Essays on Exchange Rate Volatility and Optimal Monetary Policy

by

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Abstract

This thesis consists of three essays on exchange rate behavior and optimal monetary policy in open economy.

The first essay proposes a framework to explain why the nominal and real exchange rates are highly volatile and seem to be disconnected from macroeconomic fundamentals. Two types of foreign exchange traders, rational traders and noise traders with erroneous stochastic beliefs, are introduced into a dynamic general equilibrium model with sticky prices. The presence of noise traders creates deviations from uncovered interest parity. As a result, exchange rates can diverge significantly from fundamental values. Combined with local currency pricing and consumption smoothing behavior in an infinite horizon model, the presence of noise traders can help to explain the "exchange rate disconnect puzzle".

The second essay explores the optimal monetary policy response to domestic and foreign technology shocks in an open economy with vertical structure of production and trade. Through the vertical linkage in production, any stage-specific productivity shock in one country has a trans-border spillover effect on the other country via vertical trade. So when choosing the optimal monetary rules, each monetary authority should respond to both home and foreign productivity shocks. Another finding is that the flexible exchange rate can not replicate the flexible price equilibrium even under producer currency pricing due to price stickiness in multiple stages. Finally, the exchange rate in such an environment will be more stable than that of an economy without vertical structure of production and trade.

The third essay analyzes the determination of monetary policy in a world with a dollar standard, defined here as an environment in which all traded goods prices are set in US dollars. This generates an asymmetry whereby exchange rate pass-through into the US CPI is zero, while pass-through to other countries will be positive. I find that in such an economy, the US is essentially indifferent to exchange rate volatility in setting monetary policy, while the rest of the world places a high weight on exchange rate volatility. More importantly, in a Nash equilibrium of the monetary policy game between the US and the rest of the world, the preferences of the US dominate. Despite this, the US loses from the dollar's role as an
international currency due to the absence of exchange rate pass-through even though US preferences dominate world monetary policy.

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Summary

This thesis consists of three essays on exchange rate behavior and optimal monetary policy in open economy.

A central puzzle in international macroeconomics over the last 20 years is that real exchange rates are volatile and persistent. Furthermore, the exchange rate seems to "have a life of its own", being disconnected from other macroeconomic variables. The answer to this puzzle will help us to understand whether a fixed exchange rate regime will be more desirable in an open economy. The first essay proposes a framework to explain why the nominal and real exchange rates are highly volatile and seem to be disconnected from the macroeconomic fundamentals. Two types of foreign exchange traders, rational traders and noise traders with erroneous stochastic beliefs, are introduced into the dynamic general equilibrium framework of the new open economy macroeconomic literature. The presence of noise traders creates deviations from the uncovered interest parity. As a result, exchange rates can diverge significantly from the fundamental values. Combined with local currency pricing and consumption smoothing behavior in an infinite horizon model, the presence of noise traders can help to explain the "exchange rate disconnect puzzle". Then it is shown that the excess exchange rate volatility caused by the presence of noise traders can be reduced by the 'Tobin tax' type of exchange rate policies.

The second essay explores the optimal monetary policy response to domestic and foreign technology shocks in an open economy with vertical structure of production and trade. That is, countries use imported intermediate goods as an input to produce export goods. In other words, countries are linked sequentially in the production of final goods via trade. Thus, any stage-specific productivity shock in one country has a trans-border spillover effect on the other country via vertical trade. So when choosing the optimal monetary rules, each monetary authority should respond to both home and foreign productivity shocks. Another finding is that the flexible exchange rate can not replicate the flexible price equilibrium even under producer currency pricing due to price stickiness in multiple stages. Finally, the exchange rate in such an environment would be more stable than that of an economy without
vertical structure of production and trade. These findings suggest that the changes in the trade pattern in the global economy over the last thirty years might affect the international optimal monetary policy rules and values of exchange rate flexibility. So the monetary policy maker should take into account the impact of the changes in the trade pattern when making decisions.

The third essay analyzes the determination of monetary policy in a world with a dollar standard, defined here as an environment in which all traded goods prices are set in US dollars. This generates an asymmetry whereby exchange rate pass-through into the US CPI is zero, while pass-through to other countries will be positive. I show that monetary policy in such a setting does seem to accord with popular discussion. In particular, the US is essentially indifferent to exchange rate volatility in setting monetary policy, while the rest of the world places a high weight on exchange rate volatility. More importantly, in a Nash equilibrium of the monetary policy game between the US and the rest of the world, the preferences of the US dominate. That is, the equilibrium is identical to one where the US alone chooses world monetary policy. Despite this, I find surprisingly that the US loses from the dollar’s role as an international currency. Even though US preferences dominate world monetary policy, the absence of exchange rate pass-through means that US consumers are worse off than those in the rest of the world, where exchange rate pass-through operates efficiently. Finally, the conditions for a dollar standard to exist is derived.
Chapter 1

Noise Traders and the Exchange Rate Disconnect Puzzle

1.1 Introduction

A central puzzle in international macroeconomics over the last 20 years is that real exchange rates are volatile and persistent. Furthermore, as Flood and Rose (1995) have elegantly documented, the exchange rate seems to “have a life of its own”, being disconnected from other macroeconomic variables. For example, Mussa (1986), Baxter and Stockman (1989) and Flood and Rose (1995) all find that both nominal and real exchange rates are highly volatile, especially when compared to macroeconomic fundamentals, such as relative price level, consumption and outputs. Exchange rate volatility also varies substantially over time. Obstfeld and Rogoff (2000a) state this kind of “exceedingly weak relationship between the exchange rate and virtually any macroeconomic aggregates” as the “exchange rate disconnect puzzle”.

This irregularity casts some doubts on the traditional monetary macroeconomic model of exchange rates, which assumes that purchasing power parity (PPP) holds. With PPP, the “expenditure-switching” effect of exchange rate changes will lead to substitution between domestically-produced goods and internationally-produced goods. It implies that exchange rate volatility will be transferred to macroeconomic fundamentals. Nevertheless, empirical evidence \(^1\) indicates that nominal exchange rate changes are not fully passed through to goods prices. Motivated by this evidence, Betts and Devereux (1996, 2000) introduce local currency pricing into the baseline Redux model developed by Obstfeld and Rogoff (1995). They assume that firms can charge different prices for the same goods in home and foreign markets and that the prices are sticky in each country in terms of the local currency. This allows the real exchange rate to fluctuate, and delinks the home and foreign price levels.

Although the new open economy macroeconomic models with sticky prices, imperfect

\(^1\)See Engel (1993, 1999) and Parsley and Wei (2001) for details.
competition and local currency pricing can generate volatile exchange rate movements\(^2\), they typically predict a strong counterfactual relationship between the real exchange rate and relative consumption \(^3\). A monetary shock simultaneously raises domestic consumption (by more than it raises foreign consumption) and creates a (temporary) depreciation of home currency. Consequently, these models almost generically predict a strong positive correlation between depreciation and relative consumption, which is not observed empirically. \(^4\)

One explanation for this discrepancy might lie in the fact that the nominal exchange rate is also an asset price, and therefore will be inevitably affected by imperfections in the financial markets. These imperfections may include herd behavior, momentum investing and noise traders. Working together with sticky prices, these are all important reasons to explain why the real exchange rate persistently deviates from the level predicted by the fundamentals-based models. A large body of evidence has documented strong heterogeneity in market participants' expectations in the foreign exchange markets \(^5\). Evans and Lyons (2002) show that most of the short-run exchange rate volatility is related to order flow, which also reflects the heterogeneity in investors' expectations. Although financial economists care about high frequency data, while international macroeconomists focus more on low frequency data, it is still surprising how little the microstructure of real world foreign exchange markets has been considered in the macroeconomic theory of exchange rates.

This raises another question: if exchange rate volatility is caused by erroneous beliefs and could be reduced without incurring costs due to other macroeconomic volatilities, then floating exchange rates may be too volatile and costly from a welfare point of view. However, it is impossible to make any policy recommendations in the absence of a welfare-based model which can explain exchange rate volatility and its relationship with macroeconomic

\(^2\)A high risk aversion coefficient of household (about 5) is usually required in these models to reproduce the data's volatility of real exchange rate relative to output. See Chari, Kehoe and McGrattan (2002).

\(^3\)See, for example, Chari, Kehoe and McGrattan (2002).

\(^4\)Benigno and Thoenissen (2003) report the correlation between bilateral exchange rate and bilateral relative consumption for seven countries (Canada, France, West Germany, Italy, Japan, U.K. and U.S.) for the periods starting from 1970 until 2002. The cross-correlation varies between –0.45 and 0.42.

Therefore, our paper intends to propose a new approach to study exchange rates, that combines the macroeconomic model of exchange rates and the microstructure approach of foreign exchange markets. This approach is implemented within a specific model, where noise traders are introduced into the new open economy macroeconomic framework. The combination is helpful for understanding the behavior of exchange rates and their relationship with macroeconomic fundamentals. It also gives more rigorous microeconomic foundations to the "noise trader" approach and enriches the new open economy macroeconomic framework with a more realistic setting of the microstructure of foreign exchange market. In addition, it provides a well-defined framework for policy evaluations, especially for those policies that are designed to control non-fundamental volatilities.

We adapt the overlapping-generation noise trader model of De Long et al. (1990). Two types of foreign exchange traders are introduced into the general equilibrium framework. One type is the "rational/informed trader", which has rational expectations about future investment returns, while the other type cannot forecast the future returns correctly and is called the "noise trader".

The results from the model show that when the number of noise traders increases, so does the exchange rate volatility. Nevertheless, the volatilities of macroeconomic fundamentals (except for the net foreign assets) are completely independent of the noise component on the foreign exchange market. Therefore, our model can generate a relative volatility of real exchange rate to output close to the data, even for a low risk aversion coefficient. Moreover, since in this model nominal and real exchange rate fluctuations can be generated by erroneous belief of noise traders, our model does not predict a strong comovement of exchange rates and fundamentals. Therefore, it is possible to explain the "exchange rate disconnect puzzle" by the approach suggested in this paper.

The basic intuition behind our results is as follows. The heterogeneity in beliefs among foreign exchange traders creates the basis for trading volume and deviations from the uncovered interest parity. Arbitrage does not eliminate the effect of noise here because noise
itself creates risk: short-horizon investors must bear the risk that they may be required to liquidate their positions at a time when asset prices are pushed even further away (by noise traders) from the fundamental values than when investment was made. Therefore, exchange rates can diverge significantly from the fundamental values. The greater the number of noise traders, the more volatile will be the exchange rates.

However, why is the exchange rate volatility not transferred to macroeconomic fundamentals? Normally, there are two channels through which the exchange rate affects macroeconomic variables: the expenditure-switching effect and the wealth effect (through firms’ profits). Under the assumption of local currency pricing, the expenditure-switching effect is eliminated as the relative price of home and foreign goods does not change. Although the wealth effect still exists, it turns out to be quite small quantitatively. This is because the wealth effect of exchange rate change is spread out over current and future periods through intertemporal consumption smoothing, and so tends to be very small.

Many economists have suggested that increasing the trading cost on the foreign exchange market might reduce the exchange rate volatility. To understand the effect of this kind of exchange rate policies, the size of the noise component is endogenized by introducing a heterogenous entry cost for noise traders. Only noise traders having entry costs that are sufficiently low will choose to enter the foreign exchange market. We find that given the number of potential noise traders, increasing the entry cost will reduce exchange rate volatility. We also analyze a ‘Tobin tax’ type of exchange rate policy suggested by Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995) in an extension of the baseline model. We find that a Tobin tax will decrease the exchange rate volatility, however, the size of the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

The microstructure of the exchange rate market in this paper follows the noise trader literature, especially the work of Jeanne and Rose (2002), which also focuses on the relationship between exchange rate volatility and noise traders. However, the macroeconomic part of their model is a simple monetary model of exchange rates with PPP. Neither nominal
rigidities nor pricing to market is considered. Moreover, intertemporal optimizing agents and profit maximizing firms are not considered in their model. Another feature of their model is that it is a partial equilibrium model without explicit welfare specifications for households, so rigorous policy analysis is impossible.

This paper is also closely related to the new open economy macroeconomic literature. The paper that is closest, in spirit, to our analysis of exchange rate disconnect puzzle is Devereux and Engel (2002). They stated that the key ingredients to explain the exchange rate disconnect puzzle include: local currency pricing to eliminate the expenditure-switching effect, a special structure of international pricing and product distribution to minimize the wealth effect, incomplete international financial markets, and stochastic deviations from the uncovered interest parity. The analysis in this paper differs in the following aspects. First, more microeconomic foundations of noise traders are explored. Both noise traders and rational traders in our model are risk averse and utility maximizing agents, therefore, policy analysis is possible in our model. Second, we show that, the wealth effect of exchange rate changes may be quite small, quantitatively, in an infinite horizon model. Therefore, the exchange rate disconnect puzzle can be explained even without a specific assumption of production and distribution structure to remove wealth effects.

This paper is organized as follows. In Section 2, we construct a model that embeds noise traders into a new open economy macroeconomic framework. Both the exogenous entry and endogenous entry specifications are explored. Section 3 features of the solution to the model are discussed. Section 4 gives the results of the model. Section 5 extends the baseline model to analyze the implications of Tobin tax. The paper concludes with a brief summary and suggestions for subsequent research.

1.2 The model

The world economy consists of two countries, denominated by home and foreign. Each country specializes in the production of a composite traded good. Variables in the foreign country are denoted by an asterisk. In addition, a subscript $h$ denotes a variable originating
from the home country; a subscript $f$ denotes a variable used in the foreign country.

This model is analogous to most new open economy macroeconomic models except for the foreign exchange market. Each country is populated by a large number of atomistic households, a continuum of firms that set prices in advance, and a government (a combined fiscal and monetary authority). However, we assume that home and foreign households can only trade nominal bonds denominated in their domestic currency. Although home households cannot access the international bond market, the foreign exchange traders can carry out international bond trading to maximize their utility. Thus, besides the infinitely lived household, a second type of representative agent is introduced into the model, namely, the foreign exchange trader, who lives in an overlapping-generation demographic structure. Hereafter, a superscript $H$ denotes households and a superscript $T$ stands for traders. In the foreign country, for simplicity, it is assumed that only one type of representative agent is present; the foreign household.\footnote{Introducing foreign exchange trader into the foreign country will not change the main results.}

### 1.2.1 Households, Firms and Government

The lifetime expected utility of the home representative household is:

$$
\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_H^t)^{1-\rho}}{1-\rho} + \frac{1}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} L_t^{1+\psi} \right] \right\} 
$$

Subject to

$$
P_t C_H^t + B_{t+1} + M_t = W_t L_t + \Pi_t + M_{t-1} + T_t + B_t (1 + r_t)
$$

where $C_H^t$ is the time $t$ composite consumption of home households, composed by a continuum of home goods and foreign goods; both are of measure 1. Let $C_T^t$ denote the composite consumption of traders, then $C_T^t + C_H^t = C_t$, where $C_t$ is the composite consumption of the home country and is given by:

$$
C_t = \left[ \omega \gamma C_{h,t}^{2\gamma-1} + (1-\omega) \gamma C_{f,t}^{2\gamma-1} \right]^{\gamma^{-1}}
$$
where \( C_{h,t} = \left( \int_0^1 C_{h,t}(i)^{-\theta} di \right)^{1-\theta}, \) \( C_{f,t} = \left( \int_0^1 C_{f,t}(j)^{-\theta} dj \right)^{1-\theta}, \) and the weight \( \omega \in (0,1) \) determines the home representative agent's bias for the domestic composite good. Note that \( \theta \) is the elasticity of substitution between individual home (or foreign) goods and \( \gamma \) is the elasticity of substitution across home and foreign composite goods.

\( P_t \) is a consumption based price index for period \( t \), which is defined by:

\[
P_t = \left[ \omega P_{h,t}^{1-\gamma} + (1 - \omega) P_{f,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}
\]

where \( P_{h,t} = \left( \int_0^1 P_{h,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \) and \( P_{f,t} = \left( \int_0^1 P_{f,t}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \).

In each period every household is endowed with one unit of time, which is divided between leisure and work. His income is derived from the labor income \( W_t L_t \), profits from domestic goods producers (which is assumed to be owned by domestic households) \( \Pi_t \), interest received on domestic bonds \( I_t (1 + r_t) \) and lump-sum government transfer \( T_t \). Solving the household's problem, the optimality conditions can be written as:

\[
\left( \frac{M_t}{P_t} \right)^\psi = \frac{(C_t^H)^\rho}{1 - \frac{1}{1+r_t+1}}
\]

\[
\eta L_t^\psi = \frac{W_t}{P_t (C_t^H)^\rho}
\]

\[
\beta E_t \frac{(C_t^H)^\rho}{(C_{t+1}^H)^\rho} \frac{P_t}{P_{t+1}} = \frac{1}{1 + r_{t+1}}
\]

The first order conditions of the foreign households are entirely analogous, except that foreign household's consumption is denoted by \( C_t^f \), as there is only one type of representative agent in the foreign country.

We assume firms have linear technologies, for each home good \( i \), \( y_t(i) = L_t(i) \). It is also assumed that, due to high costs of arbitrage for consumers, each individual monopolist can price discriminate across countries. Furthermore, as in Betts and Devereux (1996) and Chari, Kehoe and McGrattan (2002), we assume local currency pricing: firms set prices (separately) in the currencies of buyers. Finally, prices are assumed to be set one period in advance and cannot be revised until the following period. That is, the home monopolist sets \( P_{h,t}(i) \) and
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$P_{h,t}^*(t)$ optimally at the end of period $t-1$, and these prices cannot be changed during time $t$.

Appendix A.1 gives the derivation of the optimal pricing schedule of firms. The firms will just set the price so that it equals to a mark-up over the expected marginal cost and a risk premium term arising from the covariance of the firm's profits with the marginal utility of consumption:

$$
P_{h,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t W_t C_t \right]}{E_{t-1} \left[ D_t C_t^* \right]} \quad \text{and} \quad P_{h,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[ D_t W_t C_t^* \right]}{E_{t-1} \left[ D_t S_t C_t^* \right]}$$

where $D_t$ and $D_t^*$ denote the pricing kernels households used to value date $t$ profits. Because all home firms are assumed to be owned by the domestic households, it follows that in equilibrium $D_t$ is the intertemporal marginal rate of substitution in consumption between time $t-1$ and $t$:

$$D_t = \beta \frac{(C_t^H)^{-\rho}}{(C_{t-1}^H)^{-\rho}} \frac{P_{t-1}}{P_t}$$

$D_t^*$ is defined analogously. $S_t$ is the nominal exchange rate at time $t$.

The home government issues the local currency, has no expenditures, and runs a balanced budget every period. The nominal transfer is then given by:

$$T_t = M_t - M_{t-1}$$

The stochastic process that describes the evolution of the domestic money supply is:

$$M_t^* = \mu_t M_{t-1}^*$$

where $\epsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$ is a normally distributed random variable. The stochastic process of money supply in the foreign country is entirely analogous. Also, the home monetary shock and the foreign monetary shock are assumed to be independently distributed, that is, $\text{Cov}(\epsilon_\mu, \epsilon_\mu^*) = 0$. 
1.2.2 Foreign Exchange Market

Foreign Exchange Traders

Following closely the work of De Long et al. (1990) and Jeanne and Rose (2002), the foreign exchange traders are modelled as overlapping generations of investors who decide how many one-period foreign nominal bonds to buy in the first period of their lives. Traders have the same taste, but differ in their abilities to trade in the foreign bond market. Some of them are able to form accurate expectations on risk and returns, while others have noisy expectation about future returns. The former are referred as the "rational trader" and the latter as the "noise traders". Hereafter, the informed trader is denoted by a superscript $I$ and the noise trader is denoted by a superscript $A$.

Two specifications of the model are developed. In the first specification, the number of incumbent noise traders is exogenously determined. In the second one, the traders have to pay a fixed entry cost to trade on the foreign exchange market. The introduction of an entry cost helps to endogenize the noise component of the market. This makes the policy analysis possible as policy makers can affect the composition of traders through the entry cost.

In the foreign exchange market, at each period, a generation of foreign exchange traders is born. The continuum of the traders is indexed by $i \in [0,1]$. Assuming that in each generation of traders, $N_I$ of them are rational traders, and $1 - N_I$ are noise traders. The timing of the model is illustrated in Figure 1.1.

Figure 1.1: Timing of Model

\[
\begin{array}{ccc}
  t & t+1 \\
  \text{Action 1} & \text{Action 2} & \text{Action 3} \\
\end{array}
\]

Action 1: Time $t$ foreign exchange trader $i$ is born; Time $t$ shocks and nominal interest rates are revealed; The time $t$ born trader $i$ decides if he should enter the foreign bond market.
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Action 2: He decides the number of foreign currency bonds $B^*_{h,t+1}(i)$ to purchase based on his expectation about future exchange rate $S_{t+1}$. To finance his purchase, he borrows $B^*_{h,t+1}(i)S_t$ from the home bond market.

Action 3: Time $t + 1$ exchange rate $S_{t+1}$ is revealed, so the return of his investment in terms of home currency is realized, which equals to $S_{t+1}B^*_{h,t+1}(i)(1 + r^*_{t+1})$. He pays back the principle and interest of his borrowing($B^*_{h,t+1}(i)S_t(1 + r_{t+1})$), gets the excess return, consumes, and dies.

Let $\varphi_t^i$ denote the dummy variable characterizing the market-entry condition of period $t$ born foreign exchange trader $i$. If $\varphi_t^i = 0$, trader $i$ will not enter the foreign bond market and if $\varphi_t^i = 1$, he will enter. At the beginning of period $t$, trader $i$ will enter the market as long as the expected utility of entering the market is higher than that of not entering:

$$E^i_t(U_t^i | \varphi_t^i = 1) \geq E^i_t(U_t^i | \varphi_t^i = 0) \quad (1.2.14)$$

A foreign exchange trader who has entered the foreign bond market maximizes a mean-variance utility function:

$$\max_{B^*_{h,t+1}(i)} E^i_t(C^T_{t+1}(i)) - \frac{a}{2} Var^i_t(C^T_{t+1}(i)) \quad (1.2.15)$$

Subject to

$$P_{t+1}C^T_{t+1} = [B^*_{h,t+1}(i)(1 + r^*_{t+1})S_{t+1} - B^*_{h,t+1}(i)S_t(1 + r_{t+1})] - P_{t+1}c_i \quad (1.2.16)$$

where $B^*_{h,t+1}(i)$ denotes the amount of one-period foreign currency bonds held by trader $i$ from time $t$ to time $t + 1$, $a$ is the absolute risk aversion coefficient, the cost $c_i$ reflects the costs associated with entering the foreign bond market for trader $i$.

The entry costs may include tax, information costs for investment in the foreign bond market, and other costs when investing abroad.\footnote{These costs may be modelled in many ways. In this paper, the entry costs are assumed to be resource-consuming in the sense that it consumes the composite consumption good.} To formalize this heterogeneity, here we follow the specification used by Jeanne and Rose (2002). Rational traders are assumed to have a larger stock of knowledge about the economy and thus, do not need to invest in the
acquisition of information. Their entry costs are therefore zero. For noise traders, they do not have a natural ability to acquire and process the information about the economy and therefore have to pay an entry cost that is greater than zero.

Although the preferences of the noise traders are the same, the noise traders are assumed to be distinguished from each other by their entry costs. Without loss of generality, the noise traders are indexed by increasing entry costs:

\[ c_i = \tilde{c} \left\{ \frac{i}{1 - N_I} \right\}^\alpha \text{ for } i \in [0, 1 - N_I] \]  

(1.2.17)

where \( \alpha > 0 \) is the curvature parameter and \( \tilde{c} \) is the parameter characterizing the scale or level of the entry cost of the noise traders. Thus, the noise trader at the left end of the continuum (\( i \) near 0) tends to have a lower entry cost and the noise trader towards the right end of the continuum (\( i \) near \( 1 - N_I \)) has a higher entry cost.

**Optimal demand for foreign bond**

Once the traders have decided to enter the market, the optimal demand for foreign bonds of each type of traders can be derived. Substituting Equation 1.2.16 into Equation 1.2.15, gives:

\[
\max_{B^*_{h,t+1}(i)} E_t \left[ \frac{B^*_{h,t+1}(i)}{P_{t+1}} \right] S_t (1 + r_{t+1}) \rho_{t+1} - c_i - \frac{\alpha}{2} \text{Var}_t \left[ \frac{B^*_{h,t+1}(i)}{P_{t+1}} S_t (1 + r_{t+1}) \rho_{t+1} - c_i \right] 
\]

(1.2.18)

where \( \rho_{t+1} = \left[ \frac{S_{t+1} (1 + r_{t+1})}{S_t (1 + r_{t+1})} \right] - 1 \) is the excess return.

We now discuss the information structure of traders. Specifically, we make the following assumptions about the subjective distribution over \( \rho_{t+1} \). The rational traders can predict \( \rho_{t+1} \) correctly; while the noise traders cannot predict the future excess return correctly. That is, for informed traders:

\[
E_t[\rho_{t+1}] = E_t[\rho_{t+1}] \\
\text{Var}_t[\rho_{t+1}] = \text{Var}_t[\rho_{t+1}] 
\]

(1.2.19)

(1.2.20)
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For noise traders, following the work of De Long et al. (1990), we assume:

\[ E^N_t[\rho_{t+1}] = E_t[\rho_{t+1}] + v_t \]  
\[ \text{Var}_t^N[\rho_{t+1}] = \text{Var}_t[\rho_{t+1}] \]  
\[ \text{Var}(v_t) = \lambda \text{Var}(s_t) \quad \text{where} \quad \lambda \in (0, +\infty) \]

where \( v_t \) is assumed to be \( i.i.d \) and normally distributed with zero mean. \( \lambda \) can be considered as a parameter characterizing the relative magnitude of noise traders' erroneous beliefs to exchange rate volatility.

From Equations 1.2.21 and 1.2.19, it can be seen that, compared to the rational trader's expectation, the noise traders' expectation of \( \rho_{t+1} \) based on time \( t \) information is biased from the true conditional expectation by a random error. Nevertheless, noise traders can correctly forecast the conditional variance of the exchange rate. From Equation 1.2.23, another assumption is made that the unconditional variance of \( v_t \) is proportional to the unconditional variance of the exchange rate itself. This assumption helps to tie down the scale of the volatility of noise traders' erroneous beliefs.\(^8\)

Solving Equation 1.2.18, the optimal bond holding of trader \( i \) is given by:

\[ B^*_t[\rho_{t+1}] = \frac{E_t^i[\rho_{t+1}]}{a_t \frac{s_i}{p_t+1}(1 + \tau_t+1)\text{Var}_t[\rho_{t+1}]} \]

Therefore, informed traders and noise traders differ in their optimal bond holding. Also, from Equation 1.2.24, the lower the expected excess return, the higher the risk (excess return volatility) and the risk coefficient, the less bond traders (both rational traders and noise traders) will hold. Thus, the traders account for risk when taking positions on assets. At the margin, the return from enlarging one's position in an asset that is mispriced (the expected excess return) is offset by the additional price risk (the volatility of excess return) that must be borne.

\(^8\)The logic behind this assumption is that the bias in noise traders' expectation must be related to the volatility of the exchange rate itself, otherwise noise traders might expect the future exchange rate to be volatile even under a fixed exchange rate regime.
Chapter 1. Noise Traders and the Exchange Rate Disconnect Puzzle

Equilibrium condition of the foreign exchange market

Analysis with no entry costs  We first analyze a simple case where $c = 0$. Thus, all the noise traders will enter the market and the noise component of the market is exogenously determined by the number of existing noise traders $(1 - N_I)$ on the market. So the aggregate demand for foreign bonds by foreign exchange traders of the home country can be denoted as:

$$B_{h,t+1}^* = N_I B_{h,t+1}^{I*} + (1 - N_I) B_{h,t+1}^{N*}$$

$$= \frac{E_t \left[ \frac{S_{t+1} (1 + r_{t+1})}{S_t (1 + r_{t+1})} - 1 \right]}{a \frac{S_t}{P_{t+1} (1 + r_{t+1})} Var_t (\rho_{t+1})} + (1 - N_I) \sigma_t$$

$$\Rightarrow E_t \left[ \frac{S_{t+1} (1 + r_{t+1})}{S_t (1 + r_{t+1})} - 1 \right] + (1 - N_I) \sigma_t - a \frac{S_t}{P_{t+1} (1 + r_{t+1})} Var_t (\rho_{t+1}) B_{h,t+1}^* = 0 \quad (1.2.26)$$

Endogenous entry of noise traders  We now endogenize the composition of traders who enter the market in each period by introducing positive entry costs for noise traders. The entry decision for informed traders is trivial. They bear no entry cost and always enter the foreign bonds market in equilibrium. A noise trader, however, enters if and only if Equation 1.2.14 is satisfied. As shown in Appendix B.2, for trader $i$, this condition takes the form:

$$d < l E_i \sigma_t$$

$$\Rightarrow c_i \leq \frac{[E_t (\rho_{t+1})]^2}{2 a Var_t (\rho_{t+1})} \equiv GB_t^N \quad (1.2.27)$$

where $GB_t^N$ is the gross benefit of entry for noise traders. It increases with the expected excess return and decreases with the conditional time $t + 1$ exchange rate volatility. Note that in our general equilibrium setting, both terms are functions of the number of incumbent noise traders.

Let $c_i^* = GB_t^N$ be the cut-off value of entry cost. From Equation 1.2.27, for noise trader $i$, if $c_i \leq c_i^*$, $\varphi_i = 1$; if $c_i \geq c_i^*$, $\varphi_i = 0$. The number of incumbent noise traders $n_t$ is then given by:

$$n_t = \left( \frac{c_i^*}{c} \right)^{\frac{1}{\sigma}} (1 - N_I) = \left( \frac{[E_i (\rho_{t+1})]^2}{2 a Var_t (\rho_{t+1}) c} \right)^{\frac{1}{\sigma}} (1 - N_I) \quad (1.2.28)$$
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Apparently, the number of active noise traders on the market increases with the square of the expected excess return and the number of existing noise traders, and decreases with the entry cost, the risk aversion coefficient \( a \), and the excess return volatility. The economic intuition behind Equation 1.2.28 is as follows. The presence of more active noise traders creates higher expected excess return and incentives for other noise traders to enter the market, however, the extra volatility brought about by their entry will reduce the gross benefit of entry for noise traders. In equilibrium, the two effects balance and no more noise traders will enter.

Substituting Equation 1.2.28 into \( B_{h,t+1}^* = N_I B_{h,t+1}^{I,*} + n_t B_{h,t+1}^{N,*} \), we can derive the equilibrium condition of the foreign bond market when the entry decision of traders is endogenized:

\[
E_t \left[ \frac{S_{t+1}(1 + r_{t+1})}{S_t(1 + r_{t})} - 1 \right] + \frac{(\frac{a^2}{2})^{\frac{1}{2}}(1 - N_I)v_t}{N_I + (\frac{a^2}{2})^{\frac{1}{2}}(1 - N_I)} - a \frac{S_t(1 + r_{t+1})}{P_{t+1} \left[ N_I + (\frac{a^2}{2})^{\frac{1}{2}}(1 - N_I) \right]} Var_t(\rho_{t+1})B_{h,t+1}^* = 0
\]

Equations 1.2.26 and 1.2.29 represent the interest parity conditions in this economy. Note that the uncovered interest parity does not hold in this model. The last two terms in Equations 1.2.26 and Equation 1.2.29 show the deviation from the uncovered interest parity when noise traders are present in the market. This deviation consists of two parts: the expectation error of the noise traders, and the risk premium term, since the foreign exchange traders are risk averse.

In our model, as in De Long et al. (1990), the noise traders can "create their own space": the uncertainty of the noise traders' expectations over the future exchange rate increases the risk borne by informed traders engaged in arbitrage against noise traders. The aversion to this risk will severely limit arbitrage, especially in an overlapping-generation framework. Short-horizon investors must bear the risk that they may be required to liquidate their positions at a time when asset prices are pushed even further away (by noise traders) from the fundamental values than when the investment was made. Therefore, as shown in Section 3, exchange rate can diverge significantly from the fundamental values.
1.2.3 Equilibrium Condition

Equilibrium for this economy is a collection of 26 sequences \((P_t, P_t^*, P_{h,t}, P_{h,t}^*, P_{f,t}, P_{f,t}^*, C_t, C_t^T, C_t^H, C_t^*, C_t^*, C_{h,t}, C_{h,t}^*, C_{f,t}, C_{f,t}^*, S_t, r_t, r_t^*, D_t, D_t^*, W_t, W_t^*, B_t, B_t^*, B_{h,t}, B_{f,t}^*, L_t, L_t^*)\) satisfying 26 equilibrium conditions. They include the six household optimality conditions (Equations 1.2.5, 1.2.6, 1.2.7 and their foreign counterparts), the definition of the price indexes (Equation 1.2.4 and its foreign analogy), the definition of the pricing kernel (Equation 1.2.10 and its foreign analogy), the interest parity conditions (Equation 1.2.26 or Equation 1.2.29), the four individual demand equations, the four pricing conditions, and the four market clearing conditions for the bonds and goods markets:

\[
B_{t+1} = S_t B_{h,t+1}^* \quad \forall t \tag{1.2.30}
\]

\[
B_t^* + B_{h,t}^* = 0 \tag{1.2.31}
\]

\[
L_t = C_{h,t} + C_{h,t}^* \tag{1.2.32}
\]

\[
L_t^* = C_{f,t} + C_{f,t}^* \tag{1.2.33}
\]

Finally, the budget constraint of the foreign exchange traders.

\[
P_t C_t^T = B_{h,t}^* (1 + r_t^*) S_t - B_{h,t}^* S_{t-1} (1 + r_t) \quad \text{Exogenous Entry} \tag{1.2.34}
\]

\[
P_t C_t^T = [B_{h,t}^* (1 + r_t^*) S_t - B_{h,t}^* S_{t-1} (1 + r_t)] - P_t \sum_{i=0}^{n_t} c_t \quad \text{Endogenous Entry} \tag{1.2.35}
\]

And the home country aggregate consumption equation:

\[
C_t = C_t^H + C_t^T \quad \text{Exogenous Entry} \tag{1.2.36}
\]

\[
C_t = C_t^H + C_t^T + \sum_{i=0}^{n_t} c_t \quad \text{Endogenous Entry} \tag{1.2.37}
\]
Then the above two equations, the budget constraints of the home households 1.2.2, Equation 1.2.30 and its one-period lag can be combined to get the national budget constraint of the home country:

\[ P_tC_t = W_tL_t + \Pi_t + S_tB_{h,t}^*(1 + \tau_t^*) - S_tB_{h,t+1}^* \]  

(1.2.38)

where

\[ \Pi_t = \omega \left[ (P_{h,t} - W_t) \left( \frac{P_{h,t}}{P_t} \right)^{-\gamma_t} C_t + (P_{h,t}^*S_t - W_t) \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\gamma} C_t^* \right] \]  

(1.2.39)

1.3 Model Solution

The model can be solved by log-linearization around a non-stochastic, symmetric steady state (as described in Appendix A.3), where net foreign assets are zero, all prices are equal, and the exchange rate is unity. Given the log-linearized system, the deviations of the exchange rate and the macroeconomic variables from their \( t - 1 \) expectations are solved in terms of exogenous money supply shocks and the expectation error shocks.

1.3.1 Log-linearization

The log-linearization of the model is quite standard, except for the interest rate parity conditions (Equations 1.2.26 and 1.2.29). Since the non-linearities of the interest parity equations are important for understanding the dynamics of the economy, especially for the exchange rate, the variance term and expectation error term will be kept through second-order approximation when the parity equations are log-linearized.

9The financial market are incomplete in our model since the home and foreign household only have access to non-state contingent domestic currency nominal bonds. If there are no foreign exchange traders, then there is a unit root in the net foreign assets in this kind of model. However, when the foreign exchange traders are present, the net foreign assets are zero at the steady state. Please see Appendix A.4) for detail.

10Hereafter, \( \dot{x}_t = \log(x_t) - \log(\bar{x}) \), \( dx_t = x_t - \bar{x} \), where \( \bar{x} \) is the non-stochastic steady state value of variable \( X_t \).

11The detailed model solution, including the log-linearization of the system and the derivation of equations, is given in Appendix A.4).
Linearizing the interest parity condition for the exogenous entry specification (1.2.26) gives:

\[ \hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr^*_{t+1}) + (1 - N_I)v_t - a\frac{(1 + \hat{r})S}{P}Var_t[s_{t+1}]dB^*_h,t+1 \]  
(1.3.1)

Linearizing the interest parity condition for the endogenous entry specification (1.2.29) gives:

\[ \hat{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr^*_{t+1}) + \frac{1}{N_I}n_t v_t - a\frac{(1 + \hat{r})S}{P N_I}Var_t[s_{t+1}]dB^*_h,t+1 \]  
(1.3.2)

where \( n_t \), the number of incumbent noise traders is given by:

\[ n_t = \frac{(E_t(s_{t+1}) - \hat{s}_t - \beta(dr^*_{t+1} - dr_{t+1}) + v_t)^2 (1 - N_I)}{2aVar_t(s_{t+1})} \]  
(1.3.3)

Similar to Equations 1.2.26 and 1.2.29, Equations 1.3.1 and 1.3.2 show that the biased expectation of noise traders causes a stochastic deviation from uncovered interest rate parity. This deviation is composed of two parts: the noise traders’ expectation errors and a risk premium term. The former, as discussed intensively by Devereux and Engel (2002), is different from the traditional risk premium term that arises from the risk aversion of households. It captures the fluctuations in the exchange rate due to the variation of noise traders’ misperceptions. As one would expect, the greater the number of noise traders, the greater is the impact of the noise traders’ expectation error on the exchange rate. For example, when a positive expectation error shock occurs, the noise trader will have a higher demand for foreign bonds and foreign currency, which leads to a domestic currency depreciation. Therefore, this term tends to increase exchange rate volatility.

In contrast to Devereux and Engel (2002), there is also a risk premium term in the parity condition because of the assumption that traders are risk averse. Intuitively, when exchange rate volatility increases, traders would not hold the foreign bonds unless compensated for bearing the extra risk. Consequently, the price of the foreign currency (risky asset) should fall. Thus, the risk aversion of traders (both informed traders and noise traders) will tend to reduce exchange rate volatility.

\[ \text{Hereafter, the curvature parameter of entry costs } \alpha \text{ is set to be equal to 1. The model can be easily extended to the case where } \alpha > 1 \text{ or } 0 < \alpha < 1, \text{ and the main results will not change.} \]
Note that from the log-linearization of the pricing equation of firms, we may get the price index for the home and foreign country:

\[ p_t = \frac{1}{2} (E_{t-1}[\hat{w}_t] + E_{t-1}[\hat{w}^*_t] + E_{t-1}[\hat{s}_t]) \]

(1.3.4)

\[ \hat{p}^*_t = \frac{1}{2} (E_{t-1}[\hat{w}_t] - E_{t-1}[\hat{s}_t] + E_{t-1}[\hat{w}^*_t]) \]

(1.3.5)

Equations 1.3.4 and 1.3.5 establish that in an expected sense, PPP holds. This is not surprising, as the prices can fully adjust to all shocks after one period.

Solve for \( T - 1 \) Expectations Taking a linear approximation of the budget constraint of home household\(^{13}\), using the pricing indexes (Equations 1.3.4 and 1.3.5), the relationship between \( c_t \) and \( c^H_t \), and the fact that (as will hold in equilibrium) in an expected sense, any initial change in net foreign assets is persistent, gives:

\[ E_{t-1}(c^H_t - \hat{c}_t) = E_{t-1}(\frac{dC^T_t}{C_t}) + (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + \frac{2}{PC}(\frac{1}{\beta} - 1)dB_t \]

(1.3.6)

Use \( \bar{r} = \frac{1}{\beta} - 1 \) and the fact that \( \frac{dC^T_t}{C_t} = 0\(^{14}\)\), we get:

\[ E_{t-1}(c^H_t - \hat{c}_t) = (1 - \gamma)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + \frac{2\bar{r}}{PC}dB_t \]

(1.3.7)

Equation 1.3.7 shows that the relative home consumption increases in changes of the initial net foreign assets position and decreases in the expected terms of trade, as long as the elasticity of substitution between home and foreign composite goods \( \gamma \) is greater than 1.

From the linear approximation of the goods market clearing conditions (Equations 1.2.32 and 1.2.33), and the labor supply equation 1.2.6 and its foreign equivalent, using price indexes (Equations 1.3.4 and 1.3.5), and taking expectations at \( t - 1 \), gives:

\[ E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) = \frac{\rho}{1 + \psi \gamma} E_{t-1}(c^H_t - \hat{c}_t) \]

(1.3.8)

\(^{13}\)After the home money market equilibrium condition \( M_t = M_{t-1} + T_t \) and the profit condition 1.2.39 are imposed.

\(^{14}\)This is because the traders' income is derived from the product of the foreign currency bond holding \((B^H_{n,t+1})\) and the excess return \((\rho_{t+1})\). At the steady state, both the bond holding and the excess return are equal to zero. If we log-linearize the budget constraint of the traders (Equation 1.2.16) around the steady state, \( dC_T = 0 \).
Equations 1.3.8 and 1.3.7 give a relationship between the initial net foreign assets and the expected relative consumption:

\[ E_{t-1}(c_t^H - c_t^*) = \frac{2R}{PC} \sigma dB_t \]  

(1.3.9)

where \( \sigma = 1 - \frac{(1-\gamma)\rho}{1+\psi} \). An increase in the home country's net foreign assets leads to an expected increase in the home relative consumption.

Finally, hereafter, we assume that the elasticity of the money demand \( \epsilon = 1 \).\(^{15}\) In equilibrium, given the random walk assumption of money supply process and the log money utility function, a very convenient property is that the nominal interest rate will be constant. This is because, if the log of the money stock follows a random walk, so does the log of the term \( P_t(C_t^H) \). Using this fact and pricing indexes (Equations 1.3.4 and 1.3.5), from the linear approximation of the money demand function and its foreign equivalent we could get:

\[ E_{t-1}(\tilde{m}_t - \tilde{m}_t^*) = \rho E_{t-1}(c_t^H - c_t^*) + E_{t-1}(\tilde{s}_t) \]  

(1.3.10)

Therefore, in an expected sense, the exchange rate is consistent with the standard monetary model.

1.3.2 Model 1: Exogenous Entry

Hereafter, let \( x_{t+j} = x_{t+j} - E_{t-1}(x_{t+j}), j \geq 0 \) denote the deviation of a variable from its date \( t - 1 \) expectation. Then, the log-linearized home household's budget constraint minus its \( t - 1 \) expectation gives:

\[ c_t^H - c_t^* + \frac{2}{P\tilde{C}} dB_{t+1} = \tilde{s}_t \]  

(1.3.11)

The right-hand side of Equation 1.3.11 represents the relative wealth effect of an unanticipated shock to the exchange rate through firms' profits. This relative wealth increase will then be spread between an increase in relative home consumption and net foreign assets accumulation.

\(^{15}\)An estimate of the consumption elasticity of the demand for money (equal to \( \frac{1}{\epsilon} \) in the model) is very close to unity, as reported by Mankiw and Summers(1986).
Using Equation 1.3.9 (updated to period \( t + 1 \)) and Equation 1.3.11, we may establish that:

\[
(c_t^H - c_t^\pi) + \frac{\sigma}{\rho} E_t(c_{t+1}^H - c_{t+1}^\pi) = \delta_t
\]  

(1.3.12)

This equation gives a relationship between current relative consumption, expected period \( t + 1 \) relative consumption, and the unanticipated shock to the exchange rate. It represents the constraints on these three variables implied by the intertemporal current account.

Then, substituting the log-linearized intertemporal optimality equations into the interest parity condition, we may obtain the consumption-based interest parity condition:

\[
\rho E_t(c_{t+1}^H - c_t^\pi) + E_t(p_{t+1} - \bar{p}_t) = \rho E_t(c_{t+1}^\pi - c_t^\pi) + E_t(p_{t+1}^\pi - p_t) \\
+ E_t(\delta_{t+1}) - \delta_t + (1 - N_f)\nu_t - a(\frac{1 + \sigma}{\rho})^S Var_t[s_{t+1}^\pi]dB_{t+1}^\sigma
\]  

(1.3.13)

where the left-hand side is the domestic nominal interest rate and the first two terms on the right-hand side represent the foreign nominal interest rate. Using the price indexes (Equations 1.3.4 and 1.3.5) and subtracting Equation 1.3.13 from its \( t - 1 \) expectation, we may get the relationship between current relative consumption and anticipated future relative consumption.

\[
E_t(c_{t+1}^H - c_{t+1}^\pi) = (c_t^H - c_t^\pi) - \frac{1}{\rho}[\delta_t - (1 - N_f)\nu_t + a(\frac{1 + \sigma}{\rho})^S Var_t[s_{t+1}^\pi]dB_{t+1}^\sigma]
\]  

(1.3.14)

In this equation, expected consumption growth in the home country decreases in response to an unanticipated exchange rate depreciation, since this generates an unanticipated real depreciation, and therefore reduces the home country’s real interest rate. From Equation 1.3.13, a positive shock to foreign exchange traders’ expectations of the future exchange rate will increase the home real interest rate and lead to an increase in expected consumption growth of the home country. The last term in Equations 1.3.13 and 1.3.14, which denotes a risk premium term due to the risk-aversion of the foreign exchange traders, tends to reduce the real interest rate. Therefore, it has a negative effect on the expected home relative consumption growth.
Finally, the relation between relative money supply and relative consumption can be derived from the money demand equations:

\[ m_t - m^*_t = \rho (c_t - c^*_t) \]  \hspace{1cm} (1.3.15)

Putting Equations 1.3.15, 1.3.12, 1.3.14 and 1.3.11 together, we can get a system of equilibrium conditions that characterizes \( \{ s_t, c_t, c^*_t, dB_{t+1} \} \). We may solve for the deviation of the exchange rate from its \( t-1 \) expectation (\( \bar{s}_t \)) in terms of the exogenous money shock and expectation error of the noise traders.

\[ \bar{s}_t = (m_t - m^*_t) \frac{1 + \frac{\rho}{\rho + \frac{\sigma}{\phi}} V_{ar_t(s_{t+1})}}{\rho + \frac{\sigma}{\phi} + \phi V_{ar_t(s_{t+1})}} + \frac{\frac{\sigma}{\phi}}{\rho + \frac{\sigma}{\phi} + \phi V_{ar_t(s_{t+1})}} (1 - N_f) v_t \]  \hspace{1cm} (1.3.16)

where

\[ \phi = \frac{a(1+\bar{r})SC \sigma}{2} \]  \hspace{1cm} (1.3.17)

From Equation 1.3.16, the variance of the future exchange rate deviation, \( V_{ar_t(s_{t+1})} \) can be solved. Let \( V_{ar_t(s_{t+1})} \equiv V_s \). \( V_s \) is given by the following implicit function:

\[ V_s = \frac{(1 + \frac{\sigma}{\phi} + \frac{2}{\rho + \frac{\sigma}{\phi} + \phi V_s})^2}{1 - \left( \frac{\frac{\sigma}{\phi}}{\rho + \frac{\sigma}{\phi} + \phi V_s} \right)^2 (1 - N_f)^2 \lambda} \]  \hspace{1cm} (1.3.18)

Note that the coefficient \( \phi \) is associated with the risk-aversion of traders. The higher the risk aversion coefficient, the lower will be the exchange rate volatility. For both types of traders, their aversion to risk prevents exchange rate volatility from increasing too much. To see this, it can be shown that as long as \( \rho > 1 \), the numerator \( \left( \frac{1 + \frac{\sigma}{\phi} + \frac{2}{\rho + \frac{\sigma}{\phi} + \phi V_s}}{\rho + \frac{\sigma}{\phi} + \phi V_s} \right)^2 \) on the right-hand side of Equation 1.3.18 is decreasing in \( V_s \) and the denominator \( 1 - \left( \frac{\frac{\sigma}{\phi}}{\rho + \frac{\sigma}{\phi} + \phi V_s} \right)^2 (1 - N_f)^2 \lambda \) is increasing in \( V_s \).

Can the exchange rate display 'excess volatility' in this model? When \( \rho = 1 \), the coefficient of \( (m_t - m^*_t) \) is exactly 1 in Equation 1.3.16. Therefore, with no noise traders, the exchange

\[ \text{Notice that } m_t = \varepsilon_{m,t}, \text{ and } m^*_t = \varepsilon_{m,t}^*. \text{ We use } m_t \text{ and } m^*_t \text{ for notational convenience.} \]

\[ \text{Since } \bar{s}_t \text{ is linear in } \bar{m}_t, \bar{m}^*_t \text{ and } v_t \text{ and the monetary shocks and expectation error shocks are normally distributed with zero mean and constant variance, } V_{ar_t(s_{t+1})} = V_{ar(s_{t+1})} = \text{constant } \equiv V_s \]
rate volatility will be equal to that of the fundamentals. If noise traders are present on the market, the exchange rate volatility may be much higher than the fundamental volatility, even when $\rho = 1$.

What are the responses of the macroeconomic fundamentals such as consumption, labor and wage to the exogenous monetary shocks and expectation error shocks? From the log-linearized goods market clearing condition, labor supply condition and the money demand condition,

$$\bar{w}_t = \frac{\psi}{2\rho}(\bar{m}_t + \bar{m}_t^*) + \bar{n}_t$$  \hspace{1cm} (1.3.19)

$$\bar{l}_t = \frac{1}{2\rho}(\bar{m}_t + \bar{m}_t^*)$$  \hspace{1cm} (1.3.20)

$$\hat{c}_t = \frac{1}{\rho} \bar{m}_t \hspace{1cm} \tilde{c}_t = \frac{1}{\rho} \bar{m}_t^*$$  \hspace{1cm} (1.3.21)

Therefore, the volatilities of the macroeconomic fundamentals are only decided by the volatility of the relative monetary shock and the values of the parameters, but not by the volatility of the expectation error and the number of incumbent noise traders in the market.

Note that from Equations 1.3.11 and 1.3.15, the net foreign assets are given by:

$$d\bar{B}_{t+1} = \frac{\bar{B}_t}{2}[\frac{\bar{s}_t}{\rho} - \hat{c}_t - \bar{m}_t - \tilde{c}_t]$$  \hspace{1cm} (1.3.22)

Thus, the volatility of the net foreign assets will be affected by the number of incumbent noise traders.

**1.3.3 Model 2: Endogenous Entry**

The endogenous entry case is similar to the exogenous entry case except for the interest parity equation. Substituting the log-linearized intertemporal optimality conditions into the endogenous interest parity condition (Equation 1.3.2), we may get the consumption-based interest parity condition:

\[
\rho E_t(\hat{c}_{t+1} - \hat{c}_t) + E_t(p_{t+1} - \hat{p}_t) = \rho E_t(\tilde{c}_{t+1} - \tilde{c}_t) + E_t(p_{t+1} - \hat{p}_t) \\
+ E_t(s_{t+1}) - \delta_t + \frac{1}{N_t} n_t v_t - a \frac{(1 + \hat{r})\tilde{s}}{P N_t} Var_t[\tilde{s}_{t+1}] dB_{n,t+1}
\]  \hspace{1cm} (1.3.23)
Chapter 1. Noise Traders and the Exchange Rate Disconnect Puzzle

Using the price indexes, we may find a condition analogous to Equation 1.3.14:

\[ E_t(c_{t+1} - \delta_t) = (c_H - \delta_t) = \frac{1}{\rho}n_t + a(1 + \bar{r})Var_t(s_{t+1})dB_{h,t+1} \] (1.3.24)

where

\[ n_t = \frac{\{E_t(s_{t+1}) - \delta_t + \nu_t\}^2 (1 - N_I)}{2aVar_t(s_{t+1})} \] (1.3.25)

Equations 1.3.15, 1.3.12, 1.3.11 and Equations 1.3.24, 1.3.25 give the solution of the endogenous entry model, and the derivation is entirely analogous to Equation 1.3.16:

\[ \delta_t = (\bar{m}_t - \bar{m}_t) \frac{1 + \bar{r} + \bar{p}Var_t(s_{t+1})}{\rho + \bar{p} + \bar{p}Var_t(s_{t+1})} + \frac{\bar{p}}{\rho + \bar{p} + \bar{p}Var_t(s_{t+1})} \frac{\{E_t(s_{t+1}) - \delta_t + \nu_t\}^2 (1 - N_I)}{2aVar_t(s_{t+1})} \] (1.3.26)

where

\[ \phi' = \frac{\alpha(1 + \bar{r})\delta C \sigma}{2N_I} \] (1.3.27)

Analogous to the exogenous entry case, when \( \rho = 1 \) and no noise traders are present (\( N_I = 1 \), \( \delta_t = (\bar{m}_t - \bar{m}_t) \)), exchange rate volatility will be identical to that of the fundamental. When there are noise traders in the foreign exchange market, the exchange rate may diverge significantly from the fundamental values. The expression for net foreign assets, consumption, labor, and wage are exactly the same as in the exogenous case.

1.4 Results

Equations 1.3.16 and 1.3.26 are too complicated to be solved analytically, so the numerical undetermined coefficient method described in Appendix A.7 is used to solve for \( \delta_t \), \( V_s \) and \( E_t \).

Table 1.1 gives the parameter values that are used in the numerical simulation. We choose \( \beta = 0.94 \), which produces a steady state real interest rate of six percent, about the average long-run real return on stocks. The parameters \( \eta \) and \( \psi \) are set so that the elasticity of labor supply is 1 and the time devoted to work is one quarter of the total time in the steady

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\(^{18}\)Here we use the fact that nominal interest rate are constant and \( \delta_t = \delta_t - E_{t-1}(\delta_t) \).
Table 1.1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exogenous Case</strong></td>
<td></td>
</tr>
<tr>
<td>Preferences</td>
<td>$\beta = 0.94$, $\rho = 2$, $\epsilon = 1$, $\eta = 58.2$, $\psi = 1$</td>
</tr>
<tr>
<td>Final goods technology</td>
<td>$\theta = 11$, $\gamma = 1.5$, $\omega = 0.5$</td>
</tr>
<tr>
<td>Money Growth Process</td>
<td>$corr(\epsilon_{\mu}, \epsilon_{\mu}^*) = 0$, $\sigma_{\epsilon_{\mu}}^2 = \sigma_{\epsilon_{\mu}}^2 = 0.01$</td>
</tr>
<tr>
<td>Foreign exchange traders</td>
<td>$\bar{c} = 0$, $N_I \in [0, 1]$, $\lambda = 1, 1.5$</td>
</tr>
<tr>
<td>Steady State Values</td>
<td>$\mu_{ss} = \mu_{ss}^* = 1$, $M_{ss} = M_{ss}^* = 2$</td>
</tr>
<tr>
<td><strong>Endogenous Case</strong></td>
<td></td>
</tr>
<tr>
<td>Foreign exchange traders</td>
<td>$\bar{c} &gt; 0$, $N_I \in [0, 1]$, $\lambda = 1.5$</td>
</tr>
</tbody>
</table>

*Other parameters in the endogenous case are the same as in the exogenous case.

The business cycle literature has a wide range of estimates for the curvature parameter $\rho$. Chari, Kehoe, and McGrattan(2002) set $\rho = 5$ to generate a high volatility of the real exchange rate. In our model, a high exchange rate volatility can be obtained without high risk aversion of households, so it is set to equal 2. For the final goods technology parameters, the elasticity of substitution between domestic produced goods $\theta$ is set to 11 following Betts and Devereux(2000). This gives a wage-price mark-up of about 1.1, which is consistent with the finding of Basu and Fernald(1994). The elasticity of substitution between home goods and foreign goods $\gamma$ is set to be 1.5, following Chari, Kehoe, and McGrattan(2002) and Backus, Kehoe, and Kydland(1994). 19

1.4.1 Exogenous Case

We first solve for the exogenous entry case. Tables A.1 and A.2 illustrate the results of the simulations, for different values of $\lambda$. The first ten rows show the changes in the volatilities of the exchange rate and the net foreign assets when the number of noise traders increases from 0 to 1. The last three rows report the volatilities of the macroeconomic fundamental variables, given the calibrated parameter values.

19Note that, other parameters, such as the money supply process, number of informed traders on the market, and entry costs, are not fully calibrated. However, this will not affect the main conclusions of the paper.
Chapter 1. Noise Traders and the Exchange Rate Disconnect Puzzle

From Tables A.1 and A.2, three important findings are: First, the exchange rate volatility increases when the number of noise traders increases, while the volatilities of macroeconomic fundamentals remain constant. Moreover, the exchange rate volatility is much higher than that of the macroeconomic fundamentals. Second, from the functional form of \( s_t \), listed in the first column of Tables A.1 and A.2, the impact of fundamental monetary shocks on the exchange rate (coefficient of \( m_t \) or \( m_t^{*} \)) decreases in the number of noise traders. Meanwhile, the effect of expectation error on exchange rate (coefficient of \( u_t \)) increases when more noise traders are present on the market. Third, exchange rate volatility is higher when the magnification coefficient \( \lambda \) increases.

Therefore, the critical implication is that 'disconnection' does exist between the exchange rate and the macroeconomic fundamentals in this model. Thus, the presence of noise traders in the foreign exchange market, combined with local currency pricing, generates a degree of exchange rate volatility that may be much higher than that of the underlying fundamental shocks. In other words, the “exchange rate disconnect puzzle” may be explained by the approach suggested in this paper.

To understand intuitively why the disconnect puzzle can be solved in such a model, we may examine the case without noise traders. Obviously, the presence of local currency pricing tends to remove the expenditure-switching or substitution effects of exchange rate movements. Nevertheless, with just local currency pricing, the dynamic model will not generate a highly volatile exchange rate and the disconnection, as an exchange rate shock also affects the home real interest rates through the interest rate parity condition.

Rewriting Equation 1.3.14 by omitting the expectation error and the risk premium term, gives:

\[
\rho E_t(c_{t+1} - \hat{c}_{t+1}) = \rho (c_t - \hat{c}_t) - \delta_t \tag{1.4.1}
\]

Together with Equation 1.3.12, Equation 1.4.1 illustrates why exchange rate volatility is limited without noise traders. When a depreciation of home currency occurs, the domestic currency value of foreign sales will increase, giving rise to an increase in home wealth. Equation 1.3.12 indicates that this positive wealth effect increases both current and future relative
consumption in the home country.

Meanwhile, as an arbitrage condition, the interest parity condition (Equation 1.4.1) implies that a depreciation of home currency today will reduce the relative real interest rate in the home country and change the path of consumption, so that the current home consumption will increase, relative to the expected future consumption (holding foreign consumption constant). If the change in exchange rate is large and a disconnection between consumption and exchange rate is needed (i.e., the change in current consumption has to be small), Equation 1.4.1 suggests that the expected future consumption has to drop a lot. Nevertheless, Equation 1.3.12 implies that the future relative consumption of the home country should increase when a depreciation of home currency occurs. Therefore, with no noise traders, the only possible way to explain the difference between exchange rate volatility and the fundamental volatility is by introducing a high value of $\rho$, which is exactly the mechanism emphasized by Chari, Kehoe, and McGrattan (2002).

When the noise traders are introduced into the interest parity condition, we can see from Equation 1.3.14 that now a large increase of exchange rate and a small change in current consumption do not necessarily imply a large drop of expected future consumption, because the presence of the expectation errors and the risk premium term of noise traders also drive wedges between the home and foreign real interest rates. This could be called the “level effects” created by the noise traders.

The presence of noise traders also creates a “volatility effect”, which is due to the assumption that the volatility of $v_t$ itself is proportional to the exchange rate volatility. This assumption “magnifies” the response of the exchange rate to the expectation error of noise traders. When the nominal exchange rate volatility increases, so does the expectation error volatility, which further increases the exchange rate volatility until the system reaches an equilibrium where $\text{Var}(v_t) = \lambda \text{Var}(s_t)$. Thus, $\lambda$ is the parameter characterizing this magnification effect. The higher $\lambda$, the higher is the exchange rate volatility.

Therefore, volatile exchange rates can be obtained in this model. Still, why is the high volatility not transferred to other macroeconomic variables (except for the net foreign assets)?
Normally, there are two channels through which the exchange rate affects other macroeconomic variables: expenditure-switching effects and wealth effects. Since the prices of the import goods are assumed to be sticky in terms of the local currency, the relative price of home-produced goods to foreign-produced goods will remain unchanged when the exchange rate changes. Therefore, the expenditure-switching channel is completely shut down in our model.\(^\text{20}\)

With regard to the wealth effect, from Equation 1.3.12, the increase in wealth that comes from an unexpected depreciation will be spread between an increase in relative home consumption and the net foreign assets accumulation. From Equation 1.3.15, however, the increase in relative home consumption is limited by the relative money shocks due to the real balance effect. Therefore, the net foreign assets will absorb most of the wealth increase. This is actually shown in Tables A.1 and A.2, when the volatility of the nominal exchange rate increases, so does the volatility of the net foreign assets. However, the magnitude of the volatility of the net foreign asset and the expected future relative consumption are small quantitatively, especially when compared to that of the exchange rate. That implies the wealth effect is also quite small quantitatively. From Equation 1.3.12,

$$E_t((\tilde{c}^{f}_{t+1} - \tilde{c}^r_{t+1})) = \frac{r}{\sigma} [\tilde{s}_t - (\tilde{c}^f_t - \tilde{c}^r_t)]$$

(1.4.2)

It can be seen that the volatility of the change in expected future consumption is quantitatively small because \(\frac{r}{\sigma}\) is small given reasonable parameter values.\(^\text{21}\) The economic intuition is that the consumption-smoothing behavior of infinitely lived households limits the wealth effect in this model. When a shock leading to an exchange rate depreciation occurs, the households increase their holdings of net foreign assets. This increase will be spread over

\(^{20}\)In another paper, the assumption of local currency pricing is relaxed and the baseline model is analyzed under producer currency pricing or a mixture of local currency pricing and producer currency pricing (i.e., the exchange rate pass-through is between 0 and 1). As expected, we find that with positive exchange rate pass-through, the volatilities of macroeconomic fundamentals (consumption, labor and wage) depend on the exchange rate volatility. The higher the exchange rate pass-through, the higher is the correlation between exchange rate volatility and the fundamental volatilities.

\(^{21}\)For current calibration, \(\tilde{c}^r = \frac{0.06}{1.4} = 0.0429\). Recall that \(\sigma = 1 - \frac{(1-\gamma)p}{1+\gamma}\), so as long as the elasticity of substitution between home and foreign goods \(\gamma\) is greater than 1, \(\sigma\) is greater than 1. Thus, \(\tilde{c}^r < 0.06\).
many future periods because the households want to smooth their future consumption. The
increase in the expected consumption of next period is then quite small. Therefore, the more
risk averse are the households (the higher $\rho$), the bigger will be $\sigma = 1 - \frac{(1-\gamma)\rho}{1+\phi\gamma}$ (suppose that $\gamma > 1$), and the smaller will be the wealth effect.

Moreover, in the monetary model of exchange rate without noise traders, the monetary
shocks lead to movements in both macroeconomic fundamentals and exchange rates, as shown
by the following equations:

$$\Delta r_t = \frac{\gamma}{\rho + \phi}(\tilde{m}_t - \tilde{m}^*_t)$$

(1.4.3)

$$\Delta c^*_t - \tilde{c}^*_t = \frac{1}{\rho}(\tilde{m}_t - \tilde{m}^*_t)$$

(1.4.4)

Therefore, it generically predicts a strong comovement and a high and positive correlation
between the exchange rate and relative consumption. From empirical evidence, however,
there is no clear path in the observed cross-correlation. Chari, Kehoe and McGrattan (2002)
find that this correlation is negative for U.S. and Europe while it ranges between small and
positive to somewhat negative for other country pairs.

Nevertheless, in our model, since exchange rate movements can be generated by the ex­
pectation error shocks, our model does not predict a strong comovement of the exchange rate
and relative consumption. The functional form of $\tilde{s}_t$ (listed in the first column of Tables A.1
and A.2 shows that the exchange rate can move even when the realization of the fundamen­
tals shocks are equal to zero. Furthermore, as shown by the last column of Table A.2, the
cross-correlation between the exchange rate and relative consumption decreases when more
noise traders are present on the foreign exchange market. Intuitively, this is because the in­
troduction of noise traders generate deviations from the uncovered interest parity condition
and thus breaks the link between the real exchange rate and relative consumption. Therefore,
we may get a small and positive correlation in our model.

---

22With no traders on the foreign exchange market, Equation 1.3.16 could be rewritten as Equation 1.4.3.
23For example, in Chari, Kehoe and McGrattan (2002) the correlation is equal to 1.
1.4.2 Endogenous Entry

The exogenous entry specification gives important implications of the model, however, a natural question is what can the monetary authorities do to get rid of the excess volatility in the nominal exchange rate? So in this section, we consider ways to endogenize the entry of noise traders, which will help to evaluate the implications of policies that target the non-fundamental risk.

Table A.3 illustrates the simulation result of the endogenous entry specification: First, the exchange rate disconnection still holds in this specification. Second, given the number of noise traders in the market: $1 - N_j$, increasing the entry cost $\hat{c}$ (within a reasonable domain of $\hat{c}$) will reduce the exchange rate volatility. The first finding is not surprising. In Equation 1.3.24, as in Equation 1.3.14, the presence of noise traders generates a wedge between home and foreign real interest rates. This wedge, by analogue, is also composed of two parts, the expectation error of incumbent noise traders and the risk premium term. The only difference is that now the number of incumbent noise traders is endogenously decided, and as is the expectation error part. Nevertheless, this does not alter any of the theoretical analysis in Section 1.4.1. This wedge creates the “level effects” and the “volatility effects”, which in turn imply a degree of exchange rate volatility that is much higher than the fundamental volatility. Meanwhile, the expenditure-switching effect is eliminated because of the LCP pricing behavior. The wealth effect is quantitatively small because of the households’ consumption smoothing behavior in an infinite horizon model. Therefore, exchange rate volatility will not be transferred to the macroeconomic fundamentals except for the net foreign assets.

The second finding is quite interesting and has important policy implications. Although the model is complicated and can only be solved numerically, this result is quite intuitive. The higher the entry cost, the fewer noise traders will enter the market and therefore fewer noise components will be present. Thus, it shows that the exchange rate policies that aim at eliminating the non-fundamental risk can be justified theoretically. It also suggests possible

\[ \hat{c} \in (0, 0.25], \text{ as the steady state consumption in this model is 0.25.} \]
approaches the monetary authorities may apply to reduce the excess exchange rate volatility, to discourage the entrance of noise traders by increasing the entry cost or to 'educate' the market to reduce the number of noise traders on the foreign exchange market. Furthermore, it suggests monetary authorities could reduce this kind of excess exchange rate volatility by commitments to low exchange rate volatility. In this way, the volatility of expectation error of noise traders will be reduced and so will the exchange rate volatility. A self-contained equilibrium with low exchange rate volatility would be then established.

1.5 An Extension - Tobin Tax

Tobin (1978) and Eichengreen, Tobin and Wyplosz (1995) suggest that an international transaction tax on purchases and sales of foreign exchange would be one way to “throw sand in the wheels of super-efficient financial vehicles”. They argue that a transaction tax might diminish excess volatility. Even a small transaction tax would deter the fast round trip into a foreign money market.25

A Tobin tax is different from the entry cost we analyzed in the benchmark model. First, it is a common cost for both rational and noise traders. Second, it is not a fixed cost, but increases with the amount of foreign currency bond traded. In this extension, we extend the benchmark model to include a transaction tax to analyze the implication of the Tobin tax on exchange rate volatility in our model. When a transaction tax is imposed, the trader i's problem can be written as:

\[
\max_{B_{t+1}(i)} E_t^i(C_{t+1}(i)) - \frac{a}{2} Var_t^i(C_{t+1}(i))
\]

25 A small transaction tax would be a negligible consideration in long-term portfolio or direct investments in other economies. Relative to ordinary commercial and transportation costs, it would be too small to have much effect on commodity trade.

26 The transaction cost could not be modelled as linear in \( B_{t+1}(i) \), because this would imply that trader \( i \) will gain when selling foreign bonds. Thus, we assume a convex transaction cost.
Subject to\(^{27}\)

\[ P_{t+1}C_{t+1}^T = [B_{h,t+1}^*(1+r_{t+1})S_{t+1} - B_{h,t+1}^*(i)S_t(1+r_{t+1})] - P_{t+1}c_t - P_{t+1}\tau \frac{B_{h,t+1}^*(i)^2}{2} \]  

(1.5.2)

where \(\tau\) is the rate of the transaction tax on foreign bond trading. Solving the traders' problem, we could get:

\[ B_{h,t+1}^* = \frac{E_t[\rho_{t+1}]}{S_t(1+r_{t+1}) + a \frac{S_t}{P_{t+1}(1+r_{t+1})} Var_t[\rho_{t+1}]} \]  

(1.5.3)

From Equation 1.5.3, it can be seen that the introduction of a Tobin tax reduces the bond trading of both types of traders. This is quite intuitive, as foreign exchange traders will tend to trade less foreign currency bonds when there is a tax on transactions.

When there are only transaction costs, it can be shown that the traders will always choose to enter the market. This is because the transaction cost is convex in the bonds traded, the traders can always choose to hold a small amount of foreign bonds and get a positive expected utility, regardless of how large is \(\tau\). Therefore, similar to the benchmark model, two cases are analyzed in this extension. In the exogenous entry case, we focus on the transaction cost only. In the endogenous entry case, we assume that noise traders have to pay two costs to trade in the foreign exchange market: the transaction cost and a fixed information cost as in previous sections. However, the informed traders only need to pay a Tobin tax. The analysis of the second case will help to understand the role of the Tobin tax in the economy.

Using Equations 1.5.3, we could get the interest parity condition when there exists a transaction tax in the foreign exchange market. For the exogenous entry case:

\[ G_t = E_t(s_{t+1}^\gamma) - \beta(dr_{t+1} - dr_{t+1}) + (1 - N_t)\nu_t - \frac{PrdB_{h,t+1}^*}{S_t(1+r_{t+1})} - a \frac{(1+r_{t+1})S_t}{P_{t+1}} Var_t[s_{t+1}] dB_{h,t+1}^* \]  

(1.5.4)

When the noise traders have to pay both the transaction cost and the fixed information cost, the gross benefit of entry for noise traders can be derived:

\[ GB = \frac{[E_t(N(\rho_{t+1}))]^2}{2aVar_t(\rho_{t+1}) + 2\tau \frac{P_{t+1}^2}{S_t(1+r_{t+1})}} \]  

(1.5.5)

\(^{27}\)We assume that the Tobin tax is a real tax and is resource-consuming in the sense that it consumes the composite consumption good.
Chapter 1. Noise Traders and the Exchange Rate Disconnect Puzzle

From Equation 1.5.5, it can be seen that the Tobin tax reduces the gross benefit of entry for noise traders. Therefore, increasing the transaction cost will deter the noise traders from entering the market. Given that, we could get the interest parity condition for the endogenous entry case:

\[ \tilde{s}_t = E_t(s_{t+1}) - \beta (dr_{t+1} - dr_{t+1}^*) + \frac{1}{N_I} n_t v_t - \frac{\bar{P} \tau dB_{h,t+1}^*}{N_I \bar{S}(1 + \bar{\tau})} - a \frac{(1 + \bar{\tau}) \bar{S}}{PN_I} Var_t[s_{t+1}^*] dB_{h,t+1}^* \] (1.5.6)

where \( n_t \), the number of incumbent noise traders is given by:

\[ n_t = dnt \approx \frac{\{E_t(s_{t+1}) - \tilde{s}_t - \beta (dr_{t+1}^* - dr_{t+1}) + v_t\}^2 (1 - N_I)}{2aVar_t(s_{t+1}) + \frac{2P^2}{\bar{S}^2(1 + \bar{\tau})} \bar{\tau}} \] (1.5.7)

Analogous to Equations 1.3.1 and 1.3.2, Equations 1.5.4 and 1.5.6 give the deviation from the uncovered interest parity. This deviation is composed of three parts. Besides the expectation error term and the risk premium term, there is an extra term that comes from the transaction tax. Even in the absence of noise traders, this term still exists. As emphasized by Eichengreen, Tobin and Wyplosz (1995), this term creates room in the interest parity condition and expands the autonomy of monetary policies.

To find out if the introduction of the Tobin tax will reduce excess exchange rate volatility, we solve the extended model by the approach described in Section 1.3.²⁸ Then solution of the exogenous entry model is given by:

\[ \tilde{s}_t = (m_t - m_t^*) \frac{1 + \frac{\phi}{\rho} + \frac{\phi}{\rho} Var_t(s_{t-1}) + \xi \tau}{\rho + \frac{\phi}{\rho} + \phi Var_t(s_{t-1}) + \xi \tau} + \frac{\phi}{\rho + \frac{\phi}{\rho} + \phi Var_t(s_{t-1}) + \xi \tau} (1 - N_I) v_t \] (1.5.8)

where

\[ \phi = \frac{a(1 + \bar{\tau}) \bar{S} \sigma}{2 \bar{\tau}} \quad \xi = \frac{\bar{P}^2 \bar{C} \sigma}{2 \bar{S}(1 + \bar{\tau}) \bar{\tau}} \] (1.5.9)

²⁸ The only equation that has been changed besides the interest parity condition is the home country aggregate consumption equation, which now becomes:

\[ C_t = C_t^H + C_t^T + \tau B_{h,t+1} \] Exogenous Entry

\[ C_t = C_t^H + C_t^T + \tau B_{h,t+1} + \sum_{i=0}^{n_t} c_i \] Endogenous Entry

Once we log-linearize the above equations around the steady state, the log-linearized equation remain unchanged.
From Equation 1.5.8 we can solve for the exchange rate volatility $V_s$:

$$ V_s = \frac{\left(\frac{1+\frac{\phi}{\rho}V_s + \xi \tau}{\rho + \frac{\phi}{\rho}V_s + \xi \tau}\right)^2}{1 - \left(\frac{\phi}{\rho + \frac{\phi}{\rho}V_s + \xi \tau}\right)^2} \left[Var(m_t) + Var(m_t^*)\right] (1.5.10) $$

It can easily be shown that if the other variables are kept constant, the numerator on the right-hand side of Equation 1.5.10 decreases in $\tau$, the rate of transaction tax, while the denominator increases in $\tau$. Since Equation 1.5.10 is an implicit function of $V_s$, we solve it numerically to get the relationship between $V_s$ and $\tau$, which is given in Table A.4. It can be seen that the higher the rate of the transaction tax, the lower is the exchange rate volatility.

Intuitively, this is because the introduction of the transaction tax reduces the bond trading. In our model, when an exchange rate change occurs, the real balance effect prevents the current consumption from increasing/decreasing more than the changes in the relative real money supply, so the bond holding of households will absorb most of the wealth effect caused by the exchange rate change. If the bond trading is deterred by the transaction tax, in equilibrium, the exchange rate volatility must decrease.

For the endogenous entry model, we can derive the solution analogously using Equation 1.5.6:

$$ \bar{s}_t = (\bar{m}_t - m_t) \left(1 + \frac{\phi}{\rho} + \frac{\phi}{\rho} Var_t(s_{t+1}) + \xi' \tau + \frac{\phi}{\rho + \frac{\phi}{\rho} Var_t(s_{t+1}) + \xi' \tau} \right) \frac{1}{1 - \frac{\phi}{\rho + \frac{\phi}{\rho} Var_t(s_{t+1}) + \xi' \tau} N_I \rho_t \rho_t} (1.5.11) $$

where

$$ n_t = \frac{(E_t(s_{t+1}) - \bar{s}_t + v_t)^2}{2aVar_t(s_{t+1}) + \frac{2\rho^2}{3(1+\tau)^2} \sigma} (1.5.12) $$

$$ \phi' = \frac{a(1 + \tau)}{2N_I \bar{SC} \rho_t} \sigma \quad \xi' = \frac{\rho^2 \bar{SC}}{2N_I \bar{S}(1 + \tau)} \frac{\sigma}{\bar{r}} (1.5.13) $$

Solving Equation 1.5.11 numerically by the approach described in Appendix A.7, we find that exchange rate volatility also decreases in the transaction tax, as in the exogenous case. The results are given in Table A.4.

---

\(^{20}\) The derivation of Equation 1.5.12 is analogous to that of Equation 1.3.25.
In the endogenous entry case, the transaction cost reduces the exchange rate volatility through two channels. First, as in the exogenous case, it reduces the bond trading of both types of traders, which in turn decreases the exchange rate volatility. Second, as shown by Equation 1.5.5, the Tobin tax reduces the gross benefit of entry for noise traders, which consequently reduces the noise component of the foreign exchange market. Therefore, the mechanism through which the transaction cost affects the exchange rate volatility is different when the noise component on the market is endogenously determined. The effect of the Tobin tax will thus be different as well. This can be seen from Table A.4, for the same level of increase in $\tau$, the decrease in the exchange rate volatility in the endogenous entry case is larger than that in the exogenous entry case. This finding has important policy implications. It shows that the impact of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

1.6 Conclusions and Subsequent Research

In this paper we present a model of exchange rate determination which combines the new open economy macroeconomics approach and the noise trader approach for exchange rate behavior. This model emphasizes the interaction of the macroeconomic fundamentals of exchange rate and the microstructure channel through which exchange rates are determined. The latter is often ignored by conventional macroeconomic research on exchange rate and the literature on policy evaluation. Therefore, our work has important implications for understanding exchange rate behavior and exchange rate policies.

Two important and promising findings from this model are: 1. Models that take both the macroeconomic and microeconomic factors of exchange rate determination into consideration can explain the "exchange rate disconnect puzzle". 2. The exchange rate volatility caused by irrational market behavior or non-fundamental shocks could be controlled by exchange rate policies. We analyze two kinds of policies. One focuses on the entry cost of noise traders, while the other is a 'Tobin tax' type of exchange rate policy. We find that both policies
can reduce the exchange rate volatility. However, the effect of a Tobin tax on exchange rate volatility depends crucially on the structure of the foreign exchange market and the interaction of the Tobin tax with other trading costs.

Thus, subsequent research should focus on the policy implication of this model. What kind of exchange rate regime is better when non-fundamental shocks to the exchange rates are present – flexible or fixed? If the real exchange rate volatility is primarily affected by non-fundamental factors and most exchange rate volatility is useless, then would the fixed exchange rate regime or a single currency area be better than the flexible exchange regime? This model could also be used to evaluate the welfare implications of exchange rate policies such as the Tobin tax or other policies that discourage the entry of noise traders. These policies are discussed widely, but due to the lack of a welfare-based model which can explain exchange rate volatility and its relationship with macroeconomic fundamentals, they have not been evaluated on a welfare basis. The presentation of explicit utility and the profit maximization problem in our model allows for the possibility of answering these questions based on rigorous analysis.

Although this model could help to explain the exchange rate disconnect puzzle, it is still not a fully developed model that could be used to explain all empirical features of exchange rate. To explain the persistence of the real exchange rate, more persistent price setting or 'sticky' information of traders would be needed. For example, the information structure of the noise traders could be changed so that the expectation error is more persistent.
Chapter 2

Optimal Monetary Policy with Vertical Production and Trade

2.1 Introduction

The debate on optimal monetary policy has been at the heart of international macroeconomics for many years.\(^1\) Friedman (1953) and later Mundell (1961) argue that the flexible exchange rate can act as an efficient mechanism for dealing with country-specific shocks when the adjustment of domestic price levels is sluggish. But recent studies of monetary policy in utility-based open economy models have reached varying conclusions about the desirability of flexible exchange rates and the way monetary authorities respond to foreign shocks.

Obstfeld and Rogoff (2002) argue that an inward-looking monetary policy, in which the monetary authorities respond solely to their domestic shocks delivers the best possible outcome, and flexible exchange rate can replicate the flexible price equilibrium. However, Devereux and Engel (2003) show that the optimal monetary policy and exchange rate regime choice critically depend on the currency of export pricing. If prices are set in the currency of producers (PCP), and the pass-through from exchange rate to consumer prices is complete, then the flexible exchange rate is a central part of the optimal monetary policy. But if the prices are set in the currency of buyers (LCP), and do not respond to movements in exchange rates, then the monetary authorities should keep exchange rate fixed, so they should respond to both home and foreign shocks and the optimal monetary policy cannot replicate the flexible price equilibrium. Corsetti and Pesenti (2001) have a similar conclusion. They analyze how the degree of exchange rate pass-through to prices affect the optimal monetary

\(^1\)This chapter is based on the joint work with Kang Shi.
policy and show that the optimal monetary regime is pegged exchange rate in the extreme case where the exchange rate has no impact on consumer price.

Tille (2002) emphasizes the importance of the nature and sources of shocks on the optimal monetary policy design. He shows that the value of exchange rate flexibility is much smaller when shocks are sector-specific and argues that the sectoral structure of the economy and the source of shocks significantly affect the international monetary policy and its welfare implication.

Therefore, these literatures suggest that the presence of local currency pricing (or incomplete exchange rate pass-through) and sectoral shocks may significantly change the existing wisdom on international monetary policy based on PCP pricing.

However, a notable feature of these literatures is that they focus on an environment where all the trade in goods between countries occurs in one stage. In reality, countries can trade not only finished goods but also intermediate goods, even in more stages. We will follow Hummels et al. (1998) and use the term "vertical trade" to describe this vertical structure of production and trade. More specifically, vertical trade occurs when a country uses imported intermediate goods as an input to produce export goods. This definition captures the idea that countries are linked sequentially in the production of final goods (or finished goods).

Hummels et al. (1998) analyze data from 10 OECD countries and find a strong statistical correlation between the increase in the share of vertical trade in total trade and the rise of the share of trade in GDP. The increase in the vertical trade are found to account for more than 25 percent of the increase in the total trade in most OECD countries. In some smaller countries such as Canada and Netherlands, the share of vertical trade in total trade approaches 50 percent. Feenstra(1998), Hummels et al. (2001), and Yi (2003) also argue that the vertical structure has been a more and more important feature of today’s global production and trade. Huang and Liu (1999) show that a vertical chain of production can generate different monetary transmission mechanism in a closed economy.

However, the vertical structure of production and trade has yet been remarkably overlooked in the new open economy macroeconomics literature. Obstfeld and Rogoff (2000b)
and Devereux and Engel (2004) introduce the trade of intermediate goods, but limit the trade of finished goods and focus on the importance of the relative price adjustment of intermediate goods. Nevertheless, the vertical structure of production and trade will influence the international transmission mechanism of productivity shocks and thus affect the way monetary authorities respond to country-specific and stage-specific productivity shocks and desirability of flexible exchange rate in optimal monetary policy. Therefore, in this paper we will explore optimal monetary policy in an open economy with vertical production and trade.

To address this question, we introduce two stages of production and trade into a standard two-country general equilibrium model with sticky prices. There are two vertical stages in each country, one is the finished goods stage, the other one is the intermediate goods stage. Each stage in each country has a stage-specific productivity shock. At each stage, there is a continuum of firms, each firm produces differentiated goods. The production of finished goods requires a basket of distinct variety of domestic intermediate goods and a basket of distinct variety of imported intermediate goods. The production of intermediate goods only needs labor. To highlight the impact of this trade pattern on the optimal monetary policy, we maintain the assumption that firms set price in the currency of producers (PCP) and thus the pass-through of exchange rate changes to import prices is complete. Our model is simple enough to be solved analytically, so the policy evaluation will be based on rigorous welfare comparison. Nevertheless, it incorporates the main feature of the vertical structure of production and trade we want to emphasize in this paper and we find that this trade pattern does have an important impact on the decision of optimal monetary policies in open economies.

The main findings of this paper can be described as follows: First, when both finished goods and intermediate goods are tradable, any stage-specific productivity shock in one country has a positive trans-border spillover effect on the other country via vertical production and trade. This effect changes the way monetary authority reacts to other country's productivity shock, and each monetary authority should respond positively and partially to both home and foreign productivity shocks. This is different from Obstfeld and Rogoff (2002) and
Devereux and Engel (2003), where the optimal monetary policy based on PCP requires monetary authorities to respond only to domestic shocks. Second, a vertical production structure generates multiple price stickiness. Thus, flexible exchange rate cannot adjust the terms of trade to the efficient level in both stages simultaneously. So the flexible price equilibrium cannot be replicated by flexible exchange rate in our model, even in the situation with PCP pricing and complete exchange rate pass-through. Finally, we find that the exchange rate volatility in this economy is lower than that would be obtained in an economy without vertical structure of production and trade. This implies that the multiple stages of production and trade lead to a more integrated global economy so that smaller exchange rate adjustments are needed in response to the country-specific shocks.

This paper is closely related to Devereux and Engel (2003). Devereux and Engel (2003) investigates how the price setting affects the optimal monetary policy and exchange rate flexibility, we adopt their approach to derive optimal monetary rules. Our departure is that we allow for vertical production and trade, so that we can explore the difference in the international transmission mechanism of productivity shocks and the limitation of the exchange rate as an adjustment mechanism for nominal rigidity in such an environment.

As to the emphasis on multiple stages of production and trade, this paper is also related to Huang and Liu (2003). They try to reconcile the controversy of the welfare consequence between PCP and LCP by modelling multiple stages of production and trade. Our analysis differs because we allow for stage-specific shocks and focus on the reaction of optimal monetary policy to these shocks.

Devereux and Engel (2004) also build a new open economy macroeconomic model with intermediate goods trade. They develop a view of exchange rate policy as a trade-off between the desire to smooth fluctuations in real exchange rates (so as to ensure international risk sharing) on one hand, and the need to allow flexibility in the nominal exchange rate (so as to facilitate relative price adjustment) on the other hand, and optimal nominal exchange rate volatility will reflect these competing objectives. In our paper, as emphasized, we introduce vertical structure of production and trade into the new open macroeconomic framework.
Therefore, we focus on the international transmission mechanism of stage-specific productivity shocks via trade and the impact of multiple-stage price stickiness on the desirability of flexible exchange rate.

This paper is organized as follows. Section 2 presents the model. Section 3 gives the solution of the model and also solves the flexible price equilibrium as a benchmark. Section 4 analyzes the optimal monetary policy response to stage-specific productivity shocks and its welfare implication. Section 5 concludes.

2.2 Basic Model

The world consists of two countries of the same size, denoted as the home country and the foreign country. Each country has one unit of population, they derive utility from aggregated consumption (composed of home finished goods and foreign finished goods), real balance, and leisure. There are two stages of production in each country, one is the finished goods stage, the other is the intermediate goods stage. Note that an important assumption which differ our model from the literature is that we assume both goods are tradable. Each stage has a stage-specific productivity shock. At each stage, there is a continuum of firms indexed on the interval [0,1]. Each firm produces differentiated goods and therefore has some monopolistic power. The production of finished goods requires a basket of distinct variety of domestic intermediate goods and a basket of distinct variety of imported intermediate goods. The production of the intermediate goods only requires labor. Figure B.1 gives the structure of the economy. All firms set prices before the realization of the shocks, and the prices are in the currency of producer.

For simplicity, we abstract from any dynamics by considering a single period model with uncertainty. The structure of events within the period is as follows. First, before the period begins, households can trade in the bond market for a full set of nominal state-contingent bonds. Then the monetary authorities choose optimal monetary rules, given the cross-country risk-sharing rule, taking into account the way in which firms set prices, as well as the dis-
distribution of stochastic productivity shocks. Following this, firms set prices, given the state-contingent discount factors, the expected demand, and the expected marginal costs. After the realization of stochastic shocks, households work and choose their optimal consumption baskets, production and consumption take place, and the exchange rate is determined.

The detailed structure of the economy in the home country is described below. The foreign country is entirely analogous. From now on, foreign variables and foreign currency prices will be indicated by an asterisk. In addition, a subscript $h$ denotes a variable originating from the home country; a subscript $f$ denotes a variable used in the foreign country.

### 2.2.1 Household

The representative households in the home country maximizes the following expected utility\(^2\):

$$
U = E\left( \frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{M}{P} - \eta L \right)
$$

(2.2.1)

where

$$
C = 2C_h^{1/3}C_f^{1/3} \quad \quad C_h = \left[ \int_0^1 C_h(i)^{\lambda - 1} i^{\lambda - 1} \right]^{1/\lambda}
$$

(2.2.2)

Here $C$ is the aggregate consumption, $C_h$ is the sub-aggregate consumption of a continuum of home finished goods indexed by $[0,1]$, $C_f$ is the sub-aggregate consumption of a continuum of imported foreign finished goods. $M$ is the real money balances, and $L$ is the home labor supply. $\lambda > 1$ is the elasticity of substitution between differentiated home(foreign) finished goods. $\rho$ is the inverse of the intertemporal elasticity of substitution and is assumed to be greater than 1. From Appendix B.1, we can derive the CPI price index and individual demand for finished goods $i$ in the domestic market and in the foreign market, respectively

$$
P = P_h^{1/\lambda}P_f^{1/\lambda}
$$

(2.2.3)

$$
C_h(i) = \frac{1}{2} \left( \frac{P_h(i)}{P_h} \right)^{-\lambda} \left( \frac{P_h}{P} \right)^{-1} C
$$

(2.2.4)

---

\(^2\)The adoption of this utility function will give us a closed form solution. It is also used in Obstfeld and Rogoff (2002), Devereux and Engel (2003), and Devereux, Shi and Xu (2003).
Home and foreign households can trade a full set of state-contingent nominal bonds, thus, the budget constraint of the home households for a particular state of the world $z$ is written as:

$$P(z)C(z) + M(z) + \sum_{\xi \in Z} q(\xi)B(\xi) = W(z)L(z) + \Pi(z) + B(z) + M_0 + T(z)$$

That is, consumers derive income from the labor income $W(z)L(z)$, the payoff of the state-contingent securities $B(z)$, the profits from their ownership of all home firms $\Pi(z)$, the initial money balance $M_0$, and the lump-sum transfer from the government $T(z)$. They choose how many state-contingent bonds to purchase before the period begins, with $q(\xi)$ and $B(\xi)$ representing the price and holding, respectively, of a security paying off 1 unit of home currency in state $\xi \in Z$, where $Z$ is the set of states. Then the households will choose the money holding, the consumption and the labor supply. It is assumed that the government repays any seignorage revenue through the lump transfer, so that $M_0 - M(z) + T(z) = 0$.

The specific money supply process will be discussed in later sections.

The trade in state-contingent nominal assets across countries will lead to the following optimal risk-sharing arrangement:

$$\frac{C^*-\rho}{P} = \Gamma \frac{C^{*-\rho}}{SP^*},$$

where $S$ is the nominal exchange rate, and $P^* = P_h^{\frac{1}{2}}P_f^{\frac{1}{2}}$ is the foreign price level. $\Gamma$ is the state-invariant weight and equals 1.\textsuperscript{3} Equation (2.2.7) implies that one dollar can get the same marginal utility of consumption across countries. Therefore, the real exchange rate is equal to the ratio of marginal utilities of consumption across countries. In addition, household

\textsuperscript{3} $\Gamma$ represents the ratio of the Lagrange multiplier associated with the home household's budget constraint to the Lagrange multiplier associated with the foreign household's budget constraint. It is also a condition capturing the initial distribution of wealth. Devereux and Engel (2003) shows that $\Gamma$ equals 1 in a symmetric equilibrium.
optimization problem gives rise to the money demand equation:

\[ M = \chi PC^\rho, \]  

(2.2.8)

and the implicit labor supply function:

\[ W = \eta PC^\rho. \]  

(2.2.9)

Equation (2.2.8) and (2.2.9) imply that the nominal wage is proportional to the amount of money in circulation.

2.2.2 Finished goods stage

There is a continuum of firms indexed by \( i \in [0, 1] \) in the finished goods stage in the home country. Each firm \( i \) produces home finished goods \( Y(i) \) out of home and foreign intermediate goods according to the following production function:

\[ Y(i) = \theta_F X_h(i) X_f(i)^{\frac{1}{2}} \]  

(2.2.10)

where \( \theta_F \) is the finished goods stage specific shock in the home country and \( X_h(i) (X_f(i)) \) is a basket of distinctive variety of intermediate goods produced in the home (foreign) country.

Cost minimization  Each finished goods producer \( i \) takes the prices of intermediate goods as given, so the unit cost to produce finished goods \( i \) can be derived as:

\[ \Lambda = \frac{P_h^{\frac{1}{2}} (SP_{fh}^2)^{\frac{1}{2}}}{\theta_F} \]  

(2.2.11)

where \( P_h \) is the price index of home intermediate goods denominated by home currency, and \( P_{fh}^2 \) is the price index of foreign intermediate goods which are sold in the home country, denominated by foreign currency. From the cost minimization problem, we can derive finished goods producer \( i \)'s demand for the basket of distinct variety of home and foreign intermediate goods:

\[ X_h(i) = \frac{1}{2} \left( \frac{P_h}{\Lambda} \right)^{-1} Y(i) \]  

(2.2.12)

\(^4\text{Here, we assume the production of finished goods requires no labor input. This assumption helps us to solve the model analytically. However, the result will not change even if we allow for labor inputs in this stage.}\)
Chapter 2. Optimal Monetary Policy with Vertical Production and Trade

\[ X_f(i) = \frac{1}{2} \left( \frac{S \hat{P}_h}{\Lambda} \right)^{-1} Y(i) \]  \hspace{1cm} (2.2.13)

**Finished goods price** We assume that in each country the finished goods producer \( i \) sets its price in the currency of producer, thus the law of one price holds in each individual final good and Purchasing Power Parity holds in the CPI. From Equation (2.2.4) and (2.2.5), the total demand for home finished goods \( i \) is:

\[ Y(i) = C_h(i) + C_h^*(i) = \left( \frac{P_h(i)}{P_h^*} \right)^{-\lambda} \lambda^{-1} C \]  \hspace{1cm} (2.2.14)

Given the demand structure and the unit cost function of finished goods, we can derive the optimal pricing policies for finished good \( i \),

\[ P_h(i) = \frac{\lambda \hat{E}[\Delta C^{1-\rho}]}{\hat{E}[C^{1-\rho}]} = \lambda \frac{\hat{E}[\frac{S}{\theta_p} C^{1-\rho}] \hat{P}_h \frac{1}{\lambda} \hat{P}_h^* \frac{1}{\lambda}}{\hat{E}[C^{1-\rho}]} \]  \hspace{1cm} (2.2.15)

\[ P_h^*(i) = \frac{P_h(i)}{S} \]  \hspace{1cm} (2.2.16)

Where \( \hat{\lambda} = \frac{\lambda}{(1-\lambda)} \) is the markup for finished goods pricing. The term \( \hat{E}[\frac{S}{\theta_p} C^{1-\rho}] \) represents a risk premium term arising from the covariance of firm \( i \)'s profit with marginal utility of consumption, where the fluctuation of the exchange rate \( S \) directly affects firm's pricing decisions. Meanwhile, intermediate goods stage productivity shocks in both the home country and foreign country will affect the price of home finished goods though \( \hat{P}_h \) and \( \hat{P}_h^* \). For the home finished goods producer, the foreign intermediate goods productivity shock \( \theta_f^* \) affects the preset price in the same way as the home intermediate goods productivity shock \( \theta_f \). This implies that a positive stage specific shock in the foreign intermediate goods stage will generate a substitution from home intermediate goods to foreign intermediate goods. Meanwhile, it also has a positive trans-border spillover effect on the home finished goods.

\[ ^5 \text{For simplicity, we have used the fact that } C = C^* \text{ in the total demand function for final goods. Since the price of final goods are preset in producer currency, purchasing power parity (PPP) holds. Thus, we could derive } C = C^* \text{ from the risk-sharing condition (2.2.7).} \]
Imposing symmetry, we can drop out the subscript \( i \). The optimal pricing schedules of foreign finished goods firms can be derived analogously and are listed in Table B.1.

### 2.2.3 Intermediate goods stage

It is assumed that there is a continuum of firms indexed by \( j \in [0,1] \) in the intermediate goods stage in each country. The home intermediate goods firm \( j \) uses the following linear technology, subject to the stage-specific shock \( \theta_f \):

\[
X_h(j) = \theta_f L(j)
\]  

(2.2.17)

**Demand structure**  One basket of distinct variety of the home intermediate goods is given by:

\[
X_h = \left( \int_0^1 X_h(j) \frac{\phi-1}{\phi} \right)^{\frac{\phi}{\phi-1}}
\]  

(2.2.18)

where \( \phi \) represents the elasticity of substitution between intermediate goods in the home country. The demand for the intermediate good \( j \) in this setting can be derived as:

\[
X_h(j) = \left( \frac{\tilde{P}_h(j)}{\tilde{P}_h} \right)^{-\phi} X_h
\]  

(2.2.19)

where \( \tilde{P}_h(j) \) is the price of the intermediate good \( j \) in the home country. Then we may derive the total demand for a basket of distinct variety of home intermediate goods from the home finished goods and the foreign finished goods firms, respectively:

\[
X_h = \frac{1}{2} \left( \frac{\tilde{P}_h}{\Lambda} \right)^{-1} \int_0^1 Y(i)di = \frac{1}{2} \left( \frac{\tilde{P}_h}{\Lambda} \right)^{-1} \int_0^1 \left( \frac{P_h(i)}{P_h} \right) - \lambda \left( \frac{P_h}{P} \right)^{-1} C |di
\]  

(2.2.20)

\[
X^*_h = \frac{1}{2} \left( \frac{\tilde{S}}{\Lambda^*} \right)^{-1} \int_0^1 Y^*(i)di = \frac{1}{2} \left( \frac{\tilde{S}}{\Lambda^*} \right)^{-1} \int_0^1 \left[ \frac{P^*_f(i)}{P^*_f} \right] - \lambda \left( \frac{P^*_f}{P^*} \right)^{-1} C^* |di
\]  

(2.2.21)

Equations (2.2.20) and (2.2.21) show that the demand structure of intermediate goods would be affected by aggregate consumption \( C \), the finished goods stage shock \( \theta_F \) (or \( \theta^*_F \)) through \( \Lambda(\Lambda^*) \), and the nominal exchange rate \( S \).
Intermediate goods prices  We assume the prices of intermediate goods are preset in PCP, but we allow for pricing to market. That is, the intermediate goods producer sets two prices, one is for the sales in the domestic market, and the other is for the foreign sales. That is, the law of price may not hold in the intermediate goods stage. The two pricing policies for home intermediate goods producer $j$ can be derived from the following profit maximization problem.

$$\max_{\tilde{P}_h(j), \tilde{P}_{hf}(j)} E\Pi(j) = E\{\Omega \left( \frac{1}{2} \left( \tilde{P}_h(j) - \frac{W}{\theta_i} \right) \left( \frac{\tilde{P}_h(j)}{\tilde{P}_h} \right)^{-\phi} \int_0^1 \left( \frac{P_h(i)}{P_h} \right) - \lambda \left( \frac{P_h}{P} \right)^{-1} C \right) di \}
+ \frac{1}{2} \left( \tilde{P}_{hf}(j) - \frac{W}{\theta_i} \right) \left( \frac{\tilde{P}_{hf}(j)}{\tilde{P}_{hf}} \right)^{-\phi} \int_0^1 \left( \frac{P_f(i)}{P_f} \right)^{-\lambda} \left( \frac{P_f}{P_f} \right)^{-1} C^* \right) di\} \right)$$

This yields

$$\tilde{P}_h(j) = \phi \frac{E\left[W \frac{1}{\theta_i} \frac{1}{\theta_p} \frac{1}{\theta_p} \right]}{E\left[\frac{1}{\theta_i} \frac{1}{\theta_p} \frac{1}{\theta_p} \right]} \quad (2.2.22)$$

$$\tilde{P}_{hf}(j) = \phi \frac{E\left[W \frac{1}{\theta_i} \frac{1}{\theta_p} \frac{1}{\theta_p} \right]}{E\left[\frac{1}{\theta_i} \frac{1}{\theta_p} \frac{1}{\theta_p} \right]} \quad (2.2.23)$$

where $\Omega = \frac{1}{P^{PCP}}$ is the marginal utility of consumption of the home household, which is used as the stochastic discount factor as all the domestic firms are owned by households. $\phi = \frac{\phi}{\phi-1}$ is the markup for intermediate goods price. Note that, for the foreign finished goods producers, the price of home intermediate goods in terms of the foreign currency is $\frac{\tilde{P}_{hf}(j)}{S}$. If the intermediate goods firms can set price flexibly, then the price will simply be a fixed markup over the unit labor cost. With the sticky price, there is an additional risk premium term arising from the covariance of marginal cost with the term $\frac{1}{\theta_p} \frac{1}{\theta_p} \frac{1}{\theta_p}$ which represents the demand risk from its buyer—the finished goods producers. Thus, the productivity shock in the foreign finished goods stage will also have a positive spillover effect on the home intermediate goods as it increases the demand for these goods. Imposing symmetry, we can drop out the

---

This PCP pricing with pricing to market is used in Devereux, Shi and Xu (2003). With this assumption, we can solve the model analytically.
subscript \( j \). The optimal pricing schedules of foreign intermediate goods firms are reported in Table B.1.

### 2.2.4 Stochastic shocks

We assume the final goods stage shock \( \theta_F \) and intermediate goods stage shock \( \theta_I \) are:

\[
\theta_F = \exp(u), \quad \theta_I = \exp(v)
\]

(2.2.24)

where \( u \) and \( v \) is mean zero and normally distributed with a variance-covariance matrix

\[
\Sigma = \begin{pmatrix}
\sigma_u^2 & \sigma_{uv} \\
\sigma_{uv} & \sigma_v^2
\end{pmatrix}
\]

A similar assumption is made for the foreign productivity shocks. Our setup allows the productivity shock to be specific to a particular stage in a particular country. This nests a special cases where productivity shock are country-specific (\( u = v, u* = v* \)).

### 2.2.5 Equilibrium

The market clearing condition of finished goods market, money market and labor market are trivial. Note that the intermediate goods market clearing condition in the home country is given by:

\[
\theta_I L = X_h + X'_h = \frac{1}{2} \frac{\Lambda}{P_h} \frac{PC}{P} + \frac{1}{2} \frac{\Lambda^*}{P_{h^*}} \frac{P^*C^*}{P^*}
\]

(2.2.25)

Thus, given the stochastic processes \( \theta_F, \theta_F^*, \theta_I \) and \( \theta_I^* \), the 17 variables \( C, C^*, P, P^*, P_h, P_f, P_h^*, P_f^*, S, \Lambda, \Lambda^*, W, W^*, L \) and \( L^* \), are determined by 17 equations: risk-sharing condition (2.2.7), money demand (2.2.8) and its foreign equivalent, labor supply (2.2.9) and its foreign equivalent, home finished goods pricing equations (2.2.15) and its foreign equivalent, intermediate goods pricing equation (2.2.22), (2.2.23) and their foreign equivalents, intermediate goods market clearing condition (2.2.25) and its foreign analogy, the unit cost function of finished goods (2.2.11) and its foreign equivalent, and the two CPI price indices.
2.3 Solution

The model is log-linear and the underlying productivity shocks are log normal, so we may solve all the endogenous variables in closed form\(^7\). We focus on the variables which are determined after the shocks are realized. The nominal exchange rate can be derived from the risk-sharing condition and money clearing condition (2.2.7) and (2.2.8):

\[ S = \frac{M}{M^*} \]  

(2.3.1)

From Equation (2.2.7)-(2.2.9), we have:

\[ W = SW^* \]  

(2.3.2)

Since the law of one price and PPP holds in CPI level, we have:

\[ C = C^* = \left( \frac{M^{\frac{1}{2}}M^{*\frac{1}{2}}}{\chi P^2_h P^2_f} \right)^{\frac{1}{2}} \]  

(2.3.3)

\( P_h \) and \( P_f^* \) are set before the realization of the shocks, so the consumption in both countries are determined by the home money supply and the foreign money supply.

If all the prices can respond to the ex-post value of stochastic shocks, then clearly money will be completely neutral and the economy will be at the efficient flexible price equilibrium\(^8\).

To measure the desirability of flexible exchange rate, we will use the terms of trade and the expected utility under the flexible price equilibrium as a benchmark.

Putting the home and foreign finished goods price indices together, using the risk-sharing condition, the pricing equation of intermediate goods and the labor supply function, we may get the solution for the home and foreign consumption in the flexible price equilibrium.

\[ C = C^* = \left( \bar{\lambda} \delta \eta^2 \right)^{-\frac{1}{2b}} \left( \theta_f' \theta_f^* \right)^{\frac{1}{2b}} \]  

(2.3.4)

\(^7\)This implies that consumption, labor, prices and exchange rate all follow log normal distribution.

\(^8\)In the economy with two stages of production and trade, there are two sources of distortion associated with monopolistic competition, which lower the efficient expected utility level. However, this inefficiency cannot be eliminated by the monetary policy, so it is not our focus.
Chapter 2. Optimal Monetary Policy with Vertical Production and Trade

From the intermediate goods market clearing condition (2.2.25), we may solve for the home and foreign labor in the flexible price equilibrium.

\[ L = L^* = \frac{1}{\lambda \delta \eta} C^{1-\rho} = (\lambda \phi)^{-\frac{1}{2\rho}} \eta^{-\frac{1}{\rho}} (\theta_F \theta^*_F, \theta_I \theta^*_I)^{\frac{1-\rho}{2\rho}} \]  

(2.3.5)

Note that both the consumption and the employment are functions of geometric weighted average of productivity shocks.

The terms of trade under flexible prices in both stages can also be derived:

\[ \frac{SP^*_f}{P_h} = \frac{\theta_F}{\theta^*_F}, \quad \frac{SP^*_f}{P_h} = \frac{\theta_I}{\theta^*_I} \]  

(2.3.6)

To achieve the efficient allocation, the terms of trade in both the finished goods stage and the intermediate goods stage must be adjusted completely according to the corresponding relative productivity shocks\(^9\).

Following Obstfeld and Rogoff (1998, 2002), it is assumed that the utility derived from real balances is small enough to be neglected. From Equation (2.3.5), the expected utility for the representative consumer in the home country can therefore be measured by the following expression

\[ E\left(\frac{C^{1-\rho}}{1-\rho} - \eta L\right) = \kappa EC^{1-\rho} \]  

(2.3.7)

where \( \kappa = \frac{\lambda \phi - (1-\rho)(\lambda-1)(\delta-1)}{(1-\rho)^2} < 0 \). Since consumption has the log normal property, we may rewrite \( EC^{1-\rho} = \exp(1-\rho)(Ec + \frac{1-\rho}{2} \sigma^2_c) \), thus, maximizing the expected utility level is equivalent to maximizing \( U_0 = Ec + \frac{1-\rho}{2} \sigma^2_c \). Therefore, we can measure the expected utility level in the flexible price equilibrium as

\[ U_0(\text{flex}) = \frac{-\ln(\lambda \phi \eta^2)}{2\rho} + \frac{1-\rho}{8\rho^2} [\sigma^2_u + \sigma^2_{u^*} + \sigma^2_v + \sigma^2_{v^*} + 2\sigma_{uv} + 2\sigma_{u^*v^*}] \]  

(2.3.8)

In the absence of nominal rigidities, not surprisingly, the equilibrium is independent of the monetary policy. The expected utility level is composed by two parts: a constant function of parameters and the impact of stochastic productivity shocks on the expected utility. When

\(^9\)In the flexible price equilibrium, the relative price of finished goods to intermediate goods in each country will also respond completely to the corresponding relative productivity shocks.
$\rho = 1$, the expected utility is only affected by the mean of the log consumption. However, when $\rho > 1$, the risk-averse household will care about the uncertainty brought by the stochastic productivity shocks, and the expected utility is decreasing in the volatilities of the stochastic productivity shocks.

2.4 Optimal money rules

In this section, we study the optimal monetary policy rules in response to stage-specific productivity shocks. The independent monetary authority in each country sets the following monetary rules to maximize the expected utility of the domestic households:\(^{10}\)

\[
m = m_0 + a_1 u + a_2 u^* + a_3 v + a_4 v^* \\
m^* = m_0 + b_1 u + b_2 u^* + b_3 v + b_4 v^* 
\]

From now on, let $x = \ln X$. The policy parameter vectors $[a_1, a_2, a_3, a_4]$ and $[b_1, b_2, b_3, b_4]$ will be determined by the international monetary Nash game between two independent monetary authorities. It is assumed that they can commit to their monetary rules and the rules are announced before the firms set their prices.

As shown in Devereux and Engel (2003), the expected utility level in a stochastic environment is a function of variances and covariance terms of the log home and foreign consumption and the log exchange rate. Thus, once we solve for the consumption and the exchange rate, we may rewrite the expected utility as functions of the monetary policy parameters. In log terms, we may write Equations (2.3.1) and (2.3.3) as

\[
s - E_s = m - m^* \\
c - E_c = c^* - E c^* = \frac{1}{2\rho} (m + m^*) 
\]

\(^{10}\)The log-linear money supply rule we choose here is a general form of policy rules for the models with log normal stochastic shocks. Thus, the log money supply follows random walk. This property will give us a constant nominal interest rate and shut down the effect of the nominal interest rate on the real balance when the real balance takes log form in the utility function.
where $Ex$ denotes the conditional expectation of the variable $x$ before the period begins. The detailed derivation of the variance and covariance terms of $c$ and $s$ in terms of the monetary policy parameters are given in Appendix B.3.

We now turn to the derivation of the objective functions of monetary authorities. Using the intermediate goods market clearing condition (2.2.25), we have:

$$EL = \frac{1}{\lambda \phi \eta} EC^{1 - \rho}$$  \hspace{1cm} (2.4.5)

Thus, the objective function of the monetary authority in the home country is exactly the same as Equation (2.3.7).

From equation (2.3.7), the expected utility level depends mainly on the mean and the variance of the log consumption, so the key step is to solve for $Ec$. The detailed derivation is given in Appendix B.2.

$$Ec = \frac{-\ln(\lambda \phi \eta^2)}{2 \rho} - \frac{(2 - \rho)}{2} \sigma_c^2 - \frac{1}{4 \rho} \sigma_s^2 - \frac{1}{4 \rho} \sigma_u^2 + \sigma_{uv} + \sigma_{uv^*} + \sigma_{vu} + \sigma_{uu^*}$$

$$+ \frac{1}{4 \rho} [\sigma_{svu} - \sigma_{svu^*}] + \frac{1}{4 \rho} [\sigma_{svu} - \sigma_{svu^*}] + \frac{1}{2 \rho} [\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}]$$  \hspace{1cm} (2.4.6)

Given the log normal property of the consumption and the functional form of the expected utility (2.3.7), the optimization problem of the monetary authority is equivalent to maximizing $U_0 = Ec + \frac{1 - \rho}{2} \sigma_c^2$, so $U_0$ can be derived as

$$U_0 = \frac{-\ln(\lambda \phi \eta^2)}{2 \rho} - \frac{1}{2} \sigma_c^2 - \frac{1}{4 \rho} \sigma_s^2 - \frac{1}{4 \rho} \sigma_u^2 + \sigma_{uv} + \sigma_{uv^*} + \sigma_{vu} + \sigma_{uu^*}$$

$$+ \frac{1}{4 \rho} [\sigma_{svu} - \sigma_{svu^*}] + \frac{1}{4 \rho} [\sigma_{svu} - \sigma_{svu^*}] + \frac{1}{2 \rho} [\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}]$$  \hspace{1cm} (2.4.7)

We may rewrite the expected utility function as a function of policy parameter vectors $a$ and $b$. For the home households, consumption variance and exchange rate variance reduce utility, while the covariances of consumption and productivity shocks, a positive covariance of the exchange rate and home productivity shocks or a negative covariance with foreign productivity shocks increases utility. The optimal monetary rule will be a trade-off between these costs and benefits and the effects of monetary policies on both the consumption and the exchange rate will be considered.
A Nash equilibrium of the international monetary game between countries is characterized by the following conditions:

\[
\max_a U_0(a, b^N) \quad (2.4.8)
\]

\[
\max_b U_0^*(a^N, b) \quad (2.4.9)
\]

The objective function \( U_0^* \) is identical to \( U_0 \) since home and foreign have identical consumption and employment in equilibrium. Appendix B.3 gives the following solution to this monetary Nash game.

\[
a_1^n = \frac{3}{4}, \quad a_2^n = \frac{1}{4}, \quad a_3^n = \frac{3}{4}, \quad a_4^n = \frac{1}{4}
\]

\[
b_1^n = \frac{1}{4}, \quad b_2^n = \frac{3}{4}, \quad b_3^n = \frac{1}{4}, \quad b_4^n = \frac{3}{4}
\]

With sticky price, an optimal monetary policy response to a positive productivity shock must be expansionary, so as to shift demand to meet the potential aggregate supply. The magnitude of the response, however, depends on the expenditure switching effect of the exchange rate changes under the PCP pricing regime. As shown in Obstfeld and Rogoff (2000b, 2002), Corsetti and Pesenti (2001), Devereux and Engel (2003), with PCP pricing and full exchange rate pass-through, the optimal monetary policy requires the home monetary policy authority to respond only to the home productivity shock. That is, if there are two country-specific shocks \( \theta \) and \( \theta^* \), the policy parameter vector would be \( [a_1 = 1, a_2 = 0] \) and \( [b_1 = 0, b_2 = 1] \).

Nevertheless, the solution for the Nash game in our model implies that the optimal monetary policy requires the home monetary authority to respond positively to both home and foreign productivity shocks, though the response of the home monetary authority to domestic productivity shocks exceeds that of the foreign monetary authority. Home monetary authority responds positively to foreign productivity shocks because foreign productivity shocks have a trans-border spillover effect, which is induced by the vertical structure of production and trade. For instance, if a positive stage-specific productivity shock occurs in the foreign
finished goods stage, it not only increases the potential supply of foreign finished goods, but also increases the demand for home and foreign intermediate goods. In this sense, home intermediate goods producers will benefit from a positive foreign productivity shock. That is, any stage-specific shock in one country would affect both countries like a "world shock", so both countries should respond positively to this shock.

Meanwhile, under PCP pricing, the conventional expenditure-switching effect is still playing a role in our model. Specifically, a positive stage-specific shock in one country also generates a direct substitution effect from imported goods to domestic produced goods in this particular stage. Intuitively, the substitution effect of a home stage-specific shock on home goods should dominate its indirect positive transborder spillover effect on the foreign country, which requires a nominal exchange rate depreciation in the home country. To achieve this, the response of the home monetary authority to foreign shock should be less than that of foreign monetary authority.

From the solution to the international monetary game, we can have the following propositions.

**Proposition 1** Under the flexible exchange rate regime, the exchange rate is more stable when there is vertical production and trade.

From the optimal monetary policy rules, we may solve for the log exchange rate:

$$s = \frac{1}{2} u - \frac{1}{2} u^* + \frac{1}{2} v - \frac{1}{2} v^*$$

(2.4.10)

Given the assumption that the shocks in one country have the variance-covariance matrix

$$\Sigma = \begin{pmatrix} \sigma^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma^2 \end{pmatrix}$$

it follows that the variance of optimal exchange rate is $\sigma^2 + \sigma_{uv}$. When there is no correlation between the finished goods shock and the intermediate goods shock in one country ($\sigma_{uv} = 0$), the exchange rate volatility is just $\sigma^2$. However, when there is only one stage of production and trade and the price is set under PCP pricing $^{12}$, the optimal policy parameter vectors are $a = [1, 0]$ and $b = [0, 1]$. This implies that the variance of

$^{11}$For simplicity, we assume that $\sigma_{u}^2 = \sigma_{v}^2 = \sigma_{u^*}^2 = \sigma_{v^*}^2 = \sigma^2$, and $-\sigma^2 \leq \sigma_{uv} = \sigma_{u^*v^*} \leq \sigma^2$.

$^{12}$See Devereux and Engel (2003) for the detail derivation.
optimal exchange rate is $2\sigma^2$ if the volatilities of both home and foreign country-specific shocks are equal to $\sigma^2$ as well. Even if $\sigma_{uv} > 0$, the exchange rate volatility is still lower than that under the economy without vertical structure of production and trade. Therefore, our findings suggest that the floating exchange rate regime is more stable in a world with vertical production and trade.

The economic intuition is as follows. With vertical structure of production and trade, the production of the world import and export goods is more diversified, but each country is more integrated with other countries. Therefore, there is no need for large exchange rate changes to adjust the world economy according to the relative shocks. Our model shows that the implication of flexible exchange rate for optimal monetary policy depends on whether shocks are country or stage specific. If the policy makers are not able to observe the nature of shocks, the misconduct of monetary policy might lead to inefficient outcome and higher exchange rate volatility. Proposition 1 also suggests an empirical prediction that the increase in the vertical trade may reduce the exchange rate volatility under a floating exchange rate regime.

**Proposition 2** The optimal solution cannot replicate the flexible price equilibrium and the flexible exchange rate cannot deliver the economy to the efficient level.

When there is only one stage of trade, the conventional wisdom regarding the welfare implication of the flexible exchange rate is that it can bring the economy around the obstacle of nominal rigidities, if the prices are preset in PCP or there is complete exchange rate pass-through. That is, the optimal monetary policy can replicate the flexible price equilibrium. However, this conclusion does not hold when there is more than one stages of production and trade. The intuition is straightforward. Under PCP specification, flexible exchange rate can adjust the terms of trade to the efficient level - the level under the flexible prices equilibrium, so the world resource can be allocated efficiently. However, in a world with vertical structure of production and trade, there are two terms of trades between home and foreign country. Thus, the flexible exchange rate cannot adjust the relative prices of both finished and intermediate goods to the efficient level simultaneously. There still exists
welfare loss even when monetary authorities optimally respond to both home and foreign productivity shocks.

For instance, under the sticky price equilibrium, the term of trade in the finished goods stage is \( \frac{SP^*}{P_h} \). Since \( P_h \) and \( F_{j*} \) are both predetermined, the term of trade will be proportional to the exchange rate changes \( \frac{(\theta_P \varphi_1)^{1/2}}{(\theta_j \varphi_j)^{1/2}} \), which is different from the term of trade under the flexible price equilibrium \( \frac{\varphi}{\varphi} \). Similarly, the term of trade in the intermediate goods stage is not equivalent to its flexible price level, either.

This intuition is quite similar to the argument of Erceg, Henderson and Levin (2000). They find that when both prices and wages are sticky, the allocation with flexible prices and wages cannot be restored by the optimal monetary policy. That is, if there is multiple stickiness, one policy instrument cannot deal with all nominal rigidities efficiently. In our model, even if the exchange rate policy can adjust the relative prices between home and foreign country, there still exists misallocation between finished goods and intermediate goods.

Given the optimal monetary rules, we can compare the maximized expected utility level under the sticky price equilibrium with the expected utility level under the flexible price equilibrium:

\[
U_0(\text{flex}) - U_0(\text{sticky}) = \frac{1}{16} \left[ \sigma_u^2 + \sigma_v^2 + \sigma_u^2 + \sigma_v^2 - 2\sigma_{uv} - 2\sigma_{uv*} \right] \geq 0 \quad (2.4.11)
\]

We may use this welfare difference to measure the desirability of flexible exchange rate regime. The higher the welfare difference, the lower the value of the exchange rate flexibility in the open economy with nominal rigidities. Obviously, the vertical structure of production and trade reduces the value of exchange rate flexibility. If there are more stages of vertical production and trade, the desirability of flexible exchange rate for the optimal monetary policy will be even smaller, as more nominal rigidities are present in the economy.

Equation 2.4.11 also implies a higher correlation between finished goods shock and intermediated goods shock in each country increases the value of exchange rate flexibility. When the shocks in two different stages of one country are perfectly correlated, the optimal monetary rules can replicate the flexible price equilibrium\(^\text{13}\), but the response of monetary

\(^{13}\text{The welfare difference between the flexible price equilibrium and the sticky price equilibrium is zero.}\)
authorities to shocks are still different from that in the standard open economy literature.

Thus, our findings suggest that the introduction of vertical structure and trade does affect the international transmission mechanism of productivity shocks and the optimal monetary policy design in an open economy.

2.5 Conclusion

This paper examines the optimal monetary policy in a world with vertical production and trade by introducing two stages of production and trade into the standard utility-based open economy macroeconomic models.

We find that when both finished goods and intermediate goods are tradable, there exists trade-induced trans-border spillover effect of productivity shocks, and this may change the way monetary authorities respond to other countries' productivity shocks. We also find that the flexible exchange rate cannot bring the economy back to the efficient level even under the PCP pricing or the complete exchange rate pass-through case, and the floating exchange rate regime in our model implies more stable exchange rates than in the economy without vertical production and trade. This is quite different from the classical argument for flexible exchange desirability in optimal monetary policy in open economy, suggested by Friedman (1953), Mundell (1961) and recent new open economy macroeconomics literatures. Our findings suggest that the changes in the trade pattern in the global economy over the last thirty years might affect the international optimal monetary policy rules and values of exchange rate flexibility. So the monetary policy makers should take into account the impact of the changes in the trade pattern when making decisions.

To verify the theoretical results of the model, some empirical work can be done in the future research to check if the increase of the share of the vertical trade over the last thirty years reduces the exchange rate volatility under a floating exchange rate regime.

One advantage of this paper is that all the results can be derived analytically. Nevertheless, to make the model tractable, the number of the stages of production and trade we introduced into the economy has to be limited. In the subsequent research, with the help
of some numerical methods, more stages of production and trade can be considered and the
optimal monetary policy in a more integrated global economy can be explored. We conjecture that if the number of stages is larger and the world economy is more integrated, the exchange rate flexibility will pay a less important role in the adjustment of relative prices, so the exchange rate volatility will be even smaller. In the extreme case where the number of vertical stages is large enough, a flexible exchange rate regime will be close to a fixed exchange rate regime.
Chapter 3

Global Monetary Policy under a Dollar Standard

3.1 Introduction

If the dollar were ever displaced by the euro, [the US] .. would lose the enormous freedom it now enjoys in running macro-economic policy.

The US dollar occupies a unique role in the world economy. The dollar resembles an international currency, in the sense that it acts as a means of exchange in international goods and asset trade, a store of value in international portfolios and official foreign exchange reserves, and a unit of account in international commodity pricing. BIS estimates of foreign exchange turnover show that the dollar is used as one side of about 90 percent of daily foreign exchange rate transactions. According to Eichengreen and Mathieson (2000), 60 percent of world foreign exchange reserves are held in US dollars. Bekx (1998) estimates that over 50 percent of world exports in 1995 were denominated in US dollars, approximately four times the share of the US in total world exports. This predominance of the US dollar has been described by McKinnon (2001, 2002) as a world dollar standard. While the formation of the euro area has generated speculation about the stability of the current dollar role (e.g. Portes and Rey, 1998), at present there seems little evidence of significant change in the use of the dollar in trade and finance.

1This chapter is based on the joint work with Michael Devereux and Kang Shi.
2This share has undoubtedly gone up since the advent of the euro, because all intra-European foreign reserves held in DM's and other European currencies are no longer part of measured reserves.
How does the special role of the US dollar influence monetary policy making in the US and the rest of the world? The quotation above suggests that the US has an advantage in policy making due to the fact that the rest of world holds dollars, and sets prices in dollars. Indeed many commentators argue that there is an enormous welfare gain to the US from having its currency used so widely (e.g. Liu (2002)). The literature on the international monetary system has developed a theory of 'hegemonic stability' based on the idea that the policies of one country play a central role in maintaining the smooth working of the international monetary system (Eichengreen (1995)). According to this theory, US monetary policy may be determined without regard to international constraints, while monetary policy in the rest of the world must take account of US policies.

This paper examines the determination of optimal monetary policy in an asymmetric world economy, where the currency of one country (e.g. the US dollar) plays a predominant role in trade. While the evidence cited above illustrates the multi-dimensional role of an international currency, we focus on one particular aspect of this role - the importance of the currency in international export good pricing. We define a reference currency as one in which the prices of all world exports are pre-set. Many authors have noted (e.g. Goldberg and De Campa (2003)) that prices of imported goods sold in the US economy tend to be much less affected by exchange rate fluctuations than do imported good prices in non-US countries. This suggests that prices of a large fraction of exports to the US are pre-set in US dollar terms (which we refer to as local currency pricing, or LCP), and do not react quickly to movements of the exchange rate.

However, exports to other countries may have their prices pre-set in the currency of the original producer (producer currency pricing, or PCP), and hence import prices are more

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3In the recent international macroeconomics literature, considerable attention has been devoted to the determination of optimal monetary policy under sticky prices. See Benigno and Benigno (2003), Devereux and Engel (2003), Obstfeld and Rogoff (2002), among many other papers. But most of this literature is that it focuses on symmetric environments, where all countries are identical in economic structure.

4Bachetta and Van Wincoop (2002) and Kenen (2003) note that the US dollar is used as an invoice currency for the overwhelming majority of US imports, but for other OECD countries, imports are mainly invoiced in foreign currency.
sensitive to exchange rate movements. In this sense, the monetary policy problem will be asymmetric. The optimal monetary policy in the US will reflect the fact that there is little pass-through from exchange rates to prices of consumer goods in the US economy. On the other hand, for other countries, the optimal monetary policy will take account of high pass-through from exchange rates to prices.

How does the asymmetry in international export good pricing affect the optimal monetary policy outcome? We show that, at one level, the monetary policy context and outcomes implied by the model are quite closely in accord with popular wisdom about the position of the US in the world economy. In particular, the monetary authorities of the reference currency place a very low weight on exchange rate volatility in their monetary policy loss function. By contrast, the monetary authorities of rest of the world will be much more concerned with exchange rate volatility. This seems to well approximate the observed indifference of the US to the exchange rate in monetary policy-making. A second feature of the outcome is that the reference currency country follows a more stable monetary policy than that of the rest of the world. More importantly though, we find that the monetary policy game between the reference currency country and the rest of the world has a key sense in which the reference country acts as a 'hegemon'. A Nash equilibrium of this game is identical to one in which world monetary policy (for both countries) is determined by the preferences of the reference currency country alone. That is, the Nash equilibrium of the asymmetric game is the same as that which would obtain were the reference currency monetary authority to choose both its own and the rest of the world's monetary rules to maximize its own welfare. In this sense, the asymmetry in international pricing gives the reference country a dominant role in

5In reality, there is considerable difference between the pass-through of exchange rate changes to import goods prices and final goods prices. In this paper, we abstract from this difference. In fact, the optimal monetary policy is more focused on the pass-through to final consumer goods. It would be possible to allow for a high rate of exchange rate pass-through into import good prices in combination with low pass-through into consumer good prices, without changing the results of the paper at all. This is shown in Devereux, Engel, and Tille (1999).

6In the model, the loss function is endogenously derived from the nature of the environment facing each country.
international monetary policy determination. World monetary policy is designed according to its preferences, and, even if it could play a more explicitly dominant role (by acting as a 'Stackelberg Leader' in monetary policy determination), it would not wish to deviate from the Nash equilibrium.

For the rest of the world however, the outcome is quite different. In general their monetary authorities would wish to alter the determination of both their own and the reference country monetary rules, were they capable of playing a more dominant role. Despite this, there are no gains to international monetary policy coordination. Since the reference country enjoys the best possible outcome in a Nash equilibrium, any alternative monetary policy configuration chosen by a 'world monetary authority' is not incentive compatible, except in the trivial case where all the weight is given to reference country welfare.

Hence, our model supports the view that, under the dollar standard in international goods pricing, US monetary policy has a predominant role. A natural question to ask then is how much the US gains from this. How much better off are US residents due to the special place of the dollar in export price setting? The surprising answer is that US residents are not better off, but rather are worse off. Expected utility for residents of the reference currency country, where pass-through from the exchange rate to the CPI is zero, is lower than that of the rest of the world, where there is full pass-through. While this may seem inconsistent with the result that the US determines world monetary policy, the explanation is that the asymmetric pricing means that the welfare outcomes are asymmetric. Even if monetary policy were determined by a world social planner with equal weights on both regions, welfare of the reference country would differ from that of the rest of the world.

Why does the dollar standard hurt the US? The reason is that when export pricing is done in terms of the US dollar, it prevents an efficient response of relative prices to underlying real

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7 Our model excludes some of the factors which would be important in the overall quantitative accounting of the gains from the dollar standard. In particular, there are no offshore holdings of currency in the model, so there is no seigniorage revenue earned on foreign money holdings. Nevertheless, we can do a welfare analysis by comparing expected utility in the reference currency country and in the rest of the world, because other than the special role of the reference currency, the model is otherwise symmetric.
shocks. An efficient monetary policy will generally want to employ both expenditure level (affecting total aggregate demand) and expenditure switching (affecting the relative demand for one country’s goods) effects. When import prices do not respond to the exchange rate, monetary policy cannot be used to generate expenditure switching effects. This has a welfare cost for the residents of the reference currency economy. Hence, in our model, the dollar standard is costly for the US economy.

Where does the special role of the reference currency come from? There is a considerable literature on the determinants of ‘international currency’. An early contribution by Krugman (1984) argues that there may be multiple equilibria due to ‘snowballing effects’, whereby if one currency becomes the accepted standard, then all participants in international markets have an incentive to use this currency. On the other hand McKinnon (2003) argues that the special role of the U.S. dollar arose partly from the record of low inflation and stable monetary policy that the U.S. economy followed in the Post WWII period. In a later section of the paper, we extend the model to allow exporting firms the choice of currency in which to set prices, and investigate the conditions under which there is an equilibrium where exporters in both countries will use the currency of a single country for price setting. Our results suggest that both the Krugman multiple equilibria explanation and the McKinnon policy-determined explanation are important elements in the selection of a reference currency. We show that, in the equilibrium of the monetary policy game, the reference currency country’s monetary authority will follow a more stable (lower variance) monetary policy. As a result, this tends to lock in an equilibrium where exporters in both countries use this currency in which to set prices. But the reason that the reference currency monetary authorities follow such a rule is precisely because the currency is used as a reference in international trade pricing. This implies however that there are other equilibria where either another currency will play the role of the reference currency, or no country’s currency does, so traded goods pricing is symmetric (either LCP or PCP) across countries.

Although many other factors are likely to be important in the acceptability of an international currency, the choice of currency for pricing will remain one important channel.
Chapter 3. Global Monetary Policy under a Dollar Standard

The paper is structured as follows. The following section develops the main model, which is only a slight extension of Devereux and Engel (2003). Section 3 derives the solution of the model for given monetary policy rules. Section 4 derives the optimal rules in Nash equilibrium of a game between monetary authorities. Section 5 extends the model to allow for the endogenous choice of currency in which to set prices. Section 6 concludes.

3.2 The two-country model

We construct a simple two-country model of trade and exchange rate determination. Firms set prices in advance, by assumption 9. There is a continuum of home goods (and home population) and foreign goods (foreign population) of measure \( n \) and \( (1 - n) \) respectively. Individual home (or foreign) goods are substitutable in preferences with elasticity \( \lambda \), but there is unit elasticity of substitution across the home and foreign categories of goods. The expected utility of home agents is

\[
E\left(\frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{M}{P} - \eta L\right)
\]

where \( C = n^{-n}(1 - n)^{-(1-n)}C_h^n C_f^{1-n} \), \( C_h = (\int_0^n C_h(i)^{1-\lambda} di)^{1/\lambda} \), and \( \rho \geq 1 \).

Here \( C \) is aggregate consumption, \( C_h \) is consumption of the home sub-aggregate, \( \frac{M}{P} \) are real money balances, with \( P = P_h^n P_f^{1-n} \) being the home CPI, and \( L \) is the home labor supply. There is only a single period in which events take place 10.

The structure of events within the period is as follows. First, before the period begins, households can trade in a full set of nominal state-contingent bonds. This means that households can offset any risk that is associated with monetary policy uncertainty, as well as risk

\[\text{9Since the model has been well covered in previous papers, here we will only briefly sketch out its main elements.}\]

\[\text{10This may seem to be an extreme assumption, but in fact it is entirely innocuous, given the asset market structure. Extensions to an infinite horizon are quite trivial, and since there exists markets for risk-sharing across countries, this would leave all the results unchanged. Just so as to avoid time subscripts in the notation, we focus on a one-period problem.}\]
Chapter 3. Global Monetary Policy under a Dollar Standard

due to country-specific productivity shocks (see below). The outcome of this stage is that households will enter the period with their revenue stream governed by an optimal risk sharing rule. Then the monetary authorities choose optimal monetary rules, given the optimal risk sharing rule, but taking into account the way in which firms set prices, as well as the distribution of country-specific technology shocks. Following this, firms set prices in advance, contingent on state-contingent discount factors, and the demand and marginal conditions that they anticipate will hold. After the realization of stochastic technology shocks, households choose their optimal consumption baskets, production and consumption takes place, and the exchange rate is determined.

Trade in state-contingent nominal assets across countries will lead to the following optimal risk sharing arrangement:

$$TPC^\rho = SP^*C^{*\rho},$$

(3.2.1)

where $S$ is the nominal exchange rate, and $P^* = P_h^*n^*(1-n)$ is the foreign price level. Optimal financial markets lead to the equalization of the marginal utility of money across countries, up to a state-invariant weighting $\Gamma$. If the countries were entirely ex-ante identical, then obviously $\Gamma$ would equal unity. But given the differences in pricing policies, countries are not necessarily the same, ex-ante. In this case, $\Gamma$ will be chosen so as to reflect that different positions of the two countries in the initial competitive market in state-contingent assets. Given the structure of preferences, we can show that the that the value of $\Gamma$ will be

$$\Gamma = \frac{EC^{(1-\rho)}}{EC^{*((1-\rho)}}.$$

(3.2.2)

In addition, household optimization gives rise to the money demand rules:

$$M = \chi PC^\rho,$$

(3.2.3)

\footnote{This condition says that optimal risk sharing will equate the marginal utilities of money across countries in each state of the world. The condition is a familiar one - see for instance Chari et al. (2002). For a rigorous proof of this condition, see Devereux and Engel (2003).}

\footnote{For a proof, again see the appendix of Devereux and Engel (2003).}
and the implicit labor supply conditions given by

\[ W = \eta PC^\rho. \quad (3.2.4) \]

Since monetary policy is determined after financial markets have closed, the monetary authorities take \( \Gamma \) as given in their evaluation. We delay the discussion of optimal monetary rules until the next section.

Firms face demand for their good from consumers in both their domestic country and abroad. Firms have linear technologies, producing output from labor alone, but are subject to unpredictable (at the time of price setting) technology shocks in production. Firms can price-discriminate across national markets, and households have no ability to re-sell goods across countries. In addition, there is an asymmetric pricing structure. Home firms set prices for both the home market and the foreign market in terms of the home currency. But foreign firms set prices for export in terms of the home country currency. Hence, the foreign firms engage in LCP when selling abroad, whereas the home firms follow PCP. In this sense, the home currency is the 'reference currency' in all international trade, because all international traded goods have their prices set within this currency.

The Appendix C.1 outlines the details of the optimal pricing policies of firms. The following equations give the prices set by the representative home and foreign firm for the goods sold in home and foreign markets, respectively;

\[ P_{hh} = \lambda \frac{E(WC^{1-\rho})}{E(C^{1-\rho})} \quad (3.2.5) \]

\[ P_{fh} = \lambda \frac{E(W^*SC^{1-\rho})}{E(C^{1-\rho})} \quad (3.2.6) \]

\[ P_{hf} = \lambda \frac{E(WC^{1-\rho})}{E(C^{1-\rho})} \quad (3.2.7) \]

\[ P_{ff}^* = \lambda \frac{E(W^*SC^{1-\rho})}{E(C^{1-\rho})}. \quad (3.2.8) \]
In these equations, $\lambda$ represents the markup $\frac{1}{\lambda - 1}$, subscript $h, f$ represents the price of the home good in the foreign market etc, and $\theta$ represents the home country productivity coefficient. These equations indicate that optimal prices depend on the joint distribution of marginal cost ($\frac{w}{p}$), the exchange rate, and consumption (or aggregate demand). We assume that $\theta$ can be represented as

$$\theta = \exp(u), \quad (3.2.9)$$

where $u$ is mean zero and normally distributed. A similar assumption is made with respect to the foreign productivity shock.

An asterisk over the price means that the price is denominated in foreign currency. Hence, all home goods prices are denominated in home currency, while only foreign goods sold in foreign markets are denominated in the foreign currency. Given this convention, then the price indices for each country are as follows:

$$P = P_h^n P_f^{1-n} \quad (3.2.10)$$

$$P^* = \left[ \frac{P_h}{S} \right]^n P_f^{*1-n}. \quad (3.2.11)$$

The set of equations given by (3.2.1) and (3.2.2), in combination with (3.2.3) and (3.2.4) (with analogous conditions for the foreign economy), the pricing equations (3.2.5)-(3.2.8), and the price indices (3.2.10) and (3.2.11) give 12 equations that may be solved for the distribution of the variables $C, C^*, W, W^*, P, P^*, P_{hh}, P_{hf}, P_{fh}, P_{ff}, S,$ and $\Gamma$.

### 3.3 Solving the Model

Because the model is log-linear and the underlying technology shocks are log-normal, we may solve for the exact distribution of all endogenous variables in closed form (the details are in the Appendix C.2). The solution allows a dichotomy between variables that are determined in advance of the realization of technology shocks, i.e. $P_{hh}, P_{hf}, P_{fh}, P_{ff}$, and $\Gamma$, and variables determined after the shocks have occurred; i.e. $C, C^*, W, W^*$, and $S$. 
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The risk-sharing condition (3.2.1), in combination with the money demand equation (3.2.3) and the analogous condition for the foreign country implies a solution for the exchange rate:

\[ S = \Gamma \frac{M}{M^*}. \]  

(3.3.1)

Substituting this solution back into the money market clearing conditions then implies that

\[ C = \frac{1}{1 - \rho} \left( \frac{M}{P_{hh}P_{hh}^{1-n}} \right)^{1/2} \]  

(3.3.2)

\[ C^* = \frac{1}{1 - \rho} \left( \frac{M^*M^{*(1-n)}}{P_{hf}P_{hf}^{1-n}} \right)^{1/2}. \]  

(3.3.3)

This implies that home country consumption is independent of the realization of the foreign country money supply. This follows directly from the fact that the home country CPI is predetermined, given that both home goods and imported goods in the home market have prices pre-set in home currency. But with full exchange rate pass-through into foreign imported goods, foreign country consumption is affected by home country monetary shocks.

In log terms, we may write these equations as

\[ s - E(s) = [m - E(m)] - [m^* - E(m^*)] \]  

(3.3.4)

\[ c - E(c) = \frac{1}{\rho} [m - E(m)] \]  

(3.3.5)

\[ c^* - E(c^*) = \frac{1}{\rho} \{n[m - E(m)] + (1 - n)[m^* - E(m^*)]\} \]  

(3.3.6)

where \( E \) denotes the mathematical expectation, and small-case letters denote logarithms.

Equations (3.3.4) and (3.3.6) can be solved for the variance of the exchange rate and consumption. But first we need to set out the monetary policy rules. We make the following assumption regarding the determination of monetary policies:

\[ m = m_0 + a_1 u + a_2 u^* \]  

(3.3.7)

\[ m^* = m_0^* + b_1 u + b_2 u^* \]  

(3.3.8)

Thus, the money supply is a log linear function of the shocks in each country, where the parameters of the rules, \( a_1, a_2, \) and \( b_1, b_2, \) have yet to be determined. These rules are
perfectly general, because given that the model is log linear, and the shocks log-normal, the optimal form of monetary rules must be log-linear.

Monetary policy will be chosen to maximize expected utility for each country. In order to evaluate expected utility, it is necessary to determine expected consumption and employment. These will be affected by the stochastic structure of the model, given ex-ante optimal price setting. Using (3.2.5)- (3.2.8), along with the labor supply equations, and the risk sharing condition (3.2.1), we may set out the following two conditions which implicitly determine the mean values of $C$ and $C^*$.

\begin{align*}
1 &= \lambda \eta \Gamma^{-n} \frac{[E(C)]^n}{E(C^{1-\rho})} \\
1 &= \lambda \eta \Gamma^{-n} \frac{[E(S^{-n}C^*)]^n}{E(C^{1-\rho})} \tag{3.3.10}
\end{align*}

Using the properties of the log-normal distribution, we may re-write Equations (3.3.9) and (3.3.10) in terms of the mean and variances of log consumption and exchange rates. This gives

\begin{align*}
E(c) &= -\frac{1}{\rho} \ln(\Gamma^{-n} \lambda \eta) - \frac{2 - \rho}{2} \sigma_c^2 - \frac{n \sigma_u^2 + (1 - n) \sigma_u^2}{2 \rho} + \frac{n \sigma_{cu} + (1 - n) \sigma_{cu}^*}{\rho} \tag{3.3.11} \\
E(c^*) &= -\frac{1}{\rho} \ln(\Gamma^{-n} \lambda \eta) - \frac{2 - \rho}{2} \sigma_c^2 - \frac{n(1 - n) \sigma_s^2}{2 \rho} - \frac{n \sigma_u^2 + (1 - n) \sigma_u^2}{2 \rho} \\
&\quad + \frac{n \sigma_{cu} + (1 - n) \sigma_{cu}^*}{\rho} + \frac{n(1 - n)(\sigma_{su} - \sigma_{su}^*)}{\rho} \tag{3.3.12}
\end{align*}

Mean (log) consumption of the home country is determined only by home consumption variance, the variance of technology shocks, and the covariance of consumption with technology shocks. Equations (3.3.5) and (3.3.11) imply that both the mean and variance of home consumption is independent of foreign monetary policy. On the other hand, mean consumption of the foreign country depends both on consumption variance, the covariance of consumption with technology, and on exchange rate variance and covariance with technology shocks. Why is it that exchange rate volatility affects expected foreign consumption, but not
home consumption? This is because exchange rate volatility affects foreign import prices, and through this, the average level of pre-set prices. It will therefore affect mean consumption in the foreign country. Note that from (3.3.6) and (3.3.12), foreign consumption will clearly be influenced by both home and foreign monetary policy rules.

Since mean consumption depends on the variance and covariance properties of consumption and the exchange rate, we can derive a welfare measure for policy makers solely based on these second moments. Assume that monetary authorities in each country are concerned with the expected utility of consumption and dis-utility of labor supply, but ignore the utility of real money balances\(^{13}\). Thus, the home country monetary authority chooses its monetary rules to maximize

\[
E\left(\frac{C^{1-\rho}}{1-\rho} - \eta L\right).
\]  

(3.3.13)

From the properties of the price setting equations in the home and foreign countries, and the labor market clearing condition, we can establish that

\[
E(L) = \frac{n}{\lambda \eta} E(C^{1-\rho}) + \frac{1-n}{\lambda \eta} E(C^{*1-\rho}) \Gamma
\]

(3.3.14)

Combining (3.3.13) and (3.3.14), we may write expected home country utility as

\[
E(U) = \frac{\lambda - n(\lambda - 1)(1-\rho)}{(1-\rho)\lambda} E(C^{1-\rho}) - \frac{(1-n)(\lambda - 1)}{\lambda} \Gamma E(C^{*1-\rho})
\]

(3.3.15)

Since the log-normal distribution satisfies \(EC^{1-\rho} = \exp\{ (1-\rho)[E(c) + \frac{1-\rho}{2} \sigma_c^2] \} \), (3.3.15) ultimately depends only on the second moments of consumption and the exchange rate. These in turn depend on the monetary rules (3.3.7)-(3.3.8).

Flexible Price Equilibrium

It is useful to show the allocation that would obtain in an economy with fully flexible prices. If all prices could respond to the ex post value of technology shocks, then money would be neutral. The asymmetry in pricing would be irrelevant, because from (3.2.5)-(3.2.8), with ex-post price setting, the law of one price would hold across markets. Consumption

\(^{13}\)Obstfeld and Rogoff (2002) give a justification for this assumption.
and employment would be equalized across countries. The expressions for consumption and employment in the flexible price equilibrium are:

\[
C = C^* = (\hat{\lambda} \eta)^{-\frac{1}{\hat{\rho}}} (\theta^n \theta^* (1-n))^{\frac{1}{\hat{\rho}}}. \tag{3.3.16}
\]

\[
L = L^* = (\hat{\lambda} \eta)^{-\frac{1}{\hat{\rho}}} (\theta^n \theta^* (1-n))^{1-\frac{1}{\hat{\rho}}}. \tag{3.3.17}
\]

Productivity shocks affect consumption in each country in proportion to country size, and reduce (increase) employment in each country as \(\rho > 1\) (\(\rho < 1\)).

### 3.4 Optimal Monetary Policy

We now examine the optimal monetary rules chosen by independent monetary authorities in each country. Monetary policy is chosen with commitment, in the sense that monetary authorities choose the parameters of a monetary rule to maximize expected utility of the domestic agent, taking into account the way in which prices are set.

A natural objective of the monetary authorities would be to design optimal monetary policy so that the economy replicates the flexible price world allocation. But given the way in which prices are set, this is not possible. We show this in the following proposition.

**Proposition 1** No feasible monetary rule can replicate the flexible price world allocation.

Proof: From (3.3.16) and (3.3.17), in order to achieve the flexible price response of consumption, the home country must follow a monetary rule in which \(a_1 = n\) and \(a_2 = 1 - n\). But if the home country follows this rule, then the foreign country must follow the same rule, \(L = \hat{L}^*\). But the foreign country cannot follow this rule without violating the flexible price condition, because the foreign country's consumption is determined by the home country's monetary policy. Therefore, no feasible monetary rule can replicate the flexible price world allocation.

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14While we might anticipate that there would also be a strategic externality by which each country would attempt to use monetary policy to improve its terms of trade vis-à-vis the other country, this does not arise here because monetary rules are determined ex-ante, before private sector prices have been set. The strategic externality by which monetary policy may improve one country's welfare by influencing the terms of trade can only be effective for a *surprise* monetary shock, taking private sector expectations (i.e. prices) as given. For the same reason, there is no incentive to pursue inflation surprises in this model, because the monetary authorities follow rules with commitment. These issues are discussed in Obstfeld and Rogoff (2002).
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if it wishes to achieve the flexible price response of foreign consumption (see (3.3.6)). But in this case, neither country achieves the flexible price equilibrium response of employment. To see this, note that employment is determined by

\[ \theta L = n \frac{P}{P_{nh}} C + (1 - n) \frac{S P^*}{P_{hf}} C^* \]  
(3.4.1)

When the two countries monetary rules follow the conjectured policy, the exchange rate (3.3.4) is constant. Then the right hand side of (3.4.1) is proportional to \((\theta^n \theta^{1-n})^\frac{1}{\delta}\). But then \(L\) cannot satisfy (3.3.17). An equivalent argument holds for \(L^*\).

Monetary authorities choose their optimal rules to maximize expected utility, taking into account the determination of prices, consumption, employment, and exchange rates, but taking as given the coefficient of optimal risk-sharing \(\Gamma\). In order to define an equilibrium of the monetary policy game between countries, it is convenient to reformulate the objective functions (3.3.15) in the following way. Define expected utility in the home country as:

\[ EU(a, b) = \phi_n \Gamma \frac{\theta - 1}{\delta} X - \frac{1 - n}{\lambda} \Gamma X^* \]  
(3.4.2)

Likewise, expected utility for the foreign country monetary authority is

\[ EU^*(a, b) = \phi_{1-n} X^* - \frac{n}{\lambda} \Gamma^{-1} \frac{\theta - 1}{\delta} X \]  
(3.4.3)

where \(X\) and \(X^*\) are defined as

\[ X = \Theta \exp\left[\left(1 - \rho\right)\left(-\frac{1}{2} \sigma_c^2 - \sigma_{c*}^2 + 2 \sigma_{cu} \sigma_{c*}^2\right)\right] \]  
(3.4.4)

\[ X^* = \Theta \exp\left[\left(1 - \rho\right)\left(-\frac{1}{2} \sigma_c^2 - \sigma_{c*}^2 + 2 \sigma_{cu} \sigma_{c*}^2\right)\right] \]  
(3.4.5)

\(\phi_n, \phi_{1-n}, \text{ and } \Theta\) are constant functions of parameters\(^\text{16}\), and \(\sigma_{c}^2 = n \sigma_{c*}^2 + (1 - n) \sigma_{c*}^2\), \(\sigma_{cu} = n \sigma_{cu} + (1 - n) \sigma_{cu}^*\).\(^\text{15}\)

\(^\text{15}\)An alternative possibility is to have monetary policy rules chosen before the ex ante asset market trading. We did solve the model in that case. Although the actual form of the solutions are altered, the qualitative results (in particular the asymmetries) do not differ from those presented below.

\(^\text{16}\)In particular, \(\phi_n = \frac{1}{1 - \rho} - \frac{1}{\lambda}, \phi_{1-n} = \frac{1}{1 - \rho} - \frac{1}{\lambda}, \text{ and } \Theta = \exp\left[\left(\frac{\theta - 1}{\delta}\frac{1}{\delta}\ln(\lambda \eta)\right)\right]\)
Although home consumption is independent of the foreign monetary rule (shown above), its welfare does depend on the foreign monetary rule, because expected home country employment is affected by foreign monetary policy. Thus, the home country is not indifferent to the rule followed by the foreign monetary authority.

A special case of (3.4.2) and (3.4.3) arises when \( \rho = 1 \). Then expected employment is constant in both countries (see (3.3.14)). Therefore, the monetary authorities are concerned solely with maximizing expected utility of their own consumption. For the home country, this is equivalent to using monetary rules to maximize:

\[
\left( -\frac{1}{2} \sigma_c^2 - \frac{\sigma_u^2}{2} + \sigma_{cu} \right),
\]

(3.4.6)

By contrast, for the foreign country, in the case \( \rho = 1 \), the relevant objective function is

\[
\left( -\frac{1}{2} \sigma_c^2 - \frac{n(1-n)}{2} \sigma_u^2 + \sigma_{cu} + n(1-n)(\sigma_{su} - \sigma_{su}^*) \right).
\]

(3.4.7)

Then home utility depends (negatively) on consumption variance, but is increasing in the covariance of consumption and productivity shocks. An optimal monetary rule will trade off these costs and benefits, making consumption positively co-vary with \( u \) and \( u^* \). Because there is no pass-through into the home economy, its monetary authority is indifferent to exchange rate variance.

For the foreign country, exchange rate variance does have welfare consequences. Exchange rate variance reduces foreign utility\(^{17}\). But positive covariance of the exchange rate and home productivity shocks, or a negative covariance with foreign productivity, raises foreign utility\(^{18}\). An optimal monetary rule for the foreign country therefore has to take account of effects on both consumption and the exchange rate.

\(^{17}\)This occurs because exchange rate variance raises the mean foreign price level, for any expected value of the money stock, and hence reduces expected foreign consumption.

\(^{18}\)The intuition behind this is that the exchange rate generates an expenditure switching effect in the foreign economy. A positive home (foreign) technology shock requires a depreciation (appreciation) in the exchange rate in order to increase foreign country demand for home (foreign) goods. The equivalent channel does not work in the home country, because there is no expenditure switching at the consumer level.
Proposition 2 When $\rho = 1$, the home country is indifferent to exchange rate volatility, while the foreign country places a negative weight on exchange rate volatility.

Proof: See above discussion

In the more general case with $\rho > 1$, the home country is no longer completely indifferent to exchange rate variability. But for all reasonable parameter values and shock distributions, the home country places less weight on exchange rate variability than does the foreign country.

Table C.2 illustrates the impact of exchange rate volatility on expected utility, for each country, for various values of $\rho$, and a given calibration of other parameters. In all cases, the home country is less affected by movements in exchange rate variance.

A Nash equilibrium in the monetary game between countries is defined in the standard way, as the pair $a^n, b^n$ which solves:

$$\max_a EU(a, b^n)$$

$$\max_b EU^*(a^n, b)$$

The simple form of the model in fact allows us to solve for the exact solutions to the monetary policy rules. By observing (3.4.2) and (3.4.3), we notice an important asymmetry in the monetary policy game. The home country rules $a_1, a_2$ affect the composite expression $X^*$, defined above, but the foreign country monetary rules $b_1, b_2$, do not affect $X$. Moreover, note that $X^*$ enters linearly, with a negative sign, in both the home and foreign country’s objective functions. Hence, in choosing its monetary rules, the foreign monetary authority indirectly chooses to maximize home expected utility.

The first order conditions characterizing the Nash equilibrium can be written as (for both $a, a^*$ and $b_1, b_2$ respectively):

$$\phi_n \Gamma^{-\frac{1}{\rho}} \frac{\partial X}{\partial a} = \frac{(1 - n)}{\lambda} \frac{\partial X^*}{\partial a}$$

$$\frac{\partial X^*}{\partial b} = 0$$
Using the property of optimal risk sharing from equation (3.2.1), we may establish that

$$\Gamma = \frac{EC^{1-\rho}}{EC^{(1-\rho)}} = \Gamma_{\rho} X_{\rho} = (X_{\rho})^{\rho}. \tag{3.4.12}$$

where the second equality follows from the definition of $X$ and $X^*$. Now substituting into the first order conditions (3.4.10) and (3.4.11), we arrive at the characterization of the optimal monetary reaction functions for each country:

$$\delta a_1 = n \left\{ \lambda - (1 - \rho) [n + (1 - n)[n + \rho(1 - n)]] \right\} + n[(1 - n)(1 - \rho)]^2 b_1 \tag{3.4.13}$$

$$\delta a_2 = (1 - n)[\lambda - n(1 - \rho)] + n[(1 - n)(1 - \rho)]^2 (b_2 - 1) \tag{3.4.14}$$

$$b_1 = \frac{n(\rho - 1)}{\rho n + (1 - n)} (a_1 - 1) \tag{3.4.15}$$

$$b_2 = \frac{n(\rho - 1)(a_2 + 1) + 1}{\rho n + (1 - n)} \tag{3.4.16}$$

where

$$\delta = \lambda - n(1 - \rho)\{1 + (1 - n)\rho(1 - n) + n\}$$

Equations (3.4.13) and (3.4.14) describe the home country’s first order conditions, while (3.4.15) and (3.4.16) describe the foreign country’s conditions. The solution to (3.4.13)-(3.4.16) is a Nash equilibrium in the monetary rules.

Table C.1 describes the solution. From the Table, we may establish that a) $n \leq a_1 < 1$, $0 < a_2 \leq (1 - n)$, b) $b_1 \leq 0$, $b_2 \geq 1$, and c) $a_1 + a_2 = 1$, $b_1 + b_2 = 1$.

In the special case with $\rho = 1$, we have $a_1 = n$, $a_2 = 1 - n$, and $b_1 = 0$, $b_2 = 1$. Thus, the home country adjusts monetary policy to both the home and foreign shocks according to their weight in world GDP, and the foreign country focuses only on its own domestic shock.

Note that in this case, given our assumption that $u$ and $u^*$ are i.i.d., it follows the home country monetary variance is lower than that of the foreign country. In addition we note that the variance of the exchange rate is lower than would occur were there to be no world reference
currency. Given the Nash equilibrium described above, exchange rate variance is \(2\sigma_a^2\). It is easy to show that, if exchange rate pass-through into both home and foreign countries was complete, then the Nash equilibrium would be \(a_1 = 1, a_2 = 0, \text{ and } b_1 = 0, b_2 = 1\). In that case, exchange rate variance would be \(2\sigma_a^2\).

Table C.1 shows the solution for \(a_1, a_2\) and \(b_1, b_2\) in the more general case where \(\rho > 1\). Figure C.1 and Figure C.2 illustrate the reaction curves, for the responses to both shocks. Each is upward sloping. The same general properties of the solution described above still apply.

We now focus on the welfare outcomes of the monetary policy game. Using the solutions of Table C.1, and the description of the Nash equilibrium, we now state the following proposition:

**Proposition 3** A Nash equilibrium is identical to an outcome where the home economy determines world monetary policy rules.

The proposition says that a Nash equilibrium is asymmetric, in the sense that it gives the same allocation as if the home economy was choosing both its own and the foreign economy's monetary rules. Equivalently, in the Nash equilibrium, the foreign economy indirectly maximizes expected home country utility as well as its own utility.

The proof of the proposition is straightforward. From the objective function (3.4.2), note that \(X\) is independent of \(b_1\) and \(b_2\), and home expected utility is linear in \(X^*\). Since, in a Nash equilibrium, the foreign monetary authority chooses \(b_1\) and \(b_2\) to maximize a linear function of \(X^*\), its choice is also the optimal choice of \(b_1\) and \(b_2\) for the home economy.

Note that the proposition specifically does not hold in the reverse direction. The Nash allocations for \(a_1\) and \(a_2\) do not maximize foreign country welfare. Hence, the foreign country experiences negative welfare externalities in a Nash equilibrium.

The key ingredient in this asymmetry is that home consumption is independent of foreign monetary rules. As a result, the foreign monetary policy influences home utility only to the extent that it influences expected employment in the home country. Since the monetary rules \(b_1\) and \(b_2\) that maximize foreign utility are identical to those which minimize expected home
employment, these rules are then the optimal rules from both the home and foreign country perspective.

This also means that the monetary rules governing the world economy are identical to those that would hold were the home country a ‘Stackelberg Leader’, choosing its monetary policy in advance, taking account of the reaction of the foreign country.

Figures C.1 and C.2 illustrate the equilibrium in terms of the reaction curves (3.4.13)-(3.4.16) for both \( a_1, b_1 \) and \( a_2, b_2 \), respectively. For \( \rho > 1 \), the home country’s reaction curve slopes upward in both Figures. The foreign country’s reaction curve is also upward sloping, and steeper than that of the home country. Point N represents the Nash equilibrium. Since N is a global optimum for the home country, its iso-utility lines can be illustrated as converging to a maximum at N. For the foreign economy, point F represents the global optimum, equivalent to the allocation that would obtain if the foreign monetary authority could choose both home and foreign monetary policy rules. The foreign iso-utility lines converge to a maximum at F. Utility of the foreign country at N is less than at F.

In Figure C.1, relative to the Nash equilibrium, the foreign country would like to increase \( b_1 \) towards zero, and increase \( a_1 \). Hence, it would like the home country to react more to its own shock, and for itself to react less (in absolute terms) to the home shock. If the foreign economy could act as a Stackelberg leader, it would choose point S.

Figure C.2 illustrates the determination of \( a_2, b_2 \). Again, the points N and F represent the global optimal allocations for the home and foreign country respectively, with the first being the Nash equilibrium. If the foreign economy could act as a Stackelberg leader, it would reduce the weight placed on its own productivity shock in its monetary rule, which would have the effect of reducing the home country \( a_2 \) coefficient.

A corollary of the proposition is that there is no gain from international monetary policy coordination, except in the trivial case where the social welfare function used for coordination places all weight on the home economy welfare. The Nash equilibrium is therefore efficient. While the foreign economy could gain from an allocation chosen by an equal weighting world monetary authority, the home economy would lose. In Figures C.1 and C.2 we could illustrate
a contract curve, or set of efficient monetary coefficients, indexed by different weights on home and foreign welfare in evaluating the world optimum. The Nash equilibrium is at one end of this curve, where the weight on home utility is one.

This equilibrium of the monetary policy game has many features that seem to resemble the description of US monetary policy under the de facto world ‘dollar standard’. It is widely acknowledged that the US pays little attention to the exchange rate in its monetary policy. But compared to many other countries, the US follows a more stable path of monetary policy. More importantly, the US does have an advantage over the rest of the world in setting monetary policy, due to the special role of the dollar. In our model, this advantage is quite extreme in the sense that world monetary policy completely reflects US preferences. The situation has some parallels in the historical literature on ‘Hegemonic Stability’ of the International Monetary System (see Eichengreen (1995) for instance). While this theory was specifically designed to interpret the stability of fixed exchange rate systems, in this model, the US acts as a hegemon, but within a decentralized world Nash equilibrium with flexible exchange rates. Nevertheless, just as in the traditional theory, our model predicts that the hegemon has no interest in international consultation in monetary policy making, and would not support a move towards international monetary cooperation.

Much popular discussion of the role of the US dollar in the world economy goes even further than this. It is frequently asserted that US residents gain from the role of the US dollar as the international reference currency. There is a wide range of popular explanations for how these gains might come about - some argue that the role of dollar allows the US to run current account deficits without limit, that it gives it the ability to dictate world monetary policy, or that it allows it to dominate the world oil market. By contrast, most economists (e.g. Krugman (1999)) estimate that the gains that the US gets from the dominance of the dollar are modest, mainly accounted for by seigniorage revenue on offshore dollar holdings, and are a very small percentage of total US fiscal revenue.

We now address the question of the welfare gains to a reference currency. In our model, there are no offshore currency holdings, so the primary source of benefit is not present.
Nevertheless, the fact that the outcome of the monetary policy game is asymmetric means that welfare levels are different for the home and foreign countries. Since the rest of the model is perfectly symmetric, the difference in welfare gives an exact measure of the gains from having an international currency.

It might be thought that this question has already been answered by Proposition 2. The role of the reference currency leads the home country to be placed on one end of the utility contract curve. It would then seem that the reference currency country is always better off. But this conclusion is incorrect. Since the game itself is asymmetric, welfare levels would differ even if each country's preference were given equal weight in world monetary policy making. In order to assess the gains to having a dominant currency, we must compare levels of expected utility between the home and foreign countries.

This comparison gives a surprising result.

**Proposition 4** In a Nash equilibrium, expected utility for the home country is always lower than that of the foreign country.

Proof: See Appendix C.3.

Although the home country's preferences dominate world monetary policy making, home country residents are actually worse off than those of the foreign country, in the equilibrium of the monetary policy game. The ownership of an international reference currency bestows costs rather than benefits - residents of the reference country have lower expected utility.

Although the result may seem surprising, the explanation is quite intuitive. The absence of exchange rate pass-through into the home economy inhibits the usefulness of monetary policy. As is described above, an ideal monetary policy rule is one which achieves both expenditure level effects and expenditure switching effects. The foreign country can use both channels in designing a monetary rule, because the exchange rate affects relative prices. The home country can't do this - since the exchange rate does not affect the demand for home goods relative to foreign goods, monetary policy can affect only the level of spending. Given the absence of pass-through into the home economy, home output is not adjusted efficiently
to home and foreign technology shocks. As a result, expected utility is lower than that of the foreign country, where output can be affected by the exchange rate.

3.5 Endogenous Currency Pricing

So far it has just been assumed that the home currency is used as a reference for international pricing. In principle, this decision should be endogenous. The set of forces leading to the adoption of an international ‘vehicle’ currency have been discussed extensively in the literature on international monetary economics (see Matsuyama et al. (1991) McKinnon (2002), Krugman (1984), Rey (2001)). Many factors, such as economic size, history, capital flows, and economic policy may be part of the explanation. Moreover, the presence of ‘network externalities’ in the choice of standard may give rise to multiple equilibria. Krugman (1984) notes that while economic size is likely to be an important factor, there may also be a ‘snowballing’ effect, whereby even if countries are of similar size, if one currency becomes acceptable in exchange then all countries will have an incentive to support this outcome. This suggests that the US dollar standard may be due to historical accident as much as current fundamentals. McKinnon (2002) takes a different view however. He stresses the importance of US monetary policy, arguing that the US dollar’s role as a world currency resulted from low and stable US inflation rates in the post-WWII international system.

In this section, we present a brief analysis of the determination of the reference currency for international trade. We illustrate the sense in which the asymmetric pricing outcome examined in the previous section can be an equilibrium of the model where the currency of pricing is endogenous.

We assume that firms can choose which currency they would like to set their price in. They do this taking into account that whatever their choice, they will then choose a nominal economic size does not play any significant role in our model, because a) there are no non-traded goods, so all countries are fully open, and b) each country produces a measure of goods equal to its population, so the terms of trade is independent of size.

Our analysis doubtless omits many important factors that determine the role of an international currency, but highlights one potentially important factor, within the context of this model.
price to maximize expected discounted profits. In addition to this however, the firm incurs a cost of adjusting prices, ex-post. We assume that these costs arise only when the price facing consumers is adjusted. We might think of these as menu-changing costs, or customer resistance costs, that require management services on the part of the firm. If the firm sets the price in the local currency of the buyer, it will never face these costs, as the price will be independent of the state of the world. But if the price is set in the exporting firms' own currency, prices facing the foreign consumer will be dependent upon the exchange rate. We handle this in the following simple way. Assume that if the firm sets prices in the consumers currency (LCP), then it faces no additional cost. But if it sets prices in its own currency, then it faces a fixed (nominal) cost given by \( \delta \)\(^{21}\). This is thought of as a cost of ex-post adjustment that comes from the exchange rate pass-through into the importing countries CPI. The presence of this fixed cost per se will therefore encourage the firm to set prices in the currency of the consumer (LCP).

On the other hand, the level of expected (discounted) profits, gross of fixed costs, will depend upon whether prices are pre-set in the producers currency or consumers currency. Using the same demand and cost structure from the model set out above, we may define the expected discounted profits on foreign sales for a home firm that sets its export price in terms of its own currency (PCP) as

\[
E[d\pi(i)^{PCP}] = E[d(P_h(i) - \frac{W}{\theta})X_h^{PCP}(i)]
\]  

where \( X_h^{PCP}(i) = \left( \frac{P_h(i)}{P_{hf}^*} \right)^{-\lambda} \frac{P_{hf}^*}{P_h}C^* \).

If the firm chooses alternatively to set its price in terms of foreign currency (LCP), it faces expected discounted profits given by

\[
E[d\pi(i)^{LCP}] = E[d(SP_{hf}^*(i) - \frac{W}{\theta})X_h^{LCP}(i)]
\]  

\(^{21}\)We think of this being part of the technology. That is, there is a technology whereby firms who wish to set export prices in their own domestic currency must incur a fixed management cost \( \delta \). Competitive 'managerial' firms provide this management services by combining home and foreign varieties in the same manner that consumers do.
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where \( X^{i)_{LC} = \left( \frac{P_{t+1}^{i}_{x}}{P_{t}^{x}} \right)^{-\lambda} \frac{P^{*}}{P^{x}} C^{*} \).

The Home country firm will set its price in its own currency if the expected profit differential from doing so exceeds the expected menu cost. Thus it follows PCP whenever

\[ E[d\pi(i)^{PCP}] - E[d\pi(i)^{LCP}] > \delta \] (3.5.3)

The sequence of actions within a period is now described as follows. First, firms choose the currency in which prices are set. Following this, the monetary authorities in each country choose their optimal rules. Then firms choose the actual prices of goods. Finally, the technology shocks are realized, and consumption, output and exchange rates are determined.

In Devereux, Engel and Storgaard (2003), it is shown that the left hand side of (3.5.3) may be approximated by the following

\[ \frac{d\pi}{d\lambda}(\lambda - 1) \left[ \frac{Var(\ln S)}{2} - Cov(\ln \frac{W}{\theta}, \ln S) \right] \] (3.5.4)

where \( d \) and \( \pi \) denote the discount factor and profits in a deterministic economy, respectively. The intuition behind this condition is straightforward. Since profits are convex (linear) in the exchange rate when the firm following PCP (LCP), a higher exchange rate variance will encourage the firm to follow PCP. But if the covariance of the exchange rate and marginal cost \( W^{*} \) is positive, expected costs will be higher under PCP. If the right hand side of (3.5.4) is positive, the firm would wish to set prices in its own currency (PCP), in the absence of menu costs of price change. Thus, the condition (3.5.3) becomes

\[ \lambda(\lambda - 1) \left[ \frac{Var(\ln S)}{2} - Cov(\ln \frac{W}{\theta}, \ln S) \right] > \frac{\delta}{d\pi} \] (3.5.5)

The equivalent condition for the foreign firm is

\[ \lambda(\lambda - 1) \left[ \frac{Var(\ln S)}{2} + Cov(\ln \frac{W^{*}}{\theta^{*}}, \ln S) \right] > \frac{\delta}{d\pi}, \] (3.5.6)

where, to maintain symmetry, we assume that the fixed cost facing the foreign firm is identical to that of the home firm.

If condition (3.5.5) (condition (3.5.6)) is not satisfied, then the home firm (foreign firm) will instead set prices according to LCP. If we define \( Z = \lambda(\lambda - 1)d\pi \left[ \frac{Var(\ln S)}{2} - Cov(\ln \frac{W}{\theta}, \ln S) \right], \)
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\[ Z = \max \left\{ 0, \lambda (\lambda - 1) \bar{d} \pi \left[ \frac{\text{Var}(\ln S)}{2} + \text{Cov}(\ln W^*, \ln S) \right] \right\}, \]

from these conditions, we can establish the following proposition.

**Proposition 5** There exists a positive menu cost \( \delta \in (Z, \bar{Z}) \) such that all home firms follow PCP, and all foreign firms follow LCP.

**Proof:** See Appendix C.4.

The proposition says that there exists a positive menu cost \( \delta \) such that the asymmetric pricing structure outlined in the previous section is an equilibrium. In this equilibrium, the home firm will choose PCP, while the foreign firm will choose LCP. Following this, the monetary authorities choose their optimal rules in exactly the same way described in the previous section. The key intuition is that the way in which the monetary rules are set acts so as to lock in the asymmetric pricing policies of home and foreign firms. In an equilibrium of the monetary policy game outlined in Section 3.4, the home country's monetary policy rule tends to target both home and foreign productivity shocks. Since they are independent of each other, the home country's money supply is less volatile than that of the foreign country. As a result, \( \text{Cov}(\ln W^*, \ln S) \) is less than \( -\text{Cov}(\ln W^*, \ln S) \), because in the simple form of the model, the variability of the wage rate is completely determined by the variance of the domestic money stock. Thus, for relatively small menu costs of price change, it is more likely that the home firm will wish to set its price in its own currency, while the foreign firm will wish to set its price in the home currency. This is a variant of the result of Devereux, Engel, and Storgaard (2003), which shows that firms would wish to set their export prices in the currency of the country which had the lowest variance of money growth.

While the proposition establishes that the outcome of the previous section is an equilibrium of a game in which firms choose the currency of pricing, in general there will be other equilibria of this game. For instance, if all firms choose LCP, then from the results of Devereux and Engel (2003), and Devereux, Engel and Storgaard (2003), the optimal monetary rules chosen by home and foreign countries will in fact support global LCP as an outcome. Likewise, because the model is entirely symmetric, if Proposition 5 holds, then there must be
an alternative equilibrium where, if foreign firms follow PCP, and home firms choose LCP, this is supported as an equilibrium in the monetary policy game.

These results suggest that both the Krugman (1984) multiple equilibrium hypothesis, and the McKinnon (2002) fundamentals hypothesis, may be part of the explanation for the dollar standard. Given the presence of asymmetric pricing, the endogenous decisions of monetary authorities respond in a certain way so as to confirm the pricing decisions of firms. Nevertheless, there may be other equilibria which would also be self-confirming in the sense that they would induce different monetary policy rules.

3.6 Conclusions

An almost universal characteristic of the international monetary system is the role of a dominant currency. In the classical theory of 'hegemonic stability', the economic policies of the dominant currency determine the stability of the international system, typically by adherence to the 'rules of the game' in a fixed exchange rate system. But in the decades since floating exchange rates, the US dollar has remained a pre-eminent currency in international trade and finance - leading to a de facto dollar standard. This paper has extended the recent literature on monetary policy in sticky-price general equilibrium models to allow for such a dollar standard. We found that the equilibrium has many of the attributes of popular discussion of the predominance of US monetary policy in the world economy. In particular, a decentralized world of floating exchange rates acts so as to maximize the welfare of the US. But, in sharp contrast to popular discussion, we found that this situation brings no net benefits to the US, when compared to welfare of the rest of the world. US residents are worse off in the situation of having a dollar standard, compared to residents of the rest of the world.
Bibliography


Appendix A

Appendices of Chapter 1

A.1 Optimal Pricing Schedule of Firms

All goods are imperfect substitutes in consumption, so each individual firm has some market power determined by the parameter $\theta$. Taking prices for all individual goods as given, the optimal demand function of the consumer for each individual good can be derived, which implies that in each period the consumer allocates a given level of total consumption among the differentiated goods.

$$C_{h,t}(i) = \omega \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^\theta \left( \frac{P_t}{P_{h,t}} \right)^\gamma C_t$$

The price setting problem of monopolist $i$ is to maximize expected profit conditional on $t-1$ information, by choosing $P_{h,t}(i)$ and $P_{h,t}^*(i)$. That is, firm $i$ solves

$$\max_{P_{h,t}(i), P_{h,t}^*(i)} E_{t-1} \left\{ D_t [P_{h,t}(i)C_{h,t}(i) + S_t P_{h,t}^*(i)C_{h,t}^*(i) - W_t L_t] \right\}$$

subject to

$$L_t(i) = C_{h,t}(i) + C_{h,t}^*(i)$$

and the downward-sloping demand functions for $C_{h,t}(i)$ and $C_{h,t}^*(i)$, as in Equation A.1.1 and the foreign analogue. Note that $P_{h,t}(i)$ and $P_{h,t}^*(i)$ are denoted in the home and foreign currency, respectively. Using the fact that all prices are preset at time $t-1$ and applying symmetry, we can derive the optimal pricing schedule of firms.

A.2 Entry Condition of Noise Traders

This appendix derives the entry condition (Equation 1.2.27) for noise traders to enter the foreign bond market. The noise trader $i$ will enter the foreign bond market if and only if Equation 1.2.14 holds.

If trader $i$ does not enter the market, his expected utility is given by:

$$E_t^i(U_t^i \mid \varphi_t^i = 0) = 0$$
While if he enters, it is given by:

\[
E_t^i(U_t^i \mid \varphi_t^i = 1) = \max_{B_{h,t+1}^* (i)} \left\{ E_t^i \left[ B_{h,t+1}^* (i) S_t (1 + r_{t+1}) \right] - \frac{a}{2} \text{Var}_t^i \left[ \frac{B_{h,t+1}^* (i) S_t (1 + r_{t+1})}{P_{t+1}} - \rho_{t+1} \right] \right\}
\]

Substituting the optimal demand for foreign bonds of noise traders, \( B_{h,t+1}^* (i) = \frac{E_t^N [\rho_{t+1}]}{a \text{Var}_t^i (1 + r_{t+1}) \text{Var}_t^i [\rho_{t+1}]} \), into the above equation, we may establish that Equation A.2.2 is equivalent to:

\[
(\text{A.2.2})
\]

\[
\varphi_t^i = 1 \iff E_t^i \left[ \frac{E_t^N [\rho_{t+1}]}{a \text{Var}_t^i [\rho_{t+1}]} - c_t \right] - \frac{a}{2} \text{Var}_t^i \left[ \frac{E_t^N [\rho_{t+1}]}{a \text{Var}_t^i [\rho_{t+1}]} \right] \geq 0
\]

By the property of noise traders' subjective expectation of \( \rho_{t+1} \), Equation A.2.3 can be rewritten as:

\[
\varphi_t^i = 1 \iff \left[ (E_t^N [\rho_{t+1}])^2 - c_t \right] - \frac{(E_t^N [\rho_{t+1}])^2}{2a(\text{Var}_t^i [\rho_{t+1}])^2} \text{Var}_t^i [\rho_{t+1}] \geq 0
\]

or:

\[
\varphi_t^i = 1 \iff c_t \leq \frac{(E_t^N [\rho_{t+1}])^2}{2a(\text{Var}_t^i [\rho_{t+1}])}
\]

\[
(\text{A.2.4})
\]

**A.3 A Symmetric Steady State**

In a non-stochastic steady state, all shocks are equal to zero. Hereafter, steady state values are marked by overbars.

As the consumption is constant at the steady state, the steady state world interest rate \( r \) is tied down by the intertemporal optimality equation (Equation 1.2.7):

\[
\bar{r} = \bar{r}^* = \frac{1 - \beta}{\beta}
\]

\[
(\text{A.3.1})
\]

From the pricing equation, at the steady state, all the prices are equal and steady state exchange rate \( \bar{S} = 1 \). Then the steady state excess return \( \bar{\rho} = \frac{\bar{S}(1 + \bar{r}^*)}{\bar{S}(1 + \bar{r})} - 1 = 0 \). From Equation 1.2.24, we will have \(^1\)

\[
B_{h,t+1}^* (i) = 0 \ \forall i \in [0, 1]
\]

\[
(\text{A.3.2})
\]

The economic intuition behind A.3.2 is that traders are not going to hold foreign bonds because they know that the excess return will be zero, and thus no trade takes place. The only way that no trade will occur in equilibrium is for the uncovered interest parity to hold.

\(^1\)Note that since \( u_t = 0 \), only rational traders are present on the market.
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Therefore, from the bond market clearing condition, net foreign assets are zero.

\[ B = B^* = 0 \] (A.3.3)

The steady state values of other variables are straightforward. Since \( B = 0 \), a closed-form solution exists for the steady state, in which the countries have identical outputs, consumption and real money holdings:

\[ \bar{L} = \bar{L}^* = \bar{C} = \bar{C}^* = \left( \frac{\theta \eta}{\theta - 1} \right)^{-\frac{1}{\rho + \psi}} \] (A.3.4)

\[ \frac{\bar{M}}{\bar{P}} = \frac{\bar{M}^*}{\bar{P}^*} = \left( \frac{\bar{C}}{\bar{C}^*} \right)^{\frac{1}{1 - \beta}} \] (A.3.5)

A.4 Model Solution

The full model can be described by the 26 equilibrium conditions listed in Section 2.3. To solve the model, we take a log-linearization around the initial non-stochastic steady state described in Appendix C. Given the log-linearized system, the deviations of the exchange rate and the macroeconomic variables from their \( t - 1 \) expectations are solved in terms of exogenous money supply shocks and the expectation error shocks.

A.4.1 Log-linearized System

Money demand function

\[ \hat{m}_t - \hat{p}_t = \frac{\rho}{\epsilon} \hat{c}^H_t - \frac{\beta}{\epsilon} \hat{d} \hat{r}_{t+1} \]

\[ \hat{m}_t^* - \hat{p}_t^* = \frac{\rho}{\epsilon} \hat{c}^*_t - \frac{\beta}{\epsilon} \hat{d} \hat{r}^*_t \] (A.4.1)

Labor supply function

\[ \hat{w}_t = \psi \hat{t}_t + \rho \hat{c}^H_t + \hat{p}_t \]

\[ \hat{w}_t^* = \psi \hat{t}_t^* + \rho \hat{c}^*_t + \hat{p}_t^* \] (A.4.2)

Euler equations

\[ -\beta \hat{d} \hat{r}_{t+1} = \rho \hat{c}^H_t - \rho E_t \hat{c}^H_{t+1} + \hat{p}_t - E_t \hat{p}_t^* \]

\[ -\beta \hat{d} \hat{r}^*_t = \rho \hat{c}^*_t - \rho E_t \hat{c}^*_t + \hat{p}_t^* - E_t \hat{p}_t^* \] (A.4.3)

Home household budget constraint (\( \omega = \frac{1}{2} \))

\[ \hat{p}_t + \hat{c}_t^H = \frac{1}{2} (p_{h,t} + \hat{c}_{h,t}) + \frac{1}{2} (p_{h,t}^* + \hat{c}_{h,t}^*) + \hat{s}_t + \frac{1}{\beta} dB_t - \frac{1}{\beta} dB_{t+1} \] (A.4.4)

Pricing Equations

\[ \hat{p}_{h,t} = E_{t-1} [\hat{w}_t] \quad p_{h,t}^* = E_{t-1} [\hat{w}_t^*] - E_{t-1} [\hat{s}_t] \quad p_{f,t} = E_{t-1} [\hat{w}_t^*] + E_{t-1} [\hat{s}_t] \quad p_{f,t}^* = E_{t-1} [\hat{w}_t^*] \] (A.4.5)
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Price Indexes \( (\omega = \frac{1}{2}) \)

\[
\hat{p}_t = \frac{1}{2} \hat{p}_{h,t} + \frac{1}{2} \hat{p}_{f,t} \quad \hat{p}_t^* = \frac{1}{2} \hat{p}_{h,t}^* + \frac{1}{2} \hat{p}_{f,t}^*
\]

(A.4.6)

Individual Goods Demand

\[
c_{h,t} = -\gamma (\hat{p}_{h,t} - \hat{p}_t) + \hat{c}_t \\
c_{h,t}^* = -\gamma (\hat{p}_{h,t}^* - \hat{p}_t^*) + \hat{c}_t
\]

(A.4.7)

\[
c_{f,t} = -\gamma (\hat{p}_{f,t} - \hat{p}_t) + \hat{c}_t \\
c_{f,t}^* = -\gamma (\hat{p}_{f,t}^* - \hat{p}_t^*) + \hat{c}_t
\]

(A.4.8)

Market Clearing Conditions

\[
dB_{t+1} = \tilde{S} dB_{h,t+1}^*
\]

(A.4.9)

\[
\hat{d}_t = \frac{1}{2} \hat{c}_{h,t}^* + \frac{1}{2} \hat{c}_{f,t}^* \\
\hat{d}_t^* = \frac{1}{2} \hat{c}_{f,t} + \frac{1}{2} \hat{c}_{f,t}^*
\]

(A.4.10)

Home Country Aggregate Consumption (for both exogenous entry and endogenous entry cases)

\[
\hat{c}_t = \hat{c}_t^* + \frac{dC_t^T}{C}
\]

(A.4.11)

Budget Constraints of Traders

Exogenous Entry Case:

The budget constraint of foreign exchange traders is given by:

\[
P_{t+1} C_{t+1}^T = [(1 + r_{t+1}) S_{t+1} - (1 + r_{t+1}) S_t] = B_{h,t+1}^* S_t (1 + r_{t+1}) \rho_{t+1}
\]

(A.4.12)

Linearizing the above equation around \( B_{h}^* = 0 \) and \( \rho = 0 \) gives \( dC_{t+1}^T = 0 \).

Endogenous Case:

The budget constraint of foreign exchange traders is given by

\[
P_{t+1} C_{t+1}^T = B_{h,t+1}^* [(1 + r_{t+1}) S_{t+1} - (1 + r_{t+1}) S_t] - P_{t+1} \sum_{i=0}^{n_t} c_t = B_{h,t+1}^* S_t (1 + r_{t+1}) \rho_{t+1} - P_{t+1} \sum_{i=0}^{n_t} c_t
\]

Linearizing the above equation around \( B_{h}^* = 0, \rho = 0 \) and \( n = 0 \) gives \( dC_{t+1}^T = 0 \).

Interest Parity Conditions

Before linearizing the interest parity equation, an approximation \(^2\) is used to rewrite the excess return in log-terms:

\[
\rho_{t+1} = \frac{S_t}{S_t (1 + r_{t+1})} - 1 \approx \ln \left[ \frac{S_t (1 + r_{t+1})}{S_t (1 + r_{t+1})} \right] = s_{t+1} + \ln(1 + r_{t+1}) - s_t - \ln(1 + r_{t+1})
\]

(A.4.13)

\(^2\)If \( \xi \) is small enough, then \( \ln(1 + \xi) \approx \xi \). This approximation of the excess return is widely used in the finance literature.
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Exogenous Case: Using A.4.14, Equation 1.2.26 can be rewritten as:

\[ s_t = E_t(s_{t+1}) + \ln(1 + r_{t+1}^*) - \ln(1 + r_t) + (1 - N_I)v_t - \frac{S_t}{P_{t+1}}(1 + r_{t+1})Var(s_{t+1})B_{h,t+1}^* \]  
(A.4.15)

Linearizing the above equation around the steady state, but using the second-order approximation to approximate the variance term (as the second order terms are important for understanding the dynamics of the model), we have

\[ \dot{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + (1 - N_I)v_t - a \frac{(1 + r^*)S}{P} Var[s_{t+1}]dB_{h,t+1}^* \]  
(A.4.16)

Endogenous Entry Case:

Using A.4.14, Equation 1.2.29 can be rewritten as:

\[ s_t = E_t(s_{t+1}) + \ln(1 + r_{t+1}^*) - \ln(1 + r_t) + \frac{n_t}{N_I} v_t - a \frac{B_{h,t+1}^*}{P_{t+1}(N_I + n_t)} Var[s_{t+1}] \]  
(A.4.17)

Linearizing above equation around the steady state, but using the second-order approximation to approximate \( \frac{n_t}{N_t + n_t} v_t \) and the variance term, we have

\[ \dot{s}_t = E_t(s_{t+1}) - \beta(dr_{t+1} - dr_{t+1}^*) + \frac{1}{N_I} v_t - a \frac{(1 + r^*)S}{PN_I} Var[s_{t+1}]dB_{h,t+1}^* \]  
(A.4.18)

where

\[ n_t = \frac{E_t[s_{t+1}] - s_t + \ln(1 + r_{t+1}^*) - \ln(1 + r_t) + v_t}{2a \text{Var}[s_{t+1}]} 1 - \frac{1}{N_I} \]

\[ \approx \frac{E_t(s_{t+1}) - s_t - \beta(dr_{t+1}^* - dr_{t+1}) + v_t}{2a \text{Var}[s_{t+1}]} (1 - \frac{1}{N_I}) \]  
(A.4.20)

Money supply processes:

\[ m_{t+1} = \hat{m}_t + \varepsilon_{\mu,t} \quad m_{t+1}^* = \hat{m}_t^* + \varepsilon_{\mu,t}^* \]  
(A.4.21)

where \( \varepsilon_{\mu,t} \sim N(0, \sigma_{\mu}^2) \) and \( \varepsilon_{\mu,t}^* \sim N(0, \sigma_{\mu}^2) \).

Note that at the steady state \( S = 1 \). Here we also use the fact that approximating \( \ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) \) around the steady state gives:

\[ \ln(1 + r_{t+1}^*) - \ln(1 + r_{t+1}) = -\beta(dr_{t+1}^* - dr_{t+1}) \]  
(A.4.19)
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A.4.2 Derivation of Equations

Derivation of Equations 1.3.14 and 1.3.24 From the log-linearized intertemporal optimality conditions A.4.3, we have:

\[ \rho(c_t^H - c_t^*) - \rho E_t(c_{t+1}^H - c_{t+1}^*) + E_{t-1}(s_t) - E_t(s_{t+1}) = -\beta(dr_{t+1} - dr_{t+1}^*) \] (A.4.22)

Equation A.4.22 minus its datet - 1 expectation gives:

\[ \rho(c_t^H - c_t^*) - \rho E_t(c_{t+1}^H - c_{t+1}^*) - E_t(s_{t+1}) = -\beta(dr_{t+1} - dr_{t+1}^*) \] (A.4.23)

Taking \( t-1 \) expectation of Equation A.4.16 and using the fact that at \( E_{t-1}(V_t) = \text{constant} \),

give:

\[ E_{t-1}(s_t) = E_{t-1}(s_{t+1}) - \beta E_{t-1}(dr_{t+1} - dr_{t+1}^*) - a \frac{(1 + \bar{\rho}) S}{\bar{\rho}} \text{Var}[s_{t+1}]E_{t-1}[dB_{h,t+1}^\tau] \] (A.4.24)

Equation A.4.16 minus Equation A.4.24 gives:

\[ -\beta(dr_{t+1} - dr_{t+1}^*) = s_t - E_t(s_{t+1}) - (1 - N_I)v_t + a \frac{(1 + \bar{\rho}) S}{\bar{\rho}} \text{Var}[s_{t+1}]dB_{h,t+1}^\tau \] (A.4.25)


The derivation of Equation 1.3.24 is analogous. The only difference is that when deriving the analogy of A.4.24, we conjecture that \( E_{t-1}(n_tV_t) = \text{constant} \). Since this term only affects the level of exchange rate and we are interested in the exchange rate volatility, it could be assumed to equal 0.

Derivation of Equations 1.3.16 and 1.3.26 To derive Equation 1.3.16, first substituting Equation 1.3.14 into Equation 1.3.12

\[ (c_t^H - c_t^*) = \frac{[1 + \frac{\sigma}{\rho}]s_t - \frac{\sigma}{\rho} (1 - N_I)v_t + \frac{a(1 + \bar{\rho})S}{\bar{\rho}} \text{Var}[s_{t+1}]dB_{h,t+1}^\tau}{1 + \frac{\sigma}{\bar{\rho}}} \] (A.4.26)

Substituting A.4.26 into 1.3.15,

\[ m_t - m_t^* = \frac{\rho}{1 + \frac{\sigma}{\bar{\rho}}} [\frac{(1 + \frac{\sigma}{\rho})s_t - \frac{\sigma}{\rho} (1 - N_I)v_t + \frac{a(1 + \bar{\rho})S}{\bar{\rho}} \text{Var}[s_{t+1}]dB_{h,t+1}^\tau] \] (A.4.27)

Note that from Equations 1.3.11 and 1.3.15, \( dB_{t+1} = \frac{\rho c}{\frac{\rho}{2}} [s_t - \frac{1}{\bar{\rho}}(m_t - m_t^*)] \). Using the fact that \( dB_{t+1} = \frac{\rho c}{\frac{\rho}{2}} [s_t - \frac{1}{\bar{\rho}}(m_t - m_t^*)] \), we could get:

\[ (m_t - m_t^*)[1 + \frac{a(1 + \bar{\rho})S}{\rho} \text{Var}[s_{t+1}]^\tau] = \frac{\rho \bar{\rho} + \frac{\sigma}{\rho} + \sigma}{\rho + \bar{\rho}} + \frac{a(1 + \bar{\rho})S}{\rho \bar{\rho}} \text{Var}[s_{t+1}^\tau]s_t \]

\[ - \frac{\sigma}{\rho} (1 - N_I)v_t \] (A.4.28)

\[ \text{At this stage, we conjecture that } \text{Var}[s_{t+1}] = \text{Var}[s_{t+1}] = \text{constant} = V_s. \text{ This conjecture is verified in Section 3.2.} \]

\[ \text{From 1.3.25 and the functional form of } s_t (\text{Equation A.7.1}), \text{our conjecture can be easily verified.} \]
Appendix A. Appendices of Chapter 1

Rewriting Equation A.4.28 gives Equation 1.3.16. The derivation of Equation 1.3.26 is entirely analogous.

A.5 The simulation of $\text{Var}(v_t) = \lambda \text{Var}(s_t)$

First, for a given distribution of fundamentals, $L_0(\epsilon_{\mu,t}, \epsilon_{\mu,t}^*)$, the variance of the exchange rate when the expectation errors of the noise traders are zero can be calculated. It can denoted as $\sigma_{s_0}^2$.

Then, we assume that the stochastic expectation error $v_t$ is given by:

$$v_t = \sqrt{\lambda \sigma_{s_0}^2} \epsilon_t$$

(A.5.1)

where $\epsilon_t$ is a random variable which satisfies the following three conditions:

$$\text{Cov}(\epsilon_t, \epsilon_{\mu,t}) = \text{Cov}(\epsilon_t, \epsilon_{\mu,t}^*) = 0 \quad \sigma_{\epsilon}^2 = 1$$

(A.5.2)

Equation A.5.1 implies $\sigma_{s_1}^2 = \lambda \sigma_{s_0}^2$. Given A.5.1, and the distribution of fundamentals $L_0(\epsilon_{\mu,t}, \epsilon_{\mu,t}^*)$, the variance of exchange rate: $\sigma_s^2$ can be computed. Let it be denoted as $\sigma_{s_1}^2$

Compare $\sigma_{s_1}^2$ and $\sigma_{s_0}^2$, if

$$|\sigma_{s_0}^2 - \sigma_{s_1}^2| \leq \epsilon, \quad \epsilon \to 0$$

(A.5.3)

The procedure stops at this point, otherwise, we will redefine the stochastic process of $v_t$ as:

$$v_t = \sqrt{\lambda \sigma_{s_1}^2} \epsilon_t$$

(A.5.4)

Notice that now $\sigma_{s_0}^2 = \lambda \sigma_{s_1}^2$. Using A.5.4 and $L_0(\epsilon_{\mu,t}, \epsilon_{\mu,t}^*)$, unconditional exchange rate volatility could be computed again and would be called $\sigma_{s_2}^2$. If $|\sigma_{s_1}^2 - \sigma_{s_2}^2| \leq \epsilon$, and $\epsilon \to 0$, the procedure stops here, otherwise, the procedure described above will be repeated to get $\sigma_{s_1}^2, \ldots, \sigma_{s_n}^2, \sigma_{s_{n+1}}^2$ until $\sigma_{s_{n+1}}^2 - \sigma_{s_n}^2 \leq \epsilon$.

A.6 Entry Condition of Traders with Tobin Tax

When the traders only need to pay transaction cost to trade in the foreign exchange market, trader $i$ will enter the market if and only if:

$$E_i(U_i \mid \varphi_i^1 = 1) \geq E_i(U_i \mid \varphi_i^1 = 0) = 0$$

(A.6.1)

$^6 \text{Cov}(\epsilon_{\mu,t}, v_t) = \text{Cov}(\epsilon_{\mu,t}^*, v_t)$ must be equal to zero, as $v_t$ is some noise and should not have any fundamental content.
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Or

\[
E_t^i \left\{ \max_{B^*_h,t+1(i)} \left\{ E_t^i \left[ \frac{B^*_{h,t+1}(i) S_t(1 + r_{t+1})}{P_{t+1}} \rho_{t+1} - \frac{B^*_{h,t+1}(i)^2}{2} \right] - \frac{a}{2} Var_t^i \left[ \frac{B^*_{h,t+1}(i) S_t(1 + r_{t+1})}{P_{t+1}} \rho_{t+1} \right] \right\} \right\} \geq 0
\]

(A.6.2)

Substituting \( B^*_{h,t+1} = \frac{E_i[\rho_{t+1}]}{S_t(1 + r_{t+1}) + \rho_{t+1} Var_t[\rho_{t+1}]} \) into the above equation, it can be shown that Equation A.6.2 is equivalent to:

\[
\zeta \left\{ \frac{P_{t+1}}{S_t(1 + r_{t+1})} + a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) Var_t(\rho_{t+1}) \right\} - \tau - a Var_t(\rho_{t+1}) \left( \frac{S_t(1 + r_{t+1})}{P_{t+1}} \right)^2 \geq 0
\]

(A.6.3)

where

\[
\zeta = \frac{[E_i(\rho_{t+1})]^2}{2 \left[ \frac{P_{t+1}}{S_t(1 + r_{t+1})} + a \frac{S_t}{P_{t+1}} (1 + r_{t+1}) Var_t(\rho_{t+1}) \right]^2} \geq 0
\]

(A.6.4)

It can be shown that the terms in the big bracket of Equation A.6.3 are equal to:

\[
\tau + \left[ \frac{S_t(1 + r_{t+1})}{P_{t+1}} \right]^2 Var_t(\rho_{t+1}) \geq 0
\]

(A.6.5)

Therefore, regardless of how large is the rate of Tobin tax (\( \tau \)), the traders will always enter the foreign bond market. This is because the transaction cost is convex in the bond traded, the trader can always choose to hold a small amount of foreign bonds and get a positive expected utility.

When the noise traders has to pay two costs to trade on the foreign exchange market, for noise trader \( i \), he will enter the market if and only if:

\[
E_t^i \left\{ \max_{B^*_h,t+1(i)} \left\{ E_t^i \left[ \frac{B^*_{h,t+1}(i) S_t(1 + r_{t+1})}{P_{t+1}} \rho_{t+1} - \frac{B^*_{h,t+1}(i)^2}{2} \right] - \frac{a}{2} Var_t^i \left[ \frac{B^*_{h,t+1}(i) S_t(1 + r_{t+1})}{P_{t+1}} \rho_{t+1} \right] \right\} \right\} \geq 0
\]

(A.6.6)

Following the steps in Appendix A.2, we could get the following entry condition for noise trader \( i \):

\[
\varphi_i^t = 1 \iff c_i \leq \frac{[E_t^N(\rho_{t+1})]^2}{2a Var_t(\rho_{t+1}) + 2\tau \left( \frac{P_{t+1}}{S_t(1 + r_{t+1})} \right)^2} \equiv GB
\]

(A.6.7)

A.7 Numerical Undetermined Coefficient Method

This section gives details for the undetermined coefficient method used to solve for the functional form of \( s_t \) in Equation 1.3.26.

First, guess a functional form for \( s_t \):

\[
s_t = a_0 + a_1 \tilde{m}_t + a_2 \tilde{m}_t^2 + a_3 v_t + a_4 \tilde{m}_t v_t + a_5 \tilde{m}_t^2 v_t + a_6 v_t^2 + a_7 \tilde{m}_t v_t + a_8 \tilde{m}_t^2 v_t + a_9 \tilde{m}_t \tilde{m}_t^2 \]

(A.7.1)
Given that, we could get $E_t(s_{t+1})$ and $Var_t(s_{t+1})$ easily. Using the facts that $\tilde{m}_t = \varepsilon_{\mu,t}$, $\tilde{m}_t^* = \varepsilon_{\mu,t}^*$ and $Cov(\varepsilon_{\mu,t}, \varepsilon_{\mu,t}^*) = 0$, and that $v_t$ is noise and should not have any fundamental content, gives:

$$Cov(\tilde{m}_t, \tilde{m}_t^*) = Cov(\tilde{m}_t, v_t) = Cov(\tilde{m}_t^*, v_t) = 0$$  \hspace{1cm} (A.7.2)

Therefore,

$$E_t(s_{t+1}) = a_4 \sigma^2_{\varepsilon_{\mu}} + a_5 \sigma^2_{\varepsilon_{\mu}^*} + a_6 \sigma^2_v$$  \hspace{1cm} (A.7.3)

To get the conditional variance of the exchange rate, the properties of the normally distributed variables and the fact that the three random variables are independent are used.

$$Var_t(s_{t+1}) = a_4^2 Var(\tilde{m}_t) + a_5^2 Var(\tilde{m}_t^*) + a_6^2 Var(v_t) + a_4^2 Var(\tilde{m}_t^2) + a_5^2 Var(\tilde{m}_t^*)^2$$
$$+ a_6^2 Var(v_t)^2 + 2a_4 a_5 Var(\tilde{m}_t v_t) + a_4 a_6 Var(\tilde{m}_t^* v_t) + a_5 a_6 Var(v_t^2)$$
$$+ \text{Covariance terms}$$

$$= a_4^2 \sigma^2_{\varepsilon_{\mu}} + a_5^2 \sigma^2_{\varepsilon_{\mu}^*} + a_6^2 \sigma^2_v + 2a_4 a_5 \sigma^2_{\varepsilon_{\mu} \varepsilon_{\mu}^*} + 2a_4 a_6 \sigma^2_{\varepsilon_{\mu} v_t}$$
$$+ 2a_5 a_6 \sigma^2_{\varepsilon_{\mu}^* v_t} + a_4^2 \sigma^2_{\varepsilon_{\mu}^*} + a_5^2 \sigma^2_{\varepsilon_{\mu}^* v_t} + a_6^2 \sigma^2_v$$

$$\hspace{1cm} (A.7.5)$$

That is, $Var_t(s_{t+1}) = Var_t(s_{t+1}) = \text{constant} \equiv V_s$ and $E_t(s_{t+1}) = E(s_{t+1}) = \text{constant} \equiv E_s$. Using the parameterized $E_s$ and $V_s$ from Equations A.7.3 and A.7.5, we might solve for $\tilde{s}_t$ from Equation 1.3.26 given any exogenous shocks $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. To test if our initial guess is a good guess, we can do simulations and regress the $\tilde{s}_t$ we get from above process on $\tilde{m}_t$, $\tilde{m}_t^*$ and $v_t$. If the coefficients ($a$'s) are close enough to the initial guess, the process is stopped. Otherwise, the above procedure will be repeated. This method is actually an undetermined coefficient method, and is also known as the “parameterized estimation approach” in numerical methods.

\[\text{Notice that, if } x_t \text{ is normally distributed with variance } \sigma^2, \text{ then}\]

\[E[(x_t)^{2k}] = (2k - 1)!(\sigma^2)^k \hspace{1cm} E[(x_t)^{2k+1}] = 0 \hspace{1cm} \text{where } k = 1, 2, \ldots, n \]  \hspace{1cm} (A.7.4)
Table A.1: Exogenous Case ($\lambda = 1, \alpha = 2$)

<table>
<thead>
<tr>
<th>No. of Noise Trader</th>
<th>$\hat{s}_t$</th>
<th>$\text{Var}(\hat{s}_t)$</th>
<th>Increase of $\text{Var}(\hat{s}_t)$</th>
<th>$\text{Var}(dB_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\hat{s}_t = 0.9562\hat{m}_t - 0.9562\hat{m}_t^*$</td>
<td>0.0183</td>
<td>0.00%</td>
<td>2.4861E-04</td>
</tr>
<tr>
<td>0.1</td>
<td>$\hat{s}_t = 0.9562\hat{m}_t - 0.9562\hat{m}_t^* + 0.0912v_t$</td>
<td>0.0184</td>
<td>0.84%</td>
<td>2.4861E-04</td>
</tr>
<tr>
<td>0.2</td>
<td>$\hat{s}_t = 0.9561\hat{m}_t - 0.9561\hat{m}_t^* + 0.1824v_t$</td>
<td>0.0189</td>
<td>3.43%</td>
<td>2.7598E-04</td>
</tr>
<tr>
<td>0.3</td>
<td>$\hat{s}_t = 0.9560\hat{m}_t - 0.9560\hat{m}_t^* + 0.2736v_t$</td>
<td>0.0198</td>
<td>8.06%</td>
<td>3.2484E-04</td>
</tr>
<tr>
<td>0.4</td>
<td>$\hat{s}_t = 0.9559\hat{m}_t - 0.9559\hat{m}_t^* + 0.3647v_t$</td>
<td>0.0211</td>
<td>15.27%</td>
<td>4.0097E-04</td>
</tr>
<tr>
<td>0.5</td>
<td>$\hat{s}_t = 0.9557\hat{m}_t - 0.9557\hat{m}_t^* + 0.4557v_t$</td>
<td>0.0231</td>
<td>26.06%</td>
<td>5.1497E-04</td>
</tr>
<tr>
<td>0.6</td>
<td>$\hat{s}_t = 0.9553\hat{m}_t - 0.9553\hat{m}_t^* + 0.5464v_t$</td>
<td>0.0260</td>
<td>42.31%</td>
<td>6.8646E-04</td>
</tr>
<tr>
<td>0.7</td>
<td>$\hat{s}_t = 0.9548\hat{m}_t - 0.9548\hat{m}_t^* + 0.6367v_t$</td>
<td>0.0307</td>
<td>67.71%</td>
<td>9.5470E-04</td>
</tr>
<tr>
<td>0.8</td>
<td>$\hat{s}_t = 0.9540\hat{m}_t - 0.9540\hat{m}_t^* + 0.7263v_t$</td>
<td>0.0385</td>
<td>110.68%</td>
<td>1.4083E-03</td>
</tr>
<tr>
<td>0.9</td>
<td>$\hat{s}_t = 0.9523\hat{m}_t - 0.9523\hat{m}_t^* + 0.8141v_t$</td>
<td>0.0538</td>
<td>194.14%</td>
<td>2.2895E-03</td>
</tr>
<tr>
<td>1.0</td>
<td>$\hat{s}_t = 0.9482\hat{m}_t - 0.9482\hat{m}_t^* + 0.8964v_t$</td>
<td>0.0916</td>
<td>400.69%</td>
<td>4.4702E-03</td>
</tr>
</tbody>
</table>

Consumption

\[ \text{Var}(\hat{c}_t) = \text{Var}(\hat{c}_t^*) = 0.0025 \]

Home wage

\[ \text{Var}(\hat{w}_t) = 0.0163 \]

Home Labor

\[ \text{Var}(\hat{L}_t) = 0.0013 \]
Table A.2: Exogenous Case ($\lambda = 1.5$, $a = 2$)

<table>
<thead>
<tr>
<th>No. of Noise Traders</th>
<th>$\delta_t$</th>
<th>$\text{Var}(\delta_t)$</th>
<th>Increase of $\text{Var}(\delta_t)$</th>
<th>$\text{Var}(dB_{t+1})$</th>
<th>$\text{Corr}(\delta_t, c_t - c_t^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\delta_t = 0.9562m_i - 0.9562m_i^*$</td>
<td>0.0183</td>
<td>0.00%</td>
<td>2.3979E-04</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.1</td>
<td>$\delta_t = 0.9562m_i - 0.9562m_i^* + 0.0912v_t$</td>
<td>0.0185</td>
<td>1.26%</td>
<td>2.5308E-04</td>
<td>0.9937</td>
</tr>
<tr>
<td>0.2</td>
<td>$\delta_t = 0.9561m_i - 0.9561m_i^* + 0.1824v_t$</td>
<td>0.0192</td>
<td>5.23%</td>
<td>2.9502E-04</td>
<td>0.9747</td>
</tr>
<tr>
<td>0.3</td>
<td>$\delta_t = 0.9559m_i - 0.9559m_i^* + 0.2736v_t$</td>
<td>0.0206</td>
<td>12.58%</td>
<td>3.7265E-04</td>
<td>0.9422</td>
</tr>
<tr>
<td>0.4</td>
<td>$\delta_t = 0.9557m_i - 0.9557m_i^* + 0.3645v_t$</td>
<td>0.0228</td>
<td>24.77%</td>
<td>5.0127E-04</td>
<td>0.8948</td>
</tr>
<tr>
<td>0.5</td>
<td>$\delta_t = 0.9553m_i - 0.9553m_i^* + 0.4553v_t$</td>
<td>0.0265</td>
<td>44.85%</td>
<td>7.1327E-04</td>
<td>0.8301</td>
</tr>
<tr>
<td>0.6</td>
<td>$\delta_t = 0.9546m_i - 0.9546m_i^* + 0.5455v_t$</td>
<td>0.0329</td>
<td>80.00%</td>
<td>1.0845E-03</td>
<td>0.7441</td>
</tr>
<tr>
<td>0.7</td>
<td>$\delta_t = 0.9532m_i - 0.9532m_i^* + 0.6344v_t$</td>
<td>0.0459</td>
<td>150.75%</td>
<td>1.8314E-03</td>
<td>0.6295</td>
</tr>
<tr>
<td>0.8</td>
<td>$\delta_t = 0.9494m_i - 0.9494m_i^* + 0.7191v_t$</td>
<td>0.0803</td>
<td>339.33%</td>
<td>3.8224E-03</td>
<td>0.4737</td>
</tr>
<tr>
<td>0.9</td>
<td>$\delta_t = 0.9351m_i - 0.9351m_i^* + 0.8331v_t$</td>
<td>0.2183</td>
<td>1094.09%</td>
<td>1.1790E-02</td>
<td>0.2830</td>
</tr>
<tr>
<td>1.0</td>
<td>$\delta_t = 0.9024m_i - 0.9024m_i^* + 0.8047v_t$</td>
<td>0.5695</td>
<td>3014.60%</td>
<td>3.2060E-02</td>
<td>0.1691</td>
</tr>
</tbody>
</table>

Consumption

| $\text{Var}(\tilde{c}_t) = \text{Var}(\tilde{c}_t^*) = 0.0025$ | $\text{Var}(\tilde{w}_t) = 0.0163$ | $\text{Var}(\tilde{l}_t) = 0.0013$ |

Home wage

Home Labor
Table A.3: Endogenous Case ($\lambda = 1.5, a = 2$)

<table>
<thead>
<tr>
<th>$N_I = 0.1$</th>
<th>$\bar{c} = 0.1$</th>
<th>$\bar{c} = 0.15$</th>
<th>$\bar{c} = 0.25$</th>
<th>$\bar{c} = \infty$</th>
<th>$N_I = 0.2$</th>
<th>$\bar{c} = 0.1$</th>
<th>$\bar{c} = 0.15$</th>
<th>$\bar{c} = 0.25$</th>
<th>$\bar{c} = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Var(\bar{s}_t)$</td>
<td>0.7381</td>
<td>0.3697</td>
<td>0.1197</td>
<td>0.0168</td>
<td>$Var(\bar{s}_t)$</td>
<td>0.2505</td>
<td>0.0728</td>
<td>0.0299</td>
<td>0.0171</td>
</tr>
<tr>
<td>$Var(dB_{t+1})$</td>
<td>4.23E-02</td>
<td>2.12E-02</td>
<td>6.80E-03</td>
<td>2.11E-04</td>
<td>$Var(dB_{t+1})$</td>
<td>1.43E-02</td>
<td>4.10E-03</td>
<td>1.60E-03</td>
<td>2.199E-04</td>
</tr>
<tr>
<td>$Mean(n)$</td>
<td>0.1741</td>
<td>0.1216</td>
<td>0.0931</td>
<td>0</td>
<td>$Mean(n)$</td>
<td>0.1743</td>
<td>0.1663</td>
<td>0.1749</td>
<td>0</td>
</tr>
<tr>
<td>$N_I = 0.4$</td>
<td>$\bar{c} = 0.1$</td>
<td>$\bar{c} = 0.15$</td>
<td>$\bar{c} = 0.25$</td>
<td>$\bar{c} = \infty$</td>
<td>$N_I = 0.5$</td>
<td>$\bar{c} = 0.1$</td>
<td>$\bar{c} = 0.15$</td>
<td>$\bar{c} = 0.25$</td>
<td>$\bar{c} = \infty$</td>
</tr>
<tr>
<td>$Var(\bar{s}_t)$</td>
<td>0.0313</td>
<td>0.0239</td>
<td>0.0199</td>
<td>0.0173</td>
<td>$Var(\bar{s}_t)$</td>
<td>0.021</td>
<td>0.0201</td>
<td>0.0193</td>
<td>0.0173</td>
</tr>
<tr>
<td>$Var(dB_{t+1})$</td>
<td>1.60E-03</td>
<td>1.10E-03</td>
<td>8.00E-04</td>
<td>2.246E-04</td>
<td>$Var(dB_{t+1})$</td>
<td>8.00E-04</td>
<td>7.00E-04</td>
<td>6.00E-04</td>
<td>2.256E-04</td>
</tr>
<tr>
<td>$Mean(n)$</td>
<td>0.2938</td>
<td>0.2834</td>
<td>0.2629</td>
<td>0</td>
<td>$Mean(n)$</td>
<td>0.3067</td>
<td>0.2888</td>
<td>0.2604</td>
<td>0</td>
</tr>
<tr>
<td>$N_I = 0.8$</td>
<td>$\bar{c} = 0.1$</td>
<td>$\bar{c} = 0.15$</td>
<td>$\bar{c} = 0.25$</td>
<td>$\bar{c} = \infty$</td>
<td>$N_I = 0.99$</td>
<td>$\bar{c} = 0.1$</td>
<td>$\bar{c} = 0.15$</td>
<td>$\bar{c} = 0.25$</td>
<td>$\bar{c} = \infty$</td>
</tr>
<tr>
<td>$Var(\bar{s}_t)$</td>
<td>0.0173</td>
<td>0.0171</td>
<td>0.0169</td>
<td>0.0173</td>
<td>$Var(\bar{s}_t)$</td>
<td>0.0173</td>
<td>0.0173</td>
<td>0.0173</td>
<td>0.0173</td>
</tr>
<tr>
<td>$Var(dB_{t+1})$</td>
<td>3.00E-04</td>
<td>3.00E-04</td>
<td>2.00E-04</td>
<td>2.00E-04</td>
<td>$Var(dB_{t+1})$</td>
<td>2.00E-04</td>
<td>2.00E-04</td>
<td>2.00E-04</td>
<td>2.00E-04</td>
</tr>
<tr>
<td>$Mean(n)$</td>
<td>0.1494</td>
<td>0.14</td>
<td>0.1259</td>
<td>0</td>
<td>$Mean(n)$</td>
<td>0.0076</td>
<td>0.0072</td>
<td>0.0065</td>
<td>0</td>
</tr>
</tbody>
</table>

**Consumption**

$Var(\bar{c}_t) = Var(c_t^*) = 0.0025$

**Home wage**

$Var(\bar{w}_t) = 0.0163$

**Home Labor**

$Var(\bar{l}_t) = 0.0013$

---

*a* Here we only consider a reasonable range of $\bar{c}$: $\bar{c} \in (0,0.25]$, as the steady state consumption in this model is 0.25.
Table A.4: The impact of Tobin Tax

<table>
<thead>
<tr>
<th>Tobin Tax</th>
<th>( Var(s_t) ) (Exogenous Case, ( \lambda = 1.5, a = 2 ))</th>
<th>( N_f = 0 )</th>
<th>( N_f = 0.1 )</th>
<th>( N_f = 0.2 )</th>
<th>( N_f = 0.3 )</th>
<th>( N_f = 0.4 )</th>
<th>( N_f = 0.5 )</th>
<th>( N_f = 0.6 )</th>
<th>( N_f = 0.7 )</th>
<th>( N_f = 0.8 )</th>
<th>( N_f = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = 0 )</td>
<td>0.5695</td>
<td>0.2183</td>
<td>0.0803</td>
<td>0.0329</td>
<td>0.0265</td>
<td>0.0228</td>
<td>0.0206</td>
<td>0.0192</td>
<td>0.0183</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 0.5% )</td>
<td>0.5623</td>
<td>0.2139</td>
<td>0.0793</td>
<td>0.0328</td>
<td>0.0264</td>
<td>0.0227</td>
<td>0.0205</td>
<td>0.0192</td>
<td>0.0183</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 1% )</td>
<td>0.555</td>
<td>0.2096</td>
<td>0.0783</td>
<td>0.0326</td>
<td>0.0263</td>
<td>0.0227</td>
<td>0.0205</td>
<td>0.0192</td>
<td>0.0182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 1.5% )</td>
<td>0.5478</td>
<td>0.2054</td>
<td>0.0772</td>
<td>0.0324</td>
<td>0.0262</td>
<td>0.0226</td>
<td>0.0204</td>
<td>0.0191</td>
<td>0.0182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau = 2.5% )</td>
<td>0.5334</td>
<td>0.1971</td>
<td>0.0753</td>
<td>0.0321</td>
<td>0.0260</td>
<td>0.0225</td>
<td>0.0203</td>
<td>0.0191</td>
<td>0.0181</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tobin Tax</th>
<th>( Var(s_t) ) (Endogenous Case, ( \lambda = 1.5, a = 2, \bar{c} = 0.1 ))</th>
<th>( N_f = 0.001 )</th>
<th>( N_f = 0.1 )</th>
<th>( N_f = 0.2 )</th>
<th>( N_f = 0.3 )</th>
<th>( N_f = 0.4 )</th>
<th>( N_f = 0.5 )</th>
<th>( N_f = 0.6 )</th>
<th>( N_f = 0.7 )</th>
<th>( N_f = 0.8 )</th>
<th>( N_f = 0.99 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau = 0 )</td>
<td>1.2723</td>
<td>0.7381</td>
<td>0.2505</td>
<td>0.0576</td>
<td>0.0313</td>
<td>0.021</td>
<td>0.0178</td>
<td>0.0173</td>
<td>0.0173</td>
<td>0.0173</td>
<td>0.0173</td>
</tr>
<tr>
<td>( \tau = 0.5% )</td>
<td>1.2543</td>
<td>0.7152</td>
<td>0.2131</td>
<td>0.0404</td>
<td>0.0250</td>
<td>0.0201</td>
<td>0.0178</td>
<td>0.0173</td>
<td>0.0171</td>
<td>0.0173</td>
<td>0.0173</td>
</tr>
<tr>
<td>( \tau = 1% )</td>
<td>1.2375</td>
<td>0.6934</td>
<td>0.1747</td>
<td>0.0276</td>
<td>0.0212</td>
<td>0.0194</td>
<td>0.0178</td>
<td>0.0173</td>
<td>0.0169</td>
<td>0.0173</td>
<td>0.0173</td>
</tr>
<tr>
<td>( \tau = 1.5% )</td>
<td>1.2203</td>
<td>0.6709</td>
<td>0.1230</td>
<td>0.0222</td>
<td>0.0198</td>
<td>0.0191</td>
<td>0.0177</td>
<td>0.0171</td>
<td>0.0168</td>
<td>0.0172</td>
<td>0.0172</td>
</tr>
<tr>
<td>( \tau = 2.5% )</td>
<td>1.1856</td>
<td>0.6243</td>
<td>0.0239</td>
<td>0.0185</td>
<td>0.0186</td>
<td>0.0181</td>
<td>0.0172</td>
<td>0.0166</td>
<td>0.0165</td>
<td>0.0171</td>
<td>0.0171</td>
</tr>
</tbody>
</table>
Appendix B

Appendices of Chapter 2

B.1 Price index and individual demand

The aggregate consumption in the home country is composed by home finished goods and foreign finished goods:

\[ C = 2C_h^{\frac{1}{2}} C_f^{\frac{1}{2}} \]  
(B.1.1)

Thus, the consumption-based price index \( P \), which is the minimum nominal expenditure to purchase one unit of aggregate consumption, can be found by solving the following minimization problem:

\[
\min_{(C_h, C_f)} \quad Z = P_h C_h + P_f C_f \\
\text{s.t} \quad 2C_h^{\frac{1}{2}} C_f^{\frac{1}{2}} = 1
\]  
(B.1.2)

So the CPI price index is given by:

\[ P = P_h^{\frac{1}{2}} P_f^{\frac{1}{2}} \]  
(B.1.4)

Taking the prices for the home finished goods and foreign finished goods as given, the consumer allocates a given level of aggregate consumption among the home and foreign sub-aggregate finished goods:

\[
\max_{\{C_h, C_f\}} \quad C = 2C_h^{\frac{1}{2}} C_f^{\frac{1}{2}} \\
\text{s.t} \quad P_h C_h + P_f C_f = PC
\]  
(B.1.6)

Thus, we have

\[ C_h = \frac{1}{2} \frac{PC}{P_h}, \quad C_f = \frac{1}{2} \frac{PC}{P_f} \]  
(B.1.7)

Similarly, given the definition of the sub-aggregate consumption of the home and foreign finished goods \( C_h = \int_0^1 C_h(i)^{\frac{1}{\lambda-1}} di \) and \( C_f = \int_0^1 C_f(i)^{\frac{1}{\lambda-1}} di \) the price indexes for sub-aggregate home and foreign finished goods are given by:

\[ P_h = \left[ \int_0^1 P_h(i)^{1-\lambda} di \right]^{\frac{1}{\lambda-1}}, \quad P_f = \left[ \int_0^1 P_f(i)^{1-\lambda} di \right]^{\frac{1}{\lambda-1}} \]  
(B.1.8)
Then the optimal demand for individual finished goods can be derived as:

\[
C_h(i) = \left[ \frac{P_h(i)}{P_h} \right]^{-\lambda} C_h, \quad C_f(i) = \left[ \frac{P_f(i)}{P_f} \right]^{-\lambda} C_f
\]

(B.1.9)

The price index and individual demand for intermediate goods can be derived analogously.

### B.2 Expected welfare

**Solving for \( E_c \)** First, substituting the optimal pricing schedules \( \tilde{P}_h \) and \( \tilde{P}_f \) into the definition of the price indices \( P_h \) and \( P_f \), we have \(^1\)

\[
P_h = (\lambda \hat{\phi})^{\frac{1}{2}} \frac{[E(\frac{s^1}{\theta^1} C^{1-\rho})][E(\frac{W}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}} [E(\frac{W^*}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}{[E C^{1-\rho}][E(\frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}} [E(\frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}
\]

(B.2.1)

\[
P_f^* = (\lambda \hat{\phi})^{\frac{1}{2}} \frac{[E(\frac{s^1}{\theta^1} C^{1-\rho})][E(\frac{W}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}} [E(\frac{W^*}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}{[E C^{1-\rho}][E(\frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}} [E(\frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}
\]

(B.2.2)

Putting the above two equations together, we have

\[
P_h P_f^* = (\lambda \hat{\phi})^{\frac{1}{2}} \frac{[E(\frac{W}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}} [E(\frac{W^*}{\theta_f} \frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}{[E C^{1-\rho}]^2}
\]

(B.2.3)

Using the labor supply function to eliminate the predetermined prices on both sides, we have

\[
1 = \lambda \hat{\phi} \eta^2 \frac{[E(\frac{SC}{\theta_f \theta_f})]^{\frac{1}{2}} [E(\frac{C}{\theta_f \theta_f})]^{\frac{1}{2}} [E(\frac{s^1}{\theta^1} C^{1-\rho})]^{\frac{1}{2}}}{[E C^{1-\rho}]^2}
\]

(B.2.4)

Using the properties of log-normal distribution, taking log of (B.2.4), we have

\[
0 = \log[\lambda \hat{\phi}] + \frac{1}{2} [EC + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 - \sigma_{su} - \sigma_{sv} - \sigma_{cu} - \sigma_{cv} + \sigma_{cs} + \sigma_{uv}]
\]

\[
+ \frac{1}{2} [EC + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 + \frac{1}{2} \sigma_u^2 - \sigma_{cu} - \sigma_{uv}]
\]

\[
+ \frac{1}{2} [EC + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 + \frac{1}{2} \sigma_u^2 - \sigma_{cv} - \sigma_{uv}]
\]

\[
+ \frac{1}{2} [EC + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 + \frac{1}{2} \sigma_u^2 + \frac{1}{2} \sigma_v^2 + \sigma_{cu} - \sigma_{cu} - \sigma_{cs} - \sigma_{su} + \sigma_{sv} + \sigma_{uv} - \sigma_{cu} - \sigma_{uv} - \sigma_{cu} - \sigma_{uv}]
\]

\[
- 2[(1 - \rho)EC + \frac{(1 - \rho)^2}{2} \sigma_c^2]
\]

(B.2.5)

\(^1\)For simplicity, we have used the fact that \( C = C^* \) here.
The next step is to derive the mean of the log consumption in terms of variance and covariance of the log consumption, the log exchange rate, and the productivity shocks.

\[
Ec = \frac{-\ln[\hat{\phi}^2]}{2\rho} - \frac{(2 - \rho)\sigma_c^2}{2} - \frac{1}{4\rho}\sigma_s^2 - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u*}^2 + \sigma_v^2 + \sigma_{v*}^2 + \sigma_{uv} + \sigma_{uv*}]
\]

\[
+ \frac{1}{4\rho}[\sigma_{su} - \sigma_{su*}] + \frac{1}{4\rho}[\sigma_{sv} - \sigma_{sv*}] + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu*} + \sigma_{cv} + \sigma_{cv*}] 
\]  

(B.2.6)

**Solving for EL**  The intermediate goods market clearing condition implies

\[
\theta_i L = X_h + X_h^* = \frac{1}{2} \frac{\Lambda}{P_h} + \frac{1}{2} \frac{P^*}{P^*_f} 
\]  

(B.2.7)

Substituting \( \tilde{P}_h, \tilde{P}_f, P_h \) and \( P_f \) into the above equation, we have

\[
L = \frac{1}{\lambda^2} \frac{PC\tilde{P}_h^{1/2}(S\tilde{P}_f^{1/2})}{\theta_I \theta_f} \frac{E(\tilde{P}_h^{1/2}C^{1-\rho})}{E(\tilde{P}_f^{1/2}C^{1-\rho})} \frac{E(C^{1-\rho})}{E(\tilde{P}_h^{1/2}S^{1/2}C^{1-\rho})} 
\]  

(B.2.8)

Using the labor supply function to eliminate the predetermined terms, and then taking expectation, we have

\[
EL = \frac{1}{\lambda\phi} C^{1-\rho} 
\]  

(B.2.9)

**Expected welfare**  We may rewrite the expected utility of the home country as

\[
EU = \frac{\lambda\phi - (1 - \rho)(\lambda - 1)(\phi - 1)}{(1 - \rho)\lambda\phi} C^{1-\rho} 
\]  

(B.10.10)

Since \( C^{1-\rho} = \exp(1 - \rho)(Ec + \frac{1-\rho}{2}\sigma_c^2) \), the monetary authority’s problem is equivalent to maximize \( U_0 = Ec + \frac{1-\rho}{2}\sigma_c^2 \),

\[
U_0 = \frac{-\ln[\hat{\phi}^2]}{2\rho} - \frac{1}{2}\sigma_c^2 - \frac{1}{4\rho}\sigma_s^2 - \frac{1}{4\rho}[\sigma_u^2 + \sigma_{u*}^2 + \sigma_v^2 + \sigma_{v*}^2 + \sigma_{uv} + \sigma_{uv*}]
\]

\[
+ \frac{1}{4\rho}[\sigma_{su} - \sigma_{su*}] + \frac{1}{4\rho}[\sigma_{sv} - \sigma_{sv*}] + \frac{1}{2\rho}[\sigma_{cu} + \sigma_{cu*} + \sigma_{cv} + \sigma_{cv*}] 
\]  

(B.2.11)
B.3 Optimal money rules

Variance and covariance terms  Given the money rules and the solution for the exchange rate and the consumption (2.4.1)-(2.4.4), we have

\[
\sigma_c^2 = \frac{1}{4\rho^2}[(a_1 + b_1)^2\sigma_u^2 + (a_2 + b_2)^2\sigma_v^2 + (a_3 + b_3)^2\sigma_u^* + (a_4 + b_4)^2\sigma_v^* \\
2(a_1 + b_1)(a_3 + b_3)(\sigma_{uv} + 2(a_2 + b_2)(a_4 + b_4)\sigma_{uv^*})]
\]  

(B.3.1)

\[
\sigma_s^2 = \frac{1}{2\rho}[(a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_v^2 + (a_3 - b_3)^2\sigma_u^* + (a_4 - b_4)^2\sigma_v^* \\
2(a_1 - b_1)(a_3 - b_3)\sigma_{uv} + 2(a_2 - b_2)(a_4 - b_4)\sigma_{uv^*}]
\]  

(B.3.2)

\[
\sigma_{su} = (a_1 - b_1)\sigma_u^2 + (a_3 - b_3)\sigma_{uv}, \quad \sigma_{su^*} = (a_2 - b_2)\sigma_u^2 + (a_4 - b_4)\sigma_{uv^*}
\]  

(B.3.3)

\[
\sigma_{sv} = (a_3 - b_3)\sigma_v^2 + (a_1 - b_1)\sigma_{uv}, \quad \sigma_{sv^*} = (a_4 - b_4)\sigma_v^2 + (a_2 - b_2)\sigma_{uv^*}
\]  

(B.3.4)

\[
\sigma_{cu} = \frac{1}{2\rho}(a_1 + b_1)\sigma_u^2 + \frac{1}{2\rho}(a_3 + b_3)\sigma_{uv}, \quad \sigma_{cu^*} = \frac{1}{2\rho}(a_2 + b_2)\sigma_u^2 + \frac{1}{2\rho}(a_4 + b_4)\sigma_{uv^*}
\]  

(B.3.5)

\[
\sigma_{cv} = \frac{1}{2\rho}(a_3 + b_3)\sigma_v^2 + \frac{1}{2\rho}(a_1 + b_1)\sigma_{uv}, \quad \sigma_{cv^*} = \frac{1}{2\rho}(a_4 + b_4)\sigma_v^2 + \frac{1}{2\rho}(a_2 + b_2)\sigma_{uv^*}
\]  

(B.3.6)

Substituting the above expressions into Equation (B.2.11), we may express the objective functions of monetary authorities as functions of policy parameters.

Solution to the Nash game  Since the objective functions of the home and the foreign monetary authorities are identical, the reaction functions are given by:

\[
\frac{\partial U_0(a_i,b_i^N)}{a_i} = 0, \quad \forall i = 1, 2, 3, 4
\]  

(B.3.7)

\[
\frac{\partial U_0(a_i^N,b_i)}{b_i} = 0, \quad \forall i = 1, 2, 3, 4
\]  

(B.3.8)

Thus, from these 8 reaction functions, we may derive the solution to the international monetary game. Substituting these optimal policy parameters into Equation (B.2.11), we may get the maximized expected utility level under the sticky price equilibrium.

\[
U_0(\text{sticky}) = \frac{-\ln[\lambda\hat{\eta}\rho^2]}{2\rho} + \frac{1 - \rho}{8\rho^2}[\sigma_u^2 + \sigma_v^2 + \sigma_u^* + \sigma_v^2 + 2\sigma_{uv} + 2\sigma_{uv^*}]
\]

\[-\frac{1}{16\rho^2}[\sigma_u^2 + \sigma_v^2 + \sigma_u^2 + \sigma_v^2 - 2\sigma_{uv} - 2\sigma_{uv^*}]
\]  

(B.3.9)

From Equation (B.3.9) and (2.3.8), Equation (2.4.11) is obvious.
Table B.1: The Optimal Price Policies for Foreign Firms$^a$

$$
\begin{align*}
\hat{P}_f^* &= \frac{\lambda \frac{1}{\lambda-1} \frac{E[X^{1-\eta}]}{\beta^1}}{\hat{p}_{f}^S} \\
\hat{P}_f^* &= \frac{E[X^{1-\eta}]}{\beta^1} \\
\hat{P}_{fh}^* &= \frac{\phi}{\phi-1} \frac{E[W^{1-\eta}]}{\beta^1}
\end{align*}
$$

$^a$ The prices with asterisk are in term of foreign currency.

Figure B.1: The Structure of the Economy
Appendix C

Appendices of Chapter 3

C.1 Optimal pricing setting

The optimization problem of each firm is to maximize the discounted expected profits, taking the individual demand function as given. Home firms set both the domestic price and export price in the currency of the producer (PCP). The home firm $i$’s problem is then:

$$
\max_{P_{hh}, P_{hf}} E[d \pi(i)] = \max_{P_{hh}, P_{hf}} E[d((P_{hh}(i) - \frac{W}{\theta})X_h(i) + (P_{hf}(i) - \frac{W}{\theta})X^*_h(i))] \quad (C.1.1)
$$

Where $d = PC^{-\rho}$ is the stochastic discount factor, $X_h(i) = nC_h(i)$ is the total sales of firm $i$ to home residents and $X^*_h(i) = (1-n)C^*_h(i)$ is the total sales to foreign residents.

Foreign firms set both the domestic and the export price in the currency of the consumer (LCP). The foreign firm $i$’s problem is:

$$
\max_{P_{hh}, P_{hf}^*} E[d^* \pi^*(i)] = \max_{P_{hh}, P_{hf}^*} E[d^*((P_{hh}(i) - \frac{W^*}{\theta^*})X_f(i) + (P_{hf}^*(i) - \frac{W^*}{\theta^*})X^*_f(i))] \quad (C.1.2)
$$

Substitute the risk-sharing condition 3.2.1, the labor supply function 3.2.4 and its foreign equivalent into the first order conditions derived from home and foreign firms’ optimization problem, we can derive the optimal pricing policies of firms (3.2.5)-(3.2.8).

C.2 Model solution

Solving for $Ec$  From the price index (3.2.10) and pricing equations (3.2.5) and (3.2.6), we have

$$
P_{hh}^n P_{hf}^{1-n} = \frac{E[(\frac{WC}{\theta})^n][E(\frac{W^*SC^{1-\rho}}{\theta^*})]^{1-n}}{E(C^{1-\rho})} \quad (C.2.3)
$$

Using the risk-sharing conditions (3.2.1) and (3.2.4) and its foreign equivalent, taking out the predetermined terms, we have Equation (3.3.9) of the paper:

$$
1 = \hat{\lambda}\eta \Gamma^{1-n}[E(\frac{C}{\theta})]^n[E(\frac{C}{\theta})]^{1-n}
$$

Now using the fact that the solution for consumption and exchange rate will be log-normal, and taking logs, we may get the expected (log) consumption (3.3.11):

$$
Ec = -\frac{1}{\rho} \ln[\hat{\lambda}\eta \Gamma^{1-n}] - 2 - \frac{\rho}{2} \sigma_c^2 - \frac{n\sigma_u^2 + (1-n)\sigma^2_{u^*}}{2\rho} + \frac{n\sigma_{cu} + (1-n)\sigma_{cu^*}}{\rho}
$$
Similarly, we can derive Equations (3.3.10) and (3.3.12).

**Solving for EL and EU**  
Home goods market clearing condition implies

\[ \theta L = n \frac{P C}{P_{h h}} + (1 - n) \frac{P^* C^*}{P^*} \]  
(C.2.4)

Substituting the pricing equations (3.2.5) and (3.2.7) into (C.2.4), we get

\[ L = n \frac{P C}{\theta} \frac{E(C^{1-\rho})}{\lambda E\left(\frac{W C^x}{\theta}\right)} + (1 - n) \frac{S P^* C^*}{\theta} \frac{E(C^{1-\rho})}{\lambda E\left(\frac{W C^x}{\theta}\right)} \]  
(C.2.5)

Using the labor supply equation (3.2.4) and risk sharing condition (3.2.1), and taking expectation, we can get Equation (3.3.14) of the paper:

\[ EL = \frac{n}{\lambda \eta} EC^{1-\rho} + \frac{1 - n}{\lambda \eta} E(C^{1-\rho}) \]  
(C.2.6)

Similarly, we can get:

\[ EL^* = \Gamma^{-1} \frac{n}{\lambda \eta} E(C^{1-\rho}) + \frac{1 - n}{\lambda \eta} E(C^{1-\rho}) \]  
(C.2.6)

Then we can get the expected home country utility function (3.3.15) and its foreign equivalent:

\[ EU^* = \frac{\lambda - (1 - n)(\lambda - 1)(1 - \rho)}{(1 - \rho)\lambda} E(C^{1-\rho}) - \frac{n(\lambda - 1)}{\lambda} \Gamma^{-1} E(C^{1-\rho}) \]  
(C.2.7)

**Calculating the variances and covariances**  
From Equations (3.3.4)-(3.3.6) and monetary policy rule (3.3.7) and (3.3.8), we can solve for the variances and covariances terms in Equation (3.3.11) and (3.3.12).

\[ \sigma_s^2 = (a_1 - b_1)^2 \sigma_u^2 + (a_2 - b_2)^2 \sigma_{u^*}^2 \]  
(C.2.8)

\[ \sigma_c^2 = \frac{1}{\rho^2} [a_1^2 \sigma_u^2 + a_2^2 \sigma_{u^*}^2] \]  
(C.2.9)

\[ \sigma_{c^*}^2 = \frac{1}{\rho^2} [(na_1 + (1 - n)b_1)^2 \sigma_u^2 + (na_2 + (1 - n)b_2)^2 \sigma_{u^*}^2] \]  
(C.2.10)

\[ \sigma_{cu}^2 = \frac{1}{\rho} a_1 \sigma_u^2, \quad \sigma_{cu^*}^2 = \frac{1}{\rho} a_2 \sigma_{u^*}^2 \]  
(C.2.11)

\[ \sigma_{c^*u}^2 = \frac{1}{\rho} [na_1 + (1 - n)b_1] \sigma_u^2, \quad \sigma_{c^*u^*}^2 = \frac{1}{\rho} [na_2 + (1 - n)b_2] \sigma_{u^*}^2 \]  
(C.2.12)
\[ \sigma_{su} = (a_1 - b_1)\sigma_u^2, \quad \sigma_{su^*} = (a_2 - b_2)\sigma_u^2. \]  
(C.2.13)

Using the relationship

\[ EC^{1-\rho} = \exp \left\{ (1 - \rho) [E(c) + \frac{1 - \rho}{2}\sigma_c^2] \right\} \]  
(C.2.14)

we can express the expected home and foreign expected utility as functions of monetary policy parameters \((a_1, a_2, b_1, b_2)\).

### C.3 Proof of Proposition 4

When \( \rho = 1 \), using Equations (3.4.6) and (3.4.7), given the optimal monetary rules that \( a_1 = n, a_2 = 1 - n, b_1 = 0 \) and \( b_2 = 1 \), the expected utility for the home country and foreign country are, respectively

\[ EU = -\frac{n(1-n)}{2}[\sigma_u^2 + \sigma_{u^*}^2] \quad \text{(C.3.15)} \]

\[ EU^* = -\frac{n(1-n)^2}{2}[\sigma_u^2 + \sigma_{u^*}^2] \quad \text{(C.3.16)} \]

Thus, \( EU < EU^* \).

When \( \rho > 1 \), using equation (3.4.2) and (3.4.3) and the fact that \( \Gamma = \left( \frac{\bar{X}}{\bar{X}^*} \right)^\rho \), we can simplify \( EU \) and \( EU^* \) as:

\[ EU = \frac{\lambda - (\lambda - 1)(1 - \rho)\Gamma X^*}{(1 - \rho)\lambda} \]  
(C.3.17)

\[ EU^* = \frac{\lambda - (\lambda - 1)(1 - \rho)X^*}{(1 - \rho)\lambda} \]  
(C.3.18)

Since \( \frac{\lambda - (\lambda - 1)(1 - \rho)}{(1 - \rho)\lambda} \) is negative, to prove \( EU < EU^* \) is equivalent to prove \( \Gamma > 1 \). We denote

\[ \bar{X} = -\frac{1}{2}\sigma_c^2 - \frac{1}{2}\sigma_u^2 + \sigma_{cu} \]  
(C.3.19)

\[ \bar{X}^* = -\frac{1}{2}\sigma_{c^*}^2 - \frac{n(1-n)}{2}\sigma_{s^*}^2 - \frac{1}{2}\sigma_u^2 + \sigma_{cu} + n(1-n)(\sigma_{su} - \sigma_{su^*}) \]  
(C.3.20)

Thus,

\[ X = \Theta \exp[(1 - \rho)\bar{X}] \]  
(C.3.21)

\[ X^* = \Theta \exp[(1 - \rho)\bar{X}^*] \]  
(C.3.22)
Appendix C. Appendices of Chapter 3

Since \( p > 1 \), to prove \( \Gamma > 1 \) is equivalent to prove \( \bar{X} < \bar{X}^* \). Substituting the optimal monetary rules for the general case listed in Table C.1 into Equations (C.3.19) and (C.3.20), we may have

\[ \bar{X}^* - \bar{X} = A \sigma_u^2 + A^* \sigma_u^2 \]  

Where

\[ A = \frac{(1-n)(a_1 - b_1)}{2 \rho^2 (n(\rho - 1) + 1)} [a_1 + n^2(\rho - 1)^3 + n(\rho - 1)(\rho + (\rho - 1)(1 - a_1))] \]  

\[ A^* = \frac{(1-n)(a_2 - b_2)}{2 \rho^2 (n(\rho - 1) + 1)} [a_2 - (1-n)(n(\rho - 1)^2 + 2\rho - 1) - a_2 n(\rho - 1)^2 - \rho n(n(\rho - 1)^2 + 2\rho - 1)] \]  

Given the properties of the optimal policy coefficients (\( n \leq a_1 < 1, b_1 < 0, a_1 - b_1 > 0, 0 < a_2 \leq 1 - n \) and \( a_2 - b_2 < 0 \)), we can show

\[ A > 0, \quad A^* > 0 \]  

That is, \( \bar{X} < \bar{X}^* \) and \( X > X^* \). Therefore,

\[ \Gamma = \left( \frac{X}{X^*} \right)^\rho > 1 \]  

Thus, \( EU < EU^* \) when \( p > 1 \). Q.E.D.

C.4 Proof of Proposition 5

We can prove Proposition 5 in two steps. First, we prove that the two conditions under which the home firms choose PCP and the foreign firms follow LCP can be satisfied. That is:

\[ \frac{1}{2} \sigma_s^2 - \text{Cov}(\ln \frac{W}{\theta}, s) > Z \]  

\[ \frac{1}{2} \sigma_s^2 + \text{Cov}(\ln \frac{W^*}{\theta^*}, s) < Z \]  

where \( Z = \frac{1}{\theta^*} \ln S \), \( s = \ln S \). Then we prove there exists a positive menu cost \( \delta \) such that both conditions hold simultaneously.

Step 1 To prove that both (C.4.28) and (C.4.29) can be satisfied is equivalent to prove the following inequality:

\[ \text{Cov}(\ln \frac{W}{\theta}, s) < -\text{Cov}(\ln \frac{W^*}{\theta^*}, s) \]
Using the labor supply function $W = \eta PC^p$ and money demand function $M = \chi PC^p$, we can rewrite Equation (C.4.30) as

$$\text{Cov}(s, m - u) < -\text{Cov}(s, m^* - u^*)$$  \hspace{1cm} (C.4.31)

Given the monetary policy rules (3.3.7) and (3.3.8), (C.4.31) becomes:

$$\text{Cov}[(a_1 - b_1)u + (a_2 - b_2)u^*, (a_1 - 1)u + a_2 u^*] < -\text{Cov}[(a_1 - b_1)u + (a_2 - b_2)u^*, b_1 u + (b_2 - 1)u^*]$$  \hspace{1cm} (C.4.32)

Using the property that $u$ and $u^*$ are i.i.d, (C.4.32) could be rewritten as:

$$(a_1 - b_1)(a_1 + b_1 - 1)\sigma_u^2 + (a_2 - b_2)(a_2 + b_2 - 1)\sigma_{u^*}^2 < 0$$  \hspace{1cm} (C.4.33)

From the optimal monetary rules listed in Table C.1, we have $a_1 + a_2 = 1$, $b_1 + b_2 = 1$, $n \leq a_1 < 1$, $b_1 \leq 0$, $0 < a_2 \leq (1 - n)$ and $b_2 \geq 1$, this implies

$$(a_1 - b_1)(a_1 + b_1 - 1) = (a_2 - b_2)(a_2 + b_2 - 1) < 0$$  \hspace{1cm} (C.4.34)

Thus, we show that the two conditions (C.4.28) and (C.4.29) hold.

**Step 2** We need to show there exists a positive menu cost $\delta$ (or $Z$) such that Equations (C.4.28) and (C.4.29) hold. Defining the left-hand side term in Equations (C.4.28) and (C.4.29) as $Z_1$ and $Z_2$, respectively, and using Equation (C.4.28) and the properties of optimal policy parameters ($a$'s and $b$'s), we have

$$Z_1 = \frac{1}{2}[(a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_{u^*}^2] - [(a_1 - b_1)(a_1 - 1)\sigma_u^2 + (a_2 - b_2)a_2\sigma_{u^*}^2]
\quad + \frac{(a_1 - b_1)(a_1 - 1)\sigma_u^2 + (a_2 - b_2)(a_2 - b_2 - 1)\sigma_{u^*}^2}{2} > 0$$  \hspace{1cm} (C.4.35)

From the proof in Step 1, we have $Z_2 < Z_1$. Therefore, there must exist a positive $Z \in (\max\{0, Z_2\}, Z_1)$ such that both Equations (C.4.28) and (C.4.29) hold. Q.E.D.
Table C.1: The optimal monetary rule in Nash game

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\rho &gt; 1$</th>
<th>$\rho = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$\frac{\rho n + (1-n)\delta_1 - n(\rho-1)\delta_2}{\rho n + (1-n)\delta_2}$</td>
<td>$n$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$\frac{\rho n + (1-n)\delta_3}{\rho n + (1-n)\delta_3}$</td>
<td>$1-n$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$\frac{\rho n + (1-n)\delta_2}{\rho n + (1-n)\delta_2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$\frac{\rho n + (1-n)\delta_2 + n(\rho-1)\delta_3}{\rho n + (1-n)\delta_2}$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Where $\delta = \lambda - n(1-\rho)\{1 + (1-n)\rho(1-n) + n\}$

$\delta_1 = n \{\lambda - (1-\rho)[n + (1-n)\rho(1-n) + n]\}$

$\delta_2 = n[(1-n)(1-\rho)^2]$  

$\delta_3 = (1-n)\{\lambda - n(1-\rho)\}$

and $\delta_1 + \delta_3 = \delta$

Table C.2: The weight on exchange rate volatility in monetary policy decision

($\rho > 1$, $n = 0.5$, $\sigma_u^2 = \sigma_u^2 = 0.0004$, $\lambda = 1.1$ )

<table>
<thead>
<tr>
<th>Weight</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 2$</th>
<th>$\rho = 3$</th>
<th>$\rho = 4$</th>
<th>$\rho = 6$</th>
<th>$\rho = 8$</th>
<th>$\rho = 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>-0.04</td>
<td>-0.114</td>
<td>-0.341</td>
<td>-0.683</td>
<td>-1.710</td>
<td>-3.2</td>
<td>-12.190</td>
</tr>
<tr>
<td>Foreign</td>
<td>-0.23</td>
<td>-0.364</td>
<td>-0.716</td>
<td>-1.182</td>
<td>-2.456</td>
<td>-4.184</td>
<td>-13.825</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.185</td>
<td>0.317</td>
<td>0.476</td>
<td>0.579</td>
<td>0.696</td>
<td>0.765</td>
<td>0.882</td>
</tr>
</tbody>
</table>

a. The weight on exchange rate volatility in monetary policy decision for home and foreign monetary authorities are measured by

$$\frac{\partial EU}{\partial \sigma_2^2} = -\frac{(1-n)}{\lambda} \Gamma \frac{\partial X^*}{\partial \sigma_2^2} < 0, \quad \frac{\partial EU^*}{\partial \sigma_2^2} = \frac{1}{1-\rho} - \frac{(1-n)}{\lambda} \frac{\partial X^*}{\partial \sigma_2^2} < 0$$

where $\frac{\partial X^*}{\partial \sigma_2^2} > 0$, and $\Gamma$ is endogenously determined by equation 2.2.
Figure C.1: The reaction curves for $a_1$, $b_1$ ($\rho = 4$, $n = 0.5$ and $\lambda = 6$)

Figure C.2: The reaction curves for $a_2$, $b_2$ ($\rho = 4$, $n = 0.5$ and $\lambda = 6$)