
\]

\[
\begin{array}{ccc}
P & & \quad L & & \quad R \\
CM to Cm & CM to Em & CM to Am \\
Cm to CM & Em to CM & Am to CM
\end{array}
\]

Here, one can see that when a Riemannian UTT applies a given transposition to a triad of one mode, it applies the inverse transposition (mod 12) to the other mode. Hook regards this as “an explicit representation of Riemann’s well-known harmonic dualism.”

In a case where compound neo-Riemannian operations occur, UTTs can provide a simpler description. Example 7a shows a neo-Riemannian Klangnetz, a network of triadic relationships in which adjacent nodes are related exclusively by one of the mode-reversing transformations L, P, or R. Within this space, arrows trace paths of L and P moves (moving from southwest to northeast), as well as R and P moves (moving from northwest to southeast). In this space, the transformation from C major to E major would be defined in neo-Riemannian-Wechsel terms by LP (using left-to-right orthography). Similarly, the transformation from G minor to Bb minor would be defined in neo-

---

10 Adapted from Fig. 3 in Hyer, “Reimag(in)ing Riemann,” 119.
Riemannian-\textit{Wechsel} terms by RP. Hook’s notation can express these compound operations as single UTTs.

Example 7a: A Klangnetz (after Hyer), and various neo-Riemannian operations

More significantly, alternating \textit{Wechsel} operations can be replaced by reiterations of single UTTs. This is demonstrated in Example 7b.

Example 7b: Simplified paths, using UTTs, through the Klangnetz of Example 7a
The top part of Example 7b shows a series of triads produced by alternating L and P, completing a closed chain within the neo-Riemannian Klangnetz. Each L and P can be replaced by a single UTT, but not the ones shown for L and P in Example 6 (respectively <-, 4, 8> and <-, 0, 0>). Rather, the reiteration of the single UTT <-, 4, 0> can produce the same series of triads: <-, 4, 0> changes C major to E minor, E minor to E major, E major to G# minor, and so on. Likewise, in the lower half of the example, each R and P is replaced by the single UTT <-, 9, 0>, whose reiteration produces the same series of triads as the alternation of R and P. In both cases, Hook’s model provides a simpler, more compact analysis of the chord changes taking place.

It should be noted, as Hook indeed points out, that from any given triad to another the UTT-transformation is not uniquely determined, in the sense that one argument of the UTT is undetermined (either m or n, depending on whether the UTT is mode-preserving or mode-reversing). For example, the transformation from C major to E minor could be the Riemannian Leittonwechsel-transformation, <-, 4, 8>, but it could also be <-, 4, n>, where n is any pc-interval. On the one hand, this feature gives us the analytical flexibility that we found lacking in connection with Example 1, above. On the other hand, this flexibility also invites analytical ambivalence. For unless we have some analytical reason to assert, for instance, that the same UTT also transforms E minor back to C major, the value of n cannot be determined.
Hook addresses this issue by drawing attention to those subgroups of UTTs that are simply transitive. If we restrict the possibility of triadic transformations to the UTTs in one such subgroup, then there is only one possible way to analyze the succession of any two triads. Hook proves that any simply transitive UTT group contains 24 UTTs each of the form \(<+, n, an>\) or \(<-, n, an + b>\), as \(n\) ranges from 0 to 11, and \(a\) and \(b\) are integers mod 12, such that \(a^2 = 1 \pmod{12}\) and \(ab = b \pmod{12}\); the condition \(a^2 = 1\) is satisfied only for \(a = 1, 5, 7, 11\). Since the values for \(a\) and \(b\) distinguish one such subgroup from another, Hook labels the simply transitive groups with the expression \(K(a,b)\); with each value for \(a\), those allowable for \(b\) are automatically limited, yet are different in each case. Example 8 names all the simply transitive groups \(K(a,b)\), and derives the UTTs that belong to one of them, \(K(1,1)\).

Example 8: The simply transitive (sub)groups \(K(a,b)\), where \(a^2 = 1\) and \(ab = b\), resulting in \(<+, n, an>\) and \(<-, n, an + b>\), as \(n\) ranges from 0 to 11, and \(a\) and \(b\) are integers mod 12

\[
\begin{align*}
(a = 1) & \cdot K(1,0), K(1,1), K(1,2), \ldots, K(1,11) \\
(a = 5) & \cdot K(5,0), K(5,1), K(5,2), K(5,3), K(5,6), K(5,9) \\
(a = 7) & \cdot K(7,0), K(7,1), K(7,2), K(7,3), K(7,4), K(7,5), K(7,6), K(7,7), K(7,8), K(7,9), K(7,10) \\
(a = 11) & \cdot K(11,0), K(11,1), K(11,2), K(11,3), K(11,4), K(11,5), K(11,6) \\
\end{align*}
\]

\(K(1,1), a = 1\) and \(b = 1\). For each \(n = 0, 1, 2, \ldots, 11\), the mode-preserving member of the group is \(<+, n, an> = <+, n, n>\), and the mode-reversing member of the group is \(<-, n, an + b> = <-, n, n + 1>\). The resulting twenty-four members of the \(K(1,1)\) simply transitive subgroup are:

\[
\begin{align*}
<+, 0, 0>& \quad <-, 0, 1> \\
<+, 1, 1>& \quad <-, 1, 2> \\
<+, 2, 2>& \quad <-, 2, 3> \\
<+, 3, 3>& \quad <-, 3, 4> \\
\end{align*}
\]

---

11 According to David Lewin, a group of operations (STRANS) is said to be simply transitive on a given family of elements (S) when the following condition is satisfied: “Given any elements s and t of S, then there exists a unique member OP of STRANS such that OP(s) = t.” David Lewin, Generalized Musical Intervals and Transformations (New Haven: Yale University Press, 1987): 157.

The mode-preserving members of the $K(1,1)$ subgroup are simply the twelve transpositions, while each of the mode-reversing members transposes the roots of all major triads by $n$ and the roots of all minor triads by $n + 1$. These mode-reversing members are not as familiar as the Riemannian transformations; for instance, $<-, 10, 11>$ transforms D major to C minor, and C minor to B major.

The simply transitive subgroups of UTTs can have other attractive analytical properties. Firstly, if $a = 1$, then their members commute. This can be proven by combining members of the $K(1,b)$ group of UTTs. First, it is clear that any two mode-preserving members of $K(1,b)$ commute: $<+, m, n><+, n, n> = <+, m+n, m+n> = <+, n, n><+, m, m>$. So do any two mode-reversing members: $<-, m, n+b><-, n, n+b> = <-, m+n+b, m+b+n> = <-, n, n+b><-, m, m+b>$. And so do any pair of mode-preserving and mode-reversing members: $<+, m, m><-, n, n+b> = <-, m+n, m+n+b> = <-, n, n+b><+, m, m>$. 

Secondly, if $a = 1$ and $b$ is odd, then the simply transitive subgroup can be generated (that is, every member can be derived) via the repeated application of at least one of its mode-reversing operations. Specifically, any UTT of the form $<-, m, n>$ where
$m + n$ is equal to 1, 5, 7, or 11, can generate all the members of the subgroup. Such cases make it possible to construct a generalized interval system to measure and compare “distances” between triads, a topic to be considered in Chapter 4 of this thesis. Example 9a shows how the $K(1,1)$ subgroup is completely generated by its mode-reversing member $<-, 2, 3>$, while Example 9b shows how another mode-reversing member, $<-, 4, 5>$, generates a cycle that does not include all members of the subgroup.

Example 9a: A cycle of $<-, 2, 3>$ completely generates the $K(1,1)$ subgroup

---

Example 9b: A cycle of <-, 4, 5> does not

As a whole, Hook's theory of UTTs elegantly folds various types of triadic transformation into a more comprehensive model. The next chapters will demonstrate the applicability of this model through the analysis of chord progressions in musical passages from two works by Gavin Bryars. Specifically, I will demonstrate how a special K(1,b) simply transitive subgroup can offer understanding and insight into some of Bryars's compositional practices. I will then suggest further uses of Hook's theory, which were not explicit in his article, and apply them to a third work by Bryars.
Chapter 2: Triadic Transformation in Bryars’s Second String Quartet

Born in 1943 in Yorkshire, England, Gavin Bryars is often associated with musical minimalism, and for composing music that is “almost soundless.” To date, Bryars’ portfolio contains a wide variety of compositions, including three operas, a film score, numerous choral and orchestral works, electronic music, and a selection of chamber music, including three string quartets. Among his more well-known works to date are those that are programmatic, such as “The Sinking of the Titanic” (1969) and “Jesus’ Blood Never Failed Me Yet” (1971). Bryars began his musical career as a jazz bassist and improviser and has collaborated with such jazz artists as Bill Frisell, Charlie Haden, and Holly Cole. As a result, many of his works contain characteristics of the jazz vocabulary and style. This chapter rationalizes and analyzes the pitch structure underlying his Second String Quartet, which was composed for the Balanescu Quartet in 1990.

To date, critics have mostly limited their study of Bryars to the social, cultural and political aspects of his work, paying little attention to his compositional procedures. One analytical survey of Bryars's music describes “progression from one chord ... to the next by ...way of an enharmonic pivot" as a "veritable fingerprint" of the composer's mature style. This interpretation could suggest hearing the chords in the context of keys, where one tonal area is “pivoting” to another. I find such prolongational tonality

---

difficult to hear even locally in works such as the Second String Quartet (Example 1). A more detailed analysis conducted by Richard Bernas goes to the other extreme.\textsuperscript{17} He asserts that much of the “harmony” found in the recent works of Bryars is the result of “polyphonic rather than harmonic relationships.”\textsuperscript{18} Furthermore, many of Bryars’s harmonic progressions involve voice leading that is exclusively parsimonious (in which every voice moves by common tone, semitone, or whole tone), often accounting for Bryars’s smooth transitions between non-diatonically related chords.\textsuperscript{19} Moreover, citing Bryars’s experience as a jazz double-bass player and improviser, Bernas claims that Bryars’s music consists of simpler chord progressions that are “obscured or enriched” by non-harmonic tones.\textsuperscript{20} With this I agree, and acknowledge that this conception could be used to analyze Bryars’s melodies and, more specifically, those tones that seem to stand “outside” of any given harmonic/triadic context. Consider Example 10, a passage from Bryars’s First String Quartet (1985). Bernas calls attention to the C-to-C scales played by the violins. They are constantly being inflected with different accidentals, beginning with a flattened scale-degree 2, followed by a flattened scale-degree 5, then flattened scale-degrees 2 and 5 together, and so on. Bernas says, “the permutation of these scales, winding slowly up and down over a fairly static ground of pedal Cs and harmonized in blissfully non-functional ways, creates the most bewildering and hypnotic experience in Bryars’s recent music.”\textsuperscript{21} Though I find Bernas’s scalar analysis plausible, his dismissing

\textsuperscript{18} Ibid, 34.
\textsuperscript{19} Douthett and Steinbach, “Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition,” 241-263.
\textsuperscript{20} Bernas, “Three Works by Gavin Bryars,” 35.
\textsuperscript{21} Ibid, 40.
of the harmony as "blissfully non-functional" is rather passive. In this regard, I seek a more systematic basis for these types of progressions, a basis in which Bryars's harmonic writing could be heard as structurally coherent.

Example 10: Gavin Bryars, First String Quartet (1985), mm. 118-126

There are many passages in Bryars’s music where at least one of the instruments provides a harmonic support that is almost exclusively triadic (for example, the Cello Concerto (1995), “After the Requiem” (1990), and the String Quartets 2 and 3 (1998)). However, as noted above, many of these progressions are not traditionally tonal, and there are questions about whether a neo-Riemannian framework is adequate and appropriate.

To explore the possibility of understanding this music with UTTs, let us now examine the Second String Quartet in more detail. Its single movement is divided into six major sections. Changes of section, indicated on the score by double bar-lines, are marked principally by changes of texture. Example 11 diagrams the structural organization of the work as a whole, while emphasizing the formal symmetry of sections 2 through 6. The first and fourth sections present very similar textures and chords. Each begins with the same three-note chord \{Eb, Bb, F\} (the chord that also ends the work), which then proceeds into a harmonic progression that is untraditional but that changes root and mode in a way that can indeed be described elegantly in terms of UTTs.

Example 11: Formal organization of the Second String Quartet (1990)
Example 12: The first section (mm. 1-20) of Bryars’s Second String Quartet
In the first section, shown in Example 12, a repeated rhythm of an eighth-note followed by a long duration announces the change from one triad to the next. Three voices participate in this aspect of the texture; the fourth voice (initially Violin I, later Violin 2, and then Viola) moves more freely, in or out of the prevailing triad. The first triad presented in this way is F♯ minor (m. 3), and the root of the chord that follows (B♭ minor) is 4 semitones above it. Reading the opening chord (mm. 1-2) as Eb minor, as suggested by the resolution of the F to G♭ between violin II and violin I, I can hear an ascending-fifth root-progression between the first and third chords with no change of mode. This hearing is confirmed by the subsequent music: two chords after the B♭ minor triad there is an F minor triad, followed two chords later by a C minor triad that is followed two chords later by a G minor triad. The same ascending-fifth progression of minor triads is demonstrated briefly between the fourth and sixth chords of the section (F♯ minor and C♯ minor). Beginning on the seventh chord of the section (C minor), another progression becomes apparent, in which the roots of the chords change alternately by major and minor thirds (+4 semitones, +3 semitones), and the modes of the chords alternate between major and minor. Since this second progression maintains the transposition by ascending-fifth that is apparent in the earlier part of this passage, I am inclined to regard all these triadic changes as part of a single, coherent transformational system.

Such a system is evident in one of the special groups of UTTs that Hook labels K(a,b) described earlier. The ascending-fifth relationship that occurs between every second chord can be labeled by the UTT <++, 7, 7>, a mode-preserving transformation that
transposes the roots of both major and minor triads by 7 semitones. The mode reversals and root changes by alternating major and minor third that occur between the final four chords of the section can all be labeled $<-, 3, 4>$, a mode-reversing transformation that transposes the roots of major triads by 3 semitones, and the roots of minor triads by 4 semitones. Of the $K(a,b)$ subgroups of the UTT group, only $K(1,1)$ contains both $<+, 7, 7>$ and $<-, 3, 4>$. All of the members of this subgroup were listed in Example 8 (pp. 11-12).

It is easy to see that this set of operations includes an identity element and inverses. The identity operation is $<+, 0, 0>$. Every mode-preserving operation has an inverse within the set (for example, the inverse of $<+, 7, 7>$ is $<+, 5, 5>$), and every mode-reversing member has an inverse within the set (for example, the inverse of $<-, 3, 4>$ is $<-, 8, 9>$). The set of operations is also closed, as can be observed in Example 13, which analyzes the entire first section using only members of $K(1,1)$.

**Example 13: Analysis of first section using UTTs from $K(1,1)$ subgroup**

\[ \begin{align*}
\text{E}^m & \quad \text{F}^m \\
\text{B}^m & \quad \text{F}^m \\
\text{F}^m & \quad \text{C}^m \\
\text{C} & \quad \text{E}^m \\
\text{G} & \quad \text{B}^m \\
\text{B} & \quad \text{Bm}
\end{align*} \]

(compare Example 14)

Here, one may observe how the product of any two transpositions results in another
transposition; for instance, <+, 8, 8> and <+, 11, 11>, compose to <+, 7, 7>. Similarly, the product of any two mode-reversing operations makes a transposition. For example, <-, 3, 4>^2 results in <+, 7, 7>, and, between the last three chords of this section, the composition of <-, 3, 4> with <-, 0, 1> results in <+, 4, 4>, the inverse of the <+, 8, 8> that transforms the third chord of the section to the fourth. More generally, Example 13 asserts that during this passage there is a characteristic transformation, <+, 7, 7>, which is articulated into two successive mode-preserving operations during the first half of the passage, and into two successive mode-reversing operations during the second half of the passage, such that all operations belong to K(1,1).

Example 14: Analysis of mm. 21-76 of Bryars's Second String Quartet

(compare Example 13)
With this analysis in mind let us now look at the harmonic progression used in
Rehearsals A and B of the second section. Example 14 provides a reduction of the
progression as it occurs between measures 21 and 76. It begins on an E minor triad that is
arpeggiated in the cello. This chord lasts for two measures, as does each of the following
chords. I hear this passage as an exposition of the generative power of the UTT $\langle-,3,4\rangle$,
which was introduced towards the end of the first section. The analysis below the score
shows that essentially all the chords of the passage are produced by the repeated
application of this UTT. In effect, the reiteration is now asserting $\langle-,3,4\rangle$, instead of
$\langle+,7,7\rangle$, as the characteristic gesture of the piece (even though the latter UTT links
every alternate chord throughout the passage).

There are only two anomalies, indicated by broken-lined boxes in the example.
Before the opening E minor triad proceeds by $\langle-,3,4\rangle$ to Ab major, there appear C
minor and C major triads. Example 15 shows how this succession can be analyzed
transformationally using operations in the $K(1,1)$ group that were exposed in the first
section. E minor proceeds to C minor by the same transposition, $\langle+,8,8\rangle$, that changed
Bb minor to F# minor near the beginning of the first section. The following alternation of
C minor and C major involves the UTT $\langle-,0,1\rangle$ (and its inverse, $\langle-,11,0\rangle$) that
changed B major to B minor at the end of the first section. We also hear the UTT $\langle-,7,
8\rangle$, which implicitly connects E minor to C major, as a preparation for subsequent events
(and also its inverse, $\langle-,4,5\rangle$).
After E minor returns, as shown in Example 14, the reiteration of \([-, 3, 4]\) generates successive chords in the section up through B\(\flat\) minor, and from F minor to the end of the passage. The \([-, 3, 4]\) chain is broken by F\(\sharp\) minor, which follows B\(\flat\) minor (by \([+, 8, 8]\) ), and which is highlighted by the second broken-lined box on the example. That chord also initiates a nearly exact repetition of the chord-series that appeared in the first section of the quartet, as shown by the brackets below Examples 13 and 14. I did not hear the first section as generated by \([-, 3, 4]\), but its repetition in this context suggests that a \([-, 3, 4]\) chain underlies it. This possibility is strengthened by the one difference between the two passages, which is circled on both examples: the substitution of A major for C\(\sharp\) minor. (Chord-substitution is identified by Bernas as characteristic of Bryars’s harmonic writing.\(^{22}\)) As shown in Example 16a, A major relates by \([-, 3, 4]\) to the preceding F minor and succeeding C minor, and it relates to the original C\(\sharp\) minor by the same K(1,1) UTT, \([-, 7, 8]\), that transformed E minor to C major at the beginning of this section.

\(^{22}\) Ibid, 35.
Example 16a: The substitution of A major for C# minor during Reh. B restarts the <-, 3, 4> chain

Example 16b: The F♯ minor stands for D major in the <-, 3, 4> chain

This interpretation suggests a way of understanding the F♯ minor chord that interrupts the otherwise unbroken <-, 3, 4> chain. Example 16b gives a transformational network with the same graph as Example 16a, but with different triads as the contents of the nodes. It asserts that F♯ minor can be heard, via a K(1,1) transformation, to stand for a D major triad that would continue the <-, 3, 4> chain, exactly analogous to the relation of C# minor and A major in Example 16a.
Considering this substitution, then, it is possible to hear the reiteration of the UTT $\langle -, 3, 4 \rangle$ throughout Example 14, strongly confirming our reading of its presence during the first section. The series exposes 20 out of the 24 possible triads in the complete cycle. It suggests comparison with a 19-chord cycle, involving the UTT $\langle -, 9, 8 \rangle$, in the Scherzo of Beethoven’s Ninth Symphony (first cited by Cohn, and also referenced in Hook’s article). Although $\langle -, 3, 4 \rangle$ does not produce familiar tonal successions, it is a cognate of the Scherzo’s mediant transformation, $\langle -, 9, 8 \rangle$, in two senses: it changes mode while alternating root transpositions of minor and major thirds; and its square is transposition by interval class 5.

Example 17: The fourth section of Bryars’s Second String Quartet (mm. 177-186)

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Finally, let us consider the fourth section of the Second Quartet. The score is shown in Example 17 and my analysis in Example 18. This section, too, nearly replicates the chord progression from the first section. The difference is the second chord (m. 178), which is D major rather than F♯ minor. This is precisely the substitution that I intuited for the F♯ minor during Rehearsal B, as analyzed in Example 16b. The remainder of the fourth section repeats the corresponding chords of the first section. But keeping in mind the substitution just confirmed, we can now hear F♯ minor and C♯ minor as substituting for D major and A major, respectively. Accordingly, it is possible to understand the passage as generated entirely by a repeated <-, 3, 4>, the same UTT that generated the chord succession in mm. 21-76.

Example 18: Analysis of the fourth section using UTTs from K(1,1)

\[ <+,8,8> <+,11,11> <+,8,8> <+,11,11> <-,3,4> <-,3,4> <-,0,1> <-,3,4> <+,7,7> <+,7,7> <+,7,7> <+,7,7> <+,7,7> <+,4,4> \]

DM B↑m F♯m Fm C♯m Cm EM Gm BM Bm

Sub for DM sub for AM

<-, 3, 4> path created by chord substitutions in fourth section:

\[ B↑m \quad DM \quad Fm \quad AM \quad Cm \quad EM \quad Gm \quad BM \]

(compare examples 12 & 13)

This analysis demonstrates that Bryars's harmonies are not to be dismissed simply as "non-functional" sonorities. Supported by Hook's theory, it shows that their
“blissfulness” can instead be conceived as the result of a coherent, simply transitive transformational system. One particular transformation is established as a characteristic gesture, and all chord changes in the music result from the action of the operations in this system.
Chapter 3: Characteristic Transformations Carried Over: Examining Bryars’s “After the Requiem”

One year before the composition of the Second String Quartet, Bryars composed “The Cadman Requiem” for the Hilliard Ensemble, in memory of his friend, the sound engineer Bill Cadman. Shortly thereafter, a colleague of Bryars suggested an instrumental piece inspired by this work. The result was “After the Requiem” (1990), composed for a contemporary variation of the string quartet: two violas, cello, and an electric guitar whose timbre is manipulated by effects such as distortion and delay, as well as a volume pedal in order to minimize the attack. Bryars felt that the electric guitar “blended particularly well with the low strings.” The title of the work has multiple meanings for Bryars, specifically:

“...in the musical sense of being based on it (“The Cadman Requiem”), in the chronological sense of following on from it, and in the spiritual sense of representing that state which remains after mourning is (technically) over.”

Compositionally, “After the Requiem” holds many characteristics in common with the Second String Quartet. Firstly, it is a slow-tempo, single-movement work, whose sections are clearly marked on the score by rehearsal letters and changes of texture. Example 19 provides a graphical breakdown of the work. Unity is achieved via thematic and harmonic similarities (as opposed to actual section lengths), while the overall form is binary. More specifically: the opening and central passages (Rehearsals A and G respectively) consist of quotations taken from “The Cadman Requiem”; the entire

26 Ibid.
27 The harmonic organization that articulates the sections will be discussed in detail below.
harmonic progression from Rehearsals B through E is restated in condensed form at Rehearsal H; and Rehearsals F and J are brief, corresponding passages consisting primarily of an alternation of D major (or F♯ minor) and C minor chords.

Example 19: Formal organization of "After the Requiem" (1990)

![Diagram of formal organization of "After the Requiem"]

A second way in which this piece recalls the Second String Quartet is that long passages progress through series of triads without traditional fifth- or Riemannian-relations, and are therefore conducive to analysis using UTTs. For instance, the harmonic progression used in Rehearsal B (shown later in Example 20) can be analyzed using the UTT $\langle -, 3, 4 \rangle$ throughout – the same UTT that was the characteristic gesture in the Second String Quartet.

Harmonic progressions in other sections, however, cannot be so simply interpreted. Consider, for example, the first four chords in Rehearsal D: B♭ minor, E♭ minor, G♭ major and C minor (mm. 56-64). Riemannian theory can provide some insight into such a progression. For example, B♭ minor to E♭ minor can be labeled as LR, while E♭ minor to G♭ major can be interpreted as an R-transformation. But what of the final transformation, which includes a change of root by augmented-fourth as well as a change
of mode? Can Hook’s UTTs provide as coherent an analysis on such progressions as with the Second String Quartet? More specifically, can UTTs help us to comprehend Bryars’s harmonic writing even when the progressions involved are seemingly inconsistent? It was these questions, as well as the return of the characteristic <-, 3, 4> chain, that motivated the choice of this work and the analysis that follows.

To understand the harmonic content, it is important to see that certain events throughout “After the Requiem” can support hearing an underlying, consistent tonality, despite the non-diatonic chord progressions so prevalent throughout the work. For example, the opening of Rehearsal A, a direct quotation from “The Cadman Requiem,” suggests the key of A minor, primarily because of two events: the melodic motion <F4, E4> over the held dyad <A2, E3> in mm. 10-11, which sounds like the $\text{b}6-5$ motion characteristic of minor mode, and the similar material in mm. 19-21, where the F4 falls to the minor-third scale degree (C4). Following this A minor chord, there is a D minor-seventh chord in second inversion (mm. 22-23), which is then followed by a B7 chord (m. 24). This chord (more exactly, a B$^7\#11$) resolves two measures later to E major – the dominant of A – which further confirms the A tonality suggested earlier. The bass line throughout this section also supports an A tonic, as depicted in Example 20.

Yet how might one understand the transition from Rehearsal A into Rehearsal B? Indeed one might even question how to understand the chords of Rehearsal B as rooted triads.
Example 20: Tonal interpretation of bass line during Rehearsal A (mm. 10-27)

Such an interpretation is facilitated by referring to the reprise of this music later in the piece. Recall that Rehearsal H (mm. 127-154) acts as a condensed harmonic reprise of the entire progression used in Rehearsals B through E. As noted earlier, Bryars often enriches his harmonies with non-chord tones. As a result, the chords in the earlier sections are not always clear. However, in Rehearsal H, the electric guitar is instructed to improvise on chords whose names are notated in the score. This additional notation helps to clarify the chords used in Rehearsals B through E. I will therefore base my harmonic interpretations throughout the analysis on the composer’s chord labels.

Following the notation in Rehearsal H, then, we can understand the harmonic progression of Rehearsal B to begin with an $E_{b}$ minor chord, and to proceed as indicated on Example 21. With the exception of the fifth and final harmonies (F minor and C minor respectively), there is a consistent alternation of chord inversions throughout the progression: every minor chord is in second inversion, and every major-seventh chord is in first inversion.
Example 21: Harmonic progression of Reh. B (mm. 28-41 – chord labels taken from Reh. H)

Example 22 models these chords by their roots and qualities, as I did to analyze progressions in the Second String Quartet, and analyzes the changes of chords using UTTs that belong to the simply transitive $K(1,1)$ subgroup. The example reduces the three major-seventh chords to triads, and shows that Rehearsal B contains the same $<-3,4>$ UTT-chain found in the quartet. Again, the reiteration of this UTT produces a transposition by perfect fifth, or $<+,7,7>$, between alternate chords, so these harmonic progressions manifest aspects of traditional progressions in a non-traditional way.

Example 22: Analysis of harmonic progression in Reh. B (mm. 28-41) using UTTs from $K(1,1)$
By interpreting each major-seventh chord as a minor triad superimposed over a major triad, it becomes possible to show an additional way in which Bryars maintains the fifth-relationship, \(<+, 7, 7>\), between alternate chords in the progression. (Because the major-seventh chords all appear in first inversion, the root of the component minor triad is highlighted in the bass.) Example 23 incorporates the minor triads in these major-seventh chords into the \(K(1,1)\) transformational network. The mode-preserving UTT-chain that results \((<+, 8, 8><+, 11, 11> = <+, 7, 7>)\) was seen in the opening measures of the Second String Quartet. In fact, beginning from the initial \(Eb\) minor triad, every second chord is the same as that used in the first section of the Second String Quartet. Furthermore, we can see from the bottom of the example how the transposition by 7 is also maintained in the bass voice, as a result of the particular inversions chosen, via the combination \(T_1\) and \(T_6\).\(^{28}\)

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\(^{28}\) Of course, the transformation suggested in the bass voice occurs between singletons, and not triads. I am merely pointing out the transpositional relationships as they compare with the other UTTs in the passage.
Rehearsal C, consisting of only four chords (see Example 24), begins with a slight increase in tempo, as well as a change in texture. The first harmony is E major-seventh. We focus the analysis on its triadic base, E major, which proceeds from C minor, the final chord of Rehearsal B, via <-, 3, 4>. As a result, it also continues the <+, 7, 7> chain (A major to E major = <+, 7, 7>) and all of the other UTT-relationships that were shown in Example 23. However, the E major chord is transformed differently, as the next chord in the section is A major. The resulting transformation, <+, 5, 5>, the inverse of the characteristic <+, 7, 7>, suggests a dominant-to-tonic progression in A major, reinforcing my initial impressions of the tonic pitch class.

Example 24: Harmonic progression from Reh. B into Rehearsal C (mm. 42-55)

Let us now consider how the entire progression of Rehearsal C supports the developing harmonic structure initiated earlier in the piece. Example 24 shows a parallelism in the transformational relationships shared between the chords in the section. Specifically, the chords group into <+, 5, 5> pairs, while the first and last, as well as the second and third chords, are related by <-, 4, 5>. This mode-reversing member of the

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29 The repeated sixteenth-note texture is reminiscent of that used in “The Cadman Requiem”.

35
K(1,1) group is the inverse of the UTT that provided the chord-substitutions in the analysis of the Second String Quartet: $<-7,8>$ (see Examples 16a and b). The resulting transformation sounds traditional because it has the same effect on major triads as the *Leittonwechsel* (L) transformation, which is the UTT $<-4,8>$. 

Accordingly, all the chords in Rehearsal C can be understood as representing tonic and dominant harmonies of A major. The top portion of Example 25 demonstrates, displaying $C\#$ minor and $G\#$ minor as substitutes, via $<-4,5>$, of A major and E major respectively.

Example 25: Reinterpreting Rehearsal C (and mm. 24-29) as a result of characteristic chord-substitutions.
Furthermore, this interpretation enables us to understand the transition from the final harmonies of Rehearsal A into Rehearsal B (mm. 24-29). As the succession of the first two chords in this earlier passage, B major to E major, is accomplished by $<+, 5, 5>$, the next transformation from E major to $\text{Eb}$ minor potentially involves a $K(1, l)$ UTT, $<-, 11, 0>$, that has not been heard before. However, the $\text{Eb}$ minor can be interpreted as the $<-, 4, 5>$ substitute for B major. If we do so, we may now see the progression of Rehearsal C as being a direct transposition of measures 24-29 by $<+, 5, 5>$.

The prominence of $<+, 5, 5>$ and its close formal and musical association with $<+, 7, 7>$ suggests that it, like its inverse, might be regarded as the square of a mode-reversing member of $K(1, l)$. Specifically, just as the UTT $<+, 7, 7> = <-, 3, 4>^2 = <-, 8, 9>^2$, the UTT $<+, 5, 5> = <-, 2, 3>^2 = <-, 9, 10>^2$. These are, of course, the only two mode-reversing members (along with their respective inverses) that can, when squared, result in a direct transposition of 5 or 7 — a result that seems to be favored by Bryars.

These intuitions are borne out by the chord progression from Rehearsal C into Rehearsal D, which, following a G.P. in measure 55, continues with the same sixteenth-note texture that was used throughout Rehearsal C. Example 26 analyzes the progression from the final chord of Rehearsal C (G$\#_f$ minor) to the opening three chords of Rehearsal D, which are set apart from the next chords by a temporary break in the established texture (m. 58, beat 4, to m. 60).
Transformational relationships have been labeled on Example 26 specifically to emphasize the presence of the UTT $\leftarrow, 2, 3$. The transformation from the final chord in Rehearsal C to the first chord in Rehearsal D is $\langle +, 2, 2 \rangle$. Following this, the transformation from the first to the second chord of Rehearsal D is $\langle +, 5, 5 \rangle$, or $\leftarrow, 2, 3^2$. Together, these transformations produce a fourth instance of $\langle +, 7, 7 \rangle$ – following the three that were shown in Example 23 – between the final chord of Rehearsal C and the second chord in Rehearsal D ($\langle +, 2, 2 \rangle \langle +, 5, 5 \rangle = \langle +, 7, 7 \rangle$). The transformation from the second to the third chord of Rehearsal D is also $\leftarrow, 2, 3$. The lower branch of the network in Example 26 suggests an even more economical hearing. If one considers the $Bb$ minor chord as a substitute – via the $\leftarrow, 4, 5$ UTT that was heard during Rehearsal C – for $G^b$ major, then the $\langle +, 2, 2 \rangle \leftarrow, 2, 3^2$ path from $G^b$ minor to $E^b$ minor can be conceived as two moves by $\leftarrow, 9, 10$, the inverse of $\leftarrow, 2, 3$.

Following the opening chords of Rehearsal D, presented in Example 26, less familiar transformations are introduced. The complete harmonic progression of this
section is given in Example 27a, which analyzes the chord progression using only members of $K(1,1)$.

Example 27a: Harmonic progression of Rehearsal D (mm. 56-81)

Example 27b: Substitution-network embedded in progression of Rehearsal D

Example 27c: Interpreting less familiar UTTs through chord-substitution in progression of Rehearsal D

The initial $Bb$ minor chord is analyzed as a $<-4,5>$ substitution for an implied $G$ major chord that progresses via $<-9,10>$ to $Eb$ minor. The progression from $G$ major to the penultimate chord of the section (C minor) is taken directly from Section II of "The
Cadman Requiem” entitled “Caedmon Paraphrase (Bede)” (mm. 148-167). This specific sequence of chords is initiated by the brief change in texture that was mentioned above (Example 26). Following this, short exchanges of sixteenth-note arpeggiation occur between the violas (for example, m. 62, m. 64, and m. 66) – another reference to the corresponding passage in “The Cadman Requiem.” The direct harmonic quotation involves K(1,1) UTTs that we have not previously encountered. However, a pair of familiar <-, 2, 3>s (identified by the upward-facing square-bracket below Example 27a) produces a <+), 5, 5> from D major to G major. Another <-, 2, 3> from this G major, skipping over a chord just as the <+), 5, 5> did, leads to the tonic at the end of the section. Example 27b shows how the central chord in the <-, 2, 3> chain (E minor) can be related to the final tonic via the same network that is used at the beginning of the passage. Despite the intervening chords, this connection helps to realize the dominant-tonic, fifth-relationship present throughout the work. Example 27c demonstrates how a second chord-substitution can offer some insight into the less familiar transformations that occur between some of the other chords in the section. If we read Gb major as a <-, 4, 5> substitution for an implied Bb minor (the reverse of the section’s initial substitution) then it becomes possible to hear Bb minor, C minor and D major as the result of more familiar transformations: <+), 7, 7>, <+), 2, 2> and <-, 3, 4>.

The penultimate chord in the section is approached by its own “dominant” (G major to C minor). Within the K(1,1) group, the only possible transformation is <-, 5, 6>, which properly defines a dominant to tonic transformation in the minor mode, where the major-dominant harmony is transposed up a perfect fourth and there is a change of mode.
Example 27c shows that such an interpretation is appropriate in the sense that $<-, 5, 6>$ is also manifested from the first C minor in the passage back to its predecessor, G$\hat{b}$ major (noted by an asterisk and reversed arrow).

The UTTs that were characteristic of earlier progressions and substitutions make the transition into Rehearsal E. This section begins with a G$\hat{b}$ minor chord in root position, followed by an F major-seventh chord in first inversion – manifesting the UTT $<-, 8, 9>$, the inverse of the characteristic $<-, 3, 4>$. The progression from the final chord of Rehearsal D (A minor) to these first two harmonies of Rehearsal E suggests the possibility of a substitution taking place. Specifically, one could regard the G$\hat{b}$ minor as substituting for E major via the UTT $<-, 7, 8>$, or $<-, 4, 5>$, and the F major as substituting for A minor via $<-, 4, 5>$. Then the transition from Rehearsal D into Rehearsal E may be interpreted as a tonic-dominant-tonic progression (as shown in Example 28).

Example 28: Tonic-dominant relations connecting Reh. D & E, resulting from chord substitutions
Similar to the progression of Rehearsal D, Rehearsal E presents a variety of harmonic transformations. The complete harmonic progression of this section is given in Example 29a, along with the K(1,1) UTTs that transform each chord to its successor.

**Example 29a: Harmonic progression of Rehearsal E (mm. 82-92)**

![Diagram of harmonic progression]

**Example 29b: UTTs between alternate chords in the progression of Rehearsal E**

![Diagram of UTTs]

*(compare Example 27a)*

**Example 29c: Transformational network of Rehearsal D in progression of Rehearsal E**

![Diagram of transformational network]
Despite the aforementioned variety, there is a return of two familiar transformations: $<-, 8, 9>$, the inverse of the characteristic $<-, 3, 4>$, and $<-, 6, 7>$, the inverse of $<-, 5, 6>$, which appeared twice in the progression of the previous section. Considering the UTTs linking alternate chords, we can recognize in Example 29b another instance of the $<+, 5, 5><-, 2, 3>$ chain that was found in Example 27a, only this time in retrograde. The example also draws attention to $<+, 11, 11>$ – the same transformation that relates the final chord of Rehearsal D with the first chord of Rehearsal E – as well as $<+, 3, 3>$ – the inverse of the transformation that relates C minor to A minor in the end of both Rehearsals D and E. Example 29c asserts a reprise of the transformational network first exposed in Example 27b, again involving the final chord of the section.

This analysis has demonstrated that the UTT family that was postulated for the Second String Quartet was not isolated to that work, but appears prominently in “After the Requiem” as well. Together these analyses show Bryars’s tendency to relate alternate chords by somewhat traditional means, while elaborating these relationships by intermediate harmonies. Supported by the analysis of the Second String Quartet, certain UTTs throughout the analysis of “After the Requiem” are further emphasized by the presence of their inverses. In such cases, these UTTs draw upon the characteristic nature of the simply transitive subgroup in which I am working.
Chapter 4: UTT-Spaces

In conceptualizing a particular musical space, it often happens that we conceptualize along with it, as one of its characteristic textural features, a family of directed measurements, distances, or motions of some sort.\textsuperscript{39}

In Generalized Musical Intervals and Transformations (henceforth GMIT), David Lewin formalizes the idea of a generalized interval system (GIS) for a space of (musical) objects. According to Lewin, every GIS consists of a family of elements, a group of intervals, and a function that maps those elements to the intervals, that are subject to the conditions that: (A) the interval from one element to a second element, followed by the interval from the second to a third element, is equal to the interval from the first to the third; and (B) for any given interval from any given element there is a unique element.\textsuperscript{31} The epigram above characterizes the intuition which led him to the idea in its most general sense. Of course, the concept of “distance” (and its measurement) within a musical framework is not new. For example, musicians are quite comfortable with using the interval (in the traditional sense of the word) as a way of describing a measured distance between two pitches. However, the idea of distance is usually limited to spaces of pitches or pitch-classes. How might one describe a distance between musical objects other than pitch classes? For example, what is the “distance” between the triads C major and $A_b$ minor? Under what conditions can one discuss or conceptualize a directed measurement within a given musical space? Lewin’s intuitions answer these questions by theorizing distance, within any predefined space (or family) of elements, as a collection

\textsuperscript{39} Lewin, \textit{GMIT}, 16.
\textsuperscript{31} For a formal definition of a GIS, please refer to Lewin, \textit{GMIT}, 26.
of intervals ("of some sort") that adheres to the restrictions of a mathematical group. Specifically, his condition (B) means that a GIS can be defined on a given space when the set of transformations that function within that space constitute a simply transitive group: that is, when only one transformation in the set will map a given element of the space to another.

Since GMIT, theorists have benefited from Lewin's intuitions, and his concept of a GIS has served to strengthen various analyses. For example, in "Uniform Triadic Transformations," Hook refers to Lewin's idea of a GIS and acknowledges its potential applicability to any of the 24 simply transitive subgroups of UTTs. However, he does not develop a GIS involving UTTs. In this chapter, I will explore certain ways that the concept of UTTs can be expanded within a framework that is loosely based on Lewin's GIS. I will then use these ideas to analyze significant portions of a third work by Bryars.

Formally, each member of a simply transitive group of UTTs on the 24 major and minor triads can stand for an interval in a GIS involving the same triads. For example, within K(1,1), the transformation from C major to B♭ minor can only be achieved via the UTT <-, 10, 11>. Therefore, if we restrict transformations to members of the K(1,1) subgroup, we can use this UTT to analyze the aforementioned chord change, even if there is no instance of that UTT, <-, 10, 11>, being applied to a minor triad.

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33 Lewin, _GMT_, 157-158.
Earlier in this paper, I explained how the repeated application of certain mode-reversing members of the $K(1,1)$ subgroup can generate all the members of that group. Example 9a (page 13) showed how the UTT $\langle-, 2, 3\rangle$ can generate the complete $K(1,1)$ subgroup in this manner. Example 30 recreates this space, showing the generation of the complete group of 24 major and minor triads via the repeated application of this same UTT. Here, the label of the node $\langle+, 0, 0\rangle$ in Example 9a has been replaced by the chord label C major. By comparing Examples 9a and Example 30, we can see how powers of $\langle-, 2, 3\rangle$ act as intervals within the space of the 24 major and minor triads.

Example 30: A transformational space generated entirely by the UTT $\langle-, 2, 3\rangle$
For example, Eb major resides six clockwise-steps from C major on Example 30. Therefore, the transformation \(<+, 3, 3>\) can be expressed, in consideration of our established GIS, as \(<-, 2, 3>^6\). Similarly, there are nine clockwise-steps from Ab major to F# minor. This transformation is achieved by the UTT \(<-, 10, 11>\), or \(<-, 2, 3>^9\).

The cycle in Example 30 may be regarded as a single-path network, in the sense that there is only one arrow entering, and one arrow leaving each node. It has a single generator in the sense that each of these arrows is labeled with the UTT \(<-, 2, 3>\). In such a case, it seems sensible, then, to refer to the distance between any two nodes in Example 30 as the number of times you have to move by the generating interval to get to the goal. All pathways move strictly clockwise by iterating \(<-, 2, 3>\). As such, there is no move (in this single-pathway space conception) as \(<+, 3, 3>\) or \(<-, 10, 11>\), since there is no arrow connecting, for instance, C major and Eb major, or Ab major and F# minor; instead, one has \(<-, 2, 3>^6\) and \(<-, 2, 3>^9\) respectively.

The spaces that were created in Examples 9 and 30 bear a striking resemblance to the traditional pc-clock, where the generating transformation is \(T_1\), corresponding to the smallest interval (the semitone) between pitch classes. However, in the case of Example 30, although \(<-, 2, 3>\) is clearly a generating transformation analogous to \(T_1\), it is not clear in what sense it could be considered “smallest.” The problem originates with Hook’s model, which combines major and minor triads into a single space, but which still distinguishes them. In pitch-class space, all objects are of the same type, and the transformation that corresponds to the generating interval doesn’t alter the type. However, in the UTT-spaces we are considering, the transformation that corresponds to a
generating interval does change the type of object. Worse, it often transposes the roots of major and minor triads differently, which contradicts our intuition, drawn from the pc-space, of how interval-distance works. Therefore, Hook's model requires developing new intuitions about triadic distance, in particular, learning to hear triad successions not simply as mode- and root-changes, but with reference to the reiteration of \( \langle -, 2, 3 \rangle \)s. In this regard, the long, unbroken chains of \( \langle -, 3, 4 \rangle \)s in Bryars's music (see, for instance, Example 14, page 22) are intended to help do just that – to give a sense of how distant, say, C minor is from G major, when the only way to get there is by repeating the UTT \( \langle -, 3, 4 \rangle \).

Accepting this notion of distance suggests some further adaptations of pc-space concepts to UTT-governed triadic space. Let us consider, then, two-dimensional networks, in which each dimension is generated by a different interval. For example, in his article “Dual Interval Space in Twentieth-Century Music” (2003), Stephen Brown presents a two-dimensional space of pitch classes in which each dimension is measured by a different (non-zero) pc-interval.\(^{35}\) Example 31 reproduces Brown’s Example 8a, where a Dual Interval Space is constructed from pc-intervals 3 and 4.\(^{36}\) Here, the generating interval-transformation of the X-axis is 3, and the generating interval-transformation of the Y-axis is 4. Within this space, Brown is able to relate set classes [0148] and [01369] by flipping about the Y=X diagonal (represented by the dotted line). Brown’s space is effectively a Tonnetz of the type investigated by neo-Riemannian


\(^{36}\) Ibid, 41.
theorists. We have already encountered another multidimensional space, of triads, in Hyer’s PLR-network (Example 7a, page 9). It should be noted that within such spaces, simple transitivity cannot be maintained, as each element involved has multiple locations.

Example 31: Brown’s Dual Interval Space, relating set classes [0148] and [01369]

Using Hook’s UTTs, it is also possible to structure the space of major and minor

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triads as a multi-path space, where each dimension is generated by a different UTT. Such a structure, however, is optimal if the two UTTs commute and at least one of them generates the entire family of triads. Example 32 demonstrates the importance of the first condition, by attempting to construct a two-dimensional graph using two UTTs, $X = <-, 4, 5>$ and $Y = <-, 1, 6>$, that do not commute.

Example 32: The problem of creating a two-dimensional space with non-commuting UTTs

The node to which the arrow is directed must be expressible as $<-, 4, 5><-, 1, 6>$ – that is, moving right from $<+, 0, 0>$ then up – and as $<-, 1, 6><-, 4, 5>$ – moving up from $<+, 0, 0>$ then right. But these two products are not the same, so they cannot be represented as a single node, and, therefore, the graph is not well-formed.

By choosing UTTs that commute, we can successfully construct a consistent two-
dimensional graph. Example 33 shows a generic model for creating such a graph using UTTs $X$ and $Y$. In this space, the node located diagonally up and to the right of $<+, 0, 0>$ is labeled to show the stipulation that $X$ and $Y$ commute ($XY = YX$). If this stipulation is satisfied, then every other node of the graph can be labeled as shown — by the product of the UTTs that it takes to get to that node from the referential node labeled $<+, 0, 0>$.

Example 33: The generic model for creating a two-dimensional UTT-network

As a realization of this model, let us construct a two-dimensional UTT-network using the UTTs $X = <-, 4, 5>$ and $Y = <-, 2, 3>$ (the UTTs that were used in Example 9, pages 13 and 14). The resulting space is shown in Example 34. Although any two commuting UTTs would serve equally well, I have two reasons for these specific choices. First, the resulting network resembles the Riemannian Tonnetz, in that the UTT that labels the node at position $XY = YX$ is $<+, 7, 7>$, a direct transposition by $T_7$ — the same result achieved when constructing a traditional Riemannian Tonnetz, from the intervals $X$
= 3 and \( Y = 4 \), as Brown does (Example 31). Secondly, both UTTs belong to the same simply transitive subgroup that Hook calls \( K(1,1) \), the very group containing both Bryars's characteristic substitute-transformation, \(<-, 4, 5>\), and \(<-, 2, 3>\), introduced in the analysis of "After the Requiem." Both also relate specifically to a third piece by Bryars entitled "A Man in a Room, Gambling," to be discussed below.

**Example 34**: An abstract UTT-net, generated by \{\(<-, 4, 5>, <-, 2, 3>\)\} as \( \{X, Y\} \)

```
<+,6,6>  <-,T,E>  <+3,3>  <-,7,8>  <+0,0>  <-,4,5>  <+9,9>  <-,1,2>

<-,3,4>  <+8,8>  <-,0,1>  <+5,5>  <-,9,T>  <+2,2>  <-,7>  <+E,E>

<+,1,1>  <-,5,6>  <+T,T>  <-,2,3>  <+7,7>  <-,E,0>  <+4,4>  <-,8,9>

<-,T,E>  <+3,3>  <-,7,8>  <+0,0>  <-,4,5>  <+9,9>  <-,1,2>  <+*,6,6>

<+,8,8>  <-,0,1>  <+5,5>  <-,9,T>  <+2,2>  <-,6,7>  <+E,E>  <-,3,4>

<-,5,6>  <+T,T>  <-,2,3>  <+7,7>  <-,E,0>  <+4,4>  <-,8,9>  <+1,1>
```

In the space of Example 34, any triad may be associated with the \(<+, 0, 0>\) position to produce a network including all 24 triads. For instance, Example 35 places A major into the \(<+, 0, 0>\) position. In this space, we can see that the nodes in the rightmost column correspond to the nodes in the leftmost column, because, as shown in Example 9b, \(<-, 4, 5>\) applied eight times is the same as \(<+, 0, 0>\). In the other dimension, there is
a similar identity of rows (not shown in the example) because \((-, 2, 3)^{24} = (+, 0, 0)\).

Thus both dimensions curve back on themselves, making the whole network a torus.

**Example 35:** A portion of the triadic UTT-net generated by \(\{-, 4, 5\}, \{-, 2, 3\}\) as \((X, Y)\)

Within this space, any triadic relationship can be represented as a path made up of a series of vertical and horizontal steps. Of course, because \((-, 2, 3)\) alone generates all 24 major and minor triads, paths within this space can exclude horizontal steps, exposing the fact that there can be multiple ways to get from one node to another. For example, the transformation of A major to D\(_\#\) minor can be expressed as \((-, 2, 3)^{17}\) (refer back to Example 30, page 46), totaling seventeen *upward* steps within the space. This path can be significantly reduced to seven *downward* steps, via \((-, 2, 3)^7\). However, by combining dimensions, this same transformation can follow a path of two nodes to the right and one node down, reducing the total steps even further to three. I will represent this path
accordingly as $(+2, -1)$, which corresponds to $<-6, 7> = <-4, 5^2<-2, 3>^1$. The same series of moves also transforms G major to C♯ minor and C major to F♯ minor, as well as E minor to B major and D minor to A major. Therefore, we can see that the specific series of moves defined as $(+2, -1)$ will always achieve the same transformational effect: when starting from any major triad, reverse the mode and transpose the root by $T_6$; when starting from any minor triad, reverse the mode and transpose the root by $T_7$. In general, we may now conceptualize triadic distances in Example 35 on the total number of steps between two nodes: the fewer the number of steps, the shorter, or "smaller," the distance between triads.

I have already shown how Hook's $K(1,1)$ simply transitive subgroup of UTTs organizes numerous triadic transformations in two works by Gavin Bryars, both of which were composed in the same year. In approaching another work, then, it seems reasonable to look for manifestations of the same simply transitive subgroup. Let us consider from this perspective an excerpt from "A Man in a Room, Gambling," composed two years later (1992). The final "programme" of this work is composed for string quartet and pre-recorded voice. The quartet's arrangement consists primarily of three of the four instruments continuously arpeggiating various triads, most of which are not related by traditional tonality or fifth relations. While an analysis using $K(1,1)$ UTTs shows this passage to have a greater diversity of triadic transformations than the other passages I have analyzed, a larger transformational pattern does emerge.

The opening eleven measures expose a series of UTT-relationships that recur throughout most of the work. The following break in the texture (mm. 12-13) highlights
the UTT-chain of this opening progression, which turns out to serve as a harmonic “model” for other progressions that occur throughout the piece. I will therefore refer to these later progressions as “sequences”. The model and four sequences that follow are shown in Example 36. Chords in each of these progressions always present a UTT-chain of the form: <-, 4, 5>, <+, m, m>, <-, 2, 3>, <-, n, n + 1>, <-, 2, 3>. The unchanging UTTs involved in the chain are those that were used to generate the UTT-space of Example 35, and have been shown to be characteristic of earlier works by Bryars. The UTTs in the second and fourth positions vary throughout the piece, but they can be understood as belonging to the same group as <-, 2, 3> and <-, 4, 5>, and they always retain their respective mode-preserving or mode-reversing characteristics. I will now demonstrate how these triadic progressions can correspond to specific paths through the UTT-network created in Example 35.

Example 36: Harmonies and UTTs which define the harmonic progressions in “A Man in a Room, Gambling”
Each triad within the progressions of Example 36 lasts for two measures.\textsuperscript{38} Sequence 1 begins in measure 15, immediately following the model, and lasts through measure 26. Its repetition of the first two chords in the model (A major and C\# minor respectively) suggests that it will restate the opening progression. However, the second transformation, from the second chord to the third, is different than in the model: C\# minor to C minor = <+, 11, 11>. Nevertheless, the third transformation, as well as the last, is the same as that in the model, although the fourth transformation is not.

Sequence 2 begins when the cello takes over the melody in measure 23, following a two-measure break in the voice. Since measures 15-17 began with a repetition of the UTT series that opened the piece, one may naturally hear the <-, 2, 3> from E minor to G major in measures 23-26 as the end of this repetition. The following triads in measures 25-34, <G major, D minor, F major, Ab major, C minor>, do not seem to repeat the UTT-chain of the model – but, remarkably, they present the fixed UTTs of that chain in retrograde, overlapping with the end of the previous progression. That is, the <<<-, 4, 5>, X, <-, 2, 3>, Y, <-, 2, 3>> series is retrograded to <<<-, 2, 3>, W, <-, 2, 3>, Z, <-, 4, 5>>.

Loosely speaking, the triad-chain of Sequence 2 (mm. 23-34) is an RI-chain of that found in Sequence 1.\textsuperscript{39} Note that Sequence 2 begins with a minor triad and not a major one – a striking confirmation that the K(1,1) UTTs, and not other possible triadic transformations, are at work here.

\textsuperscript{38} The final harmony in the model appears in mm. 11 and 14, interrupted by the break of mm.12-13.
\textsuperscript{39} Lewin, GMT, 180-188.
Aurally, the beginning of Sequence 3 is not much emphasized, and a more perceptible moment occurs at measure 37, with the transformation from E major to G♯ minor. However, the characteristic transformation between these two chords, <-, 4, 5>, as well as the overall retention of the UTT-chain throughout the remainder of this progression supports hearing it begin in measure 35. Sequence 4 is between mm. 51-62, starting when the first violin resumes sixteenth-note arpeggiation, and the voice says “let’s begin…” Like Sequence 2, it presents the fixed UTTs of the model in retrograde.

Between the third and fourth sequences, there are two chords that I have not included in the progressions: G minor (mm. 47-48) and B♭ major (mm. 49-50). These four measures should be considered a transition between Sequences 3 and 4, while still supporting characteristic transformations. Example 37 shows that the chords can be understood to be related by <-, 4, 5> from the last chords of Sequence 3 and the first chord of Sequence 4, respectively. Also, they themselves are related by <-, 2, 3>, and extracting them from the work would leave a <-, 2, 3>-transformation connecting Sequences 3 and 4.

Example 37: Analysis of transition between Sequences 3 and 4

**Note** that the cello in mm.37-38 contains a B#3, which appears to contradict my labeling of those measures as G♯ minor. But both the first violin and the viola arpeggiate the complete G♯ minor triad, supporting its presence, and so the cello’s “wrong note” functions as a chromatic neighbor to the melodic C#4 that enters in measure 39.
In measures 59-65, overlapping with the last two chords of Sequence 4, there is an incomplete restatement of the model transposed by $T_1: [<B_b]$ major, D minor, $F_b$ minor, A major. The arrival of A major in measure 65 (the same harmony which opens the work) is emphasized by a textural change, and introduces the end of the work. For the present purposes, it suffices simply to note the presence of this partial sequence. Thus, Examples 36 and 37 provide a complete account of the harmonic form of the piece.

Although the repetitions of the characteristic $<-, 4, 5>$ and $<-, 2, 3>$ UTTs within each progression are striking, especially since sequences begin with either major or minor triads, this analysis leaves open the question of how the variable UTTs – the second and fourth in each sequence – function. The triadic space constructed in Example 35 can provide an interesting answer. In Example 38, I have recreated the UTT-net of Example 35, again placing A major into the $<+, 0, 0>$ position of the central node. Within this space, it is possible to trace a path of the harmonic model within the space. The five UTTs in the series are shown as five steps in the path, labeled by numbers 1 to 5. Each step comprises motion along one or both directions, and since each dimension is generated by the reiteration of a single UTT, each motion can be expressed as the number of reiterations of that UTT. Each $<-, 4, 5>$, or its inverse, corresponds to one step to the right or left on the X-axis, and each $<-, 2, 3>$, or its inverse, corresponds to one step up or down on the Y-axis. For example, step 1, the transformation from A major to $C_b$ minor ($<-, 4, 5>$) is analyzed as $(+1, 0)$, signifying one move to the right along the X-axis and no moves along the Y-axis. Similarly, the transformation from $C_b$ minor to F minor ($<+, 4, 4>$) is analyzed as $(+3, +1)$, signifying three moves to the right along the X-axis and
one move up along the Y-axis. Below the grid, the move-breakdown-by-axis is provided for each of the five moves. The entire progression involves 8 moves on the X-axis, and 4 moves on the Y-axis.

Example 38: UTT-net for harmonic model

![Diagram of UTT-net]

MODEL:  
X-moves = 8  
Y-moves = 4  
Total = 12

Examples 39a-d traces paths for Sequences 1 to 4 in the same space, and tallies the number of X- and Y-moves. The only difference between the UTT-net in Example 38 and those of Example 39 is the particular triad used to initiate the space (that which is

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41 There are other possible ways to trace paths than those given. The reason for choosing these will be evident shortly. Specifically, within the given UTT-nets, the pathway for Sequence 1 may be achieved in ten moves. The exception here is made in move 4: D# major to E minor ((+3, 0)). In the case of Sequence 2, the pathway may be achieved in six moves: G major to D minor ((−1, 0)).
input into the \(<+, 0, 0>\) node). In each case, the totals of the individual moves are \(1 + n + 1 + m + 1\), or, more specifically, \((+1, 0) + n + (0, +1) + m + (0, +1)\). Of course, this structure reflects the structure of the UTT-chain that defines our harmonic model and sequences, where the transformations in the first, third, and final positions are fixed and unchanging, and the transformation in the first position differs from those in the third and final, which are identical. However, it is very striking that, even though X- and Y-moves are not the same intervals at all, the model and every sequence can be analyzed into twelve total X- or Y-moves. In other words, even though the second and fourth UTTs in each sequence differ from those in the other, and so the six-triad series varies considerably from sequence to sequence, the sequences exhibit perfect consistency in this way. An even more striking consistency is that the (varying) second and fourth UTTs in each sequence always involve four and five X- and Y-moves, respectively, or vice versa, so that the twelve moves are always expressed as \(1 + 4 + 1 + 5 + 1\) or its retrograde.

This chapter was initiated by Hook’s suggestion to treat a simply transitive UTT group as a generalized interval system. I adhered to Lewin’s conditions for constructing a transformational space and generated a single-path UTT-network in which various triadic relationships may be explored. I then incorporated a second dimension to this space, and showed how various pathways can enable the conception of shorter distances between triads. Using this space, I demonstrated how the various harmonic progressions in Bryars’s “A Man in a Room, Gambling” may be understood as related and unified. Considering the fact that any two commuting UTTs can create two-dimensional networks, the question remains as to whether or not another simply transitive group of
UTTs would yield a more convincing analytical outcome. To explore such a question would involve exposing the work to the remaining 23 simply transitive subgroups of UTTs; a task beyond the scope of this thesis. Furthermore, in remaining consistent with the earlier analyses in this paper, I can speculate that Bryars was composing with particular “transformations” in mind – transformations that can now be accurately defined by specific UTTs. Supported by Hook’s theory, I was able to define Bryars’s “transformations” as members of the K(1,1) simply transitive group of UTTs – a group that provides traditional transposition between triads of the same mode, as well as mode-reversing operations that enable smooth transitions between chords.

Example 39a: UTT-net for Sequence 1

\[
\begin{align*}
\text{SEQ.01: } & \quad \text{X-moves = 9} & \quad \text{Y-moves = 3} & \quad \text{Total = 12} \\
1 & \quad (+1,0) & \quad 2 & \quad (+3,-1) & \quad 3 & \quad (0,+1) & \quad 4 & \quad (-5,0) & \quad 5 & \quad (0,+1)
\end{align*}
\]
Example 39b: UTT-net for Sequence 2

SEQ.02: X-moves = 4
Y-moves = 8
Total = 12

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 & 5 \\
(0, +1) & (-2, -3) & (0, +1) & (-1, +3) & (+1, 0) \\
1 & 5 & 1 & 4 & 1
\end{array}
\]

Example 39c: UTT-net for Sequence 3

SEQ.03: X-moves = 7
Y-moves = 5
Total = 12

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 & 5 \\
(+1, 0) & (-3, -1) & (0, +1) & (-3, -2) & (0, +1) \\
1 & 4 & 1 & 5 & 1
\end{array}
\]
Example 39d: UTT-net for Sequence 4

SEQ.04: \[ \begin{align*} \text{X-moves} &= 7 \\
\text{Y-moves} &= 5 \\
\text{Total} &= 12 \end{align*} \]
Chapter 5: Afterthoughts and Conclusion

Earlier in this paper, I made an analogy between a UTT-cycle and the traditional pc-clock. In further considering their similarities, one might wonder whether one could convincingly derive UTT-classes. Analogous to interval classes, a UTT-class would be a set to which a UTT and its inverse would belong. When listening to a pair of triads, we could say (as we do when listening to pcs) that, whatever order they are in, they manifest the same class of UTT. Such a conception is fairly intuitive when working with the mode-preserving members that are equal to transpositions. For example, the transformation from D major to B major is <+, 9, 9> within the K(1,1) family; the transformation from B major to D major is <+3, 3>. Just as we say the unordered pair of pcs B and D manifests ic 3, we could say that the unordered pair of triads B major and D major manifests the UTT-class <+3, 3>, which includes <+3, 3> and <+9, 9>.

The possibility of a UTT-class could allow further extensions to Hook’s theory. For example, using the progressions defined in “A Man in a Room, Gambling,” (page 55) in conjunction with a UTT-cycle that includes all 24 major and minor triads, it could be possible to create UTT-class vectors. Example 40 shows one way to do this. First (Example 40a), it shows a UTT-cycle graph generated by the UTT <-0, 1>. The position of each UTT is numbered, also indicating the associated power of <-0, 1>. The UTTs belonging to the same class are located exactly opposite to each other with respect

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42 The particular UTT in the present example was chosen for two reasons: it is completely cyclic, and its direct comparison to the traditional pc-clock, whose smallest increment is the semitone (<-, 0, 1><-, 0, 1> = <+0, 1, 1>). In regards to the second reason, every second increment in the complete <-0, 1>-cycle is equivalent to transposition by 1 – noted by the white dots on the clock face in Example 40a.
to the diagonal shown from $<+, 0, 0>$ to $<+, 6, 6>$. That is, two UTTs $<-, 0, 1>$ and $<-, 0, 1>''$ belong to the same class if $m + n = 0$, mod 24.

**Example 40a:** A UTT-cycle generated by $<-, 0, 1>$

**Example 40b:** Constructing a UTT-class vector

9. $<-,4,5>$ 17. $<-,8,9>$ 22. $<+,E,E>$ 5. $<-,2,3>$ 10. $<+,5,5>$

$\text{AM}$ $\text{C}_m$ $\text{FM}$ $A^b\text{M}$ $\text{Bm}$ $\text{DM}$

8. $<+,4,4>$ 13. $<-,6,7>$ 20. $<+,T,T>$ 1. $<-,0,1>$

$\text{C}_m$ $\text{FM}$ $A^b\text{M}$ $\text{Bm}$ $\text{DM}$

5. $<-,2,3>$ 12. $<+,6,6>$ 15. $<+,7,8>$

$\text{FM}$ $A^b\text{M}$ $\text{Bm}$ $\text{DM}$

7. $<-,3,4>$ 12. $<+,6,6>$

$A^b\text{M}$ $\text{Bm}$ $\text{DM}$

5. $<-,2,3>$

$\text{Bm}$ $\text{DM}$
Example 40b shows all the UTTs involved between pairs of chords in the harmonic model of Example 36. Each is labeled by the corresponding position number (power of \(<-, 0, 1>) on Example 40a. With six triads in the model, there are fifteen UTTs (just as a hexachord has an interval-class multiplicity of fifteen).

Let us now define a UTT-class vector as a list of 12 digits. Each digit specifies the number of instances of the UTT-class corresponding to that power of \(<-, 0, 1>)

Following the procedure taken in Example 40b, Example 41 shows the UTT-class vectors for all five progressions given in Example 36.

Example 41: UTT-class vectors for the five progressions from Example 36

<table>
<thead>
<tr>
<th>(T&lt;)</th>
<th>(T&lt;)</th>
<th>(T&lt;)</th>
<th>(T&lt;)</th>
<th>(T&lt;)</th>
<th>(T&lt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt;-,0&gt;)</td>
<td>(&lt;+,1&gt;)</td>
<td>(&lt;-,2&gt;)</td>
<td>(&lt;-,3&gt;)</td>
<td>(&lt;-,4&gt;)</td>
<td>(&lt;-,5&gt;)</td>
</tr>
<tr>
<td>1.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3.</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4.</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>5.</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

The five vectors in Example 41 provide specific information about the harmonic progressions involved in “A Man in a Room, Gambling.” Each column represents a different UTT-class, ranging from 1 to 12. As noted in the example, the boxed-in columns define specific transpositional amounts that occur within each of the progressions. For example, the harmonic model from Example 36 contains one instance of the “T<1>” UTT-class, one of the “T<2>” class, no “T<3>”, one “T<4>”, one “T<5>”, and two
“T<sub>68</sub>”. This T-class multiplicity appears in every second digit in the UTT-vector 
<110130212112>, and specifically relates to the transformations A major to Ab major 
(<+, 11, 11> = <+ , 1, 1> = UTT-class 2), B minor to C<sup>4</sup> minor (<+, 2, 2> = <+ , 10, 10> = 
UTT-class 4), C<sup>4</sup> minor to F minor (<+, 4, 4> = <+ , 8, 8> = UTT-class 8), A major to D 
major (<+, 5, 5> = <+ , 7, 7> = UTT-class 10), F minor to B minor and Ab major to D 
major (<+, 6, 6> = UTT-class 12). Note that in all cases, the UTT-relations between 
triads have been reduced to their corresponding UTT-class (meaning, the UTT that 
resides on the right-hand side of the UTT-clock in Example 40a). The same T-class 
information can be extracted from the remaining vectors as they describe the sequences 
of Example 36. For example, the vectors show that progressions 1 through 4 each contain 
one pair of triads that are related by T<sub>1</sub> (A major to Ab major, C<sup>4</sup> minor to C minor, G 
major to Ab major, and E major to Eb major), while only the first progression contains a 
pair of triads related by T<sub>5</sub> (A major to D major). In effect, the vectors provide a 
reference for comparing adjacent, as well as non-adjacent, transpositionally-related chord 
progressions.<sup>43</sup>

Example 42a shows the UTT-class vector for one of Bryars’s <-, 3, 4>-chains 
(from Example 22, page 33), and Example 42b provides the complete cycle generated by 
<- , 3, 4>. Of course, the vector shows the highest number under UTT-class 7, which 
includes the UTTs <-, 3, 4> and its inverse, <- , 8, 9>. It shows the second highest 

<sup>43</sup>Note that the UTT-class vectors given above have been derived from the <-, 0, 1> cycle. It is possible to 
generate vectors using any of the completely cyclic UTTs, however some of the “T”-classes would be 
situated differently within the vectors. By using powers of <-, 0, 1>, all of the mode-preserving UTTs 
between <+ , 0, 0> and <+ , 6, 6> are restricted to the right hand side of the clock face.
number under UTT-class 10, which includes the UTTs \(\langle+, 5, 5\rangle\) and its inverse, \(\langle+, 7, 7\rangle\).

Example 42a: UTT-class vector for a \(\langle-, 3, 4\rangle\)-chain (taken from Example 22)

\[
\begin{array}{l}
\langle+, 7, 7\rangle & \langle+, 7, 7\rangle & \langle+, 7, 7\rangle & \langle+, 7, 7\rangle & \langle+, 7, 7\rangle \\
E^b_m & GM & B^b_m & DM & Fm & AM & Cm \\
\langle-, 3, 4\rangle & \langle-, 3, 4\rangle & \langle-, 3, 4\rangle & \langle-, 3, 4\rangle & \langle-, 3, 4\rangle & \langle-, 3, 4\rangle \\
\end{array}
\]

\[
\begin{array}{cccccccc}
(T_1) & (T_2) & (T_3) & (T_4) & (T_5) & (T_6) \\
\langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle & \langle-, 2, 3\rangle & \langle+, 2, 3\rangle \\
\langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle & \langle-, 3, 4\rangle & \langle+, 3, 4\rangle \\
\langle 0 \rangle & \langle 0 \rangle & \langle 4 \rangle & \langle 3 \rangle & \langle 0 \rangle & \langle 1 \rangle & \langle 6 \rangle & \langle 0 \rangle & \langle 0 \rangle & \langle 5 \rangle & \langle 2 \rangle & \langle 0 \rangle \\
\end{array}
\]

Example 42b: UTT-cycle generated by \(\langle-, 3, 4\rangle\)
In such a case, the UTT-vector supports the sequential nature of the progression as a whole, as there is a different multiplicity of each UTT-class that is present. More specifically, the vector is following the path around the <-, 3, 4> cycle, and the amounts of each vector entry refer to steps along that path. Therefore, if one more chord were added to the end of this chain (specifically, E major), then each UTT-class present in the vector would increase by one, and a new UTT-class would become active (specifically, UTT-class 1, which includes <-, 0, 1>—the next step in the <-, 3, 4>-cycle—and its inverse, <-, 11, 0>), and so on.

Though potentially interesting, the concept of a UTT-vector raises the concern of how one may learn to “hear” the mode-reversing transformations, as they affect major and minor triads differently. This remains a question for further research.

Throughout this paper, I have used Julian Hook’s theory of uniform triadic transformations to account for triadic changes in three works by Gavin Bryars. In particular, I have shown that specific members of the K(1,1) simply transitive subgroup are characteristic of nearly every passage. I have demonstrated Bryars’s tendency to distort traditional harmonic progressions with intermediate chords that, although they do not participate in traditional tonal progressions, nevertheless maintain a consistent flow and structure. As a result, I can suggest ways of interpreting, and hearing, these passages that may not have been initially evident. Next, I have extended the applicability of UTTs by constructing a multidimensional space in which various triadic relationships may be interpreted and understood. In conjunction with the laws of simple transitivity, these UTT-networks can offer a way of interpreting “distance” between major and minor triads.
simultaneously. Within such a space, I was able to account for the seemingly unrelated chord progressions in “A Man in a Room, Gambling,” and reveal harmonic relationships that could have otherwise gone unnoticed. I believe that UTT-networks could similarly account for various other types of contemporary, unorthodox chord progressions in any musical genre that makes extensive use of triads, including popular music and jazz. In conclusion, with the support of Hook’s theory, I have clarified many of Bryars’s allegedly “non-functional” harmonic progressions, giving evidence that they are well-planned and structurally, as well as “blissfully,” coherent.


