Dynamic Capital Structure and Partial Adjustment Frameworks

by

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Abstract

I compare ordinary least squares and the Kalman filter as possible estimation tools for partial adjustment models of dynamic capital structure. I find that the latter is more suited, as it can handle forms of endogeneity and measurement error. While the firms in my sample adjust their capital structure at different rates, I do not find any evidence that this adjustment is made according to a time-varying target. Furthermore, I construct a simple theoretical dynamic model with fixed capital adjustment costs. By investigating time series generated by this model, I conclude that partial adjustment frameworks may be inadequate for estimation. When fixed costs are present, estimates of a firm’s speed of capital adjustment are biased.
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Chapter 1

Introduction

1.1 Overview of Results

Studies like Flannery [4] and Roberts [12], use variants of partial adjustment models to conclude that firms do, in fact, adjust to some target leverage over time. Using industry leverage, book-to-market equity, profitability and collateral as explanatory variables, I do not find that the firms in my sample adjust to a dynamic target that is determined by these factors. I cannot reject the hypothesis that the firms adjust their leverage to a time-invariant constant.

Via a comparison of estimates produced by ordinary least squares on the one hand and the Kalman filter on the other hand my study also shows that both endogeneity and measurement error are likely present in empirical studies of leverage and may obfuscate results. These issues have been previously raised by Roberts [12] and Titman and Wessels [14].

Partial adjustment models may be the wrong framework altogether when the firm incurs a fixed cost whenever it adjusts its capital structure. I develop a simple dynamic tradeoff model with fixed adjustment costs, solve for the optimal policies and then simulate leverage time series. I demonstrate that adjustments become less and less frequent as adjustment costs increase. Using my simulated time series, I find that the estimated speed of capital adjustment 1) exhibits a downward bias and 2) appears to paradoxically increase with rising adjustment costs. These findings emphasize that partial adjustment models likely fare poorly when factors like fixed costs prevent a firm from freely adjusting its debt-equity mix every time period. Thus, knowing a firm's adjustment cost structure is important, and should be incorporated into empirical work.
1.2 Related Literature

The work by Modigliani and Miller [10] provides an early theoretical framework for analyzing capital structure, though under a set of somewhat restrictive assumptions. One of their main findings is that firm value is independent of capital structure in a tax-free world. In the presence of taxes, maximum firm value is achieved under a 100 percent debt financing policy, as this creates tax shields for the firm. In reality, firms financed exclusively by debt are rarely observed.

The static trade-off theory advances bankruptcy costs as one reason why firms would not want to rely on an all-debt capital structure. In this framework, costs of financial distress deter the firm from using too much debt, which results in the optimal capital structure being a mix of debt and equity. Static tradeoff also implies that a target leverage level exists.

Pointing out the limitations of the static tradeoff model, Hennessy and Whited [6] formulate an elaborate dynamic tradeoff model. They show that under their setup there is no target leverage.

There exists a number of empirical investigations into the determinants of leverage. Frank and Goyal [5] find factors which are relevant in explaining leverage. They list industry leverage, the book-to-market ratio, collateral, profitability, dividends and size as their most important variables for explaining capital structure. On the other hand, Titman and Wessels [14] call into question that leverage is related to explanatory factors. Using a factor-analytic approach to take into account the latency of explanatory variables, they find no evidence that expected growth, non-debt tax shields, volatility or collateral have any influence on capital structure.

The issue of variable latency that Titman and Wessels raise is an important one, as correlated measurement errors of the independent and dependent variables may cause spurious correlation. In this spirit, Roberts [12] presents a dynamic model of leverage, in which firms adjust to a time-varying target. In his model, Roberts uses the Kalman filter to recover the unobserved state variables and he allows for correlated errors between leverage and its determinants. Roberts finds that firms do adjust to a moving target, which is determined by the marginal tax rate, bankruptcy probability, firm size, investment opportunities and industry leverage. Across industries, the significance of the respective coefficients varies to a great extent, which implies that his specification of target leverage is highly industry-dependent. Roberts also confirms the presence of both measurement errors and endogeneity in his model specification.

Leary and Roberts [9] point out the issue of capital adjustment costs. In an interesting application of duration analysis they find that firms' financing behaviour is in
fact consistent with the presence of adjustment costs.

Baker and Wurgler [1] present leverage as resulting from firms timing the market. They find that low leverage firms raised funds when their market value was high, while high leverage firms raised funds when their market value was low. They also document persistence in this effect, which suggests that firms, after having timed the market, adjust their capital structure only very gradually.
Chapter 2

Empirical and Theoretical Models

2.1 Empirical Model

The empirical model setup in this paper is related to that in Roberts [12]. However, Roberts develops his model in continuous time, while I will work in discrete time. Roberts' formulation allows him to derive a closed-form solution of the leverage process, which he then uses as the basis for an empirical state-space specification. While estimating continuous-time parameters from discrete time data is certainly possible, its structure is rather involved. Furthermore, the economic basis for specifying a continuous-time framework is unclear. We can certainly picture leverage evolving in continuous time - especially for frequently traded firms. Each time the value of market equity changes, so does the leverage value (assuming it is not based on book equity values). However, it seems somewhat of a stretch to think of firms continuously adjusting their capital structure, at the very least because of transaction costs. Thus, despite the mathematical appeal of a continuous-time model of leverage, I work within the more tractable discrete time setting.

While my study contains a much smaller sample, I fit parameters for each firm rather than for an industry as a whole. Instead of analyzing entire sectors' speeds of mean reversion, I investigate adjustment speeds of individual firms. I construct target leverage with the four factors that Frank and Goyal find to be the most important determinants of a company's leverage. These factors are industry leverage, book-to-market, profitability and collateral, which together capture 28 percent of the variation in leverage in Frank and Goyal's study. The construction of all variables is outlined in Appendix B.
2.1.1 Constant Mean Model

Model Setup

The first model stipulates that leverage returns to a time-invariant target, which for simplicity, I set to zero. Actual leverage is transformed to a mean-zero process with the following AR(1) dynamics:

\[
\begin{align*}
    l_t^* &= \varphi [l_{t-1}^* - \mu^*] + \eta_t \\
    &= (\varphi + 1)l_{t-1}^* - \varphi \mu^* + \eta_t \\
    &= \kappa l_{t-1}^* + \eta_t
\end{align*}
\]

where * indicates a latent variable measured with error, \(l_t^*\) is demeaned actual leverage, \(\mu^*\) is targeted leverage (here a constant 0), and \(\varphi\) and \(\kappa\) are the parameters measuring the speed of leverage adjustment. At the end of every period, the firm examines how far it is from its desired leverage level, and adjusts by an amount \(\varphi [l_{t-1}^* - \mu^*]\). Thus, in (2.1), \(\kappa = 0\) signifies full adjustment to the target from one period to the next, and \(\kappa = 1\) signifies no adjustment at all, which corresponds to unit root behaviour of \(l^*\). Since negative values of \(\kappa\) imply an overcorrection beyond the target, we can expect the estimate of \(\kappa\), \(\hat{\kappa} \geq 0\).

The AR(1) model of equation (2.1) is a convenient starting point for taking a first look at the time-series properties of leverage. However, while tractable, this approach is also naïve in that it ignores any cross-sectional effects beyond a time-invariant mean on leverage. Furthermore, following the work of Fischer, Heinkel and Zechner [3], rather than readjusting their capital structure every period, firms may instead decide to adjust only if a certain threshold has been crossed. This issue will not be dealt with in the empirical part of this paper; the constant mean assumption will be relaxed, however.

All observed variables are assumed to be measured with error. Measurement error can stem from accounting data being an imperfect reflection of true market values, from mixing book and market values, and from inaccurate data themselves. In the presence of measurement error, the Kalman filter provides a way to recover the state of a system based upon certain observed variables that are measured with error. Going forward, I assume that all error terms are distributed as Gaussian white noise, which results in a "nice" likelihood function. It is well possible that this assumption is violated in the data. For instance, accounting depreciation may systematically under- or overstate true economic depreciation, depending on which accounting method is chosen. The Kalman filter can still be used in such instances via quasi-maximum likelihood estimation.

The state-space representation of an AR(1) process with a constant target leverage
of 0 comprises the following state and measurement equations:

\[ l_t^* = \kappa l_{t-1}^* + \eta_t \]  
\[ y_t = l_t^* + \varepsilon_t \]

(2.2)  
(2.3)

with \( E(\eta') = \sigma_I^2 \) and \( E(\varepsilon') = \sigma_Y^2 \). The Kalman recursions, as well as the likelihood function, are given in general form in Appendix C.

In order to examine the time series behaviour described above, I choose five companies in different industry segments, namely Coca Cola, DuPont, J.C. Penney, Kellogg and Southwest Airlines. The companies are chosen on the basis of being in different industries and not having any missing observations over a long enough time period. In an extension where a larger panel of firms is considered, this approach would obviously no longer be feasible. In this case, the Kalman Filter can be adapted to handle missing observations, for instance via the EM algorithm of Dempster [2]. While complete data over a long enough time horizon is certainly desirable, restricting oneself to companies fitting this criterion potentially induces a source of sample selection bias.

**Estimation Results**

Table 1 (Section A.1) presents a comparison of OLS estimates of the AR(1) process and the maximum likelihood estimates using the Kalman filter. The estimates of the adjustment speed parameter \( \kappa \) are quite large, ranging from 0.84 for J.C. Penney to 0.94 for Kellogg. This suggests that adjustment to a mean-zero target is rather slow; the corresponding half-lives range from 3.94 quarters to 11.52 quarters. Where the filter detects measurement error, its variance \( \sigma_Y^2 \) is generally small in absolute terms, and also relative to the state noise. With the exception of Kellogg, measurement error is insignificant.

The Kalman filter estimates of \( \kappa \) are all slightly bigger than under OLS, which could be explained by the presence of measurement error. An AR(1) model is a classical example of where measurement error causes attenuation bias, i.e. the slope coefficient estimate is biased toward zero. However, since measurement error is small (and not even present in the Southwest Airlines case), the small differences in estimates may be partly attributed to the effect of initializing the Kalman filter at the unconditional mean for the state variable projections. Using the Kalman filter, we would not reject a unit root hypothesis of no adjustment at all for Coke, Kellogg and Southwest Airlines.
2.1.2 Time-Varying Industry Mean Model

Model Setup

In this section, the assumption of a constant level of target leverage is relaxed. The firm is now allowed to adjust its capital structure according to a time-varying target. This target is given by industry leverage, which Frank and Goyal [5] find to be the single most important determinant of a company's capital structure. The underlying assumption of using only industry leverage as the target is that everything that could possibly affect a firm's desired leverage level is reflected by the industry leverage. Thus, firm-specific factors are excluded under this specification. Keeping the notation from the previous section, the state space framework is now given by:

\[
\begin{pmatrix}
I_t^* \\
\mu_t^*
\end{pmatrix} = \begin{pmatrix}
\kappa & 1 - \kappa \\
0 & \phi
\end{pmatrix} \begin{pmatrix}
I_{t-1}^* \\
\mu_{t-1}^*
\end{pmatrix} + \eta_t
\]

(2.4)

\[
y_t = \begin{pmatrix}
I_t^* \\
\mu_t^*
\end{pmatrix} + \varepsilon_t
\]

(2.5)

where

\[
Q = E(\eta_t') = \begin{pmatrix}
\sigma_{I}^2 & \sigma_{I\mu} \\
\sigma_{\mu I} & \sigma_{\mu}^2
\end{pmatrix} \quad \text{and} \quad R = E(\varepsilon_t') = \begin{pmatrix}
\sigma_{\varepsilon}^2 & 0 \\
0 & \sigma_{\varepsilon}^2
\end{pmatrix}
\]

I assume that target leverage \(\mu_t^*\) (here with industry leverage as its only determinant) also follows an AR(1) process. The measurement error matrix \(R\) is diagonal. To consider the possibility of endogeneity, the state noise matrix \(Q\) contains (potentially non-zero) covariance terms. This accounts for endogeneity that is caused by omitting relevant variables, for instance. It does not address the issue of simultaneity that is inherent in any estimation involving leverage. In some cross-sectional specifications it is not clear whether changes in the explanatory variables cause a change in leverage, or whether a change in leverage might be causing changes in the explanatory variables. The dynamic structure my model partially alleviates this problem, however, since the explanatory variables are lagged. The interpretation that changes in current leverage cause changes in last period's explanatory variables is illogical.

I choose 4-digit SIC code to compute industry leverage figures. The company under examination is excluded from the computation. This is because at the 4-digit SIC level, industry leverage is based on a comparatively small sample, so that including the company in question could lead to biased results. Furthermore, it makes intuitive sense to exclude the company in question, as it will adjust its leverage according to what all other companies (but not itself) in the industry is doing.
A company is included in industry leverage if all COMPUSTAT items are available for a quarter. This leads to situations where companies could be included during one year and excluded during the next year. Due to the small number of companies in the 4-digit sample, all companies with available data are included, even if they cease to exist during the time frame of this investigation.

Estimation Results

Table 2 (Section A.2) presents estimation results for both standard OLS and the Kalman filter. Under OLS, the estimated reversion parameter $\hat{\kappa}$ is greater than under the constant mean specification for all five companies. The speed of adjustment to a time-varying industry target is slower than the adjustment to a constant mean; firm leverage appears to be quite sticky. The OLS autocorrelation coefficient of industry leverage, $\hat{\phi}$, ranges from a low of 0.70 in the case of JCP (SIC 5311: Department Stores) to a high of 0.95 for Kellogg (SIC 2040: Grain mill products).

By and large, the Kalman filter estimates of both $\hat{\kappa}$ and $\hat{\phi}$ do not differ by much from the OLS estimates. A notable exception is the industry leverage AR(1) estimate for JCP, $\hat{\phi}$, which increases from 0.70 (OLS) to 0.91 (Kalman Filter). Here, the filter picks up significant measurement error, which causes the attenuation bias in the coefficient. The OLS variance estimate, $\hat{\sigma}_\mu^2 = 0.002$ is decomposed into a state noise variance of $\hat{\sigma}_\mu^2 = 0.0007$ (standard error of 0.0002) and a measurement noise variance of $\hat{\sigma}_{\gamma\mu}^2 = 0.0006$ (standard error of 0.0002), both of which are statistically significant.

The Kalman filter estimates significant state noise variances for all five companies for both company leverage and target leverage. For company leverage, significant measurement error as measured by $\hat{\sigma}_{\gamma\mu}^2$ is only found for Kellogg's, while for industry leverage, it is only found in the aforementioned case of JCP. Nonetheless, significant state noise correlation exists in almost all cases, with the exception of COKE. It is highest in the case of JCP with a correlation coefficient of 0.56. This suggests that endogeneity is indeed a problem under this specification. Hence, using OLS for estimating (2.4) without properly accounting for endogenous explanatory variables is problematic.

In summary, from a statistical point of view, there are several interesting results when comparing OLS estimation to a state space specification that is estimated via the Kalman filter. The latter estimation technique clearly performs better in addressing the issue of endogeneity. From an economic perspective, adjusting to an industry target is sluggish; in the case of three companies (namely COKE, K, LUV), unit root behaviour of leverage cannot be ruled out, at least when using the Kalman filter.
2.1.3 Time-Varying Target Model

Model Setup

The previous section is a special case of a fairly general target leverage specification. While adjusting to an industry target is certainly sensible, Frank and Goyal [5] find that in the cross-section, a number of factors are important determinants of leverage. In their study, they report industry leverage, book-to-market equity, amount of collateral and profitability to be the most important ones. The following state-space framework incorporates these variables into the specification of a time-varying target.

To estimate the target leverage coefficients, I assume that a single set of 'universal' coefficients determines a dynamic target in all industries. Every period, all five companies adjust towards their respective target at individual speeds. In order to make the numerical optimization of the estimation more tractable, a simplified error structure is assumed, namely a diagonal state noise matrix, and no measurement noise. The previous section showed that correlated errors can indeed be a problem, so in a more complete treatment, the aforementioned constraints on state and measurement noise should be relaxed to account for endogeneity and measurement error. Using the Kalman recursions for estimation essentially amounts to OLS by maximum likelihood. The state-space framework comprises the following equations:

\[
\begin{align*}
\mathbf{y}_t &= \begin{pmatrix} l_t^* \\ \mu_t^* \end{pmatrix} = \begin{pmatrix} \kappa & 1 - \kappa \\ 0 & \phi \end{pmatrix} \begin{pmatrix} l_{t-1}^* \\ \mu_{t-1}^* \end{pmatrix} + \eta_t \\
\mu_t^* &= \beta_1 x_{1t}^* + \beta_2 x_{2t}^* + \beta_3 x_{3t}^* + \beta_4 x_{4t}^*
\end{align*}
\] (2.6) (2.7)

The (demeaned) determinants of the target leverage \( \mu^* \) are given by:

\[
\begin{align*}
x_1^* &= \text{industry leverage} \\
x_2^* &= \text{book-to-market} \\
x_3^* &= \text{profitability} \\
x_4^* &= \text{collateral}
\end{align*}
\]

The following system incorporates the target leverage determinants into the state-space specification. As before, target leverage determinants follow an AR(1) process.
\[
\begin{pmatrix}
  l^*_i \\
  x_{1t} \\
  x_{2t} \\
  x_{3t} \\
  x_{4t}
\end{pmatrix}
= \begin{pmatrix}
  \kappa \\
  0 \\
  0 \\
  0 \\
  0
\end{pmatrix}
\begin{pmatrix}
  (1 - \kappa)\beta_1 \\
  (1 - \kappa)\beta_2 \\
  (1 - \kappa)\beta_3 \\
  (1 - \kappa)\beta_4 \\
  \phi_1 \\
  \phi_2 \\
  \phi_3 \\
  \phi_4
\end{pmatrix}
\begin{pmatrix}
  l^*_{t-1} \\
  x_{1t-1} \\
  x_{2t-1} \\
  x_{3t-1} \\
  x_{4t-1}
\end{pmatrix}
+ \eta_t \quad (2.8)
\]

with state noise matrix

\[
Q = E(\eta\eta') = \begin{pmatrix}
  \sigma_1^2 & 0 & 0 & 0 & 0 \\
  0 & \sigma_2^2 & 0 & 0 & 0 \\
  0 & 0 & \sigma_3^2 & 0 & 0 \\
  0 & 0 & 0 & \sigma_4^2 & 0 \\
  0 & 0 & 0 & 0 & \sigma_5^2
\end{pmatrix} \quad (2.9)
\]

Let the vector \( \beta' \) denote the target leverage parameters and the vector \( \alpha_i = (\kappa, \phi, \text{vec}(Q')) \) denote the coefficients for firm \( i \). Maximizing the joint likelihood is equivalent to maximizing the sum of log-likelihoods:

\[
\sum_{i=1}^{5} LL_i = \sum_{i=1}^{5} LL(\alpha_i, \beta | y_i) \quad (2.10)
\]

Estimation Results

Table 3 (Section A.3) contains the parameter estimates of this specification. The values of the reversion parameters \( \kappa \) dropped notably for DuPont and JCP, to 0.80 and 0.82, respectively. These firms now seem to adjust their capital structure much more quickly. For the other firms, the values of \( \kappa \) do not vary much as compared with the previous specification in Section 2.1.2. The AR(1) coefficients of the explanatory variables exhibit a wide range; some processes are stationary while others are not. There is also considerable variation in the values of the autoregression coefficient \( \hat{\phi} \) for the same explanatory factors across firms, so that not much can be said regarding the behaviour of these processes.

The modelled target leverage structure does not fare well. The coefficient on collateral, \( \hat{\beta}_4 \) is the only one that is marginally significant. None of the other coefficients are individually significant at any conventional significance level; a likelihood ratio test for joint significance produces a p-value of 0.565. Thus, there is no statistical evidence that these five firms adjust to a dynamic target that is comprised of industry leverage,
book-to-market, profitability and collateral. It is impossible to reject the constant-mean hypothesis; Frank and Goyal’s factors do not play any role in determining these companies’ capital structures.

Nonetheless, it is worthwhile to take a closer look at whether the target leverage coefficients’ signs are even sensible. The marginally significant collateral coefficient is positive (0.53, s.e. 0.32), which is in line with expectations. Collateral reduces the deadweight costs of financial distress and also decreases the probability of bankruptcy itself, which is in the interest of entrenched managers.

The estimated coefficient on industry leverage, $\beta_1$, is 0.20 (s.e. 0.21). This suggests that the industry model specification, which assumed $\beta_1 = 1$, is inaccurate. The positive sign is expected; it makes intuitive sense that firms adjust in the same direction as the industry.

The book-to-market coefficient $\hat{\beta}_2$ is also positive at 0.018 (s.e. 0.074); normally, theories predict this coefficient to be negative, which is also what Baker and Wurgler [1] find in their study. However, with a standard error four times the size of the coefficient itself, little can be said regarding the population parameter’s true sign.

Finally, the profitability coefficient, $\hat{\beta}_3$ is negative with a value of -0.93 (s.e. 1.20). While theory predicts a positive relationship, it is a well-known fact that some successful and highly profitable companies rely on little to no debt.

### 2.1.4 Summary of Empirical Results

On the basis of the previous analyses, I cannot reject the constant mean model for company leverage. Frank and Goyal’s factors did not play any vital role in explaining the companies’ capital structures. Estimations via a partial adjustment framework produce slow leverage adjustments. Measurement error and endogeneity are encountered; they obfuscate results when using OLS as the estimation method. The Kalman filter produces more reliable estimates in this case.

### 2.2 Theoretical Model of Dynamic Capital Structure

The previous section has produced sluggish adjustments to a target leverage ratio. Clearly, there are factors that may impede firms from making adjustments every period. Leary and Roberts [9] point out adjustment costs as one possible candidate. The presence of such impeding factors is apt to make estimations based on partial adjustment frameworks unreliable. In order to investigate this further, I develop a simple theoretical model in this section, which I then use to simulate a firm’s financing choices.
along with the ensuing leverage path.

Kraus and Litzenberger's [8] single period model serves as the foundation for my model; I use an infinite horizon extension with slight modifications. In a dynamic setting, firms trade off bankruptcy costs with the benefits of debt financing. The goal is to first extract the firm's optimal financial policies in scenarios with and without adjustment costs, and then to simulate leverage time series based on these policies. We can then examine these time series via the framework of the previous section in order to ascertain how well a partial adjustment model fares in the presence of costly debt policy adjustments, and if the model's predictions are still valid.

My hypothetical firm has an entitlement to a perpetually lived project, which generates cashflows (EBIT) of $X_s$ in state $s$. The firm can take on 1-period debt each period. Default occurs when cash flows in any given period are insufficient to cover next period's face value repayment $F$. In default, the firm incurs bankruptcy costs $cX_s$ ($0 < c < 1$). Debtholders receive debt repayments $y$ according to:

$$y_s = \begin{cases} 
F_s & F_s \leq X_s \\
(1-c)X_s & F_s > X_s 
\end{cases} \tag{2.11}$$

In order to limit awkward notation, I avoid explicit time subscripts. This period's state always carries the subscript $s$, and next period's state carries the subscript $j$.

The current market value of debt with face value $F_j$ maturing next period is:

$$B_s = E_s [y_j] = \frac{1}{1+r} \sum_j p_{j|s} y_j \tag{2.12}$$

where $r$ is the risk-free rate and $p_{j|s}$ denotes the transition probability of the Markov chain governing the EBIT process.

The entire debt face value repayment $F_s$ is tax-deductible. All earnings are paid out as dividends:

$$D_s = (1-\tau)(X_s - F_s) + B_s - 1_{(F_j \neq F_s)} \alpha(\Delta F) \tag{2.13}$$

$$\Delta F = F_j - F_s$$

$\tau$ is the corporate tax rate, and the last term in (2.13), $1_{(F_j \neq F_s)} \alpha(\Delta F)$, denotes the adjustment cost, which is applied whenever the firm alters the amount of debt it carries. Consistent with the earlier definition of corporate bankruptcy, dividend payments are:

$$\begin{cases} 
D_s & \text{if } (X_s - F_s) \geq 0 \\
0 & \text{if } (X_s - F_s) < 0 
\end{cases} \tag{2.14}$$
The market value of equity in state \( s \), \( Z_s \), is comprised of the current period’s dividend plus next period’s discounted expected value of equity:

\[
Z_s = \max \{0, D_s + \frac{1}{1+r} \sum_j p_{js} Z_j\}
\]  

(2.15)

The market value of the firm is thus \( V_s = B_s + Z_s \). However, managers act in the best interest of shareholders; they will maximize equity value by adjusting debt levels in each period to take advantage of the tax shield. Below is the optimality equation for the managers’ optimization problem:

\[
Z_s^* = \max_{\{F_j\}} \left\{ \max \{0, D_s + \frac{1}{1+r} \sum_j p_{js} Z_j^*\} \right\}, \quad \forall s \in S \tag{2.16}
\]

The state space \( S \) is given by \( S = \{X, F, \Delta\} \), where \( \Delta \in \{0,1\} \) denotes the bankruptcy indicator. The combination of EBIT, the face value of debt that needs to be repayed this period, and whether the firm is still liquid thus constitutes a state. \( \Delta = 1 \) is a terminating state: the firm is bankrupt and will remain so forever. Investors are myopic in the sense that they do not see the continuing value of the EBIT stream and hence will not allow debt renegotiation. The firm is shut down in the case of bankruptcy, and partial debt repayments are made according to (2.11). The state variable \( \Delta \) is endogenous in the sense that it is determined by the combination of \( X_s \) and \( F_s \) (\( \Delta = 1 \) if \( F_s > X_s \)). Hence, the dimensionality of the state space is given by the dimensionality of \( X \times F \).

### 2.2.1 Optimal Policy without Adjustment Costs

In the absence of adjustment costs, the firm is not hampered to attain its optimal capital structure each period. In solving for the optimal policy, the definition of bankruptcy coupled with the principal deductibility assumption greatly simplifies the computational complexity of the dynamic program. The firm will always choose a debt level such that \( F_s \in \{X_s\} \). The reasoning is similar to that in Kraus and Litzenberger [8] for the static case: if the firm decides on a debt level \( F_s = X_s - \varepsilon \), it can increase its debt level to \( X_s \) and thus raise its tax savings without affecting the bankruptcy penalty. The same reasoning applies if \( F_s = X_s + \varepsilon \): the firm can choose the next higher EBIT value as the debt face value without changing bankruptcy costs in the event of a default.

Figure 2.1 shows a sample EBIT path over 100 time periods. Details of the Markov chain governing this process are given in Appendix D. As outlined in the appendix, the transition probability matrix is set up to induce both mean reversion and a certain
degree of persistence in the process. Both features are reasonable for mature firms, for instance.

![Sample EBIT Path over 100 periods](image)

Figure 2.1: Sample EBIT Path over 100 periods

In every state, the firm observes an EBIT value along with the face value of last period’s debt, and chooses an optimal debt policy according to

$$ F^*_s(X_s, F_s) = \arg \max \left\{ \max \left\{ 0, D_s + \frac{1}{1 + r} \sum_j p_{j|s} Z^*_j \right\} \right\}, \ \forall s \in S \tag{2.17} $$

The optimal policy for each state as given above in (2.17) is found by iterating on (2.16) using the value iteration algorithm as in Puterman [11], for example. Additional parameter values are 0.3 for the bankruptcy cost parameter $c$, 0.1 for the risk-free rate $r$ and 0.4 for the corporate tax rate $\tau$. A graphical representation of the optimal policy for the sample realization in Figure 2.1 is shown in Figure 2.2.

The solid line represents EBIT, while the dotted line corresponds to the optimal policy for next period’s debt face value. Since nothing inhibits the firm from changing its capital structure, the state space for the no-adjustment cost case is reduced to just one dimension. The previous period’s debt face value is irrelevant: only current EBIT values drive the optimal policy, which is why the policy so closely follows the EBIT process.

The implication of the bankruptcy cost definition is evident: In the case of a bankruptcy, shareholders would bear prohibitively high indirect bankruptcy costs in the form of the foregone future EBIT stream (the proportional bankruptcy costs of 0.3
x EBIT are negligible in comparison). As a consequence, the firm chooses a policy of extreme debt conservatism. Already, and without adjustment costs, the optimal policy exhibits a certain degree of persistence. In each of the two lowest states, the firm avoids carrying any debt. The firm's managers will steer clear of a potential bankruptcy by selecting the highest possible debt face value that will still guarantee liquidity next period.

### 2.2.2 Optimal Policy with Fixed Adjustment Costs

Adjustment costs change the flavour of the analysis. Now the firm can no longer move freely between debt values, since it incurs a fixed cost penalty whenever it does so. This cost penalty is applied to increases as well as to decreases in the firm's debt holdings. Even more stickiness in the firm's debt policy ensues: the firm will change its debt levels only if the benefits of doing so exceed the costs. Leary and Roberts [9] illustrate this point by setting policies (they do not solve for them) consistent with various adjustment cost schemes and then simulating leverage time series based on these policies. They compare simulated time series in the presence of fixed costs, proportional costs, and convex costs. I extend their work by examining within the fixed cost regime how a firm's optimal financial policy responds to different adjustment costs levels. Furthermore, I take a close look at how an increase in fixed adjustment costs affects the estimated reversion parameter of a partial adjustment framework.

Figure 2.3 confirms our intuition that changes in capital structure become less frequent as the costs of doing so increase. The sample EBIT path along with the optimal
debt policy when the adjustment cost $\alpha(\Delta F) = 20$ illustrates this point.

![Figure 2.3: EBIT (solid) and Optimal Policy $F_j$ (dotted) with $\alpha(\Delta F) = 20$](image)

The optimal policy exhibits a number of stretches where, despite changes in EBIT, no changes to the debt level are made. The costs of responding to changes in EBIT outweigh the benefits, and the firm chooses to hold the same amount of debt as during the previous period. As before, the firm is highly conservative, and bankruptcy is always avoided.

In Figure 2.4, adjustment costs are higher yet with $\alpha(\Delta F) = 80$. This extreme case shows that adjustments are rare; most of the time, the firm chooses to carry no debt at all. When changes in debt level do occur, they are of a large magnitude, which is in line with substantial fixed adjustment costs. Bankruptcy is still ruled out due to its high indirect cost.

### 2.3 Implications for Partial Adjustment Models

Partial adjustment frameworks are frequently encountered in empirical work - Flannery [4], Roberts [12], and to some extent Welch [15] serve as examples. These frameworks implicitly rest on the assumption that firms adjust their capital structure every period. As can be seen above, this assumption is violated in the presence of fixed adjustment costs. Depending on the costs' magnitude, adjustments may be very infrequent. Nonetheless, when an adjustment is made, it is by the full amount. This is antipodal to the underlying assumption of partial adjustment models: frequent adjustments by
not necessarily the full amount. Thus, we can expect these models to perform poorly under a fixed adjustment cost regime.

As a first step in the analysis, let’s compare sample leverage paths for the no-adjustment cost case and the $\alpha(\Delta F) = 20$ case. The two time series are shown in Figure 2.5 below.

![Figure 2.5](image)

**Figure 2.5:** Sample Leverage with $\alpha(\Delta F) = 0$ (solid) and $\alpha(\Delta F) = 20$ (dotted)

The effects of the different policies on leverage are clearly visible: The dotted line (reflecting the case of $\alpha(\Delta F) = 20$) exhibits substantial jumps, while other segments of the graph show comparatively little variation. These observations can be attributed
to the fixed adjustment costs, which induce large jumps or periods of debt adjustment inactivity. The two effects diminish as adjustment costs are reduced, as the solid line (no adjustment costs) illustrates.

It is difficult to conjecture by visual inspection alone how these different leverage paths affect the reversion parameters $\kappa$ or $\phi$ in a partial adjustment framework. I now turn to estimating these parameters based on simulated time series. Recall that in the constant mean case, $\kappa = 1$ means no adjustment, while $\kappa = 0$ means full adjustment. The respective values for $\phi$ under a time-varying target specification are 0 and -1.

Figure 2.6 depicts the estimates of the reversion parameter of a constant mean model for various values of fixed adjustment costs. I simulate 250 leverage time series of 100 periods for every adjustment cost value in increments of 2. I then estimate equation (2.1) via least squares. The reversion parameter $\hat{\kappa}$ in Figure 2.6 is the mean of the values obtained in the 250 estimations.

Figure 2.6 depicts a paradoxical relationship between the estimated mean reversion parameter $\hat{\kappa}$ and fixed adjustment costs. Even when no adjustment costs are present and the firm adjusts its capital structure essentially every period, the estimate of $\hat{\kappa} \approx 0.75$ would suggest a rather sluggish adjustment with a half-life of about 2.4 time periods. As adjustment costs increase, the estimate of $\hat{\kappa}$ shows an overall decreasing trend. The partial adjustment framework now suggests that the firm is adapting its capital structure at a faster pace, when in fact the frequency of adjustments has decreased! Large, albeit infrequent jumps in leverage exert this downward bias on the reversion
parameter.

The result of estimating a time-varying target model of the form

\[ l_t = \phi[l_{t-1} - \mu_t] + l_{t-1} + \eta_t \]

where target leverage \( \mu_t = \beta \text{EBIT}_t \), is depicted in Figure 2.7.

![Figure 2.7: Estimation Result for \( l_t = \phi[l_{t-1} - \mu_t] + l_{t-1} + \eta_t \)](image)

This model fares better in the sense that the estimated reversion parameters \( \phi \) are lower in value. Using EBIT as a time-varying explanatory variable produces a faster estimated speed of adjustment, which is more consistent with the firm’s actual behaviour. Rising fixed costs and the ensuing choppy adjustment policy still confound the partial adjustment model, however. As in the constant-mean case, high cost values produce seemingly faster adjustments.

In the presence of fixed adjustment costs coupled with a pronounced policy of bankruptcy avoidance, partial adjustment frameworks can produce some very misleading conclusions about how a firm’s capital structure is adjusted over time. The actual value of the reversion parameter \( \phi \) is around -1 in the absence of adjustment costs. The empirical estimates are still well above this value. As adjustment costs rise and changes to capital structure become increasingly rare, \( \hat{\kappa} \) decreases, implying that the firm is actually adjusting faster, when in fact it is not. These results suggest that a careful examination of a company’s adjustment cost structure should be undertaken before using a partial adjustment model to investigate leverage dynamics.

It is well possible that the slow adjustment speeds obtained in the empirical section are in fact consistent with dynamic rebalancing under fixed adjustment costs. In the
case of JCP, for instance, the empirical estimates of $k \approx 0.8$ are not materially different from what was obtained by estimating a constant mean model from simulated data.
Chapter 3

Conclusion

On the basis of the different empirical model specifications I investigate in Part 2.1, I cannot conclude that the five firms are chasing a moving leverage target. The most extensive model presented in Section 2.1.3 does not provide enough statistical evidence to reject the null hypothesis of a constant mean model for the firms in the sample. Industry leverage, book-to-market equity, profitability, and collateral are irrelevant as explanatory variables of a leverage target. This is in line with Titman and Wessels, but contrary to Roberts, who finds that firms are in fact adjusting to an evolving target.

I compute estimates by both OLS and the Kalman filter. The case of JCP illustrates how seriously biased coefficient estimates can be when measurement error is in fact present. Endogeneity is also an issue, as evidenced by the usually significant state error correlation in the industry leverage model in Section 2.1.2. While this does not change parameter estimates by much in absolute terms, standard errors are generally reduced by accounting for correlated errors. The Kalman filter thus is the more appropriate tool for dynamic capital structure investigations.

With the help of a simple dynamic tradeoff model, I confirm that fixed adjustment costs lead to periods of inactivity in a firm's quest to optimize its capital structure. Furthermore and more importantly, in situations where a firm faces fixed adjustment costs and wants to avoid bankruptcy, the adjustment speeds obtained via a partial adjustment framework are biased downwards and increase with adjustment costs. Inferences drawn from these estimates may be meaningless.

This paper could be extended by allowing for a bigger sample in the empirical part, though this would inevitably introduce a missing data problem. The Kalman filter can deal with this issue, albeit at the cost of inflated standard errors. In addition, the time-varying target specification in 2.1.3 should be investigated via the Kalman filter, as not controlling for measurement error and endogeneity may be problematic when using maximum likelihood least squares. On the theoretical side, the following
modifications would help to establish the robustness of my results: 1) Generalization of the EBIT process, 2) Relaxation of the bankruptcy condition along with interest-only tax deductibility, and 3) Investigation of other forms of adjustment costs (e.g. proportional and convex costs).
Bibliography


Appendix A

Tables

A.1 Table 1: Constant Target Estimation

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Table A.1: Constant Target Estimation

By setting target leverage $\mu^*$ to a constant mean of 0, state and measurement equations simplify to an AR(1) process with measurement error:

\[
l_t^* = \kappa l^*_{t-1} + \eta_t
\]

\[
y_t = l_t^* + \varepsilon_t
\]

with $E(\eta^*) = \sigma^2_{\eta}$ and $E(\varepsilon^t) = \sigma^2_{\varepsilon}$. OLS and Maximum Likelihood Estimates using the Kalman Filter are given in the table above. Standard errors are in parentheses. N is the number of quarters.
Ticker Glossary:
COKE - Coca Cola
DD - DuPont (E.I.)
JCP - J.C. Penney
K - Kellogg
LUV - Southwest Airlines
### A.2 Table 2: Time-varying Industry Target Estimation

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Standard errors are in parentheses. State space framework is below. Target leverage $\mu_t^*$ is given by industry leverage.

$$
\begin{align*}
\left( \begin{array}{c}
I_t^* \\
\mu_t^*
\end{array} \right) &= \left( \begin{array}{cc}
\kappa & 1 - \kappa \\
0 & \phi
\end{array} \right) \left( \begin{array}{c}
I_{t-1}^* \\
\mu_{t-1}^*
\end{array} \right) + \eta_t \\
\quad y_t = \left( \begin{array}{c}
I_t^* \\
\mu_t^*
\end{array} \right) + \epsilon_t
\end{align*}
$$

where $Q = E(\eta\eta') = \left( \begin{array}{cc}
\sigma_\eta^2 & \sigma_{\eta\mu} \\
\sigma_{\eta\mu} & \sigma_\mu^2
\end{array} \right)$ and $R = E(\epsilon\epsilon') = \left( \begin{array}{cc}
\sigma_\epsilon^2 & 0 \\
0 & \sigma_{\epsilon\epsilon}'^2
\end{array} \right)$

$\hat{\sigma}_1^2$ and $\hat{\sigma}_\mu^2$ are state noise variances; $\hat{\sigma}_{\eta\mu}^2$ and $\hat{\sigma}_{\mu\mu}^2$ are measurement error variances. $\hat{\rho}_{1\mu}$ is the correlation between the two state noises. $\hat{\phi}$ is the AR(1) coefficient of industry leverage.

Table A.2: Time-varying Industry Target Estimation
### A.3 Table 3: Time-varying Target Estimation by ML-OLS

**Company-specific Parameters**

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**Target Leverage Coefficients**

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Standard errors are in parentheses. State space framework is given below.

\[
y_t = \begin{pmatrix} \eta_t \\ \mu_t \end{pmatrix} = \begin{pmatrix} \kappa & 1 - \kappa \\ 0 & \phi \end{pmatrix} \begin{pmatrix} l_{t-1} \\ \mu_{t-1} \end{pmatrix} + \eta_t
\]

where \( Q = E(\eta_t \eta_t') \) is diagonal with variances \( \sigma^2_1, \sigma^2_2, \sigma^2_3, \sigma^2_4 \). LR test of joint significance of target leverage coefficient gives: \( P(\chi^2(4) > 2.96) = 0.565 \).

Target leverage is \( \mu_t = \beta_1 \ast \text{industry lev} + \beta_2 \ast B/M + \beta_3 \ast \text{profitability} + \beta_4 \ast \text{collateral} \). \( \phi_1 \) to \( \phi_4 \) are the AR(1) coefficients of these variables.

Table A.3: Time-varying Target Estimation by ML-OLS
Appendix B

Construction of Variables

Quarterly COMPUSTAT data is used to construct the variables used in this paper.

\[
\text{Leverage} = \frac{\text{Book Debt Value}}{\text{Book Debt Value} + \text{Market Equity Value}} = \text{Items} \frac{45 + 51}{45 + 51 + 14 \times 61}
\]

\[
\text{Book-to-Market} = \frac{\text{Book Debt Value} + \text{Market Equity Value}}{\text{Total Assets}} = \text{Items} \frac{45 + 51 + 14 \times 61}{44}
\]

\[
\text{Profitability} = \frac{\text{Operating Income before Depreciation}}{\text{Total Assets}} = \text{Items} \frac{21}{44}
\]

\[
\text{Collateral} = \frac{\text{Total Inventories} + \text{PPE}}{\text{Total Assets}} = \text{Items} \frac{38 + 42}{44}
\]
Appendix C

Kalman Filter

The state-space framework in this paper takes the following simple form (see Hamilton [7]):

\[
\begin{align*}
\xi_{t+1} &= F\xi_t + \eta_{t+1} \\
y_t &= \xi_t + \epsilon_t
\end{align*}
\]  
(C.1)  
(C.2)

where (C.1) is the state equation with state vector \( \xi \), (C.2) is the measurement equation with observation vector \( y \), \( F \) is a matrix of coefficients, \( \eta \) and \( \epsilon \) are Gaussian white noise sequences with \( E[\eta]\eta^T] = Q \), \( E[\epsilon\epsilon^T] = R \) for \( t = \tau \), 0 otherwise.

The distribution of \( y_t|y_{t-1} \) is then given by:

\[
f_{y_t|y_{t-1}}(y_t|y_{t-1}) = (2\pi)^{-\frac{1}{2}} |P_{t|t-1} + R|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(y_t - \hat{\xi}_{t|t-1})'(P_{t|t-1} + R)^{-1}(y_t - \hat{\xi}_{t|t-1})\right\}
\]  
(C.3)

where the 1-period linear forecast \( \hat{\xi} \) and its mean squared error \( P \) are:

\[
\begin{align*}
\hat{\xi}_{t+1|t} &= F\hat{\xi}_{t|t-1} + FP_{t|t-1}(P_{t|t-1} + R)^{-1}(y_t - \hat{\xi}_{t|t-1}) \\
P_{t+1|t} &= F[P_{t|t-1} - P_{t|t-1}(P_{t|t-1} + R)^{-1}P_{t|t-1}]F' + Q
\end{align*}
\]  
(C.4)  
(C.5)

Maximum Likelihood then maximizes the log-likelihood, which can be constructed from all \( T \) observations as follows:

\[
LL = \sum_{t=1}^{T} \ln f_{y_t|y_{t-1}}(y_t|y_{t-1})
\]  
(C.6)
Appendix D

Markov Chain Specification

The possible states for the Markov chain are $S = \{0, 10, \ldots, 90\}$. The transition probability matrix $P$ introduces persistence and mean reversion into the EBIT process. The transition probabilities for the upper 5 states is the mirror image of those for the lower 5 states:

$$
P = \begin{pmatrix}
0.25 & 0.35 & 0.4 & 0 & 0 & \ldots & 0 \\
0.25 & 0.35 & 0.4 & 0 & 0 & \ldots & 0 \\
0 & 0.25 & 0.35 & 0.4 & 0 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & 0.4 & 0.35 & 0.25 & 0 \\
0 & \ldots & 0 & 0 & 0.4 & 0.35 & 0.25 \\
0 & \ldots & 0 & 0 & 0 & 0.4 & 0.35 & 0.25
\end{pmatrix}
$$

The steady-state distribution $\pi$, given by $\pi P = \pi$ s.t. $\sum_j p_{ij} | s \rangle = 1$ is:

$$
\pi = \begin{pmatrix}
0.0135 & 0.0405 & 0.0864 & 0.1383 & 0.2213 & 0.2213 & 0.1383 & 0.0864 & 0.0405 & 0.0135
\end{pmatrix}
$$

Starting states for simulations of the chain are selected according to this steady-state distribution.