THE ROLE OF EXPECTATIONS AND EXPECTATION ERRORS IN BUSINESS CYCLES

by

LILIA VLADIMIROVNA KARNIZOVA

Diploma of Higher Education, Yaroslavl State University, 1995
M.A., Central European University, 1998

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Abstract

What drives business cycles? Traditional explanations, based on policy interventions and supply side changes, have been found empirically incomplete. This thesis examines the relative contribution of changes in beliefs to business cycles from theoretical and empirical perspectives.

The first essay evaluates quantitatively the U.S. investment boom and bust of the late 1990s and early 2000s. Revisions of optimistic beliefs are commonly viewed as the key determinant of investment during this period. Yet, can this view explain consistently the joint behaviour of consumption, investment and employment? A standard real business cycle model with technology shocks performs relatively well in capturing the boom, but very poorly in explaining the bust. Beliefs about future technology are introduced into this model by enriching the economy information set and identified from a discrepancy between the model and the data. The augmented model can only account for the joint behaviour of aggregate variables when expectations about the future are more pessimistic during the boom and more optimistic during the recession.

The second essay derives and tests a necessary condition for beliefs about future technology to be an independent source of business cycles. The essay's premise is that expectations are rational, but current and past realizations of technology do not fully summarize information relevant for forecasting future technology. This premise necessarily implies long-run predictability of technology shocks. Measures of total factor productivity, orthogonal to monetary and fiscal policy shocks, provide empirical support for this premise in the U.S. post World War II period. Macroeconomic variables help to forecast future realizations of TFP growth up to two years.

The third essay asks whether changes in beliefs due to extra information about the future (news shocks) are different from changes in beliefs due to extrinsic uncertainty (sunspot shocks). The essay incorporates news into linear rational expectations models with unique and multiple equilibria. Based on general forms of solutions and numerical simulations of a New Keynesian model, the essay demonstrates that news and sunspots have distinct predictions for dynamic properties of equilibria. Since the differences can be quantitatively significant, these predictions can be used to separate news and sunspots empirically.

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Dedication

To my grandmother Margarita for her affection and firm belief in my abilities, to my parents Vladimir and Ludmila for their encouragement and to my loving husband Michael for his patience, support and understanding, I dedicate this work.

Chapter 1

Introduction

Fluctuations in economic activity are wide-spread phenomena in modern economies. Yet, up to date, there is no widely accepted explanation for their occurrence. Traditional explanations, such as unexpected changes in policy, technology or preferences have been found empirically incomplete.[1] This thesis belongs to the line of research that evaluates the importance of changes in expectations or beliefs as a potential independent source of business cycles.

The objective of the thesis is to investigate the role of changes in beliefs driven by news shocks. News shocks represent pieces of information that help to predict future changes in economic environment. The simplest examples would be announcements of changes in government regulations. Any major tax or trade reform are discussed publicly before their acceptance and implementation.

News shocks capture a temporal separation between the time agents learn about future changes in economic fundamentals and the time these changes take place. Such temporal separation is the main departure of the thesis from most of the literature. Typically, agents are assumed to learn values of fundamental shocks at the time of their realizations. It is through the intertemporal learning, effects of beliefs on current economic decisions are extended beyond their conventional effects, which work through the current and past fundamentals. In the thesis, effects of news shocks are examined from theoretical and

[1] See, for example, Cochrane [1994].

empirical perspectives.

The thesis consists of three essays, contained in Chapters 2, 3 and 4. The second and third Chapters focus on news about future technological change and their empirical implications. The stochastic nature of technological change is inherently associated with forecasting difficulties. These difficulties are perhaps even more pronounced at a time of rapid technological change, when the past may not be an entirely good predictor of the future. In other words, factors other than the history of current and past realizations of technology may affect agents' forecasts about the direction of future technological change. These factors are captured by news shocks.

Chapter 2 asks whether optimistic beliefs and their downward revisions about the direction of future technological change were at the heart of an explanation for a particular cycle: the investment boom and bust in the US in the last decade. Overly optimistic expectations and consequent revisions are commonly viewed as a key determinant of investment during this period. This Chapter examines whether this view can be captured by a standard equilibrium business cycle model.

To evaluate the role of expectation revisions formally, prospective technological change is taken as a potential source of general optimism. The analysis is based on a growth model with exogenous stochastic aggregate and investment-specific technological changes. The Chapter proposes solutions to two conceptual problems. The first problem is how to model independent effects of expectations about future technology. The second problem is how to construct empirical measures for unobserved expectations.

To evaluate the model's performance during the 1994 – 2003 period, the simulated paths for quarterly consumption, investment and hours worked are compared with the actual data. The main question is whether the model economy, subject to estimated realizations of shocks, exhibits a boom and subsequent recession of the magnitude and duration observed in the data.

Chapter 3 investigates whether the US data are consistent with news shocks about future technology in the longer time period. News shocks provide a convenient way of capturing changes in agents' beliefs about future technology. Errors in beliefs may be at the heart of explanations for some historical episodes, as argued in Chapter 2. More generally, errors in forecasting technological change may be an independent source of aggregate fluctuations. This Chapter formulates and implements an empirical test of a necessary condition for existence of news about future technology. The test exploits an empirical implication of news for conditional forecasts of exogenous technology shocks in an environment with optimizing agents and rational expectations. If technology shocks are anticipated in advance, macroeconomic variables should help to predict future technology shocks. The absence of news (or technology shock unpredictability hypothesis) is tested using statistical methods developed in the finance literature on predictability of asset returns.

Chapter 4 is more methodological. Understanding what role changes in beliefs play in generating business cycles is impossible without a precise definition of such changes. News shocks is not the only way through which changes in beliefs can play an independent role in business cycles. Sunspot shocks, also known as animal spirits and self-fulfilling prophecies, is another alternative. A number of previous studies have explored effects of sunspots and news shocks separately. The goal of that Chapter is to understand the similarities and differences between the two types of changes in beliefs. The first contribution of this Chapter is to formalize a notion of news shocks. News shocks are modelled as outcomes of learning process, based on exogenous signals. The main implication of news shocks is their effects on conditional forecasts of future fundamental shocks. It is described how to define the joint process for news and fundamental shocks that correctly represents innovations to agents' information set. The second contribution is to propose a computationally simple framework for solving linear rational expectations models with news. Solution methods are derived for models with unique and multiple equilibria. The third contribution is to compare news and sunspot shocks along several dimensions, on the basis of general forms of solutions and numerical simulations of a New Keynesian model.

Chapter 2

Investment Boom and Bust: a General Equilibrium Perspective of the US Experience from 1994 to 2003

2.1 Introduction

Strong investment growth in the US during the 1990*s* ended abruptly in the fourth quarter of 2000. Investment continued to decline for the next two years and contributed significantly to the economic recession in 2001. The reversal in investment expenditures, accompanied by a stock market crash, has led many analysts to interpret the investment bust as a result of firms' reassessment of previously optimistic expectations about future economic conditions.[1] The hypothesis that the investment bust was driven by expectation revisions has not been formally examined in the literature. The principal contribution of this Chapter is to provide a quantitative assessment of this hypothesis using a general equilibrium framework. The main question is whether a standard business cycle model, augmented with expectation revisions, can adequately capture the joint behaviour of in-

[1] An extract from a speech of William Poole, the president and chief executive officer of the Federal Reserve Bank of St. Louis, illustrates this interpretation: "Business investment last year was driven by a reassessment of long-term prospects in certain sectors, especially telecom, and by adjustment to excess capacity resulting from the prior investment boom." (Poole [2002, *p*.12])

vestment, consumption and hours worked over the 1994 – 2003 period.

To evaluate the role of expectation revisions formally, prospective technological change is taken as a potential source of general optimism. The analysis is based on a growth model with exogenous stochastic aggregate and investment-specific technological changes.[2] Prior to pursuing the quantitative analysis, two conceptual problems must be addressed. The first problem is how to model independent effects of expectations about future technology. The second problem is how to construct empirical measures for unobserved expectations. A further contribution of this Chapter is to propose solutions to these problems.

To address the first problem, expectation formation is modelled along the lines of Beaudry and Portier [2004*a*]. The modelling approach builds upon the idea that at a time of rapid technological change, the past may not be an entirely good predictor of the future. To formalize this idea, households and firms are assumed to receive commonly observed exogenous signals. These signals are believed to be correlated with the unknown future states of technology. Specifically, technological change is characterized as a vector stochastic process driven by zero mean independent technology impulses. The signals are believed to contain information about future realizations of these impulses.[3] Since the analysis focuses on a particular historical episode, it is not necessary to impose long-run rationality.[4] In this sense, the modelling approach can accommodate "irrational exuberance" as well as perfectly rational behaviour. The Chapter demonstrates that this framework extends the effects of expectations about future technology on current economic decisions beyond their conventional effects, which work through the current and past changes in technology. The additional role of expectations is introduced without relying on strategic complementarity or multiple equilibria.[5] Furthermore, the focus of

[2] The benchmark model is a variant of the model described by Fisher [2003] and Greenwood, Hercowitz and Krusell [2000]. Investment-specific change is introduced to capture effects of falling relative investment prices on capital accumulation.

[3] Technically, the information assumption implies that conditional expectations about future technology impulses are not necessarily equal to unconditional expectations. In contrast, the conventional approach always equates the two.

[4] That is, it is not necessary for the signals to be correlated with future states of technology.

[5] In the models with sunspots, strategic complementarity can lead to self-fulfilling expectations about future paths of the economy. See Benhabib and Farmer [1994] or Farmer and Guo [1994].

this Chapter is on expectations about future economic fundamentals, rather than about nonfundamental, or sunspot, shocks.

To address the second problem, a novel method is devised to extract expectations from the data. The method exploits equilibrium cross-equation restrictions of the model and imposes minimal assumptions about the process of expectation formation. In this model, two variables summarize all effects of expected future technology impulses. The first variable, *aggregate technology prospects*, is the discounted sum of expected future realizations of impulses to aggregate technological change. The second variable, *investment technology prospects,* is the discounted sum of expected future realizations of impulses to investment-specific technological change. By construction, the technology prospects capture effects of expectations, which are independent of the current and past changes in technology. This Chapter establishes that technology prospects can be treated as unobserved state variables. The estimation procedure is designed to uncover these unobserved states. Two elements are central for identification. The first element is the independence of impact coefficients of equilibrium decision rules from the exact nature of the expectation process. These coefficients provide identification restrictions. The second element is the existence of empirical measures for realized technological change. These measures enable the separation of the effects of expectations, which arise because of the modified information structure, from the conventional effects, which operate through changes in the current and past state of technology. If at most one type of technology prospect is allowed to be present in the data, estimates of technology prospect are identified uniquely by use of generalized least squares. If both aggregate and investment prospects are allowed to be present in the data, the corresponding series can only be identified jointly.[6] In this case, the estimated series can be interpreted as a summary statistic characterizing the overall beliefs about prospective technology change.

To evaluate the model's performance during the 1994 – 2003 period, the simulated

[6]The inability to separate aggregate and investment prospects occurs because of the linear dependence of impact coefficients in the decision rules.

paths for quarterly consumption, investment and hours worked are compared with the actual data. The main question is whether the model economy, subject to estimated realizations of shocks, exhibits a boom and subsequent recession of the magnitude and duration observed in the data.[7]

The overall conclusion from the quantitative analysis is that the intuitively plausible story of overinvestment due to optimistic beliefs is surprisingly difficult to reconcile with the observed macroeconomic variables in the context of the standard business cycle model. This conclusion is supported with several findings. First, the benchmark model, which relies on technological change only, explains part of the economic boom in the 1990s. However, technological change appears to play a very limited role in accounting for the investment bust. The simulated economy captures the slowdown in 2000, but predicts positive investment growth in 2001-2002. Second, the model augmented with expectations about future technology impulses captures both the boom and the bust in investment. The simulated investment series explains eighty two percent of the stochastic variation in the investment growth rates. However, to rationalize the joint behaviour of investment, consumption and hours, the model must rely on revisions of expectations from being more pessimistic during the boom towards being more optimistic during the recession.[8] Such an interpretation is at odds with the evidence discussed in Section 2.2 of this Chapter.

The direction of the estimated expectation changes can be attributed to substitution mechanisms embedded in the standard model. This model predicts a fall in investment in anticipation of future improvement in aggregate or investment-specific technology. The intratemporal substitution possibilities imply that consumption and investment necessarily move in the opposite directions in response to expected technological change.[9] The interaction between wealth and substitution effects determines whether investment booms

[7]The event study approach to analyzing business cycles has been introduced by Hansen and Prescott [1993]. These authors examined the role of technology shocks in explaining the 1990-1991 US recession. In contrast to this paper, the model of Hansen and Prescott relied only on the conventional effects of expectations about future technology, operated through changes in the current and past states of technology.

[8]This result holds for both aggregate and investment technology prospects.

[9]This property has also been pointed out by Cochrane [1994] and Beaudry and Portier [2004*a*].

occur in anticipation of technological improvement. Expected higher future return on capital creates incentives to increase investment and decrease consumption. At the same time, expected higher wealth creates incentives to increase consumption and decrease investment. When the conventional parameters are considered, the wealth effect dominates, and investment falls. Thus, the investment boom in the standard model can be generated only by expected adverse technological change. The effects of expectations about aggregate technological change can be reversed by changing the intratemporal elasticity of substitution or by introducing capital adjustment costs. However, these modifications are not sufficient for the standard model augmented with expectation revisions to account for the joint behaviour of investment, consumption, and employment during the $1994 - 2003$ episode. Based on estimated series for technology and expectations, the model predicts 2001-2002 growth rates of consumption that are more than twice as high as those observed in the data.

The rest of the Chapter is structured as follows. Section 2.2 gives a descriptive account of the investment boom and bust. Section 2.3 introduces the model and evaluates the role of aggregate and investment-specific technological change. Section 2.4 describes modifications of the information assumption, outlines the estimation methodology and investigates the role of expectation revisions. Section 2.5 explores changes in the intertemporal elasticity of substitution and capital adjustment costs. Section 2.6 discusses why an open economy framework would unlikely help to resolve puzzling characteristics of the US experience in 1994-2003. Section 2.7 concludes.

2.2 A Story of Overinvestment

This section provides informal support for the hypothesis of expectation revisions being the key determinant of the investment bust. The evidence is based on extracts from government publications, industry case studies and stock market prices.

The boom and bust in investment is illustrated in Figure 2.1. The figure plots per capita real private fixed investment, expressed as a percentage change from the previous year. The shaded area corresponds to the official dates of the 2001 recession. The measure of investment includes investment in nonresidential structures and equipment as well as residential investment. The figure also plots total private consumer spending. In contrast to investment outlays, the growth rate of consumption remained surprisingly strong during the economic downturn.[10] This pattern of investment and consumption growth during the 2001 recession was unusual from the perspective of the post World War II US business cycle. Typically, both consumption and investment decline during the recession, and a fall in investment precedes a fall in output.

The recession itself has been acknowledged to be unusual. For example, *The Economist* [2001, *p*.26] writes:

> In contrast to the post-war norm the expansion was not "murdered" by the Federal Reserve. The contraction started with an investment bust, as firms that had radically overinvested during the boom years of the late 1990s suddenly cut back.[11]

The downturn in investment has often been interpreted as resulting from a correction of investment decisions made during the preceding boom. For example, according to Governor Bernanke [2003, *p*.5],

> As we can see, in retrospect at least, the year 2000 was one of re-evaluation, particularly for high-tech investment. Though the evidence is strong that high-tech investments have greatly enhanced technology in the economy, by 2000 many managers had apparently become concerned that the long-term profit potential of their investments in computers and communications equipment was smaller than they had expected ... and sometimes the productivity enhancements were less than anticipated.

A downward revision of previously optimistic expectations about future economic conditions has been proposed as one of the leading explanations. Three extracts from the official government publications illustrate this view:

[10] The series are from the Bureau of Economic Analysis (BEA). They are expressed in 2000 chained dollars and converted into per capita terms.

[11] See also Krugman [2002], Stock and Watson [2003], and Bernanke [2003].

> Overly optimistic expectations of future growth in demand, which were reflected in inflated stock prices, led businesses to invest in new plant and equipment at levels that appear excessive in hindsight.
>
> (*The Congressional Budget Office* [2002, *p*.24])

> Some businesses, especially in the information and communications technology sector, may have overestimated the potential of the "New Economy" and therefore overinvested in productive capacity.
>
> (*The Council of Economic Advisers* [2002, *p*.40])

> ... the magnitude by which these categories had increased in preceding years, together with abruptness of their downturn, suggests that firms may have been too optimistic about the immediate profitability of some types of high-tech capital; as these expectations were revised, business viewed their previous investment as more than sufficient to meet anticipated demand.
>
> (*Board of Governors of the Federal Reserve System* [2001, *p*.15])

The hypothesis of expectation revisions being the key determinant of the investment bust has plausible foundations. Empirical evidence points to the existence of excess capacity, at least in the telecom sector. By one estimate, 97 percent of fiber-optic capacity was unutilized in 2002 (Gordon [2003]). In addition, the econometric analysis of industry data reveals that industries that invested more during the boom in the late 1990s also cut their capital expenditures more during the bust in 2000-2002.[12]

The Internet and the 1996 Telecommunications Act are two likely contributors to the general optimism about the future. The Internet gave a rise to the notion of the "New Economy", which referred to new ways of organizing and conducting business.[13] The proliferation of dotcoms and e-commerce was "touted as a new industrial revolution" (Gordon

[12] McCarthy [2003].

[13] "The Internet has become a powerful symbol of society's expectations about the future - a future of fast-moving, disruptive technology that is shifting the terrain not only in business, but also in politics and culture... Because it is such a low-cost communications technology, the Internet holds the promise of drastically reducing transactions costs." Lohr [1999]

[2003]). However, expectations of high consumer demand did not materialize in the case of on-line trading, and failed dotcoms have become a legacy. Couper, Hejkal and Wolman [2003] provide support for the idea that the 1996 Telecommunications Act may have played a leading role in the telecom boom and bust. Designed to promote competition and innovation, the 1996 Act spiked "tremendous optimism" about development of new services at lower prices. Many firms believed that the dominant market share would go to companies with the newest technologies and biggest networks. A slow implementation process for the legislation led to unfulfilled beliefs about "meaningful competition."

Expectations about the economic outlook were arguably reflected in stock market prices and analysts' long-term earnings projections. The S&P 500 stock market average more than tripled from January 1995 to August 2000. The rise of stock market valuations was even more dramatic for the Internet-based companies. By one estimate, market participants had to expect extraordinary returns of 30 to 40 percent above the cost of capital for a significant period in the future (Ofek and Richardson [2002]). The investment bust was accompanied by a stock market crash and downward revisions of earnings forecasts. The S&P 500 index fell by 44 percent from its peak value in August 2000 to its turning point in February 2003.

The story of overinvestment due to optimistic expectations appears to be intuitively plausible. The rest of the Chapter attempts to evaluate whether this story can by adequately captured by the standard business cycle model.

2.3 Benchmark Model with Technology Shocks

This section describes the benchmark model under the conventional information assumptions. The section also investigates whether the US experience during the 1994-2003 period can be explained by changes in technology alone.[14] The model is a growth model with exogenous stochastic aggregate and investment-specific technological changes. Investment-

[14] Effects of expectations which are independent of the current and past changes in technology are examined in Section 2.4.

specific change is incorporated to capture the effects of a rapid decline in the relative investment prices on capital accumulation.

2.3.1 Model Description

To introduce the model in the simplest way, this subsection describes assumptions about preferences, technology and information structure. The economy can be easily decentralized as a standard sequence of markets equilibrium.

Preferences. A representative household has preferences over consumption C_t and leisure L_t with the expected life time utility[15] at date τ defined by

$$U = E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\ln C_t + \eta L_t\right] \tag{2.1}$$

Here β is the discount factor, $0 < \beta < 1$, η is a positive scalar, and $E_\tau x_t = E\left[x_t | \Omega_\tau\right]$ is the expectation operator conditional on all variables dated τ and earlier. The household is endowed with one unit of time, allocated between leisure and work:

$$L_t + N_t = 1 \tag{2.2}$$

Technology. Consumption and investment goods are produced from capital and labor inputs using Cobb-Douglas production function with capital share α, $0 < \alpha < 1$:

$$\begin{aligned} C_t &= K_{c,t}^{\alpha} \left(A_t N_{c,t}\right)^{1-\alpha} \\ I_t &= V_t K_{i,t}^{\alpha} \left(A_t N_{i,t}\right)^{1-\alpha} \end{aligned} \tag{2.3}$$

Variable A_t is aggregate technological change. Improvements in this technology benefit production of both consumption and investment goods. Variable V_t is investment-specific technological change. Benefits from this technology can be gained only through capital accumulation. The stochastic process followed by each technology is modelled on the basis of empirical measures, described below. The aggregate technology is assumed to follow a

[15] Alternative preferences are considered in Section 2.5

logarithmic random walk with a drift:

$$A_t = \gamma_a A_{t-1} \exp(a_t), \quad \gamma_a > 1, \; A_0 > 0 \text{ given} \tag{2.4}$$

$$a_t = \varepsilon_{a,t} \tag{2.5}$$

This specification implies that any change in aggregate technology has a permanent effect on its level. Investment-specific technology is modelled as fluctuating around a deterministic trend:

$$V_t = V_0 \gamma_v^t \exp(v_t), \quad \gamma_v > 1, V_0 > 0 \tag{2.6}$$

$$v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_{v,t} \tag{2.7}$$

The trend in investment-specific technology reflects the increased efficiency in production of investment goods, first emphasized by Greenwood, Hercowitz and Krusell [2000]. Throughout the Chapter, stationary components of aggregate and investment-specific technology, a_t and v_t, are referred as technology shocks, and stochastic building blocks of the shocks, $\varepsilon_{a,t}$ and $\varepsilon_{v,t}$, are referred as technology impulses. A vector sequence of technology impulses $\varepsilon_t = [\varepsilon_{a,t}, \varepsilon_{v,t}]'$ is assumed to be independent, identically distributed and uncorrelated at all lags and leads:

$$E\varepsilon_t = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; E\varepsilon_t \varepsilon_t' = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}, \; E\varepsilon_t \varepsilon_\tau' = 0, \; \tau \neq t \tag{2.8}$$

Capital and labour inputs are perfectly mobile across sectors. The aggregate capital stock K_t becomes productive after one period and depreciates at rate δ, $0 < \delta < 1$:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{2.9}$$

where $K_t = K_{c,t} + K_{i,t}$.

Information. The structure of the economy is common knowledge. Before making their consumption and production decisions, consumers and firms observe the current and past realizations of technology impulses. These impulses define the information set of the economy. Since impulses are independent and uncorrelated, all future impulses are unpredictable and $E[\varepsilon_{t+j}|\Omega_t] = 0$, $j > 0$.

2.3.2 Solution

At an optimum, the value of the marginal product of each input is equalized across sectors. The resource constraint can be used to define real aggregate output Y_t :

$$C_t + \frac{I_t}{V_t} = K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha} \equiv Y_t \tag{2.10}$$

Here $N_t = N_{c,t} + N_{i,t}$ defines the aggregate labour input.

Allocations of a rational expectations equilibrium $\{C_t, N_t, L_t, I_t, Y_t, K_{t+1}\}_{t=0}^{\infty}$ maximize the expected utility of the household (2.1) subject to the technological constraints $(2.9, 2.10)$, given the information structure, the exogenous processes for technology $\{A_t\}_{t=0}^{\infty}$ and $\{V_t\}_{t=0}^{\infty}$ and the initial conditions for the aggregate capital stock $K_0 > 0$. The stochastic processes for real wages, real interest rates, investment and capital goods prices that decentralize this economy can be obtained through either the marginal product conditions or the household's marginal rates of substitutions.

Along the equilibrium path, hours worked are covariance stationary. Consumption and aggregate output grow at the rate $\Upsilon_t = A_t \gamma_v^{\frac{\alpha}{1-\alpha} t}$. Capital and investment grow at a faster rate, $\gamma_v^t \Upsilon_t$, reflecting the investment-specific technological change. A covariance stationary represetation of the model is obtained by applying the relevant trend transformations.[16] The model is then solved using a log-linear approximation of the corresponding stationary representation around the unique non-stochastic steady state.

2.3.3 Identification of Technology and Parameter Choice

In a decentralized version of the model, the relative price of investment is inversely related to investment-specific technological change. The empirical measure of this change can be backed out from the investment price series. Thus, the relative investment price series is computed as a ratio of price deflators for real investment and consumption.[17]

The aggregate technology shock is computed as the Solow residual, with capital share equal to 0.32. Empirical counterparts for aggregate output, capital and labour inputs are

[16] The transformations are defined as $c_t = \frac{C_t}{\Upsilon_t}$, $y_t = \frac{Y_t}{\Upsilon_t}$, $i_t = \frac{I_t}{\Upsilon_t \gamma_v^t}$, $k_{t+1} = \frac{K_{t+1}}{\Upsilon_t \gamma_v^t}$, $n_t = N_t$.

[17] Price deflators are for chained 2000 dollar quantity series produced by the BEA.

computed as follows. Real aggregate output is constructed using the approach advocated by Greenwood, Hercowitz and Krusell [2000]. The procedure is first to compute nominal output as the sum of nominal series on consumption and investment, and then to use the consumption deflator to convert nominal output measure into real. Capital input is based on the BLS[18] annual index for capital input from equipment, structures, and rental residential capital for the nonfarm business sector. The annual series is interpolated into quarterly using real fixed investment. Labour input is based on the BLS index of nonfarm business hours. To be consistent with the model, the BLS index is converted into per-capita terms using the population trend. The level of hours is normalized to 0.21 in the first quarter of 1994.[19] The initial values for the technology are chosen to match the steady state level of the capital stock and the capital to consumption ratio with the actual data observed in the first quarter of 1994.

Parameters describing technology are estimated from the empirical measures. Parameter estimates for the investment-specific technology shock are computed using 1981 : 1 − 2003 : 4 sample. The initial date is chosen to reflect the start of a decline in relative investment prices, illustrated in the upper left panel of Figure 2.2. Parameter estimates of aggregate technology are computed using 1967 : 2 − 2003 : 4 sample. The initial date is determined by the data availability. The estimated parameters are reported in Table 2.1. Measures of technology shocks and impulses for the simulation period of 1994 : 1 − 2003 : 4 are plotted in Figure 2.2. For the ease of interpretation, the relative investment price series and aggregate technological change are expressed as indexes normalized to 100 in 1994:1.

The rest of the model parameters are assigned the following values. The discount factor β is set to yield the annual risk-free real interest rate of 3 percent. The rate of capital depreciation is set to 0.044. The parameter governing the disutility of leisure is chosen to match the steady state value of hours worked.[20]

[18] Bureau of Labor Statistics.

[19] This value is derived on the basis of the BLS series for average work hours.

[20] The main conclusion of the Chapter about the contribution of changes in technology and expectations

2.3.4 Contribution of Technology

The contribution of technological change to explaining the US experience in the 1994-2003 is assessed by comparing sample paths of the simulated and real data. The question is how well the model economy can account for the joint behaviour of consumption, investment and hours, given the estimated measures of technology.

The initial period in the model coincides with 1994:1. Parameters of the model are chosen so that the model economy is on the balanced growth path in the initial period, and the initial stock of capital coincides with the observed capital stock. Figure 2.3 reproduces series for the empirical growth rates of consumption and investment from Figure 2.1, and adds the simulated series generated with two technology shocks.

From the graphical illustration, technological change appears to be an important contributor to the economic boom in the late 1990s, and even to a slowdown in early 2000. However, the collapse in investment is a puzzling feature from the perspective of the model. Given the estimated process for technology, the model predicts positive investment growth through 2001-2003.

The graphical analysis is complemented by formal statistics in Table 2.2. For each series, the explanatory power is measured by a correlation coefficient and an R^2 from an OLS regression of the actual data on its simulated counterpart and a constant. For consumption and investment, the data are expressed as one-period growth rates. For hours, deviations from the steady state are compared. The R^2s measure stochastic variation of the actual data explained by the simulated data. The average contribution of technology is measured as the average of the R^2s for individual variables. The formal statistics confirm the graphical illustration. There is a substantial positive correlation between the model and the data. During 1994-2000, the model can explain 41 and 24 percent of variation in growth rates of consumption and employment. Once the years 2000-2003 are added, the explanatory power drops to 29 and 8 percent respectively.

to explaining the U.S. experience from 1994 to 2003 is robust to a trend stationary specification of aggregate technology and an AR(1) specification for investment specific shock.

Overall, the simulation results support the view that factors other than the economic fundamentals must have been at work during the 2000-2002 investment bust. The results also confirm findings from empirical literature on investment. Standard econometric models have been shown to have a great difficulty in explaining the drop in investment.[21]

2.3.5 Correction for Capacity Utilization

It has been long recognized that unobserved variations in inputs contaminate the Solow residual as a measure of technology.[22] It has also been established[23] that variable capital capacity utilization can provide a powerful amplification and propagation mechanism. This subsection explores whether the correction for capacity utilization can help to explain the investment bust.

The benefit of capital utilization is the ability to adjust capital input in response to changes in economic conditions. The benefit is introduced through a change of the production function:

$$Y_t = (u_t K_t)^{\alpha} (A_t N_t)^{1-\alpha} \tag{2.11}$$

The cost of a more intensive utilization is modelled as increased rate of capital depreciation. The cost is parameterized by the function

$$\delta_t = \frac{\theta}{\omega} u_t^{\omega}, \omega > 1 \tag{2.12}$$

This functional form implies that depreciation is an increasing convex function of capital utilization. Given the average depreciation rate and other parameters of the model, the elasticity of depreciation with respect to utilization can be pinned down by the steady state relations.[24] To make the steady state of the model invariant to the level of capital utilization, parameter θ is chosen so that in the steady state the level of utilization is equal to one. The parameters of the model imply $\omega = 1.34$.

[21] See references by Bernanke [2003].
[22] Hall [1990] , Burnside, Christiano and Eichenbaum [1996].
[23] King and Rebelo [1999].
[24] See Greenwood, Hercowitz and Krusell [2000].

A measure of aggregate technology corrected for capacity utilization is derived on the basis on the modified production function (2.11). The Board of Governors index of industry capacity utilization is used as a proxy for capital utilization. To get the unitary value of utilization in the steady state, the series is re-scaled by its mean over the existing sample. This measure is certainly not ideal. Yet, its usage provides an indication whether the mechanism of variable utilization can change conclusions of the benchmark model with respect to the investment bust.

Once the simulation results are repeated with the new measure of technology shock, the model series exhibit excessive volatility. To slow down the speed of resource reallocation, adjustment costs for capital accumulation are introduced into the model:

$$K_{t+1} = (1 - \delta(u_t)) K_t + Q\left(\frac{I_t}{K_t}\right) K_t \tag{2.13}$$

with adjustment costs function defined by

$$Q\left(\frac{I_t}{K_t}\right) = \phi_1 + \frac{\phi_2}{1-\psi}\left(\frac{I_t}{K_t}\right)^{1-\psi} \tag{2.14}$$

Parameters ϕ_1 and ϕ_2 are selected to make the steady state invariant to the degree of adjustment costs ψ.[25] The degree of adjustment costs is estimated by the simulated method of moments. Higher adjustment costs in the model decrease volatility by decreasing the magnitude of investment responses. Thus, the standard deviation of the growth rate of investment is used to estimate the elasticity of adjustment cost. The estimation procedure suggests $\psi = 0.23$.

The explanatory power of technology shocks in the model with variable utilization are reported in table 2.2, under the rows with "Utilization" title. The model performance worsens relative to the benchmark case. Furthermore, the main conclusion about the limited role of technology in explaining the investment bust remains unchanged. The simulation results suggest that refinements on utilization measures are unlikely to be a promising avenue for understanding the US experience of the last decade. After all, the

[25] Specifically, $\phi_2 = (\gamma_k - 1 + \delta)^{\psi}$, $\phi_1 = \frac{\psi}{1-\psi}(1 - \delta - \gamma_k) - \phi_2$, and $\gamma_k = \gamma_a \gamma_b^{1/(1-\alpha)}$ is the growth rate of capital.

introduction of variable utilization leads to even stronger implied growth rates of aggregate technology. The strong productivity growth, in turn, creates incentives for capital accumulation. The rest of the Chapter abstracts from capital utilization.

2.4 Model with Technology Prospects

The simulation results from the previous section demonstrated the great difficulty the benchmark model has in explaining the investment boom. The objective of this section is to investigate the role of expectation revisions. The analysis proceeds in several steps. First, the information structure of the model is modified and compared with the conventional approach. Second, the estimation procedure for uncovering technology prospects is outlined. Third, the procedure is implemented for aggregate technology prospects. Finally, implications for investment technology prospects are discussed.

2.4.1 Information Structure Assumption

Investment, consumption and employment decisions are forward-looking. In equilibrium, these decisions depend on expectations about the entire future paths of aggregate and investment-specific technology shocks.[26] To understand implications of information assumptions on equilibrium decisions, it is useful to consider a state space representation of the joint process for technology shocks $z_t = \begin{bmatrix} a_t & v_t \end{bmatrix}'$ with impulses $\varepsilon_t = \begin{bmatrix} \varepsilon_{a,t} & \varepsilon_{v,t} \end{bmatrix}'$:

$$\begin{aligned} \zeta_{t+1} &= A_z \zeta_t + B_z \varepsilon_{t+1} \\ z_t &= C_z \zeta_t \end{aligned} \tag{2.15}$$

Here matrices A_z, B_z and C_z are defined as follows

$$A_z = \begin{bmatrix} 0 & \underset{1\times 2}{0} \\ \underset{2\times 1}{0} & A_v \end{bmatrix}, \; B_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; C_z = \begin{bmatrix} 1 & \underset{1\times 2}{0} \\ 0 & C_v \end{bmatrix} \tag{2.16}$$
$$A_v = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}, \; C_v = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

[26] Recall that technology shocks are referred to stationary components of technological change. Thus, expectations about technology shocks are well defined.

Vector $\zeta_t = \begin{bmatrix} a_t & v_t & v_{t-1} \end{bmatrix}'$ concisely summarizes the relevant realized technology fundamentals. The state space representation is convenient for deriving the conditional expectations of future technology shocks:

$$E\left[z_{t+j}|\Omega_t\right] = C_z A_z^j \zeta_t + \Sigma_{i=1}^j C_z A_z^{j-i} B_z E\left[\varepsilon_{t+i}|\Omega_t\right] \tag{2.17}$$

The formula for conditional expectations is obtained by iterating on (2.15) and applying the expectation operator conditional on the economy's information set Ω_t. Two terms contribute to the conditional expectations. The first term on the right hand side of (2.17) summarizes effects of the realized technology fundamentals ζ_t. The second term captures effects, which are independent of the realized technology fundamentals. The independent effects are determined by the information assumption with respect to predictability of future technological impulses.

Under the conventional approach, future technological impulses are unpredictable. The second term in (2.17) vanishes, and expectations about future technology shocks are completely determined by the history of the current and past realizations of technology shocks. Consequently, changes in expectations have an impact on the current decisions only through the current and past changes in technology. The poor empirical performance of the benchmark model in explaining the bust suggests that this information assumption may limit the role of expectations from the start.

Modification of the information assumption about the unpredictability of future technological impulses introduces the additional role of expectations through the second term in (2.17). The approach builds upon the idea that at a time of rapid technological change, the past may not be an entirely good predictor of the future. To formalize this idea, households and firms are assumed to receive commonly observed exogenous signals. The signals are believed to contain information about future realizations of technological impulses. If the signals were available to the modeller, it would have been possible to determine dynamic correlations between the signals and technological impulses, and to use these correlations to construct measures of conditional expectations. This approach is not feasible in practice, as the structure of the aggregate information set is unknown. The modelling approach in this Chapter does not rely on the knowledge of the distribution for signals. Instead, the approach focuses on the outcomes of the learning process. It is assumed that aggregate expectations of future technological impulses, conditional on the current information set, may deviate from the unconditional ones. Formally, for every period t there possibly exists some future date $j > 0$ such that $E[\varepsilon_{t+j}|\Omega_t] \neq 0$.

Since the analysis focuses on a particular historical episode, it is not necessary for the signals to be correlated with future states of technology.[27] In this sense, the modelling approach can accommodate irrational exuberance as well as perfectly rational behaviour. Irrational exuberance can be formalized as a situation when the signals used in the formation of the conditional expectations are in fact uninformative about the future states of technology. Rational behaviour can be modelled as a signal extraction problem based on the knowledge of dynamic correlations between the signals and states of technology.[28]

The advantage of this parsimonious approach to modelling beliefs is the ability to define a notion of optimism about the future. In this framework, optimism corresponds to above average expectations about the future, or to the case $E[\varepsilon_{t+j}|\Omega_t] > E[\varepsilon_{t+j}]$ for some future date $j > 0$. If beliefs are not perfectly validated, and the actual technology turns out not as high as expected, then there is a precise sense in which optimism was excessive ex-post.

A potential weakness of the modelling strategy is its inability to answer why and how errors in expectation could occur. This property does not seem to be a very strong restriction, at least for understanding the role of aggregate expectation errors in the US experience during 1994-2003. Recent theoretical and empirical contributions appeal to potential difficulties in aggregating heterogenous beliefs across agents in the economy.

Theoretically, in an environment with heterogeneous agents, equilibrium asset prices

[27] In a longer time span, this way of modelling of expectations would require signals and future realizations of impulses to be correlated. A comparison between values of signals and ex-post realizations of technology impulses would eventually reveal the degree of correlation. In the absence of the correlation, it would be useless to include the signals into forecasts. Chapter 3 investigates the predictability of aggregate technology.

[28] An example of the rational signal extraction problem in a general equilibrium model with expectation errors can be found in Beaudry and Portier [2004*a*].

are a weighted average of beliefs about asset payoffs.[29] In this environment, the process of beliefs aggregation is hindered by short sale restrictions on stocks. When selling short, a seller does not own a stock, but is committed to repurchase this stock some time in the future. Thus, shorting gives a way through which investors can bet on their anticipation of a decline in the stock's price. If short sales are restricted, beliefs of more pessimistic investors tend not to be incorporated into asset prices. Furthermore, potentially erroneous beliefs can arise as equilibrium phenomena even in models with rational expectations and no short sale restrictions. Work on rational herding and information cascades[30] provides theoretical foundations for this area.

Empirically, Ofek and Richardson [2002, 2003] argue that the market for Internet stocks in the late 1990s had a limited capacity for short sales. Relative to non-Internet firms, Internet-based companies had higher short interests, higher borrowing costs for shorting and stronger violation of put-call parity in the options markets. Internet-related equity markets were also more heterogenous in their composition of market participants. In these markets, retail investors were more active on Internet-related equity markets than institutions. Ofek and Richardson advocate a view that existing short sale constraints on Internet stocks prevented beliefs of more pessimistic investors to be properly aggregated into Internet asset prices. They also hypothesize that expiration of lockup periods[31] for a large number of companies in the spring and latter half of 2000 loosened these constraints, led to an increased number of markets and eventually to a fall in asset prices.

2.4.2 Solution

The model is solved using a modification of Blanchard and Kahn's[32] algorithm discussed in details in Chapter 4. In this model, two variables θ_t^a and θ_t^v summarize all effects of

[29] See, for example, Lintner [1969], Miller [1977], Jarrow [1981].

[30] See, for example, Banerjee [1992], Zeira [1994] and Beaudry and Gonzalez [2004].

[31] It is a standard arrangement to restrict shares from sale for a certain period of time after an initial public offering without the written consent of the underwriter. These lockup periods can be interpreted as the most strict form of short sales (Ofek and Richardson [2003]).

[32] Blanchard and Kahn [1980].

expectations about the future technological impulses on equilibrium decisions:[33]

$$\theta_t^a = \sum_{j=1}^{\infty} \lambda^{-j} E\left[\varepsilon_{a,t+j} | \Omega_t\right] \tag{2.18}$$

$$\theta_t^v = \sum_{j=1}^{\infty} \lambda^{-j} \left[C_v \Sigma_{i=1}^{j} A_v^{j-i} C_v' E\left[\varepsilon_{v,t+i} | \Omega_t\right]\right] \tag{2.19}$$

Here $\lambda > 1$ is the unstable eigenvalue of the model and matrices $C_v = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $A_v = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$. Variables θ_t^a and θ_t^v have economic interpretation of the discounted sums of expected future realizations of technology impulses. These variables are called aggregate and investment technology prospects.

The equilibrium process for consumption, investment and hours, expressed as deviations from the steady state, can then be written as linear functions of the expanded state vector, which includes capital, realized technology and technology prospects:

$$\begin{bmatrix} \hat{c}_t \\ \hat{\imath}_t \\ \hat{h}_t \end{bmatrix} = M_k \hat{k}_t + M_\zeta \zeta_t + \begin{bmatrix} \pi_{ca} \\ \pi_{ia} \\ \pi_{ha} \end{bmatrix} \theta_t^a + \begin{bmatrix} \pi_{cv} \\ \pi_{iv} \\ \pi_{hv} \end{bmatrix} \theta_t^v \tag{2.20}$$

Several properties of the decision rules are of interest. First, matrices M_k and M_ζ, describing impacts of capital stock and realized technology shocks are identical to the benchmark model of Section 2.3. Second, under the conventional information assumption, technological prospects are absent ($\theta_t^a = 0$, $\theta_t^v = 0$) and expectations about future technology affect current decisions only through the realized technology fundamentals. Matrix M_ζ summarizes these effects. With the expanded information set, expectations play the additional role. This role is captured by vectors π_a and π_v. Third, impact coefficients for technology prospects, π_a and π_v are independent of the process for expectations. Finally, vectors π_a and π_v are linearly dependent.[34] In this model, there is only one channel through which future technological opportunities have a direct effect on endogenous decisions. This channel is summarized by intertemporal Euler equation. Future technological opportunities affect future income. At the end, the source of changes in future income turns out not to matter for qualitative predictions of the model.

[33] This representation is feasible only for the models with one endogenous state variable.

[34] The last two properties are proven in Chapter 4, Section 4.4.4.

2.4.3 Identification of Technology Prospects

The empirical identification of unobservable technology prospects exploits equilibrium restrictions of the model. Intuitively, the estimation strategy consists of two steps. First, a variation in consumption, investment and hours, not accounted for by the realized technology fundamentals and capital is isolated from the data. Then this residual variation (new transformed data) is used to find realizations of expectations that minimize the residual distance between the model and the data, and satisfy the equilibrium cross equation restrictions.

The starting point of the estimation is the equilibrium decision rules. Suppose that true measures of technology shocks and capital stock were at hand. Then the part of consumption, investment and hours worked not accounted by the realized fundamentals would have been described by $\mathbf{y}_t = \begin{bmatrix} \tilde{c}_t & \tilde{\imath}_t & \tilde{h}_t \end{bmatrix}' - M_k\hat{k}_t - M_\zeta \zeta_t$.[35] Further, if the model were true and observed measures of consumption, investment and hours were perfect, any of the three elements of vector $\mathbf{y}$ would have identified the effects of technology prospects perfectly, implying the deterministic relations between the observable variables. In practice, these deterministic relations do not hold, due to measurement errors or abstractions of the model. The problem of deterministic relations can be eliminated by augmenting decision rules by variable-specific error terms e_jt :

$$\mathbf{y}_{j,t} = \pi_{j,a}\theta_t^a + \pi_{j,v}\theta_t^v + e_{jt},\ j \in \{c, i, h\} \tag{2.21}$$

Then each element of vector $\mathbf{y}$ would still be an informative indicator of technology prospects, if these prospects are in fact present in the data.

Variables θ_t^a and θ_t^v are not observed directly. In evaluating their empirical contribution, the challenge is how to construct estimates of these unobserved series. Linear dependence of impact vectors π_a and π_v hinders the problem of technology prospects identification. This subsection describes how technology prospects can be estimated when at most one type of technology prospect is present in the data. Then it addresses how the

[35] Variables with tildas correspond to empirical equivalents of deviations from the steady state.

results should be interpreted if both aggregate and investment technology prospects were present in the data.

For concreteness, let us consider prospective changes in aggregate technology. The key property of the relation (2.21) is that a realization of θ_t^a affects consumption, investment and hours simultaneously, and only at time t. Furthermore, the impact coefficients are known, once parameters of preferences and technology are assigned their numerical values. These properties suggest that the discounted sum of expectations can be recovered as parameters of the generalized regression model. Specifically, let a series of the discounted sum of expectations be summarized by a vector $\beta = \left[\begin{array}{ccc} \theta_1^a & \dots & \theta_T^a \end{array} \right]'$ where T is the sample size. Consider T variables that consist of decision rule coefficients for every time period. Specifically, variable $d_{i,t}$ for time t for investment series is a vector of length T, with a value π_{im} in period t and zero otherwise. A matrix of regressors collects all time variables $\mathbf{X}_j = \left[\begin{array}{ccc} d_{j,1} & \dots & d_{j,T} \end{array} \right], j \in \{c, i, h\}$. Then the generalized regression model

$$\begin{bmatrix} \mathbf{y}_c \\ \mathbf{y}_i \\ \mathbf{y}_h \end{bmatrix} = \begin{bmatrix} \mathbf{X}_c \\ \mathbf{X}_i \\ \mathbf{X}_h \end{bmatrix} \beta + \begin{bmatrix} e_c \\ e_i \\ e_h \end{bmatrix} \tag{2.22}$$

can be used to recover β coefficients. The residuals e in this model may pick up measurement errors in vector y. They might also be interpreted more generally as capturing all the movements and co-movements in the transformed series for consumption, investment and hours that the model cannot explain. There is no reason to expect that variances of residuals will be the same across variables or that there is no cross and auto-correlation between them. The residuals are assumed to be zero mean, $E\left[\mathbf{e}_j|\mathbf{X}\right] = \mathbf{0}$ and uncorrelated with regressors. They are allowed to be heteroskedastic, cross and autocorrelated.[36] The slope coefficients β can be consistently estimated by the ordinary least squares. Parameter estimates robust to the covariance structure can be obtained using Prais-Winsten regression.

To implement the procedure, empirical analogues are constructed for deviations of

[36] Specifically, for $j \in \{c, i, h\}$, $E\left(e_{jt}e_{jt}'\right) = \sigma_j^2$, $E\left(e_{jt}e_{jt-1}'\right) = \rho_j$, $E\left(e_{jt}e_{jt-k}'\right) = 0$, $E\left(e_{jt}e_{lt}\right) = \sigma_{jl}$, $E\left(e_{jt}e_{jt}\right) = 0$ otherwise.

consumption, investment, hours worked and capital from the model's steady state values. This is achieved by applying the trend transformations implied by the model to the observed data. The estimation period is 1994 : 1 − 2003 : 4, so the generalized model contains 120 observations in total.[37] The presence of technology prospects does not alter intra-temporal equilibrium relations of the model. Thus, measures of technology shocks constructed previously remain valid in this framework.

2.4.4 Prospects of Aggregate Technology

This section implements the econometric procedure under the assumption that beliefs are formed about prospective changes in aggregate technology only. The simulation results are presented first. Then the estimates of technology prospects and their interpretation are discussed.

Contribution of Expectations

The contribution of technological prospects to explaining the US experience in the 1994-2003 is assessed by comparing sample paths of the simulated and real data. The simulated series are obtained from the model augmented with the estimated series for technology shocks and aggregate technology prospects. The resulting series for consumption and investment are plotted on Figure 2.4. This figure is a direct analogue of Figure 2.3 for the model with changes in technology only.

Remarkably, aggregate technology prospects account for the investment slump very well. For the total period, the model can explain 82 percent of variation in growth rate of investment, as shown in the first row of Table 2.3. Expectations alone (see the first row of panel B of the table) can account for 32 percent. Aggregate technology prospects also explain a large part of variation in hours. The model does less well with respect to consumption.

[37] It should be noted that capital is endogenous variable in the model, and hence it must respond to changes in the states. The transformation operates under the premise that the empirical series for capital stock provide a reasonable approximation of the true state of capital.

An alternative illustration of the contribution of technology shocks and expectations are given in Figure 2.5. Here the sample paths for consumption, investment and hours are compared in terms of indexes (levels normalized to 100 in 2000 : 4). The first column corresponds to the simulated paths in response to aggregate technology prospects only. The second column corresponds to effects of changes in aggregate and investment-specific technology. The last column describes the model behaviour with technology shocks and technology prospects. The figure illustrates the explanatory power of technology prospects for the investment series.

The simulation results are consistent with the view that the seeds of the investment bust were planted during the boom, and that the reassessment of capital needs led businesses to cut back on their investment expenditures. The estimated series for technology prospects, however, are at odds with the common perception about overoptimism. The investment bust in the model economy occurs not because the beliefs are revised downward. Instead, a reversion to optimism that triggers the bust. I further discuss the reasons for this result.

Estimates of Aggregate Technology Prospects

The estimates of aggregate technology prospects are plotted on the top panel of Figure 2.6. The solid line is the series from the Prais-Winsten regression, and the dotted line corresponds to the OLS estimates. The plot also gives asymptotic confidence intervals.[38] Surprisingly, the regression coefficients, once linked together, produce an interesting time series. In every period, the implied point estimate captures the part of expectations about the future technology that is uncorrelated with the realized technology fundamentals. Two characteristics of the series have a meaning: the sign of the series and the direction of change. When the realization is negative, the future looks bleaker relative to the historical average. In other words, the technology growth is expected to be lower than the average. When a value of the series is positive, the future looks brighter. Beliefs become more pessimistic if the series falls. The estimates in Figure 2.6 imply that the

[38] The asymptotic Wald statistic has a value of 1.47e+07, implying the joint significance of coefficients.

joint behaviour of consumption, investment and hours in the context of the model, are consistent with aggregate expectations becoming more pessimistic through the late 1990s. During the recession, perceptions of the future changed towards optimism.

To get some perspective on interpreting the estimates, Figure 2.6 also plots realizations of the aggregate technology growth rates (solid line), along with their historical average (dotted line). Interestingly, the range of values for the estimated series is higher than the range of the actual growth rates. To understand the nature of estimates, it is illustrative to consider how effects of expectations propagate through the economy.

Model Responses to Technology Prospects

Figure 2.7 plots the dynamics of the model in the extreme case when beliefs about favourable change in either aggregate (solid lines) or investment specific (dotted lines) technology are completely not materialized. In period zero, the economy is at the steady state. In period one, households and firms start to believe that in period five the growth rate of aggregate technology will be one percent higher. The beliefs are not updated until period five. In that period agents learn that their beliefs are not fulfilled.

On impact, the "good" news about future technology creates a boom in consumption, but a fall in investment, hours and output. There are two competing forces in the model: wealth and interest rate effects. On the one hand, when households expect a technological improvement, they understand that they will be wealthier in the future. The consumption-smoothing motive induces incentives to increase consumption and decrease saving in anticipating the technology change.

As consumption and leisure are normal goods, the demand for these goods tends to move in the same direction. In particular, whenever consumption demand increases, so does the demand for leisure. This creates incentives for household to work less. On the other hand, future consumption paths are evaluated at stochastic interest rates. A technological improvement increases the marginal product of capital in the future. A raise in the interest rates makes saving more attractive and induces households to decrease consumption today. In equilibrium, the two effects interact in a complex manner. When conventional parameters are considered, the wealth effect dominates. With lower work effort, higher consumption is possible only at the expense of lower investment. On the flip side, "bad" news about future technology (or pessimistic beliefs) leads to a boom in investment and output. These dynamic responses are the key to understanding the nature of the estimates.

Along the transition path output and investment continue to fall, due to below average capital stock and hours of work. When the beliefs are unrealized, the direction of response is reversed. Households work more and consume less to build up the stock of capital.

2.4.5 Prospects of Investment Specific Technology

Prospects of aggregate and investment good technology should generally induce different incentives. Intuitively, it seems harder to rationalize the boom in response to expectations about technology improvement in the production of investment goods. After all, improvements are associated with falling relative prices. The prospective price declines imply that buying and holding investment goods is costly. This reasoning would seem to prompt firms to postpone investment purchases until the time when capital goods become cheaper.

In the model, the dynamic responses to unrealized future changes in investment good technology are qualitatively similar to the responses to future changes in aggregate technology. This is shown by the dotted lines on Figure 2.7. In fact, the dynamic responses are exactly proportional to each other.[39] As a result, the implementation of the estimation procedure for the case of technology prospects about investment goods leads to a series for expectations proportional to the ones derived for aggregate technology prospects. The overall qualitative effect of prospective changes in production of investment good is then identical to the effects of prospective changes in aggregate technology.

[39]This property is linked to a linear dependence of impact vectors π_a and π_b.

2.4.6 Two Types of Technology Prospects

In the case of one endogenous state variable, the impact coefficients are proportional to each other, and two unobserved series cannot be identified separately. It is conceivable that prospective changes in both aggregate and investment good technology may have been important during the episode. It that case, the regression coefficients from the pooled regression can be interpreted as a summary statistics characterizing the overall beliefs about the future. For example, when coefficients π_a are used in (2.22), the estimated series is a linear combination of the two discounted sums, $\theta_t^v = \theta_t^a + a\vartheta_t^b$. In the benchmark model, coefficient $a = 6.12$, so that θ_t^v can still be interpreted as capturing, in the context of the model, the aggregate beliefs about technology prospects. The results on importance of expectations still remain valid, and so do the simulation results. However, the inability to separate the two sums leads to the inability to interpret the estimates as a discounted future sum of impulses of a particular technology shock.

2.5 Incentives to Save and Capital Accumulation

The results from the previous section suggest that revisions in expectations may have been important for understanding the US investment boom and bust in 1994-2003. However, the model implies that to rationalize the behaviour of macroeconomic variables, aggregate beliefs about the future had to be more pessimistic during the boom, and more optimistic during the recession. This section illustrates that the directions of changes in expectations can be reversed if households are patient enough to wait for the benefits of future technology gains. The sensitivity of the model dynamics to changes in the intertemporal elasticity of substitution is evaluated first. Then the estimation and simulations are repeated for the modified model.

2.5.1 Effects on Model Dynamics

To evaluate effects of changes in the intertemporal elasticity of substitution on the dynamic responses of the model to future improvements in technology, the model is modified by considering two alternative classes of preferences.

Multiplicatively Separable Preferences The first class of preferences is the balanced growth preferences,[40]

$$u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} L_t^{1-\xi}, \ \sigma > 0, \sigma \neq 1, \xi > 0 \tag{2.23}$$

Here σ is the inverse of the labour-constant intertemporal elasticity of substitution in consumption, and ξ is the inverse of consumption constant elasticity of labour supply. These parameters are closely linked together. The labour supply elasticity cannot be assigned independently. Rather, its value is pinned down by the share of time devoted to the market. Further, there are concavity restrictions on preference parameters.[41] Numerical simulations reveal that the magnitude of the decline of investment in response to expected future technology gains becomes smaller. Yet, for a range of values of σ consistent with joint restrictions, the investment dynamics cannot be reversed. Thus, with balanced growth preferences, a high intertemporal elasticity of substitution of consumption alone is not sufficient to generate investment booms.

Additively Separable in Leisure Preferences The second class of preferences breaks the dependence of the labour supply elasticity on the steady state hours worked. The momentary utility is of the form[42]

$$u(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \Upsilon_t^{1-\sigma} \eta \frac{L_t^{1-\xi}}{1-\xi}, \ \sigma > 0, \sigma \neq 1 \tag{2.24}$$

To be consistent with balanced growth, the value of leisure must be increasing over time at the rate of consumption growth. The trend can be rationalized by separating the market and nonmarket sectors, as done by Chari, Kehoe and McGrattan [2002]. However, such an interpretation requires equalization of σ and ξ. This is the restriction maintained in the

[40] See King, Plosser and Rebelo [1989].

[41] A concavity of the utility function requires $(1-\sigma)(1-\xi) > 0$, $(1-\xi) \leq \sigma$.

[42] In its stationary form, $u(c_t, L_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + v(L_t)$, this utility function is popular in monetary and international economics.

simulations. With additively separable preferences, η is chosen to match the steady state works worked.[43]

The dynamic responses of the model are plotted on Figure 2.8. The dynamics are computed for values of $\xi = \sigma = 0.2$. The dynamics confirm the intuition about the role of intertemporal elasticity of substitution. In response to unrealized news about improvement in aggregate technology, the economy experiences a boom in investment, hours and output in anticipation of aggregate technology gains. When the beliefs are not validated, investment, hours and output decline. The responses are smaller in magnitude than in the benchmark case. However, the relatively high willingness of consumers to substitute consumption across time is not sufficient to change the direction of response to investment good technology shock. As in the benchmark case, the good news about technology of investment good leads to a fall in investment.

2.5.2 Contribution of Expectations and Technology Shocks

The estimation and simulation results of Section 2.4 are repeated for additively separable preferences with σ and ξ equal to 0.2. The higher intertemporal elasticity of substitution intensifies the responsiveness of consumption to changes in real interest rate. As in the case of variable capacity utilization, the simulated sample paths exhibit excessive volatility. The adjustment costs in capital (2.14) are introduced to bring the model in line with the data. The simulated method of moments procedure suggests $\psi = 0.18$. The estimates of aggregate technology prospects are plotted in Figure 2.9.

The series reflects the conventional perception of the overall optimism of the 1990s. Based on statistics reported in Table 2.3, the overall performance of the model with high intertemporal elasticity of substitution and adjustment costs is on a par with the benchmark model. In this case, expectations play an even more important role. Alone, they can explain almost 70 of variation in investment growth rates.

Figure 2.10 presents the simulation results with and without aggregate technology

[43] To keep the steady state invariant to values of the intertemporal elasticity of substitution, the discount factor is adjusted accordingly.

prospects. The model with only technology shocks again performs very poorly in explaining the recession. In addition, it has a greater difficulty in accounting for the investment boom on the basis of realized technology fundamentals. As evident from the bottom left panel of the figure, aggregate technology prospects account for the investment slump very well. However, a puzzle of missing consumption emerges. The model predicts a consumption boom in 2001-2002 of a magnitude far exceeding that found in the data.

2.5.3 Interpretation

The reason the modified model attributes the investment boom to the reassessment of optimistic beliefs is the assumption of a relatively high willingness of households to substitute consumption across time. Empirically, the intertemporal elasticity of substitution is difficult to pin down. The estimates fall in a wide range. Values above unity, corresponding to logarithmic utility, are less prevalent, although they do exist.[44] It is possible that the intertemporal elasticity of substitution was relatively high during the recent historical episode, due to changes in the demographic composition of the population.[45] Yet, even in this case, the reliance on the high intertemporal elasticity of substitution to explain the US experience is problematic. The model's predictions for consumption are difficult to reconcile with the data.[46]

2.6 The US and the World Economy

This section briefly discusses whether open economy considerations could help to solve the puzzles noted above. This discussion shows the empirical and theoretical challenges associated with such analysis. However, a more thorough examination of this issue is left

[44] For example, in a recent work Chari, Kehoe and McGrattan [2002] find that in the context of an open economy model with monopolistic competition and sticky prices, a value of five is required to reproduce the real exchange rate volatility of the data.

[45] Relative to the past, the proportion of senior citizens is much larger. Longer life expectancy and prospects of retirement for the baby-boom generation may have increased the aggregate willingness to substitute over time.

[46] In addition, technological trend must be present in the utility function, which is not a very desirable feature of the model.

for future research.

Business cycle models with conventional preferences and production functions have difficulties in generating economic booms in investment and output in response to anticipated technological improvement. In a closed economy, investment must be completely financed by domestic saving. This restriction does not hold for a country that is open to trade. That country can borrow from abroad to support its productive investment. In other words, foreign borrowing and lending separates consumption and investment decisions. This separation may, in principle, help to generate investment booms in anticipation of future technological improvement. However, pursuing this avenue of research in application to the US experience over the period 1994-2003 faces several empirical and theoretical challenges.

First, the US was not the only country that had a surge in business investment in the second half of the 1990s and a decline in 2000-2001. Strong investment growth was observed in a number of OECD economies, as documented by Pelgrin, Schich and de Serres [2002]. In Canada, Denmark, Greece, the United Kingdom and Sweden, growth rates of real business investment considerably surpassed growth rates of real GDP. As in the US, investment in information technology equipment and structures was the major contributor to growth of both investment and real output in the late 1990s.[47] In all the G7 countries (Canada, France, Germany, Italy, Japan, the UK and the US) growth of information technology capital input per capita jumped to double-levels after 1995.[48]

This surge in business investment is only partly explained by basic economic fundamentals, such as cost of capital, depreciation rate, financial market measures and changes in output. Pelgrin, Schich and de Serres [2002] came to this conclusion based on panel cointegration analysis for gross business investment for 18 OECD countries. Their empirical results "would tend to support the view that investment had exceeded its steady-state level, not least in the United States."[49] As in the US, information technology and overall

[47] Jorgenson [2004].
[48] Tables 8 and 9 in Jorgenson [2004].
[49] Pelgrin, Schich and de Serres [2002, p.2].

business investment weakened in 2000-2001.[50]

Second, the world as a whole experienced an economic slowdown in 2001. According to the IMF, annual world output growth declined from 4.7 percent in 2000 to 2.5 percent in 2001.[51] A slowdown in economic activity occurred in many countries and regions. In 2001, there was also a substantial fall in the volume of the world trade of goods and services. While trade volume grew by 12.4 percent in 2000, it declined by 0.2 percent in 2001. Advanced economies were major contributors to the fall in the global trade.[52] Further, data on industrial production, business and consumer confidence appeared synchronized on the economic slowdown across industrial countries.[53] .

Looking at the US in an international context during 1994-2003 thus requires a simultaneous explanation of the world-wide economic boom and slowdown. Theoretical difficulties with this approach can be illustrated in a simple two-region two-period single good example from Obstfeld and Rogoff [1998, $p.31-38$].[54] In this example, an anticipation of domestic technological improvement in the future leads to an increase in the world interest rate, which stimulates savings and discourages investment in the foreign country. For certain preferences and production functions, investment in the home country increases.[55] The domestic investment boom is financed by foreign borrowing. That is, the home country has a current account deficit. If the model is extended beyond two periods, then unrealized expectations about future technological improvement would lead to a fall in investment in the home country and a current account surplus. Such dynamic adjustment would be difficult to reconcile with investment booms in many OECD countries and the recent persistent current account deficit in the US.

In the same two-country model, expectations can be formed about future improvement

[50]Figure 1.8. in IMF [2002, *p*.22] illustrates this fact.

[51]Data in this paragraph are from Table 1.1 in IMF [2002, *p*.6].

[52]Exports and imports of these countries, which account for a large part of the global trade, fell by 1.3 and 1.5 percents in 2000. In comparison, exports and imports of these countries grew by 11.7 and 11.6 percents in 2001.

[53] See Figure 1.2 in IMF [2002, *p*.3] and Figure 1 in Doyle and Faust [2002, *p*.428].

[54]The US economy plays an important role in determination of the world interest. Thus, a small open economy framework appears inappropriate for characterizing its behaviour.

[55]However, a fall in domestic investment because of the higher world interest rate is theoretically possible.

in global, not a country-specific, technology. In that case, the model at the aggregate level behaves as a single closed economy. Such economy faces exactly the same wealth and interest rats trade-offs as discussed above. Under conventional preferences and production functions, world investment falls in anticipation of future improvement in global technology.

Overall, it appears unlikely that international dimensions of the 1994-2003 historical episode would help to resolve puzzling characteristics of the US experience. This conclusion may be reinforced by the fact that the trade share in the US GDP still remains relatively small.[56]

2.7 Conclusion

The Chapter started by reviewing the popular perception that the investment bust of 2000-2002 was a result of a reassessment of optimistic expectations about the future formed during the preceding boom. To evaluate this view formally, the standard macroeconomic model was extended to incorporate beliefs about future changes in technology, unrelated to realized technology fundamentals. Equilibrium restrictions of the model were used to construct empirical measures of the unobserved technology shocks and expectations. The simulation paths of the model economy were then compared with the actual series.

The overall conclusion from this work is that the intuitively plausible story of overinvestment due to optimistic beliefs is difficult to reconcile with the observed macroeconomic variables within the class of models considered in this Chapter. Under a conventional parametrization, investment, consumption and employment are explained quite well, but only if one accepts that people were pessimistic during the boom, but optimistic during the recession. This findings can be attributed to qualitative properties of the model: prospects of technological improvement lead to a recession in investment. What is more surprising is that versions with qualitatively plausible investment responses still have difficulty in explaining the data. A puzzling occurrence of investment collapse is replaced by a puzzle of

[56] Measured by the total volume of exports and imports, trade share was twenty five percent of the US GDP in 2000 (BEA).

missing consumption boom. The predicted growth rates of consumption more than twice exceed those observed in the data.

If revisions of optimistic expectations did in fact play a role during the US investment boom and bust, a natural further question is to ask what alternative class of models could better reflect (both theoretically and empirically) this hypothesis. Exploring complementarity between capital and technology may provide a promising research direction.

For an illustration, let us retain the assumption of a single, perfectly mobile across sectors capital good. In the extreme case of perfect complementarity between capital and effective labour (Leontief aggregate production function), the improved technology can be benefitted from only when the higher capital stock is in place. More generally, increasing the degree of complementarity between these two inputs should boost incentives for capital accumulation in anticipation of forthcoming technological improvement. In the benchmark model, investment increases in response to good news about future aggregate technology when the degree of complementarity between capital and technology is sufficiently high. It turns out that the required degree of complementarity in this model implies implausibly high volatility in labour shares. It is conjectured that a multi-sector economy with capital stock disaggregated into equipment and structures may help to elevate the prediction for labour shares and yet to retain a qualitative prediction for investment boom. More precise specification of the economy as well as it empirical ability to explain the aggregate data during 1994-2003 are left for future research.

Chapter 3

News Shocks and Predictability of Total Factor Productivity

3.1 Introduction

The stochastic nature of technological change is inherently associated with forecasting difficulties. Errors in beliefs about future technology may be at the heart of explanations for some historical episodes. More generally, errors in forecasting future technological change may be an independent source of aggregate fluctuations. News shocks provide a convenient way of capturing changes in agents' beliefs about future technology. They are defined as exogenous variables that help to forecast future realizations of technology shocks.[1] The main question of this Chapter is whether the US post World War II experience is consistent with the presence of news.

Empirical evidence on existence of news shocks is extremely limited. Beaudry and Portier [2004*a*] use a method of indirect inference to explore how well their general equilibrium model with noisy news can explain business cycle statistics. Their parameter estimates imply that technological impulses can be anticipated four quarters in advance, and that news shocks are very informative. In another paper, Beaudry and Portier [2004*b*] attempt to recover news shocks from a vector autoregression. The news shock is identified

[1] More formally, news shocks are correlated with future realizations of technology shocks, but uncorrelated with current and past realizations of technology shocks.

by movements in stock prices unrelated to the current change in total factor productivity (TFP). This shock is highly correlated with long-run changes in TFP. The current study is complementary to the work of Beaudry and Portier. It provides some additional support for their structural estimation using a different approach.

This Chapter formulates and implements an empirical test of a necessary condition for existence of news about future technology. The test exploits an empirical implication of news for conditional forecasts of exogenous technology shocks in an environment with optimizing agents and rational expectations. If technology shocks are anticipated in advance, macroeconomic variables should help to predict future technology shocks. The absence of news (or technology shock unpredictability hypothesis) is tested using statistical methods developed in the finance literature on predictability of asset returns.

Empirical tests are based on measures of total factor productivity. Previous studies have criticized earlier TFP measures as contaminated by changes in exogenous demand or factor prices. For example, Hall [1990] found a high correlation between TFP measures and oil prices and military spending. Evans [1992] could not reject exogeneity of TFP relative to money, nominal interest rates and government spending. TFP measures used in this Chapter are free from some of the previous criticism. In particular, these measures can no longer be predicted from changes in government spending or oil prices.

The main empirical evidence is consistent with the existence of news shocks about future technology.[2] A number of macroeconomic variables, including consumption, investment and stock market prices, have predictive content for future TFP growth. Average cumulative growth of TFP can be forecast up to two years ahead (and sometimes even further). Statistical results are also economically significant, as conditional forecasts can explain on average 20% of variation of TFP growth. Overall, the results provide foundations for further examination of the role of news shocks in business cycles.

The rest of the Chapter is organized as follows. Section 3.2 derives the test hypothesis of technology shock unpredictability and outlines two methods of its implementation. Section

[2] Alternative interpretations of results are discussed in Section 3.5.

3.3 describes the data and checks whether the proposed measures of technology shocks are exogenous to other observed economic shocks. Section 3.4 summarizes empirical results. Section 3.5 discusses implications for policy and model evaluation and concludes.

3.2 A Necessary Condition for Existence of News about Future Technology

This Chapter exploits an empirical implication of news for predictability of technology shocks in an environment with optimizing agents and rational expectations. For a given technological measure, absence of news becomes a refutable hypothesis. The test of this hypothesis can be implemented without a direct measure of news, as agents in the economy would reveal the presence of news through their actions. This section derives the hypothesis of technology shock unpredictability and describes two methods of its implementation.

3.2.1 Hypothesis Formulation

Under rational expectations, conditional forecasts about future technology shocks are based on the time series properties of these shocks. A formulation of the statistical hypothesis thus requires a description of these properties. It is assumed that technological change $A_t = \exp(Z_t)$ follows a logarithmic random walk with a positive drift:[3]

$$Z_t = \gamma + Z_{t-1} + \varepsilon_t, \ \gamma > 0 \tag{3.1}$$

$$E(\varepsilon_t) = 0, \ E\varepsilon_t^2 = \sigma_\varepsilon^2 \tag{3.2}$$

White noise impulses ε_t have permanent effects on the level of technology. The drift γ is the average growth rate of aggregate technology. It captures technological improvements over time. The actual growth rate is stochastic. In any period it deviates from the average growth rate γ by amount ε_t:[4]

$$g_t \equiv \Delta Z_t = Z_t - Z_{t-1} = \gamma + \varepsilon_t \tag{3.3}$$

[3] This specification is consistent with technology measures used in this study.

[4] All growth rates in this Chapter are approximated by log-differences, denoted by Δ.

Using the same terminology as in the rest of the thesis, the stationary process g_t is referred to as a technology shock.

The test hypothesis is formulated for the average growth rate of technology, computed over $k \geq 1$ periods ahead (or the average future technology shock over k periods):

$$g_{t+1,k} = \frac{1}{k}(Z_{t+k} - Z_t) = \gamma + \frac{1}{k}\sum_{j=1}^{k} \varepsilon_{t+j} \tag{3.4}$$

The forecast of $g_{t+1,k}$ based on the period t information set Ω_t is:

$$E[g_{t+1,k}|\Omega_t] = \gamma + \frac{1}{k}\sum_{j=1}^{k} E[\varepsilon_{t+j}|\Omega_t] \tag{3.5}$$

Virtually no a priori information is available to suggest how far in advance technology impulses could be anticipated.[5] If several news shocks exist, related to different forecasting horizons, then their effects should be captured in a cumulative growth rate. Focus on the average cumulative growth rate facilitates a comparison between different horizons.

If all technology impulses are unpredictable (i.e. if there are no news shocks), then forecasts of $k-$period ahead average technology growth is time-invariant for any horizon k. This property constitutes the null hypothesis of technology shock unpredictability:

$$H_0: \quad E[g_{t+1,k}|\Omega_t] = \gamma \tag{3.6}$$

An alternative hypothesis is that some variables in the information set Ω_t help to forecast the future average technology growth:

$$H_1: \; E[g_{t+1,k}|\Omega_t] = \gamma_t \tag{3.7}$$

Hence, there is a time variation in the conditional forecasts.

To disprove the time-invariance of forecasts, it is sufficient to find variables contained in the period t information set that help to predict the future average technology growth. The idea behind the empirical methodology is to construct linear forecasts of $g_{t+1,k}$ based

[5] The only indirect evidence is provided by Beaudry and Portier [2004a]. Based on the moment estimation of a general equilibrium model, they conclude that four quarters may be a reasonable period of anticipations.

on variables from Ω_t and test whether they are constant over time. The empirical implementation of the hypothesis is based on two methods, developed in the finance literature for testing predictability of asset returns. The first method is based on a so-called long horizon regression of the average $k-$period ahead technology growth on forecasting variables. The second method is based on vector autoregressions ($VARs$). The next two subsections outline both methods.

3.2.2 Long-Horizon OLS Regression

The simplest way to construct a linear forecast for $g_{t+1,k}$ is based on a long-horizon regression.[6] This regression corresponds to an ordinary least squares (OLS) regression of the average technology growth rate over $k-$period ahead on a $n \times 1$ vector of forecasting variables x_t :

$$g_{t+1,k} = \gamma_k + b_k' x_t + u_{t+k} \tag{3.8}$$

The null hypothesis of technology shock unpredictability is equivalent to the hypothesis that all slope coefficients in regression (3.8) are equal to zero, $b_k = 0_{n\times 1}$. This hypothesis can be tested using asymptotic forms of t and F statistics. The OLS estimator of regression coefficients is asymptotically consistent. The statistical inference, however, requires a covariance matrix correction.

The error term u_{t+k} is an element of period $t+k$ information set. Since the data for growth rates are sampled more finely than the forecasting horizon, there is a problem of serially correlated residuals. The problem is easiest to see under the null hypothesis. In this case, the forecast error is equal to the average of future technological impulses $u_{t+k} = \frac{1}{k}\sum_{j=1}^{k} \varepsilon_{t+j}$. It is correlated with $(k-1)$ previous error terms.[7] The serial correlation is absent only when the forecasting interval exactly equals the frequency of data sampling $(k=1)$. Under the alternative hypothesis, the error term u_{t+k} can be arbitrarily serially correlated if forecasting variables in vector x_t do not capture all variation

[6] The method was proposed by Fama and French [1988, 1989] to evaluate predictability of asset returns.
[7] It is easy to verify that $E\left(\eta_{t+k}\eta_{t+k-i}\right) = \frac{1}{k^2}\left(k-i\right)\sigma_{\varepsilon}^2$ for -$(k-1) \le i \le k-1$.

in the conditional mean. To overcome the problem of serial correlation and potential heteroskedasticity, the asymptotic covariance matrix is corrected with the Newey-West estimator. For horizon k, $2k+1$ lags are used to perform the correction. The marginal significance of statistics is determined on the basis of bootstrap values, to account for the sample size.

3.2.3 VAR-based Approach

An alternative method estimating linear forecasts is based on vector autoregressions.[8] This method has three advantages over long-horizon OLS regressions. First, it permits forecasts for any possible horizon using the estimates of unconditional covariances and variances without actually measuring data over a long horizon. Second, it allows for endogenous feedback among variables, and thus eliminates a possible estimation bias present in OLS regressions. Third, the method employs the full covariance structure, not only one-equation error term variance.

Vector autoregressions consist of TFP growth and an $n \times 1$ vector of forecasting variables:

$$\begin{bmatrix} \Delta Z_t \\ x_t \end{bmatrix} = \mathbf{c} + \mathbf{B}(L)\,\Delta Z_{t-1} + \mathbf{A}(L)\,x_{t-1} + \mathbf{u}_t \tag{3.9}$$

Here $\mathbf{c}$ is a $(n+1) \times 1$ vector of constants, $\mathbf{u}_t$ is $(n+1) \times 1$ vector white noise,

$$E(\mathbf{u}_t) = 0, E\left(\mathbf{u}_t \mathbf{u}_\tau'\right) = \begin{cases} \Sigma_u, & \text{for } t = \tau \\ 0, & \text{otherwise} \end{cases}$$

with Σ_u an $(n+1) \times (n+1)$ symmetric positive definite matrix. $\mathbf{B}(L)$ and $\mathbf{A}(L)$ are matrix lag polynomials. The number of lags is assumed to be sufficient to summarize all dynamic correlations of between regressors.

Forecasting variables have no predictive power for TFP growth if all coefficients on lagged x are equal to zero in the TFP growth equation (the first equation of the VAR):

$$\Delta Z_t = \gamma + \beta(L)\,\Delta Z_{t-1} + \alpha(L)\,X_{t-1} + u_t \tag{3.10}$$

[8] This methodology was developed and applied to test stock market returns predictability by Hodrick [1992] and Patelis [1997] among others.

The joint significance of coefficients in $\alpha(L)$ can be tested using an $F-$test.[9]

VAR estimates can be used to construct projection coefficients analogous to the ones in long-horizon OLS regressions.[10] Specifically, a linear projection of $g_{t+1,k}$ on the i^{th} forecasting variable $y_t(i) \in \{g_t, x_t\}$ for horizon k can be consistently estimated by

$$b_k(i) = \frac{1}{k} \frac{e_1' \left[\hat{\Gamma}_1 + ... \hat{\Gamma}_k\right] e_i}{e_i' \hat{\Gamma}_0 e_i}, i = 1, ..., n+1 \tag{3.11}$$

where $\hat{\Gamma}_j$ is the OLS estimate of the j^{th} order autocovariance matrix corresponding to a companion form VAR(1) representation of (3.9), and e_j is $(r \times 1)$ indicator vector which takes a value of one at the cell that corresponds to the position j.[11] These projection coefficients are used in Section 3.4.2 to compare forecasts implied by the two methods.

Other useful statistics measure the explanatory power of the VAR analogues of long-horizon OLS regressions. The R^2 from the implied regression of $g_{t+1,k}$ on forecasting variable $y_t(i)$, controlling for other variables in the VAR, is the ratio of the explained sum of squares of the average $k-$period ahead growth rate $g_{t+1,k}$ to its total sum of squares,

$$R_k^2(i) = \frac{\| b_k(i) y_t(i) \|^2}{\| g_{t+1,k} \|^2} \tag{3.12}$$

where $\| \cdot \|$ denotes the Euclidean norm.[12] The subscript k refers to the forecasting horizon k. This measure differs from the R^2 implied by the VAR. A consistent estimate of (3.12) is given by

$$R_k^2(i) = b_k(i)^2 \left(\frac{e_i' \hat{\Gamma}_0 e_i}{\frac{1}{k^2} e_1' V_k e_1} \right) \tag{3.13}$$

where

$$V_k = k\hat{\Gamma}_0 + \sum_{j=1}^{k-1} (k-j) \left[\hat{\Gamma}_j + \hat{\Gamma}_j{}'\right] \tag{3.14}$$

is the unconditional sample variance of $\mathbf{d}_{k,t} \equiv (\mathbf{y}_{t+1} + ... + \mathbf{y}_{t+k})$, vector $\mathbf{y}_t' \equiv [\Delta Z_t, x_t']$ and $g_{t+1,k} = \frac{1}{k} e_1' \mathbf{d}_{k,t}$.

[9]This is essentially a Granger causality test. Variable X fails to Granger-cause g if for all $s > 0$ the mean squared error (MSE) of a forecast of g_{t+k} based on $(g_t, g_{t-1}, ...)$ is the same as the MSE of a forecast of g_{t+k} that uses both $(g_t, g_{t-1}, ...)$ and $(x_t, x_{t-1}, ...)$.

[10]Derivations of $VAR-$based projection and R^2 coefficients are described in Hodrick [1992].

[11]For example, $e_1' = \begin{bmatrix} 1 & 0 & 0 & ... & 0 \end{bmatrix}$, $e_2' = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \end{bmatrix}$, etc.

[12]That is, $\| x_t \| \equiv \left(\Sigma_{j=1}^T x_t^2\right)^{1/2}$.

3.3 Data Description

This section describes the data and checks whether the proposed measures of technology shocks are exogenous to other observed economic shocks.

3.3.1 Measures of Technology Shocks

Testing predictability of technology shocks requires availability of their measures. This Chapter exploits growth accounting[13] implications in measuring aggregate technological change as total factor productivity. TFP is defined as output per unit of combined inputs. This notion captures the overall efficiency with which inputs are transformed into outputs. Thus, measured TFP growth reflects the change in output not accounted for by changes in observed inputs.

Under assumptions of constant returns to scale, perfect competition and observability of all relevant inputs, TFP provides a measure of technological change. Technology shocks then correspond to TFP growth. A test of the null hypothesis of technology shock unpredictability is equivalent to the null hypothesis of no predictability of TFP growth. Measures of technology shocks are unaffected by potential presence of news. Several authors criticized earlier TFP measures as being contaminated by factors unrelated to a true shift in the aggregate production function.[14] Measures used in this study are free from some of the previous objections, as argued in the next section.

Predictability of TFP is investigated for annual and quarterly data. The annual series are taken from the Bureau of Labor Statistics (BLS).[15] The broadest data coverage includes the private business (PB) and nonfarm private business (NF) sectors of the US economy. These measures are currently available from 1949 to 2002.

The quarterly series are constructed as analogues to the BLS annual measures. Each measure is defined as a ratio of real GDP over the combined input quantity index for the

[13] Solow [1957].

[14] See, for example, Hall [1990], Evans [1992], Burnside, Eichenbaum and Rebelo [1996].

[15] The BLS uses a term multifactor, rather total factor productivity to describe the same concept. The BLS methodology is described in details in BLS [1997].

corresponding sector. Output, labour and capital inputs are sector specific. Real GDP measures are from the Bureau of Economic Analysis (BEA). The BLS annual index for labour input controls for the quality of labour force. Unfortunately, these measures do not exist at quarterly frequency. Therefore, labour input is measured by the BLS total hours. Quarterly series for capital input are obtained by interpolating the BLS annual measures of capital services using the BEA series for aggregate investment. Capital and labour inputs are aggregated using Tornquist index and cost shares of labour and capital as weights. Labour shares are assumed to be constant during a year and equal to the annual labour share in cost BLS index for the corresponding sector. Capital shares are computed as one minus labour shares.[16]

Basic descriptive information about annual and quarterly measured TFP growth are summarized in Table 4.1. Growth rates are approximated by log-differences and expressed in percentage terms. The table reports average growth rates, values of their $t-$statistics and standard deviations over sample periods (columns 2 and 3). The quarterly measures imply a little higher annual growth rates of 1.57% and 1.33% for private and non-farm private sectors. The degree of correlation between the BLS series and annual aggregators of the quarterly measures is relatively high: 0.75 and 0.71 for private business and non-farm private business sectors. The last column of Table 4.1 supports the technology shocks specification (3.3). The first order autoregressive coefficients are insignificant for all TFP measures.

3.3.2 Exogeneity of TFP to Observed Shocks

Previous studies have argued that the null hypothesis of technology shock unpredictability may fail simply because TFP-based measures are contaminated by exogenous changes in aggregate demand or factor price movements. Hall [1990] has stressed that any candidate measure of technology shock must be uncorrelated with any variable unrelated to shifts in the production function. He has rejected this invariance property for the Solow residuals in

[16] These measures of TFP are closely correlated with the Solow residuals with the constant capital share of 0.32 and 0.31 for private business and nonfarm private business sectors: $SR_t = Y_t \left[K_t^{\alpha} H_t^{1-\alpha}\right]^{-1}$.

US industries. These measures were correlated with oil prices, military spending and the political party of the president. Evans [1992] has tested the unpredictability hypothesis using VAR specifications similar to (3.9). According to his results, money, nominal interest rates, and government spending had substantial predictive power for impulses to Solow-Prescott[17] residuals.

This section applies exogeneity tests along the lines of Hall and Evans to the proposed measures of TFP. In contrast to the previous findings, the measures fair much better relative to exogeneity tests.

It is rather challenging to construct exogenous shocks. However, even without their direct measures, the econometric exogeneity of TFP to other shocks can be tested using variables believed to reflect effects of those shocks. Four variables are used to proxy for other economic shocks. Changes in real government expenditures (GOV) and military spending (DEF) capture changes in fiscal policy. Changes in relative price of oil (OIL), measured by the ratio of producer price index for crude petroleum over implicit GDP deflator, capture changes in factor supplies. Hamilton[18] has put forward institutional, historical and statistical arguments to confirm that oil price changes were exogenous with respect to the US economy over the period 1948-1972. He has also established[19] that the same arguments are no longer valid for post-1973 data. Thus, exogeneity of TFP to oil prices is tested only for the 1948-1972 sample. Monetary policy shocks (RRM) are measured by an indicator proposed by Romer and Romer [2003]. Conventional measures of monetary policy, such as the Federal Reserve's funds rate, incorporate information about future economic development available to central banks. Romer and Romer try to isolate anticipatory policy movements by controlling for the Federal Reserve's forecasts of output and inflation.[20]

[17] Prescott [1986].

[18] See, for example Hamilton [1983, 1985].

[19] Hamilton [2003].

[20] The RRM indicator is a residual from an OLS regression of the Federal Reserve's intended funds rate changes, constructed by Romer and Romer, on the Federal Reserve's internal forecasts of inflation, real output and unemployment.

Table 4.2 summarizes results of exogeneity tests. Each panel corresponds to a particular measure of TFP. Each row in the panel is associated with a variable X proxying for another exogenous shock. The monetary policy shock RRM is measured in levels, and all other variables are measured in log-differences.

The first exogeneity test investigates the invariance properties of the proposed TFP measures. To test for the absence of a contemporaneous correlation between TFP growth and a proxy for an exogenous shock X, the regression coefficient of the TFP growth on the variable X is computed.[21] The inference is based on the $t-$test, as suggested by Hall [1990]. The $p-$values for the tests are reported in column 6. The null hypothesis of no correlation is rejected when a $p-$value falls below the desired level of significance. For example, $p = 0.01$ for the regression coefficient of the annual TFP growth for private business sector on RRM means that the null hypothesis of no correlation can be rejected at significance levels of 1% and higher. Using another interpretation, $p = 0.70$ for the regression coefficient of the quarterly TFP growth for the same sector on RRM means that the empirical $t-$statistic can be observed with probability 0.70. Based on the reported statistics, the invariance of TFP to observed measures of other shocks cannot be rejected for all quarterly measures of TFP at the conventional levels of significance. For annual measures, only a correlation with the monetary policy indicator cannot be rejected.

The second exogeneity test investigates the Granger-causality in bivariate relations between a TFP growth and a variable X. The test is computed on the basis of a VAR specification, which includes a constant and lagged values of the TFP growth and X. Annual specifications use one lag, and quarterly specifications use four. Column 2 gives p-values for the F test that X variable fails to Granger cause the TFP growth (i.e. that all coefficients on lagged values of X in the TFP growth equation are zero).[22] If a TFP measure is exogenous to other shocks, then a $p-$value should be large. The null is rejected if the $p-$value falls below the desired level of marginal significance. Column 4 reports

[21] Each regression also includes a constant term.

[22] The test statistic F has a $F(m, T-2m-1)$ distribution, where T is the sample size, m is the number of lags used in a VAR.

the R^2 coefficients for TFP growth equations. Granger causality tests are known to be sensitive to whether specifications are run in growth rates or levels. The robustness of results is checked for specifications that include logarithms of TFP and one of the three variables GOV, DEF or OIL. Since RRM is a stationary process, this specification is not applicable to this variable. The test results are given in the last column of Table 4.2. Column 3 reports the $p-$values for reverse Granger causality tests that TFP growth have no predictive power for the $X-$variable.

According to the Granger causality tests, no measure of TFP can be predicted from the past values of GOV, DEF or OIL. The conclusion applies to both log-difference and log-levels specifications. These variables also have little explanatory power, as measured by R^2.

The monetary policy indicator RRM deserves special attention. It helps to forecast all measures of TFP, but cannot itself be forecast from the past values of TFP.[23] In bivariate relations, Granger causality may arise because of the forward-looking behaviour of a forecasting variable or omitted information. While the RRM indicator was derived by controlling for the Federal Reserve's internal forecasts, it is still possible that it contains some anticipatory movements. The anticipatory policy movements would represent the additional information about the future of the economy available to the Federal Reserve and not included in the forecasts of output or inflation. In particular, monetary shocks may reflect the presence of news about future technology.[24]

If the monetary measure captures news about technological impulses up to n periods ahead, then this measure should be uncorrelated with technological impulses $n+1$ periods ahead. Evans [1992] has tested this possibility by varying values of n and dismissed it on the ground that nominal variables predicted his measures of TFP up to seven quarters ahead. Column 5 reports the highest lag $N \geq 0$[25] for which X has some predictive power

[23]The null hypothesis of technology shock unpredictability is rejected at 10% significance for annual and 1% significance levels for quarterly data.

[24]King and Plosser [1984] and Litterman and Weiss [1985] use a similar argument of a forward-looking behaviour to explain why nominal variables Granger-cause real variables.

[25]Values of N are increased from zero incrementally until the null hypothesis of no predictability of TFP

at 5% significance level for the TFP growth rate in the regression:

$$\Delta Z_t = \gamma + \beta(L)\Delta Z_{t-1} + \alpha(L) X_{t-1-N} + u_t \tag{3.15}$$

For quarterly specifications, the monetary policy indicator RRM helps to forecast the TFP growth up to two quarters.

To summarize, the proposed measures of TFP fare much better against the exogeneity tests than other measures of shocks at both annual and quarterly frequencies. In the rest of the Chapter, the null hypothesis of technology shock unpredictability is tested on the basis of endogenous variables.

3.3.3 Forecasting Variables

The list of forecasting variables that are used to evaluate the predictability of TFP includes several real and nominal measures of the US economic activity. Real measures (in 2000 chained dollars) of GDP for private business and non-farm private business sectors $(OUTP^*)$, private consumption of nondurable goods $(CNDR)$ and services $(CSER)$, private residential investment $(IRES)$, private nonresidential fixed investment in equipment $(IEQP)$ and structures $(ISTR)$ are from the BEA. Employment indices for private business and nonfarm private business sectors $(EMPL^*)$, and consumer price index for all items (CPI) are from the BLS. Inflation measure $(PCPI)$ is computed as log first-difference of the CPI index. The M1 stock of money $(M1)$, the effective federal funds rate $(FFRT)$ and the University of Michigan index of consumer sentiment $(CEXP)$ are from the database of the Federal Reserve Bank of St.Louis. The interest rate spread $(SPRD)$ is computed as a difference between 10 year treasury bonds and the federal funds rate. The stock market price $(STKM)$ is the S&P's common stock price index from the DRI Economics database.

All series are seasonally adjusted. Quarterly series, when not available directly, are computed as averages of the corresponding monthly series. Annual series, when not

cannot be rejected.

available explicitly, are computed as averages of the corresponding quarterly series.

3.4 Empirical Results

3.4.1 Exploratory Data Analysis

Table 4.3 presents an exploratory analysis of the predictive content of each forecasting variable relative to TFP measures. The first panel gives results for quarterly and the second - for annual series. Columns 3, 4, 6 and 7 report asymptotic p-values for bivariate Granger causality tests of X relative to a TFP measure for private (PB) or nonfarm private (NF) business sectors. The tests are based on the same VAR representation as in section 3.3.2. Annual specifications use one lag, and quarterly specifications use four. In growth rates specifications all variables except for $FFRT$, $PCPI$ and $SPRD$ are computed as log-differences. $\Delta FFRT$, $\Delta PCPI$ and $\Delta SPRD$ correspond to the first differences of these variables. In log-level specification, results for CPI index, rather than inflation, are reported in the row for $(P)CPI$. Npb and Nnf are the highest lags for which ΔX has a predictive power for the TFP growth at 5% significance level. Output and employment series are for the same sector as the corresponding TFP measure.

Table 4.3 indicates that the null hypothesis of technology shock unpredictability is rejected for both quarterly and annual TFP measures at the conventional levels of significance. The conclusions are generally robust between log-difference and log-levels specifications. Only a change in inflation does not help to predict quarterly TFP growth. However, CPI Granger causes TFP in quarterly log-level specifications. The results are reversed for annual specifications. There is a variation in the number of periods for which each variable X has any forecasting content. Due to institutional or technological reasons, some variables may react to changes in economic conditions faster than others.

3.4.2 Predictability Regressions

Table 4.4 reports test results of the null hypothesis of technology shock unpredictability based on OLS regressions (3.8). For a forecasting horizon k, reported in column 1, the dependent variable is an average k-period ahead TFP growth for the corresponding sector.

Three model specifications are considered, which differ by the choice of regressors. Each specification includes a constant and three forecasting variables. Model 1 captures consumption and investment decisions. It uses growth rates of consumption of services, residential investment and nonresidential investment in equipment. Variables in model 2 include the change in the federal funds rate, growth rates of the stock market index and consumption of services. The first two variables are supposed to capture the forward-looking behaviour of agents. The present value theory of stock prices implies that movements in the stock market reflect changing expectations about future earning of publicly traded corporations. Earnings are linked to firms' productivity. Existence of news about future technological change are likely reflected in the stock market price. The federal funds rate is believed to capture the anticipatory information available to the Fed. Model 3 is similar to model 2, except that the consumption measure is replaced by the growth rate of the sectoral employment.

To facilitate a comparison across specifications, all models are run over the same sample period of 1955-2002. The dates are constrained by the availability of the federal funds rate series. As computing the $k-$period ahead growth rates requires a loss of the first k observations, the actual sample for each OLS regression is adjusted accordingly. That is, when k is equal to two quarters, the regression sample is $1955:3-2002:4$. Thus, each series has $T-k$ observations, where $T=112$ for quarterly and $T=48$ for annual data.

For each model $j\in\{1,2,3\}$ three statistics are reported: $F[j]$, $p[j]$ and $R^2[j]$. The first is the F statistic for the joint hypothesis that all slope coefficients in regression (3.10) are equal to zero. The statistic is computed with the Newey-West covariance matrix, and it is asymptotically distributed as $\chi^2(T-k,3)$. The test inference is based on the bootstrap $p-$values. The last statistic is the R^2 coefficient from the OLS regression.

Overall, the test results reject the null hypothesis of technology shock unpredictability, both at quarterly and annual frequencies. For quarterly series, the evidence is overwhelmingly against the null: all $p-$values are less than 1% marginal significance level.

Predictability horizons for annual measures differ across the models. Models 1 and 2 imply predictability for one or two year horizons, while variables in model 3 have a forecasting content up to four years. Since these models differ only in one variable, it may be conjectured that employment changes may have a special impact on TFP in the medium run. This can be consistent with endogenous growth theories that emphasize the learning-by-doing channel.[26] The statistical significance is also quantitatively important, as the R^2 coefficients for annual specifications of model 3 range from 0.28 to 0.39. However, the same specification is the least successful in explaining the variation in the TFP growth in the short run, up to four quarters. On the basis of the R^2 measures, model 1 gives the best forecast for quarterly frequency.[27]

Table 4.5 reports test results for the null hypothesis of technology shock unpredictability based on VARs (3.9). Three model specifications are considered, which differ by the choice of regressors. Forecasting variables are the same as in the OLS regressions, with an exception that lags of each variable and lags of TFP growth are included. Table 4.5 consists of two panels. Panel 1 corresponds to quarterly, and Panel 2 to annual data. The upper part of each panel reports the long-horizon R^2 coefficients implied by the VAR (3.12). While these statistics are uniformly higher than the ones in Table 4.4, their values are not adjusted for the increased number of regressors. The bottom part of each panel reports two F statistics for the joint hypothesis of zero coefficients on forecasting variables. F_2 excludes, but F_1 includes lagged TFP growth. For these statistics, the asymptotic $p-$values are reported in brackets.

Overall, the test results based on vector autoregressions lead to a similar conclusion as the test results based on OLS regression. In all specifications, the null hypothesis technology shock unpredictability is rejected.

Figures 3.1 and 3.2 give another perspective on the test results. Figures plot the actual growth rate of the private business sector TFP (dotted lines), along with predicted values

[26] See, for example, Copper and Johri [1999] or Chang, Gomes and Schorfheide [2002].

[27] Specifications that include the current TFP growth into regressions have also been investigated, and the conclusions are similar.

based on OLS (dark blue lines) or VAR (light blue lines) regressions. For a VAR, the implied projection coefficients are constructed using (3.11). Each figure has three panels, which corresponds to different forecasting horizons. Forecasts for Figure 3.1 are derived on the basis of model 1. Forecasts for Figure.3.2 are derived on the basis of model 3. Figures indicate a large similarity between the OLS and VAR forecasts. They also illustrate how well the predictive series capture variations in actual growth rate.

3.5 Conclusion

This Chapter examined whether the US post World War II experience was consistent with news about future technological change. It tested the hypothesis that in the absence of news forecasts of future technology shocks had to be time-invariant. Using measures of total factor productivity, the hypothesis of technology shock unpredictability was rejected.

One interpretation of the empirical results is that econometric endogeneity of TFP is due to news shocks. The test results are equally consistent with predictions of endogenous growth theories. TFP measures reflect the joint effects of many factors, possibly including research efforts, skills of the work force, organizational practices and capital investment. Learning by doing is one channel through which increased inputs or output today can influence future level of TFP. The empirical results indicate that forecasting properties of changes in consumption and employment with respect to TFP growth are important at different horizon. This suggests that both explanations for TFP endogeneity are likely to operate in practice. Ideally, we would like to separate their respective contributions. However, the question is left for future research.

Even without controlling for endogenous TFP determination, the results of TFP growth predictability have two interesting implications. The first is related to model evaluation, and the second- to policy analysis.

News shocks represent variables that help to forecast future technological change. From the perspective of model evaluation, omission of news leads to a misspecification of expectation formation. Differences in model predictions with and without news can be quite substantial.[28] We may never know what variables constitute news, as these shocks aggregate information dispersed across agents in the economy.[29] In principle, R&D input or changes in demand can be uncorrelated with current but correlated with future technological change. If these factors are not modelled explicitly, their effects may be captured by news shocks. Boileau and Normandin [2002*a*] describe a method for model evaluation without direct measures of news shocks. Their method exploits the property, studied in this Chapter, that endogenous actions reveal presence of news.

On the policy side, TFP measures are now produced by statistical agencies of the United States, Canada and a number of other countries. Forecasts of TFP growth have become an important input into economic planning. For example, the Congressional Budget Office incorporates TFP growth into its budget projections. The results of this Chapter suggests that forecasts of TFP growth can be improved on the basis of macroeconomic variables, even though the exact forces of TFP determination may not be clear. Improvement of TFP growth forecasts should facilitate policy design.

[28] See Boileau and Normandin [2002*a*, 2002*b*].

[29] "Summing over consumers, aggregate consumption can reveal information about future aggregate activity, although neither consumers in the economy nor economists who study it can name what the crucial pieces of information are." Cochrane [1994, *p*.350]

Chapter 4

News and Sunspot Shocks in Linear Rational Expectations Models

4.1 Introduction

Understanding what role changes in beliefs play in generating business cycles is impossible without a precise definition of such changes. This Chapter compares two conceptually different approaches to modelling beliefs. The first approach exploits the possibility that agents can learn about future changes in economic environment in advance, but their anticipations may occasionally be incorrect. Beliefs of this type are represented by news shocks. The second approach attributes changes in beliefs to extrinsic uncertainty. Beliefs of this type are called sunspots. A number of previous studies have explored effects of sunspots and news shocks separately. However, this is the first work that discusses their similarities and differences.

The first contribution of this Chapter is to formalize a notion of news shocks. News shocks are modelled as outcomes of learning process, based on exogenous signals. The signals are uncorrelated with past and current realizations of fundamental shocks, but correlated with future ones.[1] This definition is closest to the one of Beaudry and Portier [2004a].

[1] Fundamental shocks represent stochastic changes in preferences and production technologies.

News shocks can be alternatively represented as perfectly anticipated impulses with different timing of their realization, as in King and Plosser [1985] or Love and Lamarche [2001]. The modelling strategy in this Chapter facilitates a comparison between changes in beliefs due to anticipations and extrinsic uncertainty.

The main implication of news shocks is their effects on conditional forecasts of future fundamental shocks. The Chapter describes how to define the joint process for news and fundamental shocks that correctly represents innovations to agents' information set. Two assumptions are important. First, actual realizations of exogenous impulses must be observable. Thus, eventually agents must learn whether their original forecasts were true or not. Second, the learning problem is about exogenous variables. This property enables a separation of the learning or signal extraction problem from determination of equilibrium endogenous variables.

The second contribution of this Chapter is to propose a computationally simple framework for solving linear rational expectations models with news. The framework adopts existing algorithms of Blanchard and Kahn [1980], Sims [2001] and Lubik and Schorfheide [2002]. There are several advances relative to the previous literature. Business cycle models are usually solved numerically, as analytical solutions exist for only very restrictive specifications. The proposed framework allows one to find model equilibria with multiple news shocks easily. Solution methods used in other studies are either not discussed or somewhat complex.[2] Further, their attention is focused on news shocks about only one fundamental shock. The Chapter also extends the previous work to incorporate news shocks into models with multiple equilibria. This extension enables an explicit comparison between two approaches to modelling changes in beliefs.

The third contribution of this Chapter is to compare news and sunspot shocks along several dimensions. Both types of shocks represent changes in beliefs, independent of current and past fundamentals, without departing from the rational expectations assump-

[2]For example, the methods, proposed here, are simpler than the deterministic path method used by Love and Lamarche [2001].

tion. Sunspots exist only in models with multiple equilibria, while news shocks can be incorporated into all economic environments. The two types of shocks have different predictions for impact effects on endogenous variables and autocovariance properties of equilibria. In models with unique equilibria, the dynamic effects of news shocks are perfectly determined. In models with multiple equilibria, dynamic effects of sunspot shocks are perfectly determined, while dynamics of news are restricted, but not determined without extra assumptions. News shocks enrich dynamic correlation of model solutions, as agents generally start acting upon new information about the future immediately.

The proposed solution techniques are applied to an example of the New Keynesian model. In this model, a single parameter determines whether the underlying economy is regular or irregular. The indeterminacy arises when the central bank is not aggressive enough towards inflation. The example is simple enough to derive some analytical results. Numerical simulations of the model demonstrate that differences between news and sunspot shocks with respect to impulse dynamics and autocovariance properties of equilibria can be quantitatively significant. These differences can be used to identify which type of changes in beliefs (if any) are consistent with data in the context of a specific stochastic dynamic general equilibrium model.

This Chapter is organized as follows. Section 4.2 discusses a characterization of linear rational expectations models and introduces fundamental, news and sunspot shocks. Section 4.3 derives solutions for models with unique and multiple equilibria. Section 4.4 compares news and sunspot shocks, with a particular emphasis on their effects of model equilibria. Section 4.5 applies the proposed framework to a New Keynesian model. Section 4.6 outlines a possible empirical approach for separation news and sunspot shocks and concludes.

4.2 Characterization of the Models

Linear rational expectations models are commonly written as a system of first order stochastic difference equations:[3]

$$\mathbf{A}E_t\mathbf{y}_{t+1} = \mathbf{B}\mathbf{y}_t + \mathbf{C}z_t \tag{4.1}$$

where $\mathbf{y}_t$ is a vector of endogenous variables and z_t is a vector of exogenous disturbances. Matrices of coefficients $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are typically non-linear functions of model parameters for preferences and technologies. When matrix $\mathbf{A}$ is singular, the system (4.1) contains purely "static" relations between endogenous variables.

Models considered in this Chapter admit three types of exogenous stochastic disturbances: fundamental, news and sunspot shocks. *Fundamental shocks* describe changes in preferences and production technologies. These shocks directly affect endogenous variables through equilibrium conditions (4.1). *News shocks* represent exogenous variables that change agents' beliefs about future realizations of fundamental shocks. They influence endogenous variables only through conditional forecasts of future fundamental shocks. *Sunspot shocks* represent changes in agents' beliefs due to extrinsic uncertainty. They influence equilibrium endogenous variables without any direct appearance in equilibrium conditions. Sunspot shocks play a role only in models with indeterminacy.

The economic structure of models, including all parameters and joint distributions of exogenous shocks, is assumed to be common knowledge. The period t information set Ω_t is shared by all agents. The information set contains current and past realizations of all shocks and endogenous variables. Although the requirement of the common information set may be too strong for some applications, it is maintained here to explore differences between news and sunspot shocks in the simplest environment. Further, expectations are rational. Agents' subjective beliefs about any variable x_{t+j}, held in period t and denoted by $E_t x_{t+j}$, coincide with mathematical expectations conditional on the information set Ω_t.

[3] See, for example, Blanchard and Kahn [1980] or King and Watson [1998]. This first order system is not restrictive, as models with more lags, lagged expectations or expectations of variables farther in the future, can be accommodated by expanding a vector of endogenous variables.

Formally, $E_t x_{t+j} = E\left[x_{t+j}|\Omega_t\right]$, for any j and t.

While there exists a variety of algorithms for solving linear rational expectations models,[4] most of this Chapter works with methods that employ endogenous expectational errors as a solution device. Expectational errors define changes in agents's beliefs about a subset of endogenous variables $\mathbf{x}_t \subseteq \mathbf{y}_t$ between two consecutive periods:

$$\eta_t \equiv \mathbf{x}_t - E_{t-1}\mathbf{x}_t \tag{4.2}$$

Thus, expectational errors depend on both endogenous variables and their expectations, which are generally unknown before the models are solved. The rationality of expectations requires that future expectational errors are unpredictable. That is, $E_t\eta_t + 1 = 0$ for all t.

It is straight forward to put a linear rational expectations model into an alternative representation that emphasizes expectational errors:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi\eta_t, \ t \geq 0 \tag{4.3}$$

where y_t is a $n \times 1$ expanded vector of endogenous variables, which now also includes agents' expectations, z_t is a $l \times 1$ vector of exogenous stochastic disturbances and η_t is a $f \times 1$ vector of expectational errors, defined by (4.2). For example, this representation can be obtained from (4.1) by replacing agents' beliefs $E_t\mathbf{y}_{t+1}$ with a vector $\xi_t \equiv E_t\mathbf{y}_{t+1}$, endogenous variables $\mathbf{y}_t$ with $\xi_{t-1} + \eta_t$ and adding equations related to expectation formation $\mathbf{y}_t = \xi_{t-1} + \eta_t$. Matrices of coefficients Γ_0 and Γ_1 are $n \times n$, Ψ is $n \times l$, and Π is $n \times f$. Initial conditions for values of y are recorded in a vector y_{-1}.

This Chapter uses three specific formats conforming to representation (4.3). Each format complies with requirements of algorithms for solving linear rational expectations models proposed by Blanchard and Kahn, Sims or Lubik and Schorfheide.[5] These algorithms are adopted to solve the models incorporating news shocks.

[4] See King and Watson [1998] and Klein [1997] among others.

[5] The details of the algorithms are given in Blanchard and Kahn [1980], Sims [2001] or Lubik and Schorfheide [2003]. There references are implied implicitly, whenever the authors' names appear in this Chapter.

4.2.1 Shocks and Their Expectations

Fundamental Shocks

The models define environments with k fundamental shocks. These shocks represent stochastic changes in preferences and production technologies. Examples include changes in tastes, marginal product of labour, monetary or fiscal policy, among others. Both current and expected future fundamental shocks may appear in equilibrium equations (4.3).

Fundamental shocks are assumed to be stationary and invertible. To derive implications of news shocks for properties of model solutions, fundamental shocks are parameterized by the following recursive system:[6]

$$\begin{aligned} \zeta_{t+1} &= A\zeta_t + B\varepsilon_{t+1} \\ Z_t &= C\zeta_t \end{aligned} \tag{4.4}$$

Note that, in the tradition of business cycle literature, the term "shock" is used to describe a possibly persistent process. Each fundamental shock is driven by its own impulse, and all impulses are collected in a $k \times 1$ vector ε_t. Impulses are vector white noise and covariance matrix Σ_ε:

$$E\varepsilon_t = 0,\ E\varepsilon_t\varepsilon_t' = \underset{k\times k}{\Sigma_\varepsilon},\ E\varepsilon_t\varepsilon_\tau' = \underset{k\times k}{0} \text{ for } \tau \neq t \tag{4.5}$$

A $q \times 1$, $(q \geq k)$, vector of states ζ_t contains current and possibly past realizations of fundamental shocks. Vector ζ_0 summarizes initial conditions in period zero. Matrix A is $q \times q$, and matrices B and C have the following format:

$$\underset{k\times q}{C} = \begin{bmatrix} \underset{k\times k}{I} & \underset{k\times(q-k)}{0} \end{bmatrix},\ \underset{q\times k}{B} = \begin{bmatrix} \underset{k\times k}{I} \\ \underset{(q-k)\times k}{0} \end{bmatrix} \tag{4.6}$$

Model solutions generally depend on conditional expectations of future realizations of fundamental shocks. The recursive representation (4.4) gives a convenient way for computing these expectations. Iterating the system (4.4) yields the conditional expectations as

[6] Any stationary VARMA process can be easily put into this format.

linear functions of the current and past realizations of fundamentals ζ and expected future fundamental impulses ε:

$$E\left[Z_{t+j}|\Omega_t\right] = CA^j\zeta_t + \Sigma_{i=1}^{j} CA^{j-i}BE\left[\varepsilon_{t+i}|\Omega_t\right],\ j \geq 1 \tag{4.7}$$

News Shocks

News shocks constitute the main departure of this Chapter from most of the literature. Typically, agents are assumed to learn values of fundamental shocks at the time of their realizations. In contrast, agents in the economies considered here learn about fundamental shocks in advance. News shocks represent exogenous variables that change agents' beliefs about future realizations of fundamental shocks. These shocks capture a temporal separation between a period fundamental shocks are revealed to the agents in the model economy and a period fundamental shocks are realized. The main difference between news and fundamental shocks is their effect on equilibrium endogenous variables. New shocks are relevant in equilibrium only through their ability to improve forecasts about future fundamental shocks.

News shocks have been previously considered in the context of particular business cycle models. The definition of news shocks used in this Chapter is closest to the one of Beaudry and Portier [2004a]. It also provides an alternative interpretation for the notion of perfectly anticipated shocks, employed by King and Plosser [1984] and Love and Lamarche [2001].

Definition of news shocks News shocks can be thought of as exogenous signals about future fundamental impulses. These shocks are represented by a stationary vector process s_t with mean $\bar{s}$ and covariance matrix Σ_s. This process is defined by its correlation properties with fundamental impulses ε. For any period $t \geq 0$:

$P1$. News shocks are linearly informative about future realizations of fundamental impulses up to T periods ahead, $0 < T < \infty$. That is, the relation between s and ε is linear, and

$$E\left[s_t\varepsilon'_{t+j}\right] = \Phi_j \neq 0 \text{ for any } 1 \leq j \leq T \tag{4.8}$$

$$E\left[s_t\varepsilon'_{t+j}\right] = 0 \text{ for any } j > T \tag{4.9}$$

$P2$. News shocks are uncorrelated with current and past realizations of ε :

$$E\left[s_t\varepsilon'_{t-j}\right] = 0, \text{ for any } j \geq 0. \tag{4.10}$$

A focus on news about future impulses is without a loss of generality. News shocks could have been alternatively defined by their correlation properties with fundamental shocks Z_t, rather than their impulses. As long as news shocks are uncorrelated with current and past values of fundamental shocks, this approach would have been equivalent. Indeed,

$$E\left[s_t Z'_{t+j}\right] = \underbrace{E\left[s_t Z'_t\right]}_{=0}\left(CA^j\right)' + \sum_{i=1}^{j} E\left[s_t\varepsilon'_{t+i}\right]\left(CA^iB\right)', \text{ for any } t \geq 0 \text{ and } j \geq 1 \tag{4.11}$$

Thus, there exists a correspondence between $E\left[s_t Z'_{t+j}\right]$ and $E\left[s_t\varepsilon'_{t+j}\right]$. Defining news shocks by $P1$ and $P2$ has an advantage of making a role of news in model solutions more transparent.

In all derivations in this Chapter impulses to every shock from period $t+1$ to $t+T$ are associated with its own news. In this case, vector s_t has a dimension $kT \times 1$ with

$$\underset{kT\times 1}{s_t} = \begin{bmatrix} s_t^1 \\ s_t^2 \\ \vdots \\ s_t^T \end{bmatrix}, \text{ and } \underset{k\times 1}{s_t^j} = \begin{bmatrix} s_{1,t}^j \\ s_{2,t}^j \\ \vdots \\ s_{k,t}^j \end{bmatrix} \tag{4.12}$$

The notation $s_{i,t}^j$ indicates a news shock about a realization of $\varepsilon_{i,t+j}$, an impulse for a fundamental shock i $(1 \leq i \leq k)$ in period $t+j$, observed in period t. When news shocks are correlated only with a subset of impulses between periods $t+1$ and $t+T$, vector s_t has less than kT elements. The easiest way to proceed in this case is to solve the model with kT separate news shocks, under the assumption that the extra elements in s_t are uncorrelated with any values of fundamental impulses.

While this Chapter focuses on the reduced form implications of news shocks, it may be interesting to know that news shocks can always be parameterized by the following moving average representation:

$$s_t = \bar{s} + \mathbf{S}\tilde{\varepsilon}_t + \sum_{j=-\infty}^{0} F_j e_{t+j} \tag{4.13}$$

$$\underset{kT\times kT}{\mathbf{S}} = \mathbf{\Phi}\Sigma_{\tilde{\varepsilon}}^{-1},\ \mathbf{\Phi} \equiv E\left[s_t\tilde{\varepsilon}_t'\right] = \begin{bmatrix} \Phi_1 & \Phi_2 & \ldots & \Phi_T \end{bmatrix} \tag{4.14}$$

Vector $\tilde{\varepsilon}_t$ collects fundamental impulses from period $t+1$ to $t+T$,

$$\underset{kT\times 1}{\tilde{\varepsilon}_t} = \begin{bmatrix} \varepsilon_{t+1} \\ \vdots \\ \varepsilon_{t+T} \end{bmatrix},\ E\left[\tilde{\varepsilon}_t\tilde{\varepsilon}_t'\right] = \Sigma_{\tilde{\varepsilon}} = \begin{bmatrix} \Sigma_\varepsilon & 0 & \vdots & 0 \\ 0 & \Sigma_\varepsilon & \vdots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \vdots & \Sigma_\varepsilon \end{bmatrix} \tag{4.15}$$

Vector e_t reflects the notion that news shocks may be imperfect indicators of future fundamental impulses. It consists of orthonormal innovations,[7] uncorrelated with vector ε_t at all lags and leads. Matrix $\mathbf{\Phi}$ is composed of cross- and autocovariance coefficients between new shocks and fundamental impulses. Finally, matrices F_j define possible cross- and autocorrelation of news. It is illustrative to consider several specific examples. For simplicity, all examples deal with a single impulse.

Example 1.

Love and Lamarch [2001] study technology impulses that are perfectly anticipated T periods ahead. In this case, there is only one signal, $s_t^T = \varepsilon_{t+T}$. In terms of (4.13), $\bar{s} = 0$, $\mathbf{S} = \begin{bmatrix} 0 & \iota_T \end{bmatrix}$, $F_j = 0$, and ι_T is a $T \times 1$ vector with values of one as the last element and zeros everywhere else.

Example 2.

In this example, there are two independent news. The first conveys information about ε_{t+1} and the second - about ε_{t+2}. The representation (4.13) is:

$$\begin{bmatrix} s_t^1 \\ s_t^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+2} \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \cdot \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \tag{4.16}$$

[7] That is, $Ee_t = 0$, $E_t\left[e_te_t'\right] = I$, and all elements of e_t are uncorrelated at all lags and leads.

Example 3.

In this example, there is a single news shock, correlated with the fundamental impulse one period ahead. This news shock is serially correlated:

$$s_t = \varepsilon_{t+1} + \sigma_e \sum_{j=0}^{\infty} \rho_e^j e_{t-j}, \ \ |\rho_e| < 1 \tag{4.17}$$

Implications The main implication of news shocks is their effects on agents' beliefs about future fundamental impulses. The best linear forecast of future fundamental impulses, based on the history of news shocks, is a linear function of these shocks:

$$E\left[\varepsilon_{t+j}|s_t, s_{t-1}, ...\right] = \delta\left(j\right)\tilde{s}_t, \ j \geq 1 \tag{4.18}$$

Vector $\tilde{s}_t$ summarizes the history of news shocks relevant for forecasting future impulses:

$$\tilde{s}_t = \left[\begin{array}{cccc} s_t' & s_{t-1}' & \dots & s_{t-T}' \end{array}\right]' \tag{4.19}$$

and the coefficient row vector $\delta\left(j\right)$ is uniquely determined by:

$$\delta\left(j\right) = E\left[\varepsilon_{t+j}\tilde{s}_t'\right] \cdot \left(E\left[\tilde{s}_t\tilde{s}_t'\right]\right)^{-1}, \ j \geq 1 \tag{4.20}$$

Coefficients $\delta\left(j\right)$ simply reflect the fact that rational agents update their expectations upon arrival of new information, taken into account the quality of this information. As fundamental impulses are assumed to be i.i.d., expression (4.18) also defines the best linear forecast based on the whole information set Ω_t. Let ξ_t^j denote such forecast for values of $\varepsilon_{t+j} : \xi_t^j \equiv E\left[\varepsilon_{t+j}|\Omega_t\right] = E\left[\varepsilon_{t+j}|\tilde{s}_t\right]$. Further, let vector ξ_t^ε combine expected values of all impulses from $t+1$ to $t+T$:

$$\xi_t^\varepsilon = \left[\begin{array}{c} \xi_t^1 \\ \vdots \\ \xi_t^T \end{array}\right] \tag{4.21}$$

By construction, the expected values are linear functions of $\tilde{s}_t$:

$$\xi_t^\varepsilon = \boldsymbol{\Delta}\tilde{s}_t \tag{4.22}$$

where matrix $\mathbf{\Delta}$ depends on the characterization of both the covariance matrix of fundamental impulses and the parameterization of news shocks.

$$\mathbf{\Delta} \equiv \left[\begin{array}{cccc} \delta(1)' & \delta(2)' & \ldots & \delta(T)' \end{array} \right]' \tag{4.23}$$

As an illustration, matrix $\mathbf{\Delta}$ takes the following values for examples 1-3 :

$$\mathbf{\Delta}^{ex1} = \left[\begin{array}{cccc} 0 & 0 & \vdots & \iota_T' \\ \ldots & \ldots & \vdots & \ldots \\ 0 & \iota_T' & \vdots & 0 \\ \iota_T' & 0 & \vdots & 0 \end{array} \right], \quad \mathbf{\Delta}^{ex3} = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_e^2 \left[1 - \rho_e^2\right]^{-1}}$$

and

$$\mathbf{\Delta}^{ex2} = \left[\begin{array}{cccc} \frac{\sigma_\varepsilon^2 \sigma_2^2}{\sigma_1^2\sigma_2^2 + \sigma_\varepsilon^2(\sigma_1^2+\sigma_2^2)} & 0 & 0 & \frac{\sigma_\varepsilon^2 \sigma_1^2}{\sigma_1^2\sigma_2^2 + \sigma_\varepsilon^2(\sigma_1^2+\sigma_2^2)} \\ 0 & \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_2^2} & 0 & 0 \end{array} \right], \quad \tilde{s}_t = \left[\begin{array}{cccc} s_t^1 & s_t^2 & s_{t-1}^1 & s_{t-1}^2 \end{array} \right]'$$

Conditional expectations of future fundamental shocks become functions of current and past realizations of these shocks and agents' beliefs about future impulses:

$$E\left[Z_{t+j}|\Omega_t\right] = CA^j\zeta_t + V(j)\underbrace{\mathbf{\Delta} s_t}_{\xi_t^\varepsilon}, \; j \geq 1 \tag{4.24}$$

It is an important property that matrices $V(j)$ are invariant to any parameters of news shocks, except for the number of forecasting periods T. For any $j \geq 1$:

$$\underset{k\times kT}{V(j)} = \left[\begin{array}{cccc} CH_jB & CH_{j-1}B & \ldots & CH_{j-T}B \end{array} \right], \; H_i = \left\{ \begin{array}{ll} A^{i-1}, & \text{if } i \geq 1 \\ \underset{(k\times k)}{0} & \text{otherwise} \end{array} \right. \tag{4.25}$$

The Appendix A describes how to compute these matrices recursively.

Representation (4.24) expresses conditional expectations of future fundamental shocks in terms of variables observed in period t. Without news shocks, expectations of future fundamental shocks are completely determined by the history of their realizations. Impulses become unpredictable, and the second term in (4.24) disappears. News shocks introduce a role of agents' beliefs, which is independent of the current and past fundamentals. Representation (4.24), however, does not make clear that ζ_t and s_t are correlated over time.[8] It is convenient to break this correlation and derive an alternative formulation in terms of expectation revisions innovations.

Innovation Representation for Fundamental and News Shocks The joint representation for fundamental and news shocks is based on the notion of *expectation revisions.* An expectation revision μ_t^j defines an update in the expected value of ε_{t+j} between periods $t-1$ and t :

$$\mu_t^j \equiv E\left[\varepsilon_{t+j}|\Omega_t\right] - E\left[\varepsilon_{t+j}|\Omega_{t-1}\right], j \geq 0 \tag{4.26}$$

The rationality of expectations implies that μ_t^j constitutes a martingale difference sequence.[9] The assumption that realizations of fundamental impulses are observable, combined with the rationality of expectations, leads to a recursive update of beliefs:

$$\begin{aligned} \varepsilon_t &= \xi_{t-1}^1 + \mu_t^0 \\ \xi_t^j &= \xi_{t-1}^{j+1} + \mu_t^j, \ 1 \leq j \leq T \\ \xi_t^T &= \mu_t^T \end{aligned} \tag{4.27}$$

The modified process for fundamental shocks substitutes fundamental impulses ε_t in terms of conditional expectations ξ_t^ε and expectation revisions μ_t. The new representation takes the form:

$$\begin{aligned} \begin{bmatrix} \zeta_{t+1} \\ \xi_{t+1}^\varepsilon \end{bmatrix} &= A_m \begin{bmatrix} \zeta_t \\ \xi_t^\varepsilon \end{bmatrix} + B_m \mu_{t+1} \\ Z_t &= \begin{bmatrix} C & \underset{(kT\times kT)}{0} \end{bmatrix} \begin{bmatrix} \zeta_t \\ \xi_t^\varepsilon \end{bmatrix} \end{aligned} \tag{4.28}$$

where

$$\mu_t = \begin{bmatrix} \mu_t^0 \\ \mu_t^1 \\ \vdots \\ \mu_t^T \end{bmatrix}, \ A_m = \begin{bmatrix} \underset{q\times k}{A} & \underset{q\times k}{B} & 0 \\ 0 & 0 & \underset{k(T-2)\times k(T-2)}{I} \\ 0 & 0 & \underset{k\times k(T-1)}{0} \end{bmatrix}, \ B_m = \begin{bmatrix} \underset{q\times k}{B} & 0 \\ 0 & \underset{kT\times kT}{I} \end{bmatrix} \tag{4.29}$$

[8]Indeed $E\left[\zeta_{t+j}s_t'\right] = \sum_{i=1}^{j} A_\zeta^{j-i} B_\zeta E\left[\varepsilon_{t+i}s_t'\right] \neq 0$, while $E\left[\zeta_t s_{t+j}'\right] = E\left[\zeta_{t-j}s_t'\right] = 0$,

[9]Indeed, by the law of iterated expectations $E\left[\mu_{t+1}^j|\Omega_t\right] = E\left[E\left[\varepsilon_{t+j}|\Omega_{t+1}\right] - E\left[\varepsilon_{t+j}|\Omega_t\right]|\Omega_t\right] = 0$.

Matrices A and B are $(q+kT) \times (q+kT)$ and $(q+kT) \times k(T+1)$. The covariance structure of expectation revisions can be computed from the process for news shocks, using definitions $\mu_t^j = \delta(j)\,\tilde{s}_t - \delta(j+1)\,\tilde{s}_{t-1}$ for $j \geq 1$, and $\mu_t^0 = \varepsilon_t - \delta(1)\,\tilde{s}_{t-1}$.

Alternative Interpretation of News Shocks Under the rational expectations assumption, fundamental impulses can be decomposed into one-step-ahead expectation revisions innovations

$$\varepsilon_t = \mu_t^0 + \mu_{t-1}^1 + \ldots + \mu_{t-T}^T \tag{4.30}$$

Variable μ_t^j can be alternatively interpreted as impulses to fundamental shocks with different timing of their realizations. Under this interpretation, μ_t^0 is unpredictable, but impulses μ_t^j for $j \geq 1$ are perfectly anticipated j periods in advance. King and Plosser [1984] and Evans [1992] use precisely this interpretation for technology shocks.

The interpretation of μ_t as perfectly anticipated innovations of fundamental shocks[10] is indistinguishable from their interpretation as expectation revisions due to existence of noisy signals. The latter interpretation is adopted in this Chapter to facilitate a comparison between changes in beliefs due to anticipations and extrinsic uncertainty. This interpretation can also capture a notion of ex-post mistakes.[11] If the correlation between news shocks and fundamental impulses is not perfect, then occasionally news shocks can be void of information. That is, an anticipated change in a fundamental impulse may in fact be unrealized. Such situation corresponds to the case when expectation revisions in period t offset previous beliefs: $\mu_t^0 = -\left(\mu_{t-1}^1 + \ldots + \mu_{t-T}^T\right)$. Another advantage of modelling news shocks separately is that each fundamental shock is then associated only with a single driving impulse.[12]

[10]The term "innovation" is used to indicate the unpredictability of μ_t: $E_t\mu_{t+1} = 0$.

[11]Recall that Chapter 2 explores the role of beliefs in the US investment boom and bust. Beaudry and Portier [2004*a*] develop and estimate an interesting model in which aggregate fluctuations are driven by technology shocks and noisy news.

[12]However, this impulse is not an innovation.

Sunspot Shocks

Sunspot shocks represent changes in agents' beliefs due to extrinsic uncertainty. In stochastic economic environments with multiple equilibria, identical fundamentals can be consistent with different equilibrium paths of endogenous variables. In part, these differences can be attributed to self-fulfilling beliefs of agents, which influence equilibrium prices and allocations without any direct appearance in equilibrium conditions.[13]

The notion of a "sunspot", as a randomness unrelated to fundamentals, is also referred to in the literature as "self-fulfilling prophecies" (Azariadis [1981]) and "animal spirits" (Howitt and McAffee [1992]). A number of stochastic models with indeterminacy and sunspots exist in the literature.[14]

Typically, sunspot shocks are modelled as a martingale difference sequence. Following Lubik and Schorfheide, this Chapter adopts this assumption. That is, sunspots are described by a zero mean $p \times 1$ vector v_t such that $E\left[\nu_{t+1}|\Omega_t\right] = 0$.

4.2.2 Model Formats

This subsection describes differences in model formats.

BK-format

The solution algorithm of Blanchard and Kahn requires a separation of endogenous variables into predetermined X_t and nonpredetermined P_t. Predetermined variables are known one period in advance, so that $X_{t+1} = E\left[X_{t+1}|\Omega_t\right]$ for any realizations of variables in Ω_{t+1}. Non-predetermined variables P_{t+1} equal to their expected values $E\left[P_{t+1}|\Omega_t\right]$ only if realizations of all variables in Ω_{t+1} are equal to their expectations conditional on Ω_t.

[13]The other part can be attributed to the indeterminacy of the impacts of fundamental shocks, as recently established by Lubik and Schorfheide.

[14]Benhabib and Farmer [1998] review the current research in this area.

Model dynamics are summarized by a system of linear rational expectations equations:[15]

$$\begin{bmatrix} X_{t+1} \\ E_t P_{t+1} \end{bmatrix} = \underset{n\times n}{\Gamma_1} \begin{bmatrix} X_t \\ P_t \end{bmatrix} + \underset{n\times l}{\Psi} \begin{bmatrix} Z_t \\ E_t Z_{t+1} \end{bmatrix} \tag{4.31}$$
$$X_{t=0} = X_0$$

Note that expected values of fundamental shocks are directly included in a vector of exogenous disturbances to account for news shocks. Matrix $\Psi = \begin{bmatrix} \underset{n\times k}{\Psi_0} & \underset{n\times k}{\Psi_1} \end{bmatrix}$, and matrix Ψ_1 is understood to have zero elements if expected values of stochastic disturbances do not directly appear in equilibrium equations. Although the algorithm does not treat expectational errors explicitly, the representation (4.3) can be easily obtained from (4.31) by defining y_t as $E_t \begin{bmatrix} X'_{t+1} & P'_{t+1} \end{bmatrix}'$ and adding expectational errors for nonpredetermined variables $\eta_t \equiv P_t - E_t - 1P_t$.

S-format

A solution method proposed by Sims generalizes the Blanchard and Kahn's algorithm. The method handles dynamically singular models[16] and automatically finds predetermined variables or their linear combinations. Further, it provides necessary and sufficient conditions for the existence and uniqueness of a stable solution, regardless of the serial correlation properties of exogenous disturbances. Intuitively, the uniqueness arises in the models with news shocks when expectational errors are completely determined by expectation revisions μ_t.

The S-format corresponds directly to (4.3). To accommodate news shocks, expected values $E_t Z_{t+1}$ are directly included into a vector of stochastic disturbances. As in the BK-format, $\Psi = \begin{bmatrix} \Psi_0 & \Psi_1 \end{bmatrix}$.

LS-format

Lubik and Schorfheide extend the algorithm of Sims to models with indeterminacies. They provide a full characterization of multiple equilibria. The LS-format corresponds to (4.3)

[15]The models may also have additional "static" variables that are linear functions of X_t, P_t and Z_t.

[16]That is, the method directly deals with "static" realtions between variables (non-invertible matrix Γ_0).

with a requirement of serially uncorrelated disturbances $z_t : E_t z_{t+1} = 0$. Any model in the form

$$\tilde{\Gamma}_0 \tilde{y}_t = \tilde{\Gamma}_1 \tilde{y}_{t-1} + \tilde{\Psi}_0 Z_t + \tilde{\Psi}_1 E_t Z_{t+1} + \tilde{\Pi}\eta_t,\ t \geq 0 \tag{4.32}$$

with $\tilde{n}$ endogenous variables, serially correlated fundamental shocks described by (4.4) and news shocks can be transformed into an innovation representation (4.3). To perform the transformation, the fundamental shocks and their expected values are first substituted in terms of the fundamentals ζ and the conditional expectations ξ_t^1:[17]

$$\tilde{\Gamma}_0 \tilde{y}_t = \tilde{\Gamma}_1 \tilde{y}_{t-1} + \left[\tilde{\Psi}_0 C + \tilde{\Psi}_1 CA\right]\zeta_t + \tilde{\Psi}_1 CB\xi_t^1 + \tilde{\Pi}\eta_t, t \geq 0 \tag{4.33}$$

Then a vector of endogenous variables y_t is augmented with vectors ζ_t and ξ_t^ε. The representation (4.3) is obtained by using (4.28):

$$y_t = \begin{bmatrix} \tilde{y}_t \\ \zeta_t \\ \xi_t^\varepsilon \end{bmatrix},\ \Gamma_0 = \begin{bmatrix} \tilde{\Gamma}_0 & -(\Psi_0 C + \Psi_1 CA) & -\Psi_1 CB & 0 \\ 0 & \underset{q\times q}{I} & 0 & 0 \\ 0 & 0 & \underset{k\times k}{I} & 0 \\ 0 & 0 & 0 & \underset{k(T-1)\times k(T-1)}{I} \end{bmatrix} \tag{4.34}$$

$$z_t = \mu_t,\ \Gamma_1 = \begin{bmatrix} \tilde{\Gamma}_1 & 0 \\ 0 & A_m \end{bmatrix},\ \Psi = \begin{bmatrix} \underset{\tilde{n}\times l}{0} \\ B_m \end{bmatrix},\ \Pi = \begin{bmatrix} \underset{\tilde{n}\times f}{\tilde{\Pi}} \\ \underset{(q+kT)\times f}{0} \end{bmatrix},\ l = (k+1)T$$

4.3 Model Solutions

A solution to a linear rational expectations model is a stochastic process $\{y_t, \eta_t\}_{t=0}^{\infty}$ that satisfies the system (4.3) for all realizations of fundamental, news and sunspot shocks. A form of a stationary solution depends on whether the model economy is regular or irregular. Using the terminology of Farmer [1997], a regular economy has a unique stationary equilibrium, while an irregular economy possesses multiple equilibria. This section presents stationary solutions for models with news shocks for both types of economies. It also converts model solutions into state space innovations representations.

[17] Recall that $Z_t = C\zeta_t$ and $E_t Z_{t+1} = CA\zeta_t + CB\xi_t^1$.

4.3.1 Regular Economies

A unique stationary equilibrium of a regular economy is typically forward looking. It can be represented in the form of present-value-relations. In this form, endogenous variables of the model depend on the expected future path of fundamental shocks:

$$y_t = \Theta_y y_{t-1} + \Theta_0 Z_t + \Theta_1 E\left[Z_{t+1}|\Omega_t\right] + \Theta_s \sum_{j=2}^{\infty} \Theta_f^{j-2} \Theta_z E\left[Z_{t+j}|\Omega_t\right] \tag{4.35}$$

This form of the solution is derived by adopting formulas of Blanchard and Kahn, and Sims. The formulas are modified to account for the presence of fundamental shocks expectations among exogenous disturbances. Matrices in (4.35) are obtained by collecting the relevant terms for current and expected future fundamental shocks. The resulting formulas for matrices are presented here for convenience. In the following derivations, the number of unstable roots of the dynamic deterministic structure of the model is denoted by m.[18]

BK-solution

Blanchard and Kahn's algorithm is based on the eigenvalue decomposition of Γ_1 :

$$\Gamma_1 = J \Lambda J^{-1} \tag{4.36}$$

In this decomposition, matrix Λ is diagonal, with eigenvalues of $\mathbf{\Gamma}_1$ ordered by their increasing absolute values. Thus, all eigenvalues in Λ_1 are on or inside the unit circle, and eigenvalues of Λ_2 are outside the unit circle. Matrix of eigenvectors J and matrix of coefficients Ψ are decomposed using stable-unstable roots distinction:

$$\Lambda = \begin{bmatrix} \underset{(n-m)\times(n-m)}{\Lambda_1} & 0 \\ 0 & \underset{m\times m}{\Lambda_2} \end{bmatrix}, \quad J = \begin{bmatrix} \underset{(n-m)\times(n-m)}{B_{11}} & \underset{(n-m)\times m}{B_{12}} \\ \underset{m\times(n-m)}{B_{21}} & \underset{m\times m}{B_{22}} \end{bmatrix} \tag{4.37}$$

$$J^{-1} = \begin{bmatrix} \underset{(n-m)\times(n-m)}{C_{11}} & \underset{(n-m)\times m}{C_{12}} \\ \underset{m\times(n-m)}{C_{21}} & \underset{m\times m}{C_{22}} \end{bmatrix}, \quad \Psi \equiv \begin{bmatrix} \underset{(n-m)\times l}{\gamma_1} \\ \underset{m\times l}{\gamma_2} \end{bmatrix}$$

[18] The number of unstable roots is the number of either eigenvalues of matrix Γ_1 or generalized eigenvalues for matrices Γ_0 and Γ_1 that are greater than one in absolute values.

To conform with the format (4.35), a vector of endogenous variables y_t includes X_{t+1} and P_t. Based on the eigenvalue decomposition, matrices in (4.35) are computed as follows:

$$\underset{n\times n}{\Theta_y} = \begin{bmatrix} B_{11}\Lambda_1 B_{11}^{-1} & \underset{(n-m)\times m}{0} \\ -C_{22}^{-1}C_{21} & \underset{m\times m}{0} \end{bmatrix}, \; \underset{n\times k}{\Theta_0} = \begin{bmatrix} \gamma_{11} \\ 0 \end{bmatrix} + \Theta_s K_1,$$

$$\underset{(n-m)\times m}{H} = -\left(B_{11}\Lambda_1 C_{12} + B_{12}\Lambda_2 C_{22}\right) C_{22}^{-1} \tag{4.38}$$

$$\underset{(n-m)\times 2k}{\gamma_1} = \begin{bmatrix} \underset{(n-m)\times k}{\gamma_{11}} & \underset{(n-m)\times k}{\gamma_{12}} \end{bmatrix}, \; \underset{m\times 2k}{K} = C_{21}\gamma_1 + C_{22}\gamma_2 \equiv \begin{bmatrix} \underset{m\times k}{K_1} & \underset{m\times k}{K_2} \end{bmatrix} \tag{4.39}$$

$$\underset{n\times k}{\Theta_1} = \begin{bmatrix} \gamma_{12} \\ 0 \end{bmatrix} + \Theta_s\Theta_z, \; \underset{n\times m}{\Theta_s} = \begin{bmatrix} H\Lambda_2^{-1} \\ -C_{22}^{-1}\Lambda_2^{-1} \end{bmatrix}, \; \underset{m\times m}{\Theta_f} = \Lambda_2^{-1}, \; \underset{m\times k}{\Theta_z} = [K_2 + \Theta_f K_1]$$

If equilibrium equations of the model in (4.3) do not directly depend on expected values of fundamental shocks, matrices γ_{12} and K_2 have only zero elements.

Many macroeconomic models have a special format, in which equilibrium equations for predetermined variables, such as capital accumulation, do not directly depend on the expected future fundamental shocks. In this case, the submatrix γ_{12} is zero, and the impact of expected value $E_t Z_{t+1}$ can be included directly into the infinite sum:

$$y_t = \Theta_y y_{t-1} + \Theta_0 Z_t + \Theta_s \sum_{j=1}^{\infty} \Theta_f^{j-1}\Theta_z E\left[Z_{t+j}|\Omega_t\right] \tag{4.40}$$

S-solution

The algorithm of Sims is based on a QZ decomposition of square matrices Γ_0 and Γ_1 : [19]

$$\Gamma_0 = Q'\Lambda Z', \; \Gamma_1 = Q'\Omega Z' \tag{4.41}$$

In this decomposition, QQ′ =ZZ′ = $\underset{n\times n}{I}$ and matrices Λ and Ω are upper-triangular. All matrices are possibly complex, and the $'$ symbol indicates transposition and complex conjugation. These matrices are ordered and partitioned so that a $m\times 1$ subvector w_2, t of a

[19] A QZ decomposition is not unique. However, the generalized eigenvalues generally are.

vector $w_t \equiv Z' y_t$ is purely explosive.

$$\Lambda = \begin{bmatrix} \underset{(n-m)\times(n-m)}{\Lambda_{11}} & \Lambda_{12} \\ 0 & \underset{m\times m}{\Lambda_{22}} \end{bmatrix}, \ \Omega = \begin{bmatrix} \underset{(n-m)\times(n-m)}{\Omega_{11}} & \underset{(n-m)\times m}{\Omega_{12}} \\ 0 & \underset{m\times m}{\Omega_{22}} \end{bmatrix}, \ \underset{n\times n}{Q} = \begin{bmatrix} \underset{(n-m)\times n}{Q_1} \\ \underset{n\times n}{Q_2} \end{bmatrix} \tag{4.42}$$

$$Z = \begin{bmatrix} \underset{n\times(n-m)}{Z_1} & \underset{n\times m}{Z_2} \end{bmatrix}$$

As shown by Sims, the stationary solution to (4.3) is unique if and only if the row space of $Q_1\Pi$ is contained in that of $Q_2\Pi$.[20] In that case, there exists a $(n-m)\times m$ matrix Φ such that

$$Q_1\Pi = \Phi Q_2\Pi \tag{4.43}$$

Based on the QZ decomposition, matrices in (4.35) are computed as follows:

$$\begin{aligned}
\underset{n\times n}{\Theta_y} &= Z_1\Lambda_{11}^{-1}\begin{bmatrix} \Omega_{11} & (\Omega_{12}-\Phi\Omega_{22}) \end{bmatrix} Z', \ \underset{(n-m)\times m}{H} = Z\begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}(\Lambda_{12}-\Phi\Lambda_{22}) \\ 0 & I \end{bmatrix} \\
\underset{n\times k}{\Theta_0} &= H\cdot\begin{bmatrix} Q_1-\Phi Q_2 \\ 0 \end{bmatrix}\cdot\Psi_0, \ \underset{m\times k}{K_1} = \Omega_{22}^{-1}Q_2\Psi_0, \ \underset{m\times k}{K_2} = \Omega_{22}^{-1}Q_2\Psi_1 \\
\underset{n\times k}{\Theta_1} &= H\cdot\begin{bmatrix} Q_1-\Phi Q_2 \\ 0 \end{bmatrix}\cdot\Psi_1 - H_2K_1, \ H = \begin{bmatrix} \underset{n\times(n-m)}{H_1} & \underset{n\times m}{H_2} \end{bmatrix} \\
\underset{n\times m}{\Theta_s} &= -H_2\Theta_f^{-1}, \ \underset{m\times m}{\Theta_f} = \Omega_{22}\Lambda_2^{-1}, \ \underset{m\times k}{\Theta_z} = K_2+\Theta_f K_1
\end{aligned} \tag{4.44}$$

If equilibrium equations of the model in (4.3) do not directly depend on expected values of fundamental shocks, matrices Ψ_1 and K_2 have only zero elements. In this special case, the solution takes the form (4.40).

State Space Formulation

Model solutions can be specialized further to take into account the evolution of fundamental shocks and the information structure. Using the expressions for conditional expectations (4.24), the solution (4.35) can be expressed in terms of the observed fundamentals and

[20] Sims's result generalizes the Blanchard and Kahn's uniqueness condition that the number of unstable toors in the BK-format must be equal to the number of non-predetermined variables.

news shocks:

$$y_t = \Theta_y y_{t-1} + \Theta_\zeta \zeta_t + \Theta_e \underbrace{\mathbf{\Delta} s_t}_{\xi_t^\varepsilon} \tag{4.45}$$

with

$$\Theta_\zeta = \Theta_0 C + \Theta_1 CA + \Theta_s \sum_{j=2}^{\infty} \Theta_f^{j-2} \Theta_z CA^j \tag{4.46}$$

$$\Theta_e = \Theta_1 V(1) + \Theta_s \sum_{j=2}^{\infty} \Theta_f^{j-2} \Theta_z V(j)$$

It is then straightforward to obtain an innovation state space representation of the solution:

$$\mathcal{X}_{t+1} = \mathcal{A}\mathcal{X}_t + \mathcal{B}\mu_{t+1} \tag{4.47}$$

$$\mathcal{Y}_t = \mathcal{C}\mathcal{X}_t$$

where

$$\mathcal{X}_t = \begin{bmatrix} y_t \\ \zeta_t \\ \xi_t^\varepsilon \end{bmatrix}, \ \mathcal{A} = \begin{bmatrix} \Theta_y & \Theta_\zeta \ \ \Theta_e \\ 0 & A_m \end{bmatrix}, \ \mathcal{B} = \begin{bmatrix} \underset{n\times kT}{I} \\ B_m \end{bmatrix}, \ \mathcal{Y}_t = y_t, \ \mathcal{C} = \begin{bmatrix} \underset{n\times n}{I} & \underset{n\times(q+kT)}{0} \end{bmatrix} \tag{4.48}$$

This state space representation can be easily adjusted to accommodate differences across elements of Z_t with regards to the forecasting horizon. It can also be used for model simulation and evaluation.

4.3.2 Irregular Economies

Irregular economies are characterized by the multiplicity of stationary equilibria. In these economies, extrinsic stochastic disturbances can influence equilibrium model dynamics. This section adopts a full characterization of multiple equilibria, derived by Lubik and Schorfheide, to models with news shocks. The key insight is that their characterization can be applied essentially without any modifications. Once the model is expressed in the LS-format, exogenous expectation revisions μ_t play exactly the same role as fundamental innovations in the original paper.

The approach of Lubik and Schorfheide utilizes a notion of expectational errors as a solution device. Intuitively, a stationary solution is obtained by finding expressions for endogenous expectational errors. The existence of a stationary solution imposes the following restrictions on the relation between expectational errors η_t and innovations μ_t:[21]

$$\underset{m\times l}{F}\ \underset{l\times 1}{\mu_t} + \underset{m\times f}{G}\ \underset{f\times 1}{\eta_t} = \underset{m\times 1}{0} \tag{4.49}$$

Equations (4.49) simply require that effects of expectational errors offset the impact of μ_t on the explosive variables in the transformed system. Expressions for matrices F and G depend on whether an eigenvalue or a QZ decompositions are used in solving the models. Computational details for both cases are provided below.

Sims has established that a necessary and sufficient condition for the existence of a stationary solution is for the column space of F to be contained in the column space of G. His result implies that a solution may exist even if the rows of G are linearly dependent. A potential singularity of matrix G means that equations (4.49) generate only $r \leq m$ independent restrictions for expectational errors. To accommodate this potential singularity, Lubik and Schorfheide use a singular value decomposition of matrix G :

$$G = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \underset{m\times m}{U}\ \underset{m\times f}{D}\ \underset{f\times f}{V'} = \underset{m\times r}{U_1}\,\underset{r\times r}{D_{11}}\,\underset{r\times f}{V_1'} \tag{4.50}$$

Here D_{11} is a diagonal matrix, and U and V are orthonormal matrices. The full set of solutions for expectational errors is obtained from μ_t and v_t :

$$\eta_t = (H_\mu + V_2 M)\,\mu_t + V_2 M_v v_t \tag{4.51}$$

$$H_\mu \equiv -V_1 D_{11}^{-1} U_1' F \tag{4.52}$$

where coefficients of a $(f - r) \times l$ matrix M and a $(f - r) \times p$ matrix M_v are unrestricted. Matrix V_2 is $f \times (f - r)$. For regular economies, equations (4.49) uniquely determine expectational errors from exogenous expectation revisions. In that case, $f = r$ and V_2 is empty.

[21]Recall that m defines the number of unstable roots of the deterministic system.

For irregular economies, there are three properties of (4.51) worth noting. First, equations (4.49) do not provide enough restrictions to uniquely determine η_t by values of μ_t. A solution to (4.3) admits non-fundamental uncertainty, summarized by a $p \times 1$ ($p \geq f - r$) vector of sunspot shocks v_t. Second, the impact of sunspot shocks on expectational errors is not completely determined, since there no restrictions on elements of $\tilde{M}_v$. However, a reduced form sunspots, a $(f - r) \times 1$ vector v_t^* :

$$v_t^* \equiv M_v v_t \tag{4.53}$$

has a unique impact on η_t. Third, the impact of expectation revisions μ_t on endogenous expectational errors is indeterminate without additional assumptions. This is because the elements of M are unrestricted. One auxiliary assumption that allows to select a particular equilibrium is the orthogonality of the impacts of expectations revisions and sunspot shocks.[22] This assumption is equivalent to setting all elements of M to zero.[23]

Based on representation (4.34), any stationary solution takes the form

$$y_t = \Theta_y y_{t-1} + \Theta_\mu \mu_t + \Theta_v v_t^* \tag{4.54}$$

where v_t^* is a vector of a reduced form sunspot shocks. Computation of impact matrices is addressed next.

Solution based on an eigenvalue decomposition

When matrix Γ_0 is invertible, the system (4.34) can be rewritten as:

$$y_t = \Gamma_1^* y_{t-1} + \Psi^* \mu_t + \Pi^* \eta_t \tag{4.55}$$

with $\Gamma_1^* = \Gamma_0^{-1}\Gamma_1$, $\Psi^* = \Gamma_0^{-1}\Psi$, $\Pi^* = \Gamma_0^{-1}\Pi$. A solution can be obtained by applying an eigenvalue decomposition to matrix Γ_1^* :

$$\Gamma_1^* = J\Lambda J^{-1} \tag{4.56}$$

[22]Note that the orthogonality between these impacts is not generally the same as the orthogonality between the shocks themselves.

[23]See Lubik and Schorfheide for more details.

with matrices J and Λ defined as in (4.37). Matrices Ψ^* and Π^* are decomposed using stable-unstable roots distinction:

$$\Psi^* = \begin{bmatrix} \underset{(n-m)\times k(T+1)}{\gamma_1} \\ \underset{m\times k(T+1)}{\gamma_2} \end{bmatrix}, \ \Pi^* = \begin{bmatrix} \underset{(n-m)\times f}{\pi_1} \\ \underset{m\times f}{\pi_2} \end{bmatrix} \tag{4.57}$$

Based on the eigenvalue decomposition, matrices in (4.49) and (4.54) are:

$$\begin{aligned} &F = C_{21}\gamma_1 + C_{22}\gamma_2, \ G = C_{21}\pi_1 + C_{22}\pi_2 \\ &\Theta_y = \Gamma_1^*, \ \Theta_\mu = \Psi^* + \Pi^* \left(H_\mu + V_2 M\right), \ \Theta_v = \Pi^* V_2 \end{aligned} \tag{4.58}$$

Under determinacy, $V_2 = 0$, $\Theta_v = 0$ and the last term in Θ_μ drops out.

In deriving a stationary solution, there is an additional requirement that the initial value of the transformed explosive variables is zero. This requirement translates into the condition

$$C_{21}y_t + C_{22}y_t = 0 \tag{4.59}$$

for $t = -1$. As effects of expectational errors on explosive variables are eliminated by construction, the same condition holds for any other values of t. If matrix Γ_1^* contains any unstable roots, a solution with matrices in (4.58) cannot be used directly to compute theoretical moments of the model. An alternative representation for matrix Θ_y takes into account (4.59) explicitly:

$$\Theta_y = \begin{bmatrix} B_{11}\Lambda_1 C_{11} & B_{11}\Lambda_1 C_{12} \\ B_{21}\Lambda_1 C_{11} & B_{21}\Lambda_1 C_{12} \end{bmatrix} \tag{4.60}$$

As long as condition (4.59) is satisfied for initial values, the two representations are identical for model simulations.

Solution based on a QZ decomposition

More generally, a model solution can be obtained by applying a QZ decomposition to matrices Γ_0 and Γ_1 in (4.34). Matrices Q and Z are defined in exactly the same way as in (4.41) and (4.42). Matrices F and G again correspond to an unstable block:

$$F = Q_2\Psi, \ G = Q_2\Pi \tag{4.61}$$

Based on the QZ decomposition, matrices in (4.54) are computed as follows:

$$\Theta_y = Z_1 \Lambda_{11}^{-1} \Omega_{11} Z_1', \ \Theta_\mu = \Psi + \Pi \left(H_\mu + V_2 M \right), \ \Theta_v = \Pi V_2 \tag{4.62}$$

4.4 Comparison of News and Sunspot Shocks

News and sunspot shocks are interpreted as stochastic changes in agents' beliefs. Such changes arise for two different reasons. With news shocks, beliefs are affected because agents possess extra information that helps to forecast future fundamental impulses. With sunspots, beliefs are affected by extrinsic uncertainty. This section discusses similarities and differences between news and sunspot shocks, with a particular emphasis on their effects on model solutions.

4.4.1 Alternative Modelling of Beliefs

This subsection elaborates on the interpretation of news and sunspot socks as changes in beliefs. News shocks alter conditional expectations of fundamental impulses and shocks. Through these conditional expectations, news shocks influence equilibrium endogenous variables. A different perspective on news shocks is obtained by examining forecast revisions of endogenous variables. In regular economies, expectational errors are uniquely pinned down by μ_t. When shocks are persistent, but included into vector y_t (i.e. when the model is expressed in LS−format), $\eta_t = H_\mu \mu_t$. More generally, expectational errors are defined by the sum of discounted future expectation revisions of fundamental shocks. Looking for simplicity at the case when expected values of fundamental shocks do not directly enter equilibrium equations, the expectational errors are related to future forecasts of Z :

$$F\eta_{t+1} = \sum_{j=1}^{\infty} \Omega_{22} M^j \Omega_{22}^{-1} F \left[E_{t+1} Z_{t+j} - E_t Z_{t+j} \right] \tag{4.63}$$

Forecast changes of future fundamental shocks are determined by expectation revisions μ_t. It is straight forward to verify that even in the case of serially correlated fundamental shocks expectational errors are linear combinations of μ_t. That is, condition (4.49) holds.

Sunspot shocks capture extrinsic uncertainty. In principle, these shocks can represent any stochastic variables, not incorporated into the model. Yet, the sunspots are often interpreted as changes in beliefs that lead to a revision of forecasts. The following justification for this interpretation is based on the exposition of Lubik and Schorfheide.

The expectational errors vector η_t consists of forecast errors for a subset of endogenous variables $x_t \in y_t$:

$$\eta_t \equiv x_t - \xi^x_{t-1},\ \xi^x_t \equiv E_t x_{t+1} \tag{4.64}$$

Suppose that forecast revisions in x_t between periods $t-1$ and t are in part attributed to a sunspot vector v^x_t, so that

$$x_t = \left[\xi^x_{t-1} + v^x_t\right] + \eta^x_t \tag{4.65}$$

$$\eta_t = \eta^x_t + v^x_t \tag{4.66}$$

The term is brackets in (4.65) corresponds to the revised forecast, and η^x_t denotes the error of this revised forecast. Suppose further that the dimension of v^x_t coincides with the dimension of the expectational errors vector $(p = f)$, and v^x_t is restricted to be unpredictable, $E_t v^x_{t+1} = 0$. Then the beliefs shocks can be directly included into the model specification. In other words, they can be treated as any other exogenous shock. By definition (4.66), the impact of the beliefs shocks on endogenous variables coincides with the impact of the expectational errors:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi v^x_t + \Pi \eta^x_t,\ t \geq 0 \tag{4.67}$$

Then restrictions (4.49) change to

$$\left[\begin{array}{cc} F & G \end{array}\right] \left[\begin{array}{c} \mu_t \\ v^x_t \end{array}\right] + G\eta^x_t = 0 \tag{4.68}$$

The expectational errors of the revised forecast, η^x_t, are expressed as a linear function of the structural shocks and the beliefs shocks:

$$\eta^x_t = H_\mu \mu_t - \underset{f\times r}{V_1} \underset{r\times f}{V_1'} v_t \tag{4.69}$$

The overall expectational errors η_t are easily recovered as

$$\eta_t = \eta_t^x + v_t^x = H_\mu \mu_t + V_2 V_2' v_t \tag{4.70}$$

since by construction, $I - V_1 V_1' = V_2 V_2'$. It is clear, by comparing (4.70) with a general formula (4.49), that impacts of the structural shocks and beliefs shocks are orthogonal. Effects of belief shocks can be distinguished from each other only in a special case when matrix G provides no restrictions $(r = 0)$. Usually this property does not hold, $r > 0$ and different realizations of beliefs shocks can generate the same equilibrium dynamics. For example, a New Keynesian model discussed the last section of this Chapter, has a one-dimensional indeterminacy. It is impossible to distinguish whether a sunspot shock was associated with a revision in output or inflation forecast. In any case, interpreting sunspots as beliefs shocks is useful for developing the economic intuition about the effects of these shocks.

4.4.2 Existence

News shocks are defined by their correlation properties with fundamental shocks. Thus, the former cannot exist in isolation from the latter. News shocks are easily incorporated into both regular and irregular economies, as established in this Chapter.

Sunspot shocks affect endogenous variables only in models with indeterminacy. A number of such models exist in the literature. Two biggest theoretical concerns with respect to these models is how a particular equilibrium is selected and how it is implemented. The empirical plausibility of models with indeterminacy is also an open research question. For example, several versions of real business cycle models with increasing returns require rather high degree of increasing returns to labour input (Schmitt-Grohé [1997]). They also may require an upward sloping labour demand curve (Benhabib and Farmer [1994], Farmer and Guo [1994]). Furthermore, there exists an observational equivalence result between a standard real business cycle model with flexible parameterization of technology and an externality model with increasing returns and sunspot fluctuations (Kamihigashi

[1996]). This result implies that these two classes of models cannot be distinguished based on calibration and estimation of reduced form relations. In defence, empirical evidence on increasing returns is mixed (Farmer [1997]). Theoretically, problems with a high degree of increasing returns and an upward sloping labour demand curve can be eliminated by incorporating variable capacity utilization (Benhabib and Wen [2002]). Finally, progress has been made in developing structural estimation methods for general equilibrium models without restricting parameter space to regions of determinacy (Lubik and Schorfheide [2004]).

4.4.3 Relation to Fundamental Shocks

News shocks are defined as correlated with future, but uncorrelated with current and past fundamental impulses. Forecastability of future impulses is the main implication of news shocks. Focus on this implication alone allows a reinterpretation of news shocks as fundamental innovations with different period of realization, as argued in section 4.2.1.

Sunspot shocks can be modelled as uncorrelated with fundamental impulses (or shocks) without any loss of generality. Suppose that, in contrast, sunspot shocks are correlated with exogenous expectation revisions, so that $cov\,(v_t \mu_t') = \Phi$. Then vector v_t can be represented as

$$v_t = \Phi \Sigma_{\mu}^{-1} \mu_t + u_t, \;\; E_t u_{t+1} = 0 \tag{4.71}$$

Expectational errors $\eta_t = H_{\mu}\mu_t + V_2 M_v v_t$ are equivalent to

$$\eta_t = \left(H_{\mu} + V_2 M\right)\mu_t + V_2 M_v u_t \tag{4.72}$$

with $M = M_v \Phi \Sigma_{\mu}^{-1}$. The two expressions lead to identical series for expectational errors.

Suppose that sunspots are correlated with past realizations of μ_t. Then values of v_{t+1} are predictable from values of μ_{t-j+1}, which violates the requirements that expectational errors and sunspot shocks are martingale difference sequences. Finally, suppose that sunspots are correlated with some future realization of impulses. Then $cov\left(v_t \varepsilon_{t+j}'\right) \neq$

0. If all impulses treated as unpredictable in the model solution, this would violate the rationality of expectations. Values of v_{t-j+1} would help to predict ε_{t+1}.

4.4.4 Effects on Equilibrium Dynamics

Both news and sunspot shocks introduce effects of beliefs that are independent of realized fundamental shocks. This property follows directly from the form of solutions (4.45) and (4.54). In irregular economies, effects of news shocks are incorporated in both Θ_y and Θ_μ, as expectations of future fundamental are included into a vector of endogenous variables. There are several interesting properties of solutions.[24]

First, in regular economies, impacts of endogenous variables, Θ_y, and realized fundamentals, Θ_ζ, are invariant to the presence of news shocks. That is, their coefficients are identical in models with and without news. Further, impacts of expected impulses, Θ_e, depend only on the parameterization of fundamental shocks and the number of periods with news T. Thus, representation (4.45) can be used for model simulation under different realizations of beliefs.

Second, impacts of present values of fundamental shocks on endogenous variables are linearly dependent in irregular economies with two characteristics. These models have a single unstable root ($m = 1$), and their solution is expressed in the form (4.40). Models in Chapter 2, for example, possess these characteristics. In this case, the overall impact of the present value of fundamental shocks can be broken into a sum of individual shocks contributions. If $\Theta_z = \left[\begin{array}{ccc} c_1 & \dots & c_k \end{array} \right]$ and $\Theta_f = \phi$, then

$$\underset{n\times 1}{\Theta_s} \sum_{j=1}^{\infty} \phi^{j-1} \underset{1\times k}{\Theta_z} E\left[Z_{t+j}|\Omega_t\right] = c_1\Theta_s \sum_{j=1}^{\infty} \lambda^{j-1} E_t Z_{1,t+j} + c_2\Theta_s \sum_{j=1}^{\infty} \lambda^{j-1} E_t Z_{2,t+j} + \qquad (4.73)$$
$$+ \dots + c_k\Theta_s \sum_{j=1}^{\infty} \lambda^{j-1} E_t Z_{k,t+j}$$

Impact vectors of the present values of future expectations for fundamental shocks are proportional to each other, as Θ_s is a $n \times 1$ vector and coefficients c_j, $1 \leq j \leq k$, are

[24]Some of these properties have been already discussed in the context of particular real business cycle models in Chapter 2.

scalars. Thus, there exists an identification problem of separating beliefs about different shocks. A problem of this kind and one possible solution have been already encountered in Chapter 2.

Third, news shocks are relevant for model solutions only through their effects on expectations and expectation revisions of future fundamental impulses. From the point of view of agents, different processes of news shocks that lead to the same sequence of beliefs are identical. The simplest specification is to equate news shocks with fundamental impulses perfectly anticipated prior to their realization. However, an alternative interpretation of news shocks as noisy signals helps to develop some economic intuition about the role of these shocks.

Fourth, impacts of fundamental and news shocks in regular economies are unique. However, in irregular economies, their impacts are restricted, but not uniquely determined without extra assumptions. Impacts of sunspot shocks are defined uniquely only in the case of reduced form sunspots.

Finally, news and sunspot shocks generally impose distinct cross-equation restrictions in an irregular economy that incorporates both types of shocks.[25] This property is evident from expressions for impact matrices Θ_μ and Θ_v in (4.58) and (4.62). Impact matrix of expectation revisions is a linear combination of impact matrix of sunspots V_2 and impact matrix of expectation revisions under the orthogonality restriction H_μ. The New Keynessian model from section 4.5 demonstrates that qualitative differences in impact responses can be quantitatively significant. These differences can be used to distinguish the two ways of modelling beliefs. A multivariate framework is the key to a possible identification procedure.

4.4.5 Dynamic Properties of Model Equilibria

News shocks enrich autocovariance properties of equilibrium endogenous variables. Faced with the information about the future, rational agents generally start acting upon this

[25] A model with a single dynamic variable is one exception. In this case, impact coefficient of a news shock can be chosen to be identical to the one of a reduced form sunspot shock.

information immediately. After the fundamental shocks are realized, agents learn about the correctness of their prior information and adjust their actions accordingly. Decisions, made between a time between the news is learned and the time fundamental shocks are realized, introduce a moving average component into a model solution.

While there is no correlation between expectation revisions, expected values are correlated. In other words, beliefs are persistent. Every piece of news at least in part is carried forward into the future until the realizations of fundamental shocks are observed. The persistence of beliefs is evident from the definition (4.27), which can be solved for in terms of expectation revisions:

$$\begin{bmatrix} \varepsilon_t \\ \xi_t^{\varepsilon} \end{bmatrix} = \begin{bmatrix} 1 & L & L^2 & \vdots & L^{T-1} & L^{T-1} \\ 0 & 1 & L & \vdots & L^{T-3} & L^{T-1} \\ 0 & 0 & 1 & \vdots & & L^{T-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & 0 & 0 & \vdots & 1 & L \\ 0 & 0 & 0 & \vdots & 0 & 1 \end{bmatrix} \mu_t \tag{4.74}$$

If both ε_t and ζ_t are substituted for into solution (4.45), the moving average part of the solution becomes apparent. For regular economies, the number of moving average lags is equal to the number of periods with news. For irregular economies, the existence of a moving average part can be seen more clearly if realized fundamentals and beliefs are separated from the rest of endogenous variables:

$$\tilde{y}_t = \Theta_{y1}\tilde{y}_{t-1} + \Theta_{y2}\zeta_{t-1} + \Theta_{y3}\xi_{t-1}^{\varepsilon} + \Theta_{\mu}\mu_t + \Theta_v v_t^* \tag{4.75}$$

An ability of i.i.d. sunspots to generate highly persistent endogenous movements in output has been emphasized by a number of authors.[26] However, the richer autocovariance structure of equilibria is not linked to sunspots shocks per se. Indeterminacy also alters the propagation of fundamental shocks, as shown by Lubik and Schorfheide [2004]. In fact, structural estimation of a general equilibrium model may favour a version of indeterminacy with altered fundamentals, rather than a version with sunspots.

[26] See, for example, Farmer and Guo [1994], Benhabib and Farmer [1994].

4.5 Example of New Keynesian Model

The section introduces news shocks into a New Keynesian model analyzed by Lubik and Schorfheide. In this model, a single parameter determines whether the underlying economy is regular or irregular. The indeterminacy arises when the central bank is not aggressive enough towards inflation. The solution techniques described in this Chapter are applied to the model. The example is simple enough to derive some analytical results. Further examination of impulse response functions and theoretical moments helps develop the economic intuition about differences between news and sunspot shocks.

4.5.1 Model Description

The model describes a closed economy with nominal rigidities. Monopolistically competitive firms face a downward sloping demand curve for their differentiated products and produce output only with variable labour input. Prices are sticky due to nominal price adjustment costs or infrequent Calvo-type price setting. Households smooth their consumption streams by purchasing nominal government bonds. The central bank affects the economy through an interest rate rule. For simplicity, the only fundamental source of uncertainty is a monetary policy shock.[27]

The log-linearized reduced-form of the model consists of three equations: (i) intertemporal Euler equation, governing optimal consumption allocations, (ii) expectational Phillips curve and (iii) monetary policy rule:

$$\text{Euler equation} \quad E_t x_{t+1} + \sigma E_t \pi_{t+1} = x_t + \sigma R_t \tag{4.76}$$

$$\text{Phillips curve} \quad \beta E_t \pi_{t+1} = \pi_t - \kappa x_t \tag{4.77}$$

$$\text{Monetary policy rule} \quad R_t = \rho R_{t-1} + (1-\rho)\,\psi \pi_t + m_t \tag{4.78}$$

Aggregate output x_t, inflation π_t, and nominal interest rate R_t are expressed as log-deviations from a unique steady state. The parameter $0 < \beta < 1$ is the discount factor,

[27] This model has become a standard benchmark in the monetary literature. Details can be found, for example, in Clarida, Galí and Gertler [1999] or Woodford [2003].

$\sigma > 0$ is the intertemporal elasticity of substitution, $\kappa > 0$ is related to the degree of price adjustment costs, and $\psi \geq 0$ measures the elasticity of the interest rate response to inflation. Policy parameter $0 \leq \rho < 1$ reflects the central bank's preferences for interest rate smoothing.

The monetary policy shock is modelled as a white noise process with variance σ_ε^2 :

$$m_t = \varepsilon_t \tag{4.79}$$

To incorporate news shocks, it is assumed that agents receive an exogenous signal s_t about monetary policy impulse in the next period. This signal can be thought of as a revelation of future policy intentions. Focus on a single signal makes a comparison with a sunspot shock[28] more transparent. For concreteness, the news shock is described as:

$$s_t = \varepsilon_{t+1} + e_t \tag{4.80}$$

Process e_t is white noise with variance σ_e^2. It is uncorrelated with monetary policy impulses at all lags and leads. The information assumption implies that expected impulse for $t+1$ is

$$E_t \varepsilon_{t+1} = \phi s_t, \ \phi = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_e^2} \tag{4.81}$$

and all other future fundamental impulses are unpredictable.

It is well known that this model without news shocks exhibits indeterminacy if the central bank is not aggressive enough towards inflation. Introduction of news shocks does not alter this property. The next two sections present analytical solutions for a special case of no persistence in monetary policy rule. Cases of determinacy ($\psi \geq 1$) and indeterminacy ($\psi < 1$) are addressed separately. Then the properties of solutions are examined numerically for a more general case ($\rho > 0$).

[28]Looking ahead, the model admits only one reduced form sunspot shock in the indeterminacy region.

4.5.2 Analytical Characterization of Solutions

Solution Under Determinacy ($\psi > 1$)

Elimination of the interest rate from (4.76), using the monetary policy rule (4.78) with $\rho = 0$, leads to a two-dimensional system in the format (4.31):

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1+\frac{\kappa\sigma}{\beta} & \sigma\left(\psi - \frac{1}{\beta}\right) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}}_{\Gamma_1} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \underbrace{\begin{bmatrix} \sigma & 0 \\ 0 & 0 \end{bmatrix}}_{\Psi} \begin{bmatrix} m_t \\ E_t m_{t+1} \end{bmatrix} \tag{4.82}$$

Expectations of the future monetary policy shock do not directly appear in the dynamic equilibrium equations $(4.76 - 4.78)$. To conform with the general format (4.31), matrix Ψ_0 is set to $\begin{bmatrix} 0 & 0 \end{bmatrix}'$. In this model, there are no predetermined variables. With $\psi > 1$, the number of unstable roots of Γ_1 is two $(m = 2)$, and the solution is unique. The solution takes the form:

$$\begin{bmatrix} x_t & \pi_t & R_t \end{bmatrix}' = \Theta_0 m_t + \Theta_1 E_t \varepsilon_{t+1} = \Theta_0 \mu_t^0 + \Theta_1 \mu_t^1 + \Theta_0 \mu_{t-1}^1 \tag{4.83}$$

with

$$\Theta_0 = \frac{1}{1+\kappa\sigma\psi} \begin{bmatrix} -\sigma \\ -\sigma\kappa \\ 1 \end{bmatrix}, \quad \Theta_1 = -\frac{\sigma}{(1+\kappa\sigma\psi)^2} \begin{bmatrix} 1+\kappa\sigma(1-\beta\psi) \\ \kappa(1+\beta+\kappa\sigma) \\ \psi\kappa(1+\beta+\kappa\sigma) \end{bmatrix} \tag{4.84}$$

An unanticipated monetary contraction, represented by an increase in μ_t^0, leads to a fall in output and inflation, and an increase in nominal interest rate. The contractionary effects dissipate after one period, when all variables return to their original steady states. The parameter κ governs the inflation-output trade-off. The response of output exceeds the response of the inflation when $\kappa < 1$. The endogeneity of the monetary policy rule implies that the interest rate increases by less than the size of the unexpected monetary policy shock.

An anticipated monetary contraction, represented by an increase in μ_t^1, triggers agents' reactions before an actual change in monetary policy shock. Foreseeing the future contraction, firms reduce their prices, if they have an opportunity to so do. When the actual monetary contraction occurs, there is an additional downward adjustment of prices. Output generally falls, unless the central bank is aggressive enough towards inflation. If $\psi > \frac{1}{\beta}(1 + \frac{1}{\kappa\sigma})$, an anticipation of the monetary contraction has a stimulative effect on output. To gain the intuition for this result, it is helpful to look at the behaviour of the real interest rate:

$$r_t = \frac{(1+\kappa\sigma)\,\kappa\sigma}{1+\kappa\sigma\psi} m_t - (\psi - 1)\,\frac{\sigma\kappa\,(1+\beta+\kappa\sigma)}{(1+\kappa\sigma\psi)^2} E_t\varepsilon_{t+1} \tag{4.85}$$

The contemporaneous real interest rate falls in response to an anticipated monetary contraction. This is because the increase in nominal interest rate is not high enough to compensate for the fall in inflation, without any actual change in the monetary policy shock. However, the real interest rate is expected to increase in the future, as the coefficient in front of expected impulse is negative. Thus, the relative importance of the contemporaneous fall in the real interest rate versus its expected increase in the future determines what happens to output. While an increase in output is an interesting theoretical possibility, it is unlikely to occur in practice. Typical estimates of the interest rate elasticity are rarely exceed two.[29]

Solution Under Indeterminacy ($\psi \leq 1$)

To solve the model in the case of indeterminacy and $\rho = 0$, the system is transformed into a LS-format. First, the interest rate is eliminated using the monetary policy rule. Second, output and inflation are substituted for their expected values ξ_t ($\xi_t = \begin{bmatrix} \xi_t^x & \xi_t^\pi \end{bmatrix}'$, $\xi_t^x \equiv E_t x_{t+1}$, $\xi_t^\pi \equiv E_t\pi_{t+1}$) and expectational errors η_t ($\eta_t = \begin{bmatrix} \eta_t^x & \eta_t^\pi \end{bmatrix}'$, $\eta_t^x \equiv x_t - E_{t-1}x_t$, $\eta_t^\pi \equiv \pi_t - E_{t-1}\pi_t$). By the rationality of expectations:

$$x_t = \xi_{t-1}^x + \eta_t^x, \pi_t = \xi_{t-1}^\pi + \eta_t^\pi \tag{4.86}$$

[29]For a reference, with β close to one, logarithmic preferences and κ set to a commonly used value 0.5, ψ must exceed three for output to increase.

Third, the monetary policy shocks is substituted for by its expected value ξ_t^1 and expectation revision innovations μ_t. The resulting system with uncorrelated disturbances is:

$$\underbrace{\begin{bmatrix} 1 & \sigma & -\sigma & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\Gamma_0} \begin{bmatrix} \xi_t^x \\ \xi_t^\pi \\ m_t \\ \xi_t^1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \sigma\psi & 0 & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\Gamma_1} \begin{bmatrix} \xi_{t-1}^x \\ \xi_{t-1}^\pi \\ m_{t-1} \\ \xi_{t-1}^1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\Psi} \mu_t + \underbrace{\begin{bmatrix} 1 & \sigma\psi \\ -\kappa & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\Pi} \eta_t \tag{4.87}$$

When $0 \leq \psi < 1$, the model has only one unstable eigenvalue λ_2.[30] Condition (4.49) provides only one equation for determining two endogenous expectational errors. Thus, the model admits one reduced form sunspot shock.

When $\rho = 0$, a full characterization of model solutions can be derived analytically:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 + \frac{\kappa\sigma}{\beta} & \sigma\left(\psi - \frac{1}{\beta}\right) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma \\ 0 \end{bmatrix} \underbrace{\left(\mu_{t-1}^0 + \mu_{t-2}^1\right)}_{\varepsilon_{t-1}} + \eta_t \tag{4.88}$$

As in (4.51), the expectational errors η_t depend on exogenous expectation revisions μ_t and the sunspot shock $\grave{v_t}$, with the following impact matrices:

$$H_\mu = \frac{\kappa\sigma}{d^2} \begin{bmatrix} -\kappa\lambda_2 & b\frac{\kappa\lambda_2}{2(1+\kappa\sigma\psi)} \\ \lambda_2 - 1 - \kappa\sigma\psi & b\frac{\kappa(\lambda_2-1-\kappa\sigma)}{2(1+\kappa\sigma\psi)} \end{bmatrix}, \; V_2 = \frac{1}{d} \begin{bmatrix} \lambda_2 - 1 - \kappa\sigma\psi \\ \kappa\lambda_2 \end{bmatrix}, \; M = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \tag{4.89}$$

where coefficients a_1 and a_2 are unrestricted, $d = \sqrt{(\kappa\lambda_2)^2 + (\lambda_2 - 1 - \kappa\sigma\psi)^2}$, $b = -1 - \beta - \kappa\sigma + 2\beta l_2$, $l_2 = \frac{1}{2}\sqrt{\left(\frac{1+\kappa\sigma}{\beta} - 1\right)^2 + \frac{4\kappa\sigma}{\beta}(1-\psi)}$. It can be shown that $b < 0$ and $1 + \kappa\sigma\psi < \lambda_2$ for any values of $\kappa \geq 0$, $\sigma \geq 0$ and $\psi < 1$.

A reduced form sunspot shock has a positive impact on both expectational errors. Inflationary sunspot beliefs are validated with the actual increase in inflation and output. Nominal interest rate increases, to offset a rise in inflation. However, the actual and expected real interest rate declines, which stimulates current output.

[30] $\lambda_2 = \frac{1}{2}\left(1 + \frac{\kappa\sigma+1}{\beta}\right) + \frac{1}{2}\sqrt{\left(\frac{1+\kappa\sigma}{\beta} - 1\right)^2 + \frac{4\kappa\sigma}{\beta}(1-\psi)}$, and $\lambda_1 = \frac{1}{2}\left(1 + \frac{\kappa\sigma+1}{\beta}\right) - \frac{1}{2}\sqrt{\left(\frac{1+\kappa\sigma}{\beta} - 1\right)^2 + \frac{4\kappa\sigma}{\beta}(1-\psi)}$.

The orthogonality assumption between impacts of μ_t and v_t ($a_1 = a_2 = 0$) selects one particular solution among infinitely many. This solution predicts a somewhat counter-intuitive increase in inflation in response to unanticipated monetary contraction. As argued by Lubik and Schorfheide, it is more attractive to use another solution as a benchmark for the case of indeterminacy. This solution preserves qualitative properties of the model under determinacy and prevents abrupt changes in endogenous expectation formation, as the economy crosses the boundary that separates determinacy and indeterminacy regions. Under this so called continuity solution the impact matrix of expectation revisions on endogenous variables takes the form:

$$H_\mu = -\frac{\sigma}{(1+\kappa\sigma\psi)^2}\begin{bmatrix} (1+\kappa\sigma\psi) & 1+\kappa\sigma(1-\beta\psi) \\ \kappa(1+\kappa\sigma\psi) & \kappa(1+\beta+\kappa\sigma) \end{bmatrix} \tag{4.90}$$

It is clear that both output and inflation fall in response to an unanticipated increase in interest rate. Other solutions are obtained by varying coefficients of matrix M.

Overall, the analytical solutions illustrate that impacts of the expectation revisions μ_t and the sunspot shock v_t can differ not only in the magnitude, but also in qualitative sign predictions.

4.5.3 Numerical Properties of Solutions

Quantitative properties of the model are best illustrated in the context of impulse response functions and theoretical moments. This section examines numerically how these objects change in response to different assumptions about news shocks in regions of determinacy and indeterminacy.

Model Parameterization

There are eight parameters in the model: $\{\beta, \sigma, \kappa, \psi, \sigma_v, \sigma_\varepsilon, \rho, \sigma_e\}$. Values of all parameters, except for the standard deviation of signal's noise σ_e, are based on the estimates of Lubik and Schorfheide [2004]. Lubik and Schorfheide estimate a slightly richer specification of the model for the US data. Their results indicate that the US monetary policy after 1982 is consistent with determinacy, while the policy in the pre-Volcker period, $1960:1-1979:2$, is not. Parameter values obtained for the two subsamples used in the current simulations. Numerical exercises are not meant to represent a serious calibration. Rather, this is the first step towards understanding how properties of equilibria change under alternative ways of modelling beliefs.

The unitary interest rate elasticity separates regions of determinacy and indeterminacy. Two values of ψ are considered: 2.19 for determinacy and 0.85 for indeterminacy. The degree of interest rate smoothing affects autocorrelation properties of variables. Due to endogenous persistence, the model with indeterminacy requires a low persistence of the monetary policy shock. Thus, varying the degree of persistence facilitates a comparison across equilibrium paths. With $\rho = 0$ and $\rho = 0.6$, values of $k = 0.77$ and $\sigma^{-1} = 1.45$ are used. With $\rho = 0.85$, $\kappa = 0.58$ and $\sigma^{-1} = 1.86$. The discount factor and standard deviations of monetary policy and sunspots shocks are fixed during the simulations: $\beta = 0.99$, $\sigma_{\varepsilon} = 0.2$ and $\sigma_{v} = 0.2$.

The standard deviation of the noise is controlled implicitly by the choice of the informativeness of the news shocks, value of ϕ. Given ϕ, $\sigma_e = \sqrt{\frac{1-\phi}{\phi}}\sigma_{\varepsilon}$. With $\phi = 0$, all policy shocks are unanticipated.[31] With $\phi = 1$, all policy shocks are anticipated in advance. Value of $\phi = 0.5$ corresponds to an intermediate case, with both anticipated and unanticipated policy shocks.

Impulse Response Functions

Figure 4.1 plots impulse responses of output, inflation and interest rate to unanticipated and anticipated (and realized) monetary contraction for a model in the determinacy region ($\psi = 2.19$).[32] For the ease of comparison, a unitary increase in policy shock is considered. Alternatively, impulse responses to a one-standard deviation of the policy shock could be plotted. The relative magnitude of dynamics (but not their qualitative properties) will then depend on the fraction of anticipated shock.

[31] Technically, the noise variance is infinite in this case. A direct focus on the implication for expectation computations bypasses an issue of stationarity.

[32] Recall that an anticipated policy shock, μ_t^1 is proportional to a news shock s_t.

Figure 4.1 illustrates how model dynamics vary with the degree of interest rate smoothing. As ρ does not alter the direction of responses, the economic intuition behind the dynamic adjustment is similar to one discussed for the analytical solution.

Figure 4.2 plots impulse responses of output, inflation and interest rate corresponding to three solutions in the indeterminacy region ($\psi = 0.85$ and $\rho = 0.6$). The bottom panel depicts an adjustment to an inflationary sunspot shock. As in the case of no interest rate smoothing, this shock leads to an increase in output, inflation and interest rate.

While the dynamics of a reduced form sunspot shock are unique, the propagation of monetary policy and news shocks depends on free parameters in matrix M. The top two panels of Figure 4.2 illustrate that indeterminacy can alter impulse responses to these shocks in a significant way. The panels plot responses under three sets of parameters. Under the orthogonality assumption, $M = [0, 0]$. The model predicts an increase in inflation and interest rate in response to both unanticipated and anticipated contraction. Under the continuity assumption, parameters of M are chosen to mimic impulse responses of the model with $\psi = 1$.[33] The continuity solution has an advantage that small deviations in ψ do not drastically change properties of expectational errors. By construction, the qualitative responses to monetary policy shocks are similar to the ones reported in Figure 4.1. Finally, under the modified dynamics assumption, $M = [-2.75, 0.99]$. The first element is chosen to illustrate a possible fall in interest rate along the adjustment path. The second element is chosen so that an impact of an anticipated monetary policy shock mimics the one of a sunspot shock. Effectively, an anticipated impulse has an extra degree of freedom in potentially fitting the data.

With many theoretical possibilities for impulse responses under indeterminacy, which may be relevant empirically? Lubik and Schorfheide [2004] describe an estimation procedure that does not a priori constrain model parameters to regions of determinacy or indeterminacy. Their empirical results for the model without news shocks yield a substan-

[33]Formally, M is chosen to minimize the discrepancy between model responses for $\psi < 1$ and $\psi = 1$ using a least squares criterion. $M(\psi) = \left[B_2(\psi)' B_2(\psi)\right]^{-1} B_2(\psi)' \left[B_1(\psi = 1) - B_1(\psi)\right]$, where $B_1(\psi) = \Psi^*(\psi) + \Pi^*(\psi) H_\mu(\psi)$, $B_2(\psi) = \Pi^*(\psi) V_2(\psi)$ and $B_1(\psi = 1) = 0$.

tial degree of uncertainty about the effects of an unanticipated policy shock on inflation and interest rate. In particular, both a fall and an increase of inflation relative to its steady state and a fall of interest rate during the propagation stage are potentially possible.[34]

Selected Moments

Table 3.1 reports statistics for comovement and persistence of output, inflation and interest rate under alternative parameterization of the model. For a reference, the first row contains statistics for the actual data. As in Lubik and Schorfheide, output is log real chained-weighted per capita GDP, detrended with HP filter over the period $1955:1-1998:4$. Inflation is the annualized percentage change of the seasonally adjusted consumer price index, and nominal interest rate is the effective average Federal Funds rate. All data are quarterly, for $1960:1-1979:2$ sample.[35] Other statistics are theoretical moments derived for various model specifications. For every model specification, theoretical moments are reported for individual and joint shocks.

In the first three panels, the interest rate smoothing is 0.6, $\psi = 0.85$ for indeterminacy and $\psi = 2.19$ for determinacy. All other parameters are common across the three panels. The last panel corresponds to a determinacy model with a higher interest rate smoothing 0.85.[36] This specification also differs in the number of policy anticipation periods. The news shock is assumed to be informative about a policy impulse not one, but three periods ahead: $s_t = \varepsilon_t + 3 + e_t$. This assumption is made to facilitate a comparison of dynamic properties across determinacy and indeterminacy regions. Models with indeterminacy typically imply a richer autocovariance structure, as they suppress fewer autoregressive roots than their counterparts in determinacy region. News shocks trigger an immediate economic adjustment, due to nominal frictions. The further ahead the policy is anticipated, the longer is the period of adjustment. In the context of real business cycle models, dynamic responses to anticipated technological change have been found to have important

[34] See Figure 3 on page 208 in Lubik and Schorfheide [2004].

[35] The data are from the FRED database of the Federal Reserve Bank of St.Louis. Per capita values are derived with the total US population.

[36] As described in section 4.5.3, $\psi = 2.19$, $\kappa = 0.58$, $\sigma^{-1} = 1.86$.

effects on autocovariance properties of equilibria.[37]

Implications of news shocks for theoretical moments are investigated by changing the informativeness of news shocks ϕ. This parameter, reported in the first column, governs the decomposition of the monetary policy into unanticipated and anticipated components. It also governs the variance of each component: $var\left(\mu_t^0\right) = (1-\phi)\,\sigma_\varepsilon^2$ and $var\left(\mu_t^1\right) = \phi\sigma_\varepsilon^2$. For Determinacy II model, $var\left(\mu_t^0\right) = (1-\phi)\,\sigma_\varepsilon^2$ and $var\left(\mu_t^3\right) = \phi\sigma_\varepsilon^2$. In all experiments, the variance of ε_t is kept at the same value of 0.2. Numerical exercises in this subsection are in the spirit of Cochrane [1998]. Cochrane examines measured output effects of monetary policy shock in the context of a VAR, by varying an anticipated fraction of the shock. His results suggest that a distinction between unanticipated and anticipated policy shocks is as important for the output effects as the variable selection or shock orthogonalization assumptions.

There are several interesting observations about Table 3.1. Under model indeterminacy, monetary policy and news shocks can outperform sunspots and unexpected policy shocks or sunspots alone. This statement is illustrated by a model with modified dynamics, in which the impact of anticipated shocks mimics the one of the sunspots. Cross and autocorrelation statistics reported in the second row of panel 2 are closer to the data than those in the first row. As was mentioned earlier, there is an additional degree of freedom in selecting responses to anticipated shocks, relative to the sunspots. Thus, a better match is perhaps not as surprising. A more just approach seems to compare a model with determinacy and news shocks to a model with indeterminacy and sunspots. Statistics reported for two specifications under determinacy are not as strongly favourable relative to statistics for indeterminate models with sunspots.[38] Even in this case, however, the results suggest that effects of news shocks may be strong enough to overturn a positive autocorrelation of the interest rate implied by the policy rule.[39]

[37] See Love and Lamarche [2001].

[38] Compare first rows in panels 1 and 2 with the second rows of panels 3 and 4,

[39] See the negative coefficients in last column for Determinacy I model.

Summary of Solution Comparison

The main conclusion from numerical simulations is that news and sunspot shocks (i) generate distinct impulse dynamics and (ii) have different effects on autocovariance properties of endogenous variables. Simulation parameters, however, have not been chosen optimally. A precise comparison has to be taken with a caution. An interesting question is whether models news shocks can be empirically distinguished from sunspots. A possible direction in addressing this question is discussed in the conclusion.

4.6 Conclusion

This Chapter introduced news shocks as representing stochastic changes in agents' beliefs about future fundamental impulses. It proposed a simple framework for solving linear rational expectations models with news. It further compared news shocks with an alternative way of modelling changes in beliefs - sunspots. The two ways of modelling beliefs (through news and sunspot shocks) had distinct implications for impulse dynamics and autocovariance structure of endogenous variables. These differences were quantitatively significant in the New Keynesian model.

Differences in cross-equation restrictions and autocovariance properties of equilibria, induced by news and sunspot shocks, can be utilized to determine which type of beliefs, if any, is favoured by data in the context of a particular stochastic dynamic general equilibrium model. To compare news and sunspots, a model needs to possess regions of determinacy and indeterminacy, depending on parameter values. An indeterminacy region is necessary for existence of sunspot shocks. However, in this region, dynamics of news shocks are not completely pinned down without additional assumptions. In other words, news shocks have more degrees of freedom, relative to sunspots, in potentially explaining data. A more just approach seems to compare a model's version with news shocks and determinacy region of the parameter space with a version of the same model with sunspots and indeterminacy region of the parameter space.

Versions of the model with news and sunspot shocks would have some parameters that could be identified only in regions of determinacy or indeterminacy. Indeterminacy creates additional parameters, associated with a distribution of sunspot shocks and propagation of structural shocks. An introduction of news shocks leads to additional parameters for a forecasting horizon and variances of exogenous expectation revisions. An identification problem for parameters across the regions of determinacy and indeterminacy can be handled by extending an econometric procedure of Lubik and Schorfheide [2004] to incorporate news shocks.

Lubik and Schorfheide show how to construct the loglikelihood function for regions of determinacy and indeterminacy of the parameter space. They resolve the difficulty of parameterizing multiplicity under indeterminacy using a full characterization of equilibria derived in their earlier work (Lubik and Schorfheide [2003]). These additional parameters are identified only in the region of determinacy. A Bayesian estimation achieves this partial identification by leaving the prior density unchanged for the directions of the parameter space in which the likelihood function is flat. It updates the prior density for the directions for which the data are informative. Lubik and Schorfheide demonstrate that, based on the posterior densities, it is possible to learn whether the data are more consistent with determinacy or indeterminacy regions of the parameter space.

The estimation procedure of Lubik and Schorfheide exploits two sources of information about model parameters. The first source comes from cross-equation restrictions implied by a rational expectations solution. The second source comes from serial correlation properties. Numerical simulations of the New Keynesian model indicate that both sources of information are sensitive to the presence of fundamental, news and sunspot shocks. Further, news shocks are generally associated with endogenous propagation. As models with indeterminacy usually have richer dynamic structures, an omission of propagation mechanisms from model specifications tends to bias posterior density towards indeterminacy. Incorporating news shocks would, in principle, lead to a more just model comparison across determinacy and indeterminacy regions.

Differences in cross-equation restrictions and autocovariance properties of equilibria, induced by news and sunspot shocks, suggest that the two types of beliefs can be separated empirically, conditional on a particular model. An application of the estimation procedure only requires a model with economically plausible dynamic responses of news and sunspots shocks.[40]

[40]Recall, for example, the difficulty of standard business cycle models to generate investment and output booms in anticipation of future technological improvement, discussed in Chapter 2.

Chapter 5

Concluding Remarks

This thesis examined the role of expectations and expectation errors in business cycles. It emphasized the possibility that agents could learn about future changes in economic conditions in advance, but their anticipations could occasionally be incorrect. Changes in beliefs due to extra information about the future were defined as news shocks. Theoretical and empirical implications of these shocks were investigated.

One attractive feature of news shocks is that they provide a convenient way of modelling potentially erroneous beliefs. This property was exploited in Chapter 2. The goal of the Chapter was to assess a common view that revisions in overly optimistic expectations were the key determinant of US investment in 1994-2003. Chapter 2 also addressed a challenging problem of empirical identification of unobserved changes in beliefs. The problem was resolved with an estimation method that minimized the discrepancy between the model and the data.

The empirical results showed that the US experience could be explained in the context of standard business cycle model augmented with beliefs only if expectations about the future were more pessimistic during the boom and more optimistic during the recession. The standard model was unable to capture expectation-driven booms and recessions even qualitatively. Simple modifications of the model which predicted an investment boom in response to anticipations of technological improvement had difficulty in capturing the joint behaviour of consumption, investment and employment. These results raised further research questions of how news shocks are propagated in theoretical and actual economies.

Chapter 3 pursued an empirical investigation of the US data to determine whether they were consistent with the presence of news shocks about future technological change. The analysis was based on predictability of total factor productivity. While being econometrically exogenous to observed measures of shocks, TFP growth was forecastable from main macroeconomic series. These empirical results are consistent with (albeit not indicative of) existence of news. They provide foundations for further examination of the role of beliefs in business cycles.

Understanding dynamic effects of news shocks in theoretical environments requires an ability to solve models with these shocks. Chapter 4 described solutions to linear rational expectations models. Solutions were derived for models with unique and multiple equilibria. Chapter 4 compared news shocks with an alternative way of modelling beliefs through sunspots by examining properties of equilibria. These conceptually different ways of modelling beliefs, independent of current and past fundamentals, have distinct predictions for dynamic properties of the models. An interesting research question is how to exploit these differences to identify which types of changes in beliefs are more important (if any) in the data. A possible direction for such identification is outlined in the conclusion to Chapter 4.

To conclude, changes in beliefs are an intuitively attractive source of aggregate fluctuations. Investigation of their importance relative to other alternative sources faces many conceptual, theoretical and empirical challenges. Some of these challenges are addressed in this thesis. Yet, many open research questions remain.

Tables

Table 2.1: Parameters of Technology

γ_a	$\sigma_a^2 * 10^4$	γ_v	ρ_1	ρ_2	$\sigma_v^2 * 10^4$
1.0036	1.0830	1.0038	1.2866	-0.4347	0.0938
[0.0009]		[0.0001]	[0.1421]	[0.1391]	

Note: Standard errors are in brackets.

Table 2.2: Explanatory Power of Technology Shocks

	R_c^2	R_i^2	R_h^2	R_{av}^2	cor(c)	cor(i)	cor(h)
1.A. Joint Contribution of Shocks: 1994:2-2003:4							
Benchmark	0.2284	0.0661	0.6763	0.3236	0.4780	0.2571	0.8224
Utilization	0.1622	0.0004	0.5439	0.2355	0.4028	0.0191	0.7375
1.B. Joint Contribution of Shocks: 1994:2-2000:4							
Benchmark	0.3467	0.2011	0.9677	0.5052	0.5888	0.4485	0.9837
Utilization	0.3052	0.0720	0.9800	0.4524	0.5524	0.2683	0.9899
2. Contribution of Technology in Benchmark Model							
Aggregate	0.2842	0.0357	0.4630	0.2610	0.5331	0.1890	0.6805
Investment	0.0074	0.0189	0.7127	0.2463	0.0860	0.1375	0.8442

Note: For each variable, R^2 is obtained from an OLS regression of the actual on the simulated data and a constant. R_{av}^2 is the average of individual R^2s. The last three columns define correlations between the simulated and actual series

Table 2.3: Explanatory Power of Beliefs and Technology Shocks

	R_c^2	R_i^2	R_h^2	R_{av}^2	cor(c)	cor(i)	cor(h)
1. Joint Contribution of Shocks: 1994:2-2003:4							
Benchmark	0.1085	0.8200	0.9482	0.6256	-0.3294	0.9055	0.9738
Substitution	0.1479	0.8202	0.8837	0.6173	-0.3846	0.9057	0.9400
2. Contribution of Beliefs Only							
Benchmark	0.0525	0.3220	0.8982	0.4242	0.2291	0.5675	0.9477
Substitution	0.0134	0.6993	0.7487	0.4871	0.1156	0.8362	0.8653

Note: For each variable, R^2 is obtained from an OLS regression of the actual on the simulated data and a constant. R_{av}^2 is the average of individual R^2s. The last three columns define correlations between the simulated and actual series

Table 3.1: Cross and Autocorrelations in New Keynessian Model

	ϕ	cor(x,π)	cor(x,r)	cor(π,r)	$\rho(x)$	$\rho(\pi)$	$\rho(r)$
Data: 1960:Q1 - 1979:Q2							
data		0.06	0.38	0.85	0.83	0.97	0.97
1. Indeterminacy: Continuity Assumption							
u + ss	0.00	0.75	-0.21	0.49	0.36	0.70	0.87
u + a	0.50	0.97	-0.45	-0.22	0.48	0.41	0.22
u only	0.00	0.98	-0.95	-0.87	0.34	0.31	0.49
a only	1.00	0.95	-0.02	0.26	0.61	0.47	0.06
ss only		0.81	0.58	0.95	0.46	0.85	0.97
2. Indeterminacy: Modified Dynamics							
u + ss	0.00	0.84	-0.04	0.51	0.37	0.67	0.91
u + a	0.50	0.64	0.11	0.83	0.37	0.85	0.95
u only	0.00	0.95	-0.43	-0.13	0.36	0.52	0.68
a only	1.00	0.95	-0.02	0.26	0.61	0.47	0.06
ss only		0.81	0.58	0.95	0.46	0.85	0.97
3. Determinacy I							
u only	0.00	1.00	-1.00	-1.00	0.27	0.27	0.27
u + a	0.50	0.95	-0.40	-0.11	0.42	0.42	-0.13
a only	1.00	0.94	0.11	0.45	0.64	0.53	-0.35
4. Determinacy II							
u only	0.00	1.00	-1.00	-1.00	0.48	0.48	0.48
u + a	0.50	0.95	-0.16	0.14	0.54	0.61	0.50
a only	1.00	0.96	0.57	0.75	0.69	0.71	0.50

Note: Table reports correlation and first order autocorrelation coefficients foroutput, inflation and interest rate, conditional on effects of unanticipated andanticipated monetary policy shocks as well as on a sunspot shock under various specifications. Refer to main text for description.

Table 4.1: Total Factor Productivity: Descriptive Statistics

Sector	γ	σ_ε	ρ
Annual Measures: 1949-2002			
Private Business (PB)	1.368	1.834	0.049
	[5.484]		[0.350]
Non-farm Private Business(NF)	1.188	1.831	0.067
	[4.768]		[0.481]
Quarterly Measures: 1949:1-2002:4			
Private Business (PB)	0.393	0.933	0.066
	[6.191]		[0.969]
Non-farm Private Business (NF)	0.333	0.963	0.095
	[5.088]		[1.395]

Note: γ = average growth rate (in percents) and σ_ε= standard deviation of growth rate of a total factor productivity measure. An autocorrelation coefficient ρ is from an OLS regression of TFP growth rate on its lag and a constant. T-statistics are in brackets. Annual measures are from the BLS. Quarterly measures are described in the main text.

Table 4.2: Test of TFP Exogeneity to Other Shocks

X, sample	X⇒ΔZ	ΔZ⇒X	R^2(Z)	N	p(corr)	X*⇒Z
1. Private Business Sector: Quarterly						
ΔDEF, 49:1-02:4	0.82	0.01	0.06	0	0.11	0.60
ΔGOV, 49:1-02:4	0.96	0.05	0.05	0	0.19	0.80
ΔOIL, 49:1-72:4	0.41	0.30	0.13	0	0.32	0.45
RRM, 69:1-96:4	0.00	0.48	0.16	2	0.70	
2. Non-farm Private Business Sector: Quarterly						
ΔDEF, 49:1-02:4	0.73	0.04	0.05	0	0.15	0.46
ΔGOV, 49:1-02:4	0.85	0.17	0.04	0	0.29	0.62
ΔOIL, 49:1-72:4	0.53	0.10	0.09	0	0.35	0.49
RRM, 69:1-96:4	0.01	0.62	0.15	2	0.48	
3. Private Business Sector: Annual						
ΔDEF, 1949-2002	0.73	0.00	0.00	0	0.84	0.74
ΔGOV, 1949-2002	0.60	0.00	0.01	0	0.34	0.46
ΔOIL, 1949-1972	0.89	0.03	0.02	0	0.66	0.76
RRM, 1969-1996	0.07	0.06	0.13	0	0.01	
4. Non-farm Private Business Sector: Annual						
ΔDEF, 1949-2002	0.39	0.02	0.02	0	0.95	0.54
ΔGOV, 1949-2002	0.91	0.01	0.00	0	0.25	0.56
ΔOIL, 1949-1972	0.88	0.01	0.00	0	0.19	0.95
RRM, 1969-1996	0.06	0.06	0.14	0	0.02	

Note: Column 2 reports asymptotic p-values for bivariate Granger causality tests of X relative to growth rate of TFP. The null hypothesis of exogeneity is α(L)=0 in the regression $\Delta Z_t = \gamma_0 + \beta(L)\Delta Z_{t-1} + \alpha(L)X_{t-1} + w_t$. The null is rejected if a p-value is below the desired marginal significance level. Column 3 reports p-values for reversed causality tests. The autoregressions are computed with four lags for quarterly and one lag for annual specifications. Column 4 reports the R^2s for the TFP growth autoregressions. N is the highest lag for which variable X has predictive power for the TFP growth rate at 5 percent significance level. p(corr) is the probability value for a hypothesis of no contemporaneous correlation between ΔZ and X. The last column reports p-values for bivariate Granger causality tests when forecasting variable and TFP are in log-levels.

Table 4.3: Bivariate Granger Causality Tests

X, sample	Npb	ΔX⇒ΔPB	X⇒PB	Nnf	ΔX⇒ΔNF	X⇒NF
		1. Quarterly Frequency				
OUTP* , 49:1-02:4	3	0.00	0.00	3	0.00	0.00
EMPL* , 49:1-02:4	5	0.00	0.00	5	0.00	0.00
CNDR , 49:1-02:4	0	0.06	0.01	2	0.03	0.00
CSER , 49:1-02:4	1	0.00	0.00	1	0.00	0.00
IEQP , 49:1-02:4	3	0.00	0.00	3	0.00	0.00
ISTR , 49:1-02:4	1	0.04	0.01	3	0.01	0.00
IRES , 49:1-02:4	2	0.00	0.00	2	0.00	0.00
M1 , 59:2-02:4	0	0.08	0.04	0	0.05	0.02
(P)CPI, 49:1-02:4	0	0.48	0.01	0	0.80	0.01
FFRT , 54:4-02:4	4	0.00	0.00	5	0.00	0.00
SPRD , 54:4-02:4	2	0.00	0.00	4	0.00	0.00
STKM , 49:1-02:4	1	0.00	0.00	1	0.00	0.00
CEXP , 78:2-02:4	1	0.05	0.10	0	0.10	0.09
		2. Annual Frequency				
OUTP* , 1949-2002	1	0.00	0.61	1	0.00	0.50
EMPL* , 1949-2002	1	0.00	0.92	1	0.00	0.97
CNDR , 1949-2002	0	0.08	0.31	0	0.05	0.23
CSER , 1949-2002	0	0.48	0.16	0	0.63	0.12
IEQP , 1949-2002	1	0.00	0.87	1	0.00	0.72
ISTR , 1949-2002	1	0.01	0.28	1	0.01	0.36
IRES , 1949-2002	0	0.76	0.71	0	0.59	0.90
M1 , 1960-2002	0	0.17	0.20	0	0.20	0.20
(P)CPI, 1949-2002	1	0.00	0.63	1	0.00	0.57
FFRT , 1955-2002	1	0.00	0.00	1	0.00	0.00
SPRD , 1955-2002	1	0.00	0.00	1	0.00	0.00
STKM , 1949-2002	0	0.99	0.24	0	0.91	0.22
CEXP , 1979-2002	0	0.42	0.15	0	0.71	0.20

Note: Columns 3, 4, 6 and 7 report asymptotic p-values for bi-variate Granger causa- lity tests of X relative to a TFP measure for private (PB) or nonfarm private (NF) business sectors. Specifications are run with four lags of each varible for quar- terly and one lag of each variable for annual observations. The hypothesis of no predictablity of TFP corresponds to zero coefficients on lags of X in a regression of TFP on its lags, lags of X and a constant. The null is rejected if a p-value is below the desired significance level. Npb and Nnf are the highest lags for which ΔX has a predictive power for ΔZ at 5 percent significance level. Output and employment series are for the same sector as the corresponding TFP measure. CPI index, rather than inflation is used in log-level specification for (P)CPI.

Table 4.4: Long-Horizon OLS Test of TFP Pedictability

k	$F[1]$	$p[1]$	$R^2[1]$	$F[2]$	$p[2]$	$R^2[2]$	$F[3]$	$p[3]$	$R^2[3]$
			1. Private Business Sector : 55:1-02:4						
1 q	49.38	0.00	0.20	32.40	0.00	0.14	20.59	0.00	0.09
2 q	42.28	0.00	0.23	52.88	0.00	0.23	35.86	0.00	0.15
3 q	38.24	0.00	0.26	67.48	0.00	0.21	35.71	0.00	0.16
4 q	26.19	0.00	0.22	68.89	0.00	0.22	47.89	0.00	0.20
5 q	20.16	0.01	0.18	60.62	0.00	0.20	53.38	0.00	0.22
			2. Private Non-farm Sector : 55:1-02:4						
1 q	54.05	0.00	0.21	32.14	0.00	0.16	24.28	0.00	0.12
2 q	38.90	0.00	0.25	51.90	0.00	0.23	39.86	0.00	0.18
3 q	35.80	0.00	0.27	57.17	0.00	0.22	40.18	0.00	0.19
4 q	26.25	0.00	0.24	65.86	0.00	0.22	60.98	0.00	0.23
5 q	20.93	0.01	0.20	62.58	0.00	0.21	72.47	0.00	0.26
			3. Private Business Sector : 1955-2002						
1 y	13.71	0.07	0.27	48.78	0.00	0.54	47.72	0.00	0.34
2 y	10.87	0.20	0.16	40.06	0.01	0.38	24.34	0.03	0.28
3 y	11.37	0.25	0.17	16.46	0.13	0.17	27.55	0.02	0.31
4 y	10.83	0.28	0.18	14.83	0.19	0.10	31.95	0.03	0.36
5 y	11.47	0.27	0.19	6.60	0.59	0.06	18.47	0.14	0.37
			4. Private Non-farm Sector : 1955-2002						
1 y	14.10	0.05	0.28	53.57	0.00	0.60	55.38	0.00	0.39
2 y	10.18	0.23	0.15	37.58	0.02	0.41	31.21	0.01	0.32
3 y	10.09	0.26	0.16	16.43	0.12	0.18	28.73	0.02	0.34
4 y	9.79	0.33	0.18	16.48	0.20	0.10	37.85	0.02	0.38
5 y	13.04	0.25	0.18	7.87	0.52	0.06	20.28	0.11	0.38

Note: Table reports test results for TFP growth rate predictability based on OLS regressions of an average k-period ahead growth rate of TFP on growth rates of forecasting variables and a constant. The null hypothesis of no predictability corresponds to zero regression coefficients on forecasting variables. Columns 2, 5 and 8 are values of F statistics. Columns 3, 6 and 9 are bootstrap p-values for the F test. The null hypothesis fails if a p-value is below the desired level of significance. Columns 4, 7 and 10 are the R^2s for OLS regressions. See the main text for model specifications. The number of bootstrap iterations is 999.

Table 4.5: Long-Horizon VAR Test of TFP Pedictability

	R^2[PB,1]	R^2[PB,2]	R^2[PB,3]	R^2[NF,1]	R^2[NF,2]	R^2[NF,3]
			1. Quarterly Frequency : 55:1-02:4			
1 q	0.29	0.30	0.23	0.31	0.30	0.27
2 q	0.31	0.36	0.26	0.34	0.37	0.31
3 q	0.31	0.36	0.27	0.33	0.37	0.31
4 q	0.27	0.34	0.28	0.29	0.35	0.32
5 q	0.23	0.29	0.26	0.26	0.31	0.30
F_1	4.34	4.49	3.21	4.95	4.50	3.97
p	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
F_2	92.00	95.28	67.83	102.11	92.91	81.37
p	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
			2. Annual Frequency : 1955-2002			
1 y	0.28	0.56	0.42	0.28	0.61	0.47
2 y	0.23	0.38	0.33	0.24	0.43	0.38
3 y	0.16	0.24	0.22	0.16	0.26	0.25
4 y	0.11	0.16	0.15	0.11	0.18	0.17
5 y	0.09	0.13	0.12	0.09	0.15	0.13
F_1	3.97	12.53	6.48	4.07	15.56	7.78
p	[0.01]	[0.00]	[0.00]	[0.01]	[0.00]	[0.00]
F_2	5.19	16.28	8.53	5.38	20.41	10.33
p	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Note: Table reports test results for TFP predictability based on VARs that include a constant and lagged values of TFP growth rate for private business (PB) or private non-farm business (NF) sectors and forecasting variables. Four lags are used in quarterly specifications and one in annual. R^2[sector,model] is the implied R^2 for the long-horizon regression. F_1 and F_2 are F statistics, reported with their asymptotic p-values in brackets, for the joint hypothesis of zero coefficients on forecasting variables. F_1 includes, but F_2 excludes lags of TFP growth. The null hypothesis fails is a p-values is below the desired level of significance. See the main text for model specifications.

Figures

Notes to Figures

Figure 2.1: Data are in percentage change from the previous year. The shaded area corresponds to the NBER dates for the 2001 recession. Investment = the NIPA real private fixed investment. Consumption = the NIPA real private aggregate consumption spending. Both series are in per capita, expressed in 2000 chained dollars.

Figure 2.2: The aggregate technology shock is computed as the Solow residual with capital share of 0.32. The relative investment prices is the ratio of price deflators for investment and consumption measures. Shocks are stationary transformations of technology measures. Impulses are residuals from time series processes for shocks.

Figure 2.3: Dotted lines are actual data. Solid lines are data simulated with the benchmark model and measures of aggregate and investment specific technology.

Figure 2.4: Dotted lines are actual data. Solid lines are data simulated with the augmented benchmark model and measures of technology shocks and beliefs.

Figure 2.5: Dotted lines are actual data. Solid lines are data simulated with the augmented benchmark model and measures of technology shocks and beliefs. Data are expressed as indices, with value 100 in 2000:4.

Figure 2.6: The top panel plots estimated technology prospects (in levels). The estimates are derived from regression coefficients in 2.22 on the basis of the augmented benchmark model. The bottom panel plots the growth rate of aggregate technology (solid line) and its average (dotted line).

Figure 2.7, Figure 2.8: Impulse responses correspond to an experiment when news received in period one about a positive realization of technology shock in period five are followed by no actual change in technology.

Figure 2.9: Estimated technology prospected are derived from regression coefficients in 2.22 on the basis of the substitution model.

Figure 2.10: Dotted lines are actual data. Solid lines are data simulated with the substitution model and measures of technology shocks and beliefs.

Figure 3.1: For each forecasting horizon k, the actual growth rate is the $k-$period ahead average growth rate of TFP for private business sector. Predicted series are derived on the basis of OLS and VAR long-horizon regressions. Forecasting variables include growth rates of real private consumption of services, residential fixed investment and nonresidential fixed investment in equipment.

Figure 3.2: For each forecasting horizon k, the actual growth rate is the $k-$period ahead average growth rate of TFP for private business sector. Predicted series are derived on the basis of OLS and VAR long-horizon regressions. Forecasting variables include the change in the federal funds rate, the growth rate of stock market index and real private consumption of services.

Figure 4.1: Figure depicts impulse responses of output, inflation and nominal interest rate to a unitary increase in unanticipated or anticipated policy shock under various assumptions about interest rate smoothing in the region of determinacy $(\psi = 2.19)$. For $\rho = 0$ and $\rho = 0.6$, the parameters are $k = 0.77$, $\sigma^{-1} = 1.48$. For $\rho = 0.85$, $\kappa = 0.58$, $\sigma^{-1} = 1.86$.

Figure 4.2: Figure depicts impulse responses of output, inflation and nominal interest rate to a unitary increase in unanticipated or anticipated policy shock and a unitary increase in a reduced form sunspot shock under various assumptions about rational expectational errors in the region of determinacy $(\psi = 0.85,\ \rho = 0.6)$. $M = [0,0]$ under orthogonality assumption, $M = [-1.41, -1.58]$ under continuity and $M = [-2.75, 0.99]$ under modified dynamics.

Figure 2.1: Real Private Aggregate Investment and Consumption

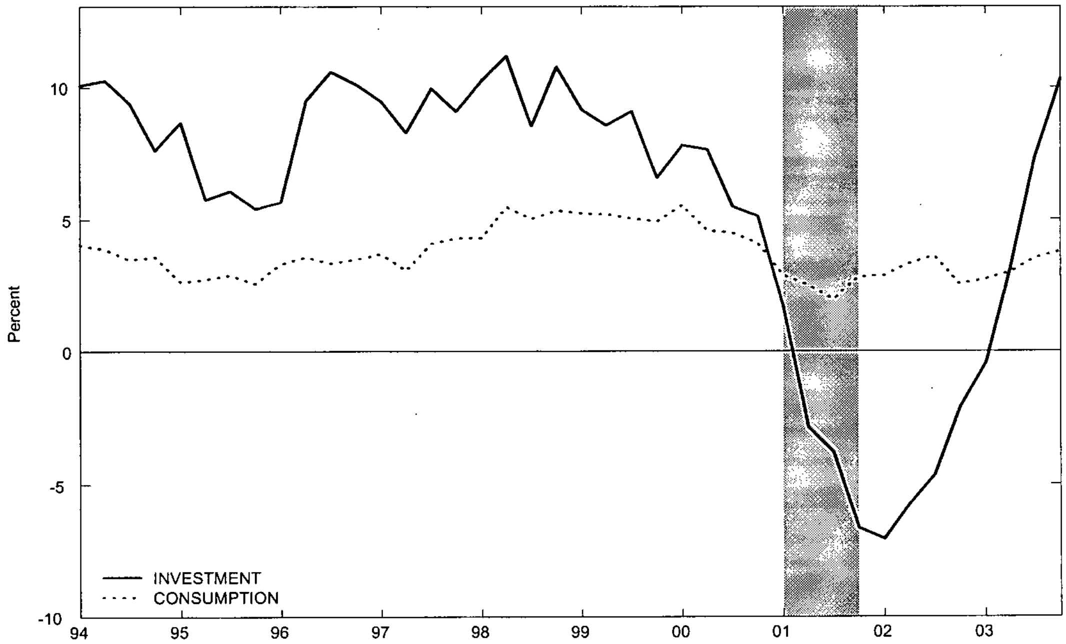


Figure 2.2: Measures of Technology and Relative Investment Price

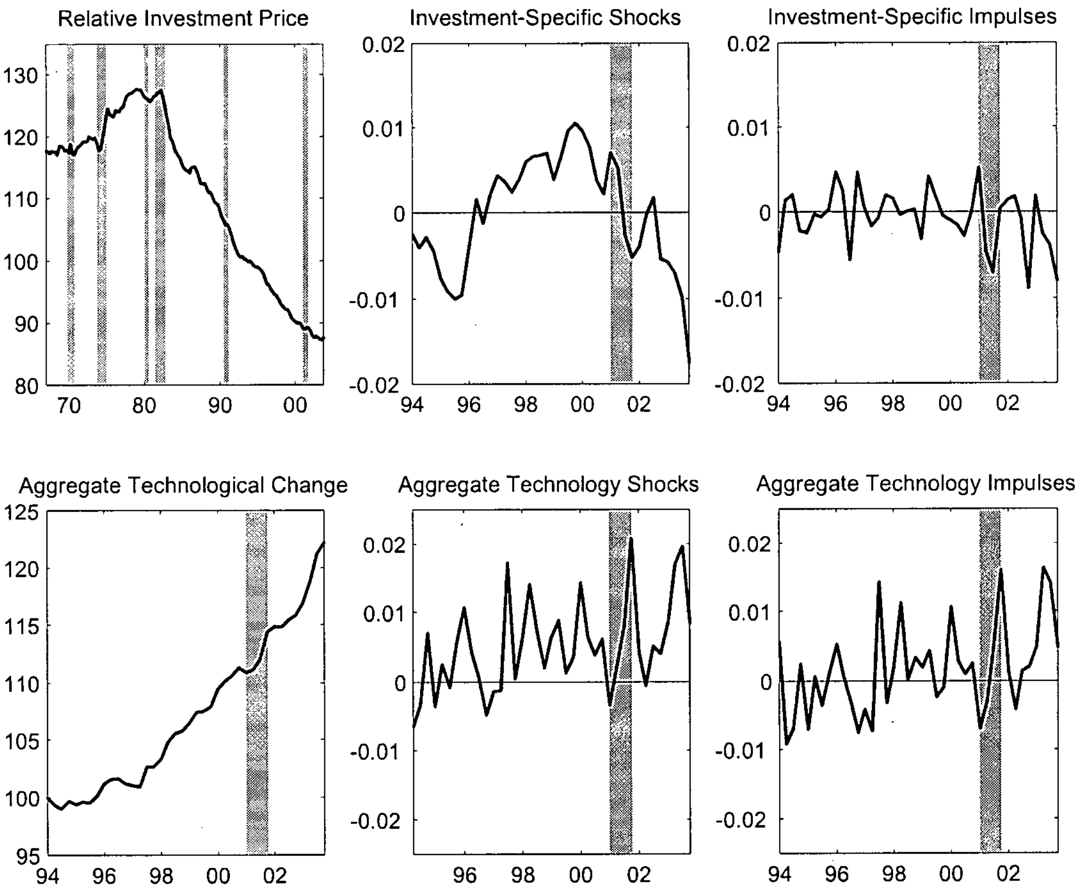


Figure 2.3: Technology Fundamentals Play a Limited Role in the Investment Bust (Benchmark Model Simulation)

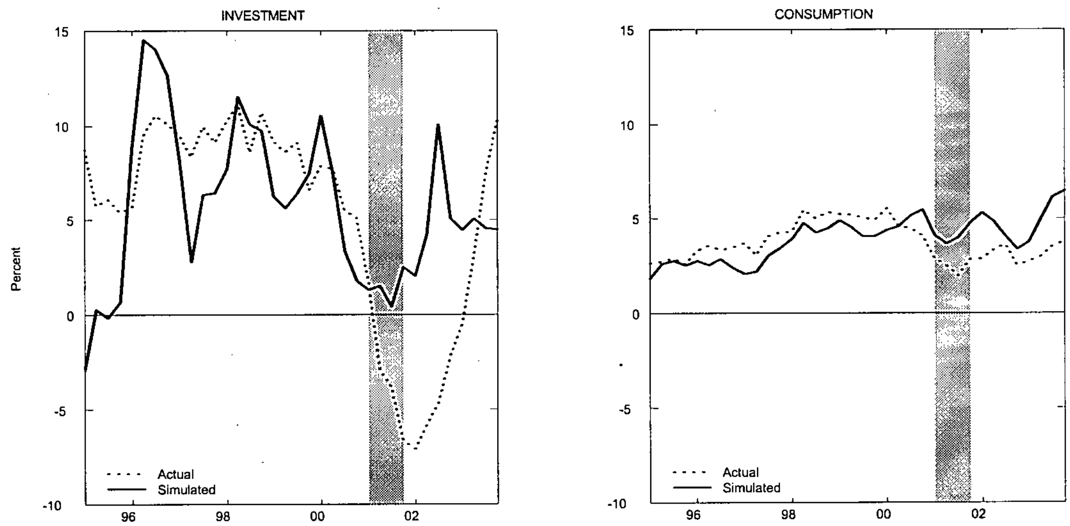


Figure 2.4: Technology Prospects with Technology Fundamentals Match Quantity Series (Augmented Benchmark Model Simulation)

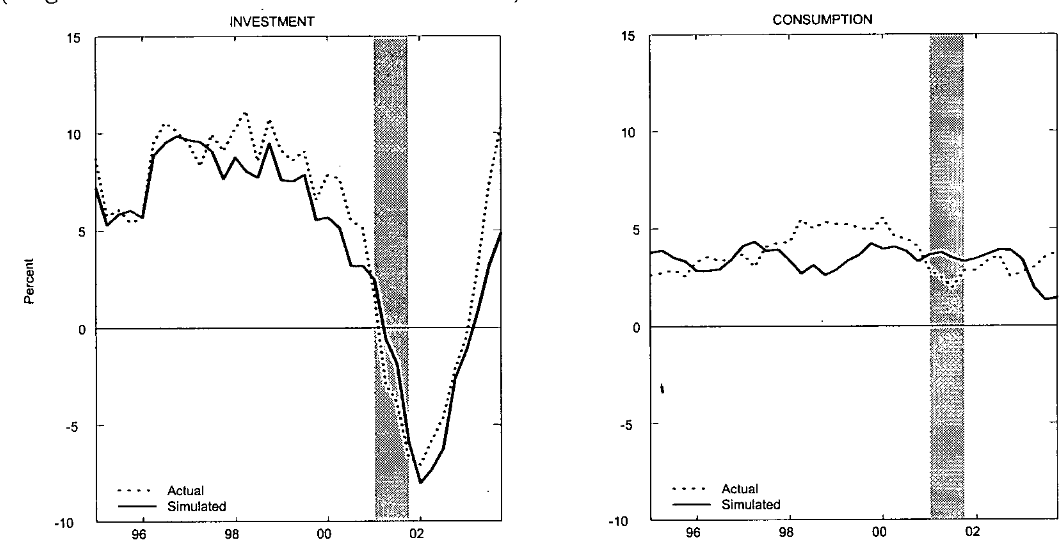


Figure 2.5: Indexes of Simulated and Actual Data (Benchmark Model with Technology Prospects)

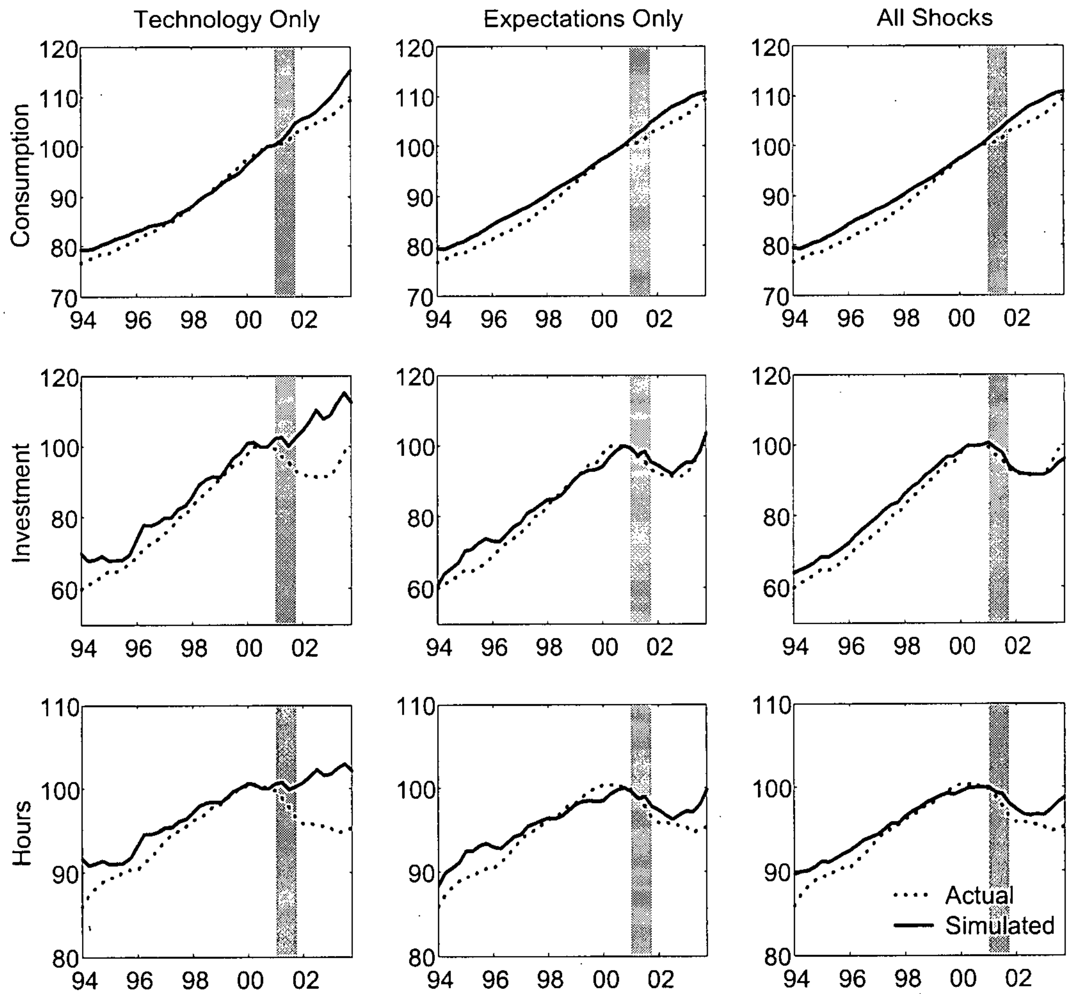


Figure 2.6: Estimates of Technology Prospects Implied by the Benchmark Model Contradict the Hypothesis of Optimistic Expectation Revisions

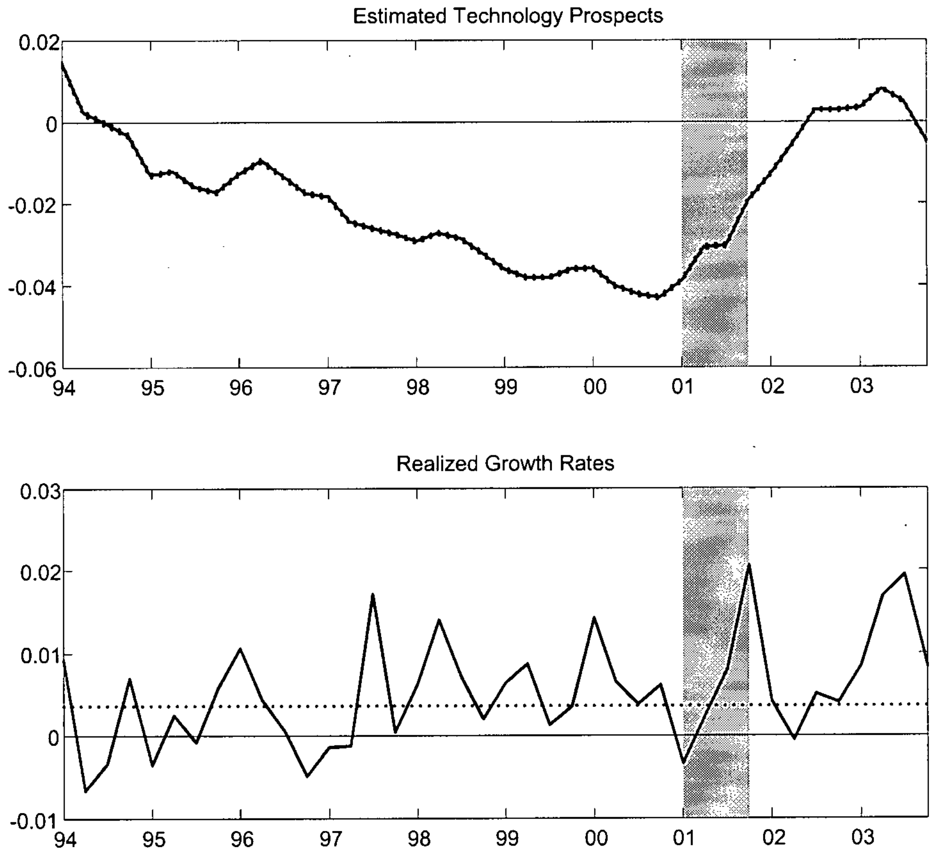


Figure 2.7: Response to Unrealized Good News About Future Technology (Augmented Benchmark Model)

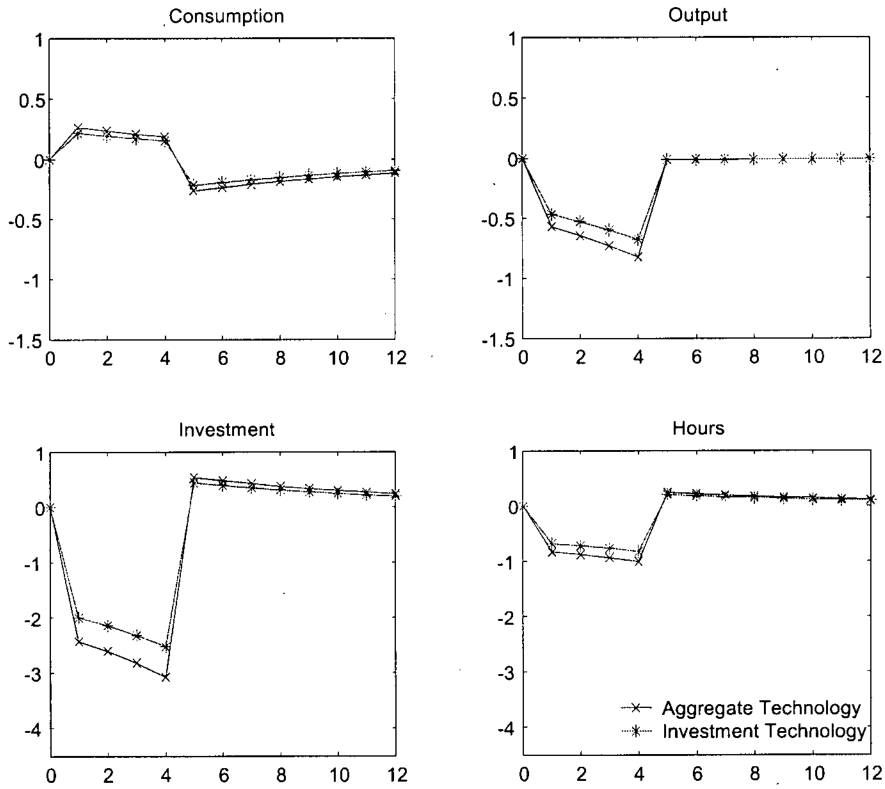


Figure 2.8: Response to Unrealized Good News About Future Technology (Substitution Model)

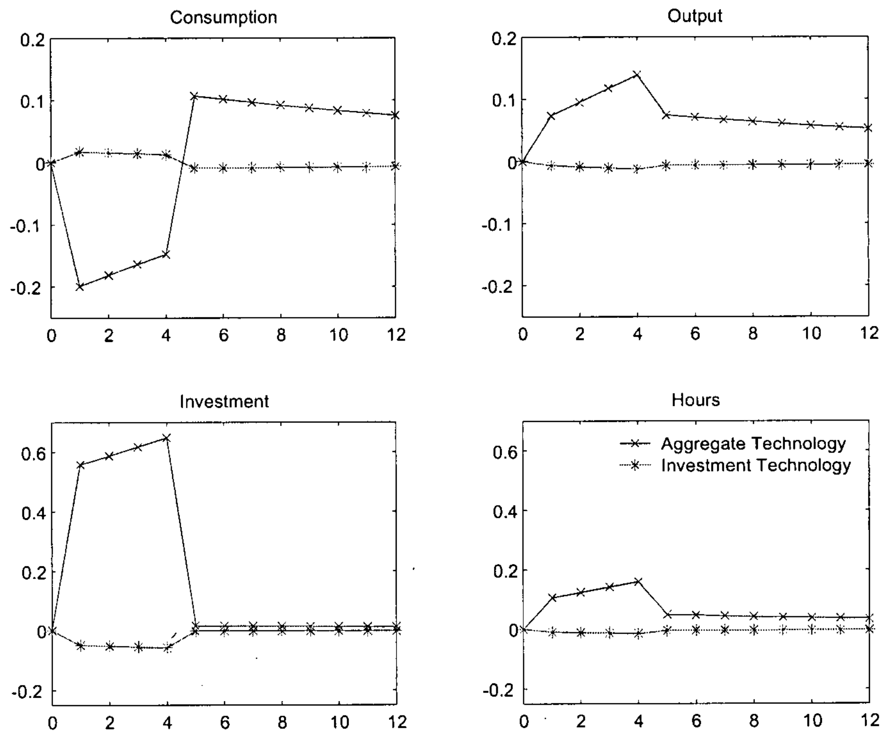


Figure 2.9: Estimates of Technology Prospects Implied by Substitution Model

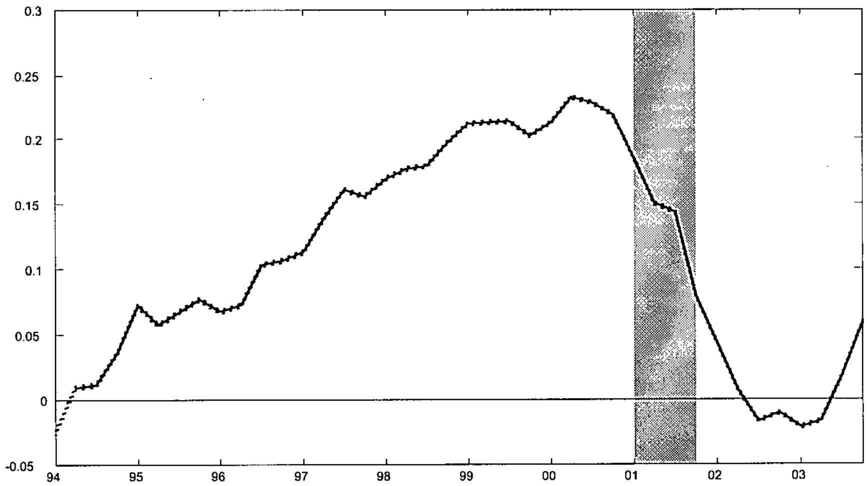


Figure 2.10: Technology Prospects with Technology Shocks Imply Missing Consumption Boom (Simulation of Subsitution Model with Adjustment Costs)

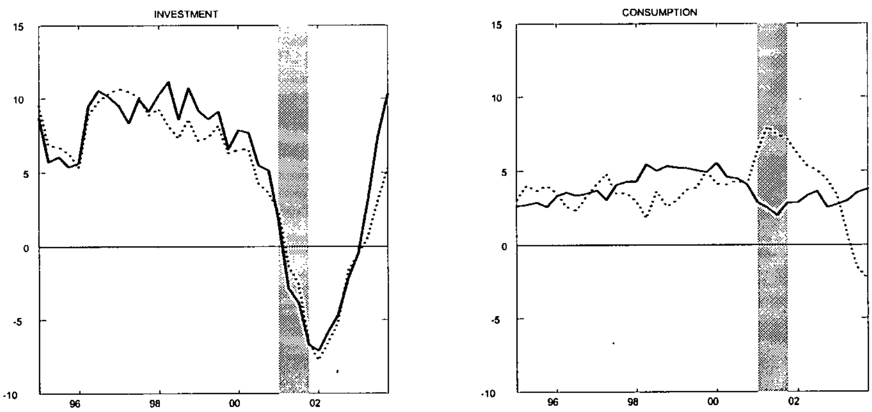


Figure 3.1: Actual and Predicted $k-$period Ahead Average TFP Growth Rates for Private Business Sector: Quarterly

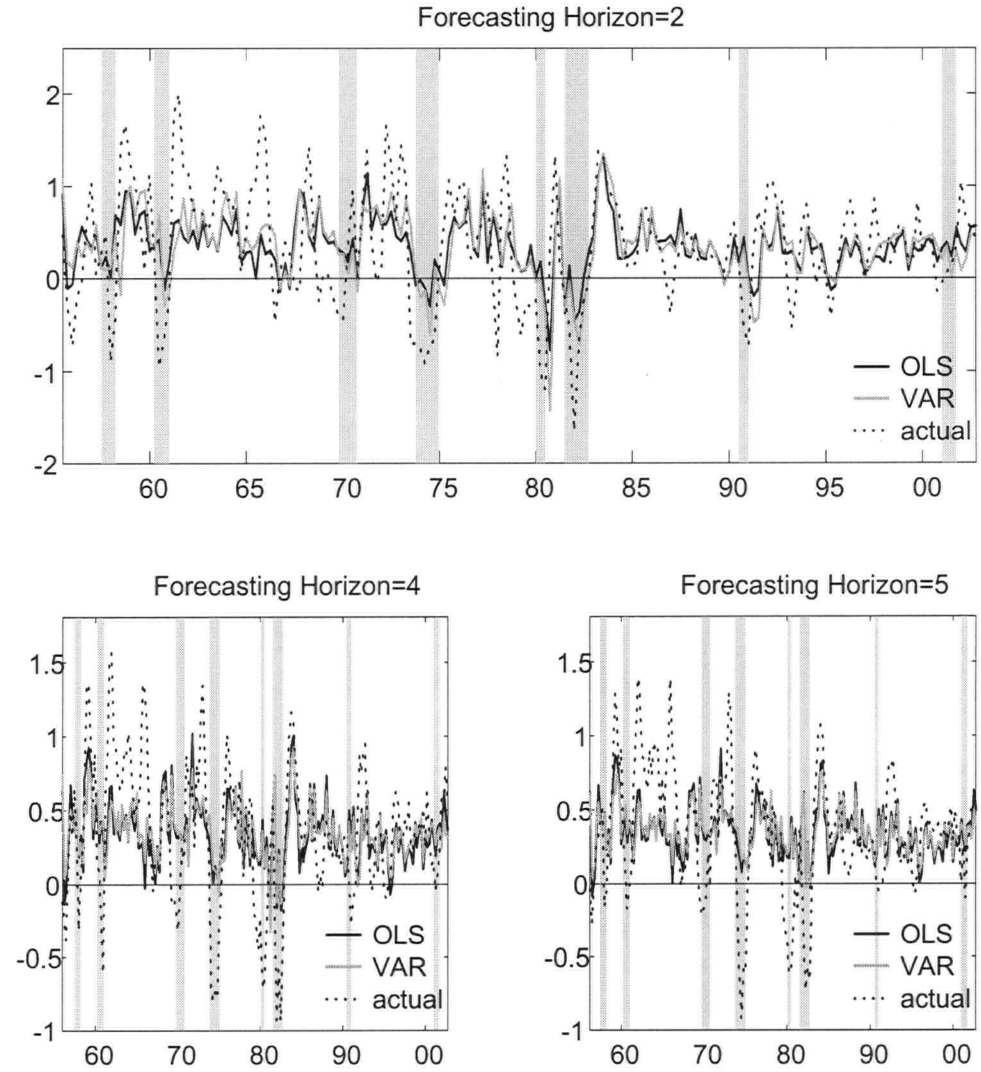


Figure 3.2: Actual and Predicted $k-$period Ahead Average TFP Growth Rates for Private Business Sector: Annual

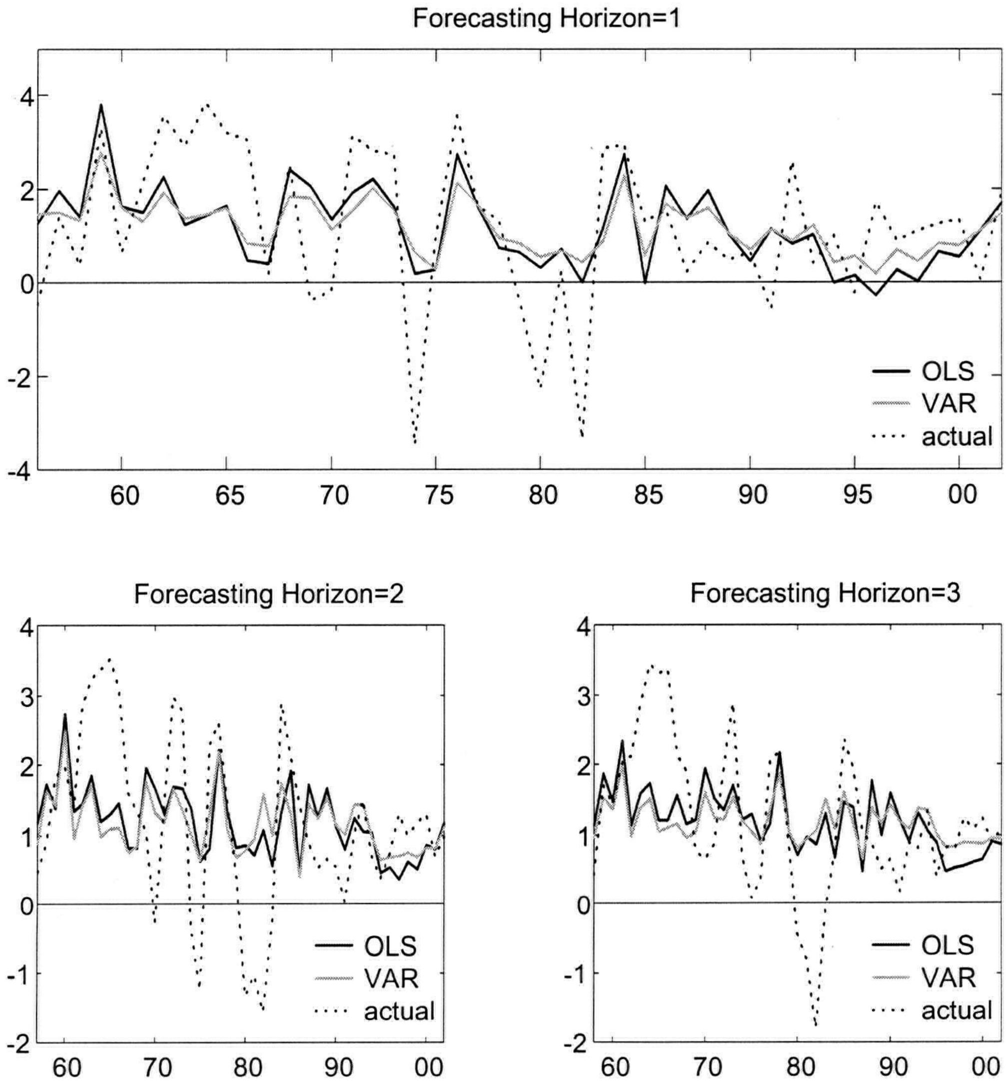


Figure 4.1: Impulse Responses to Monetary Policy Shocks: Determinacy

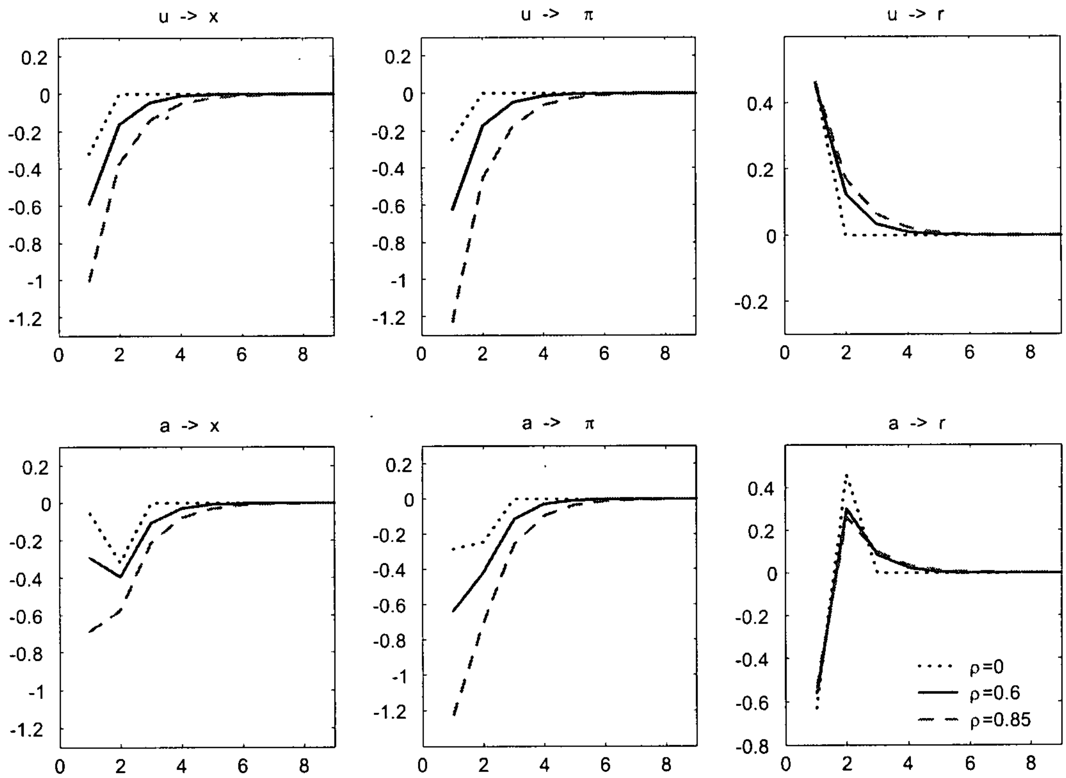
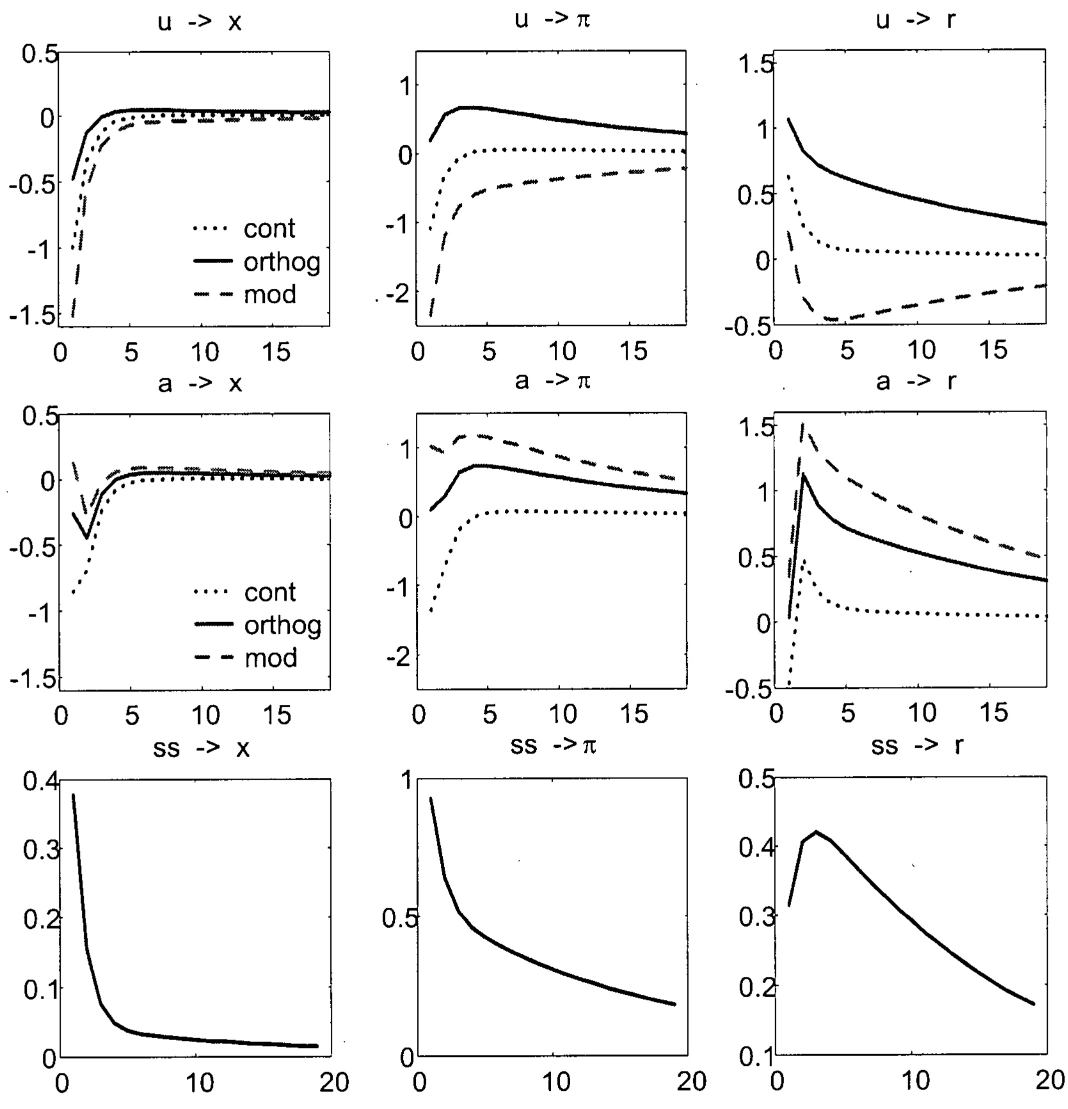


Figure 4.2: Impulse Responses to Monetary Policy and Sunspot Shocks: Indeterminacy



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Appendix A

Recursive Computation of Matrices $V(j)$

Matrices $V(j)$, $j \geq 1$, can be computed recursively as follows:

$$V(j) = \underbrace{\left[\begin{array}{ccc} C & \ldots & C \end{array} \right]}_{T \text{ times}} \left[\begin{array}{cccc} W_j^1 & 0 & \ldots & 0 \\ 0 & W_j^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & W_j^T \end{array} \right] \left[\begin{array}{cccc} B & 0 & \ldots & 0 \\ 0 & B & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & B \end{array} \right]$$

$$W_1 = \theta$$

$$W_j = \mathbf{A} W_{j-1} + \mathbf{I}^j \theta$$

$$\underset{kT \times k}{\theta} = \left[\begin{array}{c} \underset{k \times k}{I} \\ \underset{k(T-1) \times k}{0} \end{array} \right], \underset{kT \times kT}{\mathbf{A}} = \left[\begin{array}{cccc} A & 0 & \ldots & 0 \\ 0 & A & \ldots & 0 \\ \ldots & \ldots & \ldots & 0 \\ 0 & 0 & \ldots & A \end{array} \right]$$

$$\underset{kT \times kT}{\mathbf{I}} = \left[\begin{array}{ccccc} 0 & 0 & \ldots & 0 & 0 \\ I & 0 & \ldots & 0 & 0 \\ 0 & I & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & I & 0 \end{array} \right], \underset{kT \times k}{W_j} = \left[\begin{array}{c} W_j^1 \\ W_j^2 \\ \ldots \\ W_j^T \end{array} \right]$$

Matrices I are identity matrices with dimension $(k \times k)$, and zero entries refer to the comfortable matrices with zero elements.